WAVE MOTION PRODUCED BY LINEAR WAVE GENERATORS

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Prepared for David Taylor Model Basin

under

Contract Nonr 1610(02)

Technical Report No. 1

Colorado A & M College
Department of Civil Engineering
Fort Collins, Colorado

December 1956

ABSTRACT

Taking the potential function for waves emanating from a point source, equations were developed that permit ready calculation of the water surface in a rectangular wave basin having several finite length displacement type wave generators in series along the walls of the basin. The linearized wave theory is utilized throughout. A discrete wave amplitude and frequency is assumed for each generator, but no two generators necessarily put out the same wave.

The theory, extended to several finite length generators each considered as a source of a periodic function, will be presented in future reports now under preparation. Calculations for circular basins will also be reported.

WAVE TRAINS PRODUCED BY LINEAR WAVE GENERATORS

The developments described herein have been directed to providing ready means for calculating the characteristics of wave trains produced by wave generators which act by producing volume displacements along a linear interval. It has been accepted that the utility of the final results is of paramount importance, and for this reason approximations have been used whenever computation procedures could be simplified by their adoption.

A word of explanation may be worthwhile concerning the procedure followed in this development. A first simple fundamental solution of the hydrodynamic equations was sought which could be used as a basis for constructing the more elaborate cases required for the present purposes. This simple solution relates to the train of waves diverging from an isolated point source in water of any depth when the disturbance at the source is of a simple, harmonic type. The case of a wave generator of length L was then obtained by summing the effects produced by simple solutions of uniform strength and distributed uniformly along the length of the wave generator. The amplitude of the wave train produced is related to the displacement produced by the wave generator. The solution thus obtained represents the behavior of an isolated generator from which waves are free to progress in any direction.

A different case is presented, however, by a wave generator operating in an experimental wave basin since the walls of the basin interfere with the progress of the waves in certain directions. It is proposed to account for these boundaries by using the method of images. In this way the effect of operating the wave generator adjacent to a wall, in a corner, within a rectangular strip or within a rectangular basin can be found. In all these cases the same set of simple formulas can be used.

NOTATION

Symbol	Definition	Units
A	represents an amplitude constant	L
A_0	a wave amplitude at the radius r from an isolated point source	L
A	the amplitude of wave motion produced at the point r, by two, in phase, point sources	L
C	the celerity of a wave, the speed of propagation of the waves	L/T
C_1	a distance from a source to a point at which a wave height is to be computed	L
D	the variable volume displaced by a wave generator	L^3
D_1	a displacement computed at the radius r_0	L^2
D_0	the displacement computed at the origin	L^2
D_m	the maximum value reached by the variable displacement D_0	L^3
g	acceleration of gravity	L/T^2
h	depth of a basin	L
J_0 and Y_0	Bessel's functions of order zero, as tabulated in Reference (6)	
L	length of a wave generator	L
n	a quantity determined from Eq. (8) or (10)	L^{-1}
Q_m	the maximum volume displacement produced by a linear wave generator	L^3
r	radius	L
R	the distance from the center of a linear wave generator to the point at which a wave amplitude is to be computed.	L
S	separation of two sources	L
t	time	T
T	the period of a wave. The time interval between passage of crests.	T
u	velocity, positive in the direction of r.	L/T
w	unit weight of the liquid	F/L^3

Notation (continued)

		Units
x, y	coordinates, see Fig. 2	L
z	a coordinate which is zero at the water surface and measured positive upward	L
α	angle between a line passing through two sources and a radial line drawn from one of them. An angle between the length of a long wave generator and a line drawn outward from the center of the wave generator.	---
β	phase angle separation	
ζ	the height of the surface of a wave above the undisturbed level	L
η	a distance from the center of a linear wave generator, as shown in Fig. 2	L
λ	the wave length. The distance between successive crests	L
ϕ	velocity potential	
σ	$= \frac{2\pi}{T}$	T^{-1}

WAVES PROPAGATED FROM A POINT SOURCE

As a basis for the development of equations for wave motions produced by a linear wave generator it will be useful to have available the expressions relating to the waves emanating from a point source, because the formula for the more complicated case can then be built up by the process of superposition. The following development relates to waves emanating from a point source.

Consider the velocity potential,

$$\phi = \frac{Ag}{\sigma}\left[J_o(nr)\operatorname{Sin}\sigma t - Y_o(nr)\operatorname{Cos}\sigma t\right]\operatorname{Cosh} n(z+h) \quad ...(1)$$

This expression satisfies LaPlace's equation (Ref. 7, p 76)

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad ...(2)$$

To be acceptable for the purpose desired it must also satisfy the conditions (Ref. 7, pp 73 and 74.)

$$\frac{\partial\phi}{\partial z} = 0 \quad \text{when} \quad z = -h$$

$$\beta = \frac{1}{g}\frac{\partial\phi}{\partial t} \quad \text{on the free surface} \quad ...(3)$$

$$\frac{\partial\beta}{\partial t} = -\frac{\partial\phi}{\partial z} \quad \text{on the free surface}$$

From (1) by differentiation:

$$\frac{\partial\phi}{\partial z} = \frac{Ag}{\sigma}\left[J_o(nr)\operatorname{Sin}\sigma t - Y_o(nr)\operatorname{Cos}\sigma t\right] n \operatorname{Sinh} n(z+h) \quad ...(4)$$

Since $\frac{\partial\phi}{\partial z}$ is the vertical component of velocity and this expression becomes zero at the bottom, when $z = -h$, the first condition is satisfied.

The second condition yields

$$\beta = A\left[J_o(nr)\operatorname{Cos}\sigma t + Y_o(nr)\operatorname{Sin}\sigma t\right]\operatorname{Cosh} nh \quad ...(5)$$

To satisfy the third condition with

$$\frac{\partial \beta}{\partial t} = A\sigma\left[-J_0(nr)\,\mathrm{Sin}\,\sigma t + Y_0(nr)\,\mathrm{Cos}\,\sigma t\right]\mathrm{Cosh}\,nh \quad \ldots(6)$$

and

$$-\frac{\partial \phi}{\partial z} = -\frac{Ag}{\sigma}\left[J_0(nr)\,\mathrm{Sin}\,\sigma t - Y_0(nr)\,\mathrm{Cos}\,\sigma t\right] n\,\mathrm{Sinh}\,nh \quad \ldots(7)$$

It is required that

$$A\sigma\,\mathrm{Cosh}\,nh = \frac{Ag}{\sigma}\,n\,\mathrm{Sinh}\,nh$$

By rearrangement and multiplication of both sides of the expression by the factor h this relation can be put into the form

$$\frac{h\sigma^2}{g} = nh\,\tanh\,nh \quad \ldots(8)$$

This is the basic relation connecting the values of σ and n . These two quantities are associated with the period of the wave, its wave length and its speed of propagation, or celerity, as will be seen later. If the period is to be specified by selecting a value for σ the corresponding value of n can be determined immediately through the use of Fig. 1.

At a distance from the source sufficient to make the quantity nr large compared to unity, the Bessel functions $J_0(nr)$ and $Y_0(nr)$ take the approximate forms: (Ref. 6, p 202)

$$J_0(nr) = \sqrt{\frac{2}{\pi nr}}\,\mathrm{Cos}\left(nr - \frac{\pi}{4}\right)$$

$$Y_0(nr) = \sqrt{\frac{2}{\pi nr}}\,\mathrm{Sin}\left(nr - \frac{\pi}{4}\right) \quad \ldots(9)$$

If $nr > 1$.

Under these conditions a wave length can be specified since

$$n = \frac{2\pi}{\lambda} \quad \ldots(10)$$

If a wave length λ is chosen, the value of n obtained from this relation can be substituted into Eq. (8) to find the quantity σ which will, in turn, determine the period, from the relation

$$\sigma = \frac{2\pi}{T}, \quad \text{or} \quad T = \frac{2\pi}{\sigma}$$

Eq. (9) can be used to establish some additional important relationships. Under the condition stated above, Eq. (5) for the wave height takes the form

$$\beta \cong A \frac{2}{\sqrt{\pi n r}}\left[\operatorname{Cos}\left(nr - \frac{\pi}{4}\right)\operatorname{Cos}\sigma t + \operatorname{Sin}\left(nr - \frac{\pi}{4}\right)\operatorname{Sin}\sigma t\right]. \; \operatorname{Cosh} nh$$

If $nr \gg 1.$

and then can be written

$$\beta = A \frac{2}{\sqrt{\pi n r}}\left[\operatorname{Cos}\left(nr - \sigma t - \frac{\pi}{4}\right)\right] \operatorname{Cosh} nh \qquad \ldots(11)$$

If $nr \gg 1.$

If attention is fixed on a certain part of the wave, this form of the expression shows that r must increase with time if the same phase position is to be maintained. This follows from the requirement that if $\cos(nr - \sigma t - \frac{\pi}{4})$ is to have a fixed value, the quantity $(nr - \sigma t - \frac{\pi}{4})$ must likewise have a fixed value. To meet these requirements set

$$\left(nr - \sigma t - \frac{\pi}{4}\right) = k$$

where the value of the constant k is chosen to give the quantity $\cos(nr - \sigma t - \frac{\pi}{4})$ the desired value. Then it follows that this phase position will travel outward at a rate determined from the relation:

$$nr = \sigma t + k + \frac{\pi}{4}$$

Suppose, for example, one chooses to watch the crest. This will require that the cosine term will have a maximum value and this imposes the relation

$$nr - \sigma t - \frac{\pi}{4} = m\,2\pi$$

where m is some whole number. By differentiation of this expression the relation is obtained,

$$\frac{dr}{dt} = \frac{\sigma}{n}$$

which can be interpreted as the rate at which the crest progresses. This is the wave velocity C. This relation can be combined with expressions (8) and (10) to obtain a formula for the wave propagation velocity:

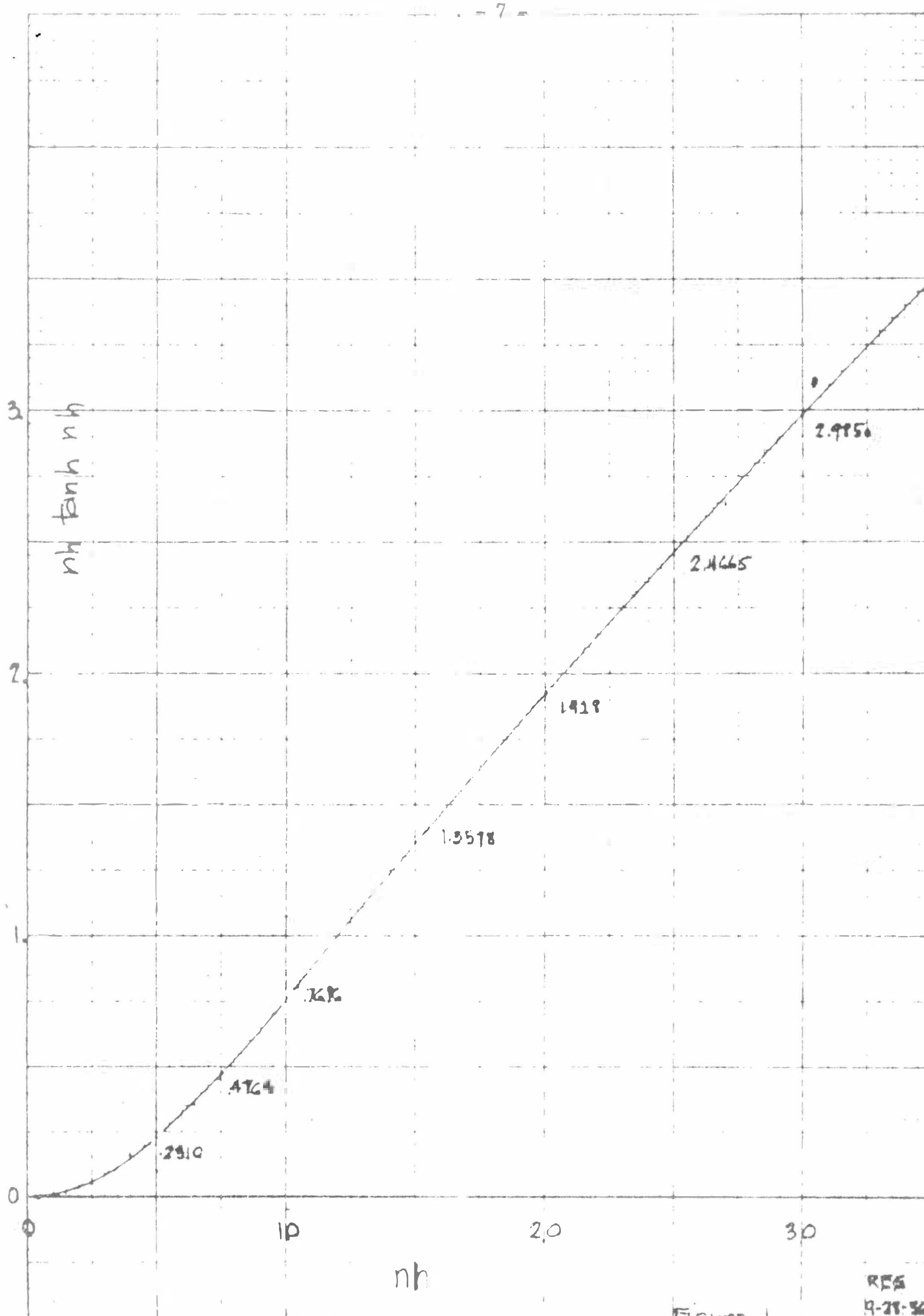

Figure 1.

$$c^2 = \frac{g\lambda}{2\pi} \tanh \frac{2\pi h}{\lambda} \qquad \text{...(12)}$$

If $nr \gg 1$.

Although these relations are approximates, a scrutiny of the roots of the J_0 and Y_0 Bessel functions will indicate that they should hold closely enough for most engineering purposes beyond two wave-lengths distance from the source.

In order to produce waves of a specific amplitude it is necessary to know how much volumetric displacement is needed at the source to maintain them. The displacement volume is given by an integral of the type

$$D_1 = 2\pi r_0 \int\int_{-h}^{0} u\,dz\,dt$$

where $u = -\frac{\partial \phi}{\partial r}$

This can be evaluated in the form

$$D_1 = \frac{2\pi r_0 A g}{\sigma^2}\left[J_0'(nr_0)\cos\sigma t + Y_0'(nr_0)\sin\sigma t\right]\sinh nh \qquad \text{..(13)}$$

Since the product $(nr_0)J_0'(nr_0)$ approaches zero as (nr_0) approaches zero and the product $(nr_0)Y_0'(nr_0)$ approaches $(2/\pi)$ as (nr_0) approaches zero the value of D_1 converges toward its value at the origin:

$$D_0 = \frac{4Ag}{n\sigma^2} \sin\sigma t \ \sinh nh \qquad \text{...(14)}$$

The maximum value of the displacement D_m is reached when $\sin t = 1$. Then:

$$D_m = \frac{4Ag}{n\sigma^2} \sinh nh \qquad \text{...(15)}$$

The value of A may then be obtained in terms of the maximum displacement as

$$A = \frac{n\sigma^2 D_m}{4g \sinh nh} \qquad \text{...(16)}$$

DIFFRACTION PATTERN PRODUCED BY TWO SOURCES

In some cases it is desired to propagate a wave train of limited width and this may be accomplished with some effectiveness by making use of the possibilities of interference. Suppose, for example, that two equal, in phase sources are operating at a separation S of one-half a wave length.

It can be expected that the wave motion in the direction of the line joining the sources would be almost completely annulled because the waves from the two sources would be of almost equal amplitude and would be 180 degrees out of phase. Along a normal to this line, drawn from a point midway between the sources, however, the wave motion would be enhanced because the waves from the two sources would be nearly in phase. The wave crests would be nearly circular in form but their height would vary in each quadrant from a maximum to nearly zero. In this way a very definite concentration of the wave motion into a portion of the surface area can be accomplished. It will be of interest to develop the case of the two sources somewhat more fully.

If represents an angle between a radius drawn from one of the sources and the line which passes through both of them, a point at radius r from one source will lie at the distance C_1 from the other source where, by the cosine law

$$C_1 = \sqrt{r^2 + S^2 - 2rS\cos\alpha}$$

If r is large compared to the separation S the square of S may be discarded so that approximately

$$C_1 \cong r\sqrt{1 - \frac{2S}{r}\cos\alpha} \qquad \text{If } r \gg S.$$

Since the quantity S/r is, under these conditions, small compared to unity it will be permissible to expand the radical by the binominal theorem and, again as an approximation, discard all of the terms except the first two, then approximately

$$C_1 \cong r - S\cos\alpha \qquad \text{If } r \gg S.$$

$$\text{Since } (r - C_1) = S\cos\alpha \qquad \text{If } r \gg S. \qquad \ldots(17)$$

The angular phase separation is

$$\frac{2\pi(r - C_1)}{\lambda} = \frac{2\pi S}{\lambda}\cos\alpha \qquad \text{If } r \gg S. \qquad \ldots(18)$$

The amplitude of a resultant wave formed by the superposition of two waves of amplitude A_0 and separated by the phase angle β is, again by the cosine law

$$A\alpha = \sqrt{2}\,A_0\sqrt{1 + \cos\beta} \qquad \ldots(19)$$

The combination of (18) and (19) yields

$$A\alpha = \sqrt{2}\,Ao\sqrt{1 + \cos\left(\frac{2\pi S\cos\alpha}{\lambda}\right)} \quad \text{If } r \gg S. \qquad ...(20)$$

The following table shows a computation of wave heights following a circular path with its center at a point midway between the two sources. The computation extends through one quadrant. The other three quadrants are similar.

TABLE 1.

Computation of relative wave amplitudes produced by two, in phase, sources one-half wave length apart. The amplitude A at the radius r and angle α is expressed in terms of A. The amplitude produced at the radius r by one of the sources. The amplitude Ao can best be computed by use of formulas (29) or (31).

α	$\cos\alpha$	$\frac{2\pi S}{\lambda}\cos\alpha$	$\cos\left(\frac{2\pi S\cos\alpha}{\lambda}\right)$	$\frac{A\alpha}{Ao}$
0	1.000	3.1416	-1.0000	0.0000
10°	.9848	3.0938	-0.9989	0.0469
20°	.9397	2.9522	-0.9821	0.1892
30°	.8660	2.7206	-0.9127	0.4178
40°	.7660	2.4065	-0.7418	0.7187
50°	.6428	2.0194	-0.4338	1.0640
60°	.5000	1.5708	0.0000	0.4142
70°	.3420	1.0744	+0.4763	1.7182
80°	.1736	0.5454	-0.8549	1.9261
90°	.0000	0.0000	-1.0000	2.0000

Note: $\frac{2\pi S}{\lambda} = \frac{2\pi\lambda}{2\lambda} = \pi$

WAVE MOTION PROPAGATED FROM A LONG WAVE GENERATOR

It is desired to find the wave pattern produced by a linear wave generator of length L, as shown in Fig. 2, if it produces a volume displacement Q_m distributed uniformly throughout its length, but sinusoidal in time, with a period T. To find this pattern the differential displacement $\frac{Q_m}{L} d\eta$, originating in each element of length $d\eta$, will be treated as the displacement producing a wave propagated from a center, as previously described, and the effect of these differential disturbances will be integrated over the length L to find the effect of the whole wave generator. The results obtained will apply to an isolated generator from which waves can progress freely in all directions. It will be shown later how these results can be adapted for computations of the performance of wave generators installed in tanks.

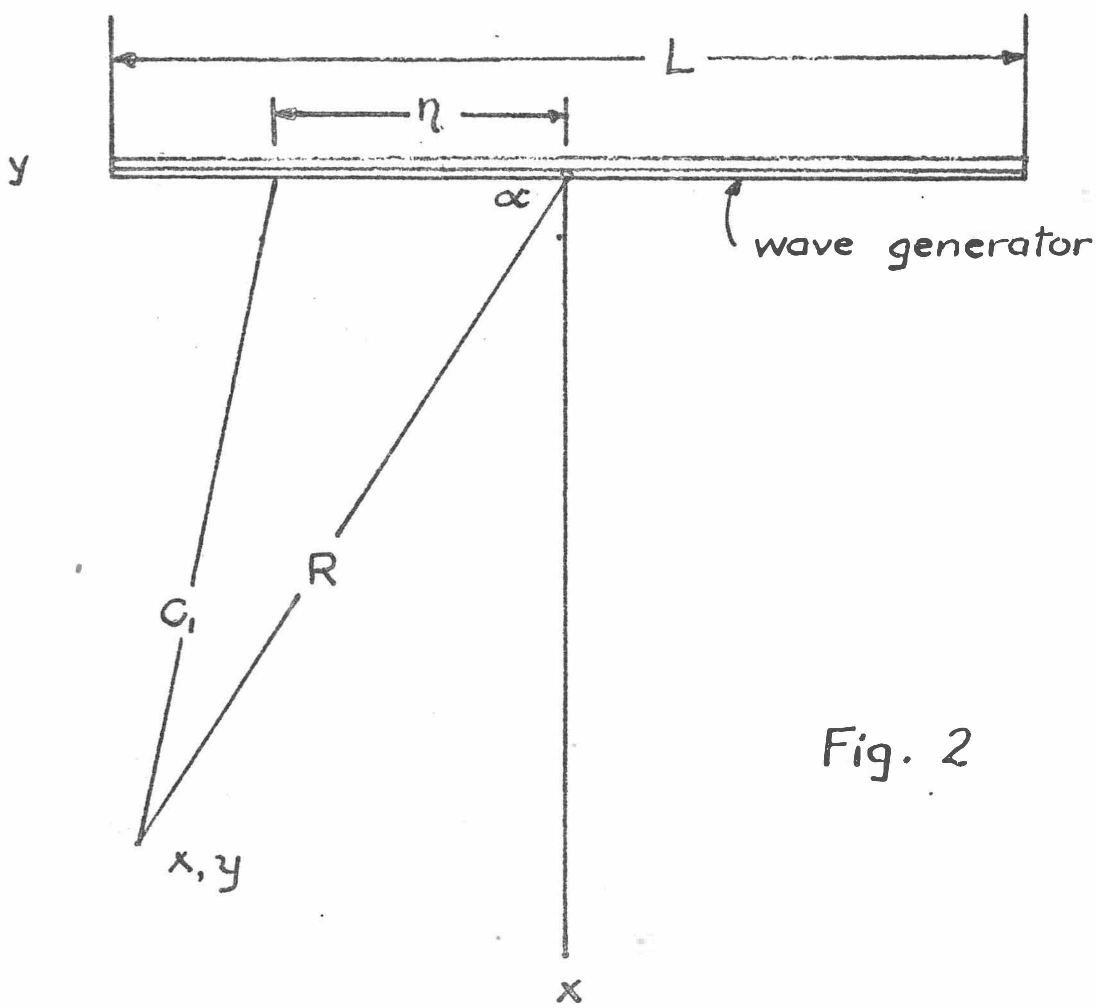

Fig. 2

If Eq. (16) is substituted into Eq. (5) an expression is obtained for the waves propagated from a center, in terms of the generating displacement. This expression takes the form:

$$\beta = \frac{n\sigma^2 Dm}{4g}\left[J_0(nr)\cos\sigma t + Y_0(nr)\beta n\sigma t\right]\coth nh \quad \ldots(21)$$

Then the increment of wave height at the distance C_1 from an element of displacement $\frac{Q_m}{L}\,dn$, originating at $y = n$, as shown in Fig. 2, is

$$d\beta = \frac{4\sigma^2 Qm}{4gL}\left[J_0(nC_1)\cos\sigma t + Y_0(nC_1)\sin\sigma t\right]\coth nd\, d\eta \ldots(22)$$

If R is large compared to $\frac{L}{2}$ then R is large compared to η and, to a first approximation, the cosine law

$$C_1^2 = R^2 + \eta^2 - 2R\eta\cos\alpha \quad \ldots(23)$$

can be expressed as

$$C_1 = \sqrt{R^2 - 2R\eta\cos\alpha} \quad \ldots(24)$$

by discarding η^2 as small compared to R^2 . Then by using the binomial theorem and discarding all but the first two terms

$$C_1 = R(1 - \frac{\eta}{R}\cos\alpha)$$

then

$$(R - C_1) = \eta\cos\alpha \quad \ldots(25)$$

If one neglects η as being small compared to R and refers phase positions to the phase position at the center of the wave generator, then approximately:

$$d\beta \simeq \frac{n\sigma^2 Qm}{4gL}\left[J_0(nr)\cos\left(\sigma t - \frac{2\pi\eta\cos\alpha}{\lambda}\right) + Y_0(nr)\sin\left(\sigma t - \frac{2\pi\eta\cos\alpha}{\lambda}\right)\right]\coth nh\, d\eta. \ldots(26)$$

and

$$\beta = \frac{n\sigma^2 Qm\coth nh}{4gL}\int_{-\frac{L}{2}}^{+\frac{L}{2}} J_0(nR)\cos\left(\sigma t - \frac{2\pi\eta\cos\alpha}{\lambda}\right)d\eta$$

$$+\frac{n\sigma^2 Q_m \operatorname{Coth} nh}{4gL}\int_{-\frac{L}{2}}^{+\frac{L}{2}} Y_o(nR)\operatorname{Sin}\left(\sigma t-\frac{2\pi\eta\operatorname{Cos}\alpha}{\lambda}\right)d\eta \quad \ldots(27)$$

or

$$\beta \simeq \frac{n\sigma^2 Q_m \operatorname{Coth} nh}{4g}\;\frac{\lambda}{2\pi L\operatorname{Cos}\alpha}\Big[-J_o(nR)\operatorname{Sin}\left(\sigma t-\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)$$

$$+J_o(nR)\operatorname{Sin}\left(\sigma t+\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)$$

$$+Y_o(nR)\operatorname{Cos}\left(\sigma t-\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)$$

$$-Y_o(nR)\operatorname{Cos}\left(\sigma t+\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)\Big]$$

If use is made of Eq. (8) and the well known formulas for the sine and cosine of the sum of two angles, this relation can be put in the form:

$$\beta \simeq \frac{n^2 Q_m}{4}\,\frac{\operatorname{Sin}\left(\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)}{\left(\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)}\Big[J_o(nR)\operatorname{Cos}\sigma t+Y_o(nR)\operatorname{Sin}\sigma t\Big] \quad \ldots(28)$$

The maximum amplitude is:

$$\beta_m \simeq \frac{n^2 Q_m}{4}\,\frac{\operatorname{Sin}\left(\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)}{\left(\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)}\sqrt{(J_o(nR))^2+(Y_o(nR))^2} \quad \ldots(29)$$

If (nR) is large compared to unity:

$$\sqrt{(J_o(nR))^2+(Y_o(nR))^2} \simeq \sqrt{\frac{2}{\pi(nR)}} \quad \ldots(30)$$

And approximately

$$\beta_m = \frac{n^2 Q_m}{4}\,\frac{\operatorname{Sin}\left(\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)}{\left(\frac{\pi L}{\lambda}\operatorname{Cos}\alpha\right)}\sqrt{\frac{2}{\pi(nR)}} \quad \ldots(31)$$

Valid if $R \gg \frac{L}{2}$

$(nR) \gg 1.$

The ratio $\dfrac{\sin\left(\frac{\pi L}{\lambda}\cos\alpha\right)}{\left(\frac{\pi L}{\lambda}\cos\alpha\right)}$ can be read from Fig. 3.

When waves from several generators are to be added it will be advantageous to keep the sine and cosine terms separate because this will permit a simpler evaluation of resultant amplitudes and phase positions than would be possible otherwise. The approximate form required for this purpose is:

$$\beta \simeq \frac{n^2 Q_m}{4} \frac{\sin\left(\frac{\pi L}{\lambda}\cos\alpha\right)}{\left(\frac{\pi L}{\lambda}\cos\alpha\right)} \sqrt{\frac{2}{\pi(nR)}} \left[\cos\left(nR - \frac{\pi}{4}\right)\cos\sigma t + \sin\left(nR - \frac{\pi}{4}\right)\sin\sigma t\right] \qquad \ldots(32)$$

Valid if $R \gg \frac{L}{2}$

$(nR) \gg 1.$

If it is suspected that R is not sufficiently large to make this formula yield as close an approximation as desired a test can be made by dividing the wave generator into two parts, computing for each part separately and superimposing the results. If the computation made in this way does not agree well with the result obtained by computing for the wave generator as a whole it indicates that the ration of R to $\frac{L}{2}$ is not large enough in the computation based upon the whole length. In such cases the computation based upon the halves gives the preferred results. The accuracy of the computations could be improved still further by dividing the wave generator into four or more parts and summing the results obtained from computations based upon each part separately, but it is believed that a need for such computations will arise only rarely, if at all, in practice.

LINE SOURCE GENERATORS IN A RECTANGULAR BASIN

The case of a wave generator operating near a wall, as shown in Fig. 4, can be obtained from the solution for the isolated case if a second wave generator, having the characteristics of the first, is located where the image of the first generator would be formed if the wall were a mirror.

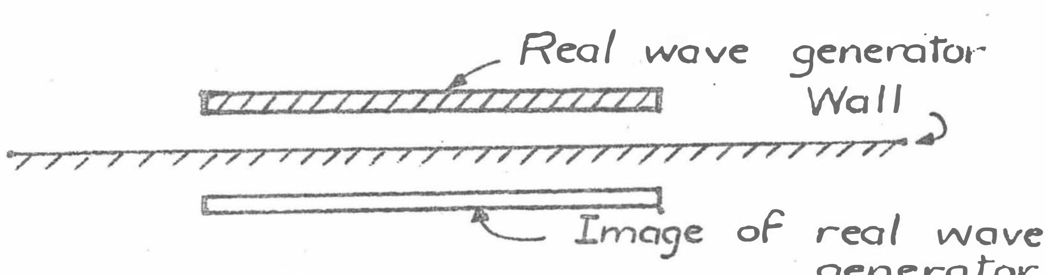

Fig. 4. Wave generator near a wall.

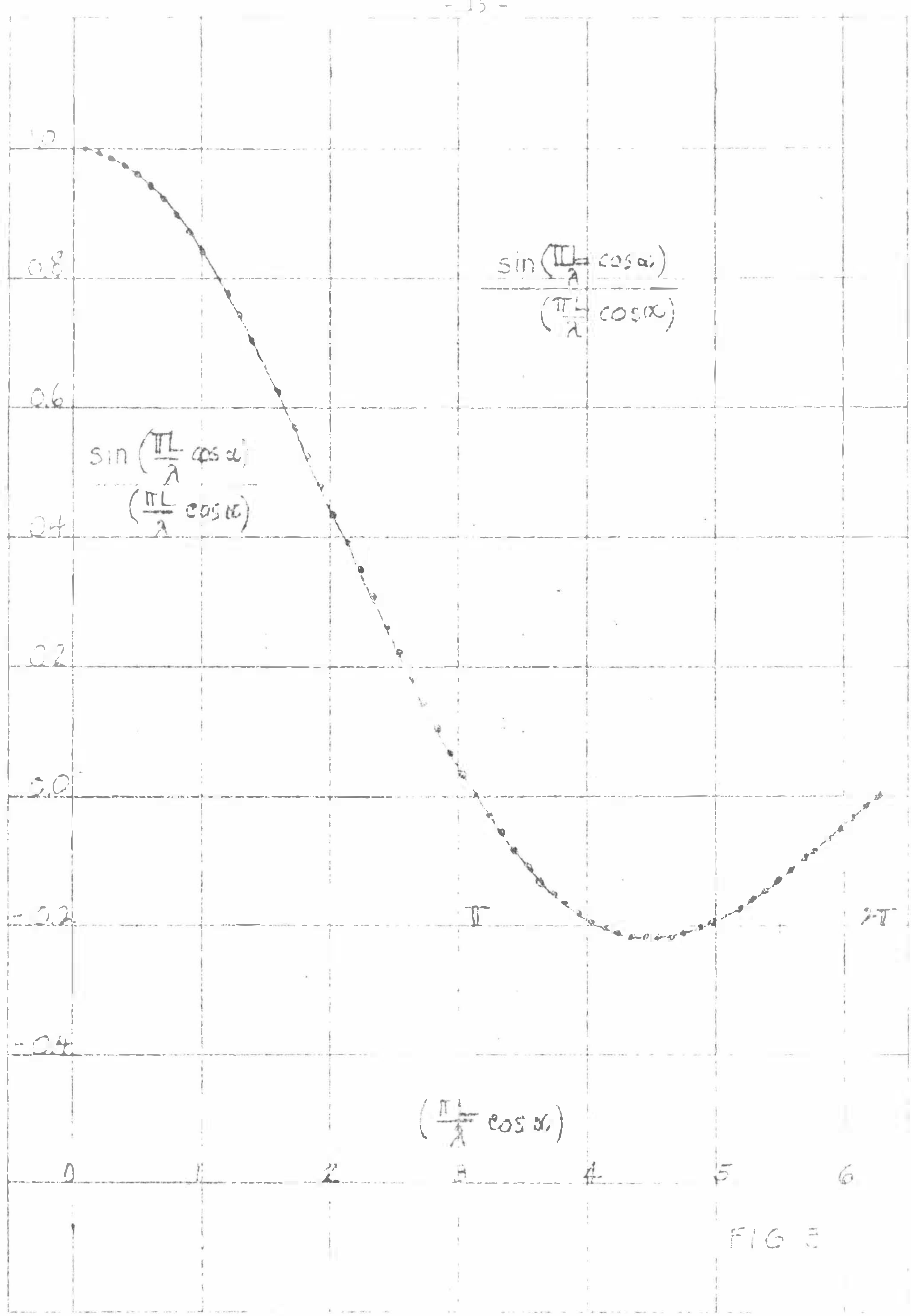
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
$\frac{\sin\left(\frac{\pi L}{\lambda}\cos\alpha\right)}{\left(\frac{\pi L}{\lambda}\cos\alpha\right)}$
π
2π
$\left(\frac{\pi L}{\lambda}\cos\alpha\right)$
0
1
2
3
4
5
6

FIG 3

The boundary condition at the wall requires that there be no normal component of velocity at its surface. Two identical wave generators would produce this condition along the line midway between them if they operated in a water surface area of unlimited extent. Two solutions for the isolated case, used in this way, will therefore reproduce the boundary condition imposed by the wall.

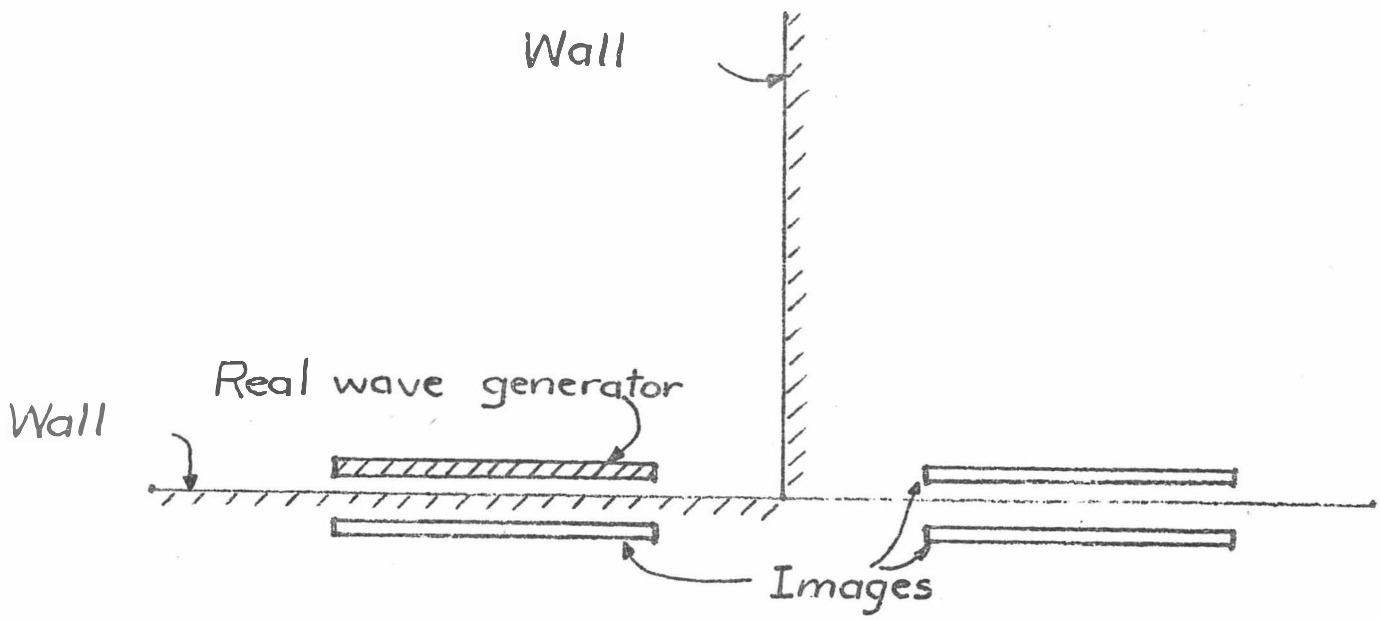

Fig. 5. Wave generator in a corner.

The case of a wave generator in a corner may be reproduced by the arrangement of images shown in Fig. 5. This will insure that there will be no normal component of velocity at either wall.

The case of a wave generator in a rectangular strip is a little more complicated. The condition is shown in Fig. 6. Here the first images in wall (1) will result in the proper boundary conditions being met along wall (1) but will not do for wall (2). If the real generator, its image in the end, and the two first images in wall (1) are now imaged in wall (2), the required conditions along wall (2) will be met, but at the expense of a slight interference with the boundary condition at wall (1).

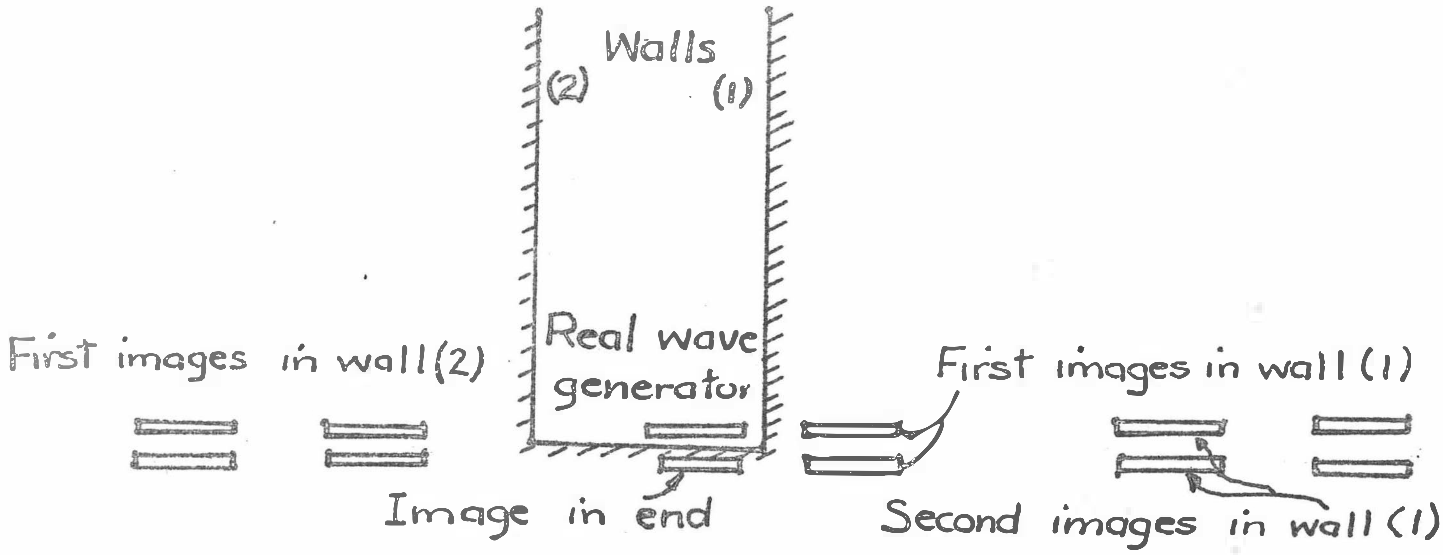

Fig. 6. Wave generator in a rectangular strip.

To remedy this one can introduce the second images in wall (1). These will completely restore the boundary conditions at wall (1) but in turn, upset them slightly at wall (2). Continuation of this process leads to an infinte series of terms. The series is generally, however, rapidly convergent.

If the strip of Fig. 6 had an upper end to convert it into a rectangular tank the real wave generator and its images as described could be imaged in the far end to meet the boundary conditions at the far end. Successive imagings in the two ends would then permit satisfaction of the boundary conditions at the ends without upsetting the boundary conditions at the walls. This process leads to a doubly infinite series of which, generally, only a few terms are needed to obtain a close approximation.

These descriptions have assumed that there is no absorption of energy at the wall. If energy absorbers are arranged along a wall it is believed permissible to consider that wall absent. To the approximation contemplated herein, it is permissible to compute the wave motion by superposing the effects of the real wave generator and its images. For this purpose Eqs. (28) and (3) should be used to obtain the sine and cosine amplitude separately. When all of these have been obtained the maximum amplitude can be computed by taking the square root of the sums of the sine and cosine amplitudes squared.

REMARKS

The developments described herein imply that the wave height is very small compared to the wave length. This limitation is present in nearly all treatments of surface waves because of the mathematical difficulties which beset attempts to treat waves of finite height. Investigators of exceptional ability, among them Stokes, Rayleigh, Gerstner, Rankine, Levi-Civita and Michell, have been able to extend the analyses of wave motion to some cases in which the wave height is some finite part of the wave length. The results of many of these investigations are summarized in paragraphs 250 and 251 of Lamb's Hydrodynamics, Ref. 3.

It is found that the celerity of wave propagation increases somewhat with wave height and that, for irrotational surface waves, the crests grow sharper and the troughs flatter as the wave height increases. The investigations of Stokes and Michell indicate that such waves can attain an extreme height of 0.142 where the crests become sharp and include an angle of 120^{0}. For this extreme form the wave velocity is 1.2 that for waves of infinitesimal height. Gerstner's rotational waves can, apparently, have sharper crests than the irrotational waves.

Because of the mathematical difficulties mentioned, the contributions which analytical developments can make to wave experimentation work may be expected to be of the nature of first approximations, and this statement is especially true if the formulas to be used are simple enough to keep the computations from becoming burdensome. Some final adjustments of the wave generators on an experimental basis may therefore be needed to compensate for the shortcomings of the computed wave patterns.

ACKNOWLEDGMENT

These developments have had the benefit of a review by Mr. Lucien Duckstein, graduate student at Colorado A & M College.

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3) Hydrodynamics, by Lamb, Dover Publications, New York, 1945. (Chapter VIII treats tidal waves, and Chapter IX treats surface wave motion.)

4) Diffraction of Water Waves by Breakwaters, by J. A. Putnam and R. S. Arthur. Transactions of the American Geophysical Union, Vol. 29, No. 4, August 1948. (Treats diffraction of waves by a semi-infinite breakwater. A comparison of computed and laboratory data is included.)

5) Diffraction of Water Waves Passing Through a Breakwater Gap, by Frank L. Blue, Jr. and J. W. Johnson. Transactions of the Americal Geophysical Union, Vol. 30, No. 5, October 1949. (Applies wave interference concepts as employed in light diffraction to evaluate the diffraction pattern of waves passing through a gap in a breakwater. Comparisons are made between theoretical and experimental diffraction coefficients for deep and shallow water cases.)

6) British Association Mathematical Tables VI., Bessel Functions Part 1. Cambridge University Press, 1950.

7) Waves, by C. A. Coulson, Sixth Edition, 1952, Oliver and Boyd Ltd., London. (Chapter V treats waves in liquids.)

8) Gravity Waves. Chapter 14, Diffraction of Water Waves by Breakwaters. This chapter is by John H. Carr and Marshall E. Stelzriede. The compilation on Gravity Waves is published as National Bureau of Standards Circular 521, issued November 28, 1952. For sale by Superintendent of Documents, Washington, D. C. (Treats diffraction of waves passing through a gap in a breakwater by use of elliptic cylinder coordinates and Mathieu functions. This development they credit largely to Morse and associates and refer to it as the Morse-Rubenstein Theory. They compare the results of this theory with the Penney-Price development and compare both with the results of experiments. The Penney-Price development is stated to be reasonably accurate only for gaps of over two wave lengths width. The Morse-Rubenstein method appears to work best for narrow gaps. Advantages to be gained by working with both methods are described.)

9) Theory of Bessel Functions, Watson, Cambridge University Press, Second Edition, 1952.