Tropical Cyclone Motion: Environmental Interaction plus a Beta Effect

by
Greg J. Holland

P.I. William M. Gray

Department of Atmospheric Science
Colorado State University
Fort Collins, Colorado

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Colorado State University

Fort Collins, Colorado

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* Permanent affiliation: Bureau of Meteorology, Department of Science and Technology, Melbourne, Australia

ABSTRACT

The dynamics of tropical cyclone motion are investigated by solving the vergent barotropic vorticity equation on a beta plane. Two methods of solution are presented: a direct analytic solution for a constant basic current, and a simple numerical solution for a more general condition. These solutions indicate that cyclone motion can be accurately prescribed by a non linear combination of two processes: 1) an interaction between the cyclone and its basic current (the well known steering concept), and 2) an interaction with the earth's vorticity field which causes a westward deviation from the pure steering flow. The nonlinear manner in which these two processes combine together with the effect of asymmetries in the steering current raise some interesting questions on the way in which cyclones of different characteristics interact with their environment, and has implications for tropical cyclone forecasting and the manner in which forecasting techniques are derived.

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1. Introduction

There has been a recent surge of interest and some controversy on the variation of tropical cyclone motion from that of the basic environmental flow, or steering current (defined as the residual after a mean, or predefined symmetric cyclone has been removed by various techniques). As Anthes (1982) has noted, observations indicate that Northern Hemispheric cyclones generally move to the left of, and slightly faster than the basic flow, whereas numerical modelling results seem to indicate that cyclones move to the right of the flow. The observed leftward motion deviation was first intimated by Hubert (1959) who interpreted the bias in a number of early barotropic forecasts as a predilection for the cyclone to move to the left of the geostrophic wind at 500 mb. Confirmation was provided by George and Gray (1976) and Gray (1978) in an extensive rawinsonde compositing study which showed that cyclones typically moved 15° to the left of, and 20% faster than the basic current, and more recently by Brand _et al_. (1981) and Chan and Gray (1982). Chan and Gray also provide evidence that cyclones in the Southern Hemisphere move to the right of the basic current.

These observations seem to be at variance with the classical Rossby beta drift hypothesis and numerical modelling results incorporating surface friction. Rossby (1948) showed that a cyclonic vortex on a beta plane should experience a net poleward acceleration arising from the meridional gradient in coriolis force, or beta. This has since been amply confirmed by theoretical and numerical modeling studies by; for example, Adem (1956), Adem and Lezama (1960), Kasehara and Platzman (1963), Madala and Piacsek (1975), Anthes and Hoke (1975) and Kitade (1980). Yet, since cyclones typically track westward, this poleward drift should produce a deflection to the right of the basic current (or left in the Southern Hemisphere) not to the left (right) as has been observed. Further complications have been introduced by Kuo (1969) and Jones (1977) who showed that surface frictional drag will also cause a cyclone to move to the right of the basic current.

In this paper we attempt to solve this dichotomy and provide an explanation for the observed motion. To do this, we use a modified Rankine vortex which is superimposed on an invariant basic state flow, then derive the vortex motion from the vergent barotropic vorticity equation on a beta plane. By using this simplified approach we are able to directly derive relationships between the cyclone velocity and its basic current for a number of circumstances.

2. Formulation of the Problem

a) Coordinate System

We shall use the cylindrical coordinate system shown in Fig. 1 in which: radius, r, is positive outwards; azimuth, θ, is measured counterclockwise from due north; the radial and azimuthal components of the windspeed are given by u and v, the direction and speed of the basic state flow by α and V_B, and its northward and eastward components by V_N and V_E.

b) Symmetric Cyclone

For the symmetric cyclone we use a modified Rankine vortex

$$v_s = C/r^x \qquad r > r_m, \quad 0 \leq x < 0.8 \tag{1}$$

$$u_s = -\gamma v_s \qquad \gamma \geq 0 \text{ (Northern Hemisphere)}, \quad \gamma \leq 0 \text{ (Southern Hemisphere)} \tag{2}$$

where the subscript s denotes symmetric components, C, x, and γ are constants which define the strength, shape and mean convergence into the cyclone and r_m is the radius to maximum winds.

Equation (1) was first proposed by Hughes (1952) as a modification to Depperman's (1947) application of a pure Rankine vortex ($x = 1$) to the low level azimuthal winds in a tropical cyclone. As has been shown by Hughes (1952), Riehl (1954, 1963), Gray and Shea (1973), x typically lies between 0.4 and 0.6 and in extremes may range between 0 and 0.8. Equation (2) provides a cyclone scale convergence in which γ is the tangent of a constant mean inflow angle which Gray (1981) and others

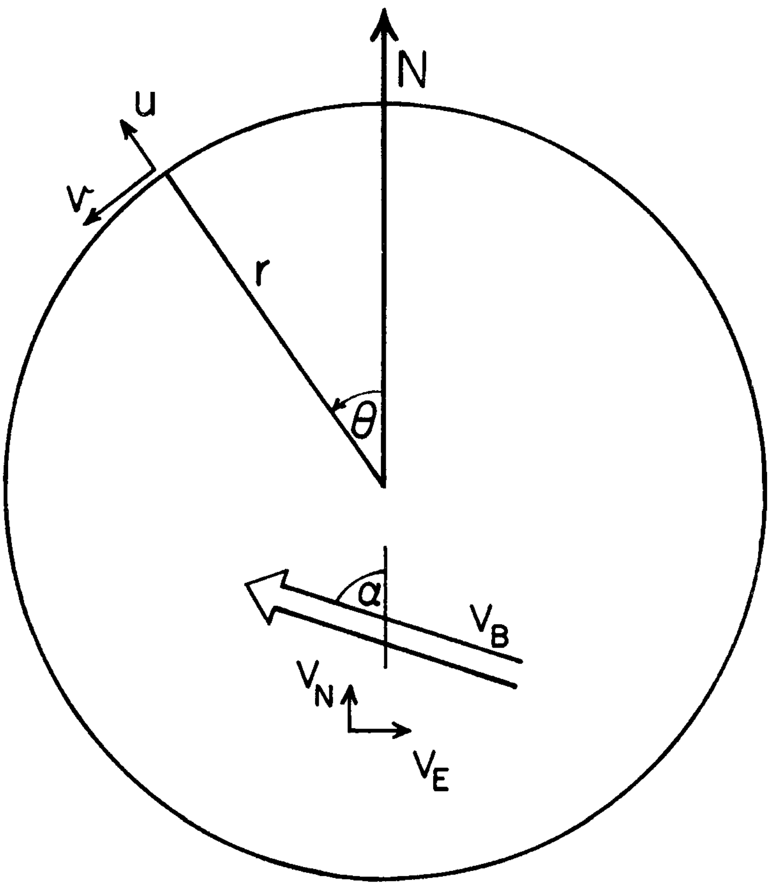

Fig. 1. The coordinate system used in this paper. The symbols are described in the text.

have shown to be a reasonable first approximation in the inflow layer (up to 500 mb).

The vorticity at a given radius outside the radius of maximum winds for this symmetric cyclone is

$$\zeta_s = \frac{(1-x)\ v_s}{r} \tag{3}$$

c) Cyclone Motion

Equation (3) tells us that if a cyclone is moving in a particular direction, then the cyclonic vorticity must be increasing in that direction. Hence under ideal circumstances we can find the cyclone motion as a solution of the vorticity equation, which in pressure coordinates is

$$\frac{\partial \zeta}{\partial t} = -\underset{\sim}{V} \cdot \nabla (\zeta + f) - \omega \frac{\partial \zeta}{\partial p} - (\zeta + f) \nabla \cdot \underset{\sim}{V} + \underset{\sim}{k} \cdot (\frac{\partial \underset{\sim}{V}}{\partial p} \times \nabla \omega)$$

$$+ \underset{\sim}{k} \cdot (\nabla \times \underset{\sim}{F}) \tag{4}$$

Where all terms, which have their usual meaning, are defined in the appendix. Unfortunately, in the cyclone environment the vertical advection, tilting, and frictional terms are, at best, difficult to calculate. Hence, we shall simply neglect these terms at this stage and provide áposteriori justification in later sections. We then have the barotropic, vergent vorticity equation in cylindrical coordinates

$$\frac{\partial \zeta}{\partial t} = - u \frac{\partial \zeta}{\partial r} - \frac{v}{r} \frac{\partial \zeta}{\partial \theta} - v_n \beta - \frac{\zeta + f}{r} \left(\frac{\partial r u}{\partial r} + \frac{\partial v}{\partial \theta}\right) \tag{5}$$

where the northward component of the total wind speed v_n is given by

$$v_n = u \cos\theta - v \sin\theta \tag{6}$$

We now assume that the cyclone will move towards the region of maximum vorticity change, then the direction of motion will be given by

$$\frac{\partial}{\partial \theta} \left(\frac{\partial \zeta}{\partial t}\right)' = 0 \tag{7}$$

and the speed of motion by

$$V_c = - \left[\left(\frac{\partial \zeta}{\partial t}\right)' / \frac{\partial \zeta_s}{\partial r}\right]_{\theta_m} \tag{8}$$

where θ_m is the solution to Eq. (7), and the prime indicates that the azimuthally symmetric terms, which cannot contribute to the motion, have been deleted. These two relations will not hold for $x = 1$, at which stage the cyclone has no vorticity outside the radius of maximum winds (Eq. 3). But they are valid for our presumed range of $0 \leq x < 0.8$.

We shall also assume that the basic flow is constant with time. Thus, the basic current may advect the vortex through Eq. (5) but the vortex may not interact with and change the basic state. This is a valid assumption in that we are primarily interested in the short term motion and Adem (1956) has shown that the non linear feedback is unimportant over a time scale of 6-12 hours.

3. Direct Analytic Solutions

a) The Circular Rossby Wave

For our first case we take a convergent vortex on a beta plane with no basic flow. Thus on substituting Eqs. (1), (2), (3) into the vorticity Eq. (5), using Eq. (6) and neglecting all azimuthally symmetric terms we arrive at

$$\left(\frac{\partial \zeta}{\partial t}\right)' = \beta v_s \left(\sin\theta + \gamma\,(2 - x)\cos\theta\right) \tag{9}$$

Using Eq. (7), the direction of motion is then

$$\theta_m = \tan^{-1}\left[1/(\gamma\,(2 - x))\right] \tag{10}$$

and from Eq. (8) the vortex speed is given by

$$V_c = \frac{\beta r^2}{1 - x^2}\left(\sin\theta_m + \gamma\,(2 - x)\cos\theta_m\right) \tag{11}$$

The direction and speed of motion are thus independent of the strength of the vortex (given by C in Eq. 1) but depend on the shape (x) and the amount of convergence (γ). The speed of motion is also radius dependent.

If we remove the convergence ($\gamma = 0$) then

$$\theta_m = 90^\circ$$

$$V_c = \frac{\beta r^2}{(1 - x^2)} \tag{12}$$

and the only asymmetric vorticity changes are due to the advection of earth vorticity by the azimuthal winds. Hence, the vortex drifts due westward in a circular analogy to the classical Rossby wave (Rossby, 1939).

In a convergent cyclone (positive γ in the Northern Hemisphere, negative γ in the Southern Hemisphere), the radial inflow introduces an additional asymmetric vorticity change, cyclonic on the poleward, and anticyclonic on the equatorward side. Figure 2 shows the resulting effect on the cyclone for typical parameters of $x = 0.5$, $r = 300$ km and $\beta = 2.15 \times 10^{-11}\ m^{-1}s^{-1}$ (correspond to a latitude of 20^{o}). We see that increasing convergence turns the cyclone poleward and increases its speed. However, for reasonable deep layer inflow angles (up to $10-20^{o}$) the speed will remain essentially constant while the direction may change considerably. These results are in qualitative agreement with those of Anthes and Hoke (1975).

b) The Rossby Drift

Rossby (1948) showed that an isolated vortex on a β plane will experience a meridional acceleration due to the variation in Coriolis force. Specifically, in the absence of compensating pressure gradient forces the stronger Coriolis force on the poleward versus the equatorward side produces a poleward force on a cyclone which is directly related to the cyclone rotation rate and radial extent, and to the magnitude of β. Adem (1956) combined this drift and the circular Rossby wave effect to show that a cyclone will drift to the northwest in the absence of a defined basic current, and a number of numerical model results have since confirmed this (Anthes and Hoke, 1975; Madala and Piacsek, 1975; Kitade, 1980). Since most tropical cyclones move to the west, the premise has therefore been that this Rossby drift should cause them to move to the right of the steering current in the Northern

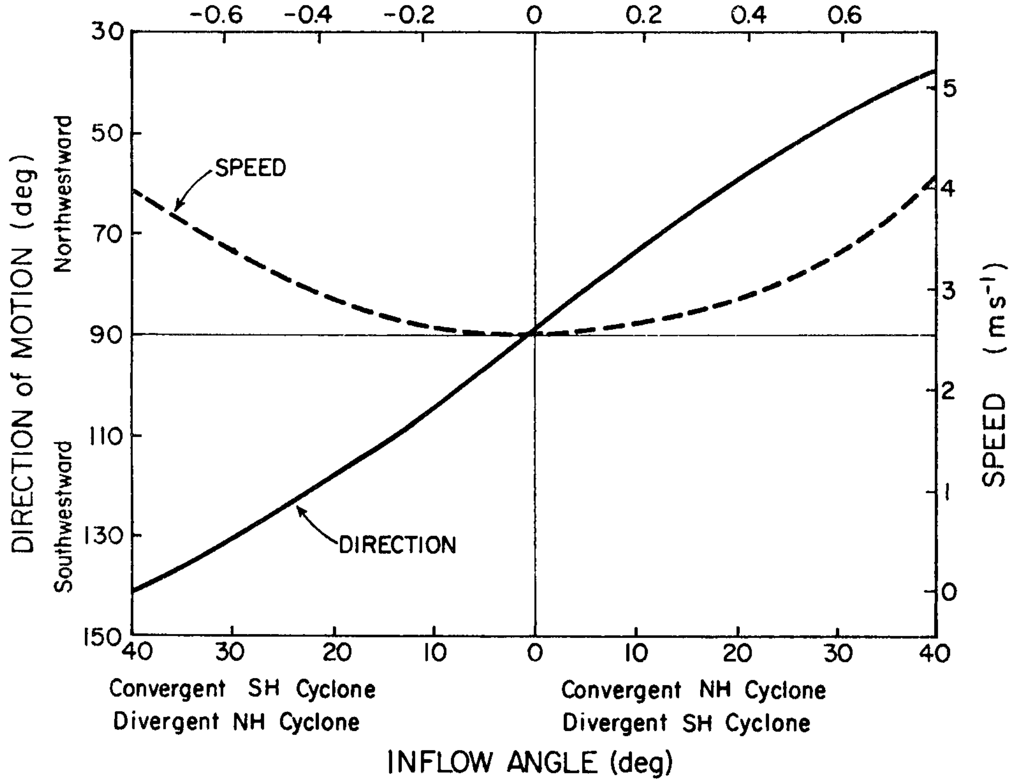

Fig. 2. The change in speed and direction of motion with degree of vergence in a tropical cyclone with no basic current.

Hemisphere; a premise which has been confirmed by the theoretical work of Rossby (1948) and Adem and Lezama (1960). We shall show that differing definitions of the basic current render this premise incorrect for direct observations.

Imagine a symmetric, nonvergent vortex drifting westward on a Northern Hemispheric beta plane as described in section 3a. Then the time rate of change of vorticity, given by Eq. (9) but viewed in a Lagrangian sense following the cyclone center, will be as shown in Fig. 3. (The assignment of units is arbitrary and will vary according to β, the strength and shape of the vortex.) These Lagrangian vorticity changes, and the distortion of the cyclone prescribed by Eq. (11), will concomitantly produce two opposing circulations, counterclockwise to the

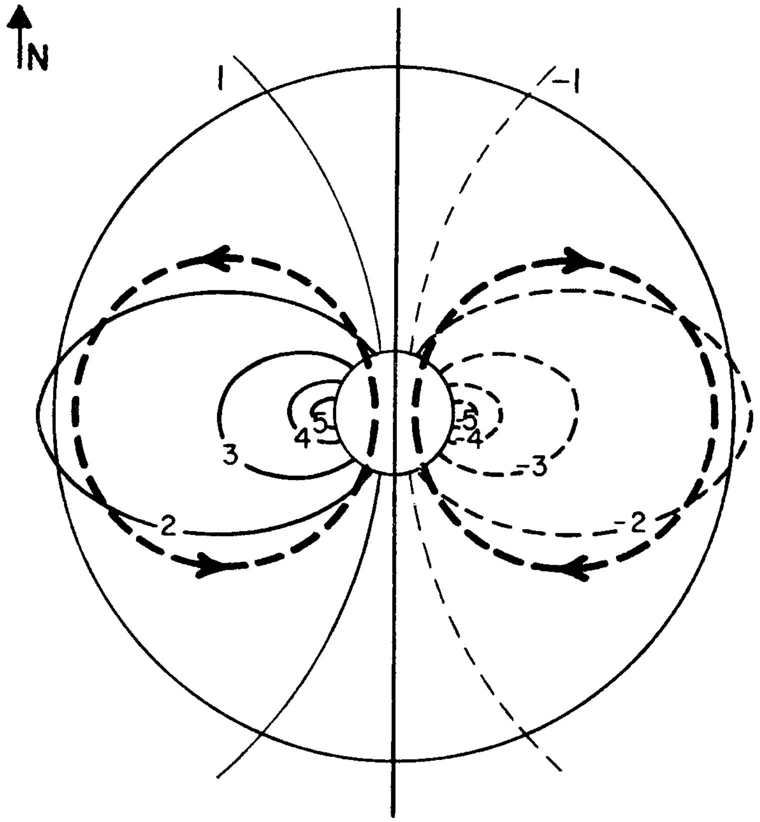

Fig. 3. Field of $\partial\zeta/\partial t$, in arbitrary units, centered on a symmetric, nonvergent Northern Hemispheric cyclone which is drifting westwards on a beta plane with no basic flow. Heavy dashed lines show the induced secondary circulation as a result of the vorticity changes.

west and clockwise to the east, as shown schematically in Fig. 3. The resultant southerly wind over the vortex center will advect it poleward. Introducing convergence into the cyclone will rotate the field in Fig. 3 in a clockwise direction so that the steering current will be from a more southwesterly direction.

A similar argument has been advanced by Anthes (1982) and some interesting examples of this effect may be seen in the work of Adem (1956), Morikawa (1962) and Kitade (1980). Adem showed that a symmetric, isolated nonvergent cyclone on a beta plane would initially move westwards. Then, as the second order circulations formed, the cyclone turned and moved northwestward under the combined poleward drift and circular Rossby wave effects. Morikawa initialized a geostrophic point vortex in a linear westerly basic current and then examined the distortion in the basic current during a 48 hours integration of a nonvergent barotropic model. The initially uniform basic current soon developed a large amplitude wave structure with southerly flow over the vortex center (Morikawa's Fig. 5). Kitade started with a realistic three dimensional tropical storm on a beta plane with no basic current and then integrated for eighty hours with a vergent quasi-barotropic model. The resultant trajectory is shown in Fig. 4, together with several plots of the basic current at 650 mb (averaged over a 10^{o} lat/long square centered on the cyclone) which has been generated as a result of the above effect. Since the cyclone is convergent in the low levels, this southwesterly basic current and the tracking of the cyclone to the left are in good agreement with our previous discussion. Kitade also describes three other experiments using a baroclinic model which produced similar results.

Thus Rossby's poleward drift is not really a force on the cyclone, rather it is an imposed basic flow. In this regard the definition of a basic current in the theoretical studies is quite different to that of a forecaster. The theoreticians start with a symmetric vortex, superimpose a basic current then calculated the beta effect to arrive at a westward and poleward beta drift after the basic current has distorted. The forecaster extracts a symmetric vortex from his wind analysis to get a basic current which already includes the poleward Rossby drift; thus leaving only the westward beta drift, as we shall show in the following section.

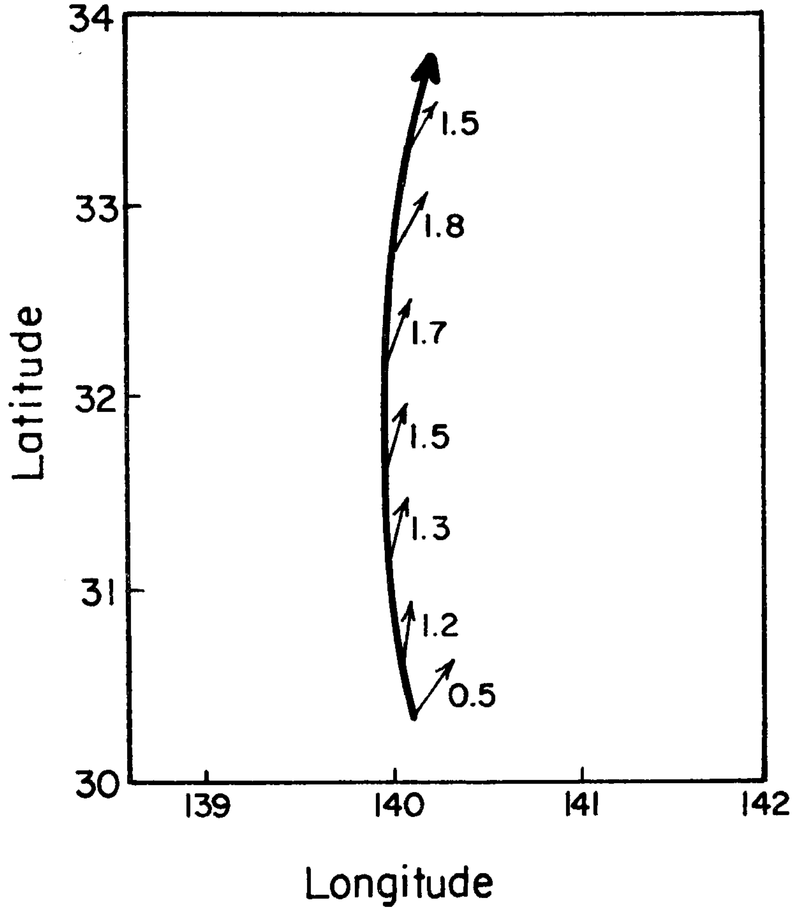

Fig. 4. Reconstruction of the trajectory of a tropical cyclone (heavy line) during an integration of a vergent quasi-barotropic model by Kitade (1980). The arrows and numerals indicate the direction and speed (m s^{-1}) of the steering current at 10 hourly intervals.

c) Constant Basic State

If we introduce a nonsheared, straight environmental flow, V_B, at an angle α to due north (Fig. 1), then the radial and tangential wind components become

$$v = v_s - V_B \sin\chi \tag{13}$$

$$u = -\gamma v_s + V_B \cos\chi \tag{14}$$

where $\chi = \theta - \alpha$. Since the environmental flow contains no vorticity the relative vorticity is still given by Eq. (3), and the asymmetric part of Eq. (5) becomes

$$\left(\frac{\partial \zeta}{\partial t}\right)' = \frac{1-x^2}{r^2} v_s V_B \cos\chi + v_s \beta \left(\sin\theta + \gamma (2 - x) \cos\theta\right) \tag{15}$$

Substituting Eq. (15) into Eqs. (7), (8) and solving them gives the direction and speed of motion

$$\theta_m = \tan^{-1} \left[\frac{\frac{1 - x^2}{r^2} V_B \sin\alpha + \beta}{\frac{1 - x^2}{r^2} V_B \cos\alpha + \gamma (2 - x)\beta}\right] \tag{16}$$

$$V_c = V_B \cos(\theta_m - \alpha) + \frac{\beta r^2}{1 - x^2} (\sin\theta_m + \gamma (2 - x) \cos\theta_m) \tag{17}$$

If we revert to an f plane ($\beta = 0$) then $\theta_m = \alpha$, $V_c = V_B$ and the cyclone moves exactly with the environmental flow, a result that has previously been obtained by Adem and Lezama (1960). For $V_B = 0$ we regain Eq. (10), (11). Thus the deviation from the environmental flow arises solely from the superimposed westward movement of the cyclone as it interacts with the earth vorticity field in the manner described in section 3a. Simplistically, the motion described by Eqs. (16), (17) may therefore be considered as the addition of two vectors: one aligned along and equal to the basic flow, V_B, and another beta effect, V_β, pointing westward with its exact alignment and length defined by Eqs. (10), (11). Four schematic examples for a nonvergent cyclone in either hemisphere are given in Fig. 5. Since the beta effect is constant for a given cyclone the deviation angle, and to a lesser extent the speed deviation (note the cos ($\theta_m - \alpha$) term in Eq. 17) vary according to the speed and direction of the basic current. We shall discuss the importance of these effects in section 6.

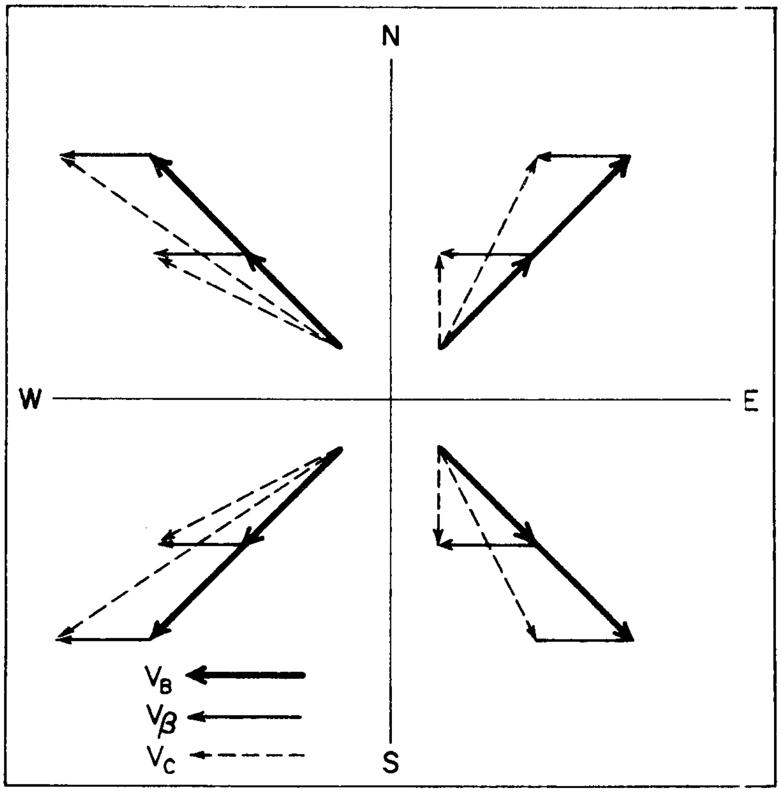

Fig. 5. A schematic illustration of the effects of adding a constant beta effect (V_β) to basic currents (V_B) of different directions and speeds to produce a resultant cyclone motion (V_c).

Introducing convergence will turn the beta effect vector, $\underset{\sim}{V}_\beta$, more polewards as described in section 3a and Fig. 2. The actual track and speed deviations as a function of α for some typical Northern Hemispheric parameters of $\gamma = 0.2$, $V_B = 5$, $10\ m\ s^{-1}$, $x = 0.5$, $r = 300$ km and $\beta = 2.15 \times 10^{-4} m^{-1} s^{-1}$ are shown in Fig. 6. We see in Fig. 6a that eastward moving cyclones have the largest track deviations and are also quite sensitive to the basic flow speed. (The two curves in Fig. 6a are from a family of curves which converge to a straight line as V_B

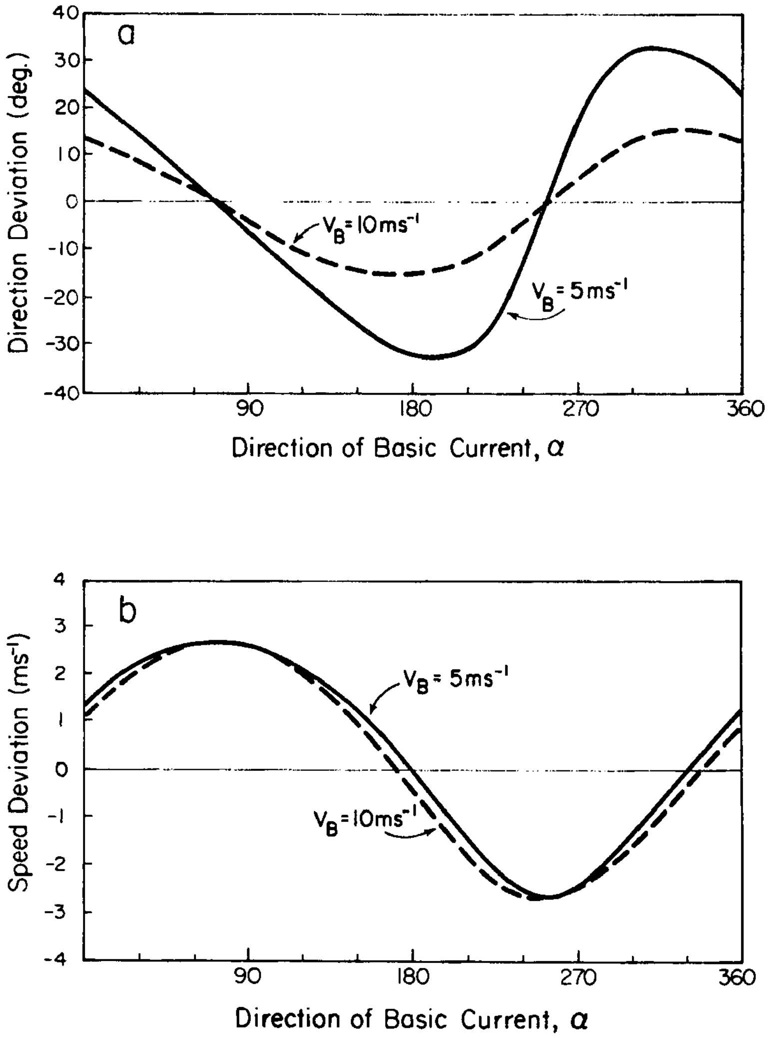

Fig. 6. Track (a) and speed (b) deviation from the basic flow for typical Northern Hemisphere cyclone parameters and two basic current speeds. A positive deviation means to the left of, and faster than the basic current.

approaches infinity and to a sawtooth curve as V_B approaches zero.) The speed deviations in Fig. 6b show a much smaller sensitivity to changes in the environmental speed. These curves should be taken as a general indication only as individual tracks and speeds depend on the choice of different parameters. We shall discuss these aspects further in section 6.

d) The Effective Radius

Equations (16) and (17) indicate that the motion for a given set of basic flow and cyclone parameters is radius dependent. Thus the cyclone is being continually distorted with different parts attempting to move at different velocities. Adem and Lezama (1960), calculated the resultant velocity by integrating over the entire cyclone domain (which they predefined) and thus arrived at an effective radius which is six or seven tenths of the radial extent of the symmetric cyclone. Physically this may be interpreted as follows. We know that the cyclone has a high rotational stability in the inner few hundred kilometers, as is evidenced by its ability to remain quasisymmetric and maintain its identity even under quite adverse conditions. (From the vorticity Eq. (5) point of view, this means that any distortion will immediately introduce circular advective terms which quickly act to retain the quasisymmetrical shape.) In the outer regions of the cyclone this stability is weak and considerable distortion may occur. Hence, even though the cyclone interacts with the environment to some extent at all radii, one effective radius will dominate in determining its motion. As shown schematically in Fig. 7, this will create an envelope which must be at large enough radius to maximize the interactions with the basic flow and β effects, but also small enough to ensure sufficient rotational stiffness to advect the cyclone center along with it.

This physical hypothesis has two interesting corollaries. One is that the outer region beyond the envelope will distort with time and thus interact with and modify the basic state, as we discussed in the explanation of the Rossby drift in Section 3b. The second relates to the observed oscillatory motion of a cyclone about is mean path.

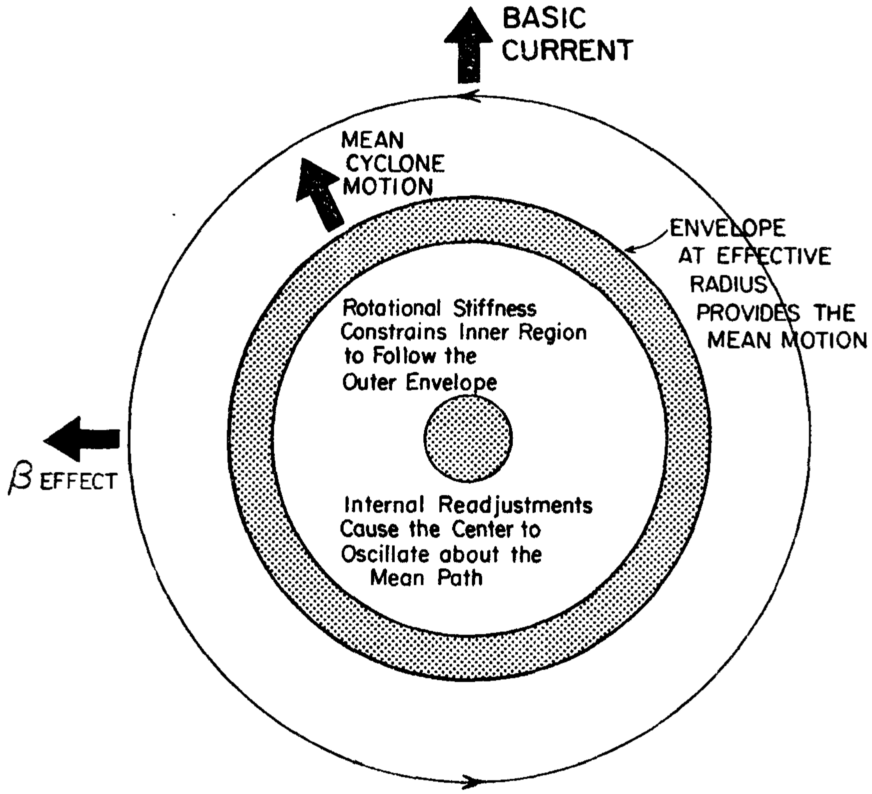

Fig. 7. A simple schematic of cyclone motion on a beta plane under a basic current.

Yeh (1950) and Kuo (1950) described this oscillatory motion as arising from the atmospheric equivalent of the Magnus force on a rigid rotating cylinder. This is an aerodynamic force resulting from the interaction between the rotating cylinder and its basic current, and is perpendicular to the current, being to the right of a cyclone in the Northern Hemisphere and to the left in the Southern Hemisphere. The resulting trajectory is a trochoidal oscillation about the mean path with a period and amplitude determined by the solid body rotation of the vortex and the speed of the basic current. Our hypothesis offers an alternative explanation. The cyclone center is constrained to move with the outer envelope by its rotational stiffness. But the contact is not a rigid one, rather the center has a limited freedom to move within these outer region constraints. For example, our simple model, Eq. (16), dictates that the center move to the right of the outer envelope for a southerly basic current in the Northern Hemisphere. This tendency for rightward movement and subsequent adjustment back to the mean path would provide a short period oscillation. However in the real world there must be a plethora of mechanisms (for example, convective and frictional asymmetries) to distort and deflect the central region and result in an oscillatory trajectory within the constraints of a slowly varying outer envelope.

4. Observations

Extensive rawinsonde compositing studies of Northern Hemispheric tropical cyclone motion in relation to the basic flow which have been reported by George and Gray (1976), Gray (1978) and Chan and Gray (1982) are in good agreement with our analytic results. As an example, some pertinent NW Pacific findings from Chan and Gray are summarized in Table 1, together with some previously unpublished results.

Considering first the speed stratifications in Table 1A, we see that the deviation angle decreases consistently with increasing speed while the speed deviation remains almost constant. Table 1B indicates that northeastward moving cyclones deviate more to the left and are slower relative to the basic current than those moving to the north and west. The small deviation angle for the northeastward moving typhoon is due to the dominating effect of the concomitant increase in basic current speed. There is no apparent reason for the seemingly anomolous motion of the westward moving typhoon, though it is noteable that westward moving Atlantic hurricanes (not shown) move to the right of the basic current, in good agreement with the theory.

Table 1C contains previously unpublished results for recurving cyclones in the NW Pacific. The four composites are homogeneous in that they contain the same cyclones at different periods of their life cycle. These cyclones all started moving northwestward, reached supertyphoon (950 mb or less) intensity at or within 1 day before recurvature, and then filled after recurvature. The mean directions of motion are given in brackets for each stratification. We see once again that as the cyclone turns poleward and recurves the deviation angle increases and the speed deviation decreases and becomes positive.

TABLE 1

General deviations of composite NW Pacific tropical cyclones from their basic currents after Chan and Gray (1982). The basic current is an azimuthal and vertical average over 700–500 mb and 5–7^{o} latitude radius from the center. Directions of cyclone motion are given in parentheses.

Cyclone Type	Basic Current Speed ($m\ s^{-1}$)	Direction Deviation (^{o})	Speed Deviation ($m\ s^{-1}$)
(A)			
Slow	2.4	30	-0.6
Moderate	4.3	23	-0.9
Fast	9.5	18	-0.6
(B)			
Westward	3.8	20	-2.4
Northward	4.4	23	-0.9
Northeastward	7.3	19	0.2
(C)			
Intensifying Tropical Storm (θ_m=67)	4.5	10	-0.5
Intensifying Typhoon Prior to Recurvature (θ_m=61)	4.0	19	-0.8
Intensifying Typhoon Near Recurvature(θ_m=48)	4.6	31	-0.4
Filling Typhoon At or After Recurvature(θ_m=359)	6.1	37	0.7

The average effective radii of interaction were also calculated for the stratifications in Table 1B, 1C. This was done by solving Eqs. (16) and (17) using the known basic current and cyclone velocity and varying x from 0.4 to 0.6 and δ from 0.2 to 0.4. The effective radii varied from 200-400 km and were typically in the range 250-300 km, thus providing tentative justification for our deletion of the tilting term in Eq. (5). However, these values cannot be unequivocably accepted as typical of all cyclones. Rather, a more thorough investigation is needed to determine the effects of, among other things, differing environments, cyclone size, intensity and convergence.

Brand et al. (1981) compared ten years of Northwest Pacific tropical cyclone data with the direction of the geostrophic basic current derived from 500 mb operational analyses. Their results, which are summarised in Fig. 8, indicate that low latitude cyclones move to the right of this basic current and that high latitude cyclones move to the left. Since low latitude cyclones typically move westward while those at higher latitudes tend to move more poleward and eastward, these results are also in good agreement with the analytical results in Fig. 6. Brand et al. also showed that the variance from these mean deviations was a maximum between 20-30°N and decreased poleward and equatorward of this region. A large part of this variance pattern will be due to the greater variety of storm directions in the subtropics compared to those in equatorial and higher latitude regions. But variations in size, intensity and convergence, and (as discussed later in section 6c) different characteristics of the basic current could also contribute to much of this behavior.

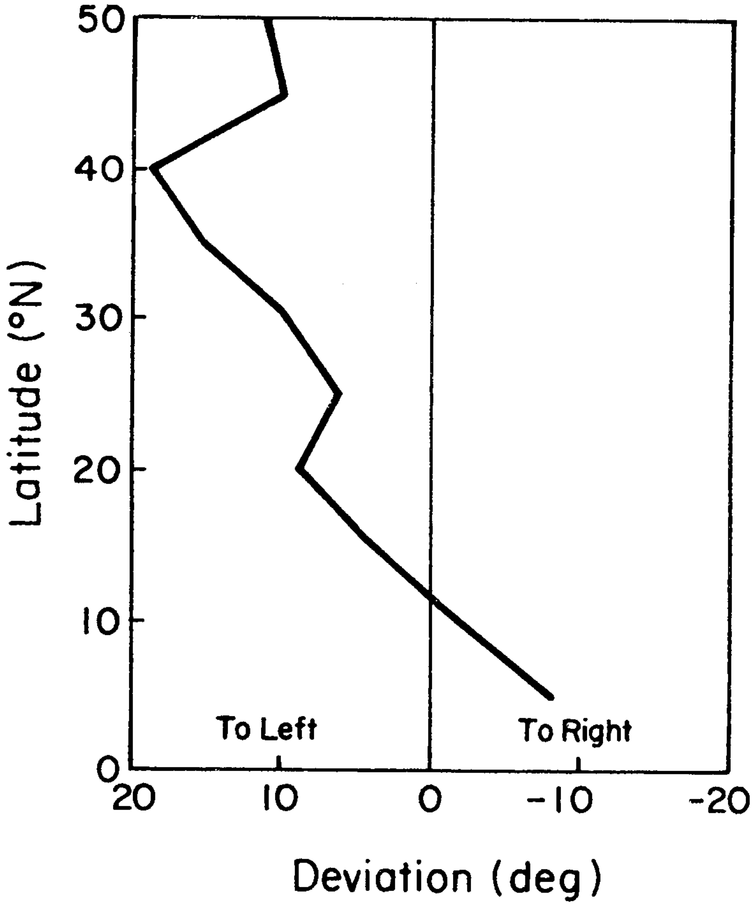

Fig. 8. Deviation of Northwest Pacific tropical cyclones from their geostrophic basic current (after Brand et al., 1981).

5. General Iterative Solutions

a) The Model

The assumption of a constant basic state in section 3c provided us with an analytic solution for the resulting direction and speed of the cyclone, which, as we showed in the previous section, provided close agreement with general observations. But, Eqs. (16) and (17) have only limited applicability to individual situations, in which the basic state flow is often far from uniform. Hence, in this section we derive a more general method of solution which can simulate complex tropical cyclone motions quite well.

To do this we specify the basic state flow by a wind at the cyclone center, $\overline{V}_B$, meridional and zonal components of vorticity, $\zeta_o = -\frac{\partial V_E}{\partial y}$, $\zeta_1 = \frac{\partial V_N}{\partial x}$, and meridional and zonal components of vergence $\delta_o = \frac{\partial V_N}{\partial y}$, $\delta_1 = \frac{\partial V_E}{\partial x}$ (where V_E, V_N are the zonal and meridional components of the basic state flow). We still specify the symmetric cyclone by Eqs. (1), (2), then

$$V_N = \overline{V}_N - \zeta_1 r \sin\theta + \delta_o r \cos\theta \tag{18}$$

$$V_E = \overline{V}_E - \zeta_o r \cos\theta - \delta_1 r \sin\theta \tag{19}$$

$$v = v_s - V_N \sin\theta - V_E \cos\theta \tag{20}$$

$$u = -\gamma v_s + V_N \cos\theta - V_E \sin\theta \tag{21}$$

$$\zeta = \frac{v_s}{r}(1-x) + \zeta_o + \zeta_1 \tag{22}$$

After substituting Eqs. (18) to (22) into Eq. (5), dropping the symmetric terms and some tedious algebra we get

$$\left(\frac{\partial \zeta}{\partial t}\right)' = \underset{\text{(A)}}{V_B \cos(\theta-\alpha)\,\frac{v_s}{r^2}\,(1-x^2)}$$

$$+\ \beta\left[\underset{\text{(B)}}{v_s(\gamma(2-x)\cos\theta+\sin\theta)} + \underset{\text{(C)}}{r(\zeta_1\sin\theta-(2\delta_o+\delta_1)\cos\theta)}\right] \tag{23}$$

Equation (23) states that the asymmetric vorticity changes around the cyclone arise from the advection of vorticity associated with the symmetric cyclone by the basic current (term A), the advection and vergence of earth vorticity by the symmetric cyclone circulation (term B), and the advection and vergence of earth vorticity by the asymmetric component of the basic current (term C). Next substituting Eq. (23) into Eq. (7) gives

$$\begin{aligned} & r^3\beta[\zeta_1\cos\theta_m + (2\delta_o+\delta_1)\sin\theta_m] \\ & + r^2\beta v_s[\cos\theta_m - \gamma(2-x)\sin\theta_m] \\ & + r(1-x^2)v_s[(\zeta_o-\zeta_1)\cos2\theta_m + (\delta_1-\delta_o)\sin2\theta_m] \\ & - \bar{V}_B(1-x^2)v_s\sin(\theta_m-\alpha) = 0 \end{aligned} \tag{24}$$

$$\begin{aligned} V_c = {} & V_B\cos(\theta_m-\alpha) + \frac{\beta r^2}{(1-x^2)}[\gamma(2-x)\cos\theta_m + \sin\theta_m] \\ & + \frac{\beta r^3}{(1-x^2)}[\zeta_1\sin\theta_m - (2\delta_o+\delta_1)\cos\theta_m] \end{aligned} \tag{25}$$

The only terms in Eqs. (24) and (25) which cannot be specified or calculated directly from available data are the direction and speed of motion, θ_m, V_c, and the effective radius at which the cyclone interacts with the basic current.

The major problem lies in determining the effective radius. A climatological definition such as given in Table 1 would probably be quite poor in individual cases since this radius is almost certainly a complex function of many different parameters. However, if we make the assumption that the effective radius changes slowly over short periods of time then we can calculate it for a past time when we know all other cyclone parameters, then use this value to predict the cyclone motion from present data. The solution technique is then as shown in Table 2. For case study or forecast applications, the model is first 'initialized' by determining the effective radius from immediate past data. Then present data are incorporated and the future cyclone motion is forecast using the same effective radius. For 'what if?' scientific applications a standard cyclone is defined at the beginning of the sequence.

This simple model, consisting of Eqs. (24) and (25) and the solution procedure in Table 2, has been programmed into a HP41C calculator for ease and convenience of use. It has two modes of operation. In the forecast or case study mode, data at up to eight equidistant grid points surrounding the center are provided, together with the cyclone position. The model then extracts the symmetric vortex and either initializes an effective radius or produces a forecast speed and direction together with positions at any specified time interval. In the second, research or interactive mode any parameters may be

TABLE 2

Method of solution for Eqs. (24) and (25).

INITIALIZE CYCLONE ON PAST DATA	PRODUCE FORECAST FROM PRESENT DATA
Known:	Update:
1)Total wind field in a specified layer	1)Total wind field in specified layer
2)Cyclone intensity and motion	2)Cyclone intensity
3)Latitude	3)Latitude
Calculate: $\beta, \gamma, x, \zeta_o, \zeta_1, \delta_o, \delta_1, \alpha, V_B$	Calculate: $\beta, \gamma, x, \zeta_o, \zeta_1, \delta_o, \delta_1, \alpha, V_B$
Initialize cyclone by using θ_m and V_c to specify effective radius, r_E, from Eqs. 24 and 25	Forecast new cyclone motion θ_m, V_c by using effective radius from initialization step and iteratively solving Eqs. 24 and 25

TABLE 3

Ten hour model and persistence forecast errors (km) using the data in Figs. 4 and 9. Root mean square errors are shown in brackets.

		ERROR	
		MODEL	PERSISTENCE
Barotropic Case (Fig. 4)	ZONAL ERROR	0.22 (3.75)	1.56 (7.81)
	MERIDIONAL ERROR	-0.67 (4.64)	-4.22 (4.95)
Baroclinic Case (Fig. 9)	ZONAL ERROR	8.68 (24.69)	-8.89 (18.31)
	MERIDIONAL ERROR	6.11 (29.88)	6.11 (37.18)

defined as constant or changed at will. The model then adjusts to these parameters and produces a forecast cyclone motion.

As a simple test of the models viability we use the two experiments from Kitade (1980) shown in Fig. 4 and Fig. 9. For each case we took the cyclone and basic state velocities at time t-10 hours, to calculate the effective radius, then used this effective radius with the basic state velocity at time t to produce a forecast position at time t + 10 hours. For the barotropic case (Fig. 4) we set $x = 0.5$ and $\gamma = 0.2$ throughout the experiment. For the baroclinic case (Fig. 9) we set $\gamma = 0.2$ throughout but varied x from 0.5 at the beginning to 0.65 at the end to emulate the intensification of the cyclone from a maximum wind of 20 $m\ s^{-1}$ to 45 $m\ s^{-1}$ (see Kitade, 1980, Fig. 5b).

The resultant forecast errors, shown in Table 3, indicate that the model can accurately reproduce the results of much more complex numerical models, despite the rather rapidly varying basic state flow in the baroclinic case. A number of experimental runs with the model are used to aid discussion in the following section, and its possible use in forecasting is discussed in section 6.

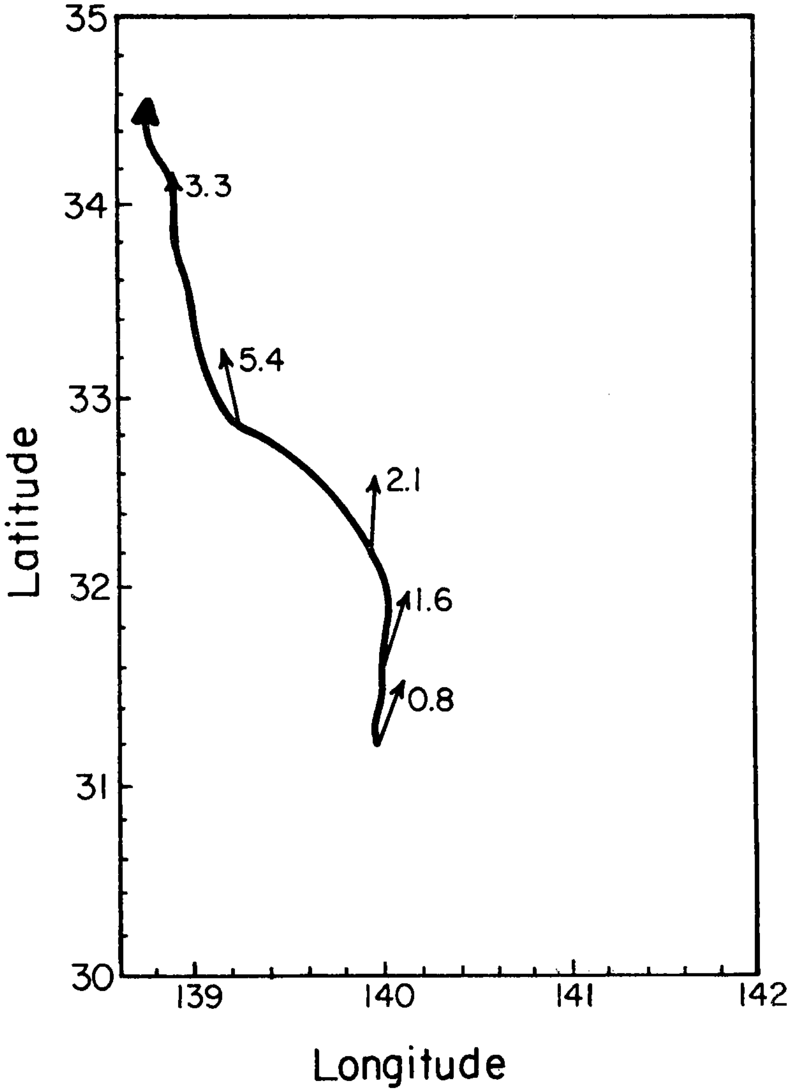

Fig. 9. Reconstruction of the trajectory (solid line) of a tropical cyclone from baroclinic experiment #1 of Kitade (1980). The arrows and numerals indicate the direction and speed ($m\ s^{-1}$) of the steering current at 10 hourly intervals.

6. Discussion

a) The Stability of Cyclone Motion

We define the motion of a cyclone to be stable when it reacts in a damped manner to basic current variations or shortlived asymmetries, and unstable when it overreacts to such variations. Then the imposition of the beta effect and asymmetries in the basic flow can result in a stable or unstable configuration, depending on the direction of the basic current.

Consider first the direction deviation curves in Fig. 6a. Convergent Nothern Hemispheric cyclones under a west to northwestward basic current ($30^o < \alpha < 120^o$ say) tend to converge towards a west-northwest direction ($\theta \simeq 70^o$ for the cyclone in Fig. 6a). Hence, they have a damped response to variations in the direction of the basic current; for the example in Fig. 6a a <u>20^o northwestward basic current veering from $\alpha = 80^o$ to 60^o produces only a 13^o change in the direction of cyclone motion</u>. Northwestward moving cyclones are also stable for changes in the speed of the basic current. A north to eastward, or south to southeastward moving cyclone however, is quite unstable with respect to changes in this basic current speed. Again referring to Fig. 6a, we can see that <u>doubling the speed of the northeastward ($\alpha = 315^o$) basic current from 5 to 10 m s^{-1} results in a 15-20^o change in cyclone direction</u>. By comparison to westward moving cyclones, the imposition of the beta effect causes cyclones which are moving to the southeast through northeast ($220 < \alpha < 300$ say) to diverge away from an east-southeasterly direction. Hence, they undergo unstable overreactions to changes in the direction of the basic current. This instability increases for lower basic current speeds; the $V_B = 5 \text{ m s}^{-1}$ curve in Fig. 6a indicates that a <u>20^o change in eastward basic current direction from $\alpha = 250^o$ to 270^o produces a 40^o change in the cyclone direction</u>.

In this simple theory we have neglected possible asymmetric effects of vertical vorticity advection, tilting, friction, asymmetric convergence, etc. In doing this we assumed that, while such effects are quite large in the region of the eyewall and may result in significant short period oscillations, they are of secondary importance at the effective radius envelope, which defines the longer period motion. We can now qualify these assumptions by examining the stability of the cyclone motion to additional asymmetric perturbations.

The azimuthal variations of rate of vorticity change at an assumed effective radius of 300 km for the typical cyclone described in section 3c are shown in Figs. 10, 11 and 12. A cyclone with a strong, sharp vorticity change maximum will have a stable motion since large additional asymmetric effects will be required to move this maximum and thus change the direction of motion. By comparison, a cyclone with a relatively flat vorticity change curve may be quite unstable to even minor perturbations. Figures 10, 11 and 12, then, indicate that westward moving cyclones are more stable than northward (and southward) or eastward moving cyclones, and that the stability increases in all cases as the basic current increases. For low wind speeds (Fig. 10) eastward moving cyclones are highly unstable to minor perturbations in any direction whereas those moving westward will only respond to perturbations very close to the direction of motion. Notice also in Figs. 11 and 12 that the addition of the beta effect results in a different response to perturbations on either side of the storm motion.

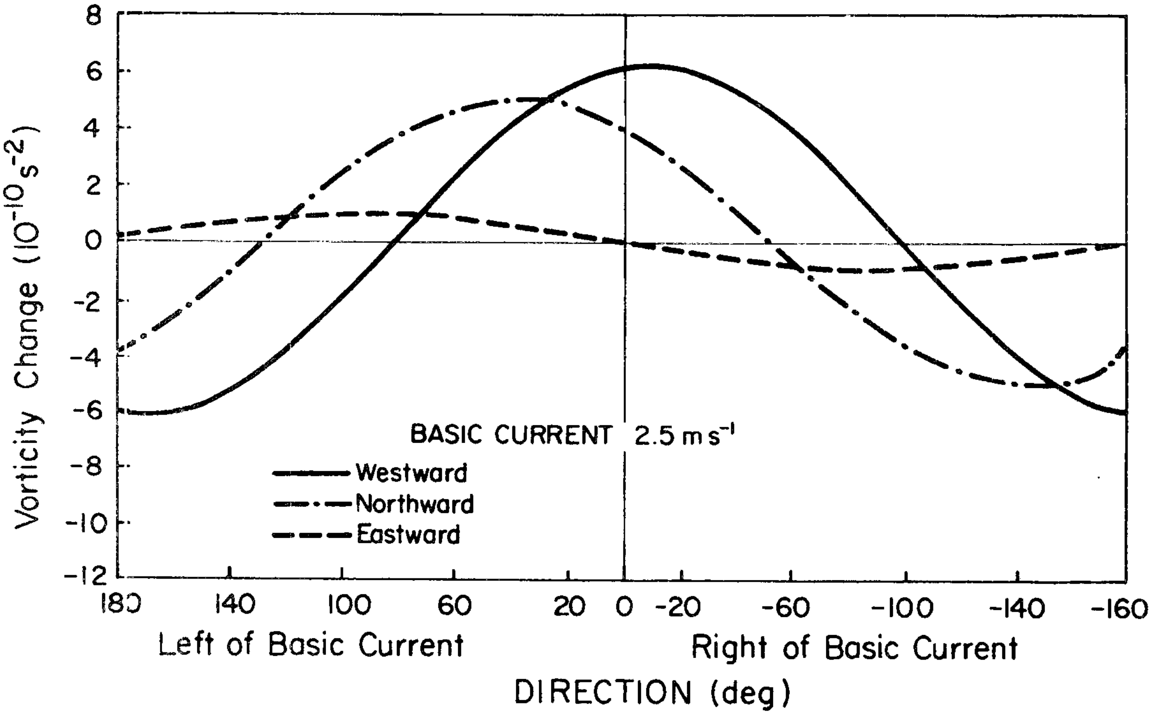

Fig. 10. Azimuthal variations of rate of vorticity change at the effective radius of 300 km for the typical cyclone described in section 3c and a basic current speed of 2.5 m s^{-1}.

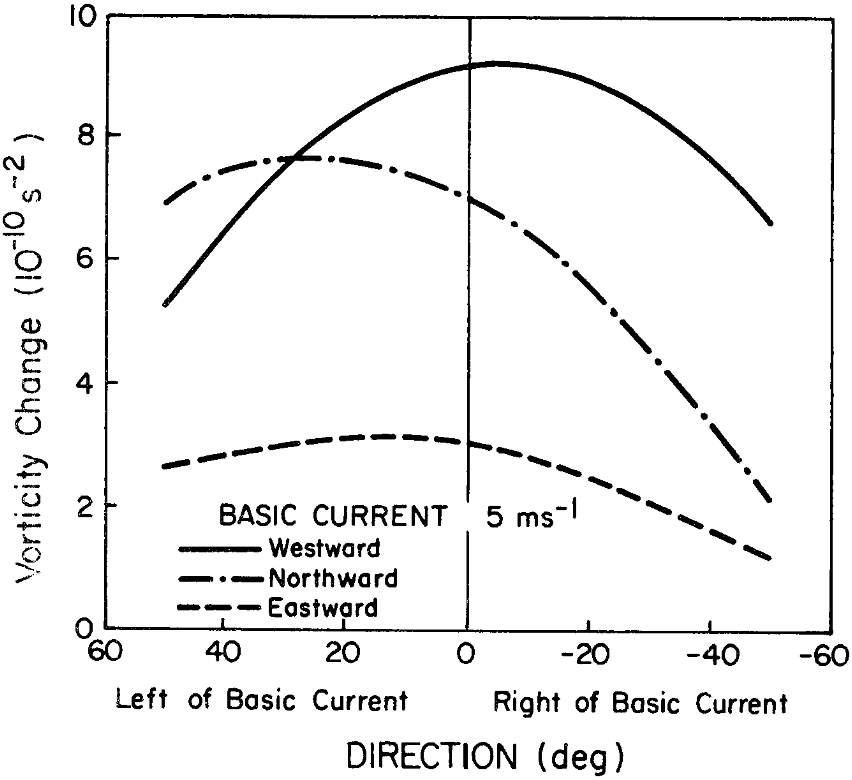

Fig. 11. As for Fig. 10 but for a basic current speed of 5 m s^{-1}.

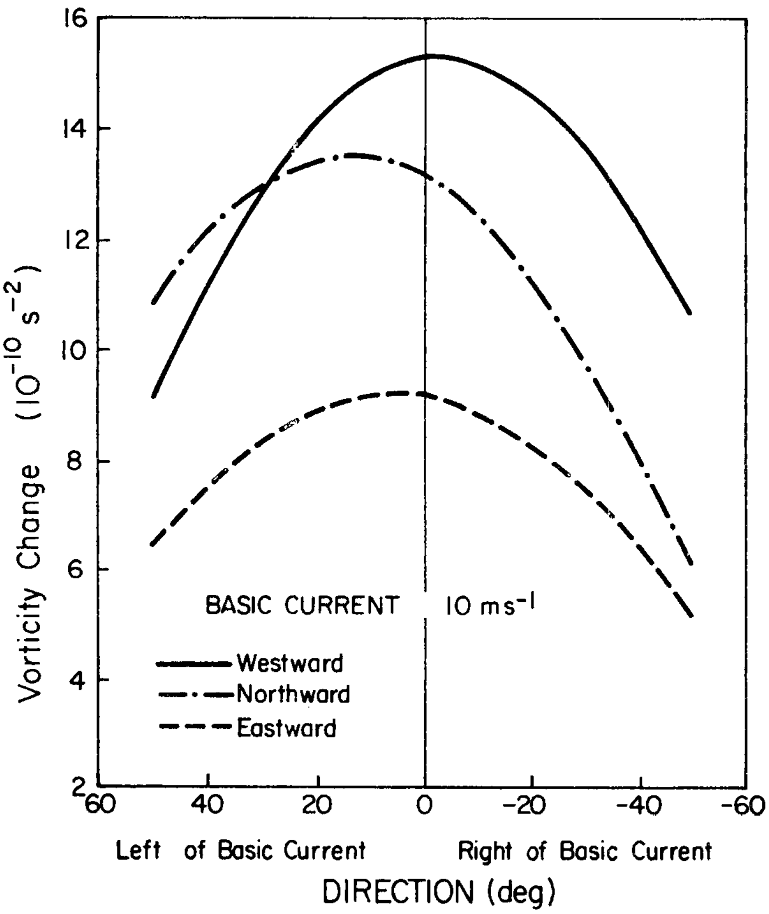

Fig. 12. As for Fig. 10 but for a basic current speed of 10 m s^{-1}.

Westward moving Northern Hemispheric cyclones will respond more to a given perturbation on the right hand compared to the left hand side. Northward and eastward moving storms will respond more to perturbations on the left hand side.

Thus, cyclones moving on a west-northwestward track in the Northern Hemisphere (or on a west-southwestward track in the Southern Hemisphere) are, so to speak, in the groove; substantial perturbations or changes in the basic current are required to produce even moderate variations in their direction of motion. By comparison, northward, southward, and especially eastward moving cyclones are quite unstable and may undergo large oscillations for small imposed perturbations. Eastward moving cyclones are rare in the Northern Hemisphere. But they are quite common in the eastern Australian/Southwest Pacific region (Holland, 1981) where, coincidentally, there is also a high proportion of erratic cyclones.

b) The Problem of Recurvature

Two simple experiments with the model described in section 5 provide a more general example of these effects in relation to the problem of cyclone recurvature. We use the idealized basic current analysis in Fig. 13 (from which the symmetric cyclone has been removed) and keep it constant over two long term integrations for typhoons Bill and Nancy. The basic current data are provided to the model at a radius of 6° latitude from the cyclone center, then, as described in section 5, the model derives an analytic approximation to these data over the whole cyclone domain, superimposes the predefined symmetric cyclone and produces a forecast speed and direction of motion by an iterative solution of Eqs. (24) and (25).

Typhoon Bill has its shape defined by $x = 0.5$, mean convergence by $\gamma = 0.2$, and an effective radius of 250 km. At first it moves to the right of the basic current, then as this current veers through a west-northwestward direction of $\theta = 70^{\circ}$, Bill changes to a leftward deviation. Thus, the long, relatively straight track and penetration deep into the midlatitude trough before eventual recurvature. Once recurvature has started Bill moves into a stronger and stronger northeastward basic current. This rapidly diminishes the leftward beta effect deviation and ensures a sharp recurvature and uniform northeastward progression. Typhoon Nancy is nonvergent ($\gamma = 0$) but

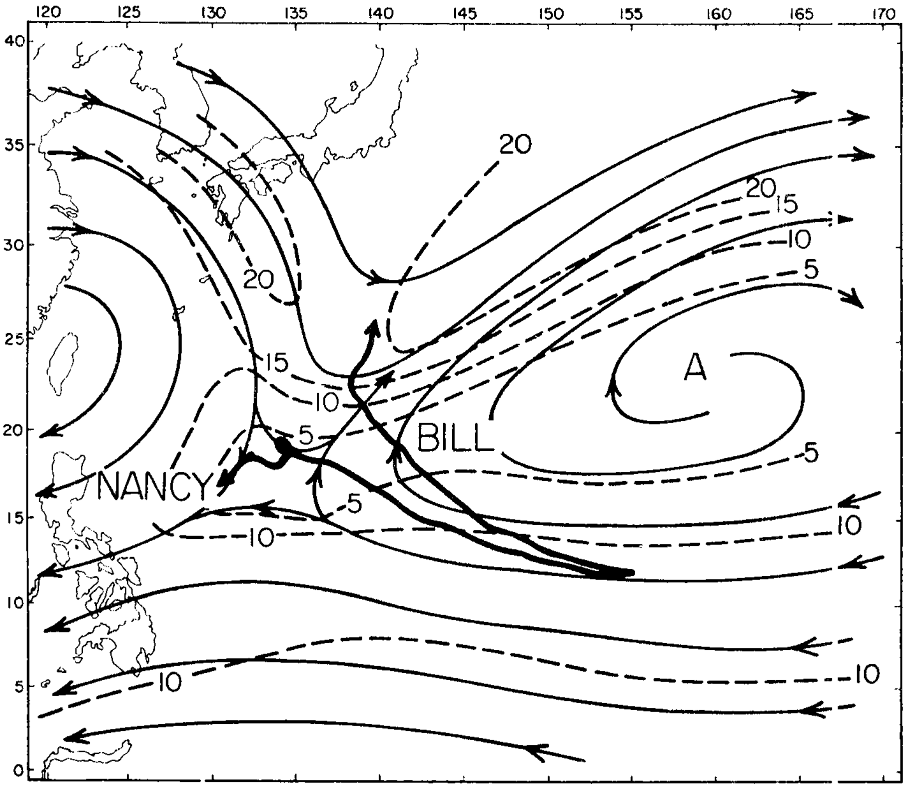

Fig. 13. Tracks of idealized typhoons Bill and Nancy relative to a constant basic state field of motion.

otherwise exactly the same as Bill. As a result it moves slightly to the left of the basic current and slower than Bill, misses much of the effect of the midlatitude trough and, after a brief vascillation in the col region, continues on its westward path.

These results illustrate three points: 1) that the motion stability of westward moving cyclones enables them to resist recurvature; 2) as a cyclone begins to move poleward, it becomes unstable and recurvature is accomplished rapidly; and 3) in a possible recurvature situation, superficially similar cyclones with initially identical environmental flows may choose very different paths.

c) Defining a Steering Current

The term 'steering current' has been classically defined as the resultant wind over the cyclone center after the symmetric components have been removed. Inconsistencies with the actual usage of the term, however, pose two major problems.

The first problem is that, as has been noted by Brand _et al_. (1981), the methods of removing the symmetric cyclone and the domain over which the current is averaged are as multifarious as the people who use them For example, in the recent literature we find George and Gray (1976) using an azimuthally averaged wind field over a domain from 1-7^{o} latitude radius from the center, Brand _et al_. using a geostrophic steering current over a 63 km domain, and Chan and Gray (1982) using an azimuthal average over 5-7^{o} latitude radius from the center. To some extent such differences are unavoidable and probably of minor consequence (for example, the height field approach with geostrophic winds should give a similar result to using the wind field directly). However, the differing, seemingly randomly chosen, domain sizes merely confuse the issue and make intercomparisons difficult. We have attempted to circumvent this problem in this paper by using an analytic approximation to the complete basic current so that the actual winds can be interpolated to the effective radius for use in determining the cyclone motion. For more general applications on azimuthal average over a concentric domain (200-400 km) from the center (which contains the effective radius and corresponds to the stippled region in Fig. 7) would be ideal. If data problems prevent such an average then perhaps some form of interpolation to this radius should be used.

The second problem is that, even with a consistently defined steering current, the basic current asymmetries (which have been averaged out) may have an important effect on the cyclone motion. This is well illustrated by the results in Table 4, which contains the solutions to Eqs. (24) and (25) for the motion of the typical Northern Hemispheric cyclone described in section 3c under differing basic state configurations. We see that the concept of a 'steering current' is indeed a poor one; the cyclone moves by an interaction with the basic current and may react considerably to asymmetries in these winds.

We have, of course, neglected any nonlinear interactions in Eqs. (24) and (25). Nevertheless, some interesting physics may be seen in the results in Table 3. We have already shown that the cyclone motion under a constant basic state is comprised of an advection of the cyclone's vorticity field by the basic current and a beta affect. Introducing a cyclonic wind shear moves the maximum vorticity advection to the right front quadrant of the cyclone and also slightly increases the beta effect (since the southerly component of the basic current is weaker on the western and stronger on the eastern side of the cyclone). But the vorticity advection is largest so the cyclone turns to the right and slows down. Introducing an anticyclonic wind shear does the opposite and turns the cyclone to the left in a similar fashion, though we note that the concomitant weakening of the beta effect reduces the leftward deviation considerably in the northeastward moving case. For a downstream speed divergence the cyclone experiences a stronger vorticity advection in the front and speeds up; downstream convergence slows the cyclone down. Downstream confluence weakens the beta effect and thus turns the cyclone to the right and slows it down. By companison,

TABLE 4

Motion of a typical Northern Hemispheric cyclone using Eqs. (24) and (25) for differing basic currents which average to two constant 'steering currents'. The numbers in brackets indicate deviations from the motion under a constant basic state configuration, a positive number indicates a leftward deviation or higher speed.

CONFIGURATION OF BASIC CURRENT	BASIC CURRENT TOWARDS NORTHWEST				BASIC CURRENT TOWARDS NORTHEAST			
	STEERING CURRENT		CYCLONE MOTION		STEERING CURRENT		CYCLONE MOTION	
	α	V_B ($m\ s^{-1}$)	θ_m	V_c ($m\ s^{-1}$)	α	V_B ($m\ s^{-1}$)	θ_m	V_C ($m\ s^{-1}$)
Constant	045	5	054(0)	6.8(0)	315	5	347(0)	3.8(0)
Cyclonic Wind Shear of 5 $m\ s^{-1}$/450 km	045	5	033(-21)	5.9(-0.9)	315	5	306(-41)	2.6(-1.2)
Anticyclonic Wind Shear of 5 $m\ s^{-1}$/450 km	045	5	069(15)	7.0(0.2)	315	5	353(6)	3.5(-0.3)
Downstream Speed Divergence of 5 ms^{-1}/450km	045	5	051(-3)	11.7(4.9)	315	5	328(-19)	7.7(3.9)
Downstream Speed Convergence of 5 ms^{-1}/450km	045	5	078(24)	1.9(-4.9)	315	5	019(32)	1.5(-2.3)
Downstream Confluence Angle of 10^{o}	045	5	052(-2)	6.3(-0.5)	315	5	337(-10)	3.3(-0.5)
Downstream Diffluence Angle of 10^{o}	045	5	062(8)	7.5(0.5)	315	5	003(16)	4.7(0.9)

downstream diffluence increases the beta effect, turns the cyclone to the left and speeds it up. Finally, introducing an anticyclonic shear, downstream speed divergence or confluence stabilizes the cyclone motion, cyclonic shear, downstream speed convergence or diffluence destablize it.

d) Forecasting Implications

Of all the problems in tropical cyclone forecasting, motion is probably the most important. This is evidenced by the considerable effort that has been expended on developing the current wide range of statistical and numerical techniques (see eg., W.M.O. 1979). Yet, in any situation these techniques are best characterized by the diversity of answers that they provide on whither the cyclone will go. Forecasters cope with this diversity by recalling from experience which techniques work best in specific circumstances or using statistical techniques to stratify the forecast techniques (see eg., Neumann and Pelissier, 1981). But, by comparison there have been very few serious attempts in the past generation at discovering why, and how a cyclone moves, especially with regard to the differing ways that cyclones of different characteristics may interact with their basic flow. Perhaps simple studies such as the one outlined here can provide some of the answers; they can certainly ask some questions. For example, much of the lack of progress in improving motion forecasts in recent years has been blamed on the deterioration of, or ultimate uncertainty in the data base, particularly in low latitudes (Neumann, 1975; Bell, 1981). Yet, it is equally possible that the remarkable success of simple persistence and climatological techniques in these latitudes (see, e.g., Neumann and Pelissier, 1981) is as much related to our demonstrated motion stability of westward moving cyclones. Even with perfect data, sophisticated techniques which do not incorporate this stability will be hard pressed to beat persistence. By comparison, we would expect persistence and climatology to perform poorly for the more unstable northward and eastward moving storms, and Neumann and Pelissier confirm this to be so.

Many statistical techniques currently incorporate a number of stratifications (region, westward/eastward moving storm, slow/fast storm). We hope that the results presented here, or further refinements thereof will provide a solid quantitative basis for stratifying future statistical techniques. We also consider the basic approach described in section 5 of using a quasianalytic technique initialized on immediate past data has some potential for forecasting and plan further research in this regard.

7. Conclusions

Cyclone motion has been investigated via analytic and quasi-analytic numerical solutions of the vergent barotropic vorticity equation on a beta plane. By using this approach we have shown that the observed deviations of cyclone motion from its basic, or 'steering' current may be explained as an additional west to northwestward component resulting from an interaction between the convergent cyclone and the meridional gradient in earth vorticity. In this regard the cyclone may be compared to a circular, convergent Rossby wave. We have also shown that the poleward drift of a cyclonic vortex on a beta plane, described by Rossby (1948), is actually a poleward flow through the cyclone. It is thus incorporated in the basic flow by definition.

Some preliminary implications of the results of this theory to determining cyclone motion have also been presented. In particular, we have indicated that the direction and speed of the basic current, asymmetries in this basic current, and the degree of convergence into the cyclone may be important in defining its relative motion. For example, we have shown that westward moving cyclones are quite stable and are only marginally effected by fluctuations in the basic flow, whereas northward and eastward moving cyclones are unstable and may undergo large motion changes for a similar range of fluctuations. The weaker the basic current speed the more erratic these cyclone fluctuations will be.

We plan further research to expand this simple theoretical approach, to quantitatively determine the applicability of the above conclusions to individual tropical cyclones, and to examine the feasibility of improved forecasting techniques based on the results.

APPENDIX

LIST OF SYMBOLS

c	Constant which defines the cyclone intensity
f	Coriolis parameter
F	Frictional dissipation
$\underset{\sim}{k}$	Unit vector in the local vertical
p	Pressure
r	Radius
r_E	Effective radius
r_m	Radius of maximum winds
t	Time
u	$= dr/dt$ Radial wind component
u_s	Symmetric radial wind component
v	$= r\ d\theta/dt$ Azimuthal wind component
v_s	Symmetric azimuthal wind component
v_n	$= dy/dt$ Northward component of the total wind
$\underset{\sim}{V}$	Wind velocity
V_B	Basic current speed
V_β	Beta effect component of cyclone speed
V_c	Cyclone speed of motion
V_E	Eastward component of the basic current
V_N	Northward component of the basic current
x	1. Eastward ordinate 2. Constant which defines the shape of the azimuthal wind profile
y	Northward ordinate
α	Angle the basic current tenders with due north
β	$= \frac{\partial f}{\partial y}$ Meridional variation of Coriolis parameter

γ	Tangent of the constant inflow angle
δ_o	$= \frac{\partial V_N}{\partial y}$ Meridional component of vergence
δ_1	$= \frac{\partial V_E}{\partial x}$ Zonal component of vergence
θ	Azimuth, measured counterclockwise from due north
θ_m	Direction towards which cyclone is moving
ζ	Vertical component of relative vorticity
ζ_s	Symmetric part of ζ
ζ_o	$= - \frac{\partial V_E}{\partial y}$ Meridional contribution to ζ
ζ_1	$= - \frac{\partial V_N}{\partial x}$ Zonal contribution to ζ
ω	$= \frac{dp}{dt}$ Vertical motion in pressure coordinates

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