

FOLIO
TA7
C6
CER-70-71-38
CP 2

LIBRARIES
COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO

FLOOD ROUTING THROUGH STORM DRAINS

FINAL REPORT

Prepared for:
Federal Highway Administration
Washington, D.C.

PROJECT: UNSTEADY FREE-SURFACE FLOW
IN A STORM DRAIN

Under Contract No. CPR-11-3584

by V. Yevjevich and A.H. Barnes
Department of Civil Engineering

December 1970

HYDROLOGY and WATER RESOURCES PROGRAM
COLORADO STATE UNIVERSITY
Fort Collins, Colorado

CER 70-71VY-AHB38

FINAL REPORT FOR THE PROJECT

"UNSTEADY FREE-SURFACE FLOW IN A STORM DRAIN"

The following four hydrology papers, taken as a whole, represent the final report for the project "Unsteady Free-Surface Flow in a Storm Drain". The history of the project is described in detail in the Introduction of (Part I) Hydrology Paper No. 43. The original assignment of the project sponsored by the Federal Highway Administration, formerly called the Bureau of Public Roads, is given in Appendix 1 of the same paper. Paper No. 43 also presents the basic material produced from the project and contains summarized versions of the other three hydrology papers.

Part II, Hydrology Paper No. 44, is related mainly to the experimental research facilities, the instrumentation used in carrying out the experiments, and the typical experimental results.

Part II, Hydrology Paper No. 45, gives results of the analysis of geometric and hydraulic parameters, which in turn define the various coefficients in the partial differential equations of gradually varied free-surface unsteady flow.

Part IV, Hydrology Paper No. 46, is mainly concerned with various aspects of finite difference numerical integration schemes in solving either the two partial differential equations or the equivalent four characteristic ordinary differential equations of gradually varied free-surface unsteady flow.

Apart from this report, all pertinent original data in the form of output microfilms, decks of computer cards, and similar original data have been deposited with the Office of Research of the Federal Highway Administration.

December 1970

Fort Collins, Colorado

V. Yevjevich
V. Yevjevich
A. H. Barnes
A. H. Barnes

FLOOD ROUTING THROUGH STORM DRAINS

by

V. Yevjevich and A. H. Barnes

Hydrology and Water Resources Program
Department of Civil Engineering
Colorado State University

PART I (Hydrology Paper No. 43)

SOLUTION OF PROBLEMS OF UNSTEADY FREE SURFACE FLOW IN STORM DRAINS

Introduction
General Procedures of Investigation
Theoretical Considerations
Physical Waves, Experimental Works and Results
Computation of Waves by Analytical Methods
Comparison of Results of Computed and Observed Waves in Subcritical Flow
Simplified Methods of Flood Routing
Analysis of Errors in the Solution of Problems
Conclusions, Limitations and Recommendations

PART II (Hydrology Paper No. 44)

PHYSICAL FACILITIES AND EXPERIMENTS

Introduction
Experimental Storm Drain System
Instrumentation and Its Calibration
Data Recording System
Experimental Test Conditions and Typical Results
Experimental Errors
Evaluation of Experimental Facilities

PART III (Hydrology Paper No. 45)

EVALUATION OF GEOMETRIC AND HYDRAULIC PARAMETERS

Introduction
Geometric Relations for the Circular Experimental Conduit
Hydraulic Resistance of the Experimental Conduit
Energy Losses at Junction Boxes
Velocity Distribution Coefficients
Boundary Conditions of Experimental Conduit
Initial Conditions for Experiments and Numerical Integrations
Conclusions and Recommendations

PART IV (Hydrology Paper No. 46)

NUMERICAL COMPUTER METHODS OF SOLUTION

Introduction

Integration of Partial Differential Equations by Finite Differences

Integration of Characteristic Differential Equations by Finite Differences

Comparison of Three Finite Difference Schemes of Numerical Integration

Conclusions and Recommendations

by

V. YEVJEVICH and A. H. BARNES

November 1970

HYDROLOGY PAPER
COLORADO STATE UNIVERSITY

Fort Collins, Colorado

FLOOD ROUTING THROUGH STORM DRAINS

Part I

SOLUTION OF PROBLEMS OF UNSTEADY
FREE SURFACE FLOW IN STORM DRAINS

by

V. YEVJEVICH and A. H. BARNES

November 1970



HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
Fort Collins, Colorado

Several departments at Colorado State University have substantial research and graduate programs oriented to hydrology. These Hydrology Papers are intended to communicate in a fast way the current results of this research to the specialists interested in these activities. The papers will supply most of the background research data and results. Shorter versions will usually be published in the appropriate scientific and professional journals, or presented at national or international scientific and professional meetings and published in the proceedings of these meetings.

The investigations leading to this paper on unsteady free-surface flow in a long storm drain were supported by the U.S. Bureau of Public Roads, Federal Highway Administration during 1960-1970 and by the Public Health Service during 1962-1964. The research was conducted in the Hydrology and Water Resources Program of Civil Engineering Department at Colorado State University, Fort Collins, Colorado, U.S.A.

The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Federal Highway Administration or the Public Health Service.

Dr. Arthur T. Corey, Professor, Agricultural Engineering Department

Dr. Robert E. Dils, Professor, College of Forestry and Natural Resources

Dr. Vujica Yevjevich, Professor, Civil Engineering Department

FLOOD ROUTING THROUGH STORM DRAINS

Part I

SOLUTION OF PROBLEMS OF UNSTEADY FREE SURFACE FLOW IN STORM DRAINS

by

V. Yevjevich and A. H. Barnes

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO 80521

ACKNOWLEDGMENTS

The writers of this paper gratefully acknowledge the support and cooperation of the U. S. Bureau of Public Roads, Federal Highway Administration, in the research on flood movements through long storm drains conducted from 1960 to 1970. The writers also acknowledge the U. S. Public Health Service, National Institute of Health, for their additional support during 1962-1964.

The initiative, cooperation and support given by Mr. Carl F. Izzard to this project on flood movement through storm drains is particularly acknowledged. Mr. Izzard, presently Director, Office of Development, Federal Highway Administration, U. S. Department of Transportation, was Chief, Hydraulic Research Division, U. S. Bureau of Public Roads at the start of the project. Further acknowledgment is extended to Mr. Charles F. Scheffey, Director, Office of Research, Federal Highway Administration, for his cooperation and encouragement. Dr. Dah-Cheng Woo, Senior Hydraulic Engineer, Federal Highway Administration, has cooperated extensively with this project. His reviews and suggestions pertaining to all reports, theses and other documents produced on the project have been particularly helpful.

Mr. George Smith, Associate Professor of Civil Engineering at Colorado State University, has worked closely with the writers in the design, construction, testing, and operation of the physical research facilities during most of this project. Several graduate students, either through their thesis research or through direct work, contributed to the project. The theses and reports are listed in the bibliography of this paper under "Internal References". Dr. Shih-Tun Su, Post Doctoral Fellow, Civil Engineering Department, Colorado State University, using existing data, assisted writers in finishing this paper.

TABLE OF CONTENTS

Chapter		Page
Acknowledgments		iii
Abstract		x
1	INTRODUCTION	1
	1.1 Review of Methods for Routing Floods Through Storm Drains.	1
	1.2 Objectives of Investigation.	1
	1.3 General Initial Conditions Assumed for the Analysis.	2
	1.4 General Physical Constraints in This Study	2
	1.5 Various Potential Applications	2
	1.6 General Approach for Investigations.	2
	1.7 Various Aspects of Investigation	3
	1.8 Justification of Investigations in This Study.	4
	1.9 Practical Relevance of Investigations.	4
	1.10 Brief Historical Review of the Project	5
	1.11 Organization of This Paper	5
2	GENERAL PROCEDURES OF INVESTIGATION.	6
	2.1 Specific Aims.	6
	2.2 Experimental Phase	6
	2.3 Numerical Integration.	7
	2.4 Comparison of Physical and Analytical Waves.	7
	2.5 Relation of this Study to Other Problems of Storm Drainage	7
3	THEORETICAL CONSIDERATIONS	9
	3.1 Derivation of the Two Partial Differential Equations for Unsteady Free-Surface Flow in Conduits	9
	3.2 Discussion of Basic Assumptions Used in Derivation of the Two Partial Differential Equations.	13
	3.3 Derivation of Characteristic Equations	14
4	PHYSICAL WAVES, EXPERIMENTAL WORK AND RESULTS.	18
	4.1 Experimental Facilities.	18
	4.2 Results of Steady Flow Measurements.	19
	4.3 Physical Waves	22
5	COMPUTATION OF WAVES BY ANALYTICAL METHODS	23
	5.1 Basic Equations.	23
	5.2 Methods of Solution.	23
	5.3 Initial and Boundary Conditions.	26
	5.4 Comparison of Three Finite-Difference Schemes of Numerical Integration	27
6	COMPARISON OF RESULTS OF COMPUTED AND OBSERVED WAVES IN SUBCRITICAL FLOW	29
	6.1 Methods of Comparison.	29
	6.2 Methods of Qualitative Comparison.	30
	6.3 Results of Qualitative Comparisons	32
	6.4 Quantitative Comparison of Results by Depth-Versus-Time Relations.	33
	6.5 Quantitative Comparison of Results by Depth-Versus-Distance Relations.	35
	6.6 General Comparison of Results by Wave-Peak-Depth Versus Distance and Time.	35
7	SIMPLIFIED METHODS OF FLOOD ROUTING.	47
	7.1 General Definitions and Descriptions of Simplified Methods	47
	7.2 Flood Routing Based on Simplified Partial Differential Equations	48
	7.3 Basic Properties of Simplified Methods	48
	7.4 Muskingum Simplified Method.	50
	7.5 Time-Lag Routing Simplified Method	51
	7.6 Non-Dimensionality Approach.	51

TABLE OF CONTENTS - (Continued)

Chapter		Page
8	ANALYSIS OF ERRORS IN THE SOLUTION OF PROBLEMS.	58
	8.1 Errors in Geometric Parameters.	58
	8.2 Errors in Hydraulic Parameters.	60
	8.3 Errors in Computations.	65
	8.4 Errors in Experimental Observations.	66
	8.5 Total Errors.	68
9	CONCLUSIONS, LIMITATIONS AND RECOMMENDATIONS.	69
	9.1 Conclusions.	69
	9.2 Limitations in the Developed Results.	69
	9.3 Recommendations for Future Research.	70
	References.	71
	Appendix 1.	72
	Appendix 2.	74
	Appendix 3.	80
	Appendix 4.	101
	Appendix 5.	104

LIST OF FIGURES AND TABLES

Figure		Page
3.1	Definition diagram for derivation of continuity and momentum equations of unsteady free-surface flow in an open-channel	9
3.2	Forces acting on the incremental volume of the channel	10
3.3	Geometric elements of a channel with free surface flow	10
3.4	Forces acting on an incremental slice in free-surface flow in a channel	11
3.5	Net of characteristic curves C_+ and C_- in (x,t) -plane.	15
3.6	Domain of dependence and range of influence of characteristic curves in the (x,t) -plane	17
4.1	General scheme of the experimental conduit with water supply and removal.	18
4.2	A view from upstream of the circular storm conduit on the hillside of the outdoor laboratory at Colorado State University Engineering Research Center	18
4.3	A view from downstream of circular storm conduit and the inclined rails	18
4.4	Scheme of the general data recording system	19
4.5	Relationship of the Darcy-Weisbach friction factor f to Reynolds number R_e	19
4.6	The junction box used in the model study.	20
4.7	Velocity distribution coefficients as related to the Reynolds number	20
4.8	Velocity distribution coefficients as related to the depth of flow in a circular cross section	21
4.9	Location of the critical depth of a circular cross section free outfall	22
4.10	A recorded inflow discharge hydrograph to the main conduit	22
4.11	A recorded inflow discharge hydrograph from a lateral conduit	22
5.1	Definition graph for the finite-difference scheme	23
5.2	Network of specified intervals for the solution of characteristic equations	25
5.3	Rectangular grid for the solution by the system of specified intervals, Δt and Δx : subcritical flow (upper graph), critical flow (center graph), and supercritical flow (lower graph)	26
5.4	Hypothetical inflow hydrograph of a Pearson Type III function, Eq. 5.32, with the selected parameters: $Q_b = 6.21$ cfs, $Q_0 = 8.00$ cfs, $t_p = 100.00$ sec., and $t_q = 150.0$ sec	27
5.5	Comparison of diffusing scheme (D_i), Lax-Wendroff scheme (A_L), and the specified intervals scheme of method of characteristics (C) in reproducing the steady initial conditions along the conduit, at the distance $x = 796.7$ ft.	28
6.1	Type of comparison of computed and observed waves for a qualitative comparison of visual inspection with depth versus time for given positions.	29
6.2	Type of comparison of computed and observed waves for qualitative comparison by visual inspection, with depth versus distance for given times	30
6.3	Type of comparison of computed and observed wave peak depths as functions of distance or time, for a qualitative comparison by visual inspection	31
6.4	Definitions of basic magnitudes of the wave-depth hydrograph.	33
6.5	Quantitative comparison of five parameters measuring differences between the computed and observed wave-depth hydrographs. The case of no corrections for shifts in time of observed hydrographs	38

LIST OF FIGURES AND TABLES - (Continued)

Figure		Page
6.6	Quantitative comparison of five parameters measuring differences between the computed and observed wave-depth hydrographs. The case of corrections made for the shifts in time of observed hydrographs	38
7.1	The hydrograph of the upstream reach end, left graph, and the computed hydrograph of the downstream reach end, right graph, by the time-lag simplified flood routing method	51
7.2	Maximum depth versus distance for Run No. 200 of Table 7.1.	53
7.3	Attenuation of wave peak for a theoretical hydrograph in subcritical flow	54
7.4	Maximum depth vs. distance for Run No. 97 of Table 7.6.	54
7.5	Maximum depth vs. distance for Run No. 98 of Table 7.6.	54
7.6	Maximum depth vs. distance for Run No. 99 of Table 7.6.	54
7.7	Maximum depth vs. distance for Run No. 100 of Table 7.6	55
7.8	Maximum depth vs. distance for Run No. 45 of Table 7.7.	55
7.9	Maximum depth vs. distance for Run No. 18 of Table 7.7.	55
7.10	Attenuation of wave peak for theoretical hydrograph in supercritical flow	57
8.1	Relative errors in parameters of circular cross section versus relative error in depth.	58
8.2	Definition sketch for the relation of circular and elliptical cross sections.	59
8.3	Percent difference in area versus eccentricity and depth.	59
8.4	Hypothetical inflow hydrograph of the Pearson Type III function, Eq. 5.32, with the selected parameters: $Q_b = 6.21$ cfs, $Q_o = 8.00$ cfs, $t_p = 100.00$ sec, and $t_g = 150.0$ sec.	62
8.5	Effect of changes in friction factor, f , at various positions along the channel: (1) $f = 0.010$, (2) $f = 0.012$, (3) $f = 0.014$, and (4) $f = f(R_e)$	62
8.6	Effect of Δx on hydrographs at various positions along the conduit: (1) $\Delta x = 40.91$ ft, (2) $\Delta x = 20.45$ ft, (3) $\Delta x = 10.23$ ft, and (4) $\Delta x = 5.12$ ft, at three locations of conduit $x = 50.0$ ft (upper graph), $x = 410.0$ ft (center graph), and $x = 771.7$ ft (lower graph)	65
Tables		
6.1	Quantitative comparison by five parameters of computed and observed waves at given three conduit positions, with no corrections for shifts in observed waves	33
6.2	Quantitative comparison by five parameters of computed and observed waves at given six conduit positions, with no corrections for shifts in observed waves	34
6.3	Quantitative comparison by five parameters of computed and observed waves at given three conduit positions, with corrections of observed peak depths shifted in time	36
6.4	Quantitative comparison by five parameters of computed and observed waves at given six conduit positions, with corrections of observed peak depths shifted in time	37
6.5	Qualitative comparison of computed and observed depth-versus-distance wave profiles	39
6.6	Qualitative comparison of computed and observed depth-versus-distance wave profiles	40
6.7	Qualitative comparison of computed and observed depth-versus-distance wave profiles	41
6.8	Qualitative comparison of computed and observed depth-versus-distance wave profiles	42

LIST OF FIGURES AND TABLES - (Continued)

Tables	Page
6.9	Summary of data on CSU experimental waves 43
6.10	Summary of data on Wallingford experimental waves 43
6.11	Comparison of data for wave-peak versus distance and time for CSU experimental data 44
6.12	Comparison of data for wave-peak versus distance and time for Wallingford experimental data 45
7.1	Conditions of different runs in the subcritical regime. 56
7.2	Effect of absolute values of Q_b and Q_o in subcritical flow for $t_p = 50$ sec., $t_g = 100$ sec. 53
7.3	Effect of relative values of Q_b and Q_o in subcritical flow for $t_p = 50$ sec., $t_g = 100$ sec. 53
7.4	Effect of absolute value of t_g in subcritical flow for $Q_b = 10$ cfs, $Q_o = 10$ cfs. 53
7.5	Effect of diameter in subcritical flow for $Q_b = 10$ cfs, $Q_o = 10$ cfs, $t_p = 50$ sec., $t_g = 100$ sec 53
7.6	Conditions of Runs 97, 98, 99, and 100. 54
7.7	Conditions of different runs in the supercritical regime. 56
7.8	Effect of the relative value of t_p and t_g in supercritical flow for $Q_b = 25$ cfs, $Q_o = 10$ cfs 55
7.9	Effect of the absolute value of t_g in supercritical flow for $Q_b = 25$ cfs, $Q_o = 10$ cfs. 55
7.10	Effect of the absolute values of Q_b and Q_o in supercritical flow for $t_p = 50$ sec., and $t_g = 100$ sec 55
7.11	Effect of absolute and relative values of Q_b and Q_o in supercritical flow for $t_p = 50$ sec, $t_g = 100$ sec. 57
7.12	Effect of diameter in supercritical flow for $Q_b = 40$ cfs, $Q_o = 20$ cfs, $t_p = 50$ sec and $t_g = 100$ sec 57
8.1	Geometry of the experimental conduit. 60
8.2	Theoretical effect of bottom irregularity on water surface profiles 61
8.3	Slope deviations of the experimental conduit. 61
8.4	Percentages of peak depth to conduit diameter at various positions, x , along the conduit and various values of f 63
8.5	The percentage ratios of the time of the peak depths to the rising time of inflow hydrograph, at various positions along the conduit, and for several values of friction factor 63
8.6	Difference in peak depths computed from f as the function of Reynolds number and three values of f , in percent of conduit diameter. 63
8.7	Differences in the occurrence of peak depths, computed from f as the function of Reynolds number and for each of three constant values of f in percent of the inflow hydrograph rising time 63
8.8	Differences in peak depths, in percent of conduit diameter computed for various values of α and β , and for $\alpha = 1.00$ and $\beta = 1.00$ 64

LIST OF FIGURES AND TABLES - (Continued)

Tables		Page
8.9	Differences in times of peak depths in percent of conduit diameter for various values of α and β and for $\alpha = 1.00$ and $\beta = 1.00$	64
8.10	Difference in y_p computed from various sizes of Δx (in percent of conduit diameter D).	66
8.11	Difference in $T_{p_{tp}}$ computed from various sizes of Δx (in percent of $T_{p_{tp}}$)	66
8.12	Estimate of instrumentation errors	67

ABSTRACT

This first part of a four-part series of hydrology papers on flood routing through storm drains presents results of experimental studies in a 3-ft diameter, 822-ft long storm conduit and theoretical studies of the unsteady free-surface flow. The derivation of the two quasi-linear hyperbolic partial differential equations of gradually varied free-surface unsteady flow based on the principles of conservations of mass and momentum are given. The numerical integrations of differential equations by the specified interval scheme of the method of characteristics, the diffusing scheme, and the Lax-Wendroff scheme are discussed; the method of characteristics is selected for the practical integration procedure whenever the complete differential equations are used. Experimental and analytical investigations of the geometric and hydraulic parameters that define the coefficients of the two differential equations are summarized. Particular attention is given to geometric characteristics of a circular conduit, to hydraulic resistance, to velocity distribution coefficients, and to junction box energy losses. The initial and boundary conditions are experimentally studied and are expressed mathematically for the numerical solutions. The analytically computed waves are then compared with the experimentally observed waves by using the same initial and boundary conditions. Qualitative and quantitative comparisons are given for depth hydrographs at different positions, for depth wave profiles at different instants in time, and for the peak-depth versus both position and time. From a practical point of view, good agreement is indicated by these comparisons. The applicabilities of simplified routing methods are also compared in general terms to the solutions obtained by solving the two partial differential equations of unsteady flow. The errors in conduit geometric parameters, in hydraulic parameters, in numerical computations, and in experimental observations are analyzed and discussed.

FLOOD ROUTING THROUGH STORM DRAINS

Part I

SOLUTION OF PROBLEMS OF UNSTEADY FREE SURFACE FLOW IN STORM DRAINS

by

V. Yevjevich* and A. H. Barnes**

Chapter 1

INTRODUCTION

1.1 Review of Methods for Routing Floods Through Storm Drains

The predominant methods used in the design of storm drains employ the steady free-surface flow approach at the selected flood peak discharge. The design flood peak of a given probability of exceedence is computed by a hydrologic procedure for a given drain section between two water inlets. The drain is designed in such a way that the design flood discharge passes through it as the free-surface flow at the largest drain flow capacity. The flood peak discharge is usually determined by the hydrologic rational method of flood estimations. Simple computations for a free-surface steady and uniform flow through a storm drain determine the basic geometric dimensions of a drain between its two inlet points. For a circular drain its diameter is the basic dimension for the given invert slope.

Because floods are mainly free-surface waves moving through storm drains, the steady-flow design approach does not take into account several factors. First, storm drains have storage capacities which attenuate the floods. Second, unsteady flows through various drain sections mutually interact; furthermore, an extensive grid of storm drains with small slopes, located in urban and combined urban-highway drainage systems, significantly affects storm flood attenuation during a local heavy storm of limited areal coverage. Third, drain conduits during storm floods have non-negligible dynamic effects in flood wave modifications as they progress along the drain. They may, under particular conditions, pass to bores (surges).

The use of approximate flood routing methods based on the simplified unsteady flow approach of storm drain design currently practiced, are the Chicago method and others. They take into account the attenuating storage effect; however, they usually neglect the dynamic effects. In other words, they mainly treat the kinematic aspects of a wave. Because some flood waves may amplify and pass to bores under given conditions, the simple flood routing methods based on the storage equation, even with some corrections, can not reproduce and take into account various aspects of dynamic effects, especially the celerities of various parts of waves as the wave evolves, movement

of flood waves upstream along a long drain, and similar effects.

The advent of digital computers enhances the feasibility of using the most complete mathematical equations for the computation of unsteady free-surface flow in a storm drain. Besides, progress in urban hydrology and the improved understanding of precipitation phenomena enable a better prediction of storm flood hydrographs which enter the storm drain system. These two factors encourage applying unsteady free-surface flow equations in their most reliable mathematical forms to storm drain design.

In summary, the historical development of methods for design of storm drains may be classified in three broad classes:

A. Steady-flow approach of storm drain design, the logical consequence of the rational type formulas in predicting storm flood hydrograph peaks.

B. Simplified unsteady flow approach of storm drain design mainly based on the storage equation or its modified forms of kinematic waves, the logical consequences of unit hydrograph approach in predicting storm flood hydrographs, of limitations in accuracy of hydraulic properties of storm drains, and of available computational devices and numerical techniques in the precomputer era.

C. Unsteady free-surface flow approach of storm drain design, based on the complete equations of conservation of mass (continuity) and momentum of unsteady free-surface flow, advanced methods of predicting flood hydrographs, the use of rapid digital computers with adequate storage capacity, and the advanced numerical methods of integration of differential equations.

The potential of using improved flood routing methods through storm drains by this third approach has inspired the investigation presented in this paper.

1.2 Objectives of Investigation

This study of the hydrodynamics of unsteady free-surface flow in a storm drain has the primary objective

* Professor-in-Charge of Hydrology and Water Resources Program, Dept. of Civil Engineering, Colorado State University.

**Associate Professor of Civil Engineering, Colorado State University.

of adding the information for developing flood routing procedures adopted to a digital computer and verified by hydraulic model tests. The ultimate objective is to provide results which permit the development of working design methods applicable to various situations where drains are used for removal of storm water in the gradually varied free-surface unsteady flow.

Other objectives are the analytical study of various aspects of basic mathematical equations of unsteady free-surface flow; detailed analysis of hydraulic and geometric parameters of storm drains simulated by a long smooth circular conduit; comparison between the physical waves simulated and observed in a smooth circular conduit and the numerically integrated waves with the same initial and boundary conditions; study of particular boundary conditions and losses at singularities along a storm drain, and similar objectives.

1.3 General Initial Conditions Assumed for the Analysis

The following are the general initial conditions of a storm drain assumed for the study. The depth of water in a storm drain is shallow prior to storm inflow, as a steady low-flow regime. Storm inflow hydrographs along the drain are given either as simple hydrographs of any shape, or as composed hydrographs of successive individual storm hydrographs. Each inlet point (i) has an inflow discharge hydrograph, $Q_i(t)$, with the shape, peaks, and time of peaks of the hydrograph differing from inlet to inlet. These hydrographs depend on the catchment area of each inlet, storm characteristics, and the direction and speed of storm movement.

1.4 General Physical Constraints in This Study

To simplify basic and applied research, physical constraints of a storm drain are assumed as follows:

1. The storm drain consists of a single continuous conduit.
2. The inflows of storm water occur at discrete points of inlets located along the storm drain.
3. The inflow discharge at any inlet has a negligible momentum in the conduit direction.
4. The conduit is circular.
5. The conduit dimensions change only at manholes and conduit junction boxes, open to atmospheric pressure. The junction boxes might or might not be inlet points. No inlet occurs outside the junction boxes.
6. The conduit has both the matching crown-lines and the matching invert lines at junction boxes except in the case where a drop junction box occurs. Hydraulic characteristics of drop junction boxes should be known as boundary conditions in the form of the outflow rating curve for the outlet of upstream conduit, though this case is not analyzed in this study.
7. The head loss function of transitions at junction boxes is known in relation to the main conduit discharge, the inlet discharge if any, and a water stage. This head loss is a singular resistance at junction boxes.
8. The slope of the storm drain under field conditions is constant between junction boxes. The slope

changes occur only at junction boxes. Points at which slopes change are equivalent to junction boxes without inflows. However, the slope of the long circular conduit is kept constant for a set of experiments and computations in this study.

9. The slopes may vary in both subcritical and supercritical flows, reaching three to five percent.

10. The alignment of the storm drain is straight, even though small deflection may occur at junction boxes in the field; the head loss due to deflection is included in the general singular loss of the junction boxes.

11. The flow regime is a free-surface water motion during all the movement of a storm flood through the drain.

12. The outflow at the end of the storm drain is generally free, or although some controlled end condition for wave movements without inflows are also studied. The outflow stage-discharge relation (rating curve or family of rating curves) is assumed as known.

13. The Darcy-Weisbach f-factor for steady state conditions is used to define the flow resistance.

1.5 Various Potential Applications

The free-surface unsteady water movement through pipes, tunnels, various types of storm drains and all other conduits, either of circular or any other shape, is applicable to many problems. Some of these are:

1. The problem of removing rainfall water from highway and urban areas through storm drains.
2. The problem of free-surface wave movement along water power tunnels and conduits, which involves computing the hydrograph transformation along the tunnel with free-surface flow.
3. The problem of using tunnels and conduits for storage under conditions of unsteady free-surface water flow.
4. The problem of handling the passage of flash-floods of small water courses through diversion tunnels or conduits.
5. The problem of propagating of water waves from the sea, lakes, estuaries or rivers with these waves entering the storm drains, etc.

1.6 General Approach for Investigations

The problem of unsteady free-surface flow in a storm drain in this study is investigated as follows:

1. Analysis of the hydrodynamic aspects of the problem is pursued with a minimum of basic assumptions and neglected factors.
2. Assumptions and neglected factors are discussed under conditions of removing floods through storm drains.
3. Simplifications matching the accuracy of available data, with the precision of their results, are introduced when their evaluation is possible.

Development of equations. Basic hydrodynamic equations are derived in this text, although these derivations repeat many classical studies. This is done for two reasons, to relate them to the computation of unsteady free-surface flow in a storm drain,

and to adapt these analytical expressions to specific characteristics of storm drains. Mathematical expressions that describe unsteady free-surface flow are based on a set of assumptions which means a difference between mathematically computed surface waves and physical waves is unavoidable. All methods for computing unsteady free-surface flow are only approximations, so the extent an approximation agrees with real flow is a basic question to answer for each method. There is little value in discussing merits of a method without determining its degree of approximation.

Selection of method for solving equations. Selecting a method for computing unsteady free-surface flow should depend on an economically justified degree of approximation, and on the risk and uncertainties involved. Two questions should be answered in principle, what is the degree of approximation for each method, and what degree of approximation is justified? To investigate methods for computing unsteady free-surface flow in a storm drain, two different approaches may be pursued. Applied research may be started by using a simple method giving an approximation of low accuracy. Then the method is improved by adding the previously neglected factors, so as to proceed from lower-degree approximation to a higher-degree approximation. This approach was widely used by many previous investigators. Another approach is using the most complete hydrodynamic equations as the closest mathematical approximation of the physics of the unsteady free-surface flow. Any computation of unsteady flow by these equations is assumed to be the highest-order approximation attainable at the present status of applied mathematics in fluid mechanics. Then neglecting some factors and simplifying the initial and boundary conditions, and quantities which describe these conditions, the lower-order approximations are derived. As the accuracy of computations decreases by an increase of simplification and neglect of factors, the practical optimization problem is in determining the order of approximation as the compromise between the accuracy and the economy of data procurement and computations.

Methodology used. The methodology followed in this second approach is:

1. Regardless of which mathematical expressions are used to describe unsteady free-surface flow, several assumptions are always made. They introduce the first departure between the real flow and the theoretical flow, described analytically. Effects of these assumptions are rarely discussed by investigators.
2. The two partial differential equations, usually called the De Saint-Venant equations of unsteady free-surface flow, as the continuity equation and the momentum equation, are presented in their most general form. The physics of the unsteady free-surface flow is stressed, clearly showing variables and quantities that are significant parts of equations and have effects on flow patterns.
3. These general equations are adapted for suitable computations of flood movement along a long storm drain.
4. Initial and boundary conditions are discussed whenever they affect the computation method.

5. Methods of integrating the two partial differential equations are particularly discussed in view of using the most adequate numerical computational methods and digital computers.

6. Specific hydraulic problems related to storm drains are studied in detail with the objective of testing the analytical methods of flood routing in comparison with the observed physical waves.

1.7 Various Aspects of Investigation

Physical and analytical waves. When a flood is produced in a storm drain and observed both at its beginning and its end, it will be called "the physical wave". When a initial flood wave at the storm drain entrance is assumed and the flood wave is computed at the end of this storm drain by an appropriate integration procedure, it will be called "the analytical wave". Comparing the most accurately computed analytical waves with the most accurately observed physical waves is one of the basic objectives of this study. This comparison is analyzed with regard to errors in basic data, errors inherent to methods used, and tolerable errors.

General solutions of unsteady-flow differential equations. Analytical solution in closed forms of the two partial differential equations for unsteady free-surface water flow is not possible for conduits and channels under natural conditions. As soon as simplifications which permit analytical solutions are introduced, the departures from real physical conditions may be so large that in most cases the results become invalid. Thus, methods of approximate integrations of simplified differential equations have been imposed.

One hundred years, from 1871 to 1970, of applying the two basic partial differential equations of unsteady free-surface flow for purposes of computing waves along canals, channels, and conduits have resulted in a variety of methods of solution, graphical, numerical, and analytical, with a wide range of degrees of approximation to exact solutions. Because the amount of work to be done in the past by numerical methods was very large, graphical methods of various types have dominated the field until recently. These tedious and time consuming graphical procedures have been often replaced in practice by the flood routing methods based either on the simple continuity equation, or on it and on a simplified momentum equation. A large number of these approximate methods have been developed and used [1]*, and most of them are still being used.

Recent developments. Two relatively recent developments have greatly influenced the computations of unsteady free-surface flow: (1) many new and appropriate numerical procedures came into being, mainly based on using a finite differences method of integrating the partial or characteristic differential equations, and (2) electronic computers (digital, analog, hybrid) of varying characteristics become available, which could do large computations at a relatively low cost. Innovation and constant progress within these developments have made possible the use of procedures known, but considered impractical 20 to 50 years ago. Among these procedures of integration are both the method of finite differences in solving

* The number or numbers inside the brackets designate references, and sometime their pages, given at the end of this paper.

the two partial differential equations of unsteady free-surface flow under complex conditions, and the method of finite differences applied to the characteristic differential equations as an equivalent set to the partial differential equations.

Two types of equations used by finite differences approach. Using finite differences, as the ratios of differences instead of derivatives, in the graphical or numerical methods of integrating the partial differential equations by their equivalent characteristic differential equations is usually called "the method of characteristics". The phrase "method of finite differences" will be used for both cases when the ratios of finite differences replace the partial derivatives in the partial differential equations or when the ratios of finite differences replace the derivatives of characteristic ordinary differential equations.

Junction problem. The selection of the flood routing method in the case of the junction of two or more drains is often a relevant problem. Inflow hydrographs at various inlets along adjacent storm drains may have different time lags of peak discharge occurrences, with different rising times. Water may temporarily flow in both upstream and downstream directions in a set of storm drains in an area. Junctions of storm drains create interdependence of unsteady flow in a system of storm drains. Flood routing methods, such as those based on the simple storage equation which cannot easily take these conditions into consideration, are not theoretically feasible for treating the unsteady free-surface flow in storm drains. The use of the two partial differential equations is an ideal approach for this complex and realistic flood routing case.

Selection of solution method for attaining a given accuracy of computations. Estimation of inlet hydrographs along storm drains is subject to errors. Predictions of design storms also have a limited accuracy when compared with the real storms of a given probability. There is a limit of accuracy economically justified in flood routing. This accuracy should correspond to the precision of the basic data used, particularly to the accuracy of computed inflow hydrographs along storm drains. The greater this accuracy, the more the accurate flood routing method is justified.

1.8 Justification of Investigations in This Study

Investment in storm drainage. There is a continuous increase in the total activity of urban and highway drainage for removal of storm flood waters. The investment in storm drains, both urban or highway, or combined urban-highway, in the United States is estimated at about two billion dollars per year, and these annual expenditures increase with an increase of time. Simultaneously with this progressively increasing activity and investment in storm drainage, pressure is building up to reduce cost by keeping the total risk of overflowing urban areas and highways below a given probability level. Therefore, it is reasonable to expect an increase in public and private pressure for better methods of storm drain design, particularly for the more accurate techniques of optimization between investment, maintenance, basic risk, and all uncertainties.

Basic risk and uncertainties. Basic risk is defined as the probability of a storm drain not being able to evacuate the storm floods. It is a direct result of stochastic character of floods and all other

random factors in storm drain design. Uncertainties result from various errors in determining design floods of given probabilities, errors inherent in the estimation of hydraulic factors of storm drain, inaccuracies of methods used for routing floods through storm drains, and similar sources of errors. Basic risk plus all uncertainties comprise the concept of total risk as it is used in practice. While the basic risk can not be changed except by various flood control works, uncertainties may be decreased by a better prediction of design storm floods, by a better knowledge of hydraulic properties of storm drains, and by more reliable methods of routing floods through storm drains.

The justification of investigations undertaken in this study are mainly two-fold, a decrease in uncertainties in hydraulic properties of storm drains, and a development of flood routing methods for storm drains, which would enable a better optimization between cost and risk, provided these methods of flood routing through storm drains are more reliable than the presently used methods.

Combined urban-highway drainage system. Storm drainage problems studied separately for urban areas and separately for highway drainage, in urban areas with important highways crossing them, are usually not economical. The optimum solution of cost and risk is in a combined urban-highway storm drainage system. Reliable flood routing methods can contribute to better planning of these integrated storm drainage systems. As many large metropolitan areas lie in low slope terrains, the slopes of interconnected storm drains are small over large distances. An integrated storm drainage system requires flood routing methods that can solve problems of storm flood water moving through a grid of interconnected storm drains.

Pollution control and storm drainage. The pollution control of rivers, lakes, estuaries, and seas increasingly puts constraints on the quality of drainage water from urban and highway areas. The general tendency at present is to separate the storm drainage system from the sewage drainage system. This new requirement on the quality of urban and highway storm drainage water affects the various concepts in planning their surface water drainage. Reliable flood routing methods through storm drains and storage systems imposed by flood control, would undoubtedly contribute to more economical and less uncertainties-taking solutions of storm drainage systems as it concerns the water quality.

1.9 Practical Relevance of Investigations

Several aspects of these investigations are of practical relevance. It is expected that results of this study would increase the confidence of designers in using mathematical equations in the form of the two partial differential equations of unsteady free-surface flow, or their equivalent characteristics differential equations, in comparison with the present use of simplified equations, mainly either of simple storage equation or only of kinematic wave equations. Practical relevance of investigation is in showing the degrees or limits of accuracy inherent to any analytical method of solving flood routing problems. Relevance of this study is seen also in its basic direction of using exclusively the inexpensive finite differences integration method made possible by fast digital computers. As in any other systematic research approach, the by-product is the detection of various problems needing more study and better solutions by pointing out the needs and potentials for new research results of practical relevance.

1.10 Brief Historical Review of the Project

These investigations of unsteady free surface flow in a storm drain have been initiated by the original description of research project entitled "Unsteady Surface Flow in a Storm Drain", outlined by Mr. Carl F. Izzard, at that time Chief, Hydraulic Research Division, U.S. Bureau of Public Roads, and presently Director, Office of Development, Federal Highway Administration, U. S. Department of Transportation. This description is given as Appendix 1 to this paper.

The first-year investigation was limited to general and analytical studies as the basis for an advanced research program in subsequent years. As a sequence, a report also entitled "Unsteady Free Surface Flow in Storm Drain" by V. Yevjevich, Professor of Civil Engineering, Colorado State University, as a general analytical study, was submitted to the U.S. Bureau of Public Roads, Hydraulic Research Division in June 1961. This report outlined problems in detail, set-up basic mathematical techniques, discussed the initial and boundary conditions, and selected the general approach to be followed in the next phases of the research program. The above study led to the decision to build an experimental physical research facility on a hillside in the Outdoor Laboratory of the Engineering Research Center at Colorado State University, Fort Collins, Colorado, as described in Part II, Hydrology Paper No. 44, of this sequence of hydrology papers.

Since beginning, five M.S. and one Ph.D. thesis have been written from this project. Several progress reports with investigation results have been submitted to the U.S. Bureau of Public Roads.

The sequence of these four hydrology papers, including this one as the first paper, is a comprehensive description of research activities and of final results. These four papers are all entitled "Flood Routing Through Storm Drains", and the four parts are: Part I, Solution of Problems of Unsteady Free Surface Flow in Storm Drains; Part II, Physical Facilities and Experiments; Part III, Evaluation of Hydraulic and Geometric Parameters; and Part IV, Numerical Computer Methods of Solution. These four papers represent the final report and the termination of the research project.

The review of literature on unsteady free surface flow in channels has been undertaken by V. Yevjevich in 1959-60 under a U.S. Geological Survey project. That review was later published under the title "Bibliography and Discussion of Flood-Routing Methods and Unsteady Flow in Channels", as U.S. Geological Survey Water Supply Paper 1690, U.S. Government Printing Office, Washington 1964. During the ensuing period and during the project life the newest liter-

ature on unsteady free surface flow has been continuously reviewed and used in the project.. Some of these newest references are given in Appendix 2.

1.11 Organization of This Paper.

Though a sequence of four hydrology papers represents the results of this study on flood routing through storm drains, this first paper, Part I, No. 43, is conceived as presenting all material related to the study. However, the material presented in detail, in Parts II, III and IV, or hydrology papers 44, 45, and 46, is only presented in this paper in summarized form. The organization of the material is such that paper No. 43 is independent of the other three papers, except when referring additional details in the other three.

The organization of this paper is as follows. Chapter 2 presents general procedures of investigation, with the underlying approaches to the problem solution.

Chapter 3 gives detailed derivation and discussions of the two partial differential equations of gradually varied free-surface unsteady flow, and derivations and discussions of the corresponding four ordinary characteristic equations. The hypotheses underlying the development of these equations are also presented.

Chapter 4 refers to experimental facilities, which is condensed material from Hydrology Papers Nos. 44 and 45.

Chapter 5 is related to the study of finite-difference numerical integration schemes of unsteady flow equations; it basically represents a condensation of information from Hydrology Paper No. 46, and partly from No. 45.

Chapter 6 gives comparisons of the computed and observed waves. It is the major chapter of this paper treating both the qualitative and quantitative comparisons of these waves. Appendices 3, 4, and 5, containing a large number of graphs, support the discussion in this Chapter 6.

Chapter 7 discusses in a summarized form, the simplified methods of flood routing, particularly emphasizing their relations to more accurate computations by the use of complete differential equations of flood waves moving through storm drains.

Chapter 8 is a summary of the effects of individual errors, both in observing physical waves and in computing analytical waves. It also includes the effect of some simplifications in the coefficients of partial differential equations on the evolution of flood waves.

Chapter 9 gives basic conclusions, limitations in the developed methods, and recommendations for further research.

GENERAL PROCEDURES OF INVESTIGATION

2.1 Specific Aims

The general procedures of the investigations described in this hydrology paper and in the three following papers, comprising a series of four papers are outlined in the subsequent text. Before the procedures are presented the specific aims of the research conducted are delineated as follows.

(1) To supply results of basic and applied research regarding flood routing methods of gradually varied free-surface unsteady flow in a storm drain using the most complete one-dimensional partial differential continuity and momentum equations. These results may enable the research sponsor or another agency to develop a set of practical design methods in the future for flooding routing through storm drains. Each design method should be a feasible procedure for given conditions of storm floods and drainage system characteristics. The design methods based on unsteady flow approach, expected to be developed in the future on the basis of information supplied by this four-part sequence of hydrology papers, should cover the range of flow conditions necessary to compute the depth, the velocity, or the discharge hydrographs at any point, and/or the wave profiles of depth, velocity or discharge along a system of storm drains at any time.

(2) To better understand flow phenomena, through basic and applied research, of gradually varied free-surface unsteady flow in conduits, especially in storm drains. This understanding may supply pertinent information for developing practical criteria that decision makers can use for storm drain design.

(3) To conduct analytical and experimental studies related to various hydrodynamic aspects of gradually varied free-surface unsteady flow in prismatic channels and conduits that could affect flood routing methods developed for storm drains using the unsteady flow approach.

(4) To produce information for improving existing design flood routing methods by using digital computers and Fortran programming, on any available digital computer manufactured in the United States.

After considering the specific aims the general procedure for doing the research on free-surface unsteady flow in storm drains was divided into three parts:

- (1) Experimental hydraulic studies.
- (2) Analytical studies using numerical digital computer integration procedures.
- (3) Comparison of numerically integrated analytical waves with the experimentally observed corresponding waves, and the study of various aspects related to this comparison.

2.2 Experimental Phase

The experimental hydraulic studies were mainly conducted in a moveable conduit, 3 ft in diameter and 822 ft long. The conduit could be moved across a hillside, as described in summary form in Chapter 4 of this paper and in detail in Hydrology Paper No. 44.

(1) The experimental phase of the project consisted of studying, measuring and observing certain aspects of flood waves and conduit. The boundary roughness of the experimental conduit was studied under conditions of free-surface steady flow with the objective of determining whether the conduit may be considered as hydraulically smooth, and if so, how the Darcy-Weisbach friction factor is related either to the Reynolds number, or to the depth of flow. This phase of study is briefly described in Chapter 4 of this paper and in detail in Chapter 3 of Hydrology Paper No. 45. Results were then used as input data for numerical integration of flow equations by a digital computer.

(2) The relationship among the head loss, the discharges of two joining storm drains and the water levels at junction boxes of the storm drains was studied. This experimental research considered only a given junction box with two different inlet positions of the lateral drains. This phase is briefly described in Chapter 4 of this paper and in detail in Chapter 4 of Hydrology Paper No. 45. Results summarized in head loss equations were then used as input data for the numerical integration of flow equations.

(3) The relationship between the depths of free-surface flow and the discharge at the conduit outlet, as downstream boundary conditions was studied. Flow rating curves were considered in the steady flow for different conduit conditions, such as the free outflow or the controlled outflow, to be used as the downstream boundary condition for the numerical integration of the flow equations. This phase is briefly described in Chapter 5 of this paper, and in detail in Chapter 6 of Hydrology Paper No. 45.

(4) Velocity distributions in the experimental conduit flowing partly full were measured to determine as accurately as feasible the velocity distribution coefficients, α and β , for their use as input data into the numerical integration of flow equations. This phase of the study is briefly described in Chapter 4 of this paper and in detail in Chapter 5 of Hydrology Paper No. 45.

(5) Flow phenomena were observed and followed to assess the degree of development of particular problems of physical waves, such as secondary waves, third dimensional oscillations, and similar. An analytical reproduction of flow phenomena is essential for the comparison of physical waves and analytical waves numerically integrated by using a digital computer. These secondary phenomena are not usually taken into account in the one-dimensional partial differential equations of unsteady flow.

(6) The waves were studied at several slopes. For reasons of cost, only the smooth boundary conduit and one diameter were used, so the slope was a variable, while the conduit diameter and its roughness were kept as constants.

(7) Simulated floods were studied and observed in the 3 ft diameter, 822 ft long storm conduit by introducing inflow hydrographs either at the entrance of the conduit, or at both the entrance and at several

inlet points along it. The experimental procedure was to accurately record the inflow hydrographs by appropriate devices, or the discharge as a function of time. Then the movement and development of flood waves along the storm conduit and at the conduit outlet were also accurately recorded. The recorded inflow hydrographs were used as input data for the computerized numerical integration of flow equations. The recorded hydrograph at any point along the storm drain served as a basis for comparing it with the hydrograph determined for the same points by the numerical integration of flow equations.

Certain flow phenomena involving free-surface unsteady flow were not studied, such as the amplification of flood waves in channels of steep slopes, the instability of flow when the pipe is flowing nearly full, the passage from supercritical to subcritical flow through the hydraulic jump, the instability of the position of hydraulic jump in unsteady flow, and similar problems.

2.3 Numerical Integration

The reason for developing methods of integrating unsteady flow equations by numerical finite-difference schemes on a digital computer was to investigate the feasibility of using the two partial differential equations of gradually varied free-surface unsteady flow as the basic mathematical equations for routing floods through storm drains. Or rather, the hypothesis was tested that drains best can be designed by the unsteady flow approach instead of the steady flow approach. However, in this study both the dynamic and the kinematic aspects of free-surface wave movements through storm drains were taken into account, instead of just the kinematic aspects of some simplified unsteady-flow approaches, as is sometimes the case.

The influence of certain parameters in the two partial differential equations were investigated by the numerical integration of these equations for given inflow hydrographs and storm drain characteristics. The advantages of using computerized numerical integration of the two partial differential equations of free-surface unsteady flow are: (a) economy of computation; (b) rapidity of predicting or computing the evolution of flood waves along the storm drain; and (c) improved accuracy in flood wave routing, particularly if the background data are sufficient and accurate.

The computerized numerical integrations and the experimental hydraulic studies were carried out simultaneously. The results obtained by the experimental work were used in developing proper computer programs to simulate numerically physical waves as closely as practically feasible under given conditions.

Two approaches are used for the numerical integration of the two partial differential equations. The first approach uses finite-difference explicit schemes as applied to the two partial differential equations of unsteady flow, with the study of effects of increments, or finite differences of time and length on the numerical integrations. The second approach of integration applies the finite-difference numerical schemes of integration to the four characteristic ordinary differential equations, derived from the two partial differential equations. Computations from this method are compared with the observed experimental or physical waves.

Additional problems may be solved by using the results of this study. The effects of certain parameters on the flood wave propagating along a storm drain can be found which parameters define some coefficients of the two partial differential equations of gradually varied free-surface unsteady flow.

Reliable and accurate computer programs of numerical integration schemes can be developed so that some hydrodynamic problems, which are not yet solved, can be studied in the future. One of these problems is the criterion to use when a wave in a storm drain either does not change or amplifies under different wave and conduit characteristics.

Also, the computer programs presented for the numerical integration of the two partial differential equations could be used effectively by other researchers to assess the accuracy and reliability of different existing simple design flood routing methods, or to develop new simplified design flood routing methods for storm drains.

2.4 Comparison of Physical and Analytical Waves

Comparisons are essential to assess how well the numerical integration of differential equations of unsteady flow, called the analytical waves, by the most reliable numerical methods, approximate the observed or physical waves. The comparison of physical waves with the analytical waves obtained by the numerical integration of the complete equations indicates how the basic hypotheses in developing differential equations and the various errors in parameters affect the accuracy in mathematically describing and reproducing the corresponding physical waves.

The closer the analytical waves are to the physical waves, or the more reliable the numerically integrated waves are, the more valuable the two partial differential equations become for assessing the accuracy of a large number of simplified design flood routing methods, either in existence or still in a state of continuing development, under the various conditions of flood hydrographs, channel or conduit characteristics, and their boundary and initial conditions.

2.5 Relation of this Study to Other Problems of Storm Drainage

To drain highways, urban areas, and airports during storm precipitation, and to avoid their flooding for the given probability of occurrence of rainfall intensity and duration by using large storm drains, four problems should be properly solved. The inflow hydrographs into inlet points of the drainage system along the highways, or streets or airports must be accurately determined. Curb inlets must be well designated so they will not impede the free and desired conveyance of flood waters into the drainage system. The primary and secondary storm drains must be designed to avoid flooding highways, streets, and airports for all hydrographs smaller than the selected design inflow hydrographs. The water from the outlet of the main storm drain must be evacuated by either gravity flow or by pumps. The research of this study is limited, through basic and applied research, to methods of solving the problem of primary and secondary storm drains. The design inflow hydrographs, the geometry of curb inlets, and the types of outflow conditions at main storm drain outlets are assumed to be known in this study.

The gradually varied free-surface unsteady flow in storm drains is studied under two conditions. The first of these conditions is that main storm drains are often vertically positioned as close to the highway or street surface as feasible, to minimize the overburdening and to avoid the cost of reinforcing the storm drain. This means a decrease of the available water head between the flooded highway or street and the drain outlet, thus minimizing the advantage of an eventual use of drains flowing under pressure.

The second condition is that the maximum discharge for free-surface flow in conduits is approximately at the depth which is 0.9 of the conduit diameter. If the same discharge should be conveyed in a full flowing conduit, or under pressure, the slope of the energy line must exceed the bottom slope of the drain. This greater slope might not be available in most cases, even with a flooded highway, street, or airport.

Two approaches may be used to determine the dimensions of a storm drain in the unsteady flow method of flood routing through storm drains. First, the method is so simplified that the diameter or other basic cross-section dimensions can be computed directly and explicitly. Second, the dimensions of a drain are first assumed, then the design storm flood is routed through that drain. If the dimensions come out to be either smaller or larger than the best use of the assumed dimensions, say 0.9 of the diameter at the flood peak depth, new dimensions are assumed and the analysis is repeated until the right dimensions are obtained. This approach needs repeated routing, some of which require larger and some smaller dimensions than the dimensions assumed. When the two partial differential equations are used as the basis for flood routing, only this second approach seems possible and feasible. The first approach is, however, a goal that should not be neglected in future research.

Flood routing methods, generally used at present, start with any given flood hydrograph at a given point along the conduit and determine, dividing the hydrograph by a unit time Δt in many parts, the transformed hydrograph at a distance position x downstream or even upstream from this initial position. Since some design storms are often assumed as one-peak hydrographs, sometimes they may be well approximated by analytical expressions. Though the analytical integration of the two partial differential equations of unsteady flow are not the subject of this study, a potential practical, storm drain flood routing method may be developed on the basis of routing flood parameters (say the three parameters of an analytical equation for the flood hydrographs) along a long prismatic conduit, instead of routing the Δt -elements of the hydrographs. Although this method is not studied, it is briefly discussed in this paper in Chapter 7. If it is shown by future research to be practically feasible, it may be used for rapid computations in preliminary design, or it may be a way of determining initial dimensions with more accurate flood routing methods to follow. The determination of the feasibility of such a method to storm drain computations, however, is outside the scope of this study.

It is expected that future studies will likely produce a set of approximate design methods for computing unsteady flow in storm drains. Only through the comparison of either waves observed in hydraulic experiments and/or waves observed in nature can the necessary conditions and the accuracy of each new or existing design method be evaluated for its applicability. For this comparison to be valid the most accurate available numerical computation based on digital computers and the unsteady-flow approach to design of storm drains must be used.

THEORETICAL CONSIDERATIONS

3.1 Derivation of the Two Partial Differential Equations for Unsteady Free-Surface Flow in Conduits

Derivation of continuity equation (law of conservation of mass). At time t the cross-section area of unsteady free-surface flow at the section x is A , 1-2 in Fig. 3.1. At the same time t and the section $x + \Delta x$ the area is $A + \frac{\partial A}{\partial x} \Delta x$, with Δx an incremental length. The mass of water between these two sections (slice 1-2-3-4-1 in Fig. 3.1) is $\rho A \Delta x + \frac{1}{2} \rho \frac{\partial A}{\partial x} (\Delta x)^2$. By neglecting the second-order differential term for $\Delta x \rightarrow dx$, the mass is $\rho A \Delta x$. Assuming that the lateral outflow or inflow is q , discharge per unit length of conduit, with q positive for inflow and negative for outflow, the change of mass with time is

$$\frac{d}{dt}(\rho A \Delta x) = \rho A \frac{d(\Delta x)}{dt} + \rho \Delta x \frac{dA}{dt} = \rho q \Delta x \quad (3.1)$$

for an incompressible fluid with ρ as a constant water density.

In considering the velocity v , of an individual particle moving between the two cross sections for the time interval Δt , then $\Delta x = v \Delta t$. Integrating Eq. 3.1 over a cross section A , by using $\Delta x = v \Delta t$, the following integrals are obtained in passing from the particle velocity v , to the average cross section velocity V ,

$$\frac{1}{VA} \iint_A v dA = 1, \text{ and } \frac{1}{A} \iint_A \frac{\partial v}{\partial x} dA = \frac{\partial V}{\partial x} \quad (3.2)$$

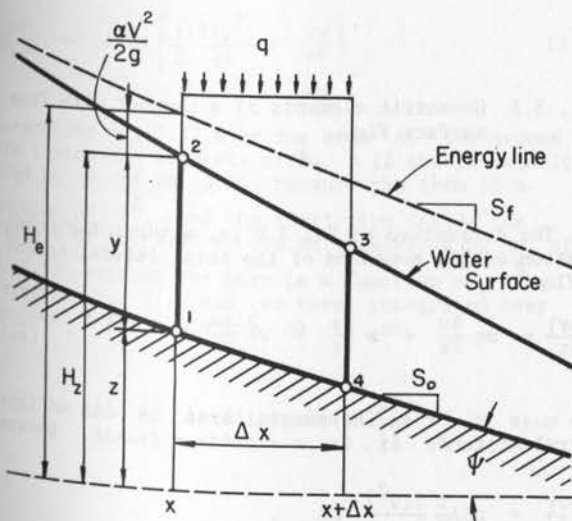


Fig. 3.1. Definition diagram for derivation of continuity and momentum equations of unsteady free-surface flow in an open channel.

The derivative $d(\Delta x)/dt$ for a moving individual particle, for Δx only a function of t , and $\Delta t \rightarrow 0$, is

$$\frac{d(\Delta x)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{(v + \frac{\partial v}{\partial x} \Delta x) \Delta t - v \Delta t}{\Delta t} = \frac{\partial v}{\partial x} \Delta x. \quad (3.3)$$

The derivative dA/dt is

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{\partial A}{\partial x} \frac{dx}{dt} = \frac{\partial A}{\partial t} + v \frac{\partial A}{\partial x} \quad (3.4)$$

For the non-prismatic channels, the derivative in x is,

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial y} \frac{\partial y}{\partial x} + \left(\frac{\partial A}{\partial x} \right)_y,$$

in which the last term is the measure of the change of cross-section area for a given y . This term can be neglected in the case of storm drains which are assumed to be prismatic conduits between any two inlet points, because a longitudinal section of drain is studied for the movement of unsteady free-surface flow.

Equation 3.1, in passing from the particle velocity to the average cross-section velocity, gives

$$A \frac{\partial V}{\partial x} + V \frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \quad (3.5)$$

or

$$\frac{\partial (VA)}{\partial x} + \frac{\partial A}{\partial t} - q = 0, \quad (3.6)$$

in which V is the mean velocity in a cross-section, V and A are the dependent variables, and x and t are the independent variables. Equation 3.6 is sometimes also written in the following form

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \quad (3.7)$$

with the discharge $Q = VA$, and A and Q now being dependent variables. Because $\partial A / \partial t = B \partial y / \partial t$, with B the width at the surface cross-section and y the depth of water above the bottom of the cross-section, Eq. 3.6 becomes

$$A \frac{\partial V}{\partial x} + VB \frac{\partial y}{\partial x} + B \frac{\partial y}{\partial t} - q = 0 \quad (3.8)$$

Any of the above four equations, 3.5 through 3.8, may be used in practical applications.

Derivation of momentum (dynamic) equation. Newton's second law, the law of momentum, reads in a given direction as

$$\frac{d(mv)}{dt} = F, \quad (3.9)$$

in which m is the mass, v is the velocity of an individual particle, and F is the resultant force of all forces acting on the particle. Replacing the movement of a particle by the movement of an elementary slice of water between sections x and $x + \Delta x$ of Fig. 3.1, and replacing the particle velocity v by the mean velocity V in the cross-section, the following velocity distribution coefficients must be introduced

$$\alpha = \frac{1}{AV^3} \iint_A v^3 dA, \text{ and } \beta = \frac{1}{AV^2} \iint_A v^2 dA. \quad (3.10)$$

Coefficients α and β depend on velocity distributions in a cross-section A , and consequently depend on the shape, area, roughness and the mean flow velocity of a cross-section.

Equation 3.9 is applied along the direction of the conduit bottom, as shown in Fig. 3.1. All acting forces, external and internal, are projected in this direction. The tangential component, $G \sin \psi$, of gravity force along the bottom, taking the positive sign with the direction of flow as shown by Fig. 3.2, is $\rho g A \Delta x \sin \psi$. Friction force, F_f , with the head loss ΔH_f along the conduit bottom, is $F_f = -\rho g A S_f \Delta x$, with $\lim_{\Delta x \rightarrow 0} \Delta H_f / \Delta x = dH_f / dx = S_f$ for $\Delta x \rightarrow 0$ being the friction slope at x .

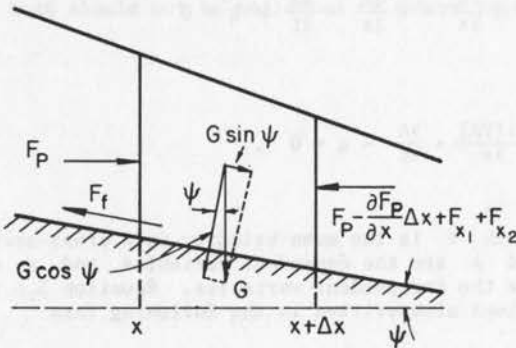


Fig. 3.2 Forces acting on the incremental volume of the channel.

Pressure forces, and other magnitudes as shown in Figs. 3.2, 3.3, and 3.4, can be expressed, assuming the hydrostatic pressure distributions along the verticals, as follows:

$$F_P = \int_0^y \rho g (y - \eta) B_\eta d\eta$$

and

$$\frac{\partial F_P}{\partial x} = \int_0^y \rho g \frac{\partial y}{\partial x} B_\eta d\eta + \int_0^y \rho g (y - \eta) \frac{\partial B_\eta}{\partial x} d\eta,$$

because y and B depend on x while η is independent of it. Since

$$\int_0^y B_\eta d\eta = A, \text{ and } \Delta x \int_0^y \rho g (y - \eta) \frac{\partial B_\eta}{\partial x} d\eta = (F_1 + F_2),$$

with F_1 , F_2 , and F_3 pressure forces defined in Fig. 3.4, then

$$\frac{\partial F_P}{\partial x} \Delta x = \rho g A \frac{\partial y}{\partial x} \Delta x - (F_1 + F_2),$$

so that the resultant pressure force in the horizontal direction, normal to the cross-section, is

$$F_P - [F_P + \Delta x \frac{\partial F_P}{\partial x} + (F_1 + F_2)] = \Delta F_P,$$

and the pressure force in the direction of the bottom slope is given by

$$\Delta F_P = -\rho g A \frac{\partial y}{\partial x} \Delta x \cos \psi.$$

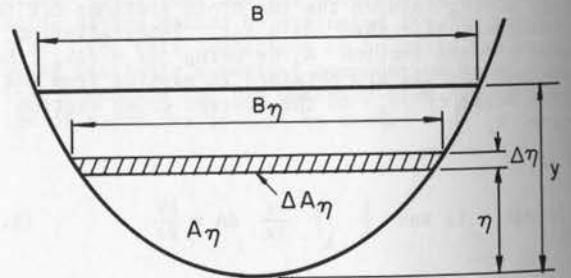


Fig. 3.3 Geometric elements of a channel with free surface flow.

The derivative of Eq. 3.9 is, without the consideration of the momentum of the total lateral inflow or outflow,

$$\frac{d(mv)}{dt} = mv \frac{\partial v}{\partial x} + m \frac{\partial v}{\partial t} + v^2 \frac{\partial m}{\partial x} + v \frac{\partial m}{\partial t}. \quad (3.11)$$

The mass m of an incremental area ΔA and an incremental distance Δx is $m = \rho \Delta A \Delta x = \rho v \Delta A \Delta t$. Because

$$mv \frac{\partial v}{\partial x} = \rho \Delta A \frac{1}{2} \frac{\partial (v^2)}{\partial x} \Delta x,$$

$$m \frac{\partial v}{\partial t} = \rho \Delta A \frac{\partial v}{\partial t} \Delta x,$$

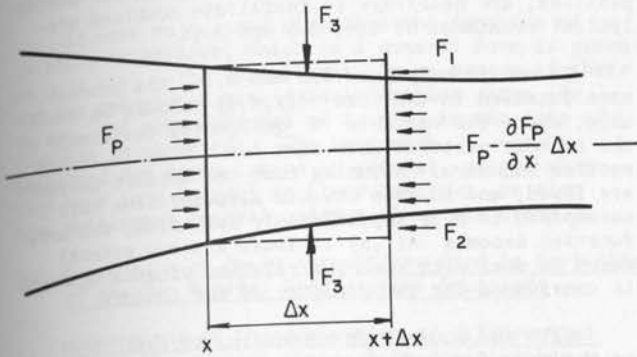


Fig. 3.4 Forces acting on an incremental slice in free-surface flow in a channel.

$$\frac{\partial m}{\partial x} = \rho \Delta A \frac{\partial(\Delta x)}{\partial x} = 0, \text{ as } \Delta x \text{ is independent of } x, \text{ and}$$

$$\frac{\partial m}{\partial t} = \rho \Delta A \frac{\partial(\Delta x)}{\partial t} = 0, \text{ as } \Delta x \text{ is independent of } t, \text{ which gives}$$

$$v \frac{\partial m}{\partial t} = \rho \Delta A \frac{1}{2} \frac{\partial(v^2)}{\partial x} \Delta x, \quad (3.11)$$

Eq. 3.11 now becomes

$$\frac{d(mv)}{dt} = \rho \Delta A \Delta x \left[\frac{1}{2} \frac{\partial(v^2)}{\partial x} + \frac{\partial v}{\partial t} \right]. \quad (3.12)$$

Integrating Eq. 3.12 over the area A , the second term inside the brackets of Eq. 3.12 should be multiplied by β of Eq. 3.10, because the term is a function of v^2 , and the first term inside the brackets of Eq. 3.12 should be multiplied by α of Eq. 3.10, because the term is a function of v^3 . Therefore, Eq. 3.12 has two terms integrated over the area A in replacing ΔA by dA ,

$$\iint_A \rho \frac{\partial v}{\partial t} \Delta x \, dA = \beta \rho \Delta x \frac{\partial V}{\partial t},$$

and

$$\frac{1}{2} \iint_A \rho \frac{\partial(v^2)}{\partial x} \Delta x \, dA = \frac{\alpha \rho \Delta x}{2} \frac{\partial(V^2)}{\partial x},$$

The momentum of the total lateral inflow or outflow, $q \Delta x$, is

$$\rho \iint_A q \Delta x \, dA = \rho \iint_A q v^2 \Delta t \, dA = \beta \rho q \Delta x V,$$

because this term is a function of v^2 , and β of Eq. 3.10 takes care of the velocity distribution across the area A . Equation 3.9 then becomes, by including the momentum of the total lateral inflow or outflow,

$$\begin{aligned} \frac{\alpha \rho A \Delta x}{2} \frac{\partial(V^2)}{\partial x} + \beta \rho A \Delta x \frac{\partial V}{\partial t} + \beta \rho q \Delta x V \\ = \rho g A \Delta x \sin \psi - \rho g A \Delta x S_f - \rho g A \Delta x \frac{\partial y}{\partial x} \cos \psi. \end{aligned} \quad (3.13)$$

For the approximations $\sin \psi \approx \tan \psi = S_0$ with S_0 the bottom slope, and $\cos \psi \approx 1$ for small slopes, Eq. 3.13 changes to

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{\alpha V^2}{2g} \right) + \frac{\beta}{g} \frac{\partial V}{\partial t} + \frac{\partial y}{\partial x} - S_0 + S_f \\ + \frac{\beta V q}{gA} = 0. \end{aligned} \quad (3.14)$$

The slope of Fig. 3.1 is $S_0 = -\partial z / \partial x$. The energy line gradient is

$$\frac{\partial H_e}{\partial x} = \frac{\partial}{\partial x} \left(y + z + \frac{\alpha V^2}{2g} \right). \quad (3.15)$$

The general form of the momentum equation is then

$$\frac{\partial H_e}{\partial x} + \frac{\beta}{g} \frac{\partial V}{\partial t} + S_f + \frac{\beta V q}{gA} = 0. \quad (3.16)$$

Designating the terms in Eq. 3.16 by

$$S_e = \frac{\partial H_e}{\partial x}, \text{ the slope of the total energy line,}$$

$$S_a = \frac{\beta}{g} \frac{\partial V}{\partial t}, \text{ the local acceleration slope, and}$$

$$S_q = \frac{\beta V q}{gA}, \text{ the slope due to lateral inflow or outflow, Eq. 3.16 becomes}$$

$$S_e + S_a + S_f + S_q = 0. \quad (3.17)$$

Multiplying Eq. 3.17 by Δx , with $\Delta H_e = S_e \Delta x$, $\Delta H_a = S_a \Delta x$, $\Delta H_f = S_f \Delta x$, and $\Delta H_q = S_q \Delta x$, then

$$\Delta H_e + \Delta H_a + \Delta H_f + \Delta H_q = 0, \quad (3.18)$$

or the sum of all slopes or of all head changes along Δx - length is zero. Equation 3.14 is sometimes given in the form

$$\alpha V \frac{\partial V}{\partial x} + \beta \frac{\partial V}{\partial t} + g \frac{\partial y}{\partial x} = g(S_0 - S_f) - \frac{\beta V q}{A} \quad (3.19)$$

Names and physical meanings of different terms in the two partial differential equations. The four terms in Eq. 3.5 when multiplied by $\Delta x \Delta t$ give the dimension of the volume. In the order of sequence, they have the following physical meanings: (1) wedge storage which results from the difference of depths at the beginning and the end of the elementary reach Δx by a change of area A along the conduit; (2) prism storage; (3) storage of rate-of-rise of level resulting from the rate-of-change of area A with time, and (4) storage, positive or negative, resulting from the lateral inflow or outflow.

The six terms in Eq. 3.14, in their order of sequence have the following physical meanings: (1) rate-of-change of velocity head, also called the dynamic head, the velocity-head term, the energy grade-line inclination, and the instantaneous energy gradient, which is the slope created by the change of velocity head along the conduit; (2) acceleration term as the ratio of accelerations, or the ratio of the change of velocity with time and the acceleration of gravity, also called the acceleration-head term, the velocity-hydrograph inclination, or the localized acceleration gradient; (3) rate-of-change of depth, or the depth-taper or the depth-change term, which is the slope created by the change of depth along the conduit; (4) bottom slope; (5) friction slope, and (6) part of the gradient on the energy line created by the lateral outflow or inflow. Equation 3.14 is differently expressed in various publications: dimensionless as in Eq. 3.14, or with dimensions of head, acceleration, momentum, energy, or in some other dimensions.

Selection of dependent variables. The two partial differential equations of unsteady free-surface flow are simplest in the case when the dependent variables are the mean velocity V and the depth y , with length x and time t being independent variables. In order to get a discharge hydrograph at a place x of the drain, the depth hydrograph $y(t)$, and velocity hydrograph $V(t)$ are first obtained. Then the area hydrograph $A(t)$ is determined from the depth hydrograph. The discharge hydrograph is then $Q(t) = V(t) A(t)$.

If the discharge hydrographs at different places should be the final result of flow routing through a storm drain, it might be convenient in some cases to use discharge Q and depth y as dependent variables, instead of velocity V and depth y . The computational procedure time using a digital computer (programming and computation time) might be somewhat longer in using Q and y than in using V and y as dependent variables.

Condition for the combined use of the two partial differential equations. The continuity equation involves the cross-section area, while the momentum equation is based on the rate-of-change of energy line, or of water surface position, plus the dynamic head. For irregular conduits with changing bottom slope and irregular cross section shape and area, the joint use of these two partial differential equations begins to create the first complexity in the mathematical analysis. The rate-of-change of cross-section characteristics, as related to the bottom position, and

the rate-of-change of bottom slope with distance, when expressed in analytical form, generally enable the joint use of the two equations. Some assumptions and simplifications for cross-section, and for bottom position, are necessary to facilitate combined analytical treatment of Eqs. 3.5 and 3.14.

To use jointly Eqs. 3.5 and 3.14, the general area function in the form $A(y, x, t)$ should be available, where the variable t designates the change of the contour position with time for a movable cross-section boundary. Assuming that the conduit contours are fixed, and in some movable alluvial beds this assumption is only approximately satisfied, the area function becomes $A(y, x)$. There are two general cases to deal with when the relation of area to depth is considered for the geometry of the conduit:

(1) The conduit is prismatic, so that $A(y)$, or the area for a given y is independent of x . The simplest equation is $A = By$, a rectangular prismatic channel or conduit, with constant width B , which is usually used for the theoretical analysis of unsteady free surface flow in channels. A fit to natural channels of a power function, $A = py^s$, is often feasible with p and s constants. The area of the circular drains of a given diameter D is an arccosine function of the depth, and for a given y the area is independent of x .

(2) The geometric description of the conduit or the channel in a non-prismatic case is given by a function $A(y, x)$. The power function $A = py^s$ is applicable for some channels, with p and s being functions of x . The converging and diverging circular conduits belong to this category of conduits. This dependence of area to y and x coordinates should not be mixed with the change of area along the length of conduit in unsteady flow.

The joint use of Eq. 3.5 and 3.14 cannot be accomplished unless the relation of A and y is defined along the conduit. A storm drain is assumed to be prismatic conduit between the two successive manholes or junction boxes, so that $A(y)$ is valid for that reach, or the area is independent of x for a given y . In this case

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial y} \frac{\partial y}{\partial t} = B \frac{\partial y}{\partial t} \quad (3.20)$$

and

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial y} \frac{\partial y}{\partial x} = B \frac{\partial y}{\partial x} \quad (3.21)$$

with

$$B(y) = \partial A / \partial y \quad (3.22)$$

Introducing the expressions of Eqs. 3.20 and 3.21 into Eq. 3.5, it becomes

$$\frac{A}{VB} \frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} + \frac{1}{V} \frac{\partial y}{\partial t} - \frac{q}{VB} = 0 \quad (3.23)$$

so that now both Eqs. 3.23 and 3.14, which describe the gradually varied free-surface unsteady flow, are given in dimensionless form.

It must be assumed also that the function of q is known in advance, which in a general form is given as $q(y, x, t)$. The variable t is necessary if there are any changes in time (slowly opening gates, or valves, and slow breaches of levees, in the case of channels or conduits). The term with q can be neglected for storm drains in general, except when they have long lateral spillway outflows, or also some continuous lateral inflows along the drain.

3.2 Discussion of Basic Assumptions Used in Derivation of the Two Partial Differential Equations

Assumptions in the derivation of differential equations. The general approach in deriving Eqs. 3.23 and 3.14 starts with the gradually varied unsteady free-surface water movement. This implies that the partial derivatives, $\partial V/\partial x$, $\partial V/\partial t$, $\partial y/\partial x$, and $\partial y/\partial t$ represent relatively small changes, in order that this basic assumption of gradually varied flow can be justified. It can be stated that the ambiguities arise mainly from the subjective interpretations by various investigators of the term "gradually varied." As far as the writers know, none of a myriad of investigations of unsteady free-surface flow since De Saint-Venant's time, in this last hundred years, has attempted to quantify the term "gradually varied unsteady flow".

Nevertheless, the basic assumptions underlying the development and the application of Eqs. 3.23 and 3.14 are:

(1) Vertical acceleration can be neglected in comparison with the horizontal acceleration, or the vertical acceleration normal to the conduit in comparison with the acceleration along the conduit is very small, because of the gradual change of depth and discharge with time and distance. The steeper a wave, the less justified becomes this assumption. It is not applicable in the case of water surges, in the form of bores and sudden wave depressions.

(2) Unsteady flow as gradually varied has the hydrostatic pressure distribution along a vertical.

(3) Flow patterns in vertical planes parallel to the longitudinal axis of the channel are the same. This assumption in the case of curvilinear channels means that the vertical surfaces parallel to the longitudinal axis have the same flow pattern. The influence of the channel sides and their curvatures on flow patterns can be neglected. This assumption is equivalent to stating that the flow is two-dimensional, with third dimension effects negligible.

(4) Velocity distribution along a vertical in unsteady flow is the same as the velocity distribution in steady flow for the same water depth. It means that the velocity distribution coefficients α and β in Eq. 3.14 are constants for given values of discharge, depth, and velocity, or the unsteady flow does not influence these coefficients. Since this assumption depends on the rate-of-change of velocities with time and distance, it is justified only in the case of a gradually varied flow.

(5) Friction resistance in unsteady flow is the same as the friction resistance in steady flow. This assumption is justified only if the rate-of-change of velocities with respect to time and distance is small.

(6) Conduit slope is so small that $\cos \psi$ can be replaced by unity and $\sin \psi$ by $\tan \psi$.

Effects of these assumptions. There are no data in the literature that show the numerical individual effects of the above assumptions on the computed waves along the flow channel. Evidence is lacking for the justification of these six assumptions in terms of the specific characteristics of a wave, of a channel, and of a lateral inflow or outflow. Only global comparisons between the observed waves and the computed waves by using Eqs. 3.23 and 3.14, and usually with $\alpha = \beta = 1$, are available in literature, mainly in the most rudimentary form. The total influence of all of the above factors is relatively small in the case of very gradually varied unsteady flows. It is, therefore, justified to neglect the effect of six assumptions for this particular case.

It is expected that the assumption of a negligible vertical acceleration produces departures between the mathematical waves and the physical waves, which increases with an increase of rate-of-change of hydrograph. A relation $D = f(dQ/dt)$, with D the departure and dQ/dt the time rate-of-change of discharge hydrograph, would give a general picture of how the first assumption influences the computed wave movement. The second assumption is implicitly included in the effect of the first assumption, and its effect will also increase with an increase of rate-of-change of discharge hydrograph.

Mathematical methods available for the computation of unsteady free-surface flow are more accurate either for a very gradually varied flow, in the case where the two De Saint-Venant partial differential equations are used, or for a steep surge in the case where the available equations for traveling bores and steep depressions are used, than is the case for a steep wave which is between these two extremes.

The effects of the third assumption are negligible in straight line conduits, provided that the inlet conditions and lateral inflows do not create the persistent lateral oscillations of the body of water in conduit during the flood wave passage.

The effects of differences for velocity distribution coefficients and flow resistance factors between the unsteady and steady flow patterns are difficult to assess without accurate experiments and basic studies. The steeper a wave is, the more influence the constantly changing boundary layer in unsteady flow has on the flow resistances and velocity distributions. These differences must increase with an increase of the rate-of-change of the discharge hydrograph.

The effects of the sixth assumption are very small for storm drains with moderate slopes, say up to 5%.

The effects of the preceding assumptions are not simple to investigate in detail, even for a research conduit sufficiently long and with a large diameter. Although some discussions are given in other papers about effects of individual assumptions, the comparison between physical recorded waves in the research conduit and computed analytical waves by digital computer produces a general picture of the effects caused by all six assumptions. But even in computer analysis, various other errors mask an accurate detection of the resulting effects.

3.3 Derivation of Characteristic Equations

Definition of characteristics. The two partial differential equations for gradually varied unsteady free-surface flow in conduits, with the two dependent variables (V, y) and the two independent variables (x, t) , have the general form

$$A_1 \frac{\partial V}{\partial x} + B_1 \frac{\partial V}{\partial t} + C_1 \frac{\partial y}{\partial x} + D_1 \frac{\partial y}{\partial t} + E_1 = 0, \quad (3.24)$$

and

$$A_2 \frac{\partial V}{\partial x} + B_2 \frac{\partial V}{\partial t} + C_2 \frac{\partial y}{\partial x} + D_2 \frac{\partial y}{\partial t} + E_2 = 0, \quad (3.25)$$

with the coefficients $A, B, \dots$, and E being functions of V, y, x , and t besides being functions of conduit system properties.

Equations 3.24 and 3.25 are linear in relation to partial derivatives, but the coefficients are also functions of dependent variables. Therefore, Eqs. 3.24 and 3.25 are called quasi-linear partial differential equations. For other data on the characteristic curves see reference [1]. The derivation in this study closely follows the derivation of characteristic curves derived by Courant and Friedrichs [2].

Equations 3.23 and 3.14, in order to be comparable with Eq. 3.24 and 3.25, must have the forms

$$\frac{A}{VB} \frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} + \frac{1}{V} \frac{\partial y}{\partial t} - \frac{q}{VB} = 0 \quad (3.26)$$

and

$$\frac{\alpha V}{g} \frac{\partial V}{\partial x} + \frac{\beta}{g} \frac{\partial V}{\partial t} + \frac{\partial y}{\partial x} + S_f - S_o + \frac{\beta V q}{gA} = 0. \quad (3.27)$$

In this case

$$A_1 = \frac{A}{VB}, B_1 = 0, C_1 = 1, D_1 = \frac{1}{V}, E_1 = -\frac{q}{VB}$$

$$A_2 = \frac{\alpha V}{g}, B_2 = \frac{\beta}{g}, C_2 = 1, D_2 = 0, \text{ and}$$

$$E_2 = S_f - S_o + \frac{\beta V q}{gA}$$

Since E_1 and E_2 are not zero, Eq. 3.26 and 3.27 are nonhomogeneous nor are they reducible. Furthermore, the roles of dependent and independent variables are not interchangeable. In other words, the hodograph transformation of the (x, t) -plane into the (V, y) -plane is not applicable. The solution of Eqs. 3.26 and 3.27 gives the two functions $V(x, t)$ and $y(x, t)$.

Because the waves are gradually varied free-surface movements, it can be assumed that Eqs. 3.24 and 3.25 or Eqs. 3.26 and 3.27, as well as all coefficients in these equations, are continuous and possess as many continuous derivatives as may be required. It is also assumed that the condition

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2} \quad (3.28)$$

is not satisfied. This last assumption is true because from Eqs. 3.24 through 3.27, the inequality for Eq. 3.28 is

$$\frac{Ag}{\alpha V^2 B} \neq 0 \neq 1 \neq \infty \quad (3.29)$$

A linear addition of Eqs. 3.24 and 3.25, with λ_1 and λ_2 the linear multipliers, respectively, gives

$$(\lambda_1 A_1 + \lambda_2 A_2) \frac{\partial V}{\partial x} + (\lambda_1 B_1 + \lambda_2 B_2) \frac{\partial V}{\partial t} + (\lambda_1 C_1 + \lambda_2 C_2) \frac{\partial y}{\partial x} + (\lambda_1 D_1 + \lambda_2 D_2) \frac{\partial y}{\partial t} + (\lambda_1 E_1 + \lambda_2 E_2) = 0 \quad (3.30)$$

For the (x, t) -plane, assume a curve is given as $t(x)$. Then dt/dx is the tangent or the direction of this curve with $V(x, t)$ and $y(x, t)$ the solutions of Eqs. 3.24 and 3.25, or Eqs. 3.26 and 3.27. For this case, the total differentials are

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial t} dt \quad (3.31)$$

and

$$dy = \frac{\partial y}{\partial x} dx + \frac{\partial y}{\partial t} dt \quad (3.32)$$

If λ_1 and λ_2 are selected to satisfy the condition

$$\frac{dt}{dx} = \frac{\lambda_1 A_1 + \lambda_2 A_2}{\lambda_1 B_1 + \lambda_2 B_2} = \frac{\lambda_1 C_1 + \lambda_2 C_2}{\lambda_1 D_1 + \lambda_2 D_2}, \quad (3.33)$$

Eq. 3.30 can be written as

$$(\lambda_1 A_1 + \lambda_2 A_2) dV + (\lambda_1 C_1 + \lambda_2 C_2) dy + (\lambda_1 E_1 + \lambda_2 E_2) dx = 0 \quad (3.34)$$

The derivatives of V and y in Eq. 3.30 are combined in Eq. 3.34 so that the derivatives are in the same direction, dt/dx . This direction is called the characteristic direction and the corresponding curves in the (x, t) -plane are called the characteristic curves.

Equation 3.33 gives the ratio λ_1/λ_2 as

$$-\frac{\lambda_1}{\lambda_2} = \frac{A_2 dt - B_2 dx}{A_1 dx - B_1 dt} = \frac{C_2 dt - D_2 dx}{C_1 dt - D_1 dx}, \quad (3.35)$$

which can also be expressed as

$$a(dt)^2 - 2b dx dt + c(dx)^2 = 0 \quad (3.36)$$

with $a = A_1 C_2 - A_2 C_1$, $2b = A_1 D_2 - A_2 D_1$ and

$$c = B_1 D_2 - B_2 D_1$$

Equation 3.36 has two distinct roots for hyperbolic partial differential equations giving $b^2 - ac > 0$, excluding the case when all three coefficients (a, b, and c) vanish. Also $a \neq 0$ is satisfied in Eqs. 3.26 and 3.27 because $a = A_1 C_2 - A_2 C_1 = A/VB - \alpha V/g$, except for $V = \sqrt{gA/\alpha B}$, or for $V = C$, or when the water velocity is equal to the celerity of a small disturbance in quiescent water with hydraulic depth, D, equal to A/B .

Designating the slope of the characteristic curves by

$$\xi = \frac{dt}{dx} \quad , \quad (3.37)$$

Eq. 3.36 becomes

$$a\xi^2 - 2b\xi + c = 0 \quad , \quad (3.38)$$

which has two different real solutions, ξ_+ and ξ_- , (when $\xi_+ \neq \xi_-$), or

$$\xi_+ = \left(\frac{dt}{dx} \right)_+ \quad , \quad \text{and} \quad \xi_- = \left(\frac{dt}{dx} \right)_- \quad . \quad (3.39)$$

Because a, b, and c are functions of V, y, x, and t, so are the slopes of the characteristic curves or $(dt/dx)_+ = \xi_+(V, y, x, t)$ and $(dt/dx)_- = \xi_-(V, y, x, t)$. The two parts of Eq. 3.39 are two separate ordinary differential equations of the first order. They define two one-parameter families of characteristic curves, usually called just "characteristics", such as C_+ and C_- curves in the (x, t)-plane belonging to the solutions V(x, t) and y(x, t). These two families represent a curvilinear coordinate net and are shown in Fig. 3.5.

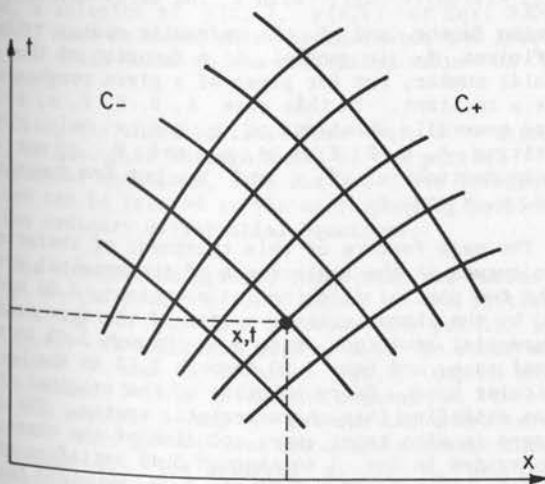


Fig. 3.5. Net of characteristic curves C_+ and C_- in (x, t)-plane.

Derivation of four characteristic ordinary differential equations. The four equations, 3.26, 3.27, 3.31, and 3.32, with four unknowns, $\partial V/\partial x$, $\partial V/\partial t$, $\partial y/\partial x$, and $\partial y/\partial t$, can be written into a single matrix equation:

$$\begin{bmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{bmatrix} \begin{bmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial t} \\ \frac{\partial y}{\partial x} \\ \frac{\partial y}{\partial t} \end{bmatrix} = \begin{bmatrix} -E_1 \\ -E_2 \\ dV \\ dy \end{bmatrix} \quad (3.40)$$

Solving Eq. 3.40, the four derivatives are determined to be

$$\frac{\partial V}{\partial x} = \frac{\Delta_1}{\Delta} \quad , \quad \frac{\partial V}{\partial t} = \frac{\Delta_2}{\Delta} \quad , \quad \frac{\partial y}{\partial x} = \frac{\Delta_3}{\Delta} \quad , \quad \text{and} \quad \frac{\partial y}{\partial t} = \frac{\Delta_4}{\Delta} \quad , \quad (3.41)$$

with

$$\Delta = \begin{vmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{vmatrix} \quad , \quad (3.42)$$

and Δ_1 , Δ_2 , Δ_3 , and Δ_4 are obtained by replacing in Eq. 3.42 the first, second, third, and fourth column, respectively, by the column on the right side of Eq. 3.40.

A unique solution for the derivatives of Eq. 3.41 exists only if the determinant $\Delta \neq 0$. When the directions ξ_+ and ξ_- are such that the determinant Δ is zero there is no unique solution along these directions. It is initially assumed that the first derivatives of V and y in x and t have finite values in the (x, t)-plane. If the determinant Δ vanishes along directions ξ_+ and ξ_- then determinants Δ_1 , Δ_2 , Δ_3 and Δ_4 must also vanish; therefore, $\partial V/\partial x = 0/0$, $\partial V/\partial t = 0/0$, $\partial y/\partial x = 0/0$ and $\partial y/\partial t = 0/0$. Expanding Eq. 3.42 with $\Delta = 0$ it becomes $a(dt)^2 - 2b dx dt + c(dx)^2 = 0$, or Eq. 3.38 if Eq. 3.33 is satisfied. If this latter condition is not satisfied, then $2b = A_1 D_2 - A_2 D_1 + B_1 C_2 - B_2 C_1$. Equation 3.38 then gives

$$\left(\frac{dt}{dx} \right)_+ = \xi_+ = \frac{b + \sqrt{b^2 - ac}}{a} \quad (3.43)$$

and

$$\left(\frac{dt}{dx} \right)_- = \xi_- = \frac{b - \sqrt{b^2 - ac}}{a} \quad (3.44)$$

If Eqs. 3.43 and 3.44 are two real solutions, Eqs. 3.24 and 3.25 are a system of hyperbolic equations; for two complex solutions they are elliptic and for one real solution they are parabolic.

As previously mentioned, if the determinant Δ vanishes, and because the determinants $\Delta_1, \Delta_2, \Delta_3$, and Δ_4 must vanish when Δ vanishes, the case of $\Delta_4 = 0$ with the determinant expanded gives, for example,

$$\begin{aligned} & [(A_1 C_2 - A_2 C_1) \frac{dt}{dx} - (B_1 C_2 - B_2 C_1)] \frac{dy}{dx} + (A_1 B_2 - A_2 B_1) \frac{dV}{dx} + \\ & + (A_1 E_2 - A_2 E_1) \frac{dt}{dx} - (B_1 E_2 - B_2 E_1) = 0 \end{aligned} \quad (3.45)$$

Substituting either $(dt/dx)_+ = \xi_+$ or $(dt/dx)_- = \xi_-$ of Eqs. 3.43 and 3.44, respectively, into Eq. 3.45, two ordinary differential equations in V and y along the characteristic curves ξ_+ and ξ_- are obtained.

Simultaneously solving these two ordinary differential equations, with the two unknowns V and y give the functions $V(x,t)$ and $y(x,t)$. No new relations are obtained when the determinants Δ_1, Δ_2 , and Δ_3 are made equal to zero. Using the coefficients of Eqs. 3.24 and 3.25 as given by Eqs. 3.26 and 3.27, and putting $2b = A_1 D_2 - A_2 D_1 + B_1 C_2 - B_2 C_1$ in Eqs. 3.43 and 3.44, the characteristic directions or slopes of these two latter equations become

$$\left(\frac{dt}{dx} \right)_+ = \frac{2\beta}{V(\alpha+\beta) + \sqrt{(\alpha-\beta)^2 V^2 + \frac{4A\beta g}{B}}} = \xi_+ \quad (3.46)$$

and

$$\left(\frac{dt}{dx} \right)_- = \frac{2\beta}{V(\alpha+\beta) - \sqrt{(\alpha-\beta)^2 V^2 + \frac{4A\beta g}{B}}} = \xi_- \quad (3.47)$$

The two ordinary differential equations for V and y along the characteristics curves ξ_+ and ξ_- are obtained from Eq. 3.47 by replacing dt/dx with ξ_+ and ξ_- , respectively, and introducing the coefficients from Eqs. 3.26 and 3.27. The equation becomes

$$\begin{aligned} & \left[\left(\frac{A}{VB} - \frac{\alpha V}{g} \right) \xi_+ + \frac{\beta}{g} \right] \frac{dy}{dx} + \left(\frac{A\beta}{VBg} \right) \frac{dV}{dx} + \frac{A}{VB} (S_0 - S_f + \\ & + \frac{\beta V q}{A}) \xi_+ - \frac{\beta q}{gVB} = 0 \end{aligned} \quad (3.48)$$

$$\begin{aligned} & \left[\left(\frac{A}{VB} - \frac{\alpha V}{g} \right) \xi_- + \frac{\beta}{g} \right] \frac{dy}{dx} + \left(\frac{A\beta}{VBg} \right) \frac{dV}{dx} + \frac{A}{VB} (S_0 - S_f + \\ & + \frac{\beta V q}{gA}) \xi_- - \frac{\beta q}{gVB} = 0 \end{aligned} \quad (3.49)$$

Equations 3.46 through 3.49 are called the four characteristic ordinary differential equations and are the equivalent set to the two partial differential equations, 3.26 and 3.27.

Assuming that $\alpha = \beta = 1$, and $q = 0$ (no distributed lateral inflow or outflow per unit length), then Eqs. 3.46 through 3.49 become

$$\left(\frac{dt}{dx} \right)_+ = \xi_+ = \frac{1}{V + \sqrt{gA/B}} \quad (3.50)$$

$$\left(\frac{dt}{dx} \right)_- = \xi_- = \frac{1}{V - \sqrt{gA/B}} \quad (3.51)$$

$$\left[\left(\frac{A}{VB} - \frac{V}{g} \right) \xi_+ + \frac{1}{g} \right] \frac{dy}{dx} + \frac{A}{gVB} \frac{dV}{dx} + \frac{A}{VB} (S_0 - S_f) \xi_+ = 0 \quad (3.52)$$

and

$$\left[\left(\frac{A}{VB} - \frac{V}{g} \right) \xi_- + \frac{1}{g} \right] \frac{dy}{dx} + \frac{A}{gVB} \frac{dV}{dx} + \frac{A}{VB} (S_0 - S_f) \xi_- = 0 \quad (3.53)$$

The term $\sqrt{gA/B} = C$ in Eqs. 3.50 through 3.53 is the celerity of a small disturbance in quiescent water with a cross section area A and width B .

Equation 3.26 and 3.27 are hyperbolic. Therefore, they are quasi-linear hyperbolic partial differential equations of gradually varied free-surface unsteady flow with two dependent and two independent variables.

These two sets of four equations, 3.46 through 3.49 and particularly 3.50 through 3.53, will be used predominantly in the numerical integration by the method of characteristics shown in Chapter 3, Part IV, Hydrology Paper No. 46.

General properties. The coefficients of Eqs. 3.26 and 3.27 contain, beside the variables V and y , quantities $A, B, q, \alpha, \beta, S_f$ and S_0 , and a constant, g . The Darcy-Weisbach formula for resistance losses is $S_f = (f/4R)(V^2/2g)$, with f the Darcy-Weisbach friction factor, and R the hydraulic radius. The coefficient f , in general, is a function of the Reynolds number, but for pipes of a given roughness f is a constant. In this case $A, B, R, f, \alpha, \beta$, and q are generally functions of y and x only. The quantities $A, B, R, f, \alpha, \beta, q$, and S_0 do not contain derivatives of y and V , but are functions of V, y, x and t .

The main feature of this treatment of characteristic curves is the replacement of the original system of the two partial differential equations, 3.26 and 3.27, by the characteristic system of the four ordinary differential equations, Eqs. 3.46 through 3.49 in the general case, and Eqs. 3.50 through 3.53 in the particular case. Every solution of the original system satisfies this characteristic system. The converse is also true; every solution of the characteristic system in Eqs. 3.46 through 3.49 satisfies the original system in Eqs. 3.26 and 3.27, provided that the determinant, Δ , of Eq. 3.42 does not vanish.

If the differential equations 3.26 and 3.27 are linear, then ξ_+ and ξ_- are known functions of x and t , so that Eqs. 3.46 and 3.47 are not coupled with Eqs. 3.48 and 3.49. In this case Eqs.

3.46 and 3.47 determine two families of characteristic curves, C_+ and C_- , which are independent of the solution. A linearization of Eqs. 3.26 and 3.27 introduces such a departure from real flow phenomena that this case will not be pursued further.

If $E_1 = E_2 = 0$, and if $A_1, \dots, D_2$ depend on V and y only, which would be a gross approximation of real flow conditions, the situation is similar in that the differential equations are reducible, the slopes ξ_+ and ξ_- are known functions of V and y and Eqs. 3.48 and 3.49 are independent of x and t . The same case occurs when E_1 and E_2 do not vanish but depend on V and y only. This last case is applicable to Eqs. 3.26 and 3.27 under the condition that the conduit is prismatic and the bottom slope, S_0 , is constant, because all coefficients $A_1, \dots, E_2$ may be considered as dependent only on V and y . The characteristic curves in the (V, y) -plane, designated as Γ_+ and Γ_- are the images of the characteristic curves C_+ and C_- in the (x, t) -plane, and are independent of the special solution of $V(x, t)$ and $y(x, t)$. However, these assumptions made to convert Eqs. 3.26 and 3.27 into reducible equations are already an approximation of real free-surface unsteady flow.

Because the objective of this study is to determine the effects of different assumptions or the effects of neglected factors, any restriction in the basic general differential equations would mean a departure from the study's basic approach.

The boundary conditions are of a major importance in any integration using hyperbolic differential equations. A curve, with all values of V and y in the (x, t) -plane must be known. In the most general case, and for a free-surface unsteady flow in conduits, either a velocity hydrograph, $V(t)$, or a discharge hydrograph, $Q(t)$, is known for a given x -value. In the first case, the boundary conditions are given along a constant distance line in the (x, t) -plane for which all values of $Q(t)$ are known. The initial conditions are given along a constant time line in the (x, t) -plane with the corresponding values of V and y known. As soon as the initial line is known, a solution of $V(x, t)$, $y(x, t)$ of Eqs. 3.26 and 3.27 must be found which approximates the initial line and which takes on the prescribed values of V and y .

Using the characteristic form (Eqs. 3.46 through 3.49, or Eqs. 3.50 through 3.53) of the partial differential equations, 3.26 and 3.27, the integration problem can be treated as the corresponding problem for the ordinary differential equations.

The integration process shows that the values V and y at the point $P(x, t)$ depend on the initial values along the line $t = 0$ between the points $(0, x_1)$ and $(0, x_2)$ as indicated in Fig. 3.6. In other words, the values V and y at point $P(x, t)$ depend only on the values V and y in the segment x_1 to x_2 or on the values V and y between the characteristic curve C_- through x_2 and the characteristic curve C_+ through x_1 . The interval x_1 to x_2 and the two characteristic curves, C_+ and C_- , passing through point $P(x, t)$ in Fig. 3.6, determine the domain of dependence. The values V and y at $P(x, t)$ are affected by the values V and y inside the domain of dependence and are not affected by the values outside.

On the other hand, if a point $R(x, 0)$ is selected in the initial line of Fig. 3.6, with the initial values V and y , the characteristic curves C_+ and C_- through point R determine the range of influence. Only the values V and y at the points (x, t) inside the range of influence depend on the initial values of V and y of point R ; the outside points do not.

If the values of the first and higher partial derivatives of V and y are continuous along the initial line, they are also continuous in all the points in the (x, t) -plane. If, however, there are some places of discontinuity, either at the initial line ($\partial V/\partial x$, $\partial V/\partial t$, $\partial y/\partial x$, $\partial y/\partial t$) or higher, the partial derivatives are not continuous. This means that if some disturbances exist, or the discontinuities are introduced at some points in the (x, t) -plane, then the discontinuities in derivatives occur only along characteristics passing through the discontinuity points on the initial line. Another way of expressing it is that the discontinuities in the first or higher partial derivatives of V and y propagate along the characteristic lines in the (x, t) -plane. These discontinuities propagate along one or both of the two characteristics through the point of the source of discontinuity, and they can never disappear. The discontinuities refer to the derivatives of V and y , but not to the discontinuities in V and y themselves, which propagate as surges (bores or depressions).

The characteristic form (Eqs. 3.46 through 3.49, or Eqs. 3.50 through 3.53) of differential equations 3.26 and 3.27 are especially useful for numerical solutions. If the differential equations are replaced by equations of finite differences the numerical solutions can be done easily, especially if a digital computer is used.

The characteristic curves, particularly in their simplified form, are useful in analyzing the properties of the solutions and studying the initial and boundary conditions.

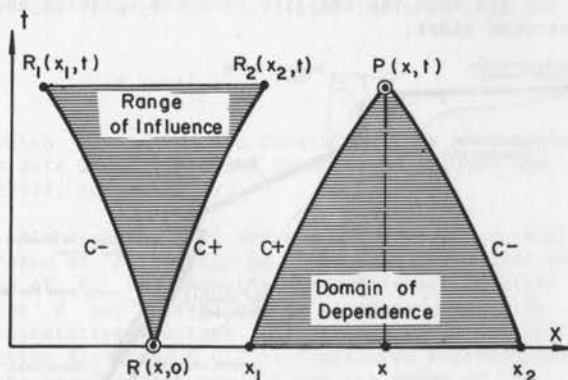


Fig. 3.6. Domain of dependence and range of influence of characteristic curves in the (x, t) -plane.

A detailed description of the experimental facilities used for this study is given in Hydrology Paper No. 44. Therefore, only a brief summary is given here.

4.1 Experimental Facilities

Circular conduit system. A pipeline 822 ft long with a 3 ft outside diameter was used as the experimental conduit. The entire 822 ft of pipe was supported on inclined rails about 20 ft apart, which permitted the pipe to be moved along the inclined plane to any slope between 0 and approximately 4 percent. Flow was introduced into the circular storm conduit by means of an inlet structure that could be adjusted to develop the desired subcritical or supercritical flow conditions at the entrance to the storm conduit and which included a baffle system for developing a uniform entrance condition with minimum downstream surface waves. This type of intake structure was developed from a special reduced model study on a much smaller scale.

Three square junction boxes for lateral inflows were installed along the storm conduit, thus dividing the conduit into four straight sections each about 200 ft long. Flow from a 12-in lateral pipe entered into each junction box at a 90-degree angle, simulating the actual junctions of storm drains. Experiments were conducted on the 822 ft conduit without the junction boxes and with the junction boxes. The outflow of free-surface flowing water was either as a free fall or as a controlled outfall.

Water supply to the 3-ft diameter storm conduit was by gravity through a 26-in underground pipeline from the nearby Horsetooth Reservoir and was conveyed into College Lake after it passed through a special collecting and energy dissipating structure. Figure 4.1 gives the general layout of this experimental facility in the outdoor laboratory of the Engineering Research Center at Colorado State University. Figures 4.2 and 4.3 show the facility from the upstream and downstream sides.

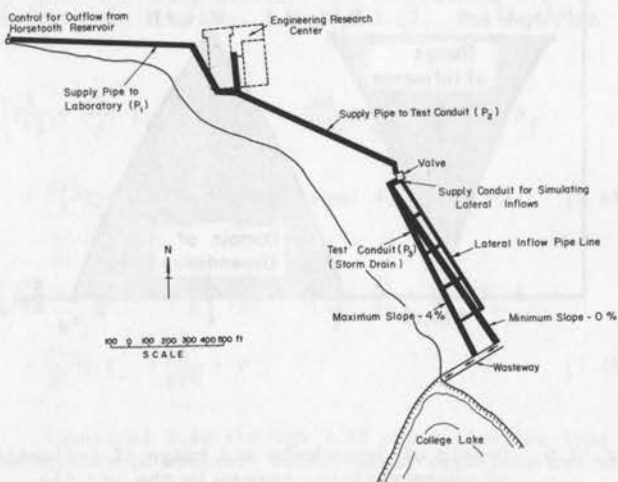


Fig. 4.1. General scheme of the experimental conduit with water supply and removal.

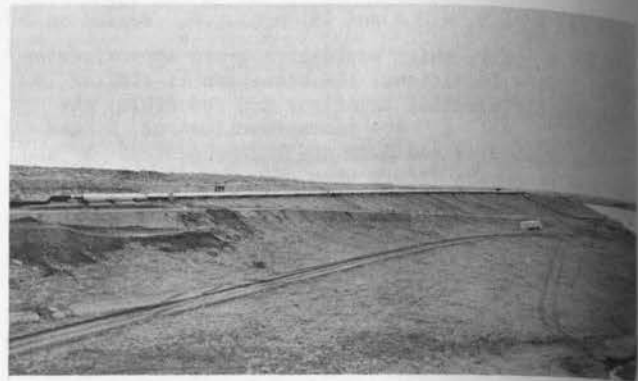


Fig. 4.2. A view from upstream of the circular storm conduit on the hillside of the outdoor laboratory at Colorado State University Engineering Research Center.

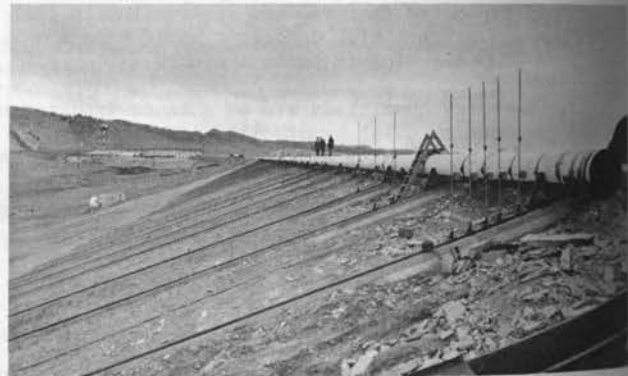


Fig. 4.3. A view from downstream of circular storm conduit and the inclined rails.

Instrumentation. Three sizes of sharp-edged circular orifice plates, based on the different orifice-to-pipe diameter ratios, were designed, calibrated, and used to measure the main inflow rate into the storm conduit. The three sizes were needed to accurately measure small, medium and large discharges through the conduit. The ratios of the diameter of orifice (d) to the diameter of pipe (D) were 0.35, 0.50, and 0.70, with area ratios of approximately $1/8$, $1/4$ and $1/2$, respectively. The calibration equations, $Q = C_d A \sqrt{2gH}$ obtained for the three orifice meters under steady flow conditions are: for $m = d/D = 0.35$, $Q_1 = 2.102 \sqrt{H}$, with $C_d = 0.606$; for $m = d/D = 0.50$, $Q_2 = 4.439 \sqrt{H}$, with $C_d = 0.627$; and for $m = d/D = 0.70$, $Q_3 = 9.783 \sqrt{H}$ with $C_d = 0.705$. In these equations C_d is the discharge coefficient, H is the differential head across the orifice in feet, A is the cross section area of the conduit in square feet, and Q is the discharge in cubic feet per second. Results indicate that each of the three discharge coefficients, C_d is constant for the Reynolds number within the range 2×10^5 to

2x10⁶. Considerations were also given to the problem of orifice calibration equations under unsteady flow conditions, but no significant deviation was found. (For more details see Hydrology Paper No. 44, Chapter 3, Pages 19-20).

Small propeller current meters with electric counters, manufactured by Ott, were selected to measure the flow velocity distributions in the storm conduit. The current meters were first carefully recalibrated by making several runs by a tow car at a constant speed over a given distance, and then by operating the tow cars at different speed covering their operational range. Good agreement was obtained between the recalibration values and those furnished by the manufacturer. (For more details see Hydrology Paper No. 44, Chapter 3, Pages 20-26).

Pressure transducers were used to measure water levels and pressure differences in various measurements. The calibration of pressure transducers was considered from these standpoints: (a) that the output voltage be zero for zero pressure input; (b) that the output voltage be a linear function of the impressed pressure differential; (c) that the output voltage did not change with time and if it does, it must be systematically checked, and (d) that the proportionality constant between the input and output be known before the observed data is interpreted. These conditions were checked for all transducers used, and with sufficient care in installation and adequate warm-up time, all transducers met the necessary conditions. (For further details see Hydrology Paper No. 44, Chapter 3, pages 26-27).

Data recording system. The outdoor circular storm conduit is located about 1,500-ft from the Colorado State University Engineering Research buildings. Data taken from the storm conduit were transformed into electric voltage signals by the pressure transducers and were then transmitted by individual shielded cables to an indoor analog-to-digital converter. The digitized information was punched onto cards or paper tape, for later computer analysis. Figure 4.4 shows the general scheme of the data recording system. (For more details see Hydrology Paper No. 44, Chapter 4, pages 28-29).

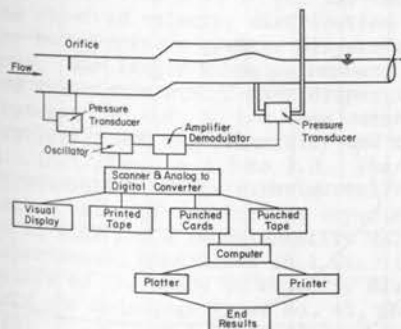


Fig. 4.4. Scheme of the general data recording system.

4.2 Results of Steady Flow Measurements

Hydraulic resistance. An experimental approach to the determination of conduit boundary roughness was investigated. The Darcy-Weisbach friction factor (f) and the corresponding Reynolds number (R_e) were computed from measured depths and discharges. Figure

4.5 represents the results in a range of depths from 0.56 to 2.6 feet, or depth-to-diameter ratios of 0.19 to 0.89. The discharges varied from 2.25 to 72.0 cfs. The corresponding Reynolds number range is from approximately 3×10^4 to 1×10^6 . The Prantl-von Karman equation

$$\frac{1}{\sqrt{f}} = a \log_{10} (R_e \sqrt{f}) + b, \quad (4.1)$$

with $a = 2.0$ and $b = 0.4$ for smooth boundary flow is also plotted in Fig. 4.5. The two points in Fig. 4.4 that extremely vary over the other points and the curve of the Prantl-von Karman equation are assumed as mistakes either in observation or in the processing of data. For data shown in Fig. 4.5, the values of the constants a and b in Eq. 4.1 are 2.075 and 0.1434, respectively, or the boundary of the conduit used in this experimental study is hydraulically smooth.

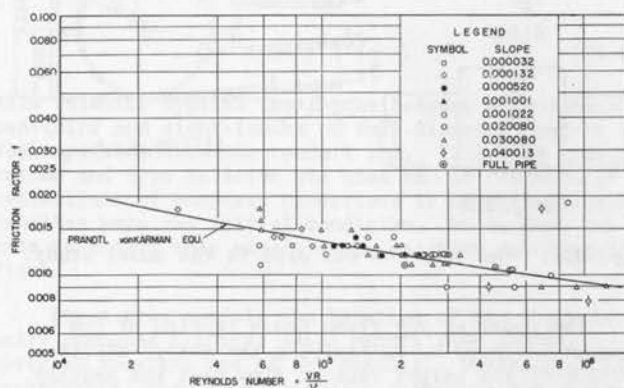


Fig. 4.5. Relationship of the Darcy-Weisbach friction factor f to Reynolds number R_e .

The evaluation of f from Eq. 4.1 for a given R_e requires an iterative procedure. To simplify computations (within practical limits of Reynolds numbers for specific conditions) it is convenient to use a simplified form of the f to R_e relation. A practical form of the expression for this relation is

$$f = c(R_e)^d, \quad (4.2)$$

in which c and d are constants; they were evaluated from data in Fig. 4.5 and found to be 0.10939 and -0.17944, respectively.

Although Fig. 4.5 shows a decrease of f with an increase of R_e it can be shown that for a limited range of R_e the changing Darcy-Weisbach friction factor f may be replaced by an average value or a representative constant. An average Darcy-Weisbach friction factor of 0.012 is considered representative of the experimental conduit for the range of R_e obtained in this study. (For details of this derivation see Hydrology Paper No. 44, Chapter 5, pages 30-31, and Hydrology Paper No. 45, Chapter 3, pages 10-15).

Junction box losses. The evaluation of energy losses accompanying lateral inflow into a main conduit with free-surface flow was accomplished through two separate experimental facilities. A plastic pipe

with a 6-in diameter was used to economically develop the basic evaluations of junction box losses. The results were then verified by the tests in the 3-ft diameter conduit. Agreement between the two systems was good, based on the Froude similitude relation.

The junction box used in the large model study was square above the half-full level of the storm conduit. Two positions of the inlet were tested, the upper and lower. The crown of the upper inlet was the same elevation as the crown of the storm conduit at its point of entry into the junction box. The invert of the lower inlet was made coincident with the invert of the storm conduit at their junction point, which was the centerline of the junction box. The arrangement of the model junction box is shown in Fig. 4.6.

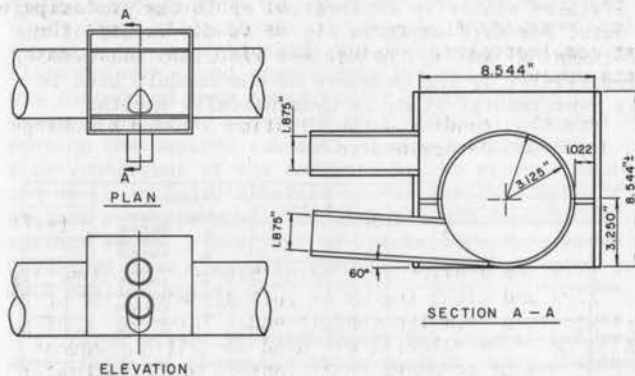


Fig. 4.6. The junction box used in the model study.

The power at any given cross section of the conduit is found by multiplying the energy at its section by the weight rate of flow past the section. The power equation can be written then as

$$P = \gamma Q E \quad (4.3)$$

or

$$P = \gamma Q \left(z + y + \frac{V^2}{2g} \right) \quad (4.4)$$

in which P is the power in foot-pounds per second, Q is the discharge in cubic feet per second, γ is the specific weight of water in pounds per cubic foot, z is the elevation of the channel bottom above the zero datum, y is depth of flow, and V is the mean velocity.

It was assumed that the power loss ratio, P_r , could be expressed as a function of the relative discharge, Q_r , of the lateral to the main conduit flow when using the upper inlet. The functional relation between the power loss ratio and the discharge ratio was found to be

$$(P_r - 0.77)(Q_r + 0.55) = -0.482 \quad (4.5)$$

for the upper inlet.

Power losses for the lower inlet were analyzed in terms of the ratio of lateral inflow discharge to the main conduit discharge, Q_r , and the ratio of the upstream depth to the conduit diameter, D_r . The power loss ratio from this procedure is

$$P_r = \frac{-2.78 + 1.71 D_r}{Q_r + 3.122 - 0.167 D_r} + 0.77, \quad (4.6)$$

for the lower inlet. (For additional details of the study of junction box head losses see Hydrology Paper No. 44, Chapter 2, Pages 6-13, and Hydrology Paper No. 45, Chapter 4, Pages 16-20).

Cross-sectional velocity distributions. To determine experimentally the velocity distribution coefficients, α and β , measurements of the distribution of conduit cross-sectional velocities were conducted. Time-average point velocities were measured by the Ott current meters. A special procedure was developed for measuring velocities and for computing the velocity distribution coefficients, as described in Hydrology Paper No. 45, Chapter 5.

It was expected that the velocity distribution coefficients would differ with changes in those parameters that determine the velocity profiles. The parameters that primarily affect the velocity profile are the geometry of the cross section, the properties of the fluid, the condition of the boundary surface (roughness), and the mean velocity. All of these variables are encompassed in the Reynolds number (VR/ν) and the Darcy-Weisbach friction factor (f).

Because the Darcy-Weisbach friction factor is related to the Reynolds number, one would expect to be able to predict α and β coefficients to the friction factor f . Since the range of the Darcy-Weisbach friction factor is small for the series of data in this study, and because the Reynolds number fluctuates within a limited range, the spread of computed α and β coefficients is apparently due to other causes.

Figure 4.7 displays the relation of α and β with the Reynolds number. These results indicate an increase of the velocity distribution coefficients with a decrease of Reynolds number. The apparent scatter of points around a smooth curve may be assumed to be a result of observational and computational errors.

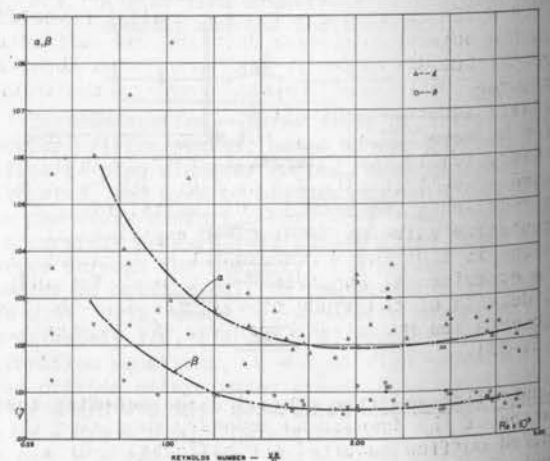


Fig. 4.7. Velocity distribution coefficients as related to the Reynolds number.

The effect of depth on the velocity distribution coefficients is presented in Fig. 4.8. This figure

indicates slightly increasing values of both β and α as depth decreases. This is expected because deviation from the mean velocity becomes greater and the friction factor becomes effectively larger as the depth decreases. At a depth of half the conduit diameter the β - coefficient has a value of approximately 1.01 and the α - coefficient has a value of approximately 1.03. At greater depths, β reduces to approximately 1.007 and α to approximately 1.022. For depths less than half the conduit diameter, both coefficients tend to increase. Data were not available for depths less than one-fourth of a diameter.

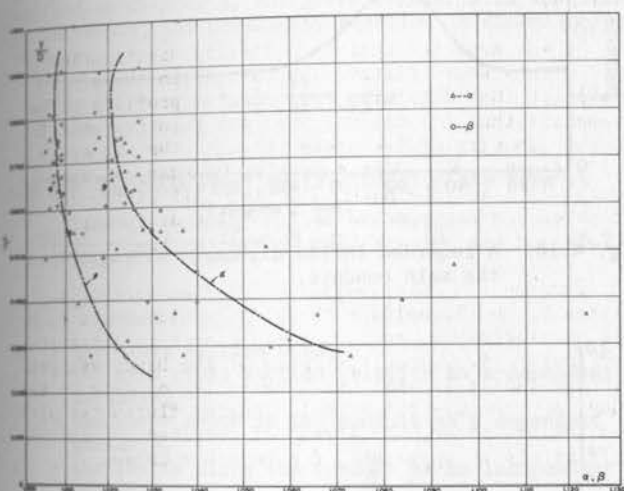


Fig. 4.8. Velocity distribution coefficients as related to the depth of flow in a circular cross section.

The results of this study are applicable to hydraulically smooth circular cross sections flowing partially full and having Reynolds numbers in the range 0.4×10^5 to 3.00×10^5 . For smaller Reynolds numbers the observed velocity distribution coefficients are greater but display a greater dispersion about a smooth curve. For larger Reynolds numbers the values tend toward invariance with lesser dispersion. The relation between α and β has been demonstrated both theoretically and experimentally and can be expressed: $(\alpha-1)/(\beta-1) = 2.3$ to 3.0 . The value of 3.0 is representative of the experimental results. A representative value of α for the experimental conditions is 1.03 and a representative value of β for the experimental conditions is 1.01. (For additional details of the study of velocity distribution coefficients see Hydrology Paper No. 45, Chapter 5, pages 21-29).

Controlled and free outfall. The mathematical simulation of the downstream boundary condition for the controlled outflow required the calibration of a terminal (end) restriction. Any geometric configuration was acceptable if it satisfied certain criteria:

1. The discharge as a function of depth could be expressed simply, such as $Q = my^n$ in which m and n are constants experimentally determined and y is the depth of flow upstream of the restrictions.
2. The restriction was not so great as to cause

the pipe to flow full under the maximum anticipated hydrograph discharge.

3. The approach-velocity distribution was symmetrical and did not differ appreciably from the undisturbed flow.

These criteria were satisfied by a constriction consisting of five 7-inch vertical wooden slats held in position by two 1/2-inch wide vertical aluminum H-sections. The clear opening was five inches between supports. The discharge could thus be controlled by varying the vertical position or by removing one or more slats.

Calibration of various combinations of openings was made by measuring the corresponding discharge and the water surface elevation approximately 20 ft upstream of the control. For the range of discharges anticipated in the unsteady flow runs, it was concluded that the best combination of openings was with the three center slats removed.

For this condition the relation between discharge and depth was determined to be

$$Q = 4.84 y^{1.35} \quad (4.7)$$

This relation applies for depths between approximately one-third and eight-tenths of full diameter, and at 20 feet upstream from the conduit end. This gate configuration and flow relation was used for all subsequent evaluations of boundary conditions in which backwater profiles were the initial condition. No attempt was made to modify this steady state relation for unsteady flows.

For a free outfall the location of the theoretically computed critical depth occurs some distance upstream from the end of the conduit. The purpose of experimental measurements was to determine the location of the critical depth. This position then served as the location of the downstream boundary. Water-surface profiles were measured for a range of discharges from 2.10 to 16.62 cfs. The channel slope ranged from 0.000032 to 0.001022 foot per foot.

Figure 4.9 presents the water-surface profiles and the locations of the computed critical depth within the range of observed end depths; the mean ratio of end-depth to critical-depth was 0.750. This ratio tended to be smaller than the mean for the lower depths.

The location of the computed critical depth from the channel end varied from less than 3.5 times the critical depth to almost 5.5 times the critical depth. A location of 4.5 times the critical depth was considered typical and was used in subsequent computations. (For details of free or controlled outflow boundary conditions see Hydrology Paper No. 44, Chapter 5, pages 30-32, and Hydrology Paper No. 45, Chapter 6, pages 30-32).

Flow regimes. The steady non-uniform flow in subcritical and supercritical regimes were established experimentally in the storm conduit. The steady non-uniform flows (backwater curves) at the hydrograph base discharge were used as initial conditions in computing the unsteady flow equations. (For details on how the initial non-uniform steady conditions were computed see Hydrology Paper No. 45, Chapter 7, pages 33-36).

The discharge and slope corresponding to the desired depth of flow in subcritical or supercritical

regimes were estimated from observations. The downstream control gate was adjusted to produce the desired type of backwater or drawdown curve. Because of the time required for steady state conditions to develop, it was not practical to adjust the downstream control until a constant depth developed throughout the length of the pipe. Thus, several conditions of non-uniform flow were established both above and below normal depth. Hook gage readings at the various piezometer locations were made at approximately 15-minute intervals until such time as the readings indicated stabilization.

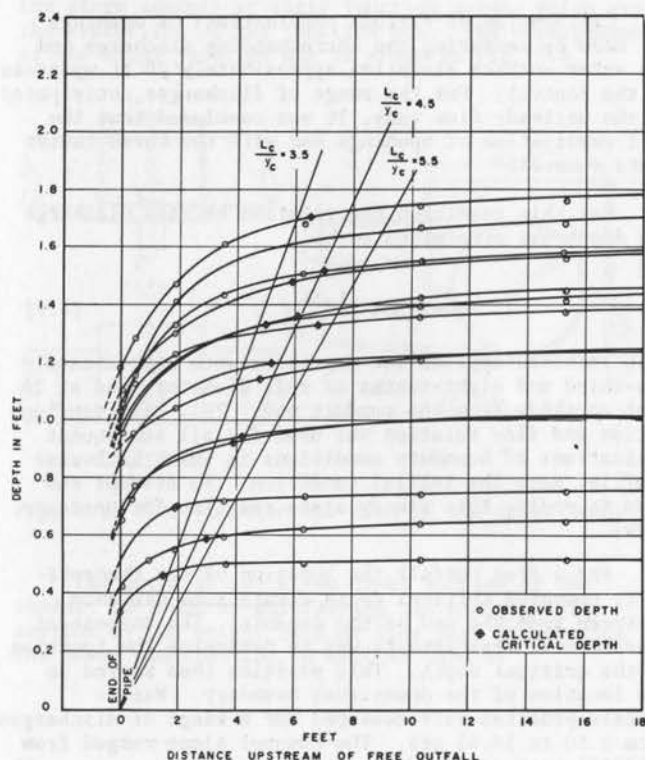


Fig. 4.9. Location of the critical depth of a circular cross section free outfall.

Water surface elevations were determined by means of hook gage readings taken in gage wells located at 16 positions along the pipe. These wells were connected to the invert of the pipe through a flexible hose. The piezometer openings were 1/16 in in diameter. At each position there were a sufficient number of openings to insure a reasonable response time for each well.

The invert slope of the pipe was carefully determined by a precise self-leveling level with an optical micrometer which permitted measurements of the invert to the nearest 1/1000 of an inch. Readings were taken every 20 ft and a least-square determination of the mean slope was computed. If the maximum deviations at any point exceeded approximately 3/100 of a foot, from the mean line, adjustments to the pipe invert were made.

4.3 Physical Waves

Inflow hydrographs. Inflow hydrographs were developed by a motor operated 26-in diameter ball-

valve at the upstream inlet of the main storm conduit and the 12-in diameter gate-valves on the lateral inflow pipelines. The initial and maximum discharges were established for a given experiment, and the valve operated at constant speed within this range. The lateral inflow discharges varied as the openings of the main valve. The discharge hydrographs were measured and recorded by pressure transducers connected across the orifices which were installed a short distance downstream of the valves. Figures 4.10 and 4.11 show the experimentally observed inflow discharge hydrographs of the main storm conduit and the lateral flow, respectively.

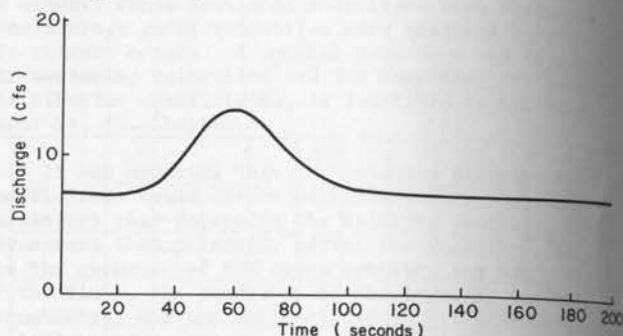


Fig. 4.10. A recorded inflow discharge hydrograph to the main conduit.

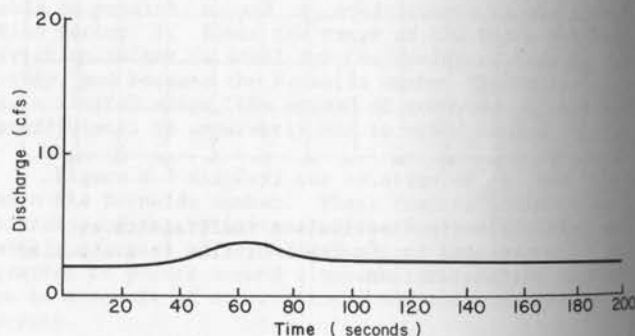


Fig. 4.11. A recorded inflow discharge hydrograph from a lateral conduit.

Wave propagation. After the generation of inflow hydrographs, the subsequent wave depths were measured at several stations downstream as the flood-wave propagated along the conduit. Waves were measured at distances of 50.00 ft, 410.00 ft, and 771.70 ft from the entrance during experiments conducted the summer of 1965 and at distances of 50.00 ft, 251.24 ft, 387.70 ft, 462.56 ft, 669.83 ft, and 771.70 ft during experiments conducted the summer of 1966. Flood wave depths were measured by pressure transducers connected between the conduit invert and a set of manometers.

The measured quantities were plotted as (1) depth versus time relations, (2) depth versus distance relations, and (3) wave peak depth versus distance and time relations. These experimentally measured physical waves were used to compare and check the analytically computed mathematical waves. Complete results of measured inflow hydrographs and physical waves are presented in Appendices 3, 4, and 5. Analysis of the comparison between physical waves and computed waves is given in Chapter 6 of this paper.

COMPUTATION OF WAVES BY ANALYTICAL METHODS

This chapter describes briefly a few finite-difference schemes used for the numerical solution of the two quasi-linear hyperbolic partial differential equations of gradually varied free-surface unsteady flow. A description of the subject is presented in more detail in Hydrology Paper No. 46.

5.1 Basic Equations

The two basic partial differential equations of gradually varied free-surface unsteady flow are derived in Chapter 3, and given as Eqs. 3.23 and 3.19. To reiterate, the continuity equation in dimensionless form is

$$\frac{A}{VB} \frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} + \frac{1}{V} \frac{\partial y}{\partial t} = \frac{q}{VB}, \quad (5.1)$$

and the momentum equation in dimensionless form is

$$\frac{\alpha V}{g} \frac{\partial V}{\partial x} + \frac{\beta}{g} \frac{\partial V}{\partial t} + \frac{\partial y}{\partial x} = (S_0 - S_f) - \frac{Vq}{Ag}, \quad (5.2)$$

in which

- A = the cross section area,
- B = the mean cross section velocity as a dependent variable,
- y = the water depth in the conduit as a dependent variable,
- x = the length along the conduit as an independent variable,
- t = the time as an independent variable,
- β = the free surface width,
- α = the energy velocity distribution coefficient,
- β = the momentum velocity distribution coefficient,
- g = the gravitational acceleration,
- S_0 = the slope of the channel invert,
- S_f = the energy gradient, and
- q = the distributed lateral inflow (or outflow) as discharge per unit length of the conduit.

The energy gradient, measuring the energy head loss along the conduit, is expressed by the Darcy-Weisbach equation in the form

$$S_f = \frac{fV^2}{8gR}, \quad (5.3)$$

in which f is the Darcy-Weisbach friction factor, R is the hydraulic radius of a partially full conduit, with $R = A/P$, and P is the wetted perimeter.

The friction factor is expressed in this study for the hydraulically smooth boundary as a function of Reynolds number, $Re = VR/\nu$, with ν the kinematic viscosity of the water.

Equations 5.1 and 5.2 generally give the closest approximations of the actual flood movement through channels and conduits for the one-dimensional unsteady flow, if the basic conditions for applying the two equations are approximately satisfied. The most important condition is that of gradual variability of the flood hydrograph; this condition is nearly always

fulfilled for storm floods entering into and moving along storm drains.

5.2 Methods of Solution

Because of the mathematical difficulties of obtaining the analytical solution in a closed form to Eqs. 5.1 and 5.2, numerical finite-difference methods of integration must be employed. For solving Eqs. 5.1 and 5.2, finite-difference approximations are made as follows. Since there are two independent variables (x,t) and two dependent variables (V,y), the designation of the time-distance locations of the variables is based on the subscripts and superscripts of the variables. The subscript refers to the distance (space) location, and the superscript refers to the time location, as shown in Fig. 5.1.

Time t

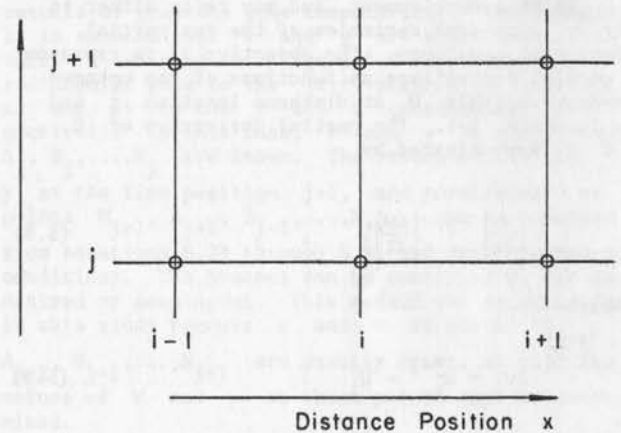


Fig. 5.1. Definition graph for the finite-difference scheme.

Thus, the depth at distance location i and at time j is y_i^j , and the four partial derivatives of Eqs. 5.1 and 5.2 may be approximated by

$$\frac{\partial V}{\partial x} \approx \frac{V_{i+1}^j - V_i^j}{x_{i+1}^j - x_i^j}, \quad (5.4)$$

$$\frac{\partial V}{\partial t} \approx \frac{V_i^{j+1} - V_i^j}{t_i^{j+1} - t_i^j}, \quad (5.5)$$

$$\frac{\partial y}{\partial x} \approx \frac{y_{i+1}^j - y_i^j}{x_{i+1}^j - x_i^j}, \quad (5.6)$$

and

$$\frac{\partial y}{\partial t} \approx \frac{y_i^{j+1} - y_i^j}{t_i^{j+1} - t_i^j}. \quad (5.7)$$

The unknown quantities in these expressions are generally the values at the incremental time locations $j+1$. Thus v_i^{j+1} and y_i^{j+1} are the unknown values for the time $j+1$, and the distance i . With the two equations of unsteady flow, these two unknowns may be solved for simultaneously. This is an explicit scheme procedure in that the conditions at a later time, $j+1$, are determined directly from the conditions at the preceding time, j .

Two approaches of the explicit schemes, the diffusing scheme and the Lax-Wendroff scheme as applied to the two partial differential equations, were used and as discussed in this study. A third approach used in this study, is the specified intervals scheme as applied to the four ordinary characteristic differential equations, as equivalent to the two partial differential equations. This latter approach will be called the specified intervals scheme of the method of characteristics.

Diffusing scheme. The diffusing scheme evolves from an approximation regarding the partial derivatives with respect to time. It is assumed the dependent variables are known for all positions at time j . These dependent variables will be designated as U in this development, and may refer either to V or y dependent variables of the two partial differential equations. The objective is to represent the partial derivatives as functions of the unknown dependent variable U at distance location i and time location $j+1$. The partial derivative of U in t is approximated by

$$\left(\frac{\partial U}{\partial t}\right)_i \approx \left(\frac{\Delta U}{\Delta t}\right)_i, \quad (5.8)$$

in which

$$\Delta U_i = U_i^{j+1} - U_i^j. \quad (5.9)$$

Expressing U_i^j as an average,

$$U_i^j \approx \frac{U_{i+1}^j + U_{i-1}^j}{2}, \quad (5.10)$$

then

$$\Delta U_i = U_i^{j+1} - \frac{U_{i+1}^j + U_{i-1}^j}{2}, \quad (5.11)$$

and finally the ratio of finite differences of this partial derivative is

$$\begin{aligned} \left(\frac{\Delta U}{\Delta t}\right)_i &= \frac{U_i^{j+1} - \frac{U_{i+1}^j + U_{i-1}^j}{2}}{\Delta t} \\ &= \frac{2U_i^{j+1} - U_{i+1}^j - U_{i-1}^j}{2\Delta t}. \end{aligned} \quad (5.12)$$

Similarly, the partial derivative with respect to the distance x is approximated by

$$\left(\frac{\partial U}{\partial x}\right)_i \approx \left(\frac{\Delta U}{\Delta x}\right)_i, \quad (5.13)$$

in which

$$\left(\frac{\Delta U}{\Delta x}\right)_i = \frac{1}{2} \left[\frac{U_{i+1}^j - U_i^j}{\Delta x} + \frac{U_i^j - U_{i-1}^j}{\Delta x} \right], \quad (5.14)$$

so that

$$\left(\frac{\Delta U}{\Delta x}\right)_i = \frac{1}{2\Delta x} (U_{i+1}^j - U_{i-1}^j). \quad (5.15)$$

It is to be noted that Eq. 5.12 is an approximation for $\partial U/\partial t$ at the position $i, j+1/2$; and Eq. 5.15 is an approximation for the position i, j .

Lax-Wendroff scheme. To eliminate some of the deficiencies of the diffusing scheme, the Lax-Wendroff finite-difference scheme was investigated. The summary of the scheme is as follows. It is assumed that all functions are continuous and contain as many continuous derivatives as required. It is also assumed that products of first-order partial derivatives and any derivative of S_f in x and t are negligible quantities.

$$\text{The expressions } \frac{\partial A}{\partial t} = B \frac{\partial y}{\partial t} \text{ and } \frac{\partial A}{\partial x} = B \frac{\partial y}{\partial x}$$

relate A , B , and y . Therefore, the equation of continuity reduces to

$$\frac{\partial y}{\partial t} = -\frac{A}{B} \frac{\partial V}{\partial x} - V \frac{\partial y}{\partial x}. \quad (5.16)$$

The intended application of the Taylor series, requires the use of second-order partial derivatives. Thus,

$$\frac{\partial^2 y}{\partial t^2} = -\frac{A}{B} \frac{\partial^2 V}{\partial x \partial t} - V \frac{\partial^2 y}{\partial x \partial t}, \quad (5.17)$$

and

$$\frac{\partial^2 y}{\partial x \partial t} = -\frac{A}{B} \frac{\partial^2 V}{\partial x^2} - V \frac{\partial^2 y}{\partial x^2}. \quad (5.18)$$

The momentum equation, 5.2, rewritten here in the form

$$\frac{\partial V}{\partial t} = -\frac{\alpha}{\beta} V \frac{\partial V}{\partial x} - \frac{g}{\beta} \frac{\partial y}{\partial x} - \frac{g}{\beta} (S_f - S_0) \quad (5.19)$$

gives

$$\frac{\partial^2 V}{\partial x \partial t} = -\frac{\alpha}{\beta} V \frac{\partial^2 V}{\partial x^2} - \frac{g}{\beta} \frac{\partial^2 y}{\partial x^2}. \quad (5.20)$$

Hence, Eq. 5.17 becomes

$$\frac{\partial^2 y}{\partial t^2} = \left(\frac{\alpha}{\beta} + 1\right) \frac{AV}{B} \frac{\partial^2 V}{\partial x^2} + \left(\frac{gA}{\beta B} + V^2\right) \frac{\partial^2 y}{\partial x^2}. \quad (5.21)$$

Equation 5.19 then gives

$$\frac{\partial^2 V}{\partial t^2} = -\frac{\alpha}{\beta} V \frac{\partial^2 V}{\partial x \partial t} - \frac{g}{\beta} \frac{\partial^2 y}{\partial x \partial t} \quad (5.22)$$

Substituting Eqs. 5.18 and 5.20 into Eq. 5.22 yields

$$\frac{\partial^2 V}{\partial t^2} = \left[\left(\frac{\alpha}{\beta} \right)^2 V^2 + \frac{g}{\beta} \frac{A}{B} \right] \frac{\partial^2 V}{\partial x^2} + \left(\frac{\alpha}{\beta} + 1 \right) \frac{g}{\beta} V \frac{\partial^2 y}{\partial x^2} \quad (5.23)$$

Again designating U as the symbol for any dependent variable V or y , the first and second partial derivatives with respect to x are approximated by

$$\frac{\partial U}{\partial x} = \frac{U_{i+1}^j - U_{i-1}^j}{2\Delta x} \quad (5.24)$$

and

$$\frac{\partial^2 U}{\partial x^2} = \frac{U_{i+1}^j - 2U_i^j + U_{i-1}^j}{(\Delta x)^2} \quad (5.25)$$

Similar equations are obtained with respect to t . Thus, recurrence relations for finding approximate solutions to V and y in Eqs. 5.21 and 5.23 are

$$\begin{aligned} y_i^{j+1} &= y_i^j - \frac{\Delta t}{2\Delta x} \left[\left(\frac{A}{B} \right)_i^j (V_{i+1}^j - V_{i-1}^j) + (V_i^j y_{i+1}^j - y_{i-1}^j) \right] \\ &+ \frac{1}{2} \left(\frac{\Delta t}{\Delta x} \right)^2 \left\{ \left(\frac{\alpha}{\beta} \right)_i^j + 1 \right\} \left(\frac{A}{B} \right)_i^j (V_i^j V_{i+1}^j - 2V_i^j + V_{i-1}^j) \\ &+ \left[\frac{g}{\beta} \left(\frac{A}{B} \right)_i^j + (V_i^j)^2 \right] (y_{i+1}^j - 2y_i^j + y_{i-1}^j) \end{aligned} \quad (5.26)$$

and

$$\begin{aligned} V_i^{j+1} &= V_i^j - \frac{\Delta t}{2\beta\Delta x} \left[\alpha V_i^j (V_{i+1}^j - V_{i-1}^j) + g(y_{i+1}^j - y_{i-1}^j) + 2g\Delta x (S_f - S_o) \right] \\ &+ \frac{1}{2} \left(\frac{\Delta t}{\Delta x} \right)^2 \left[\left(\frac{\alpha}{\beta} \right)_i^2 (V_i^j)^2 + \frac{g}{\beta} \left(\frac{A}{B} \right)_i^j \right] (V_{i+1}^j - 2V_i^j + V_{i-1}^j) \\ &+ \left(\frac{\alpha}{\beta} + 1 \right) \frac{g}{\beta} V_i^j (y_{i+1}^j - 2y_i^j + y_{i-1}^j) \end{aligned} \quad (5.27)$$

The previous equations are based on the neglect of both the products of the first-order partial derivatives and the derivatives of S_f . For those cases in which these products of the first-order partial derivatives and the derivatives of S_f cannot be disregarded, difference equations analogous to Eqs. 5.26 and 5.27 may be derived.

Specified intervals scheme of the method of characteristics. The two partial differential equations of gradually varied free-surface unsteady flow, Eqs. 5.1 and 5.2, when transformed give the four ordinary characteristics differential equations. Their development is shown in Chapter 3. The equations with $\alpha = \beta = 1$, and $q = 0$ (Eqs. 3.50 to 3.53), are the initial equations. To reiterate, they are

$$\xi_+ = \left(\frac{dt}{dx} \right)_+ = \frac{1}{V + \sqrt{gA/B}} \quad (5.28)$$

$$\xi_- = \left(\frac{dt}{dx} \right)_- = \frac{1}{V - \sqrt{gA/B}} \quad (5.29)$$

$$\left\{ \left(\frac{A}{VB} - \frac{V}{g} \right) \xi_+ + \frac{1}{g} \right\} \frac{dy}{dx} + \frac{A}{gVB} \frac{dV}{dx} + \frac{A}{VB} (S_o - S_f) \xi_+ = 0, \quad (5.30)$$

and

$$\left\{ \left(\frac{A}{VB} - \frac{V}{g} \right) \xi_- + \frac{1}{g} \right\} \frac{dy}{dx} + \frac{A}{gVB} \frac{dV}{dx} + \frac{A}{VB} (S_o - S_f) \xi_- = 0, \quad (5.31)$$

with the symbols the same as those defined for Eqs. 5.1 and 5.2. These four dependent equations form the basis for numerical solutions in the method of characteristics, in this case by the specified intervals numerical integration scheme.

There are several procedures that may be used and these procedures may be broadly divided into two categories, the grid system and the specified intervals system. In the second category, which is used in this study, the dependent variables V and y are known functions of the independent variables x and t either as initial conditions at $t = 0$ or as the results of previous time computations. For example, it is assumed that V and y are known along distance x at time t . Figure 5.2 represents the rectangular grid in the (x, t) -plane with intervals Δx and Δt in the x and t coordinates, respectively. In this case, V and y at points $M_j, A_j, B_j, \dots, N_j$ are known. The values of V and y at the time position $j+1$, and particularly at points $M_{j+1}, A_{j+1}, B_{j+1}, \dots, N_{j+1}$, can be computed from equations 5.28 through 5.31 and from the boundary conditions. The process can be continued as far as desired or meaningful. This method was selected for in this study because x and t at points $M_{j+1}, A_{j+1}, B_{j+1}, \dots, N_{j+1}$ are exactly known, so only the values of V and y at these points must be determined.

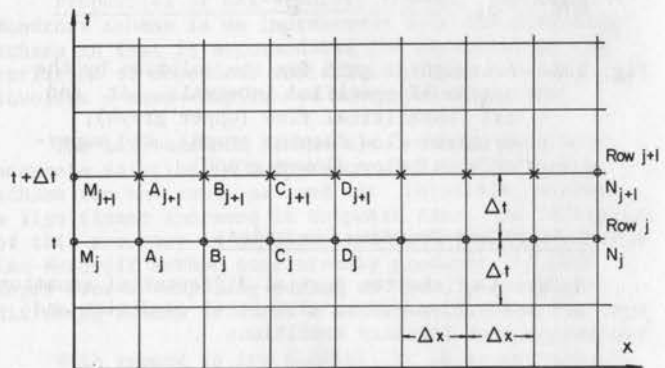


Fig. 5.2. Network of specified intervals for the solution of characteristic equations.

In this method, V and y at point P on the (x, t) -plane of Fig. 5.3 are to be computed from the initial conditions or from previous values of V and y at points A , B , and C using two assumptions:

(a) Δt is sufficiently small so that the parts of the characteristics between P and R and between P and S are considered straight lines, and points R and S fall within the interval AB , and

(b) the slope of the straight line PR at point P is equal to the positive characteristic direction at the position C, (ξ_+) , and the slope of the straight line PS at point P is equal to the negative characteristic direction at the position C, (ξ_-) .

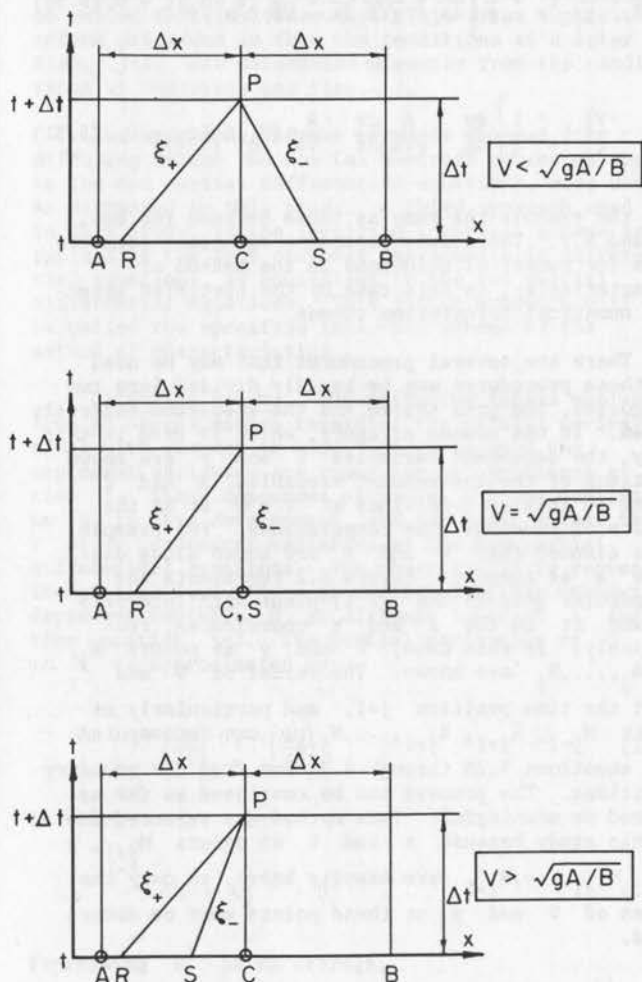


Fig. 5.3. Rectangular grid for the solution by the system of specified intervals, Δt and Δx : subcritical flow (upper graph), critical flow (center graph), and supercritical flow (lower graph).

5.3 Initial and Boundary Conditions

Solution of the two partial differential equations Eqs. 5.1 and 5.2, requires an initial condition and two independent boundary conditions.

Initial conditions. The necessary initial conditions for unsteady free-surface flow are that all velocities and depths of water along the channel must be known at a given time. In this study, it was assumed that the discharge was constant throughout the reach at the initial time. Thus, the problem can be treated as steady non-uniform flow. Velocities and depths along the channel were then determined by computations of conventional backwater or drawdown surface profiles, depending on the downstream control conditions. This procedure used the standard step method.

Boundary conditions. One of the two boundary conditions is the discharge-time relation existing at the inlet cross section of the channel under study. This relation can be either expressed in a mathematical form, or given as discrete points of discharge at selected intervals of time.

The other boundary condition imposed on the problem is that of a discharge-versus-depth relation at the downstream end, characterized either by a control structure or by the critical depth at a free outfall. This is the boundary condition that must exist for subcritical flow of the base discharge.

If the base discharge is in the supercritical range or on a supercritical slope the boundary condition must be expressed at the inlet end. This function may take the form of a discharge-versus-depth relation. This condition, in combination with the condition of a discharge-versus-time relation is somewhat difficult to visualize physically; however, it is a necessary condition because the characteristic directions both have a positive slope and thus there is no influence of the downstream conditions on the upstream conditions.

The following discussion presents a detailed analysis of these boundary conditions. Arbitrary inflow hydrographs were investigated to test and verify the computer program and also to provide results for evaluating the significance of variations in the hydraulic parameters.

The boundary condition at the upstream inlet is given by an inflow hydrograph, $Q(t)$, with no limitation on the shape of the hydrograph. A hypothetical hydrograph, with a Pearson Type III distribution and four parameters, was selected for evaluating the effect of variations in the parameter, shown by Fig. 5.4. Thus, the inflow Q at time t designated by $Q(t)$ may be described by

$$Q(t) = Q_b + Q_0 e^{-(t-t_p)/(t_g-t_p)} (t/t_p)^{t/(t_g-t_p)}, \quad (5.32)$$

in which Q_b is the constant base flow, Q_0 is the difference between the base flow and the peak flow, t_p is the time from the beginning of storm runoff to peak discharge, and t_g is the time from the beginning of the storm runoff to the center of mass of storm runoff, G . One hydrograph with arbitrary values of Q_b , Q_0 , t_p , and t_g was used in this portion of the study. The shape along with these arbitrary values of parameters are shown in Fig. 5.4.

The boundary conditions at the downstream outlet are generally given by a stage-discharge relation. In this portion of the study a free outfall at the end of the conduit was assumed. For the free outfall a critical flow at the downstream end exists, with

$$\frac{V}{\sqrt{gA/B}} = 1, \quad (5.33)$$

in which A is the cross section area and B is the top width of the downstream boundary.

It was also assumed that critical depth occurs at a distance of 4.5 times the critical depth from the

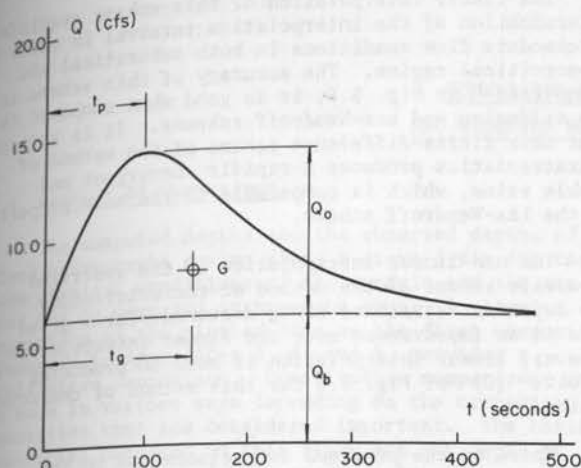


Fig. 5.4. Hypothetical inflow hydrograph of a Pearson Type III function, Eq. 5.32, with the selected parameters: $Q_b = 6.21$ cfs, $Q_0 = 8.00$ cfs, $t_p = 100.00$ sec., and $t_g = 150.0$ sec.

end. This assumption was also applied to the unsteady case, with critical depth computed from the base discharge, Q_b . Therefore, the total distance x_L from the inlet to the downstream boundary is determined by

$$x_L = x_F - 4.5 y_c \quad (5.34)$$

in which x_F is the total length of the conduit and y_c is the critical depth for discharge Q_b .

For the case wherein the downstream end of the channel is restricted, it is necessary to know the relationship between the discharge and the corresponding depth. This relationship can generally be expressed as:

$$Q = m y^n \quad (5.35)$$

in which "m" and "n" are constants. These constants may be estimated from previous experience or computed from measured discharges and depths. In this study, these constants were experimentally determined for specific outlet geometry.

5.4 Comparison of Three Finite-Difference Schemes of Numerical Integration

Criteria for comparison of schemes. Comparison of the three finite-difference schemes, diffusing, Lax-Wendroff, and the specified intervals scheme of the method of characteristics, for numerical integration and computer solution and the eventual selection of the most desirable for particular application depend on simplicity, stability, accuracy, flexibility, and the resulting computer time.

The simplicity of a particular scheme is related to both the algebraic description of its numerical algorithm and the computer programming involved. Simplified algebra, however, does not necessarily infer simplicity in the computer algorithm. The stability of a solution infers that the process will converge to a real solution. This criterion is satisfied in the case of solving the De Saint-Venant equations if the mesh size $\Delta t/\Delta x$ ratio is less than dt/dx , for any part of the (x,t) -plane used in the integration solutions. The accuracy of a method in this study infers that the algorithm will reproduce the initial

conditions for the steady state boundary conditions. As a corollary, the algorithm should be able to compute the steady state conditions from any arbitrary initial conditions. If the algorithm satisfies this criterion, it may be inferred there will be good agreement between the computed and the observed quantities. The flexibility of a computer algorithm depends on the range of conditions the algorithm will accommodate. For the unsteady flow solutions, it is desirable that the algorithm provide for all conditions of depth, velocity, and discharge within expected physical ranges. Generally, this must include both the subcritical and the supercritical conditions. Since numerical procedures at some stage require interpolations, a computer decision is required to determine the appropriate interpolation.

Properties of diffusing scheme. The diffusing scheme is the simplest of the three to develop and is represented in algebraic form. The stability of this scheme is assured provided the ratio of $\Delta t/\Delta x$ does not exceed the absolute maximum value of dt/dx at any point in the (x,t) -plane.

The accuracy of the scheme, however, suffers during eventual periods of supercritical flow. This is because the characteristics intersect at a relatively great distance from the solution point. Accuracy is further limited because the dependent variables are assumed to vary linearly within the interval of $2\Delta x$. Thus, if the actual value of a dependent variable at a given x -position is more than the interpolated value, the computed value at the later time will be less than the true value. This produces a dampening effect in time at a fixed location. The greater the curvature of the free surface the more pronounced is this effect.

To reduce this effect the physical size of Δx may be reduced, but this increases the computer time by the square of the number n of distance intervals, Δx . Subsequent comparisons indicate the diffusing scheme requires more computer time than the other two schemes.

Properties of Lax-Wendroff scheme. The Lax-Wendroff scheme is an improvement over the diffusing scheme in that it accommodates the curvature in the variation of dependent variables. This, however, involves a more complicated numerical algorithm.

The Lax-Wendroff scheme also results in a more accurate solution in comparison with the diffusing scheme for the same Δx and Δt intervals, without a significant increase in computer time. An indication of this accuracy is demonstrated in Fig. 5.5. The Lax-Wendroff method consistently produces the same depth over a very long period of time, whereas, the diffusing scheme produces a consistent change.

With regard to its flexibility in accommodating a wide range of flow conditions, the Lax-Wendroff scheme possesses the same inherent limitations as the diffusing scheme. Thus, by the Lax-Wendroff scheme the further the intersection of the two characteristic curves from the solution point, the less accurate the solution.

Properties of specified intervals scheme of the method of characteristics. Inherent complications in the specified intervals scheme of the method of characteristics are justified by its superior accuracy. Using this scheme, the points of solutions are at the intersections of characteristic curves, rather than at any point within the domain of dependence.

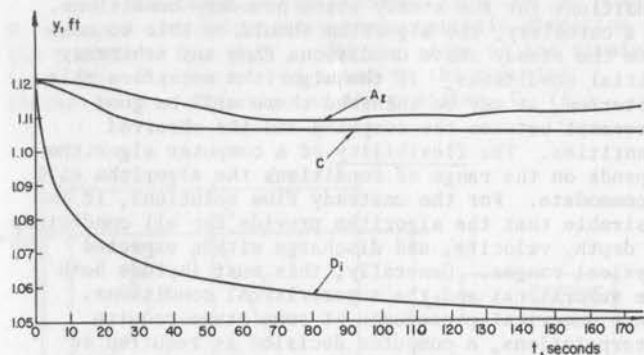


Fig. 5.5. Comparison of diffusing scheme (D_i), Lax-Wendroff scheme (A_L), and the specified intervals scheme of method of characteristics (C) in reproducing the steady initial conditions along the conduit, at the distance $x = 796.7$ ft.

The linear interpolation of this scheme requires determination of the interpolation interval in order to accommodate flow conditions in both subcritical and supercritical regime. The accuracy of this scheme is demonstrated in Fig. 5.5; it is good when compared to the diffusing and Lax-Wendroff schemes. It is apparent that this finite-difference scheme of the method of characteristics produces a rapidly convergent and stable value, which is comparable to the same property of the Lax-Wendroff scheme.

The non-linear interpolation in the specified intervals scheme of the method of characteristics for dependent variables along distances for a given time is an improvement over the linear interpolation. However, linear interpolation is used in producing results (C) of Fig. 5.5 for this method of characteristics.

Based on the previous comparisons and on the results shown in Fig. 5.5, it was decided to use the specified intervals scheme of the method of characteristics on subsequent computations of analytical waves.

COMPARISON OF RESULTS OF COMPUTED
AND OBSERVED WAVES IN SUBCRITICAL FLOW

6.1 Methods of Comparison

The computed depths and the observed depths of flood hydrographs moving along a storm drain for the same physical conditions of the conduit and the same wave may be compared in numerous ways. A visual inspection of the plotted data as the first comparison presented in Appendices 3, 4, and 5, provides a qualitative comparison. Quantitative comparisons may be made in various ways depending on the comparison quantities that are considered important. The basic quantities compared in this study are the depth of flow as a function of time at fixed locations, and the depth of flow as a function of distance at fixed instants of time. The computed waves have the same initial and boundary conditions as the observed waves. The computed waves are obtained by numerical integration of the analytical equations of the specified intervals scheme of the method of characteristics, described briefly in Chapter 5 of this paper and in detail in Hydrology Paper No. 46. The observed waves are those physically produced in the 822-ft long conduit. Comparisons in this chapter refer only to the waves in the subcritical flow regime.

The comparison of computed and observed waves in supercritical flow should not be different from the subcritical flow, provided the two boundary conditions at the inlet of the conduit for the supercritical flow are properly defined. Therefore, any difference between comparisons in supercritical and subcritical flow would be only a measure of how the assumed depth-to-discharge relation at the inlet for the supercritical flow reflects the real physical relation.

The wave property experimentally observed in this study is the depth as a function of time at selected locations along the conduit. A qualitative comparison of the computed depth versus time and the observed depth versus time, for a representative case, is presented in Fig. 6.1. In visual inspection the conclusion regarding the degree of agreement is a subjective decision. A quantitative comparison depends on the consideration of not only the difference in the depths at a given instant of time but also the difference in time for a given depth. A test of agreement could then be a statistic expressing the difference in depth at an instant, or a difference in time for a constant depth, or a combination of depth and time differences.

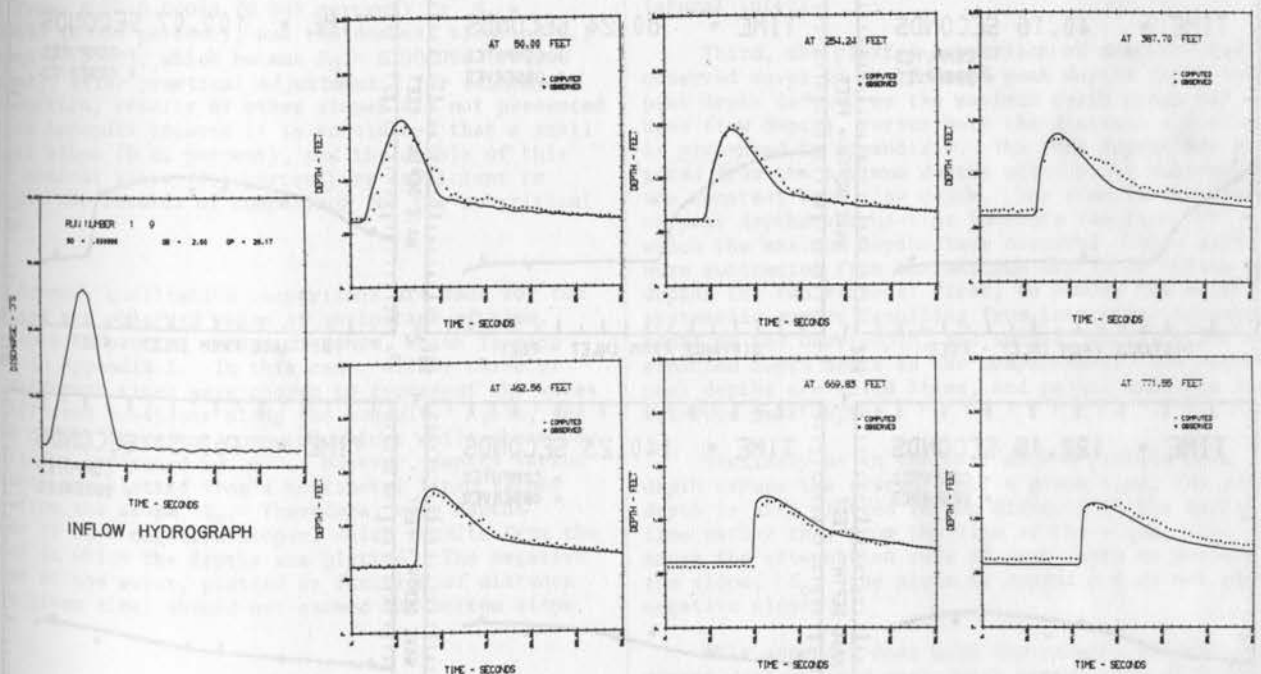


Fig. 6.1. Type of comparison of computed and observed waves for a qualitative comparison by visual inspection with depth versus time for given positions.

Another comparison was made between the computed and observed wave depths along the conduit at successive instants of time. The observed values are obtained from basic data on flood hydrographs at different positions of the conduit. Figure 6.2 presents the typical experimental values, and the corresponding computed curves. These values and curves represent instantaneous water surface profiles; differences between the depths at each position may be used as a comparison. The difference between locations of the same depth for computed and observed waves also can be used as a comparison. However, this comparison does not appear to be reasonable in the majority of the cases presented in Appendix 3 because of additional errors involved in the interpolation of wave profiles along the conduit.

A third comparison is made between the maximum or peak depths of computed and observed waves by considering them as a function of both time and distance. The maximum depth of a wave for a given conduit and for given boundary conditions is perhaps the most meaningful comparison for the design engineer, because this depth eventually determines the conduit dimension for a given design hydrograph. A typical comparison of the computed and observed maximum depth as functions of distance and time is presented in Fig. 6.3. Appendix 4 presents these comparisons for some experimental conditions for the experimental data obtained at the Colorado State University Engineering Research Center.

This chapter refers only to the general comparison of computed and observed flood waves. Both the computed and observed waves are subject to errors. The general comparison then integrates the effects of all sources of errors. Chapter 8 of this paper presents a systematic discussion of errors, and effects of some of the simplifications in the coefficients of the two partial differential equations of gradually varied free-surface unsteady flow.

6.2 Methods of Qualitative Comparison

The observed experimental data on physical waves, subsequently called the observed data, and the numerically computed data by integrating the analytical equations, subsequently called the computed data, are compared in three ways.

First, waves in the form of depth hydrographs, encountered at different points along the conduit, are compared. Plots showing the computed and observed depths of waves at given locations, as a function of time, are shown as the first graph of each run in Appendix 3. The graphs in Appendix 3 consist of wave plots at each position that depth measurements were taken, as well as a plot showing the inflow hydrograph as discharge versus time. The solid curves are the computed depths and the plus sign, +, indicates an observed depth (discrete points delineate the observed depth hydrographs).

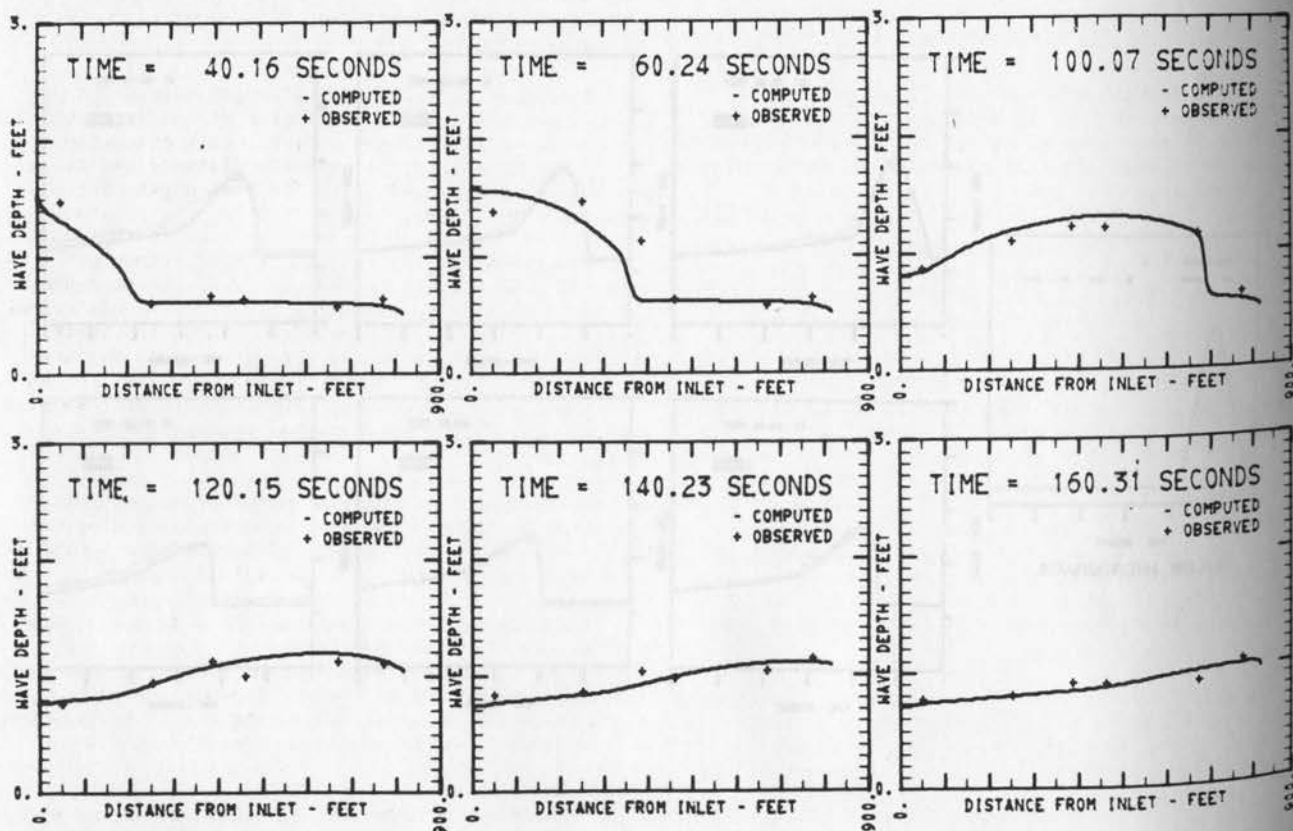


Fig. 6.2. Type of comparison of computed and observed waves for qualitative comparison by visual inspection, of depth versus distance for given times.

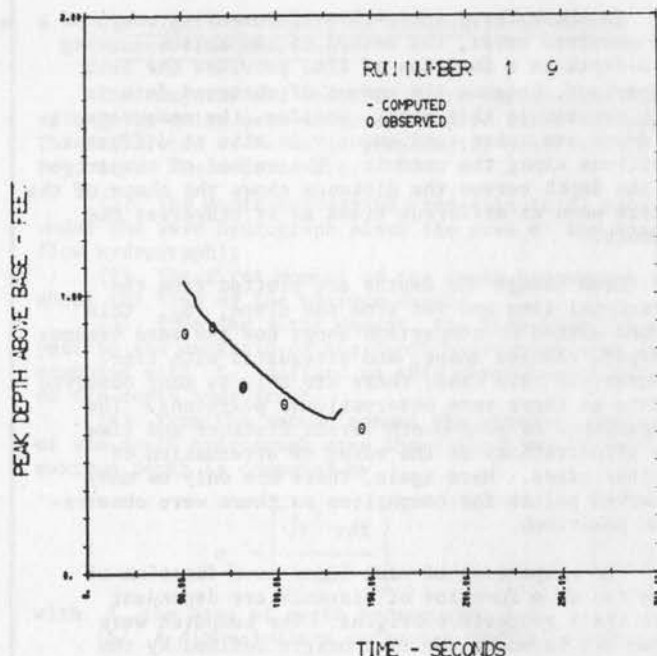
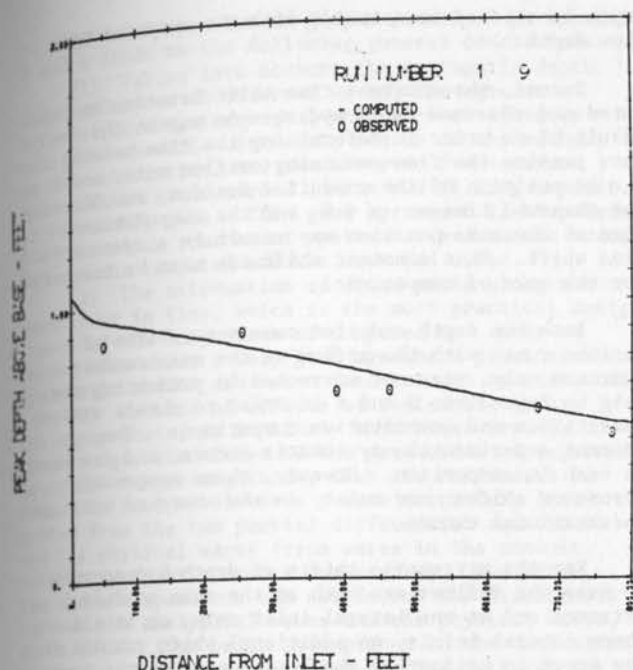


Fig. 6.3. Type of comparison of computed and observed wave peak depths as functions of distance or time, for a qualitative comparison by visual inspection.

Appendix 3 gives the results only of those runs of computed and observed waves that were conducted at the two conduit slopes, the nominal slope of 0.05 percent (0.0005) which actually varied after adjustment from $S_0 = 0.00048$ (0.048 percent) to $S_0 = 0.000550$ (0.055 percent), and the nominal slope of 0.1 percent (0.0010), which became $S_0 = 0.000990$ (0.0990 percent), after practical adjustment. For economy of reproduction, results of other slopes are not presented in this Appendix because it is considered that a small nominal slope (0.05 percent), and the double of this small nominal slope (0.1 percent) are sufficient to indicate the results of comparison for the subcritical regime.

Second, qualitative comparisons are made for the computed and observed waves at an instant of time. The depth is plotted versus distance, which is also shown in Appendix 3. In this case, either three or six different times were chosen to represent the waves at different positions along the conduit. Again, the solid curves represent computed values while observed points are indicated by $\odot$. However, depths versus distance are plotted from a horizontal line, rather than from the slope S_0 . Therefore, some depths appear to have negative slopes, which results from the manner in which the depths are plotted. The negative slopes of the waves, plotted as function of distance for a given time, should not exceed the bottom slope.

Appendix 3 gives the same results of depth versus distance for given instants of time of computed and observed waves, for the same runs and the same slopes as those given in the comparison of depths versus time of the computed and observed waves at given positions. Introductory remarks at the beginning of Appendix 3 explain these runs, and there presentation. The results of Appendix 3 refer to three particular sets of runs: (1) when the main conduit is used only with an

inflow hydrograph at its entrance; (2) when the inflow is at the main conduit entrance and at one of the three lateral inlets; and (3) when the inflow is at the main conduit entrance and at each of the three lateral inlets.

Third, the plotted comparison of computed and observed waves in the form of peak depths (with the peak depth defined as the maximum depth minus the base flow depth), versus both the distance and time, is presented in Appendix 4. The peak depths are computed from the maximum depths attained by subtracting the constant base flow depth. The time in the plots of peak depths-versus-time indicate the times at which the maximum depths have occurred. Base depths were subtracted from the maximum depths to obtain peak depths for two reasons: first, to remove the shift systematic errors resulting from inaccurate measurements of the base depth, and second, to allow for an expanded depth scale in the comparisons. The computed peak depths are solid lines, and points indicate the observed peak depths.

Similarly as in the case of the plotted wave depth versus the distance for a given time, the peak depth is also plotted versus distance in the horizontal line rather than from the line of the slope, S_0 . Because the attenuation rate of peak depth is greater than the slope, S_0 , the plots in Appendix 4 do not show negative slopes.

This appendix does give the results of peak depth versus distance and peak depth versus time of computed and observed waves for the same runs and the same slopes as those given in Appendix 3. Introductory remarks at the beginning of Appendix 4 also explain the runs, and their presentation. These runs refer to the same three cases: (1) inflow at the main conduit entrance only; (2) inflow at the main conduit entrance and at one of the three lateral inlets; and (3) inflow at the main conduit entrance and at each of the three lateral inlets.

In these three approaches of comparing computed and observed waves, the method of comparison showing wave-depth as a function of time provides the best comparison, because the amount of observed data is much greater in this case. Besides, the measurements of depth are taken continuously in time at different positions along the conduit. The method of comparison by the depth versus the distance shows the shape of the entire wave at different times as it traverses the conduit.

Even though the depths are plotted from the horizontal line and not from the slope, S_0 , this second method of comparison shows how the wave becomes steeper, changes shape, and attenuates with time. However, in this case, there are only as many observed points as there were observational positions. The comparisons of peak-depth versus distance and time are illustrations of the rates of attenuation of various waves. Here again, there are only as many observed points for comparison as there were observation positions.

The comparison of wave forms as a function of time and as a function of distance are dependent upon their respective origins. The computed wave forms are based on the time origin defined by the start of the inflow hydrograph at the beginning of the conduit reach or at $x = 0$.

The observed wave forms were the consequence of an inflow hydrograph as recorded at the measuring orifice. The orifice was upstream of the assumed beginning of the channel, and in a full conduit flow. This was necessary because of the baffles and transitions from the closed conduit flow to the conduit free-surface flow. However, this approaching length introduced the effect of a time shift in the boundary condition of inflow hydrograph, roughly equal to this conduit section length divided by the mean velocity. This condition also tended to modify the hydrograph shape due to the change in storage in the free-surface portion of this transition section. No direct quantitative evaluation of these effects was attempted. The comparisons that are made attempt to reduce the difference in the time origin by shifting the computed depth-time wave so that the peak depths of the computed and observed peaks coincide.

6.3. Results of Qualitative Comparisons

A review of various graphs in Appendices 3 and 4 reveal some patterns that warrant a brief discussion, though the discussion is based on visual inspection.

Many depth hydrographs (depth versus time) at given conduit positions show significant shifts both vertically (shift in positions of base flow depth) and horizontally (shift in time of the peak) in the case of both the inflow at the main conduit entrance only, and the inflows at the main conduit entrance and at one or at all three lateral inlets.

Two main reasons can be cited for these differences for the subcritical flow regime and the inflow hydrograph only at the main conduit entrance. First, the constant vertical depth difference between computed and observed hydrographs results from an error in the measurement of the base flow depth. It was also possible that the transducer output was not zero for the base flow depth. This resulted in a constant shift of all observed depths, but this is easy to eliminate, provided the observed base flow

depth is shifted to coincide with the computed base flow depth.

Second, the constant time shift between the computed and observed depth hydrographs may be the result of an error in determining the time between the wave passing the flow measuring orifice meter and the $x = 0$ position of the conduit. Besides, coordinating the observed time at $x = 0$ and the computational time at the same position may result in a systematic time shift. This constant shift can also be corrected for the sake of comparison.

Both the depth and time constants shifts of various runs, with the inflow at the main conduit entrance only, were not corrected in presenting the data in Appendices 3 and 4 in order to obtain various qualitative and quantitative comparisons. They represent experimental, systematic errors, and are kept as such in comparison. However, these systematic errors or shifts are mainly in the observed waves as observational errors.

For the systematic shifts at depth hydrographs in case the inflows are both at the main conduit entrance and at one lateral inlet only, or at all three lateral inlets, an additional shift results from the error in estimating the heat losses at the junction boxes. Because the differences of head loss at junction boxes between the computed and observed waves accumulate in the case of three lateral inlets, and because they are superposed to the systematic shifts of the base flow depth and the time, the systematic error-difference should be either larger at the upstream part of the conduit and smaller at the downstream part, or the opposite, depending on whether the systematic errors in the estimates of head losses are of the same or opposite sign as the base flow depth and time shifts.

Similarly, for the shifts in depth hydrographs at given positions, the shifts both in depth and time are identifiable for the wave profiles as the depth versus distance representation for various times. Appendix 3 clearly demonstrates these various shifts. However, it is not easy to determine which of the two shifts predominates and should be taken into account. The difficulty of determining the shifts is compounded by the way of plotting depths versus distance, since they are plotted from the horizontal line rather than from the corresponding slope line (S_0).

Similar systematic shifts are shown for the comparison of peak depths versus distance or peak depths versus time between the computed and observed waves (Appendix 4), as it was shown for the two previous qualitative comparison by visual inspection. However, these shifts for peak depths seem to be relatively smaller, on the average, than for the total depth hydrographs or total wave profiles along the conduit. This should be expected since the constant shift in the base flow depth and in time should have the least relative effect on the maximum depths, and therefore on the peak depths.

The hypothesis that the systematic transverse oscillations of the wave in the free-surface conduit flow, or that standing wave phenomenon has occurred along the conduit, thus producing the systematic depth shifts, must be assigned a smaller probability than the unaccounted shifts in the voltage of pressure transducers. No significant transverse oscillations were observed, however.

Visual inspection of graphs presented in Appendices 3 and 4 leads to the following general conclusions:

(1) Taking into account the systematic depth and time shifts in observed waves, the agreement between the computed waves, which are obtained by using the specified intervals scheme of the method of characteristics, and the observed free-surface waves, which are recorded in the 822 ft long conduit, is reasonably good. This good agreement is surprising due to various sources of systematic and random errors in both the computed and the observed waves.

(2) The attenuation of flood peaks along the conduit or in time, which is the most practical design aspect of the comparison of computed and observed waves, also shows a very good agreement provided the systematic shifts are taken into account or corrected.

(3) Because of effects of systematic errors in the experiments, it is likely that these errors, on the average, are either of the same order of magnitude or of the larger magnitude, than the difference between the analytical waves (waves accurately integrated from the two partial differential equations) and the physical waves (true waves in the conduit without systematic and random errors). Therefore, the above comparisons can not detect the differences between the analytical waves and the physical waves, which result from basic assumptions in the derivation of the continuity and momentum partial differential equations of gradually varied free-surface unsteady flow.

(4) In general, the waves computed by the integration procedure in this study, using the most complete partial differential equations, should be considered sufficiently accurate for all practical purposes of storm drain design.

6.4 Quantitative Comparison of Results by Depth-Versus-Time Relations

The computed and observed depth-versus-time waves at a given point were compared in five different ways. The definition of terms is graphically presented in Fig. 6.4. These terms are:

- (1) The depth hydrograph area (the total area under the wave hydrograph minus the area of the base flow hydrograph);
- (2) The first moment of the depth-hydrograph area about the time of the maximum depth;
- (3) From the first moment, the time from the wave peak to the centroid of the depth hydrograph area is computed with t_c defined as this characteristic time of the depth hydrograph;
- (4) From the second moment the standard deviation of the depth hydrograph area about the time of the maximum depth is computed by

$$\sigma_p = \left[\frac{\int t^2 y dt}{\int y dt} \right]^{1/2} \quad (6.1)$$

with A the area of depth hydrograph; and

- (5) A dimensionless parameter defined as the ratio

$$R = \frac{\sigma_p}{t_c} \quad (6.2)$$

The areas of depth hydrographs provide a bulk comparison without regard to the distribution of the depth hydrographs. The first moment about the peak is a measure of symmetry; t_c is an indication of

Table 6.1. Quantitative comparison by five parameters of computed and observed waves at given three conduit positions, with no corrections for shifts in observed waves.

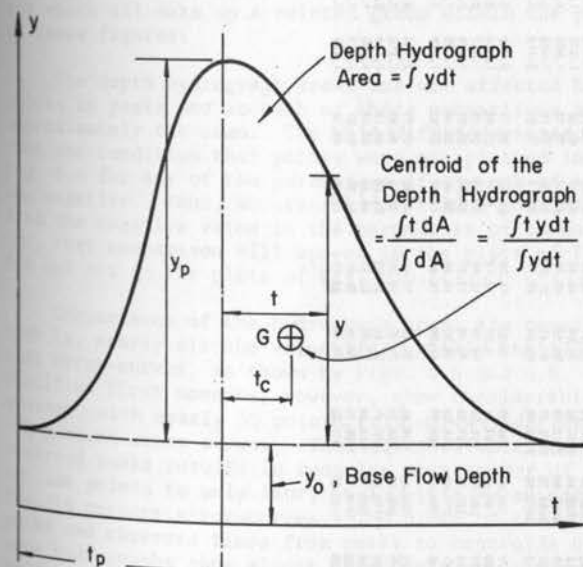


Fig. 6.4. Definitions of basic magnitudes of the wave-depth hydrograph.

R/N	POSITION	TOTAL AREA OF WAVE FORM ABOVE BASE (FT-SEC)		FIRST MOMENT OF AREA ABOUT PEAK (FT-SEC ²)		TIME FROM PEAK TO CENTROID OF WAVE FORM (SEC)		STANDARD DEVIATION OF WAVE FORM ABOUT PEAK (SEC)		RATIO	
		OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED
B2AR1	90.00	2.89	2.31	19.87	24.33	7.39	18.84	21.48	35.51	2.913	2.849
B2AR1	410.00	1.76	1.39	13.32	14.59	7.89	18.37	7.39	21.66	.937	2.189
B2AR1	771.70	.62	.64	.87	0.91	29.62	9.19	27.92	15.96	.929	1.659
B2AR2	90.00	26.47	24.36	421.62	451.45	15.13	18.55	37.84	49.59	2.448	2.648
B2AR2	410.00	21.41	18.76	365.23	319.51	25.43	27.65	43.18	46.81	1.818	1.664
B2AR2	771.70	18.96	16.84	672.11	499.79	36.21	31.36	75.81	44.19	1.205	1.419
B2AR3	90.00	39.32	36.17	894.48	736.52	22.75	23.90	46.85	91.22	2.559	2.519
B2AR3	410.00	31.66	28.78	782.95	616.79	24.73	28.39	45.22	47.48	1.827	1.972
B2AR3	771.70	28.13	24.42	980.99	743.22	36.16	39.44	47.78	44.75	1.397	1.475
B3AR1	90.00	1.95	3.42	4.43	67.39	2.98	19.66	8.95	34.25	5.353	1.742
B3AR1	410.00	1.44	1.84	4.24	19.79	2.99	19.79	6.45	18.84	2.183	1.879
B3AR1	771.70	.36	.11	.36	.44	1.81	-3.84	1.10	4.16	1.810	-1.094
B3AR2	90.00	15.64	15.95	36.99	34.84	3.58	2.26	19.68	21.74	9.498	9.179
B3AR2	410.00	15.96	13.31	297.81	144.14	21.54	18.83	32.16	21.99	1.957	2.139
B3AR2	771.70	13.62	14.81	246.78	392.71	18.84	21.68	28.23	31.78	1.569	1.471
B3AR3	90.00	35.23	36.49	984.08	736.71	17.61	23.76	41.76	52.78	2.372	2.549
B3AR3	410.00	28.47	31.24	675.19	615.88	23.09	26.12	41.87	55.87	1.758	1.948
B3AR3	771.70	25.71	28.27	766.25	619.37	27.47	21.77	39.23	39.61	1.428	1.821
C2AR2	90.00	49.95	35.16	1424.95	691.57	29.53	19.67	53.83	49.72	1.887	2.928
C2AR2	410.00	39.23	28.33	1248.43	835.67	35.21	29.98	91.21	49.33	1.426	1.672
C2AR2	771.70	39.18	24.91	976.37	848.81	27.82	34.98	44.38	47.91	1.595	1.587
C2AR3	90.00	67.85	46.61	1945.81	488.54	22.78	18.48	48.69	39.87	2.156	5.814
C2AR3	410.00	54.89	38.59	1935.79	823.66	39.79	21.27	52.19	39.16	1.458	1.841
C2AR3	771.70	51.74	36.98	184.45	1647.98	15.48	29.12	36.71	43.62	2.514	1.477
C3AR1	90.00	8.77	18.17	86.90	111.39	9.91	18.95	24.80	31.33	2.918	2.859
C3AR1	410.00	7.16	7.64	115.52	61.86	15.44	15.86	26.56	18.72	1.872	1.999
C3AR1	771.70	4.83	6.91	48.86	119.47	12.12	18.39	17.58	26.23	1.887	1.643
C3AR2	90.00	23.44	26.59	391.42	441.29	16.46	16.72	36.41	41.89	2.339	2.916
C3AR2	410.00	19.80	21.69	289.49	479.91	17.90	22.78	31.25	39.88	1.884	1.793
C3AR2	771.70	18.98	19.61	489.35	568.72	22.37	28.80	32.73	39.67	1.681	1.378
C3AR3	90.00	36.67	39.61	497.81	505.89	13.96	9.22	33.95	35.54	2.433	5.854
C3AR3	410.00	29.83	32.59	869.56	687.67	16.99	18.85	32.21	34.89	1.918	1.899
C3AR3	771.70	32.56	31.63	996.72	819.84	18.32	25.89	32.56	37.63	1.777	1.459
D2AR1	90.00	28.87	42.99	-829.79	1367.17	-39.87	31.83	71.48	61.48	-1.817	1.932
D2AR1	410.00	29.87	39.83	384.92	1678.38	19.44	46.81	17.81	68.11	.866	1.476
D2AR1	771.70	32.90	38.19	1787.89	1218.99	97.79	46.54	64.46	62.17	1.244	1.342
D2AR2	90.00	42.75	39.67	1342.79	825.84	31.42	23.99	97.43	52.82	1.828	2.649
D2AR2	410.00	27.49	29.14	1059.87	1096.81	36.50	37.78	56.05	56.93	1.884	1.789
D2AR2	771.70	26.47	23.59	369.38	901.79	13.81	41.62	38.88	56.11	2.664	1.348
D2AR3	90.00	69.18	77.81	2225.49	1914.38	24.99	18.44	91.96	91.61	2.864	2.624
D2AR3	410.00	64.80	68.88	2286.00	1939.86	34.96	29.83	92.99	82.43	1.844	1.779
D2AR3	771.70	68.96	87.43	813.41	2029.98	6.14	36.28	96.73	91.79	4.912	1.468
D3AR1	90.00	18.33	17.99	223.49	234.99	14.98	11.79	32.47	32.38	2.228	2.748
D3AR1	410.00	11.32	14.19	192.77	223.85	16.96	19.89	27.72	29.44	1.674	1.852
D3AR1	771.70	11.63	11.87	186.51	246.43	19.96	23.78	23.14	29.34	1.487	1.413
D3AR2	90.00	55.91	58.22	1367.15	1226.96	24.25	21.67	51.53	51.58	2.125	2.448
D3AR2	410.00	42.82	51.89	1916.35	1982.56	36.19	39.18	97.16	93.45	1.584	1.782
D3AR2	771.70	41.19	48.18	1446.19	1418.86	29.39	31.46	41.77	48.89	1.645	1.994
D3AR3	90.00	67.87	74.63	1112.79	1179.32	16.48	18.68	44.79	46.86	2.732	2.908
D3AR3	410.00	82.21	83.88	1322.79	1489.78	25.34	25.43	46.87	47.23	1.858	2.818
D3AR3	771.70	84.99	88.98	981.47	748.49	11.87	12.87	32.91	37.96	5.674	5.328

Table 6.2. Quantitative comparison by five parameters of computed and observed waves at given six conduit positions, with no corrections for shifts in observed waves.

RUN NO.	POSITION (FT)	TOTAL AREA OF WAVE FORM ABOVE BASE (FT-SEC)		FIRST MOMENT OF AREA ABOUT PEAK (FT-SEC ²)		TIME FROM PEAK TO CENTROID OF WAVE FORM (SEC)		STANDARD DEVIATION OF WAVE FORM ABOUT PEAK (SEC)		RATIO	RUN NO.	POSITION (FT)	TOTAL AREA OF WAVE FORM ABOVE BASE (FT-SEC)		FIRST MOMENT OF AREA ABOUT PEAK (FT-SEC ²)		TIME FROM PEAK TO CENTROID OF WAVE FORM (SEC)		STANDARD DEVIATION OF WAVE FORM ABOUT PEAK (SEC)		RATIO				
		OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED				OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED							
1	3	50.00	35.66	755.85	486.04	21.14	15.46	45.18	40.61	2.343	5.217	1	13	50.00	89.05	80.72	799.71	1105.54	8.90	13.70	45.52	46.96	5.213	5.423	
1	3	254.24	36.64	1028.15	911.24	28.96	16.12	48.22	37.37	1.719	2.319	1	13	254.24	78.28	73.50	1330.27	1238.15	17.01	16.85	46.94	45.19	2.750	2.685	
1	3	387.70	52.82	916.09	999.01	24.87	20.08	37.37	39.12	1.503	1.949	1	13	387.70	77.96	69.97	985.06	1302.76	12.61	18.62	43.44	45.77	3.445	2.450	
1	3	462.56	31.26	28.79	1018.89	999.19	32.59	20.84	47.14	1.447	1.835	1	13	462.56	64.17	66.68	1169.12	1277.93	-2.64	19.16	39.12	44.49	-14.846	2.321	
1	3	669.83	16.23	28.79	454.55	675.76	28.91	25.28	41.21	1.478	1.611	1	13	669.83	61.00	63.85	1190.30	1498.84	19.51	23.48	42.52	46.13	2.179	1.965	
1	3	771.95	30.49	27.34	744.93	715.95	24.43	26.19	38.85	1.567	1.487	1	13	771.95	64.82	63.94	1429.16	1434.61	22.15	22.44	46.79	42.59	1.851	1.898	
1	4	50.00	54.91	53.76	1321.05	912.23	24.06	16.97	49.56	48.21	2.361	2.841	19913	50.00	77.19	72.34	945.27	877.50	12.18	12.13	44.27	44.05	3.594	3.632	
1	4	254.24	49.45	91.82	1435.05	1128.95	29.83	22.57	54.64	50.61	1.882	2.243	19913	254.24	66.94	67.84	941.46	1236.80	14.16	18.45	43.97	45.73	3.127	2.479	
1	4	387.70	50.23	46.59	1543.22	1148.74	30.73	24.65	52.39	49.43	1.705	2.015	19913	387.70	71.72	62.60	1196.63	1173.11	16.60	18.74	44.81	43.55	2.658	2.324	
1	4	462.56	42.89	44.21	1325.73	1048.69	30.91	23.54	53.39	47.49	1.724	2.018	19913	462.56	56.83	60.37	1337.21	1196.77	23.53	19.82	46.12	43.66	1.960	2.252	
1	4	669.83	31.97	42.83	680.66	1324.61	21.29	30.93	36.72	51.54	1.725	1.666	19913	669.83	50.68	58.99	1319.13	1364.93	26.03	23.14	46.16	45.31	1.773	1.958	
1	4	771.95	44.61	42.23	1211.31	1176.13	27.15	27.85	43.43	45.19	1.599	1.623	19913	771.95	55.11	57.48	1199.61	1229.68	21.77	21.39	39.49	40.52	1.814	1.894	
1	8	50.00	49.88	95.51	1415.25	1894.49	28.17	19.72	55.48	50.84	1.896	2.538	8	1	50.00	88.48	86.44	-4.59	-377.58	-1.05	-4.37	60.45	60.86	-1163.345	-13.935
1	8	254.24	47.22	49.78	1286.31	1372.70	27.24	27.58	51.15	51.89	1.878	1.853	8	1	254.24	86.83	80.46	383.55	32.72	4.46	.41	63.48	61.67	14.257	149.185
1	8	387.70	42.08	45.10	1434.11	1411.50	34.14	31.30	54.43	52.31	1.594	1.671	8	1	387.70	80.84	75.91	52.56	269.22	.66	3.95	58.28	60.39	88.747	17.229
1	8	462.56	36.77	42.88	1128.29	1393.76	31.54	32.51	53.46	52.73	1.695	1.622	8	1	462.56	67.39	74.48	636.82	374.88	9.44	5.83	58.33	60.18	6.177	11.937
1	8	669.83	30.94	36.84	910.59	1322.84	29.43	35.91	44.47	53.01	1.511	1.476	8	1	669.83	58.94	78.89	-943.80	519.53	-16.81	7.41	60.82	57.27	-5.805	7.726
1	8	771.95	33.18	36.35	1094.36	1316.13	31.21	36.21	61.83	51.35	1.207	1.418	8	1	771.95	66.34	68.29	-338.81	271.41	-4.98	3.97	58.73	51.72	-10.183	13.814
1	9	50.00	62.73	64.91	1721.31	1546.13	27.44	23.82	54.93	55.82	2.132	2.344	9	1	50.00	22.83	24.48	475.13	631.78	20.81	25.91	48.17	53.29	1.931	2.158
1	9	254.24	61.55	87.17	2095.59	1679.85	34.61	29.38	57.56	54.56	1.663	1.857	9	1	254.24	21.13	21.39	689.21	680.66	32.62	31.83	51.47	54.89	1.578	1.725
1	9	387.70	53.29	53.54	1819.89	1796.77	34.13	32.93	56.07	55.88	1.643	1.697	9	1	387.70	18.16	19.12	612.38	652.50	35.17	34.13	49.70	53.94	1.498	1.580
1	9	462.56	45.38	50.15	1795.42	1724.60	39.52	34.46	58.38	55.53	1.477	1.611	9	1	462.56	16.32	17.79	580.64	618.90	35.57	34.22	52.65	52.38	1.480	1.531
1	9	669.83	41.82	43.78	1464.22	1594.43	39.78	36.42	51.84	54.72	1.452	1.552	9	1	669.83	11.79	15.78	417.89	587.58	34.53	37.43	45.35	52.79	1.314	1.410
1	9	771.95	42.18	44.87	387.84	1626.38	9.23	36.91	36.83	52.79	4.102	1.431	9	1	771.95	13.84	14.62	332.62	499.43	24.83	34.16	37.19	46.27	1.548	1.395
19989	50.00	62.48	71.12	1287.43	668.70	28.63	9.48	48.19	41.21	2.336	4.383	9	2	50.00	78.28	78.82	2356.24	2718.66	35.26	38.39	73.43	77.55	2.218	2.320	
19989	254.24	64.77	62.24	1355.95	1026.97	32.28	16.50	45.89	48.46	2.160	2.452	9	2	254.24	65.13	63.78	3298.11	2912.81	58.64	45.51	81.89	79.69	1.601	1.751	
19989	387.70	52.21	54.86	1581.87	932.43	26.47	16.99	45.77	37.78	1.729	2.222	9	2	387.70	59.76	60.51	2537.95	2974.91	42.46	49.17	72.58	80.99	1.698	1.647	
19989	462.56	46.78	52.81	1280.36	976.14	27.37	18.49	45.99	37.57	1.680	2.132	9	2	462.56	52.57	59.17	2895.84	2981.58	35.94	48.29	81.99	81.99	1.488	1.654	
19989	669.83	37.94	39.57	634.55	496.35	16.90	12.54	35.85	31.93	2.121	2.546	9	2	669.83	47.12	52.52	2457.78	2522.28	52.16	48.82	76.98	73.54	1.476	1.531	
19989	771.95	38.85	37.37	-134.65	529.39	-3.54	14.16	31.82	35.39	-8.993	2.336	9	2	771.95	45.12	47.91	1756.41	1851.44	38.93	38.62	63.39	63.51	1.628	1.644	
1	10	50.00	94.47	89.68	3724.92	3235.31	39.43	36.16	76.95	75.73	1.941	2.101	9	3	50.00	12.71	12.16	146.74	46.72	11.54	3.84	29.44	21.94	2.551	5.710
1	10	254.24	186.81	83.52	9168.98	3764.48	47.81	45.17	81.95	78.75	1.714	1.747	9	3	254.24	9.68	18.66	4.78	86.58	.58	8.12	11.76	21.19	23.621	2.618
1	10	387.70	89.32	79.12	4828.86	3863.82	54.86	48.84	84.43	81.52	1.359	1.669	9	3	387.70	9.91	9.78	89.42	91.78	8.42	9.36	14.51	20.11	1.548	2.149
1	10	462.56	82.36	76.16	4687.93	3712.39	56.92	48.78	87.84	79.72	1.529	1.635	9	3	462.56	8.86	9.18	144.78	93.52	16.35	11.38	24.95	19.14	1.473	1.849
1	10	669.83	72.68	68.67	4458.72	3571.25	61.41	51.99	86.08	79.15	1.432	1.522	9	3	669.83	7.16	7.61	68.06	88.84	9.59	11.57	19.76	18.31	2.161	1.582
1	10	771.95	62.79	68.16	1873.92	3188.68	29.84	47.71	63.14	72.81	2.116	1.526	9	3	771.95	6.78	7.52	98.89	188.79	13.32	14.47	18.16	21.64	1.365	1.496
1	11	50.00	19.11	21.17	164.67	224.85	8.62	18.98	24.84	30.15	2.789	2.848	9	4	50.00	91.57	49.79	917.58	548.25	17.68	11.81	43.89	42.67	2.581	3.875
1	11	254.24	13.74	18.63	67.49	225.91	4.91	12.12	8.95	27.46	1.824	2.265	9	4	254.24	47.63	49.46	872.47	654.24	18.32	14.39	43.11	41.71	2.394	2.890
1	11	387.70	16.12	16.77	244.13	219.56	15.15	12.49	23.43	25.16	1.547	2.814	9	4	387.70	44.86	41.92	738.18	607.23	18.39	16.48	41.18	48.83	2.908	2.491
1	11	462.56	12.89	16.88	237.78	223.58	16.12	15.87	23.54	25.91	1.448	1.825	9	4	462.56	39.47	48.13	422.99	607.94	11.81	16.43	38.46	48.18	3.497	2.439
1	11	669.83	11.16	15.61	211.93	218.82	18.91	16.88	29.41	24.44	1.555	1.521	9	4	669.83	32.81	39.07	39.99	796.62	1.23	28.88	34.98	41.88	28.524	2.181
1	11	771.95	12.90	13.38	194.65	228.35	11.99	16.81	19.48	25.63	1.618	1.525	9	4	771.95	38.91	38.18	888.63	676.37	16.92	18.11	34.94	39.67	2.166	2.361
1	12	50.00	41.4																						

symmetry. A negative t_c indicates that a majority of the depth hydrograph lies under the ascending limb, a positive t_c indicates a similar condition under the descending limb. All but two of the computed t_c values were positive, as shown in Tables 6.1 and 6.2. The standard deviation of the depth hydrograph about the peak, σ_p , is a measure of the dispersion of the wave-form about the peak, and the closeness of the computed σ_p to the observed σ_p indicates how well the computed depth hydrograph matches the observed depth hydrograph. Similarly, the ratio R measures the closeness of the computed and observed hydrographs.

These same five comparisons were also made with the observed peak depths shifted in time, so that the observed time, t_c , from the wave peak to the wave centroid was the same as for the computed value of t_c . This was done because the first and second moment comparisons are dependent upon the position of observed peak, and because the fluctuations of observed depth were such that most waves had several points that could have been specified as the wave peaks. Thus, the time to centroid of the observed and computed waves were made coincident.

In general, the time shifts necessary to equate the t_c values for computed and observed depth hydrographs were less than 5 seconds. The comparison of five parameters for the corrected times of peaks of observed waves are given in Tables 6.3 and 6.4. The shifts listed in Tables 6.3 and 6.4 are positive for a shift toward the time origin and negative for a shift away from the time origin.

Computed and observed values for all five parameters are plotted for no shift in Fig. 6.5 and for shifted peaks in Fig. 6.6. It should be noted that the points in these figures are not independent. That is, there are either three or six related points for each run which all make up a related group within the points of these figures.

The depth hydrograph areas are not affected by the shifts in peaks and so both of these comparisons are approximately the same. The only differences result from the condition that points were not plotted in Fig. 6.6 for any of the parameters if any one of them was negative. Thus, whenever a shift in a peak eliminated the negative value in the parameters of comparison, that comparison will appear in the plots of Fig. 6.6 but not in the plots of Fig. 6.5.

Comparisons of the hydrograph areas are consistent, that is, nearly all the values fall within the ± 20 percent error-curves, as shown by Figs. 6.5 and 6.6. The unshifted first moments, however, show considerable scatter, with nearly 50 points falling outside the ± 30 percent error curves. The effect of shifting the observed peaks results in reducing this number of outside points to only four, most points moved within the ± 20 percent error-curves. The plots of the computed and observed times from peaks to centroids of wave hydrographs show almost a random scatter for no shifted observed peak times, and no error for the shifted times, by definition. The comparisons show the standard deviations of wave hydrographs about peaks to be, for most data, within the ± 20 percent error-envelopes for the unshifted points, and within ± 10 percent error-envelopes for the shifted observed peaks. Similarly, comparisons of the ratios of Eq. 6.2 are scattered but they are generally within ± 30 percent error-curves for the unshifted peaks, and within ± 20 percent error-curves for the shifted peaks.

The comparisons in Tables 6.1 through 6.4 and in Figs. 6.5 and 6.6 refer only to cases that treat the inflow of water only at the main conduit entrance as presented in Appendices 3 and 4. They do not refer to any inflow through the three lateral inlets.

Results on the runs that do involve inflows through lateral inlets are not summarized in the form of quantitative comparisons of the five parameters of comparisons, because of difficulties of applying the parameters as defined in Fig. 6.4.

6.5 Quantitative Comparison of Results by Depth-Versus-Distance Relations

The computed and observed waves as seen at an instant of time were compared by measuring the deviations between the corresponding points of computed curves and the points of observed waves at the positions of observation. These differences are listed in Tables 6.5 through 6.8. They give the depth deviation either at three or at each of six positions, $x = 50.00$, 254.24, 387.70, 462.56, 669.89, and 771.55 ft; absolute average deviation of the six points for each instant in time; and the ratio of the average difference of six differences to the conduit diameter. The mean of this ratio of the average difference versus the diameter for all runs and all times is 0.0256. The standard deviation of all differences is 0.0954 ft. Thus, if the distribution of these deviations is assumed to be normal, this last figure indicates that 67 percent of the computed points will fall within 0.0954 ft of the observed points. Again, the comparisons refer to runs for inflows only through the main conduit entrance.

6.6 General Comparison of Results by Wave-Peak-Depth Versus Distance and Time.

In Appendix 4, the computed and observed wave-peak depths are shown plotted versus both distance and time. For a quantitative comparison, the five parameters as defined by Fig. 6.4 are not applicable. Therefore, it was decided to make general visual comparisons of computed and observed data, not only for the observed data in this study but also with other data, on the rate of attenuation of flood peaks along free-surface flowing circular conduits. For this purpose, data on the attenuation of flood waves in partly full pipes, obtained experimentally at the Hydraulics Research Station, Wallingford, Berkshire, England, were used [4,5].

Graphs in Appendix 4, as stated above, are comparisons of computed and observed data on peak depth attenuations with distance and time as obtained in this study. The summary of experimental data obtained at Colorado State University, used for this comparison, are given in Table 6.9. Similarly, the summary of experimental data obtained at Wallingford, and used for this comparison, are given in Table 6.10. The graphs in Appendix 5 give the observed attenuations of peak depths versus distance or time of the Wallingford data and the computed attenuations by using the specified intervals scheme of the method of characteristics. These computations were made by using the inflow hydrographs of the Wallingford Report No. Int. 31, Fig. 2 of reference [4], for the conduit conditions used in the Wallingford study (3 inch diameter and a 300 ft long). The comparison of computed waves and observed waves at Wallingford was possible only for the peak depth attenuations, because this was the only dynamic wave data available in the report.

Table 6.3. Quantitative comparison by five parameters of computed and observed waves at given three conduit positions, with corrections of observed peak depths shifted in time.

RUN NO.	SHIFT (SEC)	TOTAL AREA OF WAVE FORM ABOVE BASE (FT-SEC)		FIRST MOMENT OF AREA ABOUT PEAK (FT-SEC ²)		TIME FROM PEAK TO CENTROID OF WAVE FORM (SEC)		STANDARD DEVIATION OF WAVE FORM ABOUT PEAK (SEC)		RATIO	
		OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED
B2ARS1	3.15	2.69	2.31	28.36	24.30	10.53	10.54	22.75	30.01	2.163	2.849
B2ARS1	2.47	1.76	1.39	18.28	14.39	10.36	10.37	9.99	21.66	.964	2.289
B2ARS1	-20.42	-.02	.64	-.18	5.90	9.20	9.19	5.91	15.06	.643	1.539
B2AQS2	3.40	26.47	24.36	490.58	451.45	18.53	18.53	38.55	49.08	2.080	2.648
B2AQS2	1.25	21.41	18.76	592.08	518.91	27.65	27.66	40.90	46.01	1.479	1.664
B2AQS2	-4.84	18.56	15.94	582.18	499.75	31.36	31.36	42.08	44.19	1.342	1.439
B2AQS3	-2.38	39.32	36.17	800.87	736.52	20.37	20.36	45.74	51.22	2.245	2.515
B2AQS3	3.64	31.65	28.78	898.12	816.75	28.37	28.38	47.29	47.45	1.667	1.672
B2AQS3	-4.72	28.13	24.42	856.31	743.22	30.44	30.44	44.34	44.75	1.457	1.470
B3AQS1	16.98	1.65	3.42	32.52	67.30	19.66	19.66	21.39	34.25	1.088	1.742
B3AQS1	7.78	1.44	1.84	15.42	19.79	10.74	10.74	12.17	18.04	1.134	1.679
B3AQS1	-4.94	-.36	.11	1.43	-.44	-3.94	-3.94	3.94	4.16	-1.000	-1.254
B3AQS2	-1.32	15.64	15.95	35.39	36.04	2.26	2.26	19.48	20.74	8.612	9.179
B3AQS2	-10.50	13.96	13.31	151.19	144.14	10.83	10.83	26.38	21.98	2.436	2.330
B3AQS2	3.56	13.62	14.01	294.10	302.71	21.60	21.60	30.63	31.78	1.418	1.471
B3AQS3	3.15	33.23	36.45	689.61	736.71	20.75	20.76	43.18	52.78	2.080	2.543
B3AQS3	2.47	28.47	31.24	743.41	815.80	26.11	26.12	43.02	50.87	1.648	1.948
B3AQS3	-5.70	25.71	28.27	559.75	615.37	21.77	21.77	35.47	39.61	1.629	1.820
C2AQS2	-8.85	49.95	35.16	982.71	691.57	19.67	19.67	49.71	49.72	2.527	2.528
C2AQS2	-5.71	35.23	28.33	1039.19	835.67	29.50	29.50	46.38	49.33	1.572	1.672
C2AQS2	6.73	35.10	24.51	1212.62	846.81	34.55	34.55	48.88	47.91	1.415	1.387
C2AQS3	-12.29	67.83	46.61	711.13	488.54	10.48	10.48	44.24	39.87	4.220	3.884
C2AQS3	-14.51	54.89	38.59	1158.63	820.66	21.27	21.27	43.54	39.15	2.047	1.841
C2AQS3	13.66	50.74	35.98	1477.61	1047.96	29.12	29.12	45.91	43.02	1.577	1.477
C3AQS1	1.03	8.77	18.17	95.97	111.30	10.94	10.95	25.29	31.33	2.311	2.863
C3AQS1	-1.46	7.16	7.64	100.10	106.88	13.99	13.98	24.97	26.56	1.785	1.899
C3AQS1	8.23	4.83	6.51	88.55	119.47	18.35	18.35	22.93	26.43	1.250	1.440
C3AQS2	.85	23.44	26.39	391.68	441.23	16.71	16.72	38.42	41.89	2.299	2.516
C3AQS2	4.80	15.88	21.06	361.75	479.90	22.78	22.78	33.34	39.88	1.464	1.750
C3AQS2	6.48	18.30	19.61	527.90	565.72	28.85	28.85	37.46	39.67	1.298	1.375
C3AQS3	-4.73	35.67	39.61	328.92	365.89	9.22	9.22	32.29	35.54	3.502	3.856
C3AQS3	1.69	29.83	32.59	956.18	687.67	18.64	18.65	33.13	34.85	1.777	1.869
C3AQS3	7.57	32.56	31.63	843.06	819.04	25.89	25.89	37.34	37.63	1.442	1.453
D2ARS1	71.39	20.87	42.95	664.22	1367.17	31.82	31.83	75.25	61.48	2.364	1.932
D2ARS1	27.37	25.97	35.88	1215.69	1675.86	46.81	46.81	38.77	69.11	.828	1.476
D2ARS1	-5.45	32.98	26.13	1528.82	1218.98	46.34	46.34	68.14	62.17	1.298	1.342
D2AQS1	-7.87	42.73	35.87	1886.42	825.84	23.55	23.55	53.53	52.82	2.273	2.243
D2AQS1	-.85	27.49	29.14	1836.49	1898.51	37.71	37.78	55.46	58.93	1.471	1.563
D2AQS1	27.81	26.47	23.59	1181.35	981.73	41.61	41.62	53.81	56.11	1.293	1.348
D2AQS3	-5.85	89.18	77.91	1731.94	1514.38	19.44	19.44	49.13	50.61	2.527	2.614
D2AQS3	-5.43	64.98	65.85	1986.36	1935.86	29.53	29.53	49.18	52.43	1.664	1.775
D2AQS3	27.12	62.95	37.43	2219.86	2028.88	35.26	35.26	58.26	51.78	1.428	1.468
D3AQS1	-2.78	15.33	17.55	188.83	206.99	11.80	11.79	31.32	32.38	2.656	2.745
D3AQS1	-.66	11.52	14.08	183.19	223.85	15.90	15.89	27.33	29.44	1.719	1.852
D3AQS1	5.28	10.63	11.87	228.78	246.43	20.76	20.76	26.91	29.34	1.296	1.413
D3AQS2	-3.17	53.91	58.22	1136.27	1226.95	21.08	21.07	50.12	51.58	2.378	2.448
D3AQS2	-6.09	42.82	50.89	1268.54	1592.56	30.00	30.00	53.53	53.45	1.784	1.782
D3AQS2	6.06	41.19	45.18	1295.77	1418.65	31.46	31.46	45.71	48.89	1.453	1.554
D3AQS3	-.71	67.87	74.63	1864.49	1170.32	15.68	15.68	44.54	46.86	2.840	2.988
D3AQS3	-1.98	52.21	63.58	1223.38	1489.76	23.43	23.43	45.56	47.20	1.944	2.015
D3AQS3	1.99	54.99	58.98	691.83	748.48	12.57	12.57	33.21	37.96	2.643	3.020

Table 6.4. Quantitative comparison by five parameters of computed and observed waves at given six conduit positions, with corrections of observed peak depths shifted in time.

RUN NO.	SHIFT (SEC)	TOTAL AREA OF WAVE FORM ABOVE BASE (FT-SEC)		FIRST MOMENT OF AREA ABOUT PEAK (FT-SEC ²)		TIME FROM PEAK TO CENTROID OF WAVE FORM (SEC)		STANDARD DEVIATION OF WAVE FORM ABOUT PEAK (SEC)		RATIO		RUN NO.	SHIFT (SEC)	TOTAL AREA OF WAVE FORM ABOVE BASE (FT-SEC)		FIRST MOMENT OF AREA ABOUT PEAK (FT-SEC ²)		TIME FROM PEAK TO CENTROID OF WAVE FORM (SEC)		STANDARD DEVIATION OF WAVE FORM ABOUT PEAK (SEC)		RATIO	
		OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED			OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED	OBSERVED	COMPUTED
1 3	-7.00	35.66	36.11	486.12	486.84	13.46	13.46	39.39	43.61	2.975	3.117	1 13	4.71	89.85	86.72	1219.24	1185.54	13.69	13.78	46.19	46.96	5.373	5.429
1 3	-11.94	36.64	31.72	591.74	511.24	16.12	16.12	42.40	37.37	2.630	2.319	1 13	-1.16	78.28	73.50	1317.58	1238.15	16.89	16.85	46.89	45.19	2.783	2.683
1 3	-4.78	32.82	29.87	659.88	599.81	20.88	20.88	34.37	39.12	1.712	1.949	1 13	6.81	77.96	69.97	1451.22	1392.76	18.61	18.62	45.54	45.77	2.447	2.458
1 3	-11.75	31.26	28.75	651.62	599.19	20.84	20.84	39.94	38.21	1.916	1.853	1 13	21.80	64.17	66.68	1229.60	1277.93	19.16	19.16	43.48	44.49	2.269	2.321
1 3	-2.73	16.23	26.73	418.27	675.76	25.29	25.28	39.39	42.48	1.958	1.601	1 13	3.97	61.18	63.83	1432.29	1498.84	23.48	23.48	44.48	46.13	1.894	1.955
1 3	1.79	38.49	27.34	798.39	715.99	26.19	26.19	39.26	38.95	1.497	1.487	1 13	.39	64.82	63.94	1454.15	1434.61	22.43	22.44	41.91	42.59	1.828	1.898
1 4	-7.19	54.91	53.76	931.89	912.23	16.97	16.97	46.54	48.21	2.742	2.841	19913	-1.85	77.19	72.34	296.59	877.50	12.13	12.13	44.76	44.55	3.648	3.632
1 4	-6.46	49.45	50.12	1116.19	1128.95	22.57	22.57	51.50	56.61	2.282	2.243	19913	4.38	66.94	67.04	1254.62	1236.80	18.44	18.45	45.56	45.73	2.473	2.479
1 4	-6.37	58.25	46.59	1238.44	1148.74	24.66	24.66	49.08	49.43	1.991	2.035	19913	2.85	71.62	62.60	1343.78	1175.11	18.74	18.74	44.83	45.55	2.392	2.324
1 4	-7.37	42.89	44.21	1039.69	1040.05	23.54	23.54	49.39	47.49	2.598	2.318	19913	-3.70	56.83	60.37	1126.89	1196.77	19.83	19.82	44.35	43.66	2.236	2.232
1 4	9.65	31.97	42.83	988.62	1324.68	31.92	31.93	43.13	51.94	1.391	1.666	19913	-2.89	50.68	58.99	1172.85	1364.93	23.14	23.14	44.59	45.31	1.927	1.958
1 4	.69	44.61	42.23	1242.28	1176.13	27.89	27.89	43.86	45.19	1.575	1.623	19913	-3.37	55.11	57.48	1179.28	1229.68	21.49	21.39	39.29	49.52	1.856	1.894
1 8	-8.45	49.88	55.51	985.66	1094.49	19.72	19.72	49.47	50.84	2.559	2.538	8 1	-4.31	88.48	86.44	-385.78	-377.50	-4.36	-4.37	60.61	62.86	-13.251	-13.535
1 8	.33	47.22	49.78	1302.11	1372.78	27.57	27.58	51.33	51.19	1.862	1.853	8 1	-4.85	86.83	80.46	35.32	32.72	.41	.41	63.32	60.67	154.213	149.185
1 8	-2.84	42.18	45.18	1314.76	1411.58	31.39	31.39	52.69	52.31	1.683	1.671	8 1	2.89	80.84	75.91	283.62	269.22	3.54	3.58	58.38	60.59	16.475	17.029
1 8	.90	39.77	42.88	1162.95	1393.76	32.50	32.51	54.13	52.73	1.662	1.622	8 1	-4.41	67.39	74.48	339.29	374.88	5.84	5.83	57.78	60.08	11.469	11.937
1 8	6.47	31.94	36.84	1111.99	1322.84	35.91	35.91	48.99	55.11	1.564	1.476	8 1	23.42	58.94	78.89	436.72	519.53	7.41	7.41	59.29	57.27	7.996	7.726
1 8	-15.81	33.18	36.36	1198.88	1316.13	36.21	36.21	51.12	51.35	1.384	1.418	8 1	8.95	66.34	68.29	263.33	271.41	3.97	3.97	59.64	51.72	12.758	13.314
1 9	-3.62	62.73	64.91	1494.38	1546.13	23.82	23.82	53.21	55.82	2.254	2.344	9 1	5.89	22.83	24.48	591.16	631.78	25.89	25.98	43.12	53.29	1.662	2.258
1 9	-5.23	61.95	57.17	1779.12	1679.85	29.38	29.38	54.58	54.56	1.858	1.857	9 1	-7.79	21.13	21.39	672.56	681.66	31.83	31.83	55.98	54.89	1.622	1.725
1 9	-1.28	53.29	53.34	1795.38	1756.77	32.94	32.93	55.35	55.88	1.680	1.697	9 1	.95	18.16	19.12	619.67	652.50	34.12	34.13	50.34	53.94	1.475	1.582
1 9	-5.16	49.38	51.85	1563.76	1724.65	34.46	34.46	55.18	55.53	1.598	1.611	9 1	-1.35	16.32	17.79	598.62	608.91	34.22	34.22	51.75	52.38	1.512	1.551
1 9	7.72	41.82	43.78	1483.69	1594.43	36.42	36.42	52.34	54.72	1.437	1.532	9 1	2.89	11.79	15.78	441.21	587.58	37.42	37.43	47.68	52.79	1.272	1.413
1 9	27.71	42.18	44.87	1956.47	1626.38	36.98	36.91	51.32	52.79	1.391	1.431	9 1	18.12	13.84	14.62	472.73	499.43	34.12	34.18	44.48	46.27	1.393	1.395
19919	-11.23	62.48	71.12	586.81	668.78	9.48	9.48	44.56	41.21	4.738	4.383	9 2	5.13	70.25	78.82	2696.60	2718.66	38.39	38.39	75.89	77.55	1.977	2.328
19919	-5.78	61.77	62.24	1812.98	1826.97	16.58	16.58	43.38	48.46	2.629	2.452	9 2	-5.12	65.13	63.78	2964.32	2902.80	45.51	45.51	77.99	79.69	1.714	1.751
19919	-9.48	52.28	54.86	887.21	932.83	16.99	16.99	41.82	37.75	2.414	2.222	9 2	6.78	59.76	60.51	2938.03	2974.99	49.16	49.17	76.22	80.99	1.952	1.647
19919	-8.88	46.78	52.88	886.14	976.14	18.49	18.49	41.33	37.57	2.235	2.332	9 2	-5.46	52.57	59.87	2653.71	2981.38	50.48	50.47	78.10	80.95	1.545	1.604
19919	-4.36	37.84	39.57	471.62	496.35	12.58	12.54	34.61	31.93	2.711	2.546	9 2	-4.14	47.12	52.52	2262.69	2522.21	48.62	48.62	74.24	75.54	1.546	1.531
19919	17.78	38.88	37.37	938.82	929.38	14.16	14.16	34.65	33.19	2.447	2.356	9 2	-3.38	45.12	47.91	1742.75	1859.44	38.63	38.62	63.28	63.91	1.656	1.644
1 10	-5.37	94.47	89.68	3416.31	3233.31	36.86	36.86	74.87	75.73	2.676	2.103	9 3	-7.78	12.71	12.16	48.98	46.72	3.85	3.84	27.36	21.94	7.113	5.713
1 10	-2.74	186.81	85.52	4778.58	3764.48	49.87	49.87	80.58	78.75	1.783	1.747	9 3	7.62	9.60	18.66	77.97	86.58	8.12	8.12	8.52	21.19	1.949	2.638
1 10	-6.82	88.92	79.12	4299.83	3863.82	48.84	48.84	80.65	81.92	1.651	1.669	9 3	-6.6	9.91	9.78	92.81	90.78	9.36	9.36	14.47	20.11	1.946	2.149
1 10	-8.17	82.36	76.16	4115.13	3712.39	48.78	48.78	81.93	79.72	1.681	1.635	9 3	-8.83	8.86	9.88	91.27	93.52	10.38	10.38	20.44	19.84	1.984	1.849
1 10	-9.42	72.48	68.67	3774.72	3878.25	51.99	51.99	79.65	79.15	1.552	1.522	9 3	1.98	7.16	7.61	82.82	88.14	11.57	11.57	28.88	18.31	1.797	1.582
1 10	17.06	62.79	68.16	2895.74	3188.68	47.71	47.71	73.59	72.88	1.536	1.526	9 3	1.15	6.76	7.52	97.85	188.79	14.47	14.47	19.81	21.64	1.314	1.498
1 11	1.96	19.11	21.17	282.18	224.65	18.58	18.58	24.81	30.15	2.345	2.848	9 4	-6.98	51.57	49.79	568.88	548.25	11.62	11.61	41.79	42.67	3.794	3.875
1 11	7.21	13.74	18.63	166.57	225.91	12.12	12.12	6.52	27.46	.538	2.265	9 4	-9.92	47.63	45.46	685.72	654.24	14.48	14.39	41.60	41.71	2.889	2.898
1 11	-2.05	16.12	16.77	281.42	289.56	12.58	12.49	21.81	25.16	1.745	2.014	9 4	.88	44.86	41.92	739.34	687.23	16.39	16.48	41.11	48.83	2.508	2.491
1 11	-2.14	12.89	16.81	181.15	223.58	13.98	13.97	21.92	25.91	1.568	1.825	9 4	8.44	38.47	48.83	632.18	687.98	16.43	16.43	48.35	48.88	2.455	2.439
1 11	-2.85	11.16	15.61	179.48	218.82	16.88	16.88	27.67	24.44	1.721	1.528	9 4	18.88	32.01	39.67	884.78	796.62	21.88	21.88	48.32	41.88	2.388	2.181
1 11	4.82	12.98	13.58	216.76	228.35	16.81	16.81	22.69	25.63	1.391	1.529	9 4	-1.81	39.81	38.18	498.91	676.37	15.11	15.11	34.18	39.67	2.256	2.361
1 12	-5.39	41.36	44.32	681.83	729.88	16.47	16.47	45.88	47.77	2.737	2.981												
1 12	-5.78	36.33	39.77	722.12	798.38	19.87	19.87	46.39	45.88	2.334	2.309												
1 12	-8.88	36.92	37.98	894.84	865.37	23.83	23.83	49.52	46.13	2.150	2.103												
1 12	-2.99	38.29	36.64	786.42	914.88	24.98	24.97	44.77	47.18	1.793	1.889												
1 12	-3.48	38.16	38.16	841.38	1899.18	31.21	31.21	52.85	51.37	1.693	1.646												
1 12	-6.38	34.43	38.29	824.48	947.32	26.88	26.88	44.43	43.39	1.695	1.613												

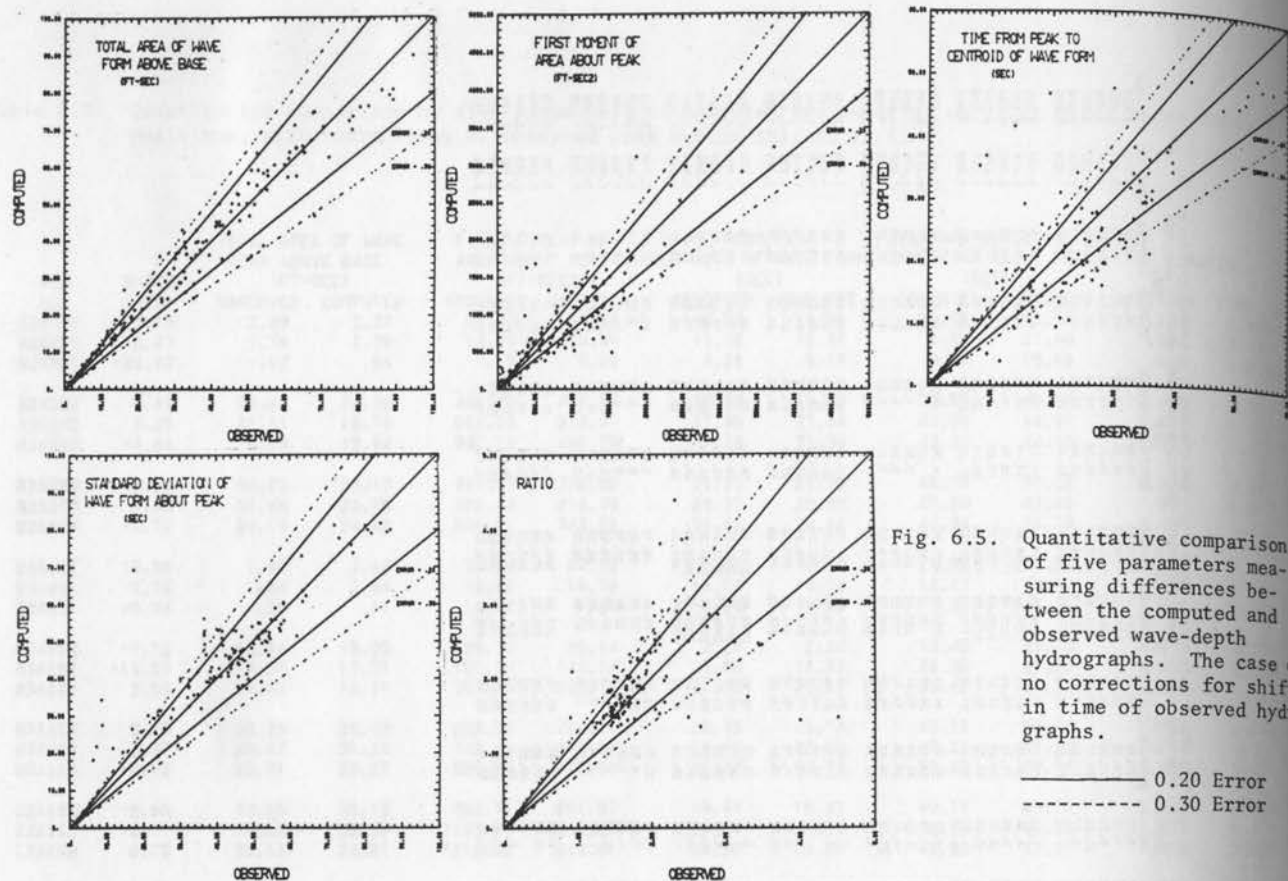


Fig. 6.5. Quantitative comparison of five parameters measuring differences between the computed and observed wave-depth hydrographs. The case of no corrections for shifts in time of observed hydrographs.

— 0.20 Error
- - - 0.30 Error

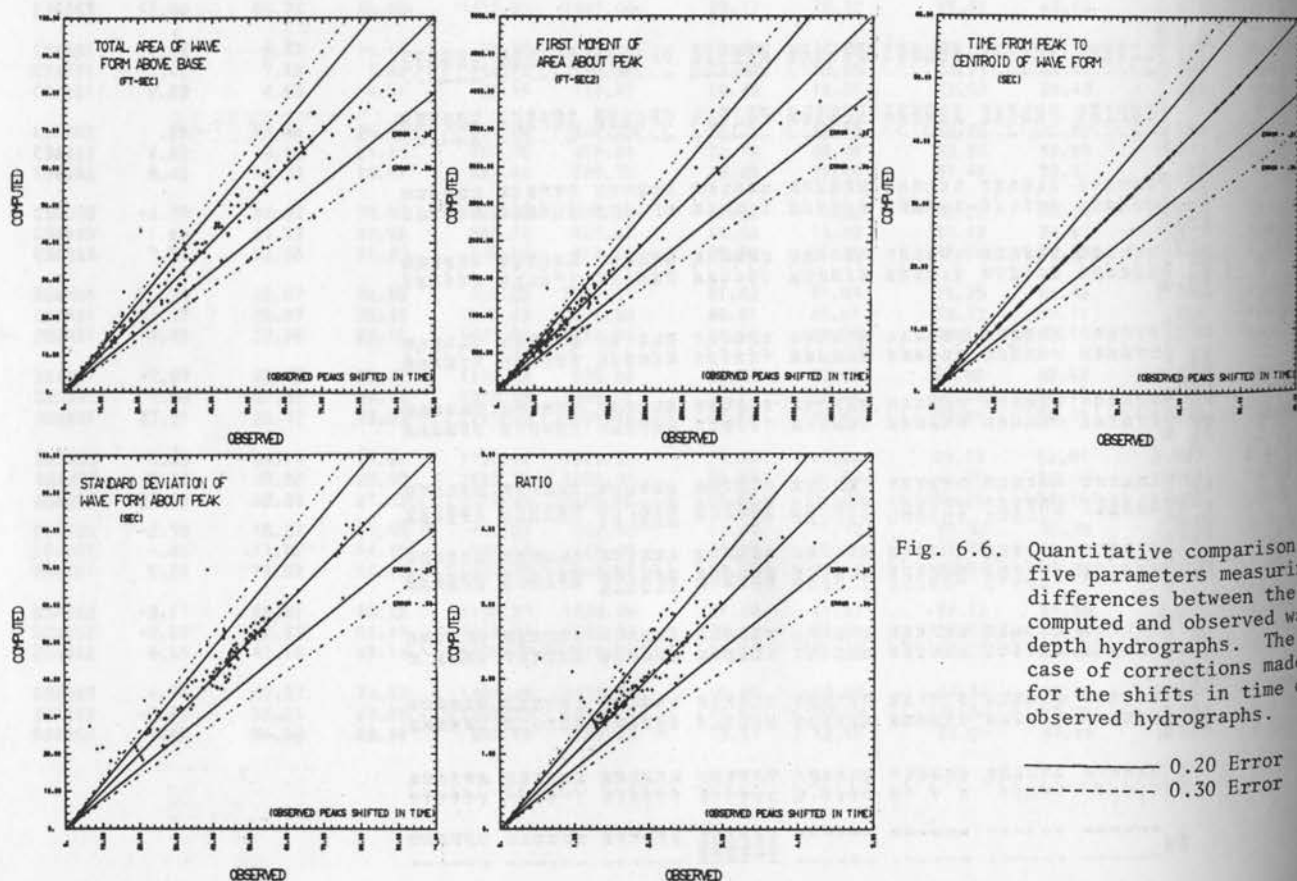


Fig. 6.6. Quantitative comparison of five parameters measuring differences between the computed and observed wave-depth hydrographs. The case of corrections made for the shifts in time of observed hydrographs.

— 0.20 Error
- - - 0.30 Error

Table 6.5. Qualitative comparison of computed and observed depth-versus-distance wave profiles.

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

RUN	TIME	POSITIONS						AVERAGE/DIA	
		1	2	3	4	5	6	AVERAGE	RATIO
B2ARS1	30.13	-.1006	-.0459	-.0688				.0718	.02452
B2ARS1	60.26	-.0866	-.0463	-.0631				.0653	.02233
B2ARS1	90.40	-.0802	-.0467	-.0514				.0594	.02031
B2ARS1	120.13	-.0856	-.0408	-.0417				.0560	.01915
B2ARS1	150.26	-.0872	-.0404	-.0581				.0619	.02116
B2ARS1	180.40	-.0864	-.0569	-.0482				.0638	.02182
B2AOS2	30.12	.0865	-.0772	-.0589				.0742	.02555
B2AOS2	60.25	-.1212	-.0676	-.0464				.0784	.02678
B2AOS2	90.01	-.0858	-.0809	-.0564				.0743	.02540
B2AOS2	120.14	-.0654	-.0396	-.0252				.1034	.03534
B2AOS2	150.26	-.0614	-.0361	-.0579				.0518	.01770
B2AOS2	180.03	-.0728	-.0327	-.0244				.0433	.01480
B2AOS2	210.15	-.0768	-.0570	-.0283				.0540	.01846
B2AOS2	240.28	-.0758	-.0437	-.0341				.0512	.01748
B2AOS2	270.04	-.0696	-.0500	-.0377				.0524	.01792
B2AOS3	30.27	.0249	-.0589	-.0588				.0475	.01624
B2AOS3	60.20	-.1064	-.0598	-.0480				.0714	.02441
B2AOS3	90.13	-.1050	-.0933	-.0513				.0832	.02844
B2AOS3	120.05	-.0714	-.0554	-.0612				.1293	.04420
B2AOS3	150.33	-.0956	-.0181	-.0539				.0559	.01909
B2AOS3	180.26	-.0959	-.0251	-.0200				.0470	.01606
B2AOS3	210.18	-.1041	-.0410	-.0151				.0534	.01824
B2AOS3	240.11	-.1030	-.0313	-.0371				.0571	.01953
B2AOS3	270.04	-.0944	-.0413	-.0330				.0562	.01922
B3AOS1	30.18	.0197	.0205	.0097				.0166	.00568
B3AOS1	60.30	.0221	.0420	.0248				.0296	.01012
B3AOS1	90.08	.0211	.0200	.0225				.0212	.00724
B3AOS2	30.31	.0100	.0199	.0168				.0156	.00531
B3AOS2	60.26	-.0207	.1048	.0185				.0746	.02548
B3AOS2	90.22	.0172	.0172	.0200				.0148	.00506
B3AOS2	120.18	.0090	.0277	.0230				.0199	.00680
B3AOS2	150.13	.0134	.0382	.0281				.0239	.00816
B3AOS2	180.09	.0115	.0265	.0239				.0206	.00705
B3AOS2	210.05	.0127	.0326	.0269				.0240	.00822
B3AOS2	240.00	.0132	.0384	.0294				.0243	.00832
B3AOS3	30.33	.0337	.0170	.0122				.0210	.00716
B3AOS3	60.32	-.0250	.0646	.0208				.0368	.01257
B3AOS3	90.32	.0410	-.0867	.0173				.0217	.00741
B3AOS3	120.31	.0328	.0175	.0070				.0158	.00539
B3AOS3	150.30	.0257	.0273	.0089				.0180	.00614
B3AOS3	180.29	.0135	.0282	.0159				.0166	.00566
B3AOS3	210.28	.0228	.0186	.0268				.0227	.00776
B3AOS3	240.28	.0254	.0257	.0247				.0253	.00863
B3AOS3	270.27	.0287	.0231	.0278				.0263	.00897

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

RUN	TIME	POSITIONS						AVERAGE/DIA	
		1	2	3	4	5	6	AVERAGE	RATIO
C2AOS2	30.21	.0606	-.2269	-.2050				.1642	.05610
C2AOS2	60.08	-.2474	.2633	-.2015				.2374	.08113
C2AOS2	90.29	-.1789	-.2694	-.1982				.2155	.07365
C2AOS2	120.16	-.1654	-.1127	-.1588				.1456	.04977
C2AOS2	150.03	-.2214	-.1632	-.1303				.1716	.05866
C2AOS2	180.24	-.2294	-.1613	-.1046				.1651	.05641
C2AOS2	210.11	-.2419	-.1726	-.1117				.1754	.05995
C2AOS2	240.32	-.2522	-.1603	-.1367				.1831	.06257
C2AOS2	270.19	-.2596	-.1799	-.1439				.1945	.06645
C2AOS3	30.11	.0965	-.1961	-.1703				.1542	.05271
C2AOS3	60.22	-.2985	.1142	-.1656				.1928	.06588
C2AOS3	90.00	-.2229	-.2400	-.1637				.2089	.07137
C2AOS3	120.11	-.2314	-.0841	-.1704				.1620	.05536
C2AOS3	150.22	-.2767	-.1163	-.0985				.1638	.05599
C2AOS3	180.01	-.2973	-.1709	-.0786				.1823	.06230
C2AOS3	210.12	-.3240	-.1888	-.0996				.2041	.06976
C2AOS3	240.23	-.3442	-.1933	-.1478				.2285	.07887
C2AOS3	270.01	-.3496	-.2309	-.1665				.2490	.08559
C3AOS1	30.09	-.0083	.0476	.0634				.0398	.01359
C3AOS1	60.17	.0795	.0667	.0753				.0738	.02523
C3AOS1	90.26	.0844	.0512	.0650				.0669	.02285
C3AOS1	120.34	.0821	.0703	.0376				.0634	.02165
C3AOS1	150.08	.0718	.0779	.0781				.0759	.02595
C3AOS1	180.16	.0749	.0407	.0625				.0594	.02038
C3AOS2	30.20	.0121	.0744	.0562				.0475	.01625
C3AOS2	60.07	.0878	-.0085	.0594				.0519	.01775
C3AOS2	90.27	.0856	.0099	.0584				.0513	.01753
C3AOS2	120.14	.0906	.0198	.0815				.0639	.02185
C3AOS2	150.01	.0861	.0369	.0673				.0634	.02168
C3AOS2	180.22	.0822	.0330	.0687				.0613	.02095
C3AOS2	210.09	.0731	.0342	.0709				.0594	.02030
C3AOS2	240.29	.0852	.0472	.0725				.0683	.02333
C3AOS3	30.05	-.0116	.0567	.0070				.0251	.00858
C3AOS3	60.10	-.0327	.0442	.0238				.0336	.01148
C3AOS3	90.15	.0070	-.0329	.0257				.0219	.00748
C3AOS3	120.20	.0286	.0186	.0190				.0220	.00753
C3AOS3	150.25	.0039	.0141	.0157				.0112	.00384
C3AOS3	180.29	-.0016	.0050	.0081				.0049	.00167
C3AOS3	210.02	.0005	.0010	.0147				.0054	.00186
C3AOS3	240.07	.0007	-.0085	.0175				.0089	.00304
D2ARS1	30.08	-.1716	-.2073	-.1852				.1881	.06427
D2ARS1	60.17	-.0503	-.4679	-.1859				.2347	.08921
D2ARS1	90.25	-.1087	-.1634	-.1863				.1528	.05222
D2ARS1	120.33	-.1856	-.0652	-.1797				.1435	.04904
D2ARS1	150.08	-.1682	-.0642	-.0763				.1829	.03517
D2ARS1	180.16	-.0254	-.0941	-.0409				.0535	.01827
D2ARS1	210.24	.0013	-.1074	-.0289				.0459	.01567
D2ARS1	240.33	.0194	-.1142	-.0516				.0617	.02110
D2ARS1	270.07	.0299	-.0233	-.0682				.0405	.01382

Table 6.6. Qualitative comparison of computed and observed depth-versus-distance wave profiles.

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -										- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -										
RUN	TIME	1	2	POSITIONS 3	4	5	6	AVERAGE	AVERAGE/DIA RATIO	RUN	TIME	1	2	POSITIONS 3	4	5	6	AVERAGE	AVERAGE/DIA RATIO	
D2A0S1	30.25	-.1168	-.1819	-.1729				.1572	.05372	1	3	20.15	.0491	-.0204	.0626	-.0277	-.0484	-.1631	.0619	.02114
D2A0S1	60.12	-.0189	-.2348	-.1771				.1436	.04906	1	3	40.29	-.0127	.0554	.0929	-.0182	-.0714	-.1777	.0714	.02439
D2A0S1	90.01	-.0563	-.1801	-.1703				.1382	.04724	1	3	60.12	-.0284	-.0903	.1375	.0425	-.0574	-.1497	.0843	.02881
D2A0S1	120.24	-.0885	-.1778	-.1909				.1524	.05208	1	3	80.27	-.0270	-.1669	-.0553	-.1580	.1734	-.1573	.1230	.04204
D2A0S1	150.13	-.0657	-.0950	-.1003				.0873	.02983	1	3	100.10	-.0268	-.1108	.0450	-.0576	-.1561	-.1287	.0875	.02990
D2A0S1	180.02	-.0812	-.1040	-.0903				.0918	.03138	1	3	120.24	.0152	-.0637	.0547	-.0522	-.2067	-.1733	.0943	.03222
D2A0S1	210.25	-.0891	-.1058	-.0646				.0865	.02957	1	3	140.07	-.0299	-.0391	.0363	-.0102	-.0382	-.1821	.0545	.01861
D2A0S1	240.14	-.0997	-.1157	-.0770				.0975	.03331	1	3	160.22	-.0107	-.0361	-.0080	-.0223	-.0647	-.1492	.0485	.01657
D2A0S1	270.03	-.1072	-.1168	-.0783				.1008	.03444	1	3	180.04	-.0223	-.0494	-.0637	-.0878	-.0550	-.1713	.0749	.02561
D2A0S3	30.26	-.1381	-.1278	-.1146				.1268	.04334	1	3	200.19	-.0336	-.1138	-.0545	-.1467	-.0740	-.1810	.1005	.03434
D2A0S3	60.21	-.1018	-.4590	-.1230				.2249	.07686	1	3	220.02	-.0489	-.1189	-.0494	-.1282	-.0809	-.1947	.1008	.03446
D2A0S3	90.17	-.0031	-.2181	-.1217				.1116	.03815	1	3	240.17	-.0247	-.0933	-.0294	-.1264	-.0745	-.1840	.0887	.03932
D2A0S3	120.12	-.0986	-.1871	-.1635				.1497	.05117	1	4	20.27	.0172	-.0391	.0000	-.0775	-.1057	-.2003	.0733	.02505
D2A0S3	150.07	-.0739	-.0080	-.0285				.0368	.01257	1	4	40.24	.0392	.0052	-.0152	-.0840	-.0824	-.2355	.0769	.02628
D2A0S3	180.02	-.1067	-.1111	-.0329				.0836	.02855	1	4	60.20	-.1036	.0789	.1280	-.0481	-.1136	-.2226	.1158	.03957
D2A0S3	210.28	-.1403	-.0889	.0142				.0811	.02772	1	4	80.16	-.0665	-.1200	-.0307	-.0920	-.1595	-.2115	.1050	.03590
D2A0S3	240.23	-.1550	-.1061	-.0422				.1011	.03455	1	4	100.12	-.0411	-.0529	.0366	-.2135	-.1848	-.0921	.1035	.03538
D2A0S3	270.19	-.1718	-.1137	-.0494				.1114	.03807	1	4	120.08	-.0117	.0049	.0496	-.1061	-.2668	-.1732	.1020	.03487
DSA0S1	30.26	-.0196	.0695	.0304				.0385	.01316	1	4	140.05	-.0260	.0061	.0202	-.0416	-.2600	-.2084	.0937	.03202
DSA0S1	60.19	-.0142	.0471	.0373				.0329	.01124	1	4	160.01	-.0060	-.0119	.0022	-.0468	-.0695	-.1683	.0508	.01735
DSA0S1	90.12	.0399	-.0063	.0066				.1176	.04018	1	4	180.28	-.0113	.0172	.0098	-.0127	-.0917	-.1732	.0527	.01800
DSA0S1	120.05	.0555	.0517	-.0272				.0448	.01531	1	4	200.24	-.0268	.0026	.0220	-.0390	-.0939	-.1848	.0615	.02102
DSA0S1	150.31	.0393	.0664	.0290				.0449	.01534	1	4	220.21	-.0322	-.0672	.0239	-.0243	-.0909	-.1579	.0561	.01916
DSA0S1	180.24	.0365	.0593	.0305				.0421	.01438	1	4	240.17	-.0293	.0088	.0088	-.0524	-.0473	-.1863	.0513	.01754
DSA0S2	30.11	-.0455	.0625	.0382				.0488	.01667	1	4	260.13	-.0290	-.0011	-.0091	-.0578	-.1156	-.1865	.0665	.02273
DSA0S2	60.22	-.0652	-.0143	.0512				.0436	.01489	1	4	280.09	-.0272	-.0068	-.0059	-.0397	-.0587	-.1928	.0552	.01886
DSA0S2	90.02	.0706	-.1254	-.0200				.0720	.02461	1	8	20.30	.1583	.0373	.0494	.1235	.0752	.0824	.0877	.02997
DSA0S2	120.13	.0356	.0366	.0234				.0299	.01021	1	8	40.27	.0517	.0863	.0737	.1215	.0313	.0848	.0749	.02559
DSA0S2	150.25	.0492	.0468	.0138				.0366	.01251	1	8	60.24	-.2494	.1219	.0016	.1223	.0932	.0804	.1798	.06145
DSA0S2	180.05	.0309	.0715	.0362				.0462	.01579	1	8	80.21	-.0236	-.0960	-.0956	.0413	.0328	.0874	.0611	.02089
DSA0S2	210.16	.0296	.0687	.0627				.0537	.01834	1	8	100.18	.0340	-.0973	-.1033	-.0700	.4233	.1236	.1419	.04850
DSA0S2	240.27	.0324	.0736	.0642				.0567	.01938	1	8	120.15	-.0313	.0086	-.0022	-.0688	-.0578	.0138	.0304	.01040
DSA0S2	270.07	.0283	.0626	.0652				.0521	.01779	1	8	140.12	.0252	.0162	.0607	.0294	-.0882	.0455	.0442	.01511
DSA0S3	30.10	-.0349	.0366	.0360				.0358	.01224	1	8	160.09	.0278	.0298	.0208	.0771	-.0656	.0276	.0408	.01393
DSA0S3	60.21	-.0327	-.0560	.0578				.0488	.01668	1	8	180.06	.0225	.0157	.0309	.0771	.0879	.0443	.0331	.01130
DSA0S3	90.01	-.0260	-.1286	-.0369				.0638	.02181	1	8	200.03	.0390	.0760	.0281	.0867	.0195	.0450	.0469	.01602
DSA0S3	120.12	-.0210	-.0240	.0186				.0212	.00726	1	8	220.01	.0199	.0567	.0412	.0925	.0020	.0518	.0440	.01594
DSA0S3	150.22	-.0164	.0050	-.0260				.0161	.00549	1	8	240.30	-.0067	.0294	.0378	.0892	.0667	.0575	.0378	.01293
DSA0S3	180.02	.0013	-.0230	.0256				.0169	.00577	1	8	260.27	-.0060	.0248	.0281	.0964	.0896	.0582	.0372	.01271
DSA0S3	210.13	-.0023	-.0098	.0084				.0027	.00093	1	8	280.26	-.0092	.0179	.0388	.0584	.0146	.0587	.0330	.01126
DSA0S3	240.23	-.0106	-.0038	.0028				.0056	.00192											
DSA0S3	270.04	-.0108	-.0220	.0111				.0148	.00500											

Table 6.7. Qualitative comparison of computed and observed depth-versus-distance wave profiles

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

RUN	TIME	POSITIONS						AVERAGE	AVERAGE/DIA RATIO
		1	2	3	4	5	6		
1 9	20.00	.0920	-.0132	.0415	.0087	-.0472	.0413	.0407	.01389
1 9	40.16	.1079	-.0212	.0449	.0073	-.0485	.0490	.0464	.01587
1 9	60.24	-.1802	.1538	.3864	.0015	-.0440	.0437	.1349	.04610
1 9	80.32	-.1113	-.0549	-.0331	.0236	-.0396	.0520	.0521	.01780
1 9	100.07	.0278	-.0711	-.0705	-.1201	.0088	.1838	.0817	.02791
1 9	120.15	-.0320	.0065	.0300	-.1211	-.0813	-.0441	.0526	.01799
1 9	140.23	.0625	.0206	.1081	.0034	-.0944	.0236	.0521	.01780
1 9	160.31	.0317	-.0077	.0676	-.0030	-.1116	.0133	.0391	.01339
1 9	180.07	.0318	-.0136	.0390	-.0011	-.0165	.0448	.0245	.00837
1 9	200.15	.0151	.0172	.0606	-.0011	-.0019	.0432	.0232	.00792
1 9	220.23	.0095	.0216	.0589	.0060	-.0378	.0533	.0312	.01065
1 9	240.31	.0016	.0031	.0665	.0080	-.0227	.0397	.0236	.00806
1 9	260.06	-.0077	-.0084	.0497	.0141	-.0172	.0374	.0224	.00767
1 9	280.14	.0042	-.0099	.0446	-.0016	-.0123	.0438	.0193	.00658
1999	20.25	.2790	.2092	.2890	.2507	.1974	.2662	.2452	.08381
1999	40.19	.1587	.2071	.2904	.2296	.1957	.2687	.2250	.07690
1999	60.13	.0552	.2825	.2866	.2290	.1959	.2665	.2193	.07494
1999	80.07	-.1671	.1603	.2940	.0073	.1967	.2657	.3152	.10771
1999	100.01	-.0321	-.0376	.0376	.1017	.7066	.2684	.1973	.06744
1999	120.26	.0421	-.0214	.0143	-.0457	.1552	.0530	.1896	.06480
1999	140.20	.0692	.0358	.1224	.0204	.0145	.1237	.0643	.02198
1999	160.14	.0798	.0372	.1480	.0188	-.0245	.1251	.0724	.02473
1999	180.08	.0593	.0439	.1307	.0768	-.0447	.1150	.0784	.02680
1999	200.02	.0684	.0307	.1317	.0562	.0949	.1154	.0829	.02832
1999	220.27	.0492	.0744	.1384	.0697	.0413	.1025	.0793	.02718
1999	240.21	.0458	.0590	.1371	.0582	.0352	.0846	.0701	.02396
1999	260.15	.0418	.0580	.1370	.0582	.0509	.0924	.0727	.02484
1999	280.09	.0432	.0406	.1272	.0469	.0407	.0931	.0653	.02231
1 10	20.16	.1979	-.0115	.0577	.0237	-.0355	.0644	.0651	.02225
1 10	40.08	.1753	.0629	.0716	.0319	-.0266	.0677	.0727	.02483
1 10	60.16	.0385	.0191	.2653	.0343	-.0286	.0647	.1417	.04344
1 10	80.00	-.1805	.1375	.1671	.2777	-.0217	.0694	.1437	.04909
1 10	100.16	-.0819	-.0420	-.0350	.0008	.0743	.0701	.1514	.05172
1 10	120.01	-.0169	.0341	-.0430	-.1172	-.0048	.0288	.01398	.01398
1 10	140.17	.0357	.0097	.0911	-.0387	-.0886	-.0305	.0490	.01676
1 10	160.01	.0229	-.0047	.0843	-.0167	-.1276	-.0205	.0461	.01576
1 10	180.17	.0148	-.0060	.0566	.0260	-.0240	.0089	.0226	.00772
1 10	200.01	.0035	.0345	.0593	.0295	.0093	.0190	.0258	.00883
1 10	220.17	-.0110	.0192	.0569	.0306	-.0221	.0252	.0275	.00940
1 10	240.01	-.0089	.0165	.0632	.0285	-.0247	.0259	.0279	.00955
1 10	260.17	-.0199	.0004	.0510	.0297	-.0198	.0371	.0265	.00904
1 10	280.01	-.0224	-.0088	.0380	.0208	-.0203	.0377	.0247	.00843
1 10	300.17	-.0222	-.0143	.0427	-.0029	-.0192	.0428	.0240	.00821
1 10	320.01	-.0244	-.0297	.0382	.0075	-.0203	.0403	.0267	.00914
1 10	340.10	-.0259	-.0314	.0137	.0035	-.0299	.0376	.0237	.00809
1 10	360.02	-.0234	-.0379	.0201	-.0144	-.0394	.0346	.0283	.00967
1 10	380.10	-.0307	-.0422	.0188	-.0244	-.0406	.0316	.0314	.01073

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

RUN	TIME	POSITIONS						AVERAGE	AVERAGE/DIA RATIO
		1	2	3	4	5	6		
1 11	20.11	.1394	-.0221	.0713	-.0022	-.0418	.0286	.0509	.01740
1 11	40.21	-.0633	.1631	.0493	.0016	-.0465	.0278	.0586	.02902
1 11	60.32	-.0593	-.1490	.1739	.2207	-.0532	.0266	.1138	.03888
1 11	80.10	.0292	-.0779	-.0379	-.1330	.1346	.0332	.0741	.02534
1 11	100.20	.0063	-.0279	.0621	-.0145	-.1646	.0356	.0518	.01771
1 11	120.31	.0424	-.0350	.0656	.0016	-.1607	.0093	.0524	.01791
1 11	140.09	.0346	-.0188	.0517	.0195	-.0538	.0101	.0314	.01074
1 11	160.19	.0164	-.0225	.0498	.0217	-.0300	.0370	.0295	.01010
1 11	180.30	.0224	-.0088	.0663	.0152	-.0434	.0357	.0318	.01888
1 12	20.18	.0726	-.0396	.1008	-.0179	-.0568	.0507	.0564	.01927
1 12	40.04	.0438	.0739	.0983	-.0114	-.0587	.0110	.0495	.01692
1 12	60.22	-.1568	-.0228	.2105	.1289	-.0498	.0927	.1102	.03767
1 12	80.00	-.1015	-.2305	-.0126	-.1204	.0192	.0390	.0872	.02980
1 12	100.25	-.0180	.0795	.0350	-.2401	.1327	.0728	.0963	.03292
1 12	120.11	.0241	-.0408	.1034	-.0015	-.2270	-.0163	.0822	.02808
1 12	140.29	-.0052	-.0471	.0036	-.0207	-.1588	-.0366	.0587	.02905
1 12	160.15	-.0133	-.0648	.0640	-.0362	-.0428	.0070	.0374	.01277
1 12	180.01	.0031	-.0147	.0763	-.0230	-.0648	.0049	.0311	.01064
1 12	200.19	-.0184	-.0355	.0797	-.0444	-.0717	.0414	.0485	.01658
1 12	220.05	-.0361	-.0553	.0892	-.0428	-.0669	.0378	.0547	.01868
1 12	240.23	-.0175	-.0585	.0691	-.0397	-.0643	.0290	.0463	.01584
1 12	260.09	-.0248	-.0565	.0767	-.0568	-.0699	.0225	.0512	.01750
1 12	280.27	-.0164	-.0534	.0609	-.0353	-.0758	.0206	.0437	.01494
1 13	20.16	.1165	-.0366	.0571	-.0347	-.0348	.0634	.0572	.01953
1 13	40.01	.1412	.1064	.0656	-.0361	-.0365	.0316	.0696	.02378
1 13	60.17	.1178	.1131	.2614	.1099	-.0716	.0033	.1128	.03856
1 13	80.03	.1193	.0236	.1994	.0228	.1467	.0526	.0941	.03215
1 13	100.19	-.0392	-.0564	.2304	-.0694	.0577	.2245	.1129	.03859
1 13	120.04	.0251	-.0855	.1314	-.0447	-.0311	.1519	.0783	.02675
1 13	140.20	.0934	-.0232	.1058	-.1210	-.1514	.0538	.0915	.03126
1 13	160.06	.0037	.0538	.1025	-.0248	-.0261	-.0198	.0518	.01770
1 13	180.22	.0373	.0465	.1146	.0159	.0040	.0089	.0378	.01293
1 13	200.07	.0370	.0453	.1126	.0257	-.0178	.0208	.0432	.01476
1 13	220.23	.0235	.0143	.1135	-.0384	.0100	.0528	.0421	.01438
1 13	240.09	.0377	.0365	.1274	-.0037	-.0106	.0553	.0452	.01545
1 13	260.25	.0385	.0076	.1049	-.0031	-.0096	.0596	.0373	.01273
1 13	280.10	.0393	.0177	.0963	-.0200	.0080	.0598	.0373	.01276
19913	20.12	.0213	-.0311	.0523	-.0113	-.0322	.0418	.0317	.01082
19913	40.23	.0568	-.0378	.0615	-.0123	-.0334	.0516	.0422	.01444
19913	60.04	.0985	.0156	.1774	-.0069	-.0333	.0529	.0641	.02191
19913	80.16	.1314	.0228	.1241	-.0068	-.0333	.0512	.0616	.02106
19913	100.20	-.0095	-.0752	.2268	-.0521	-.0380	.1594	.0935	.03196
19913	120.09	-.0163	-.0175	.1150	-.0694	-.0099	.0663	.0776	.02650
19913	140.20	.0751	-.0539	.1106	-.1131	-.1793	.0319	.0940	.03211
19913	160.02	.0534	.0021	.0642	-.0344	-.0469	.0029	.0340	.01161
19913	180.13	.0325	.0568	.0968	-.0149	-.0361	.0146	.0420	.01434
19913	200.25	.0223	.0145	.0814	.0214	-.0596	.0299	.0382	.01395
19913	220.06	.0028	.0050	.1129	.0203	.0107	.0413	.0335	.01145
19913	240.10	-.0006	-.0059	.0778	.0019	-.0033	.0522	.0236	.00805
19913	260.29	.0227	-.0126	.0781	-.0181	-.0069	.0544	.0316	.01081
19913	280.10	.0097	-.0060	.0627	-.0130	-.0148	.0548	.0268	.00916

Table 6.8. Qualitative comparison of computed and observed depth-versus-distance wave profiles.

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

RUN	TIME	POSITIONS						AVERAGE	AVERAGE/DIA RATIO
		1	2	3	4	5	6		
0 1	20.05	.0122	-.0462	.0497	-.0250	-.0620	.0500	.0395	.01349
0 1	40.11	.0716	-.0494	.0351	-.0192	-.0626	.0494	.0479	.01637
0 1	60.16	-.0996	.0620	.0442	-.0306	-.0627	.0569	.0445	.01519
0 1	80.22	.0102	-.0185	.0224	.0672	-.0637	.0554	.0529	.01809
0 1	100.27	.0291	-.0350	.0296	-.0955	.1174	.0532	.0599	.02048
0 1	120.32	.0454	-.0369	.0835	-.1141	-.1306	.1163	.0878	.03001
0 1	140.04	-.0684	.0039	.1195	-.0297	-.1455	.0520	.0698	.02386
0 1	160.10	-.0429	-.0150	.0639	-.0422	-.1621	.0441	.0617	.02108
0 1	180.15	.0342	-.0233	.0584	-.0597	-.2009	.0497	.0710	.02428
0 1	200.21	.0377	.0044	.0363	-.0397	-.1674	.0284	.0523	.01787
0 1	220.26	.0298	.0386	.0639	-.0034	-.0746	.0189	.0382	.01335
0 1	240.31	.0169	.0450	.0736	-.0056	-.0642	.0325	.0396	.01355
0 1	260.03	.0078	.0330	.0797	.0084	-.0373	.0595	.0376	.01285
0 1	280.09	.0937	.0272	.0785	.0037	-.0273	.0638	.0340	.01163
0 1	300.14	.0983	.0180	.0676	.0000	-.0282	.0768	.0332	.01133
0 1	320.20	.0160	.0177	.0586	-.0124	-.0285	.0798	.0355	.01214
0 1	340.25	.0179	.0185	.0547	-.0113	-.0348	.0705	.0333	.01137
0 1	360.30	.0172	.0025	.0543	-.0097	-.0413	.0710	.0327	.01116
0 1	380.02	.0121	-.0017	.0543	-.0080	-.0396	.0692	.0308	.01054
9 1	20.09	.0226	-.0482	.0165	.0514	-.0406	.0119	.0319	.01089
9 1	40.19	.0701	-.0459	.0211	.0527	-.0359	.0141	.0400	.01366
9 1	60.20	-.0507	.1008	.0180	.0517	-.0389	.0162	.0594	.02029
9 1	80.03	-.0066	-.0953	.1781	.0542	-.0347	.0166	.0643	.02196
9 1	100.12	.0066	-.0512	-.0308	-.0017	-.0323	.0205	.0239	.00815
9 1	120.22	.0511	-.0165	.0224	.0324	.0073	.0177	.0246	.00840
9 1	140.31	.0390	-.0262	.0363	.0302	-.1059	.0057	.0420	.01436
9 1	160.05	.0894	-.0355	.0210	.0579	-.0608	.0039	.0328	.01120
9 1	180.15	.0216	-.0015	.0190	.0603	-.0172	.0008	.0201	.00586
9 1	200.24	.0165	-.0181	.0199	.0604	-.0221	.0212	.0250	.00855
9 1	220.34	.0111	-.0180	.0346	.0556	-.0313	.0240	.0291	.00995
9 1	240.00	.0139	-.0273	.0336	.0655	-.0303	.0274	.0330	.01120
9 1	260.17	.0156	-.0340	.0266	.0583	-.0211	.0298	.0309	.01056
9 1	280.27	.0221	-.0297	.0225	.0593	-.0192	.0315	.0307	.01049
9 2	20.14	.0134	-.0497	.0220	-.0181	-.0409	-.0152	.0266	.00908
9 2	40.20	.1320	-.0439	.0210	-.0178	-.0372	-.0105	.0437	.01494
9 2	60.00	.0952	.1786	.0196	-.0184	-.0385	-.0111	.0602	.02058
9 2	80.21	-.0253	-.0357	.0910	-.0130	-.0414	-.0066	.0856	.02926
9 2	100.02	-.1095	-.1297	.0312	.0249	-.0339	-.0085	.0561	.01918
9 2	120.15	-.0760	-.0582	-.0362	-.1291	.2928	-.0046	.0995	.03399
9 2	140.29	.0543	-.0598	.0281	-.0897	-.1254	-.0155	.0621	.02124
9 2	160.09	.0170	-.0183	.0280	-.0684	-.1752	-.0519	.0599	.02048
9 2	180.23	.0511	-.0458	.0248	-.0172	-.0606	-.0759	.0472	.01614
9 2	200.03	.0070	.0237	.0284	-.0024	-.0216	-.0407	.0193	.00660
9 2	220.17	-.0008	.0212	.0464	-.0025	-.0400	-.0263	.0229	.00782
9 2	240.31	.0059	-.0035	.0568	.0233	-.0518	-.0115	.0255	.00870
9 2	260.11	.0204	-.0119	.0501	.0129	-.0212	-.0071	.0206	.00704
9 2	280.25	.0143	-.0126	.0374	.0093	-.0061	.0056	.0142	.00485
9 2	300.05	.0123	-.0190	.0234	-.0006	-.0087	.0198	.0140	.00478
9 2	320.19	.0195	-.0185	.0284	-.0066	-.0146	.0187	.0177	.00605
9 2	340.32	.0223	-.0235	.0381	.0014	-.0124	.0174	.00593	.00593
9 2	360.13	.0202	-.0203	.0264	.0043	-.0161	.0180	.0162	.00555
9 2	380.26	.0180	-.0225	.0368	.0073	-.0272	.0042	.0193	.00658

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

RUN	TIME	POSITIONS						AVERAGE	AVERAGE/DIA RATIO
		1	2	3	4	5	6		
9 3	20.16	-.0002	-.0481	.0185	-.0277	-.0652	-.0419	.0336	.01148
9 3	40.32	.0119	-.0483	.0259	-.0243	-.0616	-.0398	.0353	.01266
9 3	60.13	-.0017	-.0146	.0256	-.0230	-.0573	-.0380	.0267	.00913
9 3	80.29	.0045	-.0301	.0169	-.0375	-.0609	-.0372	.0312	.01066
9 3	100.10	.0230	-.0302	.0380	-.0430	-.0479	-.0345	.0361	.01234
9 3	120.25	.0176	-.0371	.0486	-.0203	-.1163	-.0427	.0471	.01610
9 3	140.07	.0068	-.0436	.0298	-.0196	-.0609	-.0391	.0333	.01139
9 3	160.22	.0118	-.0255	.0366	-.0142	-.0346	-.0444	.0279	.00952
9 3	180.03	.0033	-.0205	.0348	-.0168	-.0601	-.0288	.0274	.00936
9 4	20.19	.0055	-.0469	.0174	-.0118	-.0467	.0157	.0240	.00819
9 4	40.05	.0424	-.0405	.0190	-.0126	-.0507	.0189	.0307	.01048
9 4	60.23	.0405	.0441	.0231	-.0014	-.0434	.0247	.0309	.01054
9 4	80.09	-.0029	.0230	.0770	-.0333	-.0463	.0261	.0348	.01188
9 4	100.20	-.0341	-.0364	.0425	-.0166	-.0425	.0267	.0331	.01132
9 4	120.14	.0201	-.0352	.0192	-.0627	-.0655	.0609	.0439	.01502
9 4	140.32	.0531	-.0362	.0348	-.0534	-.1681	-.0076	.0589	.02011
9 4	160.18	.0327	-.0319	.0299	-.0208	-.0776	-.0387	.0386	.01319
9 4	180.14	.0180	.0159	.0564	-.0093	-.0374	-.0081	.0193	.00661
9 4	200.23	.0077	.0099	.0406	.0063	-.0464	-.0094	.0201	.00685
9 4	220.00	.0039	.0060	.0594	.0198	-.0422	.0081	.0233	.00795
9 4	240.27	.0183	-.0118	.0489	.0042	-.0163	.0304	.0203	.00694
9 4	260.13	.0156	-.0102	.0326	-.0012	-.0136	.0362	.0182	.00623
9 4	280.32	.0124	-.0116	.0244	-.0055	-.0143	.0364	.0174	.00596

THE MEAN OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .02347
 THE STANDARD DEVIATION OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .01885
 THE STANDARD DEVIATION OF ALL DIFFERENCES = .09861

Table 6.9. Summary of data on CSU Experimental Waves.

RUN	SLOPE	PROPORTIONAL BASE DEPTH H/DIA	BASE FLOW DISCHARGE (CFS)	PEAK FLOW DISCHARGE (CFS)	PEAK/ BASE	WAVE DURATION (SEC)	WAVE VOLUME (CU-FT)
02A051	.0001100	.21666	1.690	5.070	2.41075	27.000	31.215
02A052	.0001100	.21666	1.540	13.020	8.45455	67.000	382.415
02A053	.0001100	.20846	1.470	16.290	10.45156	78.000	579.665
03A051	.0001100	.37660	4.180	6.940	1.66029	24.000	41.715
03A052	.0001100	.35131	3.490	15.110	4.33086	59.000	373.435
03A053	.0001100	.38993	4.390	17.930	4.12184	85.000	707.330
02A051	.0005500	.16198	2.580	20.330	7.87984	77.000	621.690
02A052	.0005500	.16434	2.410	26.250	10.89212	95.000	985.745
02A053	.0005500	.35640	5.070	14.380	2.83629	42.000	199.295
03A051	.0005500	.36190	4.910	21.920	4.46436	61.000	552.945
03A052	.0005500	.35268	4.990	28.270	5.66533	80.000	924.525
03A053	.0005500	.14729	2.990	19.230	6.43144	69.000	595.050
02A051	.0010500	.15412	2.590	18.500	7.42800	70.000	558.800
02A052	.0010500	.17292	2.990	32.070	10.99331	97.000	1549.860
02A053	.0010500	.17216	7.610	21.450	2.81966	67.000	385.065
03A051	.0010500	.34892	6.940	33.510	4.82853	90.000	1235.195
03A052	.0010500	.33991	6.330	39.370	6.21959	104.000	1772.535
03A053	.0010500	.34482	6.990	28.071	4.11474	74.470	808.329
1 3	.0009900	.34311	7.590	32.582	4.34717	90.000	1198.515
1 4	.0009900	.20812	1.915	24.396	12.75290	85.470	896.639
1 8	.0009900	.20812	2.642	26.174	10.05925	89.000	1033.399
1 9	.0009900	.21290	1.914	30.484	33.72119	95.000	1385.264
19909	.0009900	.28128	2.084	20.326	13.59221	110.000	1348.131
1 10	.0009900	.34994	6.833	22.912	3.34329	60.000	486.610
1 12	.0009900	.34652	6.031	30.085	4.88767	80.500	993.741
1 13	.0009900	.35849	7.280	36.539	5.01391	120.000	1936.801
19913	.0009900	.36466	7.466	36.336	4.94717	110.000	1766.589
9 1	.0004800	.38817	8.439	20.779	3.82357	100.000	1664.597
9 2	.0004800	.24981	2.472	13.077	5.49213	67.000	345.416
9 3	.0004800	.25182	2.536	21.330	8.41970	94.000	990.301
9 4	.0004800	.36396	8.133	19.214	3.62284	83.000	277.084
9 4	.0004800	.36803	8.292	22.348	4.22397	107.000	1094.823

Table 6.10. Summary of data on Wallingford experimental waves.

RUN	SLOPE	PROPORTIONAL BASE DEPTH H/DIA	BASE FLOW DISCHARGE (CFS)	PEAK FLOW DISCHARGE (CFS)	PEAK/ BASE	WAVE DURATION (SEC)	WAVE VOLUME (CU-FT)
57	.0242	.4730	.04150	.0839	2.0217	67.650	1.565
58	.0242	.4730	.04150	.0846	2.0385	36.665	.852
60	.0242	.4730	.04150	.0653	1.5735	67.650	.878
61	.0242	.4730	.04150	.0653	1.5735	36.665	.476
62	.0242	.4730	.04150	.0653	1.5735	18.337	.238
83	.0222	.4730	.02740	.0882	3.2190	18.337	.618
84	.0222	.4730	.02740	.0875	3.1934	9.196	.321
85	.0222	.4730	.02740	.0754	2.7518	67.650	1.771
86	.0222	.4730	.02740	.0717	2.6168	36.665	.886
87	.0222	.4730	.02740	.0719	2.5875	18.337	.436
88	.0222	.4730	.02740	.0712	2.5985	9.196	.219
89	.0222	.4730	.02740	.0534	1.9489	67.650	.959
90	.0222	.4730	.02740	.0531	1.9380	36.665	.515
91	.0222	.4730	.02740	.0525	1.9161	18.337	.251
92	.0222	.4730	.02740	.0524	1.9124	9.196	.125
94	.0222	.2320	.03695	.0687	9.8849	67.650	2.277
98	.0222	.2320	.03695	.0552	7.9137	67.650	1.771
99	.0222	.2320	.03695	.0513	7.3813	36.665	.886
102	.0222	.2320	.03695	.0332	4.7482	67.650	.961
103	.0222	.2320	.03695	.0327	4.7050	36.665	.515
114	.0210	.4730	.01890	.0521	2.7566	67.650	1.225
115	.0210	.4730	.01890	.0532	2.8148	36.665	.686
116	.0210	.4730	.01890	.0530	2.8042	18.337	.341
117	.0210	.4730	.01890	.0531	2.8295	9.196	.171
118	.0210	.4730	.01890	.0469	2.4815	67.650	1.133
119	.0210	.4730	.01890	.0465	2.4623	36.665	.552
120	.0210	.4730	.01890	.0475	2.5132	18.337	.286
121	.0210	.4730	.01890	.0469	2.4815	9.196	.143
122	.0210	.4730	.01890	.0343	1.8148	67.650	.568
124	.0210	.4730	.01900	.0347	1.8561	36.665	.316
125	.0210	.4730	.01890	.0345	1.8254	9.196	.078
126	.0210	.2320	.03470	.0380	0.2555	44.265	.941
127	.0210	.2320	.03470	.0379	0.1638	67.650	1.225
128	.0210	.2320	.03470	.0390	0.2979	36.665	.686
130	.0210	.2320	.03470	.0388	0.2593	18.337	.341
131	.0210	.2320	.03470	.0327	6.9574	67.650	1.133
132	.0210	.2320	.03470	.0323	6.8723	36.665	.552
134	.0210	.2320	.03470	.0333	7.1873	18.337	.286
136	.0210	.2320	.03470	.0281	4.2766	67.650	.648
136	.0210	.2320	.03470	.0285	4.3617	36.665	.316
136	.0210	.2320	.03470	.0287	4.4843	18.337	.168

The peak depth versus distance and time relations of the computed and observed waves were compared by determining differences between the computed and observed points. These differences are averaged over six points in the CSU data and over five points in the Wallingford data; ratios of these average differences to the conduit diameter are also computed. The differences, average differences, and ratios of the average difference to the conduit diameter are presented in Table 6.11 for the CSU data. The table on the left shows the results of peak depth versus distance relations, and the table on the right shows the results of the peak depth versus time relations. The corresponding results for the Wallingford data are given in Table 6.12.

For the CSU data, the means of the ratio of average differences to conduit diameter for peak depths versus distance and time are 0.0177 and 0.0213, respectively. For the Wallingford data, the corresponding figures are 0.0216 and 0.0223. Similarly, the standard deviation of all differences divided by the diameter for the CSU data is 0.0253 for peak depth versus distance and 0.0316 for peak depth versus time. The corresponding figures for the Wallingford data are 0.0209 and 0.0251. The consistency of these figures show a definite reliability in computational schemes presented in this study to predict the rates of attenuation of flood peaks with distance and time.

Differences in the comparisons of the computed and observed peaks of the Wallingford data, as shown in Table 6.12 and by graphs in Appendix 5, may be explained in part by the reliability of the data from the Wallingford report. The experimental procedure at Wallingford employed a calibrated butterfly valve to obtain the inflow discharge hydrograph. Only depth recordings were reported to have been made during the tests. Test runs were made with four slopes, with four base flows, and with three or four peak discharges. The Wallingford data of the maximum depth versus distance show a rapid drop at the end of the computed curves (Appendix 5), while similar CSU data (Appendix 4) show a rise at the end point. These differences can be explained by the different ways in which the peak depths were computed. For the CSU data the base depths were subtracted from the maximum depths, and the base depths were taken from the initial condition of the M2 backwater curves, with a sharp drop at the conduit end.

The Wallingford data have only one maximum depth value for each position and particular time. These maximum depths were limited only to five points from $x = 0$ to $x = 290$ ft, because the total length of pipe was only 300 ft. The large pipe friction factors ($f = 0.027$ to 0.045) and the large length to diameter ratio ($x/D = 1200$) of the small Wallingford conduit system with a diameter of 0.264 ft, produced flow depths that, except at the outlet, were essentially normal depths throughout the conduit. Since the maximum peak depths were not measured at the outlet, and since the base flow depths were virtually a constant value, the subtraction of the base flow depths were virtually a constant value, the subtraction of the base flow depths for Wallingford data has no effect on the attenuation curves, except to somewhat lower ordinates.

In the CSU conduit system, a maximum peak depth and the corresponding time were recorded for all distance positions from $x = 0$ to $x = a$ point at the free overfall. The method of determining the peak depth values is as follows. First, the variables

Table 6.11. Comparison of data for wave-peak versus distance and time for CSU experimental data.

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

R.T.	POSITIONS						AVERAGE	AVERAGE DIA RATIO
	1	2	3	4	5	6		
B2ARS1	.0263	.0785	.0581				.0543	.0785
B2A052	.0217	-.0179	-.0260				.0135	.02461
B2A053	.0139	-.0522	.0190				.0283	.02958
B3A051	.0101	.0290	-.0447				.0279	.02954
B3A052	-.0239	-.0178	.0216				.0231	.02586
B3A053	-.0134	-.0276	-.0071				.0160	.02548
C2A052	.0996	-.0334	.0678				.0573	.01947
C2A053	.0340	.0140	.0650				.0377	.01287
C3A051	-.0452	-.0527	.0435				.0405	.01383
C3A052	-.0849	-.0359	.0229				.0734	.02454
C3A053	-.0398	-.0792	.0364				.0518	.01771
D2ARS1	.0352	-.0219	-.0014				.0195	.02666
D2A051	.0214	-.0541	.0317				.0357	.01222
D2A053	.0467	-.0800	.0122				.0463	.01582
D3A051	-.0455	-.0743	-.0188				.0462	.01579
D3A052	-.0732	-.0638	.0151				.0830	.02837
D3A053	-.0827	-.0342	.0190				.0786	.02688
1 3	-.0136	-.0289	.0272	-.0782	-.0422	-.0547	.0570	.01947
1 4	.0301	.0437	-.0121	-.0682	-.0442	-.0373	.0546	.01866
1 8	-.0825	-.0668	-.0765	-.0726	-.0154	-.0148	.0306	.03438
1 9	-.0874	.0498	-.0937	-.0496	-.0012	-.0439	.0543	.01854
19909	-.2945	-.0614	-.0906	-.0265	-.0556	-.0261	.0425	.04869
1 10	-.0048	.0162	-.0037	.0460	.0724	-.0659	.0682	.02333
1 11	-.0507	-.0634	-.0159	-.0680	-.0243	-.0173	.0386	.01323
1 12	-.0644	-.0543	-.0129	-.0189	-.0485	-.0332	.0590	.02017
1 13	.0906	.0505	.0682	-.0334	.0145	.0148	.0190	.03725
19913	.0577	.0664	.0788	-.0537	-.0324	.0237	.0836	.02858
8 1	.0391	.0395	.0576	-.0156	-.0638	-.0085	.0360	.01231
9 1	-.0253	-.0053	-.0138	.0311	-.0362	-.0088	.0187	.00641
9 2	-.0236	-.0295	-.0259	-.0841	-.0845	-.0271	.0486	.01859
9 3	-.0145	-.0036	-.0085	-.0511	-.0885	-.0413	.0336	.01147
9 4	.0188	.0448	-.0080	-.0244	-.0427	-.0398	.0271	.00926

THE MEAN OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .01760
 THE STANDARD DEVIATION OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .00890
 THE STANDARD DEVIATION OF ALL DIFFERENCES = .07398

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

R.T.	POSITIONS						AVERAGE	AVERAGE DIA RATIO
	1	2	3	4	5	6		
B2ARS1	.0207	.0748	.0547				.0521	.01771
B2A052	-.0072	-.0349	-.0242				.0221	.01756
B2A053	-.0539	-.0379	.0111				.0359	.01058
B3A051	.0177	.0287	-.0674				.0345	.01182
B3A052	-.0343	-.0431	.0199				.0324	.01139
B3A053	-.0379	-.0391	-.0186				.0318	.01088
C2A052	.0476	-.0349	.0762				.0356	.02135
C2A053	-.0590	-.0644	.0315				.0850	.02924
C3A051	-.0602	-.0406	.0451				.0486	.01662
C3A052	-.0818	-.0362	.0261				.0714	.02438
C3A053	-.0542	-.0754	.0256				.0518	.01759
D2ARS1	.0326	-.0238	.0322				.0185	.01633
D2A051	.0233	-.0519					.0374	.01280
D2A053	-.0381	-.0560					.0520	.01778
D3A051	-.0566	-.0371	-.0453				.0827	.02825
D3A052	-.0737	-.0810	.0183				.0910	.03139
D3A053	-.0233	-.0650	.0141				.0342	.00802
1 3	-.0047	-.0140	.0330	-.0631	-.0629		.0355	.01214
1 4	.0167	.0443	-.0148	-.0492	-.0213	-.01724	.0531	.01814
1 8	-.0628	.0414	-.0397	-.0181	-.0358	-.01990	.0945	.03228
1 9	-.0644	.0602	-.0561	-.0490	.0049		.0469	.01633
19909	-.2491	-.0418	-.0177	-.0541	-.0848		.0455	.04973
1 10	.0564	.0546	.0205	.0593	.0481		.0078	.03031
1 11	-.0274	-.0512	-.0078	-.0562	-.0149		.0315	.01076
1 12	-.0790	-.0388	.0019	-.0989	-.0253	-.0313	.0459	.01568
1 13	.0295	.0838	.0176	.0669	.0480	.0134	.0465	.05038
19913	.0251	.0147	.0140	-.0380	-.0279	.0201	.0133	.03531
8 1	.0516	.0560	.0812	-.0365	0.		.0488	.01668
9 1	.0037	.0048	.0071	.0513	-.0387	-.0135	.0186	.00634
9 2	.0479	-.0121	.0090	-.0785	-.0682	-.0471	.0438	.01496
9 3	-.0016	.0115	.0164	-.0465	-.0560	-.0493	.0285	.00975
9 4	.0146	.0561	.0291	.0135	0.		.0273	.00933

THE MEAN OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .02127
 THE STANDARD DEVIATION OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .01529
 THE STANDARD DEVIATION OF ALL DIFFERENCES = .09296

Table 6.12. Comparison of data for wave-peak versus distance and time for Wallingford experimental data.

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

RUN	1	2	POSITIONS 3	4	5	AVERAGE	AVERAGE/DIA RATIO
87	.0135	.0100	.0111	.0070	.0050	.0096	.03651
88	.0159	.0050	.0092	.0003	-.0003	.0063	.02379
89	.0064	.0036	.0035	.0023	.0006	.0033	.01247
90	.0149	.0106	.0090	.0063	.0041	.0106	.04009
91	.0165	.0124	.0102	.0022	.0011	.0085	.03211
92	.0051	.0120	.0191	.0015	.0000	.0077	.02921
93	.0060	.0197	.0144	.0070		.0122	.04606
94	.0052	.0074	-.0027	.0004	-.0010	.0035	.01321
95	.0077	.0050	.0037	.0059	-.0050	.0054	.02063
96	.0121	.0129	.0209	-.0017		.0119	.04505
97	.0119	.0107	.0079			.0102	.03853
98	.0060	.0049	.0050	.0000	.0042	.0059	.02245
99	-.0021	.0054	.0012	.0007	.0011	.0021	.00700
100	-.0044	-.0042	.0004			.0030	.01120
101	.0020	.0035	.0020			.0030	.01121
102	.0063	-.0004	-.0034	-.0013	-.0026	.0020	.01061
103	.0229	.0073	.0066	.0001	-.0024	.0079	.02970
104	.0115	.0140	.0065	.0039	.0050	.0082	.03101
105	.0026	.0015	-.0005	-.0007	-.0037	.0010	.00602
106	.0037	.0103	.0030	.0045	.0002	.0043	.01645
107	.0122	.0002	.0061	.0004		.0067	.02550
108	.0095	.0102	.0100	.0106		.0101	.03817
109	.0000	.0030	.0040			.0056	.02104
110	.0154	.0059	.0036			.0083	.03130
111	.0006	-.0004	.0059	.0070		.0055	.02075
112	.0002	-.0014	.0023			.0040	.01510
113	.0100	.0040	.0142			.0097	.03661
114	.0035	.0002	.0007			.0060	.02501
115	.0042	.0013	.0041	.0002		.0045	.01606
116	.0069	.0100	.0091			.0087	.03204
117	.0050	.0003				.0031	.01172
118	.0031	.0044	-.0000	-.0001		.0021	.00800
119	.0055	.0020	.0065	.0001	.0011	.0030	.01152
120	.0009	.0040	.0073	.0029	-.0025	.0053	.02001
121	-.0079	.0005	-.0019	-.0017		.0050	.01904
122	.0022	.0047	.0044	.0010		.0033	.01257
123	.0005	.0023	-.0010	-.0051		.0044	.01676
124	.0029	.0003	-.0023	-.0007		.0036	.01349
125	.0036	-.0019	-.0029			.0025	.00929
126	.0023	.0016	.0021	-.0002		.0015	.00507
127	.0045	.0025	.0000			.0026	.00904

THE MEAN OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .02164
 THE STANDARD DEVIATION OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .01124
 THE STANDARD DEVIATION OF ALL DIFFERENCES = .00552

- DIFFERENCES IN FEET BETWEEN OBSERVED AND COMPUTED POINTS -

RUN	1	2	POSITIONS 3	4	5	AVERAGE	AVERAGE/DIA RATIO
87	.0125	.0092	.0110	.0102	.0105	.0100	.04009
88	.0177	.0094	.0134	.0034		.0110	.04153
89	.0052	.0029	.0041	.0033	.0023	.0036	.01347
90	.0135	.0191	.0106	.0071	.0045	.0109	.04147
91	.0104	.0146	.0111	.0025	.0015	.0096	.03644
92	.0100	.0176	.0173	.0011	.0011	.0094	.03570
93	.0160	.0239	.0125	.0079		.0153	.05734
94	-.0067	-.0010	-.0066	.0011	.0001	.0033	.01233
95	-.0009	.0053	.0051	.0075	-.0054	.0040	.01032
96	.0083	.0161	.0203	-.0036		.0121	.04573
97	.0105	.0131	.0072			.0129	.04897
98	.0030	.0044	.0075	.0103	.0055	.0061	.02324
99	-.0076	.0050	.0019	.0009	.0012	.0033	.01251
100	-.0044	-.0029	.0025			.0032	.01224
101	.0050	.0052	.0040			.0050	.01800
102	-.0150	-.0001	.0031	.0049	.0067	.0059	.02252
103	.0110	.0030	.0090	.0030	.0007	.0049	.01862
104	-.0005	.0130	.0071	.0060	.0004	.0072	.02712
105	-.0067	-.0043	-.0010	-.0003	-.0031	.0031	.01164
106	.0003	.0094	.0022	.0050	.0001	.0035	.01340
107	.0075	-.0050	.0035	.0057	-.0027	.0051	.01910
108	.0025	.0139	.0070	.0009	.0060	.0070	.02940
109	.0001	.0005	.0035	.0053		.0066	.02502
110	.0167	.0097	.0050	.0042		.0089	.03370
111	.0050	-.0042	.0024	.0039	-.0031	.0035	.01322
112	-.0003	-.0017	-.0004			.0000	.00309
113	.0097	.0069	.0125	.0099		.0097	.03690
114	.0046	.0111	.0097			.0085	.03202
115	-.0011	-.0022	.0010	.0062		.0026	.00993
116	.0019	.0096	.0074	.0031		.0055	.02000
117	.0042	.0015				.0020	.01076
118	-.0106	-.0001	-.0039	-.0020		.0042	.01581
119	-.0030	-.0035	.0053	-.0013	.0012	.0029	.01009
120	-.0015	.0027	.0055	.0011	-.0030	.0020	.01046
121	-.0070	.0046	-.0032	-.0016		.0043	.01621
122	-.0072	-.0013	.0079	.0011		.0031	.01180
123	.0001	-.0010	-.0032	-.0064		.0027	.01017
124	.0097	.0110	-.0042	-.0001		.0064	.02437
125	-.0030	-.0064	-.0025			.0040	.01533
126	-.0027	-.0003	.0025	.0001	-.0007	.0013	.00476
127	-.0034	-.0010	.0006			.0017	.00633

THE MEAN OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .02227
 THE STANDARD DEVIATION OF THE AVERAGE DIFFERENCE/DIAMETER RATIOS = .01315
 THE STANDARD DEVIATION OF ALL DIFFERENCES = .00663

defined as the maxima were set equal to zero for all points in space. Then for each Δt , and each position, the new depth values were compared with the corresponding previous maxima, and the new maxima were set equal to the new depths whenever they were greater than the previous maxima. The time of peak depth was then recorded. Since the computed peak depths at the outlet often show a slight increase over previous values, the outfall sharp drop of the base depth was able to influence the peak depth versus distance relations. Because the base depths are subtracted at the outlet, they may be grossly in error, and they may be very small relative to the base depths subtracted upstream. The effect is that at the end of the attenuation curves a small sharp rise at their downstream ends is shown.

The difference in the computing methods of the peak depth also shows why the CSU comparisons of peak depth versus time tend to rise and even curl back at the ends, while the corresponding Wallingford data do not. In the Wallingford data, peak depths were recorded at different times, so that each time has but one peak value. In the CSU data, however, peak depths were recorded for each distance position; and the times that the peaks were reached was recorded as secondary information. Thus, two peak depths may be recorded as occurring at the same instant in time at two different positions.

The comparison plots shown in Appendices 4 and 5 are nearly all consistent and require no additional explanation. However, in the CSU data of Appendix 4, there are several experimental runs that should be viewed with caution because of limitations or errors

in data. The inflow hydrograph of Run No. B2ARS1 had a Q_p/Q_b ratio of only 2.42 and the depth transducers at $x = 410.00$ ft and $x = 771.70$ ft were virtually unable to distinguish the wave from the base flow. Also, during Run No. B3AOS1, the recording system was prematurely shutoff so that no wave was recorded at $x = 771.70$ ft. Other irregular data are found in Run No. D2ARS1. It appears that a secondary wave was generated after the main experimental wave. This would not effect the peak comparisons but other comparisons for this run must be disregarded.

The inflow hydrograph of Run No. D3AOS3 has a point on the descending limb where the discharge jumps from 30 cfs to 39 cfs and then returns to 28 cfs, after which it continues to drop uniformly. Because of its erratic nature, it would first appear that this recording was a result of a malfunction in the orifice transducer. Closer examination, however, reveals that this unusual peak is observed at all three recording stations along the conduit. Therefore, it must be concluded that this wave actually occurred in this manner.

In summary, the comparison of CSU computed and observed data, and the comparison of Wallingford observed data and the corresponding CSU computed data on flood peak depth attenuations with distance and time in partly full circular conduits show similar agreements. Even though the experimental shifts and errors significantly affect these comparisons, for all practical purposes the numerically integration solutions of the partial differential equations approximate the flood peak attenuations of observed waves extremely well.

SIMPLIFIED METHODS OF FLOOD ROUTING

7.1 General Definitions and Descriptions of Simplified Methods

After writing the momentum equation, Eq. 5.2., in its particular form, then the steady uniform, the steady nonuniform, and the unsteady flow conditions can be defined as [7, p. 287]

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{V \partial V}{g \partial x} - \frac{1 \partial V}{g \partial t}$$

in which $S_f = fV^2/8gR$. Unsteady flow conditions are usually called the kinematic flow whenever a balance between gravitational and friction forces is achieved. This means that the derivatives in Eq. 5.2., or the derivatives in the above equation, are negligible when compared to the effect of gravity (measured by S_o) and the effect of friction (measured by S_f).

Therefore, Eq. 5.2 can be reduced to the simple form

$$S_f = S_o \quad (7.1)$$

with S_f the friction slope (or the slope of the energy line) and S_o the bottom slope. Therefore, the continuity equation, Eq. 5.1, is the primary equation governing unsteady flow, provided Eq. 7.1 is satisfied, with S_f given by Eq. 5.3.

The simplified flood routing methods generally called the "storage-routing methods" are those based only on the storage differential equation, or on the continuity or mass-conservation differential equation, Eq. 5.1, using the principle that for any reach of a channel the difference between the inflow and the outflow is equal to the stored or depleted water in a given time interval.

When the space is defined that contains the inflow, outflow, and water storage change (a fixed reach of the channel), Eq. 5.1 can be expressed as

$$P - Q = \frac{dW}{dt} = A \frac{dy}{dt} \quad (7.2)$$

in which P is the inflow discharge, Q is the outflow discharge, and W is the stored volume of water, with $dW = A dy$, where A is the area of channel water surface, and y is the average depth or elevation of that area. The inflow is given as $P = f(t)$, and the storage is generally given as $W = f(y)$, or $W = f(y_1, y_2, \dots)$ with $y_1, y_2, \dots$, consecutive depths. So, if $Q = f(y)$, then, by elimination, the function $W = f(Q)$ can be determined. In this case, Eq. 7.2 has only two variables, Q and t , in the form of a differential

equation, whose integration gives $Q = f(t)$.

Equation 7.2 generally serves for the computation of relations among five functions in any small channel reach: (1) inflow hydrograph, $P = f_1(t)$; (2) outflow hydrograph, $Q = f_2(t)$; (3) stage hydrograph, $y = f_3(t)$; (4) outflow rating curve, $Q = f_4(y)$, and (5) storage function, $W = f_5(y)$, with the five variables, Q, P, y, W , and t . When three of the five functions with boundary conditions are given or the three variables can be excluded, Eq. 7.2 facilitates the computation of the relation between the two remaining variables.

The basic conditions for applying Eq. 7.2 to flood routing are: (1) the storage of a channel reach responds in less time to any unsteady inflow or outflow, than the time unit generally used for integrating Eq. 7.2 by finite differences; (2) the wave is long, so that the change of the discharge is gradual; (3) the accuracy of the basic data and the required accuracy of the results do not justify any method that takes into consideration dynamic effects in unsteady flow, and (4) the velocity and its changes along the channel reach are relatively small, so that the dynamic effect is negligible in comparison with the storage effect during wave movements.

Equation 7.2 has been applied to flood routing studies in channels, with some adaptations for the more complex discharge-storage or stage-storage relations in a channel reach. Many integration methods have been developed.

The analytical integration method was used successfully by some authors with schematic inflow hydrographs and simple linear approximate relations of storage and outflow discharge. The formulas were obtained for computing the decrease of flash-flood peaks and relatively small water-surface fluctuations. The difficulties in fitting tractable mathematical expressions to natural inflow hydrographs, and the difficulties of analytical integration when these expressions become complex, limit this integration method to specific problems.

The numerical integration of Eq. 7.2 written in finite-difference form is

$$(P - Q) \Delta t = A \Delta y = \Delta W \quad (7.3)$$

in which P, Q , and A are mean values during the time interval Δt , and the corresponding level difference Δy . Using P_1 and P_2, Q_1 and Q_2 , and A_1 and A_2 , the values at the beginning and the end of Δt , with a linear change during a sufficiently small Δt , then becomes

$$\left[\frac{P_1 + P_2}{2} - \frac{Q_1 + Q_2}{2} \right] \Delta t = \frac{A_1 + A_2}{2} \Delta y = \Delta W \quad (7.4)$$

For known P_1, P_2, Q_1, A_1 , and Δt , and for known relations of W to Q , or A to y , and Q to y , it is possible by trial and error to determine the value Q_2 . Expressing Eq. 7.4 in different ways, especially by using storage factors, $(W + Q\Delta t/2)$ and $(W - Q\Delta t/2)$, as functions of Q , a trial and error method may be replaced by direct numerical integration.

Individual differences between investigators in arranging Eqs. 7.3 and 7.4 for step by step computations may account for differences of the tabular numerical procedures. Procedures vary according to which basic difference factor ($\Delta t, \Delta y, \Delta W$, or ΔQ) is known at the beginning of the equation and which factor must be determined. The accuracy of a method depends on the accuracy of selecting the basic difference factors (Δx and Δt), apart from the accuracy involved in the use of only one differential equation.

Equations 7.2 through 7.4 may be arranged in several combinations to facilitate graphical integration of a simple differential equation. There are numerous graphical procedures, with different shortcuts, for computing an outflow hydrograph or solving other problems.

Two general approaches in graphical integration are: (1) a mass-curve procedure, that represents the given hydrograph in form of its summation or mass curve, and obtains the routed hydrograph in the form of its mass curve; and (2) a procedure that uses the inflow or outflow directly for integration. Flash floods are not suitable for routing by the mass-curve procedure, because of non-negligible dynamic effects. Also, there are many semi-graphical methods in the literature [1], which combine a numerical tabular procedure with partial graphical integration. The graphical methods are generally restricted to simple problems, because for complex problems with several routing computations, the time involved becomes economically prohibitive.

Some simplified flood routing relations, often called coefficient methods, comprise a group of procedures that approximate the complex relations existing between the volume of water stored in a channel reach and the hydraulic magnitudes (inflow and outflow discharge, stages, slope, or others). The relations, consisting of coefficients, give weight in a specific way to each variable involved, the simplest being the weight coefficients for inflow and outflow discharge and for the time of travel of a wave moving along a reach. The coefficients are empirical as determined from flood wave movements that have actually occurred, or from waves analytically determined by an accurate integration procedure of complete differential equations. The empirical coefficients account, in part, for the effect of changing inflow and outflow on the water stored in a reach.

This group of procedures can be considered as an attempt to bridge two methods, the method of the two partial differential equations and the method of storage differential equation; this group uses empirical coefficients to take care of the dynamic conditions in a reach. The principal disadvantage of the coefficient methods is their dependence on empirical coefficients which require a large body of data to provide reliable estimates.

Although flood routing methods based on the simple storage differential equation have been used ever since the routings of inflow hydrographs and flood prediction

along channels were begun, there are only a few studies which compare the flood routing methods based on a solution of the two partial differential equations to those based on the storage differential equation only. Only a few studies compare accuracy, cost, advantages and disadvantages of various methods, devices, or speedups under different conditions of waves, channel and lateral inflows or outflows.

7.2 Flood Routing Based on Simplified Partial Differential Equations

Another group of flood routing methods represents transitions between those methods that use only the storage (continuity) equation. Some terms in the two partial differential equations, mostly in the momentum equation, are omitted, or some additional assumptions are made (constant bottom slope, linear change of channel characteristics between two cross sections used, separate routings of storage component and translation component of a flood wave, and others). Examples of these transitional cases are the omission of the acceleration term, $\partial V/\partial t$, the omission of the velocity head term, $V\partial V/\partial x$, or the omission of both the acceleration and velocity terms; division of the total discharge in two parts, as discharge of steady flow plus a changing discharge caused by the unsteady flow, with some simplifications in the momentum equation, and similar simplifications. See some references related to these simplification in [1].

The basic characteristic of most of these transition methods is the use of wave celerity, so both the wave translation and the channel storage are taken into account.

Again, there is a lack of comparative studies this time between the transition methods and other methods. For certain conditions, it could be demonstrated that the transition methods would give more accurate results than methods based on the storage differential equation, at little increase in computer cost. Under some conditions, however, the transition method may give less accurate results at substantial savings in work time, compared with methods based on the two partial differential equations. There has been more effort in the past to invent or derive new methods of integrating the different types of equations in flood routing than to analyze the particular limitations and advantages of each method and to compare all of them in general.

7.3 Basic Properties of Simplified Methods

The simplified methods such as the Muskingum method, time-lag method, and similar methods based primarily on the continuity equation have certain main characteristics:

(1) For given types of flood waves and given channel characteristics these methods need information a priori on flood waves, which is used to evaluate the coefficients or various other parameters of these simplified methods.

This information on flood waves may come from one or more of the following three sources:

(a) data on observed flood waves and observed properties of free-surface flowing water conveyance structures in nature;

(b) data on observed flood waves and on properties of the experimental channels or conduits;

(c) data on conveyance structures, and data on the flood waves obtained by reliable numerical integration methods using the complete continuity and momentum partial differential equations of one-dimensional gradually varied free-surface unsteady flow. This information is based on the hypothesis that the difference between analytical waves and corresponding physical waves is much smaller than the difference between these analytical waves and the waves computed by the simplified flood routing methods.

(2) Simplified methods in general assume a reduced cost of computation compared with the more accurate methods available, to compensate for decreased accuracy in predicting flood waves.

(3) Limited accuracy of the simplified methods is justified by the corresponding limited accuracy of all data used in flood routing, particularly of data on channel, boundary, and initial conditions.

Two basic properties of simplified flood routing methods that warrant discussion are the prior information required for the evaluation of various flood waves along a given channel and conduit, and the economy of computations as a counter-balance to a decreased or imposed level of accuracy.

A discussion of properties of simplified methods can not be separated from an analysis of the objectives the simplified methods serve. Basically these methods are used either for purposes of design in determining the sizes of open channels and free-surface flowing conduits, or for prediction of flood hydrographs resulting from a given storm at known points of an existing storm drainage system. Most of the methods have been developed mainly to predict flood movements along known channels. But, in general, this objective should be considered secondary in comparison to the first objective of storm drainage design. The usefulness of any prediction can only depend on the lag between the time of the prediction after the storm occurred and the time of occurrence of flood waves at given positions. The shorter this lag the less economically attractive is the prediction. The prediction of flood waves at given positions seems to be less important than design in cases of urban, highway or airport storm drainage systems. In other words, once a drainage system has been designed by peak flow, it really is not very important that the extent of the damage can be predicted. The exception may eventually be in those cases for which the pumps evacuate the storm water, or the operation and safety of the system depend on flood movement through storm drains.

The simplified methods of flood routing through storm drains may be useful for predicting purposes, provided these methods are based on observations of flood waves in nature, observations of waves in experimental conduits or accurately computed data on waves for various expected flood hydrographs. However, for the design of new storm drains, particularly under conditions of an interconnected system of drains, the simplified methods that require a prior information on flood waves are less attractive than the explicit and accurate methods of flood routing for given initial and boundary conditions, and assumed dimensions of storm drains. Since this study is oriented toward producing information that can be used for new designs of storm drains, based on the unsteady flow approach, a discussion of simplified methods is not overly relevant. However, since the initial dimensions for a newly

designed storm drain must be first assumed, the simplified methods might produce the information needed for these assumptions. Then, the more accurate procedure developed in this study can be used for the final computations. With this in mind, three simplified methods are discussed in this chapter; the potential of each method to give a reliable first estimate of storm-drain dimensions is reviewed. The three methods are the Muskingum, time-lag, and non-dimensional methods.

Before these methods are reviewed, however, a discussion of computer economy is presented. Generally, the cost of computer time is about the same for both an accurate and a simplified method of flood routing. Usually, economy of computation in the electronic computer age does not prohibit accuracy.

For example, assume the computer time is t_i for a given unit flood routing operation, with given Δx and Δt increments, by the specified interval scheme of the method of characteristics, as proposed by this study for future computations of floods moving through storm drains. Further assume the corresponding unit operation with the same increments is t_j for a particular simplified method. Also assume that $t_j < t_i$; otherwise, the simplified method is meaningless in regard to more expensive and less accurate results than by the method of characteristics. Though the algebraic operation of the given simplified method may be cheaper in computer time, the same or similar accuracy of the simplified method in comparison with the accuracy of the method of characteristics, may often require a smaller Δx and correspondingly a smaller Δt . Let the total length of the conduit be L , and let the corresponding total integration time be T . For given Δx and Δt , the number of unit operations is

$$n = \frac{L}{\Delta x} \cdot \frac{T}{\Delta t} \quad (7.5)$$

If $\Delta x_j < \Delta x_i$ and $\Delta t_j < \Delta t_i$, then for n_j and n_i the corresponding numbers of unit operations multiplied by the unit computer time give

$$t_i n_i = \frac{LTt_i}{\Delta x_i \Delta t_i} \leq \frac{LTt_j}{\Delta x_j \Delta t_j} = t_j n_j \quad (7.6)$$

Because Δx_i and Δt_i can be taken as the largest values by preserving a given degree of accuracy, $t_i/\Delta x_i \Delta t_i$ may be smaller, or much smaller than $t_j/\Delta x_j \Delta t_j$, even though $t_j < t_i$. In other words, the computer time for a unit of operation of given Δx and Δt is not the only measure of economy. Methods must be compared on the basis of given accuracy, because if a relaxation is given for the accuracy of a simplified method, its application to the method of characteristics results in a larger Δx -value and correspondingly in a larger Δt -value, and a decrease in n_i -value.

Even if $t_{i,n_i} > t_{j,n_j}$, it still does not prove the economy of a simplified method. For development of this method, observed or computed data are necessary. In this latter case, assume that k -total flood routing runs are necessary to derive and check the coefficients of simplified method. In this case, by using the method of characteristics the necessary computer time is $k t_{i,n_i}$. Assume that the simplified method will be used for N cases, similar to floods and conduits for which the coefficients are derived and checked. Then the equality of cost gives

$$k t_{i,n_i} + N t_{j,n_j} = N t_{i,n_i}, \text{ or}$$

$$(1 - \frac{k}{N}) t_{i,n_i} = t_{j,n_j}, \quad (7.7)$$

The larger k needed, and the smaller N will be, the less attractive becomes the simplified method, even though $t_{j,n_j} < t_{i,n_i}$ for $k/N = 0$. This general analysis shows that the economy of computations is a more complex problem than just comparing t_j with t_i .

7.4 Muskingum Simplified Method.

The summary of the Muskingum method is taken from reference 9 [pp. 605-607], but different symbols are used for inflow (P instead of I) outflow (Q instead of O), and storage (W instead of S). The storage, W , in a channel reach for unsteady flow depends primarily on the inflow (P) and outflow (Q) discharges and on the geometric and hydraulic characteristics of the channel and its control features. It can be assumed that the upstream and downstream end sections of a reach have the same mean discharge and storage relations with respect to the depth of flow (y). Thus, the equations may be written that

$$P = a y_u^n, \quad Q = a y_d^n, \quad W_u = b y_u^m, \quad \text{and} \quad W_d = b y_d^m \quad (7.8)$$

in which the subscript u refers to the upstream and d to the downstream end of the reach, a and n express the constants of the depth-discharge relation at the two sections, b and m express the constants of the mean depth-storage relation of the reach, and W_u and W_d are the storages referred to the corresponding depths at the upstream and downstream ends, respectively. Eliminating y in these equations, then

$$W_u = b \frac{P}{a}^{m/n}, \quad \text{and} \quad W_d = b \frac{Q}{a}^{m/n} \quad (7.9)$$

Designating by X a dimensionless factor that defines the relative weights given to inflow and outflow in the determination of the storage volume within the reach, then the storage of the reach at any given time is expressed by

$$W = X W_u + (1-X) W_d \quad (7.10)$$

Substituting Eqs. 7.9 in Eq. 7.10 and simplifying

$$W = K [X P^r + (1-X) Q^r] \quad (7.11)$$

in which $K = b/a^{m/n}$ and $r = m/n$. In case $r = 1$ (or the linear relation of storage and discharge), Eq. 7.11 becomes

$$W = K [X P + (1-X) Q] \quad (7.12)$$

which is the basis of the Muskingum method.

Further substituting Eq. 7.12 into Eq. 7.2 gives

$$KX \frac{dP}{dt} + K(1-X) \frac{dQ}{dt} = P - Q \quad (7.13)$$

Using the approximate relations of finite differences,

$$\frac{dP}{dt} \approx \frac{P_2 - P_1}{\Delta t}, \quad \frac{dQ}{dt} \approx \frac{Q_2 - Q_1}{\Delta t}, \quad P = \frac{P_1 + P_2}{2}, \quad \text{and} \quad Q = \frac{Q_1 + Q_2}{2}$$

in Eq. 7.13 gives

$$Q_2 = C_1 P_1 + C_2 P_2 + C_3 Q_1 \quad (7.14)$$

with

$$C_1 = \frac{KX + 0.5\Delta t}{K(1-X) + 0.5\Delta t}, \quad C_2 = \frac{-KX + 0.5\Delta t}{K(1-X) + 0.5\Delta t}, \quad \text{and} \quad C_3 = \frac{K(1-X) - 0.5\Delta t}{K(1-X) + 0.5\Delta t} \quad (7.15)$$

Equation 7.14 is the final form of the Muskingum method.

In Eq. 7.12 the term KQ represents the water volume of the prismatic storage in the reach which corresponds to the outflow discharge, and $KX(P-Q)$ represents the additional storage (wedge storage) which is positive or negative depending on whether $P > Q$ or $P < Q$. In this case, K has the dimension of time, while X is a dimensionless parameter. In fact, the reach length Δx is included in the coefficient K , which can be then defined as $K = \Delta x/C$, with C representing celerity of the wave along the reach.

The application of this simplified method requires the estimation of two quantities. First, K is both a time constant, which depends on the incremental length selected, and the channel characteristics that affect resistance to flow and hence the time of wave travel. Thus, K is the incorporation of several physical factors of the system. Second, the value of X must be estimated. Since this relates to the difference in inflow and outflow it is a measure of the shape of the discharge hydrograph passing through the reach. The two coefficients, X and K then account for the necessary conditions of channel shape, slope, and roughness, and the hydrograph to be routed. However, these parameters cannot be determined without information a priori. Information on waves and on the channel is necessary to estimate the weight X for the P and Q in Eq. 7.10, and it is needed to estimate a , b , m , and n of Eqs. 7.8 in order to compute K . Inaccuracies in these estimates are usually the main sources of errors in the Muskingum

method. Only through observed waves, or through computed waves based on a more accurate procedure of computation, may K and X be determined with any exactness.

Because flood hydrographs may have different shapes, which affect the weight X , the application of the Muskingum method to the design of storm routing systems is limited. Both the thesis by Suvich [see Internal References, 11], and the paper by Harris [8] arrived at this limitation for the Muskingum method as applied to the design of storm drains. Also, it is not even evident that the computer time necessary for a given level of accuracy is less than that required by the specified intervals scheme of the method of characteristics.

7.5 Time-Lag Routing Simplified Method

In his paper [8], Harris discusses the possibility of a simplified routing method that would provide the comparable results to observed discharge hydrographs. By using the method of characteristics as a standard, after the CSU data verified the method of characteristics, he found that the "progressive average lag routing method" meets the required good agreement between discharge hydrographs for particular circular conduits. According to Harris, this method also requires considerably less computer time.

For a given hydrograph at the upstream end of the reach, as given in Fig. 7.1 (left graph), the point R of Fig. 7.1 (right graph) on the routed hydrograph at the downstream end of the reach is obtained by

$$q_r = \frac{1}{n} \sum_{j=i-n}^i q_j, \quad (7.16)$$

in which the point q_i and q_{i-1} are separated by the time increment Δt , q_r is lagged by time t_L and n is the number of ordinates of upstream hydrograph averaged to determine an ordinate of the downstream hydrograph. This is a moving-average scheme in smoothing the upstream hydrograph, and it is translated by the time lag, t_L , to obtain the downstream hydrograph.

The three parameters to be selected and used in this simplified method are: Δt , the time increment of the hydrograph; n , the number of points on the hydrograph to be averaged and t_L the time lag. These routing coefficients for a given conduit were determined by computing the routed waves by the method of characteristics for three inflow hydrographs.

The procedure by Harris was to adjust the coefficients so that the first output hydrograph was reproduced accurately by the time-lag method.

These coefficients were then used to compute the other two hydrographs by the simplified time-lag method, and then compare with the hydrographs computed by the method of characteristics. Harris states that minor adjustments were required to make all three hydrographs fit well.

The results for a series of tests on different conduit diameters includes the size, length, and shape of each conduit. It is concluded then that the "progressive average-lag method", or "moving-average time-lag method", does provide answers which are the same as calculated by the method of characteristics.

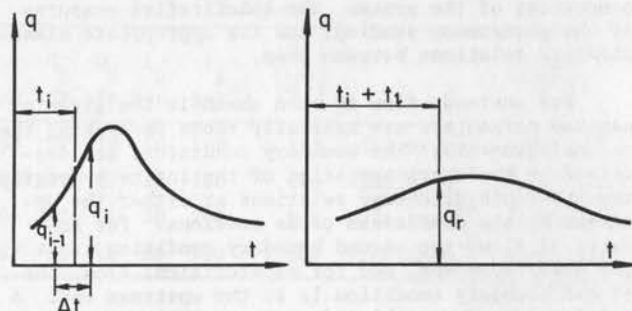


Fig. 7.1. The hydrograph of the upstream reach end, left graph, and the computed hydrograph of the downstream reach end, right graph, by the time-lag simplified flood routing method.

In summary, the simplified method proposed by Harris, and based on prior work by Dooze and the U.S. Army Corps of Engineers [8], depends on expressing the discharges of the hydrograph at a later time (further downstream) as a centered average of n number of equally time spaced discharge values of the earlier hydrograph. The time between the two hydrographs is a coefficient of the conduit properties and the reach length. The time-lag term includes the effect of the channel shape, slope, and roughness. The number n of discharge values to be averaged is the other routing coefficients for a given Δt . This number n of discharge quantities takes into account the change in the inflow hydrograph shape. Thus, the procedure appears to take into account the physical conditions determining the downstream hydrograph.

There is no procedure a priori for evaluating the routing coefficients. For a given channel, they are determined by observing physical waves and the channel, or by computing waves based on an accurate method of computation, such as those by Harris using the accurate integration scheme by the method of characteristics.

The time-lag routing method, like the Muskingum method or its variations, assumes, in general, constant values of coefficients for all stages of a hydrograph. This condition is not physically possible for shapes such as the circular section of a partly full flowing conduit. However, for modest ranges of base flow, and the inflow hydrographs of limited range, the coefficients may exhibit reasonable constancy.

The simplified methods, like the moving-average time-lag method, may offer an approximate flood routing of inflow hydrographs through storm drains for purposes of design. However, they do not permit a direct evaluation of corresponding depths. Neither do they give the maximum depth determined either as a function of position along the conduit or in time. Since the maximum depth does not occur at the same time as the maximum discharge, it is not to be expected that the routed discharge hydrograph would be an indicator of maximum depth.

7.6 Non-dimensionality Approach

Parameters of non-dimensionality. A convenient procedure for estimating discharge and depth hydrographs is by a non-dimensional generalization of functions of time and distance, boundary and initial

conditions, and other factors. This approach requires the identification of pertinent physical properties of the system, the quantitative measures of the phenomenon studied, and the appropriate dimensionless relations between them.

For unsteady flow in open channels the given or assumed parameters are basically those describing the channel geometry. The boundary conditions are described by the characteristics of the inflow hydrograph and the depth-discharge relations at either the upstream or the downstream cross sections. For subcritical flow, the second boundary condition is at the downstream end, and for supercritical flow, the second boundary condition is at the upstream end. A third group of variables describes the initial conditions.

Within each of these three basic groups are numerous variables, some of which are of primary importance and some of which may have only negligible effects on the quantity to be predicted. The three basic groups of variables are:

A. Channel Geometry.

1. Cross section shape
 - a. Circular shape, diameter, D ,
 - b. Trapezoidal shape, base width and side slope, B_o, z ,
 - c. Parabolic shape, rate of change of width with depth, k ,
2. Channel bed slope, S_o ,
3. Channel boundary roughness
 - a. Darcy-Weisbach friction factor, f ,
 - b. Velocity distribution coefficients, α and β .

B. Boundary Conditions.

1. Inflow hydrograph
 - a. Base flow, Q_b ,
 - b. Peak flow, $Q_o + Q_b$, with Q_o the maximum discharge in excess of the base flow,
 - c. Time from beginning of hydrograph to time of maximum flow, t_p ,
 - d. Volume of hydrograph, W ,
 - e. Time to center of mass of wave, t_g ,
 - f. Time duration of hydrograph, t_b ,
2. Downstream depth versus discharge relation, $y(Q)$, or
3. Upstream depth versus base discharge relation, $y(Q)$.

C. Initial Conditions.

1. Steady uniform flow with a constant depth, y_o , or

2. Steady nonuniform flow with

a. Subcritical slope or

- (1) Backwater or M1 curve, $y(x)$, or
- (2) Drawdown or M2 curve, $y(x)$.

b. Supercritical slope

- (1) S1 curve, $y(x)$, or
- (2) S2 curve, $y(x)$.

Thus, there are numerous variables to be related into a reduced number of significant dimensionless parameters describing the conditions of the phenomena.

The functions to be related to the above basic variables are:

A. Wave Profiles as a function of

1. Distance, $y(x)$,
2. Time, $y(t)$.

B. Maximum Depth as a function of

1. Distance, $y_p(x)$,
2. Time, $y_p(t)$.

C. Discharge Hydrograph at any location, $Q(t,x)$.

The significance and usefulness of these quantities depend on the reason for the analysis. For example, for the design of a storm drainage system, one important consideration is the determination of the peak depth as a function of distance. From this information the location of a change in channel size can be determined. For the problems of an existing storm drainage system related to time, the desirable information would relate the discharge to the time. The recording of existing conditions and then the prediction of a later event would permit direct control and reductions of peak flows.

The complications of the problem of non-dimensionality may be realized if one considers the number of independent dimensionless parameters that describe the unsteady flow process. According to the Buckingham Pi theorems, the number of independent dimensionless parameters necessary to describe a physical process is equal to the number of variables minus the number of dimensional categories (such as length, force, time). For the description of unsteady flow, considering all variables listed above, there would be approximately 20 variables and 2 dimensional quantities (length and time). The number of dimensionless parameters would then be approximately 18, depending on the physical conditions to be represented. The systematic correlation of these parameters requires holding all but one or two of them constant at a time and then observing or computing their effect on the quantity to be predicted.

An exploratory attempt was made to demonstrate a non-dimensional approach to the description of unsteady flow. A series of flood waves were computed by the method of characteristics for the subcritical and supercritical base flow conditions. The hypothetical inflow hydrograph was defined by a Pearson Type III distribution:

$$Q(t) = Q_b + Q_o e^{-(t-t_p)/(t_g-t_p)} (t/t_p)^{t_p/(t_g-t_p)} .$$

The shape was defined by the base flow Q_b , the maximum discharge in excess of the base flow, Q_o , the time to the maximum discharge, t_p , the time to the centroid of the inflow hydrograph, t_g , and e , the base of the Napierian logarithms. These parameters were varied over a limited range to identify their effect on the resultant maximum depths of flow.

Subcritical flow. The conditions used for the subcritical flow computer runs are listed in Table 7.1. The dependent variable computed for each of these conditions was the maximum depth of flow as a function of position along the conduit. Figure 7.2 presents a typical attenuation curve for the maximum depth for subcritical flow. For that position of the curve, relatively unaffected by the free outfall end condition, the depth may be represented by the relation $D = a + ix$, in which i is the attenuation rate.

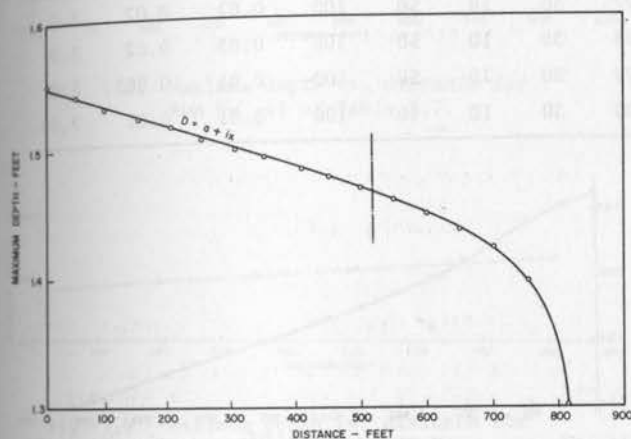


Fig. 7.2. Maximum depth versus distance for Run No. 200 of Table 7.1.

In the subcritical flow, Q_b and Q_o played a more important role in the rate of attenuation of wave than t_p and t_g . It was found that the attenuation rate varied as $(1/Q_b + Q_o)^{0.34}$, as seen in Table 7.2. The effect of the relative values of Q_b and Q_o is shown in Table 7.3; $(Q_b/Q_o)^{0.3}$ was found to be the appropriate dimensionless parameter. The effect of the relative values of t_p and t_g was negligible, but the rate of attenuation varied as $t_g^{0.1}$, as shown in Table 7.4. Table 7.5 shows the effect of the diameter; the effect of the diameter was defined by $D^{0.8}$.

Table 7.2. Effect of absolute values of Q_b and Q_o in subcritical flow for $t_p = 50$ sec., $t_g = 100$ sec.

Run	Q_b cfs	Q_o cfs	t_p sec.	t_g sec.	$(\frac{1}{Q_b + Q_o})^{0.34}$	i	$i(\frac{1}{Q_b + Q_o})^{0.34}$
200	10	10	50	100	0.360	-162.65×10^{-6}	-58.6×10^{-6}
201	5	5	50	100	0.457	-122.65×10^{-6}	-56.1×10^{-6}
202	7.5	7.5	50	100	0.398	-141.55×10^{-6}	-56.4×10^{-6}
203	9	9	50	100	0.374	-153.76×10^{-6}	-57.5×10^{-6}

Table 7.3. Effect of relative values of Q_b and Q_o in subcritical flow for $t_p = 50$ sec., and $t_g = 100$ sec.

Run	Q_b cfs	Q_o cfs	t_p sec.	t_g sec.	$(\frac{Q_b}{Q_o})^{0.3}$	i	$i(\frac{Q_b}{Q_o})^{0.3}$
204	17	10	50	100	1.173	-168.9×10^{-6}	-198×10^{-6}
205	10	17	50	100	.856	-253.4×10^{-6}	-216.5×10^{-6}
206	12	15	50	100	.935	-221.4×10^{-6}	-207×10^{-6}
207	15	12	50	100	1.069	-186.5×10^{-6}	-199.1×10^{-6}

Table 7.4. Effect of absolute value of t_g in subcritical flow for $Q_b = 10$ cfs, and $Q_o = 10$ cfs

Run	Q_b cfs	Q_o cfs	t_p sec.	t_g sec.	$(t_g)^{0.1}$	i	$i(t_g)^{0.1}$
208	10	10	50	100	1.66	-275.9×10^{-6}	-457×10^{-6}
212	10	10	40	80	1.62	-283.7×10^{-6}	-459×10^{-6}
213	10	10	30	60	1.57	-292.1×10^{-6}	-459×10^{-6}
214	10	10	60	120	1.69	-268.7×10^{-6}	-455×10^{-6}

Table 7.5. Effect of diameter in subcritical flow for $Q_b = 10$ cfs, $Q_o = 10$ cfs, $t_p = 50$ sec., and $t_g = 100$ sec.

Run	Q_b cfs	Q_o cfs	t_p sec.	t_g sec.	D ft	$D^{0.8}$	i	$i(D)^{0.8}$
208	10	10	50	100	3	2.41	-275.9×10^{-6}	-665×10^{-6}
215	10	10	50	100	5	3.62	-176.3×10^{-6}	-636×10^{-6}
216	10	10	50	100	7	4.75	-150.2×10^{-6}	-715×10^{-6}

Four dimensionless parameters were found to define the rate of attenuation of the wave peak in the subcritical flow:

$\pi_1 = i$, the rate of attenuation;

$\pi_2 = (\frac{Q_b}{Q_o})^{0.3}$, the parameter defining the relative value of base flow to the superimposed peak flow;

$\pi_3 = (\frac{1}{Q_b + Q_o})^{0.34} (t_g)^{0.1} (D)^{0.8} (g)^{0.22}$, the parameter representing the absolute values of Q_b , Q_o , t_g/D , and the gravitational acceleration, and

$\pi_4 = \frac{S_o}{f^{2.88}}$, the parameter measuring the effects of the channel slope and friction factor. Combining π_1 , π_2 and π_3 into one-dimensionless parameter gives

$$\pi_5 = \pi_1 \pi_2 \pi_3 = i \left(\frac{Q_b}{Q_0} \right)^{0.3} \left(\frac{1}{Q_b + Q_0} \right)^{0.34} (t_g)^{0.1} (D)^{0.8} (g)^{.22}$$

A plot of π_5 versus π_4 is shown in Fig. 7.3

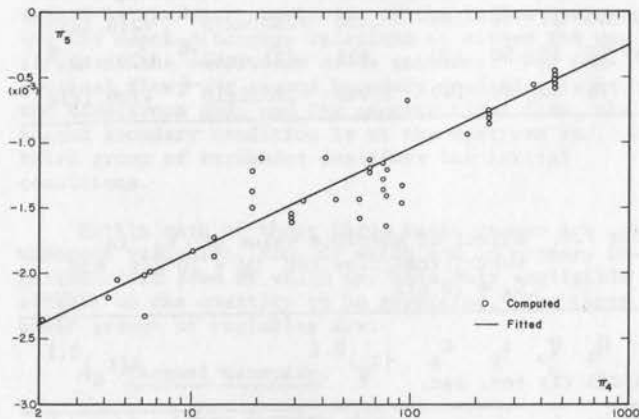


Fig. 7.3. Attenuation of wave peak for a theoretical hydrograph in subcritical flow.

Supercritical flow. The condition for transition from subcritical to supercritical flow may be estimated from the Darcy-Weisbach energy loss equation and the expression for critical flow.

$$\frac{S_o}{f} = \frac{Q^2}{8gRA^2} \quad \text{- Darcy-Weisbach}$$

$$1 = \frac{Q^2 B}{gA^3} \quad \text{- Critical flow.}$$

The ratio of these two equations results in:

$$\frac{S_o}{f} = \frac{P}{8B}$$

For a circular conduit flowing one-half full, the wetted perimeter $P = \pi D/2$ and the surface width $B = D$. Thus

$$\frac{S_o}{f} \approx 0.2$$

It is to be noted that S_o/f required for the change of flow from supercritical to subcritical flow is approximately one-tenth of S_o/f needed for change of flow from an attenuating to an amplifying wave. The waves in subcritical flow always attenuate, while the waves in the supercritical regime can either attenuate or amplify.

To determine how the profile of an attenuating wave in supercritical flow responds to parameters of slope, friction factor, and diameter, four consecutive runs were made in which the inflow hydrograph was the same for all four runs. The conditions of these runs are given in Table 7.6. Figure 7.4 shows the profile of Run 97. The profile is initially convex, it then becomes concave. Figure 7.5 shows the profile of Run 98 which depicts the effect of an increase in slope; the profile is a straight line. Figure 7.6 shows the profile of Run 99; the effect of a decrease in friction factor is the same as an increase in slope, and the profile is again a straight

line. Figure 7.7 is the profile of an attenuating wave, Run 100, routed in a conduit with a diameter of 7.0 ft, as compared with a diameter of 3.0 ft in the previous three runs. The profile still has the initial convex segment with the following concave segment, but the concavity and convexity are not very pronounced. Another type of wave profile encountered in the course of computer runs is when the depth of the base flow is near one-half full. The profile of Run 45 is shown in Fig. 7.8 and it has a pronounced concavity.

Table 7.6. Conditions of Runs 97, 98, 99, and 100

Run	Q_b cfs	Q_0 cfs	t_p sec	t_g sec	S_o	f	D ft
97	30	10	50	100	0.01	0.02	3.0
98	30	10	50	100	0.03	0.02	3.0
99	30	10	50	100	0.01	0.005	3.0
100	30	10	50	100	0.01	0.02	7.00

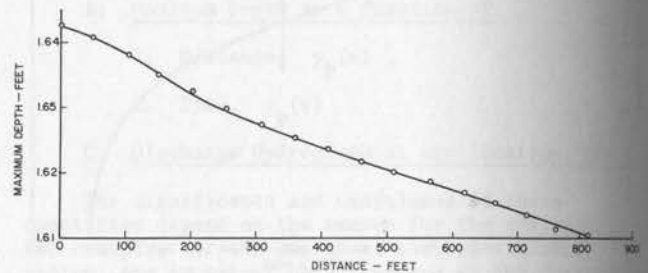


Fig. 7.4. Maximum depth vs. distance for Run No. 97 of Table 7.6.

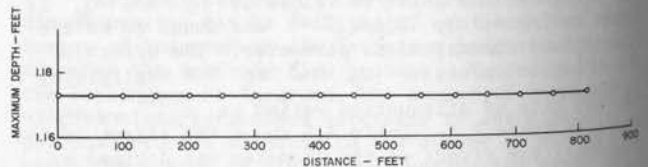


Fig. 7.5. Maximum depth vs. distance for Run No. 98 of Table 7.6.

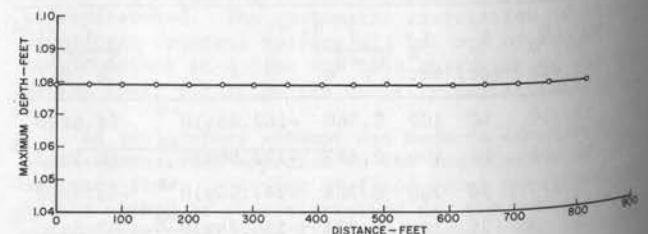


Fig. 7.6. Maximum depth vs. distance for Run No. 99 of Table 7.6.

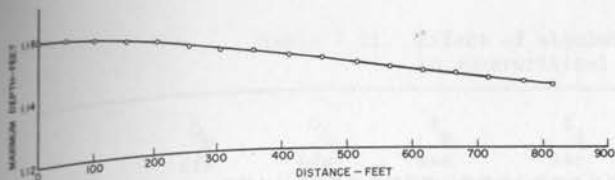


Fig. 7.7. Maximum depth vs. distance for Run No. 100 of Table 7.6.

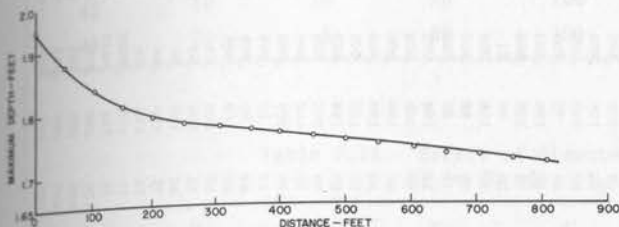


Fig. 7.8. Maximum depth vs. distance for Run No. 45 of Table 7.7.

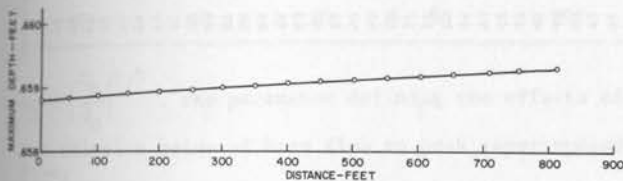


Fig. 7.9. Maximum depth vs. distance for Run No. 18 of Table 7.7.

The condition for the amplification of a wave in the supercritical regime was assumed to be $S_0/f > 2$ (or Froude number $F > 2$). The amplitude of waves grows but only slightly, as observed by comparing Figs. 7.8 and 7.9. In Fig. 7.8 the general slope of the curve is 116.2×10^{-6} , where as the general slope of the curve in Fig. 7.9 is 0.614×10^{-6} . If the above condition for amplification is satisfied, then the profile of the wave amplification is nearly a straight line in this case. Figure 7.9 shows a typical amplification of maximum wave depth.

The attenuation characteristics of a Pearson Type III function inflow hydrograph in the supercritical regime were analyzed by computational methods. The parameters of the hydrograph control the volume and steepness of the wave. The effects of given parameters of the physical system, conduit diameter D , its slope S_0 , and the friction factor f , on the attenuation rate of flood waves were investigated. The attenuation rate of the maximum depth was defined for the supercritical flow by

$$y_p = ae^{ix} \quad (7.17)$$

in which i is the parameter of the exponential attenuation rate. The parameter i in Eq. 7.17 was determined for each attenuating wave.

The conditions of each run are given in Table 7.7. Parameters defining the attenuation rate are representative of the hypothetical conditions. The use of attenuation rates beyond the computed range, however, may not be reliable.

The attenuation rate was correlated with the characteristics of the inflow hydrograph and the circular conduit. To determine the general parameters in question, computer runs were carried out which simulated actual conditions. The effect of each of the seven variables was evaluated by changing one of the variables while keeping the rest constant. It was found that the attenuation rate varied as $[(t_g - t_p)/t_g]^{0.25}$, as seen in Table 7.8.

The effect of the absolute value of t_g is shown in Table 7.9, the attenuation rate varied as

$(t_g)^{0.75}$. Tables 7.10, 7.11, and 7.12 show that $[1/(Q_b + Q_0)]^{0.47}$, $(Q_b/Q_0)^{0.5}$, and $D^{0.8}$ may be appropriate parameters.

Table 7.8. Effect of the relative value of t_p and t_g in supercritical flow for $Q_b = 25$ cfs, and $Q_0 = 10$ cfs.

Run	Q_b cfs	Q_0 cfs	t_p sec	t_g sec	$(\frac{t_g - t_p}{t_g})^{0.25}$	i	$i(\frac{t_g - t_p}{t_g})^{0.25}$
53	25	10	50	100	0.841	-116×10^{-6}	-97.8×10^{-6}
55	25	10	70	80	0.595	-170×10^{-6}	-99.5×10^{-6}
66	25	10	60	90	0.760	-129×10^{-6}	-98.0×10^{-6}
67	25	10	40	110	0.893	-111×10^{-6}	-99.0×10^{-6}

Table 7.9. Effect of the absolute value of t_g in supercritical flow for $Q_b = 25$ cfs, and $Q_0 = 10$ cfs

Run	Q_b cfs	Q_0 cfs	t_p sec	t_g sec	$(t_g)^{0.75}$	i	$i(t_g)^{0.75}$
53	25	10	50	100	31.6	-116×10^{-6}	-3665×10^{-6}
54	25	10	25	50	18.8	-174×10^{-6}	-3270×10^{-6}
63	25	10	75	150	42.8	-828×10^{-6}	-3542×10^{-6}
64	25	10	100	200	53.0	-608×10^{-6}	-3225×10^{-6}

Therefore, the attenuation characteristics of the waves in the supercritical regime of flow can be defined in terms of five dimensionless parameters:

$\pi_1 = i$, the rate of attenuation;

Table 7.10. Effect of the absolute values of Q_b and Q_0 in supercritical flow for $t_p = 50$ sec. and $t_g = 100$ sec.

Run	Q_b cfs	Q_0 cfs	t_p sec	t_g sec	$(\frac{1}{Q_b + Q_0})^{0.47}$	i	$i(\frac{1}{Q_b + Q_0})^{0.47}$
57	20	8	50	100	$\frac{1}{4.8}$	-88.3×10^{-6}	-18.4×10^{-6}
58	15	6	50	100	$\frac{1}{4.18}$	-66.2×10^{-6}	-15.8×10^{-6}
59	10	4	50	100	$\frac{1}{3.46}$	-47.1×10^{-6}	-13.6×10^{-6}

Table 7.1. Conditions of different runs in the subcritical regime

Run	Q _b cfs	Q _o cfs	t _p seconds	t _g seconds	S _o	f	D ft
200	10	10	50	100	.001	.01	3
201	5	5	50	100	.001	.01	3
202	7.5	7.5	50	100	.001	.01	3
203	9	9	50	100	.001	.01	3
204	17	10	50	100	.001	.01	3
205	10	17	50	100	.001	.01	3
206	12	15	50	100	.001	.01	3
207	15	12	50	100	.001	.01	3
208	10	10	50	100	.0005	.01	3
209	10	10	70	80	.0005	.01	3
210	10	10	40	110	.0005	.01	3
211	10	10	60	90	.0005	.01	3
212	10	10	40	80	.0005	.01	3
213	10	10	30	60	.0005	.01	3
214	10	10	60	120	.0005	.01	3
215	10	10	50	100	.0005	.01	5
216	10	10	50	100	.0005	.01	7
217	10	10	50	100	.0001	.01	4
218	10	10	50	100	.0001	.02	4
219	10	10	50	100	.0001	.03	4
220	10	10	40	90	.00005	.01	4
221	9	8	50	90	.00005	.01	4
222	9	8	70	80	.00005	.01	4
223	10	10	60	70	.00002	.005	4
224	9	8	50	80	.00002	.005	4
225	8	7	40	80	.00002	.005	4
226	10	9	50	100	.00001	.006	5
227	10	9	50	100	.00003	.006	5
228	10	9	50	100	.00004	.006	5
229	10	9	50	70	.00006	.007	3.5
230	15	10	40	80	.00007	.007	3.7
231	8	9	40	80	.00001	.006	5
232	2	4	30	50	.00001	.006	5
234	14	9	40	70	.00003	.006	5
235	5	4	30	80	.00003	.006	5
236	17	10	50	100	.00004	.006	5
237	5	7	40	80	.00004	.006	5
238	5	5	150	100	.00006	.007	3.5
239	7	4	40	70	.00006	.007	3.5
240	10	8	50	100	.0002	.02	4
241	9	11	60	90	.0002	.02	4
242	12	10	60	80	.0002	.01	4
243	13	8	50	100	.0003	.03	4
244	10	10	50	100	.0003	.03	4
250	10	10	40	80	.0004	.01	3
251	10	10	50	90	.0008	.01	3
252	4	3	50	80	.00001	.01	6
254	15	25	80	100	.001	.02	4
255	15	10	50	100	.0015	.02	4.5
256	10	9	60	100	.0005	.02	3
257	8	6	50	30	.00005	.02	7.5
258	15	10	60	120	.00001	.02	7
259	5	5	60	90	.001	.03	4
260	12	10	40	80	.0005	.03	4
261	10	10	50	100	.0002	.03	4
262	5	4	50	90	.00005	.03	4.5
263	15	12	60	90	.002	.03	3

Table 7.7. Conditions of different runs in the supercritical regime

Run	Q _b cfs	Q _o cfs	t _p sec	t _g sec	S _o	f	D ft
6	10	8	100	150	.006	.01	2.9262
7	15	12	100	150	.006	.01	2.9262
8	10	8	100	150	.007	.01	2.9262
10	10	8	100	150	.010	.01	2.9262
11	20	16	100	150	.015	.01	2.9262
12	20	16	100	150	.050	.01	2.9262
13	20	16	100	150	.030	.01	2.9262
14	20	16	100	150	.070	.01	2.9262
15	10	8	100	150	.025	.01	2.9262
16	20	16	100	150	.008	.01	2.9262
17	15	12	100	150	.015	.01	2.9262
18	10	8	50	100	.025	.01	2.9262
19	10	8	150	200	.025	.01	2.9262
20	20	16	50	100	.010	.01	2.9262
21	20	16	50	100	.008	.01	2.9262
22	20	16	25	50	.02	.01	2.9262
23	25	20	25	50	.01	.01	2.9262
24	15	12	25	50	.01	.01	2.9262
25	20	16	25	50	.05	.01	2.9262
26	20	16	10	20	.05	.01	2.9262
29	20	15	100	150	.02	.01	2.9262
30	20	15	75	100	.02	.01	2.9262
31	20	15	25	50	.02	.01	2.9262
32	20	15	10	20	.02	.01	2.9262
33	20	15	20	20	.02	.01	2.9262
34	20	16	100	150	.07	.01	2.9262
35	20	14	25	50	.06	.01	2.9262
36	20	14	35	40	.06	.01	2.9262
37	20	14	30	40	.06	.01	2.9262
38	20	14	60	80	.06	.01	2.9262
39	20	10	50	100	.005	.01	2.9262
40	10	20	50	100	.005	.01	2.9262
41	5	25	50	100	.005	.01	2.9262
42	10	10	50	100	.005	.01	2.9262
43	10	5	50	100	.005	.01	2.9262
44	20	10	50	75	.005	.01	2.9262
45	20	18	50	100	.003	.01	2.9262
47	20	10	75	100	.006	.02	2.9262
48	30	10	50	90	.01	.02	2.9262
53	25	10	50	100	.005	.02	2.9262
54	25	10	25	50	.005	.02	2.9262
55	25	10	70	80	.005	.02	2.9262
56	15	20	50	100	.005	.02	2.9262
57	20	8	50	100	.005	.02	2.9262
58	15	6	50	100	.005	.02	2.9262
59	10	4	50	100	.005	.02	2.9262
60	20	15	50	100	.005	.02	2.9262
61	10	25	50	100	.005	.02	2.9262
62	5	30	50	100	.005	.02	2.9262
63	25	10	75	150	.005	.02	2.9262
64	25	10	100	200	.005	.02	2.9262
65	25	10	10	20	.005	.02	2.9262
66	25	10	60	90	.005	.02	2.9262
67	25	10	40	110	.005	.02	2.9262
69	40	20	50	100	.01	.04	5.00
70	40	20	50	100	.01	.04	8.00
71	40	20	50	100	.01	.04	10.0
73	25	10	75	150	.04	.02	2.9262
74	25	10	75	150	.08	.04	2.9262
75	25	10	75	150	.0028	.01	2.9262
76	25	10	75	150	.0025	.01	2.9262
77	40	30	100	200	.003	.005	7.00
78	20	15	100	200	.003	.005	7.00
79	50	20	100	200	.003	.005	7.00
80	80	30	100	200	.003	.005	7.00

Table 7.11. Effect of absolute and relative values of Q_b and Q_o in supercritical flow for $t_p = 50$ sec, and $t_g = 100$ sec.

Run	Q_b cfs	Q_o cfs	t_p sec	t_g sec	$\left(\frac{Q_b}{Q_o}\right)^{0.5} \left(\frac{1}{Q_b+Q_o}\right)^{0.47}$	i	$i \left(\frac{Q_b}{Q_o}\right)^{0.5} \left(\frac{1}{Q_b+Q_o}\right)^{0.47}$
39	20	10	50	100	0.285	-48.5×10^{-6}	-13.8×10^{-6}
41	5	25	50	100	0.0903	-145×10^{-6}	-13.1×10^{-6}
42	10	10	50	100	.245	-51.3×10^{-6}	-12.5×10^{-6}
43	10	5	50	100	.396	-28.5×10^{-6}	-11.3×10^{-6}

Table 7.12. Effect of diameter in supercritical flow for $Q_b = 40$ cfs, $Q_o = 20$ cfs, $t_p = 50$ sec, and $t_g = 100$ sec.

Run	Q_b cfs	Q_o cfs	t_p sec	t_g sec	D ft	$(D)^{0.8}$	i	$i(D)^{0.8}$
69	40	20	50	100	5.0	3.62	-71.5×10^{-6}	-260×10^{-6}
70	40	20	50	100	8.0	5.27	-47.1×10^{-6}	-248×10^{-6}
71	40	20	50	100	10.0	6.31	-41.4×10^{-6}	-260×10^{-6}

$\pi_2 = \left(\frac{Q_b}{Q_o}\right)^{0.5}$, the parameter defining the effects of the relative value of base flow to peak superimposed flow;

$\pi_3 = \left(\frac{t_g - t_p}{t_g}\right)^{0.25}$, the parameter defining the effects of t_p and t_g ;

$\pi_4 = \left(\frac{1}{Q_b+Q_o}\right)^{0.47} (t_g)^{0.75} (D)^{0.8} (g)^{0.61}$, where g is gravitational acceleration, and π_4 correlates the absolute value to t_g , Q_b , and Q_o ; and $\pi_5 = S_o/f^{0.5}$, the parameter correlating the effects of channel slope and friction factor.

A new parameter was obtained by combining π_1 , π_2 , π_3 , and π_4 into $\pi_7 = \pi_1 \pi_2 \pi_3 \pi_4$, or

$$\pi_7 = i \left(\frac{Q_b}{Q_o}\right)^{0.5} \left(\frac{t_g - t_p}{t_g}\right)^{0.25} \left(\frac{1}{Q_b+Q_o}\right)^{0.47} (t_g)^{0.75} (D)^{0.8} (g)^{0.61}$$

The plot of π_7 versus π_5 is given in Fig. 7.10.

In conclusion, it is possible to approximate the attenuation rate of a flood wave peak in the supercritical flow in a plot of:

$$i \left(\frac{1}{Q_b+Q_o}\right)^{0.47} \left(\frac{Q_b}{Q_o}\right)^{0.5} (t_g)^{0.75} \left(\frac{t_g - t_p}{t_g}\right)^{0.25} (D)^{0.8} (g)^{0.61}$$

versus $\frac{S_o}{f^{0.5}}$, and in the subcritical regime of

$$i \left(\frac{Q_b}{Q_o}\right)^{0.3} \left(\frac{1}{Q_b+Q_o}\right)^{0.34} (t_g)^{0.1} (D)^{0.8} (g)^{0.22}$$

versus $\frac{S_o}{f^{2.88}}$.

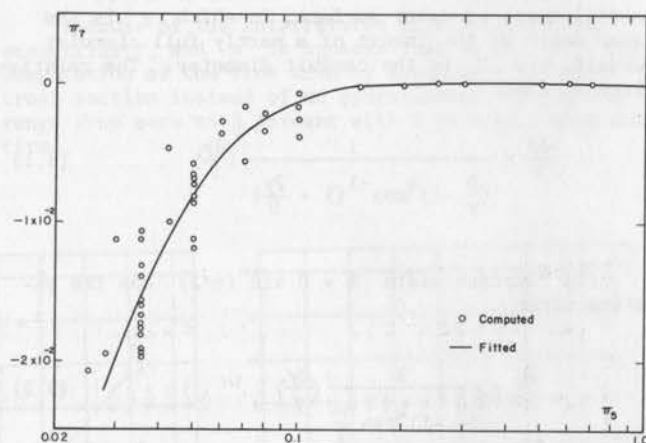


Fig. 7.10. Attenuation of wave peak for theoretical hydrograph in supercritical flow.

This analysis is only an attempt to show that the non-dimensionalization of variables in gradually varied free-surface unsteady flow has some promise in the future research. The accurate integration of hypothetical waves and conduit conditions by the method of characteristics may yield computer "experimental data", for further investigating the non-dimensional approach of flood peak attenuation.

It is not recommended that the rate of highly amplifying flood waves be studied by this approach, because the transition of the gradually varied waves to rapidly varied waves may have just the same criterion as the criterion when a wave starts to amplify. In other words, the gradually varied unsteady flow equations may be used for studying the conditions when waves start to amplify but not for studying the rate of amplification. The non-dimensionality of changes in unsteady flow regimes requires that the conduits be prismatic and very long. The local energy losses have to be incorporated into the general conduit losses.

Chapter 8

ANALYSIS OF ERRORS IN THE SOLUTION OF PROBLEMS

8.1 Errors in Geometric Parameters

The steel conduit used for free-surface unsteady flow experiments was 3 ft in diameter, 1/2-inch thick rolled-plate with a longitudinal welded joint. The 20-ft sections were butt-welded together and were supported on steel rails at approximately 20-ft spacing, not necessarily at the conduit joints. As a result of the manufacturing process, handling, field welding, and the method of support, the conduit is not perfectly circular and does not possess a straight line invert profile. The errors caused by physical departures from the mathematical geometric forms in the conduit cross section and longitudinal invert slope are defined as the errors in geometric parameters.

Errors in parameters as a function of errors in depth. The error in each of the dependent parameters can be expressed in terms of the relative error in the depth as follows:

(1) Wetted perimeter $P = D\theta/2$ has the relative error $dP/P = d\theta/\theta$, with $\theta = 2 \cos^{-1}(1 - 2y/D)$, the central angle of water surface, in which y is the water depth at the invert of a partly full circular conduit, and D is the conduit diameter. The relative error is then

$$\frac{d\theta}{\theta} = \frac{1}{\left(\frac{D}{y} - 1\right)^{1/2} \cos^{-1}\left(1 - \frac{2y}{D}\right)} \left(\frac{dy}{y}\right) \quad (8.1)$$

(2) Surface width $B = D \sin(\theta/2)$ has the relative error

$$\frac{dB}{B} = \frac{1}{\left(\frac{D}{y} - 1\right)^{1/2} \tan \frac{\theta}{2}} \left(\frac{dy}{y}\right) \quad (8.2)$$

(3) Area $A = D^2(\theta - \sin \theta)/8$ has the relative error

$$\frac{dA}{A} = \frac{1 - \cos \theta}{1 - \sin \theta} \left(\frac{d\theta}{\theta}\right) \quad (8.3)$$

(4) Hydraulic depth $y_* = A/B$ has the relative error

$$\frac{dy_*}{y_*} = \frac{dA}{A} - \frac{dB}{B} \quad (8.4)$$

(5) Hydraulic radius $R = A/P$ has the relative error

$$\frac{dR}{R} = \frac{dA}{A} - \frac{dP}{P} \quad (8.5)$$

(6) Section factor $Z = AR^{3/2}$ has the relative error

$$\frac{dZ}{Z} = \frac{dA}{A} + \frac{dR}{2R} \quad (8.6)$$

These relative errors (Eqs. 8.1 through 8.6) as functions of depth are plotted as ratios of the relative-depth error in Fig. 8.1. The relative errors in all parameters except for the wetted perimeter and hydraulic depth decrease as depth increases for a given relative depth error, dy/y . The significance of these curves is demonstrated in the calculation of friction factors and Reynolds numbers.

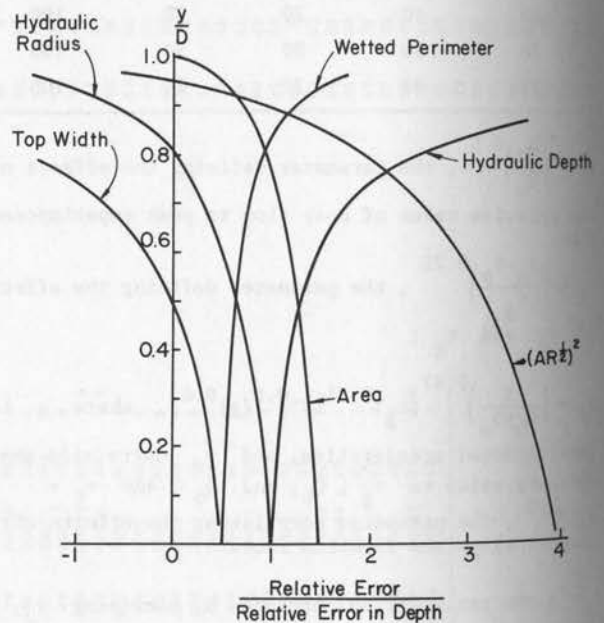


Fig. 8.1. Relative errors in parameters of circular cross section versus relative error in depth.

Errors in parameters as a function of ellipticity. Since no physical conduit has a truly geometric circular shape, it is necessary to determine the effects of deviations from circularity. An elliptical shape is assumed. Parameters describing the departure from a circular cross section are then the eccentricity and the direction of the principal axes. The eccentricity is defined as $e = \sqrt{1 - (b/a)^2}$, in which a and b are the major and minor ellipse semi-diameter, respectively. The direction of the principal axes is the angle α that the minor axis makes with the vertical as shown in Fig. 8.2.

To compare circular segments with elliptical segments the percent of difference between the two was computed for depths from the bottom up to the center of the ellipse. Eccentricity was varied in increments of 0.05 up to 0.30 and α ranged from 0

to $\pi/2$ in increments of $\pi/10$. The area of the complete ellipse was made equal to the area of the complete circle. Depth was relative to the center of the ellipse. The results of these calculations are shown in Fig. 8.3.

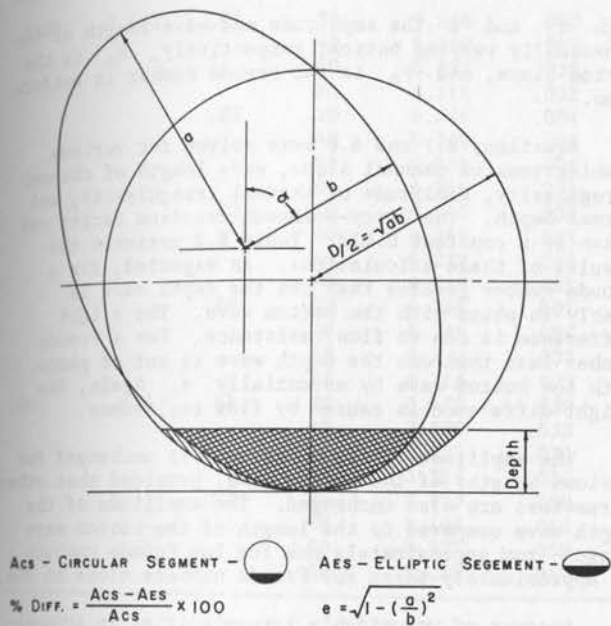


Fig. 8.2. Definition sketch for the relation of circular and elliptical cross sections.

- These calculations indicate
- (1) that the relative error in area increases with increased eccentricity;
 - (2) that the relative error in area decreases with increasing depth, and
 - (3) that the relative error in area is maximum at the vertical and horizontal positions of the principal axes and is minimum at an angular position of 45° with the horizontal.

Measurements to the nearest 0.001 inch were made of the inside diameter of the pipe at 60° intervals at cross sections spaced 40 feet apart before the inside of the conduit was painted and at intervals of 20 feet after the inside was painted. An ellipse was fitted to the three measured diameters at each cross section and its orientation determined. The results of these calculations are presented in Table 8.1. The differences between the means of each of the parameters for the two surveys are not significant on the 5 percent level. This indicates that the painting of the conduit had no effect on the internal geometry, and that doubling the number of stations did not significantly improve knowledge about the inside geometry of the conduit.

Accepting an average area of 968.41 square inches (6.725 square ft) the mean diameter of the conduit is 2.9262 feet. This figure was used for the conduit diameter in all calculations.

Because of the interrelated effects of depth, eccentricity, and α it appears that an error in the computation of the flow area by assuming a circular cross section instead of an approximated ellipse may range from zero to 3 percent with 1 percent representative.

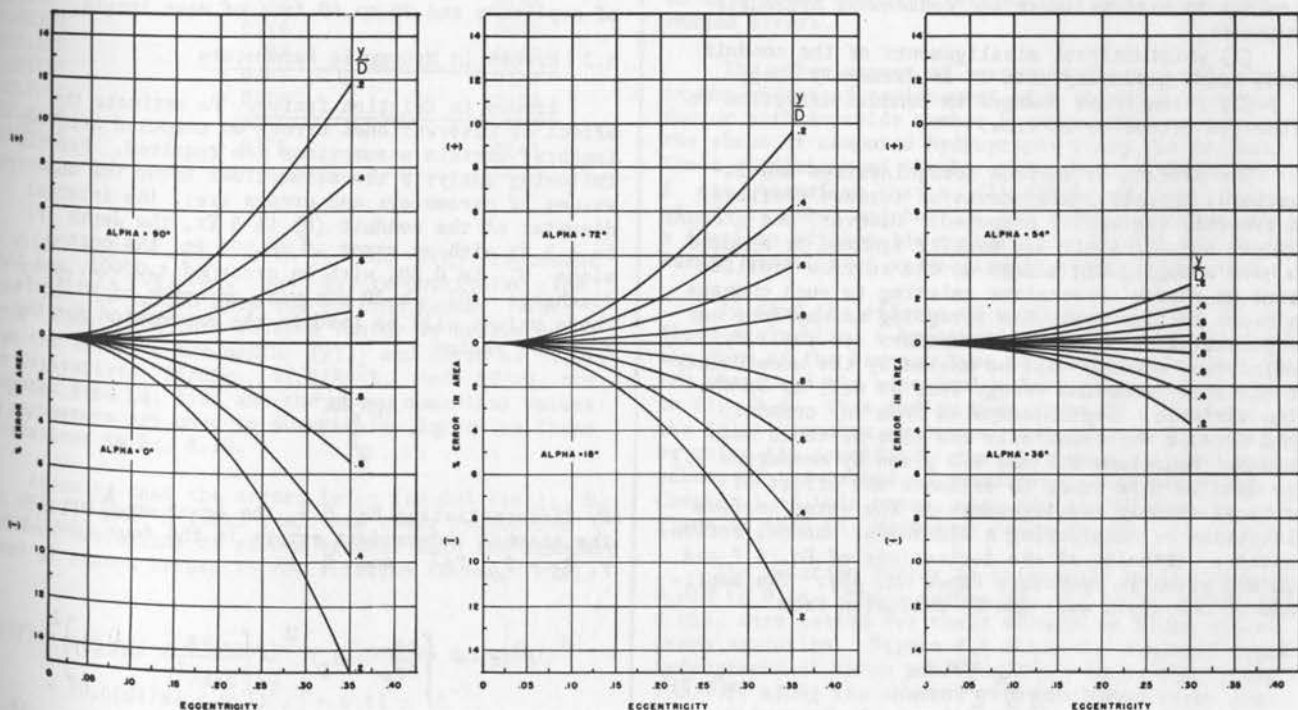


Fig. 8.3. Percent difference in area versus eccentricity and depth.

Table 8.1. Geometry of the experimental conduit.

	Number of Stations	Maximum	Mean	Minimum	Standard Deviation
Major Axis (in.)	21 41	17.869 17.913*	17.617 17.604	17.538 17.554	0.175 0.047
Minor Axis (in.)	21 41	17.626 17.680	17.516 17.510	17.435 17.430	0.0375 0.031
Eccentricity	21 41	0.176 0.175	0.1021 0.0993	0.046 0.051	0.0310 0.0244
Alpha α (degrees)	21 41	165.58 160.37	84.84 82.94	13.71 7.78	46.5 49.43
Area (sq. in)	21 41	989.5 994.9*	969.47 968.4	965.3 964.1*	3.84 3.94
Wetted Perimeter (in)	21 41	111.51 111.82*	110.373 110.314	110.13 110.07*	0.2769 0.2167
Hydraulic Radius (in)	21 41	8.87 8.89	8.7785 8.7742	8.76 8.75*	0.0183 0.0181

*occurred at same section;

The first row of figures refers to the unpainted interior of the conduit, while the second refers to the painted interior.

Errors because of vertical displacements of a circular cross section. The deviations of a given conduit from a mathematically straight alignment may be:

(1) the surface irregularities which contribute directly to viscous shear and consequent hydraulic roughness;

(2) unintentional misalignments of the conduit which occur gradually when it is extremely long;

(3) intentional changes in conduit direction to alter the direction of flow.

The effects of surface irregularities and intentional conduit realignments on surface profiles, in general, are easily computed. However, the gradual conduit misalignments are usually ignored or assumed to have a negligible effect on the surface profile. Based on energy conversions relating to such changes in cross section area, the foregoing assumptions may be justified, since energy transfers are small by definition, and may well be masked by the uncertainty of the mean turbulent energy loss as well as by the time variable. Depths computed from any commonly used formula represent only the time-distance mean values. Equations 8.7 and 8.8 given by Henderson [6] are used in this study to estimate the effect of vertical conduit misalignments on the water surface elevations by considering a sinusoidal channel-bottom profile. (Details of the derivations of Eq. 8.7 and 8.8 are given in Hydrology Paper No. 45). The amplitude of the depth wave can be evaluated from

$$\epsilon = -\frac{2\pi z_a \sin \phi}{3 S_o L} \quad (8.7)$$

in which ϵ is the ratio of amplitude to the uniform depth y_o , and ϕ is the phase angle for the depth wave. The value of ϕ is

$$\phi = \tan^{-1} \left[\frac{-3 S_o L}{2\pi y_o (1 - F_o^2)} \right], \quad (8.8)$$

with z_a and L the amplitude and wave-length of the sinusoidally varying bottom, respectively, S_o is the bottom slope, and F_o is the Froude number in uniform flow.

Equations 8.7 and 8.8 were solved for various combinations of channel slope, wave length of channel irregularity, amplitude of channel irregularity, and normal depth. The Darcy-Weisbach friction factor was taken as a constant 0.012. Table 8.2 presents the results of these calculations. As expected, for a Froude number greater than one the depth wave is nearly in phase with the bottom wave. The slight difference is due to flow resistance. For a Froude number less than one the depth wave is out of phase with the bottom wave by essentially π . Again, the slight difference is caused by flow resistance.

The amplitude of the depth wave is unchanged for various lengths of the bottom wave, provided that other parameters are also unchanged. The amplitude of the depth wave compared to the length of the bottom wave ranges from approximately one for low Froude numbers to approximately three for Froude numbers close to one.

Because of unavoidable irregularities in successive sections of the conduit and the method used to join sections, it was impossible to completely eliminate all deviations from the mean slope. Table 8.3 presents the results of mean slope determinations and the corresponding maximum and root-mean-square deviations from the slope of the least-square fit. These results show that the invert profile of the conduit had an undulating bottom with approximately 0.01 foot of amplitude and 20 to 40 feet of wave length.

8.2 Errors in Hydraulic Parameters

Errors in friction factor. To estimate the effect of observational errors on computed friction factors, certain assumptions are required. For the following analysis the assumptions about the numerical values of parameters and errors are: the internal diameter of the conduit (D) is 3 ft, the depth (y) is 1.5 ft with an error of ± 0.005 ft, the bottom slope S_o is 0.001 with an error of ± 0.0001 , and the discharge (Q) is 30 cfs with an error of ± 0.3 cfs. These values will be used in the expression for the friction factor

$$f = \frac{8g S_o R A^2}{Q^2} \quad (8.9)$$

By differentiating Eq. 8.9, the error equation for the assumed independent errors in the four quantities, y , $S_f = S_o$, RA^2 , and Q is

$$\epsilon^2(f) = \left[\frac{\partial f}{\partial S_f} \epsilon(S_f) \right]^2 + \left[\frac{\partial f}{\partial (RA^2)} \epsilon(RA^2) \right]^2 + \left[\frac{\partial f}{\partial Q} \epsilon(Q) \right]^2 \quad (8.10)$$

Table 8.2. Theoretical effect of bottom irregularity on water surface profiles.

Slope	Froude No.	z_a -ft	L-ft	ϕ -Rad	ϵy_o -ft	Slope	Froude No.	z_a -ft	L-ft	ϕ -Rad	ϵy_o -ft
.0100	2.582	.01	20	6.266	.002	.0001	.258	.03	20	3.170	.090
			40	6.249	.002				40	3.198	.090
			60	6.232	.002				60	3.227	.090
			80	6.216	.002				80	3.255	.090
		.02	20	6.266	.004			.04	20	3.170	.120
			40	6.249	.004				40	3.198	.120
			60	6.232	.004				60	3.227	.120
			80	6.216	.004				80	3.255	.120
	.03		20	6.266	.005			.01	20	3.142	.011
			40	6.249	.005				40	3.143	.011
			60	6.232	.005				60	3.144	.011
			80	6.216	.005				80	3.145	.011
	.04		20	6.266	.007			.02	20	3.142	.021
			40	6.249	.007				40	3.143	.021
			60	6.232	.007				60	3.144	.021
			80	6.216	.007				80	3.145	.021
.001	.816	.01	20	3.170	.030			.03	20	3.142	.032
			40	3.198	.030				40	3.143	.032
			60	3.227	.030				60	3.144	.032
			80	3.255	.030				80	3.145	.032
		.02	20	3.170	.060			.04	20	3.142	.043
			40	3.198	.060				40	3.143	.043
			60	3.277	.060				60	3.144	.043
			80	3.255	.060				80	3.145	.043

Table 8.3. Slope deviations of the experimental conduit

Slope	Max. Deviation, ft	Root-Mean-Square Deviation, ft
.0000052	+.0188	.0116
.0000157	+.0182	.0135
.0000303	+.0214	.0099
.0001325	+.0195	.0099
.0005197	+.0347	.0117
.0010101	+.0279	.0119
.0074578	-.0240	.0133
.0200690	+.0375	.0141

in which $\epsilon(f)$, $\epsilon(S_f)$, $\epsilon(RA^2)$, and $\epsilon(Q)$ represent errors in f , S_f , RA^2 , and Q , respectively. The term RA^2 is called here the section factor. The error $\epsilon(RA^2)$ of this section factor is evaluated from the error in the depth (y), and given as $\epsilon(y)$. The derivatives $\partial f/\partial S_f$, $\partial f/\partial(RA^2)$, and $\partial f/\partial Q$ are computed from Eq. 8.9, and the above numerical values of parameters are used to numerically determine these derivatives in Eq. 8.10.

Assuming that the errors in y (± 0.005 foot), S_f (± 0.00001), and Q (± 0.3 cfs) are their respective standard deviations of random errors, then the standard error of random errors in the friction factor, $\sigma(f)$, is

$$\sigma(f) = \{(2.681 \times 0.00001)^2 + (0.000286 \times 0.0997)^2 + (0.0001787 \times 0.3)^2\}^{1/2} = 6.61 \times 10^{-5}$$

For the representative constant friction factor of 0.012 for this conduit and the range of Reynolds numbers

used in this study, this estimated standard error of 6.61×10^{-5} represents only a 0.55 percent error. For this particular numerical case, the largest contribution of errors to the friction factor is from the errors in discharge measurement. The standard error in the friction factor of 6.61×10^{-5} may be considered as a lower boundary in practical evaluations of random errors.

The other aspect of errors in the friction factor is the effect of replacement of a changing friction factor with Reynolds number by an average factor on the shape of computed hydrographs along the conduit. These conditions of the Darcy-Weisbach friction factor f are, therefore, that a (1) single constant value of f is used, and that a (2) friction factor f is a function of Reynolds number given by the Prandtl-von Karman equation of hydraulically smooth boundaries.

To find the effects of various values of f on flood evaluation, a hypothetical inflow hydrograph in the form of the Pearson Type III function is used, with its shape and the numerical values of parameters given in Fig. 8.4. This inflow hydrograph is considered as the inlet boundary condition, and waves are integrated by using the specified intervals numerical integration scheme of the method of characteristics, described in Chapter 5 of this paper, with additional details given in Part IV, Hydrology Paper No. 46.

The friction factor in this study varied between 0.010 to 0.14. Three values of f , 0.010, 0.012 and 0.014, were tested for their effects on flood hydrograph evolution. Figure 8.5 shows the computed depth hydrographs at three positions $x = 50.0$, 410.0, and 771.7 ft along the channel with the above three constant values of f , as lines (1), (2), and (3). These results and their comparison lead to four conclusions;

(1) Differences of depth hydrographs for $f = 0.010$ and $f = 0.014$ at $x = 50.0$ ft from the inlet

are of the order of 5 percent of the channel diameter. These differences decrease with an increase of x . This decrease can be explained by both the initial conditions being a M2 curve and the downstream boundary conditions being in the critical flow at the conduit outlet.

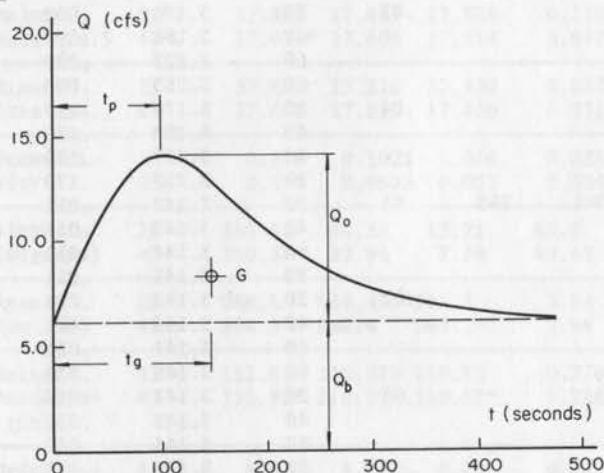


Fig. 8.4. Hypothetical inflow hydrograph of the Pearson Type III function, Eq. 5.32, with the selected parameters: $Q_b = 6.21$ cfs, $Q_0 = 8.00$ cfs, $t_p = 100.00$ sec, and $t_g = 150.0$ sec.

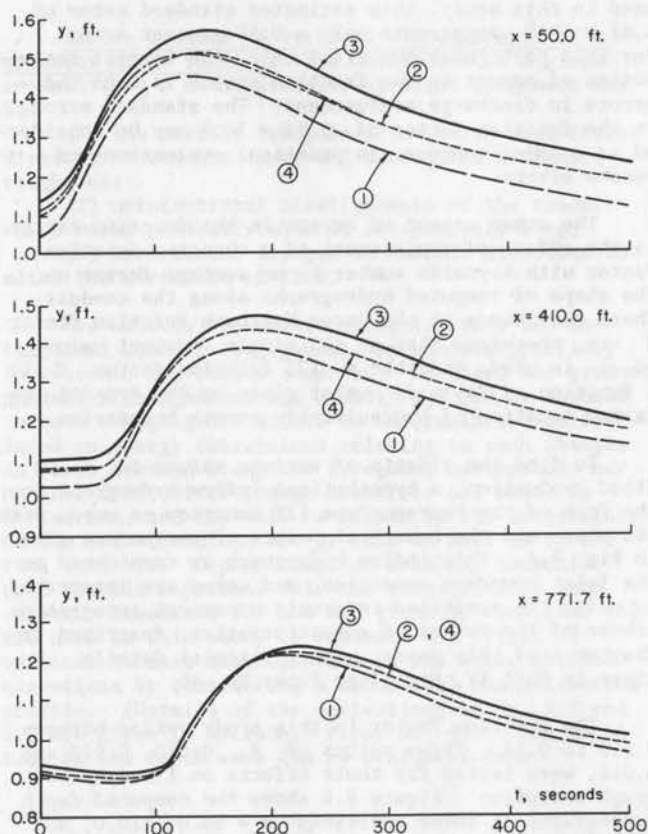


Fig. 8.5. Effect of changes in friction factor, f , at various positions along the channel: (1) $f = 0.010$, (2) $f = 0.012$, (3) $f = 0.014$, and (4) $f = f(R_e)$.

(2) The smaller the friction factor, the smaller are the depths, in general, and the peak depths, in particular, at various positions along the channel. For Fig. 8.5, the differences in peak depths between $f = 0.010$ and $f = 0.014$ in relation to the conduit diameter amount to approximately 4 percent at $x = 50.0$ ft, 3 percent at $x = 410.0$ ft, and 0.1 percent at $x = 771.7$ ft. Table 8.4 gives the percentage ratios of peak depth to conduit diameters, at various positions x along the conduit for $f = 0.010$, 0.012, and 0.014.

(3) The smaller the friction factor the earlier the peak depth occurs. The differences of the time at the peak depth for $f = 0.014$ and $f = 0.010$ are approximately 3 percent of the inflow hydrograph rising time (t_p) at $x = 50.0$ ft, 6 percent at $x = 410.0$ ft, and about 7 percent at $x = 771.7$ ft. Table 8.5 gives the times of peak depth percentages to the inflow hydrograph rising time; this shows how significant the effect of the friction factor is on the time occurrence of the peak depth.

(4) Because of downstream boundary conditions of a free fall at the outlet, there are no systematic trends in peak depths, nor are there any at the time of the peak depths along the downstream portion of the conduit within 100 ft of the outlet.

Figure 8.5 and Tables 8.4 and 8.5 give the respective properties of the depth hydrographs when the Darcy-Weisbach friction factor follows the Prandtl-von Karman resistance equation for hydraulically smooth conduits. The comparison of the results of three constant values of friction factor and the friction factor computed by this equation as a function of Reynolds number gives the following conclusions:

(1) Only a very small difference is shown between the depth hydrographs computed by using the average value of $f = 0.012$ and f as the function of R_e , for the given range of Reynolds numbers. The difference is on the order of 0.3 percent of the conduit diameter.

(2) The differences in peak depths computed by $f = 0.012$ and by f as the function of R_e are less than 0.4 percent of the conduit diameter at all positions along the conduit, as shown in Table 8.6.

(3) The differences in the time of the occurrence of peak depth computed by $f = 0.012$ and by f as the function of R_e range along the conduit from 3.9 to 8.5 percent of the rising time (t_p) of the inflow hydrograph. These differences at various positions along the conduit are shown in Table 8.7.

Errors in velocity distribution coefficients. By definition the minimum values of the two velocity distribution coefficients, α and β , are $\alpha = 1$ and $\beta = 1$. As soon as the time mean velocity at points in the cross section is not uniformly distributed, the coefficients are $\alpha > 1$ and $\beta > 1$. It was found in this study that for partly full flowing conduits, and for the depths between $0.3D - 0.8D$, with D the conduit diameter, the average coefficient values are $\alpha = 1.03$ and $\beta = 1.01$. In practice, the values $\alpha = 1$ and $\beta = 1$ are nearly always used for the numerical integration of the two partial differential equations of gradually varied free-surface unsteady flow. The investigation of the effects of various values of α and β , particularly of the assumptions $\alpha = 1$ and $\beta = 1$, on the evolution of flood waves along a long storm drain has been accomplished and is presented as follows.

The hypothetical inflow hydrograph (Fig. 8.4) and the numerical integration scheme as it was used to study the friction factor are also used to

Table 8.4. Percentages of peak depth to conduit diameter at various positions, x , along the conduit and various values of f .

f	x , position in feet along the conduit																
	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
0.010	49.95	49.62	49.28	48.94	48.59	48.23	47.86	47.48	47.07	46.64	46.16	45.63	45.03	44.36	43.56	42.39	39.76
0.012	52.11	51.72	51.32	50.91	50.50	50.08	49.64	49.19	48.70	48.18	47.61	46.99	46.29	45.50	44.51	42.99	39.81
0.014	54.01	53.56	53.11	52.65	52.18	51.70	51.20	50.68	50.12	49.52	48.88	48.17	47.38	46.47	45.29	43.48	39.86
$f(R_e)$	52.00	51.66	51.30	50.94	50.56	50.17	49.76	49.32	48.86	48.35	47.79	47.16	46.47	45.67	44.70	43.27	40.21

Table 8.5. The percentage ratios of the time of the peak depths to the rising time of inflow hydrograph, at various positions along the conduit, and for several values of friction factor.

f	x , position in feet along the conduit							
	0	50	100	150	200	250	300	350
0.010	124.03	130.47	136.90	142.78	149.29	155.89	162.63	169.05
0.012	126.21	132.55	139.51	145.26	152.38	159.07	166.14	173.19
0.014	128.20	135.06	141.31	148.00	154.87	162.16	169.38	176.23
$f(R)$	123.03	128.65	134.89	141.24	147.57	153.91	160.24	166.59

Table 8.5. Continued

x, position in feet along the conduit								
400	450	500	550	600	650	700	750	800
175.48	182.74	189.18	196.44	203.70	211.78	222.39	216.93	208.00
180.16	186.51	194.37	202.23	210.09	217.95	221.07	218.11	211.86
183.10	190.74	198.39	206.82	214.45	220.89	222.45	219.40	215.58
172.92	180.07	186.42	193.74	201.50	209.50	215.47	213.21	206.59

Table 8.6. Difference in peak depths computed from f as the function of Reynolds number and three values of f , in percent of conduit diameter.

f	x , position in feet along the conduit																
	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
0.010	-2.05	-2.04	-2.02	-2.00	-1.97	-1.94	-1.90	-1.84	-1.79	-1.71	-1.63	-1.53	-1.44	-1.31	-1.14	-0.88	-0.45
0.012	0.11	0.06	0.02	-0.03	-0.06	-0.09	-0.12	-0.13	-0.16	-0.17	-0.18	-0.17	-0.18	-0.17	-0.19	-0.28	-0.40
0.014	2.01	1.90	1.81	1.71	1.62	1.53	1.44	1.36	1.26	1.17	1.09	1.01	0.91	0.80	0.59	0.21	-0.35

Table 8.7. Differences in the occurrence of peak depths, computed from f as the function of Reynolds number and for each of three constant values of f , in percent of the inflow hydrograph rising time.

f	x , position in feet along the conduit																
	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
0.010	1.00	1.82	2.01	1.54	1.72	1.98	2.39	2.46	2.56	2.67	2.76	2.70	2.20	2.28	6.92	3.72	1.41
0.012	3.18	3.90	4.62	4.02	4.81	5.16	5.90	6.60	7.24	6.44	7.95	8.49	8.59	8.45	5.60	4.90	5.27
0.014	5.17	6.41	6.42	6.76	7.30	8.25	9.14	9.64	10.18	10.67	11.97	13.08	12.95	11.39	6.98	6.19	8.99

study the effects of the velocity distribution coefficients.

The effects of the velocity distribution coefficient α were studied for three values, 1.01, 1.02, and 1.03, while the coefficient β was kept constant at $\beta = 1.00$. The effects of the velocity distribution coefficient β were then studied for the values of $\beta = 1.005$, and 1.010, and $\alpha = 1.00$. The results can be summarized as follows:

(1) The higher a value of α the higher the depths produced. The differences in depths for various values of α and β , and for $\alpha = 1.00$ and $\beta = 1.00$, with $\alpha = 1.01, 1.02, 1.03$, and $\beta = 1.00$, increase with an increase of time. They are of the order of 0.40 percent of the conduit diameter.

(2) The differences in depths are higher at the upstream than at the downstream part of the conduit, and are on the order of less than 0.10 percent of the conduit diameter.

(3) The higher the α value, the higher are the peak depths at various positions along the channel. The differences in peak depths are less than 0.25 percent of the conduit diameter, as shown by the numbers in the first three rows of Table 8.8.

(4) The higher the α value, the later the peak depth, t_p , occurs. The differences in t_p between $\alpha = 1.01, 1.02, 1.03$ and $\beta = 1.00$, and $\alpha = 1.00$ and $\beta = 1.00$, range from 0.5 to 2.0 percent of the inflow hydrograph time parameter, t_p , as shown by the first three rows of Table 8.9.

Table 8.8. Differences in peak depths, in percent of conduit diameter computed for various values of α and β , and for $\alpha = 1.00$ and $\beta = 1.00$.

α	β	x, position along the conduit, in feet																
		0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
1.01	1.000	0.08	0.08	0.08	0.09	0.08	0.08	0.08	0.07	0.08	0.08	0.07	0.07	0.07	0.07	0.06	0.05	0.03
1.02	1.000	0.17	0.17	0.17	0.17	0.16	0.16	0.16	0.15	0.15	0.15	0.15	0.14	0.14	0.13	0.13	0.11	0.05
1.03	1.000	0.25	0.25	0.24	0.24	0.23	0.23	0.22	0.21	0.22	0.22	0.21	0.20	0.20	0.19	0.18	0.15	0.07
1.00	1.005	0.01	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.03
1.00	1.010	0.01	0.02	0.02	0.03	0.03	0.02	0.03	0.02	0.03	0.03	0.03	0.02	0.03	0.02	0.02	0.02	0.03
1.03	1.010	0.25	0.19	0.23	0.27	0.30	0.32	0.36	0.37	0.39	0.40	0.41	0.39	0.39	0.37	0.39	0.44	0.49

Table 8.9. Differences in times of peak depths in percent of conduit diameter for various values of α and β and for $\alpha = 1.00$ and $\beta = 1.00$.

α	β	x, position along the conduit, in feet																
		0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
1.01	1.000	0.72	0.63	0.09	0.70	0.35	0.71	0.69	0.51	-0.03	0.67	-0.05	0.14	0.66	0.21	0.66	0.12	0.66
1.02	1.000	0.63	1.34	0.62	0.87	1.04	1.40	1.11	0.57	0.65	1.35	0.54	0.52	1.05	0.85	1.30	0.78	0.51
1.03	1.000	1.33	1.32	1.31	1.23	1.73	1.71	1.24	1.24	1.31	2.01	1.18	1.17	1.15	1.14	1.12	1.14	1.15
1.00	1.005	0.31	0.24	-0.45	-0.17	0.02	0.05	-0.39	-0.38	-0.27	0.46	-0.33	-0.30	-0.29	-0.27	-0.26	0.02	-0.27
1.00	1.010	-0.18	-0.15	-0.73	-0.08	-0.06	-0.01	-0.26	-0.75	-0.63	0.12	-0.66	-0.62	-0.57	-0.54	0.28	-0.25	0.25
1.03	1.010	2.56	2.73	3.52	3.62	4.06	3.88	4.78	4.96	5.77	5.24	6.23	7.22	6.81	6.70	4.36	4.06	4.38

(5) The higher a value of β the higher the depths produced. The differences in depth between $\beta = 1.005$ and 1.010 and $\beta = 1.000$ decrease with an increase of both the time and distance, and are on the order of 0.08 percent of the conduit diameter.

(6) The differences in depths at the upstream part of the conduit are of the same order as the differences at the downstream part of the conduit.

(7) The higher a value of β , the higher are the peak depths produced at various positions along the conduit. The differences in peak depths are less than 0.03 percent of the conduit diameter as shown by the fourth and fifth row of Table 8.8.

(8) The higher a value of β the earlier the occurrence of the peak depth. The differences in t_p between $\beta = 1.005$, 1.010 , and $\beta = 1.000$ for $\alpha = 1.00$ range from 0.01 to 0.75 percent of the inflow hydrograph parameter t_p , as shown by the fourth and fifth row of Table 8.9.

When both $\alpha > 1.00$ and $\beta > 1.00$ are used, particularly $\alpha = 1.03$ and $\beta = 1.01$, the differences in Tables 8.8 and 8.9, the sixth row, show the largest values. The differences in peak depths between $\alpha = 1.03$ and $\beta = 1.01$ and $\alpha = 1.00$ and $\beta = 1.00$ reach 0.50 percent of the conduit diameter at $x = 800$ ft. This value is relatively negligible.

In summary, replacing the average values of $\alpha = 1.03$ and $\beta = 1.01$ of free-surface conduit flow, for the flow depths $y = 0.3 D - 0.8 D$, by minimum values of $\alpha = 1.00$ and $\beta = 1.00$, only slightly affects the evolution of flood waves along the circular storm drainage conduits.

8.3 Errors in Computations

Computational errors resulting from procedures used in this study are the product of the following:

(1) The approximation of infinitesimal variations by finite values. This is a result of assuming, in general, linear relations rather than true curvilinear relations. This is a systematic error, which is not readily determined since it may be positive or negative during different stages of the computations.

(2) Truncation of numerical values. This is due to the limited precision of any discrete-element calculator. The computer used for these studies was programmed for single precision computations using 15 decimal digits.

(3) Round-off in the printed output. The computer used for the calculations rounded off all numbers in a manner similar to a manual calculator.

The following discussion evaluates the effects of control variables in the solution of the unsteady flow equations. These equations are considered under the computational schemes with the incremental length and incremental time interval during which the integration process proceeds.

Determination of computational parameter Δt . The grid sizes of Δx and Δt in the computational schemes is limited by the characteristic directions ξ_+ , ξ_- , encountered during the integration. In order to assure that this condition exists, it is necessary that

$$\Delta t < \frac{\Delta x}{V + \sqrt{gA/B}} \quad (8.11)$$

Computations require that Δt be computed before the integration process. The incremental distance

Δx was arbitrarily established. The computed velocity (V) was based on conditions that would produce the maximum critical velocity. The geometric term gA/B , which goes to infinity for full size flow, was computed for a depth of 0.82 of the diameter.

Effect of computational parameter Δx . The method of characteristics using a specified intervals scheme gives the complete numerical solution of the free-surface unsteady flow. The accuracy of the results depends on the size of the rectangular grids Δx and Δt .

If n is the number of intervals along the conduit and x_L is the length of the conduit, then

$$\Delta x = \frac{x_L}{n} \quad (8.12)$$

with n an arbitrarily selected number. The smaller the Δx presumable the more accurate are the results. But also, the smaller the Δx , the greater the required computing time. In compromising these two conditions to satisfy the objectives of this study, several values of n for the fixed x_L were tried for the inflow hydrograph of Fig. 8.4 and the circular experimental conduit of this study.

Figure 8.6 shows the effect of the size of Δx on the depth hydrographs at three positions along the conduit. The upper graph is the depth hydrograph at a position 50.0 ft downstream from the inlet and for a Δx of 40.91, 20.45, 10.23, and 5.12 ft corresponding to n values of 20, 40, 80, and 160, respectively. The center and lower graphs are the depth hydrographs at 410.0 ft and at 771.7 ft from the inlet, respectively. The initial condition for each computation is the steady-state water surface for a free outfall.

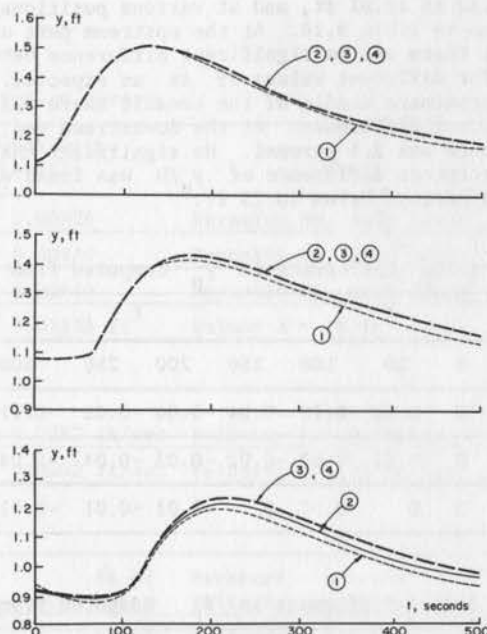


Fig. 8.6. Effect of Δx on hydrographs at various positions along the conduit; (1) $\Delta x = 40.91$ ft, (2) $\Delta x = 20.45$ ft, (3) $\Delta x = 10.23$ ft, and (4) $\Delta x = 5.12$ ft, at three locations of conduit $x = 50.0$ ft (upper graph), $x = 410.0$ ft (center graph), and $x = 771.7$ ft (lower graph).

Comparing the depth hydrographs of Fig. 8.6 with the inflow discharge hydrograph of Fig. 8.4 it was found that:

(1) The critical portion of the conduit for computing depth hydrographs is near the outlet where there is the greatest curvature of the base-flow water-surface profile. The maximum differences between the computed depths, with Δx being 40.91 and 5.12 ft, are approximately 0.3, 0.6, and 1.0 percent of the conduit diameter at 50.0, 410.0, and 771.7 ft from the inlet, respectively.

(2) There is no significant increase in accuracy over 0.005 ft or 0.15 percent of the conduit diameter when Δx is less than 10.23 ft. Therefore, a Δx equal to 10.23 ft, or n equal to 80, was selected for computations.

The peak depth y_p and the time to peak depth t_p are two important parameters describing a depth hydrograph. The required accuracy of a computed hydrograph at various positions along the conduit can be measured by the peak depth, y_p , relative to the diameter, D , of the conduit, for various lengths Δx . Also, the accuracy can be measured by the time to peak depth, t_p , relative to the peak discharge, time t_p , of the inflow discharge hydrograph of Fig. 8.4, for various lengths Δx and the same positions x .

From the selected criteria for defining the accuracy of a computed hydrograph for a given Δx , it was found that the percentage differences of y_p ,

$$\frac{(y_p)_i - (y_p)_{\min}}{D} \times 100,$$

in which the index "min" refers to the depth y_p of the smallest difference used, $\Delta x = 5.12$ ft, and the index "i" refers to depths of any other $\Delta x > 5.12$ ft, ranged from 0.0 percent to 2.1 percent for Δx ranging from 5.12 to 40.91 ft, and at various positions x , as shown in Table 8.10. At the upstream part of the conduit there was no significant difference between y_p/D for different values of Δx as expected. At the approximate middle of the conduit there was a 0.2 percent difference. At the downstream end, the difference was 2.1 percent. No significant change in the percentage difference of y_p/D was found when Δx was reduced below 10.23 ft.

In using the other parameter, T_p to define the accuracy of computed depth hydrographs with different values of Δx and various positions x , the measure of accuracy was

$$\frac{(T_p)_i - (T_p)_{\min}}{t_p} \times 100,$$

in which the indices "min" and "i" refer to $\Delta x = 5.12$ ft and all other values of Δx , respectively. It was found that there were no significant differences for values $\Delta x > 5.12$ ft, and for various positions x . The percentages were about 1.2 percent at the upstream, 2.0 percent at the middle, and 8.5 percent at the downstream part of the conduit. It was also found that there was no significant change of the percentages of T_p to t_p (which was about 1.9 percent) when Δx was reduced below 10.23 ft, as shown in Table 8.11.

Tables 8.10 and 8.11 show the percentage differences of y_p to the diameter D of the conduit, and T_p to t_p , respectively, with different values of Δx and at various positions of x . These values at even distances (0, 50, 100, ..., ft) were computed by linear interpolation from the values in the grid system; therefore, some error may have been introduced. However, the change in shape of the depth hydrograph due to varying Δx was considered to be small. Larger Δx produced a lower and later peak depth.

8.4 Errors in Experimental Observations

Errors in geometric variables. A summary of this type of error has been already presented in Section 8.1. The two sources of geometric errors, the irregularities of conduit cross section and invert slope, were discussed in Chapter 6, Hydrology Paper No. 44 for the observed results, and were analyzed in detail in Chapter 2, Hydrology Paper No. 45.

Time difference errors. A systematic error was introduced into all tests because the flow-measuring orifice was located 82.2 feet upstream of what was considered the beginning of the test conduit. For much of this distance the conduit was flowing full and providing instantaneous transmission of changes in flow. This systematic error effected primarily

Table 8.10. Difference in y_p computed from various sizes of Δx (in percent of conduit diameter D).

Δx	Distance, ft																
(ft)	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
40.91	0	-0.02	-0.16	-0.04	-0.06	-0.08	-0.11	-0.16	-0.24	-0.31	-0.41	-0.50	-0.59	-0.70	-0.94	-1.43	-2.07
20.45	0	-0.01	-0.02	-0.02	-0.03	-0.04	-0.04	-0.06	-0.10	-0.13	0.18	-0.22	-0.27	-0.39	-0.42	-0.66	-0.99
10.23	0	0	-0.01	0	-0.01	-0.01	-0.01	-0.02	-0.03	-0.04	-0.06	-0.08	-0.09	-0.11	-0.14	-0.23	-0.39

Table 8.11. Difference in T_p computed from various sizes of Δx (in percent of t_p).

Δx	Distance, ft																
(ft)	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
40.91	1.23	-0.09	0.18	0.14	-1.21	-0.36	-1.62	-2.04	-2.02	-1.81	-1.09	1.21	-0.96	-1.43	-8.47	-7.32	-3.48
20.45	-0.40	-0.09	0	0.14	0.05	-0.06	0	-0.40	-0.40	-1.81	-2.73	-0.42	-0.40	0	-3.58	-4.07	-2.04
10.23	0.41	0	0	0.14	0.05	0	0	-0.22	-0.40	0	-1.90	-0.24	-0.42	0	-1.49	-1.62	-0.41

the time correlations of observed and computed depths. A secondary effect is due to a minor modification of the inflow hydrograph between the hydrograph observed at the orifice and the actual hydrograph observed at the beginning of the conduit. The distance between the point where a free surface formed and the beginning point of the test conduit, however, provided a varying time lag between observed and computed data of from 8 to 14 seconds. That is, the time recorded by the orifice transducer for a given flow discharge was ahead of the time when that discharge actually reached the conduit test section by the amount of time it took the wave to travel the 82.2 feet. To eliminate this difference, time shifts can be estimated visually from comparison plots (depth versus time) and experimental times can be adjusted by this amount. This shift in time is discussed more fully in Chapter 6.

Instrumentation errors. Instrumentation errors were analyzed on the basis of calibration results. Since the true values of physical quantities are never exactly measured, the calibration process of each instrument was considered to be an accurate estimate of the errors in the measured quantities.

A common method of placing boundaries on the errors used the unbiased standard deviation, s , defined by

$$s = \left[\sum_{i=1}^N \frac{(q_i - q_0)^2}{N - 1} \right]^{1/2}, \quad (8.15)$$

in which q_i is the individual reading observed in a given range, q_0 is the corresponding reading from a reference curve in the same given range as q_i , and N is the total number of observations.

For a Gaussian distribution of errors at an approximately 95 percent significance level, the physical quantities measured by each instrument can be expressed as

$$q_0 = q_i \pm 2s \quad (8.16)$$

Equation 8.16 means that 95 percent of the measured quantities q_i lie within the range of $\pm 2s$ of q_0 .

Table 8.12 gives a summary of the calibration results of the orifice meters, current meters, and pressure transducers using Equation 8.16.

Reproducibility errors. An attempt was made to perform some experiments with conditions exactly the same as conditions of some selected previous runs. This was done to have some measure of the errors that were inherently reproduced in the experimental system. By running two runs under exactly the same conditions, the differences between the observed wave depths would be a measure of this type of error. If no errors were generated by the system, the observed values for both runs would be the same. This manner of comparison would not, however, be a measure of the random errors.

During the experiments, seven attempts were made to duplicate the conditions of previous runs. Only one of these runs, however, was successful. The other six could not be used in this evaluation because either the base or peak flows did not correspond to the earlier conditions or the time of one wave was longer than that of the previous run, resulting in different total water volumes. The runs with matching conditions were 010013 and 019913. The error was small.

Table 8.12. Estimate of instrumentation errors.

	s Standard Deviation	Error Bounds with 2.5% Level	Range
Orifice Meters	Small Opening	$\alpha = \alpha \pm 0.00826$	Reynolds No. $3 \times 10^5 \sim 2 \times 10^6$
	Medium Opening	$\alpha = \alpha \pm 0.00440$	Reynolds No. $2 \times 10^5 \sim 2 \times 10^6$
	Large Opening	$\alpha = \alpha \pm 0.00640$	Reynolds No. $5 \times 10^5 \sim 2 \times 10^6$
	Large Opening	$\psi = \psi \pm 0.03238 \text{ ft}^3$	Volume $8 \sim 15 \text{ ft}^3$
Current Meters	0.0141 ft/sec	$v = v \pm 0.0282 \text{ ft/sec}$	Velocity $1 \sim 8 \text{ ft/sec}$
	0.0451 ft/sec	$v = v \pm 0.0902 \text{ ft/sec}$	Velocity $8 \sim 16 \text{ ft/sec}$
Pressure Transducers	ft of 0.0002 water	$H = H \pm 0.0004 \text{ ft of water}$	Pressure $\pm 1 \text{ psi}$ Voltage $\pm 3 \text{ volts}$
	ft of 0.0003 water	$H = H \pm 0.0006 \text{ ft of water}$	Pressure $\pm 5 \text{ psi}$ Voltage $\pm 3 \text{ volts}$

8.5 Total Errors

The discrepancy between a computed value of analytical waves and the corresponding observed value from a physical wave experiment is attributable to numerous sources. These discrepancies are the result of systematic and random errors in the observational system and possible systematic errors in computational procedures. Random errors are a result of unavoidable accidental variations in the physical system.

The comparison of computed and observed waves, as discussed in Chapter 6, include both the systematic and the random errors. The systematic errors are mainly in the form of base depth shifts due to zero-reading shifts, and in the form of time shifts because of the difficulty of coordinating the zero time of computed and observed waves. Additional systematic errors result in using average or lower bounds of friction factor and velocity distribution

coefficients. All other differences between the computed and observed waves are attributable to random errors in observations and systematic errors in computations.

In previous analysis of individual sources of errors, and effects of various simplifications in hydraulic parameters, can not be simply integrated into the expression of a total error because: (1) errors are of different types, and (2) they may be dependent, with the dependence unknown. Therefore, it was not considered feasible to attempt to develop an overall formula for total errors.

It is reasonable to state that the gradually varied free-surface unsteady flow is in such a stage of development that the theory can be less effective for further contribution than a systematic analysis of all sources of errors.

CONCLUSIONS, LIMITATIONS AND RECOMMENDATIONS.9.1 Conclusions.

General conclusions, regarding this Part I, Hydrology Paper No. 43, as well as general results presented in detail in Parts II, III, and IV, Hydrology Papers 44, 45, and 46, are summarized as follows.

(1) The comparison of the solution of the theoretical equations of unsteady flow by numerical integration with accurately observed waves or the comparison of analytical and physical gradually varied free-surface waves, with the same boundary and initial conditions shows good practical agreement, provided systematic shifts in depth or in the time in some of the observed waves are considered.

(2) Observed waves in this study were subjected to a variety of sources of systematic and random errors. The systematic errors in recording the base flow depth, the recorded discharge as a function of time at the entrance into the conduit, and the energy head losses at the junction boxes are sometimes much larger than the cumulative random errors of various other sources of errors (measurements of discharge, depths, hydraulic parameters, etc.).

(3) Though the experimental conduit was 3 ft in diameter and 822 ft long, the observational errors on waves produced in this conduit, and the computational errors in the numerical finite-difference integration methods of the unsteady flow differential equations, did not permit an assessment of the differences between the physical and analytical waves, with these differences resulting from the basic hypotheses underlying the development of the continuity and momentum partial differential equations of gradually varied free-surface unsteady flow.

(4) The unique experimental facilities in this study of flow in conduits made it possible, through experimental investigations and observations, to understand various aspects of gradually varied free-surface unsteady flow.

(5) The geometric and hydraulic parameters such as area, depth, surface width, hydraulic radius, friction factor, velocity distribution coefficients, junction box losses, etc., which define the coefficients in the continuity and momentum partial differential equations, have been evaluated with sufficient validation from the experimental conduit so that a reliable numerical integration of these equations can be obtained.

(6) The recommended numerical integration procedure, which is based on experimental evidence, is the specified intervals scheme of the method of characteristics, in which case the four ordinary characteristic equations equivalently replace the two partial differential equations of unsteady flow.

(7) To simplify the coefficients of the characteristic equations, the velocity distribution coefficients, α and β , are taken as unities. This research shows that substituting constant α and β for functions of α and β on Reynolds numbers or on the depth of water in partly full conduits does not significantly affect the floods routed along the

circular conduits. Similarly, the use of a Darcy-Weisbach friction factor, $f = 0.012$, for the range of Reynolds numbers in this study, instead of using the function $f(R_e)$, did not significantly affect the routed flood waves.

(8) It is doubtful that simplified flood routing methods have any significant advantage over numerical solutions of the theoretical equations for the design of storm drains. Simplified flood routing methods may be useful in those cases when the diameter of the conduit may be obtained explicitly from the equations or graphs, as the initial assumed dimension for the more accurate computations by the method of characteristics, or when these methods substantially save computer time with only a small loss in the accuracy of the solution.

(9) Research results in this study are considered as the basic information necessary for the expected development of practical flood routing methods for storm drain design, based on the unsteady flow approach to flood routing through a storm drain by using the most complete differential equations.

(10) The only flood routing method that can meet various field conditions is a method that uses the most complete differential equations of unsteady flow. A versatile new flood routing method should be applicable under various conditions such as branching of storm drains, upstream flow under particular conditions, different boundary and initial conditions, subcritical and supercritical flows and their transitions, etc.

(11) It is expected that in the future more accurate estimations of inlet flood hydrographs will be produced and the hydraulic conditions of storm drains will become better known so that extremely accurate flood routing methods will be both required and justified in practice. For that purpose, the use of digital computers and numerical integration schemes of the complete differential equations of unsteady flow will be used for both the design of storm drains, and the eventual prediction of flood hydrographs at particular points of existing storm drainage systems.

Conclusions in more detail are given in the last chapters of Papers 44, 45, and 46.

9.2 Limitations in the Developed Results.

The proposed method of using the most complete differential equations of unsteady flow for flood routing through storm drains, particularly in applying the numerical integration by the finite-differences specified intervals scheme of the method of characteristics, has some limitations and conditions of application; they are as follows.

(1) The method is applicable only to gradually varied free-surface unsteady flow, which implies a negligible vertical acceleration, or, in general, the method is applicable to flood waves which attenuate as they progress along the conduit.

(2) Waves must be one-dimensional or the velocity components in the horizontal direction normal to the conduit axis are negligible.

(3) The hydraulic resistance and velocity distribution coefficients in unsteady flow do not deviate significantly from the resistance and coefficients in steady flow, for the same Reynolds numbers.

(4) The method can not accommodate accurately the case when the flood wave moves on a dry bed, particularly for the frontal part of the wave.

(5) The method as developed can not take into account the instability and air entrainment effects in a conduit flowing nearly full.

(6) The second boundary condition at the upper end of the conduit (the first being the inflow hydrograph) in the supercritical flow, as a discharge-versus-depth relation, is assumed usually to be the normal depth relation. This approach may depart from actual physical conditions, which makes it difficult for computed waves to correspond with physical waves, at least at the upper sections of the conduit.

(7) Unavoidable inaccuracies in estimating junction box energy losses, particularly with the lower inlet position of laterals, result in significant differences between observed and computed flood hydrographs in storm drains.

(8) Though the method of characteristics with its specified intervals scheme may be improved by nonlinear interpolations of velocities and depths along the initial time t_j , when their values at time t_{j+1} must be computed, there is still a lack of information for determining whether the interpolation should be made through three or four points for the supercritical regime; there is also insufficient information for determining what advantage four-point interpolation can produce. This is a limitation in assessing the accuracy of results under the various conditions of nonlinear interpolations.

(9) The basic limitations in applying the proposed finite-difference scheme is related to the accuracy available in describing boundary and initial conditions. The better they are defined and the closer they are to the physical boundary and initial conditions, the more reliable are the numerical solutions.

9.3 Recommendations for Future Research.

Recommendations for further experimental research are given in more detail in subchapter 8.2 of Part III, Hydrology Paper No. 45; the potential of the developed facilities for further experimental investigations is given in subchapter 7.2 of Part II, Hydrology Paper No. 44. Recommendations for further studies of numerical integration schemes of complete differential equations of unsteady flow are given in subchapter 5.2 of Part IV, Hydrology Paper No. 46. Although recommendations in detail can be found as cited, general recommendations for future research are summarized as follows.

(1) The energy head losses of certain types of singularities, which occur in storm drains, should be investigated systematically. Such energy losses occur at various junction-box structures, boxes with various

angles and diameters of joining drains, or boxes at which the direction and/or the slope of the main drain change, drop-structures for which there is a backwater effect of downstream reach on the upstream section during the peak flows, and similar types of structures along the storm drains. Of particular interest is the dissipation of additional turbulence created by the structures of concentrated energy losses along the immediate downstream section of storm drains.

(2) Investigation is necessary for the air-water interaction when the conduit flows nearly full; all instability phenomena in the passage from the free-surface flow to flow under pressure, or the opposite, should also be investigated. Because the drain dimensions are determined by the design flood hydrograph, and because the peak depth for that hydrograph should approximately coincide with the depth of the largest drain capacity in free-surface flow, which occurs in the depth range of relatively unstable flow, this air-water interaction has particular relevance to storm drain design.

(3) The peak depth of a well designed storm drain is largest immediately downstream of the junction box, or the peak depth is at the upper end of the conduit between the two junction boxes, if the flood hydrograph is in the attenuation regime. This is the case if the flow is with the Froude number approximately less than $F=2$. The second largest peak depth may be produced immediately upstream of the junction box because of the backwater produced by the energy loss at the junction box. The section downstream of the junction box is also subject to dissipation of surplus turbulence generated in the junction box; this is equivalent to additional roughness. Therefore, the critical design conditions are either in the conduit sections where both the instability of nearly full conduit flow of air-water interaction and the additional turbulence occur, or in the section where significant backwater effect occurs. These phenomena which greatly affect design conditions, need to be researched.

(4) Observations in the 3 ft diameter and 822 ft long conduit, containing both systematic and random observational and computational errors, were unable to throw light on the differences between analytical and physical free-surface conduit waves, with these differences resulting from the five hypotheses underlying the derivation of the continuity and momentum partial differential equations of gradually varied free-surface unsteady flow. This objective of studying effects of basic hypotheses should be pursued in future research with appropriate improvements in the accuracy of all observations and computations. Future research in gradually varied unsteady flow might be more relevant if the accuracy of solutions are determined rather than reworking in the new light fundamentals of the basic equations.

(5) New finite-difference schemes for integrating the quasi-linear hyperbolic partial differential equations are proposed from time to time. Thus, literature should be surveyed systematically for these various schemes; they should be then subjected to investigations, particularly if they concern stability, accuracy, versatility and flexibility of application, and economy of computer time.

REFERENCES

External References

1. Yevjevich, V. M., 1964, Bibliography and discussion of flood-routing methods and unsteady flow in channels, U.S. Geological Survey Water-Supply Paper 1690.
2. Courant, R. and Friedrichs, W. R., 1948, Supersonic flow and shock waves, Interscience Publishers, New York.
3. Lister, M., 1960, The numerical solutions of hyperbolic partial differential equations by the method of characteristics in mathematical methods for digital computers, edited by A. Ralston and H. S. Wilf, John Wiley and Sons, pp. 165-179.
4. Ackers, P. and Harrison, A. J. M., 1963, The attenuation of flood waves in part-full pipes, Hydraulic Research Station, Wallingford, Berkshire, England, Report No. INT 31, December.
5. Ackers, P. and Harrison, A. J. M., 1964, The attenuation of flood waves in part-full pipes, Proceedings, Institute of Civil Engineers (London), Vol. 28, Paper No. 6777.
6. Henderson, F. M., 1964, Steady flow in sinusoidally varying channels, Hydraulics and Fluid Mechanics, R. Silvester (Editor), McMillan Company, New York, pp. 51-67.
7. Henderson, F. M., 1966, Open channel flow, McMillan Company, New York.
8. Harris, G. S., 1970, Real time routing of flood hydrographs in storm sewers, American Society of Civil Engineers, Journal of Hydraulics Division, paper 7327, HY6.
9. Chow, Ven Te, 1959, Open channel hydraulics, McGraw-Hill Book Company, New York.

Internal References

From the Project Reports and Theses

1. Yevjevich, V. M., June 1961, "Unsteady Free Surface Flow in a Storm Drain" - General and Analytical Study. Report for the U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University. CER61-VMY38
2. Barnes, A. H., August 1965, "Predictability of Free-Surface Profiles for Steady Non-uniform Flow in a Circular Cross-section," Ph.D. dissertation, Dept. of Civil Eng., Colorado State University.
3. Frye, W. B., June 1966, "A Development of a Numerical Solution to a System of Partial Differential Equations," M.S. Thesis - Dept. of Mathematics, Colorado State University.
4. Lorah, W. L., June 1966, "Free-Surface Flow Energy Losses in a 90° Junction Box," M.S. Thesis, Dept. of Civil Eng., Colorado State University.
5. Yevjevich, V. M. and Barnes, A. H., August 1966, "Hydraulic Roughness of Free-Surface Flow in a Smooth Steel Pipe," Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University.
6. Pinkayan, S., Sept., 1966, "Solution of the Unsteady Free Surface Flow in a Storm Drain by the Method of Characteristics," Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University. CER66-SP29
7. Barnes, A. H., October 1966, "Velocity Distribution Factors in a Circular Cross-section," Report of U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University.
8. Pinkayan, S. and Barnes, A. H., Feb. 1967, "Unsteady Free-Surface Flow in a Storm Drain with Lateral Inflows," Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University.
9. Mitchell, J. S., March 1967, "Comparison of Mathematical versus Experimental Flood Wave Attenuation in Part-Full Pipes II," Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University.
10. Mitchell, J. S., April 1967, "Comparison of Mathematical versus Experimental Flood Wave Attenuation in Part-Full Pipes," M.S. Thesis, Dept. of Civil Eng., Colorado State University, also Report for U.S. Bureau of Public Roads.
11. Putrakul, S., June 1967, "Flood Routing in a Circular Section," M.S. Thesis, Dept. of Civil Eng., Colorado State University.
12. Safai, J., May 1968, "Non-dimensional Analysis of Attenuation of Flood Waves in Partly Full Circular Conduits," Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University. CER67-68JS8
13. Barnes, A. H., June 1969, "Unsteady Flow in a Storm Drainage System, Part A - Theory and Error Considerations," Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University. CER69-70AB1
14. Barnes, A. H., January 1970, "Unsteady Flow in a Storm Drainage System, Part B - Hydraulic Parameters, Boundary and Initial Conditions," Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University. CER69-70AB1

APPENDIX 1

Description of Research Project

UNSTEADY FLOW IN A STORM DRAIN

Part One

THE BROAD PROBLEM

Construction of highways in urban areas (and sometimes elsewhere) requires disposal of storm water by means of underground storm drains because property values and other considerations prohibit carrying storm water in open channels. These systems frequently include picking up storm water contributed by areas outside the right-of-way. The usual design procedure is to compute sizes of pipe by the so-called "rational method." When the highway is depressed the highway department usually attempts to exclude all water falling outside of the depressed section so that the size of the system collecting water for the highway itself (and usually requiring pumping) will be a minimum.

Storm drains for depressed highways sometimes are miles in length (West Route in Chicago for example is about six miles) this producing a watershed that is very long in relation to its width. There is good reason to doubt that the rational method is reliable in such a case, (nor for that matter has the rational method been scientifically proved in any case). A flood-routing procedure beginning with routing of overland flow to inlets is generally conceded to be the logical approach especially since digital computers would permit investigation of various storm patterns both as to time and areal distribution in testing the probable functioning of a given system and indicated modifications. Such a procedure would make it possible to know where every cubic foot of water was at any time so that opportunities for temporary storage reducing the peak load could be investigated. Major economies in initial cost might result and are worthwhile exploring since the usual storm-drain system for a depressed highway will cost around \$500,000 per mile.

To my knowledge no one has developed a procedure for routing storm water through a storm drain by any except grossly approximate methods.

The problem then is to study the hydrodynamics of unsteady flow in storm drains with the objective of developing a sound procedure adopted to a digital computer, verifying the procedure by hydraulic model tests and field measurements as may seem necessary. The ultimate purpose is to provide a working design method applicable to any situation where storm drains are used for removal of storm water. However, there are many variations possible in the physical set-up as will be shown in the following paragraphs.

Part Two

AN OUTLINE OF THE FACTORS INVOLVED IN THE

HYDRODYNAMICS ANALYSIS

The inflow hydrographs to the storm-drain system will not be considered as part of the hydrodynamics of the storm drain as that is a separate problem. It can be assumed that methods of computing inflow hydrographs will be provided. The system will also be assumed to consist of a single continuous line of pipe with inflow

from inlets, or from laterals collecting flow from a series of inlets all located on the highway right-of-way. The right-of-way may include large interchange areas in which case lateral inflow may be substantial in relation to flow in the main drain and conceivably may be large enough to require analysis as a system by itself. For purpose of analysis it may be assumed that flow entering system at any point will have no momentum in the direction of the outflow pipe.

Conduit may be circular or of any shape commonly used, either precast or monolithic concrete, generally will increase in size in downstream direction, changes in size being made at manholes open to atmospheric pressure, and crown-lines will match up except in case where a drop manhole occurs. The latter would be equivalent to a free outlet for system upstream. In large drains especially those of monolithic construction, conduit may be continuous with manhole rising at one side in which case transitions will be used for changes in size.

The slope of the main drain will change usually with breaks at manholes but could be constructed on a vertical curve. Slopes may be subcritical or supercritical and can be very steep, slopes of 3-5% sometimes occurring in main drains. The latter may produce augmented rates of discharge. A single line may involve a wide range of slopes, the usual situation involving steep slopes on upstream reaches becoming mild on downstream end. A break to a steeper slope, however, is also possible.

Alignment commonly will be straight or with relatively small deflections at manholes. Curved alignment is possible but rare. As a rule the main drain will not involve abrupt changes in direction such as 90° except at a connection to existing interceptor which case should be given special treatment which is beyond the scope of this problem.

Design criteria ordinarily provide that conduit will not flow under pressure for the design storm. But it should be possible to compute what will happen in the main drain when any part does flow full. Outflow may be either free, or subject to back pressure from stream or conduit into which drain discharges, or from water in wet well of a pumping station. In the latter case flow may be subject to surges created by sudden stoppage of pumps due to power failure.

Manholes are commonly constructed either round or square with or without a stream-lined invert conforming to invert of conduit; section through manhole normal to direction of flow will be as large or larger than cross section of conduit. Common practice is to bring all laterals in at manholes and may be at any elevation at or above flow line of main drain. The laterals for individual inlets are brought increasingly in a T or Y connection (inflow from one inlet is usually so small relative to flow in main drain that momentum in downstream direction may be neglected).

Inflow hydrographs may have a single peak, or more than one peak. A situation will also occur where a second storm follows so closely after the first that only a part of the volume from the first storm will have been discharged from the system, when the inflow from the second storm begins.

Part Three

LIMITATIONS OF ANALYTICAL STUDY FOR THE FIRST YEAR

The numerous possible variations in boundary conditions, inflow hydrographs, and outlet conditions require that the analytical study contemplated for the first year be limited so that initial solution for the more simple cases will be possible.

During the first year the study will be limited to hydrodynamic analysis of a single storm drain on straight alignment with single-peak hydrographs (not necessarily identical) introduced at discrete points along the line, and a free outlet.

The conduit shall be considered to be circular in cross section (other cross sections may be introduced if feasible), changing in size at manholes, with crown lines matched up and changes in slope at manholes but not necessarily at every manhole.

The conduit shall be considered to be smooth concrete with resistance factor Darch-Weisbach "f" varying as a function of the Reynolds Number of the flow in accordance with latest results from full-scale tests made for the Florida State Road Department and Public Roads at the St. Anthony Falls Hydraulic Laboratory. In the event this requirement complicates the solution unduly, then an average value of "f" for

each size may be used.

Only the case of free-water surface at atmospheric pressure is to be studied initially. Flows as introduced to line shall be considered to have no momentum in direction of outflow line. Both subcritical and supercritical slopes shall be studied but not as steep as to augment the rate of discharge. Disturbances created by discontinuity of boundary at manholes shall be given consideration based on assumption that manhole is an abrupt enlargement over entire periphery of conduit except at flow line and distance across manhole in direction of flow is not more than three pipe diameters.

The hydrodynamic analysis shall be made having in mind conversion of the results to solution by a digital computer. The contract will provide for employment of a consultant on machine computation to assist in that development. It is hoped that the end result will be a program whereby the outflow hydrograph for the simple case herein described may be printed out for any set of inflow hydrographs which do not cause the line to flow under pressure at any point (i.e. to flow full).

The analysis is quite likely to require experimental verification and establishment of certain constants by empirical tests. The study should outline the tests and how they should be made, but no experimental work is to be included under the initial contract.

Carl F. Izzard, Chief
August 1960 Div. of Hydraulic Research
Washington 25, D.C. U.S. Bureau of Public Roads

APPENDIX 2

GENERAL REFERENCES ON FREE-SURFACE UNSTEADY FLOW

This general list of references follows closely the method of annotation given in the "Bibliography and Discussion of Flood-Routing and Unsteady Flow in Channels," U. S. Geological Survey Water-Supply Paper 1690, U. S. Government Printing Office, Washington, D. C., 1964 (prepared by V. Yevjevich).

References on subjects devoted to the general theory of wave motion in channels as well as references describing practical methods and procedures for flood routing are included. Only references which are not included in the above mentioned bibliography are given in the following list, mainly containing the works produced or published during the decade 1960-1969.

The list comprises references to papers that are original reprints of studies made, and also references to papers that are restatements, summarizing the results of previous studies. No abstract of 145 references is given in this list. The list gives references in the chronological sequence by years, and within a particular year the sequence of references is by author's name, in alphabetical order. Thus arranged, references have been numbered consecutively. Each reference can therefore be identified by its number, author or authors, and year of publication. No index by authors or by subjects is provided. The coverage of years 1968-1970 is not as complete as is for the previous years.

BIBLIOGRAPHY

- Hayashi, T., Nougaro, J., 1959, (in French), Sur la similitude des régimes non-permanents dans les canaux découverts (On the similitude of unsteady flow regimes in open channels): Comptes rendus de l'Académie des Sciences (Paris), Sept.
- Mongiardini, V., 1959, (in Italian), La propagazione delle onde di traslazione nelle deviazioni curvilinee e rettilinee dei canali (Propagation of translation waves in the straight and curvilinear deviations of channels): L'Acqua, nos. 4-5, year XXXVII, July-August, September-October.
- Pezzoli, G., 1959, (in Italian), L'attenuazione delle onde negli alvei a fondo orizzontale (Waves attenuation in channels with the horizontal bed): VI Convegno di idraulica e costruzioni idrauliche, Padova, (VIth Congress of Hydraulics and Hydraulic Structures, Padua), 25-27 May 1959, pp. 1-5.
- Bata, G., 1960, (in Serbo-Croatian), Prolazak poplavnih talasa u prirodnim koritima (Propagation of flood waves in natural channels): Saopštenja, Hidrotehnicki Institut, Beograd (Yugoslavia), no. 20-21.
- Kartvelishvili, N. A., 1960, (in Russian), Kolebaniya urovnya nizhnego byefa v stvore GES pri sutochnom regulirovanii (The fluctuations of tailrace level at a hydroelectric power station due to daily regulations): Gidrotekhnicheskoe Stroitel'stvo, no. 11, p. 45-47.
- Keller, H. B., Levine, D. A., Whitham, G. B., 1960, Motion of a bore on a sloping beach: Journal of Fluid Mechanics, no. 7, pp. 302-316.
- Moklyak, V. I., 1960, (in Ukrainian), Do osnov razrakhunku neustalenoogo rukhu vodi u vidkritikh ruslakh (On the fundamentals of computation of unsteady water flow in open channels): Izvestiya Instituta Akad. Nauk Ukrainskoy S.S.R., tom 16, p. 68-78.
- Montuori, C., 1960, (in Italian), Immissione di una portata costante in un canale vuoto (Inflow of a constant discharge in an empty canal): Atti Fondazione Politecnica del Mezzogiorno, vol. V.
- Obrezkov, V. I., 1960, (in Russian), Opyt primeneniya sredstv vychislitel'noy tekhniki dlya raschota neustanovivshegosya dvizheniya v nizhnem byefe GES (Experiment with the application of means of computational technique for the computation of unsteady motion in the tailrace canal of a hydroelectric power station): Gidrotekhnicheskoe Stroitel'stvo, no. 3, p. 32-36.
- Phillips, O. M., 1960, On the dynamics of unsteady gravity waves of finite amplitude, Part I, The elementary interaction: Journal of Fluid Mechanics, vol. 9, part 2, October.
- U.S. Army Corps of Engineers, 1960, Routing of floods through river channels: U.S. Army Corps of Eng., Manual EM110-2-1408, March.
- U.S. Army Corps of Engrs., 1960, Flood resulting from suddenly breached dams, Miscellaneous papers no. 2-374, report 1, Feb., Smooth Channel.
- Bird, G. A., 1961, The motion of a shock-wave through a region of non-uniform density: Journal of Fluid Mechanics, vol. 11, part 2, September, pp. 180-208.
- Bock, H., 1961, Problems arising from surge and suction waves caused by a sudden shut-down of a turbine: Permanent International Assoc., 20th International Congress on Navigation, Baltimore, U.S.A., (Interior navigation, Section I, subject 2, 1961).
- Chen, T. C., 1961, Experimental study on the solitary wave reflection along a straight sloped wall at oblique angle of incidence: Department of the Army, Beach Erosion Board (U.S.A.), Technical memorandum No. R4, March.

16. Glukhovskij, B. Kh., 1961, (in Russian), Study of wave attenuation with depth by correlation analysis: *Meteorologiya i Gidrologiya* (U.S.S.R.), no. 11, pp. 22-30.
17. Hughes, B.A., Stewart, R.W., 1961, Interaction between gravity waves and a shear flow: *Journal of Fluid Mechanics*, vol. 10, part 3, May.
18. Hunt, J.N., 1961, (in French), Les ondes interfaciales d'amplitude finie (Interfacial waves of finite amplitude): *La Houille Blanche* (France), no. 4, pp. 515-531, August-September.
19. Johnson, J.W., 1961, Deficiencies in research on gravity surface waves: Council on Wave Research, U.S.A.
20. Kartvelishvili, N.A., 1961, (in Russian), The present state of the hydraulic theory of unsteady flow resulting from research in U.S.S.R.: *Izvestiya Akademii Nauk, Mekhanika i Mashinostroyeniye*, U.S.S.R.
21. Longuet-Higgins, M.S., Stewart, R.W., 1961, The changes in amplitude of short gravity waves on steady non-uniform currents: *Journal of Fluid Mechanics*, vol. 10, part 4, June.
22. Mongiardini, V., 1961, (in Italian), Considerazioni sulle soluzioni analitiche del movimento ondoso stazionario (Considerations on the analytical solutions of the stationary wave movement): *L'Energia Elettrica*, fasc. 5.
23. Starosolszky, O., 1961, (in Hungarian), A hullamzas hidraulikaja (Hydraulics of waves): *Visügyi Közlemények* (Hungary), No. 3, pp. 293-312.
24. Ursell, F., 1961, The transmission of surface waves under surface obstacles: *Proceedings of the Cambridge Philosophical Society* (England) vol. 57, part 3, pp. 638-668, July.
25. U.S. Army Corps of Engineers, 1961, Flood resulting from suddenly breached dams: *Miscellaneous Paper No. 2-374*, report No. 2, Rough Channel.
26. Escoffier, F.F., Boyd, M.B., 1962, Stability aspects of flow in open channels: *Proceedings A.S.C.E.*, vol. 88, no. HY6, November.
27. Fox, L., 1962, Numerical solutions of ordinary partial differential equations: Addison-Wesley Publishing Co., Inc., Reading, Mass.
28. Gary, J., 1962, On certain finite difference schemes for the equations of hydrodynamics: *AEC Computing and Applied Mathematics Center*, Paper NYO-9188, March.
29. Grushevsky, M.S., 1962, (In Russian), The use of an electronic digital computer for computing unsteady flow in a prismatic channel: *Transactions the U.S.S.R. State Hydrologic Institute*, No. 94, pp. 136-183.
30. Kovalenko, E.P., 1962, (In Russian) Computation of unsteady flow in channels: *Gidrotekhnicheskoye Stroitel'stvo*, U.S.S.R., No. 3, pp. 31-32.
31. Lillevang, O.J., 1962, Mean direction of waves and wave energy: *Trans. A.S.C.E.*, vol. 127, part IV, pp. 193-220.
32. Miles, J.W., 1962, On the generation of surface waves by shear flows: *Journal of Fluid Mechanics* (England), vol. 13, part 3, pp. 433-448, July.
33. Rybka, V.G., 1962, (Original in Russian), Investigation of the forms of flood waves of rain caused floods in the Prut and Stryy Rivers: *Soviet Hydrology Selected Papers*, American Geophysical Union, No. 4.
34. Sandover, J.A., 1962, Cnoidal waves and bores: *La Houille Blanche*, vol. 17, no. 3, pp. 443-455, July-Aug.
35. Shinbrot, M., 1962, Waves in shallow water: *Archive for Rational Mechanics and Analysis* (Germany), vol. 9, no. 3, pp. 234-244, January.
36. Tammekivi, W., 1962, Evaluation of methods of flood routing in open channels: *Northwestern University, M.S. Thesis*, Evanston, Illinois.
37. Wackernagel, A., 1962, Computation of flood waves from dam breaches: *Schweiz. Bauzeitung*, v. 80, no. 22, pp. 370-373, May.
38. Abbott, M.B., 1963, The solution of wave propagation problems using an iterative operator: *La Houille Blanche*, No. 5, pp. 513-524, Aug-Sept.
39. Bledermann, R., 1963, (In German), Numerical approximate procedures for the computation of reservoir emptying (Two-dimensional flow patterns): *Mitteilungen der Versuchsanstalt fuer Wasserbau und Erdbau und der Eidgenoessischen Technischen Hochschule in Zurich*, no. 61, 89 p.
40. Cunge, J.A., Wegner, M., 1963, (In French), Intégration numérique des équations d'écoulement de Barré de St. Venant par un schéma implicite de différences finies (Numerical integration of Barré de St. Venant equations by an implicit finite-difference scheme): *Sogreah*, Grenoble, France, NT. 1073, Sept.
41. Faure, J., Nahas, N., 1963, (in French), Deux problèmes de mouvements non-permanents à surface libre résolus sur ordinateur électronique: *Centre de Recherches et d'Essais de Châtou, Department Mécanique des Fluides*, HF/3/E307, p. 37, Châtou, France.
42. Gary, J., 1963, Some implicit finite difference schemes for hyperbolic systems: *New York University, Courant Institute of Mathematical Sciences*, paper NYO-10426, March.
43. Henderson, F.M., 1963, Flood waves in prismatic channels: *Proceedings ASCE*, vol. 89, Hydraulic Division, no. HY4, part 1, paper 3568, p. 39, July.
44. Kalinin, G.P., Kuchment, L.S., 1963, (in Russian) O chislennykh metodakh resheniya uravneniy Sen-Venana dlya rashota neustanovivshegosya dvizheniya vody v rekakh (On the numerical solution methods of De Saint-Venant equations for the computation of unsteady water flow in rivers): *Meteorologiya i Gidrologiya*, U.S.S.R. (Meteorology and Hydrology Journal), no. 6, p. 3-9.

45. Rantz, S.E., 1963, An empirical method of determining momentary discharge of tide-affected streams: U.S. Geological Survey Water-Supply Paper 1586-D.
46. Richtmyer, D., 1963, A survey of difference methods for non-steady fluid dynamics: National Center for Atmospheric Research, NCAR, Technical Notes 63-2.
47. Schnoor, E., 1963, (In German), Berechnung der Tidewelle in beliebig verzweigten Stromsystemen vermittels des differenzenverfahrens (Computation of tide wave in any branching stream system by the finite-difference schemes): Der Bauingenieur, no. 11, pp. 426-430, November.
48. Shipley, A.M., 1963, On measuring long waves with a tide gauge: Deutsche Hydrographische Zeitschrift, no. 3, pp. 136-140.
49. Simon, A.L., 1963, Travel time of waves in backwaters: A.S.C.E. Proceedings, Journal of the Hydraulics Division, vol. 89, no. HY6, pp. 1-13, November.
50. Supino, Giulio, 1963, (In Italian), Propagazione ondoze nei canali a marea, (Wave propagation in tidal channels): Accademia Nazionale dei Lincei, Class Physical, Mathematical and Natural Sciences; Nota I (Note I), ser. VIII, vol. XXXIV, fasc. 3 and 4, pp. 230-231; Nota II (Note II), p. 345-351.
51. Sveishi, T., 1963, Runoff estimation in storm sewer system using equivalent roughness: Japan Society of Civil Engineers, Trans. no. 91, pp. 41-54, March.
52. Vasyliiev, O.F., Godunov, S.K., Pritvits, N.A., Temnoeva, T.A., Fryazinova, I.L. and Shugrin, S.M., 1963, (In Russian), Chislennyi metod raschota rasprostraneniya dlinnykh voln v otkrytykh ruslakh i prilozhenie ego k zadache o povodke (Numerical method of computation of long wave movement in open channels and its contribution to problems of floods): Doklady Akademii Nauk USSR, (Transactions of USSR Academy of Sciences), v. 151, no. 3 pp. 525-527.
53. Zheleznyak, I.A., 1963, (In Russian), Uproshchenyi raschot dvizheniya volny popuska v nizhnem byefe GES po krivym ob'yom (A simplified computation of wave release movement in the tail-race canals of hydroelectric power plant by the volume curves): Meteorologiya i Gidrologiya, U.S.S.R., no. 3, p. 36-42.
54. Ackers, P. and Harrison, J.A.M., 1964, Attenuation of flood waves in part-full pipes: proceedings Institution of Civil Engineers, London, vol. 28, pp. 361-382, July. Also the same title: Dept. of Scientific and Industrial Research Hydraulics Research Station, Wallingford, Berkshire, England, report no. INT31, December 1963.
55. Amein, M., 1964, Bore inception and propagation by nonlinear wave theory: Proceedings of 9th Conference on Coastal Engineering, Lisbonne, June 1964, pp. 70-81.
56. Balek, J., 1964, (in Slovakian), Vypocty charakteristik povodnovyvin (Computation of wave characteristics): Vodohospodarsky Časopis, no. 2, pp. 137-160, with English summary, p. 159.
57. Bretherton, F.P., 1964, Resonant interactions between waves. The case of discrete oscillations: Journal of Fluid Mechanics, vol. 20, part 3, pp. 457-479, November.
58. Cunge, J.A., Wegner, M., 1964, (In French), Intégration numériques des équations d'écoulement de Barré de Saint-Venant par un schéma implicite de différences finies. Application au cas d'une galerie tantôt en charge tantôt à surface libre (Numerical integration of Barré de Saint-Venant's flow equations by means of an implicit scheme of finite differences. Application to the case of alternately free and pressurized flow in a tunnel): La Houille Blanche, pp. 33-39, Jan.-Feb.
59. Daubert, A., 1964, (In French), Quelques aspects de la propagation des crues (Some aspects of flood routing): La Houille Blanche, no. 3, pp. 341-346, May-June.
60. Dronkers, J.J., 1964, Tidal computations: John Wiley and Sons, Inc., New York.
61. Freeman, J.C., LeMéhauté, B., 1964, Wave breakers on a beach and surges on a dry bed: Proceedings A.S.C.E., vol. 90, no. HY2, part 1, pp. 187-216, March, (Discussion Nov. 1964).
62. Grushevskiy, M.S., 1964, (Original in Russian), The effect of flood plains on the flattening of waves of discharge releases, (according to observational data of GGI for Tvertsa River): Soviet Hydrology Selected Papers, no. 6, 1964, American Geophysical Union, pp. 579-586.
63. Higgins, R.V., Boley, D.W., Leighton, A.J., 1964, Aids to forecasting the performance of water floods: Journal of Petrol Technol., U.S.A., vol. 16, no. 9, pp. 1076-1082, September.
64. Koshcheyev, A.N., 1964, (In Russian), Calculation of long waves on continental bodies of water: Transaction of the U.S.S.R. State Hydrologic Institute, Trudy GGI, no. 113, pp. 36-81.
65. Masch, F.D., 1964, Cnoidal waves in shallow water: Proceedings of the 9th Conference on Coastal Engineering, Lisbonne, pp. 1-22, June.
66. Nougare, J., Castex, L., Bacquie, S., 1964, (In French), Influence d'une vallée latérale sur la propagation de l'onde de rupture d'un barrage (Effect of a lateral valley on the propagation of wave due to a dam breach): Comptes rendus de l'Académie des Sciences (France), tome 259, no. 16, pp. 2596-2598, 19 Octobre.
67. Ribeny, F.M.J., 1964, Flood routing with a unit hydrograph approach: The Journal of the Institution of Engineer (Australia), vol. 36, no. 1-2, pp. 9-22, January and February.
68. Thirriot, C., Bednarczyk, S., 1964, (In French), Ondulations secondaires en front d'intumescences et ondes solitaires (Secondary oscillations in front of waves and solitary waves): La Houille Blanche, no. 8, pp. 879-888, December.

69. Williams, J.A., 1964, A non-linear problem in surface water waves: University of California Hydraulic Engineering Laboratory, techn. rep. no. HEL 1-5, p. 245, October.
70. Arnborg, L., 1965, Unsteady flows in open channels: IAHR 11th International Congress, Leningrad, September.
71. Brakensiek, D.L., Comer, G.H., 1965, Re-examination of flood routing method comparison: Journal of Hydrology, Amsterdam, Netherlands, no. 3-4, pp. 225-230, Nov.
72. Bramble, J.H., 1965, Numerical solutions of partial differential equations (Flood waves in rivers): Academic Press, pp. 42-49.
73. Cioc, D., Amaftiesci, R., Finkelstein, A., 1965, (In French), Sur le mouvement non-permanent avec discontinuité dans une galeries de fuite (On the unsteady flow with a discontinuity in the tail-race gallery): IAHR 11th International Congress, Leningrad, September.
74. Escoffier, F.F., 1965, Routing flood waves apalachicola River: IAHR 11th International Congress, Leningrad, September.
75. Escande, L., Nougaro, J., Castex, L., Bacquie, S., 1965, Propagation d'une onde de crue subite a la suite de l'effacement d'un barrage (Propagation of flood wave created by dam breach): IAHR 11th International Congress, Leningrad, September.
76. Ghambarian, H.H., 1965, On waves in inclined open channels: IAHR 11th International Congress, Leningrad, vol. 1(11), pp. 1-10, September.
77. Harlow, F.H., Shannon, J.P., Welch, J.E., 1965, Liquid waves by computer: AAAS, Science, vol. 149, no. 3688, September.
78. Hayashi, T., 1965, Propagation and deformation of flood waves in natural channels: IAHR 11th International Congress, Leningrad, vol. 1 (1-12), pp. 1-7, September.
79. Ito, T., 1965, Study on flood waves by analog computer: IAHR 11th International Congress, Leningrad, vol. 3 (3-39), pp. 1-6, September.
80. Krivoshey, M.I., 1965, (Original in Russian), Review of laboratory investigations of unsteady movement of water in open channels: Soviet Hydrology Selected Papers, American Geophysical Union, no. 1, pp. 12-20.
81. Lai, C., 1965, Flows of homogeneous density in tidal reaches - Solution by the implicit method: U.S. Geological Survey, Open-File Report, Washington, D.C.
82. Lean, G. H., 1965, A particular case of steady wave propagation along a parallel-sided open channel: IAHR Journal of Hydraulic Research (Netherlands), vol. 3, no. 1, pp. 2829-2838.
83. Levin, L., 1965, (In French), Calcul approché de mouvement non-permanent avec oscillations secondaires dans les canaux trapézoïdaux (Approximate computation of unsteady flow with the secondary oscillations in the trapezoidal channels): IAHR 11th International Congress, Leningrad, September.
84. Levy, I.R., 1965, (In French), Mouvement non-permanent dans les lits instables (Unsteady flow in instable channels): IAHR 11th International Congress, Leningrad, vol. 3 (3-41), pp. 1-8, September.
85. Meijer, J.G.P., Vreugdenhil, C.B., DeVries, M., 1965, A method of computation for non-stationary flow in open-channel networks: Hydraulics Laboratory, Delft, Netherlands, pub. no. 34, September.
86. Moklyak, V.I., 1965, (Original in Russian), On the calculation of movement of release wave: Transaction of the State Hydrologic Institute (U.S.S.R.), Trudy GGI, no. 125.
87. Montuori, C., 1965, (In French), Introduction d'un débit constant dans un canal vide (Inflow of a constant flow in an empty canal): IAHR 11th Congress, Leningrad, September.
88. Morgali, J.R., Linsley, R.K., 1965, Computer analysis of overland flow: Proceedings ASCE, vol. 91, HY3, pp. 81-100, May.
89. Newman, J.N., 1965, Propagation of water waves past long two-dimensional obstacles: Journal of Fluid Mechanics, (England), vol. 23, part 1, pp. 23-29.
90. Novak, P., 1965, Model research on flood waves passing through a series of reservoirs and a river channel: IAHR 11th International Congress, Leningrad, vol. 3 (3-27), pp. 1-5, September.
91. Puzanov, A., 1965, Velocity and pressure distribution in plane positive surges: IAHR 11th International Congress, Leningrad, September.
92. Quick, M.C., 1965, River flood flows, Forecasts and probabilities: Proceedings ASCE, vol. 91, no. HY3, pp. 1-8, May.
93. Riquois, R., and Ract-Madoux, X., 1965, (In French), Intumescences observées sur le canal d'amenée de la chute d'Oraison lors des variations rapides de charge (Waves observed in a head-race canal of the cascade d'Oraison, during a rapid variation of flow): IAHR 11th International Congress, Leningrad, September.
94. Rozovsky, I.L., Yeremenko, E.V., Shabrin, A.N., and Vlasenko, Y.D., 1965, Experimental investigations of unsteady motion in open channels of rectangular cross-section rectilinear and curvilinear in the plan: IAHR 11th International Congress, Leningrad, September.
95. Rzhantzin, N.A., Rabkova, E.K., 1965, Kinematics of a stream and channel deformations in the lower reaches of hydro-projects at the daily regulation: IAHR 11th International Congress, Leningrad, September.
96. Starosolccky, O., 1965, (In French), Surge waves in the vicinity of hydroelectric power plants: IAHR 11th International Congress, Leningrad, September.

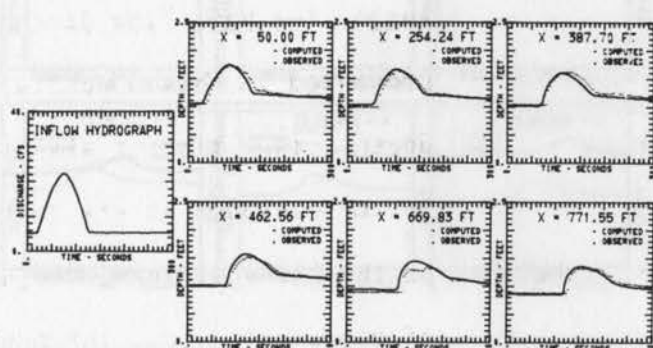
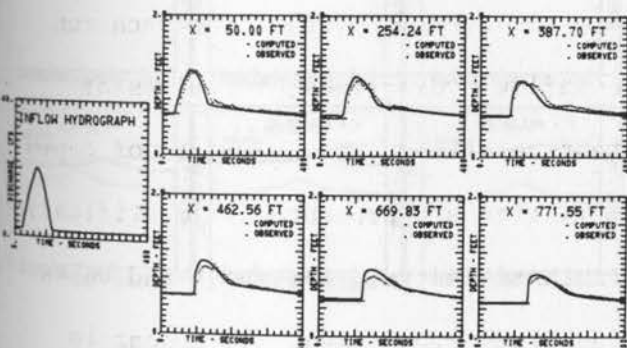
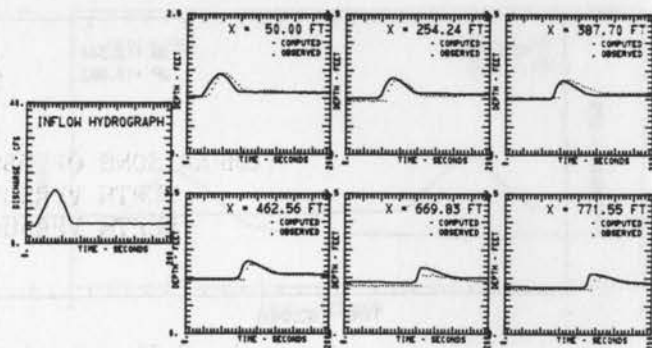
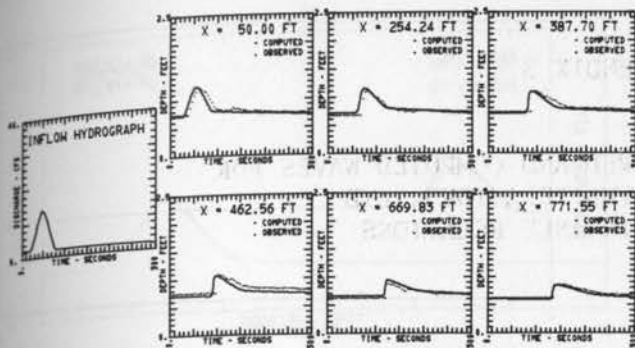
97. Strelkoff, T., Amorocho, J., 1965, Gradually varied unsteady flow in a controlled canal system: IAHR 11th International Congress, Leningrad, September.
98. Szigyarto, Z., 1965, A new method for calculating discharge wave profiles: IAHR 11th International Congress, Leningrad, September.
99. Thirriot C., Bednarczyk, S., 1965 (In French), Considération sur les ondulations secondaires en front d'intumescences dans les canaux découvertes (Note on the secondary undulations in front of open channel waves): IAHR 11th International Congress, Leningrad, September.
100. Varley, E., Cumberbatch, E., 1965, Non-linear theory of wave-front propagation: Journal of Institute of Mathematics and its Applications (England), vol. 1, no. 2, pp. 101-112, June.
101. Vasyliiev, O.F., Gladyshev, M.T., Pritvits, N.A. and Sudobicher, V.G., 1965, Numerical methods for the calculation of shock wave propagation in open channels: IAHR 11th International Congress, Leningrad, September.
102. Vasyliiev, O.F., Temnoeva, T.A., and Shugrin, S.M. 1965, (In Russian), Chislennyi metod raschota neustanovivshikhsya techeniy v otkrytykh ruslakh (Numerical computational method of unsteady flow in open channels): Izvestiya Akademii nauk SSSR, Mekhanika (Proceedings of the U.S.S.R. Academy of Sciences, Section Mechanics), no. 2, pp. 17-25.
103. Yamada, H., Okabe, J., 1965, On the resonance effect in a storm surge (Parts I and II): Bulletin of the Disaster Prevention Research Institute, Kyoto University, Japan, vol. 15.
104. Yevjevich, V.M., 1965, Rate of change of the peak for floods progressing along a channel: IAHR 11th International Congress, Leningrad, September.
105. Zierep, J., Heynatz, J.Th., 1965, (In German), Ein analytisches Verfahren zur Berechnung der nichtlinearen Wellenausbreitung (An analytic method for computation of non-linear wave propagation): Zeitschrift fuer angewandte Mathematik und Mechanik, (Germany), vol. 45, no. 1, pp. 37-46.
106. Amein, M., 1966, Streamflow routing on computer by characteristics: Journal of Water Resources Research (U.S.A.), vol. 2, no. 1, pp. 123-130.
107. Cunge, J.A., 1966, (In French), Comparaison des résultats des essais d'intumescences effectués sur le modèle réduit et sur le modèle mathématique sur canal Oraison-Manosque (Comparison of results of wave experiments on a reduced model and the mathematical model of the canal Oraison-Manosque): La Houille Blanche, no. 1.
108. Evangelisti, G., 1966, On the numerical solution of the equations of propagation by the method of characteristics: Meccanica, Journal of the Italian Association of Theoretical and Applied Mechanics, no. 1-2, Tamburini Editor, Milano.
109. Liggett, J.A., Graf, W. H., 1966, Steady and unsteady effects on discharge in a river connecting two reservoirs: Great Lakes Research Division, pub. no. 15, University of Michigan.
110. Lister, M., 1966, Numerical solution of hyperbolic partial differential equations by the method of characteristics: Mathematical Methods for Digital Computers by Anthony Ralston and Herber S. Wilf, pp. 165-179, John Wiley and Sons, Inc. Publishers, ninth printing, July.
111. Pezzoli, G., 1966, Non-linear mechanics of transitory waves in channels: Meccanica, Journal of Italian Association of Theoretical and Applied Mechanics, No. 1-2, Tamburini Editor, Milano.
112. Welch, E.J., 1966, Computer simulation of water waves: Datamation, vol. 12, no. 1, pp. 41-47, November.
113. Wood, D.J., Dorsch, R.G., Lightner, C., 1966, Wave-plan analysis of unsteady flow in closed conduits: ASCE Journal of Hydraulics Division, vol. 92, no. HY2, pp. 83-110, March.
114. Abbott, M.B., 1967, An Introduction to the Method of Characteristics: American Elsevier Publishing Co., Inc., New York.
115. Abbott, M.B., Ionescu, F., 1967, On the numerical computation of nearly horizontal flows: IAHR, Journal of Hydraulic Research, vol. 5, no. 2.
116. Anonymous, 1967, Mathematical model of Mekong River Delta-Descriptive and Operating Manual: United Nations Educational Scientific and Cultural Organization.
117. Barnes, A.H., 1967, Comparison of computed and observed flood routing in a circular section: Proceedings, International Hydrology Symposium, Fort Collins, Colorado, September.
118. Dooge, J.C., Harley, B.M., 1967, Linear routing in uniform open channels: Proceedings International Hydrology Symposium, Fort Collins, Colorado, Sept. 6-8, pp. 1-8.
119. Dracos, T.A., Glenne, B., 1967, Stability criteria for open-channel flow: Proc. ASCE, vol. 93, No. HY6, November.
120. Fletcher, A. G., Wallis, S., Hamilton, W. S., 1967, Flood routing in an irregular channel: Proc. ASCE, vol. 93, no. EM3, pp. 45-62, June.
121. Lai, C., 1967, Computation of transient flows in rivers and estuaries by the multiple-reach method of characteristics: Geological Survey Research 1967, U.S. Geological Survey Prof. Paper 575-D, PD273-D280.
122. Laitone, E.V., 1967, Shallow water waves in canals of variable section: ASCE, Journal of Waterways and Harbors Division, May, WW2, paper No. 5246, p. 237.
123. Liggett, J.A., Woolhiser, D.A., 1967, Difference solutions of the shallow-water equation: Proc. ASCE, vol. 93, no. EM2, Paper 5189, April.

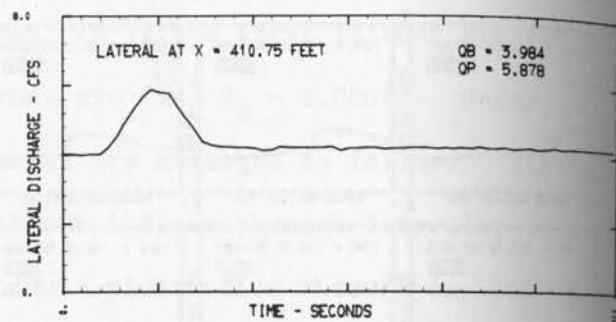
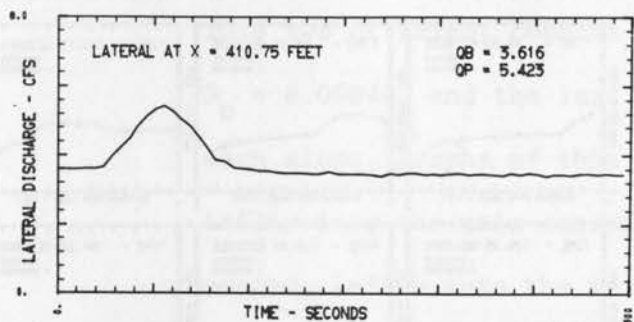
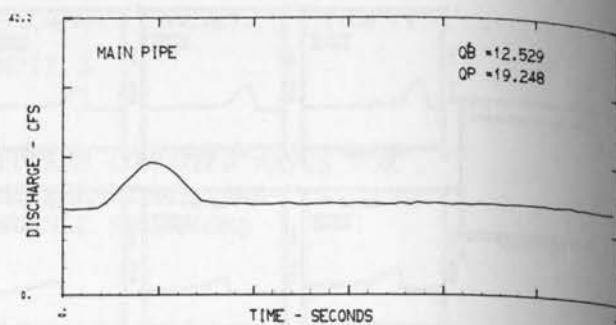
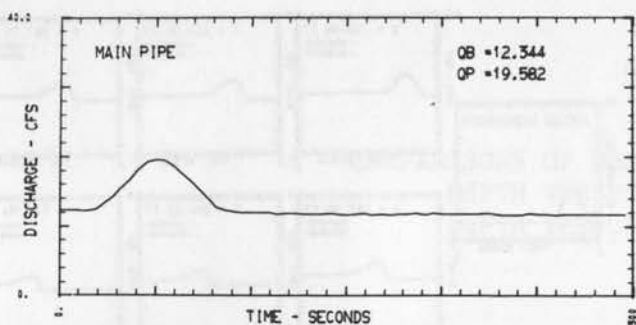
124. Liggett, J.A., Woolhiser, D.A., 1967, The use of the shallow water equations in runoff computation: Proc. 3rd Annual American Water Resources Conference, San Francisco, November 8-10.
125. Streeter, V. L., Wylie, E.B., 1967, Hydraulic Transients: McGraw-Hill Book Co., New York.
126. Woolhiser, D.A., Liggett, J.A., 1967, Unsteady one-dimensional flow over a plane-The rising hydrograph: Journal of Water Resources Research, vol. 3, no. 3.
127. Baltzer, R.A., Chintu, Lai, 1968, Computer simulation of unsteady flows in waterways; Proc. ASCE, vol. 94, HY4, paper 6048, July.
128. Liggett, J.A., 1968, Mathematical flow determination in open channels: Proc. ASCE, vol. 94, EM4, paper 6078, August.
129. Miller, R.L., 1968, Experimental determination of run-up of undular and fully developed bores: Jour. of Geo. Research, vol. 73, no. 14, July 15, p. 4497-4510.
130. Peregrine, D.H., 1968, Long waves in a uniform channel of arbitrary cross-section: Journal of Fluid Mechanics (England), vol. 32, part 2, pp. 353-365, May.
131. Balloffet, A., 1969, One-dimensional analysis of floods and tides in open channels: Proc. ASCE, vol. 95, HY4, paper 6695, pp. 1429-1451, July.
132. Chervet, A., Dallèves, P., 1969, (In French), Calcul sur ordinateur d'écoulements instationnaires dans des canaux découverts (Computer calculations of the nonstationary flow in open channels): IAHR 13th International Congress, Kyoto, Japan, 1969, vol. 1, pp. 259-266.
133. DiSilvio, G., 1969, Flood wave modification along prismatic channels: Proc. ASCE, vol. 95, HY5, paper 6777, pp. 1589-1614, Sept.
134. DiToro, D.M., 1969, Stream equations and method of characteristics: Proc. ASCE, vol. 95, SA4, paper 6723, pp. 699-703, August.
135. Dronkers, J.J., 1969, Tidal computations for rivers, coastal areas, and seas: Proc. ASCE, vol. 95, HY1, paper 6341, pp. 29-77, January.
136. Garrison, J.M., Granju, J.P., Price, J.T., 1969, Unsteady flow simulation in rivers and reservoirs: Proc. ASCE, vol. 95, HY5, paper 6771, pp. 1559-1576, September.
137. Martin, C.S., DeFagio, F.G., 1969, Open-channel surge simulation by digital computer: Proc. ASCE, vol. 95, HY6, paper 6911, p. 2049, November.
138. Mozayeny, B., Song, C.S., 1969, Propagation of flood waves in open channels: ASCE, vol. 95, HY3, paper 6561, pp. 877-892, May.
139. Nakagawa, H., Nakamura, S., Ichihashi, K., 1969, Generation and development of a hydraulic bore due to the breaking of a dam: Bulletin of the Disaster Prevention Research Institute Kyoto University (Japan), vol. 19, part 2, no. 154, November.
140. Strelkoff, T., 1969, One-dimensional equations of open-channel flow: Proc. ASCE, vol. 95, HY3, paper 6557, pp. 861-876, May.
141. Terstriep, M.L., Stall, J.B., 1969, Urban runoff by Road Research Laboratory method: Proc. ASCE, vol. 95, HY6, paper 6878, p. 1809, Nov.
142. Wylie, E.B., 1969, Control of transient free-surface flow: Proc. ASCE, vol. 95, HY1, paper 6360, pp. 347-361, January.
143. Cunge, J.A., 1970, (In French), Calcul de propagation des ondes de rupture de barrage (Computation of dam breach waves): La Houille Blanche, no. 1, pp. 25-33.
144. Harris, G.S., 1970, Real time routing of flood hydrographs in storm sewers: Proc. ASCE, vol. 96, no. HY6, June.
145. Zorne, J.J., 1970, The numerical solution of transient superical flow by the method of characteristics with a technique for simulating bore propagation: School of Civil Engineering, ERC-0370, Georgia Institute of Technology, Atlanta, Georgia (U.S.A.).

APPENDIX 3

COMPARISONS OF OBSERVED AND COMPUTED WAVES FOR DEPTH VERSUS TIME RELATIONS, AND DEPTH VERSUS DISTANCE RELATIONS

A total of 50 runs is included in Appendix 3. They are grouped into two different conduit slopes (S_o); the first 14 runs are for $S_o = 0.00048$ and the last 36 runs are for $S_o = 0.00099$. Under each slope, graphs of this appendix are arranged as follows: first, inflow into the main conduit without inflows through laterals; second, inflow into the main conduit plus inflow through one lateral, and finally inflow into the main conduit plus inflows through all three laterals. Inflow hydrographs of laterals are given in the graphs whenever there is an inflow through laterals. In each group of the same type of inflows, the graphs are arranged in the order of increasing base flow discharge in the conduit (QB). For each run presented in this appendix, the inflow hydrographs, the plots of depth versus time at six different positions, and the plots of depth versus distance at six instants in time are presented. Identification of the six positions, the six instants in time, variables and units of coordinates, and the peak flow discharges (QP), all appear in graphs and are self-explanatory.



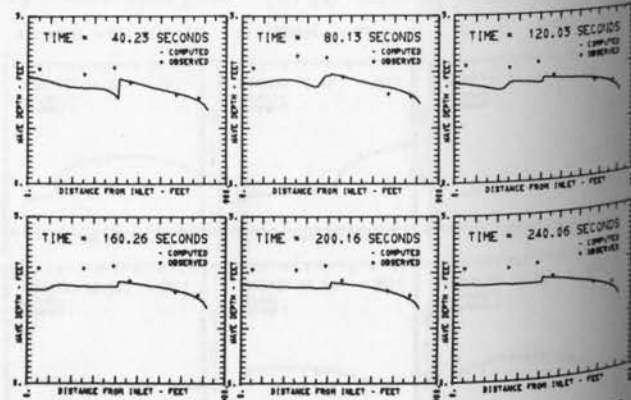
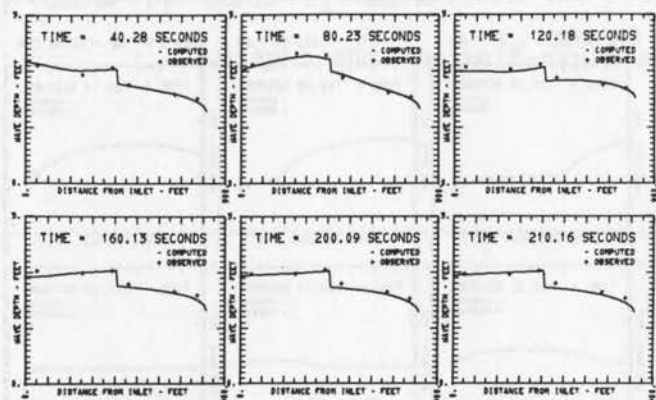
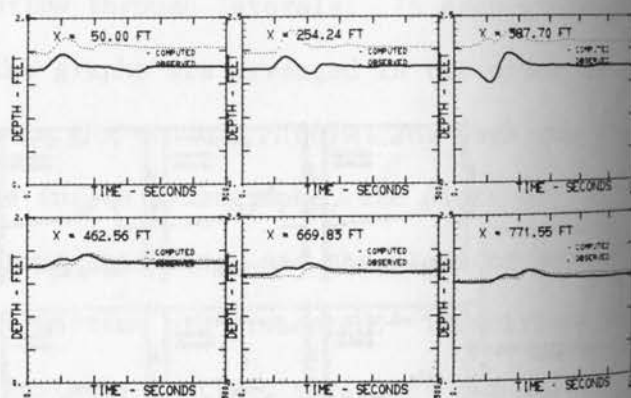
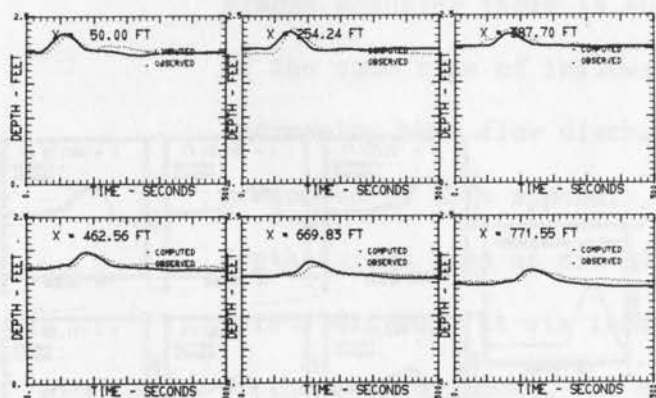


RUN NUMBER 100002

INFLOW HYDROGRAPHS

RUN NUMBER 120002

INFLOW HYDROGRAPHS

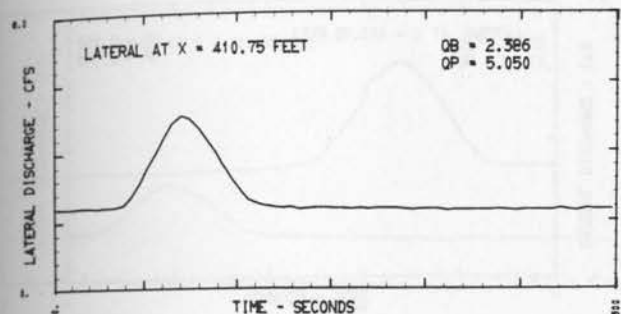
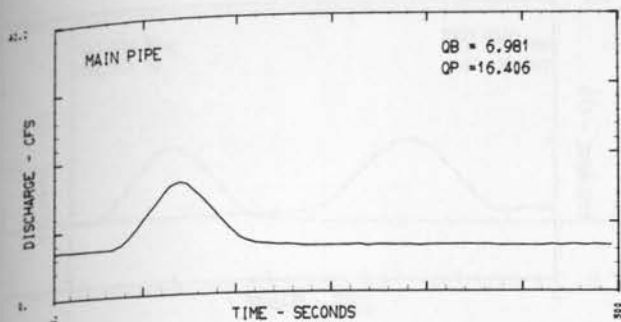


RUN NUMBER 100002

SO = .000480 QB = 12.34 CFS QP = 19.58 CFS

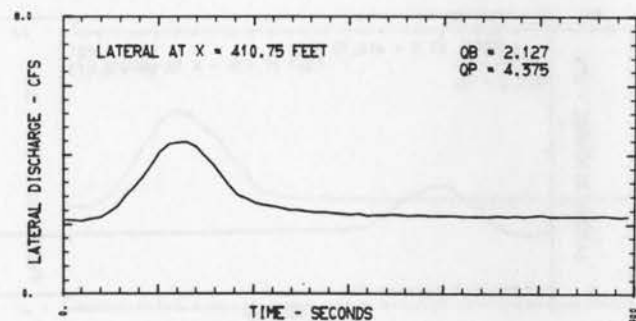
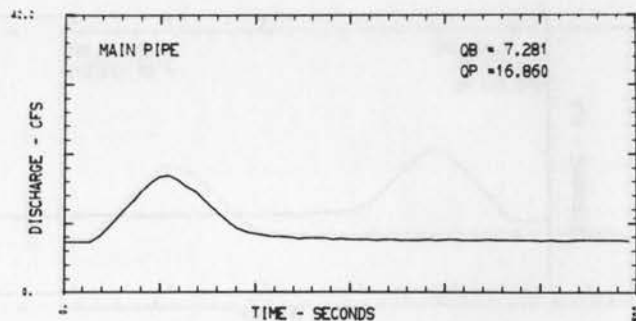
RUN NUMBER 120002

SO = .000480 QB = 12.53 CFS QP = 19.25 CFS



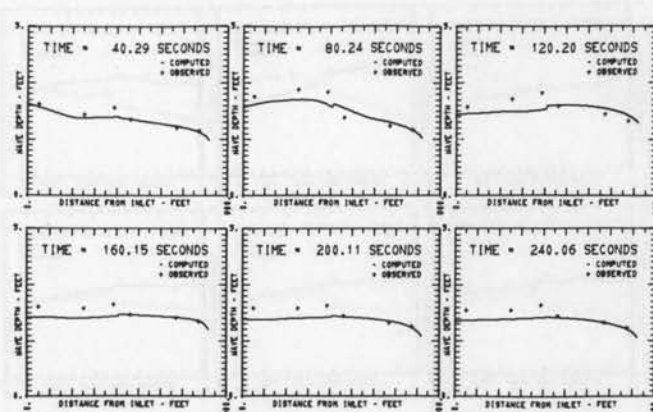
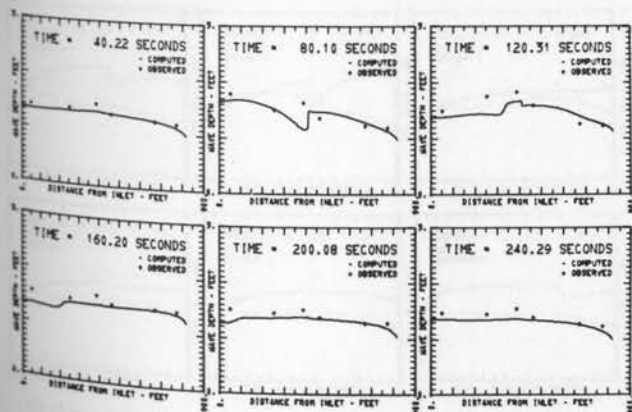
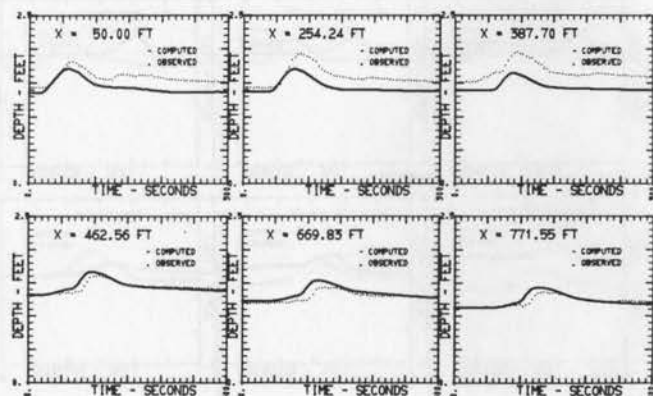
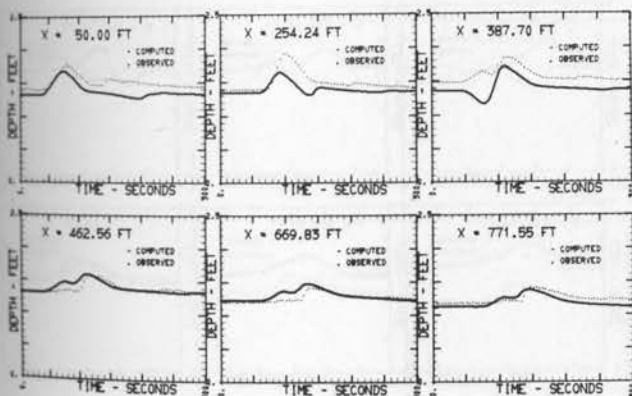
RUN NUMBER 120001

INFLOW HYDROGRAPHS



RUN NUMBER 100001

INFLOW HYDROGRAPHS

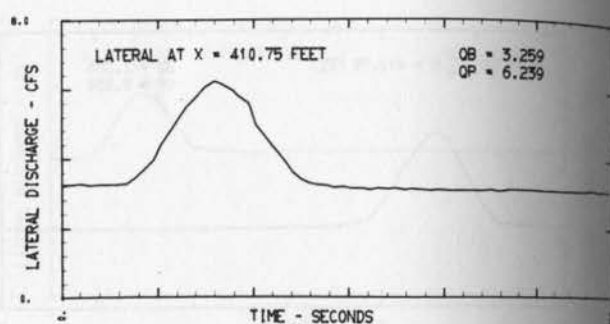
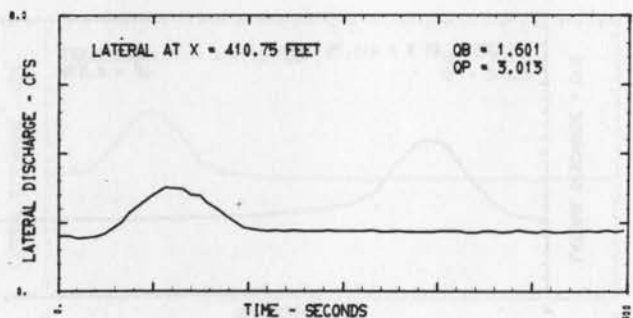
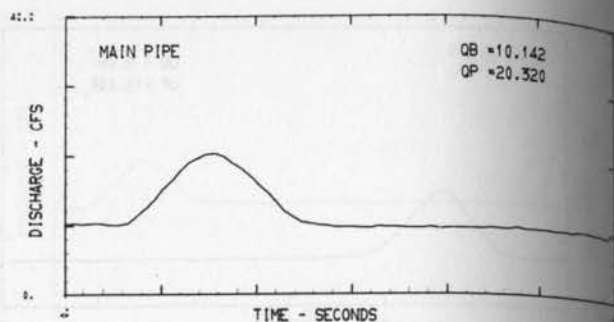
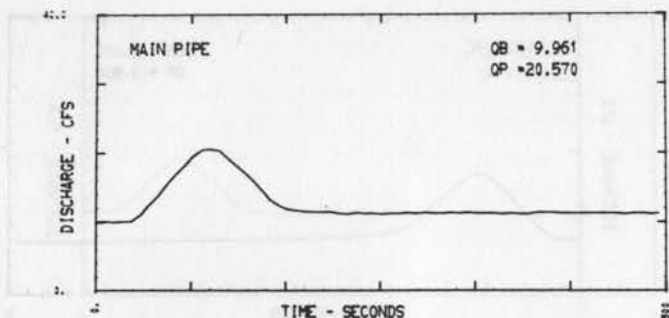


RUN NUMBER 120001

S0 = .000480 OB = 6.98 CFS QP = 16.41 CFS

RUN NUMBER 100001

S0 = .000480 OB = 7.28 CFS QP = 16.86 CFS

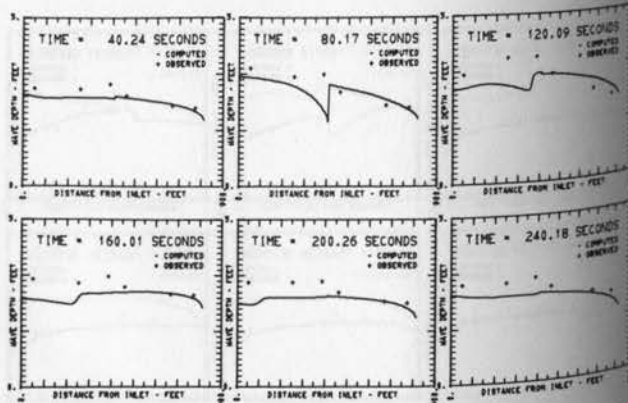
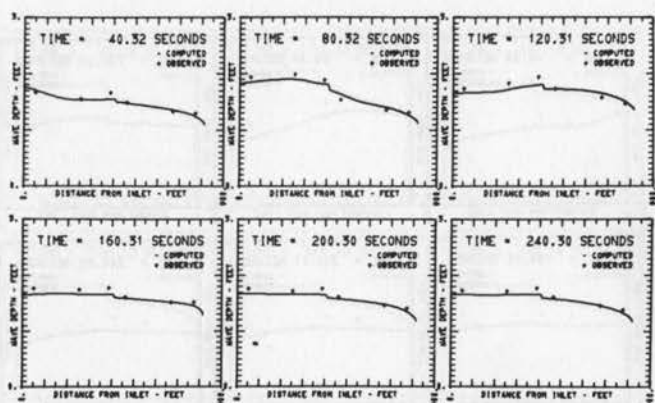
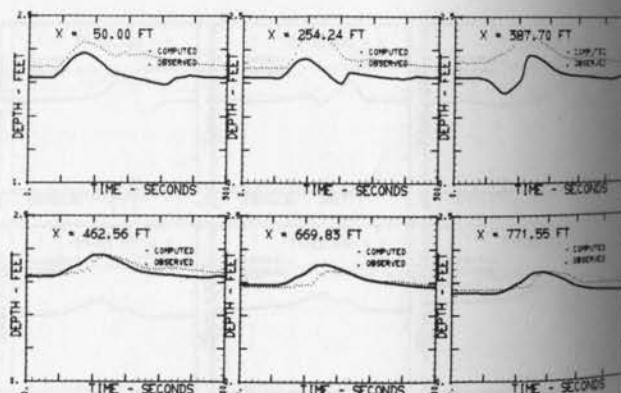
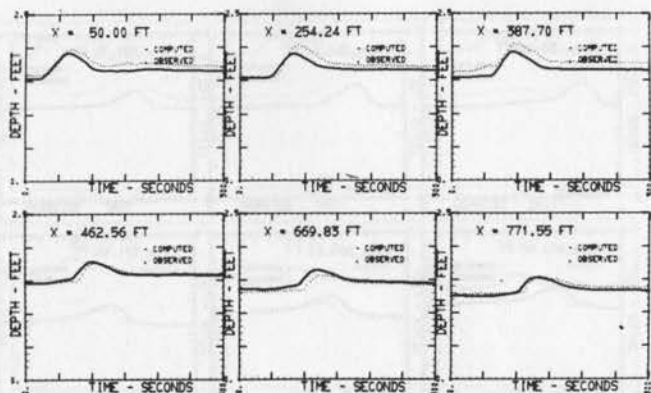


RUN NUMBER 120005

INFLOW HYDROGRAPHS

RUN NUMBER 120004

INFLOW HYDROGRAPHS

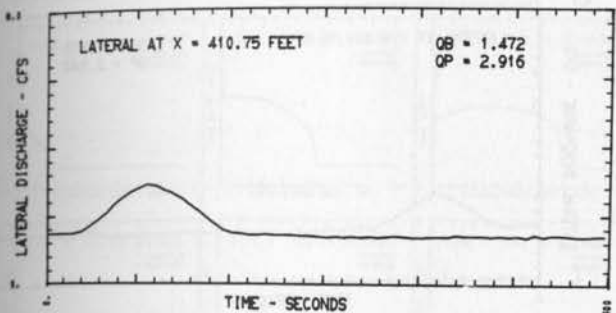
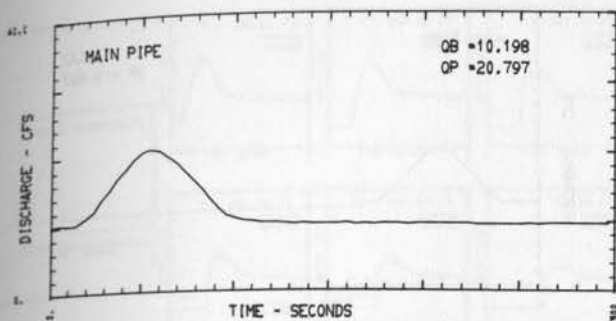


RUN NUMBER 120005

S0 = .000480 OB = 9.96 CFS OP = 20.57 CFS

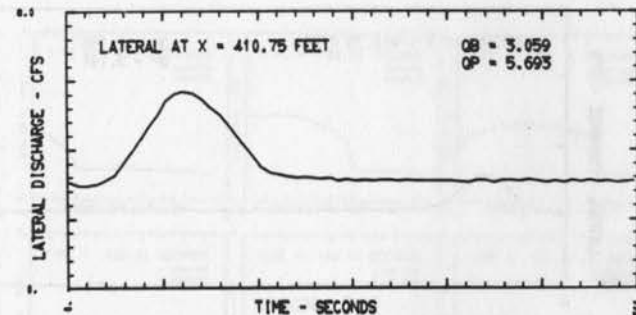
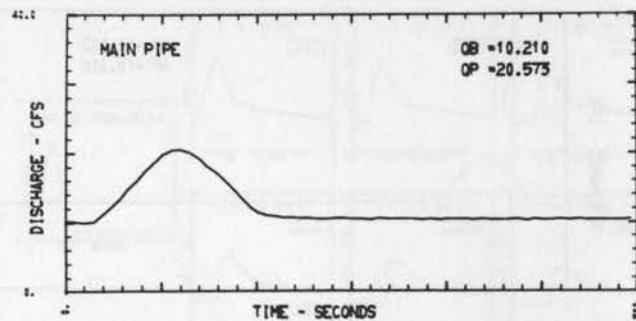
RUN NUMBER 120004

S0 = .000480 OB = 10.14 CFS OP = 20.32 CFS



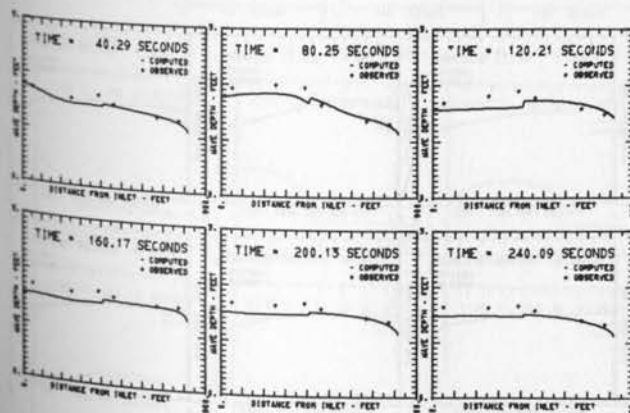
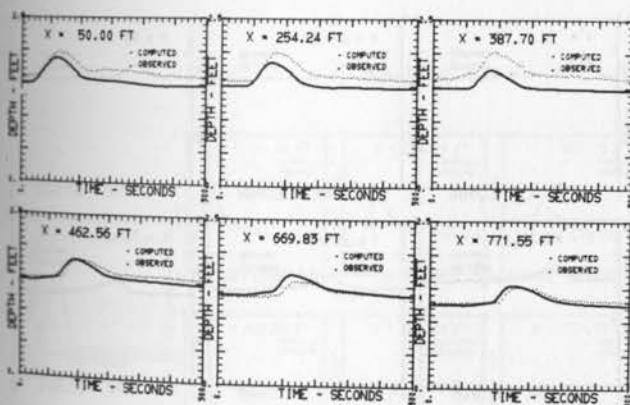
RUN NUMBER 100005

INFLOW HYDROGRAPHS



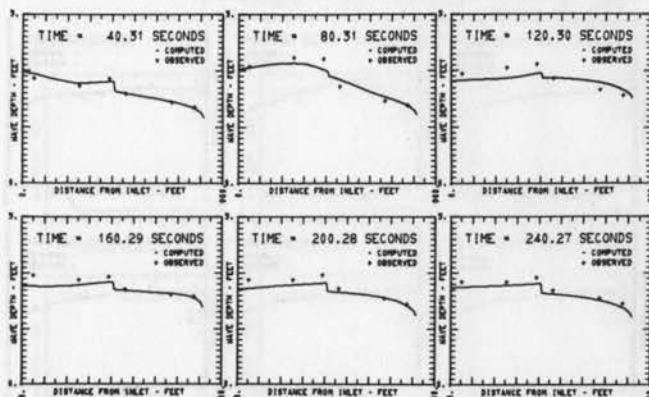
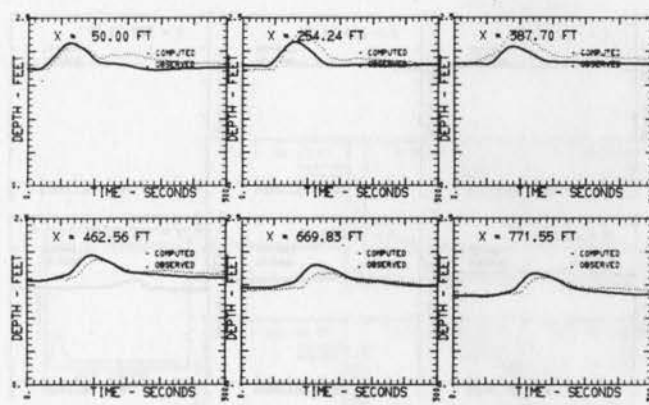
RUN NUMBER 100004

INFLOW HYDROGRAPHS



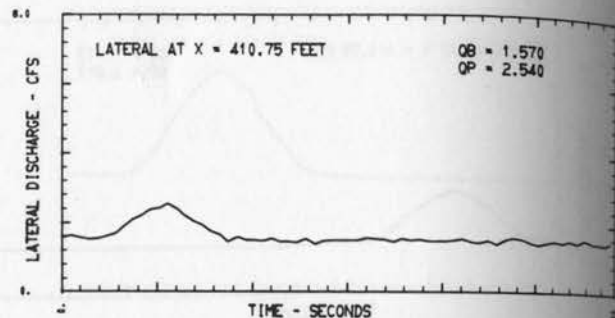
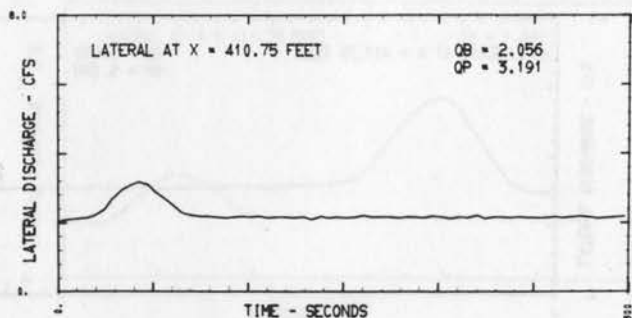
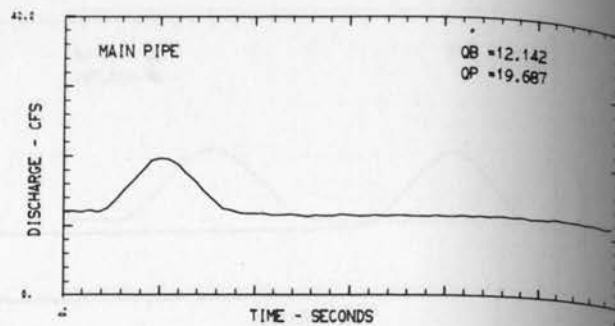
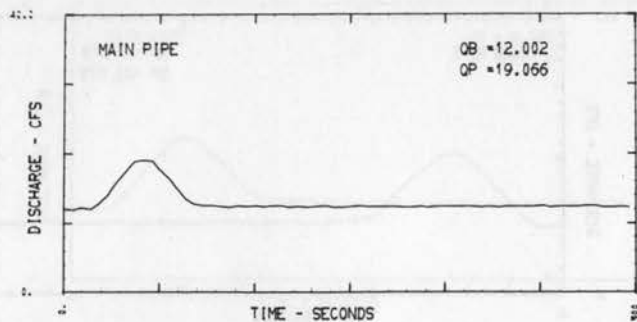
RUN NUMBER 100005

SO = .000480 QB = 10.20 CFS QP = 20.80 CFS



RUN NUMBER 100004

SO = .000480 QB = 10.21 CFS QP = 20.57 CFS

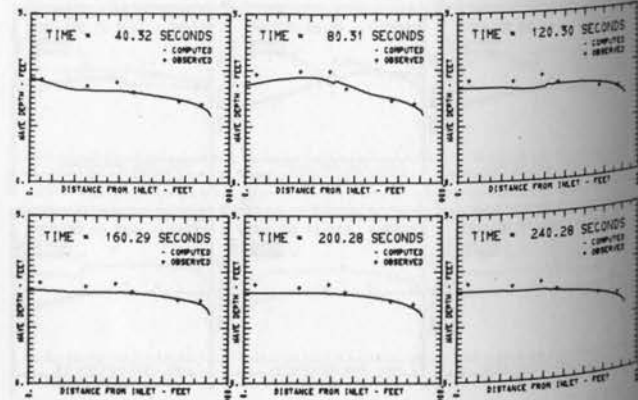
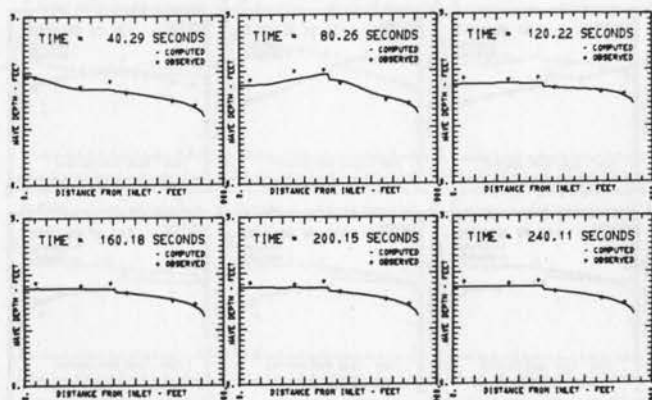
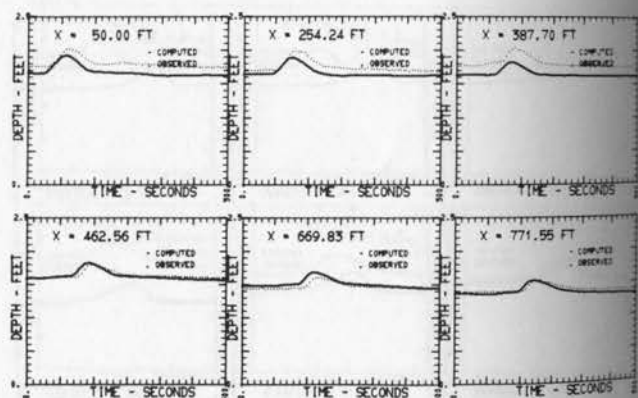
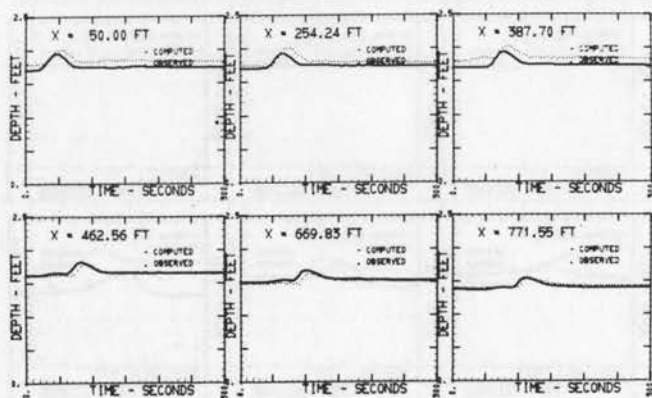


RUN NUMBER 120003

INFLOW HYDROGRAPHS

RUN NUMBER 100003

INFLOW HYDROGRAPHS

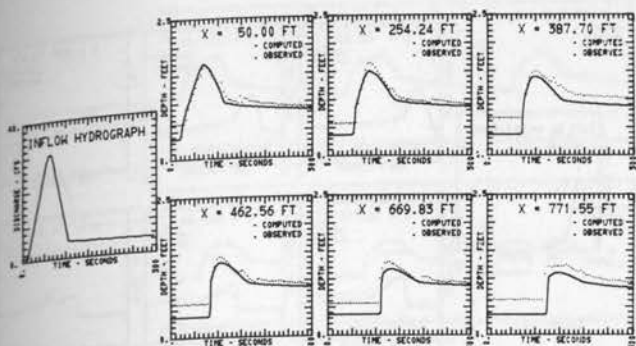


RUN NUMBER 120003

SO = .000480 OB = 12.00 CFS QP = 19.07 CFS

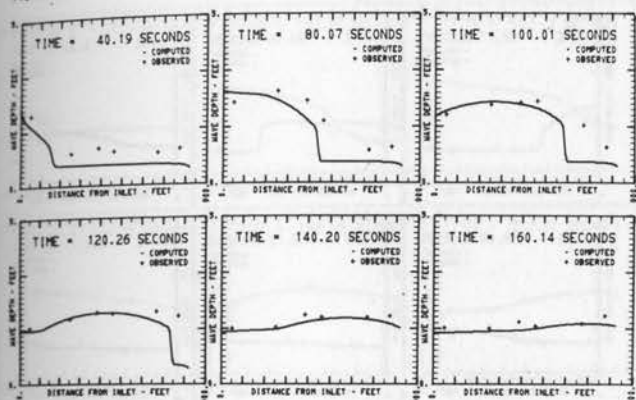
RUN NUMBER 100003

SO = .000480 OB = 12.14 CFS QP = 19.69 CFS

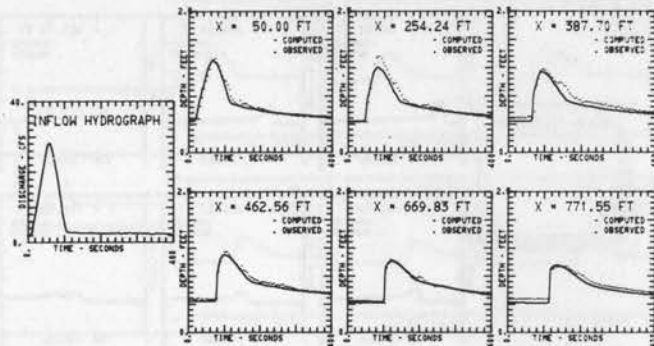


RUN NUMBER 19909

SO = .000990 QB = .90 CFS QP = 30.48 CFS

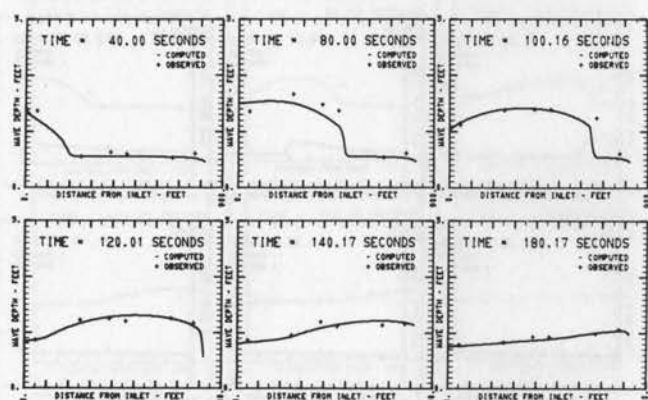


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 19909

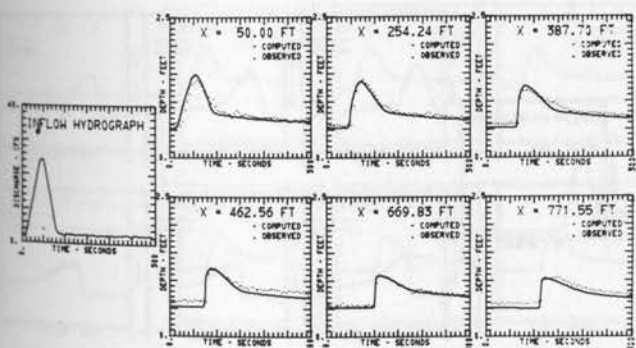


RUN NUMBER 1 10

SO = .000990 QB = 2.08 CFS QP = 28.33 CFS

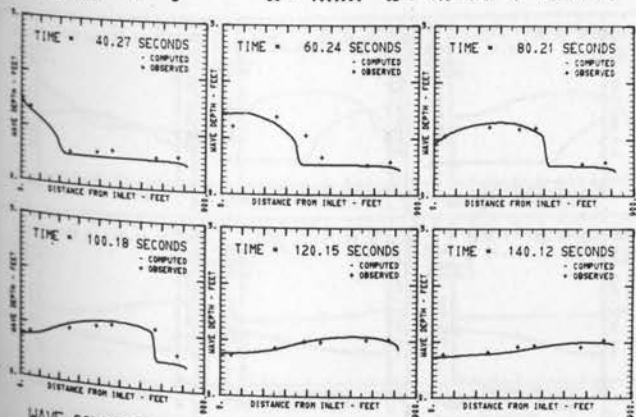


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 1 10

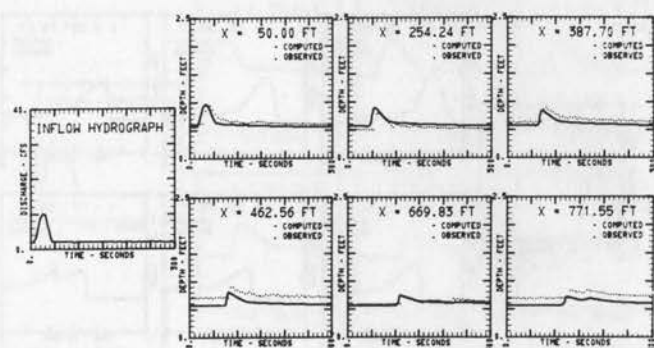


RUN NUMBER 1 8

SO = .000990 QB = 1.91 CFS QP = 24.40 CFS

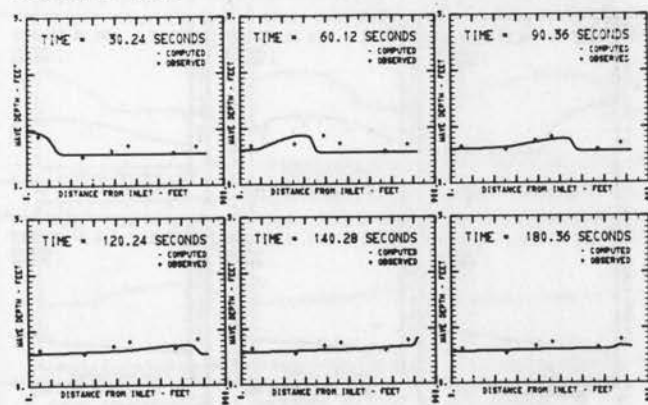


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 1 8

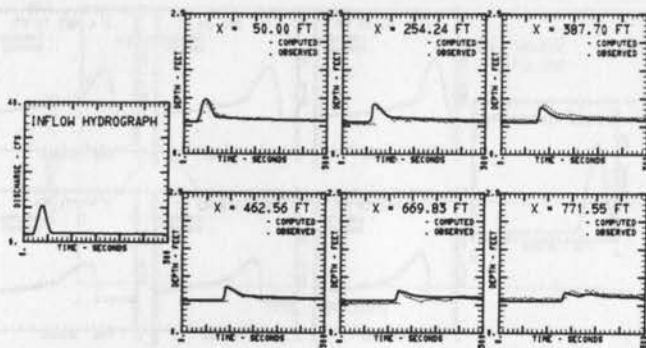


RUN NUMBER 020001

SO = .000990 QB = 2.21 CFS QP = 10.23 CFS

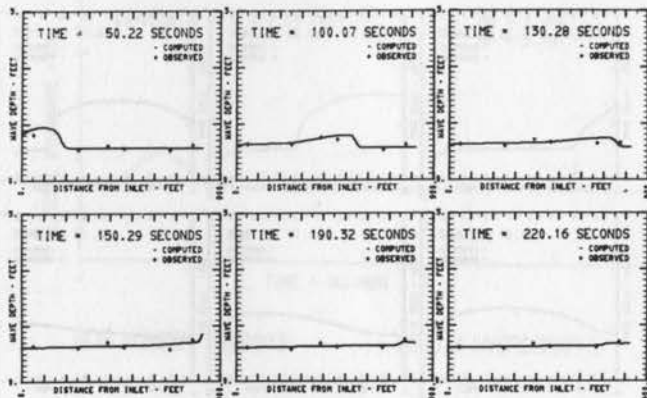


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 020001

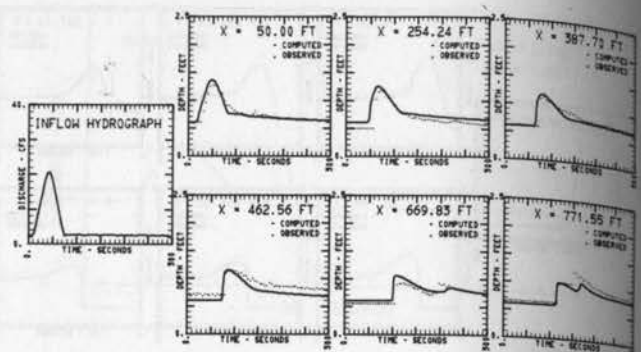


RUN NUMBER 029901

SO = .000990 QB = 2.27 CFS QP = 10.36 CFS

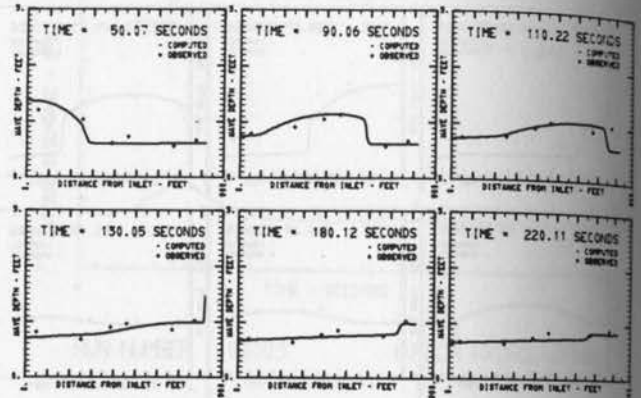


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 029901

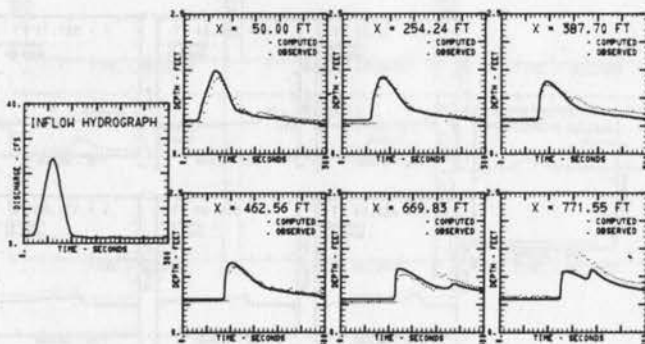


RUN NUMBER 020002

SO = .000990 QB = 2.48 CFS QP = 20.57 CFS

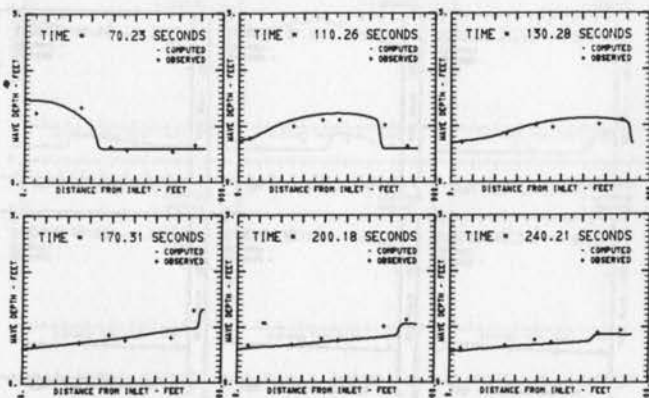


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 020002

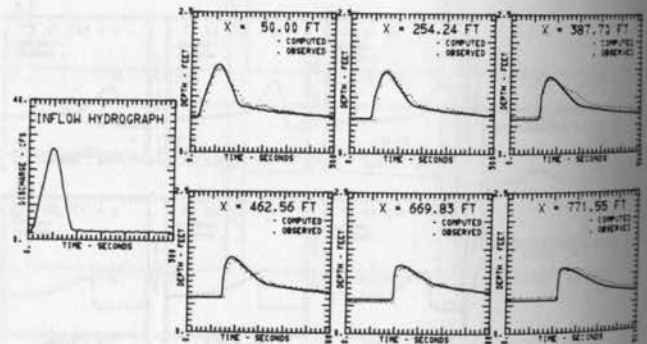


RUN NUMBER 029902

SO = .000990 QB = 2.41 CFS QP = 24.40 CFS

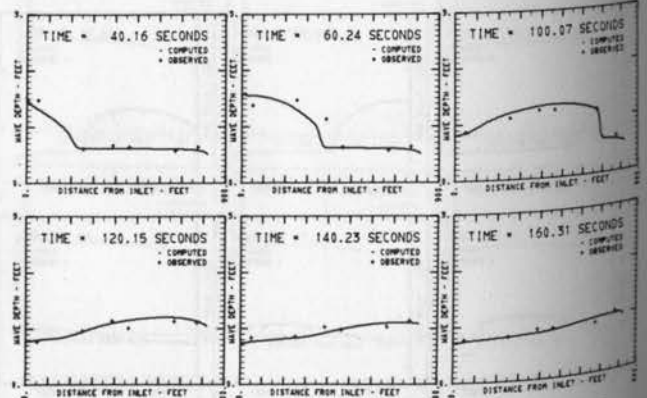


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 029902

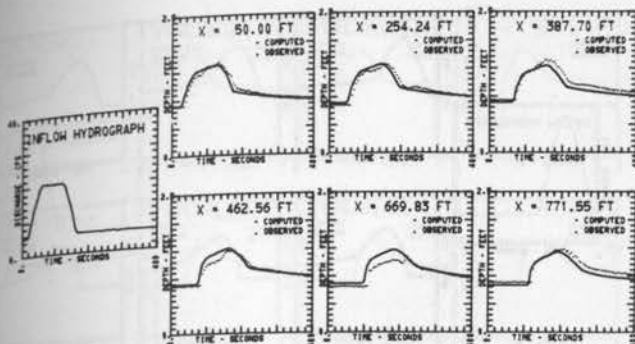


RUN NUMBER 19

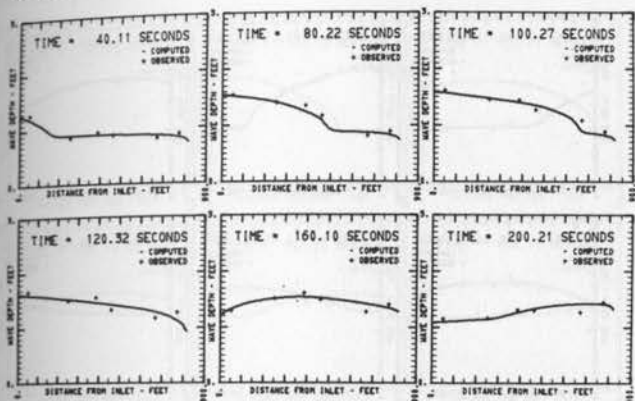
SO = .000990 QB = 2.60 CFS QP = 26.17 CFS



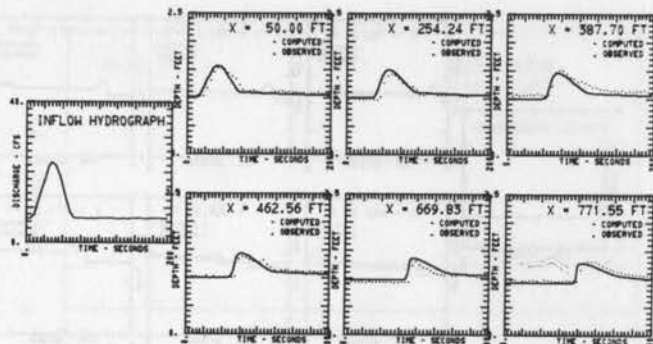
WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 19



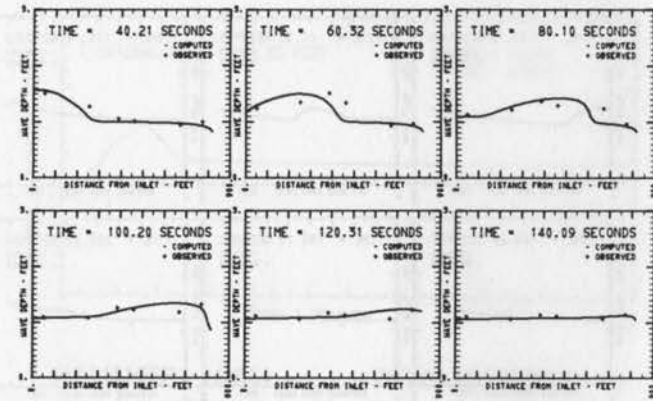
RUN NUMBER 8 1 SO = .000990 OB = 5.44 CFS OP = 20.78 CFS



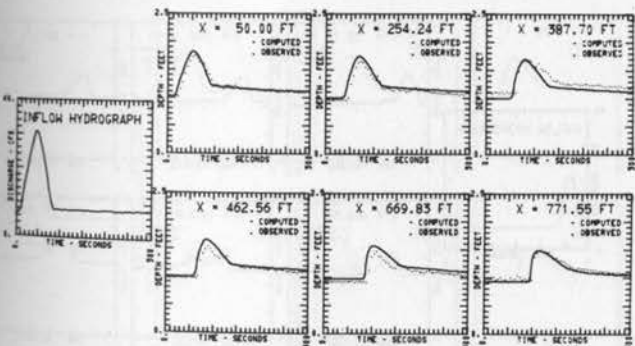
WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 8 1



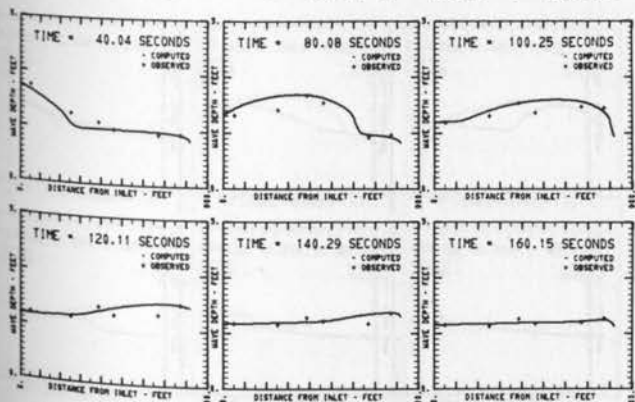
RUN NUMBER 1 11 SO = .000990 OB = 6.85 CFS OP = 22.91 CFS



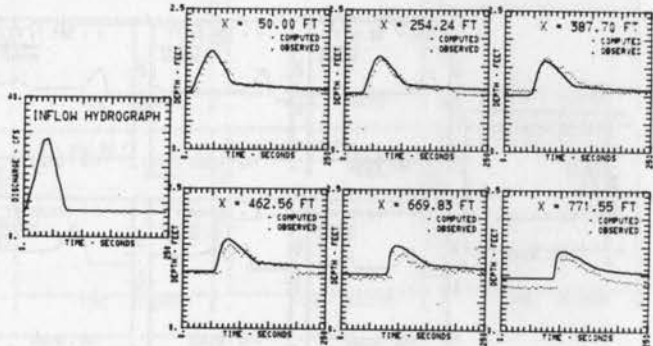
WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 1 11



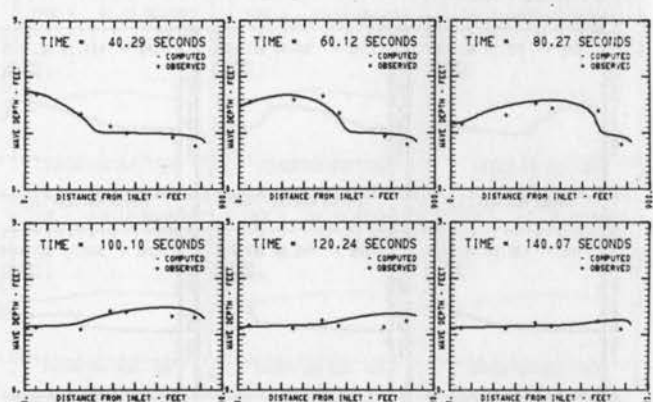
RUN NUMBER 1 12 SO = .000990 OB = 6.83 CFS OP = 30.66 CFS



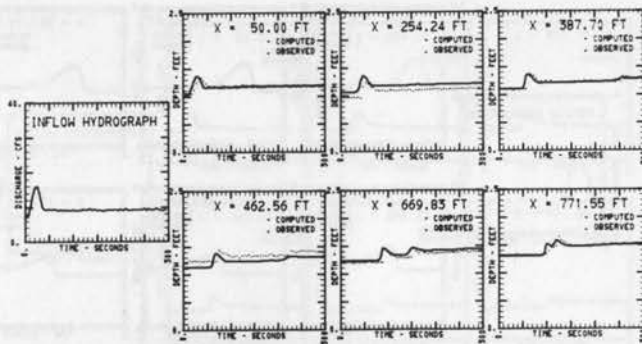
WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 1 12



RUN NUMBER 1 3 SO = .000990 OB = 6.99 CFS OP = 28.07 CFS

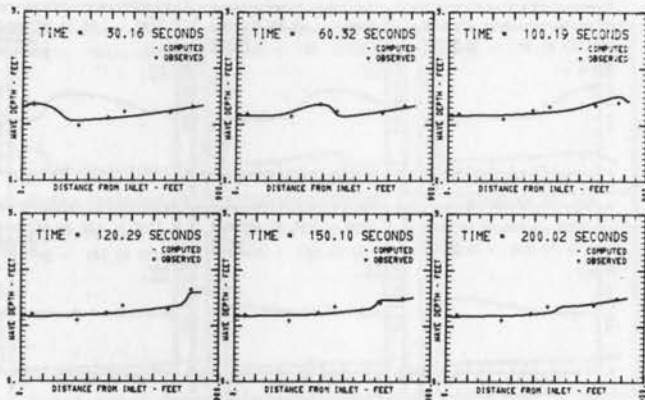


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 1 3



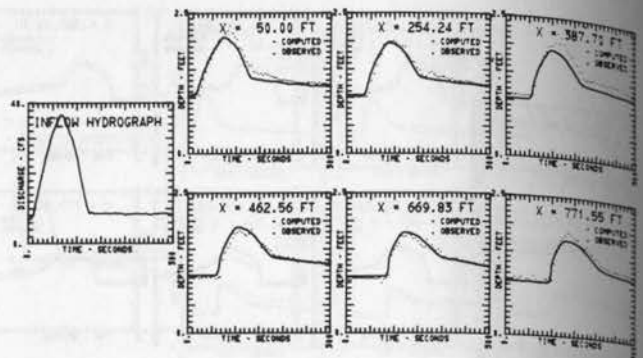
RUN NUMBER 020003

SO = .000990 OB = 7.17 CFS OP = 16.07 CFS



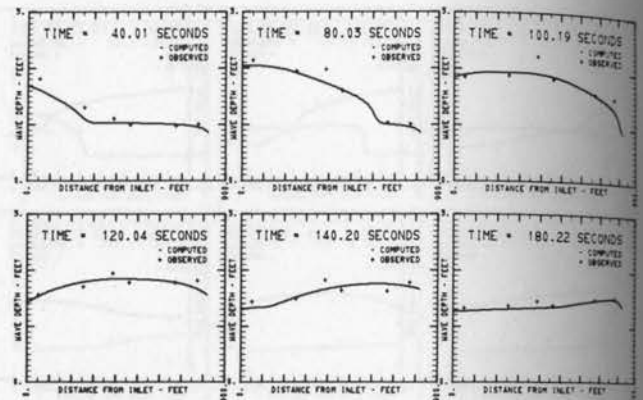
WAVE COMPARISONS AT INSTANTS IN TIME

RUN NUMBER 020003



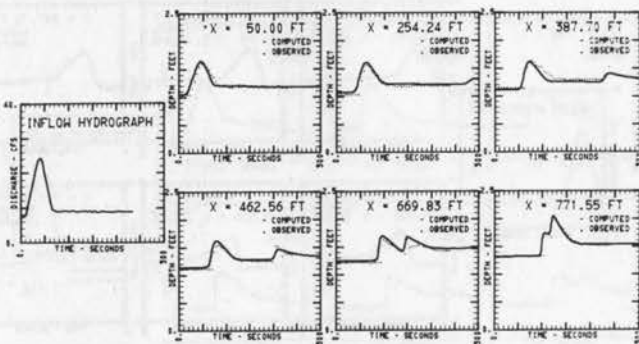
RUN NUMBER 1 13

SO = .000990 OB = 7.29 CFS OP = 36.53 CFS



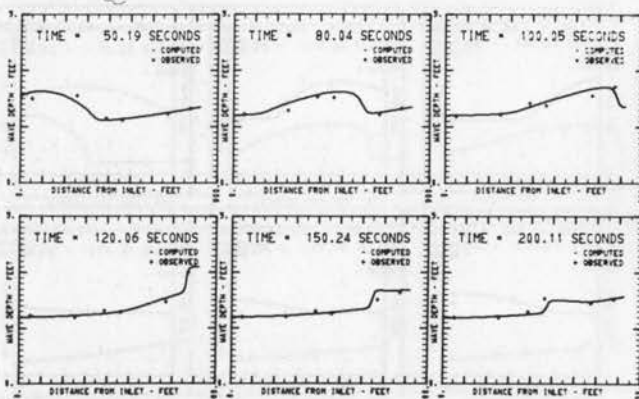
WAVE COMPARISONS AT INSTANTS IN TIME

RUN NUMBER 1 13



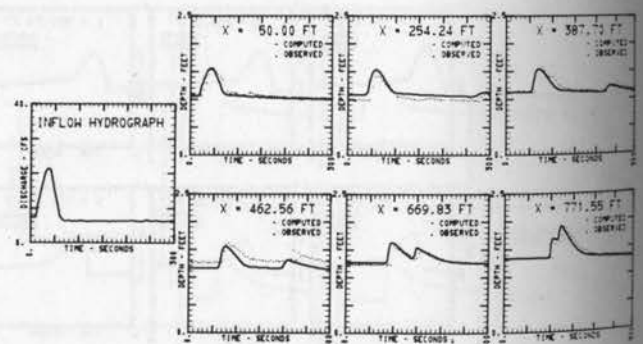
RUN NUMBER 029904

SO = .000990 OB = 7.26 CFS OP = 24.46 CFS



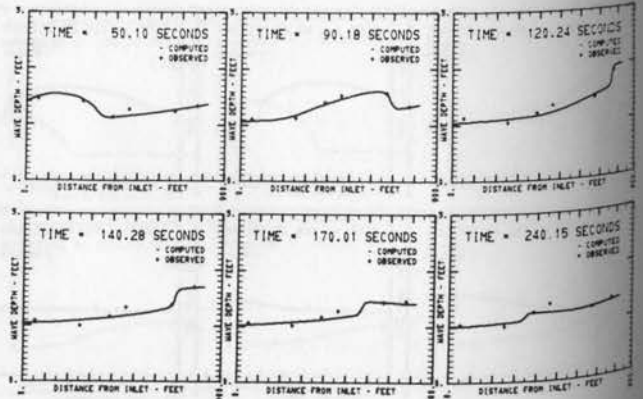
WAVE COMPARISONS AT INSTANTS IN TIME

RUN NUMBER 029904



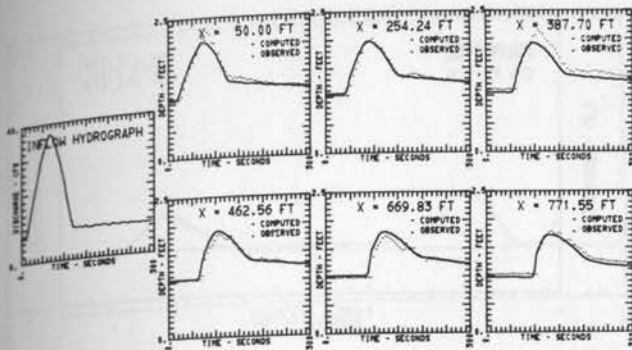
RUN NUMBER 020004

SO = .000990 OB = 7.36 CFS OP = 21.52 CFS



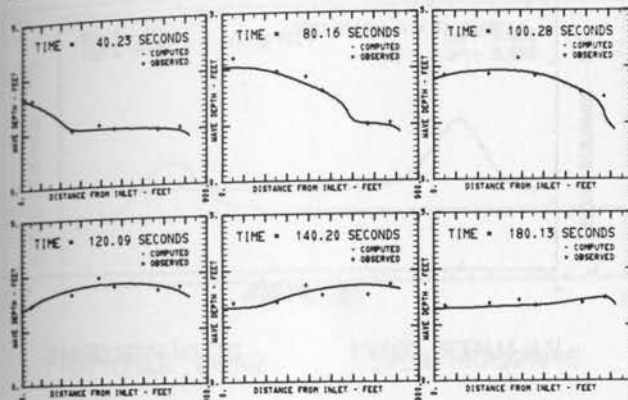
WAVE COMPARISONS AT INSTANTS IN TIME

RUN NUMBER 020004

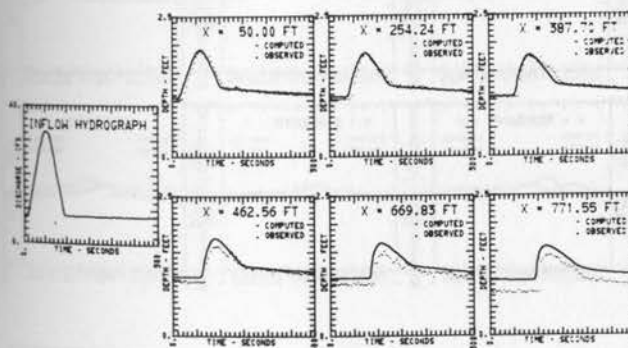


RUN NUMBER 19913

SO = .000990 OB = 7.47 CFS OP = 36.94 CFS

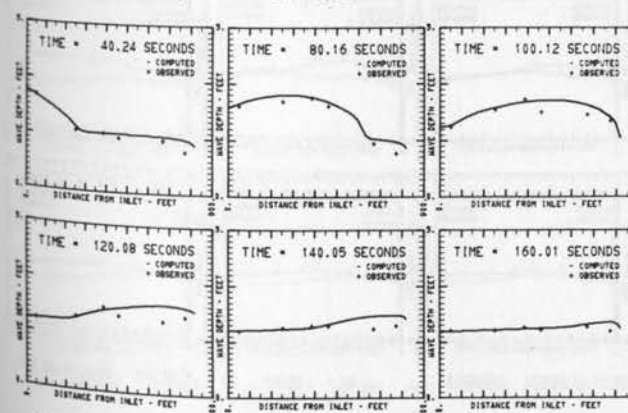


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 19913

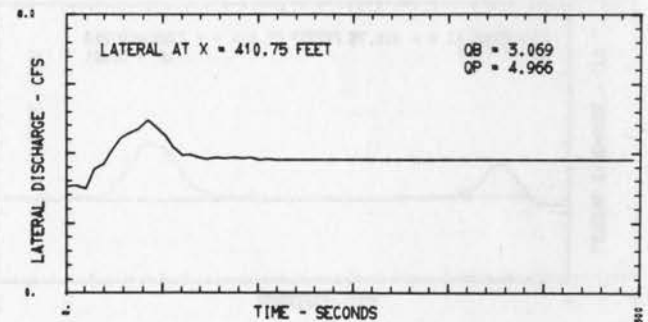
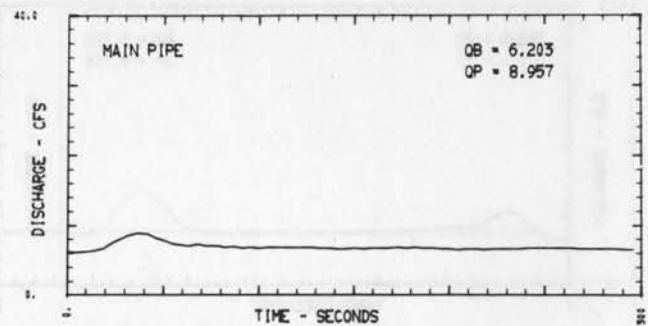


RUN NUMBER 1 4

SO = .000990 OB = 7.59 CFS OP = 32.58 CFS

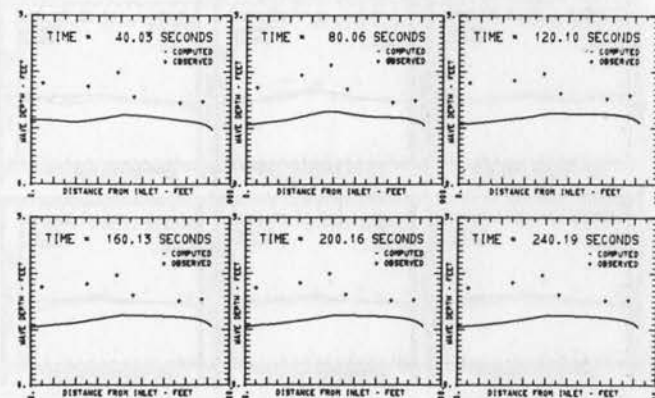
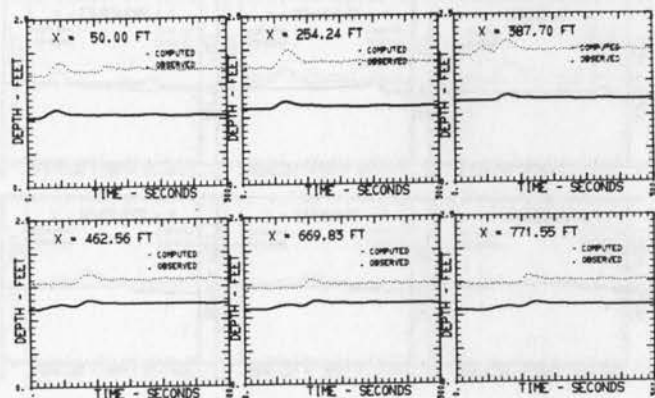


WAVE COMPARISONS AT INSTANTS IN TIME RUN NUMBER 1 4



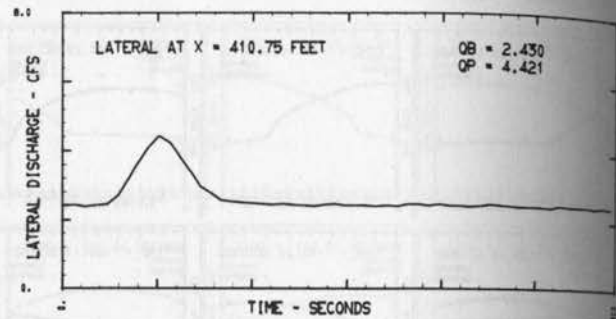
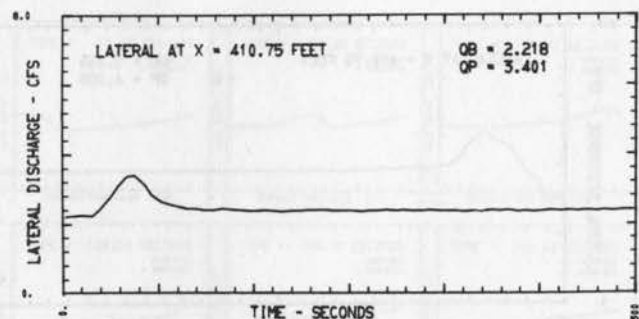
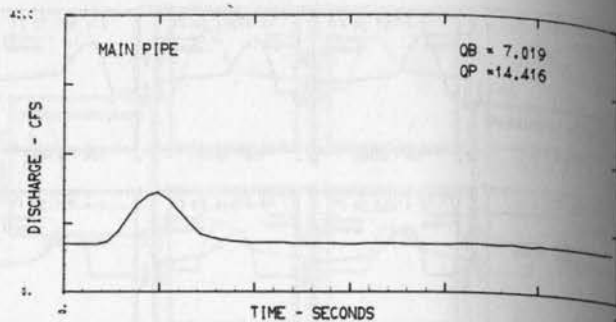
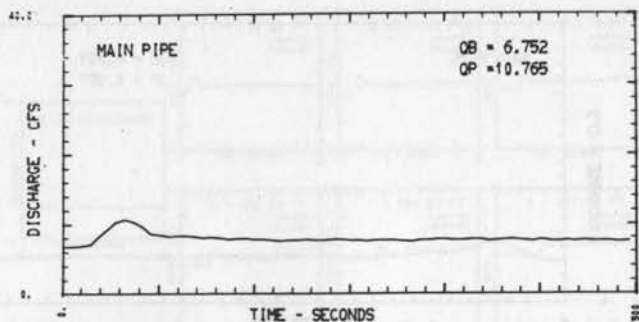
RUN NUMBER 040004

INFLUX HYDROGRAPHS



RUN NUMBER 040004

SO = .000990 OB = 6.20 CFS OP = 8.96 CFS

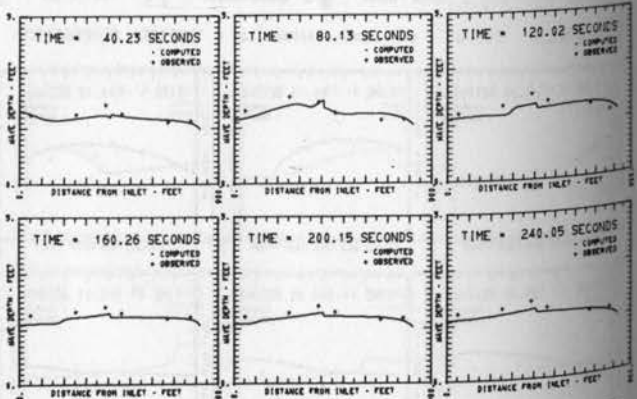
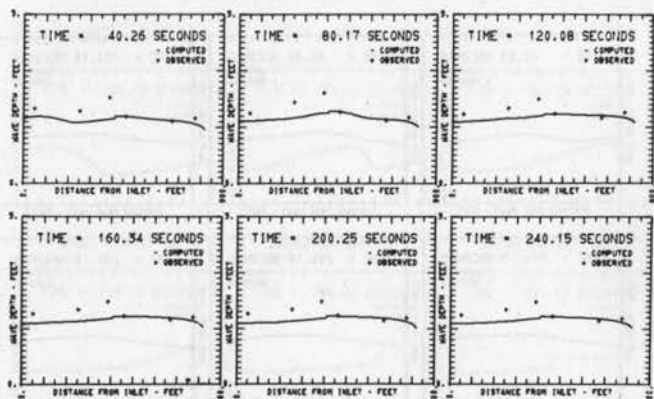
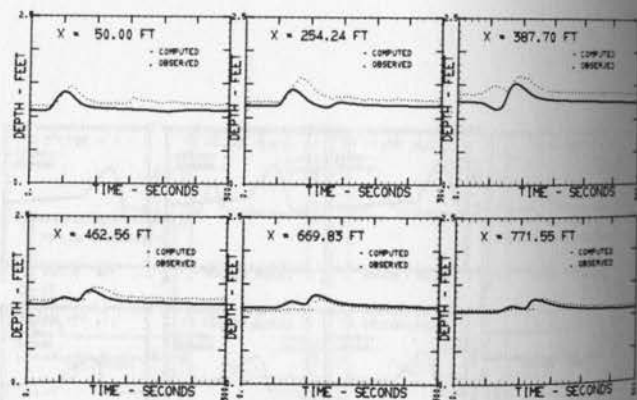
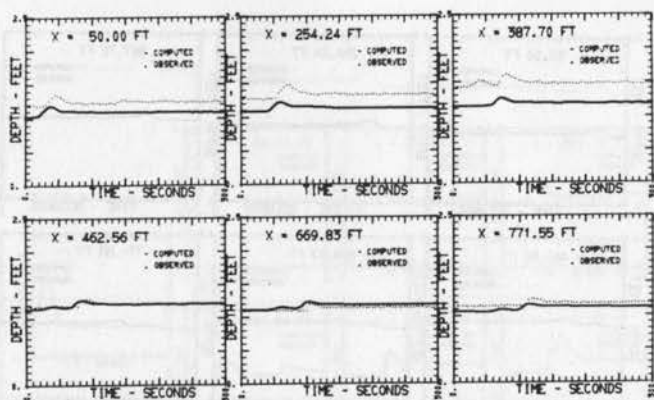


RUN NUMBER 040003

INFLOW HYDROGRAPHS

RUN NUMBER 059901

INFLOW HYDROGRAPHS

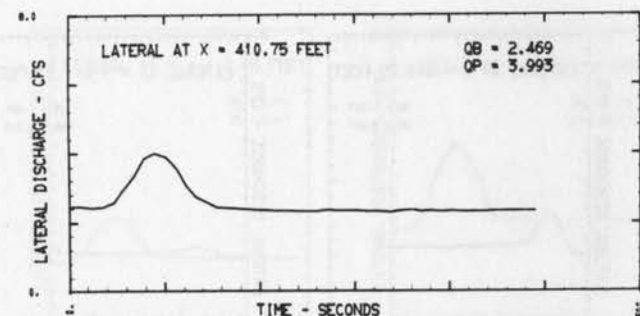
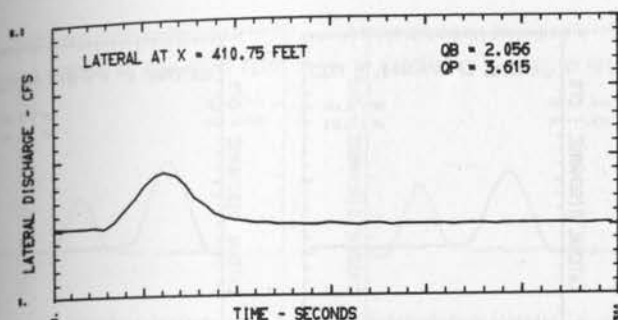
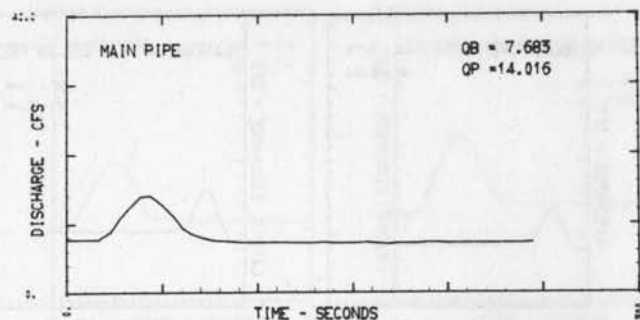
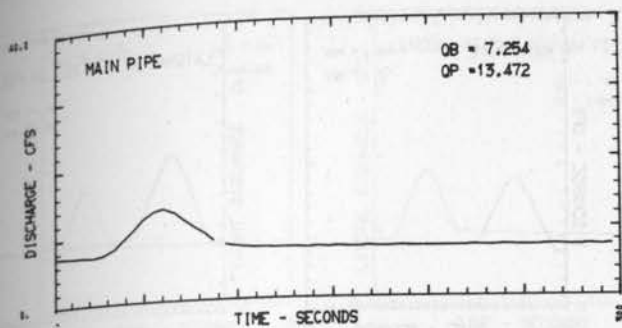


RUN NUMBER 040003

SO = .000990 OB = 6.75 CFS OP = 10.76 CFS

RUN NUMBER 059901

SO = .000990 OB = 7.02 CFS OP = 14.42 CFS

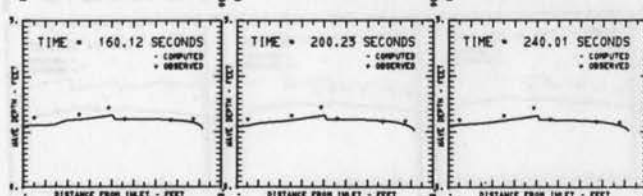
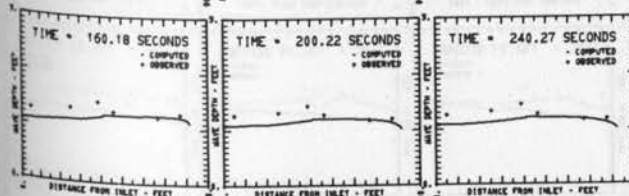
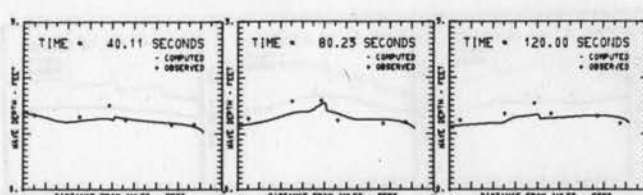
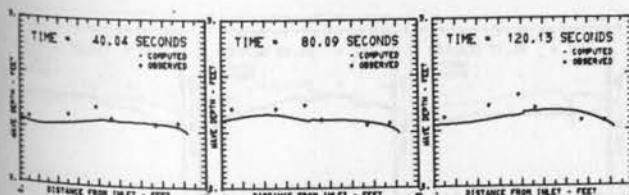
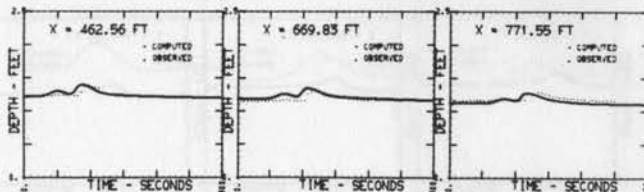
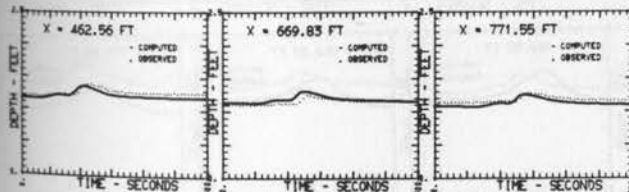
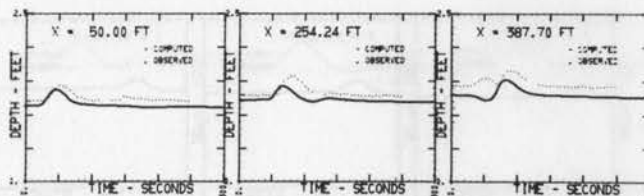
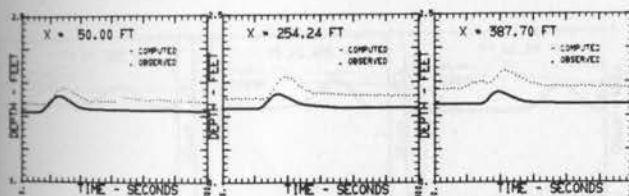


RUN NUMBER 049902

INFLOW HYDROGRAPHS

RUN NUMBER 050001

INFLOW HYDROGRAPHS

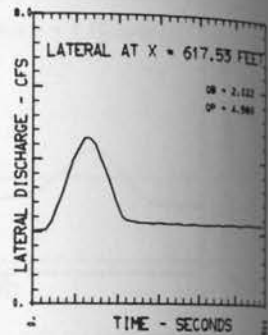
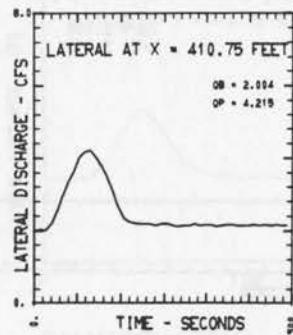
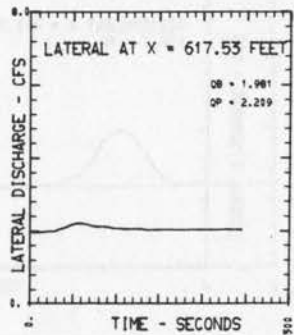
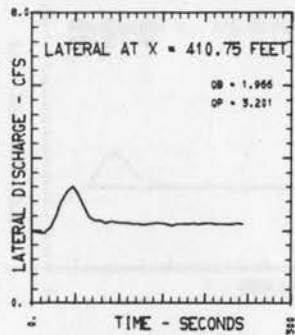
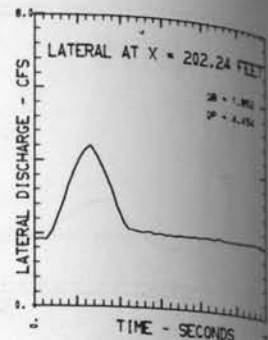
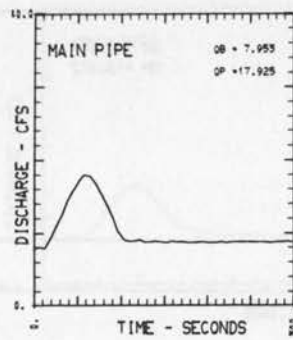
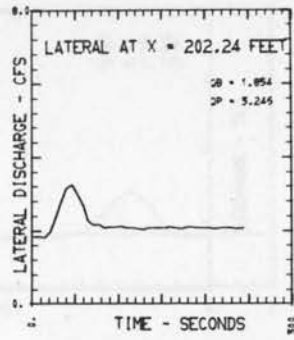
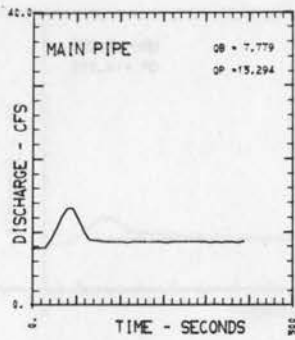


RUN NUMBER 049902

SO = .000990 OB = 7.25 CFS OP = 13.47 CFS

RUN NUMBER 050001

SO = .000990 OB = 7.68 CFS OP = 14.02 CFS

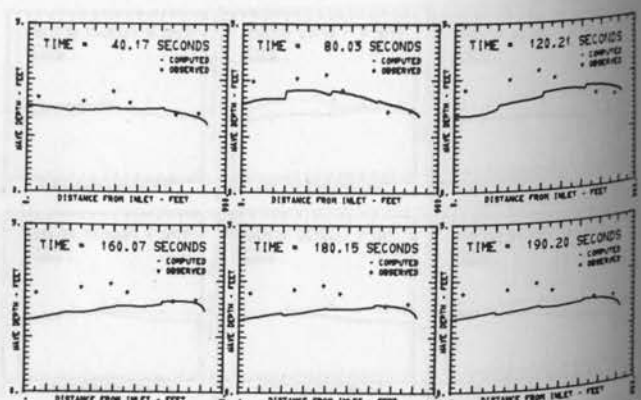
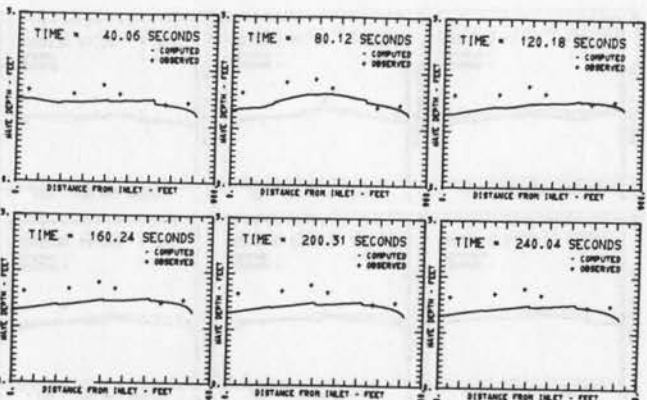
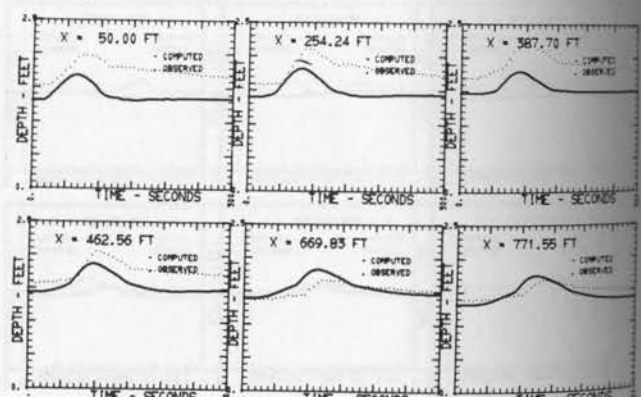
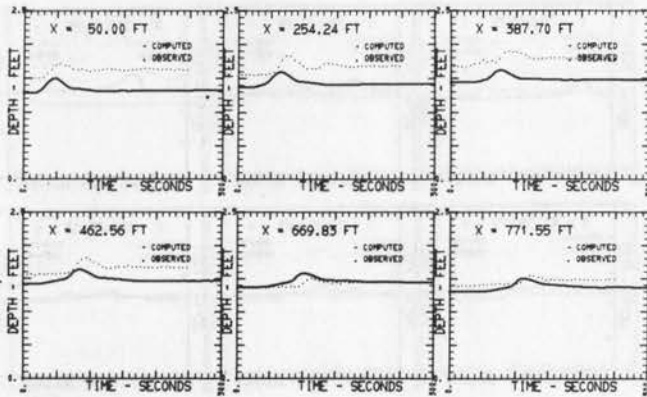


RUN NUMBER 079901

INFLOW HYDROGRAPHS

RUN NUMBER 079992

INFLOW HYDROGRAPHS

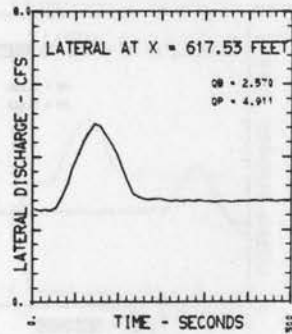
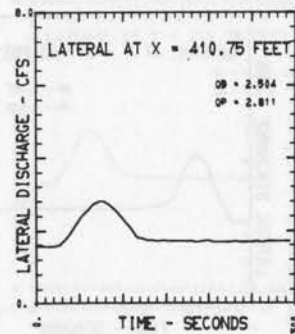
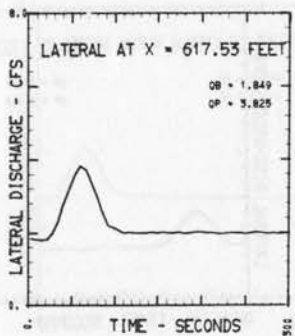
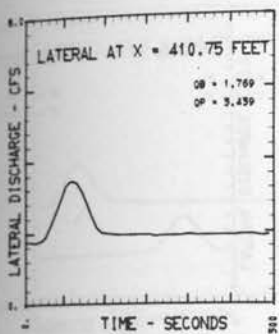
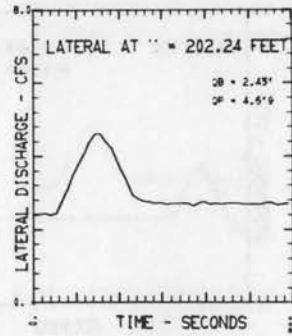
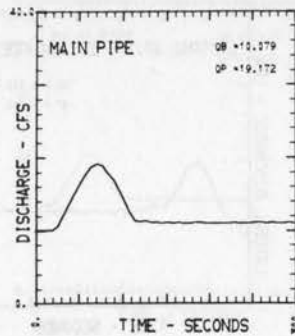
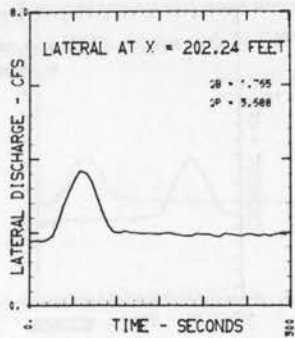
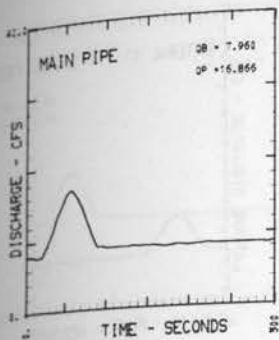


RUN NUMBER 079901

SO = .000990 OB = 7.78 CFS OP = 15.29 CFS

RUN NUMBER 079992

SO = .000990 OB = 7.95 CFS OP = 17.92 CFS

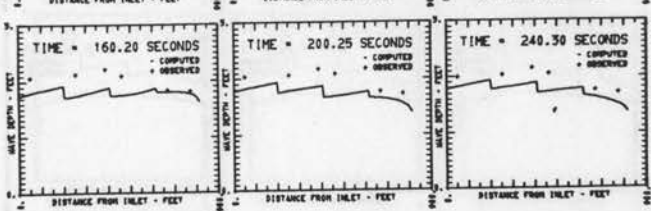
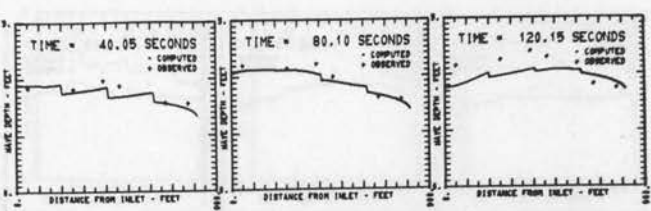
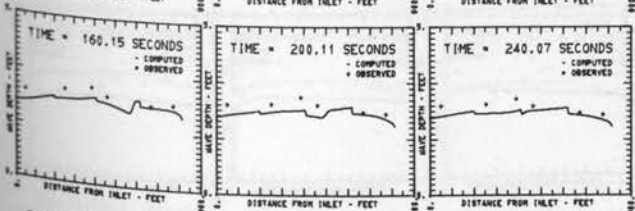
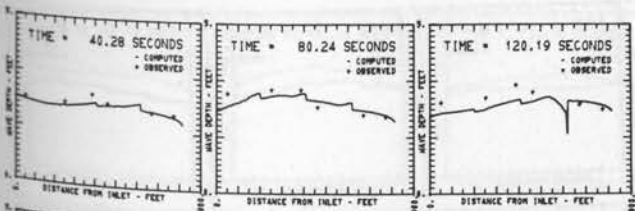
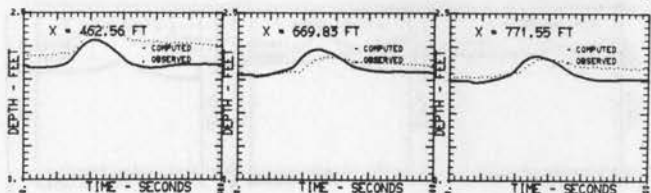
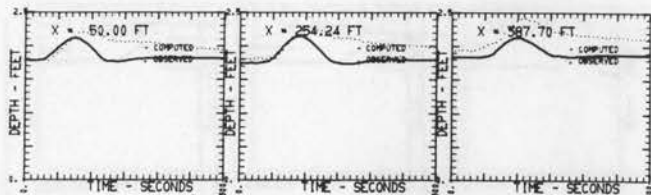
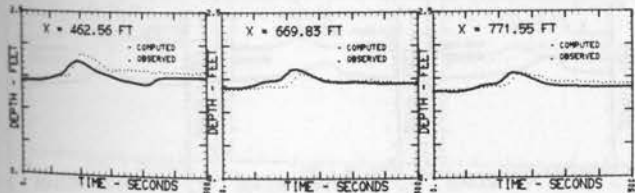
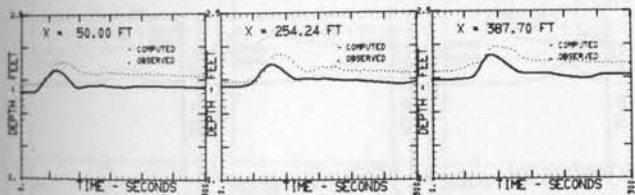


RUN NUMBER 060002

INFLOW HYDROGRAPHS

RUN NUMBER 079905

INFLOW HYDROGRAPHS

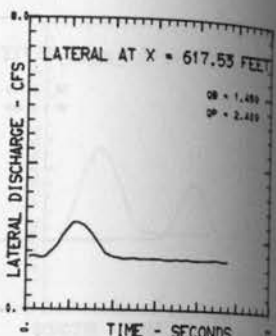
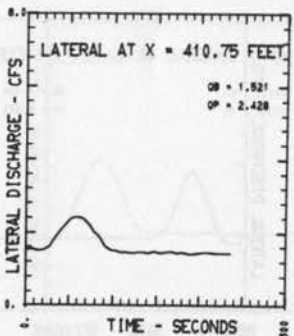
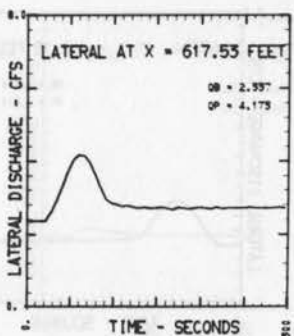
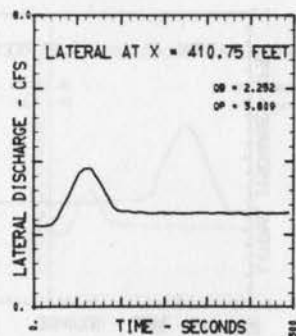
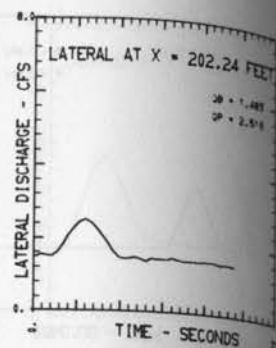
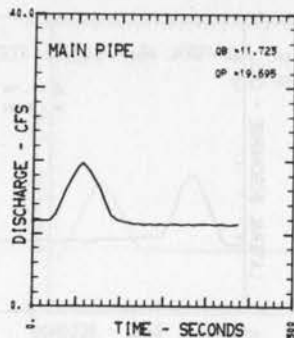
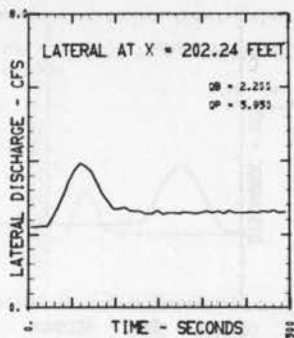
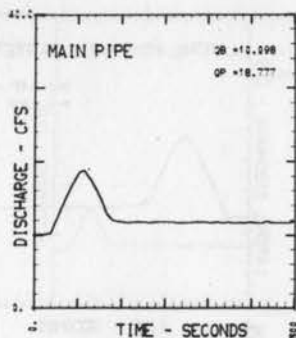


RUN NUMBER 060002

SO = .000990 DB = 7.96 CFS OP = 16.87 CFS

RUN NUMBER 079905

SO = .000990 DB = 10.08 CFS OP = 19.17 CFS

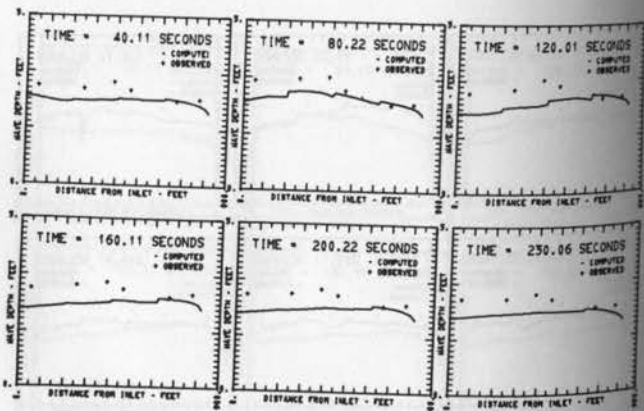
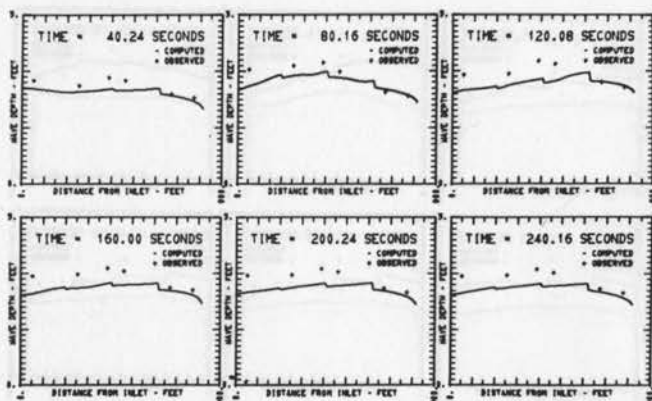
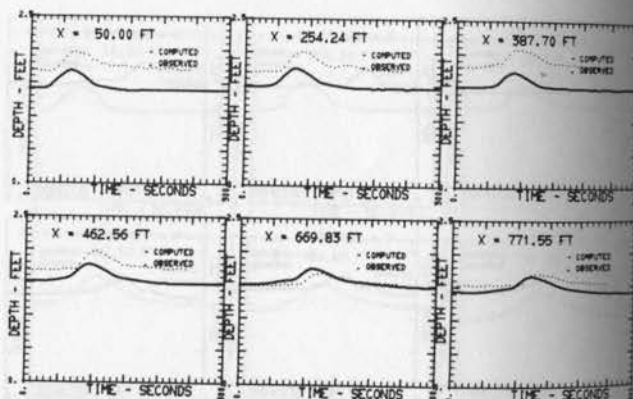
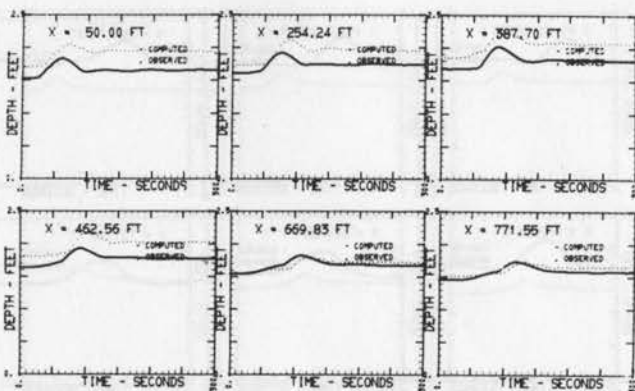


RUN NUMBER 060005

INFLOW HYDROGRAPHS

RUN NUMBER 079904

INFLOW HYDROGRAPHS

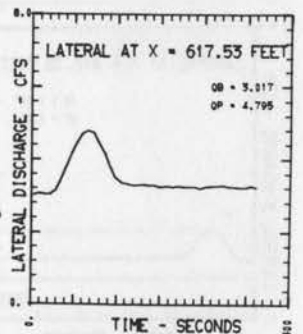
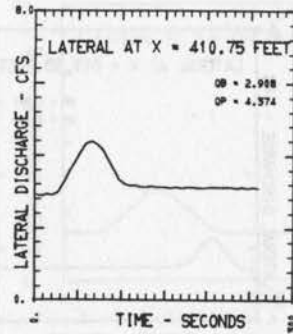
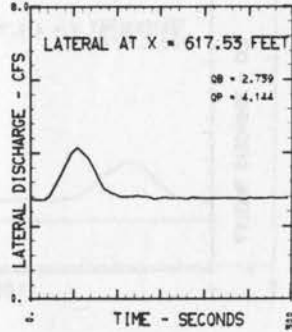
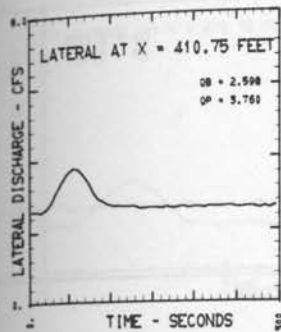
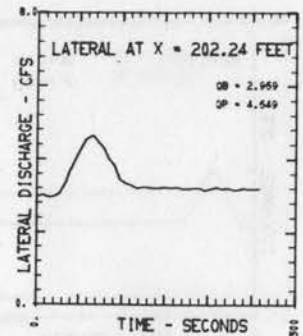
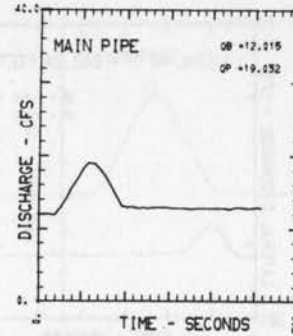
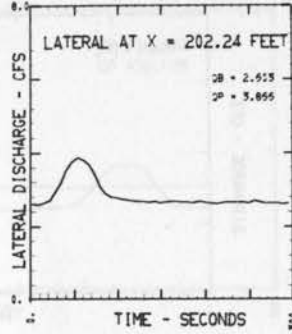
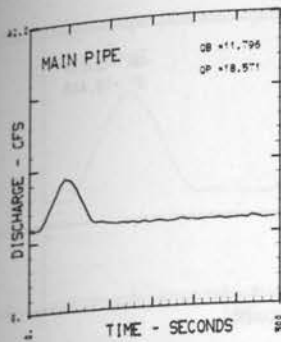


RUN NUMBER 060005

SO = .000990 OB = 10.10 CFS OP = 18.78 CFS

RUN NUMBER 079904

SO = .000990 OB = 11.72 CFS OP = 19.69 CFS

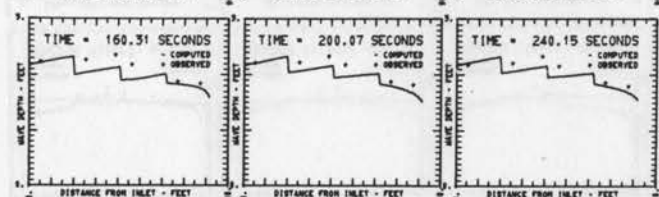
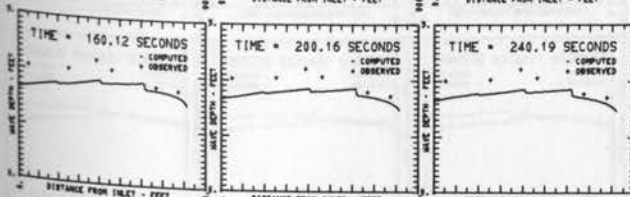
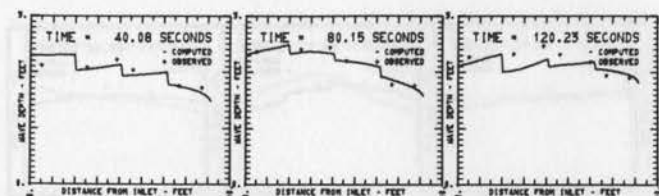
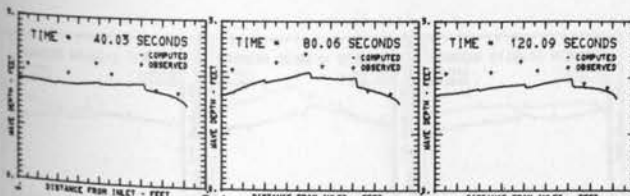
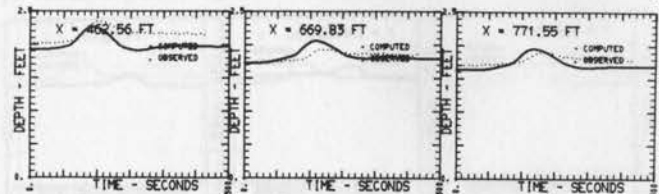
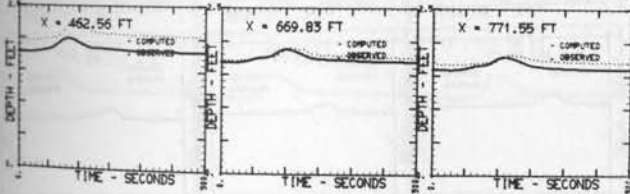
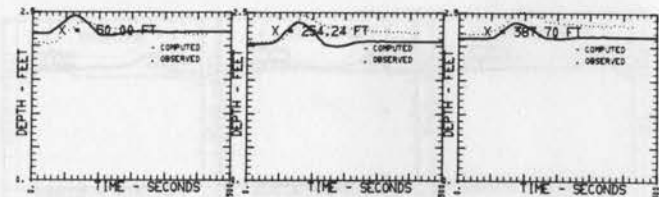
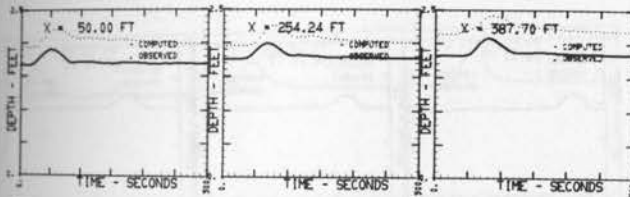


RUN NUMBER 060003

INFLOW HYDROGRAPHS

RUN NUMBER 079903

INFLOW HYDROGRAPHS

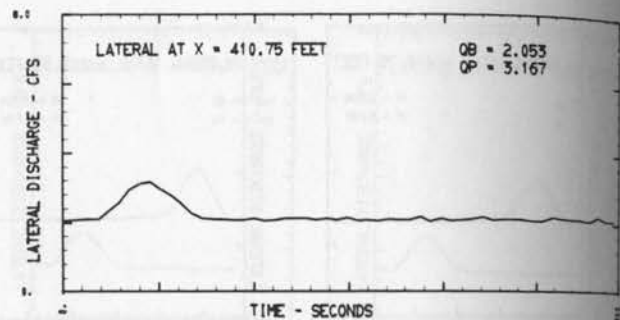
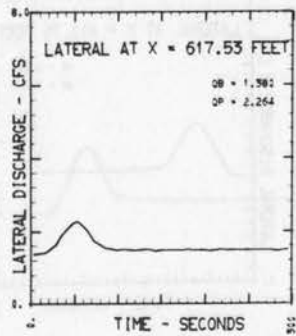
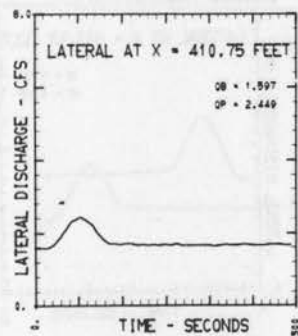
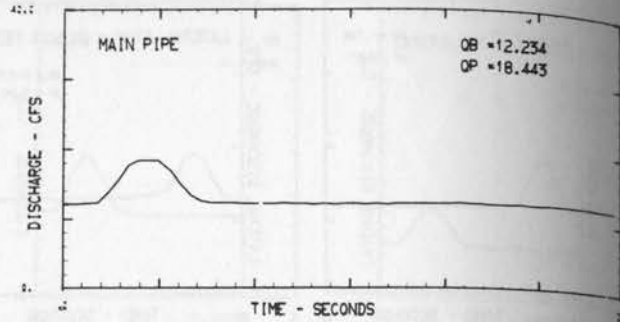
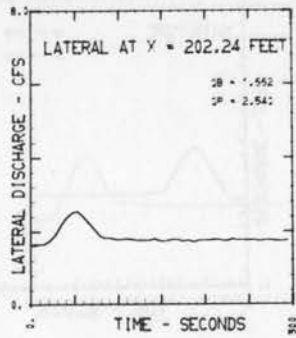
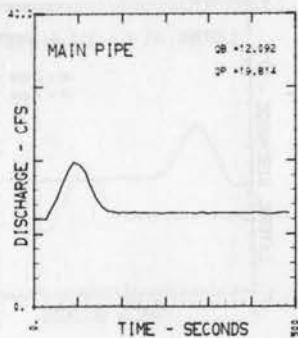


RUN NUMBER 060003

SO = .000990 OB = 11.80 CFS OP = 18.57 CFS

RUN NUMBER 079903

SO = .000990 OB = 12.02 CFS OP = 19.03 CFS

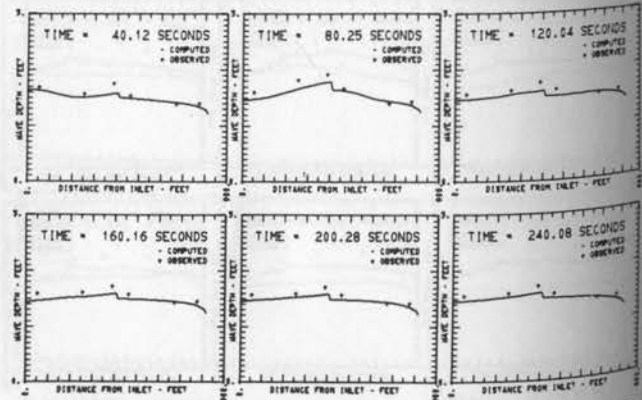
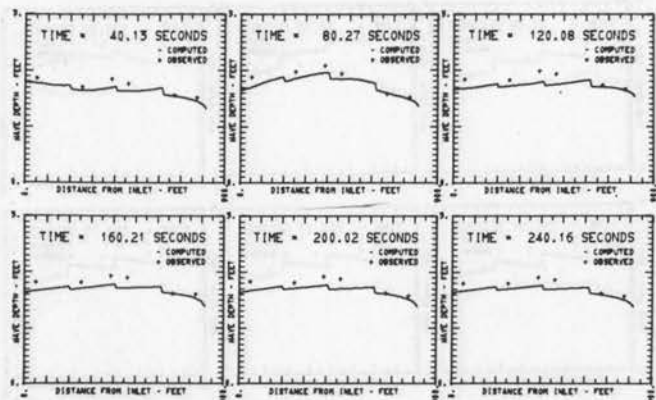
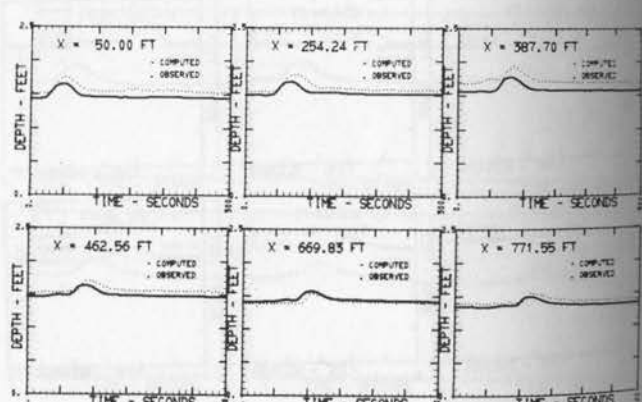
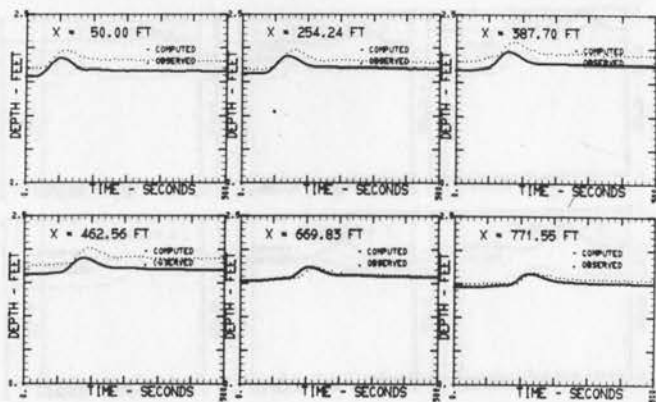


RUN NUMBER 060004

INFLOW HYDROGRAPHS

RUN NUMBER 050003

INFLOW HYDROGRAPHS

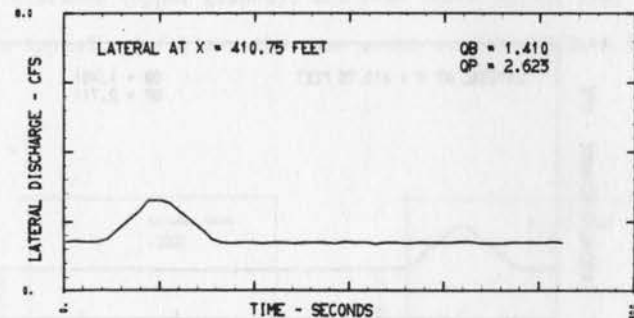
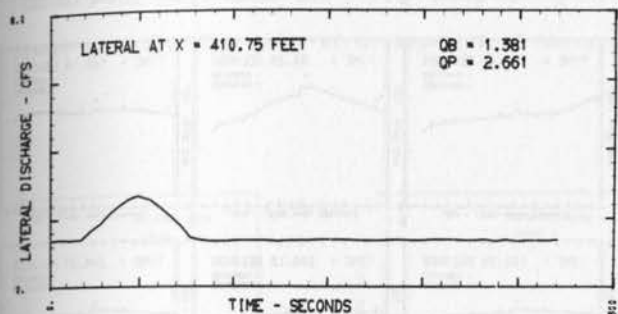
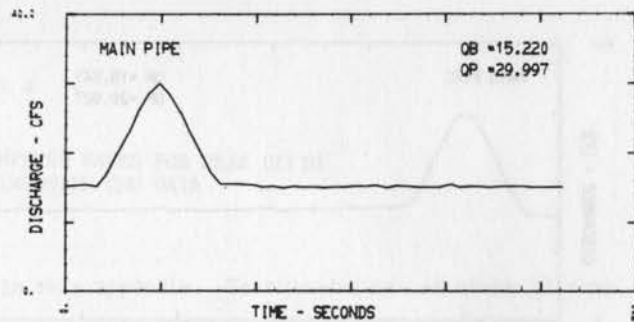
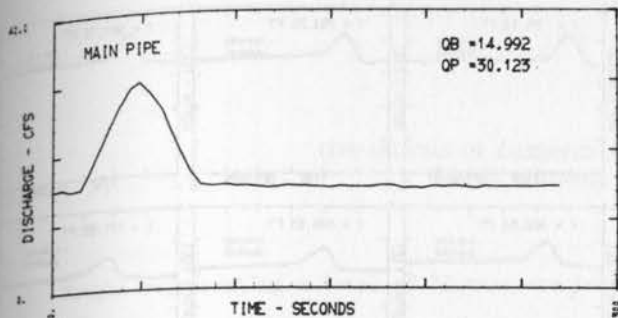


RUN NUMBER 060004

SO = .000990 OB = 12.09 CFS OP = 19.81 CFS

RUN NUMBER 050003

SO = .000990 OB = 12.23 CFS OP = 18.44 CFS

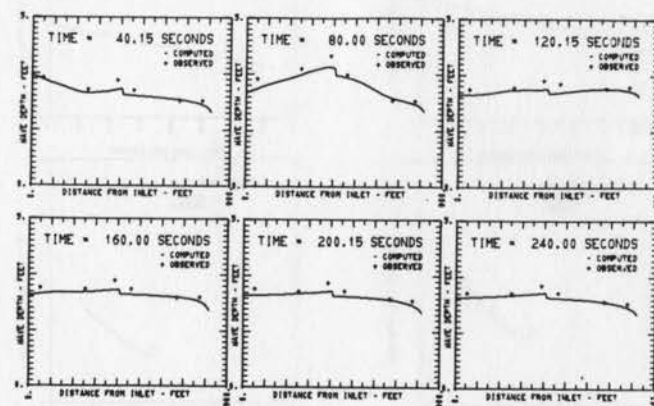
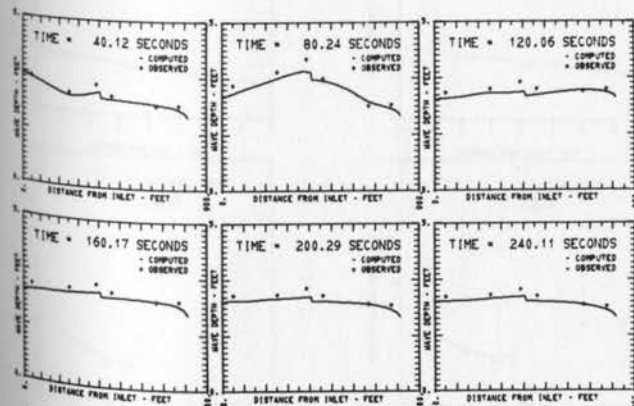
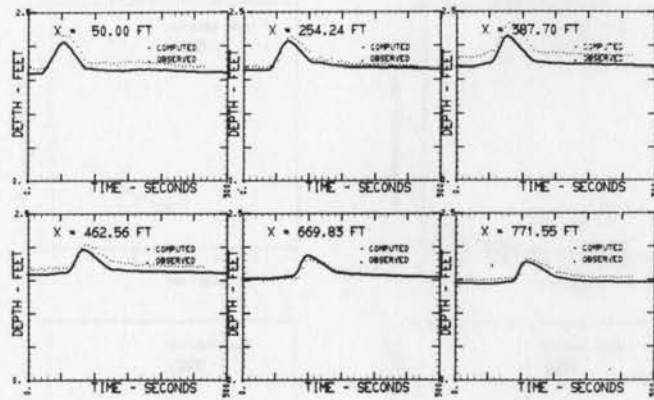
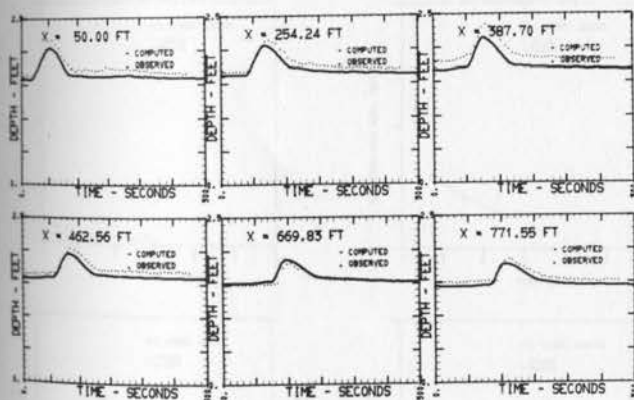


RUN NUMBER 030003

INFLOW HYDROGRAPHS

RUN NUMBER 030002

INFLOW HYDROGRAPHS

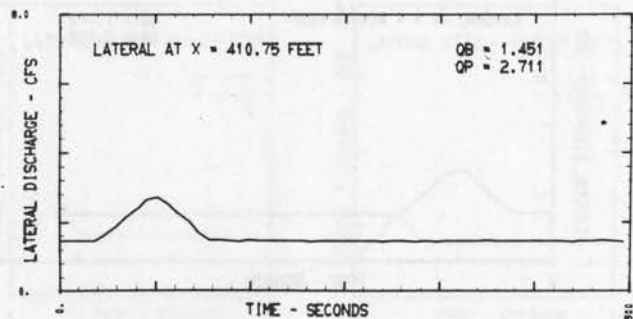
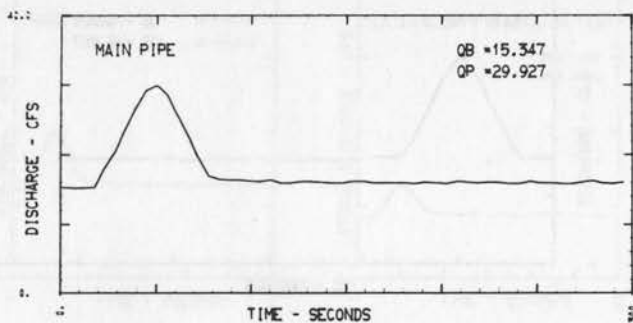


RUN NUMBER 030003

SO = .000990 OB = 14.99 CFS OP = 30.12 CFS

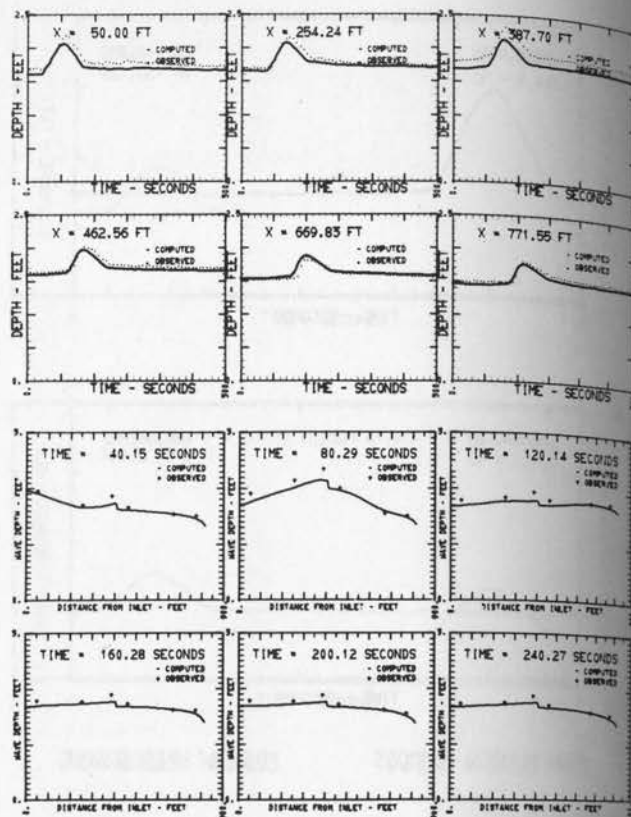
RUN NUMBER 030002

SO = .000990 OB = 15.22 CFS OP = 30.00 CFS



RUN NUMBER 030001

INFLOW HYDROGRAPHS



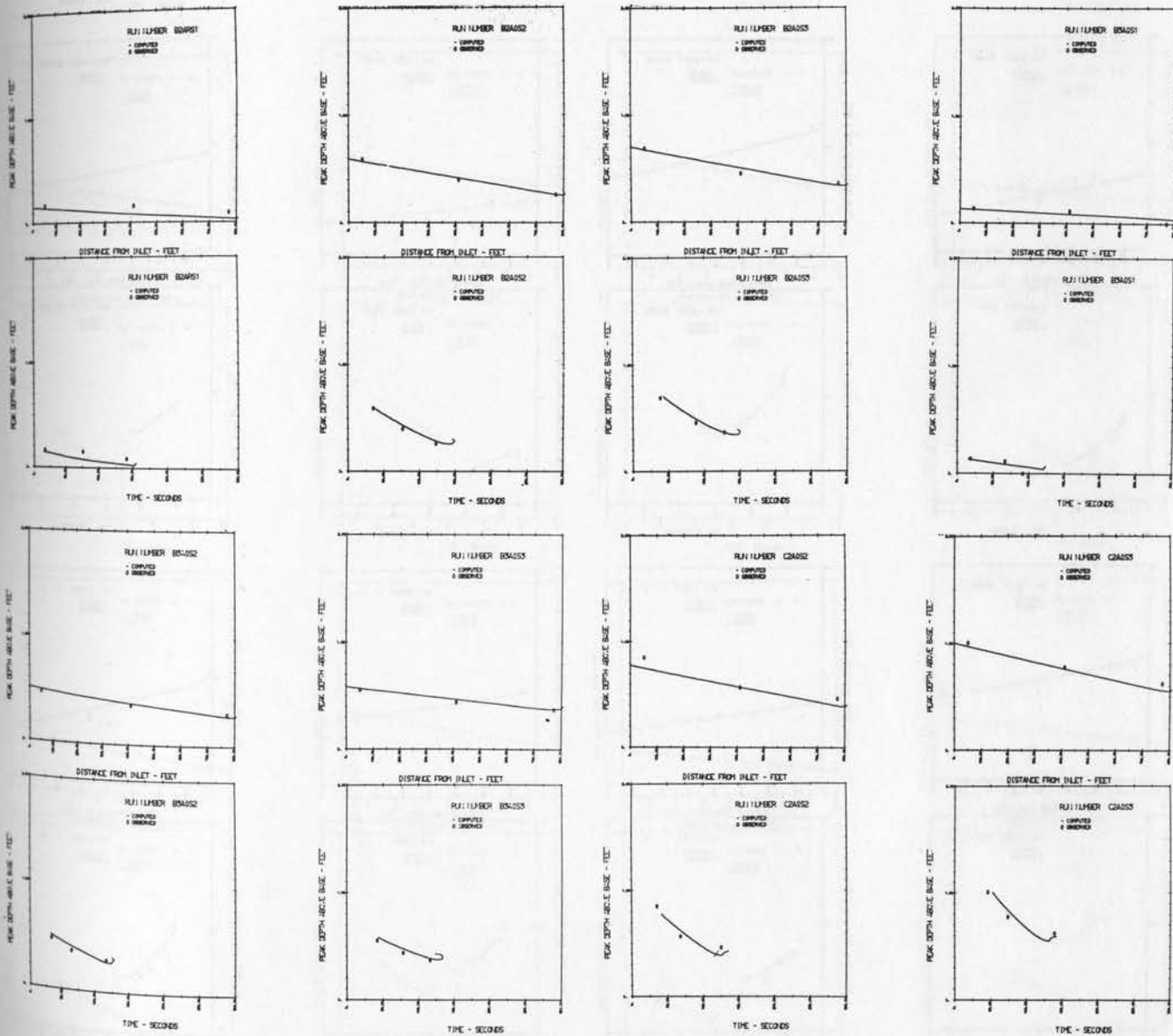
RUN NUMBER 030001

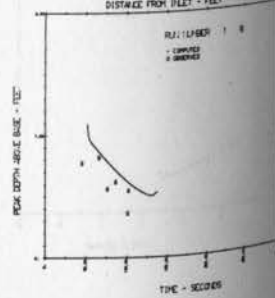
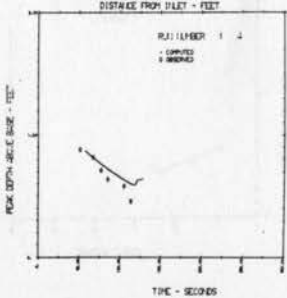
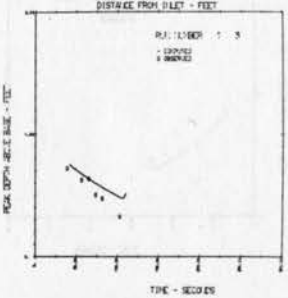
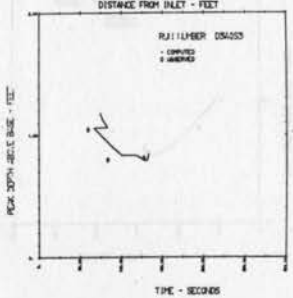
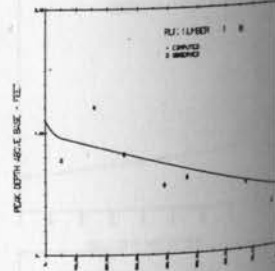
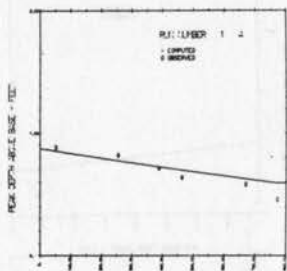
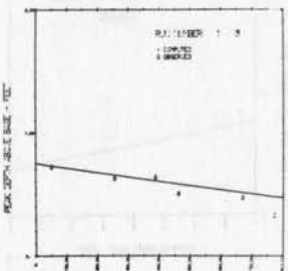
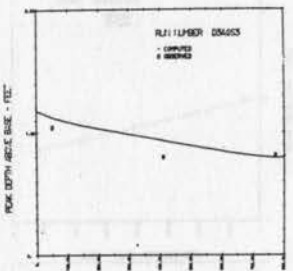
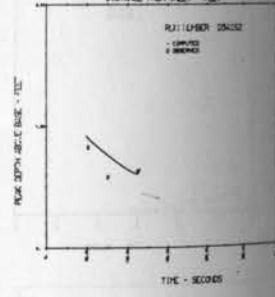
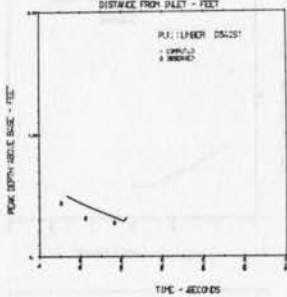
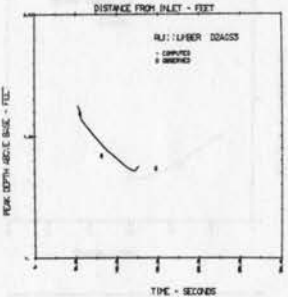
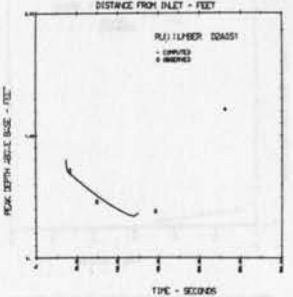
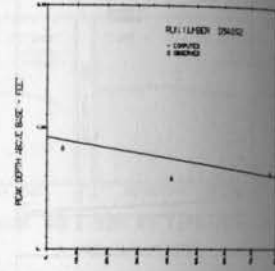
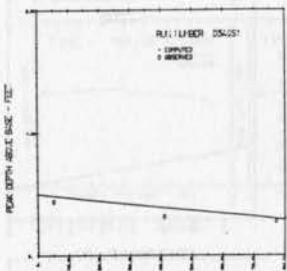
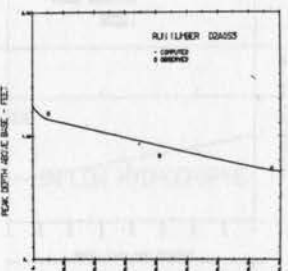
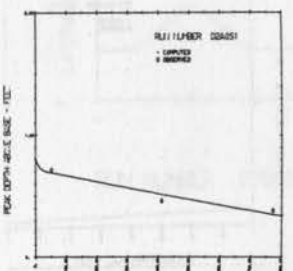
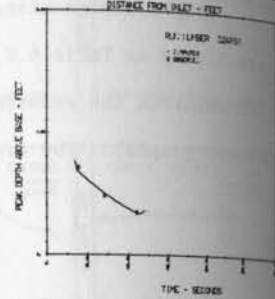
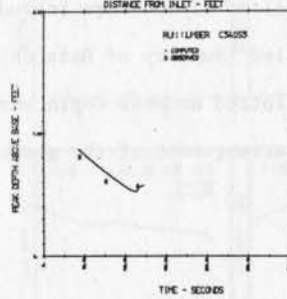
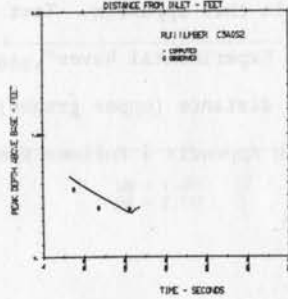
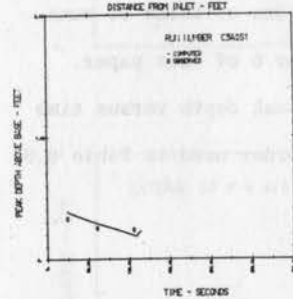
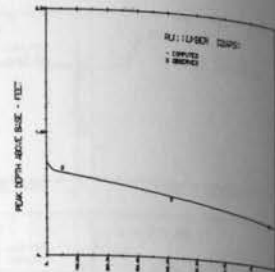
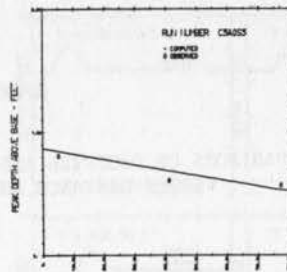
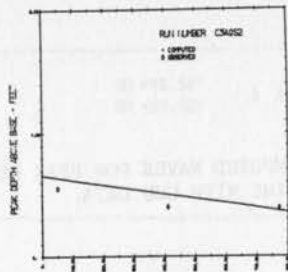
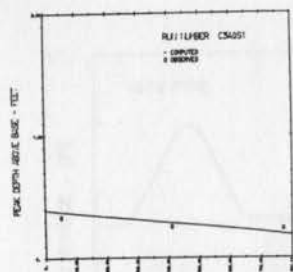
SO = .000990 QB = 15.35 CFS QP = 29.95 CFS

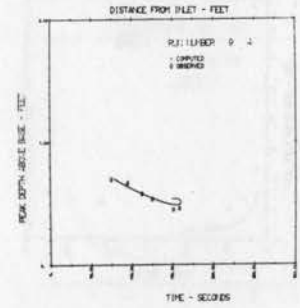
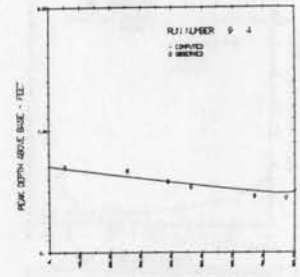
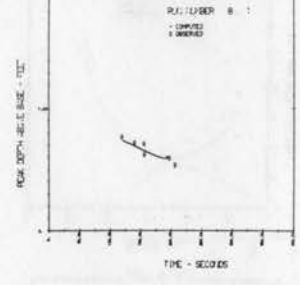
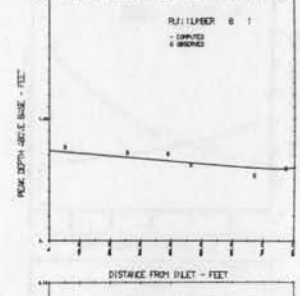
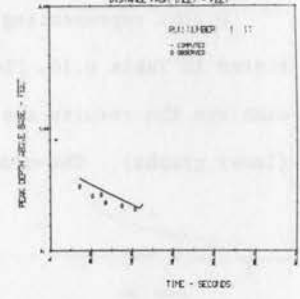
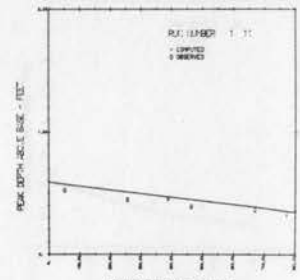
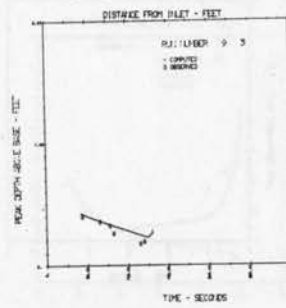
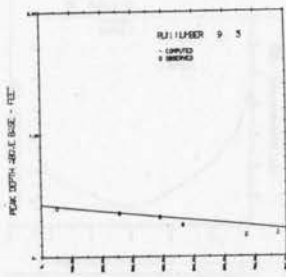
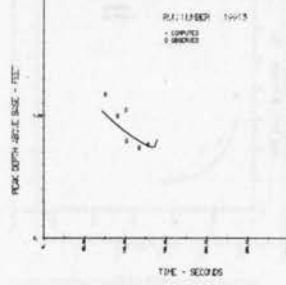
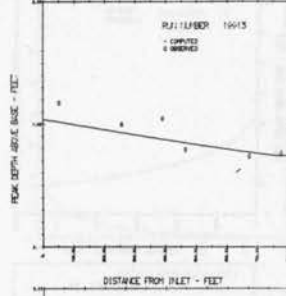
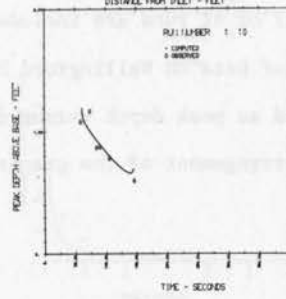
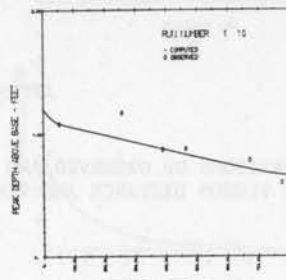
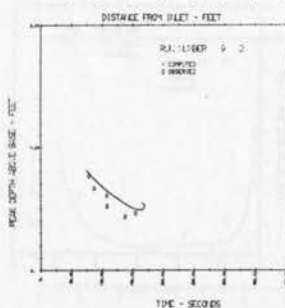
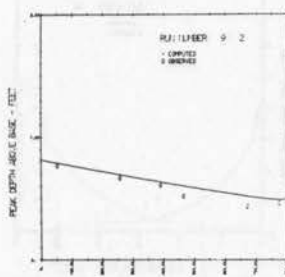
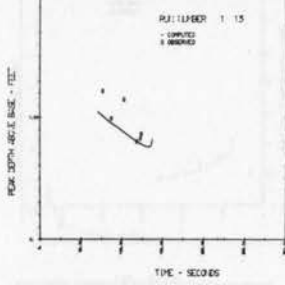
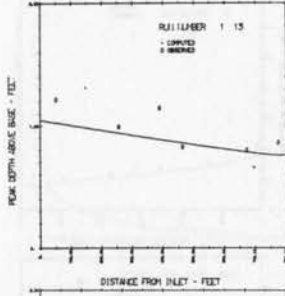
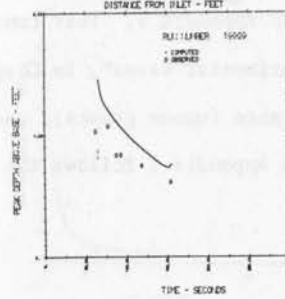
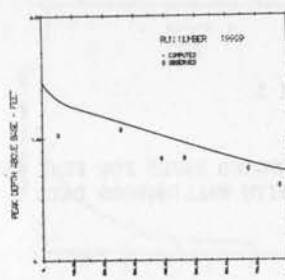
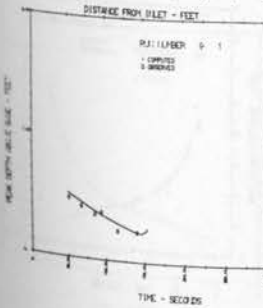
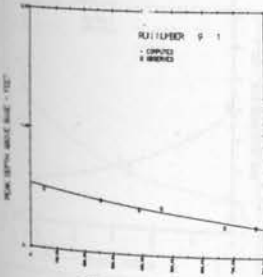
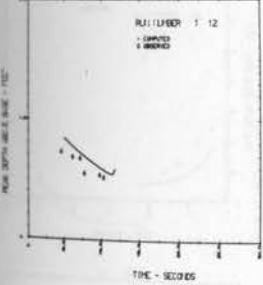
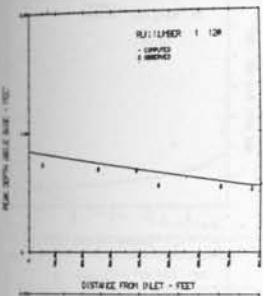
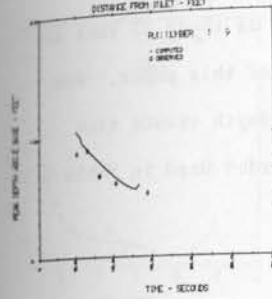
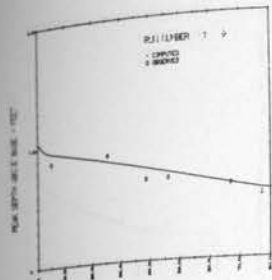
APPENDIX 4

COMPARISONS OF OBSERVED AND COMPUTED WAVES FOR PEAK DEPTH VERSUS DISTANCE AND TIME WITH CSU DATA

Graphs representing a total of 32 runs are included in this appendix. Test conditions of these 32 runs are listed in Table 6.9, entitled "Summary of Data on CSU Experimental Waves", in Chapter 6 of this paper. For each run the results are plotted as peak depth versus distance (upper graphs), and peak depth versus time (lower graphs). The order of arrangement of the graphs in Appendix 4 follows the same order used in Table 6.9.



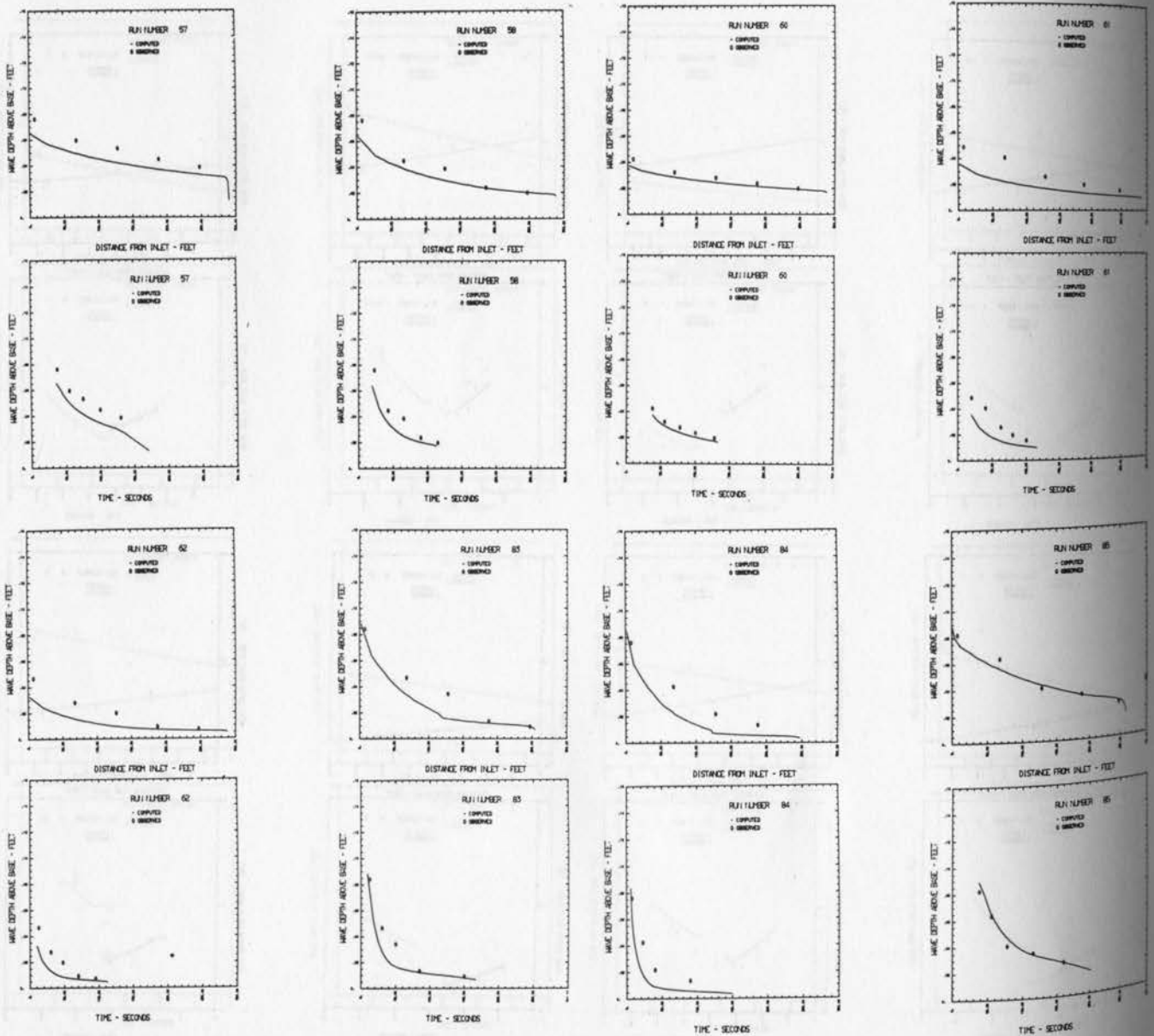


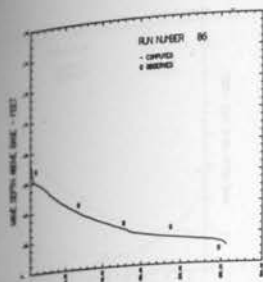


APPENDIX 5

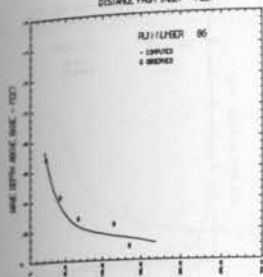
COMPARISONS OF OBSERVED AND COMPUTED WAVES FOR PEAK DEPTH VERSUS DISTANCE AND TIME WITH WALLINGFORD DATA

Graphs representing a total of 41 runs are included in Appendix 5. Test conditions of these 41 runs are listed in Table 6.10, "Summary of Data on Wallingford Experimental Waves", in Chapter 6 of this paper. For each run the results are plotted as peak depth versus distance (upper graphs), and peak depth versus time (lower graphs). The order of arrangement of the graphs in Appendix 5 follows the same order used in Table 6.10.

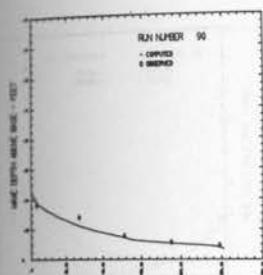




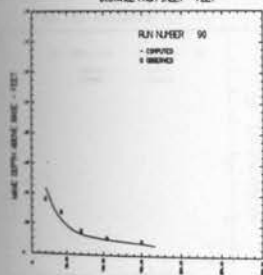
DISTANCE FROM INLET - FEET



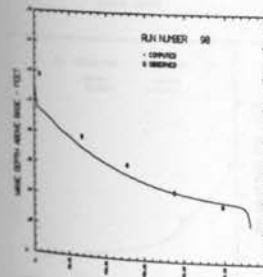
TIME - SECONDS



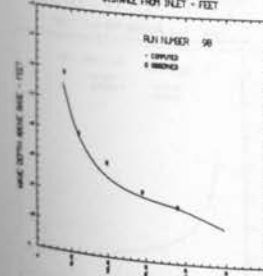
DISTANCE FROM INLET - FEET



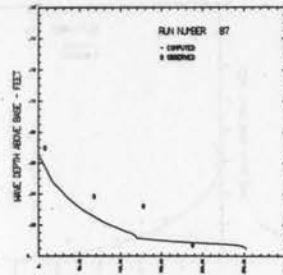
TIME - SECONDS



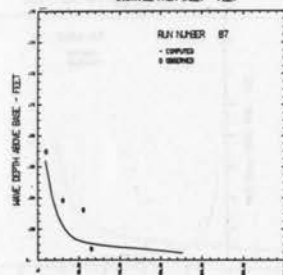
DISTANCE FROM INLET - FEET



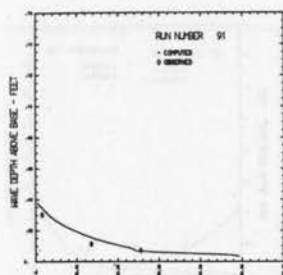
TIME - SECONDS



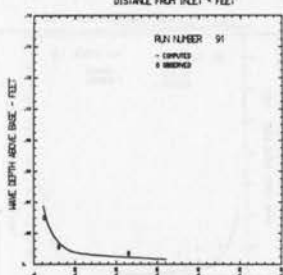
DISTANCE FROM INLET - FEET



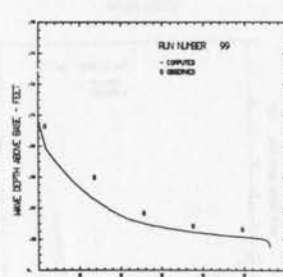
TIME - SECONDS



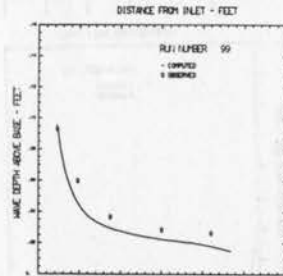
DISTANCE FROM INLET - FEET



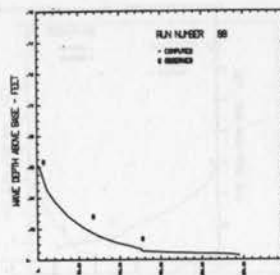
TIME - SECONDS



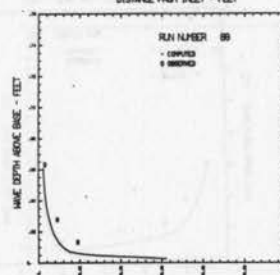
DISTANCE FROM INLET - FEET



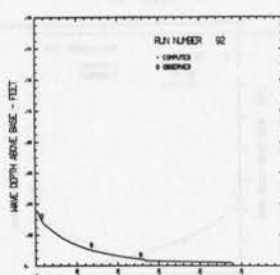
TIME - SECONDS



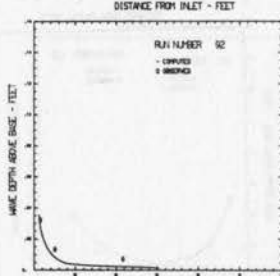
DISTANCE FROM INLET - FEET



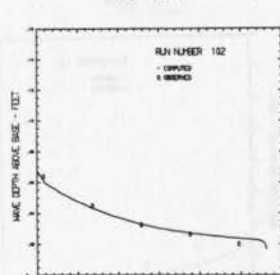
TIME - SECONDS



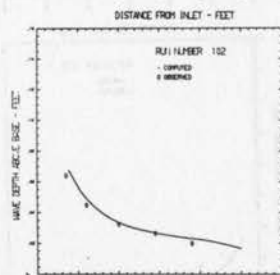
DISTANCE FROM INLET - FEET



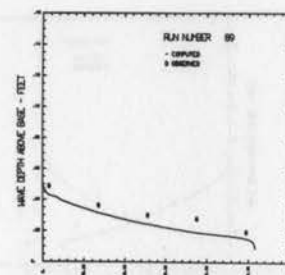
TIME - SECONDS



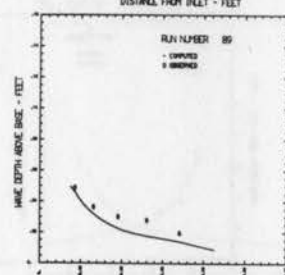
DISTANCE FROM INLET - FEET



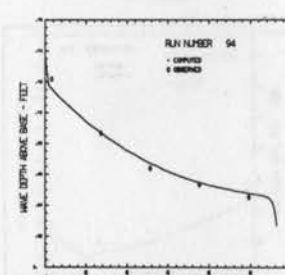
TIME - SECONDS



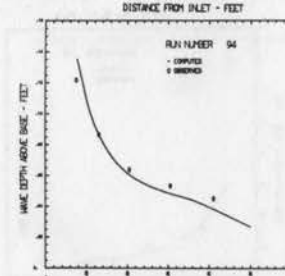
DISTANCE FROM INLET - FEET



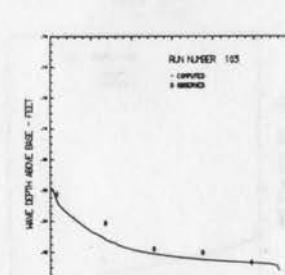
TIME - SECONDS



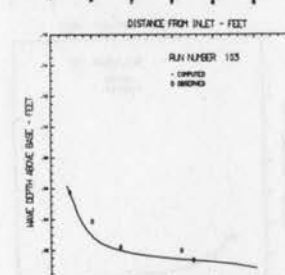
DISTANCE FROM INLET - FEET



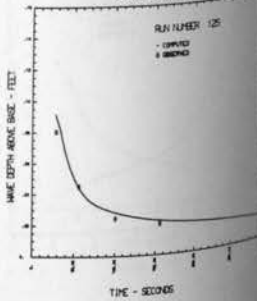
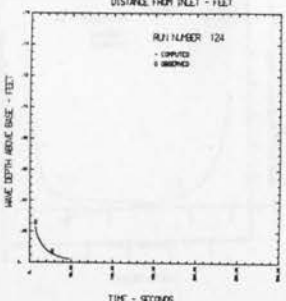
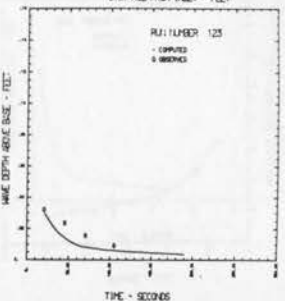
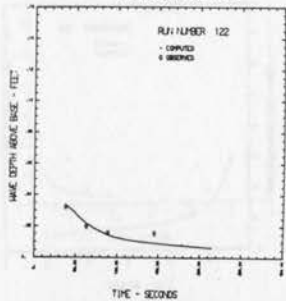
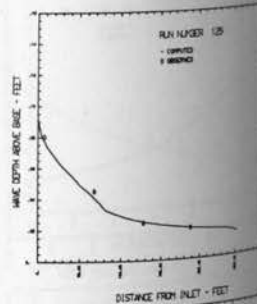
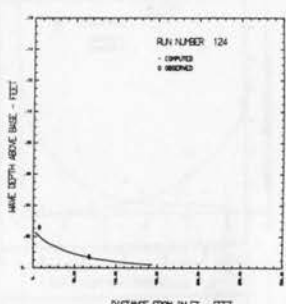
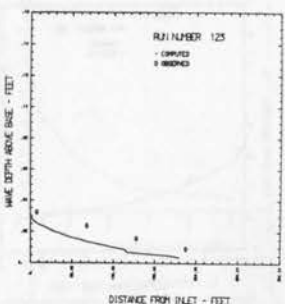
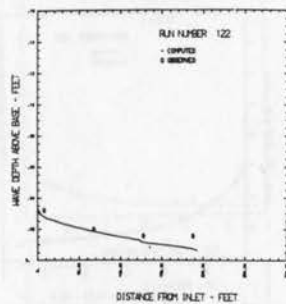
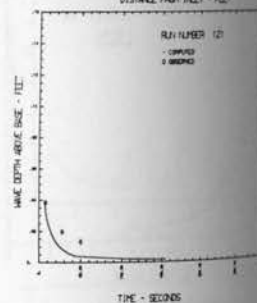
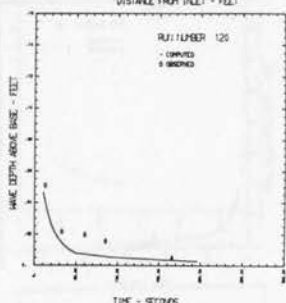
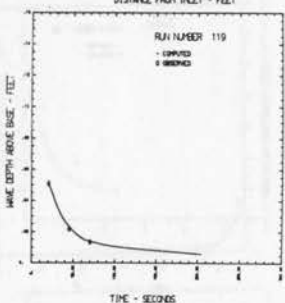
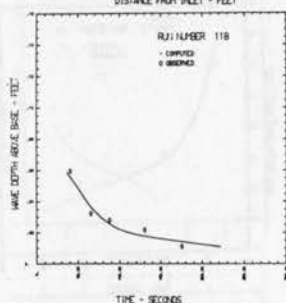
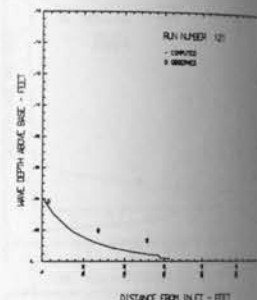
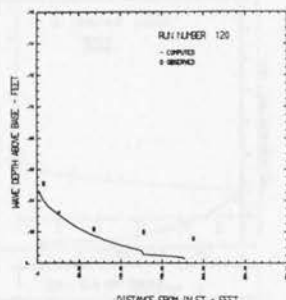
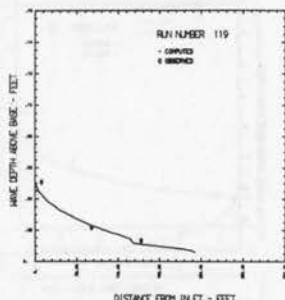
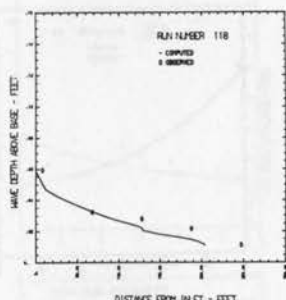
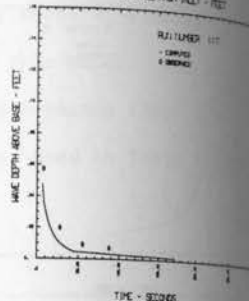
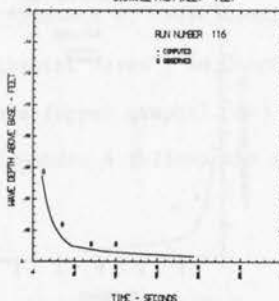
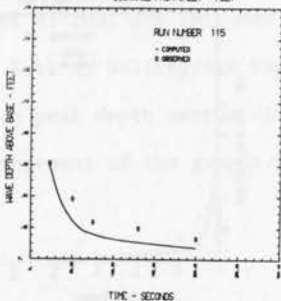
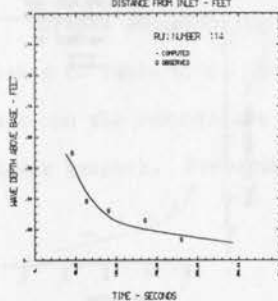
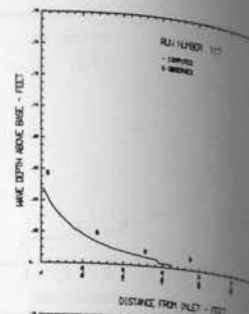
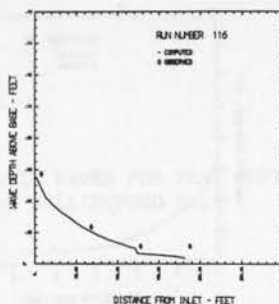
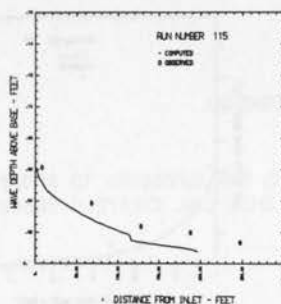
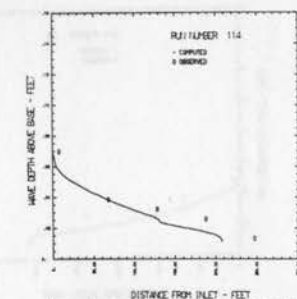
TIME - SECONDS

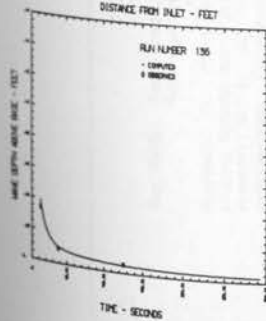
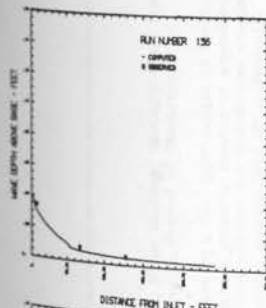
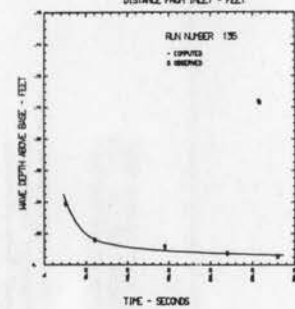
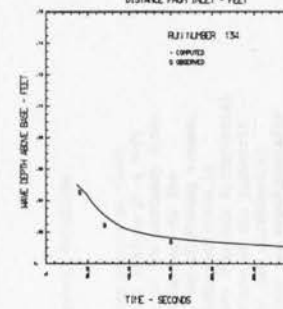
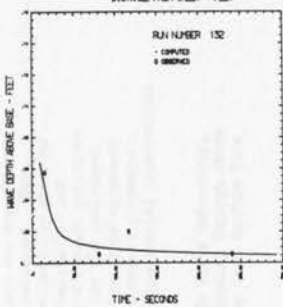
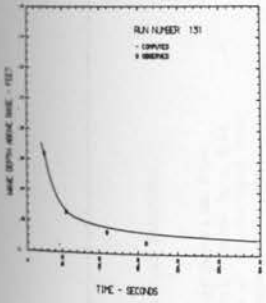
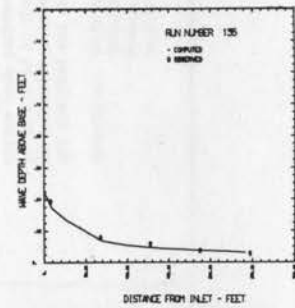
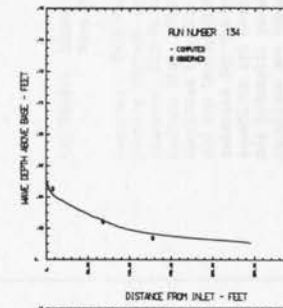
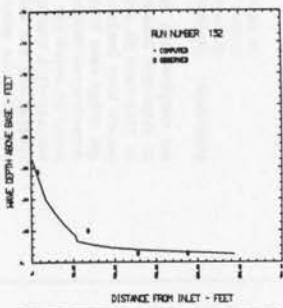
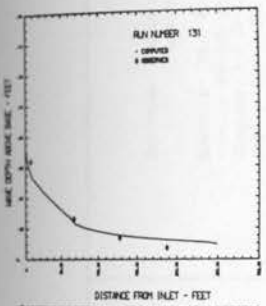
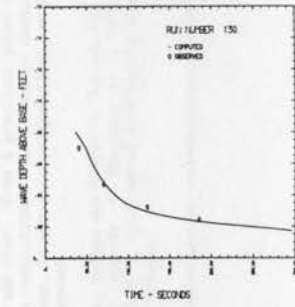
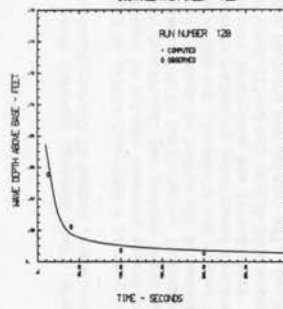
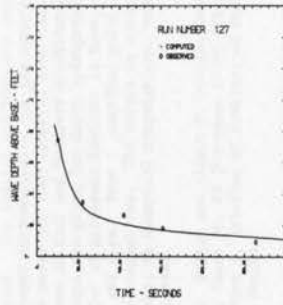
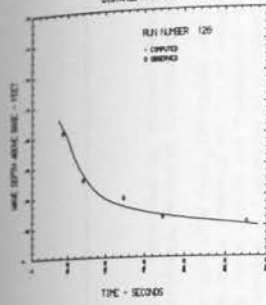
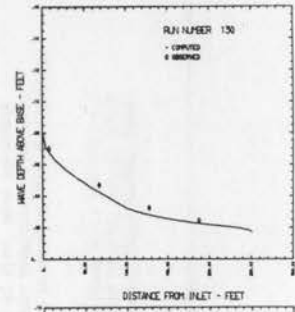
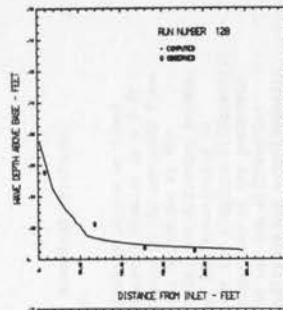
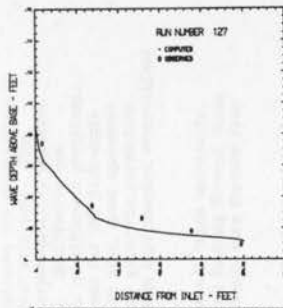
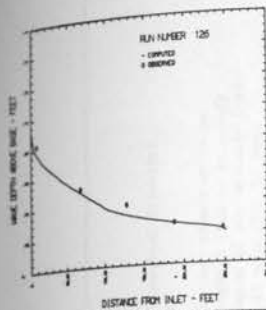


DISTANCE FROM INLET - FEET



TIME - SECONDS





Key Words: Storm Drain, Flood Routing, Unsteady Flow, Numerical Solutions, Wave Experiments, Method of Characteristics.

Abstract: This first part of a four-part series of hydrology papers on flood routing through storm drains presents results of experimental studies in a 3-ft diameter, 822-ft long storm conduit and theoretical studies of the unsteady free-surface flow. The numerical integrations of differential equations by the specified interval scheme of the method of characteristics, the diffusing scheme, and the Lax-Wendroff scheme are discussed; the method of characteristics is selected for the practical integration procedure whenever the complete differential equations are used. Experimental and analytical investigations of the geometric and hydraulic parameters that define the coefficients of the two differential equations are summarized. The initial and boundary conditions are experimentally studied and are expressed mathematically for the numerical solutions. The analytically computed waves are then compared with the experimentally observed waves by using the same initial and boundary conditions. Qualitative and quantitative comparisons are given for depth hydrographs at different positions, for depth wave profiles at different instants in time, and for the peak-depth versus both position and time. From a practical point of view, good agreement is indicated by these comparisons. The errors in conduit geometric parameters, in hydraulic parameters, in numerical computations, and in experimental observations are analyzed and discussed.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 43 (November 1970) "Flood Routing Through Storm Drains, Part I, Solution of Problems of Unsteady Free Surface Flow in Storm Drains."

Key Words: Storm Drain, Flood Routing, Unsteady Flow, Numerical Solutions, Wave Experiments, Method of Characteristics.

Abstract: This first part of a four-part series of hydrology papers on flood routing through storm drains presents results of experimental studies in a 3-ft diameter, 822-ft long storm conduit and theoretical studies of the unsteady free-surface flow. The numerical integrations of differential equations by the specified interval scheme of the method of characteristics, the diffusing scheme, and the Lax-Wendroff scheme are discussed; the method of characteristics is selected for the practical integration procedure whenever the complete differential equations are used. Experimental and analytical investigations of the geometric and hydraulic parameters that define the coefficients of the two differential equations are summarized. The initial and boundary conditions are experimentally studied and are expressed mathematically for the numerical solutions. The analytically computed waves are then compared with the experimentally observed waves by using the same initial and boundary conditions. Qualitative and quantitative comparisons are given for depth hydrographs at different positions, for depth wave profiles at different instants in time, and for the peak-depth versus both position and time. From a practical point of view, good agreement is indicated by these comparisons. The errors in conduit geometric parameters, in hydraulic parameters, in numerical computations, and in experimental observations are analyzed and discussed.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 43 (November 1970) "Flood Routing Through Storm Drains, Part I, Solution of Problems of Unsteady Free Surface Flow in Storm Drains."

Key Words: Storm Drain, Flood Routing, Unsteady Flow, Numerical Solutions, Wave Experiments, Method of Characteristics.

Abstract: This first part of a four-part series of hydrology papers on flood routing through storm drains presents results of experimental studies in a 3-ft diameter, 822-ft long storm conduit and theoretical studies of the unsteady free-surface flow. The numerical integrations of differential equations by the specified interval scheme of the method of characteristics, the diffusing scheme, and the Lax-Wendroff scheme are discussed; the method of characteristics is selected for the practical integration procedure whenever the complete differential equations are used. Experimental and analytical investigations of the geometric and hydraulic parameters that define the coefficients of the two differential equations are summarized. The initial and boundary conditions are experimentally studied and are expressed mathematically for the numerical solutions. The analytically computed waves are then compared with the experimentally observed waves by using the same initial and boundary conditions. Qualitative and quantitative comparisons are given for depth hydrographs at different positions, for depth wave profiles at different instants in time, and for the peak-depth versus both position and time. From a practical point of view, good agreement is indicated by these comparisons. The errors in conduit geometric parameters, in hydraulic parameters, in numerical computations, and in experimental observations are analyzed and discussed.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 43 (November 1970) "Flood Routing Through Storm Drains, Part I, Solution of Problems of Unsteady Free Surface Flow in Storm Drains."

Key Words: Storm Drain, Flood Routing, Unsteady Flow, Numerical Solutions, Wave Experiments, Method of Characteristics.

Abstract: This first part of a four-part series of hydrology papers on flood routing through storm drains presents results of experimental studies in a 3-ft diameter, 822-ft long storm conduit and theoretical studies of the unsteady free-surface flow. The numerical integrations of differential equations by the specified interval scheme of the method of characteristics, the diffusing scheme, and the Lax-Wendroff scheme are discussed; the method of characteristics is selected for the practical integration procedure whenever the complete differential equations are used. Experimental and analytical investigations of the geometric and hydraulic parameters that define the coefficients of the two differential equations are summarized. The initial and boundary conditions are experimentally studied and are expressed mathematically for the numerical solutions. The analytically computed waves are then compared with the experimentally observed waves by using the same initial and boundary conditions. Qualitative and quantitative comparisons are given for depth hydrographs at different positions, for depth wave profiles at different instants in time, and for the peak-depth versus both position and time. From a practical point of view, good agreement is indicated by these comparisons. The errors in conduit geometric parameters, in hydraulic parameters, in numerical computations, and in experimental observations are analyzed and discussed.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 43 (November 1970) "Flood Routing Through Storm Drains, Part I, Solution of Problems of Unsteady Free Surface Flow in Storm Drains."

PREVIOUSLY PUBLISHED PAPERS OF VOLUME 2

Colorado State University Hydrology Papers

- No. 26 "The Investigation of Relationship Between Hydrologic Time Series and Sun Spot Numbers," by Ignacio Rodriguez-Iturbe and Vujica Yevjevich, April 1968.
- No. 27 "Diffusion of Entrapped Gas From Porous Media," by Kenneth M. Adam and Arthur T. Corey, April 1968.
- No. 28 "Sampling Bacteria in a Mountain Stream," by Samuel H. Kunkle and James R. Meimann, March 1968.
- No. 29 "Estimating Design Floods from Extreme Rainfall," by Frederick C. Bell, July 1968.
- No. 30 "Conservation of Ground Water by Gravel Mulches," by A.T. Corey and W.D. Kemper, May 1968.
- No. 31 "Effects of Truncation on Dependence in Hydrologic Time Series," by Rezaul Karim Bhuiya and Vujica Yevjevich, November 1968.
- No. 32 "Properties of Non-Homogeneous Hydrologic Series," by V. Yevjevich and R.I. Jeng, April 1969.
- No. 33 "Runs of Precipitation Series," by Jose Llamas and M.M. Siddiqui, May 1969.
- No. 34 "Statistical Discrimination of Change in Daily Runoff," by Andre J. Duma and Hubert J. Morel-Seytoux, August 1969.
- No. 35 "Stochastic Process of Precipitation," by P. Todorovic and V. Yevjevich, September 1969.
- No. 36 "Suitability of the Upper Colorado River Basin for Precipitation Management", by Hiroshi Nakamichi and Hubert J. Morel-Seytoux, October 1969.
- No. 37 "Regional Discrimination of Change in Runoff," by Viboon Nimmannit and Hubert J. Morel-Seytoux, November 1969.
- No. 38 "Evaluation of the Effect of Impoundment on Water Quality in Cheney Reservoir," by J.C. Ward and S. Karaki, March 1970.
- No. 39 "The Kinematic Cascade as a Hydrologic Model," by David F. Kibler and David A. Woolhiser, February 1970.
- No. 40 "Application of Run-Lengths to Hydrologic Series," by Jaime Saldarriaga and Vujica Yevjevich, April 1970.
- No. 41 "Numerical Simulation of Dispersion in Groundwater Aquifers," by Donald Lee Reddell and Daniel K. Sunada.
- No. 42 "Theoretical Probability Distributions for Flood Peaks," by Emir Zelenhasic, November 1970.

FLOOD ROUTING THROUGH STORM DRAINS

Part II

PHYSICAL FACILITIES AND EXPERIMENTS

by

V. YEVJEVICH and A. H. BARNES

November 1970



HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
Fort Collins, Colorado

Several departments at Colorado State University have substantial research and graduate programs oriented to hydrology. These Hydrology Papers are intended to communicate in a fast way the current results of this research to the specialists interested in these activities. The papers will supply most of the background research data and results. Shorter versions will usually be published in the appropriate scientific and professional journals, or presented at national or international scientific and professional meetings and published in the proceedings of these meetings.

The investigations leading to this paper on unsteady free-surface flow in a long storm drain were supported by the U.S. Bureau of Public Roads, Federal Highway Administration during 1960-1970 and by the Public Health Service during 1962-1964. The research was conducted in the Hydrology and Water Resources Program of Civil Engineering Department at Colorado State University, Fort Collins, Colorado, U.S.A.

The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Federal Highway Administration or the Public Health Service.

Dr. Arthur T. Corey, Professor, Agricultural Engineering Department

Dr. Robert E. Dils, Professor, College of Forestry and Natural Resources

Dr. Vujica Yevjevich, Professor, Civil Engineering Department

FLOOD ROUTING THROUGH STORM DRAINS

Part II

PHYSICAL FACILITIES AND EXPERIMENTS

by

V. Yevjevich and A. H. Barnes

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO 80521

November 1970

No. 44

ACKNOWLEDGMENTS

The writers of this paper gratefully acknowledge the support and cooperation of the U. S. Bureau of Public Roads, Federal Highway Administration, in the research on flood movements through long storm drains conducted from 1960 to 1970. The writers also acknowledge the U. S. Public Health Service, National Institute of Health, for their additional support during 1962-1964.

The initiative, cooperation and support given by Mr. Carl F. Izzard to this project on flood movement through storm drains is particularly acknowledged. Mr. Izzard, presently Director, Office of Development, Federal Highway Administration, U. S. Department of Transportation, and Chief, Hydraulic Research Division, U. S. Bureau of Public Roads at the start of the project. Further acknowledgment is extended to Mr. Charles F. Scheffey, Director, Office of Research, Federal Highway Administration, for his cooperation and encouragement. Dr. Dah-Cheng Woo, Senior Hydraulic Engineer, Federal Highway Administration, has cooperated extensively with this project. His reviews and suggestions pertaining to all reports, theses and other documents produced on the project have been particularly helpful.

Mr. George Smith, Associate Professor of Civil Engineering at Colorado State University, has worked closely with the writers in the design, construction, testing, and operation of the physical research facilities during most of this project. His contribution, particularly for the material included in this paper, is acknowledged. Several graduate students, either through their thesis research or through direct work, contributed to the project. The theses and reports are listed in the bibliography of this paper under "Internal References". Acknowledgment is also given to Mr. Lawrence Wetter, Project Engineer, Soil Conservation Service, Winfield, Kansas, and Mr. Don Signor, Hydrologist, Geological Survey, Lubbock, Texas, for their contributions in the design and testing of the orifice meters, junction boxes, and the model study of inlet and outlet conditions during their M.S. graduate studies at Colorado State University. Particular acknowledgment is extended to the mechanics and other shop specialists of the Engineering Research Center, who have made it possible to construct, maintain and operate these unique facilities and the instrumentation described in this paper. Dr. Shih-Tun Su, Post Doctoral Fellow, Civil Engineering Department, Colorado State University, using existing data, assisted the writers in finishing this paper.

TABLE OF CONTENTS

Chapter		Page
	Acknowledgments	iii
	Abstract.	viii
1	INTRODUCTION	1
	1.1. Objectives in the Use of Physical Research Facilities	1
	1.2. General Conceptions for the Physical Research Facilities.	1
	1.3. Organization of Material Included in This Paper	2
2	EXPERIMENTAL STORM DRAIN SYSTEM.	3
	2.1. Description of Circular Conduit	3
	2.2. Inlet Structure	3
	2.3. Laterals and Junction Boxes	6
	2.4. Outlet Restriction Gate	12
	2.5. Water Supply and Removal.	12
3	INSTRUMENTATION AND ITS CALIBRATION.	19
	3.1. Orifice Meters.	19
	3.2. Current Meters.	20
	3.3. Pressure Transducers.	26
	3.4. Pitot Tubes	26
4	DATA RECORDING SYSTEM.	28
	4.1. Description of the System	28
	4.2. Analog-to-Digital Converter	28
	4.3. Operations and Controls	29
5	EXPERIMENTAL TEST CONDITIONS AND TYPICAL RESULTS	30
	5.1. Steady Flow Conditions.	30
	5.2. Unsteady Flow Conditions.	32
6	EXPERIMENTAL ERRORS.	39
	6.1. Errors in Geometric Variables	39
	6.2. Time-Difference Errors.	40
	6.3. Instrumentation Errors.	41
	6.4. Reproducibility Errors.	41
7	EVALUATION OF EXPERIMENTAL FACILITIES.	42
	7.1. Summary of Characteristics of Experimental Facilities	42
	7.2. Potential of Facilities for Further Experimental Investigations	42
	REFERENCES	43

LIST OF FIGURES AND TABLES

Figure		Page
2.1	The circular conduit on the hillside of the Outdoor Laboratory at Colorado State University Engineering Research Center.	3
2.2	View from the outlet of circular conduit and the inclined rails	3
2.3	General plan of storm conduit experimental facilities	4
2.4	Substructure for storm conduit experimental facilities.	5
2.5	Inlet structure, scheme no. 1	7
2.6	Inlet structure, scheme no. 2	8
2.7	Schematic diagram showing experimental set-up for model study of storm drain -- intake structure, main pipe line and outlet.	9
2.8	Junction box at 90° with the upper inlet.	10
2.9	Details of the junction box	11
2.10	Details of the junction box used in model study	13
2.11	Top view of the junction box used in the model study.	13
2.12	The side view of the laterals and the junction box used in the model study.	13
2.13	Detailed dimensions of the outlet restriction gate.	14
2.14	The view of the removed outlet restriction gate for the conditions of a free outfall.	14
2.15	General plan of the supply of water and of the storm conduit experimental facility location.	15
2.16	General scheme for water supply and removal for storm conduit experimental facilities	16
2.17	Water level in Horsetooth Reservoir for period May 1951 - September 1962.	16
2.18	Rating curve for discharge at the inlet structure of storm conduit.	16
2.19	Plan and elevation of supply line for the computation of head losses (numbers refer to pipe fittings of Table 2.2	17
3.1	Discharge coefficient C_d as function of the Reynolds Number, Re , of 36-inch O.D. pipeline for three orifice plates: (1) small (2) medium, and (3) large	19
3.2	Relation of the measured to the computed volumes for the unsteady flow conditions through the orifice meters.	20
3.3	Electrically propelled tow car.	22
3.4	Current meter tow tank. Dimensions of the tank are: 200 ft; width - 5 ft; depth - 5 ft.	22
3.5	Recording system for calibration of current meters, and tow car control switch. Strip charts records speed of tow car and revolutions per second of current meters	22
3.6	Ott meters in calibration position on tow car	22
3.7	Relay circuit chassis, electric clock, and current meter counters on tow car for simultaneous calibration of five Ott-current meters	23
3.8	Arrangement of micro-switch switches and cam plate which operate relay circuit. "Off cam" is illustrated.	23

LIST OF FIGURES AND TABLES - (Continued)

Figure		Page
3.9	Behavior of two neighboring current meters with propellers of different pitch (after Benini)	25
3.10	Behavior of two neighboring current meters with propellers of the same pitch (after Benini)	25
3.11	The calibration line for the pressure transducer No. 12180, ± 1 psi. Tested 7-23-66, Using data recording system, CD-25 No. 14490	26
3.12	The calibration line for the pressure transducer No. 14400, ± 5 psi, Tested 7-23-66, Using data recording system, CD-25 No. 17083	26
3.13	Pitot tube rake used for point velocity measurements at large discharges of supercritical flow	27
4.1	Data recording system.	28
4.2	Schematic representation of time series sampled by data recording system	28
4.3	The data format as the output from Analog-to-Digital converter	29
4.4	Schematic representation of facilities and data recording system	29
5.1	Isovels for partially full pipe flow	31
5.2	Variation of Darcy-Weisbach friction factor f with the Reynolds number	31
5.3	Location of critical depth at the free outfall of a circular cross-section	32
5.4	Estimate of discharge versus slope and depth of flow	33
5.5	An example of the discharge hydrograph at the upstream inlet of storm conduit.	33
5.6	An example of the lateral discharge hydrograph at the junction box being 410.7 feet from the inlet structure to the storm conduit	33
5.7	Observed wave forms given as depth versus time, computer plotted, run no. 090004, $S_o = 0.00048$	37
5.8	Observed wave forms given as depth versus distance, computer plotted, run no. 090004, $S_o = 0.00048$	38
5.9	Observed hydrograph peak depths versus the distance (left graph) or versus the time (right graph), computer plotted, run no. 090004, $S_o = 0.00048$	38
6.1	Definition sketch for the relation of the circular and the elliptical cross-sections.	39
6.2	Percent error in area of the storm conduit, (at the depth ratio of 0.2).	40
6.3	Invert profiles for Colorado State University experimental storm conduit (1965).	40
6.4	Invert profiles for Colorado State University experimental storm conduit (1966).	40
<u>Tables</u>		
2.1	Design details, cost, advantages, and disadvantages of the two schemes of inlet structure	6
2.2	Head losses in the supply line for storm conduit experimental facilities	18
3.1	Errors in C_d of Fig. 3.1	19
3.2	Results of unsteady flow investigation of orifice meter calibration for different area ratios.	21
3.3	Selection of propellers	21

LIST OF FIGURES AND TABLES - (Continued)

<u>Tables</u>		<u>Page</u>
3.4	Comparison of Ott-Meter calibration rating curves, $V = a + b n$, by the least square estimate of Ott and CSU calibration data for meter velocity > 1.0 fps	24
5.1	Piezometer locations.	30
5.2	Free outfall data	32
5.3(a)	Operating conditions for June 3-9, 1964	34
(b)	Operating conditions for June 18-23, 1964	34
(c)	Operating conditions for July 13-17, 1964	35
(d)	Operating conditions for August 1-8, 1964	36
5.4	Summary of data on experimental waves at CSU.	36
6.1	Conduit geometry.	39
6.2	Data for Colorado State University experimental conduit invert slopes	40
6.3	Estimate of instrumentation errors.	41

ABSTRACT

This second part of a four-part series of hydrology papers on flood routing through storm drains relates exclusively to experimental research facilities and experiments. The following subjects are presented: (a) design and construction of the experimental storm drain system as a special conduit research outdoor facility; (b) instrumentation and its calibration; (c) description of the data recording system; (d) various experimental test conditions and their typical results, and (e) discussion of experimental errors.

A large conduit, 3 feet in diameter and 822 feet long, was selected, designed, and constructed in the Outdoor laboratory at Colorado State University, Engineering Research Center, to accurately measure geometric and hydraulic characteristics, as well as the propagation of flood hydrographs. Instrumentation was selected to suit the field conditions. The calibration of the instruments was carried out to the point where there were relatively small errors. The data recording system was designed and constructed so that the output could be put either on cards or paper tapes and provide a direct input for computations on a digital computer. Only typical results of experiments carried out are described in this paper; experimental errors are reviewed in a summarized form.

FLOOD ROUTING THROUGH STORM DRAINS

Part II

PHYSICAL FACILITIES AND EXPERIMENTS

by

V. Yevjevich* and A. H. Barnes**

Chapter 1

INTRODUCTION

1.1 Objectives in the Use of Physical Research Facilities

One of the main objectives of investigating flood routing through storm drains was to compare analytical and physical waves, or to compare waves computed by integrating the two quasi-linear hyperbolic partial differential equations of gradually varied free surface unsteady flow and the corresponding free-surface waves measured in a conduit. Therefore, constructing appropriate facilities to simulate and measure the physical waves became necessary. These new facilities have the objectives:

(1) To physically simulate and measure free-surface waves at various points along the research conduit for the comparisons discussed.

(2) To measure various geometric and hydraulic characteristics of the conduit and the waves in order to analyze these characteristics, so that the analytical waves-as the inflow flood hydrographs routed along the conduit by the integration of the two De Saint-Venant partial differential equations-may come as close to the corresponding physical waves as practically feasible.

(3) To physically simulate a storm drain with lateral inflows joining the main conduit at junction boxes.

(4) To study boundary and initial conditions in the experimental conduit, so that their uses in the integration of differential equations represent the real conditions of physical waves as much as practically feasible.

1.2 General Conceptions for the Physical Research Facilities

Most hydraulic research and the development of hydraulic formulas in the past have been done on relatively small laboratory experimental facilities. Economy was the primary reason research facilities were relatively smaller than the prototypes they represented. Most hydraulic formulas and related computational methods have been developed by using the computational devices available. For economy of computations in the pre-computer age required simplified formulas and tractable computational methods to treat the often complex hydraulic problems of nature.

The advent of fast digital, analog, and hybrid computers, however, has reduced substantially the unit cost of numerical solutions of complex problems,

and methods. The availability of electronic computers has made many previous hydraulic mathematical models and corresponding computational methods obsolete. With time, these models and formulas are likely to be completely revised and upgraded. Versatile improvements of many hydraulic computational methods are progressing and are expected to come about in the near future.

One of the expected contributions of this research project on flood routing through storm drains is an improved base for replacing less reliable steady-state conditions by more reliable unsteady free-surface flow conditions in the design of storm drains. Therefore, the advanced methods, simulating natural conditions as closely as feasible, may be introduced in the form of unsteady-flow approach to the design of storm drains. Improving computational flood routing methods through storm drains is meaningful only if the accuracy of input flood hydrographs and the accuracy of geometric and hydraulic parameters in differential equations becomes much greater than their accuracy in the steady-flow peak-discharge approach for design of storm drains.

The two partial differential equations of gradually varied free-surface unsteady flow are developed from several basic assumptions. Even with these assumptions there will always be a difference between the analytically computed wave at a given position of a free-surface flowing storm drain for a given inflow flood hydrograph, and the physical wave observed at the same position and for the same inflow hydrograph. To study the geometric and hydraulic characteristics of conduits and waves in order to integrate flood waves moving through storm drains, and to compare the analytical and the physical waves, small scale facilities did not appear feasible. Relative errors that appear when various hydraulic and geometric parameters are measured in small scale facilities are usually large. To decrease these errors, and to better study various aspects of flood movement through storm drains, larger scale physical research facilities were conceived, designed, and built.

Because of the potential of various numerical computations being reliably performed by digital, analog, and hybrid computers, the science of hydraulic is now in the position of re-evaluating previously developed formulas and computational methods. When the objective becomes an assessment of the accuracy of presently available formulas and computational methods in hydraulics, not only or predominantly the development of the new hydraulic regularities and laws, the most feasible way of checking the existing mathematical hydraulic models and the computational methods is

* Professor of Civil Engineering and Professor-in-Charge of Hydrology and Water Resources Program, Civil Engineering Department, Colorado State University, Fort Collins, Colorado.

**Associate Professor of Civil Engineering, Civil Engineering Department, Colorado State University.

through facilities of sufficiently large scale. For mainly that purpose, an outdoor laboratory was built at the Engineering Research Center of Colorado State University. Several large scale facilities already have been built in this outdoor laboratory. One of them is the conduit research facility having a pipe 822 feet long with an inside nominal diameter of 3 feet. This conduit can be moved on inclined rails to obtain slopes from 0 to 4 percent. This facility, with auxiliary apparatus, represents the conduit research center and will be used primarily for the study of the free-surface water flow through storm drains, and also for future objectives. A significant study on the diffusion of tracers in a full flowing pipe has also been performed in these facilities.

It is expected that the conduit research facilities will be used for other problems, e.g. water transport of materials in pipes, pre-treatment of sewage water in drainage systems, problems in separating sewage water from storm-drain water of the same drainage system, studies of pipe roughness, studies of various diffusion problems, investigations of losses at hydraulic singularities along conduits, etc. These facilities are available to both researchers in the United States and to guests from foreign universities. These facilities are described in Chapter 2.

A detailed description of facilities, instruments, the calibration of instruments, methods, and typical experimental results was considered necessary in a special hydrology paper for two reasons.

(1) That an assessment of the value of results and conclusions in this investigation of flood routing

through storm drains can be made by those who read and study the four parts of this research project, published as the Colorado State University Hydrology Papers, Nos. 43-46.

(2) That the scientists who would like to perform various research projects at the CSU conduit research center may assess advantages, potential, and limitations in using these facilities for particular research problems.

1.3 Organization of Material Included in this Paper

The general description of various alternatives in planning the research facilities, and the details of the constructed storm drain system are described in Chapter 2. The instruments used in the experiments are described in Chapter 3, with a major emphasis on calibration of individual devices. Chapter 4 describes the data recording system which allows data taken in the field to be transmitted to an indoor analog-to-digital converter located approximately 1/4 mile from the pipeline. Chapter 4 describes how the output from the analog-to-digital converter, in the form of punched cards or tapes, may be fed directly into the computer. Various experimental test conditions and their typical results are described in Chapter 5 such as flow resistance, cross-sectional velocity distribution, box losses, initial and boundary conditions, and observations of propagated flood waves. Experimental errors caused by different sources are discussed in Chapter 6. Chapter 7 presents the general evaluation of experimental facilities; it is followed by the references (external and internal).

EXPERIMENTAL STORM DRAIN SYSTEM

2.1 Description of Circular Conduit

A pipeline 822 feet long with a 3-foot diameter was used as the experimental conduit in this study as shown in Figs. 2.1 and 2.2.



Fig. 2.1. The circular conduit on the hillside of the Outdoor Laboratory at Colorado State University Engineering Research Center.

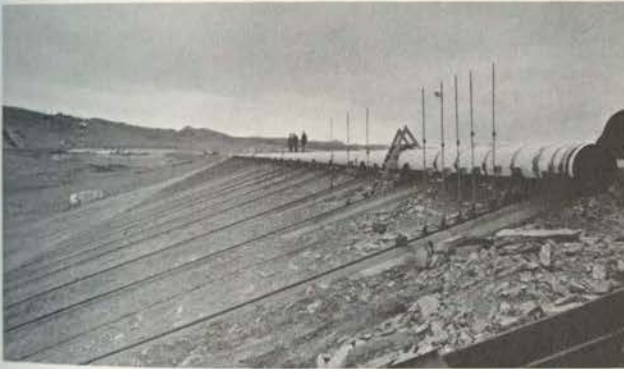


Fig. 2.2. View from the outlet of circular conduit and the inclined rails.

The entire 822 feet of pipe, which consisted of 20-foot sections, was supported on inclined rails which permitted the pipe to be moved along the inclined plane to any slope between 0 and approximately 4 percent. Figures 2.3 and 2.4 display general layout and detailed designs of the circular storm conduit.

The pipe material is 1/2 inch thick rolled steel plate with a longitudinal weld located at the crown. The approximate 20-foot lengths of the pipe were welded except at three positions where the connections were bolted. Extreme care was taken to insure that all inside welds and joints were ground smooth and that the depressions were filled with a plastic material to insure a uniformly smooth surface. The inside surface was sand blasted and painted with two coats of a rust preventative paint.

2.2 Inlet Structure

Selection of inlet structure. Flow was introduced into the circular storm conduit by means of an inlet structure. Two inlet structures were initially designed, as shown by Figs. 2.5 and 2.6. The design details of these two schemes, their cost estimates, advantages, and disadvantages are presented in Table 2.1. The advantage of lower cost and added convenience in operation led to the selection of the second scheme as the inlet structure to be constructed.

Model study of selected inlet structure. A model study was made of the selected inlet structure (scheme No. 2). The length ratio of model to prototype was 1 to 6. The objectives of this study were:

1. To investigate the problems of controlling the water profile in the main pipe line for both subcritical and supercritical flow by one or more of the following methods:
 - a. Varying the grid size, number, and position of baffles in the inlet section of the intake structure.
 - b. Varying the slope of the intake structure only.
 - c. Use of a tailgate at the point of discharge from the main pipe for subcritical flow.

2. To investigate the hydraulic performance of the inlet box for subcritical and supercritical flows.

A schematic diagram of the model study with design details is shown in Fig. 2.7. The types of baffles tested are also shown in the figure.

Conclusions of the model tests are as follows:

1. For subcritical flow the use of baffles and a tailgate is an effective method for developing uniform flow in the main conduit. It was necessary, however, to double the length of the intake structure between the flexible pipe and the main pipe line. This permitted full development of a uniform flow before entry into the main pipe.
2. By keeping the invert of the intake structure above the crown of the supply pipe, full pipe flow is always maintained at the orifice meter.
3. Varying only the slope of the intake structure is as effective as baffles in developing uniform flow in the main pipe for subcritical flow.

Description of inlet structure as constructed.

On the basis of the experimental results of the model study, the inlet structure was constructed following the design of scheme No. 2. For subcritical flow conditions it was necessary that the desired flow characteristics be developed upstream of the entrance to the storm conduit. This required that the 22-foot approach section have freedom of movement both vertically and horizontally so that it could be oriented in any given plane with the storm conduit. The flexible section of pipe was built to allow the desired freedom of orientation of the 22-foot approach section to be a straight continuation of the storm conduit in any experiments.

The movement of the storm conduit relative to the fixed portion of the intake structure created undesirable secondary currents in the approach section to the storm conduit. To re-align these transverse

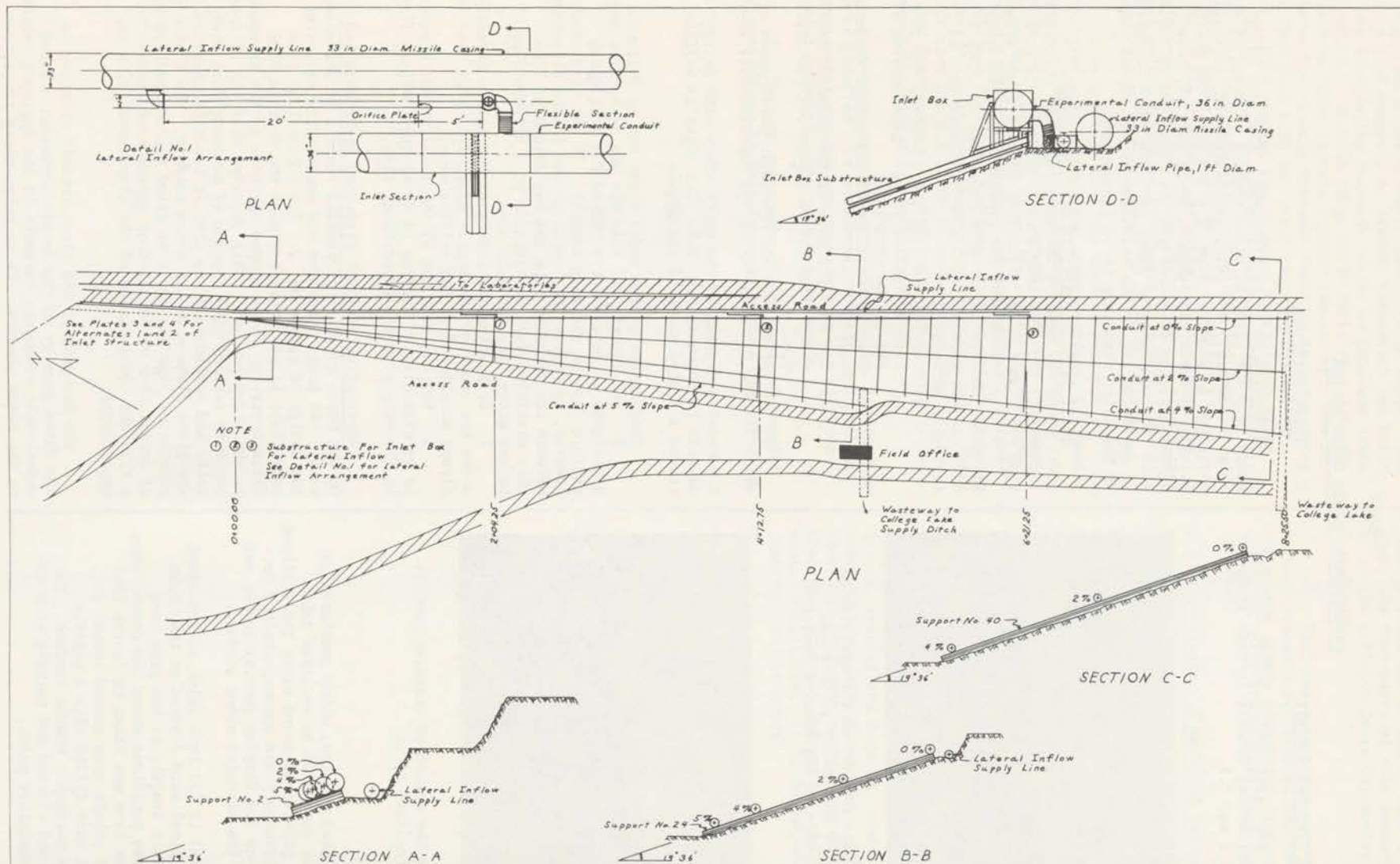


Fig. 2.3. General plan of storm conduit experimental facilities.

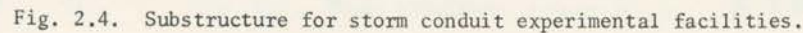


TABLE 2.1. Design details, cost, advantages, and disadvantages of the two schemes of inlet structure.

	SCHEME NO. 1	SCHEME NO. 2
Design details	(1) A 30-foot section of 26-inch diameter pipe. (2) A 5-foot transition section with guide vanes. (3) An 8-foot by 10-foot by 58-foot stilling basin with baffles at the entrance and a triangular weir at the exit. (4) A 10-foot by 10-foot catchment box. (See Fig. 2.5)	(1) An 80-foot section of 26-inch diameter pipe with orifice meter. (2) An 12.25-foot of steel pipe between the orifice meter and a length of flexible pipe. (3) A 10-foot of 26-inch diameter flexible pipe. (4) A 3-foot transition section between the flexible pipe and, (5) A 22-foot missile case with baffles. (See Fig. 2.6)
Estimate of cost	\$3850.00	\$2300.00
Advantages	(1) It provides an accurate measurement of discharge from very low to maximum discharge. (2) Flood waves - inflow hydrographs - can be easily generated in the stilling basin with the addition of a relatively expensive wave generator.	(1) It is relatively inexpensive. (2) It provides positive control of the flow phenomena without interference, that is, the water surface is not affected by the wind. (3) Because it is a single pipe system a valve can be easily installed in the system at any time if it be found desirable, which is a desirable feature for a limited water supply. (4) It permits the use of several sizes of orifice meters for measuring a wide range of flow discharges.
Disadvantages	(1) Wind will cause surface waves, which will have an adverse effect on the discharge measurements, particularly at low discharge. (2) Its cost is high. (3) Calibration of the weir may require double construction of the stilling basin. (4) It is cumbersome to operate.	(1) For supercritical flows it might not be easy for some discharges to develop desired uniform flow. (2) There is a danger of the flexible pipe collapsing during movement of the pipe system.

velocity components into the desired uniform flow it is necessary to place baffles in the approach section. The location and number of these baffles is determined by observing the flow developed for a given discharge.

To develop the desired supercritical flow conditions at the entrance to the storm conduit, it was necessary that the slope of the movable approach pipe be different than the slope of the storm conduit. To achieve this slope differential, a flexible coupling was installed at the junction of the two pipes, i.e., at the entrance to the storm conduit.

2.3 Laterals and Junction Boxes

Selection of junction box. The design of laterals for the lateral inflows into the storm conduit, and the junction boxes basically involves three factors: (1) The angle of intersection; (2) The vertical position of the lateral inlet in the cross-section

of the main conduit, and (3) The shape of junction box. If the crown of the inlet pipe is at the same elevation as the crown of the main conduit, it is referred to as the "upper inlet". If the invert of the inlet pipe is at the same elevation as the invert of the main conduit, it is referred to as the "lower inlet".

Based on recommendations from the U. S. Bureau of Public Roads, a square box with a changeable upper and lower inlet, and an angle of intersection equal to 90 degrees was selected as the junction box. Design details are shown in Figs. 2.8 and 2.9.

A model study of the selected junction box with its two positions, i.e., the upper and lower inlets, is described in the next section.

Model study of selected junction box. A hydraulic model was made to determine the relation of the power loss at the junction to other hydraulic properties.

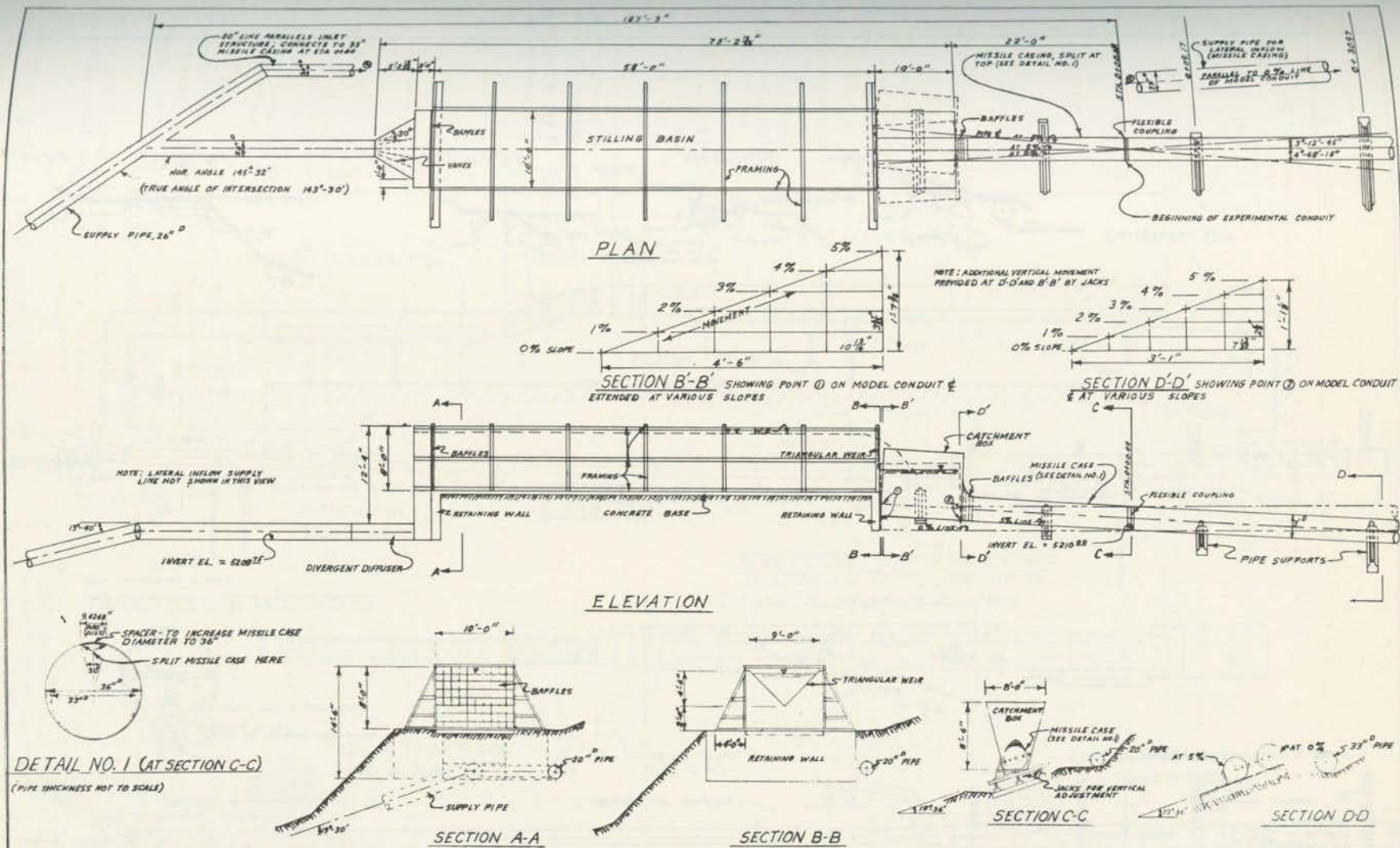
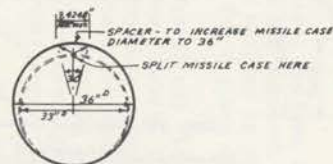
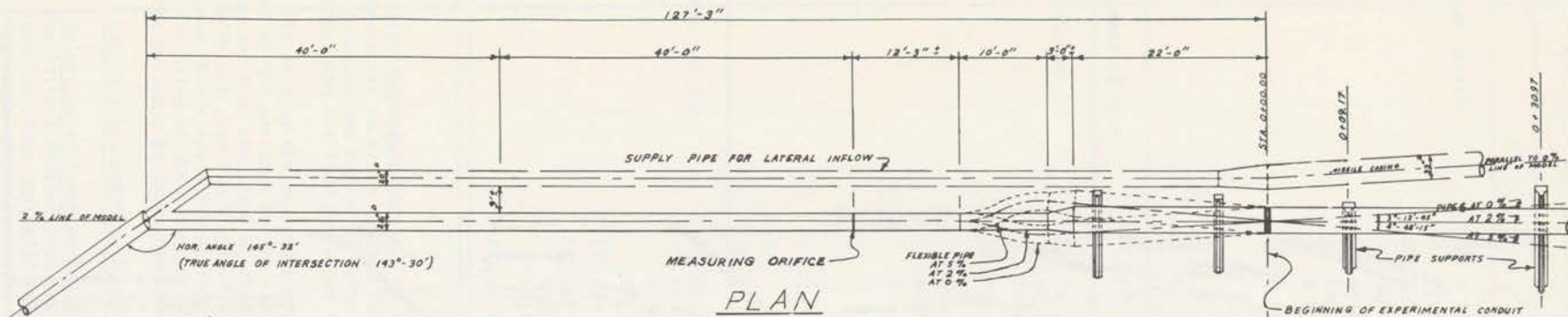


Fig. 2.5. Inlet structure, scheme No. 1.



DETAIL NO. 1 (AT SECTION D-D)

(PIPE THICKNESS NOT TO SCALE)

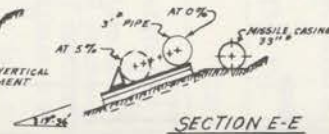
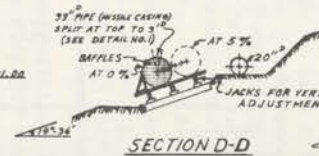
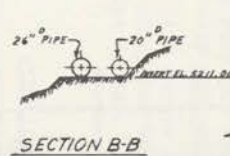
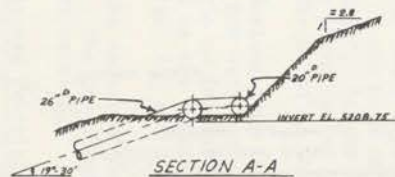
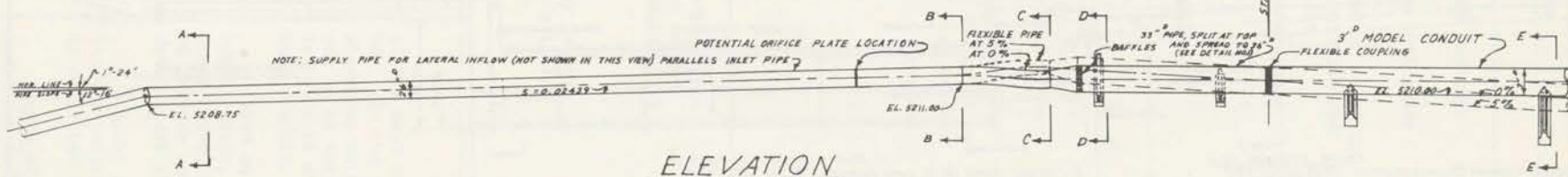
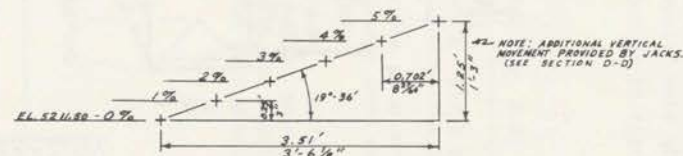
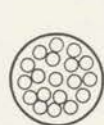
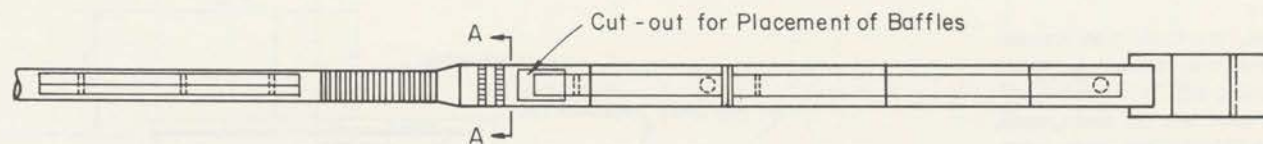
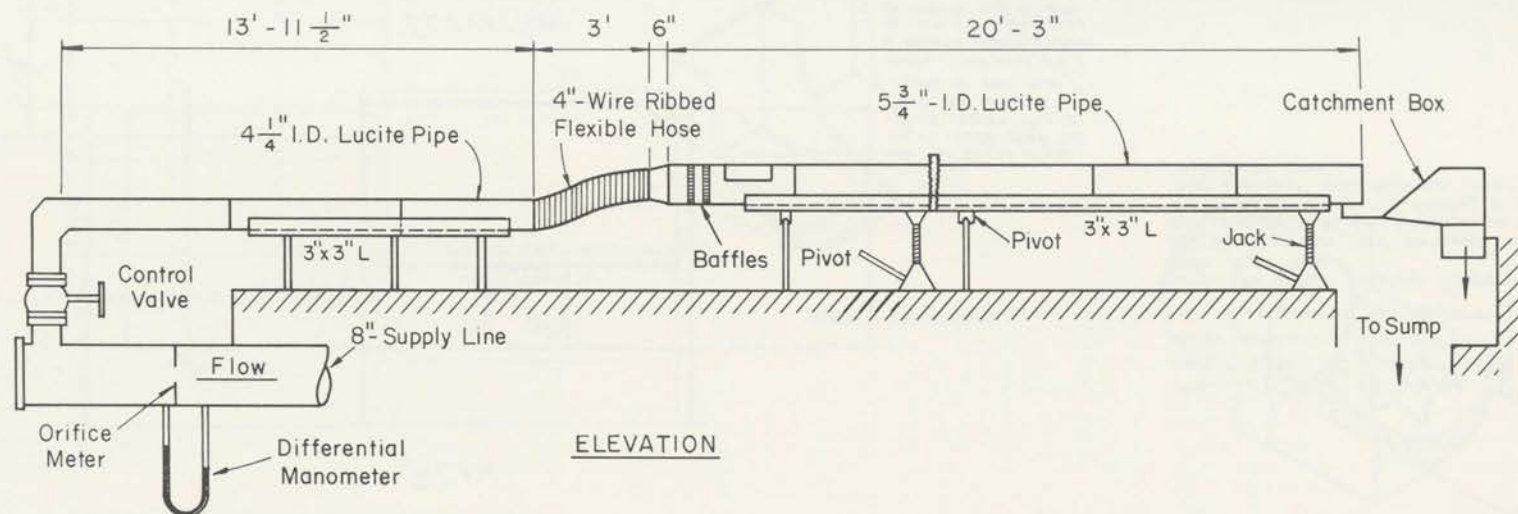
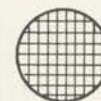


Fig. 2.6. Inlet structure, scheme No. 2.



Typical Circular Grid
Used in Baffles

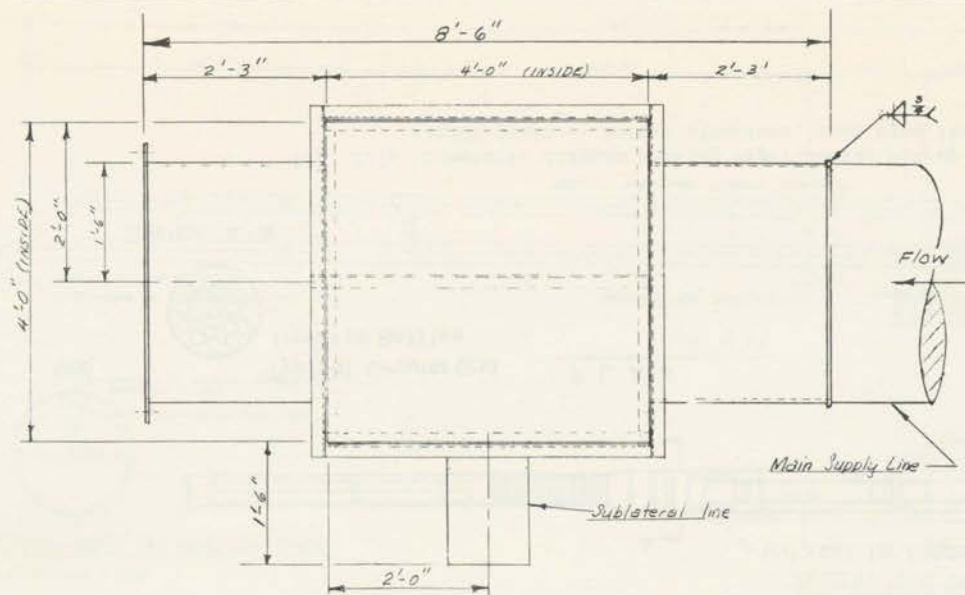
Section A-A



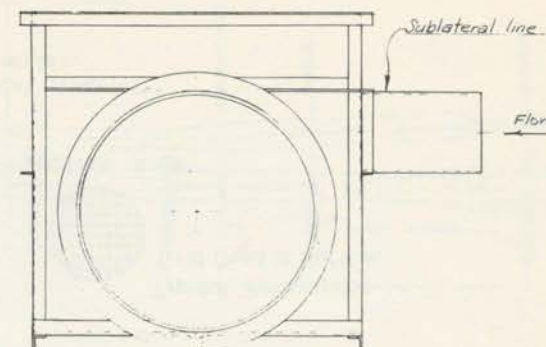
Typical Rectangular
Grid Used in Baffles

Section A-A

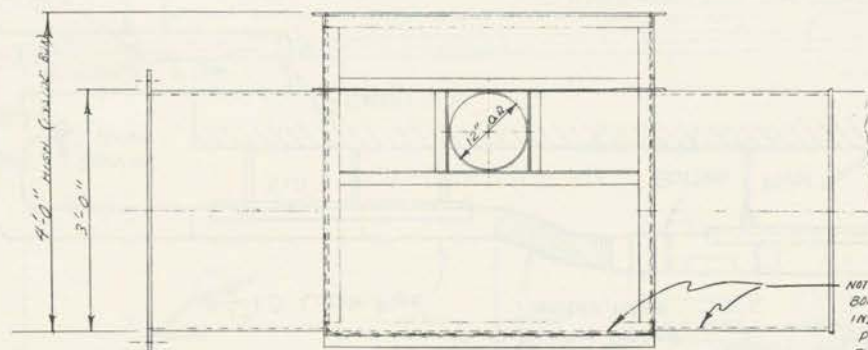
Fig. 2.7. Schematic diagram showing experimental set-up for model study of storm drain -- intake structure, main pipe line and outlet.



PLAN



END



ELEVATION

NOTE: THE BOTTOM OF THE BOX IS LOWER THAN THE INSIDE INVERT OF THE PIPE BY AN AMOUNT EQUAL TO THE PIPE WALL THICKNESS, ($\approx \frac{1}{8}$ ").

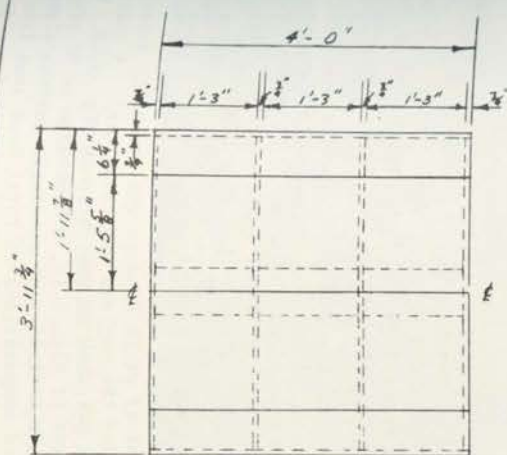
$\frac{1}{8}$ " PLYWOOD SHEET IS NEEDED TO SHIM UP FLOOR WHEN NO INSERTS ARE IN BOX.

FRAMING: 2" X 2" X $\frac{1}{4}$ " ANGLE
 SIDES & FLOOR: $\frac{3}{8}$ " PLATE
 INSIDE DIMENSION IS 4' X 4' X 4'
 PIPE 3' O.D. (SAME AS TEST CONDUIT)

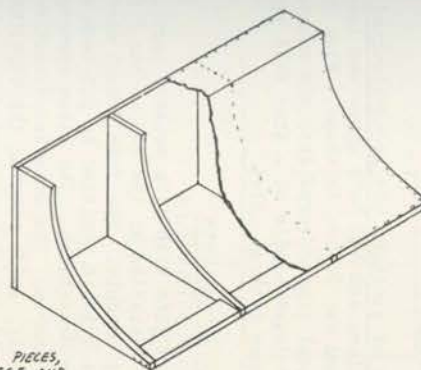
Flange: To match existing flange on main pipe line.

NOTE: Upstream joint between main line and manhole to be field welded with $\frac{3}{4}$ -in fillet weld. On Plates 8 and 9 the flanged and welded joints are noted.

Fig. 2.8. Junction box at 90° with the upper inlet.

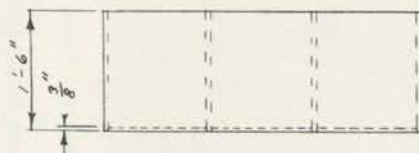


PLAN

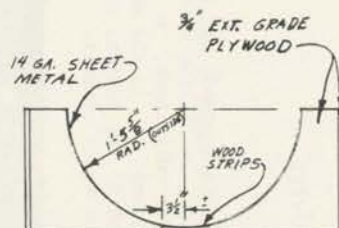


HALF - ISOMETRIC

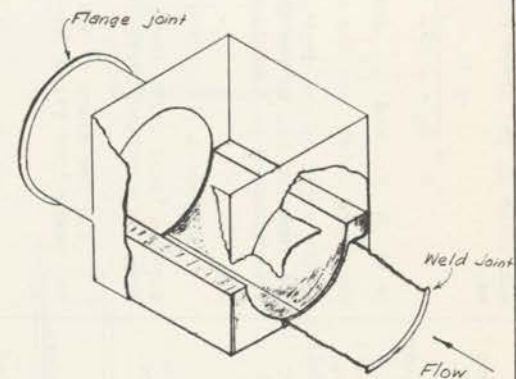
MAKE TWO IDENTICAL PIECES,
OR MAKE AS ONE PIECE AND
SPLIT ALONG $\frac{1}{2}$ ".



ELEVATION



END



BROKEN-OUT SECTION SHOWING
INSERT INSTALLED IN MANHOLE
The invert of the insert is in the
same plane as the invert of the
main pipe line. (NOTE: Insert as shown
is similar to insert in round manhole
of Plate 10.)

Fig. 2.9. Details of the junction box.

Froude modeling criteria were used with a model to prototype linear ratio of 1 to 5.618 (selected because of the available plastic pipe size).

The junction box used in the model study was square above the half-full level of the storm conduit. The two positions of the inlet were tested, the upper inlet and the lower inlet. The crown of the upper inlet was at the same elevation as the crown of the storm conduit at its point of entry into the junction box. The invert of the lower inlet was made coincident with the invert of the storm conduit at their junction point, which was the centerline of the junction box. The lower inlet had a horizontal slope of 0.05. This slope was a result of necessary structural details in the prototype. Details of the model junction box are shown in Figs. 2.10, 2.11 and 2.12.

When the upper inlet was used, the lower inlet was plugged. Depths of water were measured at 22 points along the model conduit. The rate of main flow and the rate of lateral flow were read from two calibrated manometers. The measured depths were used to calculate the cross-sectional areas. The mean velocities were then calculated by the continuity equation. Then the velocity heads were calculated. Knowing the velocity heads, depth of flow, and the elevation of the invert, energies could be calculated for each station. The power at a section in the conduit is found by multiplying the energy at the section by the weight rate of flow past the section. In other words, this power is the energy for all of the water transmitted across a section per unit time. The power equation can be written as:

$$P = Q \gamma (z + y + V^2/2g) \quad (2.1)$$

or

$$P = Q \gamma E, \quad (2.2)$$

in which P is the power in foot-pounds per second, Q is the rate of flow in cfs, γ is the specific weight of water in pounds per cubic foot, E is the energy in foot-pounds per pound, V is the mean velocity, z is the elevation of the bottom, and y is the depth of water in the model conduit.

Conclusions drawn from this model study for subcritical flow are:

1. For a given ratio of lateral to main flow there is always more power lost when the upper inlet is utilized than when the lower inlet is used.

2. The ratio of power loss to the incoming power for the upper inlet is dependent only on the ratio of lateral to main flow.

The power loss relation for the upper lateral inlet (the inlet that had its crown at the same elevation as the crown of the storm conduit) was found to be:

$$P_R = \frac{-0.482}{Q_R + 0.55} + 0.77, \quad (2.3)$$

in which P_R is the ratio of power loss in the junction to the power entering the junction, and Q_R is the ratio of lateral inflow to storm conduit inflow. This equation is valid for Q_R greater than 0.10.

For Q_R less than 0.10 the P_R -value of zero can be assumed with a maximum expected error of 3 percent in P_R .

3. The ratio of power loss to the power entering the lower inlet depends on the ratio of lateral to main discharge and the depth ratio of the junction.

The power loss relation for the lower lateral inlet (the inlet that had its invert at the same

elevation as the storm conduit's invert) was found to be:

$$P_R = \frac{-2.78 + 1.71 D_R}{Q_R + 3.122 - 0.167 D_R} + 0.77, \quad (2.4)$$

in which P_R and Q_R are the same magnitudes as for the upper inlet, and the new parameter, D_R , is equal to the ratio of the depth immediately upstream from the junction box to the diameter of the storm conduit. This equation is limited to D_R -values greater than 0.5.

4. For a given flow ratio, Q_R , the power loss for the lower inlet increases as the depth upstream from the junction increases.

5. The slope of the main conduit does not affect the power loss ratio for the subcritical flow conditions.

6. The effective friction factor downstream from the junction has a larger value than the friction factor upstream.

Description of junction box as constructed. The constructed junction box had both upper and lower lateral inlets and was square in shape. Flow from the 12 inch lateral pipe entered into the square junction box at a 90-degree angle as a free fall jet of water for the upper lateral inlet.

2.4. Outlet Restriction Gate

A restriction gate with five movable vertical wooden slats was installed at the end of the 3 ft diameter, 822 ft long conduit. The five 7-inch vertical wooden slats were held in position by 2.5-inch wide vertical aluminum H-sections. The clear opening was 5 inches between supports. The detailed dimensions of the gate is shown in Fig. 2.13. During the experiments, the flow discharge could thus be controlled by varying the vertical position or by removing one or more of the slats to give the backwater (M1) profile. For the condition of a free outfall, the gate could be completely removed as shown in Fig. 2.14. A discussion of the gate condition is given in Chapter 5, p. 30.

2.5. Water Supply and Removal

Water supply to the 3-foot diameter storm conduit was by gravity from the nearby Horsetooth Reservoir (Colorado Big Thompson Project) through a 26-inch underground pipeline as shown in Figs. 2.15 and 2.16. The conduit flow discharged into College Lake. This system permits wide variation in the discharge demand on the water supply.

The monthly maximum, average, and minimum water surface elevations of Horsetooth Reservoir from May 1951 to September 1962 were obtained from records kept at the Northern Colorado Water Conservancy District, Loveland, Colorado. These values were plotted on the graph presented in Fig. 2.17. The mean 11-year reservoir surface elevation for this period is 5382 feet above mean sea level. The differential elevation between the inlet of storm conduit and reservoir water surface provides the maximum available head. A rating curve was then prepared as indicated in Fig. 2.18. From the rating curve of Fig. 2.18, a discharge of 90 cfs was obtained for the mean 11-year reservoir surface elevation. Detailed computations of total head loss from the reservoir to the inlet of the storm conduit for a discharge of 90 cfs are presented in Fig. 2.19 and Table 2.2.

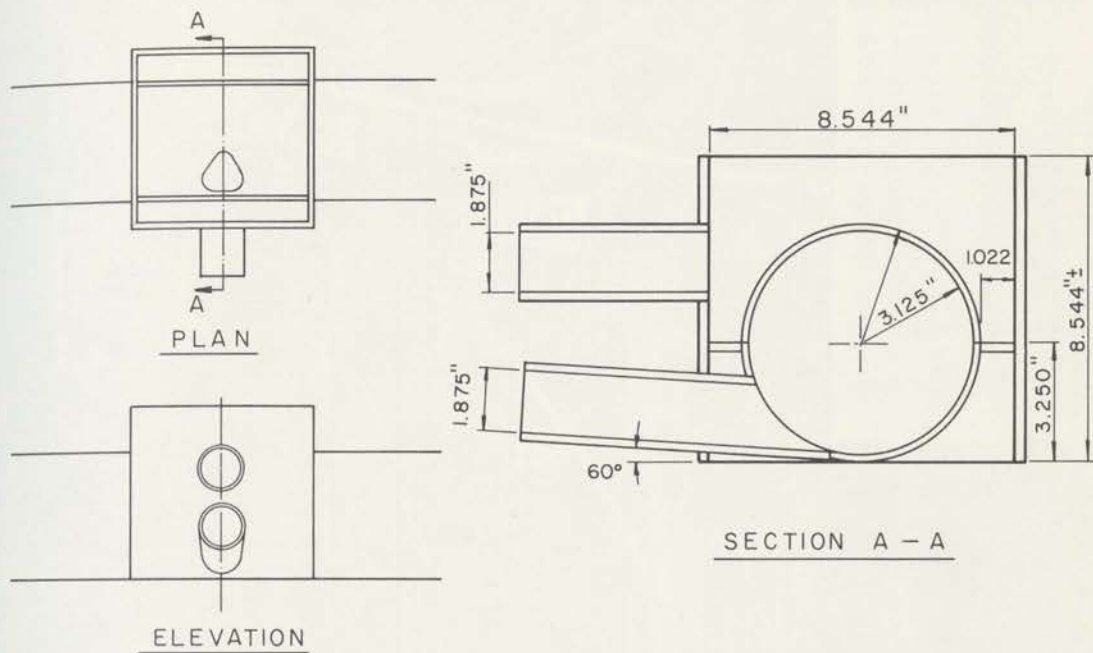


Fig. 2.10. Details of the junction box used in model study.

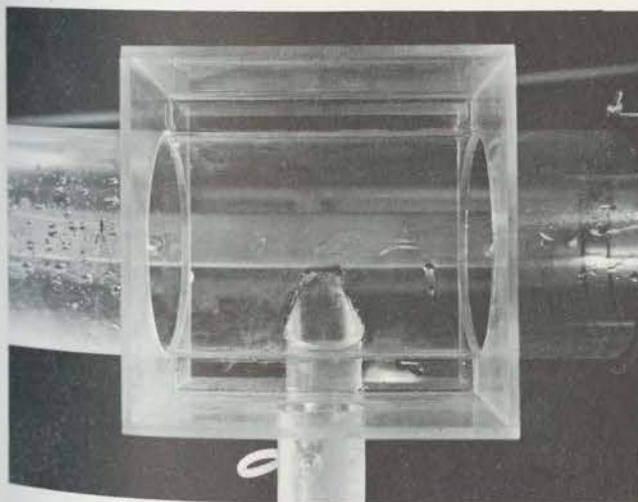


Fig. 2.11. Top view of the junction box used in the model study.

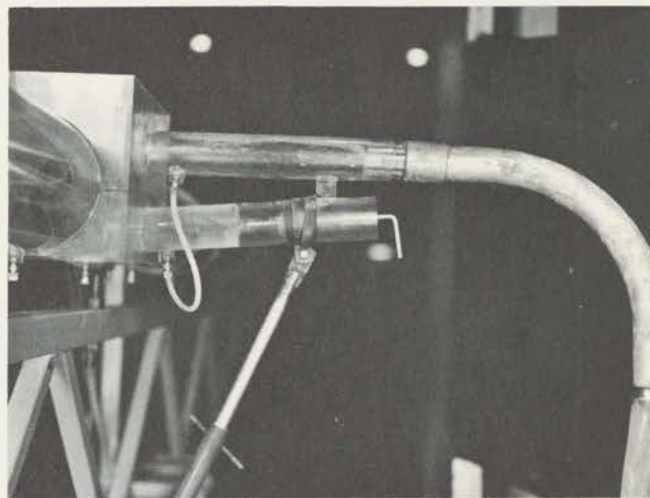


Fig. 2.12. The side view of the laterals and the junction box used in the model study.

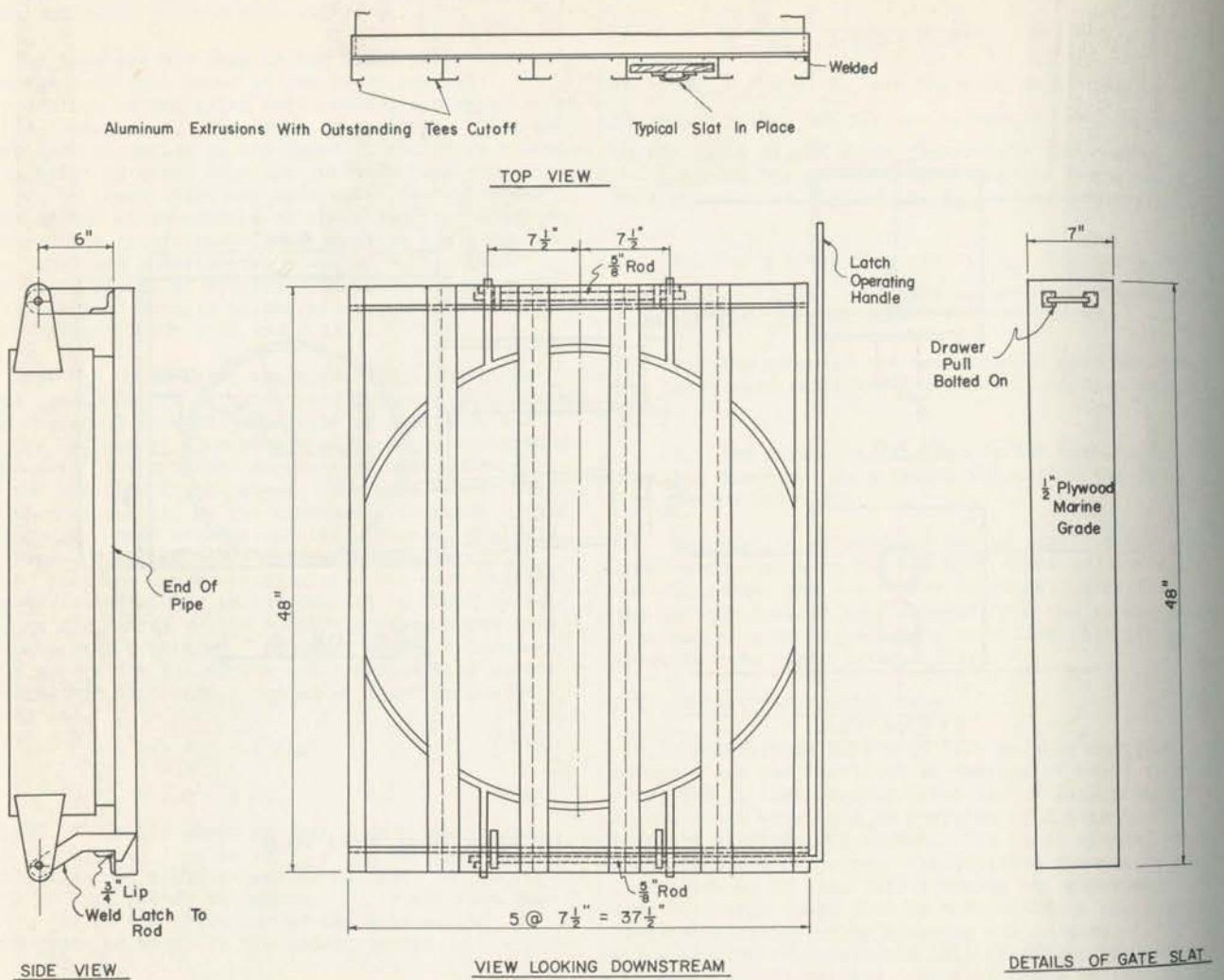


Fig. 2.13. Detailed dimensions of the outlet restriction gate.

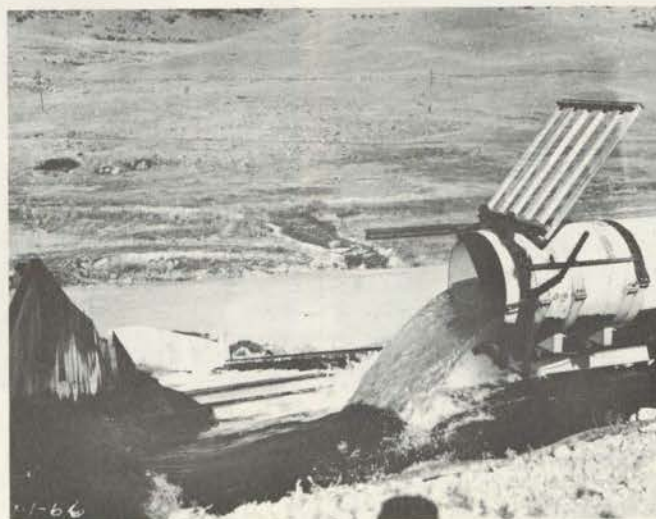


Fig. 2.14. The view of the removed outlet restriction gate for the conditions of a free outfall.

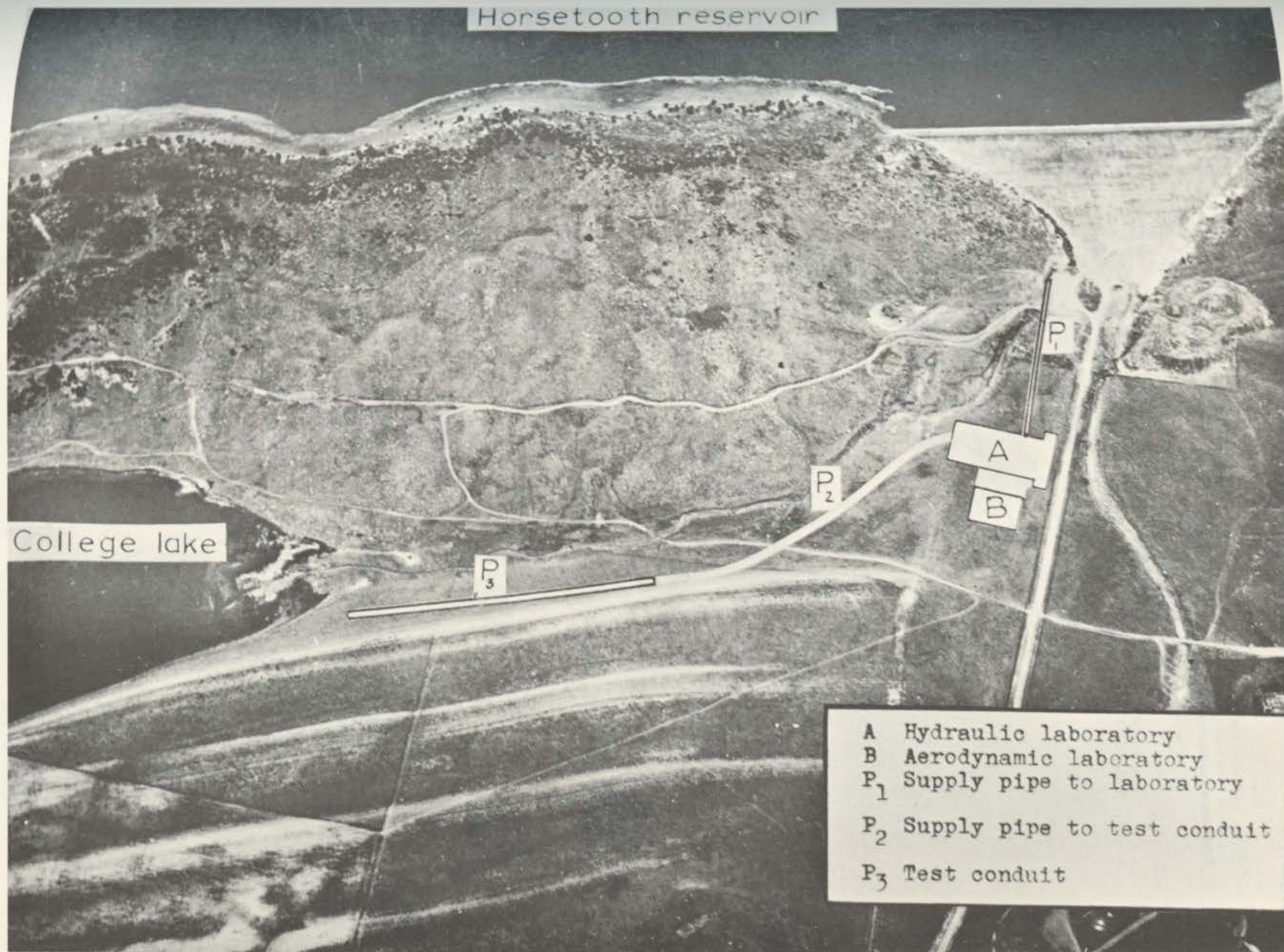


Fig. 2.15. General plan of the supply of water and of the storm conduit experimental facility location.

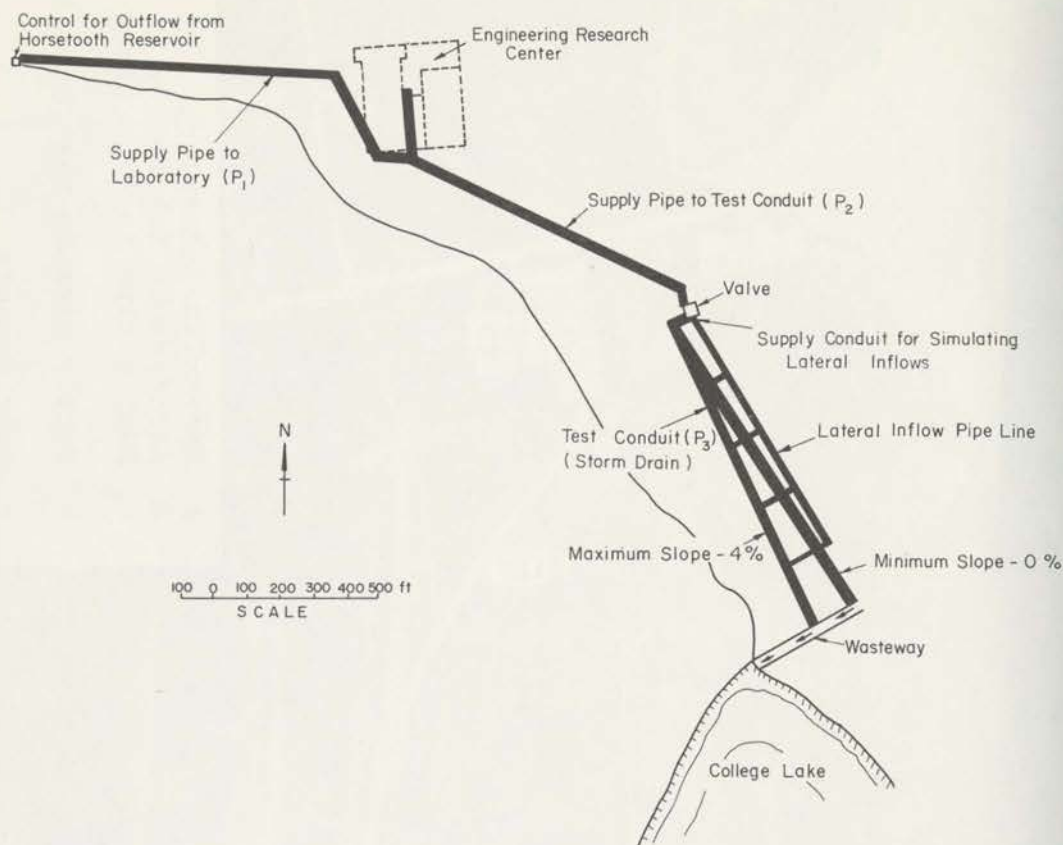


Fig. 2.16. General scheme for water supply and removal for storm conduit experimental facilities.

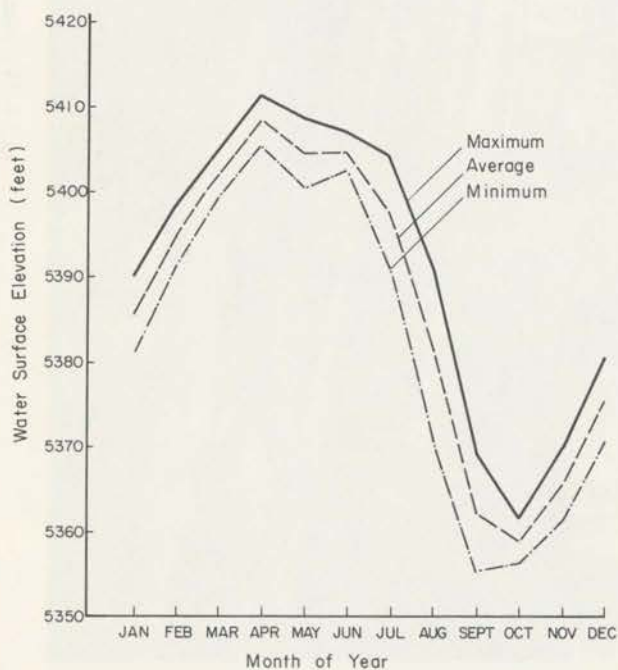


Fig. 2.17. Water level in Horsetooth Reservoir for period May 1951 ~ September 1962.

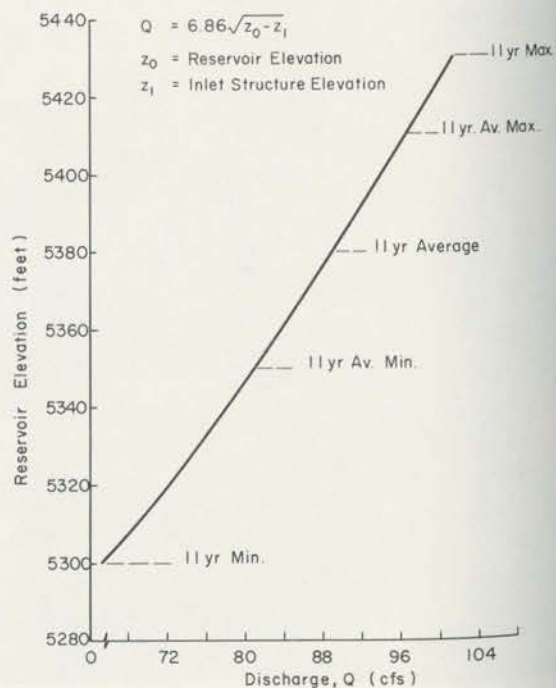


Fig. 2.18. Rating curve for discharge at the inlet structure of storm conduit.

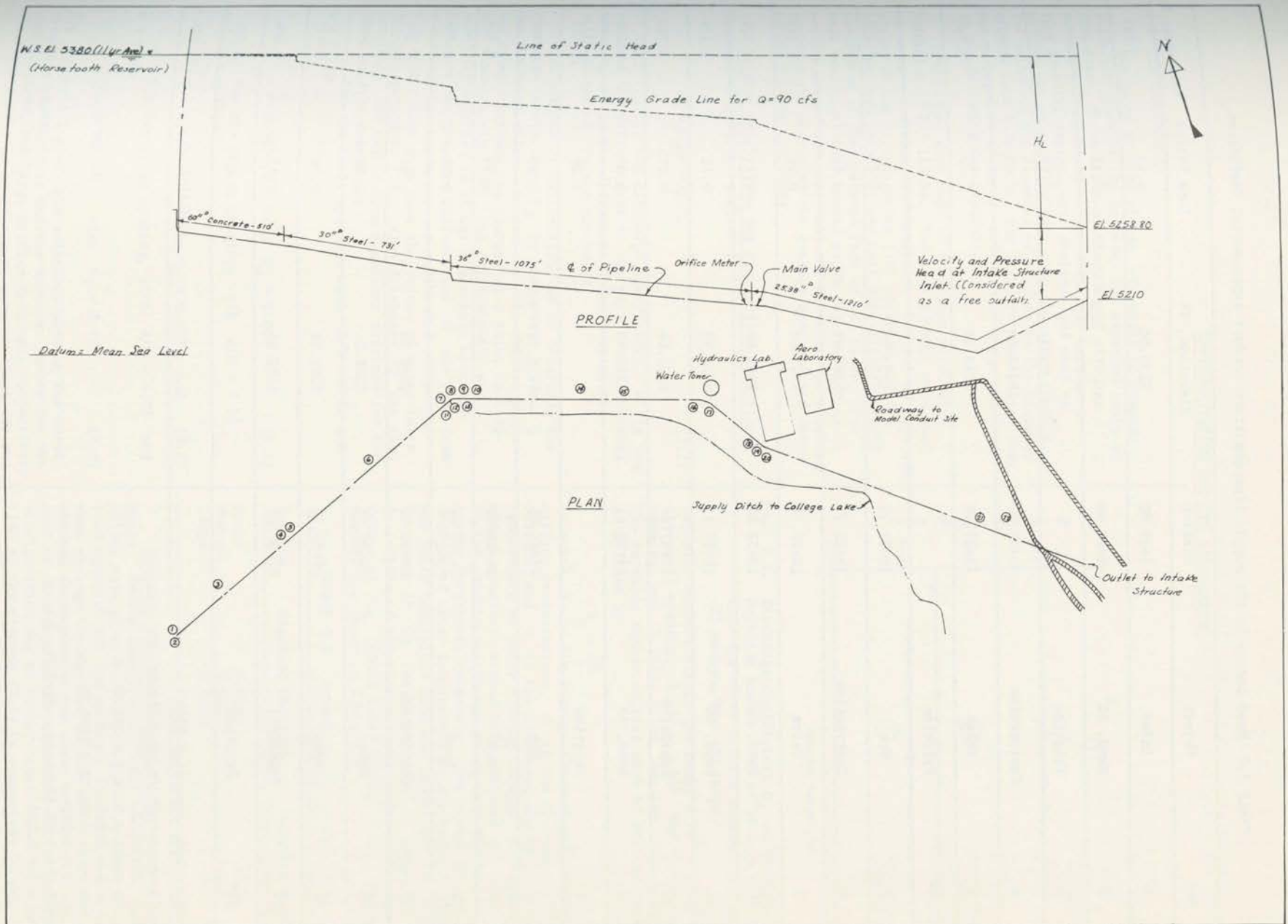


Fig. 2.19. Plan and elevation of supply line for the computation of head losses (numbers refer to pipe fittings of Table 2.2).

TABLE 2.2 Head losses in the supply line for storm conduit experimental facilities.

No.	Object	Station	Elevation, ft.	Loss Factor	Head Loss, ft.
1	Inlet	5+88.00	5270.00	0.50	0.163
2	Bend, 90°	5+91.00		0.13	0.042
3*	Friction			2.04	0.664
4	Contraction	10+90.00	5244.64	0.33	1.723
5	Valve	11+01.50	5244.64	0.25	3.186
6*	Friction			3.51	18.326
7	Wye	18+21.25	5227.00	0.30	1.566
8	Contraction	18+21.25	5227.00	0.04	0.510
9	Valve	18+29.75	5227.00	0.20	2.549
10	Bend, 45°	18+32.60	5227.00	0.12	1.529
11	Bend, 45°	18+37.11	5222.59	0.12	1.529
12	Expansion	18+37.56	5222.59	0.215	2.740
13	Bend, 40°	18+50.61	5222.39	0.08	0.201
14*	Friction			4.26	10.722
15	Wye	24+45.05	5213.27	0.30	0.755
16	Bend	26+83.75		0.10	0.252
17	Bend	27+13.75		0.10	0.252
18	Contraction	29+19.75	5205.59	0.04	0.408
19	Valve	29+29.75	5205.51	0.20	2.549
20	Bend	29+49.75	5205.52	0.10	1.020
21	T-Lateral	37+30.00	5173.65	0.03	0.306
22*	Friction			6.89	70.209
W.S. Ref. Elevation 5380				TOTAL	121.201

*3 concrete 5 ft. D - 510 ft. L.

f = 0.02

*6 steel 2.5 ft D - 731 ft. L

f = 0.012

*14 steel 3 ft. D - 1075 ft. L

f = 0.012

*22 steel 25.38 in. D - 1210 ft. L

f = 0.012

3.1 Orifice Meters

Design of orifice plates. In order to provide sufficient flexibility to cover the expected range of flow rates, three sizes of sharp-edged circular orifice plates, based on the different orifice-to-pipe diameter ratios, were designed and used to measure the main inflow rate into the storm conduit. The ratios of the diameter of orifice to the diameter of pipe were 0.35, 0.50, and 0.70, respectively. They are referred to as small, medium, and large openings in the following descriptions.

The plates were made of stainless steel with thickness of 5/8 inches. Two sets of taps were used to measure pressure differentials. One set of taps was placed at one pipe-diameter (1D) upstream and the other set was placed at one-half pipe-diameter (D/2) downstream of the upstream face of the plate. For each set, eight taps spaced at 45 degrees were installed around the pipe wall and connected to a common manifold for measuring the average upstream and downstream orifice pressures.

Orifice calibration for steady flow conditions. The general orifice equation is

$$Q = C_d mA \sqrt{2gH}, \quad (3.1)$$

in which C_d is the discharge coefficient, m is the ratio of the orifice area to the pipe cross-sectional area A , H is the differential head across the orifice, and Q is the discharge.

The purpose of calibration was to determine the discharge coefficient C_d by measuring H and Q for the known values m and A . The orifice plate was clamped between the flanges at a joint in a 26-inch diameter pipeline. The orifice plate was carefully adjusted so it was concentric with the pipe, and the pressure differential H was read from a differential manometer. A calibrated volumetric tank with a hook gage gave the volume of water for a measured period of time. Flow discharge Q was then obtained. The calibration equations obtained for the three orifice meters are:

1. For the small opening, with $d/D = 0.35$ and $m = 0.1225$,

$$Q = 2.102 \sqrt{H}, \text{ with } C_d = 0.606. \quad (3.2)$$

2. For the medium opening, with $d/D = 0.50$ and $m = 0.25$,

$$Q = 4.439 \sqrt{H}, \text{ with } C_d = 0.627, \text{ and } (3.3)$$

3. For the large opening, with $d/D = 0.70$ and $m = 0.49$,

$$Q = 9.783 \sqrt{H}, \text{ with } C_d = 0.705. \quad (3.4)$$

Fig Figure 3.1 gives the relations between the three discharge coefficients and the Reynolds number for the small, medium, and large orifice openings, respectively. The three discharge coefficients were constant for the

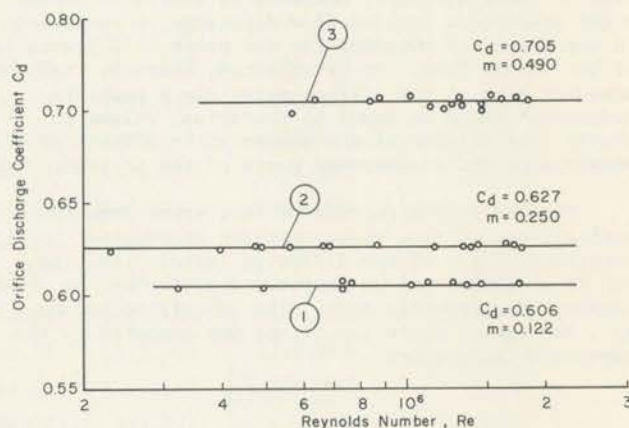


Fig. 3.1. Discharge coefficient C_d as function of the Reynolds Number, Re , of 36-inch O.D. pipeline for three orifice plates: (1) small (2) medium, and (3) large.

Reynolds numbers within the range $3 \cdot 10^5 - 2 \cdot 10^6$ for the small opening, $2 \cdot 10^5 - 2 \cdot 10^6$ for the medium opening, and $5 \cdot 10^5 - 2 \cdot 10^6$ for the large opening. The Reynolds number in this case of orifice meters is defined by

$$Re = \frac{Vd}{\nu}, \quad (3.5)$$

in which V is the average flow velocity through the orifice cross-section, d is the diameter of the orifice, and ν is the kinematic viscosity of water. Temperature of water flowing through the conduit was almost constant, around 50°F , because the water supplied to the conduit system was taken from the bottom of the nearby Horsetooth Reservoir. The effect of the small changes of water temperature on the Reynolds number was hence negligible from one experiment to another. The error in C_d can be estimated by computing the deviations of the measured points from the constant values of C_d (solid lines in Fig. 3.1). The computed results are shown in Table 3.1.

TABLE 3.1. Errors in C_d of Fig. 3.1

Orifice	Standard Deviation	Relative Error (%)
Small	0.0041	0.5
Medium	0.0022	0.3
Large	0.0032	0.4

Unsteady flow considerations for orifice measurements. Before experimental tests were made, considerations were given to the problem of orifice calibration equation for the unsteady flow phenomenon of the wave passage through the orifice meter. It was assumed discrepancies may occur between the measurement of discharge in unsteady flow with a hydrograph configuration due to either acceleration or deceleration of the flowing fluid in comparison to the measurement of discharge in steady flow through the

orifice. The accelerating phase, or the rising limb of a hydrograph, would require increased pressure to produce acceleration of flow through the orifice meter. This increase of pressure would mean an increase of the pressure differential across the orifice meter. The use of this pressure difference to compute the steady flow discharge would give a greater value than the true flow would be for the pressure differential measured under the steady flow conditions. Similarly, the discharge of the decelerating phase, or the falling limb of the hydrograph, measured in a similar manner might indicate a smaller flow discharge in comparison to the discharge obtained for the pressure differential of the steady flow. It is expected, however, that the measured flow at the orifice meter for a complete hydrograph would be equal to the actual volume in steady flow because of the compensating effects of the accelerated and decelerated parts of the unsteady flow.

The order of magnitude of this error from the unsteadiness of flow under pressure is studied experimentally. If the discharge varied with time, the total volume can be measured directly by the calibrated volumetric tank. The calculated volume, W_c , for steady state conditions was computed by the numerical integration

$$W_c = \int_0^t Q_t dt \quad (3.6)$$

in which the discharge Q_t is computed from the equation of the steady flow orifice, $Q_t = C \cdot \text{steady} \cdot H^{1/2}$. In comparing the measured volume of the unsteady flow and the calculated volume of the steady-flow orifice equation, it was possible to infer whether the orifice coefficients, C , of the unsteady flow departed from the coefficients of the steady flow.

Experimental procedure also provided data for a comparison of the total measured volume of flow through the orifice plate under unsteady flow conditions for a given period of time, and the computed volume for the steady conditions by using Eqs. 3.2 through 3.4. The volumes of rising limbs, of falling limbs, and of the total hydrograph were measured directly by volumetric tanks while the pressure differential across the orifice plate was measured by a transducer and recorded on a strip chart. Within the limitations of the testing facility the runs were carried out in such a manner as to use almost maximum volumetric tank capacity while varying the times of runs and the peak flows.

Water was supplied to the test section containing the orifice plate by a pump. The unsteady flow condition was introduced by opening and closing a butterfly valve placed in the line specifically for that purpose. The valve, operated manually, was placed about 40 feet upstream from the orifice plate. A base flow was established. The water was diverted into the volumetric tanks, and the time of the run and the pressure differential across the orifice plate recorded. The diverter, time clock, and strip-chart were started at the same time the valve was opened or closed.

Volume of flow for steady condition over a given time was computed by numerical integration. Time intervals for the integration were taken as three seconds. At each three second interval on the strip chart recording, pressure was read and the flow rate computed using the steady flow orifice coefficient. The flow rate was averaged over the three second

period and the volume for each time period determined and summed for the total. The actual computation was performed by digital computer.

Comparison of the direct measurement of volume to that computed from the transducer recorded data gave a maximum deviation of measured volume from the computed volume of 1.62 percent. Eleven of the sixteen runs or 69 percent of the runs gave deviations of measured volume from computed volume of less than one percent. No specific trend in the data was found. The results of comparing measured and computed volumes for various runs are shown in Table 3.2. Figure 3.2 shows the same results found in Table 3.2, which supports the conclusion that no significant deviation is observed because of unsteady flow conditions.

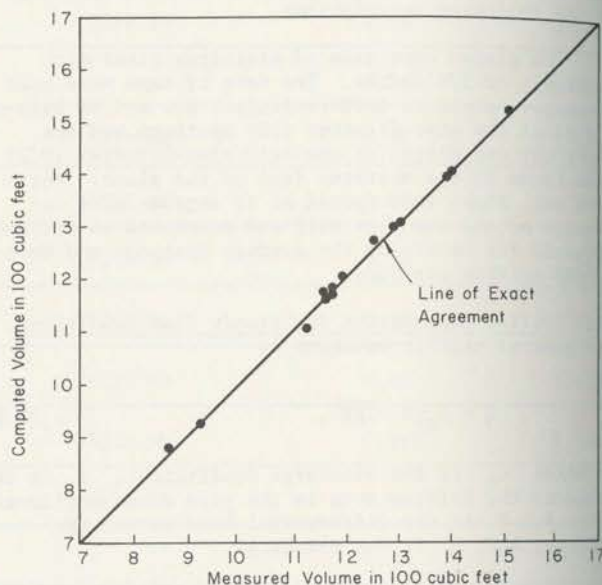


Fig. 3.2. Relation of the measured to the computed volumes for the unsteady flow conditions through the orifice meters.

In order to avoid surges of flow caused by a smaller orifice opening, 16 of the total 17 runs in Table 3.2 were made for the large orifice opening ($m = 0.49$) and only one run was made for the medium opening ($m = 0.25$). No data were taken for the small orifice opening ($m = 0.1225$).

3.2 Current Meters

Type of current meters. Propeller current meters have the following advantages as compared to cup current meters: (1) The propeller current meters produce smaller disturbances in flow, (2) The mechanical (bearing) friction is smaller in propeller current meters, and (3) The propeller current meters are less susceptible to fouling by foreign bodies carried in the flow. Based on these advantages, reliability of operation, and compact construction, the Ott propeller current meters with electric counters were selected to measure the flow velocities in the storm conduit. To cover a wide range of flow velocities, the Ott propeller current meters were equipped with interchangeable propellers with different pitches. The size and selection of the propellers and pitches are given in the Table 3.3.

TABLE 3.2. Results of unsteady flow investigation of orifice meter calibration for different area ratios

Run No.	Area Ratio	Hydrograph	Flow Rate cfs		Time of Test in sec.	Computed Volume in ft ³	Measured Volume in ft ³	Deviation of Computed From Measured Volume in %
			max	min				
3	0.49	Rising	23.704	12.111	62.99	1272.170	1252.091	-1.604
4	0.49	Rising	21.102	14.195	78.81	1408.636	1408.839	+0.015
5	0.49	Rising	22.100	9.439	59.95	1200.912	1193.531	-0.618
6	0.49	Rising	17.893	7.482	91.59	1317.455	1312.542	-0.374
7	0.49	Rising	22.261	10.053	62.25	1176.432	1160.429	-1.043
8	0.49	Rising	20.502	7.924	81.51	1397.028	1397.867	+0.060
9	0.49	Falling	21.585	6.584	59.48	1178.076	1173.244	-0.412
10	0.49	Falling	21.290	18.549	73.91	1524.571	1516.301	-0.545
11	0.49	Falling	21.290	15.265	65.07	1303.741	1297.380	-0.490
12	0.49	Falling	21.767	7.477	49.20	880.549	868.447	-1.394
13	0.49	Complete	22.188	6.765	62.61	1184.071	1173.726	-0.881
14	0.49	Complete	21.858	7.296	63.78	1107.155	1124.776	+1.567
15	0.49	Complete	22.351	8.132	61.37	1176.518	1168.864	-0.655
16	0.49	Complete	20.152	8.220	80.11	1174.248	1166.717	-0.645
17	0.25	Rising	17.041	8.305	60.07	924.271	923.561	-0.077
18	0.25	Rising	17.086	7.853	79.92	1155.196	1174.204	+1.619

TABLE 3.3. Selection of propellers

Propeller No.	Propeller Diameter (cm)	Pitch (cm)	Approx. Maximum Velocity (ft./sec)
2	5	10	3
2	3	10	3
3	5	25	8
4	5	50	16

To check the procedures used to determine the change, if any, in performance of the Ott current meters three series of tests were made using the Colorado State University calibration facilities (Figs. 3.3, 3.4, and 3.5).

In the first tests each meter propeller was tested singly. Each meter was mounted on a 3/8-inch rod attached to the carriage and towed at a depth of two feet (Fig. 3.3). The second test series involved three meters mounted on the 3/8-inch rod at a distance of 4-inches on centers. The purpose of the test was to determine interference effects, if any, on the meters operating under simulated field conditions. Because the recording system was only a single channel, it was not possible to record the performance of all three meters simultaneously. Therefore, the test procedure was the same for the single mounted meter, that is, it was necessary to make three runs with the carriage at a given speed in order to obtain the performance data at this speed on the three meters.

Since the velocities of the meters under field conditions fell on the uppermost range of a 3-range calibration chart plotted from data furnished by the Ott Laboratory, only the calibration data in this range were analyzed. In this range it was found that the theoretical curve that best fits the data is the

asymptotic limb of a hyperbola, which can be approximated by a straight line of the form $V = a + bn$. In this equation V is the velocity in feet per second and n is the number of turns per second. Values of the coefficients a and b for least-square curves fitted to the recalibration data and the Ott laboratory data were found to be essentially identical. For each set of data the accuracy of the curve fitting procedure was found by the standard error of estimate of V on n .

In view of the general agreement between the recalibration values and the manufacturer's furnished values, there did not appear to be justification for the adoption of the recalibrated values. Furthermore, the effect of spacing the meters at 4-inches center to center does not indicate any trend when compared to the single meter calibration.

For the third test series a system was devised whereby all five velocity meters could be calibrated simultaneously utilizing the mounting arrangement used in the actual testing program of the storm conduit. A bracket was clamped to the front of the test car and the field mounting frame for the velocity meters was bolted to the bracket (Fig. 3.6). The velocity meters were tested at a depth of approximately one foot. A relay circuit operated by micro-switches provided means for turning a clock and the current meter counters on and off, (Fig. 3.7), while the test car traveled at constant speed over a known distance. Stakes were driven into the ground and cam plates were clamped to the stakes, (Fig. 3.8, lower right). Activation of the micro-switch was marked on the cam and the distance between the "on cam" and the "off cam" was measured. The tow car was initially at a point near the end of the water tank. During each run, the cams and micro-switches provided a time and count record over the known distance with variation being made only in time required to traverse the distance.



Fig. 3.3. Electrically propelled tow car.



Fig. 3.5. Recording system for calibration of current meters, and tow car control switch. Strip chart records speed of tow car and revolutions per second of current meters.



Fig. 3.4. Current meter tow tank. Dimensions of the tank are: Length - 200 ft; width - 5 ft; depth - 5 ft.

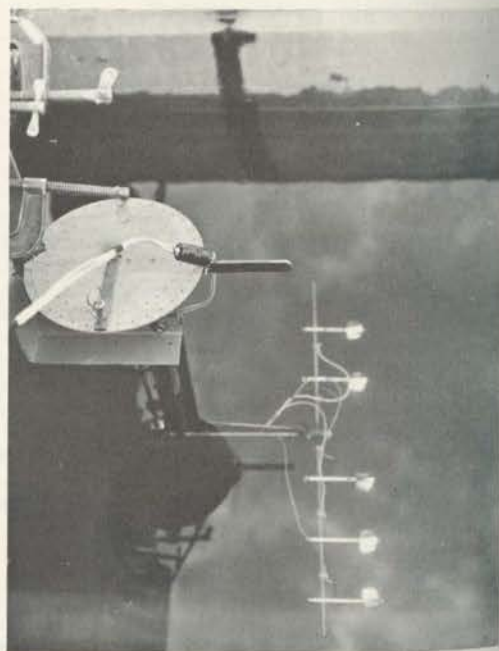


Fig. 3.6. Ott meters in calibration position on tow car.

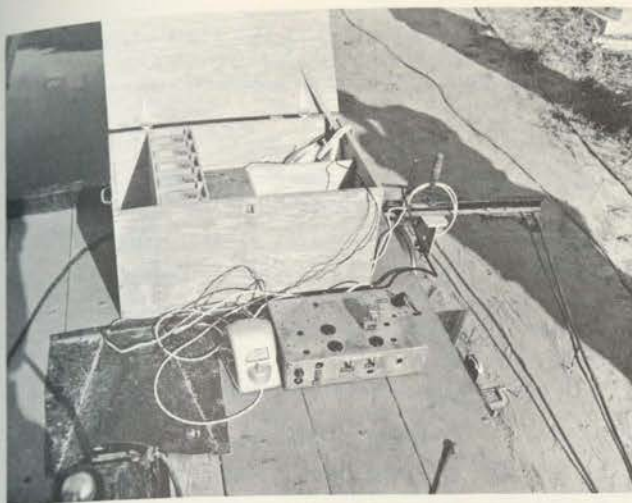


Fig. 3.7. Relay circuit chassis, electric clock, and current meter counters on tow car for simultaneous calibration of five Ott-current meters.

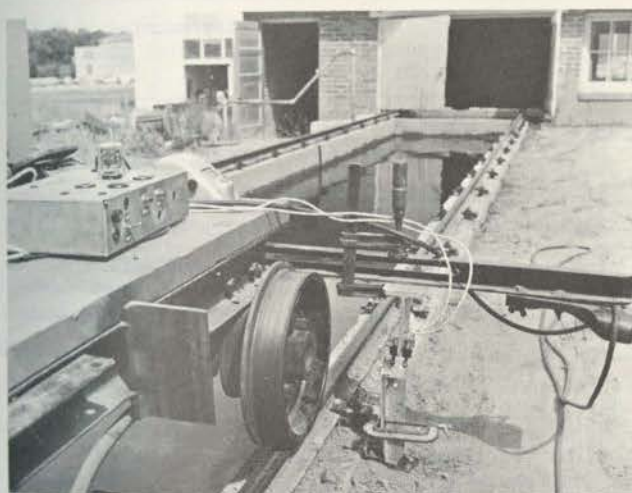


Fig. 3.8. Arrangement of micro-switch switches and cam plate which operate relay circuit. "Off cam" is illustrated.

Calibration results. The technique of calibrating the meters was found to be satisfactory, that is, the system was first carefully checked by making several runs of the tow car at a constant speed over a given distance until the meter count did not vary by more than ± 0.5 percent. The meters were then calibrated by operating the tow car at eleven different speeds covering its operational range.

The calibration results of the values of "a" and "b" as compared to the original calibration data furnished by Ott Company are presented in Table 3.4.

Mutual interference and wall proximity effects on current meter readings. Investigations of mutual interference and wall proximity effects on current meter readings have been conducted by Henn [1], Status [2], Jovanovic [3], and Benini [3]. Based on the results of the tests of these investigators it was decided that similar tests would not be needed for the Ott current meters. Of particular importance were the results of the tests made by Benini, who was

concerned with accurate measurement of flow in large diameter pipes. His calibration tests simulated the positioning of the current meters were similar to what were used by this study in the measurement of velocity distributions. Benini's rating equipment consisted of a flume 50.85 meters long (useful length between the two ends of the run 41.80 m), 2.00 meters wide and 3.00 meters deep; a carriage was operated by a cable system from a hydraulic converter enabling the required speed to be reached within a very short distance. The speed could be varied continuously between zero and 5.50 meters/second.

The experimental results obtained by Professor Benini are presented in Figs. 3.9 through 3.11. On the abscissa the axial distance between the propellers, or the distance between the propeller axis and the wall, is plotted in terms of the ratio between the distance L and the propeller diameter D . It was preferred to express the distance between the meter and the wall or between the two meters in terms of the propeller diameter rather than in absolute values, in order to extend the test results to meters with the propellers of different dimensions. The percentage deviations from the reference values are plotted as the ordinate. In Figs. 3.9 to 3.11 the representative points of the deviations have been joined by straight lines, to better emphasize the variation.

Apart from the tests carried out at the velocity of 0.5 meter/second, it can be seen that the divergences from the reference values are, with a single exception, less than 0.5 percent, and show no decisive trend in either the positive or the negative direction.

The tests reported thus demonstrate that within the range of velocities investigated the performance of a current meter is only slightly influenced by the proximity of a second current meter, or of a wall. The divergences found, referring to the behavior of meters in a region without any disturbances, are attributed rather to unavoidable experimental error, being of the same order of magnitude as, or slightly larger than, these errors. For this reason it is difficult to distinguish between experimental errors and effects caused by an adjacent meter or wall; it can be stated that, within the range investigated, the possible effect does not exceed 0.2 to 0.3 percent.

For comparison, the results of the other investigators are summarized as follows:

1. Staus [2] found that a propeller current meter records velocities that are low by 1.1 percent near the wall and by 0.8 percent when near the bottom, the errors being independent of absolute velocity.

2. Henn [1] made use of tests by the Ott firm. The Ott tests indicate that the reduction in current meter readings increases with the increasing velocity, to reach 1.2 percent for meters near the bottom and 2.1 percent for meters near the wall, at a velocity of 2.5 meter/second. Regarding the mutual interference of two nearby meters, the Ott tests indicate that in the velocity range 0.1 to 0.8 m/s, there is no significant correlation with the distance between the two meters.

3. Jovanovic [3] at the rating station at the Institute of Hydraulics in Belgrade found that in the velocity range of 0.5 to 4.5 m/s, meters near the bottom gave errors as large as 1.73 percent at a velocity of 4.5 m/s. Tests on meters placed 3 cm between the peripheries of the propellers gave errors of only 0.8 percent at 4 m/s.

TABLE 3.4. Comparison of Ott-Meter calibration rating curves, $V = a + b n$, by the least square estimate of Ott and CSU calibration data for meter velocity > 1.0 fps.

Meter No.	Prop. No.	Prop. Dia. cm.	Slope of curve b	Intercept a	Standard Deviation σ	Calibration Date	Remarks
12457	2-3	3	0.3411	0.108			From mfg. ⁽¹⁾ calibration curve for meter.
			0.338	0.143	0.0016	5-1-64	1st test series; single meter.
			0.339	0.106	0.0051	7-2-64	2nd test series-run 1; deepest of 3 meters.
			0.337	0.111	0.0059	7-13-64	2nd test series-run 2; deepest of 3 meters.
			0.340	0.112	0.0042	7-15-64	From analysis of original data.*
12457	4	5	1.6531	0.030			From mfg. calibration curve for meter.
			1.675	-0.007	0.0074	5-1-64	1st test series; single meter.
			1.648	0.049	0.0078	7-2-64	2nd test series-run 1; deepest of 3 meters.
			1.648	0.029	0.0074	7-13-64	2nd test series-run 2; deepest of 3 meters.
			1.651	0.036	0.0052	7-15-64	From analysis of original data.
			1.693	0.007	0.0429	8-24-64	3rd test series; meter mounted on field support with original meter gear replaced by a new gear.
12458	2-3	3	0.3395	0.115			From mfg. calibration curve for meter.
			0.335	0.177	0.0059	5-1-64	1st test series; single meter.
			0.336	0.153	0.0034	7-2-64	2nd test series-run 1; middle meter of 3.
			0.338	0.124	0.0019	7-13-64	2nd test series-run 2; middle meter of 3.
			0.338	0.131	0.0056	7-15-64	From analysis of original data.*
12458	4	5	1.6531	0.033			From mfg. calibration curve for meter.
			1.651	0.016	0.0104	7-2-64	2nd test series-run 1; middle meter of 3.
			1.663	0.018	0.0135	7-13-64	2nd test series-run 2; single meter of 3.
			1.642	0.051	0.0088	7-13-64	2nd test series-run 3; middle meter.
			1.650	0.040	0.0098	7-16-64	From analysis of original data.*
			1.673	-0.001	0.0385	8-24-64	3rd test series; meter mounted on field support with original meter gear replaced by a new gear.
12459	2-3**	3	0.3395	0.112			From mfg. calibration curve for meter.
			0.339	0.150	0.0050	5-1-64	1st test series; single meter.
			0.342	0.122	0.0109	7-2-64	2nd test series-run 1; shallowest of 3 meters.
			0.340	0.112	0.0141	7-2-64	2nd test series-run 2; shallowest of 3 meters.
			0.341	0.115	0.0055	7-13-64	2nd test series-run 3; shallowest of 3 meters.
12459	4	5	1.6564	0.036			From mfg. calibration curve for meter.
			1.675	0.027	0.0099	5-1-64	1st test series; single meter.
			1.668	0.022	0.0101	7-2-64	2nd test series-run 1; shallowest of 3 meters.
			1.681	0.007	0.0093	7-2-64	2nd test series; single meter.***
			1.668	0.017	0.0176	7-13-64	2nd test series-run 2; shallowest of 3 meters.
			1.654	0.043	0.0053	7-17-64	From analysis of original data*.
			1.691	0.004	0.0451	8-24-64	3rd test series; meter mounted on field support with original meter gear replaced by a new gear.

⁽¹⁾Mfg. means manufacturer's calibration results.

TABLE 3.4. Comparison of Ott-Meter calibration rating curves, $V = a + b n$, by the least square estimate of Ott and CSU calibration data for meter velocity > 1.0 fps. - Continued.

Meter No.	Prop. No.	Prop. Dia. cm.	Slope of curve b	Intercept a	Standard Deviation σ	Calibration Date	Remarks
12460	2-3	3	0.3362	0.118			From mfg. calibration curve for meter.
			0.334	0.156	0.0027	5-1-64	1st test series; single meter.
			0.335	0.127	0.0042	7-17-64	From analysis of original data*
12460	4	5	1.6466	0.033			From mfg. calibration curve for meter.
			1.666	0.023	0.0171	5-1-64	1st test series; single meter.
			1.643	0.044	0.0063	7-18-64	From analysis of original data.*
			1.700	-0.032	0.0397	8-24-64	3rd test series; meter mounted on field support with original meter gear replaced by a new gear.
12461	2-3	3	0.3378	0.112			From mfg. calibration curve for meters.
			0.333	0.156	0.0050	5-1-64	1st test series; single meter.
			0.338	0.109	0.0035	7-9-64	From analysis of original data.*
12461	4	5	1.6498	0.030			From mfg. calibration curve for meter.
			1.670	-0.008	0.0078	5-1-64	1st test series; single meter.
			1.652	0.024	0.0041	7-19-64	From analysis of original data.*
			1.674	-0.019	0.0378	8-24-64	3rd test series; meter mounted on field support with original meter gear replaced by a new gear.

* Original calibration data furnished by Ott Company. Note: This data was used in preparation of the calibration curve supplied by Ott with each meter.

** No original data was furnished for this propellor.

*** Meter shaft had been bent during operation; calibration was made after meter was required.

These results are at variance with those by Benini. Because of the different technique used by Benini in carrying out the tests, in particular by comparing calibration tests carried out successively with repeated tests, the results given by Figs. 3.9 to 3.10 are liable to less error and are therefore assumed to be reliable.

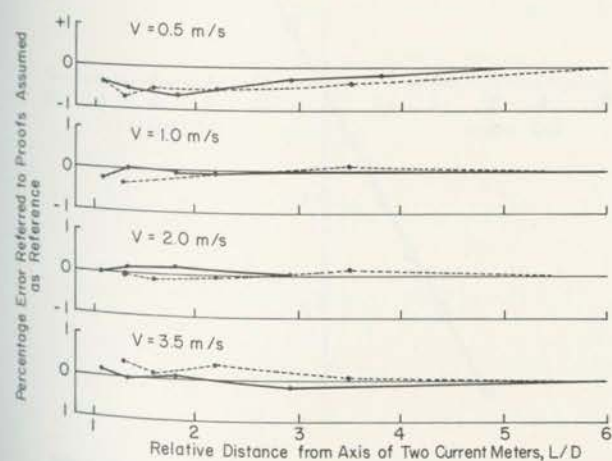


Fig. 3.9. Behavior of two neighboring current meters with propellers of different pitch (after Benini).

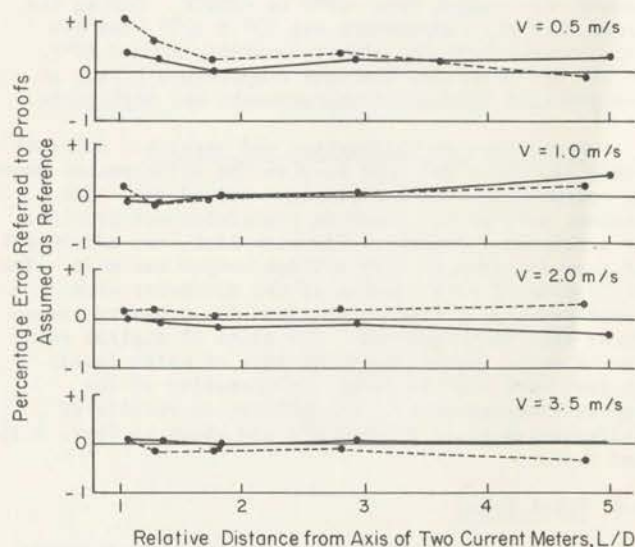


Fig. 3.10. Behavior of two neighboring current meters with propellers of the same pitch (after Benini).

In summary, since mutual interference and wall proximity effects on current meter readings are negligible in the velocity range of this project's

experimental studies, it was decided that a particular calibration of the current meters for these effects should not be made. Instead, the German code for the meter use was adopted. The code specifies that the distance between the meter axis and the wall must not be less than 0.75 times the diameter, and the distance between the two adjacent meters must be less than 1.2 times the propeller diameter.

3.3 Pressure Transducers

General considerations. The calibration of pressure transducers is considered from those standpoints: (a) that the output voltage be zero for zero pressure input; (b) that the output voltage be a linear function of the input pressure differential; (c) that it does not change with time and if so, it must be systematically checked, and (d) that the proportionality constant between the input and the output be known before the observed data can be interpreted.

The proportionality constant may be reasonably expected to include the composite effects of all elements in the data recording system. Hence, this constant should be established immediately prior to and immediately following data observation. It was not assumed this value would remain constant from run to run or from day to day, so facilities were implemented to determine this value in the field as desired without making a complete calibration check, which essentially only establishes the linearity of the transducer.

The first, second and third conditions under (a), (b), and (c) have been investigated for all transducers in use. With sufficient care in installation and adequate warm-up time, all transducers were linear and passed through zero with no hysteresis for the range of pressure differentials anticipated measurements. The manufacturer's specifications state a compensated temperature interval of these pressure transducers ranges from -65°F to $+250^{\circ}\text{F}$. During the tests, the air temperature was $75^{\circ} \pm 10^{\circ}\text{F}$, and the water temperature was almost constant, around 50°F . It is thus concluded that the temperature effect on the pressure transducer measurements was negligible.

Procedures of calibration and results. A 3-foot water manometer was used to give the differential depth in conjunction with the pressure transducer. The readout part of the pressure transducer was provided by a digital voltmeter. For each test, the adjustment of zero pressure to zero voltage output was made. The difference of water levels of the manometer with corresponding voltage output from the pressure transducer were then recorded. The plots of digital readout in volts versus the difference of water levels in feet were used to check the linearity of the pressure transducers. The calibration results of different pressure transducers are shown in Figs. 3.11 and 3.12.

3.4 Pitot Tubes

Early in the investigation of velocity distributions a single pitot tube was considered. A particular tube was calibrated and used for one vertical velocity traverse. Its calibration constant was essentially 1.00 for relatively small velocities. There was a slight difference in the velocity profile as observed by the pitot tube and the Ott current meters for the same flow conditions, but because of the time involved in using a single tube to define the complete velocity profile and because the Ott current

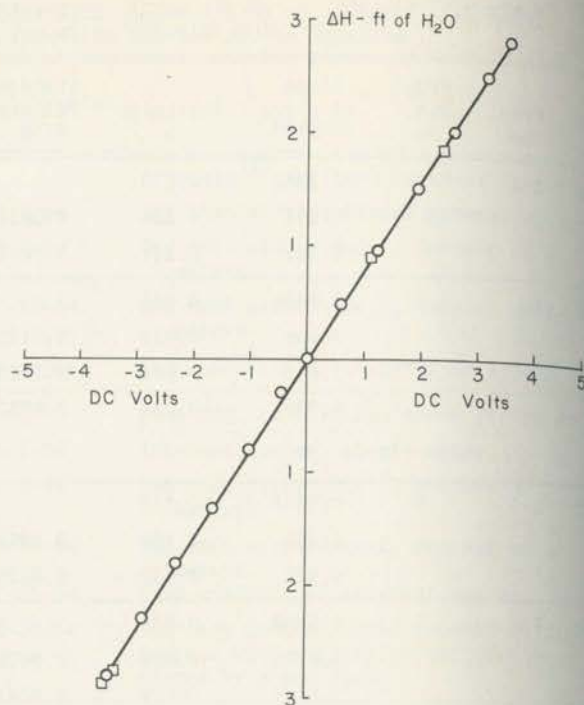


Fig. 3.11. The calibration line for the pressure transducer No. 12180, ± 1 psi. Tested 7-23-66 Using data recording system. CD-25 No. 14490

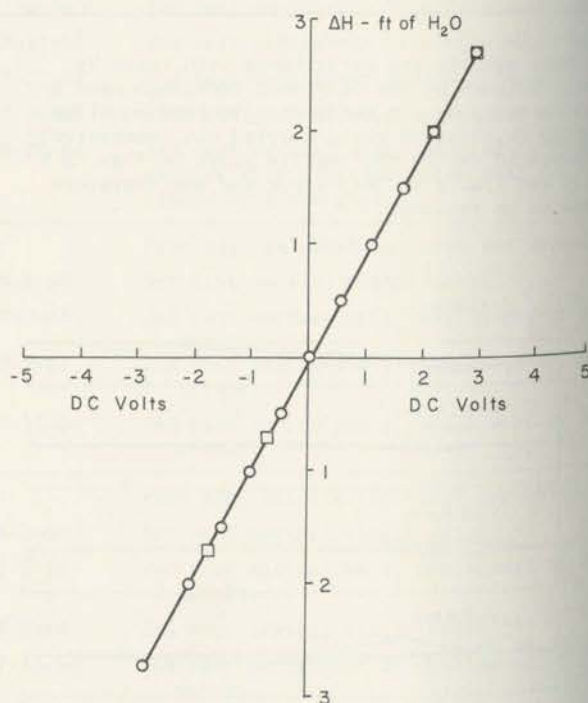


Fig. 3.12. The calibration line for the pressure transducer No. 14400, ± 5 psi. Tested 7-23-66 Using data recording system. CD-25 No. 17083

meters produced essentially the same vertical profile with a fraction of the time requirement, the use of the single tube was discontinued.

Velocities in excess of 10 feet per second, for the steeper slopes, reduce the use of the Ott current meters. In order to reduce field run times to a minimum and to reduce the drag on the velocity meters and accompanying support to a minimum, hypodermic needles (as shown in Fig. 3.13) were used to measure the total static and dynamic head. The static head was measured at the solid boundary surface. Seven of these were arranged along a revolving radial arm. Based on the use of similar tubes in air flow at high Reynolds numbers the coefficient was assumed to be constant for the range of velocities being observed. Considering this assumption and the fact that if the distribution of velocities as represented in the α and β velocity distribution coefficients only was desired, then these coefficients can be represented by the appropriate integral of the ratio of the square root of the pressure differential. The pressure differentials were detected by means of a pressure transducer whose voltage output was proportional to the pressure differential. The voltage was read and recorded automatically onto punched cards. Since the voltage was proportional to the pressure differential, the velocity distribution is identical to the distribution of the square root of the observed voltages.

Subsequent comparison of velocity distribution coefficients as measured by the Ott current meters and by the hypodermic dynamic tubes indicated an overlap within the ranges of variability of the Ott

meter results. Therefore, it did not appear desirable or necessary to calibrate the dynamic tubes for the determination of the velocity distribution coefficients.

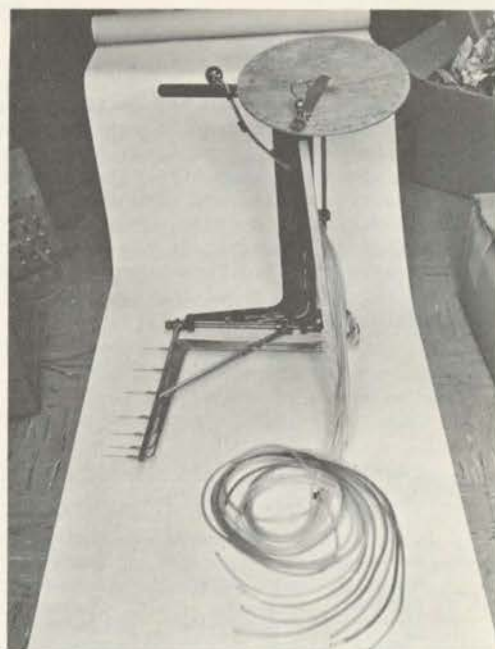


Fig. 3.13. Pitot tube rake used for point velocity measurements at large discharges of supercritical flow.

DATA RECORDING SYSTEM

4.1 Description of the System

A system for recording and transmitting field data from the outdoor storm conduit was installed. The continuous time series of physical quantities measured by the sensing device were first transferred into analog electric signals and were then digitized by an analog-to-digital converter. The output from the analog-to-digital converter in the form of punched cards or tapes were then fed into computer for further computations. Figure 4.1 shows the general outline of the Data Recording System.

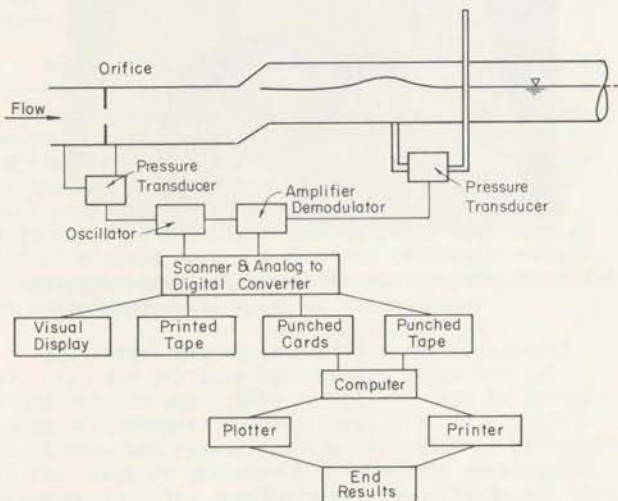


Fig. 4.1. Data recording system.

4.2 Analog-to-Digital Converter

The Systron-Donner Model 1234 Analog-to-Digital (A/D) converter was designed to accept continuous (analog) data up to 30 input channels. It sequentially scans, digitizes, and drives an output coupler to punched cards or punched paper tapes for eventual computer processing. The system samples at a rate determined by the type of output device and the number of digits per word. For punched cards, the rate is 2 words or 17 characters per second. For punched paper tape, it is 14 words or 110 characters per second. A pinboard provides the means of setting the full-scale input range of each channel to be scanned. These ranges are ± 10 , 50, 100, and 500 millivolts, 1, and 10 volts. The same pinboard allows any unused or unwanted channels to be deleted from the scan. Each sample is of 3 milliseconds (ms) duration and the next sample is taken T milliseconds later, (Fig. 4.2). The T is determined by the time required for digitizing, punching, etc; however, its duration may be varied by choice of output device and number of characters printed out per sample. For a standard 8-digit data word, with card punch output, T is 500 ms less 60 ms/digit deleted. With paper tape output, T is 70 ms less 9 ms/digit deleted.

The output from the A/D converter of "time" and "manual data" are printed out at the beginning of each scan though they could be omitted at the option of the operator.

The following is printed out for each channel:

Channel identification	2 digits
Polarity or overload	1 "
Data	4 "
Range or data ID	1 "

Q is printed out for an overload.

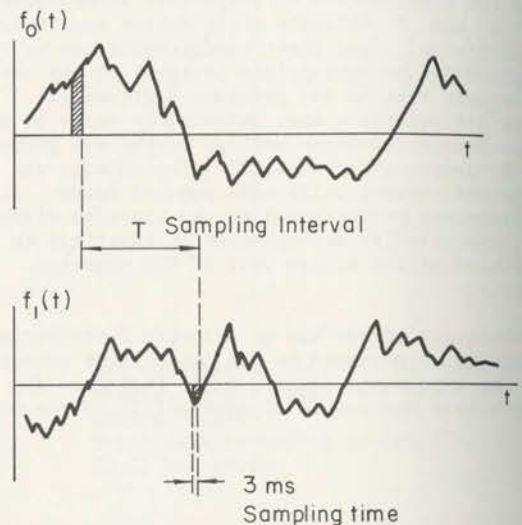


Fig. 4.2. Schematic representation of time series sampled by data recording system.

At the option of the operator the system may enter six digits of time information at the beginning of each record (scan). The indication and recording of time is in hours, minutes, and seconds. The source of each input data point is identified by a two digit channel number preceding the value of the data taken. Each four digit value is accompanied by indicators for range, polarity, and "off-scale" conditions.

The range symbol is:

	volts full scale	Symbol
10	"	3
1	"	4
0.5	"	1
0.1	"	5
0.05	"	2
0.01	"	6

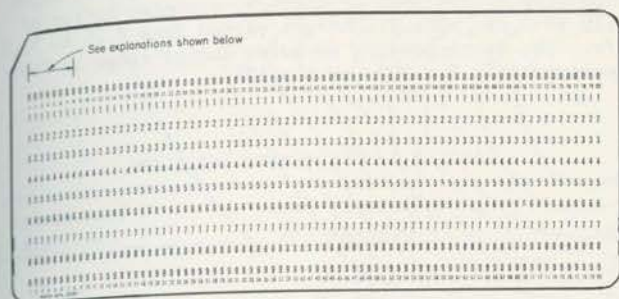
For the decade steps, the range symbol is the negative power of 10 by which the data is to be multiplied. The number 8 identifies the data as a manual entry and the number 9 identifies a time entry. For example,

05-74973 means that a value of -7.497 is read for channel 5,

14+53922 means that a value of +0.02696 is read for channel 14,

24Q10735 means that channel 24 is overloaded,
13+45329 means a time of 13 hours 45 minutes
32 seconds, etc.

The details of the data format as the output from the A/D converter is 0.1% at full scale.



	Columns (1-8)							
<u>Voltage</u>	1	2	3	4	5	6	7	8
	c	c	+	v	v	v	v	R
	channel number		sign					
<u>Time</u>	H	H	+	M	M	S	S	9
	Hours			Minutes		Seconds		
<u>Identification</u>	I	I	-	I	I	I	I	8

Fig. 4.3. The data format as the output from Analog-to-Digital Converter.

4.3 Operations and Controls

The outdoor circular storm conduit is located about 1500 feet away from the Colorado State University Hydraulic Laboratory building. Data taken from the storm drain were transformed into electric voltage signals by the pressure transducers installed in the control trailer, shown in Fig. 4.4. The transducer's slope and intercept calibrations for head versus voltage relations were determined in the following manner. First, the systems were allowed to come to equilibrium. The zero transducer voltage output was then recorded. Next, valves No. 1 and No. 3 were closed and a differential head was applied in manometer tube A. This was measured and recorded along with the corresponding transducer voltage output. Once these calibrations were obtained, all valves were opened and the systems were again allowed to return to

equilibrium. Then valves 2 and 3 were closed and the wave was started. Using this system, the deviations from the initial depths were recorded.

Flow passed through a motor-driven Rockwell Permisphere valve, then through an orifice where a constantly recording pressure transducer was connected to measure discharges. It then passed through a baffle system to dampen out surface oscillations and provide a more uniform velocity distribution, and, finally, on to what is considered the inlet of the pipe.

Waves were measured at distances of 50.00 ft, 410.00 ft, and 771.70 ft from the entrance during experiments of the summer of 1965 and at distances of 50.00 ft, 251.24 ft, 387.70 ft, 462.56 ft, 669.83 ft, and 771.70 ft during experiments of the summer of 1966. Wave heights were measured by pressure transducers connected between the pipe invert and a set of manometers, as shown in Fig. 4.4.

Voltage signals were converted from analog to digital form in the data recording system and were then punched directly onto data cards. The experimental wave could then be reconstructed by feeding the calibration and voltage cards into a digital computer which converted the voltages into depths.

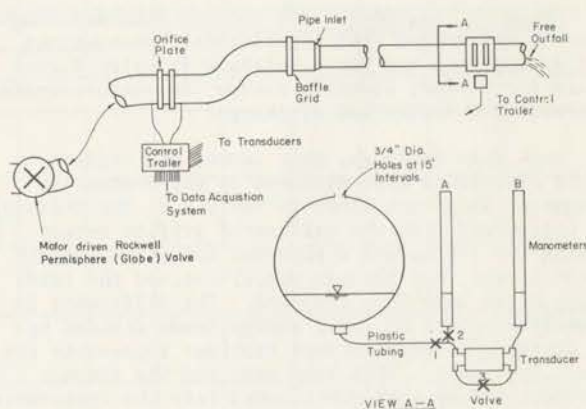


Fig. 4.4. Schematic representation of facilities and data recording system.

EXPERIMENTAL TEST CONDITIONS AND TYPICAL RESULTS

5.1 Steady Flow Conditions

Cross-sectional velocity distributions. In order to determine the velocity distribution coefficients (α and β) experimentally, measurements of the distribution of conduit cross-sectional velocities were made. Time average point velocities were measured by the Ott laboratory current meter. Five meters were mounted on a rod which was supported at the center. The rod support was at the conduit centerline and could rotate to place the meters in any angular position. The meters were spaced along the rod to sample equal circumferential areas. The meters were placed at the minimum recommended spacing distance from the conduit wall. The meter support rod was positioned at angular intervals of 10 degrees. Thus, the point velocities were observed at five radial positions and as many 10-degree intervals as required to sample the circular segment.

The input data to the computer program consisted of: (1) velocity meter identification number and propeller number; (2) velocity meter position on rod, (3) angular position of the rod, (4) time interval of revolutions, (5) number of revolutions, (6) water depth at the cross-section, and (7) measured discharge.

A typical result of the measured velocity distributions of the conduit cross-section is shown in Fig. 5.1. The properties and computation details of these velocity distribution coefficients, α and β , based on the measured velocities of conduit cross-sections are described in Part III of this four-part series of papers. This paper is the second in the series.

Boundary roughness. The conduit boundary roughness was determined from experimental observations. In this approach the Darcy-Weisbach friction factor f and the conduit Reynolds number Re were computed from measured depths and discharge.

Hook gage readings, gage zeros, and conduit invert elevations were measured at the successive piezometer locations shown in Table 5.1. The discharge was determined from the calibrated orifice meters. By knowing the depths and discharges for fixed conduit invert slopes, the average velocities and the total energy heads were then computed. The difference in successive values of total energy heads divided by the distance between conduit stations represents the energy loss rate. This loss rate and the average hydraulic radius were substituted into the resistance equation to evaluate the friction factor f , namely

$$f = \frac{4 R_{av}}{V_{av}^2} \cdot \frac{\Delta E}{\Delta x} \quad (5.1)$$

The Reynolds number for the same reach was computed, based on average velocity and hydraulic radius as the characteristic length, i.e.

$$Re = \frac{V_{av} \cdot R_{av}}{\nu} \quad (5.2)$$

A plot of these f values versus Reynolds number, Re , are presented in Fig. 5.2. The plotted points represent the results of experimental ranges of depth from 0.56 to 2.6 feet or depth-to-diameter ratios of 0.19 to 0.89. The discharges varied from 2.25 to 72.6 cfs. The corresponding Reynolds number range is from approximately 3×10^4 to 1×10^6 .

TABLE 5.1. Piezometer locations

No.	Distance from upstream end	Incremental Distances
1	20.00	
2	100.00	80.00
3	197.00	97.00
4	308.40	111.40
5	406.10	97.70
6	509.60	103.50
7	613.20	105.60
8	707.20	94.00
9	772.20	65.00
10	802.20	30.00
11	807.25	5.05
12	812.25	5.00
13	816.25	4.00
14	819.10	3.00
15	820.70	1.60
16	821.70	1.00

Controlled and free outfall. The mathematical simulation of the downstream boundary condition for the controlled outflow required the calibration of a terminal (end) restriction. Any geometric configuration was acceptable if it satisfied certain criteria:

1. The discharge as a function of depth could be expressed simply, such as $Q = m y^n$ in which m and n are constants and y is the depth of flow upstream of the restrictions.

2. The restriction was not so great as to cause the pipe to flow full under the maximum anticipated hydrograph discharge.

3. The approach-velocity distribution was symmetrical and did not differ appreciably from the undisturbed flow.

These criteria were satisfied by a restriction gate consisting of five 7-inch vertical wooden slats held in position by 2 1/2-inch wide vertical aluminum H-sections. The clear opening was five inches between supports. The discharge could thus be controlled by varying the vertical position or by removing one or more slats (Figs. 2.13 and 2.14).

Calibration of various combinations of openings was made by measuring the corresponding discharge and the water surface elevation approximately 20 feet upstream of the control. For the range of discharges anticipated in the unsteady flow runs, it was concluded that the best combinations of openings was with the center three slats removed.

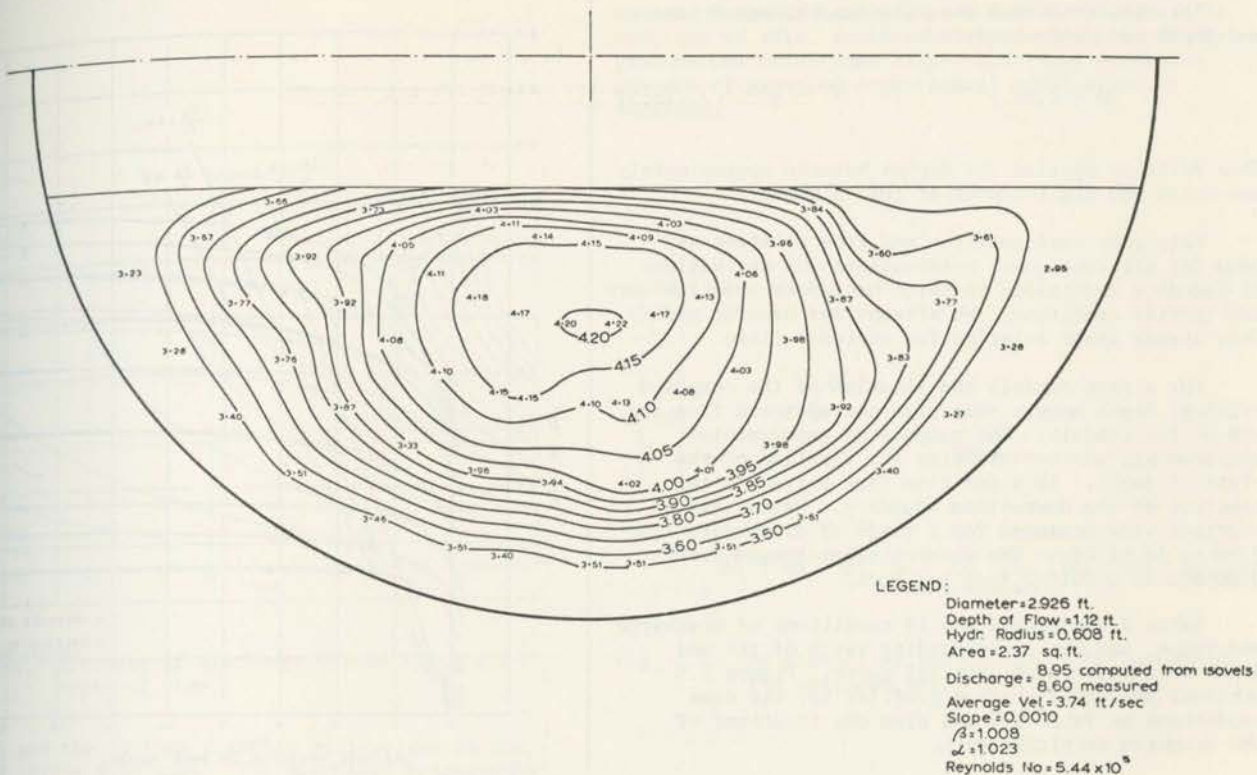


Fig. 5.1. Isovels for partially full pipe flow.

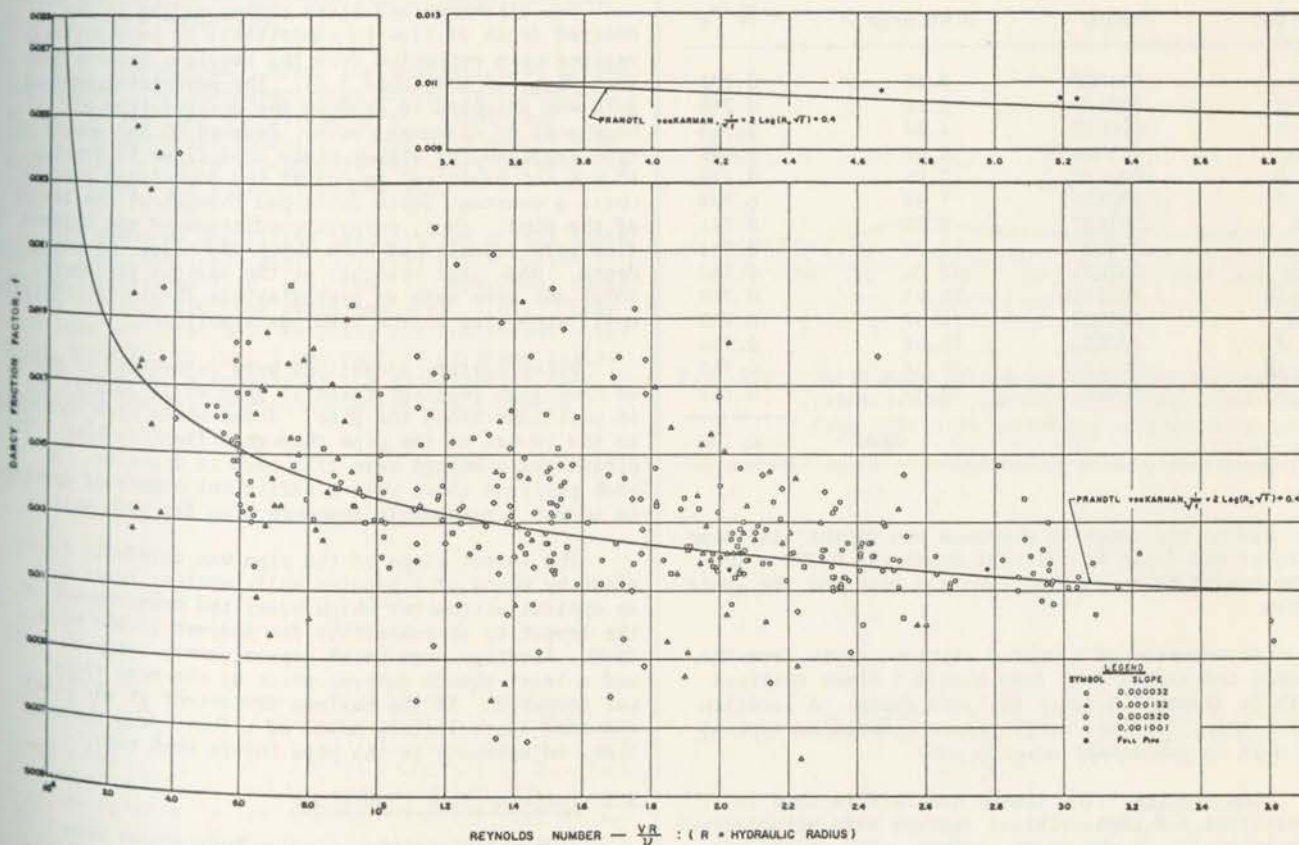


Fig. 5.2. Variation of Darcy-Weisbach friction factor f with the Reynolds number.

For this condition the relation between discharge and depth was determined to be

$$Q = 4.84 y^{1.35} \quad (5.3)$$

This relation applied for depths between approximately one-third and eight-tenths of full diameter.

This gate configuration and flow relation was used for all subsequent observations and evaluations of boundary conditions in which backwater profiles were the initial condition. No attempt was made to modify this steady state relation for unsteady flows.

For a free outfall the location of the computed critical depth occurs some distance upstream from the end of the conduit. The purpose of experimental measurements was to determine the location of the critical depth. This position then served as the location of the downstream boundary. Water-surface profiles were measured for a range of discharges from 2.10 to 16.62 cfs. The channel slope ranged from 0.000032 to 0.001022 foot per foot.

Table 5.2 presents the 14 conditions of discharge and slope, and the corresponding ratio of the end depth to the computed critical depth. Figure 5.3 presents the water-surface profiles for the same conditions as Table 5.2 and also the locations of the computed critical depth.

TABLE 5.2. Free outfall data
Diameter - 2.926 ft.

Run No.	Slope	Discharge	y_e/y_c
D1A	.001022	2.10	0.731
S2-9	.000132	3.26	0.746
S1-5	.000032	4.14	0.758
D2A	.001022	4.58	0.749
S1-6	.000032	7.96	0.776
S3-9	.000520	7.98	0.764
D3A	.001022	8.26	0.751
S1-7	.000032	11.98	0.761
D4A	.001022	12.92	0.740
S3-10	.000520	15.97	0.739
D5A	.001022	16.02	0.752
S1-8	.000032	16.04	0.726
S2-10	.000132	16.64	0.753
S1-9	.000032	19.62	0.761
Mean -			0.750

Within the range of observed end depths, the mean ratio of end depth to critical depth was 0.750. The ratio tended to be smaller than the mean for the lower depths.

The location of computed critical depth from the channel end varied from less than 3.5 times critical depth to almost 5.5 times critical depth. A location of 4.5 times critical depth was considered as typical and used in subsequent computations.

Flow regimes. The steady non-uniform flow in subcritical and supercritical regimes were established experimentally in the storm conduit. The steady non-uniform flows (backwater curves) at the hydrograph base discharge were used as initial conditions in computing the unsteady flow equations.

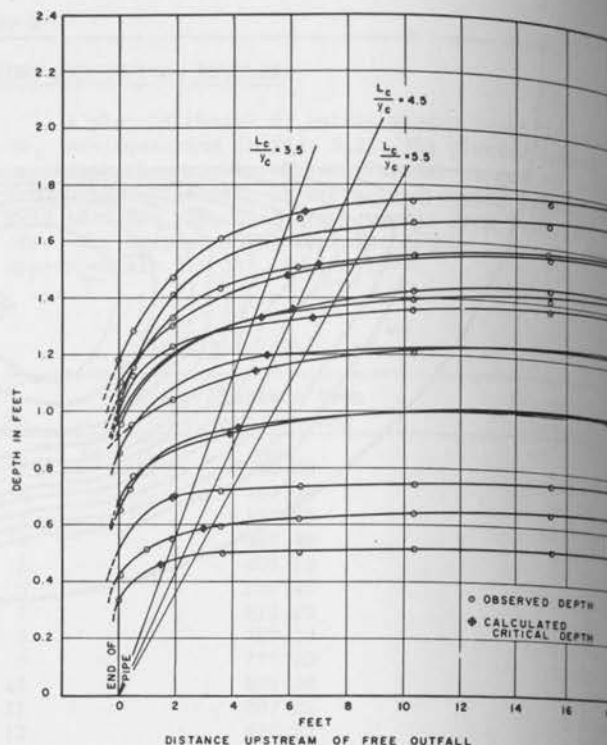


Fig. 5.3. Location of critical depth at the free outfall of a circular cross-section.

The discharge and slope corresponding to the desired depth of flow in subcritical or supercritical regimes were estimated from the previous observations (see Fig. 5.4 and Table 5.3). The downstream control gate was adjusted to produce the desired type of backwater or drawdown curve. Because of the length of time required for steady state conditions to develop, it was not practical to adjust the downstream control until a constant depth developed throughout the length of the pipe. Thus, several conditions of non uniform flow were established both above and below the normal depth. Hook gage readings at the various piezometer locations were made at approximately 15-minute intervals until such time as the readings stabilized.

Water surface elevations were determined by means of hook gage readings taken in gage wells located at 16 positions along the pipe. These wells were connected to the invert of the pipe through a flexible hose. The piezometer openings were 1/16-inch in diameter. At each position there were a sufficient number of openings to insure a reasonable response time for each well.

The invert slope of the pipe was carefully determined by means of a precise self-leveling level with an optical micrometer which permitted measurements of the invert to approximately the nearest 1/1000 of an inch. Readings were taken approximately every 20 feet and a least-square determination of the mean slope was computed. If the maximum deviations at any point exceeded approximately 3/100 of a foot, from the mean line, adjustments to the pipe invert were made.

5.2 Unsteady Flow Conditions

Inflow hydrographs. Inflow hydrographs were developed by manually manipulating a 26-inch diameter ball-valve at the upstream inlet of the main storm

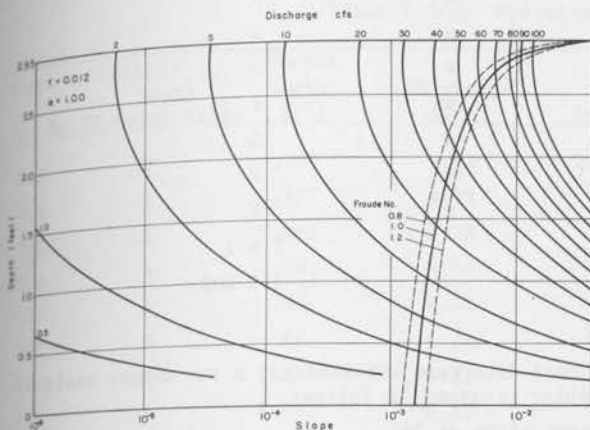


Fig. 5.4. Estimate of discharge versus slope and depth of flow.

conduit and the 12-inch diameter ball-valves on the lateral inflow pipelines. The discharge hydrographs were measured and recorded by pressure transducers connected across the orifices that were installed a short distance downstream of the valves. Figures 5.5 and 5.6 show the experimental observed discharge-inflow hydrographs of the main storm conduit and the lateral flow, respectively.

Wave propagation. After the generation of inflow hydrographs, the subsequent wave depths were measured at several stations downstream as the flood wave propagated along the conduit. Flood wave depths were measured by pressure transducers connected between the conduit invert and a set of manometers.

The measured quantities were plotted as (1) depth versus time relations (Fig. 5.17), (2) depth versus distance relations (Fig. 5.8), and (3) wave peak depth versus distance and time relations (Fig. 5.9). These relations are later used to check the analytical solutions described in Part I and Part IV of these series of four papers. A summary of the experimental test conditions for the wave measurements are given in Table 5.4.

Effort was made to produce runs which covered a maximum range of base and peak flow discharges. In addition to this, several attempts were made to reproduce the conditions of previous runs so that a measure of expected experimental error might be obtained.

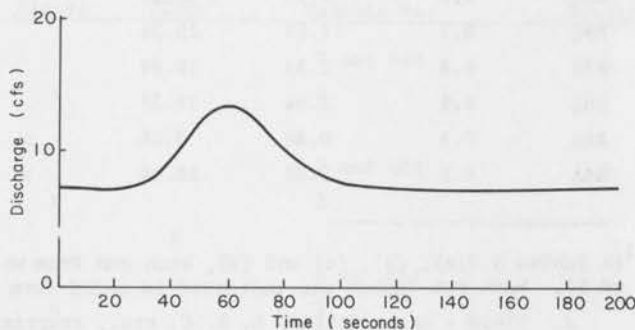


Fig. 5.5. An example of the discharge hydrograph at the upstream inlet of storm conduit.

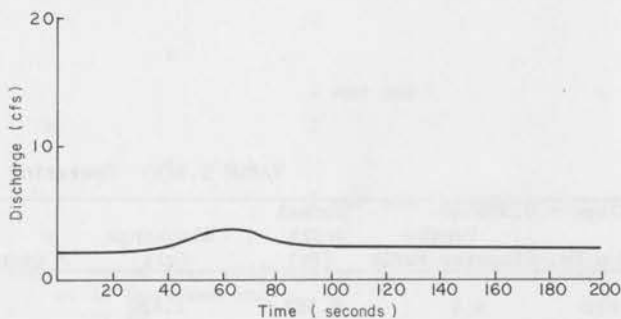


Fig. 5.6. An example of the lateral discharge hydrograph at the junction box being 410.7 feet from the inlet structure to the storm conduit.

TABLE 5.3(a) Operating conditions for June 3-9, 1964¹

Slope = 0.000132			Normal Depth (ft)	Discharge (cfs)	Gate Condition			Velocity Traverse Manhole No.	Outlet Profile
Run No.	Depth- Diameter Ratio				Open	Closed	Slats		
B2C	0.3		0.88	3.44			x	2	
B3C	0.4		1.17	4.71			x	2	
B4C	0.5		1.47	5.61			x	2	
B5C	0.6		1.76	10.08			x	2	
B6C	0.7		2.05	15.34			x	2	
B7C	0.8		2.34	18.94			x	2	
B8C	0.9		2.64	19.57			x	2	
B2A	0.3		0.88	3.26	x			2 & 3	x
B6A	0.6		2.05	16.64	x			2, 3 & end	x

¹In Tables 5.3(a), (b), (c) and (d), each run from which recorded data were obtained has a run number assigned to it. Each run number was indicated in coded form for computer programs as follows:

- Slope - using letters A, B, C, etc., starting with zero slope as "A".
- Base discharge - using numbers 1 through 9. 1 corresponding to the lowest and 9 to the highest.
- Downstream gate position - using letters A, B, C, etc. Letter A represents the free flow (no gate condition); B, gate closed with no slats; and C, D, etc., gate closed with progressively decreasing area by means of the slats.
- Lateral inflow conditions - using numbers 0 to 9. Zero indicates no lateral inflow, 1-9 will indicate the rate of inflow at each manhole.
- Unsteady flow conditions - this is either blank, R, or F, depending on whether the flow is of steady, rising, or falling discharge, respectively.
- Unsteady flow rate-of-change condition - using numbers 1-9 for progressively increasing rates of change of discharge, the longest rate indicated by 1, the highest rate by 9.

TABLE 5.3(b) Operating conditions for June 18-23, 1964

Slope = 0.000520			Normal Depth (ft)	Discharge (cfs)	Gate Conditions			Velocity Traverse Manhole No.	Outlet Profile
Run No.	Depth- Diameter Ratio				Open	Closed	Slats		
C1C	0.2		0.587	1.420			x	2	
C2C	0.3		0.88	2.040			x	2	
C3C	0.4		1.17	6.211			x	2	
C4C	0.5		1.47	7.960			x	2	
C5C	0.6		1.76	10.407			x	2	
C6C	0.7		2.05	14.097			x	2	
C7C	0.8		2.34	12.273			x	2	
C8C	0.9		2.64	18.353			x	2	
C3A	0.4		1.17	7.981	x			2, 3 & end	x
C6A	0.7		2.05	15.972	x			2, 3 & end	x

TABLE 5.3(c) Operating conditions for July 13-17, 1964

Slope = 0.001022 Depth- Run No. Diameter Ratio		Normal Depth (ft)	Discharge ¹ (cfs)	Gate Open	Condition Closed	Slats ²	Velocity Traverse Manhole No.	Outlet Profile
D1A	.2	.587	2.13	x			2 and end	x
D1B	.2	.587	2.13		x		2	
D1C	.2	.587	2.13			x		
D2A	.3	.88	4.71	x			2 and end	x
D2B	.3	.88	4.71		x		2	
D2C	.3	.88	4.71			x		
D3A	.4	1.17	8.23	x			2 and end	
D3B	.4	1.17	8.23		x		2	
D3C	.4	1.17	8.23			x		
D4A	.5	1.47	12.1	x			2 and end	x
D4B	.5	1.47	12.1		x		2	
D4C	.5	1.47	12.1			x		
D5A	.6	1.76	16.2	x			2 and end	
D5B	.6	1.76	16.2		x		2	
D5C	.6	1.76	16.2			x		
D6A	.7	2.05	20.2	x			2 and end	x
D6B	.7	2.05	20.2		x		2	
D6C	.7	2.05	20.2			x		
D7A	.8	2.34	23.5	x			2 and end	x
D7B	.8	2.34	23.5		x		2	
D7C	.8	2.34	23.5			x		
D8A	.9	2.64	25.7	x			2 and end	x
D8B	.9	2.64	25.7		x		2	
D9A-R1			5-30	Unsteady flow ³ -point gage readings at 1) Upstream end 2) MH 1 3) MH 2 4) MH 3 5) Downstream end				
D9A-F1			30-5					

- ¹ To be set at approximately this value (± 0.2)
- ² To produce maximum depth at downstream end.
- ³ To be repeated enough times to define profile of wave and times of transit.

TABLE 5.3(d) Operating conditions for August 1-8, 1964

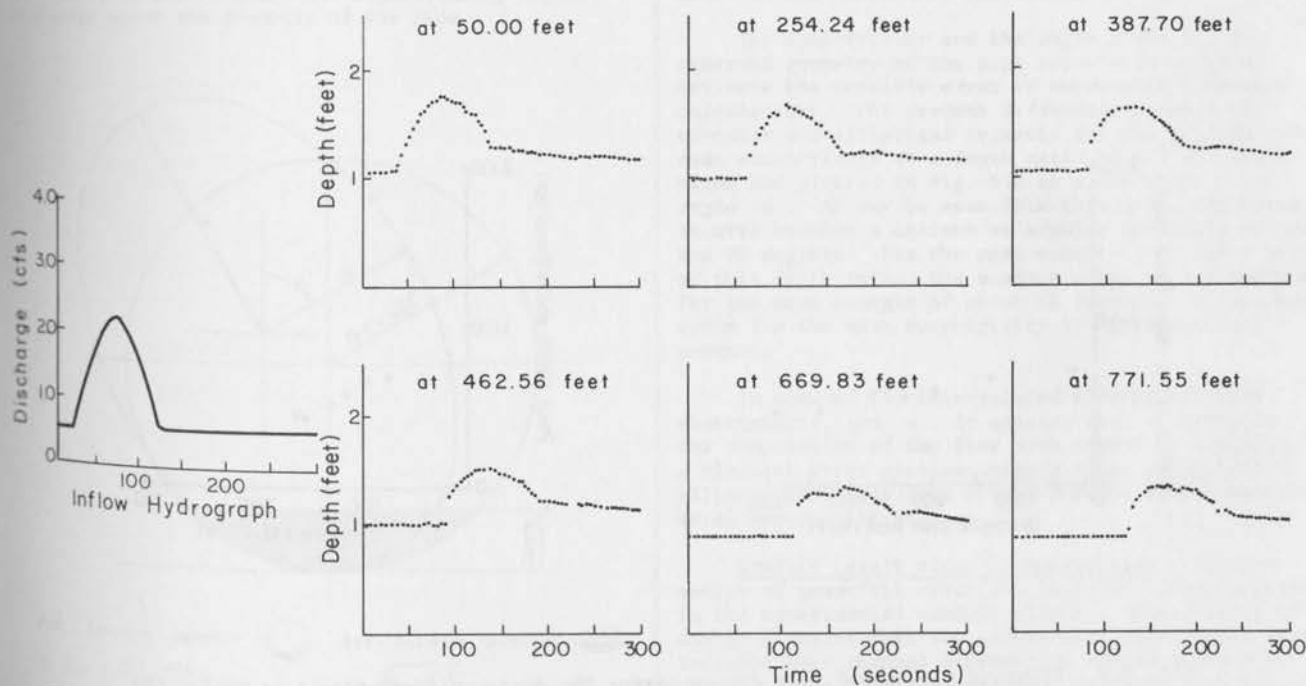
Slope = 0.0075		Normal Depth (ft)	Discharge (cfs)	Gate Condition		Velocity Traverse Manhole No.	Outlet Profile	Remarks
Run No.	Depth- Diameter Ratio			Open	Closed			
E1A	.2	.587	7.0	x		2 and end	x	
E1B	.2	.587	7.0		x	2		Note location of jump
E2A	.3	.88	16.0	x		2 and end	x	
E2B	.3	.88	16.0		x	2		Note location of jump
E3A	.4	1.17	25.0	x		2 and end	x	
E3B	.4	1.17	25.0		x	2		Note location of jump
E4A	.5	1.47	37.0	x			x	
E4B	.5	1.47	37.0		x	2		"
E5A	.6	1.76	48.0	x		2	x	"
E5B	.6	1.76	48.0		x	2		"
*E6A	.7	2.05	60.0	x			x	"
E6B	.7	2.05	60.0		x	2		"
E7A	.8	2.34	68.0	x		2	x	
E7B	.8	2.34	68.0		x	2		"
E8A	.9	2.64	70.0+	x		2	x	
E8B	.9	2.64	70.0+		x	2		"
E9A			70.0	Unsteady flow-point gage readings at 1) Upstream end 2) MH 1 3) MH 2 4) MH 3 5) Downstream end				

TABLE 5.4. Summary of data on experimental waves at CSU

Run	Slope	Base Flow Depth- Diameter Ratio	Base Flow Discharge (cfs)	Peak Flow Discharge (cfs)	Peak Discharge- Base Flow Ratio	Wave Duration (sec)	Wave Volume (cu-ft)
B2ARS1	.0001100	.21666	1.600	3.870	2.41875	27.000	31.215
B2AOS2	.0001100	.21666	1.540	13.020	8.45455	67.000	382.415
B2AOS3	.0001100	.20846	1.470	15.290	10.40136	78.000	579.665
B3AOS1	.0001100	.37660	4.180	6.940	1.66029	24.000	41.715
B3AOS2	.0001100	.35131	3.490	15.010	4.30086	59.000	473.435
B3AOS3	.0001100	.38993	4.350	17.930	4.12184	83.000	707.330
C2AOS2	.0005500	.16198	2.580	20.330	7.87984	77.000	621.699
C2AOS3	.0005500	.16494	2.410	26.250	10.89212	95.000	986.745
C3AOS1	.0005500	.36849	5.070	14.380	2.83629	42.000	199.295
C3AOS2	.0005500	.36190	4.910	21.920	4.46436	61.000	532.945
C3AOS3	.0005500	.35268	4.990	28.270	5.66533	80.000	924.520
D2ARS1	.0010300	.14729	2.990	19.250	6.43144	60.000	595.650
D2AOS1	.0010300	.15412	2.500	18.550	7.42000	70.000	538.800
D2AOS3	.0010300	.17292	2.990	32.870	10.99331	97.000	1549.860
D3AOS1	.0010300	.37216	7.610	21.450	2.81866	57.000	385.065
D3AOS2	.0010300	.34892	6.940	33.510	4.82853	90.000	1235.095

TABLE 5.4. Summary of data on experimental waves at CSU - (Continued)

Run	Slope	Base Flow Depth- Diameter Ratio	Base Flow Discharge (cfs)	Peak Flow Discharge (cfs)	Peak Discharge- Base Flow Ratio	Wave Duration (sec)	Wave Volume (cu-ft)
D5A0S3	.0010300	.33901	6.330	39.370	6.21959	104.000	1772.335
1 3	.0009900	.34482	6.992	28.071	4.01474	74.740	808.329
1 4	.0009900	.34311	7.592	32.582	4.29163	90.000	1198.544
1 8	.0009900	.20812	1.913	24.396	12.75290	85.470	890.639
1 9	.0009900	.21666	2.602	26.174	10.05925	89.000	1035.399
19909	.0009900	.21290	.904	30.484	35.72119	95.000	1385.264
1 10	.0009900	.20128	2.084	28.326	13.59221	110.000	1348.151
1 11	.0009900	.34994	6.855	22.912	3.34529	60.000	486.608
1 12	.0009900	.34652	6.831	30.655	4.48767	80.000	993.741
1 13	.0009900	.35849	7.280	36.535	5.01300	120.000	1938.801
19913	.0009900	.36464	7.466	36.936	4.94717	110.000	1766.589
0 1	.0009900	.30517	5.439	20.779	3.82037	150.000	1660.597
9 1	.0004800	.24981	2.472	13.577	5.49215	57.000	345.406
9 2	.0004800	.25152	2.336	20.338	0.01970	94.000	995.381
9 3	.0004800	.33336	5.033	15.214	3.02284	53.000	277.884
9 4	.0004800	.35885	5.292	22.349	4.22307	107.000	1090.523

Fig. 5.7. Observed wave forms given as depth versus time, computer plotted, Run No. 090004, $S_0 = 0.00048$.

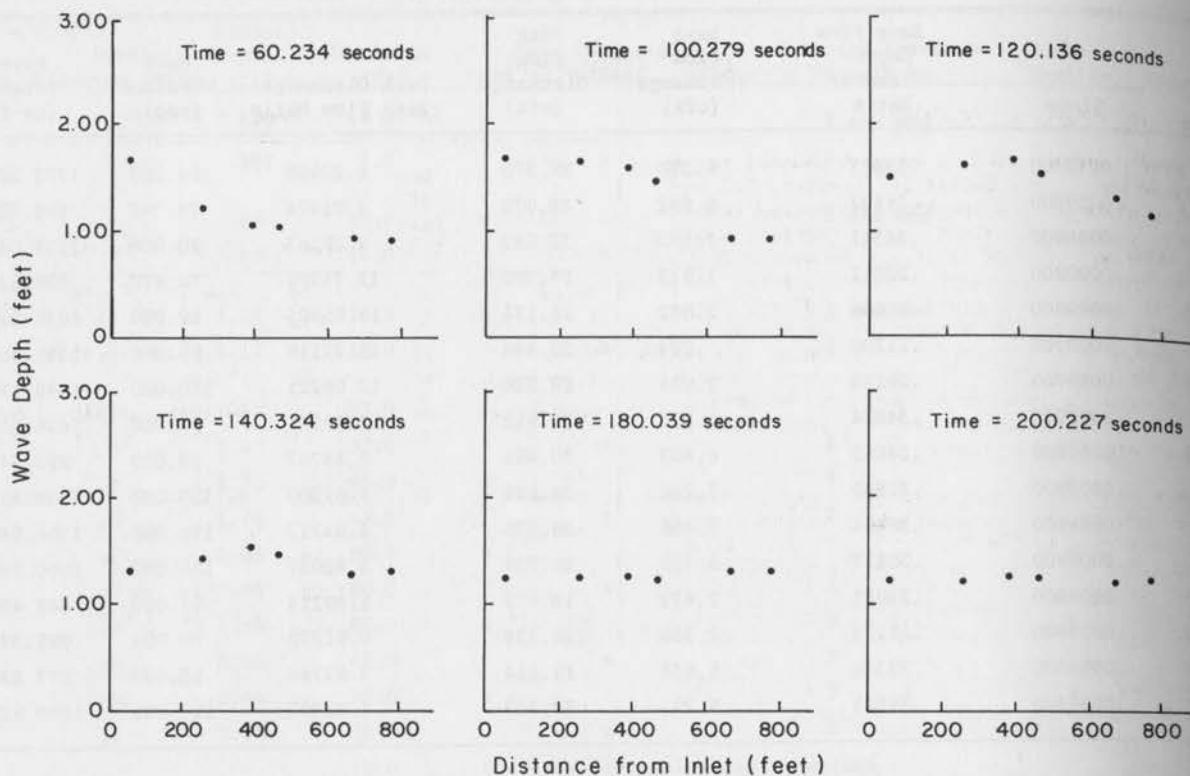


Fig. 5.8. Observed wave forms given as depth versus distance, computer plotted, Run No. 090004, $S_o = 0.00048$.

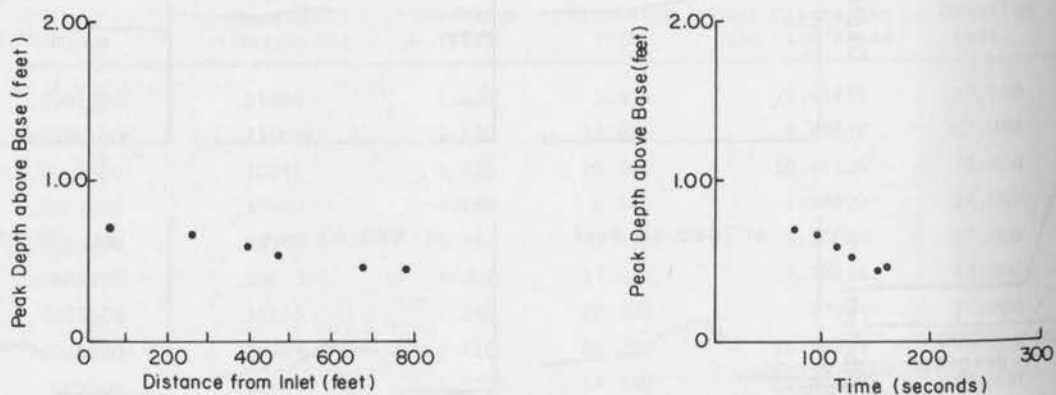


Fig. 5.9. Observed hydrograph peak depths versus the distance (left graph) or versus the time (right graph), computer plotted, Run No. 090004, $S_o = 0.00048$.

EXPERIMENTAL ERRORS

6.1 Errors in Geometric Variables

The steel conduit used as an open channel for the data analyzed herein was 3-feet in diameter, 1/2-inch thick rolled-plate with a longitudinal welded joint. The 20-foot sections were butt-welded together and were supported on steel rails at approximately 20-foot spacing, not necessarily at the conduit joints. As a result of the manufacturing process, handling, field welding, and the method of support it was not expected that the conduit would be perfectly circular or possess a straight line invert profile. The errors caused by physical departures from the mathematical geometric forms in the conduit cross-section and longitudinal invert slope are discussed here.

Conduit cross-sectional irregularities. Measurements were made on the inside diameter of the pipe at 60 degree intervals to the nearest 0.001 inch, both before and after painting the inside of the pipe. Before the inside of the pipe was painted measurements were made at cross sections spaced 20-feet apart. After painting, similar measurements were made at 10-ft. intervals. An elliptical cross-section (Fig. 6.1) was assumed and the corresponding major axis (a), minor axis (b), the direction of the principal axes (α), eccentricity ($e = \sqrt{1 - (b/a)^2}$), area, wetted perimeter, and hydraulic radius were computed based on the three measured diameters at each cross-section and its orientation angle (Table 6.1). The differences between the means of each of the parameters for the two surveys are not significant on the 5 percent level. This would indicate (1) that painting the pipe had no effect on the internal geometry, and (2) that doubling the number of stations did not significantly improve knowledge about the geometry of the pipe.

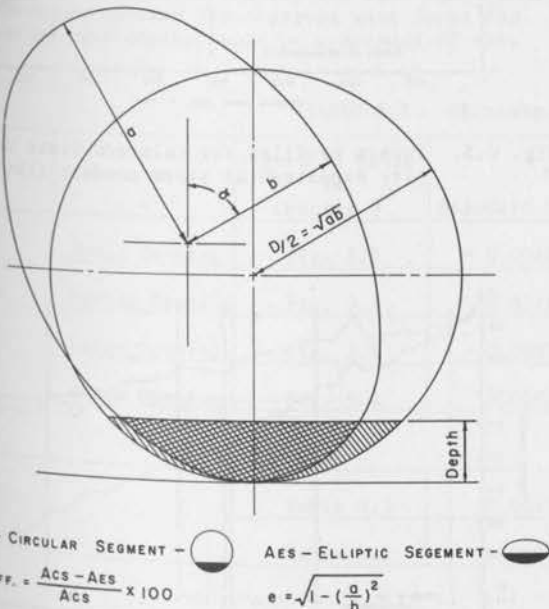


Fig. 6.1. Definition sketch for the relation of the circular and the elliptical cross-sections.

TABLE 6.1. Conduit geometry

	No. of Stations	Maximum	Mean	Minimum	Standard Deviation
Major Axis-(inches)	40	17.869	17.617	17.538	0.175
	84	17.913*	17.604	17.554	0.047
Minor Axis-(inches)	40	17.626	17.516	17.435	0.0375
	84	17.680	17.510	17.430	0.031
Eccentricity	40	0.176	0.1021	0.046	0.0310
	84	0.175	0.0993	0.051	0.0244
Alpha-(degrees)	40	165.58	84.84	13.71	46.5
	84	160.37	82.95	7.78	49.43
Area-(inch ²)	40	989.5	969.47	965.3	3.84
	84	994.9*	968.4	964.1*	3.94
Wetted Perimeter (inches)	40	111.51	110.373	110.13	0.2769
	84	111.82*	110.314	110.07*	0.2167
Hydraulic Radius (inches)	40	8.87	8.7785	8.76	0.0183
	84	8.89*	8.7742	8.75*	0.0181

* Occurred at same section

Accepting an average area of 968.41 square inches (6.725 sq ft) the mean diameter for the pipe is then 2.9262 feet. This figure was used for the pipe diameter in all subsequent calculations.

The eccentricity and the angle alpha for the observed geometry of the pipe serve as a means to estimate the possible error in subsequent hydraulic calculations. The percent difference between the circular and elliptical segments for the maximum and mean eccentricity at a depth ratio of 0.2 was determined and plotted in Fig. 6.2 as a function of the angle α . As may be seen from this plot, the error in area becomes a maximum at angular positions of zero and 90 degrees. For the mean eccentricity for a pipe of this depth ratio, the maximum error is 1.1 percent. For the mean α -angle of about 85 percent, the maximum error for the mean eccentricity is approximately 1 percent.

In view of the interrelated effects of depth, eccentricity, and α , it appears that an error in the computation of the flow area caused by assuming a circular cross section instead of an approximated ellipse, may range from 0 to 3 percent with 1 percent being representative.

Conduit invert slope irregularities. Another source of geometric error are inherent irregularities in the experimental conduit slopes. Conservation of energy dictates that for subcritical flow, depth must increase over channel depressions and decrease over channel rises. This fluctuation in depth resulted in observed depth values that differed from the corresponding theoretical depth values that had been computed assuming a perfectly constant invert slope. For computational purposes, the average values of experimental slopes were determined by running precision

level surveys along the pipe's invert and applying a least-square analysis to the elevation points obtained. This was accomplished by first adjusting the pipe to a predetermined position on the supporting rails and then leveling the pipe with a self-leveling level having an optical micrometer of at least count of 0.001 inch. The invert elevations were observed at 45 positions approximately 20 feet apart. A least-square determination of the slope and the deviations at each position was then made. If the deviations displayed a consistent or excessive trend in a given length, that portion of the conduit was readjusted, and the elevations redetermined.

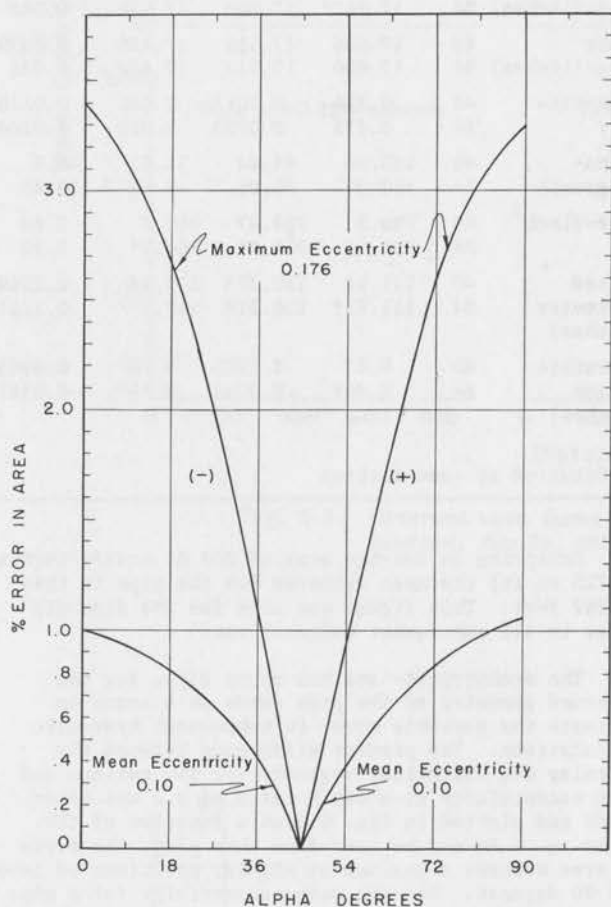


Fig. 6.2. Percent error in area of the storm conduit. (at the depth ratio of 0.2).

The standard deviations of the actual invert profile points about these least-square slopes are given in Table 6.2, and profiles of the conduit's invert for representative slopes are shown in Figs. 6.3 and 6.4. It may be concluded that for the slopes investigated, the observed depths may deviate from the ideal depths by an average of 0.01 to 0.03 ft.

6.2 Time-Difference Errors

A systematic error was introduced into all tests because the flow-measuring orifice was located 82.2 feet upstream of what was considered the beginning of the test conduit. For much of this distance the conduit was flowing full and providing instantaneous transmission of changes in flow. The distance between the point where a free surface formed and the beginning point of the test conduit, however, provided a varying

time lag between observed and computed data of from 8 to 14 seconds. That is, the time recorded by the orifice transducer for a given flow discharge led the time when that discharge actually reached the conduit test section by the amount of time it took for the wave to travel the 82.2 feet. To eliminate this difference in the comparisons, time lags were estimated visually from comparison of observed and computed waves (depth versus time) and all experimental times were adjusted by this amount.

TABLE 6.2. Data for Colorado State University experimental conduit invert slopes

Slope	Standard Deviation - Ft
0.0001110	0.0141
0.0005487	0.0141
0.001033	0.0136
0.0009930	0.0180
0.0004848	0.0213
0.0000052	0.0116
0.0000157	0.0135
0.0000303	0.0099
0.0001325	0.0099
0.0005197	0.0117
0.0010101	0.0119
0.0074578	0.0133
0.0200690	0.0141

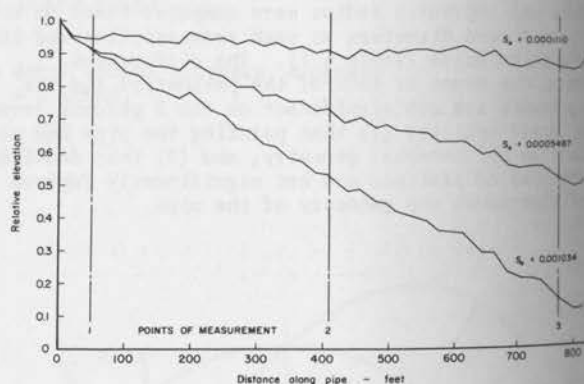


Fig. 6.3. Invert profiles for Colorado State University experimental storm conduit (1965).

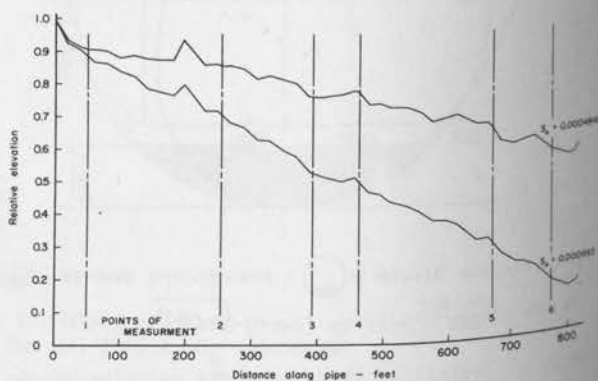


Fig. 6.4. Invert profiles for Colorado State University experimental storm conduit (1966).

6.3 Instrumentation Errors

Instrumentation errors were analyzed on the basis of calibration results. Since the true values of physical quantities can never be exactly measured, the calibration process of each instrument was considered to be an accurate estimate of the errors in the measured quantities.

A common method of giving bounds on the errors uses the standard deviation s defined by

$$s^2 = \sum_{i=1}^N \frac{(q_i - q_0)^2}{N-1}, \quad (6.1)$$

in which q_i is the individual reading observed in a given range, q_0 is the corresponding reading from a reference curve in the same given range as q_i and N is the total number of observations.

For a Gaussian distribution of errors and a significance level, the physical quantities measured by each instrument can then be expressed as

$$q_0 = q_i \pm 2s. \quad (6.2)$$

Equation 6.2 means that approximately 95 percent of the measured quantities q_i lie within the range of $\pm 2s$ of q_0 .

Table 6.3 gives a summary of the calibration results of the orifice meters, current meters, and pressure transducers (as described in Chapter 3) by using Equation 6.2.

6.4 Reproducibility Errors

As stated earlier, an attempt was made to perform some experiments having conditions exactly the same as conditions of some selected previous runs. This was done to have some measure of the errors due to reproducibility inherent in the experimental system. By running two runs under exactly the same conditions, the differences between the observed wave forms and the observed wave depths would be a measure of this

type of error. If no errors were generated by the system, the observed values for both runs would be the same. This manner of comparison would not, however, measure the random errors.

During the experiments, seven attempts were made to duplicate the conditions of previous runs. Only one of these runs, however, actually duplicated previous conditions. The other six runs could not be used in this evaluation because either the base or peak flows did not correspond to the earlier conditions or the time of one wave was longer than that of the previous run resulting in different total water volumes. The runs with matching conditions were 010013 and 019913.

To measure the differences between the two sets of observed data on the flood hydrographs, relative errors between the two areas of each of the six different observed wave forms (depth versus time) were computed. Corresponding relative errors between the two computed areas were obtained. The resulting average error of reproducibility was 5.86 percent.

Another measure of errors of reproducibility was obtained by computing the relative errors between corresponding maximum depth values at each of the six observation positions. For exactly the same inflow hydrographs, these errors should be zero. The average of the errors, however, is 8.13 percent. Again, the effect of the difference in inflow hydrographs was removed. In this case, the average relative error between the two computed conditions was 1.47 percent. The average error of reproducibility computed this way was 6.66 percent.

Observed values for the depth versus distance relations were not compared because of the influence on these depths by the experimental time shifts. An error of one or two seconds in the determination of these shifts could introduce errors as high as +0.1 foot to the observed depths. Other relations for the depth versus time data were not compared because of their dependence on the exact positioning of the observed peaks. The two errors may be combined to give an average error of reproducibility of approximately 6 percent.

TABLE 6.3. Estimate of instrumentation errors

		Figure or Table in Chapter 3	s Standard Deviation	Error Bounds with 2.5% Level	Range
Orifice Meters	Small Opening	Fig. 3.1	0.00413	$\alpha = \alpha \pm 0.00826$	Reynolds no. $3 \times 10^5 \sim 2 \times 10^6$
	Medium Opening	Fig. 3.1	0.00220	$\alpha = \alpha \pm 0.00440$	Reynolds no. $2 \times 10^5 \sim 2 \times 10^6$
	Large Opening	Fig. 3.1	0.00320	$\alpha = \alpha \pm 0.00640$	Reynolds no. $5 \times 10^5 \sim 2 \times 10^6$
	Large Opening	Fig. 3.2	0.01619 ft^3	$V = V \pm 0.03238 \text{ ft}^3$	Volume $8 \sim 15 \text{ ft}^3$
Current Meters		Table 3.1	0.0141 ft/sec	$V = V \pm 0.0282 \text{ ft/sec}$	Velocity $1 \sim 8 \text{ ft/sec}$
		Table 3.1	0.0451 ft/sec	$V = V \pm 0.0902 \text{ ft/sec}$	Velocity $8 \sim 16 \text{ ft/sec}$
Pressure Transducers			ft of	ft of	Pressure $\pm 1 \text{ psi}$
	Fig. 3.12		0.0002 water	$H = H \pm 0.0004 \text{ water}$	Voltage $\pm 3 \text{ volts}$
	Fig. 3.13		0.0003 water	$H = H \pm 0.0006 \text{ water}$	Pressure $\pm 5 \text{ psi}$ Voltage $\pm 3 \text{ volts}$

EVALUATION OF EXPERIMENTAL FACILITIES7.1 Summary of Characteristics of Experimental Facilities

(1) The large scale conduit experimental research facilities made it possible to carry out experiments with discharges ranging up to approximately 70 cubic feet per second and with depths of free-surface flow up to the full pipe diameter of 3 feet.

(2) The slope of the conduit can be varied from zero to 5 percent which covers the subcritical flow range as well as part of the supercritical range.

(3) Instrumentation was selected for maximum accuracy and reliability under field conditions.

(4) Calibration of instruments was carried to the point of reducing errors to practical, feasible minimums.

(5) A data acquisition analog-to-digital system was designed and constructed with outputs in a form ready for computation in digital computers.

(6) The facilities were designed such that all basic geometric and hydraulic properties can be measured with maximum accuracy.

(7) The facilities were designed so that accurate observations of a generated flood hydrograph at the inlet and the depth hydrographs at selected positions along the conduit, rather than for a precise reproduction of flood waves could be made.

(8) Typical experimental data are considered sufficiently accurate for analysis and synthesis of both the geometric and hydraulic conditions of the conduit and waves, as well as for comparison of numerically integrated and measured waves.

(9) Although the experimental facilities were built in the Outdoor Laboratory as described in Chapter 2, tests were conducted only during the summer months with air temperature in the range of 65°-85°F. No significant effects due to the differences in climatic conditions were observed.

7.2 Potential of Facilities for Further Experimental Investigations

Experience with the designed and constructed experimental research facilities indicates that the following subjects may be studied effectively, and eventually warrant further investigation or verification on these facilities:

(1) Friction losses in unsteady free-surface flow as compared to losses in steady uniform flow could be studied, after the further theoretical investigations have been undertaken, and the new experimental methods conceived and designed beyond all previous works.

(2) Experiments studying the movement of hydraulic bores through storm drains when the long flood waves pass to bores may have practical significance. If so, this significance can be checked through new analytical derivations made by experimental observations; for example, how the shape and friction factors of storm drains affect this movement.

(3) Experiments for better understanding the modification of flood waves in a complex conduit, such as changes in conduit cross-sections, bottom irregularities, presence of local humps or similar obstructions could be conducted.

(4) Experiments with the propagation of flood waves in a dry storm drain will contribute to a better understanding of this phenomenon.

(5) Experiments could be conducted to better understand and describe particular end boundary conditions, which can occur in practice with storm drains.

(6) Experiments of flow close to full cross-sections would permit an evaluation of conditions for full flow or intermittent full flow, free surface flow.

(7) Specially designed experiments of partially full steady flow would permit an evaluation of the air transported by the surface drag and the accompanying pressure reduction within the free space of the pipe.

REFERENCES

External References

1. Henn, W., Fundamentals of Hydrometry with Propeller Current Meters (in German). Forschungsh. Ver. dtsh. Ing., 1937, 8(385), 1-22.
2. Staus, A., Die Genauigkeit von Flugelmessungen bei Wasser Kraftanlagen. (The accuracy of current meter measurements at hydraulic power stations.) Berlin: Springer, 1926.
3. Jovanovic, S., Current Meter. Theory, rating and precision of measuring. Monographs, 6. Belgrade: Institute of Hydraulic Engineering.
4. Benini, G., Experimental Research on Current Meters Carried out in the Rating Tank (in Italian). 5th Congress on Hydraulics. Turin, 1957.

Internal References

From the Project Reports and Theses

1. Yevjevich, V. M., June 1961, "Unsteady Free Surface Flow in a Storm Drain" - General and Analytical Study. Report for the U. S. Bureau of Public Roads, Eng. Research Center, Colorado State University. CER61-VMY38.
2. Barnes, A. H., August 1965, "Predictability of Free-Surface Profiles for Steady Non-uniform Flow in a Circular Cross-Section", Ph.D. dissertation, Dept. of Civil Eng., Colorado State University.
3. Lorah, W. L., June 1966, "Free-Surface Flow Energy Losses in a 90° Junction Box", M.S. Thesis, Dept. of Civil Eng. Colorado State University.
4. Yevjevich, V. M. & Barnes, A. H., August 1966, "Hydraulic Roughness of Free-Surface Flow in a Smooth Steel Pipe", Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University.
5. Barnes, A. H., October 1966, "Velocity Distribution Factors in a Circular Cross-Section", Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University.
6. Pinkayan, S. & Barnes, A. H., Feb. 1967, "Unsteady Free-Surface Flow in a Storm Drain with Lateral Inflows", Report for U. S. Bureau of Public Roads, Eng. Research Center, Colorado State University.
7. Mitchell, J. S., April 1967, "Comparison of Mathematical versus Experimental Flood Wave Attenuation in Part-full Pipes", M.S. Thesis, Dept. of Civil Eng., Colorado State University. Also prepared as a report for U.S. Bureau of Public Roads.
8. Barnes, A. H., June 1969, "Unsteady Flow in a Storm Drainage System, Part A - Theory and Error Considerations", Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University. CER69-70AB1.
9. Barnes, A. H., January 1970, "Unsteady Flow in a Storm Drainage System, Part B - Hydraulic Parameters, Boundary and Initial Conditions", Report for U.S. Bureau of Public Roads, Eng. Research Center, Colorado State University. CER69-70AB1.

Key Words: Storm Drain, Flood Routing, Unsteady Flow, Wave Propagation.

Abstract: This second part of a four-part series of hydrology papers on flood routing through storm drains relates exclusively to experimental research facilities and experiments. The following subjects are presented: (a) design and construction of the experimental storm drain system as a special research outdoor facility; (b) instrumentation and its calibration; (c) description of the data recording system; (d) various experimental test conditions and their typical results, and (e) discussion of experimental errors.

A large conduit, 3 feet in diameter and 822 feet long, has been selected, designed, and constructed in the Outdoor Laboratory of Colorado State University to accurately measure geometric and hydraulic characteristics, as well as the propagating of flood waves. Instrumentation has been selected to suit the field conditions. The calibration of the instruments has been carried out to the point where there are relatively small errors. The data recording system was designed and constructed so that the output could be put either on cards or paper tapes and provide a direct input for computations on a digital computer. Only typical results of experiments carried out are described in this paper; experimental errors are reviewed in a summarized form.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 44 (November 1970) "Flood Routing Through Storm Drains, Part II, Physical Facilities and Experiments.

Key Words: Storm Drain, Flood Routing, Unsteady Flow, Wave Propagation.

Abstract: This second part of a four-part series of hydrology papers on flood routing through storm drains relates exclusively to experimental research facilities and experiments. The following subjects are presented: (a) design and construction of the experimental storm drain system as a special research outdoor facility; (b) instrumentation and its calibration; (c) description of the data recording system; (d) various experimental test conditions and their typical results, and (e) discussion of experimental errors.

A large conduit, 3 feet in diameter and 822 feet long, has been selected, designed, and constructed in the Outdoor Laboratory of Colorado State University to accurately measure geometric and hydraulic characteristics, as well as the propagating of flood waves. Instrumentation has been selected to suit the field conditions. The calibration of the instruments has been carried out to the point where there are relatively small errors. The data recording system was designed and constructed so that the output could be put either on cards or paper tapes and provide a direct input for computations on a digital computer. Only typical results of experiments carried out are described in this paper; experimental errors are reviewed in a summarized form.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 44 (November 1970) "Flood Routing Through Storm Drains, Part II, Physical Facilities and Experiments.

Key Words: Storm Drain, Flood Routing, Unsteady Flow, Wave Propagation.

Abstract: This second part of a four-part series of hydrology papers on flood routing through storm drains relates exclusively to experimental research facilities and experiments. The following subjects are presented: (a) design and construction of the experimental storm drain system as a special research outdoor facility; (b) instrumentation and its calibration; (c) description of the data recording system; (d) various experimental test conditions and their typical results, and (e) discussion of experimental errors.

A large conduit, 3 feet in diameter and 822 feet long, has been selected, designed, and constructed in the Outdoor Laboratory of Colorado State University to accurately measure geometric and hydraulic characteristics, as well as the propagating of flood waves. Instrumentation has been selected to suit the field conditions. The calibration of the instruments has been carried out to the point where there are relatively small errors. The data recording system was designed and constructed so that the output could be put either on cards or paper tapes and provide a direct input for computations on a digital computer. Only typical results of experiments carried out are described in this paper; experimental errors are reviewed in a summarized form.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 44 (November 1970) "Flood Routing Through Storm Drains, Part II, Physical Facilities and Experiments.

Key Words: Storm Drain, Flood Routing, Unsteady Flow, Wave Propagation.

Abstract: This second part of a four-part series of hydrology papers on flood routing through storm drains relates exclusively to experimental research facilities and experiments. The following subjects are presented: (a) design and construction of the experimental storm drain system as a special research outdoor facility; (b) instrumentation and its calibration; (c) description of the data recording system; (d) various experimental test conditions and their typical results, and (e) discussion of experimental errors.

A large conduit, 3 feet in diameter and 822 feet long, has been selected, designed, and constructed in the Outdoor Laboratory of Colorado State University to accurately measure geometric and hydraulic characteristics, as well as the propagating of flood waves. Instrumentation has been selected to suit the field conditions. The calibration of the instruments has been carried out to the point where there are relatively small errors. The data recording system was designed and constructed so that the output could be put either on cards or paper tapes and provide a direct input for computations on a digital computer. Only typical results of experiments carried out are described in this paper; experimental errors are reviewed in a summarized form.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 44 (November 1970) "Flood Routing Through Storm Drains, Part II, Physical Facilities and Experiments.

PREVIOUSLY PUBLISHED PAPERS OF VOLUME 2

Colorado State University Hydrology Papers

- No. 26 "The Investigation of Relationship Between Hydrologic Time Series and Sun Spot Numbers," by Ignacio Rodriguez-Iturbe and Vujica Yevjevich, April 1968.
- No. 27 "Diffusion of Entrapped Gas From Porous Media," by Kenneth M. Adam and Arthur T. Corey, April 1968.
- No. 28 "Sampling Bacteria in a Mountain Stream," by Samuel H. Kunkle and James R. Meimann, March 1968.
- No. 29 "Estimating Design Floods from Extreme Rainfall," by Frederick C. Bell, July 1968.
- No. 30 "Conservation of Ground Water by Gravel Mulches," by A.T. Corey and W.D. Kemper, May 1968.
- No. 31 "Effects of Truncation on Dependence in Hydrologic Time Series," by Rezaul Karim Bhuiya and Vujica Yevjevich, November 1968.
- No. 32 "Properties of Non-Homogeneous Hydrologic Series," by V. Yevjevich and R.I. Jeng, April 1969.
- No. 33 "Runs of Precipitation Series," by Jose Llamas and M.M. Siddiqui, May 1969.
- No. 34 "Statistical Discrimination of Change in Daily Runoff," by Andre J. Dumas and Hubert J. Morel-Seytoux, August 1969.
- No. 35 "Stochastic Process of Precipitation," by P. Todorovic and V. Yevjevich, September 1969.
- No. 36 "Suitability of the Upper Colorado River Basin for Precipitation Management," by Hiroshi Nakamichi and Hubert J. Morel-Seytoux, October 1969.
- No. 37 "Regional Discrimination of Change in Runoff," by Viboon Nimmannit and Hubert J. Morel-Seytoux, November 1969.
- No. 38 "Evaluation of the Effect of Impoundment on Water Quality in Cheney Reservoir," by J.C. Ward and S. Karaki, March 1970.
- No. 39 "The Kinematic Cascade as a Hydrologic Model," by David F. Kibler and David A. Woolhiser, February 1970.
- No. 40 "Application of Run-Lengths to Hydrologic Series," by Jaime Saldarriaga and Vujica Yevjevich, April 1970.
- No. 41 "Numerical Simulation of Dispersion in Groundwater Aquifers," by Donald Lee Reddell and Daniel K. Sunada
- No. 42 Theoretical Probability Distributions for Flood Peaks, Emir Zelenhasic, December 1970.
- No. 43 "Flood Routing Through Storm Drains, Part I, Solution of Problems of Unsteady Free Surface Flow in a Storm Drain", by V. Yevjevich and A.H. Barnes.

FLOOD ROUTING THROUGH STORM DRAINS

Part III

EVALUATION OF GEOMETRIC AND
HYDRAULIC PARAMETERS

by

V. YEVJEVICH and A. H. BARNES

November 1970



HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
Fort Collins, Colorado

Several departments at Colorado State University have substantial research and graduate programs oriented to hydrology. These Hydrology Papers are intended to communicate in a fast way the current results of this research to the specialists interested in these activities. The papers will supply most of the background research data and results. Shorter versions will usually be published in the appropriate scientific and professional journals, or presented at national or international scientific and professional meetings and published in the proceedings of these meetings.

The investigations leading to this paper on unsteady free-surface flow in a long storm drain were supported by the U.S. Bureau of Public Roads, Federal Highway Administration, during 1960-1970 and by the U.S. Public Health Service during 1962-64. The research was conducted in the Hydrology and Water Resources Program of Civil Engineering Department at Colorado State University, Fort Collins, Colorado, U.S.A.

The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Federal Highway Administration or the Public Health Service.

EDITORIAL BOARD

Dr. Arthur T. Corey, Professor, Agricultural Engineering Department

Dr. Robert E. Dils, Professor, College of Forestry and Natural Resources

Dr. Vujica Yevjevich, Professor, Civil Engineering Department

FLOOD ROUTING THROUGH STORM DRAINS

Part III

EVALUATION OF GEOMETRIC AND
HYDRAULIC PARAMETERS

by

V. Yevjevich and A. H. Barnes

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO 80521

November 1970

No. 45

ACKNOWLEDGMENTS

The writers of this paper gratefully acknowledge the support and cooperation of the U. S. Bureau of Public Roads, Federal Highway Administration, in the research on flood movements through long storm drains conducted from 1960 to 1970. The writers also acknowledge the U. S. Public Health Service, National Institute of Health, for their additional support during 1962-64.

The initiative, cooperation and support given by Mr. Carl F. Izzard to this project on flood movement through storm drains is particularly acknowledged. Mr. Izzard, presently Director, Office of Development, Federal Highway Administration, U. S. Department of Transportation, was Chief, Hydraulic Research Division, U. S. Bureau of Public Roads at the start of the project. Further acknowledgment is extended to Mr. Charles F. Scheffey, Director, Office of Research, Federal Highway Administration, for his cooperation and encouragement. Dr. Dah-Cheng Woo, Senior Hydraulic Engineer, Federal Highway Administration, has cooperated extensively with this project. His reviews and suggestions pertaining to all reports, theses and other documents produced on the project have been particularly helpful.

Acknowledgment is given to Mr. William L. Lorah, Hydraulic Engineer, Wright Water Engineers, Denver, Colorado, for his contributions in the experimental study of the energy losses of junction boxes during his M.S. graduate work at Colorado State University in 1966. His advisor, Professor George L. Smith of the Civil Engineering Department, is also acknowledged. Several other graduate students, either through their thesis research or through direct work, contributed to the project. These theses and reports are listed in the bibliography of Hydrology Paper No. 44 under "Internal References." Dr. Shih-Tun Su, Post-Doctoral Fellow, Civil Engineering Department, Colorado State University, using existing data, assisted the writers in finishing this paper.

TABLE OF CONTENTS

Chapter		Page
Acknowledgments		iii
Abstract.		vii
1	INTRODUCTION.	1
	1.1 Objective of Evaluation of Geometric and Hydraulic Parameters.	1
	1.2 Evaluation of Parameters Under the Conditions of Gradually Varied Free-Surface Unsteady Flow.	2
	1.3 Specifications about Geometric and Hydraulic Parameters that Define Coefficients of the Two Partial Differential Equations.	2
	1.4 Lateral Inflows.	3
	1.5 Reasons for Discussing Geometric and Hydraulic Parameters in a Separate Hydrology Paper.	3
2	GEOMETRIC RELATIONS FOR THE CIRCULAR EXPERIMENTAL CONDUIT	4
	2.1 Characteristics of a Circular Cross Section.	4
	2.2 Errors in Parameters as a Function of Errors in Depth.	4
	2.3 Errors in Parameters as a Function of Ellipticity.	5
	2.4 Physical Characteristics of the Conduit.	6
	2.5 Errors Due to Vertical Displacements of a Circular Cross Section	6
3	HYDRAULIC RESISTANCE OF THE EXPERIMENTAL CONDUIT.	10
	3.1 Expressions for the Friction Factor of Partly Flowing Conduits	10
	3.2 Observations on Hydraulic Resistance in the Experimental Conduit	10
	3.3 Analysis of Experimental Results	11
	3.4 Effect of Depth on Friction Factor	13
	3.5 Effect of Measurement Errors on Computed Friction Factors.	13
	3.6 Conclusions.	14
4	ENERGY LOSSES AT JUNCTION BOXES	16
	4.1 Definition of Losses at Junctions.	16
	4.2 Brief Review of Previous Investigations of Losses at Junction Boxes.	16
	4.3 Experimental Facilities.	16
	4.4 Energy Loss Evaluations.	17
	4.5 Effect of the Junction Boxes on Downstream Conduit Flow Resistance	20
5	VELOCITY DISTRIBUTION COEFFICIENTS	21
	5.1 Definition of Velocity Distribution Coefficients	21
	5.2 Approximate Relation Between α and β Coefficients.	21
	5.3 Evaluation of Velocity Distribution Coefficients	22
	5.4 Results of Experimental Investigations	24
	5.5 Discussion of Experimental Results	27
	5.6 Summary of Results	29
6	BOUNDARY CONDITIONS OF EXPERIMENTAL CONDUIT	30
	6.1 Definitions of Boundary Conditions	30
	6.2 Free Outfall Boundary Conditions	30
	6.3 Controlled Outfall Conditions.	31
7	INITIAL CONDITIONS FOR EXPERIMENTS AND NUMERICAL INTEGRATIONS	33
	7.1 Definition of Initial Conditions	33
	7.2 Computation Procedure and Results.	33
	7.3 Discussion of Comparison of Backwater Calculations	33
	7.4 Conclusions.	33
8	CONCLUSIONS AND RECOMMENDATIONS	37
	8.1 Conclusions.	37
	8.2 Recommendations for Further Experimental Research.	37
	REFERENCES.	58

LIST OF FIGURES AND TABLES

Figure		Page
2.1	Definition sketch for parameters of a circular cross section	4
2.2	Relative geometry of the circular cross section as it varies with relative depth	4
2.3	Geometry of circular cross section represented as relative error in parameter versus relative error in depth.	5
2.4	Definition sketch for the relation of circular and elliptical cross sections	5
2.5	Percent difference in area versus eccentricity and depth	6
2.6	Percent error in area at depth 0.2D, for the experimental conduit.	7
2.7	Definition sketch of the effect of bottom irregularities	7
3.1	Variation of Darcy-Weisbach friction factor f with Reynolds number R_e (computed from small reaches of the conduit)	11
3.2	Variation of Darcy-Weisbach friction factor f with Reynolds number R_e based on the slope representative of the entire length of experimental conduit	12
3.3	Variation of Darcy-Weisbach friction factor f with depth of flow in a circular cross section	14
4.1	Flow conditions for $S_o = 0.000537$, $Q_T = 5.56$ with lower inlet.	17
4.2	Flow conditions for $S_o = 0.000537$, $Q_T = 8.44$ with lower inlet.	17
4.3	Flow conditions for $S_o = 0.000537$, $Q_T = 3.30$ with upper inlet.	17
4.4	Flow conditions for $S_o = 0.000537$, $Q_T = 7.40$ with upper inlet.	17
4.5	Energy nomenclature	18
4.6	The P_r versus Q_T relationship for the upper inlet.	19
4.7	The P_r versus Q_T for various D_r relationship for the lower inlet	19
4.8	The relation of friction factor f versus Reynolds number R_e : (a) downstream, upper inlet, (b) downstream, lower inlet and (c) upstream, upper and lower inlets.	20
5.1	Velocity distribution coefficients versus friction factor f	22
5.2	Relation of α and β versus friction factor f	23
5.3	Isovels for partially full pipe flow	24
5.4	Velocity distribution coefficients versus Reynolds number	27
5.5	Velocity distribution coefficients versus depth of flow in a circular cross section.	29
6.1	Location of critical depth at the free outfall of a circular cross section	31
Tables		
2.1	Geometry of the experimental conduit	6
2.2	Theoretical effect of bottom irregularity on water surface profiles.	9
2.3	Slope deviations of the experimental conduit	9

LIST OF FIGURES AND TABLES - Continued

<u>Table</u>		<u>Page</u>
5.1	Reproducibility and effects of various factors on velocity distribution coefficients	25
5.2	Velocity distribution coefficients in the conduit of 2.926 feet in diameter.	26
5.3	Velocity distribution coefficients as a function of depth.	28
5.4	Velocity distribution coefficients as a function of mean velocity.	28
6.1	Observed end to critical depth ratios.	31
7.1	The root-mean-square deviations for M-1 type backwater curves with the normal depth smaller than the initial depth	34
7.2	The root-mean-square deviations for M-2 type backwater curves with the normal depth greater than the initial depth	35
7.3	Difference in computed and observed depths, root-mean-square values in feet for M-1 type curves	36
7.4	Difference in computed and observed depths, root-mean-square values in feet for M-2 type curves	36

ABSTRACT

The primary objective of this third part of a four-part series of hydrology papers on flood routing through storm drains is to present results on the investigation (experimental and analytical) of the geometric and hydraulic parameters of the experimental conduit; these parameters in turn define the coefficients in the two quasi-linear hyperbolic partial differential equations of gradually varied free-surface unsteady flow. As the accuracy of the evaluations of these parameters improves, the coefficients of the partial differential equations become more reliable. The final objective of this evaluation of parameters is to determine the extent to which the conduit geometry and hydraulic parameters influence the ability to predict the free-surface non-uniform steady or gradually varied unsteady flow, in a channel with a circular cross section.

The errors in cross section geometric parameters are analyzed in a conduit not ideally circular but approximated by an elliptical shape; errors are also analyzed when the undulations in the longitudinal slope of the conduit affect the predicted water surface profiles and thus the geometric parameters for a given water depth. The variation of hydraulic parameter of resistance, expressed by the Darcy-Weisbach friction factor, is experimentally determined and compared to the theoretical relation to Reynolds number. The friction factor may be satisfactorily expressed by the Prandtl-von Karman equation for turbulent smooth boundary. Energy losses in a 90 degree junction box are studied and relations are determined among the loss of power at the junction box, the ratio of the lateral inflow to the combined lateral and main inflow into the junction box, and the depth upstream from the junction box. The velocity distribution coefficients are shown to vary with the Darcy-Weisbach friction factor, and consequently, with the depth of flow. Boundary conditions for both controlled and free outfall are experimentally determined and approximated by power functions. Two types of steady non-uniform flow profiles, M1 and M2, are observed and analyzed as the initial conditions for the unsteady flow computations.

Part III

EVALUATION OF GEOMETRIC AND
HYDRAULIC PARAMETERS

by

V. Yevjevich* and A. H. Barnes**

Chapter 1

INTRODUCTION1.1 Objective of Evaluation of Geometric and Hydraulic Parameters

The objective of evaluating various geometric and hydraulic parameters of an experimental conduit was to arrive at sufficiently accurate knowledge of the coefficients in the two partial differential equations of gradually varied free-surface unsteady flow in a storm drain. The conduit is described in detail in Part II of this series of four parts, (Hydrology Paper No. 44). The numerical integration of these two quasi-linear, partial differential equations can not produce accurate solutions if the coefficients of these equations are not sufficiently and reliably evaluated and expressed, with well estimated constants. These two partial differential equations in dimensionless form are (see Eqs. 3.23 and 3.19, Part I, Hydrology Paper No. 43):

$$\frac{A}{VB} \frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} + \frac{1}{V} \frac{\partial y}{\partial t} = \frac{q}{VB} \quad (1.1)$$

and

$$\frac{\alpha V}{g} \frac{\partial V}{\partial x} + \frac{\beta}{g} \frac{\partial V}{\partial t} + \frac{\partial y}{\partial x} = (S_0 - S_f) - \beta \frac{Vq}{Ag} \quad (1.2)$$

The coefficients of these equations therefore, are A/VB , $1/V$, and q/VB in Eq. 1.1., and $\alpha V/g$, β/g , 1 , and $[(S_0 - S_f) - \beta Vq/Ag]$ in Eq. 1.2. Using the Darcy-Weisbach equation for energy gradients, $S_f = fV^2/8gR$, and expressing the friction factor equation as a function of Reynolds number, $f = \psi(R_e) = \psi(VR/v)$, the parameters that appear in the coefficients of Eqs. 1.1 and 1.2 are: cross section area (A), water surface width (B), hydraulic radius (R), and the bottom slope (S_0), as geometric parameters, and the mean velocity ($V = Q/A$), lateral continuous inflow or outflow as discharge per unit length (q), friction factor (f), the velocity distribution coefficients (α and β), the kinematic water viscosity (v) in the Reynolds number, and the gravitational acceleration (g), as hydraulic parameters. These parameters are constants or functions of either of the two or both of the dependent variables, the mean velocity V , and the depth y of the flow in the conduit. They may change also as the independent variables x and t

change for given values of V and y . In general, the geometric parameters of cross sections and the longitudinal profiles of natural erodible channels change with the change of these two independent variables, the distance along the channel, and the time. The kinematic viscosity is a function of water temperature. Whatever the case may be, it is necessary to evaluate as accurately as feasible the geometric and hydraulic parameters that define the coefficients in the two partial differential equations as functions of any, several, or all of the four variables V , y , x and t , as functions of temperature, or as constant parameters.

One of the main objectives of the study of flood routing through storm drains is to compare analytical waves (numerically integrated) with physical waves (observed in the experimental facilities). The purpose of evaluating the parameters presented in this hydrology paper is to improve accuracy in computing the coefficients of the two partial differential equations, so that comparison of analytical and physical waves can be as accurate as practically feasible under the conditions of experimentation and research conducted in this study with well defined experimental facilities and conditions.

The comparison of computed depths of analytical waves and the observed depths of physical waves at any point in time and space depends, therefore, on the accuracy of the geometric and hydraulic parameters. The individual effects of errors in these parameters may tend to counter balance each other in the numerical solution of partial differential equations or they may tend to accumulate. For example, an estimated friction factor higher than the actual tends to increase the initial depths, and all depths in general, while an estimate of the cross sectional area greater than actual tends to produce computed depths of flow that are less than actual depths.

Justification and verification of a theoretical hypothesis depend on the agreement between the predicted and the observed results. Any disparity between the computed and measured values must be due to basically two influences. The first influence includes limitations of the theoretical analysis of waves and its computational methods of integration. The second influence includes limitations in physical measurements and the impossibility of total precision

* Professor-In-Charge of Hydrology and Water Resources Program, Dept. of Civil Engineering, Colorado State University.

**Associate Professor of Civil Engineering, Colorado State University.

in determining the parameters that define the coefficients of partial differential equations.

The objective of the discussion in the following chapters is to provide an evaluation of the significant physical parameters (either as constants or as functions of velocity and/or depth), which enter into the coefficients of differential equations of gradually varied free-surface unsteady flow. These evaluations are based on the same experimental system in which the physical waves are observed. Thus, the discrepancy between computed and observed results could be caused by random errors rather than by systematic errors in parameter evaluations.

In any applied situation, the analyst and the designer must estimate values of the appropriate geometric parameters, friction factors, local energy losses, velocity distribution coefficients, and boundary and initial conditions. The differences between computed and observed results include both the systematic errors of estimation and the random errors of observation. The effect of systematically varying the hydraulic parameters with regard to the resulting analytical waves has been also studied and is both briefly mentioned in this paper, and discussed in detail in Part I, Hydrology Paper No. 43.

Another objective of this investigation of geometric and hydraulic parameters is to provide insight into the various sources of uncertainties in the numerical solutions of analytical waves, which result from the uncertainties in coefficients of partial differential equations. The potential experimental research that could improve knowledge about the coefficients in the two partial differential equations of unsteady flow may be also inferred from the results presented.

1.2 Evaluation of Parameters Under the Conditions of Gradually Varied Free-Surface Unsteady Flow

In general, parameters such as friction factors and velocity distribution coefficients as functions of Reynolds number for hydraulically smooth boundaries should be different in unsteady flow and steady flow, because of the differences in boundary layers of a steady and unsteady flow. These differences should be larger if a wave is less gradually varied. No attempt was made in this study to find whether there are significant differences in friction factors between gradually varied unsteady-flow conditions and corresponding steady-flow conditions. No significant difference could be detected, as discussed in Chapter 3. Because the experiments related to particular experimental facilities and instrumentation did not permit a detection of significant differences in the friction factor between the gradually varied waves and the steady flow, the influence of unsteadiness on velocity distribution coefficient was not studied either.

The results and discussions presented in the following chapters refer to steady flow conditions, under the general assumption that the relations obtained for parameters in steady flow are sufficiently accurate for gradually varied waves. Also, as it is shown in Part I, Hydrology Paper No. 43, the small relative errors in friction factor (f) and in the velocity distribution coefficients (α, β) result in still smaller relative errors in the water depths of numerically integrated partial differential equation of given waves.

1.3 Specifications about Geometric and Hydraulic Parameters that Define Coefficients of the Two Partial Differential Equations

The geometric parameters depend on the depth y (as one of the two dependent variables in partial differential equations) for a circular prismatic conduit and are constants along the conduit and in time for a given depth. The geometric parameters A , B , and R , or the area, surface width, and hydraulic radius, respectively, must be known as functions of depth for input into Eqs. 1.1 and 1.2. The bottom slope (S_0) as another geometric parameter is an arbitrary parameter. It is constant in this study for any given run, and is assumed independent of any other variable or parameter. The area (A) and the surface width (B) are functions of the depth (y) only. As the energy gradient (S_f) is a function of the hydraulic radius (R), and as $R = A/P$, with P the wetted perimeter, it is necessary to know $A(y)$ and $P(y)$ and through them $R(y)$. The errors in the measurement of depth (y), the non-circularity approximated by the ellipticity of the conduit in this study, the angle of the minor axis of the fitted ellipse with the vertical (tilting of the ellipse), the errors in the conduit diameter (D), and the undulations in the bottom slope (S_0), all affect the accuracy of the three parameter functions, $A(y)$, $P(y)$, and $R(y)$, which define the coefficients of the partial differential equations. The computation of these three functions and the errors involved are defined in this paper under evaluation of geometric parameters. Chapter 2 is concerned with the geometric parameters of the experimental conduit, particularly the discussion of effects of errors in depth, errors due to approximate ellipticity, inclination of the elliptic profiles to the vertical, errors in conduit diameter, and effects of the undulation of the invert of the straight aligned conduit.

It was necessary to obtain by experiments the relation between the friction factor and the Reynolds number, instead of using a constant friction factor of rough pipes, because the energy loss measurements in the experimental conduit have shown the conduit to be a smooth boundary pipe. The evaluation is given in Chapter 3 using the Darcy-Weisbach friction factor (f) as the measure of the smooth boundary roughness.

Apart from the resistance to flow of the smooth conduit boundary, resistances may occur in the form of additional energy losses at sudden changes in the geometry along the conduit. All losses at short structures along the conduit, which deviate from the circular prismatic conduit, may be encompassed as losses at geometric singularities. The only important singularity studied for energy losses was the junction box where the lateral inlets join the main conduit and produce additional energy losses, both because of the geometry of the junction box and because of the inflow from the lateral inlets into the main flow. These losses are studied both on a small model and in the experimental conduit and are described in Chapter 4. The experimental facilities used to determine the results are described in Part II, Hydrology Paper No. 44.

The two velocity distribution coefficients, α and β , enter into the coefficients of partial differential equations. Experimental study has been carried out in the main conduit of the experimental facility to determine both the variation of these coefficients with hydraulic parameters and their

approximate and constant representative values. The procedure and the results are described in Chapter 5.

It can be stated that no numerical solution of partial differential equations of gradually varied free-surface unsteady flow is reliable without accurate evaluation of the corresponding boundary and initial conditions. Chapter 6 and 7 describe the analysis and experimental results in establishing reliable boundary and initial conditions of the experimental conduit for the objectives of this study.

Because the temperature of water in the experimental conduit drawn from the bottom of the Horse-tooth Reservoir near the CSU Engineering Research Center was constant (about 50°F) during experiments, the variations of kinematic viscosity (ν) with temperature, and through it the variation of R_e , f , α , and β with temperature were not studied.

1.4 Lateral Inflows

The lateral inflows into storm drains, or the outflows out of them, may be either concentrated at points or distributed along them. In most cases the inflows into storm drains are concentrated at junction boxes. Also, in a majority of cases the outflow is concentrated at some main points of a drain. However, cases can be conceived where some inflows into storm drains may be continuous along a conduit (longitudinal drainage of ground water), and the outflow may be continuous either as the longitudinal seepage out of the drain or as the spillover along the top of a drain. Equations 1.1 and 1.2 are designed to deal with these continuous inflows or outflows, with q measured in discharge per unit length of a drain. This case was not investigated in this study, however. In other words, the potential physical inflows or outflows as discharges per unit length of the storm drain have not been evaluated, and the influence of errors in these

flows in the comparison between the analytical and physical waves has not been studied. Therefore, the coefficients q/VB in Eq. 1.1 and $\beta Vq/Ag$ in Eq. 1.2, with q the distributed lateral inflow, were not investigated.

The known concentrated lateral inflows are studied by considering their boundary conditions effects on Eqs. 1.1 and 1.2, without requiring the study of any new geometric or hydraulic parameter. Chapter 4 presents the influence of these concentrated lateral inflows on the energy losses at the singularity of energy losses at the junction boxes.

1.5 Reasons for Discussing Geometric and Hydraulic Parameters in a Separate Hydrology Paper

Apart from the use of the complete quasi-linear hyperbolic partial differential equations of gradually varied free-surface unsteady flow, with their derivation and discussion given in Chapter 3, Part I, Hydrology Paper No. 43, it was considered that a reliable estimate of geometric and hydraulic parameters that define coefficients of partial differential equations was necessary. The better these estimates and evaluations are, the better the correlation is between analytical waves (numerically integrated) and physical waves (observed on the experimental facilities).

This study is basically concerned with the accuracies attainable in using the most accurate parameters in flood routing through storm drains. Thus, the evaluation of all coefficients in partial differential equations is of a crucial importance. The writers of this paper consider a separate paper on this evaluation justified. The material presented shows the accuracy attained for these parameters. Further improvements in evaluating the parameters of boundary conditions are recommended.

GEOMETRIC RELATIONS FOR THE CIRCULAR EXPERIMENTAL CONDUIT

Geometric irregularity errors, which are a function of depth of flow in the conduit, and errors made in measuring this depth of flow influence the predictions of flow characteristics in an open channel. The following is an evaluation of these errors and their relative significance.

2.1 Characteristics of a Circular Cross Section

As shown in Fig. 2.1, the geometric parameters of a circular cross section which influence the flow of a free-surface liquid are defined as follows

- 1 - Diameter, D
- 2 - Depth, y
- 3 - Central angle, θ
- 4 - Wetted perimeter, P
- 5 - Surface width, B
- 6 - Area, A .

Derived parameters of significance are:

- 1 - Hydraulic depth, $y_* = \frac{A}{B}$
- 2 - Hydraulic radius, $R = \frac{A}{P}$
- 3 - Section factor from Darcy-Weisbach equation,

$$A = AR^{\frac{1}{2}}$$

Each of these parameters may be expressed as the ratio of its value at a specific depth to its value at the upper limit of depth which is the diameter of the conduit. Figure 2.2 displays these variations as a function of the depth-diameter ratio. It is interesting to note that the hydraulic radius and the section factor maximize at values greater than one, their ratios at full section values. This infers that theoretical maximum discharge would occur at less than full depth for the same energy slope. The usual theory, based on atmospheric pressure at the free surface, does not, in practice, necessarily apply at this depth; hence, prediction of flow at depth ratios near one must be based on additional considerations.

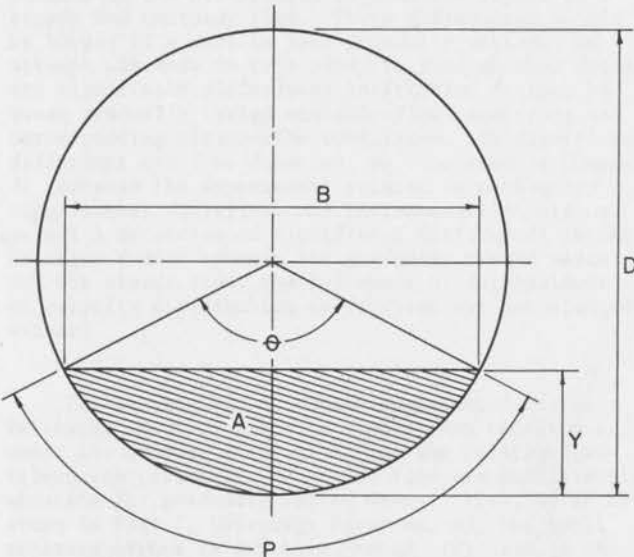


Fig. 2.1. Definition sketch for parameters of a circular cross section.

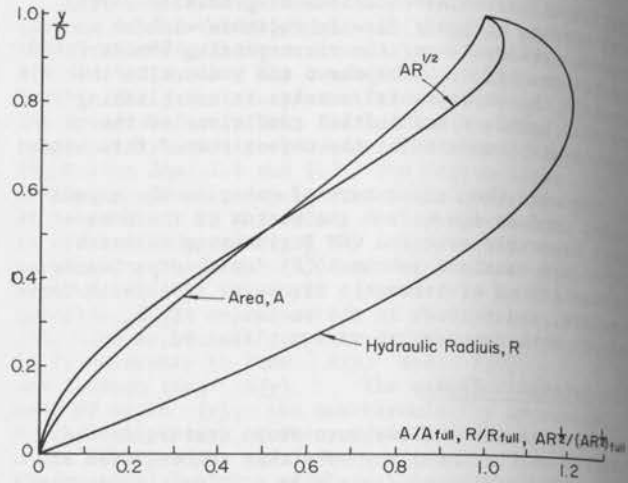


Fig. 2.2. Relative geometry of the circular cross section as it varies with relative depth.

2.2 Errors in Parameters as a Function of Errors in Depth

The relative error in each of the dependent parameters is expressed in terms of the relative error in the depth as follows:

1. Wetted perimeter defined as

$$P = \frac{D}{2} \theta \quad (2.1)$$

has the relative error

$$\frac{dP}{P} = \frac{d\theta}{\theta} \quad (2.2)$$

in which

$$\theta = 2 \cos^{-1} \left(1 - \frac{2y}{D} \right) \quad (2.3)$$

and

$$\frac{d\theta}{\theta} = \frac{1}{\left(\frac{D}{y} - 1 \right)^{\frac{1}{2}} \cos^{-1} \left(1 - \frac{2y}{D} \right)} \left(\frac{dy}{y} \right) \quad (2.4)$$

2. Surface width defined as

$$B = D \sin \frac{\theta}{2} \quad (2.5)$$

gives

$$\frac{dB}{B} = \frac{1}{\left(\frac{D}{y} - 1 \right)^{\frac{1}{2}} \tan \frac{\theta}{2}} \left[\frac{dy}{y} \right] \quad (2.6)$$

3. Area defined as

$$A = \frac{D^2}{8} (\theta - \sin \theta) \quad (2.7)$$

gives

$$\frac{dA}{A} = \frac{1 - \cos \theta}{1 - \sin \theta} \left(\frac{d\theta}{\theta} \right) \quad (2.8)$$

4. Hydraulic depth defined as

$$y_* = \frac{A}{B} \quad (2.9)$$

gives

$$\frac{dy_*}{y_*} = \frac{dA}{A} - \frac{dB}{B} \quad (2.10)$$

5. Hydraulic radius defined as

$$R = \frac{A}{P} \quad (2.11)$$

gives

$$\frac{dR}{R} = \frac{dA}{A} - \frac{dP}{P} \quad (2.12)$$

6. Section factor defined as

$$Z = AR^{\frac{3}{2}} \quad (2.13)$$

gives

$$\frac{dZ}{Z} = \frac{dA}{A} + \frac{dR}{2R} \quad (2.14)$$

These relative errors, (Eqs. 2.2, 2.4, 2.6, 2.8, 2.10, 2.12, and 2.14) being functions of depth, are plotted as ratios of the relative-depth error in Fig. 2.3. It may be seen that the relative error in all parameters except for that of wetted perimeter and hydraulic depth become less as depth increases for a given relative depth error. The significance of these curves will be demonstrated in the calculation of friction factors and Reynolds number.

2.3 Errors in Parameters as a Function of Ellipticity

Since no physical "circular" conduit has a mathematically circular shape, it is necessary to determine the effects of the physical variables. As a systematic approximation, an elliptical shape was assumed. Parameters describing the departure from the flow area in a circular cross section are then the eccentricity and the direction of the principal axes. The eccentricity is defined as

$$e = \sqrt{1 - \left(\frac{b}{a} \right)^2} \quad (2.15)$$

in which a and b are the major and minor semi-diameter, respectively. The direction of the principal axes, defined as the angle α is the angle that the minor axis makes with the vertical as shown in Fig. 2.4.

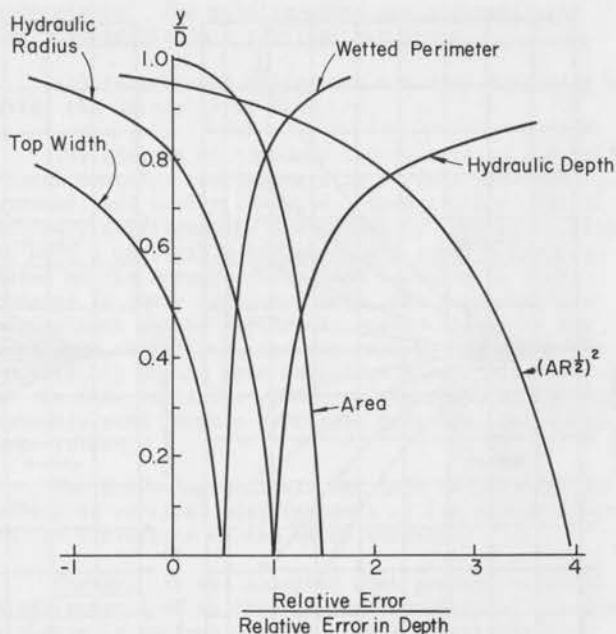
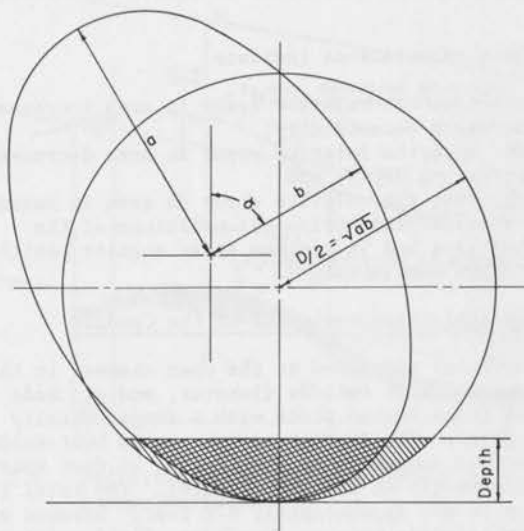


Fig. 2.3. Geometry of circular cross section represented as relative error in parameter versus relative error in depth.



Acs - CIRCULAR SEGMENT -  AES - ELLIPTIC SEGMENT - 

$$\% \text{ DIFF.} = \frac{Acs - Aes}{Acs} \times 100 \quad e = \sqrt{1 - \left(\frac{a}{b} \right)^2}$$

Fig. 2.4. Definition sketch for the relation of circular and elliptical cross sections.

To compare circular with elliptical segments the percent of difference between the two segments was computed for depths from the bottom up to the center of the ellipse. Eccentricity was varied in increments of 0.05 up to 0.30 and for values of α ranging from 0 to $\pi/2$ in increments of $\pi/10$. For all eccentricities, the area of the complete ellipse was made equal to that for the complete circle. Depth was relative to the center of the ellipse. The results of these calculations are shown in Fig. 2.5.

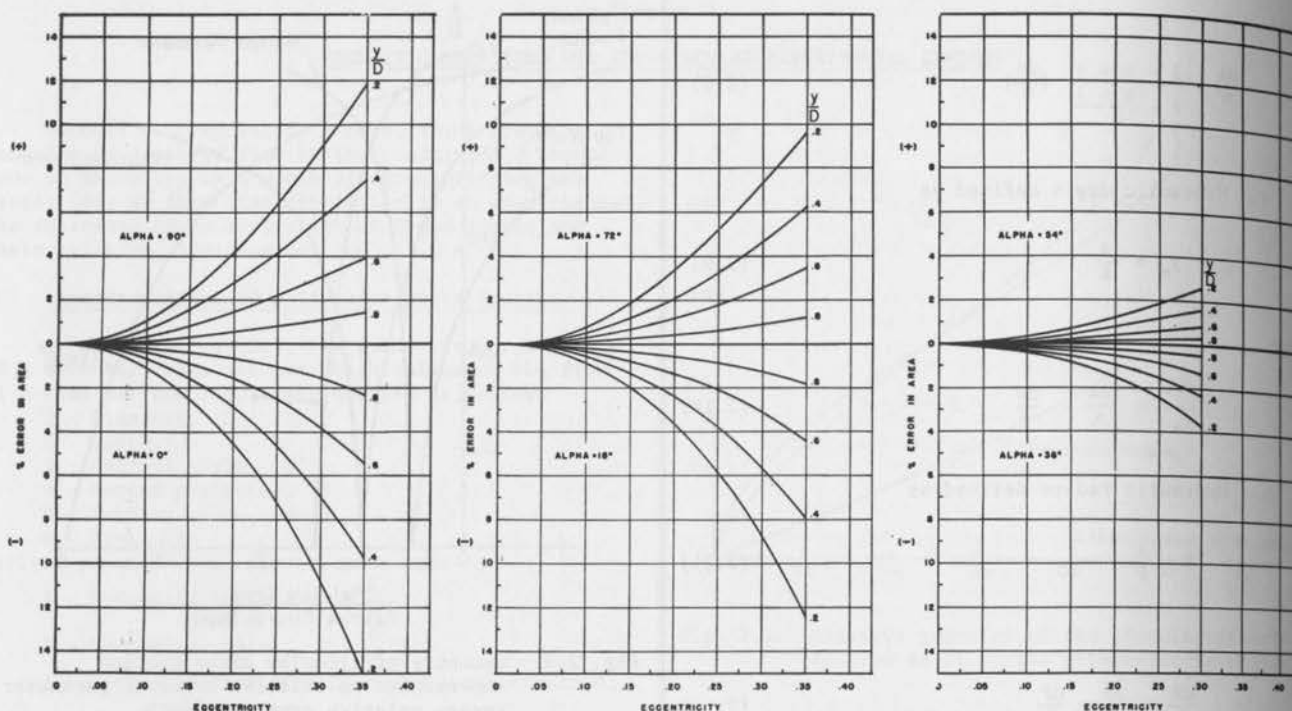


Fig. 2.5. Percent difference in area versus eccentricity and depth.

These calculations indicate

- (a) that the relative error in area increases with increased eccentricity;
- (b) that the relative error in area decreases with increasing depth; and
- (c) that the relative error in area is maximum at the vertical and horizontal positions of the principal axes and is minimum at an angular position of 45° with the horizontal.

2.4 Physical Characteristics of the Conduit

The steel pipe used as the open channel in this study had a 3 foot outside diameter, and was made of 1/2-inch thick rolled plate with a longitudinally welded joint. The 20 foot sections were butt-welded together and supported on steel rails 20 feet apart (not necessarily at the pipe joints). The total length of the pipe was approximately 822 feet. Because of the manufacturing process, handling, field welding, and the method of support it was not expected that this conduit would be perfectly circular or possess a straight line invert profile.

Measurements to the nearest 0.001 inch were made of the inside diameter of the pipe at 60° intervals. These measurements were made at cross sections spaced 40 feet apart before the inside of the conduit was painted; similar measurements were made at intervals of 20 feet after the inside of the conduit was painted. An ellipse was fitted to the three measured diameters at each section and its orientation determined.

The results of the above calculations are presented in Table 2.1. The differences between the means of each of the parameters for the two surveys are not significant on the 5 percent level. This indicates that (1) the painting of the conduit had

no effect to its internal geometry and (2) doubling the number of stations did not significantly improve knowledge about the geometry of the conduit.

Accepting an average area of 968.41 square inches (6.725 square feet) the mean diameter of the conduit is 2.9262 feet. This figure was used for the conduit diameter in all calculations.

TABLE 2.1. Geometry of the experimental conduit

	Number of Stations	Maximum	Mean	Minimum	Standard Deviation
Major Axis Inches	21	17.869	17.617	17.538	0.175
	41	17.913*	17.604	17.554	0.047
Minor Axis Inches	21	17.626	17.516	17.435	0.0375
	41	17.680	17.510	17.430	0.031
Eccentricity	21	0.176	0.1021	0.046	0.0310
	41	0.175	0.0993	0.051	0.0244
Alpha α Degrees	21	165.58	84.84	13.71	46.5
	41	160.37	82.94	7.78	49.43
Area Inches ²	21	989.5	969.47	965.3	3.84
	41	994.9*	968.4	964.1*	3.94
Wetted Perimeter Inches	21	111.51	110.373	110.13	0.2769
	41	111.82*	110.314	110.07*	0.2167
Hydraulic Radius Inches	21	8.87	8.7785	8.76	0.0183
	41	8.89	8.7742	8.75*	0.0181

*occurred at same section.

The first line of figures refers to unpainted interior of the conduit, while the second refers to the painted conduit

Eccentricity and the angle α in the observed geometry of the conduit were used to estimate possible errors in the hydraulic calculations. The percent difference between the circular and elliptical segments for the maximum and mean eccentricity at a depth ratio of 0.2 was determined (Fig. 2.5) and plotted as a function of the angle α (Fig. 2.6).

As may be seen from this plot, the error in area is maximum at an angular position of zero and 90 degrees. The maximum error for the mean eccentricity of the conduit with this depth ratio is 1.1 percent. For a mean α -angle of about 85 degrees, the maximum error for the mean eccentricity is approximately 1 percent.

For depth ratios greater than 0.2 the relative errors are less than the errors with smaller ratios. For larger eccentricities, relative errors become larger at an increasing rate. For smaller α -angles relative errors decrease to zero at approximately 45 degrees and with an absolute value which is equal to the maximum at zero degrees.

Because of the interrelated effects of depth, eccentricity and α , it appears that an error in the computation of the flow area by assuming a circular cross section instead of an approximated ellipse, may range from zero to 3 percent with 1 percent being representative.

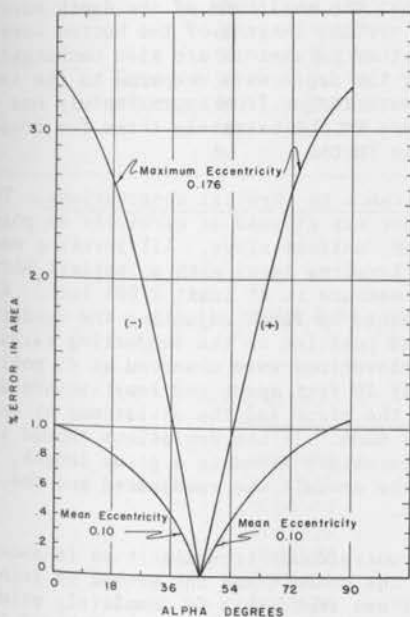


Fig. 2.6. Percent error in area at depth 0.2D, for the experimental conduit.

2.5 Errors Due to Vertical Displacements of a Circular Cross Section

Definition of Errors. The deviations of a given solid boundary from a mathematically straight alignment may be identified in three broad regions:

(a) The boundary irregularities which contribute directly to viscous shear and consequent hydraulic roughness.

(b) Misalignments of the mean boundary which occur gradually when the length of the boundary is

appreciable. The misalignments are unintentional but unavoidable in a physical situation.

(c) Intentional changes in boundary direction to alter the direction of flow.

The effects of boundary irregularities and intentional boundary realignments on surface profiles, in general, are easily computed. However, the gradual boundary misalignments are generally ignored or assumed to have a negligible effect on the surface profile. Based on the energy conversions relating to such changes in cross sectional area, the foregoing assumptions may be justified; energy transfers are small, by definition, and may well be masked by the uncertainty of the mean turbulent energy loss as well as the time variable. Thus, depths computed from any commonly used formula represent only the time-distance mean values.

The following analysis was made to estimate the effect of vertical misalignments of the channel section on the elevations of the water surface.

Theory. It was expected that gradual vertical misalignments of an open-channel boundary will cause a change in surface profile. This effect may be idealized and subsequently quantitized by considering a sinusoidal channel-bottom profile [1] (see Fig. 2.7).

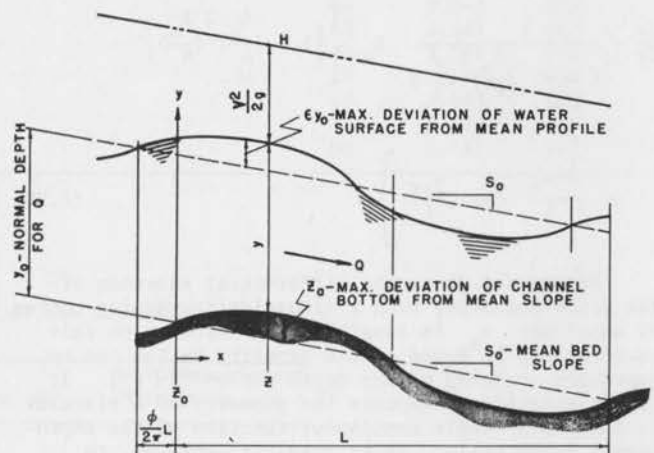


Fig. 2.7. Definition sketch of the effect of bottom irregularities.

At any section the total energy is

$$E = z + y + \frac{V^2}{2g}$$

Upon differentiation,

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{V}{g} \frac{dV}{dx}$$

or

$$-S_f = S_b + \frac{dy}{dx} + \frac{V}{g} \frac{dV}{dx} \quad (2.16)$$

in which S_f is the rate of energy loss which may be represented by

$$S_f = \frac{f}{8g} \frac{V^2}{R} = \frac{f}{8g} \frac{Q^2}{A^2 R} = \left(\frac{A_o}{A}\right)^2 \left(\frac{R_o}{R}\right) S_o \quad (2.17)$$

The derivatives of Eq. 2.16 may be expressed as

$$z = z_o - x S_o + z_a \sin \frac{2\pi x}{L}$$

$$-S_b = \frac{dz}{dx} = -S_o + \frac{2\pi}{L} z_a \cos \frac{2\pi x}{L} \quad (2.18)$$

and

$$\frac{V}{g} \frac{dV}{dx} = -\frac{Q^2 B}{g A^3} \frac{dy}{dx} = -F^2 \frac{dy}{dx}$$

$$= -F^2 \left(\frac{A_o}{A}\right)^2 \left(\frac{B_o}{B}\right) \frac{dy}{dx} \quad (2.19)$$

in which F is the Froude Number corresponding to a uniform slope S_o .

After substitution of Eqs. 2.17, 2.18 and 2.19 into Eq. 2.16 and solving for dy/dx , then

$$\frac{dy}{dx} = \frac{1}{1 - F^2 \left(\frac{A_o}{A}\right)^2 \left(\frac{B_o}{B}\right)} S_o \left\{ \left[1 - \left(\frac{A_o}{A}\right)^2 \left(\frac{R_o}{R}\right) \right] - \frac{2\pi z_a}{L} \cos \frac{2\pi x}{L} \right\} \quad (2.20)$$

Equation 2.20 is the differential equation of the depth resulting from a sinusoidally-varying bottom of amplitude z_a in length L . A solution to this equation may be found if the geometry ratios can be expressed in terms of the depth ratios (y/y_o). It is not possible to express the geometry of a circular section as a simple continuous function of the depth-normal depth ratio. It is possible, however, to achieve a solution if the depth ratio is assumed to be

$$\frac{y}{y_o} = 1 + \epsilon \sin \left(\frac{2\pi x}{L} - \phi \right) \quad (2.21)$$

in which ϵ is the ratio of amplitude to y_o , and ϕ is the phase angle for the depth wave. If this is done then

$$\frac{dy}{dx} = \frac{2\pi y_o}{L} \epsilon \cos \left(\frac{2\pi x}{L} - \phi \right) \quad (2.22)$$

From Eqs. 2.20 and 2.22, expanding the cosine of the sum and equating the coefficients of the unknown phase angle ϕ in Eqs. 2.20 and 2.22, the value of ϕ is determined for the infinite wide channel to be

$$\phi = \tan^{-1} \left[\frac{-3 S_o L}{2\pi y_o (1 - F^2)} \right] \quad (2.23)$$

The amplitude of the depth wave y_o can now be evaluated from

$$\epsilon = -\frac{2\pi z_a \sin \phi}{3 S_o L} \quad (2.24)$$

Equations 2.23 and 2.24, for the infinite wide channels, relate those quantities which are required to estimate the effect which the periodic channel irregularities have on the corresponding changes in the depth of flow. This is an approximate evaluation of effects of periodic bottom irregularities in conduits.

Calculations and results. Equations 2.23 and 2.24 were solved for various combinations of

- channel slope,
- wave length of channel irregularity,
- amplitude of channel irregularity, and
- normal depth.

The Darcy-Weisbach friction factor was taken as a constant 0.012.

Table 2.2 presents the results of these calculations. The results indicate, as expected, that for a Froude number greater than one the depth wave is nearly in phase with the bottom wave. The slight difference is due to flow resistance. For a Froude number less than one the depth wave is out of phase with the bottom wave by essentially π . Again, the slight difference is due to flow resistance.

Note that the amplitude of the depth wave is unchanged for various lengths of the bottom wave, provided that other parameters are also unchanged. The amplitude of the depth wave compared to the length of the bottom wave ranges from approximately one for low Froude numbers to approximately three for Froude numbers close to one.

Significance to physical observations. The channel invert was aligned as carefully as possible to a constant, uniform slope. All leveling was done with a self-leveling level with an optical micrometer which could measure to at least 0.001 inch. Alignment was accomplished by first adjusting the conduit to a predetermined position on the supporting rails. Then the invert elevations were observed at 45 positions approximately 20 feet apart and least-square determination of the slope and the deviations at each position was made. If the deviations showed a consistent or excessive trend in a given length, that portion of the conduit was readjusted and the elevations redetermined.

Due to unavoidable irregularities in successive sections of the conduit and the method of joining sections, it was impossible to completely eliminate all deviations from the mean slope. Table 2.3 presents the results of mean slope determinations and the corresponding maximum and root-mean-square deviations from the slope of least-square fit.

These results show that the invert profile of the conduit had an undulating bottom with approximately 0.01 foot of amplitude and 20 to 40 feet of wave length.

Equations 2.23 and 2.24 were solved using the case of an infinitely wide channel with sinusoidal bottom. This case may be considered as the limiting one for a circular cross section flowing partially full if the radius is considered to remain constant and the centerline of the section to vary sinusoidally about the mean slope. Thus, the entire section may

be considered as changing position vertically rather than only a sinusoidal change of the invert or radius.

On consideration of the results of Table 2.2 in predicting the effect on the observed water surface

profile, it may be concluded that for the slopes used in this study the observed depths may deviate from the ideal by 0.01 to 0.03 of a foot on the average. Based on the maximum deviations the water depth may differ from the ideal by 0.03 to 0.09 of a foot.

TABLE 2.2. Theoretical effect of bottom irregularity on water surface profiles.

Slope	Froude No.	z_a -ft.	L-ft.	ϕ -Rad.	ϵy_o -ft.	Slope	Froude No.	z_a -ft.	L-ft.	ϕ -Rad.	ϵy_o -ft.
.0100	2.582	.01	20	6.266	.002	.0001	.258	.03	20	3.170	.090
			40	6.249	.002				40	3.198	.090
			60	6.232	.002				60	3.227	.090
			80	6.216	.002				80	3.255	.090
		.02	20	6.266	.004			.04	20	3.170	.120
			40	6.249	.004				40	3.198	.120
			60	6.232	.004				60	3.227	.120
			80	6.216	.004				80	3.255	.120
		.03	20	6.266	.005			.01	20	3.142	.011
			40	6.249	.005				40	3.143	.011
			60	6.232	.005				60	3.144	.011
			80	6.216	.005				80	3.145	.011
		.04	20	6.266	.007			.02	20	3.142	.021
			40	6.249	.007				40	3.143	.021
			60	6.232	.007				60	3.144	.021
			80	6.216	.007				80	3.145	.021
.001	.816	.01	20	3.170	.030			.03	20	3.142	.032
			40	3.198	.030				40	3.143	.032
			60	3.227	.030				60	3.144	.032
			80	3.255	.030				80	3.145	.032
		.02	20	3.170	.060			.04	20	3.142	.043
			40	3.198	.060				40	3.143	.043
			60	3.277	.060				60	3.144	.043
			80	3.255	.060				80	3.145	.043

TABLE 2.3. Slope deviations of the experimental conduit

Slope	Max. Deviation, ft	Root-Mean-Square Deviation, ft
.0000052	+.0188	.0116
.0000157	+.0182	.0135
.0000303	+.0214	.0099
.0001325	+.0195	.0099
.0005197	+.0347	.0117
.0010101	+.0279	.0119
.0074578	-.0240	.0133
.0200690	+.0375	.0141

HYDRAULIC RESISTANCE OF THE EXPERIMENTAL CONDUIT

3.1 Expressions for the Friction Factor of Partly Flowing Conduits

Resistance to motion of open-channel flow, including the free-surface flow in conduits, due to boundary roughness can be expressed in various forms. Foremost among those commonly in use are the Chézy, Manning, Colebrook-White, Hazen-Williams, and Darcy-Weisbach equations of flow resistance.

The Committee for Hydromechanics of the Hydraulics Division of the American Society of Civil Engineers [2] recommended the use of the Darcy-Weisbach equation for future presentation of resistance data in pipes. Thus, boundary resistance in conduits is evaluated and expressed in this study in terms of the Darcy-Weisbach equation

$$S_f = \frac{f}{D} \cdot \frac{V^2}{2g} \quad (3.1)$$

in which S_f is the slope of the energy gradient, f is the Darcy-Weisbach friction factor, V is the mean flow velocity, and D is the diameter of the conduits flowing full. For channel cross sections other than circular, and for partly full circular conduits, the diameter (D) is customarily replaced by four times the hydraulic radius ($4R$). The validity of this replacement may be questioned for open-channel flow and for partly full circular conduits. However, for lack of a better length parameter describing the velocity gradients of the velocity distribution in a cross section, and hence the shear stresses, the hydraulic radius is commonly used. Equation 3.1 thus becomes

$$S_f = \frac{f}{R} \cdot \frac{V^2}{8g} \quad (3.2)$$

in which the hydraulic radius (R) is defined as the cross sectional area divided by the wetted perimeter (P), or $R = A/P$.

It has been demonstrated that the Darcy-Weisbach friction factor f is a function of the Reynolds number, the relative roughness (k/y), and the channel cross sectional shape, in which k is the absolute boundary roughness, y is the depth of flow, and k/y is the relative roughness. The form of this theoretical relation depends in turn on the range of the Reynolds number, the relative roughness, and the shape of the channel cross section. For hydraulically smooth boundaries, such as the boundary of experimental conduit used in this study, and for Reynolds number greater than 2.5×10^4 , the Prandtl-von Karman equation [4]

$$\frac{1}{\sqrt{f}} = a \log_{10} (R_e \sqrt{f}) + b \quad (3.3)$$

relates the friction factor f only in terms of the Reynolds number (R_e). The constants a and b in Eq. 3.3 are determined experimentally. Their values depend on the selection of the Reynolds number length parameter, assuming that the mean flow

velocity and the kinematic viscosity are always uniquely defined. For open channels, it is convenient to express the Reynolds number as

$$R_e = \frac{VR}{\nu} \quad (3.4)$$

in which R is the hydraulic radius of the given cross section. Equation 3.3 for open channels with smooth boundaries becomes

$$\frac{1}{\sqrt{f}} = 2 \log_{10} (R_e \sqrt{f}) + 0.4 \quad (3.5)$$

In the case of full flow through a circular cross section, the length parameter in the Reynolds number is the diameter, or $R_e^* = VD/\nu$. In this case Eq. 3.3 takes the familiar form

$$\frac{1}{\sqrt{f}} = 2 \log_{10} (R_e^* \sqrt{f}) - 0.8 \quad (3.6)$$

The evaluation of f from Eq. 3.3 for a given R_e requires an iterative procedure, and often excessive computations are needed. To simplify computations (within practical limits of Reynolds numbers for specific conditions) it is convenient to use a simplified form of the f to R_e relation. Thus, a practical form of expression for this relation is

$$f = c R_e^d \quad (3.7)$$

in which c and d are constants to be empirically determined.

The purposes of the experimental investigations of hydraulic resistance in this study were: (a) to confirm the hydraulically-smooth surfaces of the experimental conduit; (b) to develop a simplified expression for the friction factor as it is related to Reynolds number, in the form of Eq. 3.7, and (c) to determine the effects of an assumed constant friction factor on unsteady flow through storm drains and compare these with the effects of a changing friction factor related to Reynolds number in the same type of flow.

3.2 Observations on Hydraulic Resistance in the Experimental Conduit

The experimental conduit and the instrumentation used are described in Part II (Hydrology Paper No. 44) of these series of four papers on flood routing through storm drains. A brief summary is repeated in this subchapter.

The experimental facilities consisted of a 3-foot diameter, 822-foot long circular conduit with a slope adjustable between 0 and approximately 4 percent. Water surface elevations used to study hydraulic resistance are measured by hook gages

located at 16 positions along the conduit (see Table 5.1, Hydrology Paper No. 44). The invert slope of the conduit was carefully determined by means of a precise self-leveling level with an optical micrometer. Discharge for a desired depth of flow in the conduit was estimated from previous observations (see Table 5.3 Hydrology Paper No. 44) and established at the conduit inlet. The downstream outflow-control gate was adjusted to produce the desired backwater or drawdown curve. Several conditions of non-uniform flow were established, both above and below normal depth. Hook gage levels at the various piezometer locations were read at approximately 15 minute intervals until the readings stabilized. Hook gage readings, gage zeros, and invert elevations were transferred to punch cards along with the steady discharge rate and the conduit slope. Data was then analyzed by means of a digital computer.

3.3 Analysis of Experimental Results

The total energy per unit weight flowing in a partially full channel is defined as: total energy head = invert elevation + depth of flow + velocity head. The difference in successive values of total energy head divided by the distance between conduit stations represents the rate of energy loss. This loss rate $(E_2 - E_1)/\Delta x$, the average hydraulic radius, and the average of the velocity heads at the ends of the reach, $(V^2/2g)_{av}$, were then substituted into the resistance equation to evaluate the friction factor f as

$$f = \frac{4R_{av}}{(V^2/2g)_{av}} \cdot \frac{E_2 - E_1}{\Delta x} \quad (3.8)$$

The Darcy-Weisbach friction factor f was computed initially by considering the slope of the energy gradient between the successive piezometer locations. This approach gave a large number of less accurately determined friction factors. The computations were performed by the digital computer thus eliminating any subjective bias in establishing the slope of the gradient at these short distances. A plot of these f - values versus Reynolds number, however, resulted in a wide scatter of values as shown in Fig. 3.1. As expected, this was largely due to experimental errors in observation of depth, the influence of unavoidable bottom irregularities on the surface water profile, and small surface waves and pulsations, which produced substantial variation in the slopes of the energy gradient computed from the short lengths of conduit. The use of small reaches in computing the slope and other parameters of Eq. 3.8, therefore, resulted in a large scatter of points in Fig. 3.1 around the expected smooth curve.

It was apparent that the mean slope of a long reach was necessary to define the friction factor more accurately. Therefore, the values of energy heads at the piezometer locations throughout the conduit were plotted and the best estimate of the average slope was graphically determined. The least square fit of these

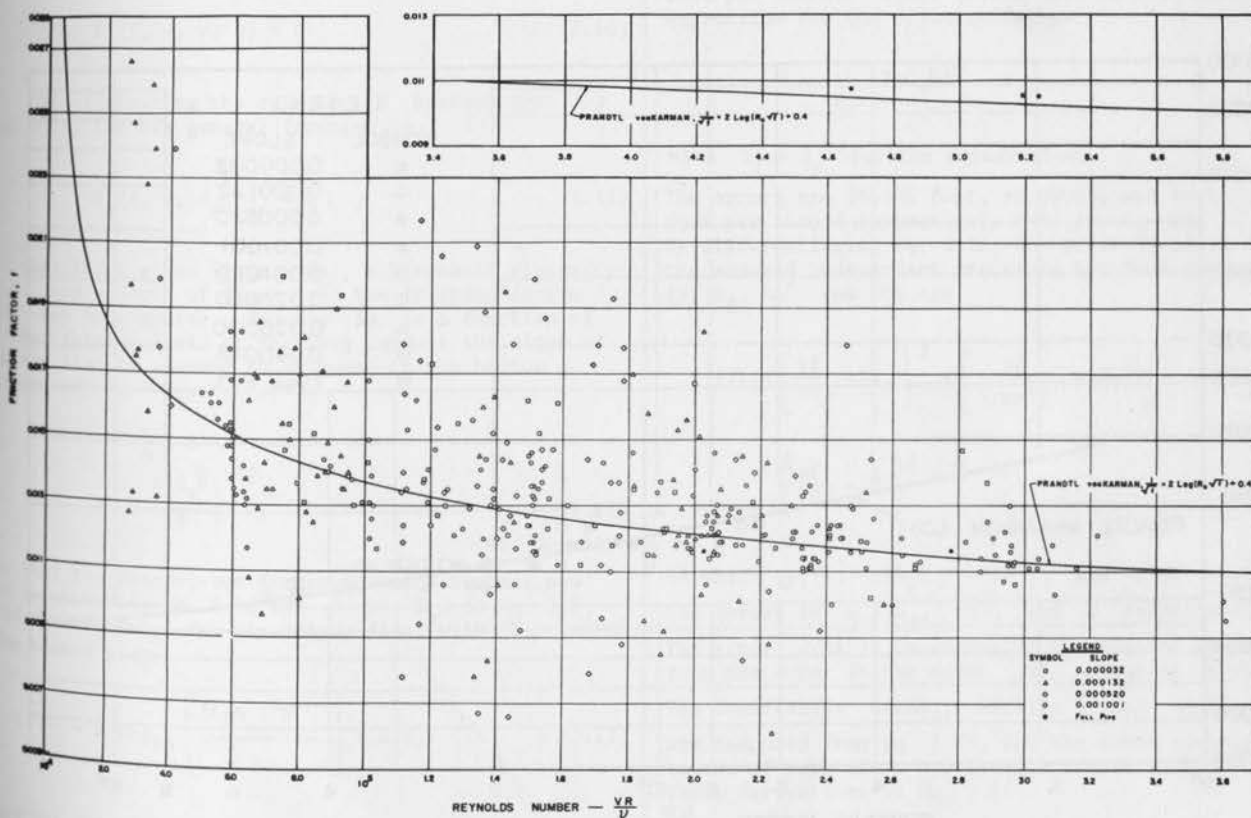


Fig. 3.1. Variation of Darcy-Weisbach friction factor f with Reynolds number R_e (computed from small reaches of the conduit).

data was not used because it would not produce the expected results in regions of high curvature of the water surface the slope of the energy gradient did not remain constant. This variation occurs because friction factor varies with Reynolds number and thus with depth. The graphically-fitted average slope of the energy gradient was taken to be the most representative slope of the particular flow conditions.

The following procedure was used to determine the average value of the hydraulic radius and the average velocity head. The depths of flow at both ends of the reach were computed based on the energy heads indicated by the uniform gradient. These two depths were then averaged and from this value the average area, average hydraulic radius, average velocity, and average velocity head were computed.

Even with these precautions and averagings, as it was expected, a scattering of points (f , R_e) remained but with much smaller deviations than in Fig. 3.1. The results of these averaging computations for the entire length of conduit are presented in Fig. 3.2. It is apparent that the points computed from the average slopes of the energy line show an improved relation of the friction factor with the Reynolds number. The points tend to cluster around the Prandtl-von Karman smooth-boundary friction-factor equation.

The plotted points represent the results in a range of depth from 0.56 to 2.6 feet, or depth-to-diameter ratios of 0.19 to 0.89. The discharges varied from 2.25 to 72.0 cfs. The corresponding Reynolds-number range is from approximately 3×10^4 to 1×10^6 .

For the data shown in Fig. 3.2, the values of the constants a and b in Eq. 3.3 are 2.075 and 0.1434, respectively. These will be compared later with the constants of 2.0 and 0.4 in the Prandtl-von Karman equation for the free-surface smooth boundary flow. For the values $a = 2.075$ and $b = 0.1434$ in Eq. 3.3 the mean absolute deviation of the points in Fig. 3.2 from the line defined by Eq. 2.3 is 0.00167, with a standard deviation of 0.0024. The two extremely departing points on the right side of Fig. 3.2 make the standard deviation greater than it really should be.

The two points in Fig. 3.2, which are much above the other points and the curve of the Prandtl-von Karman equation could not be verified as inaccuracy of observations or the ensuing data processing. It is considered to be reasonable to assume that they are mistakes either in observations or in data processing, their values being about two times the expected values on both the Prandtl-von Karman curve for their corresponding Reynolds numbers and the expected values determined from the curve fitted to all observational points. These two points are retained in Fig. 3.2 to show that mistakes are often unavoidable in experimental results, and that they should not be arbitrarily removed.

To save computing time, the constants c and d in Eq. 3.7 were evaluated and used in subsequent computer programs. The constants c and d in Eq. 3.7 used for integrating unsteady-flow equations, are estimated from data in Fig. 3.2 to be 0.10939 and -0.17944, respectively.

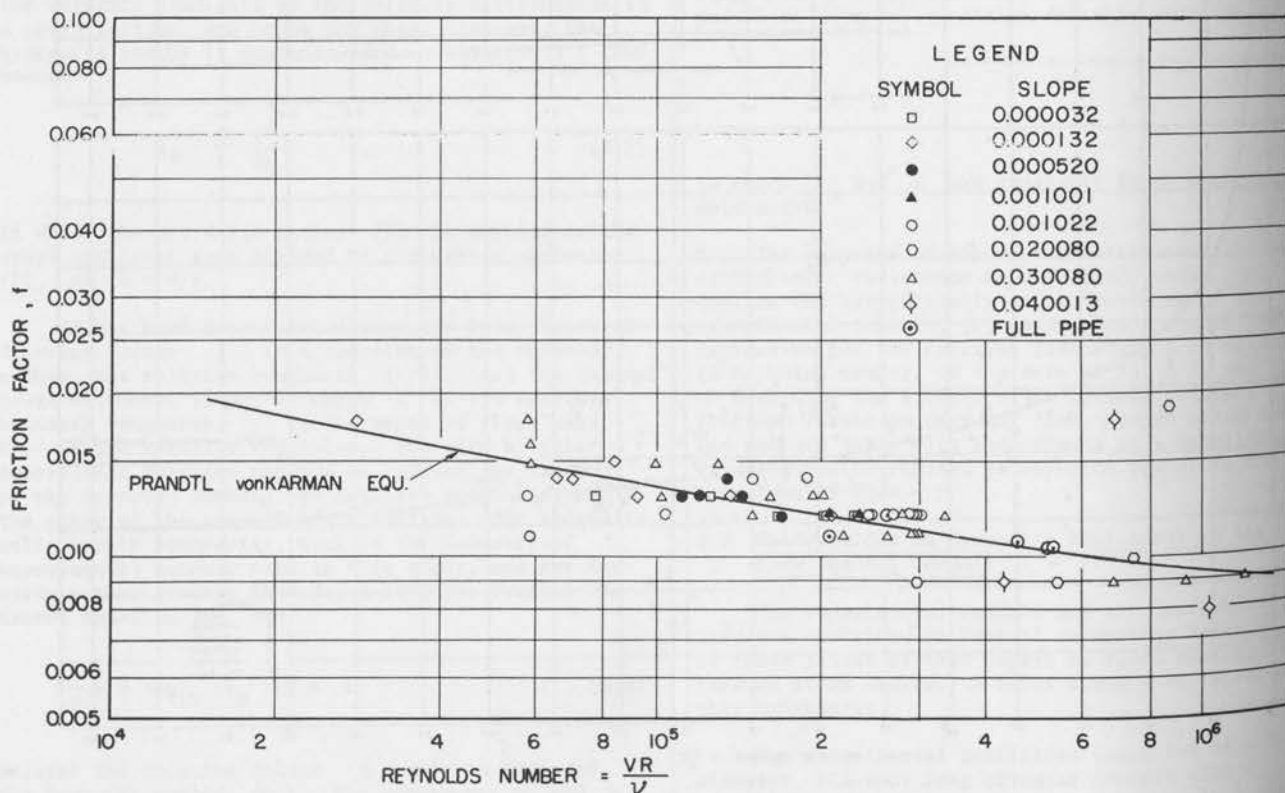


Fig. 3.2. Variation of Darcy-Weisbach friction factor f with Reynolds number R_e based on the slope representative of the entire length of experimental conduit.

Although Eqs. 3.3 and 3.7 show a decrease of f with an increase of R_e it can be shown that for a limited range of R_e the changing friction factor f may be replaced by an average value or a representative constant. An investigation, presented in Part I (Hydrology Paper No. 43), was done for the range of R_e -values observed in this study and shows a relatively small difference between the analytical waves computed by the use of the friction factor in Eq. 3.7 and by the use of a constant representative friction factor f_c .

3.4 Effect of Depth on Friction Factor

The Darcy-Weisbach friction factor, f , has been demonstrated to be a function of Reynolds number. Although it is not possible to separate the effects of velocity and geometry on the friction factor, there have been attempts in the past [3] to demonstrate the effect of depth alone on friction factor. A similar attempt was made in this study to compare results with previously published results.

The Darcy-Weisbach equation relates friction factor, depth, velocity, and slope of energy line in the general form

$$F_1(f, y, V, S_f) = 0 \quad (3.9)$$

The Prandtl-von Karman equation relates the friction factor, depth, velocity, and properties of the fluid in the form

$$F_2(f, y, V, \nu) = 0 \quad (3.10)$$

By eliminating the velocity V between Eqs. 3.9 and 3.10, the new general function is

$$F_3(f, y, S_f, \nu) = 0 \quad (3.11)$$

so that for a given slope (S_f), a kinematic viscosity (ν), and a depth of flow (y) the friction factor (f) may be computed. Because S_f is a function of f for this analysis of the long conduit the slope of energy line is assumed to be equal to the bottom slope, or $S_f = S_o$.

Equation 3.2 gives

$$V = \left(\frac{8gS_f R}{f} \right)^{1/2} \quad (3.12)$$

so that the Prandtl-von Karman equation becomes now in substituting Eq. 3.12 for V in R_e in Eq. 3.5 and for $S_f = S_o$ for the steady flow, with S_o being the bottom slope,

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left[\frac{(8gS_o)^{1/2} R^{3/2}}{\nu} \right] + 0.4, \quad (3.13)$$

and the fitted Eq. 3.3 with $a = 0.1434$ and $b = 2.075$ becomes

$$\frac{1}{\sqrt{f}} = 0.1434 \log_{10} \left(\frac{(8gS_o)^{1/2} R^{3/2}}{\nu} \right) + 2.075 \quad (3.14)$$

A plot of the results of Eqs. 3.13 and 3.14 is given in Fig. 3.3 for two extreme slopes, and these two equations are presented in the form relating the relative friction factor to the relative depth. The relative friction factor is defined as the ratio of the partial-flow Darcy-Weisbach friction factor to the full-flow friction factor of a conduit. The relative depth is defined as the ratio of the partial-flow depth to the full-flow depth (or the diameter) of a conduit.

Figure 3.3 shows that both the theoretical relation of relative friction factor to the relative depth of the Prandtl-von Karman equation, or the experimental relation obtained in this study lie appreciably below the curve proposed by Camp [3]. Figure 3.3 further shows that variations of f with depth from the curves given are equal to or lesser than the errors in the computations of f for a free-surface flow within the upper range of the conduit (upper 2/3 of the diameter). Below the lower 1/3 of the diameter an appreciable change of the friction factor occurs with a change of water depth.

3.5 Effect of Measurement Errors on Computed Friction Factors

To estimate the effect of observational errors on the computed friction factors, certain assumptions are required. For the following analysis the assumptions about the numerical values of parameters and errors are: Internal diameter of the conduit (D) is 3 feet, the depth (y) is 1.5 feet ± 0.005 foot, the bottom slope S_o is 0.001 ± 0.0001 , and the discharge (Q) is 30 cubic feet per second ± 0.3 cubic foot per second. These values will be used in the expression for the friction factor

$$f = \frac{8gS_f R A^2}{Q^2} \quad (3.15)$$

with $S_f = S_o$ for the steady flow.

The errors are ± 0.005 foot, ± 0.0001 , and ± 0.3 cubic foot per second respectively for y , S_f and Q . By differentiating Eq. 3.15, the error equation for the assumed independent errors in the four quantities (y , S_f , RA^2 and Q) is

$$\epsilon^2(f) = \left[\frac{\partial f}{\partial S_f} \epsilon(S_f) \right]^2 + \left[\frac{\partial f}{\partial (RA^2)} \epsilon(RA^2) \right]^2 + \left[\frac{\partial f}{\partial Q} \epsilon(Q) \right]^2 \quad (3.16)$$

in which $\epsilon(f)$, $\epsilon(S_f)$, $\epsilon(RA^2)$, and $\epsilon(Q)$ represent the errors in f , S_f , RA^2 , and Q respectively. The error $\epsilon(RA^2)$ is evaluated by means of Fig. 2.3 from the error in the depth (y), given as $\epsilon(y)$. The derivatives $\partial f / \partial S_f$, $\partial f / \partial (RA^2)$ and $\partial f / \partial Q$ are computed from Eq. 3.15, and the above numerical values of parameters are used to numerically determine these derivatives in Eq. 3.16.

Assuming that the above errors in y , S_f , and Q are their respective standard deviations of random errors, then the standard error of random errors in the friction factor, $\epsilon(f)$, is

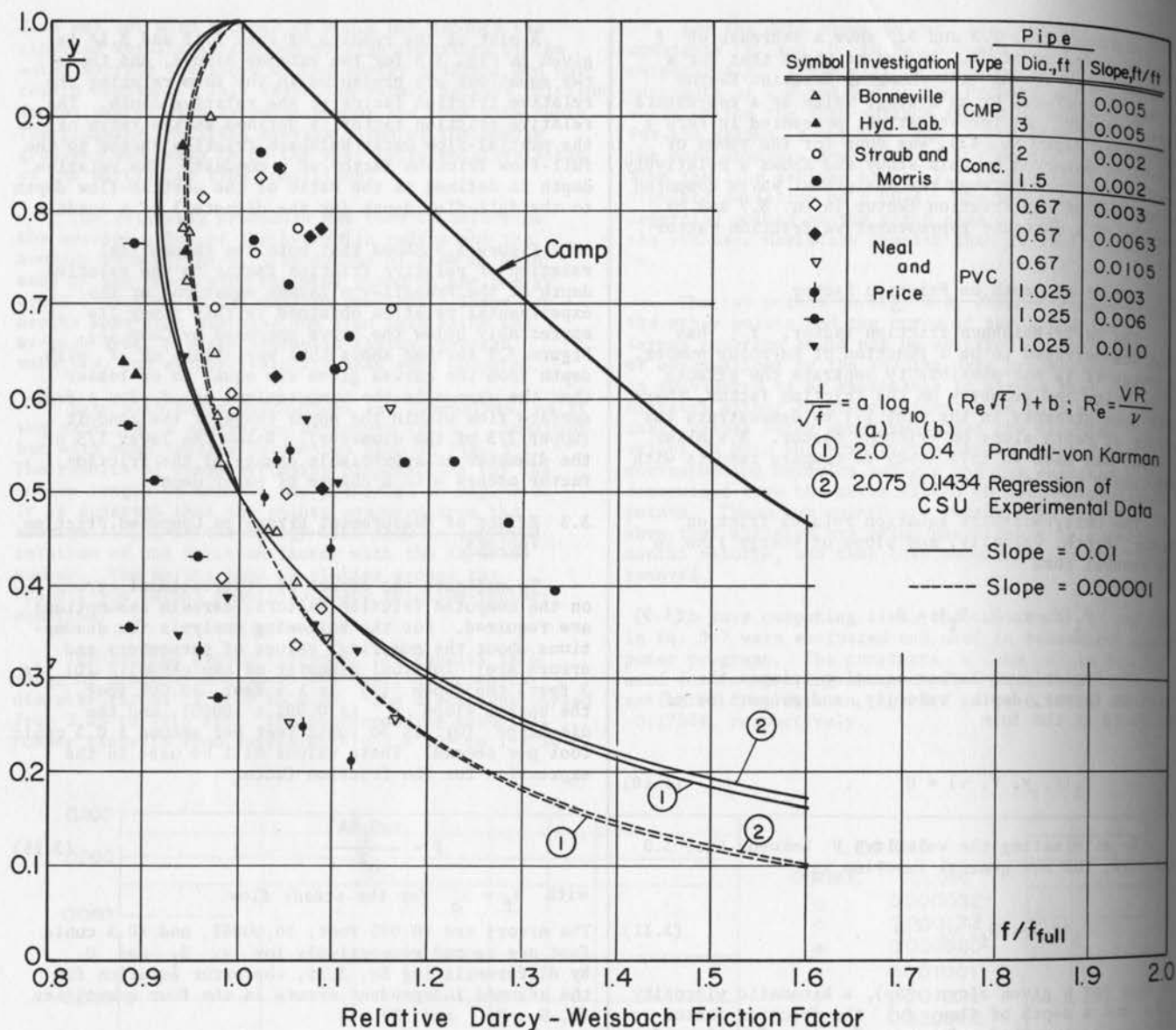


Fig. 3.3. Variation of Darcy-Weisbach friction factor f with depth of flow in a circular cross section.

$$\begin{aligned}
 \epsilon(f) &= \{(2.681 \times 0.00001)^2 + (0.000286 \times 0.0997)^2 + \\
 &\quad + (0.0001787 \times 0.3)^2\}^{\frac{1}{2}} = \\
 &= \{7.18 \times 10^{-10} + 8.12 \times 10^{-10} + \\
 &\quad + 28.5 \times 10^{-10}\}^{\frac{1}{2}} = 6.61 \times 10^{-5}
 \end{aligned}$$

For the representative constant friction factor of 0.012 for this conduit and the range of Reynolds numbers used in this study, this estimated standard error of 6.61×10^{-5} represents only a 0.55 percent error.

For the above particular numerical case it is noted that the largest contribution of error to the friction factor is by the error made in discharge measurement whereas the error in slope has the least effect. The standard error in the friction factor of

6.6×10^{-5} may be considered as a lower boundary in practical evaluations of random errors.

3.6 Conclusions

From the consideration of the experimental data it can be concluded that:

- (1) The boundary of the conduit used in this experimental study is hydraulically smooth.
- (2) Estimations of friction factors made from comparing water elevations across short reaches result in significant random errors (shown by Fig. 3.1). By using the entire conduit length (822 feet) for computing the energy slope the spread of errors is much smaller (shown by Fig. 3.2).
- (3) The friction factor may be expressed for practical applications as

$$f = 0.10939 R_e^{-0.17944} \quad (3.17)$$

for a range of Reynolds numbers from 3×10^4 to 1×10^6 .
 (4) An average Darcy-Weisbach friction factor of 0.012 is considered representative of the smoothness of the boundary of the experimental conduit for the range of R_e in this study.

(5) Roughness values for any depth in a circular cross section can be estimated from the roughness of the full conduit (shown by Fig. 3.3)

(6) The assumption of a representative constant friction factor for CSU smooth pipe may be in error by a maximum of ± 10 percent for depths in excess of one-third of the diameter shown in Fig. 3.3, lines (2), and the friction factor of a full-pipe flow is not representative of roughness for depths less than one-third full.

ENERGY LOSSES AT JUNCTION BOXES4.1 Definition of Losses at Junctions

The previous chapter considered the hydraulic energy losses resulting from the effects of the boundary roughness. This chapter considers the energy losses resulting from local changes in channel geometry and the losses accompanying energy exchange when the main flow and lateral inflows interact at junction boxes. The junction of any two storm drainage systems presents the problems of these two energy losses. The two energy losses, however, cannot be separated whenever the intersecting flows are associated with a change in boundary geometry, which is usually the case with junction boxes of storm drainage systems. Only the energy loss due to the change in boundary configuration can readily be evaluated.

The need for an experimental evaluation of these two losses at junction boxes was necessary to evaluate the validity of applying numerical solutions of unsteady flow equations along a conduit with local lateral inflow. Mathematical representation of the junction-box loss appears as a discontinuity in the application of unsteady flow equations. Thus, the junction-box loss presents a boundary condition which is satisfied by the conditions upstream and downstream of the junction box.

4.2 Brief Review of Previous Investigations of Losses at Junction Boxes

There is a limited number of references on energy losses at hydraulic junctions of various types, particularly for free-surface flow. The following brief review of several references and the material presented in this chapter are taken from the M.S. thesis by William L. Lorah entitled "Free-Surface Flow Energy Losses in a 90° Junction Box", Colorado State University, June 1966, which was done under this project on "Unsteady Free-Surface Flow in a Long Storm Drain."

In 1926 Stevens [5] developed a theoretical equation from the energy equation to express impact and eddies losses at the intersection of two full-flowing pipes. His computations assumed that eddies losses for a 90° junction are equal to the upstream velocity head in the main pipe, plus the velocity head in the lateral pipe times the ratio of lateral to main flow.

Taylor [6], in 1944, derived a relation between the upstream and downstream depths at a channel junction. His derivation was based on the momentum equation and these assumptions:

- (1) flow was parallel to the channel walls,
- (2) velocity was uniformly distributed immediately above and below the junction, and
- (3) the depths of flow entering the junction were equal. His resulting equation was

$$k_2 = \frac{\left(\frac{Q_l}{Q_d}\right)^2 \left[\frac{y_u}{y_d} - 1\right]}{4 \left(\frac{y_u}{y_d}\right)^2 \left[2 \left(\frac{Q_l}{Q_d}\right) - \left(\frac{Q_l}{Q_d}\right)^2 (1 + \cos \phi) \frac{y_u}{y_d} - 1\right]}, \quad (4.1)$$

in which k_2 is the ratio of the velocity head to depth in the branch channel, Q_l is the branch flow, Q_d is the downstream flow, y_u is the depth upstream, y_d is the depth downstream, and ϕ is the angle the branch channel makes with the main channel.

He tested a model junction in which the main and branch channels were of equal width and at the same intersecting elevation. He also tested 45° and 135° junctions for combining and dividing flow. His experimental results for a 45° combining flow agreed well with his theory but there was lack of agreement between theory and model for the 135° combining flow. This disagreement was attributed to the distortion of velocity distribution below the channel junction and to the flow not being parallel to the channel walls.

In 1950 the University of Minnesota modeled open-channel junctions for a drainage system to be used at Whiting Field Naval Air Station [7]. The channels were trapezoidal, with a maximum prototype discharge of 960 cubic feet per second. Their slopes were supercritical, which normally creates hydraulic jumps in both the main and side channels. One of the conclusions of this report was that model studies of this type of junction were necessary until better design criteria could be developed.

Discussing manhole and other junction losses in storm sewer drains in 1964, Jens and McPherson [8, page 20-31] state "There are virtually no data on which estimates of such losses can be based, other than those from the recent University of Missouri experiments."

In 1958 a study of storm drain junctions [9] was conducted at the University of Missouri. It was both an analytical and experimental investigation, involving full-pipe relation only, with the top of the junction box open to the atmosphere. This work was concerned with pressure changes across the various junctions studied. The results of this study were graphic procedures for designing full-pipe storm drain junctions. These graphs represented the empirical relations between the ratio of lateral flow to the flow leaving the junction, and the ratio of pressure head lost in the junction to the velocity head of either the main or lateral flow.

Discussing the present state of knowledge concerning channel junctions, Chow [10, page 512] states "The problem is so complicated that only a few simple and specific cases have been studied. The conclusions of such studies indicate that generalization of the problem is not possible or even desirable."

4.3 Experimental Facilities

The evaluation of energy losses accompanying lateral inflow into a main conduit having free-surface flow was accomplished on two separate experimental facilities in this study. A plastic pipe with a six-inch diameter provided a means of economically developing the basic evaluations of junction box losses. The results were then verified by a limited number of tests in the 3-foot diameter conduit. Agreement between the two systems was obtained based on Froude similarity relation. The details of the two physical systems is presented in Chapter 2, Hydrology Paper No. 44.

In the model studies of the relation of power loss at the junction to other hydraulic properties only one type of junction box configuration was studied. This junction box had two lateral inlets at different elevations above the invert of the main conduit. The two lateral inlets were studied separately. The main and lateral conduits entering the junction box were circular, having diameters of 6.250 inches and 1.873 inches, respectively. The study was limited to steady, free-surface, subcritical combining flow.

The results of this study are in the form of two equations (one for each inlet) which show the relations between the power loss in the junction, the power entering the junction, the rate of main conduit flow into the junction, the rate of lateral inflow, and the depth of flow entering the junction from the main conduit.

This particular study is one of the foundations for further study of unsteady flow through a similar prototype junction box.

The experimental facilities for the six-inch pipe are described in Hydrology Paper No. 44, Chapter 2, subchapter 2.3.

Photographs of typical flow conditions are presented in Fig. 4.1, 4.2, 4.3 and 4.4. Figures 4.1 and 4.2 show the flow conditions with the lower inlet, and Figures 4.3 and 4.4 show the flow conditions with the upper inlet for the two extreme relative discharges of lateral inflow to main conduit flow (Q_T).

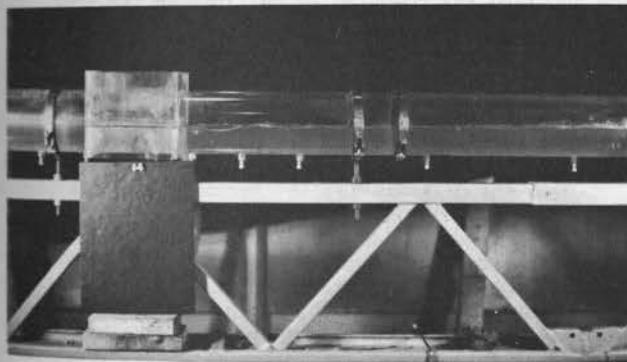


Fig. 4.1. Flow conditions for $S_0 = 0.000537$, $Q_T = 5.56$ with lower inlet.

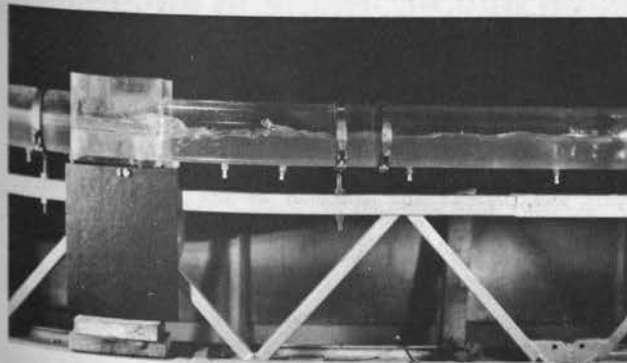


Fig. 4.2. Flow conditions for $S_0 = 0.000537$, $Q_T = 8.44$ with lower inlet.

The following conditions should be noted in these photographs:

- (1) The turbulence within the junction box.
- (2) The drop in water depth from the upstream to the downstream side of the junction box.
- (3) The persistence of turbulence, as evidenced by the entrained air and surface waves several pipe diameters downstream of the junction box.

4.4 Energy Loss Evaluations

The energy of steady, free-surface flow in any shaped channel may be defined as

$$E = z + y \cos \theta + \alpha \frac{V^2}{2g}, \quad (4.2)$$

in which E is the energy head, z is the elevation of the channel bottom above the zero datum, y is the depth of water perpendicular to the channel bottom, V is the mean velocity, α is the velocity distribution coefficient for the energy, and θ is the angle the channel bottom makes with the horizontal as shown in Fig. 4.5 for the definition of energy heads and differences. For the cases considered, θ is, within the limits of physical observation, sufficiently close to zero so that the cosine of θ is essentially one. It is also assumed that the velocity distribution coefficient α is sufficiently uniform and may safely be assumed to be one. Thus the total energy head equation may be written as

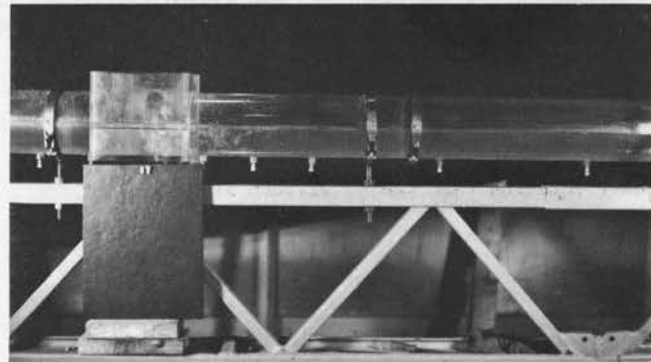


Fig. 4.3. Flow conditions for $S_0 = 0.000537$, $Q_T = 3.30$ with upper inlet.

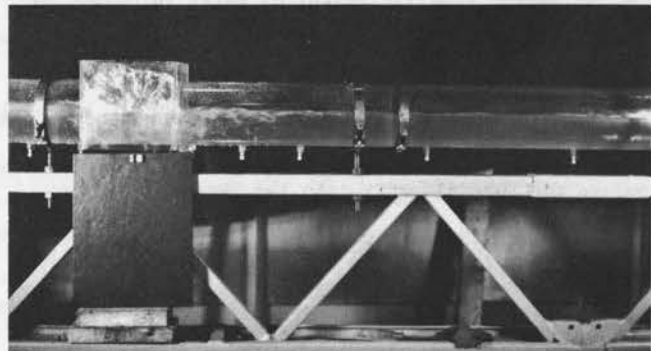


Fig. 4.4. Flow conditions for $S_0 = 0.000537$, $Q_T = 7.40$ with upper inlet.

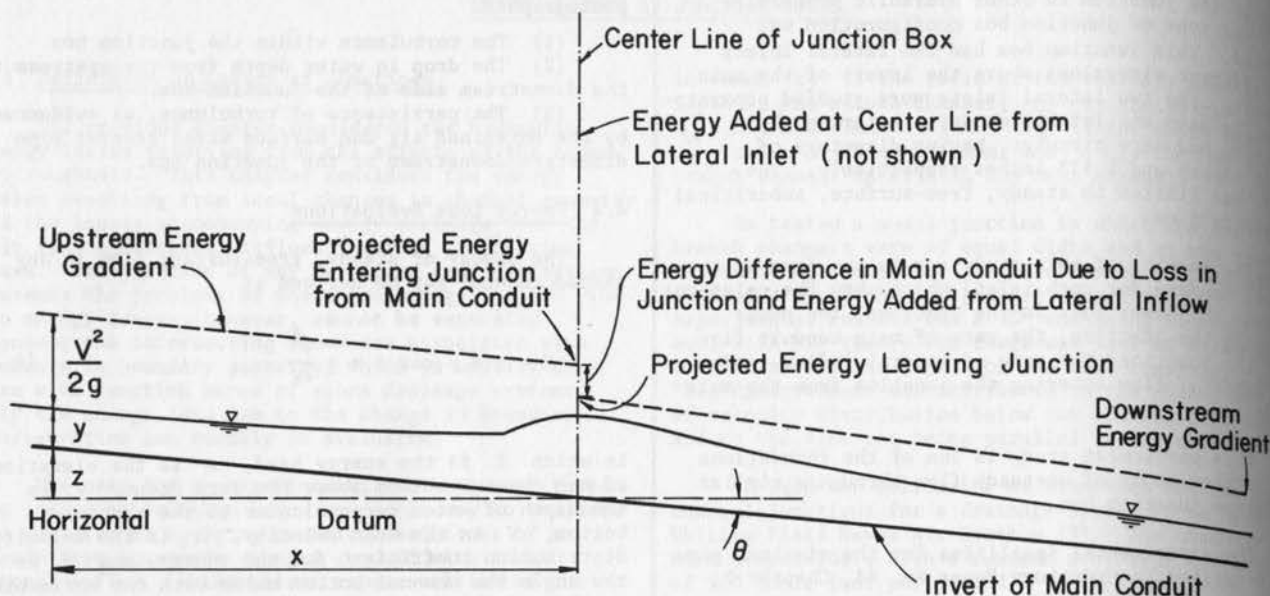


Fig. 4.5. Energy nomenclature.

$$E = z + y + \frac{V^2}{2g} \quad (4.3)$$

The power at any given cross section of the conduit is found by multiplying the energy at its section by the weight rate of flow past the section. The power equation then can be written as

$$P = \gamma QE \quad (4.4)$$

or

$$P = \gamma Q \left(z + y + \frac{V^2}{2g} \right) \quad (4.5)$$

in which P is the power in foot-pounds per second, Q is the volume rate of flow in cubic feet per second, and γ is the specific weight of water in pounds per cubic foot.

Power computations. For each set of flow conditions, power was evaluated at three sections. These sections were: the main conduit section upstream from the junction box, the main conduit section downstream from the junction box, and a lateral section immediately before the lateral entry into the junction box.

Power in the main conduit. The power entering the junction box from the main conduit was calculated by first computing the energy E at each of the stations measured. The energy grade line was then computed by taking a least-squares regression line through five of the energies computed. The energy grade line was linearly projected to the center of the junction box. The power from the main flow into

the junction was computed by multiplying the projected energy at the center of the junction box by the main flow in cubic feet per second and by the specific weight of the water.

The computation of power downstream from the junction was made in a similar manner to computations made for the upstream power.

Power contributed to the junction box by the lateral inflow was similarly computed. In this case it was necessary to consider both the difference in elevation between the lateral conduit invert and the main conduit invert, and the depth of flow in the lateral conduit or the piezometric head of the lateral conduit at full flow.

In the case of the lateral flow with the lower inlet, the inlet can be submerged or flowing partially full. As all experiments were carried out with non-submerged lateral inlets, the power of lateral inflow was determined by the conditions of full lateral pipe flow as existed a short distance upstream of the inlet. This power is then

$$P = \gamma Q_L \left(\frac{Q_L^2}{2gA_L^2} + D_L + z \right) \quad (4.9)$$

in which A_L is the area of the lateral inlet conduit and D_L is the diameter of the lateral conduit. The term z is zero since the zero datum was taken at the invert in the center of the junction box.

Using the lower inlet of the lateral inflow into the junction box and with submerged flow the power input was evaluated by

$$P = \gamma Q_L \left(\frac{Q_L^2}{2gA_L^2} + \bar{y} + z \right) \quad (4.10)$$

The term z is again zero. The depth of water in the junction box is denoted as y and was determined by averaging the depth readings immediately upstream and downstream of the junction box.

When using the upper inlet of lateral inflow into the junction box, with free-surface flow into the main conduit, the power input was evaluated by the relation

$$P = \gamma Q_L \left(\frac{v_c^2}{2g} + y_c + z_u \right) \quad (4.11)$$

in which y_c is the critical depth for the given flow conditions, v_c is the critical velocity, and z_u is the vertical distance from the datum to the invert of the upper inlet. Critical depth and critical velocity were evaluated by the expression

$$\frac{gA^3}{Q^2B} = 1 \quad (4.12)$$

in which A is the cross sectional area, B is the top width, Q is the rate of flow, and g is the gravity acceleration.

Analysis and results. In order to relate the energy losses for various flow conditions, the data were made dimensionless by expressing the power losses as ratios of power inputs in the form

$$P_r = \frac{\gamma Q_m E_m + \gamma Q_L E_L - \gamma Q_d E_d}{\gamma Q_m E_m + \gamma Q_L E_L} \quad (4.13)$$

in which P_r is the ratio of power lost in the junction box to the power entering the junction, the subscript m referring to the upstream conditions or the inflow to the junction box from the main conduit, the subscript L referring to the lateral inflow conditions, and the subscript d referring to the downstream conditions or the outflow from the junction box. The above Eq. 4.13 is thus an expression of efficiency of the given geometry and flow conditions at the junction boxes.

It was assumed that the power loss ratio P_r could be expressed as a function of the relative discharge of the lateral to the main conduit flow when using the upper inlet ($Q_r = Q_L/Q_m$). The validity of this assumption is demonstrated in Fig. 4.6. The functional relation between the power loss ratio and the discharge ratio was developed. The fitted equation to the points in Fig. 4.6 is

$$(P_r - 0.77)(Q_r + 0.55) = -0.482 \quad (4.14)$$

Equation 4.14 was used for all subsequent numerical computations for losses produced at the junction box when lateral inflow was introduced at the upper inlet. Equation 4.14 is applicable for values of Q_r between 0.1 and 6.0. Within this range the standard error in estimating P_r is 0.0218. For Q_r values less than 0.1, the power loss ratio is less than 0.03. When there is no lateral inflow and the depth of flow in the main conduit is less than one-half full, the power loss through the junction box is the same as for the straight, undisturbed conduit.

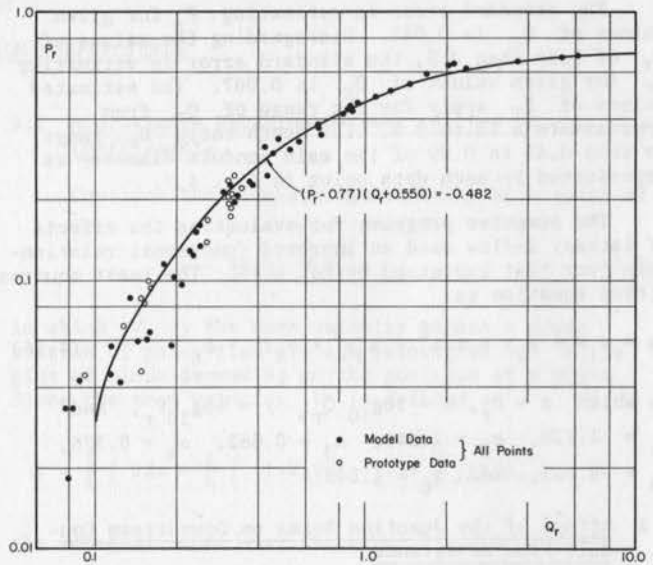


Fig. 4.6. The P_r versus Q_r relationship for the upper inlet of laterals.

Power losses for the lower inlet of lateral inflow at the junction box were analyzed in terms of the ratio of lateral inflow discharge to the main conduit discharge, and the ratio of the upstream depth to the conduit diameter. A plot of the data points is presented in Fig. 4.7. A functional relation of these data points was developed and the resultant power loss ratio is

$$P_r = \frac{-2.78 + 1.71 D_r}{Q_r + 3.122 - 0.167 D_r} + 0.77 \quad (4.15)$$

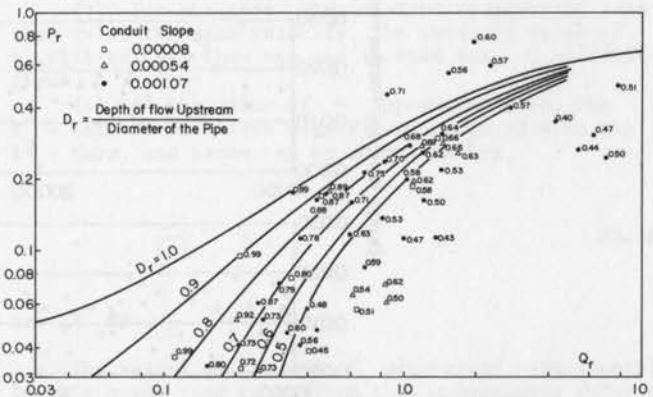


Fig. 4.7. The P_r versus Q_r for various D_r relationship for the lower inlet of laterals.

The standard error in estimating P_r for given values of Q_r is 0.082. Disregarding the values of D_r of less than 0.5, the standard error in estimating P_r for given values of Q_r is 0.067. The estimated values of P_r apply for the range of Q_r from approximate 0.15 to 0.8. The depth ratio D_r range is from 0.40 to 0.99 of the main conduit diameter as represented by each data point in Fig. 4.7.

The computer programs for evaluating the effects of lateral inflow used an improved functional relationship over that expressed by Eq. 4.15. The least squares fitted equation is

$$z = a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + a_6 \quad (4.16)$$

in which $z = D_r$, $x = \log_{10} Q_r$, $y = \log_{10} P_r$, and $a_1 = -1.129$, $a_2 = 1.045$, $a_3 = 0.662$, $a_4 = 0.376$, $a_5 = -0.753$, and $a_6 = 1.045$.

4.5 Effect of the Junction Boxes on Downstream Conduit Flow Resistance

For each of the run conditions used to develop the power loss ratios at the junction box, data were collected to evaluate the hydraulic boundary resistance downstream of the junction box. These values were consistently higher than the values of hydraulic resistance computed upstream of the junction box.

An explanation of this observation involves two factors. The first factor is the increased turbulence introduced into the main conduit flow by the lateral inflow and the junction-box geometry. All the lateral inflows studied intersected the main conduit at 90° to the main conduit centerline. The intersecting flow had to traverse a right angle turn as well as undergo a change in the particle velocities. This combined motion produces standing surface waves and an internal rotational motion superimposed on the main conduit

flow. This higher velocity near the solid boundary increases boundary shear and hence the boundary resistance. The effect persists downstream until the normal depth is approached which in this experiment was about 50 pipe diameters.

The second factor is the experimental difficulties in measuring the water surface due to the transverse waves and longitudinal surges generated by the instabilities in the junction box. In spite of the measurement difficulties, the analysis of data show consistencies in the means of junction hydraulic parameters, but with relatively wide scattering of points about the fitted curves.

The observed hydraulic friction factors of the Darcy-Weisbach equation are presented in Fig. 4.8 for the downstream main conduit section of upper and lower inlet positions, as well as for the upstream main conduit section. Within the upstream reach, with no flow disturbance produced by lateral inflow, the friction factor ranges approximately between 0.010 and 0.020. Downstream of the junction box, with lateral inflow from the upper inlet, the friction factor ranges between 0.020 and 0.030 with a tendency toward the lower end of the range. Downstream of the junction box, with lateral inflow from the lower inlet, the friction factors range approximately between 0.010 and 0.030 with a tendency to cluster around a value of 0.020.

From the preceding discussion, it is concluded that the energy losses resulting from lateral inflow into a junction box consist of two components: the immediate energy losses during the change in direction of the lateral flow and the gradual decay of turbulence in the channel downstream of the junction. The former has been evaluated and related to the significant parameters of geometry and flows. The latter has not been evaluated in a manner which would permit application to general use.

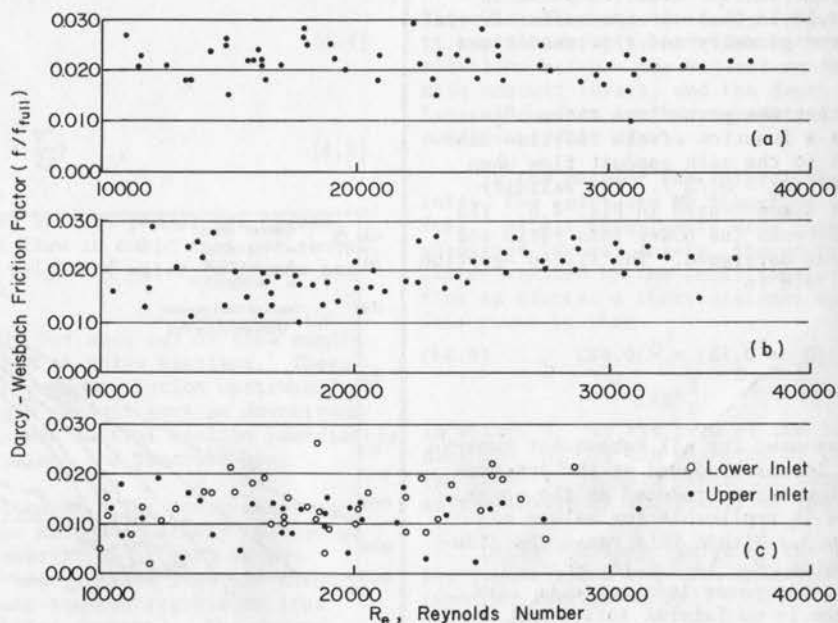


Fig. 4.8. The relation of friction factor f versus Reynolds number R_e : (a) downstream, upper inlet, (b) downstream, lower inlet and (c) upstream, upper and lower inlets.

VELOCITY DISTRIBUTION COEFFICIENTS

5.1 Definition of Velocity Distribution Coefficients

Equations which define the velocity distribution coefficients are based on momentum and energy considerations, respectively. This may be demonstrated as follows. Momentum due to a motion of an incompressible fluid may be expressed as

$$M = \int_A \rho v |v| dA, \quad (5.1)$$

in which $\rho |v| dA$ describes the mass of water passing through the area dA of a cross section in a unit of time. The integrals in the following text are over the cross sectional area A and the limit A will be deleted in these expressions.

One-dimensional considerations give

$$M = \rho \int v^2 dA. \quad (5.2)$$

An approximation to this evaluation of one-dimensional momentum flux is to represent it in terms of the mean velocity V as,

$$M = \beta \rho V^2 A. \quad (5.3)$$

The coefficient β is then defined as

$$\beta = \frac{\int v^2 dA}{V^2 A}. \quad (5.4)$$

The constant in the equation of kinetic energy per unit weight correspondingly develops into the form

$$\alpha = \frac{\int v^3 dA}{V^3 A}. \quad (5.5)$$

Thus, the general theoretical form of a velocity distribution coefficient of n -th order is represented for the velocity v at a point raised to the power n by

$$\phi = \frac{\int v^n dA}{V^n A}, \quad (5.6)$$

in which equation n takes on only integer values. For n equal to one, the coefficient ϕ is unity, which by definition is the mean velocity. Values of $n = 2$ and $n = 3$ in Eq. 5.6 give the momentum (β) and energy (α) velocity distribution coefficients, respectively.

Equation 5.6 permits the evaluation of the effect of velocity distributions in a cross section on α and β , and the computation of interrelation between α and β .

5.2 Approximate Relation Between α and β Coefficients

Consider the time-average velocity at a point as

$$v = (1+k) V, \quad (5.7)$$

in which V is the mean velocity across a cross section of point time-average velocities and k is plus or minus depending on the position of a point. Since the mean velocity V is defined as

$$V = \frac{1}{A} \int v dA = \frac{1}{A} \int (1+k) V dA = V + \frac{V}{A} \int k dA,$$

the integral $k dA$ over the area A must be zero.

When β and α are expressed in terms of k , then

$$\beta = \frac{\int v^2 (1+k)^2 dA}{V^2 A} = 1 + \frac{1}{A} \int k^2 dA, \quad (5.8)$$

and

$$\alpha = \frac{\int v^3 (1+k)^3 dA}{V^3 A} = 1 + \frac{3}{4} \int k dA + \frac{3}{A} \int k^2 dA + \frac{1}{A} \int k^3 dA.$$

Because the integral of $k dA$ is zero,

$$\alpha = 1 + \frac{3}{A} \int k^2 dA + \frac{1}{A} \int k^3 dA. \quad (5.9)$$

From Eqs. 5.8 and 5.9 the following conclusions may be derived:

(a) The larger the deviation of the point velocities from the mean, the larger are the values of the velocity distribution coefficients, α and β .

(b) For the case when maximum velocity is less than twice the mean velocity, the absolute value of k will be less than one and in that case $0 < |\int k^3 dA| < \int k^2 dA$.

(c) As the value of k approaches zero, the k^3 - term becomes less significant compared with the k^2 - term, and hence, as an approximation,

$$\frac{\alpha-1}{\beta-1} \approx 3, \quad (5.10)$$

or $\alpha \approx 3\beta - 2$.

The values of α and β determined experimentally in this study tend to confirm this approximate relation.

5.3 Evaluation of Velocity Distribution Coefficients

Equation 5.6 suggests several methods of evaluating the velocity distribution coefficients, ϕ . One method is the analytical integration of a given time-average velocity distribution function over the cross section, the other two being the graphic and numerical methods of integration using point time-average velocities observed in a specific flow by a replacement of integrals by summations.

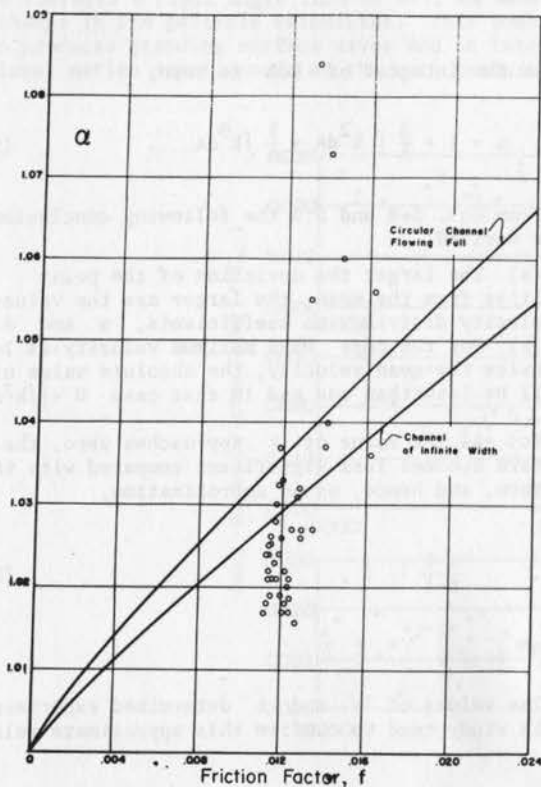
Analytical Integration Method. The analytical integration of Eq. 5.6 depends on a knowledge of the explicit function of velocity as related to position. Such equations for fully developed turbulent flow are available only for certain limiting cases of boundary configuration.

For the case of an infinitely-wide open channel with two-dimensional flow the following equation has been determined experimentally for velocity distributions outside the boundary layer [4]:

$$\frac{(v-V)}{V\sqrt{f}} = 2 \log_{10} \frac{y}{y_0} + 0.88, \quad (5.11)$$

in which v is the point velocity at position y , V is the mean velocity for depth y_0 and f is the friction factor in the Darcy-Weisbach equation. Substituting the value of v from Eq. 5.11 into Eq. 5.6, and integrating in the limits from zero to y_0 , ϕ - coefficients for values of $n = 2$ and $n = 3$ are, respectively,

$$\beta = \phi_2 = 1.0 + 0.755 f + 0.023 \sqrt{f} + \delta_1(f) \quad (5.12)$$



and

$$\alpha = \phi_3 = 1.0 + 2.263 f + 0.035 \sqrt{f} - 1.284 f \sqrt{f} + \delta_2(f), \quad (5.13)$$

in which $\delta_1(f)$ and $\delta_2(f)$ represent the negligible effects of boundary layers. In this case, the ϕ - functions are the result of integrating Eq. 5.11 from the limit of the boundary layer rather than from the solid boundary. In each case, however, the velocity distribution functions of the boundary layer have a negligible effect on the respective velocity distribution coefficients. The plots of these equations are shown in Fig. 5.1 as the computed α and β values of partly flowing conduits. Within this range of friction factors the theoretical dependencies of α and β coefficients on f are approximately linear for values of f greater than 0.004.

It is interesting to note that the ratio of $(\alpha-1)/(\beta-1)$ lies approximately between 2.5 and 2.6 as shown in Fig. 5.2. The fact that the ratio of 3.0 of Eq. 5.10 does not agree with the range 2.5 - 2.6 as developed for the logarithmic velocity distribution in an infinitely-wide channel indicates that, for that case at least, the integral of the k^3 - term must be significant as well as negative.

For the case of a full flow in a circular conduit, the following equation has been determined experimentally to be the velocity distribution function [4]:

$$\frac{v-V}{V\sqrt{f}} = 2 \log_{10} \left(\frac{y}{r_0} \right) + 1.32, \quad (5.14)$$

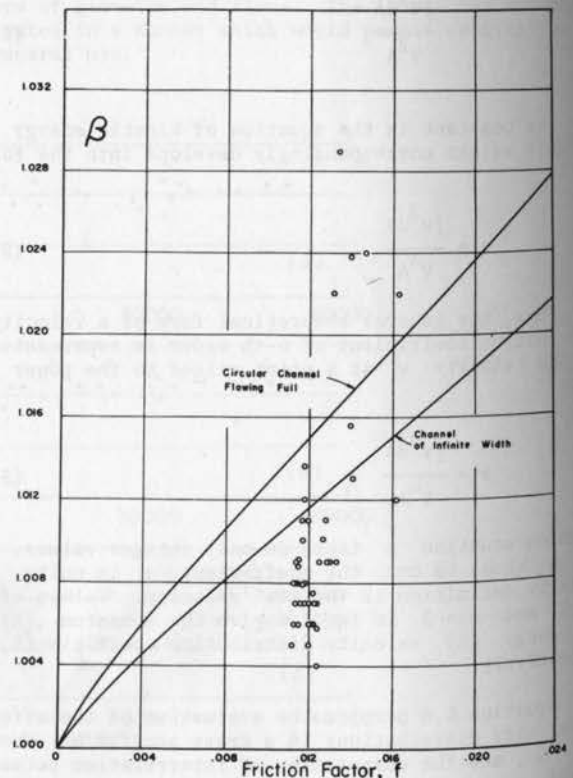


Fig. 5.1. Theoretical relation and CSU experimental points of velocity distribution coefficients versus friction factor f .

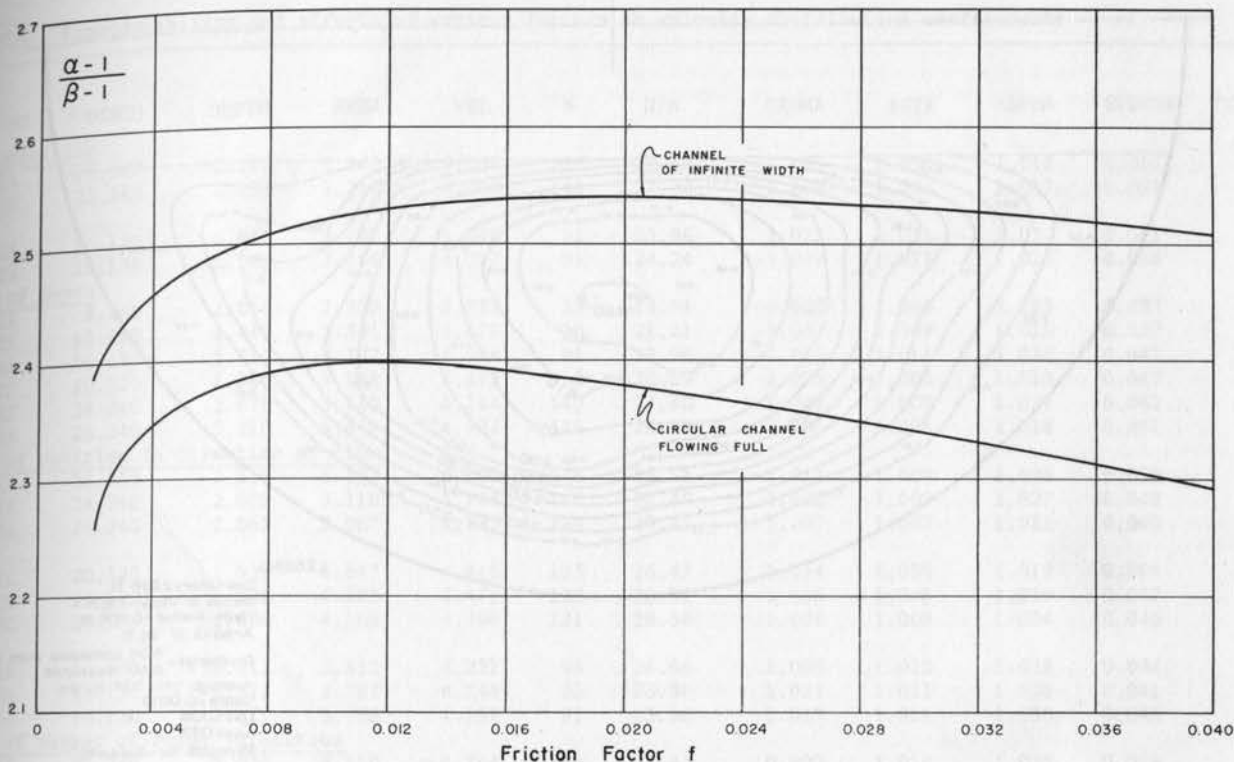


Fig. 5.2. Relation of α and β versus friction factor f .

in which the variables are defined as before, and r_0 is the radius of the conduit. Substituting the value of v from Eq. 5.14 into Eq. 5.6 and solving for ϕ within the limits of boundary thickness (δ) and r_0 one obtains respectively

$$\beta = \phi_2 = 1 + 0.034 \sqrt{f} + 0.941 f \quad (5.15)$$

and

$$\alpha = \phi_3 = 1 + 0.051 \sqrt{f} + 2.828 f - 2.685 f \sqrt{f} \quad (5.16)$$

The values of α and β from Eqs. 5.15 and 5.16 are also plotted in Fig. 5.1. These curves approach straight lines for a large range of f -values. Similarly, as in the case of an infinitely-wide open channel, α and β coefficients depend only on the friction factor f . In this case, the ratio $(\alpha-1)/(\beta-1)$ also lies approximately between 2.3 and 2.4, as shown in Fig. 5.2.

Because the velocity distributions are often non symmetrical around the mean velocity, the integral $\int k^3 dA$ may be negative if the absolute values of the negative k are much greater than the absolute values of the positive k . This is the case in the velocity distributions of both the infinitely-wide open channel and the full pipe flow as given by Eqs. 5.11 and 5.14 while using a lower limit of zero. Therefore, $(\alpha-1)/(\beta-1)$ being smaller than 3.0 is expected.

The comparison of curves of Eqs. 5.15 and 5.16 with the corresponding curves for β and α of Eqs. 5.12 and 5.13 in Fig. 5.1 indicates that for channels in which the side walls substantially affect the

velocity distributions (or when the height of sides is of the same order of magnitude as the width of the channel bottom) the k -values, on the average, are greater than the k values for infinitely-wide open channels. Hence, the α and β -coefficients are greater for full flows in circular conduits.

Because velocity distributions in partial flows through circular conduits can be considered cases which are between the velocity distributions of an infinitely-wide open channel and a full flowing circular conduit, Eqs. 5.12, 5.13, 5.15 and 5.16 give an indication and a range of the expected velocity distribution coefficients as they change with the friction factor f .

Graphic integration method. The classic method for computing velocity distribution coefficients from observed data is to plot the position of observed velocities along with the velocity at that point. The lines of equal velocity (isovels) are then drawn by interpolating between the known velocities. The area between successive incremental velocities is then determined, for example, by a planimeter. The summation of the individual areas times the mean velocity in the area taken to the appropriate power (2 or 3), provides the numerical integration of the numerator of β and α . A sample of the result is shown in Fig. 5.3.

Numerical integration method. A numerical integration method was developed around the point velocity measurement equipment. Time-average point velocities were measured by Ott current meters. Five meters were mounted on a center-supported rod which was then placed at the conduit centerline. The rod support could rotate to place the meters in any angular position. The meters were spaced along the

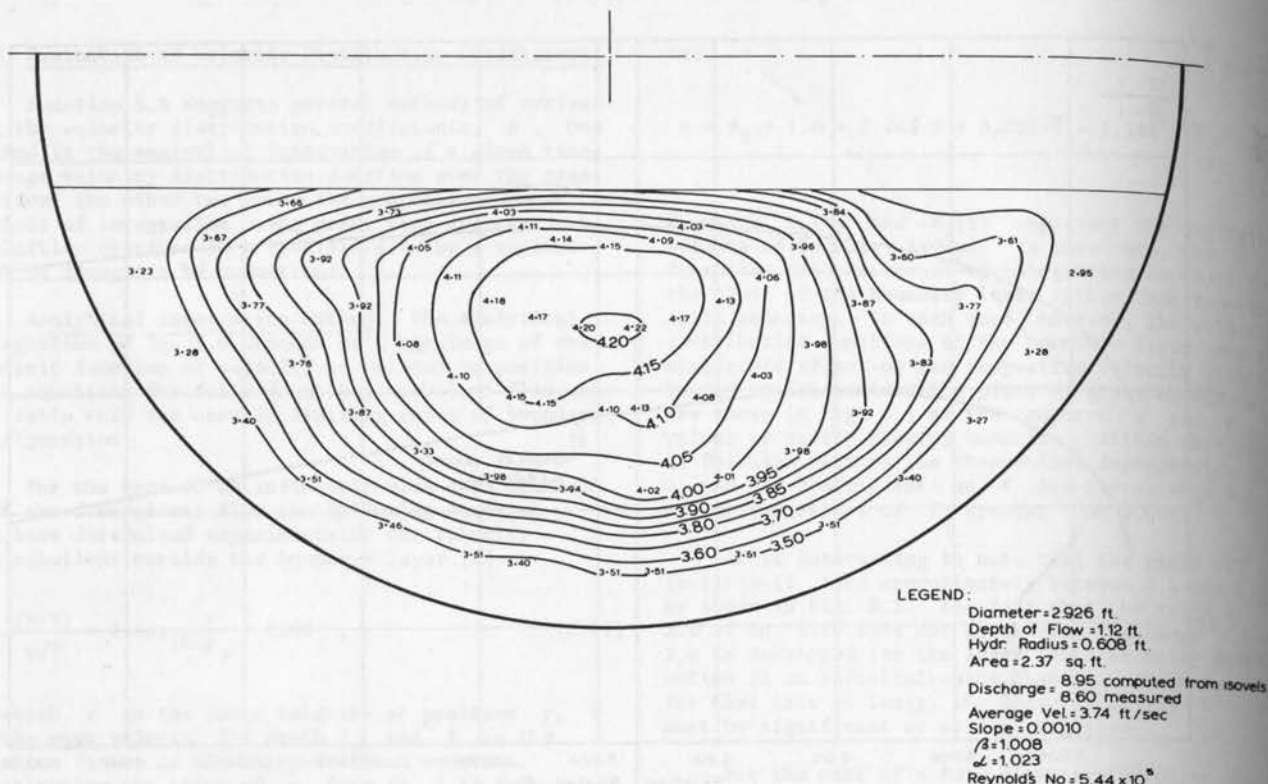


Fig. 5.3. Isovels (equal velocity lines) for partially full conduit flow.

rod to sample equal annular areas and the end meter was placed at the minimum recommended distance from the pipe wall. The support rod of the meters was positioned at angular intervals of 10 degrees and point velocities were observed at five radial positions using as many 10-degree intervals as required to sample the circular segment.

Data processing was based on the observation that velocity distributions along the radial directions and along circumferential arcs of constant radius were smooth and could be approximated by third-degree polynomials.

The computer procedure was as follows:

1. A third-degree polynomial was fitted to the observed velocities at a fixed radius, of the form

$$v_r = a_r + b_r \theta + c_r \theta^2 + d_r \theta^3 \quad (5.17)$$

2. For a given angular position, a third-degree polynomial was fitted to the computed velocities along the radial direction, of the form

$$v_\theta = a_\theta + b_\theta r + c_\theta r^2 + d_\theta r^3 \quad (5.18)$$

3. For the velocity expression at a given angular position, the integrals of $v_\theta dA$, $v_\theta^2 dA$ and $v_\theta^3 dA$ are obtained, in which dA was the area represented by a 10-degree sector to the free-water surface.

4. The results of Step 3 divided by the appropriate relation of the mean velocity and the total area resulted in α and β coefficients.

The root-mean-square difference between the observed velocities and the computed velocities (based on the polynomial fitting procedure) was computed for each case of measurement of velocity distributions of partly full conduits. The calibration equations for each current meter and propeller were included in the computer program so that the velocity was computed from the given data before polynomial fitting was begun.

5.4 Results of Experimental Investigations

Several comparisons of test conditions and observational procedures were made to identify their effects on the computed velocity distributions. These considerations were reproducibility of results, effects of depth, location along the conduit, number of point velocities, and length of time for observing the mean velocity. The results of these evaluations are presented in Table 5.1. This table gives data in the following order: run identification (RUN NO.), discharge in cubic feet per second as measured by the inflow orifice (DISCH), depth of flow in feet at the measurement cross section (DEPTH), cross sectional area in square feet (AREA), mean-velocity in feet per second based on the measured discharge (VEL), number of point velocity observations (N), average number of point observations per square foot (N/A), ratio of the mean velocity based on the measured discharge compared to the mean velocity obtained by integrating the observed velocities (GAMMA), momentum velocity distribution coefficient (BETA), energy velocity distribution coefficient (ALPHA), the root-mean square difference between the observed velocities

TABLE 5.1. Reproducibility and effects of various factors on velocity distribution coefficients

RUN NO	DISCH	DEPTH	AREA	VEL	N	N/A	GAMMA	BETA	ALPHA	STDDEV	TIME
Reproducibility											
X7MH2A	26.340	2.210	5.449	4.834	146	26.79	1.006	1.006	1.018	0.051	
X7MH2B	26.340	2.139	5.268	5.000	144	27.34	1.008	1.005	1.017	0.081	
X10M2A	16.130	1.612	3.797	4.248	91	23.96	1.025	1.011	1.030	0.041	
X10M2B	16.130	1.597	3.754	4.297	91	24.24	1.028	1.011	1.029	0.035	
Effect of Depth											
X12M2A	8.260	1.064	2.209	3.739	33	14.94	0.920	1.066	1.123	0.037	
X6MH2A	13.450	1.442	3.301	4.075	70	21.21	1.037	1.005	1.019	0.037	
X10M2A	16.130	1.612	3.797	4.248	91	23.96	1.025	1.011	1.030	0.041	
X9MH2C	20.520	1.888	4.588	4.472	122	26.59	1.025	1.006	1.019	0.047	
X8MH2X	24.240	2.079	5.110	4.744	140	27.40	1.002	1.009	1.027	0.042	
X7MH2A	26.340	2.210	5.449	4.834	146	26.79	1.006	1.006	1.018	0.051	
Effect of Position in Direction of Flow											
X8MH1A	23.930	2.110	5.192	4.609	148	28.51	1.011	1.009	1.025	0.058	
X8MH2X	24.240	2.079	5.110	4.744	140	27.40	1.002	1.009	1.027	0.042	
X8MH3A	24.240	2.063	5.067	4.783	144	28.42	1.007	1.007	1.021	0.048	
X9MH1C	20.520	1.909	4.647	4.416	123	26.47	1.014	1.006	1.019	0.094	
X9MH2C	20.520	1.888	4.588	4.472	122	26.59	1.025	1.006	1.019	0.047	
X9MH3C	20.520	1.880	4.566	4.494	121	26.50	1.024	1.008	1.024	0.046	
X10M1A	16.130	1.617	3.812	4.231	94	24.66	1.006	1.010	1.028	0.044	
X10M2A	16.130	1.612	3.797	4.248	91	23.96	1.025	1.011	1.030	0.041	
X10M3A	16.130	1.611	3.795	4.251	91	23.98	1.017	1.011	1.030	0.045	
Effect of Number of Point Velocities											
X8MH2X	24.240	2.079	5.110	4.744	140	27.40	0.999	1.010	1.028	0.029	
X8MH2Y	24.240	2.094	5.150	4.707	73	14.18	1.012	1.008	1.022	0.049	
Effect of Length of Time of Observations											
X6MH2A	13.450	1.442	3.301	4.075	70	21.21	1.037	1.005	1.019	0.037	30
X6MH2A	13.450	1.442	3.301	4.075	70	21.21	1.034	1.005	1.020	0.028	60
X6MH2A	13.450	1.442	3.301	4.075	70	21.21	1.032	1.005	1.019	0.021	90
X6MH2A	13.450	1.442	3.301	4.075	70	21.21	1.031	1.005	1.020	0.018	120
X6MH2A	13.450	1.442	3.301	4.075	70	21.21	1.031	1.005	1.019	0.018	150
X6MH2A	13.450	1.442	3.301	4.075	70	21.21	1.032	1.006	1.020	0.101	180
X8MH2X	24.240	2.079	5.110	4.744	140	27.40	1.002	1.009	1.027	0.042	30
X8MH2X	24.240	2.079	5.110	4.744	140	27.40	0.999	1.010	1.028	0.029	60
X8MH2X	24.240	2.079	5.110	4.744	140	27.40	0.999	1.009	1.027	0.029	90
X8MH2X	24.240	2.079	5.110	4.744	140	27.40	0.999	1.009	1.027	0.030	120
X8MH2X	24.240	2.079	5.110	4.744	140	27.40	0.998	1.009	1.027	0.030	150
X8MH2X	24.240	2.079	5.110	4.744	139	27.20	0.998	1.009	1.027	0.030	180
X8MH2Y	24.240	2.094	5.150	4.707	73	14.18	1.012	1.008	1.022	0.049	30
X8MH2Y	24.240	2.094	5.150	4.707	74	14.37	1.013	1.008	1.022	0.039	60
X8MH2Y	24.240	2.094	5.150	4.707	73	14.18	1.011	1.008	1.023	0.032	90
X8MH2Y	24.240	2.094	5.150	4.707	73	14.18	1.010	1.008	1.024	0.029	120
X8MH2Y	24.240	2.094	5.150	4.707	73	14.18	1.010	1.008	1.024	0.030	150
X8MH2Y	24.240	2.094	5.150	4.707	73	14.18	1.010	1.008	1.024	0.026	180

and the velocities in feet per second computed from the polynomial fits (STDDEV), and the observed time interval in seconds used to compute the mean velocity.

These results were not intended to be conclusive. They do provide, however, a measure of effect and reliability of the overall results.

The general observations on these data are:

- The root-mean-square (STDDEV), as a percent of the mean velocity, is of the order of 1 percent.
- The computed mean velocity is larger than the measured mean velocity by less than 3 percent in most cases.
- The relation between α and β coefficients conforms approximately with Eq. 5.10.

(d) The variation of the velocity distribution coefficients within any one of five effect categories is insufficient to detect a clear effect over the entire sample size. In other words, the experimental and computational errors overshadow the effect of varying the experimental conditions.

Based on the preliminary results of Table 5.1, an extended series of observations were made to relate the velocity distribution coefficients to the depth and the mean velocity. The observations were made at the midpoint of the 822 foot conduit. Each point velocity was averaged over a 60-second period. The results of these observations are presented in Table 5.2.

TABLE 5.2. Velocity distribution coefficients in the conduit of 2.926 feet in diameter.

Designation	Slope	Depth ft	Hydraulic Radius R, ft	Discharge Q cfs	Mean Velocity fps	$Re = \frac{VR}{\nu}$ $\times 10^5$	α	β	Friction Factor f
S1-4	0.000032	2.926	0.732	5.610	0.834	0.4	1.036	1.012	0.0162
S1-7		1.547	0.757	11.980	3.320	1.653	1.017	1.004	0.0124
S1-8		1.779	0.818	16.040	3.748	2.017	1.019	1.006	0.0120
S1-9		1.984	0.857	19.620	4.042	2.278	1.021	1.007	0.0118
S2-2	0.000132	1.749	0.811	10.080	2.404	1.283	1.026	1.009	0.0129
S2-3		2.064	0.869	15.340	3.026	1.736	1.020	1.007	0.0123
S2-4		2.371	0.890	18.940	3.245	1.900	1.017	1.006	0.0121
S2-5		2.630	0.873	19.570	3.073	1.765	1.018	1.006	0.0122
S2-6		1.152	0.620	4.710	1.915	0.781	1.040	1.013	0.0141
S2-9		0.903	0.512	3.260	1.848	0.622	1.060	1.024	0.0147
S2-10		1.785	0.819	16.640	3.873	2.087	1.024	1.008	0.0119
S2-10		1.936	0.849	16.640	3.524	1.968	1.021	1.007	0.0120
S3-1	0.000520	2.644	0.870	18.350	2.870	1.643	1.021	1.007	0.0124
S3-2		2.309	0.889	12.270	2.156	1.261	1.027	1.009	0.0130
S3-3		2.079	0.870	14.100	2.756	1.577	1.027	1.009	0.0125
S3-4		1.740	0.809	10.410	2.498	1.330	1.032	1.011	0.0128
S3-5		1.497	0.742	7.960	2.299	1.122	1.055	1.022	0.0132
S3-6		1.154	0.620	6.210	2.519	1.028	1.084	1.029	0.0134
S3-7		0.871	0.497	2.040	1.215	0.397	1.056	1.022	0.0163
S3-10		1.771	0.816	15.970	3.752	2.014	1.033	1.011	0.0120
D2A	0.001022	0.810	0.468	4.000	2.637	0.812	1.073	1.024	0.0140
D2B		0.817	0.471	4.000	2.605	0.807	1.037	1.016	0.0140
D3C		1.964	0.854	8.220	1.713	0.962	1.027	1.009	0.0135
D7A		1.989	0.858	23.380	4.803	2.711	1.024	1.008	0.0114
D7C		2.357	0.890	23.380	4.028	2.358	1.021	1.007	0.0116
D8B		2.166	0.880	25.620	4.800	2.779	1.024	1.008	0.0113
X6MH2A	0.001001	1.442	0.725	13.450	4.075	1.944	1.019	1.005	0.0120
X7MH2A		2.210	0.884	26.340	4.835	2.812	1.018	1.006	0.0113
X7MH2B		2.139	0.878	26.340	5.000	2.888	1.017	1.005	0.0112
X8MH1A		2.110	0.874	23.930	4.609	2.650	1.025	1.009	0.0114
X8MH2X		2.079	0.871	24.240	4.744	2.718	1.027	1.009	0.0114
X8MH24		2.094	0.873	24.240	4.707	2.703	1.022	1.008	0.0114
X8MH3A		2.063	0.869	24.240	4.783	2.734	1.021	1.007	0.0114
X9MH1C		1.909	0.844	20.520	4.416	2.452	1.019	1.006	0.0115
X9MH2C		1.888	0.841	20.520	4.472	2.474	1.019	1.006	0.0115
X9MH3C		1.880	0.839	20.520	4.494	2.481	1.024	1.008	0.0115
X10M1A		1.617	0.777	16.130	4.231	2.163	1.028	1.010	0.0117
X10M2A		1.612	0.776	16.130	4.248	2.169	1.030	1.011	0.0117
X10M3A		1.611	0.775	16.130	4.251	2.167	1.030	1.011	0.0117
X10M2B		1.597	0.772	16.130	4.297	2.182	1.030	1.011	0.0117
X12M1A		1.090	0.843	8.260	3.618	2.006	1.035	1.007	
X12M2A		1.064	0.583	8.260	3.739	1.434	1.123	1.066	
X12M2B		1.372	0.701	8.260	2.668	1.230	1.095	1.040	
X12M2C		1.066	0.583	8.300	3.747	1.437	1.024	(1)	
D3A	0.001022	1.057	0.580	8.220	3.754	1.432	1.031	1.010	0.0127
D3B		1.078	0.589	8.220	3.655	1.416	1.016	(1)	0.0127
D4C		1.903	0.843	12.920	2.790	1.666	1.019	1.006	0.0124
D5A		1.605	0.774	16.000	4.236	2.157	1.032	1.012	0.0118
D5B		1.601	0.772	16.000	4.249	2.159	1.037	1.014	0.0118
D5C		2.187	0.882	16.000	2.968	1.718	1.022	1.008	0.0122
D6A		1.855	0.834	20.510	4.562	2.503	1.025	1.009	0.0115
D6B		1.868	0.837	20.510	4.526	2.492	1.026	1.009	0.0115
D6C		2.198	0.883	20.510	3.785	2.199	1.023	1.008	0.0117

(1) Computed value less than the minimum value of one.

5.5 Discussion of Experimental Results

It was expected that the velocity distribution coefficients would differ with changes in those parameters which determine the velocity profiles. The parameters which primarily effect the velocity profile are the geometry of the cross section, the properties of the fluid, the condition of the boundary surface (roughness), and the mean velocity. All of these variables are encompassed in the Reynolds number (VR/v) and the Darcy-Weisbach friction factor (f).

Because the Darcy-Weisbach friction factor is related to the Reynolds number, one would expect to be able to correlate α and β -coefficients to the friction factor f . The experimentally observed values of α and β in this study are plotted in Fig. 5.1. Since the range of the Darcy-Weisbach friction factor is small for the series of data in this study, and because the Reynolds number fluctuates within a limited range, the spread of computed α and β coefficients is apparently due to other causes.

Figure 5.4 displays the relation of α and β with the Reynolds number. These results indicate an increase of the velocity distribution coefficients with a decrease of Reynolds number. The general trends may be represented by two curves. The apparent scatter of points around these curves may be assigned to observational and computational errors.

The following parameters of the computed velocity distribution coefficients remained essentially constant: the circular form of the section, the fluid properties (water at approximately 45°F), and

approximately the friction factor because the Reynolds number varied over a narrow range. It would follow, therefore, that the variation in α and β could be represented as a function of depth and mean velocity or slope as a first approximation. Because the effect of depth and mean velocity are included, through their product, into the Reynolds number (assuming an approximate proportionality of depth and hydraulic radius), the main relation should be between the velocity distribution coefficients and the Reynolds number.

The effect of depth on the velocity distribution coefficients is presented in Table 5.3 and Fig. 5.5. The values in Table 5.3 are grouped in ascending order of the depth-diameter ratio (y/D). The corresponding mean velocities (V) which are also listed do not arrange themselves in any discernable manner. This is probably because the mean velocity increases with the depth for a given slope and roughness, and the depth has already accounted for the effects of the mean velocity. Figure 5.5 indicates slightly increasing values of both β and α as depth decreases. This is expected because deviation from the mean velocity becomes greater and the friction factor becomes effectively larger as the depth decreases.

At a depth of half the conduit diameter β -coefficient has a value of approximately 1.01 and α -coefficient of approximately 1.03. At greater depths β reduces to approximately 1.007 and α reduces to approximately 1.022. For depths less than half the conduit diameter, both coefficients tend to increase. Data was not available for depths less than one-fourth of a diameter.

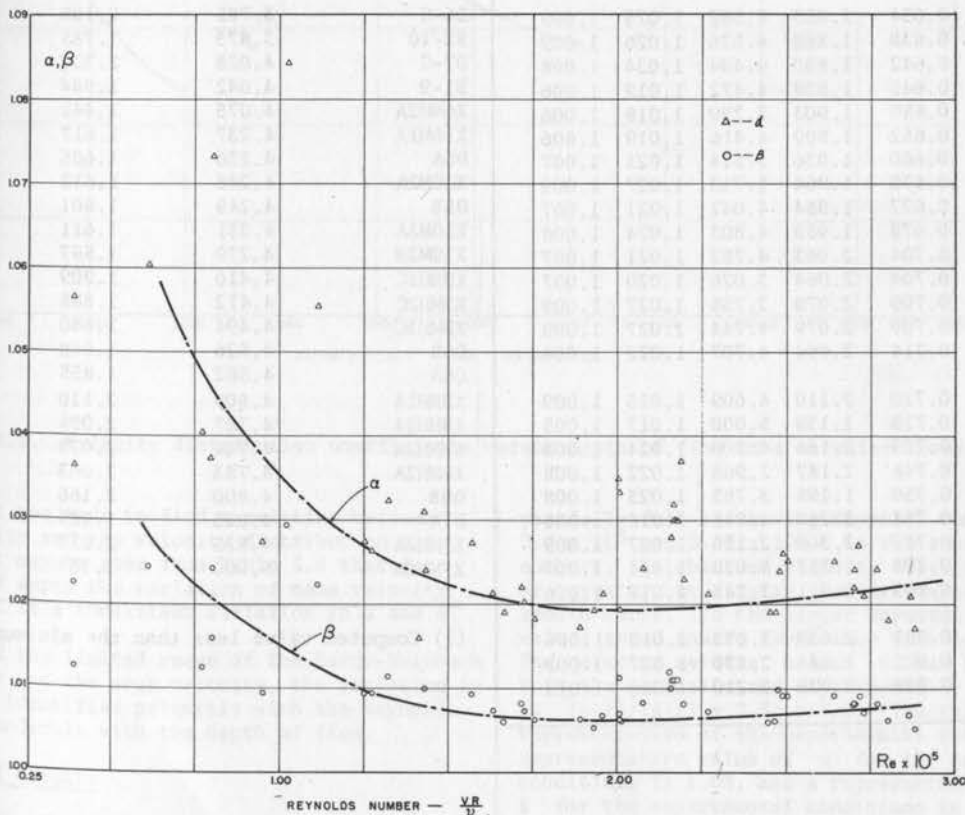


Fig. 5.4. Velocity distribution coefficients versus Reynolds number.

TABLE 5.3. Velocity distribution coefficients as a function of depth.

Range of Depth ft.	Run No.	Depth-Diameter Ratio	Depth y, ft	Velocity V, fps	α	β
0-1.0	D2A	0.276	0.810	2.637	1.073	1.024
	D2B	0.279	0.817	2.605	1.037	1.016
	S3-7	0.297	0.871	1.215	1.056	1.022
	S2-9	0.310	0.903	1.848	1.060	1.024
1.0-1.5	D3A	0.360	1.057	3.754	1.031	1.010
	X12M2A	0.363	1.064	3.739	1.123	1.066
	X12M2C	0.366	1.066	3.747	1.024	(1)
	X12M1A	0.373	1.090	3.618	1.035	1.007
	D3B	0.368	1.078	3.655	1.016	(1)
	S2-6	0.393	1.152	1.915	1.040	1.013
	S3-6	0.394	1.154	2.519	1.084	1.029
	X12M2B	0.468	1.372	2.668	1.095	1.040
	X6MH2A	0.492	1.442	4.075	1.019	1.005
	S3-5	0.510	1.497	2.299	1.055	1.022
1.5-1.8	S1-7	0.528	1.547	3.320	1.017	1.004
	X10M2B	0.545	1.597	4.297	1.030	1.011
	D5B	0.546	1.601	4.249	1.037	1.014
	D5A	0.547	1.605	4.236	1.032	1.012
	X10M3A	0.549	1.611	4.251	1.030	1.011
	X10M2A	0.550	1.612	4.248	1.030	1.011
	X10M1A	0.551	1.617	4.231	1.028	1.010
	S3-4	0.593	1.740	2.498	1.032	1.011
	S2-2	0.596	1.749	2.404	1.026	1.009
	S3-10	0.585	1.771	3.752	1.033	1.011
	S1-8	0.604	1.779	3.748	1.019	1.006
	S2-103	0.609	1.785	3.873	1.024	1.008
1.8-2.1	D6A	0.634	1.855	4.562	1.025	1.009
	D6B	0.638	1.868	4.526	1.026	1.009
	X9MH3C	0.642	1.880	4.494	1.024	1.008
	X9MH2C	0.645	1.888	4.472	1.019	1.006
	D4C	0.650	1.903	2.790	1.019	1.006
	X9MH1C	0.652	1.909	4.416	1.019	1.006
	S2-10	0.660	1.936	3.524	1.021	1.007
	D3C	0.670	1.964	1.713	1.027	1.009
	S1-9	0.677	1.984	4.042	1.021	1.007
	D7A	0.678	1.989	4.803	1.024	1.008
	X8MH3A	0.704	2.063	4.783	1.021	1.007
	S2-3	0.704	2.064	3.026	1.020	1.007
	S3-3	0.709	2.079	2.756	1.027	1.009
	X8MH2X	0.709	2.079	4.744	1.027	1.009
	X8MH24	0.714	2.094	4.707	1.022	1.008
2.1-2.5	X8MH1A	0.720	2.110	4.609	1.025	1.009
	X7MH2B	0.729	2.139	5.000	1.017	1.005
	D8B	0.739	2.166	4.800	1.024	1.008
	D5C	0.746	2.187	2.968	1.022	1.008
	D6C	0.750	1.198	3.785	1.023	1.008
	X7MH2A	0.754	2.210	4.835	1.018	1.006
	S3-2	0.787	2.309	2.156	1.027	1.009
	D7C	0.804	2.357	4.028	1.021	1.007
	S2-4	0.809	2.371	3.245	1.017	1.006
2.5-	S2-5	0.897	2.630	3.073	1.018	1.006
	S3-1	0.902	2.644	2.870	1.027	1.009
	S1-4	0.998	2.926	2.210	1.036	1.012

TABLE 5.4. Velocity distribution coefficients as a function of mean velocity.

Run No.	Velocity V, fps	Depth y, ft	α	β
S3-7	1.215	0.871	1.056	1.022
D3-C	1.713	1.964	1.027	1.009
S2-9	1.848	0.903	1.060	1.024
S2-6	1.915	1.152	1.040	1.013
S3-2	2.156	2.309	1.027	1.009
S1-4	2.210	2.926	1.036	1.012
S3-5	2.299	1.497	1.055	1.022
S2-2	2.404	1.749	1.026	1.009
S3-4	2.498	1.740	1.032	1.011
S3-6	2.519	1.154	1.084	1.029
D2B	2.605	0.817	1.037	1.016
D2A	2.637	0.810	1.073	1.024
X12M2B	2.668	1.372	1.095	1.040
S3-3	2.756	2.079	1.027	1.000
D4C	2.790	1.903	1.019	1.006
S3-1	2.870	2.644	1.027	1.009
D5C	2.968	2.187	1.022	1.008
S2-3	3.026	2.064	1.020	1.007
S2-5	3.073	2.630	1.018	1.006
S2-4	3.245	2.371	1.017	1.006
S1-7	3.320	1.547	1.017	1.004
S2-10	3.524	1.936	1.021	1.007
X12M1A	3.618	1.090	1.035	1.007
D3B	3.655	1.078	1.016	(1)
X12M2A	3.739	1.064	1.123	1.066
X12M2C	3.747	1.066	1.024	(1)
S1-8	3.748	1.779	1.019	1.006
S3-10	3.752	1.771	1.033	1.011
D3A	3.754	1.057	1.031	1.010
D6-C	3.785	2.198	1.023	1.008
S2-10	3.873	1.785	1.024	1.008
D7-C	4.028	2.357	1.021	1.007
S1-9	4.042	1.984	1.021	1.007
X6MH2A	4.075	1.442	1.019	1.005
X10M1A	4.237	1.617	1.028	1.010
D5A	4.236	1.605	1.032	1.012
X10M2A	4.248	1.612	1.030	1.011
D5B	4.249	1.601	1.037	1.014
X10M3A	4.251	1.611	1.030	1.011
X10M2B	4.279	1.597	1.030	1.011
X9MH1C	4.416	1.909	1.019	1.006
X9MH2C	4.472	1.888	1.019	1.006
X9MH3C	4.494	1.880	1.024	1.008
D6B	4.526	1.868	1.026	1.009
D6A	4.562	1.855	1.025	1.009
X8MH1A	4.609	2.110	1.025	1.009
X8MH24	4.707	2.094	1.022	1.008
X8MH2X	4.744	2.079	1.027	1.009
X8MH3A	4.783	2.063	1.021	1.007
D8B	4.800	2.166	1.024	1.008
D7A	4.803	1.989	1.024	1.008
X7MH2A	4.835	2.210	1.018	1.006
X7MH2B	5.000	2.139	1.017	1.005

(1) Computed value less than the minimum value of one.

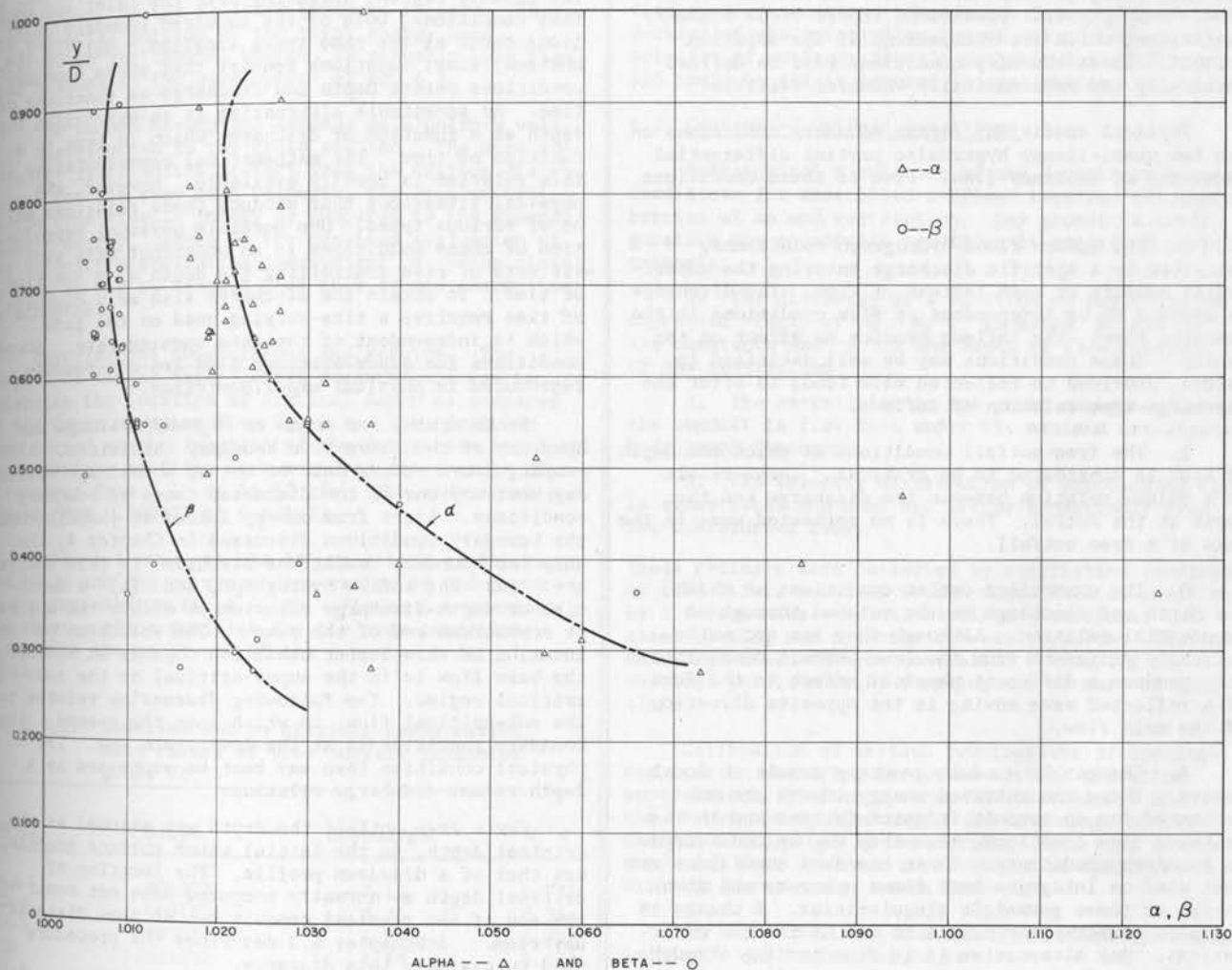


Fig. 5.5. Velocity distribution coefficients versus depth of flow in a circular cross section.

An attempt was made to find a relation between the mean velocity and the velocity distribution coefficients. It may be seen from Table 5.4 that for modest range of depth the variation of mean velocity does not result in a consistent variation in α and β .

Because of the limited range of the Darcy-Weisbach friction factor and the mean velocity, the variation in α and β are identified primarily with the Reynolds number and secondarily with the depth of flow.

5.6 Summary of Results

The results of this study are applicable to hydraulically-smooth circular cross sections flowing

partially full and having Reynolds numbers between 0.4×10^5 and 3.00×10^5 . In the smaller Reynolds numbers, the observed velocity distribution coefficients are greater but display greater dispersion about a smooth curve. In the larger Reynolds numbers the values trend toward invariance with lesser dispersion. The relation between α and β has been demonstrated both theoretically and experimentally to be expressible as $(\alpha-1)/(\beta-1) = 2.3$ to 3.0 . The value of 3.0 is representative of the experimental results. A representative value of α for the experimental conditions is 1.03 , and a representative value of β for the experimental conditions is 1.01 .

6.1 Definitions of Boundary Conditions

Adequate knowledge of boundary conditions is necessary for solving the two partial differential equations of gradually varied free-surface unsteady flow. Many physical conditions impose these boundary conditions, which are independent of the solution methods. These boundary conditions must be defined physically and mathematically whenever feasible.

Physical conditions impose boundary conditions on the two quasi-linear hyperbolic partial differential equations of unsteady flow. Five of these conditions follow:

1. The inflow flood hydrograph conditions, described by a specific discharge entering the experimental conduit at each instant of time. The discharge is assumed to be independent of flow conditions in the conduit; hence, the inflows produce no effect on the system. These conditions may be well satisfied in nature, provided no reflected wave tends to alter the discharge-time relation of inflows.
2. The free-outfall conditions at which the depth of flow is considered to be critical. These result in a unique relation between the discharge and the depth at the outlet. There is no reflected wave in the case of a free outfall.
3. The controlled-outlet conditions at which the depth and discharge may be related through an exponential relation. Although they are not mathematically different from the free-outfall conditions, they produce a different physical effect in the form of a reflected wave moving in the opposite direction of the main flow.
4. Changes in conduit geometry result in local losses. These concentrated energy losses are not accounted for in conduit friction factors and in the unsteady flow equations, thus they may be considered as boundary conditions. These boundary conditions are then used to integrate both flows upstream and downstream of these geometric singularities. A change in boundary geometry may result in one of the two alternatives. One alternative is in flow passing through the critical flow at geometric singularities. In this case the boundary conditions upstream of the boundary singularities are the same as described under (1). The downstream boundary conditions are then the same as those described under point (3). The second alternative is in the flow not passing through the critical stage. In this case the relation between the upstream and downstream depths and the corresponding energy loss at singularities must be known. This relation of energy loss to the two depths produce the boundary conditions in the solution of partial differential equations of unsteady flow.
5. A moving hydraulic jump, a bore, or a breaking wave may also be considered as boundary conditions. The previous four dictates of boundary conditions were considered to be fixed in space. Similar moving boundary conditions exist in the case of moving mechanisms along the conduit with energy dissipation. In this case, it is necessary to determine whether the jump, the bore, or the breaking wave exists at each point of the conduit in time and space. If these conditions exist, then the appropriate energy loss relations are applied as boundary conditions.

The preceding five aspects of boundary conditions have presumed that the condition of sub-critical flow exists in at least one of the regions of unsteady flow. For the condition of super-critical flow throughout the conduit region, including both the inlet and outlet flow conditions, both of the required boundary conditions occur at the same space location. Solving the unsteady flows equations require that these boundary conditions relate depth and discharge as functions of time. An acceptable alternative is an expression for depth as a function of discharge which in turn is a function of time. The mathematical representative of this relation is readily available. However, the physical situations that produce these relations may be of various types. One possible physical reproduction of these conditions is an undershot gate with its rate of rise controlling the depth as a function of time. To obtain the discharge also as a function of time requires a time-varying head on the gate, which is independent of the gate opening rate. Boundary conditions for super-critical flow are not readily reproduced in physical experimentation.

Storm drains for urban or highway drainage may have any of the above five boundary conditions. Also, pumping storm waters into or out of a drainage system may meet any one of the discussed cases of boundary conditions. Apart from energy losses at junction boxes, the boundary conditions discussed in Chapter 4, the only two boundary conditions discussed in this chapter are: (a) the inflow hydrograph, and (b) a depth-time or depth-discharge relation at either the upstream or downstream end of the reach. The selection of the location of this latter condition depends on whether the base flow is in the super-critical or the sub-critical regime. The following discussion relates to the sub-critical flow, in which case the second boundary condition is at the downstream end. The physical condition then may best be expressed as a depth versus discharge relation.

For a free outfall the depth was assumed as the critical depth, so the initial water surface profile was that of a drawdown profile. The location of critical depth as normally computed does not occur at the end of the physical conduit but at some distance upstream. Subchapter 6.2 describes the procedure used to evaluate this distance.

The second boundary condition imposed in the experiments at the downstream end was that of a restricted opening. This insured that the depth of flow was always greater than the normal depth. This produced an initial condition of a backwater surface profile. Subchapter 6.3 describes this boundary condition.

6.2 Free Outfall Boundary Conditions

Definitions and assumptions. The free outfall at the downstream end of a prismatic channel may be considered as that condition for which the total energy of flow is at a minimum for a given discharge. Mathematically this condition may be expressed by

$$\frac{Q^2 B}{g A^3} = 1, \quad (6.1)$$

in which Q is the discharge, B is the surface width, A is the cross-sectional area, and g is

the gravitational acceleration. This expression is based on two assumptions, namely, the pressure distribution is hydrostatic, and the kinetic energy may be expressed through the mean velocity.

The first assumption is invalidated in the vicinity of the free outfall because of the significant curvature of streamlines. Furthermore, the pressure at the bottom of the end cross section must be atmospheric, or the relative pressure is zero. Thus, the potential portion of the total energy head, relative to the channel bottom, is actually less than that assumed in the development of Eq. 6.1.

The second assumption depends on a uniform velocity distribution in the cross section. The more the velocity distribution differs from the uniform distribution for the same mean velocity, the greater will be the true kinetic energy as compared to the assumed kinetic energy. Based on the previous evaluation of the velocity distribution coefficient α being close to one, it may be assumed that this second assumption is acceptable.

Experimental observations and results. The purpose of taking experimental measurements was to determine the location of critical depth as compared to the depth computed from Eq. 6.1. This position then served as the location of the downstream boundary. Water-surface profiles were measured for a range of discharges from 2.10 to 16.62 cfs. The channel slope ranged from 0.000032 to 0.001022.

Table 6.1 presents the 14 conditions of discharge and slope and the corresponding ratios of end depth to the computed critical depth. Figure 6.1 presents the water-surface profiles for the same conditions along with the locations of the computed critical depths.

TABLE 6.1. Observed end to critical depth ratios. Data values of free outfall (diameter 2.926 ft).

Run No.	Slope	Discharge	y_e/y_c
D1A	.001022	2.10	0.731
S2-9	.000132	3.26	0.746
S1-5	.000032	4.14	0.758
D2A	.001022	4.58	0.749
S1-6	.000032	7.96	0.776
S3-9	.000520	7.98	0.764
D3A	.001022	8.26	0.751
S1-7	.000032	11.98	0.761
D4A	.001022	12.92	0.740
S3-10	.000520	15.97	0.739
D5A	.001022	16.02	0.752
S1-8	.000032	16.04	0.726
S2-10	.000132	16.64	0.753
S1-9	.000032	19.62	0.761
Mean =			0.750

Conclusions. Within the range of observed end depths, the mean ratio of end-depth to critical depth is 0.750. The ratios tend to be smaller than the mean ratio for the lower depths. The location of computed critical depths from the conduit end varied from less than 3.5 times the critical depth to almost 5.5 times the critical depth. A location of 4.5 times the critical depth was considered typical and was used in the subsequent computations of solution of partial differential equations of unsteady flow. This reduction of the integration length for the numerical solution of unsteady flow is probably insignificant and could be safely ignored in application.

6.3 Controlled Outfall Conditions

Mathematical simulation of the downstream boundary conditions for controlled outflows required the calibration of an end restriction. Any geometric configuration was acceptable providing it satisfied certain criteria:

1. The discharge as a function of depth can be expressed simply by $Q = my^n$, in which m and n are constants and y is the depth of flow upstream of the restriction.
2. The restriction is not great enough to cause the conduit to flow full under the maximum anticipated hydrograph discharge.
3. The velocity distribution of approaching flow is symmetrical and does not differ appreciably from the undisturbed flow.

These criteria were satisfied by restriction consisting of five 7-inch vertical wooden slats held in position by 2.5-inch wide vertical aluminum H-sections. The clear opening was 5 inches between supports. The discharge could thus be controlled by varying the vertical position or by removing one or more of the slats.

Calibration of various combinations of openings was made by measuring the water surface elevation approximately 20 feet upstream of the control and the corresponding discharge. For the range of discharges anticipated in the unsteady flow runs it was concluded that the best combinations of openings was with the three center slats removed.

For this condition the relation between discharge and depth was determined to be

$$Q = 4.84 y^{1.35} \quad (6.2)$$

This relation was applied for depths between approximately one-third and eight-tenths of the full diameter, and at 20 feet upstream from the conduit end (the 802.71 ft. station).

This gate configuration and the relation of Eq. 6.2 were used for all subsequent boundary condition evaluations in which the backwater profiles were the initial condition. No attempt was made to modify this steady state relation for the unsteady flow state.

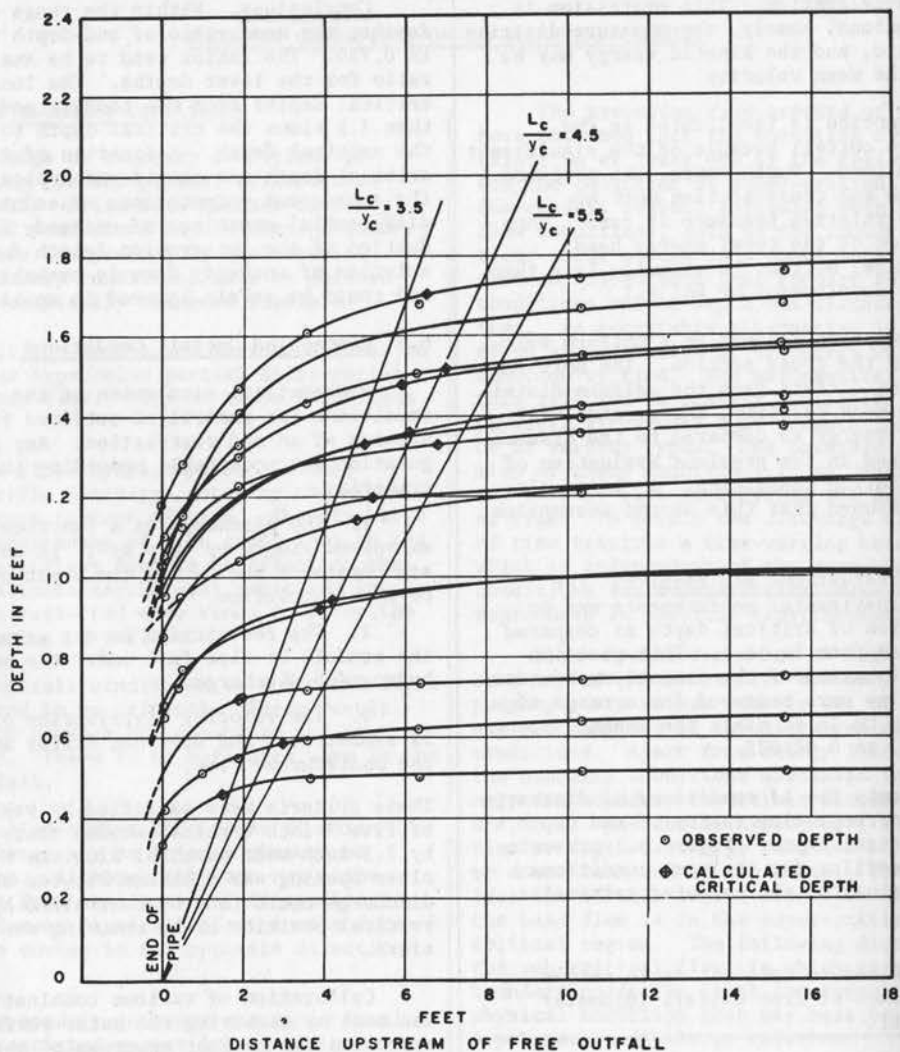


Fig. 6.1. Location of critical depth at the free outfall of a circular cross section.

7.1 Definition of Initial Conditions

The integration of the two partial differential equations of gradually varied free-surface steady flow requires that the initial values of velocity and depth be known at a given position in time and space. These values are independent of the ensuing solutions and may be arbitrarily established. Realistically, these initial conditions should be the result of physical initial conditions.

For this study the initial conditions are the non-uniform steady flow of the hydrograph base discharge. A mathematical expression for this condition is the ordinary differential equation.

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 + \frac{d}{dy} \left(\frac{\alpha Q^2}{2gA^3} \right)}, \quad (7.1)$$

in which y is the depth at the position x , x is the distance along the channel, S_o is the bed slope, S_f is the friction slope, α is the energy velocity distribution coefficient, Q is the steady discharge of the hydrograph, A is the cross-sectional area, and g is the gravitational acceleration.

The slope of the energy gradient, S_f , was evaluated by the Darcy-Weisbach equation based on the friction factor evaluation described in Chapter 3.

Comparisons were made between the computed and the observed water-surface profiles for non-uniform steady flow. The objective of these comparisons was to test the validity of the theoretical and numerical determinations of initial conditions.

7.2 Computational Procedure and Results

Determining the depth at specified positions along the conduit was accomplished by a Newton-Raphson iteration of Eq. 7.1 to a tolerance of 0.001 foot of depth. The given information included discharge Q , channel slope S_o , friction factor f , velocity distribution coefficient α , position x along the conduit of points of observed depth, and observed or critical depth at the extreme downstream position. The depth of flow in feet was observed at the following eight positions with respect to the upstream end of the conduit: 20.00, 197.92, 406.07, 509.64, 613.20, 707.71, 772.71, and 821.00.

The boundary conditions for the determination of the steady non-uniform flow as initial conditions were established:

- for the mild slope profiles at depths greater than the normal depth (M-1 type curves) the observed depth at the 802.71-foot station was used, and
- for the mild slope profiles at depths between the critical and the normal depth (M-2 type curves) the computed critical depth at a position of 4.5 times the critical depth upstream of the conduit end was used.

The root-mean-square (RMS) deviation between the observed and computed depths was computed for three different values of α (1.00, 1.02, and 1.05), and three values of f (.011, .012, and .013). The α -values were selected on:

(a) the usually assumed value of 1.00 in lieu of better information as to its true value;

(b) the value of 1.02 as a representative of the values within the range of expected depths, and

(c) the value of 1.05 as being an extreme for the flow in a uniform conduit with free-surface water flow.

The friction factors were selected according to the most reliable constant value throughout the range of expected depths and velocities, and approximately 10 percent more and less this constant value. It was to be expected that these values would include an engineering estimate of the best constant value for the friction factor of smooth conduits in the expected range of Reynolds numbers. The results of these computations are given in Tables 7.1 and 7.2.

The invert of the experimental conduit deviated from a mathematically uniform slope as indicated in Table 2.3. Since the actual depth of flow above this slightly irregular invert may not be expected to agree with the computed depth, an adjusted depth was computed at each position. The adjusted depth was based on the depth that would have occurred with the same total energy but with the invert on the mean slope. The root-mean-square deviations for the adjusted depths are given in Tables 7.4 and 7.5.

7.3 Discussion of Comparison of Backwater Calculations

The data presented in Tables 7.1 and 7.2 were analyzed in terms of the mean values of the root-mean-square deviations for each friction factor and α -coefficient. The consolidated results are presented in Tables 7.3 and 7.4. These results do not indicate any strong tendency for a smaller root-mean-square deviation for the friction factor and α -coefficients previously estimated for this conduit, i.e., 0.012, and 1.02. A representative root-mean-square deviation for the conditions observed is approximately 0.025 foot for both the M-1 and M-2 type surface profiles of backwater curves. There is a larger spread of deviations for changes in the friction factor than for changes in the velocity distribution coefficient α .

7.4 Conclusions

Based on the preceding results, it is concluded that a steady non-uniform water-surface profile can be accurately computed as the initial conditions for the solution of unsteady flow equations. It is also concluded that the evaluation of the friction factors is more important than the evaluation of velocity distribution coefficients. The solutions of these equations utilize the variation of the friction factor with the Reynolds number, as given in Chapter 3, to compute S_f of Eq. 7.1. Since the variation of velocity distribution coefficients did not produce significant differences in solutions of unsteady flow equations, subsequent computations utilize an α -value of one in Eq. 7.1.

TABLE 7.1. The root-mean-square deviations for M-1 type backwater curves with the normal depth smaller than the initial depth

Run No.	Q cfs	Normal Depth ft	Initial* Depth ft	$\alpha = 1.00$			$\alpha = 1.02$			$\alpha = 1.05$			f	Slope
				Critical Depth	RMS Dev.		Critical Depth	RMS Dev.		Critical Depth	RMS Dev.			
					Actual	Adjusted		Actual	Adjusted		Actual	Adjusted		
S3-3	14.10	1.834	2.074	1.205	0.075	0.077	1.211	0.074	0.074	1.220	0.075	0.074	0.011	0.000520
S3-5	7.96	1.269	1.554	0.896	0.034	0.034	0.900	0.034	0.034	0.907	0.034	0.034	0.011	
S3-6	6.21	1.097	1.107	0.788	0.034	0.034	0.792	0.034	0.034	0.798	0.034	0.034	0.011	
S3-7	2.04	0.594	1.028	0.446	0.016	0.014	0.448	0.019	0.016	0.452	0.019	0.016	0.011	
S3-8	1.42	0.490	0.621	0.371	0.025	0.024	0.373	0.025	0.025	0.376	0.025	0.025	0.011	
S3-3	14.10	1.893	2.074	1.205	0.066	0.065	1.211	0.067	0.065	1.220	0.066	0.065	0.012	
S3-5	7.96	1.303	1.554	0.896	0.027	0.026	0.900	0.021	0.026	0.907	0.027	0.025	0.012	
S3-7	2.04	0.608	1.028	0.446	0.017	0.016	0.448	0.017	0.016	0.452	0.018	0.016	0.012	
S3-8	1.42	0.501	0.621	0.371	0.022	0.029	0.373	0.023	0.023	0.376	0.022	0.023	0.012	0.001001
S3-3	14.10	1.950	2.074	1.205	0.061	0.058	1.211	0.059	0.058	1.220	0.060	0.058	0.013	
S3-5	7.96	1.335	1.554	0.896	0.022	0.021	0.900	0.022	0.021	0.907	0.022	0.020	0.013	
S3-7	2.04	0.621	1.028	0.446	0.018	0.015	0.448	0.018	0.015	0.452	0.018	0.015	0.013	
S3-8	1.42	0.512	0.621	0.371	0.020	0.020	0.373	0.020	0.020	0.376	0.020	0.020	0.013	
X6A	13.20	1.415	1.441	1.164	0.027	0.040	1.170	0.029	0.040	1.179	0.029	0.040	0.011	
X6B	13.20	1.415	1.757	1.164	0.016	0.032	1.170	0.017	0.032	1.179	0.016	0.033	0.011	
X6C	13.20	1.415	2.379	1.164	0.021	0.020	1.170	0.021	0.020	1.179	0.021	0.020	0.011	
X9A	20.30	1.884	2.341	1.457	0.062	0.060	1.465	0.067	0.062	1.476	0.067	0.062	0.011	
X10B	16.00	1.599	1.990	1.287	0.042	0.039	1.293	0.041	0.039	-----	-----	-----	0.011	
X12A	8.20	1.066	1.697	0.909	0.022	0.024	0.914	0.023	0.024	-----	-----	-----	0.011	
X12B	8.20	1.066	1.104	0.909	0.014	0.012	0.914	0.014	0.035	-----	-----	-----	0.011	
X6B	13.20	1.454	1.757	1.164	0.023	0.020	1.170	0.022	0.020	1.179	0.022	0.020	0.012	
X6C	13.20	1.454	2.379	1.164	0.021	0.018	1.170	0.021	0.017	0.179	0.020	0.018	0.012	
X9A	20.30	1.945	2.341	1.457	0.023	0.020	1.465	0.077	0.064	1.476	0.071	0.073	0.012	
X10B	16.00	1.646	1.990	1.287	0.050	0.042	1.293	0.048	0.042	1.303	0.047	0.041	0.012	
X12A	8.20	1.093	1.697	0.909	0.019	0.017	0.914	0.019	0.018	0.921	0.019	0.018	0.012	
X12B	8.20	1.093	1.104	0.909	0.018	0.026	0.914	0.018	0.026	0.921	0.015	0.026	0.012	
X12C	8.20	1.093	1.109	0.909	0.015	0.028	0.914	0.015	0.028	-----	-----	-----	0.012	
X13A	4.68	0.798	0.833	0.681	0.018	0.030	-----	-----	-----	-----	-----	-----	0.012	
X13B	4.68	0.798	1.079	0.681	0.013	0.022	-----	-----	-----	-----	-----	-----	0.012	
X12C	8.20	1.093	1.109	-----	-----	-----	0.914	0.015	0.028	-----	-----	-----	0.012	
X12D	8.21	1.093	1.053	-----	-----	-----	0.915	0.044	0.040	-----	-----	-----	0.012	
X6B	13.20	1.491	1.757	1.164	0.033	0.026	1.170	0.035	0.026	1.179	0.032	0.026	0.013	
X6C	13.20	1.491	2.379	1.164	0.026	0.022	1.170	0.026	0.022	0.179	0.026	0.022	0.013	
X9A	20.30	2.006	2.341	1.457	0.044	0.045	1.465	0.046	0.045	1.476	0.091	0.080	0.013	
X10B	16.00	1.690	1.990	1.287	0.060	0.054	1.293	0.060	0.053	1.303	0.059	0.053	0.013	
X12A	8.20	1.119	1.697	0.909	0.0024	0.021	0.914	0.023	0.020	0.921	0.022	0.018	0.013	
D4C	12.92	1.388	2.225	1.151	0.026	0.026	1.157	0.027	0.027	1.166	0.028	0.028	0.011	
D6C	20.51	1.883	2.253	1.465	0.040	0.040	1.473	0.040	0.040	1.484	0.041	0.042	0.011	
D1C	2.10	0.516	1.417	0.453	0.020	0.023	-----	-----	-----	0.458	0.021	0.023	0.012	
D2C	4.58	0.784	2.293	0.674	0.011	0.010	-----	-----	-----	-----	-----	-----	0.012	
D4C	12.92	1.425	2.225	1.151	0.016	0.016	-----	-----	-----	1.166	0.017	0.017	0.012	
D6C	20.51	1.945	2.253	1.465	0.025	0.024	1.473	0.025	0.024	1.484	0.025	0.023	0.012	
D1C	2.10	0.527	1.417	0.453	0.019	0.022	0.455	0.020	0.022	-----	-----	-----	0.013	
D2C	4.58	0.801	2.293	0.674	0.011	0.010	0.677	0.011	0.012	0.682	0.011	0.010	0.013	
D4C	12.92	1.462	2.225	1.151	0.012	0.011	1.157	0.012	0.011	1.166	0.011	0.011	0.013	
D6C	20.51	2.006	2.253	1.465	0.027	0.026	1.473	0.027	0.026	1.484	0.027	0.025	0.013	
D1C	2.10	0.504	1.417	-----	-----	-----	0.455	0.021	0.024	-----	-----	-----	0.011	
D2C	4.58	0.766	2.293	-----	-----	-----	0.677	0.012	0.011	-----	-----	-----	0.011	

*Observed depth at the 802.71 ft. station.

TABLE 7.2. The root-mean-square deviations for M-2 type backwater curves with the normal depth greater than the initial depth

Run No.	Q cfs	Normal Depth ft	Observed values of Initial Depth ft	$\alpha = 1.00$			$\alpha = 1.02$			$\alpha = 1.05$			f	Slope
				Critical Depth	RMS Dev.		Critical Depth	RMS Dev.		Critical Depth	RMS Dev.			
					Actual	Adjusted		Actual	Adjusted		Actual	Adjusted		
S3-4	14.41	1.863	1.768	1.218	0.040	0.036	1.225	0.040	0.036	1.234	0.040	0.036	0.011	0.000520
S3-4	14.41	1.923	1.768	1.218	0.042	0.038	1.225	0.039	0.039	1.234	0.033	0.041	0.012	
S3-6	6.21	1.125	1.107	0.788	0.026	0.025	0.792	0.025	0.024	0.798	0.026	0.025	0.012	
S3-4	14.41	1.962	1.768	1.218	0.045	0.042	1.225	0.043	0.042	1.234	0.045	0.041	0.013	
S3-6	6.21	1.152	1.107	0.788	0.021	0.019	0.792	0.021	0.024	0.798	0.020	0.019	0.013	
X6-E	13.20	1.415	1.331	1.164	0.028	0.044	1.170	0.028	0.044	1.179	0.027	0.044	0.011	0.001001
X7-A	25.90	2.301	2.122	1.656	0.056	0.051	1.664	0.065	0.059	1.677	0.065	0.052	0.011	
X7-B	25.90	2.301	1.828	1.656	0.055	0.062	1.664	0.066	0.072	1.677	0.067	0.072	0.011	
X7-C	25.90	2.301	1.831	1.656	0.072	0.074	1.664	0.065	0.070	1.677	0.067	0.074	0.011	
X8-B	23.70	2.124	1.753	1.580	0.019	0.042	1.589	0.023	0.043	1.601	0.022	0.043	0.011	
X8-C	23.70	2.124	2.023	1.580	0.027	0.036	1.589	0.027	0.034	1.601	0.026	0.034	0.011	
X9-B	20.30	1.884	1.619	1.457	0.061	0.066	1.465	0.061	0.066	1.476	0.060	0.066	0.011	
X9-C	20.30	1.884	1.832	1.457	0.061	0.061	1.465	0.061	0.061	1.476	0.065	0.060	0.011	
X10-A	16.00	1.599	1.579	1.287	0.013	0.032	1.293	0.013	0.030	1.303	0.013	0.030	0.011	
X10-C	16.00	1.599	1.448	1.287	0.023	0.043	1.293	0.023	0.043	-----	-----	-----	0.011	
X6-A	13.20	1.454	1.441	1.164	0.018	0.024	1.170	0.018	0.025	1.179	0.018	0.025	0.012	
X6-E	13.20	1.454	1.331	1.164	0.020	0.034	1.170	0.020	0.011	1.179	0.020	0.034	0.012	
X8-B	23.70	2.289	1.753	1.580	0.066	0.080	1.589	0.079	0.082	1.601	0.082	0.085	0.013	
X8-C	23.70	2.289	2.023	1.580	0.052	0.047	1.589	0.051	0.048	1.601	0.052	0.048	0.013	
X9-B	20.30	2.006	1.619	1.457	0.098	0.086	1.465	0.091	0.087	1.476	0.095	0.090	0.013	
X9-C	20.30	2.006	1.832	1.457	0.091	0.080	1.465	0.092	0.081	1.476	0.091	0.080	0.013	
X10-A	16.00	1.690	1.579	1.287	0.045	0.043	1.293	0.046	0.042	1.303	0.046	0.043	0.013	
X10-C	16.00	1.690	1.448	1.287	0.048	0.053	1.293	0.049	0.054	1.303	0.050	0.055	0.013	
X12-B	8.20	1.119	1.104	0.909	0.026	0.026	0.914	0.026	0.026	1.303	0.050	0.053	0.013	
D1B	2.10	0.504	0.464	0.453	0.026	0.037	0.455	0.026	0.036	-----	-----	-----	0.011	0.001022
D2A	4.58	0.766	0.696	-----	-----	-----	0.677	0.050	0.056	0.682	0.050	0.055	0.011	
D2B	4.58	0.766	0.724	-----	-----	-----	0.677	0.037	0.040	0.682	0.037	0.037	0.011	
D4B	12.92	1.388	1.292	1.151	0.032	0.037	1.157	0.034	0.035	1.166	0.034	0.034	0.011	
D5B	16.02	1.590	1.490	1.288	0.032	0.033	1.294	0.032	0.032	1.304	0.032	0.032	0.011	
D6B	20.51	1.883	1.737	1.465	0.038	0.038	1.473	0.039	0.040	1.484	0.038	0.038	0.011	
D7B	23.51	2.092	1.926	1.574	0.024	0.024	1.582	0.024	0.024	1.594	0.024	0.024	0.011	
D8B	25.60	2.253	2.029	1.646	0.029	0.029	1.654	0.028	0.028	1.667	0.028	0.028	0.011	
D1B	2.10	0.516	0.464	0.453	0.023	0.034	-----	-----	-----	0.458	0.020	0.036	0.012	
D2A	4.58	0.784	0.696	0.674	0.039	0.044	-----	-----	-----	0.682	0.038	0.044	0.012	
D2B	4.58	0.784	0.724	0.674	0.029	0.033	-----	-----	-----	0.682	0.032	0.033	0.012	
D4B	12.92	1.425	1.292	1.151	0.021	0.020	-----	-----	-----	1.166	0.021	0.020	0.012	
D5B	16.02	1.636	1.490	1.288	0.018	0.017	1.294	0.019	0.017	1.304	0.019	0.016	0.012	
D6B	20.51	1.945	1.737	1.465	0.025	0.023	1.473	0.025	0.023	1.484	0.024	0.023	0.012	
D7B	23.51	2.170	1.926	-----	-----	-----	1.582	0.025	0.022	1.594	0.025	0.023	0.012	
D8B	25.60	2.352	2.029	-----	-----	-----	1.654	0.024	0.018	1.667	0.021	0.018	0.012	
D1B	2.10	0.527	0.464	0.453	0.020	0.034	0.455	0.017	0.032	-----	-----	-----	0.013	
D2A	4.58	0.801	0.696	0.674	0.028	0.034	0.677	0.028	0.035	0.682	0.027	0.035	0.013	
D2B	4.58	0.801	0.724	0.674	0.029	0.025	0.677	0.031	0.025	0.682	0.026	0.023	0.013	
D4B	12.92	1.462	1.292	1.151	0.025	0.024	1.157	0.027	0.025	1.166	0.024	0.020	0.013	
D5B	16.02	1.680	1.490	1.288	0.030	0.026	1.294	0.030	0.027	1.304	0.032	0.028	0.013	
D6B	20.51	2.006	1.737	1.465	0.037	0.035	1.473	0.038	0.035	1.484	0.037	0.035	0.013	
D7B	23.51	2.250	1.926	1.574	0.049	0.048	-----	-----	-----	1.594	0.052	0.050	0.013	
S21	5.61	1.563	1.210	0.748	0.035	0.034	0.752	0.036	0.033	0.757	0.036	0.034	0.011	0.000132
S2-6	4.71	1.400	1.012	0.684	0.035	0.034	0.687	0.034	0.033	0.692	0.034	0.033	0.011	
S2-7	3.44	1.159	0.611	0.582	0.035	0.035	0.585	0.033	0.034	0.590	0.032	0.033	0.011	
S2-8	1.06	0.604	0.339	0.320	0.044	0.042	0.322	0.043	0.041	0.324	0.043	0.040	0.011	
S2-1	5.61	1.608	1.210	0.748	0.029	0.028	0.752	0.030	0.027	0.757	0.030	0.027	0.012	
S2-6	4.71	1.438	1.012	0.684	0.028	0.026	0.687	0.028	0.026	0.692	0.028	0.026	0.012	
S2-7	3.44	1.189	0.611	0.582	0.027	0.027	0.585	0.025	0.025	0.590	0.024	0.025	0.012	
S2-8	1.06	0.618	0.339	0.320	0.037	0.035	0.322	0.038	0.035	0.324	0.037	0.035	0.012	
S2-1	5.61	1.651	1.210	0.748	0.026	0.023	0.752	0.025	0.022	0.757	0.025	0.022	0.013	
S2-6	4.71	1.475	1.012	0.684	0.022	0.020	0.687	0.022	0.020	0.692	0.022	0.020	0.013	
S2-7	3.44	1.218	0.611	0.582	0.020	0.020	0.585	0.018	0.019	0.590	0.017	0.018	0.013	
S2-8	1.06	0.631	0.339	0.320	0.033	0.030	0.322	0.032	0.030	0.324	0.032	0.029	0.013	

TABLE 7.3. Difference in computed and observed depths, root-mean-square values in feet for M-1 type curves

Velocity distribution coefficient α	Friction factor f					
	0.011	0.012	0.013	0.011	0.012	0.013
	Root-mean-square values Unadjusted depth			Adjusted depth		
1.00	0.026	0.020	0.023	0.033	0.023	0.022
1.02	0.024	0.024	0.023	0.033	0.026	0.022
1.05	0.024	0.022	0.024	0.031	0.023	0.021

TABLE 7.4. Difference in computed and observed depths, root-mean-square values in feet for M-2 type curves

Velocity distribution coefficient α	Friction factor f					
	0.011	0.012	0.013	0.011	0.012	0.013
	Root-mean-square values Unadjusted depth			Adjusted depth		
1.00	0.035	0.026	0.030	0.037	0.030	0.034
1.02	0.034	0.025	0.030	0.040	0.026	0.031
1.05	0.036	0.026	0.036	0.041	0.026	0.037

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

After considering the analysis and discussion of the various evaluations of geometric and hydraulic parameters in Chapters 2 through 7, the following major conclusions are drawn:

1. Geometric parameters can be well evaluated for the circular storm drain; errors in depth measurements, errors from the assumed ellipticity and inclination of cross sections, and errors from undulation of inverts are all well within tolerable limits.

2. Because the 822-foot experimental storm conduit is a hydraulically smooth pipe, the relation between the Darcy-Weisbach friction factor (f) and the Reynolds number (Re) may be well approximated by a power function, $f = 0.10939 Re^{-0.17944}$, with the Reynolds number in the range 3×10^4 to 6×10^5 for the steady flow conditions. An average Darcy-Weisbach friction factor of 0.012 is a representative value of the conduit roughness, for this range of Re .

3. Under steady-flow conditions, for the lateral inlet with its upper position in the junction box, the power loss relation for junction box losses is

$$P_r = \frac{-0.482}{Q_r + 0.55} + 0.77,$$

and for the lateral inlet with its lower position in the junction box the power loss relation is

$$P_r = \frac{-2.78 + 1.71 D_r}{Q_r + 3.122 - 0.167 D_r} + 0.77.$$

4. The relation between the velocity distribution coefficients, α and β is approximately $(\alpha-1)/(\beta-1) = 3.0$ for the experimental results obtained under the steady-flow conditions, with representative (average) values of $\alpha = 1.03$ and $\beta = 1.01$ for the experimental conditions.

5. The relation between discharge (Q) and depth (y) at the controlled outfall of the conduit for steady flow conditions can be well approximated by an exponential function of the form $Q = my^n$. The values of the constants m and n depend on the shape and size of the arbitrary opening of the controlled outfall.

6. For conduit slope less than 0.001 the mean ratio of end depth to critical depth obtained for the conduit free-water outfall and for the steady-flow conditions is 0.750.

7. By examining the sources of experimental errors, it is possible to establish boundary and initial conditions, with the errors within tolerable limits.

8.2 Recommendations for Further Experimental Research

1. For storm drains of circular cross section the flow characteristics change significantly when the flow is at nearly full pipe because of changes in air pressure. A criterion should be developed to define the applicability of partially full flow in circular conduits when the water surface approaches full conduit.

2. The influence of changes in boundary layers in the unsteady flow regime on the estimation of friction factors and the velocity distribution coefficients suggests that further studies of such effects would be beneficial, with new ideas, different experimentation and new and appropriate instrumentation.

3. The evaluation of flow resistance in this study is limited to turbulent hydraulically smooth boundaries. Similar evaluation should be carried out for turbulent hydraulically rough boundaries.

4. Because storm drains may have sudden changes in physical boundary geometry (such as water drops with partial backwater effects of downstream depths on upstream flow, use of pumps for removing the storm water at particular points, and similar), further investigations should be conducted to properly evaluate these particular boundary conditions.

REFERENCES

1. Henderson, F. M., 1964, Steady Flow in Sinu-
soidally Varying Channels, *Hydraulics and Fluid
Mechanics*, R. Silvester (Editor), MacMillan
Company, New York, pp. 51-67.
2. Committee on Hydromechanics of the Hydraulics
Division, 1963, Friction Factors in Open Channels,
American Society of Civil Engineers, *Journal of
Hydraulics Division*, Vol. 89, HY2, pp. 97-143.
3. Camp, Thomas R., 1946, Design of Sewers to
Facilitate Flow, *Sewage Works Journal*, Vol. 18,
No. 1, pp. 3-16.
4. Rouse, Hunter, 1946, *Elementary Mechanics of
Fluids*, John Wiley & Sons, Inc., New York.
5. Stevens, J. C., 1926, Theoretical Energy Losses
in Intersecting Pipes, *Engineering News-Record*,
pp. 140.
6. Taylor, Edward H., 1944, Flow Characteristics
at Rectangular Open-Channel Junctions, *Trans-
actions, American Society of Civil Engineers*,
Vol. 109, pp. 893-903.
7. Bowers, Charles E., 1950, Studies of Open-
Channel Junctions, Part V of Hydraulic Model
Studies for Whiting Field Naval Air Station,
Milton, Florida. University of Minnesota,
St. Anthony Falls Hydraulic Laboratory, Techni-
cal Paper Number 6, Series B.
8. Chow, Ven Te, 1964, *Handbook of Applied
Hydrology*, New York, McGraw-Hill Book Company.
9. Sangster, W. M., Wood, H. W., Smerdon, E. T.,
and Bossy, H. G., 1958, Pressure Changes at
Storm Drain Junctions. The University of
Missouri Bulletin, Vol. 59, No. 35, *Engineering
Experiment Station Series No. 41*, Columbia, Mo.
10. Chow, Ven Te, 1959, *Open Channel Hydraulics*,
New York, McGraw-Hill Book Company, 680 pp.

Key Words: Storm Drain, Flood Routing, Geometry, Flow Resistance, Velocity Distribution Coefficients, Junction Box Losses, Steady Non-Uniform Flow.

Abstract: The primary objective of this third part of a four-part series of hydrology papers on flood routing through storm drains is to present results on the investigation of the geometric and hydraulic parameters of the experimental conduit. The errors in cross section geometric parameters are analyzed in a conduit not ideally circular but approximated by an elliptical shape; errors are also analyzed when the undulations in the longitudinal slope of the conduit affect the predicted water surface profiles and thus the geometric parameters for a given water depth. The variation of hydraulic parameter of resistance, expressed by the Darcy-Weisbach friction factor, is experimentally determined and compared to the theoretical relation to Reynolds number. Energy losses in a 90° junction box are studied. The velocity distribution coefficients are shown to vary with the Darcy-Weisbach friction factor, and consequently, with the depth of flow. Boundary conditions for both controlled and free outfall are experimentally determined and approximated by power functions. Two types of steady non-uniform flow profiles, M1 and M2 are observed and analyzed as the initial conditions for the unsteady flow computations.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 45 (November 1970) "Flood Routing Through Storm Drains, Part III, Evaluation of Geometric and Hydraulic Parameters."

Key Words: Storm Drain, Flood Routing, Geometry, Flow Resistance, Velocity Distribution Coefficients, Junction Box Losses, Steady Non-Uniform Flow.

Abstract: The primary objective of this third part of a four-part series of hydrology papers on flood routing through storm drains is to present results on the investigation of the geometric and hydraulic parameters of the experimental conduit. The errors in cross section geometric parameters are analyzed in a conduit not ideally circular but approximated by an elliptical shape; errors are also analyzed when the undulations in the longitudinal slope of the conduit affect the predicted water surface profiles and thus the geometric parameters for a given water depth. The variation of hydraulic parameter of resistance, expressed by the Darcy-Weisbach friction factor, is experimentally determined and compared to the theoretical relation to Reynolds number. Energy losses in a 90° junction box are studied. The velocity distribution coefficients are shown to vary with the Darcy-Weisbach friction factor, and consequently, with the depth of flow. Boundary conditions for both controlled and free outfall are experimentally determined and approximated by power functions. Two types of steady non-uniform flow profiles, M1 and M2 are observed and analyzed as the initial conditions for the unsteady flow computations.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 45 (November 1970) "Flood Routing Through Storm Drains, Part III, Evaluation of Geometric and Hydraulic Parameters."

Key Words: Storm Drain, Flood Routing, Geometry, Flow Resistance, Velocity Distribution Coefficients, Junction Box Losses, Steady Non-Uniform Flow.

Abstract: The primary objective of this third part of a four-part series of hydrology papers on flood routing through storm drains is to present results on the investigation of the geometric and hydraulic parameters of the experimental conduit. The errors in cross section geometric parameters are analyzed in a conduit not ideally circular but approximated by an elliptical shape; errors are also analyzed when the undulations in the longitudinal slope of the conduit affect the predicted water surface profiles and thus the geometric parameters for a given water depth. The variation of hydraulic parameter of resistance, expressed by the Darcy-Weisbach friction factor, is experimentally determined and compared to the theoretical relation to Reynolds number. Energy losses in a 90° junction box are studied. The velocity distribution coefficients are shown to vary with the Darcy-Weisbach friction factor, and consequently, with the depth of flow. Boundary conditions for both controlled and free outfall are experimentally determined and approximated by power functions. Two types of steady non-uniform flow profiles, M1 and M2 are observed and analyzed as the initial conditions for the unsteady flow computations.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 45 (November 1970) "Flood Routing Through Storm Drains, Part III, Evaluation of Geometric and Hydraulic Parameters."

Key Words: Storm Drain, Flood Routing, Geometry, Flow Resistance, Velocity Distribution Coefficients, Junction Box Losses, Steady Non-Uniform Flow.

Abstract: The primary objective of this third part of a four-part series of hydrology papers on flood routing through storm drains is to present results on the investigation of the geometric and hydraulic parameters of the experimental conduit. The errors in cross section geometric parameters are analyzed in a conduit not ideally circular but approximated by an elliptical shape; errors are also analyzed when the undulations in the longitudinal slope of the conduit affect the predicted water surface profiles and thus the geometric parameters for a given water depth. The variation of hydraulic parameter of resistance, expressed by the Darcy-Weisbach friction factor, is experimentally determined and compared to the theoretical relation to Reynolds number. Energy losses in a 90° junction box are studied. The velocity distribution coefficients are shown to vary with the Darcy-Weisbach friction factor, and consequently, with the depth of flow. Boundary conditions for both controlled and free outfall are experimentally determined and approximated by power functions. Two types of steady non-uniform flow profiles, M1 and M2 are observed and analyzed as the initial conditions for the unsteady flow computations.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 45 (November 1970) "Flood Routing Through Storm Drains, Part III, Evaluation of Geometric and Hydraulic Parameters."

PREVIOUSLY PUBLISHED PAPERS

Colorado State University Hydrology Papers

- No. 27 "Diffusion of Entrapped Gas From Porous Media," by Kenneth M. Adam and Arthur T. Corey, April 1968.
- No. 28 "Sampling Bacteria in a Mountain Stream," by Samuel H. Kunkle and James R. Meimann, March 1968.
- No. 29 "Estimating Design Floods From Extreme Rainfall," by Frederick C. Bell, July 1968.
- No. 30 "Conservation of Ground Water by Gravel Mulches," by A.T. Corey and W.D. Kemper, May 1968.
- No. 31 "Effects of Truncation on Dependence in Hydrologic Time Series," by Rezaul Karim Bhuiya and Vujica Yevjevich, November 1968.
- No. 32 "Properties of Non-Homogeneous Hydrologic Series," by V. Yevjevich and R.I. Jeng, April 1969.
- No. 33 "Runs of Precipitation Series," by Jose Llamas and M.M. Siddiqui, May 1969.
- No. 34 "Statistical Discrimination of Change in Daily Runoff," by Andre J. Dumas and Hubert J. Morel-Seytoux, August 1969.
- No. 35 "Stochastic Process of Precipitation," by P. Todorovic and V. Yevjevich, September 1969.
- No. 36 "Suitability of the Upper Colorado River Basin for Precipitation Management," by Hiroshi Nakamichi and Hubert J. Morel-Seytoux, October 1969.
- No. 37 "Regional Discrimination of Change in Runoff," by Viboon Nimmannit and Hubert J. Morel-Seytoux, November 1969.
- No. 38 "Evaluation of the Effect of Impoundment on Water Quality in Cheney Reservoir," by J.C. Ward and S. Karaki, March 1970.
- No. 39 "The Kinematic Cascade as a Hydrologic Model," by David F. Kibler and David A. Woolhiser, February 1970.
- No. 40 "Application of Run-Lengths to Hydrologic Series," by Jaime Saldarriaga and Vujica Yevjevich, April 1970.
- No. 41 "Numerical Simulation of Dispersion in Groundwater Aquifers," by Donald Lee Reddell and Daniel K. Sunada, June 1970.
- No. 42 "Theoretical Probability Distributions for Flood Peaks," by Emir Zelenhasic, November 1970.
- No. 43 "Flood Routing Through Storm Drains, Part I, Solution of Problems of Unsteady Free Surface Flow in a Storm Drain", by V. Yevjevich and A.H. Barnes, November 1970.
- No. 44 "Flood Routing Through Storm Drains, Part II, Physical Facilities and Experiments", by V. Yevjevich and A.H. Barnes, November 1970.

FLOOD ROUTING THROUGH STORM DRAINS
Part IV
NUMERICAL COMPUTER METHODS OF SOLUTION

By

V. YEVJEVICH and A. H. BARNES

November 1970



HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
Fort Collins, Colorado

Several departments at Colorado State University have substantial research and graduate programs oriented to hydrology. These Hydrology Papers are intended to communicate in a fast way the current results of this research to the specialists interested in these activities. The papers will supply most of the background research data and results. Shorter versions will usually be published in the appropriate scientific and professional journals, or presented at national or international scientific and professional meetings and published in the proceedings of these meetings.

The investigations leading to this paper on unsteady free-surface flow in a long storm drain were supported by the U.S. Bureau of Public Roads, Federal Highway Administration, during 1960-1970 and by the U.S. Public Health Service during 1962-64. The research was conducted in the Hydrology and Water Resources Program of Civil Engineering Department at Colorado State University, Fort Collins, Colorado, U.S.A.

The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Federal Highway Administration or the Public Health Service.

EDITORIAL BOARD

Dr. Arthur T. Corey, Professor, Agricultural Engineering Department

Dr. Robert E. Dils, Professor, College of Forestry and Natural Resources

Dr. Vujica Yevjevich, Professor, Civil Engineering Department

FLOOD ROUTING THROUGH STORM DRAINS

Part IV

NUMERICAL COMPUTER METHODS OF SOLUTION

by

V. Yevjevich and A. H. Barnes

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO 80521

ACKNOWLEDGMENTS

The writers of this paper gratefully acknowledge the support and cooperation of the U.S. Bureau of Public Roads, Federal Highway Administration, in the research on flood movements through long storm drains conducted from 1960 to 1970. The writers also acknowledge the U.S. Public Health Service, National Institute of Health, for their additional support during 1962-1964.

The initiative, cooperation and support given by Mr. Carl F. Izzard to this project on flood movement through storm drains is particularly acknowledged. Mr. Izzard, presently Director, Office of Development, Federal Highway Administration, U.S. Department of Transportation, was Chief, Hydraulic Research Division, U. S. Bureau of Public Roads at the start of the project. Further acknowledgment is extended to Mr. Charles F. Scheffey, Director, Office of Research, Federal Highway Administration, for his cooperation and encouragement. Dr. Dah-Cheng Woo, Senior Hydraulic Engineer, Federal Highway Administration, has cooperated extensively with this project. His reviews and suggestions pertaining to all reports, theses and other documents produced on the project have been particularly helpful.

Acknowledgment is given to Dr. Subin Pinkayan, Post-Doctoral Fellow and Mr. William B. Frye, Graduate Research Assistant, for their contributions in the investigations of numerical methods and solutions.

Dr. Shih-Tun Su, Post-Doctoral Fellow, Civil Engineering Department, Colorado State University, using existing data, assisted the writers in finishing this paper.

TABLE OF CONTENTS

Chapter	Page
Acknowledgments	iii
Abstract	vi
1 INTRODUCTION	1
1.1 General Classification of Partial Differential Equations	1
1.2 Continuity and Momentum Equations of Unsteady Flow	1
1.3 Methods of Solving Equations of Unsteady Flow	2
1.4 Computer Oriented Numerical Solutions	3
1.5 Objectives of Studies Presented in this Paper	4
2 INTEGRATION OF PARTIAL DIFFERENTIAL EQUATIONS BY FINITE DIFFERENCES	5
2.1 Finite-Difference Methods	5
2.2 Various Finite-Difference Schemes	6
2.3 Diffusing Scheme	8
2.4 Lax-Wendroff Scheme	8
2.5 Comparison of Solutions by the Two Schemes	10
2.6 Concluding Remarks	11
3 INTEGRATION OF CHARACTERISTIC DIFFERENTIAL EQUATIONS BY FINITE DIFFERENCES	14
3.1 Statement of Characteristic Equations	14
3.2 Various Schemes	14
3.3 Numerical Solution by the Specified Intervals System	15
3.4 Initial Conditions	17
3.5 Boundary Conditions	17
3.6 Summary of Computational Procedures	19
3.7 Effect of Variations in Computational Parameters	19
4 COMPARISON OF THREE FINITE DIFFERENCE SCHEMES OF NUMERICAL INTEGRATION	23
4.1 Criteria for Comparison	23
4.2 Properties of Diffusing Scheme	23
4.3 Properties of Lax-Wendroff Scheme	23
4.4 Properties of Specified Intervals Scheme of the Method of Characteristics	24
5 CONCLUSIONS AND RECOMMENDATIONS	25
5.1 Conclusions	25
5.2 Recommendations	25
References	25
Appendix 1 Computational Details of Diffusing Scheme	26
Appendix 2 Computational Details of Lax-Wendroff Scheme	33
Appendix 3 Computational Details of Finite-Difference Specified Intervals Scheme of the Method of Characteristics	40

LIST OF FIGURES AND TABLES

Figure		Page
2.1	Definition graph for the finite-difference scheme.	5
2.2	Comparison of Lax-Wendroff scheme (A_w) and the diffusing scheme (D_i) in reproducing the steady initial conditions along the conduit, at the distance $x = 796.7$ ft	11
2.3	Study of effects of the size of Δx and Δt intervals (measured by n , the number of Δx intervals over the length $L = 822$ ft), on the predicted depth y at $x = 735.8$ ft	12
2.4	Comparison of the hydrographs at the position $x = 409.1$ computed with the Lax-Wendroff scheme without the assumption (A_w) and with the assumption (A_{wo}) of products of partial derivatives or the derivatives of S_f in x and t being negligible.	12
2.5	The same comparison as in Fig. 2.4, except at the position $x = 797.8$ ft	13
3.1	Network of characteristics in the method of grid system for the solution of unsteady flow equations.	14
3.2	Network of specified intervals for the solution of characteristic equations.	15
3.3	Rectangular grid for the solution by the system of specified intervals, Δt and Δx : subcritical flow (upper graph), critical flow (center graph), and supercritical flow (lower graph)	15
3.4	Hypothetical inflow hydrograph of the Pearson Type III function, Eq. 3.25, with the selected parameters: $Q_b = 6.21$ cfs, $Q_o = 8.00$ cfs, $t_p = 100.00$ sec, and $t_g = 150.0$ sec	18
3.5	Upstream boundary conditions: subcritical flow (upper graph), critical flow (central graph), and supercritical flow (lower graph).	18
3.6	Downstream boundary conditions for the subcritical flow, with x_L the computational conduit length	19
3.7	Effect of Δx on hydrographs at various positions along the conduit: (1) $\Delta x = 40.91$ ft, (2) $\Delta x = 20.45$ ft, (3) $\Delta x = 10.23$ ft, and (4) $\Delta x = 5.12$ ft, at three locations of conduit $x = 50.0$ ft (upper graph), $x = 410.0$ ft (center graph) and $x = 771.7$ ft (lower graph)	20
3.8	Characteristics of the depth hydrograph with T_p the time at peak depth, and y_p the peak depth	21
3.9	Relations between n and Δx and the computer time T , required for CDC 6600 computer	22
4.1	Effect of characteristic slopes.	23
4.2	Comparison of diffusing scheme (D_i), Lax-Wendroff scheme (A_L), and the specified intervals scheme of method of characteristics (C) in reproducing the steady initial conditions along the conduit, at the distance $x = 796.7$ ft	24
Table		
2.1	Various finite-difference schemes.	7
2.2	Substitutions.	10
3.1	Difference in y_p computed from various sizes of Δx (in percent of conduit diameter D)	22
3.2	Difference in T_p computed from various sizes of Δx (in percent of t_p)	22

ABSTRACT

This fourth part of a four-part series of hydrology papers on flood routing through storm drains presents computer-oriented numerical methods for solving the two quasi-linear hyperbolic partial differential equations known as the De Saint-Venant equations of gradually varied free-surface unsteady flow. Formulation and description of various finite-difference schemes based on explicit methods include the "unstable", diffusing, upstream differencing, leap frog, and Lax-Wendroff schemes. Stability and convergence are examined for these various schemes of the explicit method. Using various criteria of comparison, the specified intervals scheme of the method of characteristics, the Lax-Wendroff scheme, and the diffusing scheme are compared. Of the above explicit schemes in using the finite-difference ratios in the two partial differential equations, it is found that the Lax-Wendroff scheme with the second-order interpolation for dependent variables is the most accurate stable scheme. The specified intervals scheme of the method of characteristics, using either the first-order or second-order interpolations for the dependent variables, is also discussed. It is concluded that this scheme, based on the method of characteristics and using the second-order interpolations, is the most accurate numerical integration scheme of all those studied. Flow charts, computer programs, variable conversion tables, and sample inputs and outputs, for the three numerical computer schemes, the diffusing scheme, the Lax-Wendroff scheme, and the specified intervals scheme of the method of characteristics, used in the solution of the De Saint-Venant equations, are given in appendices 1 through 3.

FLOOD ROUTING THROUGH STORM DRAINS

Part IV

NUMERICAL COMPUTER METHODS OF SOLUTION

by

V. Yevjevich* and A. H. Barnes**

Chapter 1

INTRODUCTION

1.1 General Classification of Partial Differential Equations

Partial differential equations of physical processes fall within one of three forms, depending on the character of the coefficients of the partial derivatives. The equations expressing the one-dimensional gradually varied free-surface unsteady flow result in what is termed the hyperbolic form of partial differential equations. These equations are characterized by the initial conditions of the dependent variables being known, given, or independently evaluated at all distance positions for the time selected as zero, the boundary conditions being independently established at two distance locations, and the process being continued indefinitely in time within the established boundary conditions. As time increases, the effect of the initial conditions becomes less influential as the boundary conditions dominate the process.

The hyperbolic partial differential equations contrast the elliptic differential equations in which the process is not time dependent. In this case the initial conditions are the boundary conditions and are independent of time. A typical process described by this form is a two-dimensional temperature distribution in a thin plate with prescribed boundary conditions along the edges.

The third type of partial differential equations are parabolic equations, with the solution requirements being similar to the hyperbolic form. The simplest parabolic equation is the one-dimension heat-flow equation.

In subsequent text only the hyperbolic partial differential equation for gradually varied free-surface unsteady flow are discussed.

1.2 Continuity and Momentum Equations of Unsteady Flow

The two basic quasi-linear hyperbolic partial differential equations of gradually varied free-surface unsteady flow are derived in Chapter 3, Part I, Hydrology Paper No. 43, as Eqs. 3.23 and 3.19, and are reproduced here in their final dimensionless forms. The continuity equation is

$$\frac{A}{VB} \frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} + \frac{1}{V} \frac{\partial y}{\partial t} = \frac{q}{VB} \quad (1.1)$$

and the momentum equation is

$$\frac{\alpha V}{g} \frac{\partial V}{\partial x} + \frac{\beta}{g} \frac{\partial V}{\partial t} + \frac{\partial y}{\partial x} = (S_o - S_f) - \beta \frac{Vq}{Ag} \quad (1.2)$$

in which

A = the cross-section area,
V = the mean cross-section velocity as a dependent variable,
y = the water depth in the conduit as a dependent variable,
x = the length along the conduit as an independent variable,
t = the time as an independent variable,
B = the water surface width,
 α = the energy velocity distribution coefficient,
 β = the momentum velocity distribution coefficient,
g = the gravitational acceleration,
 S_o = the slope of the conduit invert,
 S_f = the energy gradient, and
q = the distributed lateral inflow (or outflow) as discharge per unit length of the conduit.

The energy gradient, measuring the energy head loss along the conduit, is expressed in this study by the Darcy-Weisbach equation in the form

$$S_f = \frac{fV^2}{8gR} \quad (1.3)$$

in which f is the Darcy-Weisbach friction factor, R is the hydraulic radius of a partially full conduit, with $R = A/P$, and P is the wetted perimeter.

The friction factor (f) is expressed as a function of Reynolds number, $Re = VR/\nu$, with ν the kinematic viscosity of the water.

Equations 1.1 and 1.2 generally give the closest approximations of the actual flood movement through channels and conduits, if the basic conditions for applying the two equations are approximately satisfied. The most important condition is that of gradual variability of the flood hydrograph; this condition is nearly always fulfilled for storm floods entering into and moving along storm drains.

* Professor-In-Charge of Hydrology and Water Resources Program, Dept. of Civil Engineering, Colorado State University.

** Associate Professor of Civil Engineering, Colorado State University.

1.3 Methods of Solving Equations of Unsteady Flow

All methods available in literature for solving Eqs. 1.1 and 1.2 may be grouped into analytical, graphical, and numerical procedures. The numerical procedures depend on the computational devices available.

Analytical solutions. The partial differential equations 1.1 and 1.2 have a friction slope, S_f , proportional to the square of the velocity or to the square of the discharge. Because their coefficients are functions of dependent variables (V , y), they are non-linear differential equations of the hyperbolic type. Because of the inherent mathematical difficulties of these non-linear and non-homogeneous equations, there is no way to carry out the analytical integration in closed form, unless many simplifications are introduced.

The classical approach, first performed by De Saint-Venant, neglects friction resistance and assumes the channel to be horizontal with wide rectangular cross sections. These assumptions deviate so much from the reality of flood-wave movement in channels and conduits that the wave characteristics resulting from analytical integration are generally not comparable with true wave characteristics. This classical approach by means of analytical integration is an extreme; it may be considered to be a rough approximation, and, in accuracy, can be compared with some of the very simple integration procedures of flood routing that are based on the water storage ordinary differential equation.

The use of analytical integration makes it necessary to approximate and simplify both the initial conditions and the boundary conditions by analytical expressions, which are used in Eqs. 1.1 and 1.2. The inflow hydrograph as the boundary condition, and the wave profile along the conduit, as the initial condition, must be mathematically approximated by considering them to be either symmetrical or asymmetrical waves, with functions of bell-shaped curves (gamma-functions, and others). The channel conditions may be represented by the cross section area or width as functions of water depth and distance along the conduit, with a roughness coefficient usually a constant, and the bottom slope being either a constant or a function of distance. The lateral inflow and outflow are taken as constant or are approximated by simple functions of channel and lateral flow characteristics, and of time.

The great diversity in shape and roughness of natural channels, free-surface flow conditions and the complexity of the pattern of the lateral inflows and outflows tend to complicate the analytical expressions that approximate these conditions to the extent that the analytical integration of the two partial differential equations becomes impossible. In summary, the two partial differential equations for unsteady flow can be integrated analytically, with expressions for wave evolution, by rather restrictive and very simplifying conditions, which generally are not acceptable for the solution of current practical problems.

For some discussions and abstracted references about the analytical solutions of simplified conditions for flood routing through conduits and channels, as well as of graphical and numerical solutions, see the "Bibliography and Discussion of Flood-Routing Methods

and Unsteady Flow in Channels" [1]*, and the general reference list in Appendix 2 of Hydrology Paper No. 43 (Part I of this series of four papers).

Graphical solutions. The graphical solutions of equations for free-surface unsteady flow may be characterized by the following procedure. The celerity of the disturbance in the distance-time reference plane (x, t) - plane, is computed from the simplified wave relation

$$\frac{dx}{dt} = V \pm \sqrt{gy_*} \quad (1.4)$$

in which V is the mean velocity of flow, y_* is the hydraulic depth (A/B) in any cross-sectional shape, and g is the gravitational acceleration.

The term $C = \sqrt{gy_*}$ is usually referred to as the celerity of a small disturbance moving in a quiescent water of a channel. The terms $V + \sqrt{gy_*}$ and $V - \sqrt{gy_*}$ are called either the wave velocity [2, p. 540], or the celerity of a small disturbance in the moving fluid [1, p. 10]. This latter term will be used in this paper when Eq. 1.4 is discussed or used. If the first derivative, dt/dx , in the (x, t) - plane is used as the measure of the celerities of disturbances in the moving water, then the inverse of Eq. 1.4 should be used as

$$\frac{dt}{dx} = \frac{1}{V \pm \sqrt{gy_*}} \quad (1.5)$$

In case of the circular conduit in which flood waves move with gradually varied free-surface flow, y_* should be replaced by $y_* = f(y)$, a function of water depth.

In the discussion to follow the two directions of Eq. 1.5 will be referred to as the characteristic directions, which are first derivatives of characteristic curves, defined in Chapter 3, Part I, Hydrology Paper No. 43. Along the characteristic curves, the wave phenomenon may be expressed by the two ordinary differential equations with two dependent variables as unknowns. Thus, starting from the known values of the dependent variables (V and y) at two locations in time (t) and position (x), the direction of the characteristics may be graphically plotted. From these plots, the location of the intersections in time and position can be determined. With the known time (t) and position (x) a finite difference solution to the two ordinary differential equations gives the corresponding dependent variables (V and y). Repeating the procedure, the integration proceeds along the time scale for the given length of channel or conduit.

This procedure has been used extensively by Parmakian in his book on waterhammer analysis [3]. Akers and Harrison presented a similar analysis for free-surface unsteady flow in a circular channel in their paper on attenuation of flood waves in partially full pipes, [4].

The limitations of graphical procedures are immediately evident when one considers the effect of

*[] Reference numbers refer to the list of references at the end of this paper.

various parameters, initial and boundary conditions, in a given problem. Thus the graphical solution has limited application at present because of the labor involved, except perhaps for the visualization of the digital computer schemes and the results to be presented.

Numerical solutions. Various numerical procedures have been used in the past. The excessive number of calculations in order to progress the solution in time, however, has limited the application of these solutions.

The two partial differential equations, 1.1 and 1.2, are usually approximated by the two finite-differences equations, replacing the increments (dx, dt, dv, dy) by the finite differences $(\Delta x, \Delta t, \Delta v, \Delta y)$. At the same time the partial derivatives are replaced by ratios of finite differences: $\partial v / \partial x$ by $\Delta v / \Delta x$, $\partial v / \partial t$ by $\Delta v / \Delta t$, $\partial y / \partial x$ by $\Delta y / \Delta x$, and $\partial y / \partial t$ by $\Delta y / \Delta t$. With Δx and Δt given, Δv and Δy are changes of dependent variables which occur for these finite differences.

The basic characteristics of the above finite-difference approximations are: (1) the accuracy depends on the size and relation of finite differences Δt and Δx ; (2) the smaller the Δx , the more involved the computation work, but also the greater the accuracy may be, and (3) the values of dependent variables computed for the end of a Δt become the initial values for the next Δt .

With the development of electronic computers, which provide fast and relatively inexpensive computations, the past drawbacks in economy of performing the operations of the finite-differences method of integration are largely eliminated. The method is highly favored inasmuch as it is the most accurate of all practical methods of flood routing in channels and conduits. The advent of new numerical schemes helped this progress in the use of numerical methods of solution by digital computers.

The results of integration are given for two dependent variables as functions $V = F_1(x, t)$ and $y = F_2(x, t)$. These two functions represent surfaces in the space (V, x, t) and (y, x, t) . If there is any discontinuity in the four partial derivatives of Eqs. 1.1 and 1.2, these discontinuities propagate along the channel, and the projection of the position of discontinuities at surfaces F_1 and F_2 in the (x, t) -plane produces lines that are called "characteristics", or "characteristic lines". These lines are usually curves, but in application may be replaced by straight lines along the finite differences Δx and Δt .

The simplified characteristic lines are usually given in the form

$$dx = (V \pm \sqrt{gy_*}) dt, \quad (1.6)$$

and

$$d(V \pm 2\sqrt{gy_*}) = g(S_0 - S_f)dt, \quad (1.7)$$

which are equivalent to Eqs. 1.1 and 1.2. The hydraulic depth (y_*) should be expressed as a function of y for the free-surface flow in circular conduits.

Equations 1.6 and 1.7 are usually numerically integrated by replacing dx and dt with Δx and Δt , and $d(V \pm 2\sqrt{gy_*})$ with $\Delta(V \pm 2\sqrt{gy_*})$. Several numerical procedures have been developed for these approximations in the finite-differences form.

Certain features of the method of numerical integration by characteristics are important for applicability in practical cases in flood routing by finite differences: (1) the long wave is assumed to be composed of many elementary waves in the form of small surges so that for the time Δt and the reach Δx , the velocity change, Δv , and height change, Δy , are considered as discontinuities traveling with celerities $V \pm \sqrt{gy_*}$ (providing only a rough approximation in the case of long flood waves, where the friction forces are not negligible); (2) the straight-line characteristics are used as approximations instead of curve-line characteristics for Δx and Δt , and (3) some complexity of procedure when friction factors, channel slope, sudden changes of cross section, bifurcations, junctions, and similar changes, are to be taken into consideration.

With the advent of computers and new numerical schemes, numerical integration by finite differences of Eqs. 1.6 and 1.7 has become economical. The general applicability of various electronic computers (analog, hybrid, digital) to the numerical integration either of Eqs. 1.1 and 1.2, or of Eqs. 1.6 and 1.7, is discussed in the next subchapter.

Concluding remarks. All three methods -- analytical, graphical, and numerical -- by finite differences applied either to partial differential equations or to characteristic differential equations, when applicable, give sufficiently accurate results if the methods are extended to their limits of accuracy. These methods can be successfully applied to the analysis of particular waves that have been observed. The practical prediction of wave movement, however, requires a considerable amount of work, especially when the network of drains is complex.

The mathematical difficulties of analytical integration of the two partial differential equations, the need for a large amount of data for the graphical methods, the accompanying drawbacks of time-consuming procedures and the cost in applying the approximate methods of numerical integration have provided incentive for developing simpler, but generally less accurate, flood-routing methods [1]. Since the objective of this study is to produce research results that lead to practical methods in using complete Eqs. 1.1 and 1.2, or Eqs. 1.6 and 1.7, in routing flood hydrographs through storm drains, the only acceptable integration methods from both economic and accuracy standpoints are numerical methods by finite differences and the use of electronic computers. This paper is, therefore, concerned only with these latter methods.

1.4 Computer Oriented Numerical Solutions

The obvious conclusion to the dilemma of excessive repetitive calculations and the limit of manual computations is the use of electronic computers. Three possibilities exist for the solution of the problem equations.

One type of computer is the analog computer in which the mathematical functions are simulated by suitable amplifiers, potentiometers or other electronic elements. The combination of these elements simulate the mathematical equations of the physical phenomenon.

This technique is particularly desirable for a physical system with fixed parameters and repetitive operations. This analog system permits an evaluation of the effect of variations in boundary conditions. A disadvantage of the analog solution would be the problems of generating the geometric and hydraulic parameters at each stage in the computations.

The hybrid electronic computer permits continuous evaluation of the differential equations by analog and evaluates the required parameters by digital computation. Thus, a continuous solution can be obtained with the geometric and hydraulic parameters evaluated by direct computation. The availability of such computers is still limited, but hybrid computers may become the best computational device for unsteady flow. The programming is specialized and not readily usable by most programmers. For these reasons the more conventional digital computer has been generally used and will be discussed exclusively in this paper.

The digital computer presents the advantage of rapid arithmetical operations and a relatively simple and versatile programming capability. The basic limitation is that integration cannot be expressed as a continuous function as is done in the analog computer. This requires that any integration of an equation or a set of equations be represented by a series of discrete elements. The approximation to the correct integration would be expected to improve as the size of the discrete elements decreased and their number increased. This is an acceptable assumption for many integration processes. However, it cannot be assumed that it is correct for all cases. This is due to the effect of round-off and truncation errors within the computer. For this study it has been assumed that the functions to be integrated are "well

behaved" and may be reasonably integrated by the assumption of discrete increments of the variables of integration.

There are a large variety of numerical integration procedures available for the solution of the St-Venant partial differential equations of gradually varied free-surface unsteady flow. One method of categorization of these basic procedures is to consider solutions depending on the two partial differential equations of 1.1 and 1.2 of the phenomena, in the other method solutions depend on the ordinary differential equation forms, Eqs. 1.6 and 1.7, of the same equations. How the forms of the ordinary differential equations are derived from the partial differential equations is shown in Chapter 3 of Part I, Hydrology Paper No. 43.

1.5 Objectives of Studies Presented in this Paper

The objectives of this paper are to present only the results of studies concerning the numerical solutions by various finite-differences schemes, either for the case of the two partial differential equations, 1.1 and 1.2, or for the case of the four characteristic equations, 1.6 and 1.7. Chapter 2 analyzes the applicability of various finite-difference schemes in the numerical solution of the two partial differential equations. Chapter 3 analyzes the various finite-difference schemes in the numerical solution of the four characteristic equations. The applicability of various schemes is discussed at the end of each of these two chapters. Chapter 4 is a comparison of the best finite-difference schemes in the case of numerical solution of partial differential equations and numerical solution of characteristic equations. Chapter 5 presents the conclusions and recommendations for further research.

INTEGRATION OF PARTIAL DIFFERENTIAL

EQUATIONS BY FINITE DIFFERENCES

2.1 Finite-Difference Methods

The finite-difference methods of numerical integration to be discussed refer to the partial differential equations of gradually varied free-surface unsteady flow. Because these equations do not permit a closed analytical solution, approximate numerical methods of integration must be employed. Since all numerical integration methods are fundamentally finite-difference procedures some distinctions between various methods or schemes are appropriate.

For this presentation, the term "finite-difference method" will refer to the approximation to the partial derivatives as the ratios of differences of finite values of the dependent variables at fixed uniform intervals. The ratios of finite differences will approach the partial derivatives as the intervals or differences become smaller. The basic definition of a partial derivative in x of a two-variable function, $f(x, y)$, is

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\Delta x \rightarrow 0} \left[\frac{f(x + \Delta x, y) - f(x, y)}{\Delta x} \right] \quad (2.1)$$

Using the right side of this equation, the partial derivative may be approximated as nearly accurate as desired by selecting a small difference Δx .

For solving De Saint-Venant equations 1.1 and 1.2 difference approximations are made as follows. Since there are two independent variables and two dependent variables, designation of the time-distance locations of the variables will be based on the subscripts and superscripts of the variables. The subscript will refer to the distance (space) location, and the superscript to the time location as shown in Fig. 2.1.

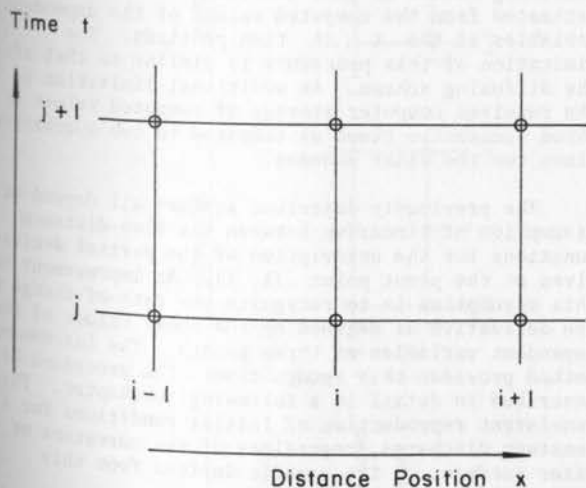


Fig. 2.1. Definition graph for the finite-difference scheme.

Thus, the depth at distance location i and at time location j is designated as y_i^j . The four partial derivatives of Eqs. 1.1 and 1.2 may be approximated by

$$\frac{\partial V}{\partial x} \approx \frac{V_{i+1}^j - V_i^j}{x_{i+1}^j - x_i^j} \quad (2.2)$$

$$\frac{\partial V}{\partial t} \approx \frac{V_i^{j+1} - V_i^j}{t_i^{j+1} - t_i^j} \quad (2.3)$$

$$\frac{\partial y}{\partial x} \approx \frac{y_{i+1}^j - y_i^j}{x_{i+1}^j - x_i^j} \quad (2.4)$$

and

$$\frac{\partial y}{\partial t} \approx \frac{y_i^{j+1} - y_i^j}{t_i^{j+1} - t_i^j} \quad (2.5)$$

The unknown quantities in these expressions are generally the values at the incremental time locations, $j+1$. Thus V_i^{j+1} and y_i^{j+1} are the unknown values.

With the two equations of unsteady flow, these two unknowns may be solved for simultaneously. This procedure is referred to as an explicit scheme in that the conditions at a later time, $j+1$, are determined directly from the conditions at the preceding time, j . Other explicit schemes are presented in the next subchapter.

Another manner of expressing the partial derivatives with respect to the distance position is

$$\frac{\partial V}{\partial x} \approx \frac{V_{i+1}^{j+1} - V_i^{j+1}}{x_{i+1}^{j+1} - x_i^{j+1}} \quad (2.6)$$

and

$$\frac{\partial y}{\partial x} \approx \frac{y_{i+1}^{j+1} - y_i^{j+1}}{x_{i+1}^{j+1} - x_i^{j+1}} \quad (2.7)$$

The partial derivatives in the case of Eqs. 2.6 and 2.7 are described in terms of the independent variable x along the incremental time locations. Therefore, there are four unknowns of V and y , at two distance locations at a given incremental time location. The two equations of unsteady flow at a given point in time and distance are insufficient for

the solution. However, if a system of simultaneous equations are developed for each point, there will be as many equations as the total number of unknowns. A simultaneous solution of this set then results in the desired solution. This scheme is referred to as the implicit solution since all solutions are directly interrelated. No attempt was made to use this method, however, because of the limits in solving equations for the dependent variables at an unlimited number of distance locations.

A physical and, consequently, mathematical limitation to either an explicit or implicit scheme is imposed by the direction a disturbance travels in the time-distance reference plane. The directions of a disturbance are commonly referred to as the characteristic directions and are defined by Eq. 1.5. The two expressions for dt/dx of Eq. 1.5 represent the two directions the disturbances propagate along.

If one considers these directions as emanating from a single given point in the time-distance plane, where a disturbance occurred, the region x and t between these two directions is affected by the disturbance. This region is the "region of influence". If one considers the disturbances as having occurred at two different locations in the time-distance plane, two of the four directions will intersect. The region bounded by this intersection is the "domain of dependence." The dependent variables in this region are functions of all their previous values within this region. As a corollary, the values of dependent variables outside this region do not affect the values of V and y inside this region.

Thus, the directions of the disturbance or characteristic directions in the (x, t) - plane divide the time-distance plane into a region wherein solutions from given conditions are possible, and a region in which solutions are theoretically impossible. It is necessary to consider this in any finite-difference method of integrating the two partial differential equations. The general criterion to be applied is that

$$\frac{dt}{dx} \approx \frac{1}{V \pm \sqrt{g A/B}} \quad (2.8)$$

in which V and A/B are the average values for the specified finite differences, Δx and Δt . The criteria of Eq. 2.8 is valid for all values of the dependent variables in the solution. The nearer the two points in the (x, t) - plane are, the more nearly the numerical solution will approach the true solution.

2.2 Various Finite-Difference Schemes

Equations 2.2 through 2.5 present the simplest approximation by the finite-difference expressions to the partial derivatives. A wide variety of schemes, usually more sophisticated than Eqs. 2.2 through 2.5, have been developed by various authors to provide better accuracy and to maintain the stability of the solutions with minimum computational work.

Richtmeyer [5] presented six schemes with their corresponding truncation errors. These schemes are presented in Table 2.1. This table displays the computational template of the (x, t) - plane, the approximation to the partial derivatives, and the order of the truncation error $O(\Delta)$, due to the approximation where Δ is the symbol of increment, either Δx or Δt .

Substituting these approximations into the basic equations results in a pair of equations with two unknowns, velocity and depth, at the end of the time interval.

The "unstable scheme" is inherently unstable. It is presented to demonstrate the simplest scheme, and to permit comparison of stable schemes with this basic scheme.

The diffusing scheme is the simplest stable scheme. It offers two approaches for computation. One approach consists of the staggered scheme as presented in Table 2.1. It uses known values of V and y at the $i-1$ and the $i+1$ distance positions at time t to compute the dependent variables at the distant position i , at time $t + \Delta t$. This approach determines values at all locations defined by $i+j$ equal an even number. The other approach is to advance one Δx and thus compute the dependent variables at each intersection. This approximately doubles the computational time but produces results at one-half the intervals of the first method.

In order for the diffusing scheme to be stable, it is necessary that

$$\frac{\Delta t}{\Delta x} \leq \left| \frac{1}{V \pm \sqrt{g A/B}} \right|$$

be a condition throughout the computation. As the flow progresses into the super-critical range, this condition is less likely to be fulfilled unless an arbitrary reduction in Δt is made. An additional limitation of this scheme is the assumed linearity of the dependent variables within the interval from $i-1$ to $i+1$.

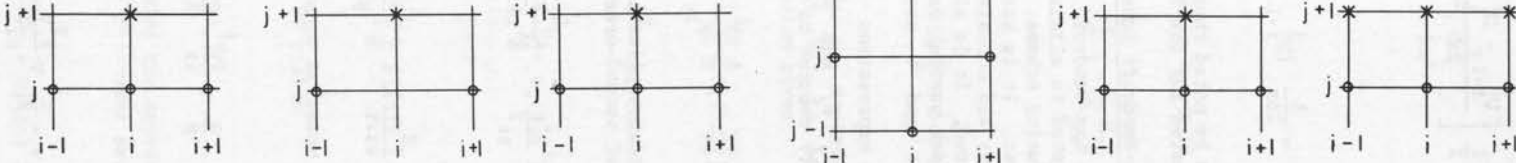
The upstream differencing scheme is similar to the diffusing scheme. The computer programming, however, is somewhat more involved because of the necessity of deciding which representation of the distance derivative to use for each computation. For this reason this scheme was not investigated in this study.

The leap-frog scheme is an improvement over the diffusing scheme in that the time derivative is estimated from the computed values of the dependent variables at the $t - \Delta t$ time position. The limitation of this procedure is similar to that of the diffusing scheme. An additional limitation is the required computer storage of computed values at three successive times as compared to two successive times for the other schemes.

The previously described schemes all depend on an assumption of linearity between the time-distance junctions for the description of the partial derivatives at the pivot point (i, j) . An improvement to this assumption is to recognize the rate-of-change of the derivative as defined by the known values of the dependent variables at three points. The Lax-Wendroff method provides this recognition. The procedure is described in detail in a following subchapter. The consistent reproduction of initial conditions for a constant discharge, regardless of the curvature of the water surface, is the benefit derived from this method.

The implicit scheme requires the solution of a system of simultaneous equations equal in number to the number of distance intervals plus one. Two of

Table 2.1 Various finite-difference schemes

	UNSTABLE	DIFFUSING	UPSTREAM DIFFERENCING	LEAP FROG	LAX WENDROFF	IMPLICIT
COMPUTATIONAL TEMPLATE	× Unknown ○ Known ↑ t → x 					
PARTIAL DERIVATIVE APPROXIMATION	$\frac{\partial U}{\partial x} \approx \frac{U_{i+1}^j - U_{i-1}^j}{2\Delta x}$	$\frac{U_{i+1}^j - U_{i-1}^j}{2\Delta x}$	$\frac{U_{i+1}^j - U_i^j}{\Delta x}$ or $\frac{U_i^j - U_{i-1}^j}{\Delta x}$	$\frac{U_{i+1}^j - U_{i-1}^j}{2\Delta x}$	Depends on form of partial differential equation.	$\frac{U_{i+1}^{j+1} - U_{i-1}^{j+1} + U_{i+1}^j - U_{i-1}^j}{4\Delta x}$
	$\frac{\partial U}{\partial t} \approx \frac{U_i^{j+1} - U_i^j}{\Delta t}$	$\frac{U_i^{j+1} - \frac{U_{i+1}^j + U_{i-1}^j}{2}}{\Delta t}$	$\frac{U_i^{j+1} - U_i^j}{\Delta t}$	$\frac{U_i^{j+1} - U_i^{j-1}}{2\Delta t}$		$\frac{U_i^{j+1} - U_i^j}{\Delta t}$
TRUNCATION ERROR	$O[\Delta^2]$	$O[\Delta^2]$	$O[\Delta^2]$	$O[\Delta^3]$	$O[\Delta^3]$	$O[\Delta^3]$

these equations involve the boundary conditions. This system was not used because of the number of equations that needed to be solved simultaneously, for an arbitrarily long conduit.

All but one of the above schemes are explicit. Two of the schemes, the diffusing scheme and the Lax-Wendroff scheme, are used in this study to solve the De Saint Venant equations. These solutions provide good accuracy and require only reasonable computer time. The diffusing and Lax-Wendroff schemes are summarized in the following two subchapters.

2.3 Diffusing Scheme

The diffusing scheme evolves from the following approximation to the partial derivatives with respect to time. The schemes in Table 2.1 is the definition graph for the location of significant variables. It is assumed that the dependent variables are known for all positions at time j . The dependent variable will be designated as U in this development, and it may refer either to the V or y dependent variables of the two partial differential equations. The objective is to represent the partial derivatives as functions of the unknown dependent variable U at distance location i and time location $j+1$. The partial derivative of U with respect to t is approximated by

$$\left(\frac{\partial U}{\partial t}\right)_i \approx \left(\frac{\Delta U}{\Delta t}\right)_i, \quad (2.9)$$

in which

$$\Delta U_i = U_i^{j+1} - U_i^j. \quad (2.10)$$

Expressing U_i^j as an average

$$U_i^j \approx \frac{U_{i+1}^j + U_{i-1}^j}{2}, \quad (2.11)$$

then

$$\Delta U_i = U_i^{j+1} - \frac{U_{i+1}^j + U_{i-1}^j}{2}, \quad (2.12)$$

and finally the finite difference approximation to this partial derivative is

$$\begin{aligned} \left(\frac{\Delta U}{\Delta t}\right)_i &= \frac{U_i^{j+1} - \frac{U_{i+1}^j + U_{i-1}^j}{2}}{\Delta t} \\ &= \frac{2U_i^{j+1} - U_{i+1}^j - U_{i-1}^j}{2\Delta t}. \end{aligned} \quad (2.13)$$

Similarly, the partial derivative with respect to the distance x is approximated by

$$\left(\frac{\partial U}{\partial x}\right)_i \approx \left(\frac{\Delta U}{\Delta x}\right)_i, \quad (2.14)$$

in which

$$\left(\frac{\Delta U}{\Delta x}\right)_i = \frac{1}{2} \left[\frac{U_{i+1}^j - U_i^j}{\Delta x} + \frac{U_i^j - U_{i-1}^j}{\Delta x} \right], \quad (2.15)$$

so that

$$\left(\frac{\Delta U}{\Delta x}\right)_i = \frac{1}{2\Delta x} (U_{i+1}^j - U_{i-1}^j). \quad (2.16)$$

It is to be noted that both partial derivatives are approximated for the location i, j .

2.4 Lax-Wendroff Scheme

The Lax-Wendroff finite difference scheme was investigated to eliminate some of the deficiencies of the diffusing scheme. The summary of the scheme is as follows. It is assumed that all functions are continuous and contain as many continuous derivatives as required. It is also assumed that products of first-order partial derivatives, and any derivative of S_f in x and t are negligible quantities.

The expressions $\frac{\partial A}{\partial t} = B \frac{\partial y}{\partial t}$ and $\frac{\partial A}{\partial x} = B \frac{\partial y}{\partial x}$ relate A , B , and y . Therefore, the equation of continuity reduces to

$$\frac{\partial y}{\partial t} = -\frac{A}{B} \frac{\partial V}{\partial x} - V \frac{\partial y}{\partial x}. \quad (2.17)$$

The intended application of the Taylor series requires the use of second-order partial derivatives. Thus,

$$\frac{\partial^2 y}{\partial t^2} = -\frac{A}{B} \frac{\partial^2 V}{\partial x \partial t} - V \frac{\partial^2 y}{\partial x \partial t}, \quad (2.18)$$

and

$$\frac{\partial^2 y}{\partial x \partial t} = -\frac{A}{B} \frac{\partial^2 V}{\partial x^2} - V \frac{\partial^2 y}{\partial x^2}. \quad (2.19)$$

The momentum equation, 1.2, is rewritten here in the form

$$\frac{\partial V}{\partial t} = -\frac{\alpha}{\beta} V \frac{\partial V}{\partial x} - \frac{g}{\beta} \frac{\partial y}{\partial x} - \frac{g}{\beta} (S_f - S_0), \quad (2.20)$$

which gives then

$$\frac{\partial^2 V}{\partial x \partial t} = -\frac{\alpha}{\beta} V \frac{\partial^2 V}{\partial x^2} - \frac{g}{\beta} \frac{\partial^2 y}{\partial x^2}. \quad (2.21)$$

Hence, Eq. 2.18 becomes

$$\frac{\partial^2 y}{\partial t^2} = \frac{A}{B} \frac{1}{\beta} \left(\alpha V \frac{\partial^2 V}{\partial x^2} + g \frac{\partial^2 y}{\partial x^2} \right) + \frac{VA}{B} \frac{\partial^2 V}{\partial x^2} + V^2 \frac{\partial^2 y}{\partial x^2}$$

or

$$\frac{\partial^2 y}{\partial t^2} = \left(\frac{\alpha}{\beta} + 1\right) \frac{AV}{B} \frac{\partial^2 V}{\partial x^2} + \left(\frac{g}{\beta} \frac{A}{B} + V^2\right) \frac{\partial^2 y}{\partial x^2} \quad (2.22)$$

Equation 2.20 then gives

$$\frac{\partial^2 V}{\partial t^2} = -\frac{\alpha}{\beta} V \frac{\partial^2 V}{\partial x \partial t} - \frac{g}{\beta} \frac{\partial^2 y}{\partial x \partial t} \quad (2.23)$$

Substituting Eqs. 2.19 and 2.21 into Eq. 2.23 yields

$$\frac{\partial^2 V}{\partial t^2} = -\frac{\alpha}{\beta} V \left(-\frac{\alpha}{\beta} V \frac{\partial^2 V}{\partial x^2} - \frac{g}{\beta} \frac{\partial^2 y}{\partial x^2} \right) - \frac{g}{\beta} \left(-\frac{A}{B} \frac{\partial^2 V}{\partial x^2} - V \frac{\partial^2 y}{\partial x^2} \right)$$

or

$$\frac{\partial^2 V}{\partial t^2} = \left[\left(\frac{\alpha}{\beta}\right)^2 V^2 + \frac{g}{\beta} \frac{A}{B} \right] \frac{\partial^2 V}{\partial x^2} + \left(\frac{\alpha}{\beta} + 1\right) \frac{g}{\beta} V \frac{\partial^2 y}{\partial x^2} \quad (2.24)$$

Putting U as the symbol for any dependent variable V or y , then for any $U(x, t)$ and a fixed x , a Taylor series expansion gives

$$U(t+\Delta t) = U(t) + \Delta t \frac{\partial U}{\partial t} + \frac{(\Delta t)^2}{2} \frac{\partial^2 U}{\partial t^2} + O[(\Delta t)^3] \quad (2.25)$$

in which both $\partial U / \partial t$ and $\partial^2 U / \partial t^2$ are functions of t . Similarly, for a fixed t ,

$$U(x+\Delta x) = U(x) + \Delta x \frac{\partial U}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 U}{\partial x^2} + O[(\Delta x)^3] \quad (2.26)$$

and

$$U(x-\Delta x) = U(x) - \Delta x \frac{\partial U}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 U}{\partial x^2} - O[(\Delta x)^3] \quad (2.27)$$

Subtracting Eq. 2.27 from Eq. 2.26 yields

$$\frac{\partial U}{\partial x} = \frac{U(x+\Delta x) - U(x-\Delta x)}{2\Delta x} + O[(\Delta x)^3] \quad (2.28)$$

Adding Eq. 2.27 and Eq. 2.26 yields the approximation of the second-order partial derivative of U with respect to x

$$\frac{\partial^2 U}{\partial x^2} = \frac{U(x+\Delta x) - 2U(x) + U(x-\Delta x)}{(\Delta x)^2} + O[(\Delta x)^4] \quad (2.29)$$

Substituting V and y for U , respectively, and using Eqs. 2.17, 2.20, 2.22, and 2.24 for the appropriate partial derivatives with respect to t in Eq. 2.25 produces

$$V(t+\Delta t) = V(t) - \frac{\Delta t}{\beta} \left[\alpha V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} + g(S_f - S_o) \right]$$

$$+ \frac{(\Delta t)^2}{2} \left[\left(\frac{\alpha^2 V^2}{\beta^2} + \frac{g}{\beta} \frac{A}{B} \right) \frac{\partial^2 V}{\partial x^2} + \left(\frac{\alpha}{\beta} + 1 \right) \frac{g}{\beta} V \frac{\partial^2 y}{\partial x^2} \right] + O[(\Delta t)^3] \quad (2.30)$$

and

$$y(t+\Delta t) = y(t) - \Delta t \left(\frac{A}{B} \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} \right) + \frac{(\Delta t)^2}{2} \left[\left(\frac{\alpha}{\beta} + 1 \right) \frac{AV}{B} \frac{\partial^2 V}{\partial x^2} + \left(\frac{A}{B} \frac{g}{\beta} + V^2 \right) \frac{\partial^2 y}{\partial x^2} \right] + O[(\Delta t)^3] \quad (2.31)$$

Let j index the t intervals and i index the x intervals. Referring to Eqs. 2.28 and 2.29, the first and second partial derivatives with respect to x are approximated by

$$\frac{\partial U}{\partial x} = \frac{U_{i+1}^j - U_{i-1}^j}{2\Delta x} \quad (2.32)$$

and

$$\frac{\partial^2 U}{\partial x^2} = \frac{U_{i+1}^j - 2U_i^j + U_{i-1}^j}{(\Delta x)^2} \quad (2.33)$$

Thus, recurrence relations for finding approximate solutions to V and y in Eqs. 2.30 and 2.31 are

$$y_i^{j+1} = y_i^j - \frac{\Delta t}{2\Delta x} \left[\left(\frac{A}{B} \right)_i^j (V_{i+1}^j - V_{i-1}^j) + V_i^j (y_{i+1}^j - y_{i-1}^j) \right] + \frac{1}{2} \left(\frac{\Delta t}{\Delta x} \right)^2 \left\{ \left(\frac{\alpha}{\beta} + 1 \right) \left(\frac{A}{B} \right)_i^j V_i^j (V_{i+1}^j - 2V_i^j + V_{i-1}^j) + \left[\frac{g}{\beta} \left(\frac{A}{B} \right)_i^j + (V_i^j)^2 \right] (y_{i+1}^j - 2y_i^j + y_{i-1}^j) \right\} \quad (2.34)$$

and

$$V_i^{j+1} = V_i^j - \frac{\Delta t}{2\beta\Delta x} \left[\alpha V_i^j (V_{i+1}^j - V_{i-1}^j) + g(y_{i+1}^j - y_{i-1}^j) + 2g\Delta x(S_f - S_o) \right] + \frac{1}{2} \left(\frac{\Delta t}{\Delta x} \right)^2 \left\{ \left[\left(\frac{\alpha}{\beta} \right)^2 (V_i^j)^2 + \frac{g}{\beta} \left(\frac{A}{B} \right)_i^j \right] (V_{i+1}^j - 2V_i^j + V_{i-1}^j) + \left(\frac{\alpha}{\beta} + 1 \right) \frac{g}{\beta} V_i^j (y_{i+1}^j - 2y_i^j + y_{i-1}^j) \right\} \quad (2.35)$$

For those cases in which the products of the first order partial derivatives and the derivatives of S_f cannot be disregarded, difference equations analogous to Eqs. 2.34 and 2.35 may be derived by appropriate substitutions of relations from Table 2.2 into Eqs. 2.25, 2.26, and 2.27.

TABLE 2.2
Substitutions

The substitutions in the following equations are:

$$M = \frac{(1 - \frac{2y}{D})}{\sqrt{\frac{y}{D} (1 - \frac{y}{D})}}, \text{ with } D \text{ the conduit diameter;}$$

$$N = \frac{1}{D} \left\{ \frac{B}{\cos^{-1}(1 - \frac{2y}{D})} - \frac{A}{\sqrt{\frac{y}{D} (1 - \frac{y}{D})} [\cos^{-1}(1 - \frac{2y}{D})]^2} \right\};$$

$$\frac{\partial B}{\partial x} = M \frac{\partial y}{\partial x}, \quad \frac{\partial B}{\partial t} = M \frac{\partial y}{\partial t}, \quad \frac{\partial R}{\partial x} = N \frac{\partial y}{\partial x}, \quad \text{and} \quad \frac{\partial R}{\partial t} = N \frac{\partial y}{\partial t};$$

$$\frac{\partial^2 y}{\partial x \partial t} = \frac{\partial V}{\partial x} (-2 \frac{\partial y}{\partial x} + \frac{A}{B^2} \frac{\partial B}{\partial x}) - \frac{A}{B} \frac{\partial^2 V}{\partial x^2} - V \frac{\partial^2 y}{\partial x^2};$$

$$\frac{\partial^2 y}{\partial t^2} = - \frac{\partial V}{\partial x} \frac{\partial y}{\partial t} - \frac{A}{B^2} \frac{\partial B}{\partial t} - \frac{A}{B} \frac{\partial^2 V}{\partial x \partial t} - V \frac{\partial^2 y}{\partial t^2} - \frac{\partial V}{\partial t} \frac{\partial y}{\partial t};$$

$$\frac{\partial^2 V}{\partial x \partial t} = - \frac{\alpha}{\beta} (\frac{\partial V}{\partial x} \frac{\partial V}{\partial x} + V \frac{\partial^2 V}{\partial x^2}) - \frac{g}{\beta} \frac{\partial^2 y}{\partial x^2} - \frac{\alpha}{\beta} \frac{f}{8} (\frac{2RV \frac{\partial V}{\partial t}}{R^2} - V^2 \frac{\partial R}{\partial x});$$

and

$$\frac{\partial^2 V}{\partial t^2} = - \frac{\alpha}{\beta} (\frac{\partial V}{\partial x} \frac{\partial V}{\partial t} + V \frac{\partial^2 V}{\partial x \partial t}) - \frac{g}{\beta} \frac{\partial^2 y}{\partial x \partial t} - \frac{\alpha}{\beta} \frac{f}{8} (\frac{2RV \frac{\partial V}{\partial t}}{R^2} - V^2 \frac{\partial R}{\partial t}).$$

2.5 Comparison of Solutions by the Two Schemes

Comparing the solutions of both water depth and water velocity at various times and distances would be redundant. Since the analytical and physical waves will be compared by their water depths at a given position, solutions of y alone are considered. In this analysis, comparison is made for the theoretical dimensions of the experimental conduit, approximately 3 feet in diameter and 822 feet long. In the subsequent plots of these solutions of y let A_w be the solutions with all the derivative terms, and A_{wo} be the solutions without the terms consisting of the product of the first order derivatives and the derivatives of the energy slope, and D the solutions based on the diffusing scheme.

An important criterion of any numerical solution is the ability to repeat the values of y given at the initial conditions as best as possible over a period of time under a constant discharge. Under this steady flow, a critical x position is that which is near the downstream end of the pipe. Figure 2.2 shows the plots of y versus t at $x = 796.7$ ft using the Lax-Wendroff Scheme developed in the previous subchapter, and the method based on the

diffusing scheme. In these two methods the total number n of x intervals used was 160, or $\Delta x = L/n = 822/160$. It is to be noted that after 175 seconds the maximum drops are about 0.01 and 0.07 ft for A_w and D_1 schemes, respectively.

Another important criterion in a numerical solution is stability. Paraphrasing material from the Journal of Mathematics and Physics [6] stability is related to the difference between the exact solution of the difference equations and the numerical solution of these equations. This difference may be called the round-off error. In the Journal stability is defined in terms of the growth of round-off errors. That is, strong stability exists if the over-all error due to round-off errors does not grow, and weak stability exists if single round-off errors do not grow. Strong and weak instability occurs if neither of the above is true. Also stated is the assumption that weak stability implies strong stability. Thus, stability is a measure of error propagation.

The first series of tests studying the measure of error propagation was that of strong stability under a constant discharge or steady flow. That is, for both the Lax-Wendroff method and the method based on the diffusing scheme, an error of 0.001 feet was added to the initial condition at each x partition point. Simultaneously, these schemes were run over a period of time using the correct initial conditions, and these same conditions, plus the induced error were used as the starting lines. In both cases the induced error did not grow but approached zero with the developed scheme tending to zero at a faster rate.

Some effects were observed in the second series of tests with reference to weak stability, as the induced error was added only to the middle partition point. Using 81 partition points and observing the solutions of y at $x = 4n - 3$ and $t = 2n - 1$, it was found that the developed solution took 225.3 seconds to zero out to five decimal places, and the diffusing scheme took 520.9 seconds.

Of more importance in the matter of stability is the third series of tests studied. This time the constant discharge input hydrograph was replaced by a varying hypothetical input hydrograph. An error of 0.001 feet was added to the initial conditions at the 81st point of a total of 160 partition points in both the Lax-Wendroff scheme and the diffusing scheme. The solutions of y for the same t and x partition points were the same as those observed for the second series of tests. After 180.9 seconds the error at point $i = 5$ was 0.00001, and the error at the other points has zeroed out to 5 decimal places using the Lax-Wendroff scheme. The diffusing scheme solutions did not show an induced error growth either; this time the error did not stop at zero but became negative.

Thus, these series of tests indicate that both the diffusing scheme and the Lax-Wendroff scheme are stable with the latter showing the greater stability.

The next consideration regarding comparisons of solutions using the hypothetical flood input hydrograph, is that of the effect of interval size. In both the Lax-Wendroff scheme and the diffusing scheme $\Delta t = \Delta x/4z$, where z is the initial discharge divided by the initial area (A). This is done to insure that Δt will be small enough to fall within the domain of dependence. Figure 2.3 shows the plots

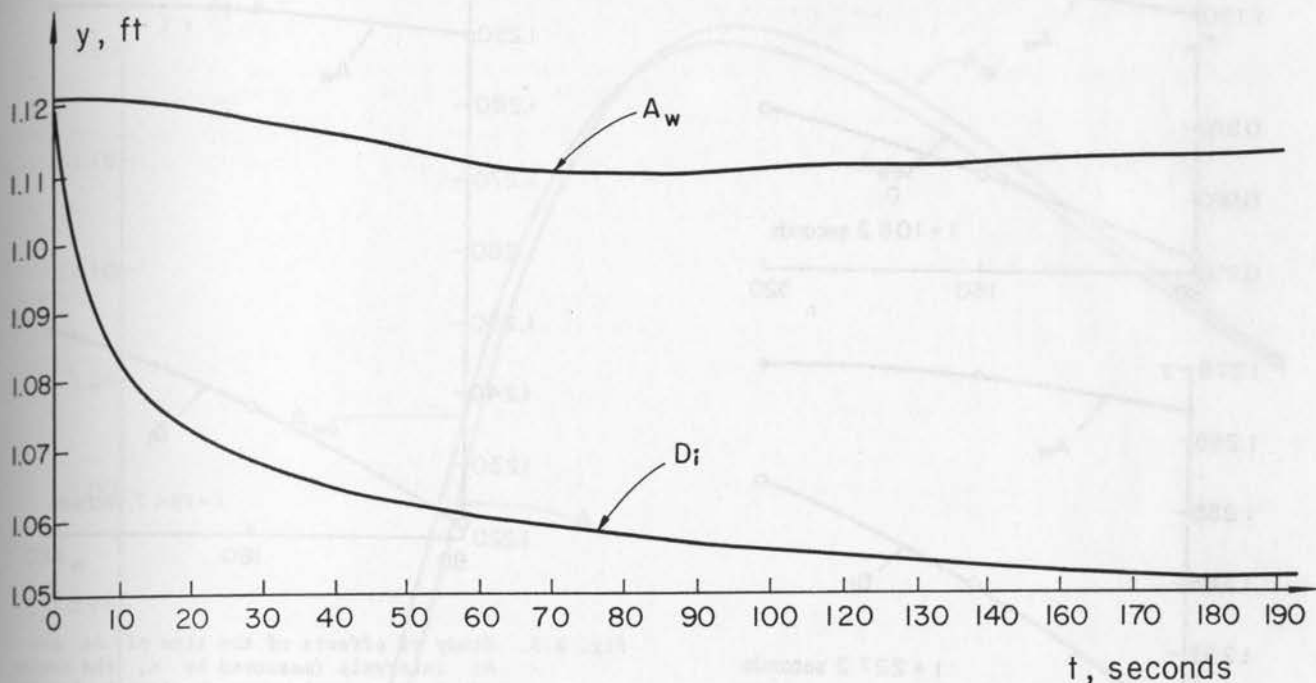


Fig. 2.2. Comparison of Lax-Wendroff scheme (A_w) and the diffusing scheme (D_i) in reproducing the steady initial conditions along the conduit, at the distance $x = 796.7$ ft.

of y in feet at $x = 735.8$ ft versus the number n of Δx intervals used ($n = 80$, $n = 160$, and $n = 320$) for both schemes and for three different times. The entire length of 822 ft of the conduit was divided by n to obtain the corresponding Δx . From top to bottom in Fig. 2.3, the given times t represent y rising (upper graph), y near maximum (central graph), and y falling (lower graph). The effects of the size of the Δx intervals are noticeable, and, thus, the corresponding size of Δt intervals are also noticeable, when comparing the diffusing scheme to the Lax-Wendroff scheme. Since the error in the Taylor series expansion is on the order of $(\Delta t)^3$, in which Δt is a function of Δx , the difference in y due to different Δx sizes is not as profound in the Lax-Wendroff scheme solutions as in the diffusing scheme. Figure 2.3 also shows the underestimation by the diffusing scheme similarly shown before in Fig. 2.2 in the study of ability of this scheme to repeat the initial condition under a constant input discharge.

The last consideration in this comparison of solutions involves the Lax-Wendroff scheme but with the assumption (A_{w0}), or without this assumption (A_w), that all products of first-order partial derivatives and any derivative of S_f are negligible.

Using the same hypothetical input hydrograph, Figs. 2.4 and 2.5 show plots of the depth y versus time t at positions $x = 409.1$ ft, and $x = 797.8$ ft, respectively. These figures give the comparisons of results for the developed Lax-Wendroff scheme (A_w) and the simplified scheme with the above assumption (A_{w0}). The difference occurs in the computed hydrographs when the first-order partial derivatives are

such that the assumption becomes less valid. That is for example, $\partial y / \partial t$ is negligible only until the computed water wave reaches a particular x position and causes an increase in y .

2.6 Concluding Remarks

Among the finite-difference schemes, the Lax-Wendroff scheme is considered as superior not only to the diffusing scheme but to all others investigated for the purpose of flood routing through storm drains under the conditions of application of Eqs. 1.1 and 1.2. Taking into account all six schemes, either discussed briefly or analyzed, it is concluded that the Lax-Wendroff scheme is an optimal scheme between the accuracy in the results produced and the computer time necessary for the corresponding numerical solutions. It is, therefore, considered as the feasible numerical computational scheme whenever a gradually varied free-surface unsteady flow is computed directly by numerically integrating the two partial differential equations stated in Chapter 1 as Eqs. 1.1 and 1.2.

For benefit to other investigators and users, the computational procedures and programs are reproduced here in the two appendices.

Appendix 1 gives the computation details of the diffusing scheme and Appendix 2 gives the computation details of the Lax-Wendroff scheme. Each appendix contains the following items, (1) Flow chart; (2) Computer program, (3) Definition of variables; this gives the conversion table between the mathematical symbols used in this paper and the symbols used in Fortran language for a CDC 6600 or CDC 6400 digital computer; and (4) Sample input and output.

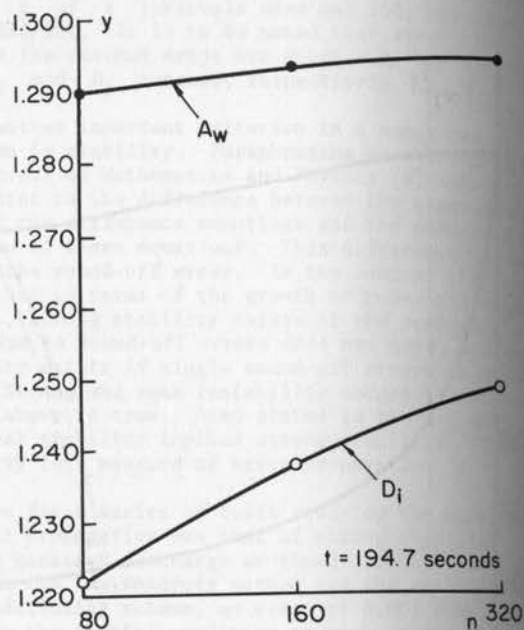
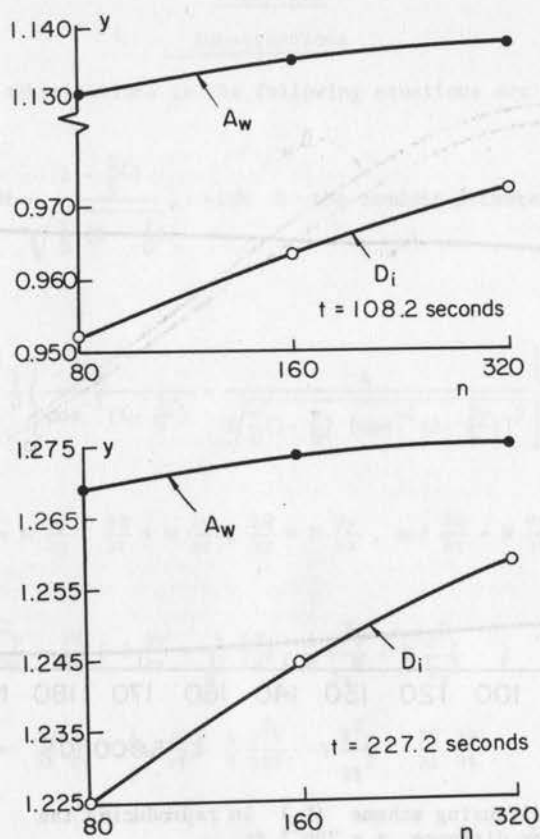


Fig. 2.3. Study of effects of the size of Δx and Δt intervals (measured by n , the number of Δx intervals over the length $L = 822$ ft), on the predicted depth y at $x = 735.8$ ft.

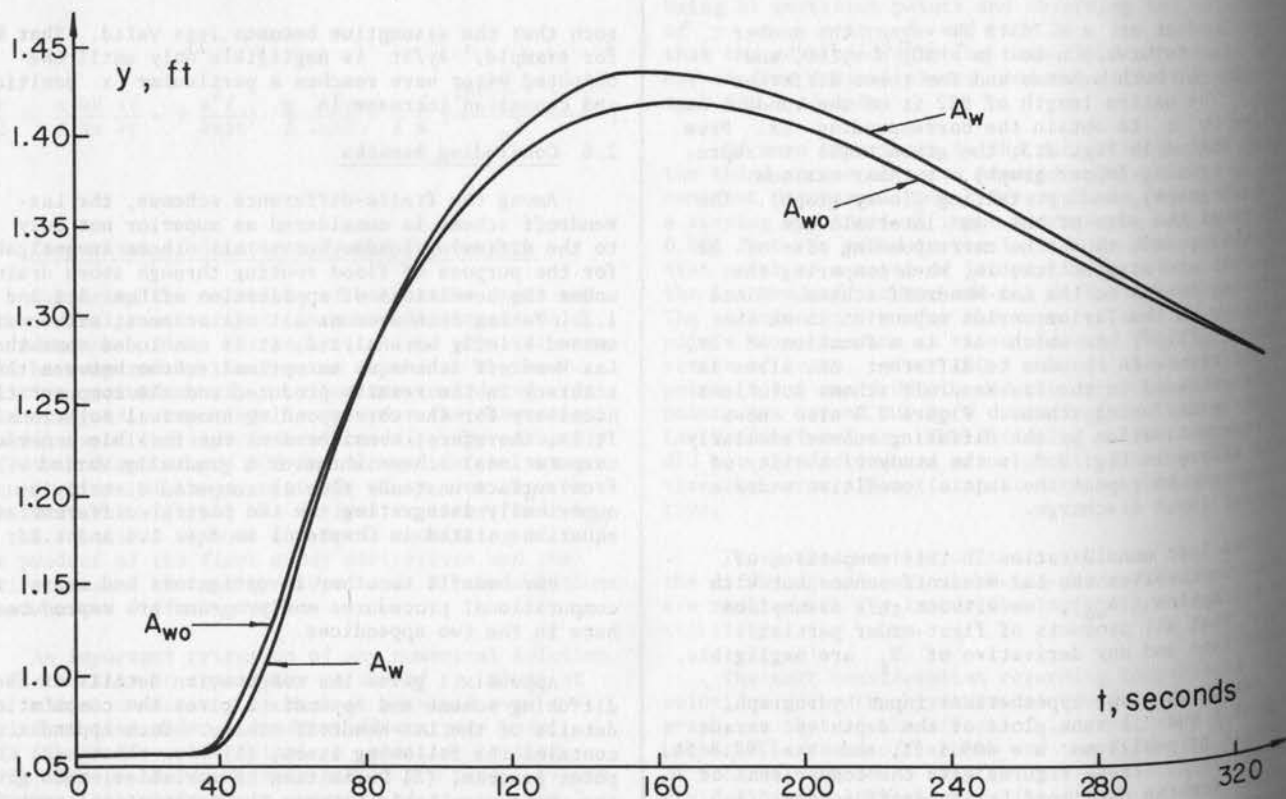


Fig. 2.4. Comparison of the hydrographs at the position $x = 409.1$ computed with the Lax-Wendroff scheme without the assumption (A_w) and with the assumption (A_{wo}) of products of partial derivatives or the derivatives of S_f in x and t being negligible.

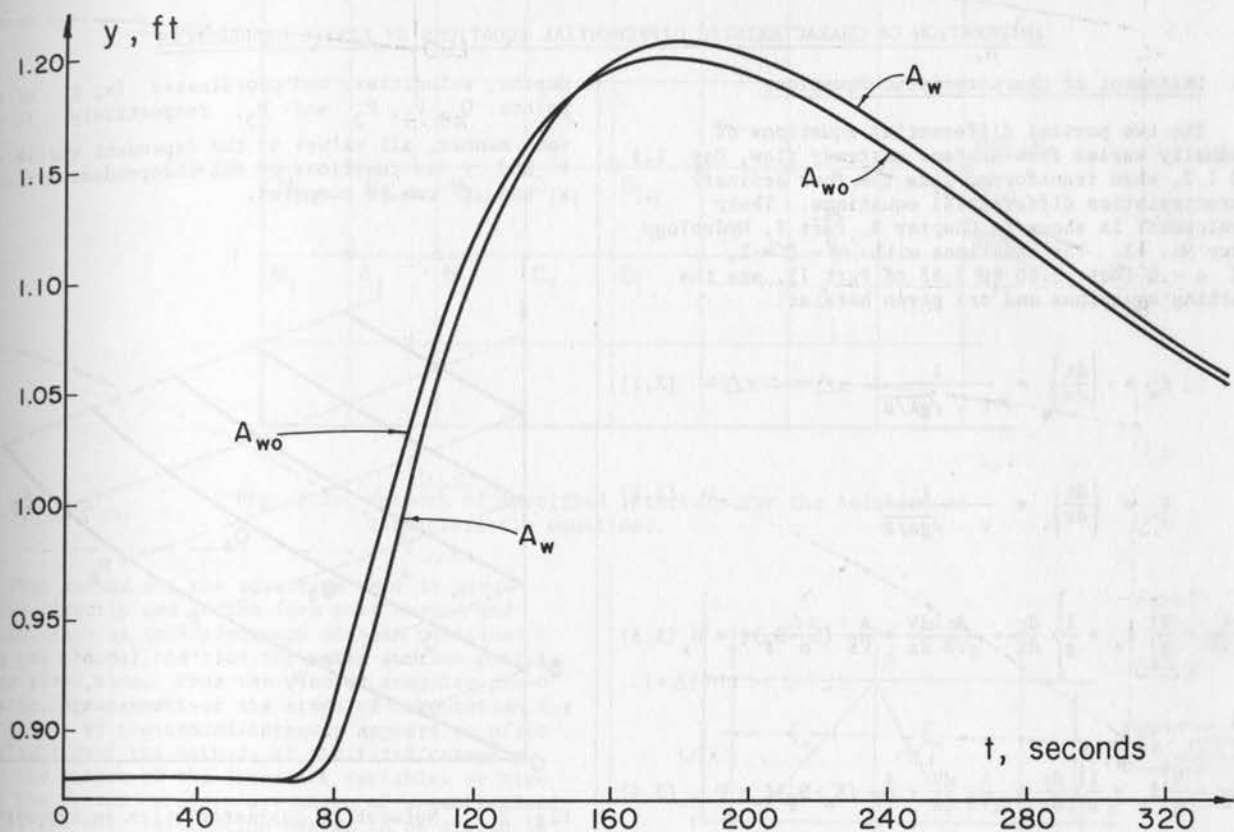


Fig. 2.5. The same comparison as in Fig. 2.4, except at the position $x = 797.8$ ft.

INTEGRATION OF CHARACTERISTIC DIFFERENTIAL EQUATIONS BY FINITE DIFFERENCES

3.1 Statement of Characteristic Equations

The two partial differential equations of gradually varied free-surface unsteady flow, Eqs. 1.1 and 1.2, when transformed give the four ordinary characteristics differential equations. Their development is shown in Chapter 3, Part I, Hydrology Paper No. 43. The equations with $\alpha = \beta = 1$, and $q = 0$ (Eqs. 3.50 to 3.53 of Part I), are the starting equations and are given here as:

$$\xi_+ = \left(\frac{dt}{dx} \right)_+ = \frac{1}{V + \sqrt{gA/B}}, \quad (3.1)$$

$$\xi_- = \left(\frac{dt}{dx} \right)_- = \frac{1}{V - \sqrt{gA/B}}, \quad (3.2)$$

$$\left\{ \left(\frac{A}{VB} - \frac{V}{g} \right) \xi_+ + \frac{1}{g} \right\} \frac{dy}{dx} + \frac{A}{gVB} \frac{dV}{dx} + \frac{A}{VB} (S_0 - S_f) \xi_+ = 0, \quad (3.3)$$

and

$$\left\{ \left(\frac{A}{VB} - \frac{V}{g} \right) \xi_- + \frac{1}{g} \right\} \frac{dy}{dx} + \frac{A}{gVB} \frac{dV}{dx} + \frac{A}{VB} (S_0 - S_f) \xi_- = 0. \quad (3.4)$$

These four dependent equations form the basis for numerical solutions in the method of characteristics. There are a variety of procedures that may be used and these procedures may be broadly divided into two categories, the grid system and the specified intervals system.

3.2 Various Schemes

The first category uses the grid system generated by the intersecting characteristics curves in the time-distance plane. In this case, solutions to the problem are made at the intersections. These intersections occur at the nonuniform spacings in both x and t directions, thus, interpolations are required in order to develop time or distance relations. These relations are commonly referred to as the Lagrangian description for the distance relations at an instant of time, and the Eulerian description for the time relations at a fixed position. This method of using grids of characteristics is based on establishing the initial characteristic curves from the initial conditions. The receding characteristic curves emanate from it. In Fig. 3.1 the initial characteristic curve ξ_0 , first determined from the inflow hydrograph and the initial steady conditions, is drawn from $x = 0$ and $t = 0$. By introducing the values of the dependent variables V and y along the initial characteristic curve ξ_0 , at the appropriate points in the computational scheme, the values of V and y as functions of the independent variables x and t are obtained at successive points. For example, the values of the depths and velocities at points Q_1 , Q_2 and Q_3 in Fig. 3.1 are obtained from the values of

depths, velocities, and coordinates (x, t) of the points Q_0 , P_1 , P_2 and P_3 , respectively. In the same manner, all values of the dependent variables V and y as functions of the independent variables x and t can be computed.

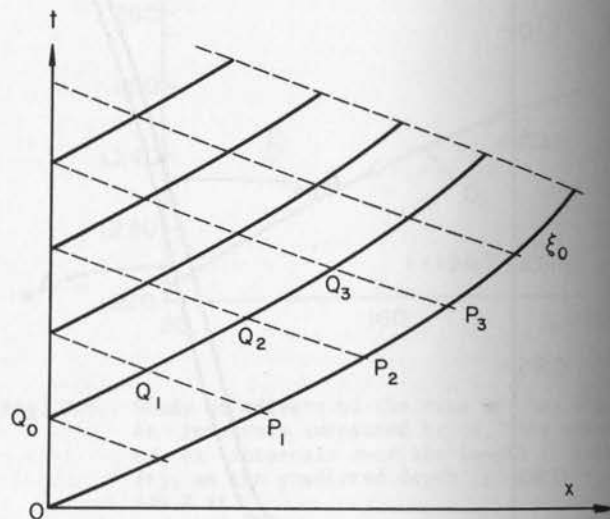


Fig. 3.1. Network of characteristics in the method of grid system for the solution of unsteady flow equations.

It is evident from the preceding brief description that the values in the solution at each intersection of characteristics must be retained in the computer for the later interpolation for fixed times and positions. No attempt was made in this study to use the method of characteristics curves. The principal reason was the need for excessive computer storage of solutions at each intersection.

The second category is the specified intervals system for independent variables. In this approach, the dependent variables V and y are known functions of the independent variables x and t either as initial conditions of $t = 0$ or as the results of previous time computations. For example, it is assumed that V and y are known along distance x at time t . Figure 3.2 represents the rectangular grid in the (x, t) -plane with intervals Δx and Δt in x and t coordinates, respectively. In this case, V and y at points M_j , A_j , $B_j, \dots, N_j$ are known. The values of V and y at time $t + \Delta t$, and particularly at points M_{j+1} , A_{j+1} , $B_{j+1}, \dots, N_{j+1}$, can then be computed from equations 3.1 through 3.4 and from the boundary conditions. In this manner, V and y at time $t + \Delta t$ at various points along distance x can also be computed. This process can be continued as far as desired or meaningful. This method was selected and used in this study because the values of x and t at points M_{j+1} , A_{j+1} , $B_{j+1}, \dots, N_{j+1}$ are exactly known, and only the values of V and y at these points must be determined.

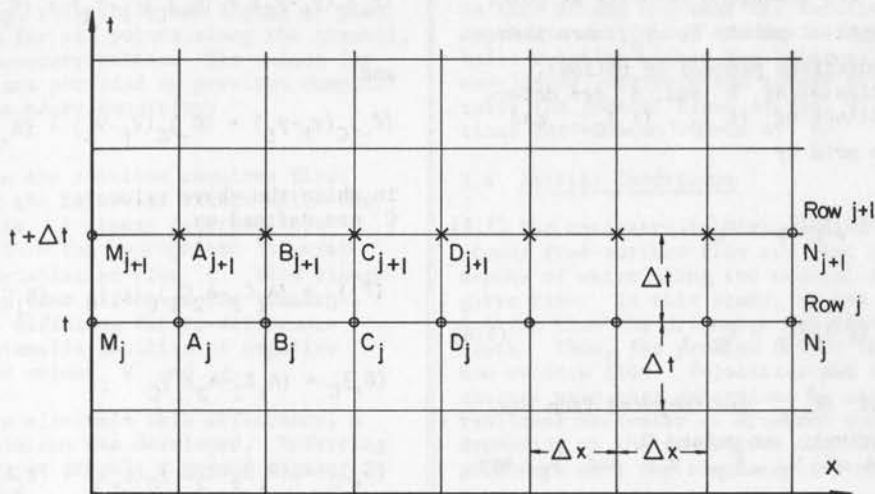


Fig. 3.2. Network of specified intervals for the solution of characteristic equations.

This method has the advantage that it gives results directly and in the form most needed and useable, such as the hydrograph at each position along the channel and also the water surface profile at any given time. From the view of computer programming, arrangement of the steps of computation for the methods of the second category appears to offer advantages over the methods of the first category. Since the values of the dependent variables at time t in the second category are known at predetermined points, the only information needed to be stored in the computer is the values of the dependent variables at time $t + \Delta t$. Therefore, this category needs computer storage of only two time lines as indicated in Fig. 3.2 and designated by j and $j+1$ rows, respectively. Values of the dependent variables V and y of row j are known and stored while the values of V and y of row $j+1$ are being computed for the next time interval. After completion of this time interval, the values of V and y of row $j+1$ are stored for computation at the next time interval; the values of V and y of row j are then printed out and the storage space is replaced by the values of row $j+1$.

3.3 Numerical Solution by the Specified Intervals System

This section discusses the numerical solution of the equations of free-surface unsteady flow by the method of characteristics with the specified time interval, Δt , and the specified distance interval Δx . In this method, V and y at point P on the (x, t) -plane of Fig. 3.3 are to be computed from the initial conditions or from previous values of V and y at points A , B , and C using two assumptions:

- Δt is sufficiently small so that the parts of the characteristics between P and R and between P and S may be considered as straight lines, and
- The slope of the straight line PR at point P is the positive characteristic direction of the position C , $(\xi_+)_C$, and the slope of the straight line PS at point P is the negative characteristic direction of the position C , $(\xi_-)_C$.

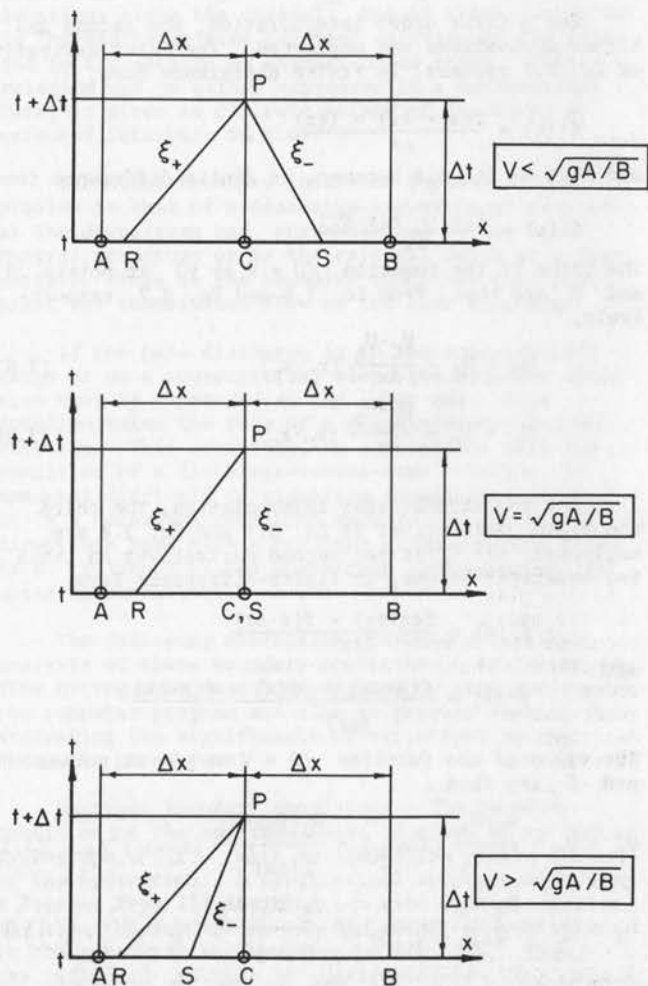


Fig. 3.3. Rectangular grid for the solution by the system of specified intervals, Δt and Δx : subcritical flow (upper graph), critical flow (center graph), and supercritical flow (lower graph).

Since x_p and t_p are known, the velocity at point P, V_p , and the depth at point P, y_p , are then computed. The computations proceed as follows.

(1) The coordinates of R and S are determined from the relations of $(\xi_+)_C$, $(\xi_-)_C$, and the geometry of the grid by

$$t_p - t_R = (\xi_+)_C (x_p - x_R), \quad (3.5)$$

and

$$t_p - t_S = (\xi_-)_C (x_p - x_S), \quad (3.6)$$

in which $(\xi_+)_C$ and $(\xi_-)_C$ are computed from Eqs. 3.1 and 3.2, respectively, at point C.

(2) The values of V_R , V_S , y_R , and y_S are determined by interpolation from the Taylor expansion, with h the symbol of either Δx or Δh , as

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \dots + 0(h^n), \quad (3.7)$$

$$\text{and} \\ f(x-h) = f(x) - hf'(x) + \frac{h^2}{2!} f''(x) + \dots + 0(h^n), \quad (3.8)$$

For a first order interpolation, the second and higher derivatives are neglected. The first derivative of Eq. 3.7 becomes, in finite difference form,

$$f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x},$$

and that of Eq. 3.8 becomes, in finite-difference form,

$$f'(x) = \frac{f(x) - f(x - \Delta x)}{\Delta x}.$$

The value of the function ($U = V$ or y) at points R and S are then, from Eq. 3.8 and Eq. 3.7, respectively,

$$U_R = U_C - \frac{U_C - U_A}{\Delta x} (x_C - x_R) \quad (3.9)$$

$$U_S = U_C + \frac{U_C - U_B}{\Delta x} (x_C - x_S) \quad (3.10)$$

For the second order interpolation, the third and higher derivatives of Eq. 3.7 and Eq. 3.8 are neglected, the first and second derivatives in these two equations become, in finite-difference form,

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

and

$$f''(x) = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{(\Delta x)^2}$$

The value of the function ($U = V$ or y) at points R and S are then

$$U_R = U_C - \frac{U_B - U_A}{2\Delta x} (x_C - x_R) + \frac{U_B - 2U_C + U_A}{2(\Delta x)^2} (x_C - x_R)^2, \quad (3.11)$$

$$U_S = U_C - \frac{U_B - U_A}{2\Delta x} (x_C - x_S) + \frac{U_B - 2U_C + U_A}{2(\Delta x)^2} (x_C - x_S)^2, \quad (3.12)$$

from which V_R , V_S , y_R and y_S may be computed knowing the V and y at points A, C, and B.

(3) Then V_p and y_p are obtained by solving simultaneously the finite-difference forms of Eqs. 3.3 and 3.4, or by

$$(F_+)_C (y_p - y_R) + (G_+)_C (V_p - V_R) + (S_+)_C (x_p - x_R) = 0 \quad (3.13)$$

and

$$(F_-)_C (y_p - y_S) + (G_-)_C (V_p - V_S) + (S_-)_C (x_p - x_S) = 0 \quad (3.14)$$

in which the above values of F , G , and S at point C are defined as

$$(F_+)_C = (A_1 C_2 - A_2 C_1)_C (\xi_+)_C - (B_1 C_2 - B_2 C_1)_C;$$

$$(G_+)_C = (A_1 B_2 - A_2 B_1)_C;$$

$$(S_+)_C = (A_1 E_2 - A_2 E_1)_C (\xi_+)_C - (B_1 A_2 - B_2 A_1)_C;$$

$$(F_-)_C = (A_1 C_2 - A_2 C_1)_C (\xi_-)_C - (B_1 C_2 - B_2 C_1)_C;$$

$$(G_-)_C = (A_1 B_2 - A_2 B_1)_C, \quad \text{and}$$

$$(S_-)_C = (A_1 E_2 - A_2 E_1)_C (\xi_-)_C - (B_1 E_2 - B_2 E_1)_C,$$

in which the above coefficients of the two general partial differential equations (Eqs. 3.24 and 3.25, Part I, Hydrology Paper No. 43) are: $A_1 = A/VB$, $A_2 = V/g$, $B_1 = 0$, $B_2 = 1/g$, $C_1 = C_2 = 1$, $D_1 = 1/V$, $D_2 = 0$, $E_1 = 0$, and $E_2 = S_f - S_0$. Solving equations 3.13 and 3.14 simultaneously,

$$y_p = \frac{\begin{vmatrix} (T_+)_C & (G_+)_C \\ (T_-)_C & (G_-)_C \end{vmatrix}}{\begin{vmatrix} (F_+)_C & (G_+)_C \\ (F_-)_C & (G_-)_C \end{vmatrix}} \quad (3.15)$$

and

$$V_p = \frac{\begin{vmatrix} (F_+)_C & (T_+)_C \\ (F_-)_C & (T_-)_C \end{vmatrix}}{\begin{vmatrix} (F_+)_C & (G_+)_C \\ (F_-)_C & (G_-)_C \end{vmatrix}} \quad (3.16)$$

in which

$$(T_+)_C = (F_+)_C y_R + (G_+)_C V_R - (S_+)_C (x_p - x_R), \quad (3.17)$$

and

$$(T_-)_C = (F_-)_C y_S + (G_-)_C V_S - (S_-)_C (x_p - x_S). \quad (3.18)$$

By these computations, velocities and depths at time $t + \Delta t$ are obtained for all points along the channel, except for the two boundary points. The values for the boundary points are provided by previous computations of the known boundary conditions.

The procedure in the solution requires first the determination of the intervals within which the points R and S lie. A linear interpolation is then performed within the appropriate interval for the dependent variables at time t . This linear interpolation has the same effect as the linear interpolation in the diffusing finite-difference scheme, namely a systematic positive or negative shift in the computed values V and y .

In an attempt to eliminate this deficiency, a second-order interpolation was developed. Referring again to Fig. 3.3 (upper graph), a second-degree polynomial of the form

$$U = a + bx + cx^2 \quad (3.19)$$

is assumed to fit the function of V and y through points A, C, and B. This is the same interpolation as in Eqs. 3.9 and 3.10, except in a different way of implementing it. If the function is centered on the location of C, then the constants are

$$a = U_C, \quad b = \frac{U_B - U_A}{2\Delta x}, \quad \text{and} \quad c = \frac{U_B - 2U_C + U_A}{2\Delta x^2} \quad (3.20)$$

Thus, the value of the function of the location of R is

$$U_R = U_C - \frac{1}{2}(UP)(U_B - U_A) + \frac{1}{2}(UP)^2(U_B - 2U_C + U_A) \quad (3.21)$$

in which

$$UP = - \frac{\Delta t}{\Delta x} \left/ \frac{dt}{dx} \right|_+ \quad (3.22)$$

The ratio of Δt to Δx is the selected grid mesh ratio and $(dt/dx)_+$ is the direction of the positive characteristic estimated from the conditions at location C.

Similarly, the value of the function at location S is

$$U_S = U_C - \frac{1}{2}(UN)(U_B - U_A) + \frac{1}{2}(UN)^2(U_B - 2U_C + U_A) \quad (3.23)$$

in which

$$UN = - \frac{\Delta t}{\Delta x} \left/ \frac{dt}{dx} \right|_- \quad (3.24)$$

This interpolation scheme offers two advantages. First, the curvature of the function at a given time is approximated. Second, it is not necessary to compute within which interval the intersection of the characteristic and the x -axis falls. The assumptions

in this scheme are that the functions of velocity and depth are continuous and may be approximated by a parabolic relation within the interval. Any other similar non-linear interpolation scheme may be designed if it suits the general types of the $V(x)$ and $y(x)$ functions for various values of t .

3.4 Initial Conditions

The necessary initial conditions for the unsteady free-surface flow are that all velocities and depths of water along the channel must be known at a given time. In this study, it was assumed that at the initial time the discharge was constant throughout the reach. Thus, the problem can be treated as a steady non-uniform flow. Velocities and depths along the channel were then determined by computations of conventional backwater or drawdown surface profiles, depending on the downstream control conditions. This procedure uses the standard step method [2, p. 265].

3.5 Boundary Conditions

The two governing partial differential equations for unsteady flow require two independent boundary conditions relating velocity and depth at certain locations along the channel. One of these conditions is the discharge-time relation existing at the inlet end to the section of channel under study. This relation can be either expressed in a mathematical form, or given as discrete points of discharge at selected intervals of time.

The other boundary condition imposed on the problem is that of a discharge-versus-depth relation at the downstream end, characterized either by a control structure or by the critical depth at a free outfall. This is the boundary condition that must exist for subcritical flow of the base discharge.

If the base discharge is in the supercritical range or on a supercritical slope the boundary condition must be expressed at the inlet end. This function takes the form of a discharge-versus-depth relation. This condition, in combination with the condition of a discharge-versus-time relation, is somewhat difficult to visualize physically; however, it is a necessary condition because the characteristic directions both have a positive slope and thus there is no influence of the downstream conditions on the upstream conditions.

The following discussion presents a detailed analysis of these boundary conditions. Arbitrary inflow hydrographs were investigated to test and verify the computer program and also to provide results for evaluating the significance of variations in the hydraulic parameters.

Upstream boundary conditions - The boundary condition at the upstream inlet is given by an inflow hydrograph, $Q(t)$, with no limitation on the shape of the hydrograph. A hypothetical hydrograph, having a Pearson Type III distribution with four parameters, was selected for evaluating the effect of variations in the parameter and is shown by Fig. 3.4. Thus, the inflow Q at time t designated by $Q(t)$ may be described by

$$Q(t) = Q_b + Q_0 e^{-\frac{(t-t_p)}{(t_g-t_p)} \frac{t}{(t_g-t_p)}}, \quad (3.25)$$

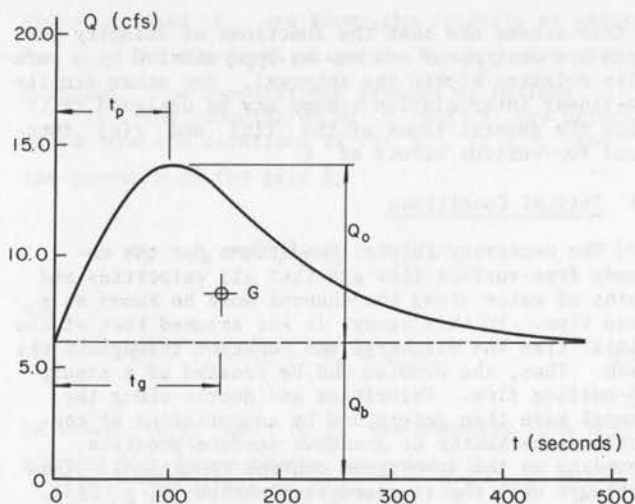


Fig. 3.4. Hypothetical inflow hydrograph of the Pearson Type III function, Eq. 3.25, with the selected parameters: $Q_b = 6.21$ cfs, $Q_o = 8.00$ cfs, $t_p = 100.00$ sec, and $t_g = 150.0$ sec.

in which Q_b is the constant base flow, Q_o is the peak flow, t_p is the time from the beginning of storm runoff to peak discharge and t_g is the time from the beginning of the storm runoff to the center of mass of storm runoff, G . One hydrograph with arbitrary values of Q_b , Q_o , t_p , and t_g were used in this study. The shape and these arbitrary values of parameters are shown in Fig. 3.4.

The depth and the velocity at the upstream boundary point P in Fig. 3.5, which is at $x = 0$ and at the time $t + \Delta t$, can be computed from initial conditions at C and B , with the boundary conditions given by the inflow hydrograph

$$AV = Q(t) \quad (3.26)$$

in which A is the cross-sectional area and V is the velocity at P .

Using the previously discussed assumptions and procedure of computing velocities and depths at other points along the channel the negative characteristic direction at point C is also given by the initial conditions. The relation between the depth y_p and velocity V_p at point P can be determined from Eq. 3.4. Substituting the boundary condition of Eq. 3.26 into Eq. 3.14 gives

$$y_p = y_s - \frac{(G_-)_C \left(\frac{Q(t)}{A} - V_s \right) + (S_-)_C (x_p - x_s)}{(F_-)_C} \quad (3.27)$$

in which A is the cross-sectional area at P and A is a function of y_p .

Solving for y_p from Eq. 3.27 and substituting y_p into Eq. 3.26 makes it possible to determine V_p . Since Eq. 3.27 is not linear in y_p , a Newton-Raphson iteration was used for its solution.

Downstream boundary conditions - The boundary conditions at the downstream outlet may generally be

given by a stage-discharge relation. In this portion of the study only a free outfall at the end of conduit was assumed. Therefore, a critical flow at the downstream end exists

$$\frac{V}{\sqrt{gA/B}} = 1 \quad (3.28)$$

where A is the cross-sectional area and B is the top width of the downstream boundary.

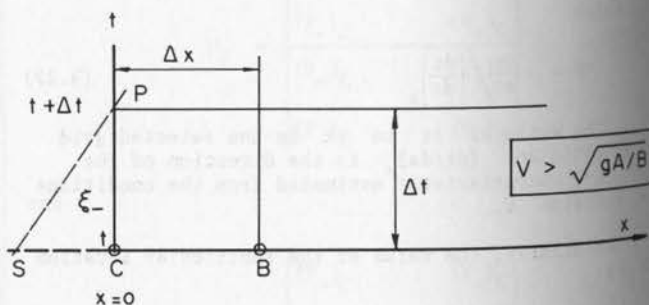
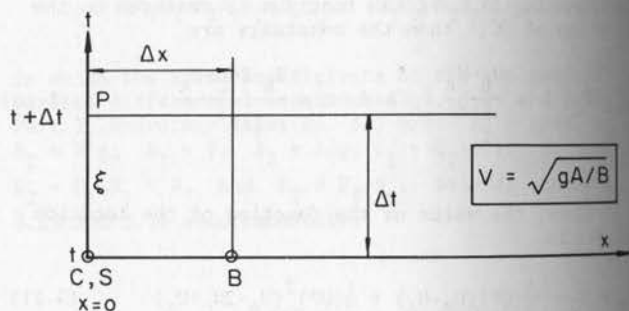
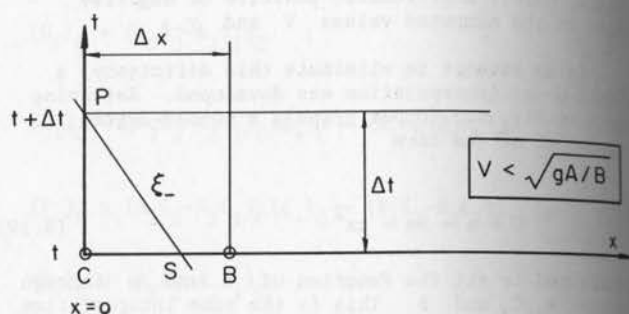


Fig. 3.5. Upstream boundary conditions: subcritical flow (upper graph), critical flow (central graph), and supercritical flow (lower graph).

Figure 3.6 shows the downstream boundary where the critical depth occurs. For the free outfall, it was assumed that critical depth occurred at a distance of 4.5 times the critical depth from the end. This assumption was also applied to the unsteady case, with critical depth computed from the base discharge, Q_b . Therefore, the total distance x_L from the inlet to the downstream boundary is determined by

$$x_L = x_F - 4.5 y_C \quad (3.29)$$

in which x_F is the total length of the channel and y_C is the critical depth for discharge Q_b .

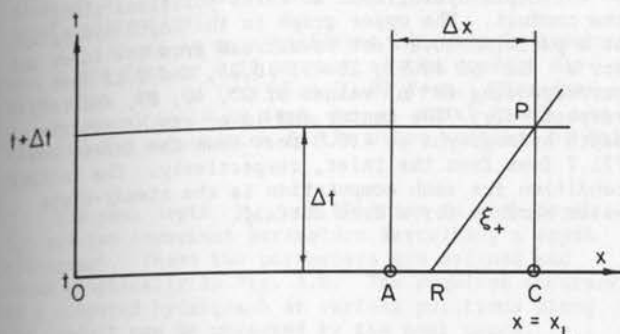


Fig. 3.6. Downstream boundary conditions for the subcritical flow, with x_L the computational conduit length.

The depth and velocity at the downstream boundary point P at time $t + \Delta t$ can be computed from the initial conditions at A and C, and from the boundary conditions given by Eq. 3.28.

Using the same assumptions and computational procedures, the initial conditions also give the relation between the depth y_P and the velocity V_P by applying Eq. 3.3. Substituting the boundary conditions of Eq. 3.28 into Eq. 3.13 results in

$$y_P = y_R - \frac{(G_+)_C(\sqrt{gA/B} - V_R) + (S_+)_C(x_P - x_R)}{(F_+)_C} \quad (3.30)$$

in which A is the cross-sectional area and B is the top width at P, with both A and B functions of y_P .

Solving y_P from Eq. 3.30 and substituting y_P into Eq. 3.16 makes it possible to determine V_P . Since Eq. 3.30 is not linear in y_P , a Newton-Raphson iteration was again used for a solution.

3.6 Summary of Computational Procedures

In solving the equations of free-surface unsteady flow, Eqs. 1.1 and 1.2 and Eqs. 3.1 and 3.4, by the system of specified intervals, the steps of computing velocity V and depth y at various times and positions along the conduit are as follows.

(1) Values of V and y at various positions along the channel for the steady-state condition of constant base flow, Q_b , are determined from a computation of the backwater curve.

(2) The upstream boundary conditions are evaluated.

(3) The downstream boundary conditions are evaluated.

(4) Values of V and y at time $t + \Delta t$ along the channel are computed from the known values of V and y at time t .

(5) Steps (2), (3), and (4) are repeated as long as desired or meaningful.

To benefit other investigators, the computational procedures and programs are reproduced in Appendix 3. Appendix 3 gives the computation details of the numerical integration method using the specified interval scheme of the method of characteristics. It includes (1) flow chart, (2) computer program, (3) definitions of variables and (4) sample input and output. Additional subroutines were developed to compute the boundary conditions for supercritical regime and for lateral inflow at specified locations.*

3.7 Effect of Variations in Computational Parameters

The discrepancy between a computed value and the observed value from a physical experiment is attributable to numerous sources of errors. These errors are generally the result of systematic and random errors in the observational system and possible systematic errors in computational procedures. Random errors are a result of unavoidable accidental variations in the physical systems. The discussion that follows will be concerned with errors in the computational procedure.

Computational errors emanating from procedures in this study are the result of:

(1) The approximation of infinitesimal variations by finite values. This is a result of assuming in general, linear relations rather than the true curvilinear relations. This is a systematic error. However, the propagation of this error is not readily determined since it may be positive or negative during different stages of the computations.

(2) Truncation of numerical values. This is due to the limited precision of any discrete-element calculator.

(3) Round off in the printed output. The printed output of any computed value from a digital computer differs from the internally generated value by the amount the value is rounded off in conversion to numeric form. The computer used for these calculations rounds off in a manner similar to manual calculators.

The following discussion evaluates the significance of the controllable variables in the solution of the unsteady flow equations. These equations are considered under the computational parameters of incremental length and incremental time interval during which the integration process proceeds.

The effect of variations in the hydraulic parameters of roughness and the velocity distribution coefficients is discussed in Part I, Hydrology Paper No. 43.

Determination of computational parameter Δt .

The grid sizes of Δx and Δt in the computational scheme, Fig. 3.2, is limited by the characteristic directions ξ_+ , ξ_- , encountered during the integration.

Referring to Fig. 3.3, in order for R to lie in the interval A-C for all conditions of flow, it is necessary that the ratio of $\Delta t/\Delta x$ be less than the value of dt/dx assumed at the location R. This condition must exist throughout the integration solution.

In order to assure that this condition exists, it is necessary that Δt be computed from

$$\Delta t = \Delta x / [V + \sqrt{gA/B}]$$

* Originals of all computer-program and punched-card decks are deposited with the Office of Research, Federal Highway Administration, U.S. Department of Transportation, Washington, D.C.

in which

- (1) V is the maximum anticipated velocity, and
- (2) A/B is a maximum for free surface flow.

Effect of computational parameter Δx . The method of characteristics using a specified intervals system gives the complete numerical solution of the free-surface unsteady flow. The accuracy of the results depends on the size of the rectangular grids Δx and Δt of Fig. 3.2. In this section only the effect of Δx is discussed; Δt will be discussed in the next section.

If n is the number of intervals along the conduit and x_L is the length of the conduit, then

$$\Delta x = \frac{x_L}{n} \quad (3.32)$$

Since x_L is assumed to be fixed, n is arbitrarily selected as any even number, thus Δx is determined. The smaller the Δx , presumably the more accurate are the results. But also, the smaller the Δx , the greater the required computing time. In compromising these two conditions to satisfy the objectives of this study, several values of n for the fixed x_L were tried.

Figure 3.7 shows the effect of the size of Δx on the depth hydrographs at three positions along the conduit. The upper graph is the depth hydrograph at a position 50.0 feet downstream from the inlet and for a Δx of 40.91, 20.45, 10.23, and 5.12 feet corresponding to n values of 20, 40, 80, and 160, respectively. The center and lower graphs are the depth hydrographs at 410.0 feet from the inlet, and 771.7 feet from the inlet, respectively. The initial condition for each computation is the steady-state water surface for a free outfall.

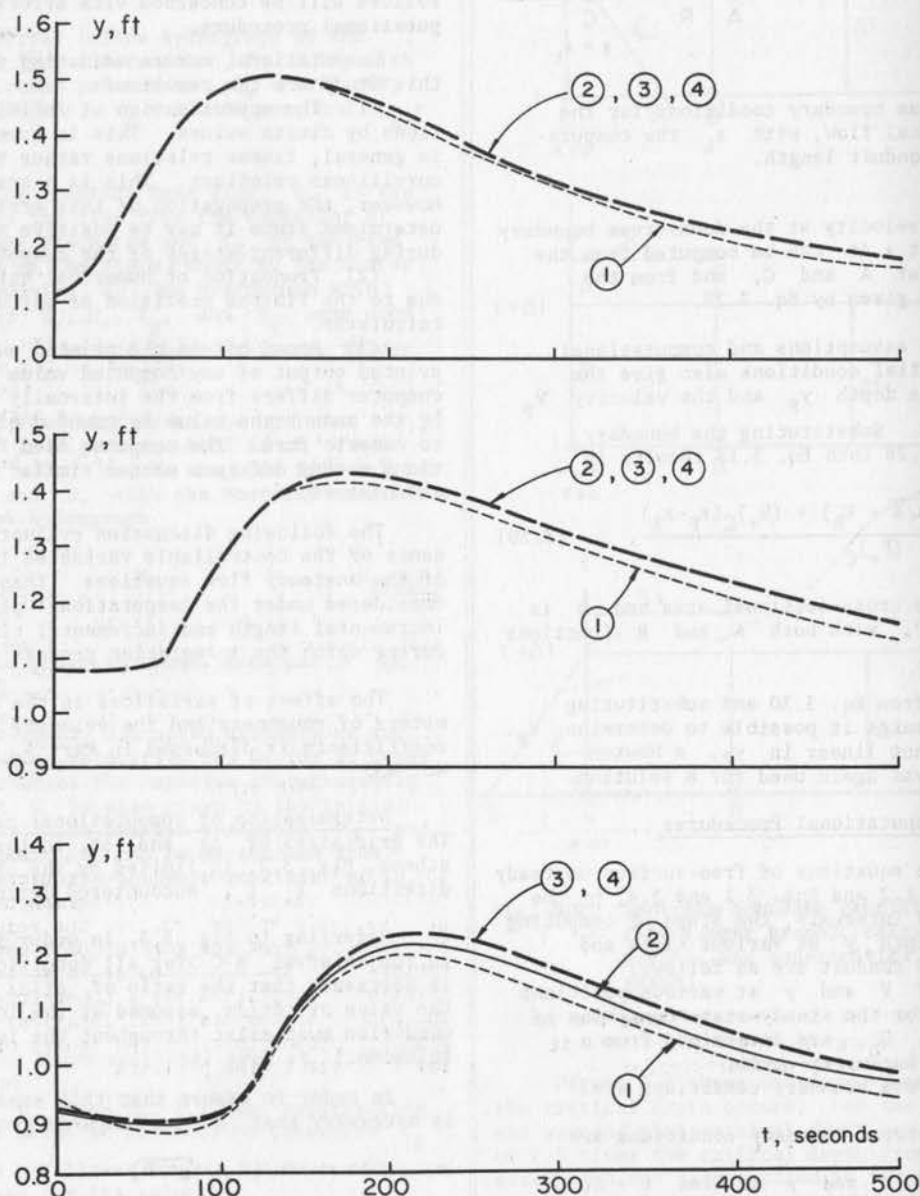


Fig. 3.7. Effect of Δx on hydrographs at various positions along the conduit; (1) $\Delta x = 40.91$ ft, (2) $\Delta x = 20.45$ ft, (3) $\Delta x = 10.23$ ft, and (4) $\Delta x = 5.12$ ft, at three locations of conduit $x = 50.0$ ft (upper graph), $x = 410.0$ ft (center graph) and $x = 771.7$ ft (lower graph).

Comparing the depth hydrographs of Fig. 3.7 with the given inflow discharge hydrograph of Fig. 3.4, it was found that:

(1) The critical portion of the conduit for computing depth hydrographs is near the outlet where there is the greatest curvature of the water surface profile. The maximum differences between the computed depths, with Δx being 40.91 and 5.12 feet, are approximately 0.3, 0.6, and 1.0 percent of the conduit diameter at 50.0, 410.0, and 771.7 feet from the inlet, respectively.

(2) There is no significant increase in accuracy over 0.005 feet or 0.15 percent of the conduit diameter when Δx is less than 10.23 feet. Therefore, a Δx equal to 10.23 feet, or n equal to 80, was selected for computation in the other portions of this study.

The peak depth y_p and the time to peak depth T_p are two important parameters describing a depth hydrograph. These two parameters are defined and shown graphically in Fig. 3.8. The required accuracy of a computed hydrograph at various positions along the conduit can be measured by the peak depth, y_p , relative to the diameter, D of the conduit, for various lengths Δx . Also, the accuracy can be measured by the time to peak depth, T_p , relative to the time to peak discharge, t_p , of the inflow discharge hydrograph of Fig. 3.4, for various lengths Δx and the same positions, x .

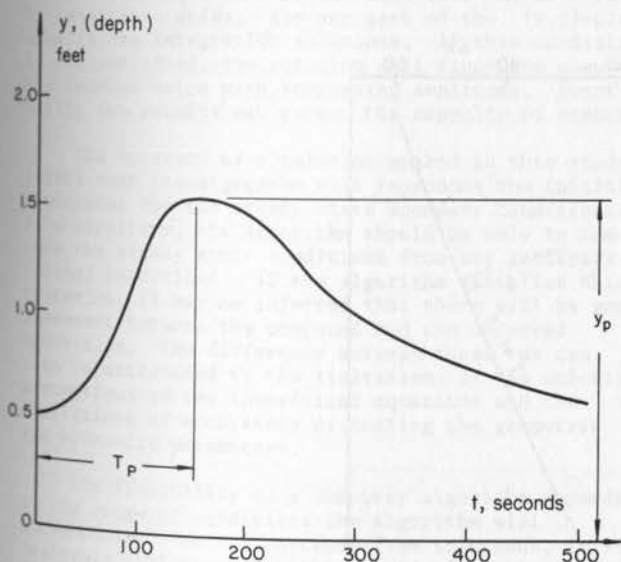


Fig. 3.8. Characteristics of the depth hydrograph with T_p the time at peak depth, and y_p the peak depth.

From the selected criteria for defining the accuracy of a computed hydrograph for a given Δx , it was found that the percentage differences of y_p ,

$$\frac{(y_p)_i - (y_p)_{\min}}{D} \times 100$$

in which the index "min" refers to the depth y_p of the smallest difference used, $\Delta x = 5.12$ ft, and the index "i" refers to depths of any other $\Delta x > 5.12$ ft, ranged from 0.0 percent to 2.1 percent for Δx ranging from 5.12 ft to 40.91 ft, and at various positions x , as shown in Table 3.1. At the upstream part of the conduit there was no significant difference between y_p/D measure for different values of Δx , as expected. At the approximate middle of the conduit there was a 0.2 percent difference. At the downstream end, the difference was 2.1 percent. No significant change in the percentage difference of y_p to D was found when Δx was reduced below 10.23 ft.

In using the other parameter, T_p , to define the accuracy of computed depth hydrographs with different values of Δx and various positions x , the measure of accuracy was

$$\frac{(T_p)_i - (T_p)_{\min}}{t_p} \times 100$$

in which the indices "min" and "i" refer to the $\Delta x = 5.12$ ft and all others Δx , respectively. It was found that there were no significant percentage differences for values $\Delta x > 5.12$ ft, and various positions x . The percentages were about 1.2 percent at the upstream, 2.0 percent at the middle, and 8.5 percent at the downstream part of the conduit. It was also found that there was no significant change of the percentages of T_p to t_p (which was about 1.9 percent) when Δx was reduced below 10.23 ft, as shown in Table 3.2.

Tables 3.1 and 3.2 show the percentage differences of y_p to the diameter D of the conduit, and T_p to t_p , respectively, with different values of Δx and various positions, x . These values at even distances (0, 50, 100, ... ft) were computed by linear interpolation from the values in the grid system of Fig. 3.2; therefore, some error may have been introduced. However, the change in shape of the depth hydrograph due to varying Δx was considered to be small. Larger Δx produced a lower and later peak depth.

As previously mentioned, the smaller the Δx , the longer the computing time required. For these particular values in the hydrograph and the specified grid system computer program, the relation between the time required for the CDC 6600 computer and the various Δx or n values is shown in Fig. 3.9. This relation is approximately a power function because the number of computational locations in the (x, t) -plane is proportional to the square of the x -positions for a constant time position.

Table 3.1. Difference in y_p computed from various sizes of Δx
(in percent of conduit diameter D)

Δx	DISTANCE, ft																
(ft)	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
40.91	0	-0.02	-0.16	-0.04	-0.06	-0.08	-0.11	-0.16	-0.24	-0.31	-0.41	-0.50	-0.59	-0.70	-0.94	-1.43	-2.07
20.45	0	-0.01	-0.02	-0.02	-0.03	-0.04	-0.04	-0.06	-0.10	-0.13	-0.18	-0.22	-0.27	-0.39	-0.42	-0.66	-0.99
10.23	0	0	-0.01	0	-0.01	-0.01	-0.01	-0.02	-0.03	-0.04	-0.06	-0.08	-0.09	-0.11	-0.14	-0.23	-0.39

Table 3.2. Difference in T_p computed from various sizes of Δx
(in percent of t_p)

Δx	DISTANCE, ft																
(ft)	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
40.91	1.23	-0.09	0.18	0.14	-1.21	-0.36	-1.62	-2.04	-2.02	-1.81	-1.09	1.21	-0.96	-1.43	-8.47	-7.32	-3.48
20.45	-0.40	-0.09	0	0.14	0.05	-0.06	0	-0.40	-0.40	-1.81	-2.73	-0.42	-0.40	0	-3.58	-4.07	-2.04
10.23	0.41	0	0	0.14	0.05	0	0	-0.22	-0.40	0	-1.90	-0.24	-0.42	0	-1.49	-1.62	-0.41

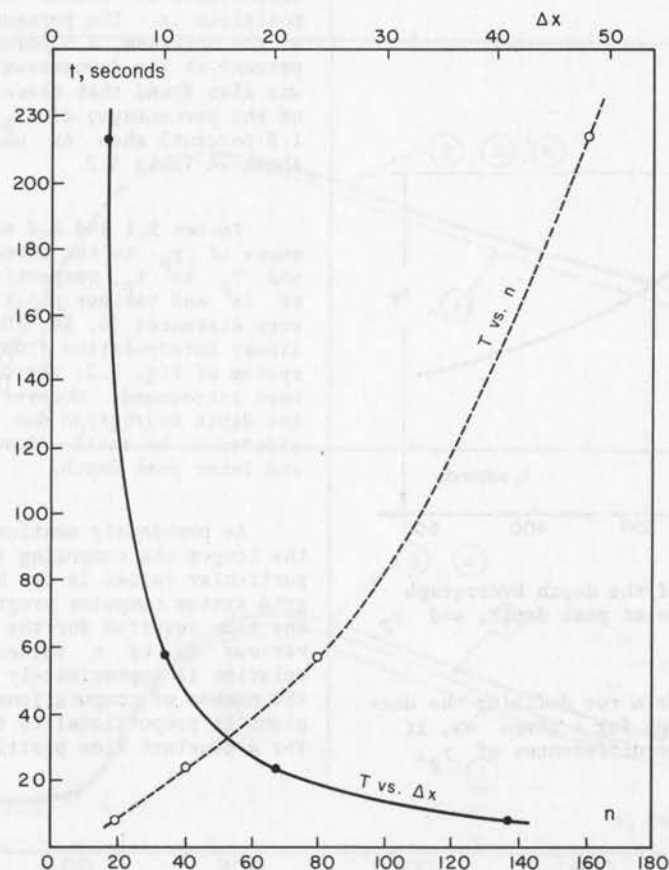


Fig. 3.9. Relations between n and Δx and the computer time, T , required for CDC 6600 computer.

COMPARISON OF THREE FINITE DIFFERENCE SCHEMES OF NUMERICAL INTEGRATION

4.1. Criteria for Comparison

The comparison of three finite-difference schemes for numerical integration and numerical computer solution and the eventual selection of the most desirable scheme for particular applications depend on simplicity, stability, accuracy, flexibility, and resulting computer time. The three schemes to be compared are: diffusing, Lax-Wendroff, and specified intervals scheme in the method of characteristics.

The simplicity of a particular scheme is related to both the algebraic description of its numerical algorithm and the computer programming involved. Generally, if the algebra is kept simple for understanding the computer programming is usually also simplified. Frequently, however, this may lead to numerous programming decisions to insure that conditions outside the range of the simplified assumptions are either included or deliberately excluded. Thus, simplified algebra does not necessarily infer simplicity in the computer algorithm.

The stability of a solution infers that the process will converge to a real solution. This criterion is satisfied in the case of solving the De Saint Venant equations if the mesh size $\Delta t/\Delta x$ ratio is less than dt/dx , for any part of the (x,t) -plane used in the integration solutions. If this condition is not satisfied, the solution will fluctuate about the correct value with increasing amplitude. Eventually, the results may exceed the capacity of computer.

The accuracy of a solution method in this study infers that the algorithm will reproduce the initial conditions for the steady state boundary conditions. As a corollary, the algorithm should be able to compute the steady state conditions from any arbitrary initial conditions. If the algorithm satisfies this criterion, it may be inferred that there will be good agreement between the computed and the observed quantities. The difference between these two can then be attributed to the limitations of the underlying assumptions of the theoretical equations and the limitations of accurately estimating the geometric and hydraulic parameters.

The flexibility of a computer algorithm depends on the range of conditions the algorithm will accommodate. For the unsteady flow solutions, it is desirable that the algorithm provide for all conditions of depth, velocity, and discharge within the expected physical ranges. Generally, this must include both the subcritical and the supercritical conditions. Since numerical procedures at some stage require interpolations, a computer decision is required to determine the appropriate interpolation.

4.2 Properties of Diffusing Scheme

The diffusing scheme is the simplest of the three compared schemes to develop and represent in algebraic form. This can be seen from Table 2.1, wherein the partial derivatives are represented as ratios of finite differences. This simplicity, of algebraic form, however, limits accuracy and flexibility.

The stability of the diffusing scheme is assured provided the ratio of $\Delta t/\Delta x$ does not exceed the

absolute maximum value of dt/dx at any point in the (x, t) -plane during the integration process.

The accuracy of the scheme may suffer during eventual periods of supercritical flow. This is because the characteristics intersect at a relatively great distance from the solution point. Figure 4.1 graphically presents this relationship. The accuracy of the diffusing scheme is further limited because the dependent variables are assumed to vary linearly within the interval of $2\Delta x$. Thus, if the actual value of a dependent variable at a given x -position is more than the interpolated value, the computed value at the same position for a later time will be less than it should be. This effect produces a dampening effect in time at a fixed location. Figure 4.1 demonstrates this effect for the depth at a location near the free-fall outlet. The greater the curvature of the free surface the more pronounced is this effect.

To reduce this effect the physical size of Δx may be reduced but this results in an increase of the computer time needed. The computer time increases by the square of the number n of distance intervals, Δx . Subsequent comparison indicate that the diffusing scheme requires more computer time than the other two schemes.

4.3 Properties of Lax-Wendroff Scheme

The Lax-Wendroff scheme is an improvement over the diffusing scheme in that it accommodates the curvature in the variation of dependent variables. This, however, involves a more complicated numerical algorithm.

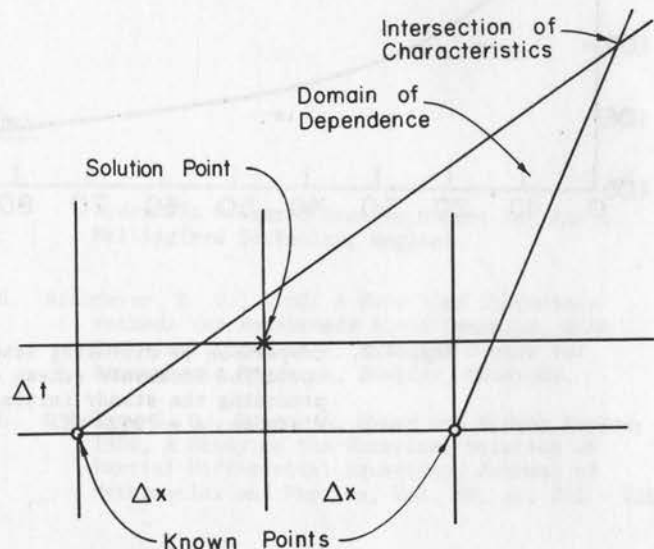


Fig. 4.1. Effect of characteristic slopes.

The Lax-Wendroff scheme results in a more accurate solution in comparison with the diffusing scheme for the same Δx and Δt intervals without a significant increase in computer time. An indication of this improved accuracy is demonstrated in Fig. 4.2. The Lax-Wendroff method consistently produces the same depth over a very large period of time, whereas, the diffusing produces a consistent change.

With regard to its flexibility in accommodating a wide range of flow conditions, the Lax-Wendroff scheme possesses the same inherent limitations as the diffusing scheme. Thus, by the Lax-Wendroff scheme the further the intersection of the two characteristic curves from the solution point, the less accurate the solution.

4.4 Properties of Specified Intervals Scheme of the Method of Characteristics

The complications inherent in the specified intervals scheme of the method of characteristics are justified because of its inherent accuracies. The

basis for this is that the points of solutions are at the intersections of characteristic curves, rather than at any point within the domain of dependence.

The linear interpolation of this scheme is made without the need of a computer decision. All flow conditions can be accommodated by this scheme.

The accuracy of this scheme is demonstrated in Fig. 4.2, and is very good when compared to the diffusing and Lax-Wendroff schemes.

It is apparent that this finite-difference scheme of the method of characteristics produces a rapidly convergent and stable value. It is comparable to the same property of the Lax-Wendroff scheme.

The non-linear interpolation of the method of characteristics for dependent variables along distances for a given time is an improvement over the linear interpolation. However, linear interpolation is used in producing results (C) of Fig. 4.2 for this method of characteristics.

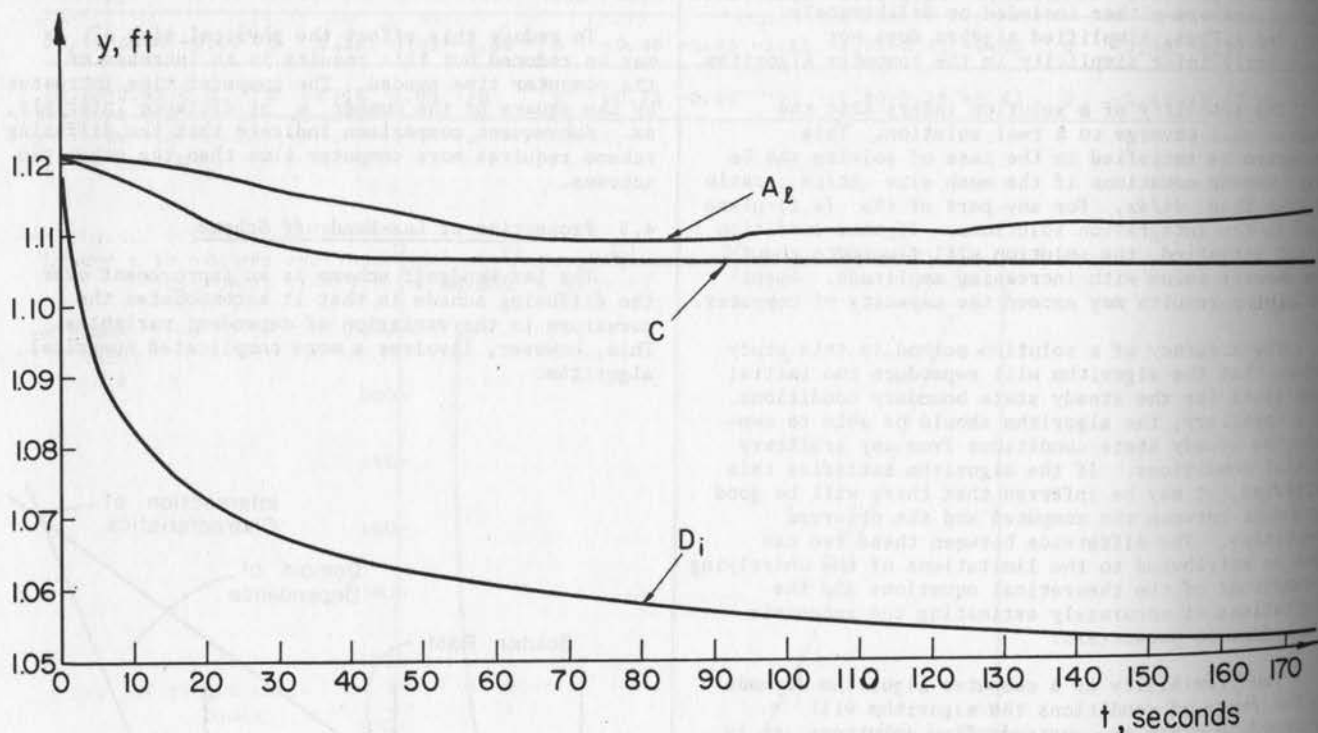


Fig. 4.2. Comparison of diffusing scheme (D_i), Lax-Wendroff scheme (A_L), and the specified intervals scheme of method of characteristics (C) in reproducing the steady initial conditions along the conduit, at the distance $x = 796.7$ ft.

CONCLUSIONS AND RECOMMENDATIONS5.1 Conclusions

1. Numerical integration solutions to the differential equations of gradually varied free-surface unsteady flow in prismatic channels and conduits have been reviewed, evaluated, and compared, both by the integration of the two partial differential equations and by their equivalent, four ordinary characteristic differential equations.

2. Numerical integration schemes, their solutions and their resulting computer programs are compared on the basis of their simplicity, stability, accuracy, flexibility, and the resulting computer time needed under given physical conditions.

3. Second-order or non-linear interpolations for dependent variables in the finite-difference schemes, for both the Lax-Wendroff scheme and the specified intervals scheme of the method of characteristics, were found to be necessary if maximum accuracy is to be obtained.

4. Solutions by the specified intervals scheme of the method of characteristics, with the second-order or non-linear interpolations for dependent variables, do not significantly require more computer time for a given accuracy comparable to the accuracy of solutions by any other scheme.

5. The Lax-Wendroff finite-difference scheme requires some particular programming considerations and adjustments in the case of supercritical flow.

6. The finite-difference specified intervals scheme of the method of characteristics with the

second-order of non-linear interpolations of dependent variables is sufficiently flexible to accommodate a large range of flow conditions.

7. Numerical integration by the specified intervals scheme of the method of characteristics with the second-order or non-linear interpolations of dependent variables in the writers' opinion should be used in general for studies of gradually varied free-surface unsteady flow.

5.2 Recommendations

Four recommendations for further studies are present in the following:

1. Other numerical integration finite-difference schemes, periodically appearing in the literature or not studied in this paper, should be investigated and compared with the recommended finite-difference specified intervals scheme of the method of characteristics. This should be done to find whether improvements in overall applicability can be attained.

2. The finite-difference specified intervals scheme of the method of characteristics may be further improved by considering the curvilinear nature of the characteristic curves. Thus, a better method of interpolation may be designed.

3. For the integration of gradually varied free-surface unsteady flow equations the use of a hybrid computer should be particularly investigated.

4. Computer times and computer costs should be systematically investigated for the most popular digital computers and for various finite-difference schemes.

REFERENCES

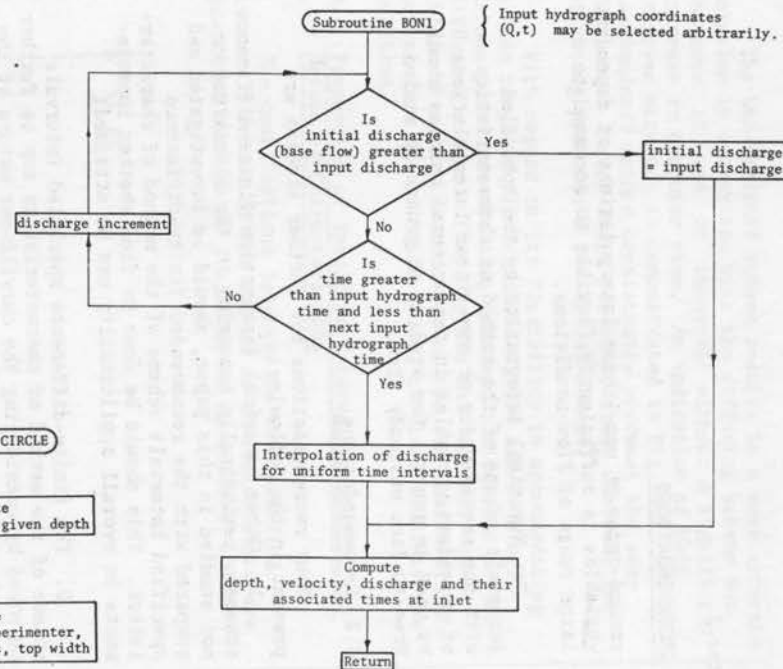
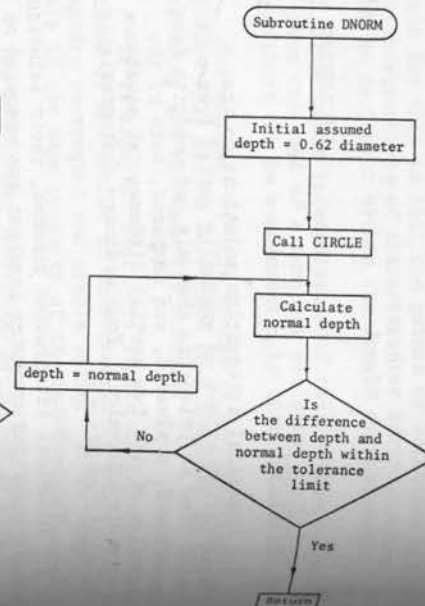
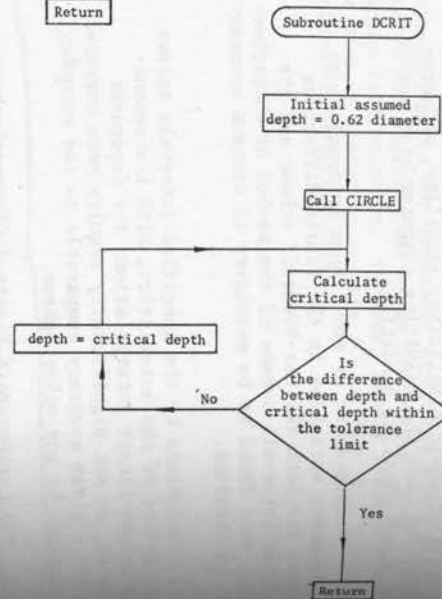
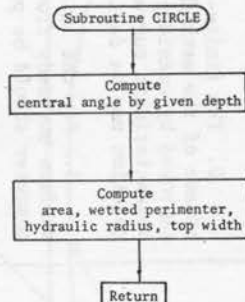
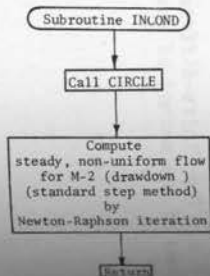
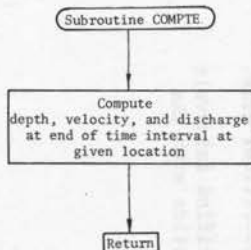
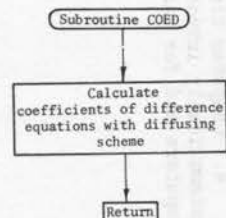
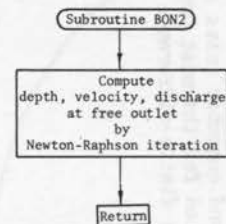
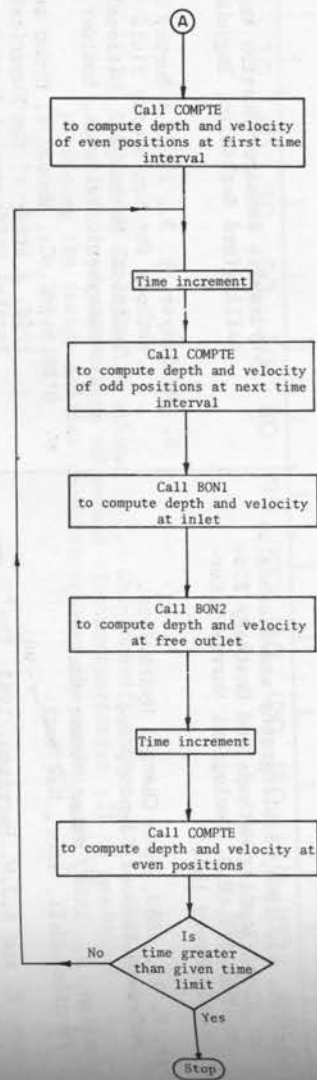
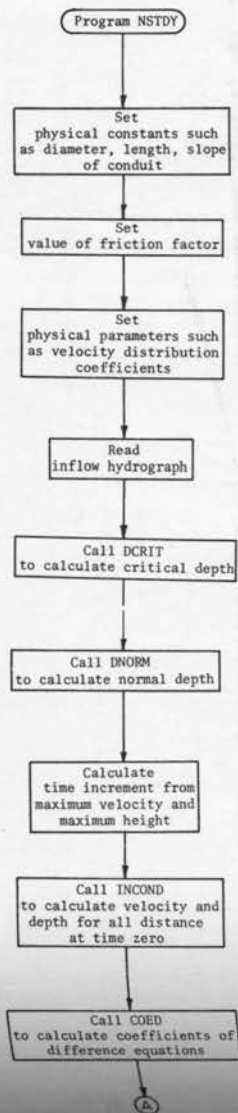
1. Yevjevich, V., 1961, Bibliography and Discussion of Flood-Routing Methods and Unsteady Flow in Channels, U.S. Geological Survey Water-Supply Paper 1690.
2. Chow, V. T., 1959, Open Channel Hydraulics, McGraw-Hill Book Company, Inc.
3. Parmakian, J., 1955, Water Hammer Analysis, Prentice-Hall, Inc., New York.
4. Ackers, P. and A.J.M. Harrison, 1963, The Attenuation of Flood Waves in Part-Full Pipes,

Hydraulic Research Station Report No. INT 31, Wallingford Berkshire, England.

5. Richtmyer, R. D., 1962, A Survey of Difference Methods for Non-Steady Fluid Dynamics, NCAR Technical Notes 63-2, National Center for Atmospheric Research, Boulder, Colorado.
6. O'Brien, G. C., Morton A. Hyman and Sidney Kaplan, 1950, A Study of the Numerical Solution of Partial Differential Equations, Journal of Mathematics and Physics, Vol. 29, pp. 223 - 251.

APPENDIX I COMPUTATIONAL DETAILS OF DIFFUSING SCHEME

A.I.I. FLOW CHART



{ Input hydrograph coordinates (Q,t) may be selected arbitrarily.

A.I.2. FORTRAN IV COMPUTER PROGRAM

MAIN PROGRAM FOR UNSTEADY FLOW BY DIFFUSING SCHEME

```

PROGRAM NSTDY(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
DIMENSION BLNK(120), PN1(400), PN2(400), PN3(400)
DIMENSION TQ(200), QI(200)
DIMENSION G(400), H(400), V(400)
DIMENSION G(330)
COMMON DN,H1,A,AP,SP,A1,C1,D1,E1,B2,AZ,C2,E2,VP,DTA,T,TH,X,DC
COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,M,V,G,UA,UT,T,TO,TF,N,FB,FC,B
COMMON H,MM,MMH,L,I,PERD,DUT,VA,IG,TU,QI,NGCD
COMMON HA,HH,VM,HT,VT,NPT,HH,G,WII
COMMON THETA,W,P,R,UEPT,VC
COMMON GMAX(400),VMAX(400),HMAX(400)
COMMON TQMAX(400),TVMAX(400),THMAX(400)
INTEGER RUN
DO 1 I=1,400
HMAX(I)=0
VMAX(I)=0
GMAX(I)=0
1 CONTINUE
C-----INPUT WHICH MAY BE ALTERED
D=2.9262
XU=0.0
XF=821.70
F=0.012
ALPHA=1.000
BETA=1.000
GR=32.175
H1=0.4*0
C-----COEFFICIENTS
C1=1.0
E1=0.0
B2=BETA/GR
C2=1.0
C-----END OF COEFFICIENTS
C-----READ INPUT HYDROGRAPH
READ (5,12) NGCD
READ (5,13) (TQ(I),QI(I),I=1,NGCD)
QI1=QI(1)
TQ0=0
TO=0
T=TO
TF=2UC
SO=0.001
N=20
N1=N+1
NPO=6
IXOX=1
PERD=120.
FNU=0.0000141
FB=0.03
FC=0
PERD=120
NT=NPG
IQ=1
C-----CALCULATION OF CRITICAL AND NORMAL DEPTH AT BASE FLOW
CALL DNORM
DX=DN
CALL DCRIT
XL=XF-XO-4.5*DC
FD=SGRTF(IG*A/B)
M=2*N
FM=M
MM=M+1
MMM=M-1
C-----CALCULATION OF DT FROM MAXIMUM VELOCITY AND MAXIMUM HEIGHT
DX=XL/F
RA=1.0/(2.0*VC)
DT=RA*DX*0.5
WRITE (6,14) DN,DC
WRITE (6,15) M,DX,DT,XO,XF,TO,TF,SO,U,F
WRITE (6,16) RA,H1,PERD,FB,FC
T1=T+DT
C-----CALCULATION OF INITIAL CONDITIONS
C-----HEIGHTS AT PARTICULAR DISTANCES FROM INLET END
CALL INCOND

```

```

DTA=DT
DO 2 I=2,M+2
C-----CALCULATION OF COEFFICIENTS AND SOLUTION OF DIFFERENCE EQUATIONS
CALL COED
CALL COMPT
2 CONTINUE
DTA=2.0*DT
K=1
3 IF (NPO-NT) 4,6,6
4 WRITE (6,17) I
WRITE (6,18)
DO 5 I=1,N1-IXOX
WRITE (6,19) I,H(I),V(I),Q(I)
5 CONTINUE
NT=0
PN1(K)=H(10)*0.5*0
PN2(K)=H(100)*0.5*0
PN3(K)=H(150)*0.5*0
K=K+1
NT=NT+1
T1=T+DTA
T=T+DTA
QA=Q(3)
HA=H(3)
VA=V(3)
QM=Q(MMM)
HM=H(MMM)
VM=V(MMM)
DO 7 I=3,MMM+2
CALL COED
CALL COMPT
7 CONTINUE
C-----CALCULATION OF INLET BOUNDARY CONDITION
CALL BON1
HX=H(MM)
C-----CALCULATION OF OUTLET BOUNDARY CONDITIONS
CALL BON2
DO 9 I=2,M+2
CALL COED
CALL COMPT
9 CONTINUE
IF (TF-T) 9,3,3
10 CONTINUE
NPG=N1/50+1
DO 10 I1=1,NPG
I1=50*I1-49
IL=I1+49
WRITE (6,20)
WRITE (6,21)
DO 10 I=I1,IL
X=(I-1)*DA
WRITE (6,22) X,HMAX(I),THMAX(I),VMAX(I),TVMAX(I),GMAX(I),TQMAX(I)
IF (I.EQ.N1) GO TO 11
10 CONTINUE
11 CALL EXIT
C-----
12 FORMAT (I3)
13 FORMAT (8F10.0)
14 FORMAT (* DNGS = *E16.8// * DCQB = *E16.8/)
15 FORMAT (* M = *15//
1 * DX = *E16.8//
2 * DT = *E16.8//
3 * XO = *E16.8//
4 * XF = *E16.8//
5 * TO = *E16.8//
6 * TF = *E16.8//
7 * SO = *E16.8//
8 * D = *E16.8//
9 * F = *E16.8//
16 FORMAT (* RA = *E16.8//
1 * H1 = *E16.8//
2 * PERD = *E16.8//
3 * FB = *E16.8//
4 * FC = *E16.8//
17 FORMAT (1M1.7HTIME IS,ELB.0.0M SEC.)
18 FORMAT (2X,3HPNT,10X,1MM,17X,1MV,17X,1MW)
19 FORMAT (1X,14,2X,E16.8,2X,E16.8,2X,E16.8)
20 FORMAT (// *1 MAXIMUM VALUES AND TIMES AT EACH LOCATION *//)
21 FORMAT (* DISTANCE MAX DEPTH TIME MAX VEL TIME MAX U
1 TIME*)
22 FORMAT (F6.2,3(4X,F6.2,2X,F7.2))
END

```

SUBROUTINE FOR COMPUTING UPSTREAM BOUNDARY CONDITION

```

SUBROUTINE BON1
  DIMENSION TG(200), QI(200)
  DIMENSION Q(400), H(400), V(400)
  DIMENSION G(330)
  COMMON DN,H1,A,AP,SP,A1,C1,D1,E1,B2,A2,C2,E2,VP,UTA,WT,HX,UC
  COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
  COMMON M,MM,MMH,L,I,PERD,DDT,VA,IG,TQ,Q1,NQCD
  COMMON HA,HH,VM,HT,VT,NPT,HH,G,QII
  COMMON THETA,W,P,R,DEPTH,VC
  COMMON QMAX(400),VMAX(400),HMAX(400)
  COMMON TQMAX(400),TVMAX(400),THMAX(400)
  IF (IG,GE,NQCD) 2,3
  QT=QI(NQCD)
  GO TO 6
  IF (T,GE,TQ(IG),AND,T,LT,TQ(IG+1)) 5,4
  IQ=IG+1
  GO TO 1
  QT=QI(IQ)+(QI(IQ+1)-QI(IQ))*(T-TQ(IG))/(TQ(IG+1)-TQ(IG))
  HN=H(1)
  THETA=2.0*ATANF((SQRTF(D*H(2)-H(2)**2)))/(D*0.5-H(2)))
  IF (THETA) 7,8,6
  THETA=6.28318+THETA
  A=0.125*(THETA-SINF(THETA))*(D*D)
  WP=D*0.5*THETA
  R=A/WP
  A2=V(2)*ALPHA/GR
  SF=.125*F*B2*V(2)*V(2)/R
  E2=SF-SO
  SQ=SQRTF(D*HN-HN*HN)
  THETA=2.0*ATANF((SQRTF(SQ))/(D*0.5-HN))
  IF (THETA) 10,11,11
  THETA=6.28318+THETA
  AX=0.125*(THETA-SINF(THETA))*(D*D)
  FH=HN-A2*(V(3)+VA-V(1)-QT/AX)-B2*DX/DT*(V(3)+QT/AX-VA-V(1))-C2*(HABO)
  1+H(3)-H(1))-4.0*DX*E2
  DAX=0.25*D*D*(1.0-COSF(THETA))/SQ
  FPH=1.0-(A2-B2*DX/DT)*(QT*DAX/(AX*AX))
  HNU=HN-FH/FPH
  IF (ABS(FHNU-HN)-0.0001) 13,12,12
  HN=HNU
  GO TO 9
  H(1)=HNU
  Q(1)=QT
  V(1)=QT/AX
  IF (H(1),LT,HMAX(1)) GO TO 14
  HMAX(1)=H(1)
  THMAX(1)=T
  IF (V(1),LT,VMAX(1)) GO TO 15
  VMAX(1)=V(1)
  TVMAX(1)=T
  IF (Q(1),LT,QMAX(1)) GO TO 16
  QMAX(1)=Q(1)
  TQMAX(1)=T
  RETURN
END

```

SUBROUTINE FOR COMPUTING CRITICAL DEPTH

```

SUBROUTINE DCRIT
  DIMENSION TG(200), QI(200)
  DIMENSION Q(400), H(400), V(400)
  DIMENSION G(330)
  COMMON DN,H1,A,AP,SP,A1,C1,D1,E1,B2,A2,C2,E2,VP,UTA,WT,HX,DC
  COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
  COMMON M,MM,MMH,L,I,PERD,DDT,VA,IG,TQ,Q1,NQCD
  COMMON HA,HH,VM,HT,VT,NPT,HH,G,QII
  COMMON THETA,W,P,R,DEPTH,VC
  THETA=2.0*ATANF((SQRTF(D*DX-DX**2)))/(D*0.5-DX))
  IF (THETA) 2,3,3
  THETA=6.28318+THETA
  A=0.125*(THETA-SINF(THETA))*(D*D)
  B=D*SINF(THETA*0.5)
  DC=DX-(B*(A**3)-ALPHA*(B*QII)**2)/UN/(3.0*(1.0+A)**2-(1.0*(A**3)))
  1+COSF(THETA*0.5))/(SINF(THETA*0.5)))
  IF (ABS(F'QC-DX)-0.0001) 5,4,4
  DX=DC
  GO TO 1
  VC=QI(1)/A
  RETURN
END

```

SUBROUTINE FOR COMPUTING DOWNSTREAM BOUNDARY CONDITION

```

SUBROUTINE BON2
  DIMENSION TG(200), QI(200)
  DIMENSION Q(400), H(400), V(400)
  DIMENSION G(330)
  COMMON DN,H1,A,AP,SP,A1,C1,D1,E1,B2,A2,C2,E2,VP,UTA,WT,HX,DC
  COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
  COMMON M,MM,MMH,L,I,PERD,DDT,VA,IG,TQ,Q1,NQCD
  COMMON HA,HH,VM,HT,VT,NPT,HH,G,QII
  COMMON THETA,W,P,R,DEPTH,VC
  COMMON QMAX(400),VMAX(400),HMAX(400)
  COMMON TQMAX(400),TVMAX(400),THMAX(400)
  HP=H(M)
  VP=V(M)
  THETA=2.0*ATANF((SQRTF(D*HP-HP*HP))/(0.5*D-HP))
  IF (THETA) 1,1,2
  THETA=THETA+6.28318
  AP=0.125*(THETA-SINF(THETA))*(D*D)
  BP=D*SINF(THETA*0.5)
  THETA=2.0*ATANF((SQRTF(D*HX-HX*HX))/(D*0.5-HX))
  IF (THETA) 4,5,5
  THETA=6.28318+THETA
  A=0.125*(THETA-SINF(THETA))*(D*D)
  B=D*SINF(THETA*0.5)
  VX=SQRTF(A*GR/B)
  CTN=COSF(THETA*0.5)/SINF(THETA*0.5)
  FORG=AP/(BP*DX)*(VX+V(MM)-V(MMM)-VM)+VP/UX*(HX+H(MM)-H(MMM)-VM)+(HBO)
  1X+H(MM)-H(MM)-H(M)) /DT
  FPRI=AP/(BP*DX*2.0*VX)*(GR-A*GR*2.0*CTN/(D*D))+VP/UX+1.0/DT
  DC=HX-FORG/FPRI
  IF (ABS(DC-HX)-0.0001) 7,6,6
  HX=DC
  GO TO 3
  H(MM)=DC
  THETA=2.0*ATANF((SQRTF(D*DC-DC*DC))/(D*0.5-DC))
  IF (THETA) 8,9,9
  THETA=6.28318+THETA
  A=0.125*(THETA-SINF(THETA))*(D*D)
  B=D*SINF(THETA*0.5)
  V(MM)=SQRTF(A*GR/B)
  Q(MM)=A*V(MM)
  I=MM
  IF (H(I),LT,HMAX(I)) GO TO 10
  HMAX(I)=H(I)
  THMAX(I)=T
  IF (V(I),LT,VMAX(I)) GO TO 11
  VMAX(I)=V(I)
  TVMAX(I)=T
  IF (Q(I),LT,QMAX(I)) GO TO 12
  QMAX(I)=Q(I)
  TQMAX(I)=T
  RETURN
END

```

SUBROUTINE FOR COMPUTING GEOMETRIC PARAMETERS OF CIRCULAR SEGMENT

```

SUBROUTINE CIRCLE
  DIMENSION TG(200), QI(200)
  DIMENSION Q(400), D(400), V(400), X(330)
  COMMON DN,H1,A,AP,SP,A1,C1,D1,E1,B2,A2,C2,E2,VP,UTA,WT,HX,DC
  COMMON DIA,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
  COMMON M,MM,MMH,L,I,PERD,DDT,VA,IG,TQ,Q1,NQCD
  COMMON HA,HH,VM,HT,VT,NPT,HH,G,QII
  COMMON THETA,W,P,R,DEPTH,VC
  THETA=2.0*ATANF((SQRTF(DIA*DEPTH-DEPTH**2)))/(DIA*0.5-DEPTH))
  IF (THETA) 1,2,2
  THETA=6.28318+THETA
  A=0.125*(THETA-SINF(THETA))*(DIA*DIA)
  WP=(DIA*0.5)*THETA
  R=A/WP
  B=DIA*SINF(THETA/2.0)
  RETURN
END

```

SUBROUTINE FOR COMPUTING COEFFICIENTS IN DIFFERENCE EQUATIONS

```

SUBROUTINE COEF
  DIMENSION TG(200), Q1(200)
  DIMENSION G(400), H(400), V(400)
  DIMENSION G1(300)
  COMMON DN,H1,A,AP,AP,A1,C1,D1,E1,B2,A2,C2,E2,VP,UTA,UT,HA,UC
  COMMON DX,XF,GR,ALPHA,BETA,S0,F,H,V,Q,UX,DT,T,TO,TF,N,Fb,FC,B
  COMMON N,NM,MM,L,I,PERD,DDT,VA,IQ,TU,Q1,NGCD
  COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
  COMMON THETA,W,P,R,DEPTH,VC
  VT=(V(I+1)+V(I-1))*0.5
  HT=(H(I+1)+H(I-1))*0.5
  THETA=2.0*ATANF((SQRTF(D*HT-HT**2)))/(D*0.5-HT))
  IF (THETA) 1+2
  THETA=6.28316+THETA
  A=0.125*(THETA-SINF(THETA))*(D*D)
  WP=D*C.5*THETA
  R=A/WP
  B=D*SINF(THETA*0.5)
  A1=A/(VT*B)
  D1=1.0/VT
  A2=VT*ALPHA/GR
  SF=1.25*F*D2*VT*VT/R
  E2=SF-S0
  RETURN
END

```

SUBROUTINE FOR COMPUTING DEPTH & VELOCITY AT END OF TIME INTERVAL

```

SUBROUTINE COMPT
  DIMENSION TG(200), Q1(200)
  DIMENSION G(400), H(400), V(400)
  DIMENSION G1(300)
  COMMON DN,H1,A,AP,AP,A1,C1,D1,E1,B2,A2,C2,E2,VP,UTA,UT,HA,UC
  COMMON DX,XF,GR,ALPHA,BETA,S0,F,H,V,Q,UX,DT,T,TO,TF,N,Fb,FC,B
  COMMON N,NM,MM,L,I,PERD,DDT,VA,IQ,TU,Q1,NGCD
  COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
  COMMON THETA,W,P,R,DEPTH,VC
  COMMON QMAX(400),VMAX(400),HMAX(400)
  COMMON TGMAX(400),TVMAX(400),THMAX(400)
  H(I)=HT-(DT/2.0*DX*H(I))*(A1*(V(I+1)-V(I-1)))+(H(I+1)-H(I-1))
  V(I)=VT-(DT/2.0*DX*V(I))*(A2*(V(I+1)-V(I-1)))+(V(I+1)-V(I-1))
  VP=V(I)
  Q(I)=V(I)*A
  IF (H(I).LT.HMAX(I)) GO TO 1
  HMAX(I)=H(I)
  THMAX(I)=T
  IF (V(I).LT.VMAX(I)) GO TO 2
  VMAX(I)=V(I)
  TVMAX(I)=T
  IF (Q(I).LT.QMAX(I)) GO TO 3
  QMAX(I)=Q(I)
  TQMAX(I)=T
  RETURN
END

```

SUBROUTINE FOR COMPUTING NORMAL DEPTH

```

SUBROUTINE DNORM
  DIMENSION TG(200), Q1(200)
  DIMENSION G(400), H(400), V(400)
  DIMENSION G1(300)
  COMMON DN,H1,A,AP,AP,A1,C1,D1,E1,B2,A2,C2,E2,VP,UTA,UT,HA,UC
  COMMON DX,XF,GR,ALPHA,BETA,S0,F,H,V,Q,UX,DT,T,TO,TF,N,Fb,FC,B
  COMMON N,NM,MM,L,I,PERD,DDT,VA,IQ,TU,Q1,NGCD
  COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
  COMMON THETA,W,P,R,DEPTH,VC
  THETA=2.0*ATANF((SQRTF(D*H1-H1**2)))/(D*0.5-H1))
  IF (THETA) 2+3
  THETA=6.28316+THETA
  A=0.125*(THETA-SINF(THETA))*(D*D)
  WP=(0.5*D)*THETA
  R=A/WP
  B=D*SINF(THETA*0.5)
  DN=H1-(WP*(F*QII*QII)/(8.0*GR*S0*N*A))/((3.0*0)/K-2.0/SINF(THETA*DN)
  1*0.5))
  IF (ABS(DN-H1)-0.0001) 7+4
  IF (D-DN) 5+5
  DN=DN*0.5
  GO TO 4
  H1=DN
  GO TO 1
  RETURN
END

```

SUBROUTINE FOR COMPUTING INITIAL CONDITION

```

SUBROUTINE INCOND
  DIMENSION TG(200), Q1(200)
  DIMENSION G(400), H(400), V(400), X(300)
  COMMON DN,H1,A,AP,AP,A1,C1,D1,E1,B2,A2,C2,E2,VP,UTA,UT,HA,UC
  COMMON DX,XF,GR,ALPHA,BETA,S0,F,H,V,Q,UX,DT,T,TO,TF,N,Fb,FC,B
  COMMON N,NM,MM,L,I,PERD,DDT,VA,IQ,TU,Q1,NGCD
  COMMON THETA,W,P,R,DEPTH,VC
  DTOL=0.00001
  IF (DN-DC) 1+1+2
  K=1
  GO TO 17
  DIN=(DN+DC)*0.5
  DEPTH=DC
  CALL CIRCLE
  VV=GB/A
  VH=(VV*VV)/(2.0*GR)
  S1=F*VH/(4.0*R)
  EE1=DC+ALPHA*VH
  D(2*N+1)=DC
  Q(2*N+1)=GB
  V(2*N+1)=VV
  NCOUNT=0
  DO 16 L=1,N
  DEPTH=DIN
  CALL CIRCLE
  HFTH=0.5*THETA
  DTHET=4.0/(DIA*SINF(HFTH))
  DAREA=0.125*DIA*DIA*(1.0-COSF(THETA))*DTNETH
  DW=0.5*DIA*DTHET
  DRA=(WP*DAREA-A*DW)/(WP*WP)
  DENG=1.0-(QB*QB)/(GR*(A**3))*DAREA
  DSLOE=F*QB*QB/(2.0*R*A*DAREA+(A**2)*DRA)/(8.0*GK*((K*A**2)**2))
  VV=QB/A
  VH=(VV*VV)/(2.0*GR)
  S2=F*VH/(4.0*R)
  SF=(S1+S2)*0.5
  EE2=DIN+ALPHA*VH
  FRATIO=(EE2-EE1+2.0*DX*(S0-SF))/(DENG+(EE2-EE1)*DSLOE/(S0-SF))
  DCOM=DIN-FRATIO
  IF (DCOM) 5+4+6
  WRITE (6,19)
  GO TO 18
  DCOM=ABS(DCOM)
  IF (ABS(DCOM-DIN)-DTOL) 15+15+7
  IF (0.82*DIA-DCOM) 8+14+14
  DIN=DCOM*0.5
  IF (0.82*DIA-DIN) 10+10+11
  DIN=DIN*0.5
  NCOUNT=NCOUNT+1
  GO TO 9
  IF (NCOUNT-20) 12+12+13
  GO TO 3
  WRITE (6,20)
  GO TO 18
  DIN=DCOM
  GO TO 3
  DIN=DCOM
  S1=S2
  EE1=EE2
  II=2*(N-L)+1
  D(II)=DIN
  V(II)=VV
  Q(II)=QB
  CONTINUE
  GO TO 18
  WRITE (6,21) K
  RETURN
C-----
19 FORMAT (* DCOM EQUALS ZERO *)
20 FORMAT (25H D2 MUCH GREATER THAN DIA)
21 FORMAT (* STOP *,13)
END

```


A.I.3. DEFINITION OF VARIABLES

NAME	DEFINITION	STATEMENT	NUMBER				
A AREA OF CIRCULAR SEGMENT		DNO 13	DCR 13	COE 15	B01 23		
ALPHA VEL. DISTRIBUTION COEF.-ENERGY		B02 22	B02 37	CIR 12			
AP AREA OF CIRCULAR SEGMENT		NST 24					
AX AREA OF CIRCULAR SEGMENT		B02 17					
A1 COEFFICIENT		R01 33					
A2 COEFFICIENT		COE 19					
B FREE-SURFACE WIDTH		COE 21	B01 26				
BETA VEL. DISTRIBUTION COEF.-MOMENTUM		UNO 16	DCR 14	COE 18	B02 23		
BP FREE-SURFACE WIDTH		B02 38	CIR 15				
B2 COEFFICIENT		NST 25					
CTN COTANGENT OF 1/2 THETA		B02 18					
C1 COEFFICIENT		NST 31					
C2 COEFFICIENT		B02 25					
D DIAMETER OF CONDUIT		NST 29					
DAREA DERIVATIVE OF AREA WITH DEPTH		NST 32					
DAX DERIVATIVE OF AREA WITH DEPTH		NST 20	INC 20	INC 62			
DC CRITICAL DEPTH		INC 29					
DCOM COMPUTED DEPTH		R01 36					
DENG DERIVATIVE OF ENGERY WITH DEPTH		DCR 15	B02 29				
DEPTH ASSUMED DEPTH OF FLOW		INC 40	INC 44				
DIN INITIAL DEPTH ASSUMED		INC 32					
DN NORMAL DEPTH		INC 14	INC 25				
DRA DERIVATIVE OF HYD. RADIUS WITH DEPTH		INC 13	INC 47	INC 49	INC 56		
DSLO DERIVATIVE OF SLOPE WITH DEPTH		INC 58					
DT TIME INCREMENT		DNO 17	DNO 21				
DTA TIME INCREMENT		INC 31					
DTHEAT DERIVATIVE OF THETA WITH DEPTH		INC 33					
DTOL TOLERANCE IN APPROX. DEPTH		NST 67					
DW DERIVATIVE OF SURFACE WIDTH WITH DEPTH		NST 75	NST 81				
DX INCREMENT IN X-POSITION		INC 28					
D1 COEFFICIENT		INC 9					
EE1 ENERGY SLOPE AT POSITION 1		NST 56	NST 65	DCR 18			
EE2 ENERGY SLOPE AT POSITION 2		COE 20					
E1 COEFFICIENT		INC 19	INC 60				
E2 COEFFICIENT		INC 38					
F DARCY-WEISBACH FRICTION FACTOR		NST 30					
FB FRICTION FACTOR COEFFICIENT		COE 23	B01 28				
FC FRICTION FACTOR EXPONENT		NST 23					
FD CELERITY OF WAVE		NST 49					
FH (2)		NST 50					
FM NUMBER OF POSITION INTERVALS		NST 59					
FNU KINEMATIC VISCOSITY		R01 34					
FORG (2)		NST 61					
FPH (2)		NST 48					
FPR1 (2)		R02 26					
FRATIO (2)		R01 37					
GR ACCELERATION DUE TO GRAVITY		B02 28					
H DEPTH OF FLOW		INC 39					
HA DEPTH OF FLOW		NST 26					
HFTH1/2 THETA		COM 12	B01 42	B02 33			
HM DEPTH OF FLOW		NST 98					
HMAX MAX. DEPTH OF FLOW		INC 27					
HN DEPTH OF FLOW-INITIAL		NST 101					
HNU DEPTH OF FLOW-COMPUTED		NST 15	COM 17	B01 46	B02 43		
HP DEPTH OF FLOW		R01 19	R01 40				
HT DEPTH OF FLOW-MEAN		R01 38					
HX DEPTH OF FLOW		R02 12					
H1 ASSUMED DEPTH		COE 11					
I (1)		NST 109	B02 31				
II (1)		NST 27	DNO 23				
		B02 41					
		NST 120	INC 61				

IL (1)	NST 121						
IO (1)	NST 53	B01 16					
IXOX PRINT OUT LIMIT	NST 46						
K (1)	NST 82	NST 93	INC 11				
M NO. OF COMPUTATION INTERVALS	NST 60						
MM NO. OF COMPUTATION POSITIONS	NST 62						
MMM NO. OF COMPUTATION INTERVALS	NST 63						
N NO. OF LENGTH INTERVALS	NST 43						
NCOUNT (1)	INC 23	INC 50					
NPG (1)	NST 118						
NPO (1)	NST 45						
NT (1)	NST 52	NST 49	NST 94				
N1 PRINTOUT INTERVAL	NST 44						
PERD (2)	NST 47	NST 51					
PN1 (2)	NST 90						
PN2 (2)	NST 91						
PN3 (2)	NST 92						
Q DISCHARGE	INC 21	INC 64	COM 15	B01 43			
QA DISCHARGE	B02 40						
QII DISCHARGE	NST 97						
QM DISCHARGE	NST 37						
QMAX MAX. DISCHARGE	NST 100						
QT DISCHARGE OF LAST POINT ON HYDRO.	NST 17	COM 23	B01 52	B02 49			
R HYDRAULIC RADIUS	B01 13	B01 18					
RA (2)	DNO 15	COE 17	B01 25	CIR 14			
SF FRICTION SLOPE	NST 66						
S0 CHANNEL BED SLOPE	INC 37	COE 22	B01 27				
S0 (2)	NST 42						
S1 INITIAL FRICTION SLOPE	R01 29						
S2 FINAL FRICTION SLOPE	INC 18	INC 59					
T TIME	INC 36						
TF TOTAL TIME OF INTEGRATION	NST 40	NST 95					
THETA CENTRAL ANGLE OF CIRC. SEGMENT	NST 41						
	DNO 10	DNO 12	DCR 10	DCR 12			
	COE 12	COE 14	B01 20	B01 22			
	B01 30	B01 32	B02 14	B02 16			
	B02 19	B02 21	B02 34	B02 36			
	CIR 9	CIR 11					
THMAX TIME TO MAX. DEPTH	COM 18	B01 47	B02 44				
TO INITIAL TIME	NST 39						
TOMAX TIME TO MAX. DISCHARGE	COM 24	B01 53	B02 50				
T00 TIME OF INITIAL DISCHARGE	NST 38						
TVMAX TIME OF MAX. VELOCITY	COM 21	B01 50	B02 47				
T1 TIME AT END OF TIME INTERVAL	NST 71	NST 96					
V VELOCITY	INC 22	INC 63	COM 13	B01 44			
	R02 39						
VA VELOCITY	NST 99						
VC CRITICAL VELOCITY	DCR 20						
VH VELOCITY HEAD	INC 17	INC 35					
VM VELOCITY	NST 102						
VMAX MAX. VELOCITY	NST 16	COM 20	B01 49	B02 46			
VP VELOCITY	COM 14	B02 13					
VT AVERAGE VELOCITY	COE 10						
VV VELOCITY OF BASE FLOW	INC 16	INC 34					
VX CELERITY OF WAVE	R02 24						
WP WETTED PERIMETER	DNO 14	COE 16	B01 24	CIR 13			
X POSITION	NST 125						
XF TOTAL LENGTH OF CHANNEL	NST 22						
XL INTEGRATION LENGTH	NST 58						
X0 INITIAL DISTANCE	NST 21						
(1) DO LOOP COUNTER OF VARIABLE							
(2) INTERMEDIATE VARIABLE							

A.I.4. SAMPLE INPUT AND OUTPUT

Format No.	SAMPLE INPUT																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
12	xxxx (Number of Discharge Time Pairs Describing Inflow Hydrograph)																								
13	(Repeated As Many Cards As Desired To Describe Hydrograph)																								

5 0 4.0 30.0 10.0 50.0 10. 80.0 4.0
200.0 4.0

SAMPLE OUTPUT

Q₁ = 7.3268337E-01
Q₂ = 6.2649370E-01
M = 40
D₁ = 2.0471738E+01
UT = 1.3579064E+00
A0 = 0.
KF = 1.2170000E+02
T0 = 0.
TF = 2.0000000E+02
SO = 1.0000000E-03
D = 2.9262000E+00
F = 1.2000000E-02
RA = 1.3266703E-01
M1 = 7.3264345E-01
PERU = 1.2000000E+02
FR = 3.0000000E-02
FC = 0.

TIME IS 4.88866471E+01 SFC.
PNT H V Q
1 1.10390403E+00 4.46511988E+00 1.00000000E+01
2 1.09920055E+00 4.44727763E+00 1.06982096E+01
3 1.08735325E+00 4.45784206E+00 1.05950130E+01
4 1.08420133E+00 4.46423678E+00 1.04855689E+01
5 1.07326912E+00 4.45193921E+00 1.03572673E+01
6 1.06585705E+00 4.43713641E+00 1.02338305E+01
7 1.05495287E+00 4.43068796E+00 1.00431943E+01
8 1.04480727E+00 4.41949922E+00 9.89022024E+00
9 1.02350033E+00 4.43302183E+00 9.56739876E+00
10 1.01640405E+00 4.56295854E+00 9.37098906E+00
11 9.94427050E-01 4.44815272E+00 8.85461230E+00
12 9.78664291E-01 4.43743271E+00 8.61090926E+00
13 9.44040389E-01 4.29367685E+00 7.91207490E+00
14 9.31030135E-01 4.23164030E+00 7.64892278E+00
15 8.93458405E-01 4.03765295E+00 6.88260576E+00
16 8.80654078E-01 3.96953420E+00 6.63654532E+00
17 8.43811666E-01 3.76187348E+00 5.93567930E+00
18 8.32876116E-01 3.69745085E+00 5.73384994E+00
19 8.01940405E-01 3.50883770E+00 5.17605128E+00
20 7.9355356E-01 3.45704272E+00 5.03043411E+00
21 7.70706660E-01 3.32999994E+00 4.63905497E+00

TIME IS 0. SFC.
PNT H V Q
1 7.32687974E-01 3.03570825E+00 4.00000000E+00
2 7.32684444E-01 3.0357083E+00 3.99999451E+00
3 7.32684233E-01 3.03571062E+00 4.00000000E+00
4 7.32684222E-01 3.03570795E+00 3.99999830E+00
5 7.3268694E-01 3.03570303E+00 4.00000000E+00
6 7.32686478E-01 3.03571790E+00 4.00000145E+00
7 7.32690126E-01 3.03574412E+00 4.00000000E+00
8 7.32687510E-01 3.03574634E+00 4.00002499E+00
9 7.32683022E-01 3.03573291E+00 4.00000000E+00
10 7.32684284E-01 3.03572057E+00 3.99997636E+00
11 7.32690012E-01 3.03570821E+00 4.00000000E+00
12 7.32679412E-01 3.03576739E+00 3.99996608E+00
13 7.32683088E-01 3.03580809E+00 4.00000000E+00
14 7.32684075E-01 3.03584913E+00 3.99995473E+00
15 7.32683075E-01 3.03580173E+00 4.00000000E+00
16 7.32682016E-01 3.03580870E+00 3.99996207E+00
17 7.3268330E-01 3.03591736E+00 4.00000000E+00
18 7.32673890E-01 3.03583339E+00 3.99997391E+00
19 7.3268034E-01 3.03595214E+00 4.00000000E+00
20 7.3268479E-01 3.03597526E+00 3.99996031E+00
21 7.32681545E-01 3.03599033E+00 4.00000000E+00

TIME IS 6.51061941E+01 SFC.
PNT H V Q
1 9.94012932E-01 3.94038347E+00 6.96356078E+00
2 9.94350055E-01 3.9772237E+00 7.96737347E+00
3 1.02487545E+00 4.06820849E+00 8.59084738E+00
4 1.02886646E+00 4.07060573E+00 8.57529174E+00
5 1.04745103E+00 4.2294032E+00 9.26748775E+00
6 1.04736885E+00 4.24300150E+00 9.24449070E+00
7 1.05888148E+00 4.37098861E+00 9.63356487E+00
8 1.05629155E+00 4.38112528E+00 9.6157381E+00
9 1.05874409E+00 4.44976202E+00 9.8879313E+00
10 1.05388997E+00 4.46889178E+00 9.7770427E+00
11 1.04762220E+00 4.51104504E+00 9.7246928E+00
12 1.04404448E+00 4.50086783E+00 9.61516311E+00
13 1.02833440E+00 4.50506222E+00 9.44792702E+00
14 1.02071392E+00 4.48768760E+00 9.31187066E+00
15 1.00376290E+00 4.46368316E+00 9.03376026E+00
16 9.94847291E-01 4.43731805E+00 8.86751218E+00
17 9.73183830E-01 4.37781511E+00 8.46310389E+00
18 9.63019117E-01 4.33917563E+00 8.26212390E+00
19 9.3527774E-01 4.23147810E+00 7.71781494E+00
20 9.23783696E-01 4.18026661E+00 7.4029216E+00
21 8.9119001E-01 4.02562303E+00 6.88525597E+00

TIME IS 1.62955490E+01 SFC.
PNT H V Q
1 9.53113623E-01 4.27004548E+00 7.25910961E+00
2 9.38411556E-01 4.2036666E+00 7.66945517E+00
3 8.95240351E-01 3.9324726E+00 6.81321281E+00
4 8.79489297E-01 3.92560505E+00 6.53375813E+00
5 8.35570546E-01 3.6483572E+00 5.72046692E+00
6 8.21067426E-01 3.61772685E+00 5.48592588E+00
7 7.8394787E-01 3.4172599E+00 4.82940903E+00
8 7.7406184E-01 3.32993698E+00 4.68080602E+00
9 7.50295486E-01 3.15349522E+00 4.27576092E+00
10 7.46451184E-01 3.13560691E+00 4.21401895E+00
11 7.35750241E-01 3.08090633E+00 4.04682816E+00
12 7.34913682E-01 3.05401791E+00 4.03425374E+00
13 7.32050092E-01 3.03590166E+00 4.00003299E+00
14 7.32634127E-01 3.0405769E+00 4.00003463E+00
15 7.32010194E-01 3.03014499E+00 4.00003331E+00
16 7.32592719E-01 3.04029902E+00 4.00003471E+00
17 7.3267053E-01 3.04044231E+00 4.00002555E+00
18 7.32220784E-01 3.0367802E+00 4.00002608E+00
19 7.32480240E-01 3.03840674E+00 4.00001542E+00
20 7.3242342E-01 3.03728126E+00 4.00001676E+00
21 7.32450247E-01 3.03764955E+00 4.00000672E+00

TIME IS 8.14777451E+01 SFC.
PNT H V Q
1 8.38347827E-01 2.85786384E+00 4.00000000E+00
2 8.5062424E-01 2.9154043E+00 4.8605184E+00
3 8.8126816E-01 3.0773317E+00 5.3830538E+00
4 8.9354501E-01 3.15971193E+00 5.59529460E+00
5 9.2952091E-01 3.48490412E+00 6.30495722E+00
6 9.3734687E-01 3.43597031E+00 6.47311899E+00
7 9.64477521E-01 3.62694917E+00 7.00390172E+00
8 9.69023062E-01 3.67077978E+00 7.22073888E+00
9 9.89777824E-01 3.84725301E+00 7.7570002E+00
10 9.9187345E-01 3.87189576E+00 7.84507918E+00
11 1.00710630E+00 4.0070343E+00 8.2950380E+00
12 1.00726665E+00 4.04173341E+00 8.34347448E+00
13 1.0172611E+00 4.12487801E+00 8.69406058E+00
14 1.0154664E+00 4.1794896E+00 8.76194155E+00
15 1.0187964E+00 4.2706104E+00 8.9324850E+00
16 1.01631398E+00 4.28117936E+00 8.9864807E+00
17 1.01472618E+00 4.34425478E+00 9.48871256E+00
18 1.0094410E+00 4.34187815E+00 9.91441616E+00
19 1.00163130E+00 4.34949627E+00 8.8552531E+00
20 9.94771830E-01 4.35449627E+00 8.74466865E+00
21 9.81021715E-01 4.3501021E+00 8.54370935E+00

TIME IS 3.25910981E+01 SFC.
PNT H V Q
1 1.0704278E+00 4.8508801E+00 1.00000000E+01
2 1.07120050E+00 4.80714946E+00 1.06256005E+01
3 1.0525571E+00 4.74353716E+00 1.01539792E+01
4 1.0396740E+00 4.64449230E+00 9.67984044E+00
5 1.00063141E+00 4.43973419E+00 9.05918300E+00
6 9.27188173E-01 4.4494380E+00 8.77595938E+00
7 9.46510939E-01 4.3041364E+00 7.94304677E+00
8 9.32500022E-01 4.23792078E+00 7.66345842E+00
9 8.91307891E-01 4.0337241E+00 6.8417798E+00
10 8.77910827E-01 3.98096003E+00 6.58225490E+00
11 8.38030025E-01 3.73028853E+00 6.03930245E+00
12 8.2744866E-01 3.6688283E+00 5.62685427E+00
13 7.9453341E-01 3.46148351E+00 5.0302087E+00
14 7.8413413E-01 3.40725856E+00 4.89152513E+00
15 7.5244649E-01 3.2421984E+00 4.4527580E+00
16 7.5795047E-01 3.21801869E+00 4.266653E+00
17 7.4457408E-01 3.1240404E+00 4.19174016E+00
18 7.2168361E-01 3.1079155E+00 4.15487404E+00
19 7.3546647E-01 3.042056E+00 4.05634896E+00
20 7.3511474E-01 3.05874434E+00 4.0440844E+00
21 7.3293041E-01 3.04349526E+00 4.0115522E+00

TIME IS 9.77732942E+01 SFC.
PNT H V Q
1 8.4408414E-01 2.82927804E+00 4.00000000E+00
2 8.4455006E-01 2.8523768E+00 4.59528985E+00
3 8.4718000E-01 2.9710042E+00 4.66922714E+00
4 8.49334725E-01 2.89229034E+00 4.70113643E+00
5 8.54887025E-01 2.92621034E+00 4.8002117E+00
6 8.5859012E-01 2.9493269E+00 4.87700639E+00
7 8.68646291E-01 3.0075490E+00 4.86418158E+00
8 8.7351464E-01 3.04588143E+00 5.17290139E+00
9 8.8825055E-01 3.14416254E+00 5.47799164E+00
10 8.9346033E-01 3.19141736E+00 5.60938107E+00
11 9.12542395E-01 3.2521879E+00 6.0200251E+00
12 9.1752837E-01 3.37370539E+00 6.15209081E+00
13 9.34665151E-01 3.4250371E+00 6.6117027E+00
14 9.40529122E-01 3.46632541E+00 6.7226730E+00
15 9.57603074E-01 3.7104661E+00 7.16206602E+00
16 9.5936413E-01 3.74548685E+00 7.26280878E+00
17 9.72540635E-01 3.87475923E+00 7.62029792E+00
18 9.7270321E-01 3.90940271E+00 7.6694270E+00
19 9.8123511E-01 4.01020301E+00 7.7125349E+00
20 9.79590276E-01 4.0275925E+00 7.94737305E+00
21 9.8388610E-01 4.12007529E+00 8.20203593E+00

TIME IS 1.1400884E+02 SEC.

PNT	H	V	Q
1	8.35070653E-01	2.87451201E+00	4.00000000E+00
2	8.36042086E-01	2.84727926E+00	4.58316175E+00
3	8.33430999E-01	2.8405332E+00	4.6125334E+00
4	8.3960224E-01	2.90604463E+00	4.04099449E+00
5	8.42372553E-01	2.9141556E+00	4.67605642E+00
6	8.43842991E-01	2.92683909E+00	4.70777663E+00
7	8.47245761E-01	2.9401088E+00	4.74556755E+00
8	8.49051305E-01	2.95319026E+00	4.79449648E+00
9	8.53522135E-01	2.9730786E+00	4.86687225E+00
10	8.55730487E-01	2.99314099E+00	4.91740934E+00
11	8.61026642E-01	3.02832718E+00	5.02948193E+00
12	8.64300132E-01	3.05352180E+00	5.09449800E+00
13	8.72090965E-01	3.10822005E+00	5.26030743E+00
14	8.75737114E-01	3.1494742E+00	5.34003169E+00
15	8.86115755E-01	3.21543316E+00	5.56579302E+00
16	8.89259525E-01	3.25116938E+00	5.65810974E+00
17	9.01307564E-01	3.34667325E+00	5.93522957E+00
18	9.04090829E-01	3.38393788E+00	6.02627191E+00
19	9.16745176E-01	3.4435273E+00	6.34102431E+00
20	9.18090757E-01	3.45843495E+00	6.42370832E+00
21	9.30231795E-01	3.66144049E+00	6.74481350E+00

TIME IS 1.3430434E+02 SEC.

PNT	H	V	Q
1	8.27870938E-01	2.91143859E+00	4.00000000E+00
2	8.28464552E-01	2.92070843E+00	4.58000621E+00
3	8.31071431E-01	2.92349081E+00	4.60323438E+00
4	8.32170885E-01	2.93357677E+00	4.62877823E+00
5	8.34409932E-01	2.93901028E+00	4.65074457E+00
6	8.35613571E-01	2.94088389E+00	4.67311383E+00
7	8.38023785E-01	2.95430359E+00	4.70116189E+00
8	8.39192835E-01	2.96470628E+00	4.72946030E+00
9	8.41178095E-01	2.98097972E+00	4.75870878E+00
10	8.429719137E-01	2.97797707E+00	4.78951976E+00
11	8.45635517E-01	2.99406819E+00	4.82685634E+00
12	8.47477927E-01	3.00324498E+00	4.8577449E+00
13	8.50415527E-01	3.01951439E+00	4.91320833E+00
14	8.51757382E-01	3.03036832E+00	4.9500389E+00
15	8.55232440E-01	3.05033633E+00	5.02644657E+00
16	8.57297801E-01	3.07952415E+00	5.07017149E+00
17	8.62364612E-01	3.11865226E+00	5.17602748E+00
18	8.63927215E-01	3.13892292E+00	5.22857506E+00
19	8.70114673E-01	3.17946948E+00	5.36741647E+00
20	8.7166381E-01	3.1946948E+00	5.42719176E+00
21	8.78879157E-01	3.27882670E+00	5.60121507E+00

TIME IS 1.46659941E+02 SEC.

PNT	H	V	Q
1	8.21941649E-01	2.94237247E+00	4.00000000E+00
2	8.22911540E-01	2.94947449E+00	4.57872478E+00
3	8.24850228E-01	2.9564181E+00	4.59771976E+00
4	8.28882692E-01	2.99448309E+00	4.61673713E+00
5	8.27905558E-01	2.99425837E+00	4.63745915E+00
6	8.28737993E-01	2.99566708E+00	4.65673391E+00
7	8.31007442E-01	2.97337849E+00	4.67492079E+00
8	8.32014101E-01	2.980803713E+00	4.68860411E+00
9	8.34100736E-01	2.99324522E+00	4.7221256E+00
10	8.35054920E-01	2.99350523E+00	4.74292680E+00
11	8.37162480E-01	2.99945012E+00	4.7693967E+00
12	8.38036580E-01	3.00639016E+00	4.79087190E+00
13	8.40178800E-01	3.01625424E+00	4.82070801E+00
14	8.4090365E-01	3.02063178E+00	4.84353908E+00
15	8.43167794E-01	3.03108231E+00	4.87922033E+00
16	8.43089545E-01	3.06893009E+00	4.90419801E+00
17	8.46280846E-01	3.06335812E+00	4.94829415E+00
18	8.46097283E-01	3.07760942E+00	4.97637308E+00
19	8.49566218E-01	3.07780802E+00	5.03253739E+00
20	8.50994138E-01	3.11454771E+00	5.06454606E+00
21	8.53159876E-01	3.14602139E+00	5.13677493E+00

TIME IS 1.62955490E+02 SEC.

PNT	H	V	Q
1	8.16939376E-01	2.94873537E+00	4.00000000E+00
2	8.17838204E-01	2.97444614E+00	4.57676337E+00
3	8.19360092E-01	2.97061807E+00	4.59335970E+00
4	8.20494984E-01	2.98235569E+00	4.60959904E+00
5	8.22291528E-01	2.98461648E+00	4.62726783E+00
6	8.23630141E-01	2.990047324E+00	4.64376094E+00
7	8.25000405E-01	2.99326948E+00	4.66260747E+00
8	8.25958242E-01	2.99445740E+00	4.67941104E+00
9	8.27782963E-01	3.00208894E+00	4.69951378E+00
10	8.28026106E-01	3.00968011E+00	4.71665472E+00
11	8.30424173E-01	3.01413649E+00	4.73811103E+00
12	8.31164707E-01	3.02148724E+00	4.75561307E+00
13	8.32910903E-01	3.0270664E+00	4.77459902E+00
14	8.33534738E-01	3.0326588E+00	4.7905139E+00
15	8.3561905E-01	3.04228506E+00	4.82137214E+00
16	8.35669029E-01	3.05148414E+00	4.83980755E+00
17	8.37297396E-01	3.06032319E+00	4.86715755E+00
18	8.37596592E-01	3.07077774E+00	4.88631017E+00
19	8.39134744E-01	3.0819937E+00	4.91710355E+00
20	8.39424524E-01	3.0939750E+00	4.93732266E+00
21	8.40736409E-01	3.1083833E+00	4.97315599E+00

TIME IS 1.79201039E+02 SEC.

PNT	H	V	Q
1	8.12700707E-01	2.99133799E+00	4.00000000E+00
2	8.13020393E-01	2.9981715E+00	4.57546804E+00
3	8.15036821E-01	2.99761048E+00	4.5845744E+00
4	8.15880281E-01	3.00231509E+00	4.6039653E+00
5	8.17457257E-01	3.00400312E+00	4.61874210E+00
6	8.18314431E-01	3.00885301E+00	4.63292180E+00
7	8.19924951E-01	3.01102543E+00	4.64911571E+00
8	8.20737146E-01	3.0162132E+00	4.66354671E+00
9	8.22389547E-01	3.0194773E+00	4.6800503E+00
10	8.2309494E-01	3.02472173E+00	4.69564202E+00
11	8.24673342E-01	3.0283632E+00	4.71399129E+00
12	8.25328350E-01	3.0334592E+00	4.72902980E+00
13	8.2684555E-01	3.03923473E+00	4.74445422E+00
14	8.27381230E-01	3.04423899E+00	4.76370106E+00
15	8.28810805E-01	3.05152203E+00	4.7820205E+00
16	8.2949494E-01	3.0576032E+00	4.7995705E+00
17	8.3050248E-01	3.06670895E+00	4.82121970E+00
18	8.3070745E-01	3.07250230E+00	4.83670176E+00
19	8.31869866E-01	3.08393321E+00	4.8594991E+00
20	8.31862246E-01	3.0836234E+00	4.8759991E+00
21	8.32851591E-01	3.10402713E+00	4.89947434E+00

TIME IS 1.9554580E+02 SEC.

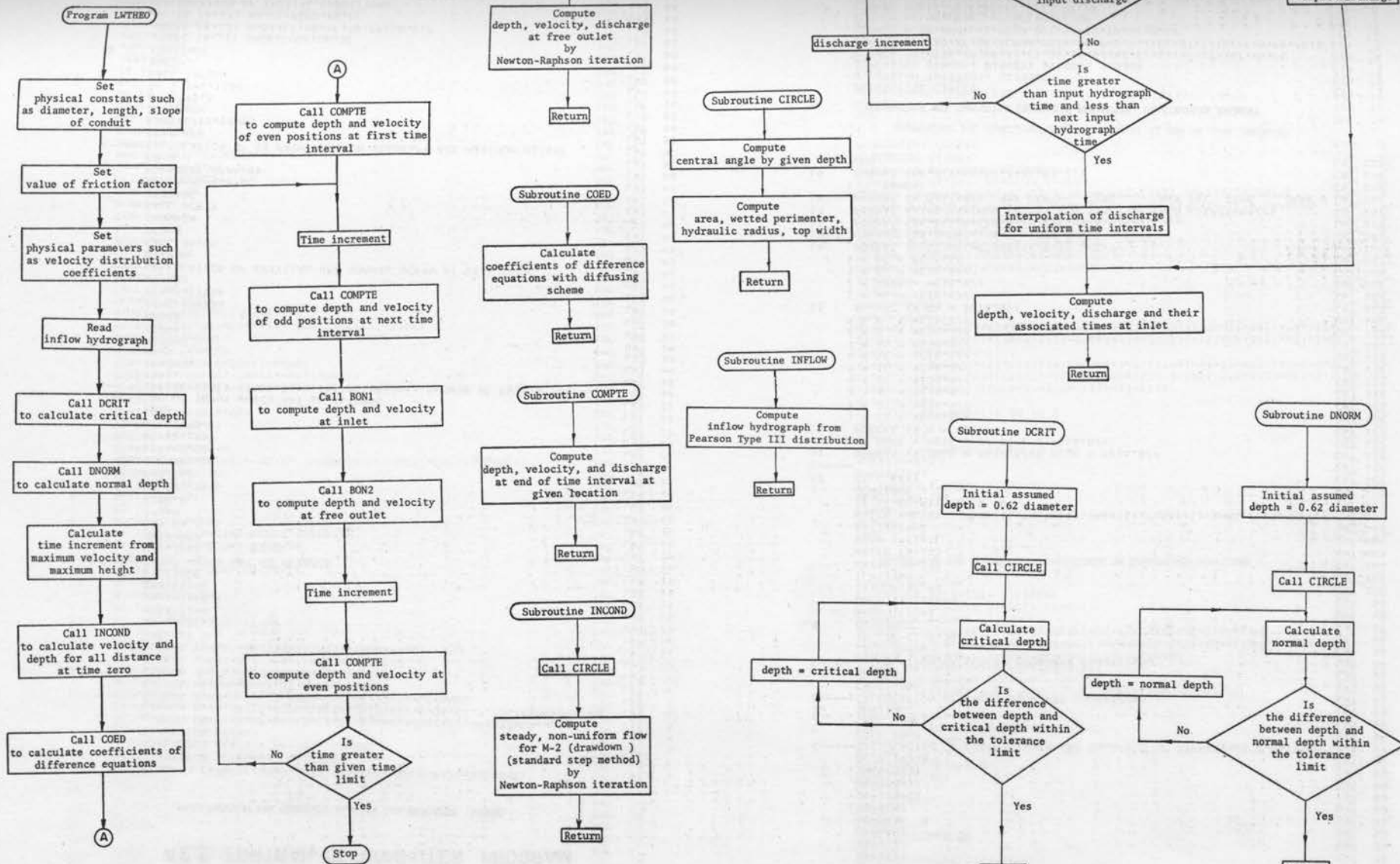
PNT	H	V	Q
1	8.09109293E-01	3.01079943E+00	4.00000000E+00
2	8.0983216E-01	3.01474454E+00	4.57431780E+00
3	8.11102653E-01	3.0127346E+00	4.58649591E+00
4	8.1192036E-01	3.01967605E+00	4.5984403E+00
5	8.13310194E-01	3.02094490E+00	4.6114665E+00
6	8.14074620E-01	3.0249825E+00	4.62367432E+00
7	8.15441360E-01	3.0208806E+00	4.63762557E+00
8	8.1621703E-01	3.0110001E+00	4.65012748E+00
9	8.1703637E-01	3.0133664E+00	4.66503212E+00
10	8.1829066E-01	3.01920481E+00	4.67781691E+00
11	8.19064730E-01	3.0242445E+00	4.69360792E+00
12	8.20202316E-01	3.03583710E+00	4.70869189E+00
13	8.2154379E-01	3.05054799E+00	4.72345400E+00
14	8.2206723E-01	3.0562876E+00	4.7386882E+00
15	8.2330202E-01	3.0614644E+00	4.75426615E+00
16	8.2363313E-01	3.06832451E+00	4.7676487E+00
17	8.2475991E-01	3.0742647E+00	4.78594394E+00
18	8.2491685E-01	3.0819668E+00	4.7991926E+00
19	8.2589196E-01	3.0841338E+00	4.8183104E+00
20	8.2582743E-01	3.0870366E+00	4.8314271E+00
21	8.2659272E-01	3.1084229E+00	4.85119929E+00

MAXIMUM VALUES AND TIMES AT EACH LOCATION

DISTANCE	MAX UERTH	TIME	MAX VEL	TIME	MAX Q	TIME
0.00	1.10	48.89	4.86	32.59	10.00	48.85
20.47	1.10	48.89	4.81	32.89	10.71	48.74
40.94	1.09	51.60	4.77	35.31	10.60	48.88
61.42	1.09	51.60	4.73	38.02	10.69	48.66
81.89	1.08	54.32	4.70	40.74	10.39	48.39
102.36	1.08	57.03	4.67	40.74	10.28	47.83
122.83	1.07	59.75	4.65	46.17	10.17	47.03
143.30	1.07	59.75	4.62	46.17	10.06	47.03
163.77	1.06	62.47	4.60	51.60	9.94	46.74
184.25	1.05	62.47	4.57	51.60	9.83	46.47
204.72	1.05	67.03	4.55	57.03	9.72	45.14
225.19	1.04	67.03	4.53	57.03	9.62	45.14
245.66	1.04	70.61	4.51	62.67	9.50	47.90
266.13	1.03	73.33	4.49	62.67	9.39	70.61
286.60	1.03	76.04	4.47	67.00	9.28	73.33
307.08	1.02	76.04	4.45	67.00	9.17	73.33
327.55	1.01	81.48	4.43	73.33	9.06	70.61
348.02	1.01	81.48	4.41	73.33	8.96	70.61
368.49	1.00	84.10	4.39	76.03	8.86	81.48
388.96	1.00	86.01	4.37	78.74	8.74	84.10
409.43	.99	89.63	4.35	81.48	8.65	86.01

APPENDIX 2 COMPUTATIONAL DETAILS OF LAX-WENDROFF SCHEME

A.2.1. FLOW CHART



SUBROUTINE FOR COMPUTING INITIAL CONDITION

```

SUBROUTINE INCOND
  DIMENSION TQ(3300), QI(3300)
  DIMENSION QI(400), D(400), V(400), X(330)
  COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,G2,A2,C2,E2,VP,DTA,QT,MB,DC
  COMMON DIA,XO,XF,GR,ALPHA,BETA,SO,F,H,V,G,DX,DT,T,TG,TF,N,FB,FC,B
  COMMON M,MM,MMH,L,I,PERD,DDT,VA,IG,TG,QI,NQCD
  COMMON HA,HH,VM,H,VT,NPT,HH,G,QB
  COMMON THETA,W,P,R,DEPTH,VC,J,HN,VN
  DTOL=0.00001
  ELEV=100.0
  XX=4.5*DC
  ELEV=ELEV+SO*XX
  IF (DN-DC) 1,1,2
  K=1
  GO TO 17
  DIN=1.75*DC
  DEPTH=DC
  CALL CIRCLE
  VV=QB/A
  VH=(VV*VV)/(2.0*GR)
  S1=F*VH/(4.0*R)
  EE1=DC+ALPHA*VH
  X(N+1)=XF-XX
  D(N+1)=DC
  Q(N+1)=QB
  V(N+1)=VV
  NCOUNT=0
  DO 16 L=1,N
  XX=XX+DX
  DEPTH=DIN
  CALL CIRCLE
  HFTH=0.5*THETA
  DTHET=4.0/(DIA*SINF(HFTH))
  DAREA=0.125*OIA*DIA*(1.0-COSF(THETA))*DTHET
  DW=0.5*DIA*DTHET
  WP=0.5*DIA*THETA
  DRA=(WP*DAREA-A*DW)/(WP*WP)
  DENG=1.0-(QB*QB/(GR*(A**3)))*DAREA
  DSLQ=-F*QB*QB*(2.0*R*A*DAREA+(A**2)*DRA)/(8.0*GR*((R*A**2)**2))
  VV=QB/A
  VH=(VV*VV)/(2.0*GR)
  S2=F*VH/(4.0*R)
  SF=(S1+S2)/2.0
  EE2=DIN+ALPHA*VH
  FRATIO=(EE2-EE1+DX*(SO-SF))/(DENG+(EE2-EE1)*DSLQ/(SO-SF))
  DCOM=DIN-FRATIO
  IF (DCOM) 5,4,6
  WRITE (6,19)
  GO TO 18
  DCOM=ABSF(DCOM)
  IF (ABSF(DCOM-DIN)-DTOL) 15,15,7
  IF (0.82*DIA-DCOM) 8,14,14
  DIN=DCOM/2.0
  IF (0.82*DIA-DIN) 10,10,11
  DIN=DIN/2.0
  NCOUNT=NCOUNT+1
  GO TO 9
  IF (NCOUNT-20) 12,12,13
  GO TO 3
  WRITE (6,20)
  GO TO 18
  DIN=DCOM
  GO TO 3
  DIN=DCOM
  S1=S2
  EE1=EE2
  II=N-L+1
  X(II)=XF-XX
  D(II)=DIN
  V(II)=VV
  Q(II)=QB
  CONTINUE
  GO TO 18
  WRITE (6,21) K
  RETURN
C-----
  FORMAT (* DCOM EQUALS ZERO *)
  FORMAT (25H 02 MUCH GREATER THAN DIA)
  FORMAT (* STOP *,13)
  END

```

SUBROUTINE FOR COMPUTING INFLOW HYDROGRAPH

```

SUBROUTINE INFLOW
  DIMENSION TQ(3300), QI(3300)
  DIMENSION QI(400), H(400), V(400)
  DIMENSION G(330)
  COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,G2,A2,C2,E2,VP,DTA,QT,MB,DC
  COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,G,DX,DT,T,TG,TF,N,FB,FC,B
  COMMON M,MM,MMH,L,I,PERD,DDT,VA,IG,TG,QI,NQCD
  COMMON HA,HH,VM,H,VT,NPT,HH,G,QB
  COMMON THETA,W,P,R,DEPTH,VC,J,HN,VN
  QO=10.0
  TP=100.0
  TG=150.0
  UU=TP/(TG-TP)
  AJ=J
  TQ(J)=(AJ-1.0)*DT
  QI(J)=QB+GO*(EXPF(-(TQ(J)-TP)/(TG-TP)))*(TG(J)/TP)**UU
  RETURN
  END

```

SUBROUTINE FOR COMPUTING DEPTH & VELOCITY AT END OF TIME INTERVAL

```

SUBROUTINE COMPIE
  DIMENSION G(330)
  DIMENSION TQ(3300), QI(3300)
  DIMENSION QI(400), H(400), V(400)
  COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,G2,A2,C2,E2,VP,DTA,QT,MB,DC
  COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,G,DX,DT,T,TG,TF,N,FB,FC,B
  COMMON M,MM,MMH,L,I,PERD,DDT,VA,IG,TG,QI,NQCD
  COMMON HA,HH,VM,H,VT,NPT,HH,G,QI
  COMMON THETA,W,P,R,DEPTH,VC,J,HN,VN
  COMMON QMAX(400),VMAX(400),HMAX(400)
  COMMON TQMAX(400),TVMAX(400),THMAX(400)
  Z1=DT/DX
  Z2=A/B
  Z3=ALPHA/BETA
  Z4=GR/BETA
  H(1)=H(1)-0.5*Z1*(Z2*(V(1+1)-V(1-1))+V(1)*(H(1+1)-H(1-1)))+.5*(Z1*COM
  171)*((Z3+1.0)*V(1)+Z2*(V(1+1)-2.0*V(1)+V(1-1))+(Z2*Z4+V(1)**2)*H(1)
  181)-2.0*H(1)+H(1-1))
  V(1)=V(1)-0.5*Z1*(Z3*V(1)*(V(1+1)-V(1-1))+Z4*(H(1+1)-H(1-1))+2.0*DCOM
  191)*Z4*E2)+0.5*Z1*(Z3*Z3*V(1)**2+Z4*Z2)*(V(1+1)-2.0*V(1)+V(1-1))
  201)+Z3+1.0)*Z4*V(1)*(H(1+1)-2.0*H(1)+H(1-1))
  Q(1)=V(1)*A
  IF (H(1).LT.HMAX(1)) GO TO 1
  HMAX(1)=H(1)
  THMAX(1)=T
  1 IF (V(1).LT.VMAX(1)) GO TO 2
  VMAX(1)=V(1)
  TVMAX(1)=T
  2 IF (Q(1).LT.QMAX(1)) GO TO 3
  QMAX(1)=Q(1)
  TQMAX(1)=T
  3 RETURN
  END

```

SUBROUTINE FOR COMPUTING COEFFICIENTS IN DIFFERENCE EQUATIONS

```

SUBROUTINE COED
  DIMENSION TQ(3300), QI(3300)
  DIMENSION QI(400), H(400), V(400)
  DIMENSION G(330)
  COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,G2,A2,C2,E2,VP,DTA,QT,MB,DC
  COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,G,DX,DT,T,TG,TF,N,FB,FC,B
  COMMON M,MM,MMH,L,I,PERD,DDT,VA,IG,TG,QI,NQCD
  COMMON HA,HH,VM,H,VT,NPT,HH,G,QI
  COMMON THETA,W,P,R,DEPTH,VC,J,HN,VN
  THETA=2.0*ATANF((SQRTF(D*H(1)-H(1)**2)))/(D/2.0-n(1))
  IF (THETA) 1,2,2
  1 THETA=6.28318*THETA
  2 A=0.125*(THETA-SINF(THETA))*(D*D)
  WP=(D/2.0)*THETA
  R=A*WP
  B=D*SINF(THETA/2.0)
  A1=A/(V(1)*B)
  C1=1.0
  D1=1.0/V(1)
  E1=0.0
  B2=BETA/GR
  A2=V(1)*ALPHA/GR
  C2=1.0
  SF=.125*F*B2*V(1)*V(1)/R
  E2=SF-SO
  RETURN
  END

```


37

[illegible]

A2.4. SAMPLE OUTPUT

(No input required)

```

TF = 2.00000000E+02
DNQB = 1.22858217E+00
DCQB = 1.0078548E+00
DNPB = 1.22858217E+00
DCPB = 1.0078548E+00
N = 20
DX = 4.0582327E+01
DT = 2.04629670E+00
XO = 0.
XF = 8.21700000E+02
TO = 0.

TIME IS 0. SEC.
PNT 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
1 1.22751153E+00 3.73669328E+00 1.00000000E+01
2 1.22725478E+00 3.73772414E+00 1.00000000E+01
3 1.22699597E+00 3.73900945E+00 1.00000000E+01
4 1.22653979E+00 3.74066024E+00 1.00000000E+01
5 1.22604705E+00 3.74259188E+00 1.00000000E+01
6 1.22553304E+00 3.74480734E+00 1.00000000E+01
7 1.22497683E+00 3.74731549E+00 1.00000000E+01
8 1.22437395E+00 3.75014432E+00 1.00000000E+01
9 1.22372664E+00 3.75327359E+00 1.00000000E+01
10 1.22304173E+00 3.75671430E+00 1.00000000E+01
11 1.22232208E+00 3.76042488E+00 1.00000000E+01
12 1.22158436E+00 3.76441100E+00 1.00000000E+01
13 1.22082589E+00 3.76868400E+00 1.00000000E+01
14 1.21995131E+00 3.77324298E+00 1.00000000E+01
15 1.21907587E+00 3.77809371E+00 1.00000000E+01
16 1.21819218E+00 3.78324298E+00 1.00000000E+01
17 1.21729814E+00 3.78869371E+00 1.00000000E+01
18 1.21629838E+00 3.79441100E+01 1.00000000E+01
19 1.21519218E+00 3.80042488E+00 1.00000000E+01
20 1.21397587E+00 3.80671430E+00 1.00000000E+01
21 1.21265131E+00 3.81327359E+00 1.00000000E+01

TIME IS 1.25777802E+01 SEC.
PNT 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
1 1.2433175E+00 4.00745144E+00 1.09089652E+01
2 1.24233490E+00 4.01093923E+00 1.02965757E+01
3 1.24039333E+00 3.78135739E+00 1.01532011E+01
4 1.2377535E+00 3.76282951E+00 1.00829123E+01
5 1.22815193E+00 3.75372160E+00 1.00403455E+01
6 1.2263453E+00 3.75043137E+00 1.00188662E+01
7 1.22512882E+00 3.75067417E+00 1.00084471E+01
8 1.22329290E+00 3.75074060E+00 1.00035452E+01
9 1.22262615E+00 3.7521188E+00 1.00012804E+01
10 1.22109743E+00 3.75239478E+00 1.00002205E+01
11 1.21922238E+00 3.77031407E+00 9.99368795E+00
12 1.21687818E+00 3.77797425E+00 9.99336699E+00
13 1.21391552E+00 3.79192231E+00 9.99311226E+00
14 1.21013645E+00 3.80753208E+00 9.99855979E+00
15 1.20526892E+00 3.82783191E+00 9.99855630E+00
16 1.19891042E+00 3.85467948E+00 9.99822386E+00
17 1.19042701E+00 3.89108931E+00 9.99792128E+00
18 1.17868937E+00 3.94273126E+00 9.99846978E+00
19 1.16076100E+00 4.02694457E+00 1.00088499E+01
20 1.12184014E+00 4.21721499E+00 1.00334580E+01
21 1.01175211E+00 4.88339999E+00 1.00747613E+01

TIME IS 2.51555604E+01 SEC.
PNT 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
1 1.26389703E+00 4.61171407E+00 1.28272130E+01
2 1.26389703E+00 4.02043707E+00 1.15537030E+01
3 1.26791237E+00 4.01158796E+00 1.12272947E+01
4 1.26694725E+00 3.93593134E+00 1.08722562E+01
5 1.25350423E+00 3.89089171E+00 1.06001932E+01
6 1.24380276E+00 3.83838915E+00 1.03955126E+01
7 1.23647535E+00 3.80459917E+00 1.02506293E+01
8 1.23032025E+00 3.78965713E+00 1.01530666E+01
9 1.22690571E+00 3.77944730E+00 1.00902771E+01
10 1.22358088E+00 3.77594859E+00 1.00514419E+01
11 1.22007414E+00 3.7780831E+00 1.00282383E+01
12 1.21711135E+00 3.78400859E+00 1.00147614E+01
13 1.2142813E+00 3.79429157E+00 1.00070869E+01
14 1.21026555E+00 3.80892381E+00 1.00027343E+01
15 1.20525563E+00 3.82877419E+00 1.00002145E+01
16 1.19877501E+00 3.85555288E+00 9.9985818E+00
17 1.19013001E+00 3.89270788E+00 9.99868014E+00
18 1.17759714E+00 3.94730051E+00 9.99873389E+00
19 1.15726467E+00 4.04421575E+00 1.00206899E+01
20 1.1228252E+00 4.26691408E+00 1.00254592E+01
21 1.01042234E+00 4.87974435E+00 1.00492244E+01

TIME IS 3.77333405E+01 SEC.
PNT 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
1 1.36443243E+00 4.86214779E+00 1.44444247E+01
2 1.40577995E+00 4.29563497E+00 1.35154404E+01
3 1.35449104E+00 4.36627755E+00 1.31220509E+01
4 1.3375047E+00 4.24790218E+00 1.25595495E+01
5 1.31619048E+00 4.16833245E+00 1.20746697E+01
6 1.29709912E+00 4.0804443E+00 1.16300218E+01
7 1.27991452E+00 4.01648354E+00 1.12436850E+01
8 1.26304211E+00 3.9527734E+00 1.09203191E+01
9 1.25251210E+00 3.90583156E+00 1.06606849E+01
10 1.24215562E+00 3.86875254E+00 1.04604382E+01
11 1.23361813E+00 3.84348484E+00 1.03117817E+01
12 1.22642768E+00 3.82907039E+00 1.02052739E+01
13 1.22005247E+00 3.82428418E+00 1.01314160E+01
14 1.21393286E+00 3.82844508E+00 1.00816784E+01
15 1.20748294E+00 3.84134711E+00 1.00491938E+01
16 1.19998430E+00 3.86387733E+00 1.00282326E+01
17 1.19049050E+00 3.89991477E+00 1.00181848E+01
18 1.17848469E+00 3.9584358E+00 1.00129816E+01
19 1.15346822E+00 4.07358420E+00 1.00408847E+01
20 1.10637086E+00 4.29852423E+00 1.00210820E+01
21 1.00969596E+00 4.87778591E+00 1.00352876E+01

TIME IS 5.03111207E+01 SEC.
PNT 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
1 1.31861795E+00 4.77676497E+00 1.68378584E+01
2 1.49634822E+00 4.46698836E+00 1.57208776E+01
3 1.43921743E+00 4.70059050E+00 1.56817287E+01
4 1.42635078E+00 4.56580420E+00 1.48614249E+01
5 1.40027274E+00 4.5005991E+00 1.41151200E+01
6 1.37743993E+00 4.41661263E+00 1.36094984E+01
7 1.3570831E+00 4.33476797E+00 1.30278762E+01
8 1.33281433E+00 4.25335657E+00 1.25210605E+01
9 1.31206383E+00 4.1754217E+00 1.20519030E+01
10 1.29281266E+00 4.1036047E+00 1.16303147E+01
11 1.27531013E+00 4.04080432E+00 1.12637385E+01
12 1.25964440E+00 3.98487517E+00 1.09555194E+01
13 1.24569914E+00 3.94968446E+00 1.07051823E+01
14 1.23314072E+00 3.92379473E+00 1.05080046E+01
15 1.22143404E+00 3.91203499E+00 1.03577673E+01
16 1.20971111E+00 3.9155784E+00 1.02462939E+01
17 1.19689162E+00 3.93428257E+00 1.01704214E+01
18 1.17421616E+00 3.99343702E+00 1.01189727E+01
19 1.15282619E+00 4.1097771E+00 1.01116177E+01
20 1.10467303E+00 4.32741493E+00 1.00589899E+01
21 1.0100437E+00 4.87862681E+00 1.00412039E+01

TIME IS 6.28889009E+01 SEC.
PNT 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
1 1.83343203E+00 4.74140525E+00 1.83078435E+01
2 1.56439295E+00 4.87348339E+00 1.76925965E+01
3 1.50793355E+00 4.93051129E+00 1.72263458E+01
4 1.51206340E+00 4.80524287E+00 1.66574607E+01
5 1.48481239E+00 4.7695940E+00 1.61534188E+01
6 1.46224606E+00 4.7003818E+00 1.5612795E+01
7 1.43951100E+00 4.63444133E+00 1.5074372E+01
8 1.41622990E+00 4.56381344E+00 1.45333699E+01
9 1.39285892E+00 4.49086474E+00 1.3994674E+01
10 1.36958417E+00 4.41639277E+00 1.34664774E+01
11 1.34658655E+00 4.34214118E+00 1.2970602E+01
12 1.32410358E+00 4.27042484E+00 1.25234941E+01
13 1.30237067E+00 4.20412515E+00 1.2122472E+01
14 1.28152962E+00 4.14046185E+00 1.1734820E+01
15 1.26152722E+00 4.10107378E+00 1.13792132E+01
16 1.24183718E+00 4.07226388E+00 1.10732941E+01
17 1.22147844E+00 4.06799058E+00 1.0855172E+01
18 1.19678711E+00 4.10053475E+00 1.0641488E+01
19 1.16451661E+00 4.19728424E+00 1.0431488E+01
20 1.11370489E+00 4.34183924E+00 1.02862345E+01
21 1.01892655E+00 4.89739257E+00 1.01737643E+01

TIME IS 7.54666811E+01 SEC.
PNT 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
1 1.98085001E+00 4.82881162E+00 1.93025499E+01
2 1.91044890E+00 5.03261191E+00 1.89418304E+01
3 1.58701486E+00 5.02571158E+00 1.85775821E+01
4 1.58056699E+00 4.93278939E+00 1.81388311E+01
5 1.55492461E+00 4.92794748E+00 1.77461561E+01
6 1.53670802E+00 4.87841255E+00 1.72476741E+01
7 1.51547683E+00 4.83713297E+00 1.68440311E+01
8 1.49425171E+00 4.7887359E+00 1.63710599E+01
9 1.47232865E+00 4.73713606E+00 1.58849014E+01
10 1.44986239E+00 4.68187507E+00 1.53867840E+01
11 1.42682953E+00 4.62362222E+00 1.48804933E+01
12 1.40323775E+00 4.56314918E+00 1.43591435E+01
13 1.37909414E+00 4.50169367E+00 1.38551322E+01
14 1.35439035E+00 4.44130424E+00 1.3350333E+01
15 1.32904766E+00 4.3853142E+00 1.28688276E+01
16 1.30264633E+00 4.33915427E+00 1.2409576E+01
17 1.27421620E+00 4.31269655E+00 1.19911620E+01
18 1.24138424E+00 4.3199829E+00 1.16161700E+01
19 1.20109247E+00 4.38373931E+00 1.12940822E+01
20 1.14571362E+00 4.54300973E+00 1.0943506E+01
21 1.04321629E+00 4.96884191E+00 1.06877794E+01

```

TIME IS 8.000000E+01 SEC.				TIME IS 1.500333E+02 SEC.			
PNT	V	Q		PNT	V	Q	
1	1.08327355E+00	4.3502275E+00	1.9054744E+01	7	1.7004444E+00	4.4732941E+00	1.82255254E+01
2	1.08327355E+00	4.3502275E+00	1.9054744E+01	8	1.08327355E+00	4.6202928E+00	1.85520726E+01
3	1.08327355E+00	4.3502275E+00	1.9054744E+01	9	1.08327355E+00	4.58975469E+00	1.85087567E+01
4	1.08327355E+00	4.3502275E+00	1.9054744E+01	10	1.08327355E+00	4.63430770E+00	1.85516948E+01
5	1.08327355E+00	4.3502275E+00	1.9054744E+01	11	1.0778023E+00	4.64189479E+00	1.85286565E+01
6	1.08327355E+00	4.3502275E+00	1.9054744E+01	12	1.07257824E+00	4.65597758E+00	1.85096738E+01
7	1.08327355E+00	4.3502275E+00	1.9054744E+01	13	1.06606435E+00	4.6770346E+00	1.84797125E+01
8	1.08327355E+00	4.3502275E+00	1.9054744E+01	14	1.05850970E+00	4.69579362E+00	1.84257742E+01
9	1.08327355E+00	4.3502275E+00	1.9054744E+01	15	1.0500527E+00	4.70161753E+00	1.83673361E+01
10	1.08327355E+00	4.3502275E+00	1.9054744E+01	16	1.04161640E+00	4.71287425E+00	1.82916306E+01
11	1.08327355E+00	4.3502275E+00	1.9054744E+01	17	1.03209666E+00	4.72569555E+00	1.82061295E+01
12	1.08327355E+00	4.3502275E+00	1.9054744E+01	18	1.02178551E+00	4.73836365E+00	1.81087791E+01
13	1.08327355E+00	4.3502275E+00	1.9054744E+01	19	1.01033131E+00	4.75274543E+00	1.80013965E+01
14	1.08327355E+00	4.3502275E+00	1.9054744E+01	20	1.00745879E+00	4.77007771E+00	1.78845474E+01
15	1.08327355E+00	4.3502275E+00	1.9054744E+01	21	1.00270221E+00	4.79246735E+00	1.77588499E+01
16	1.08327355E+00	4.3502275E+00	1.9054744E+01		1.00549226E+00	4.82232115E+00	1.76237638E+01
17	1.08327355E+00	4.3502275E+00	1.9054744E+01		1.04517070E+00	4.86255413E+00	1.74783317E+01
18	1.08327355E+00	4.3502275E+00	1.9054744E+01		1.02052020E+00	4.9175616E+00	1.73179381E+01
19	1.08327355E+00	4.3502275E+00	1.9054744E+01		1.00889908E+00	5.00385251E+00	1.71567794E+01
20	1.08327355E+00	4.3502275E+00	1.9054744E+01		1.00374510E+00	5.14930118E+00	1.69513995E+01
21	1.08327355E+00	4.3502275E+00	1.9054744E+01		1.00273776E+00	5.21086451E+00	1.68610325E+01
TIME IS 1.000224E+02 SEC.				TIME IS 1.635114E+02 SEC.			
PNT	V	Q		PNT	V	Q	
1	1.0810347E+00	5.0054922E+00	1.99990144E+01	7	1.0809092E+00	4.3793591E+00	1.75066114E+01
2	1.0810347E+00	5.0054922E+00	1.99990144E+01	8	1.06740865E+00	4.4704723E+00	1.78404374E+01
3	1.0810347E+00	5.0054922E+00	1.99990144E+01	9	1.0774010E+00	4.4653445E+00	1.78400532E+01
4	1.0810347E+00	5.0054922E+00	1.99990144E+01	10	1.06810747E+00	4.52030201E+00	1.79364802E+01
5	1.0810347E+00	5.0054922E+00	1.99990144E+01	11	1.06826708E+00	4.52840942E+00	1.7958271E+01
6	1.0810347E+00	5.0054922E+00	1.99990144E+01	12	1.06377830E+00	4.55130947E+00	1.79937248E+01
7	1.0810347E+00	5.0054922E+00	1.99990144E+01	13	1.05871437E+00	4.5762759E+00	1.8011137E+01
8	1.0810347E+00	5.0054922E+00	1.99990144E+01	14	1.05455348E+00	4.58917097E+00	1.80105987E+01
9	1.0810347E+00	5.0054922E+00	1.99990144E+01	15	1.04812715E+00	4.6107058E+00	1.80045818E+01
10	1.0810347E+00	5.0054922E+00	1.99990144E+01	16	1.04190402E+00	4.62715645E+00	1.79804808E+01
11	1.0810347E+00	5.0054922E+00	1.99990144E+01	17	1.03435385E+00	4.64581582E+00	1.79466982E+01
12	1.0810347E+00	5.0054922E+00	1.99990144E+01	18	1.02594062E+00	4.66617314E+00	1.7897758E+01
13	1.0810347E+00	5.0054922E+00	1.99990144E+01	19	1.01634378E+00	4.68448439E+00	1.78419346E+01
14	1.0810347E+00	5.0054922E+00	1.99990144E+01	20	1.00517794E+00	4.70765466E+00	1.77732382E+01
15	1.0810347E+00	5.0054922E+00	1.99990144E+01	21	1.00205096E+00	4.73570201E+00	1.76941432E+01
16	1.0810347E+00	5.0054922E+00	1.99990144E+01		1.0050729E+00	4.77054734E+00	1.76038839E+01
17	1.0810347E+00	5.0054922E+00	1.99990144E+01		1.00578970E+00	4.8150776E+00	1.75025224E+01
18	1.0810347E+00	5.0054922E+00	1.99990144E+01		1.00372602E+00	4.87492666E+00	1.73864053E+01
19	1.0810347E+00	5.0054922E+00	1.99990144E+01		1.00356594E+00	4.9175775E+00	1.72777923E+01
20	1.0810347E+00	5.0054922E+00	1.99990144E+01		1.00474538E+00	5.0588954E+00	1.71199644E+01
21	1.0810347E+00	5.0054922E+00	1.99990144E+01		1.00314891E+00	5.13806720E+00	1.70751340E+01
TIME IS 1.300002E+02 SEC.				TIME IS 1.7008923E+02 SEC.			
PNT	V	Q		PNT	V	Q	
1	1.09718734E+00	4.90557153E+00	1.9009908E+01	1	1.057049791E+00	4.26669135E+00	1.67690168E+01
2	1.0844409E+00	4.9002442E+00	1.9009944E+01	2	1.04459336E+00	4.3705536E+00	1.71035431E+01
3	1.0844409E+00	4.9002442E+00	1.9009944E+01	3	1.0587684E+00	4.34599479E+00	1.71382843E+01
4	1.07246203E+00	4.8810347E+00	1.9355342E+01	4	1.05065596E+00	4.3506133E+00	1.72544197E+01
5	1.05705944E+00	4.89292415E+00	1.92144450E+01	5	1.05337164E+00	4.40977574E+00	1.73110763E+01
6	1.04909275E+00	4.8632328E+00	1.90238412E+01	6	1.04446505E+00	4.4425208E+00	1.73881796E+01
7	1.03505444E+00	4.8014687E+00	1.88478951E+01	7	1.04691736E+00	4.4567585E+00	1.74372122E+01
8	1.02290005E+00	4.84848110E+00	1.8696737E+01	8	1.04417284E+00	4.4848438E+00	1.74770188E+01
9	1.00943229E+00	4.8482721E+00	1.84451879E+01	9	1.03960527E+00	4.50994173E+00	1.75117785E+01
10	1.00359710E+00	4.8785668E+00	1.82278774E+01	10	1.03525615E+00	4.53301738E+00	1.75301019E+01
11	1.00096396E+00	4.87411391E+00	1.8090835E+01	11	1.02948169E+00	4.55381163E+00	1.75408306E+01
12	1.0058490E+00	4.8693245E+00	1.79990835E+01	12	1.02294804E+00	4.57695757E+00	1.75392824E+01
13	1.0047143E+00	4.86490398E+00	1.77577523E+01	13	1.01305028E+00	4.60256151E+00	1.75279950E+01
14	1.0031707E+00	4.8619147E+00	1.75034178E+01	14	1.00561702E+00	4.63120519E+00	1.75061782E+01
15	1.00156835E+00	4.8624740E+00	1.7255922E+01	15	1.00423358E+00	4.65661833E+00	1.74739366E+01
16	1.00172160E+00	4.87025508E+00	1.69538198E+01	16	1.0044997E+00	4.6846823E+00	1.74302295E+01
17	1.0015640E+00	4.8701791E+00	1.66579091E+01	17	1.0035099E+00	4.75116858E+00	1.73761425E+01
18	1.00320271E+00	4.89017391E+00	1.6344367E+01	18	1.00457992E+00	4.82130799E+00	1.73097392E+01
19	1.00258131E+00	4.9232944E+00	1.60129120E+01	19	1.01037461E+00	4.93158942E+00	1.72563226E+01
20	1.00400296E+00	4.9953404E+00	1.56406157E+01	20	1.00453532E+00	5.1389375E+00	1.71440448E+01
21	1.0023830E+00	5.1218170E+00	1.52511034E+01	21	1.00343458E+00	5.2413664E+00	1.71452381E+01
TIME IS 1.257780E+02 SEC.				TIME IS 1.88666703E+02 SEC.			
PNT	V	Q		PNT	V	Q	
1	1.71941163E+00	4.73319137E+00	1.94472341E+01	7	1.93578401E+00	4.1490178E+00	1.60428432E+01
2	1.69947183E+00	4.8103212E+00	1.94803882E+01	8	1.92747091E+00	4.25516794E+00	1.64011513E+01
3	1.69362593E+00	4.79624450E+00	1.93600776E+01	9	1.93824186E+00	4.23763055E+00	1.64596775E+01
4	1.68369642E+00	4.81627431E+00	1.92735612E+01	10	1.93136139E+00	4.29731412E+00	1.68022696E+01
5	1.67247508E+00	4.82880715E+00	1.91549718E+01	11	1.93577986E+00	4.30053597E+00	1.66669584E+01
6	1.66461759E+00	4.82054486E+00	1.90158293E+01	12	1.93235375E+00	4.33830479E+00	1.6590286E+01
7	1.65223711E+00	4.83329179E+00	1.89862973E+01	13	1.93189162E+00	4.35842430E+00	1.68368848E+01
8	1.64199152E+00	4.8320975E+00	1.87332601E+01	14	1.93001255E+00	4.38267406E+00	1.69051159E+01
9	1.63002836E+00	4.8361149E+00	1.85773755E+01	15	1.92699486E+00	4.40900948E+00	1.69549142E+01
10	1.6173265E+00	4.83777457E+00	1.84096194E+01	16	1.92403754E+00	4.43194485E+00	1.70126672E+01
11	1.60508366E+00	4.83997346E+00	1.82323236E+01	17	1.91646504E+00	4.4544529E+00	1.70545662E+01
12	1.59151707E+00	4.84240153E+00	1.80471417E+01	18	1.91456494E+00	4.48477425E+00	1.70841938E+01
13	1.57699904E+00	4.84600713E+00	1.78053442E+01	19	1.91456494E+00	4.51473760E+00	1.71103304E+01
14	1.56121476E+00	4.85204155E+00	1.76463741E+01	20	1.90011408E+00	4.54679438E+00	1.7254944E+01
15	1.54362863E+00	4.86271493E+00	1.74314542E+01	21	1.9024515E+00	4.58427755E+00	1.7317648E+01
16	1.52346457E+00	4.88134733E+00	1.72063579E+01		1.900338E+00	4.62877455E+00	1.71282500E+01
17	1.49968789E+00	4.91204442E+00	1.69677870E+01		1.90246653E+00	4.68352495E+00	1.71173976E+01
18	1.47112867E+00	4.95990206E+00	1.67123404E+01		1.9411744E+00	4.75944809E+00	1.70797690E+01
19	1.43502875E+00	5.03414447E+00	1.64312973E+01		1.90954306E+00	4.80951173E+00	1.70972439E+01
20	1.38539596E+00	5.16776444E+00	1.61133192E+01		1.95180138E+00	5.1099978E+00	1.70265000E+01
21	1.68187552E+00	5.6034463E+00	1.58779542E+01		1.33124310E+00	5.7332257E+00	1.70890972E+01

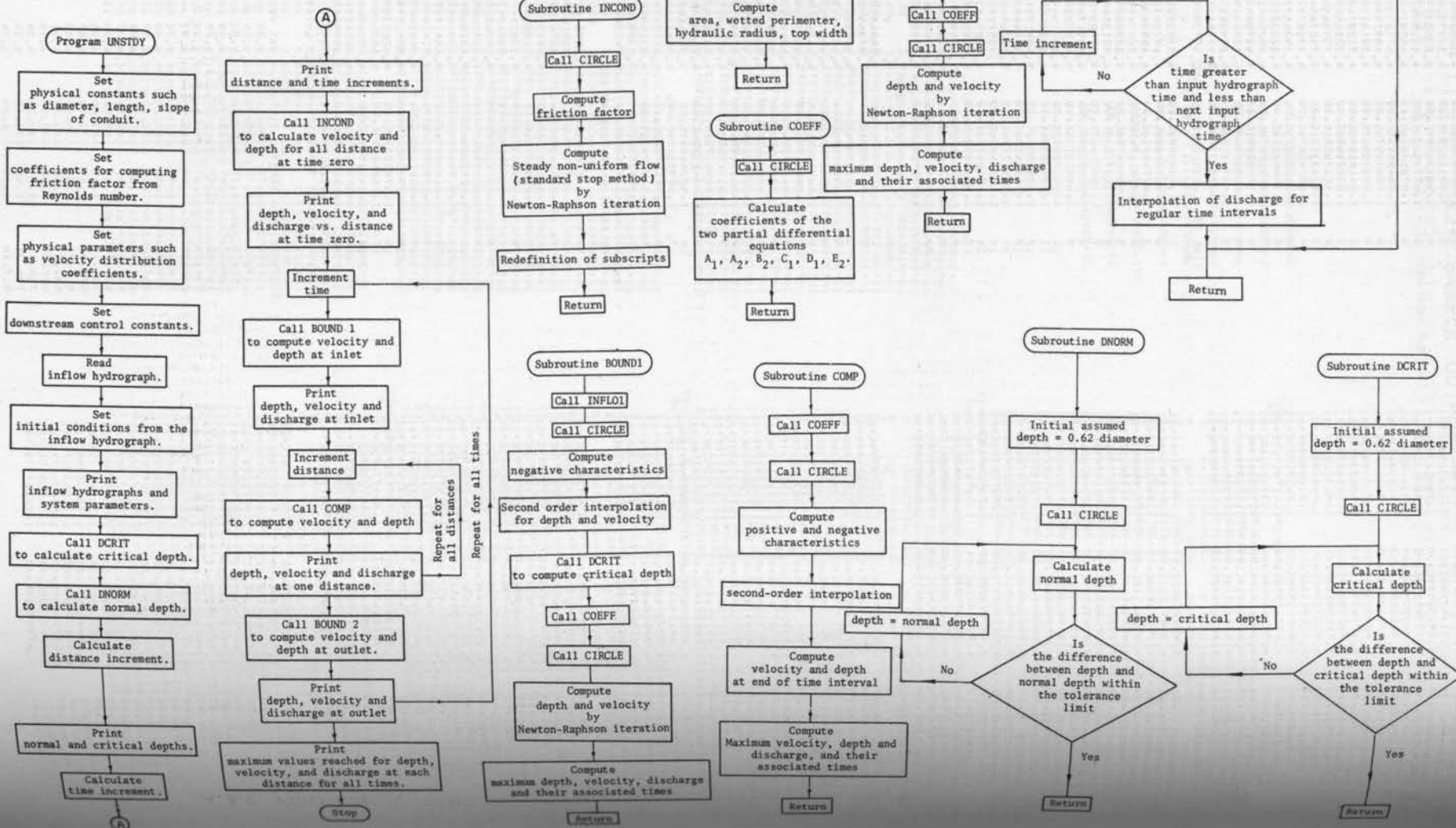
MAXIMUM VALUES AND TIMES AT EACH LOCATION

DISTANCE	MAX DEPTH	TIME	MAX VFL	TIME	MAX Q	TIME
0.00	1.73	134.16	5.00	98.53	20.00	100.62
40.86	1.70	129.97	5.03	83.85	19.74	100.62
81.72	1.70	136.24	5.03	79.66	19.52	106.91
122.57	1.69	135.24	4.96	85.95	19.37	111.10
163.43	1.68	142.54	4.98	90.14	19.21	115.30
204.29	1.67	144.64	4.95	92.24	19.04	119.49
245.15	1.66	148.94	4.93	92.33	18.89	121.59
286.01	1.66	150.93	4.92	98.53	18.73	125.78
326.87	1.65	155.17	4.90	100.62	18.59	129.97
367.72	1.64	157.27	4.89	102.72	18.44	134.16
408.58	1.63	161.41	4.88	106.91	18.30	138.35
449.44	1.63	163.51	4.87	109.01	18.16	140.46
490.30	1.62	167.70	4.86	113.20	18.02	144.64
531.16	1.61	169.80	4.86	115.30	17.89	148.83
572.02	1.59	173.49	4.87	119.49	17.76	153.02
612.87	1.58	176.09	4.88	121.59	17.63	155.13
653.73	1.56	180.24	4.91	125.78	17.61	159.31
694.59	1.54	182.34	4.96	127.87	17.39	163.50
735.45	1.51	180.28	5.04	129.97	17.29	167.69
776.31	1.45	178.19	5.18	136.24	17.15	171.88
817.16	1.33	176.09	5.74	176.09	17.15	176.09

APPENDIX 3

COMPUTATIONAL DETAILS OF FINITE-DIFFERENCE SPECIFIED INTERVALS SCHEME OF THE METHOD OF CHARACTERISTICS

A.3.1. FLOW CHART



A.3.2. FORTRAN IV COMPUTER PROGRAM

MAIN PROGRAM FOR UNSTEADY FLOW BY METHOD OF CHARACTERISTICS

```

PROGRAM UNSTDY
1 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C-----ATTENUATION ANALYSIS - CIRCULAR CROSS SECTION
C-----M1 INITIAL CONDITIONS
C-----DETERMINATION OF HYDROGRAPH AT THE SPECIFIC POINT WITH TWO CONTROLS
C----- ( AT UPSTREAM AND DOWNSTREAM ) BY THE METHOD OF CHARACTERISTICS
C-----FRICTION COEFFICIENT F VARIES WITH REYNOLDS NUMBER
DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
DIMENSION QI(200), QMAX(200)
DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
DIMENSION V(500), VDT(500), VMAX(200), X(500)
COMMON A,AR,AC,AD,AE,ALPHA,H,RC,RD,BETA,CD,CO,DO,DDT
COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DM,DDDT,DT,DTOL,DX,ED
COMMON F,FB,FC,FD,FNU,GR,I,ITD,ITOC,IXO,IXOC,J,MC,N,NGCD
COMMON NT,N1,Q,OB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
COMMON X,XE,XF,XX
C-----PHYSICAL CONSTANTS OF THE SYSTEM
DIA=2.9262
XF=821.70
SQ=0.001
C-----COEFFICIENTS FOR COMPUTING F FROM THE REYNOLDS NUMBER
FNU=0.0000141
FR=0.109394
FC=0.17944
C-----PHYSICAL PARAMETERS
GP=32.175
ALPHA=1.00
BETA=1.00
C-----DOWNSTREAM CONTROL CONSTANTS
CD=0.0
ED=1.35
C-----COMPUTATIONAL PARAMETERS
N=20
IXO=2
TF=200.
TIO=20.
DTOL=0.00001
C-----INFLOW HYDROGRAPH
READ (5,200) NGCD
READ (5,210) (TQ(I),QI(I),J=1,NGCD)
QR=QI(1)
QP=QI(2)
TP=TQ(2)
VOL=(QP-QR)*TP
QRA=QR/QP
N1=N+1
DO 10 I=1,N1
  DMAX(I)=0.0
  VMAX(I)=0.0
  QMAX(I)=0.0
10 WRITE (6,220)
   WRITE (6,270)
   WRITE (6,230) QR
   WRITE (6,250) QP
   WRITE (6,260) TP
   WRITE (6,320) QRA
   WRITE (6,240) VOL
   WRITE (6,270)
   WRITE (6,280)
   WRITE (6,290) SQ
   WRITE (6,300) ALPHA
   WRITE (6,310) BETA

```

```

WRITE (6,270)
WRITE (6,330) N,ITD,IF,110
C-----COMPUTATION OF NORMAL DEPTH AND CRITICAL DEPTH
QO=QB
CALL DCRIT
CALL DNORM
IF (DN-DC) 20+20,30
20 WRITE (6,340)
   GO TO 190
30 IF (CD) 40,50,60
40 MC=1
   XX=0.0
   DOIT=(OB/CD)**(1.0/FU)
   GO TO 60
50 MC=2
   XX=4.5*DC
   DOIT=DC
60 XF=XF-XX
   AN=N
   DX=XE/AN
   WRITE (6,150) DN,DC
C-----COMPUTATION OF DT ( TIME INCREMENT )
QO=QP
CALL DCRIT
DEPTH=DC
CALL CIRCLE
VC=QP/A
DEPTH=0.8*DEPTH
CALL CIRCLE
DTMAX=(DX*2.0*BETA)/(VC*(ALPHA+BETA)+SQRT(((ALPHA+BETA)**2)+VC*VC))
I=(4.0*BETA*GR*DM)))
DT=DTMAX*.5
CO=DT/DT
NT=TF/DT
ITO=TIO/DT
WRITE (6,360) DX,DT
C-----COMPUTATION OF VELOCITY AND DEPTH FOR ALL DISTANCES X AT TIME 0.0
DIN=(DOIT+DN)/2.0
DEPTH=DOIT
CALL INCOND
QIN=QR
T=TQ(1)
WRITE (6,370) T
WRITE (6,380)
WRITE (6,390)
DO 70 I=1,N1,IXO
  WRITE (6,400) X(I),D(I),V(I),Q(I)
70 CONTINUE
  ITOC=1
  DO 170 J=2,NT
    T=T+DT
C-----COMPUTATION OF VELOCITY AND DEPTH FOR THE INLET AT TIME T
CALL ROUND1
IF (ITOC-ITO) 90,80,90
80 WRITE (6,370) T
   WRITE (6,380)
   WRITE (6,390)
   WRITE (6,400) X(I),DDT(I),VDT(I),QDT(I)
90 IXOC=1
  DO 120 I=2,N
C-----COMPUTATION OF VELOCITY AND DEPTH AT TIME T
CALL COMP
IF (ITOC,EO,ITO,AND,IXOC,EQ,IXO) 100,110
100 IXOC=1
   WRITE (6,400) X(I),DDT(I),VDT(I),QDT(I)
   GO TO 120
110 IXOC=IXOC+1
120 CONTINUE
C-----COMPUTATION OF VELOCITY AND DEPTH FOR THE OUTLET AT TIME T
I=N1
CALL ROUND2
IF (ITOC-ITO) 140,130,140
130 ITOC=1
   WRITE (6,400) X(I),DDT(I),VDT(I),QDT(I)
   GO TO 150
140 ITOC=ITOC+1
150 DO 160 I=1,N1
   Q(I)=QDT(I)

```

```

160 D(I)=DDT(I)
170 V(I)=VDT(I)
CONTINUE
NPR=N1/50+1
DO 180 I=1,NPR
  IT=50*I-49
  IL=II+49
  WRITE (6,410)
  WRITE (6,420)
  DO 180 I=II,IL
    WRITE (6,430) X(I),DMAX(I),TDMAX(I),VMAX(I),TVMAX(I),QMAX(I),TOMAX(I)
  I(I)
  IF (I.EQ.N1) GO TO 190
CONTINUE
190 CALL EXIT
C-----
200 FORMAT (I3)
210 FORMAT (8F10.4)
220 FORMAT (1H1,2X,*INFLOW HYDROGRAPH PARAMETERS*//)
230 FORMAT (2X,*QB= *F10.5,*CFS*)
240 FORMAT (2X,*WAVE VOLUME ABOVE BASE FLOW= *F8.2,*CU FT*)
250 FORMAT (2X,*QP= *F10.5,*CFS*)
260 FORMAT (2X,*TP= *F10.5,*SEC*)
270 FORMAT (/)
280 FORMAT (2X,*SYSTEM PARAMETERS*//)
290 FORMAT (* S0 =*F10.5)
300 FORMAT (* ALPHA=*F10.5)
310 FORMAT (* BETA=*F10.5)
320 FORMAT (* QB/QP=*F10.5)
330 FORMAT (* N =*15/* IX0=*15/* IF =*F8.0/* TIO =*F10.5)
340 FORMAT (* FLOW IS SUPERCRITICAL*)
350 FORMAT (2X,*NORMAL DEPTH= *F6.4,*FT*,4X,*CRITICAL DEPTH= *F6.4,*FT*)
360 FORMAT (2X,*DX= *F8.5,*FT*,4X,*DT= *F8.5,*SEC*//)
370 FORMAT (1H1,5X,*CONDITIONS AT *9.3*SECONDS*//)
380 FORMAT (2X,*DISTANCE*.9X,*DEPTH*.8X,*VELOCITY*.
1 7X,*DISCHARGE*)
390 FORMAT (4X,*FT)*.11X,*FT)*.10X,*FPS)*.11X,*CFS*)//)
400 FORMAT (4(F10.4,5X))
410 FORMAT (///1 MAXIMUM VALUES AND TIMES AT EACH LOCATION*//)
420 FORMAT (* DISTANCE MAX DEPTH TIME MAX VEL TIME MAX Q
1 TIME*)
430 FORMAT (F8.2,3(4X,F6.2,2X,F1.2))
END

```

SUBROUTINE FOR COMPUTING INITIAL CONDITION

```

SURROUTINE INCOND
DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
DIMENSION QI(200), QMAX(200)
DIMENSION TDMAX(200), TQ(200), TOMAX(200), TVMAX(200)
DIMENSION V(500), VDT(500), VMAX(200), X(500)
COMMON A,AR,AC,AD,AE,ALPHA,H,BC,HD,BETA,CD,CO,DD,DC,DDT
COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DDIT,DT,DTOL,DX,ED
COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IX0,IXOC,J,MC,N,NQCD
COMMON NT,N1,Q,QR,QDT,QI,QIN,QMAX,QP,QQ,H,REY,S0,T,TDMAX
COMMON TF,THETA,TIO,TP,TQ,TOMAX,TVMAX,V,VDT,VMAX,VV,WP
COMMON X,XF,XF,XX
CALL CIRCLE
C CONDITION AT INITIAL POSITION
VV=QB/A
VH=VV*VV/(2.0*GR)
C COMPUTE REYNOLDS NUMBER
REY=VV*H/FNU
C COMPUTE FRICTION FACTOR
F=FB*REY**FC
S1=F*VH/(4.0*H)
EE1=DEPTH*ALPHA*VH
X(IN=1)=XF-XX
D(IN=1)=DDIT
V(IN=1)=VV
Q(IN=1)=QB
NOMAX=0

```

```

C INTEGRATION OF STADY FLOW
DO 150 L=1,N
XX=XX+DX
10 DPTH=DN
CALL CIRCLE
VV=QB/A
REY=VV*H/FNU
F=FB*REY**FC
HFTH=0.5*THETA
DTHET=4.0/(DIA*SINF(HFTH))
DAREA=0.125*DIA*DIA*(1.0-COSF(THETA))*DTHET
DW=0.5*DIA*DTHET
WP=0.5*DIA*THETA
DRA=(WP*DAREA-A*DW)/(WP*WP)
DFNG=1.0-(QB*QB/(GR*(A**3)))*DANFA
DSLO=-F*QB*QB*(2.0*H*A*DAREA*(A**2)*DIA)/(H*GR*(R*A**2)**2)
VV=QB/A
VH=VV*VV/(2.0*GR)
REY=VV*H/FNU
F=FB*REY**FC
S2=F*VH/(4.0*H)
SF=(S1+S2)/2.0
EE2=DIN+ALPHA*VH
FRATIO=(EE2-EE1+DX*(S0-SF))/(DENG*(EE2-EE1)*DSLO/(S0-SF))
C NEWTON-RAPHSON ITERATION
DCOM=DIN-FRATIO
IF (DCOM) 30,20,40
20 WRITE (6,200)
GO TO 190
30 DCOM=ABSF(DCOM)
40 IF (ABSF(DCOM-DIN)-DTOL) 130,130,50
50 IF (0.82*DIA-DCOM) 60,120,120
60 DIN=DCOM*0.5
70 IF (0.82*DIA-DIN) 80,80,90
80 DIN=DIN*0.5
NCOUNT=NCOUNT+1
GO TO 70
90 IF (NCOUNT-20) 100,100,110
100 GO TO 10
110 WRITE (6,210)
GO TO 190
120 DIN=DCOM
GO TO 10
130 IF (ABSF(DCOM-DN),LE,DTOL) 160,140
C END OF NEWTON-RAPHSON
140 DIN=DCOM
S1=S2
EE1=EE2
C REDEFINITION OF SUBSCRIPTS
II=N-L+1
X(II)=XF-XX
D(II)=DIN
V(II)=VV
Q(II)=QB
150 CONTINUE
GO TO 180
160 DPTH=DN
CALL CIRCLE
VV=QB/A
C CONSTANT CONDITIONS
DO 170 J=L,N
II=N-J+1
X(II)=XF-XX
D(II)=DN
V(II)=VV
Q(II)=QB
XX=XX+DX
170 CONTINUE
180 RETURN
190 CALL EXIT
C-----
200 FORMAT (* DCOM EQUALS ZERO *)
210 FORMAT (* INCOND DOES NOT CONVERGE*)
END

```


SUBROUTINE FOR COMPUTING DEPTH & VELOCITY AT END OF TIME INTERVAL

```

SUBROUTINE COMP                                COM 2
C-----COMPUTATION OF VELOCITY AND DEPTH AT THE TIME T*DT BY KNOWING THE COM 4
C-----VELOCITY AND THE DEPTH AT THE TIME T COM 6
DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500) COM 8
DIMENSION QI(200), QMAX(200) COM 10
DIMENSION TDMAX(200), TQ(200), TOMAX(200), TVMAX(200) COM 12
DIMENSION V(500), VDT(500), VMAX(200), X(500) COM 14
COMMON A,AB,AC,AD,AE,ALPHA,H,RC,HD,HETA,CD,CO,D,DC,DDT COM 16
COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUIT,DT,DTOL,DX,ED COM 18
COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD COM 20
COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX COM 22
COMMON TF,THETA,TIO,TP,TQ,TOMAX,TVMAX,V,VDT,VMAX,VV,WP COM 24
COMMON X,XE,XF,XX COM 26
DD=D(I) COM 28
VV=V(I) COM 30
CALL COEFF COM 32
DEPTH=D(I) COM 34
CALL CIRCLE COM 36
C POSITIVE CHARACTERISTIC COM 38
CP=(2.0*BETA)/(V(I)*(ALPHA+HETA)+SQRT(((ALPHA-BETA)**2)*V(I)*V(I)) COM 40
I=(4.0*A*BETA*GR/R)) COM 42
C NEGATIVE CHARACTERISTIC COM 44
CM=(2.0*BETA)/(V(I)*(ALPHA+HETA)-SQRT(((ALPHA-BETA)**2)*V(I)*V(I)) COM 46
I=(4.0*A*BETA*GR/R)) COM 48
UP=CO/CP COM 50
UN=CO/CM COM 52
C 2ND ORDER INTERPOLATION COM 54
DR=D(I-1)*0.5*UP*(UP-1)+D(I)*(1.0-UP**2)+D(I+1)*0.5*UP*(UP+1.) COM 56
VR=V(I-1)*0.5*UP*(UP-1)+V(I)*(1.0-UP**2)+V(I+1)*0.5*UP*(UP+1.) COM 58
DS=D(I-1)*0.5*UN*(UN-1)+D(I)*(1.0-UN**2)+D(I+1)*0.5*UN*(UN+1.) COM 60
VS=V(I-1)*0.5*UN*(UN-1)+V(I)*(1.0-UN**2)+V(I+1)*0.5*UN*(UN+1.) COM 62
FCP=AC*CP-BC COM 64
FCM=AC*CM-BC COM 66
GCP=AB COM 68
GCM=AB COM 70
SCP=AE*CP COM 72
SCH=AE*CM COM 74
TCP=FCP*DR+GCP*VR-SCP*DT/CP COM 76
TCM=FCM*DS+GCM*VS-SCH*DT/CM COM 78
C VELOCITY AND DEPTH AT END OF TIME INTERVAL COM 80
VP=(FCP*TCM-FCM*TCP)/(FCP*GCM-FCM*GCP) COM 82
DP=(TCP*GCM-TCM*GCP)/(FCP*GCM-FCM*GCP) COM 84
IF (DP-0.62*DIA) 20,10,10 COM 86
10 WRITE (6,90) X(I),T COM 88
GO TO 80 COM 90
20 DEPTH=DD COM 92
CALL CIRCLE COM 94
QDT(I)=A*VP COM 96
DDT(I)=DP COM 98
VDT(I)=VP COM 100
C MAX DEPTH, VELOCITY, AND DISCHARGE, AND THEIR ASSOCIATED TIMES COM 102
IF (DDT(I)-DMAX(I)) 40,40,30 COM 104
30 DMAX(I)=DDT(I) COM 106
TDMAX(I)=T COM 108
IF (VDT(I)-VMAX(I)) 60,60,50 COM 110
50 VMAX(I)=VDT(I) COM 112
TVMAX(I)=T COM 114
IF (QDT(I)-QMAX(I)) 80,80,70 COM 116
70 QMAX(I)=QDT(I) COM 118
TOMAX(I)=T COM 120
80 RETURN COM 122
C----- COM 124
90 FORMAT (* FLOW IS FULL AT X = *.F7.2,* T = *.F6.2) COM 126
END COM 128

```

SUBROUTINE FOR COMPUTING COEFFICIENTS IN ORDINARY DIFFERENTIAL EQUATIONS

```

SURROUTINE COEFF                                COE 2
C-----COMPUTATION OF ALL COEFFICIENTS OF THE TWO PARTIAL DIFFERENTIAL COE 4
C-----EQUATIONS COE 6
DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500) COE 8
DIMENSION QI(200), QMAX(200) COE 10
DIMENSION TDMAX(200), TQ(200), TOMAX(200), TVMAX(200) COE 12
DIMENSION V(500), VDT(500), VMAX(200), X(500) COE 14
COMMON A,AB,AC,AD,AE,ALPHA,H,RC,HD,HETA,CD,CO,D,DC,DDT COE 16
COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUIT,DT,DTOL,DX,ED COE 18
COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD COE 20
COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX COE 22
COMMON TF,THETA,TIO,TP,TQ,TOMAX,TVMAX,V,VDT,VMAX,VV,WP COE 24
COMMON X,XE,XF,XX COE 26
DEPTH=DD COE 28
CALL CIRCLE COE 30
A)=A/(VV*R) COE 32
D1=1.0/VV COE 34
A2=ALPHA*VV/GR COE 36
B2=BETA/GR COE 38
C REYNOLDS NUMBER COE 40
RFY=VV*R/FNU COE 42
C FRICTION FACTOR COE 44
F=FB*REY**FC COE 46
C ENERGY SLOPE COE 48
SF=.125*F*VV*VV/(R*GR) COE 50
E2=SF-SO COE 52
AB=A1*R2 COE 54
AC=A1-A2 COE 56
AD=-A2*D1 COE 58
AE=A1-E2 COE 60
BC=-B2 COE 62
BD=-B2*D1 COE 64
RETURN COE 66
END COE 68

```

SUBROUTINE FOR COMPUTING NORMAL DEPTH

```

SUBROUTINE DNORM                                DNO 2
DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500) DNO 4
DIMENSION QI(200), QMAX(200) DNO 6
DIMENSION TDMAX(200), TQ(200), TOMAX(200), TVMAX(200) DNO 8
DIMENSION V(500), VDT(500), VMAX(200), X(500) DNO 10
COMMON A,AB,AC,AD,AE,ALPHA,H,RC,HD,HETA,CD,CO,D,DC,DDT DNO 12
COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUIT,DT,DTOL,DX,ED DNO 14
COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD DNO 16
COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX DNO 18
COMMON TF,THETA,TIO,TP,TQ,TOMAX,TVMAX,V,VDT,VMAX,VV,WP DNO 20
COMMON X,XE,XF,XX DNO 22
DEPTH=0.62*DIA DNO 24
CALL CIRCLE DNO 26
VV=QQ/A DNO 28
C REYNOLDS NUMBER DNO 30
REY=VV*R/FNU DNO 32
C FRICTION FACTOR DNO 34
F=FB*REY**FC DNO 36
C NEWTON-RAPHSON DNO 38
DN=DEPTH-(WP-(F*(QQ**2))/(8.0*GR*SO*(R**2)*A))/((3.0*B)/R-2.0/SINF DNO 40
1THETA/2.0)) DNO 42
IF (ABS(F(DN-DEPTH)-DTOL) 30,20,20 DNO 44
20 DEPTH=DN DNO 46
GO TO 10 DNO 48
30 RETURN DNO 50
END DNO 52

```

SUBROUTINE FOR COMPUTING UPSTREAM BOUNDARY CONDITION.

```

SUBROUTINE BOUND1
C-----COMPUTATION OF VELOCITY AND DEPTH FOR X=0.0 AT THE TIME T
DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
DIMENSION QI(200), QMAX(200)
DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
DIMENSION V(500), VDT(500), VMAX(200), X(500)
COMMON A,AB,AC,AD,AE,ALPHA,H,HC,HN,RETA,CD,CO,DC,DDT
COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
COMMON NT,N1,Q,QR,QDT,QI,QIN,QMAX,OP,QQ,R,REY,SO,T,TDMAX
COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
COMMON X,XE,XF,XX
CALL INFL01
DEPTH=D(1)
CALL CIRCLE
C NEGATIVE CHARACTERISTIC
CM=(2.0*BETA)/(V(1)*(ALPHA+BETA)-SQRT(((ALPHA-BETA)**2)*V(1)*V(1)
1+(4.0*A*BETA*GR/H)))
IF (CM) 10,20,30
10 UN=CO/CM
C 2ND ORDER INTERPOLATION FOR DEPTH AND VELOCITY
DS=D(1)*0.5*UN*(UN-1.)+D(2)*(1.-UN**2)+D(3)*0.5*UN*(UN+1.)
VS=V(1)*0.5*UN*(UN-1.)+V(2)*(1.-UN**2)+V(3)*0.5*UN*(UN+1.)
GO TO 40
20 XS=X(1)
DS=D(1)
VS=V(1)
GO TO 40
30 QS=QIN
CALL DCRIT
DP=DC
GO TO 80
40 DN=D(1)
VV=V(1)
CALL COEFF
FCM=AC*CM-RC
GCM=AB
SCM=AE*CM
ASMLL=DS-(SCM*CM*DT-GCM*VS)/FCM
BSMLL=-QIN*GCM/FCM
DP1=D(1)
RN=2.0*DP1/DIA-1.0
DEPTH=DP1
CALL CIRCLE
FDPI=1.0*(BSMLL/A**2)*((DIA*(1.0-COSF(THETA))/2.0)*(1.0/SQRTF(1.0-RN**2)))
10-RD**2)))
C NEWTON-RAPHSON ITERATION
DP2=DP1-FDPI/FDPI
IF (ABSF(DP2-DP1)-DTOL) 70,70,60
60 DP1=DP2
GO TO 50
C END OF NEWTON-RAPHSON
70 DP=DP1
80 IF (DP-0.82*DIA) 100,90,90
90 WRITE (6,170) X(1),T
GO TO 160
100 DEPTH=DP
CALL CIRCLE
VP=QIN/A
DDT(1)=DP
VDT(1)=VP
QDT(1)=QIN
C MAX DEPTH, VELOCITY, AND DISCHARGE, AND THEIR ASSOCIATED TIMES
IF (DDT(1)-DMAX(1)) 120,120,110
110 DMAX(1)=DDT(1)
TDMAX(1)=T
120 IF (VDT(1)-VMAX(1)) 140,140,130
130 VMAX(1)=VDT(1)
TVMAX(1)=T
140 IF (QDT(1)-QMAX(1)) 160,160,150
150 QMAX(1)=QDT(1)
TQMAX(1)=T
160 RETURN
C-----
170 FORMAT (' FLOW IS FULL AT X = *F7.2,* T = *F6.2)
END

```

SUBROUTINE FOR COMPUTING DOWNSTREAM BOUNDARY CONDITION.

```

SUBROUTINE BOUND2
DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
DIMENSION QI(200), QMAX(200)
DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
DIMENSION V(500), VDT(500), VMAX(200), X(500)
COMMON A,AB,AC,AD,AE,ALPHA,H,HC,HN,RETA,CD,CO,DC,DDT
COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
COMMON NT,N1,Q,QR,QDT,QI,QIN,QMAX,OP,QQ,R,REY,SO,T,TDMAX
COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
COMMON X,XE,XF,XX
DEPTH=D(1)
CALL CIRCLE
C POSITIVE CHARACTERISTIC
CP=(2.0*BETA)/(V(1)*(ALPHA+BETA)+SQRT(((ALPHA-BETA)**2)*V(1)*V(1)
1+(4.0*A*BETA*GR/H)))
UP=CO/CP
C 2ND ORDER INTERPOLATION FOR DEPTH AND VELOCITY
DR=D(1)*0.5*UP*(UP-1.)+D(2)*(1.-UP**2)+D(3)*0.5*UP*(UP+1.)
VR=V(1)*0.5*UP*(UP-1.)+V(2)*(1.-UP**2)+V(3)*0.5*UP*(UP+1.)
DV=D(1)
VV=V(1)
CALL COEFF
FCP=AC*CP-BC
GCP=AB
SCP=AE*CP
CSMLL=DR-(SCP*CP*DT-GCP*VR)/FCP
DSMLL=-GCP/FCP
DP1=D(1)
10 RN=DP1*2.0/DIA-1.0
DEPTH=DP1
CALL CIRCLE
GO TO (20,30), MC
20 FD=CD*DP1**ED
FD1=CD*ED*DP1**ED-1.0
U=FD/A
FDPI=DP1-CSMLL-DSMLL*U
THETA2=THETA/2.0
DADD=(DIA/2.0)*(1.0-COSF(THETA2))*(1.0/SQRTF(1.0-RN**2))
DIIND=((A*FD1)-(FD*DADD))/(A*A)
GO TO 40
30 U=SQRTF(GR*A/B)
FDPI=DP1-CSMLL-DSMLL*U
THETA2=THETA/2.0
DIIND=(2.0/DIA)*(1.0/SQRTF(1.0-RD**2))*(1.0/U)*((DIA**2*(1.0-COSF(THETA2)
1ETA**2))/(8.0*B)-(A*(DIA/2.0)*COSF(THETA2)/H**2)))
40 FDP1=1.0-DSMLL*DIIND
C NEWTON-RAPHSON ITERATION
DP2=DP1-FDPI/FDPI
IF (ABSF(DP2-DP1)-DTOL) 60,60,50
50 DP1=DP2
GO TO 10
C END OF NEWTON-RAPHSON
60 DEPTH=DP2
IF (DEPTH-0.82*DIA) 80,70,70
70 WRITE (6,180) X(1),T
GO TO 170
80 CALL CIRCLE
DDT(1)=DEPTH
GO TO (90,100), MC
90 QDT(1)=CD*DEPTH**ED
VDT(1)=QDT(1)/A
GO TO 110
100 VDT(1)=SQRTF(GR*A/B)
QDT(1)=VDT(1)*A
C MAX DEPTH, VELOCITY, AND DISCHARGE, AND THEIR ASSOCIATED TIMES
110 IF (DDT(1)-DMAX(1)) 130,130,120
120 DMAX(1)=DDT(1)
TDMAX(1)=T
130 IF (VDT(1)-VMAX(1)) 150,150,140
140 VMAX(1)=VDT(1)
TVMAX(1)=T
150 IF (QDT(1)-QMAX(1)) 170,170,160
160 QMAX(1)=QDT(1)
TQMAX(1)=T
170 RETURN
C-----
180 FORMAT (' FLOW IS FULL AT X = *F7.2,* T = *F6.2)
END

```

802 2
802 4
802 6
802 8
802 10
802 12
802 14
802 16
802 18
802 20
802 22
802 24
802 26
802 28
802 30
802 32
802 34
802 36
802 38
802 40
802 42
802 44
802 46
802 48
802 50
802 52
802 54
802 56
802 58
802 60
802 62
802 64
802 66
802 68
802 70
802 72
802 74
802 76
802 78
802 80
802 82
802 84
802 86
802 88
802 90
802 92
802 94
802 96
802 98
802 100
802 102
802 104
802 106
802 108
802 110
802 112
802 114
802 116
802 118
802 120
802 122
802 124
802 126
802 128
802 130
802 132
802 134
802 136
802 138
802 140
802 142
802 144
802 146
802 148
802 150
802 152
802 154
802 156
802 158

```

SUBROUTINE CIRCLE
  DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
  DIMENSION QI(200), QMAX(200)
  DIMENSION TOMAX(200), TQ(200), TQMAX(200), TVMAX(200)
  DIMENSION V(500), VDT(500), VMAX(200), X(500)
  COMMON A,AR,AC,AD,AE,ALPHA,H,RC,HD,RETA,CD,CO,DC,DDT
  COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
  COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXU,IXUC,J,MC,N,NQCD
  COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,OP,OO,R,PEY,SO,T,TOMAX
  COMMON TF,THETA,TIO,TP,TO,TOMAX,TVMAX,V,VDT,VMAX,VV,WP
  COMMON X,XE,XF,XX
  C TEST TO INSURE DEPTH LESS THAN 0.82 DIA.
  IF (DEPTH) 10,20,20
  10 WRITE (6,100)
  CALL EXIT
  20 IF (DEPTH=0.82*DIA) 40,40,30
  30 WRITE (6,110)
  CALL EXIT
  40 IF (DIA/2.0-DEPTH) 60,50,70
  50 THETA=7.14159
  GO TO 90
  C SURTENDED ANGLE
  60 THETA=6.28318-2.0*ATANF((SQRTF(DIA*DEPTH-DEPTH*DEPTH))/(DEPTH-DIA/2.0))
  GO TO 90
  70 THETA=2.0*ATANF((SQRTF(DIA*DEPTH-DEPTH*DEPTH))/(DIA/2.0-DEPTH))
  IF (THETA) 80,90,90
  80 THETA=6.28318+THETA
  C ARFA
  90 A=0.125*(THETA-SINF(THETA))*(DIA**2)
  C WFTED PERIMETER
  WP=(DIA/2.0)*THETA
  C HYDRAULIC RADIUS
  R=A/WP
  C SURFACE WIDTH
  B=DIA*SINF(THETA/2.0)
  C HYDRAULIC DEPTH
  DM=A/R
  RETURN
C-----
100 FORMAT (* DEPTH IS NEGATIVE*)
110 FORMAT (* FLOW IS FULL*)
END

```

SUBROUTINE FOR COMPUTING CRITICAL DEPTH

```

SUBROUTINE DCWIT
  DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
  DIMENSION QI(200), QMAX(200)
  DIMENSION TOMAX(200), TQ(200), TQMAX(200), TVMAX(200)
  DIMENSION V(500), VDT(500), VMAX(200), X(500)
  COMMON A,AR,AC,AD,AE,ALPHA,H,RC,HD,RETA,CD,CO,DC,DDT
  COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
  COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXU,IXUC,J,MC,N,NQCD
  COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,OP,OO,R,PEY,SO,T,TOMAX
  COMMON TF,THETA,TIO,TP,TO,TOMAX,TVMAX,V,VDT,VMAX,VV,WP
  COMMON X,XE,XF,XX
  DEPTH=0.62*DIA
  CALL CIRCLF
  C NEWTON-RAPHSON
  DC=DEPTH-(R*(A**3)-ALPHA*((H*OO)**2)/GR)/(3.0*((R*A)**2)-(2.0*(A**3)
  13)*COSF(THETA/2.0))/(SINF(THETA/2.0)))
  IF (ABSF(DC-DEPTH)-DTOL) 30,20,20
  20 DPTH=DC
  GO TO 10
  30 RETURN
END

```

```

SUBROUTINE INFLO1
  C-----COMPUTATION OF THE INFLOW HYDROGRAPH
  DISCHARGES AT IRREGULAR TIME INTERVALS.
  DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
  DIMENSION QI(200), QMAX(200)
  DIMENSION TOMAX(200), TQ(200), TQMAX(200), TVMAX(200)
  DIMENSION V(500), VDT(500), VMAX(200), X(500)
  COMMON A,AR,AC,AD,AE,ALPHA,H,RC,HD,RETA,CD,CO,DC,DDT
  COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
  COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXU,IXUC,J,MC,N,NQCD
  COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,OP,OO,R,PEY,SO,T,TOMAX
  COMMON TF,THETA,TIO,TP,TO,TOMAX,TVMAX,V,VDT,VMAX,VV,WP
  COMMON X,XE,XF,XX
  AJ=J
  T=(AJ-1.0)*DT
  IQ=1
  C INTERPOLATION FOR REGULAR TIME INTERVALS
  IF (T.GE.TQ(NQCD)) 10,20
  10 QIN=QI(NQCD)
  GO TO 50
  20 IF (T.GE.TQ(IQ).AND.T.IT.TQ(IQ+1)) 40,30
  30 IQ=IQ+1
  GO TO 20
  40 QIN=QI(IQ)+(QI(IQ+1)-QI(IQ))*(T-TQ(IQ))/(TQ(IQ+1)-TQ(IQ))
  GO TO 50
  50 RETURN
END

```

```

INF 2
INF 4
INF 6
INF 8
INF 10
INF 12
INF 14
INF 16
INF 18
INF 20
INF 22
INF 24
INF 26
INF 28
INF 30
INF 32
INF 34
INF 36
INF 38
INF 40
INF 42
INF 44
INF 46
INF 48
INF 50
INF 52
INF 54

```

```

DCR 2
DCR 4
DCR 6
DCR 8
DCR 10
DCR 12
DCR 14
DCR 16
DCR 18
DCR 20
DCR 22
DCR 24
DCR 26
DCR 28
DCR 30
DCR 32
DCR 34
DCR 36
DCR 38
DCR 40
DCR 42

```


A.3.3. DEFINITION OF VARIABLES

NAME	DEFINITION	STATEMENT NUMBER(S)
A	AREA OF CIRCULAR SEGMENT	CIR 60
AB	(2)	COF 54
AC	(2)	COE 56
AD	(2)	COE 58
AE	(2)	COF 60
AJ	(1)	INF 28
ALPHA	VEL DISTRIBUTION FACTOR-ENERGY	UNS 56
AN	NUMBER OF DISTANCE INTERVALS	UNS 164
ASMLL(2)		H01 78
A1	(2)	COE 32
A2	(2)	COE 36
B	FREE SURFACE WIDTH	CIR 72
BC	(2)	COE 62
BD	(2)	COE 64
BETA	VEL DISTRIBUTION FACTOR-MOMENTUM	UNS 58
BSMLL(2)		H01 80
B2	(2)	COE 38
CD	OUTLET DISCHARGE COEFFICIENT	UNS 62
CM	NEGATIVE CHARACTERISTIC DIRECTION	H01 34 COM 46
CO	MINUS DT/DX	UNS 192
CP	POSITIVE CHARACTERISTIC DIRECTION	COM 40 H02 30
CSMLL(2)		H02 32
D	DEPTH OF FLOW AT TIME T	INC 46 INC 156 INC 180 UNS 284
DADD	DERIVATIVE OF AREA WITH DEPTH	H02 78
DAREA	DERIVATIVE OF AREA WITH DEPTH	INC 74
DC	CRITICAL DEPTH	DCR 30
DCOM	COMPUTED DEPTH	INC 104 INC 112
DD	DEPTH	H01 66 H02 42 COM 28
DDT	DEPTH OF FLOW AT TIME T+DT	H01 122 H02 118 COM 98
DENG	(2)	INC 82
DEPTH	DEPTH OF FLOW	H01 28 H01 86 H01 116 COE 28
		H02 24 H02 62 H02 108 COM 34
		UNS 176 UNS 142 UNS 204 COM 92
		INC 60 INC 166 DNO 24 DNO 46
		DCR 24 DCR 16
DIA	DIAMETER OF PIPE	UNS 38
DIN	INITIAL VALUE OF DEPTH	INC 118 INC 122 INC 136 INC 144
		UNS 202
DM	HYDRAULIC DEPTH	CIR 76
DMAX	MAXIMUM DEPTH	UNS 98 H01 132 H02 136 COM 106
DN	NORMAL DEPTH	DNO 40
DOUT	DEPTH AT OUTLET	UNS 152 UNS 160
DP	DEPTH	H01 62 H01 108 COM 84
DP1	INITIAL VALUE OF DEPTH	H01 82 H01 102 H02 58 H02 102
DP2	COMPUTED VALUE OF DEPTH	H01 98 H02 98
DR	INTERPOLATED VALUE OF DEPTH	H02 38 COM 56
DRA	(2)	INC 80
DS	INTERPOLATED VALUE OF DEPTH	H01 44 H01 52 COM 60
DSLO	DERIVATIVE OF ENERGY SLOPE WITH DEPTH	INC 84
DSMLL(2)		H02 56
DT	INCREMENT OF TIME	UNS 190
DTDET	DERIVATIVE OF THETA WITH DEPTH	INC 72
DTMAX	TIME OF MAXIMUM DEPTH	UNS 186
DTOL	MAXIMUM ERROR IN DEPTH CALCULATION	UNS 76
DUDD	(2)	H02 40 H02 90
DW	(2)	INC 76
DX	INCREMENT OF DISTANCE	UNS 166
D1	(2)	COE 34
ED	OUTLET DISCHARGE EXPONENT	UNS 64
EE1	ENERGY AT KNOWN DEPTH	INC 42 INC 148
EE2	ENERGY AT UNKNOWN DEPTH	INC 98
E2	(2)	COE 52
F	DARCY-WEISBACH RESISTANCE COEFF	INC 38 INC 68 INC 92 DNO 36
		COE 46
FB	FACTOR IN REY.NO. - DARCY FUNCTION	UNS 48
FC	FACTOR IN REY.NO. - DARCY FUNCTION	UNS 50
FCM	(2)	H01 72 COM 66
FCP	(2)	H02 48 COM 64
FD	DOWNSTREAM OUTLET DISCHARGE	H02 68
FDPI	(2)	H01 40 H02 74 H02 86
FDPI1	(2)	H01 92 H02 94
FD1	(2)	H02 70

FNU	KINEMATIC VISCOSITY	UNS 46
FRATIO(2)		INC 100
GCM	(2)	H01 74 COM 70
GCP	(2)	H02 50 COM 68
GR	ACCELERATION OF GRAVITY	UNS 54
HFTH	(2)	INC 70
I	(1)	UNS 266
II	(1)	UNS 294 INC 152 INC 176
IL	LINE PRINTING INDEX	UNS 296
IO	(1)	INF 32 INF 44
ITO	TIME INTERVAL BETWEEN OUTPUTS	UNS 196
ITOC	TIME, OUTPUT INTERVAL COUNTER	UNS 224 UNS 272 UNS 278
IXO	DISTANCE INTERVAL BETWEEN OUTPUTS	UNS 70
IXOC	DISTANCE, OUTPUT INTERVAL COUNTER	UNS 244 UNS 254 UNS 260
J	(1)	INC 158 UNS 226
L	(1)	INC 48
MC	BACKWATER PROFILE CODE	UNS 148 UNS 156
N	NUMBER OF X-INTERVALS	UNS 68
NCOUNT	ITERATION COUNTER	INC 52 INC 124
NPG	OUTPUT PAGE CONTROL	UNS 290
NQCD	NUMBER OF INPUT HYDROGRAPH POINTS	UNS 80
NT	NUMBER OF TIME INTERVALS	UNS 194
NI	NUMBER OF X-INTEGRATION LOCATIONS	UNS 94
O	DISCHARGE AT TIME T	UNS 282 INC 50 INC 160 INC 184
OB	BASE DISCHARGE	UNS 84
ODT	DISCHARGE AT TIME T+DT	H01 126 H02 122 H02 130 COM 96
OI	INPUT HYDROGRAPH DISCHARGE	UNS 82
OIN	INITIAL DISCHARGE	UNS 208 INF 18 INF 48
OMAX	MAXIMUM DISCHARGE AT TIME T	UNS 102 H01 144 H02 148 COM 118
OP	PEAK HYDROGRAPH DISCHARGE	UNS 86
OQ	DISCHARGE	UNS 134 UNS 172 H01 58
ORA	PEAK TO BASE DISCHARGE RATIO	UNS 92
R	HYDRAULIC RADIUS	CIR 68
RD	DEPTH TO RADIUS RATIO	H01 84 H02 60
REY	REYNOLDS NUMBER	INC 34 INC 66 INC 90 DNO 32
REY		COE 42
SCM	(2)	H01 76 COM 74
SCP	(2)	H02 52 COM 72
SF	FRICTION SLOPE (AVERAGE)	INC 96 COE 50
S0	INVERT SLOPE	UNS 42
S1	FRICTION SLOPE AT KNOWN DEPTH	INC 40 INC 146
S2	FRICTION SLOPE AT UNKNOWN DEPTH	INC 94
T	TIME	UNS 210 UNS 228 INF 30
TCM	(2)	COM 78
TCP	(2)	COM 76
TDMAX	TIME TO MAXIMUM DEPTH	H01 134 H02 138 COM 108
TF	FINAL TIME FOR CALCULATION	UNS 72
THETA	CENT ANG SUBTENDED BY FREE SURFACE	CIR 46 CIR 52 CIR 56
THETA2ONE-HALF THETA		H02 76 H02 88
TIO	TIME INTERVAL BETWEEN OUTPUTS	UNS 74
TP	TIME TO PEAK INPUT DISCHARGE	UNS 88
TQ	INFLOW HYDROGRAPH TIME	UNS 82
TOMAX	TIME TO MAXIMUM DISCHARGE	H01 146 H02 150 COM 120
TVMAX	TIME TO MAXIMUM VELOCITY	H01 140 H02 144 COM 114
U	VELOCITY	H02 72 H02 84
UN	(2)	H01 40 COM 52
UP	(2)	H02 34 COM 50
V	VELOCITY AT TIME T	UNS 286 INC 48 INC 158 INC 182
VC	CRITICAL VELOCITY	UNS 180
VDI	VELOCITY AT TIME T+DT	H01 124 H02 124 H02 128 COM 100
VH	VELOCITY HEAD	INC 30 INC 88
VMAX	MAXIMUM VELOCITY	UNS 100 H01 138 H02 142 COM 112
VOL	VOLUME OF HYDROGRAPH WAVE	UNS 90
VP	VELOCITY	H01 120 COM 82
VR	INTERPOLATED VALUE OF VELOCITY	H02 40 COM 58
VS	INTERPOLATED VALUE OF VELOCITY	H01 46 H01 54 COM 62
VV	VELOCITY	H01 88 H02 44 COM 30 DNO 28
		INC 28 INC 44 INC 86 INC 170
WP	WETTED PERIMETER	CIR 64
X	POSITION ALONG CHANNEL	INC 44 INC 154 INC 178
XE	WORKING LENGTH OF CHANNEL	UNS 162
XF	INITIAL LENGTH OF CHANNEL	UNS 40
XS	STARTING POSITION	H01 50
XX	POSITION ALONG CHANNEL (XX=XE-X)	UNS 150 UNS 158 INC 58 INC 186

CODES

(1) - DO LOOP COUNTERS OR VARIABLE SUBSCRIPTS
(2) - INTERMEDIATE VARIABLES

A.3.4. SAMPLE INPUT AND OUTPUT

SAMPLE INPUT

Same format as in A.1.4.

S 4.0 30.0 10.0 50.0 10. 80.0 4.0
S.0
200.0 4.0

SAMPLE OUTPUT

INFLOW HYDROGRAPH PARAMETERS

Q₀ = 4.00000CFS
Q₁ = 10.00000CFS
T_P = 30.00000SEC
BB/Q₀ = 4.0000
WAVE VOLUME ABOVE BASE FLOW = 180.00CU FT

SYSTEM PARAMETERS

SO = .00100
ALPHA = 1.00000
BETA = 1.00000
N = 20
IXO = 2
TF = 200
TIO = 20.00000
NORMAL DEPTH = .7659FT CRITICAL DEPTH = .6290FT
DX = 40.94348FT DT = 1.45574SEC

CONDITIONS AT 0.000SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.7659	2.8522	4.0000
81.8870	.7658	2.8524	4.0000
163.7739	.7658	2.8528	4.0000
245.6609	.7656	2.8535	4.0000
327.5478	.7654	2.8549	4.0000
409.4348	.7648	2.8578	4.0000
491.3217	.7637	2.8639	4.0000
573.2087	.7612	2.8768	4.0000
655.0956	.7559	2.9052	4.0000
736.9826	.7433	2.9749	4.0000
818.8695	.6290	3.7688	4.0000

CONDITIONS AT 18.925SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.9539	4.0904	7.7849
81.8870	.8362	3.3542	5.3194
163.7739	.7734	2.9100	4.1378
245.6609	.7659	2.8554	4.0049
327.5478	.7654	2.8550	4.0001
409.4348	.7648	2.8579	4.0000
491.3217	.7636	2.8640	4.0000
573.2087	.7612	2.8771	4.0000
655.0956	.7559	2.9071	4.0024
736.9826	.7427	3.0034	4.0338
818.8695	.6369	3.7940	4.0992

CONDITIONS AT 37.849SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	1.0700	4.4918	10.0000
81.8870	1.0308	4.4354	9.3859
163.7739	.9196	3.8807	7.0221
245.6609	.8053	3.1387	4.7230
327.5478	.7704	2.8920	4.0842
409.4348	.7651	2.8606	4.0062
491.3217	.7636	2.8643	4.0004
573.2087	.7612	2.8782	4.0020
655.0956	.7561	2.9125	4.0117
736.9826	.7390	3.0322	4.0441
818.8695	.6359	3.7906	4.0858

CONDITIONS AT 56.774SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	1.0404	4.0341	8.6452
81.8870	1.0060	4.3090	9.5438
163.7739	1.0442	4.3710	9.4136
245.6609	1.0048	4.3362	8.8619
327.5478	.8731	3.5932	6.0508
409.4348	.7883	3.0276	4.4214
491.3217	.7668	2.8881	4.0573
573.2087	.7616	2.8815	4.0091
655.0956	.7556	2.9160	4.0125
736.9826	.7353	3.0583	4.0500
818.8695	.6347	3.7870	4.0714

CONDITIONS AT 75.698SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8836	2.8386	4.8603
81.8870	.9667	2.4656	6.7178
163.7739	1.0199	3.9327	8.2021
245.6609	1.0257	4.1370	8.6958
327.5478	1.0230	4.3124	9.0317
409.4348	.9642	4.1384	7.9931
491.3217	.8844	3.3584	5.3101
573.2087	.7753	2.9823	4.2551
655.0956	.7568	2.9366	4.0500
736.9826	.7325	3.0785	4.0551
818.8695	.6340	3.7846	4.0621

CONDITIONS AT 94.623SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8295	2.5505	4.0000
81.8870	.8422	2.6368	4.2239
163.7739	.9002	3.0444	5.3496
245.6609	.9579	3.5015	6.7027
327.5478	.9923	3.8510	7.7364
409.4348	.9932	4.0117	8.0694
491.3217	1.0019	4.2687	8.6895
573.2087	.9129	3.8849	6.9592
655.0956	.7989	3.2460	4.8296
736.9826	.7390	3.1523	4.2044
818.8695	.5374	3.7954	4.1048

CONDITIONS AT 113.548SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8186	2.5984	4.0000
81.8870	.8261	2.6335	4.1063
163.7739	.9313	2.6343	4.1438
245.6609	.9615	2.8325	4.6823
327.5478	.9054	3.1650	5.6051
409.4348	.9484	3.5363	6.6769
491.3217	.9623	3.7555	7.2345
573.2087	.9096	3.9720	7.7310
655.0956	.9642	4.2183	8.1489
736.9826	.8394	3.8319	6.1088
818.8695	.7013	3.9941	4.9473

CONDITIONS AT 132.472SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8100	2.6368	4.0000
81.8870	.8177	2.6636	4.0942
163.7739	.9259	2.7020	4.2186
245.6609	.9290	2.6875	4.2113
327.5478	.8424	2.7629	4.4272
409.4348	.8704	2.9676	4.9761
491.3217	.9055	3.2611	5.7765
573.2087	.9322	3.5523	6.5495
655.0956	.9298	3.7297	6.8521
736.9826	.9372	4.1744	7.7537
818.8695	.8042	4.4732	7.4271

CONDITIONS AT 151.397SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8030	2.6690	4.0000
81.8870	.8100	2.6875	4.0770
163.7739	.9168	2.7079	4.1558
245.6609	.9265	2.7503	4.2911
327.5478	.8297	2.7525	4.3182
409.4348	.8342	2.7684	4.3757
491.3217	.8495	2.8817	4.6718
573.2087	.8720	3.0865	5.1890
655.0956	.9456	3.3874	5.9100
736.9826	.9662	3.6284	6.2386
818.8695	.8074	4.3099	6.5087

CONDITIONS AT 170.321SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.7973	2.6959	4.0000
81.8870	.8036	2.7086	4.0632
163.7739	.9101	2.7291	4.1407
245.6609	.9161	2.7427	4.2083
327.5478	.8246	2.7814	4.3261
409.4348	.8301	2.8077	4.4074
491.3217	.8310	2.8107	4.4187
573.2087	.8360	2.8713	4.5524
655.0956	.8431	3.0140	4.8410
736.9826	.8489	3.3117	5.3636
818.8695	.7584	4.1660	5.7630

CONDITIONS AT 189.246SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.7925	2.7188	4.0000
81.8870	.7981	2.7281	4.0537
163.7739	.9037	2.7414	4.1137
245.6609	.9102	2.7507	4.1894
327.5478	.8155	2.7742	4.2485
409.4348	.8220	2.8041	4.3419
491.3217	.8280	2.8471	4.4539
573.2087	.8272	2.8701	4.4839
655.0956	.8229	2.9294	4.5433
736.9826	.8097	3.0944	4.6919
818.8695	.7053	4.0064	5.0030

MAXIMUM VALUES AND TIMES AT EACH LOCATION

DISTANCE	MAX DEPTH	TIME	MAX VEL	TIME	MAX Q	TIME
.00	1.09	49.30	4.34	30.57	10.00	30.57
40.94	1.08	52.41	4.49	36.39	9.86	40.76
81.89	1.07	55.32	4.47	40.76	9.71	45.13
122.83	1.06	59.69	4.45	46.58	9.59	48.04
163.77	1.05	64.05	4.43	50.95	9.48	52.41
204.72	1.04	61.14	4.42	55.32	9.39	58.23
245.66	1.03	55.51	4.40	61.14	9.31	62.60
286.60	1.03	69.88	4.39	65.51	9.22	66.96
327.55	1.02	74.24	4.37	71.33	9.13	72.79
368.49	1.02	80.07	4.35	77.15	9.02	78.61
409.43	1.01	84.43	4.33	81.52	8.92	82.98
450.38	1.01	90.26	4.30	87.34	8.81	88.80
491.32	1.00	94.62	4.28	93.17	8.69	94.62
532.27	1.00	100.45	4.26	98.99	8.57	98.99
573.21	.99	105.27	4.24	103.36	8.46	104.81
614.15	.98	110.64	4.22	109.18	8.31	110.64
655.10	.97	115.44	4.22	115.00	8.21	115.00
696.04	.95	122.28	4.22	119.37	7.99	120.83
736.98	.95	128.10	4.34	125.19	8.13	126.65
777.93	.89	136.84	4.28	132.47	7.37	133.93
818.87	.87	133.93	4.48	133.93	7.44	133.93

Key Words: Finite-Difference Schemes, Unsteady Flow Equations, Method of Characteristics, Numerical Solutions of Differential Equations.

Abstract: This fourth part of a four-part series of hydrology papers on flood routing through storm drains presents the computer-oriented numerical methods on solving the quasi-linear hyperbolic partial differential equations known as De Saint-Venant equations of gradually varied free-surface unsteady flow. Formulation of various numerical finite-difference schemes either explicit schemes based on the two partial differential equations, unstable, diffusing, upstream differencing, leap frog, and Lax-Wendroff or the specified intervals scheme based on the method of characteristics is analyzed. A comparison between the specified intervals scheme of the method of characteristics, the Lax-Wendroff scheme and the diffusing scheme is discussed. Flow charts and computer programs for these various numerical methods are given in the appendices.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 46 (November 1970) "Flood Routing Through Storm Drains, Part IV, Numerical Computer Methods of Solution".

Key Words: Finite-Difference Schemes, Unsteady Flow Equations, Method of Characteristics, Numerical Solutions of Differential Equations.

Abstract: This fourth part of a four-part series of hydrology papers on flood routing through storm drains presents the computer-oriented numerical methods on solving the quasi-linear hyperbolic partial differential equations known as De Saint-Venant equations of gradually varied free-surface unsteady flow. Formulation of various numerical finite-difference schemes either explicit schemes based on the two partial differential equations, unstable, diffusing, upstream differencing, leap frog, and Lax-Wendroff or the specified intervals scheme based on the method of characteristics is analyzed. A comparison between the specified intervals scheme of the method of characteristics, the Lax-Wendroff scheme and the diffusing scheme is discussed. Flow charts and computer programs for these various numerical methods are given in the appendices.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 46 (November 1970) "Flood Routing Through Storm Drains, Part IV, Numerical Computer Methods of Solution".

Key Words: Finite-Difference Schemes, Unsteady Flow Equations, Method of Characteristics, Numerical Solutions of Differential Equations.

Abstract: This fourth part of a four-part series of hydrology papers on flood routing through storm drains presents the computer-oriented numerical methods on solving the quasi-linear hyperbolic partial differential equations known as De Saint-Venant equations of gradually varied free-surface unsteady flow. Formulation of various numerical finite-difference schemes either explicit schemes based on the two partial differential equations, unstable, diffusing, upstream differencing, leap frog, and Lax-Wendroff or the specified intervals scheme based on the method of characteristics is analyzed. A comparison between the specified intervals scheme of the method of characteristics, the Lax-Wendroff scheme and the diffusing scheme is discussed. Flow charts and computer programs for these various numerical methods are given in the appendices.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 46 (November 1970) "Flood Routing Through Storm Drains, Part IV, Numerical Computer Methods of Solution".

Key Words: Finite-Difference Schemes, Unsteady Flow Equations, Method of Characteristics, Numerical Solutions of Differential Equations.

Abstract: This fourth part of a four-part series of hydrology papers on flood routing through storm drains presents the computer-oriented numerical methods on solving the quasi-linear hyperbolic partial differential equations known as De Saint-Venant equations of gradually varied free-surface unsteady flow. Formulation of various numerical finite-difference schemes either explicit schemes based on the two partial differential equations, unstable, diffusing, upstream differencing, leap frog, and Lax-Wendroff or the specified intervals scheme based on the method of characteristics is analyzed. A comparison between the specified intervals scheme of the method of characteristics, the Lax-Wendroff scheme and the diffusing scheme is discussed. Flow charts and computer programs for these various numerical methods are given in the appendices.

Reference: Yevjevich, Vujica and Albert H. Barnes, Colorado State University, Hydrology Paper No. 46 (November 1970) "Flood Routing Through Storm Drains, Part IV, Numerical Computer Methods of Solution".

PREVIOUSLY PUBLISHED PAPERS

Colorado State University Hydrology Papers

- No. 28 "Sampling Bacteria in a Mountain Stream," by Samuel H. Kunkle and James R. Meismann, March 1968.
- No. 29 "Estimating Design Floods From Extreme Rainfall," by Frederick C. Bell, July 1968.
- No. 30 "Conservation of Ground Water by Gravel Mulches," by A. T. Corey and W. D. Kemper, May 1968.
- No. 31 "Effects of Truncation on Dependence in Hydrologic Time Series," by Rezaul Karim Bhuiya and Vujica Yevjevich, November 1968.
- No. 32 "Properties of Non-Homogeneous Hydrologic Series," by V. Yevjevich and R. I. Jeng, April 1969.
- No. 33 "Runs of Precipitation Series," by Jose Llamas and M. M. Siddiqui, May 1969.
- No. 34 "Statistical Discrimination of Change in Daily Runoff," by Andre J. Dumas and Hubert J. Morel-Seytoux, August 1969.
- No. 35 "Stochastic Process of Precipitation," by P. Todorovic and V. Yevjevich, September 1969.
- No. 36 "Suitability of the Upper Colorado River Basin for Precipitation Management," by Hiroshi Nakamichi and Hubert J. Morel-Seytoux, October 1969.
- No. 37 "Regional Discrimination of Change in Runoff," by Viboon Nimmannit and Hubert J. Morel-Seytoux, November 1969.
- No. 38 "Evaluation of the Effect of Impoundment on Water Quality in Cheney Reservoir," by J. C. Ward and S. Karaki, March 1970.
- No. 39 "The Kinematic Cascade as a Hydrologic Model," by David F. Kibler and David A. Woolhiser, February 1970.
- No. 40 "Application of Run-Lengths to Hydrologic Series," by Jaime Saldarriaga and Vujica Yevjevich, April 1970.
- No. 41 "Numerical Simulation of Dispersion in Groundwater Aquifers," by Donald Lee Reddell and Daniel K. Sunada, June 1970.
- No. 42 "Theoretical Probability Distributions for Flood Peaks," by Emir Zelenhasic, November 1970.
- No. 43 "Flood Routing Through Storm Drains, Part I, Solution of Problems of Unsteady Free Surface Flow in a Storm Drain", by V. Yevjevich and A. H. Barnes, November 1970.
- No. 44 "Flood Routing Through Storm Drains, Part II, Physical Facilities and Experiments", by V. Yevjevich and A. H. Barnes, November 1970.
- No. 45 "Flood Routing Through Storm Drains, Part III, Evaluation of Geometric and Hydraulic Parameters," by V. Yevjevich and A. H. Barnes, November 1970.