Technical Note No. 1

STRESS DUE TO TANGENTIAL FORCE

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June 30, 1962 CER62UV85

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TECHNICAL NOTE

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Synopsis

The braking force of a heavy truck and the forces of acceleration and deceleration cannot be ignored in the design of flexible pavement. In some cases these may be as high as eighty per cent or more of the wheel load. Boussinesq-Cerruti solutions (1) on the stress distribution due to vertical and tangential forces applied at a point on the straight boundary surface of a homogeneous, isotropic, and elastic solid of infinite extent are available.

In this paper the stress distribution due to tangential force for the special case of Poisson's ratio of 1/2 is presented. The general solution is too tedious for practical applications and Poisson's ratio of 1/2 is the value generally accepted in flexible pavement design.

Introduction

There are a great many methods of flexible pavement design. Some are purely empirical and others are partly theoretical. None of the methods properly takes into account the shear at the surface created by the braking force or the acceleration and deceleration forces. It is obvious why some of the empirical methods have widespread use, and why theoretical methods have to be used in conjunction with empirical coefficients. The Boussinesq-Cerruti solution for vertical force, commonly known as Boussinesq's solution, is quite well known and has been used in some of the methods. Though there are some theoretical objections in the use of the solution for the stress analysis of the multi-layer systems, field tests (2) and model tests (3) on stress distribution in flexible pavements have verified the applicability of the Boussinesq-Cerruti

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(1) Love, A. E. H., A treatise on the mathematical theory of elasticity, 4th ed., Cambridge University Press, 1952.

(2) Griffith, John M., "Development of CBR flexible pavement design method for airfields (symposium): Wheel load tests, Marietta, Ga." American Society of Civil Engineers, Transactions, v 115, 1950, pp 506-519.

(3) McMahon, T. F., and Yoder, E. J., "Design of a pressure-sensitive cell and model studies of pressures in a flexible pavement subgrade," Highway Research Board, Proceedings, v 39, 1960, pp 650-682.

solution for vertical force. The solution for tangential force, though available in mathematical form, is little known and has never been considered seriously by pavement engineers. The introduction of the effects of tangential force in the analysis of stress distribution is definitely a step in the advancement of the complex art of flexible pavement design.

Scope of work

The main work of this paper is in presenting the special case of stress distribution due to tangential force for Poisson's ratio of 1/2. No attempt is made to apply the solution to design problems.

Boussinesq-Cerruti solutions

Displacement components (4) of Boussinesq-Cerruti solutions of both cases, that of vertical force and that of tangential force are as follows:

Vertical force

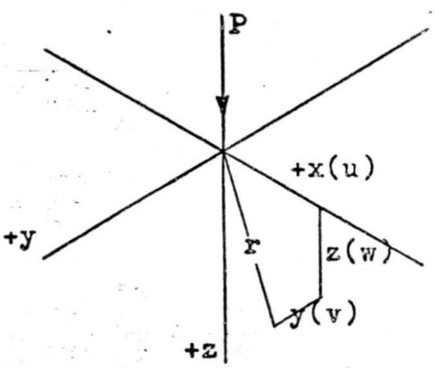

Fig. 1a

Tangential force

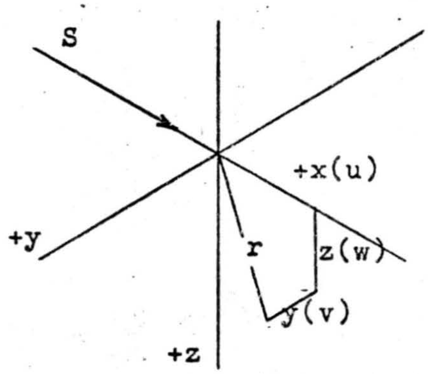

Fig. 1b

$$u = \frac{P}{4\pi\mu}\frac{xz}{r^3} - \frac{P}{4\pi(\lambda+\mu)}\frac{x}{r(z+r)}$$

$$v = \frac{P}{4\pi\mu}\frac{yz}{r^3} - \frac{P}{4\pi(\lambda+\mu)}\frac{y}{r(z+r)}$$

$$w = \frac{P}{4\pi\mu}\frac{z^2}{r^3} + \frac{P(\lambda+2\mu)}{4\pi\mu(\lambda+\mu)r}$$

$$u = \frac{S}{4\pi\mu}\left(\frac{\lambda+3\mu}{\lambda+\mu}\frac{1}{r} + \frac{x^2}{r^3}\right) - \frac{S}{2\pi(\lambda+\mu)}\frac{1}{r} + \frac{S}{4\pi(\lambda+\mu)}\left\{\frac{1}{(x+r)} - \frac{x^2}{r(z+r)^2}\right\}$$

$$v = \frac{S}{4\pi\mu}\frac{xy}{r^3} - \frac{S}{4\pi(\lambda+\mu)}\frac{xy}{r(x+r)^2}$$

$$w = \frac{S}{4\pi\mu}\frac{xz}{r^3} + \frac{S}{4\pi(\lambda+\mu)}\frac{x}{r(z+r)}$$

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(4) A. E. H. Love, op. cit.

where P = vertical force
S = tangential force
u = displacement parallel to x axis
v = " " " y "
w = " " " z "
E = modulus of elasticity
γ = Poisson's ratio

$$\mu = \frac{E}{2(1+\gamma)}$$

For $\gamma = \frac{1}{2}$.

$$\mu = \frac{E}{2(1+\frac{1}{2})} = \frac{E}{3}, \quad \lambda = \frac{\frac{E}{2}}{(1+\frac{1}{2})(1-2\times\frac{1}{2})} = \infty$$

$$\lim_{\lambda\to\infty} \frac{\lambda+3\mu}{\lambda+\mu} = 1, \quad \lim_{\lambda\to\infty} \frac{\lambda+2\mu}{\lambda+\mu} = 1$$

We have

$$u = \frac{3P}{4\pi E}\frac{xz}{r^3}$$

$$v = \frac{3P}{4\pi E}\frac{yz}{r^3}$$

$$w = \frac{3P}{4\pi E}\frac{z^2}{r^3} + \frac{3P}{4\pi E}\frac{1}{r}$$

$$= \frac{3P}{4\pi E}\left(\frac{z^2}{r^3} + \frac{1}{r}\right)$$

$$u = \frac{3S}{4\pi E}\left(\frac{x^2}{r^3} + \frac{1}{r}\right)$$

$$v = \frac{3S}{4\pi E}\frac{xy}{r^3}$$

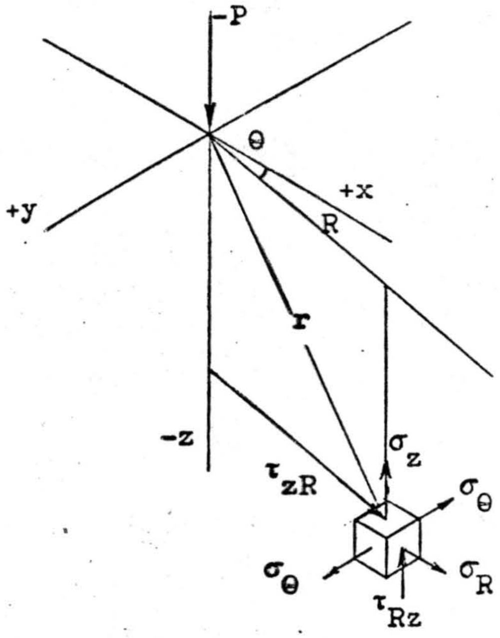
$$w = \frac{3S}{4\pi E}\frac{xz}{r^3}$$

Note the identity of the two sets if we interchange x and z and u and w.

Obviously the strain and the stress components of the two solutions are identical if we interchange x and z, and u and w, in the case of the stress components interchanging x and z in the subscripts. This is the three dimensional analogue of the solution for semi-infinite plate (5).

(5) Timoshenko, Stephan, and Goodier, J. N., Theory of elasticity, 2nd ed., McGraw Hill Book Co., New York, 1951.

Boussinesq-Cerruti solution for vertical force in cylindrical co-ordinates



For $\gamma = 1/2$ (6)

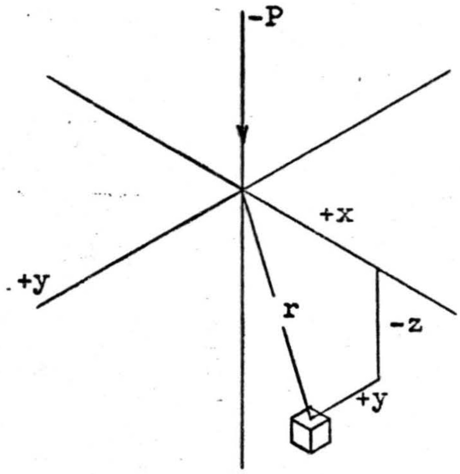
$$\sigma_R = -\frac{3}{2}\frac{P}{\pi}\frac{R^2 z}{r^5}$$

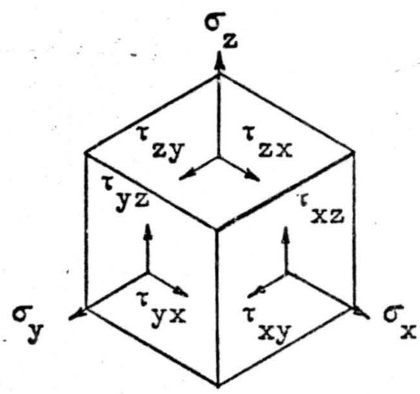
$$\sigma_\theta = 0$$

$$\sigma_z = -\frac{3}{2}\frac{P}{\pi}\frac{z^3}{r^5}$$

$$\tau_{zR} = \tau_{Rz} = -\frac{3}{2}\frac{P}{\pi}\frac{Rz^2}{r^5}$$

The stress components can be readily converted into stress components in Cartesian co-ordinates as follows:





$$\sigma_x = \sigma_R \cos^2\theta, \quad \cos\theta = \frac{x}{R}$$

$$\sigma_x = \sigma_R \frac{x^2}{R^2} = -\frac{3}{2}\frac{P}{\pi}\frac{R^2 z}{r^5}\frac{x^2}{R^2}$$

$$\sigma_x = -\frac{3}{2}\frac{P}{\pi}\frac{x^2 z}{r^5} \qquad \text{-------(1)}$$

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(6) ibid.

$$\sigma_y = \sigma_R \sin^2 \theta, \quad \sin \theta = \frac{y}{R}$$

$$= -\frac{3}{2}\frac{P}{\pi}\frac{R^2 z}{r^5}\frac{y^2}{R^2}$$

$$\sigma_y = -\frac{3}{2}\frac{P}{\pi}\frac{zy^2}{r^5} \qquad \text{-------(2)}$$

$$\tau_{xy} = \tau_{yx} = \sigma_R \sin \theta \cos \theta$$

$$= -\frac{3}{2}\frac{P}{\pi}\frac{R^2 z}{r^5}\frac{x}{R}\frac{y}{R}$$

$$\tau_{xy} = \tau_{yx} = -\frac{3}{2}\frac{P}{\pi}\frac{xyz}{r^5} \qquad \text{-------(3)}$$

$$\tau_{xz} = \tau_{zx} = \tau_{zR} \cos \theta$$

$$= -\frac{3}{2}\frac{P}{\pi}\frac{Rz^2}{r^5}\frac{x}{R}$$

$$\tau_{xz} = \tau_{zx} = -\frac{3}{2}\frac{P}{\pi}\frac{z^2 x}{r^5} \qquad \text{-------(4)}$$

$$\tau_{yz} = \tau_{zy} = \tau_{zR} \sin \theta$$

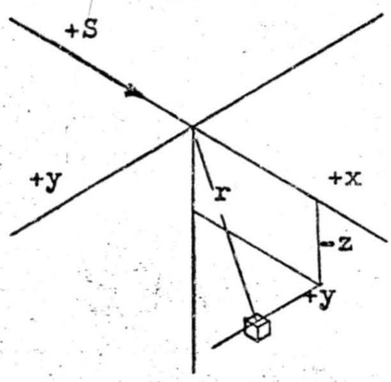
$$= -\frac{3}{2}\frac{P}{\pi}\frac{Rz^2}{r^5}\frac{y}{R}$$

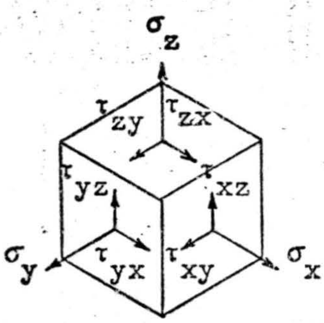
$$\tau_{yz} = \tau_{zy} = -\frac{3}{2}\frac{P}{\pi}\frac{z^2 y}{r^5} \qquad \text{-------(5)}$$

$$\sigma_z = -\frac{3}{2}\frac{P}{\pi}\frac{z^3}{r^5} \qquad \text{-------(6)}$$

The nine stress components are defined by equations (1) to (6).

Boussinesq-Cerruti solution for tangential force





For $\gamma = 1/2$

By interchanging x and z and change P to S, we have

$$\sigma_z = -\frac{3}{2}\frac{S}{\pi}\frac{z^2 x}{r^5} \qquad \text{--------(1 a)}$$

$$\sigma_y = -\frac{3}{2}\frac{S}{\pi}\frac{xy^2}{r^5} \qquad \text{--------(2 a)}$$

$$\tau_{zy} = \tau_{yz} = -\frac{3}{2}\frac{S}{\pi}\frac{zyx}{r^5} \qquad \text{--------(3 a)}$$

$$\tau_{zx} = \tau_{xz} = -\frac{3}{2}\frac{S}{\pi}\frac{x^2 z}{r^5} \qquad \text{--------(4 a)}$$

$$\tau_{yx} = \tau_{xy} = -\frac{3}{2}\frac{S}{\pi}\frac{x^2 y}{r^5} \qquad \text{--------(5 a)}$$

$$\sigma_x = -\frac{3}{2}\frac{S}{\pi}\frac{x^3}{r^5} \qquad \text{--------(6 a)}$$

The nine stress components are thus obtained for tangential force S.

Conclusion

For the case of $\gamma = 1/2$, Boussinesq-Cerruti solutions for vertical force and tangential force are identical if the coordinates x and z, and the subscripts x and z are interchanged.