Orbital Mechanics and Analytic Modeling of Meteorological Satellite Orbits

Applications to the Satellite Navigation Problem

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ABSTRACT

An analysis is carried out which considers the relationship of orbit mechanics to the satellite navigation problem, in particular, meteorological satellites. A preliminary discussion is provided which characterizes the distinction between "classical navigation" and "satellite navigation" which is a process of determining the space time coordinates of data fields provided by sensing instruments on meteorological satellites. Since it is the latter process under consideration, the investigation is orientated toward practical applications of orbit mechanics to aid the development of analytic solutions of satellite orbits.

Using the invariant two body Keplerian orbit as the basis of discussion, an analytic approach used to model the orbital characteristics of near earth satellites is given. First the basic concepts involved with satellite navigation and orbit mechanics are defined. In addition, the various measures of time and coordinate geometry are reviewed. The two body problem is then examined beginning with the fundamental governing equations, i.e. the inverse square force field law. After a discussion of the mathematical and physical nature of this equation, the Classical Orbital Elements used to define an elliptic orbit are described. The mathematical analysis of a procedure used to calculate celestial position vectors of a satellite is then outlined. It is shown that a transformation of Kepler's time equation (for an elliptic orbit) to an expansion in powers of eccentricity removes the need for numerical approximation.

The Keplerian solution is then extended to a perturbed solution, which considers first order time derivatives of the elements defining the orbital plane. Using a formulation called the gravitational perturbation function, the form of a time variant perturbed two body orbit is examined. Various characteristics of a perturbed orbit are analyzed including definitions of the three conventional orbital periods, the nature of a sun-synchronous satellite, and the velocity of a noncircular orbit.

Finally, a discussion of the orbital revisit problem is provided to highlight the need to develop efficient, relatively exact, analytic solutions of meteorological satellite orbits. As an example, the architectural design of a satellite system to measure the global radiation budget without deficiencies in the space time sampling procedure is shown to be a simulation problem based on "computer flown" satellites. A set of computer models are provided in the appendices.

TABLE OF CONTENTS

1.0 INTRODUCTION

The topic of this investigation is orbital mechanics and its relationship to the satellite navigation problem. Since the term "satellite navigation" denotes a variety of concepts, it is important to refine a definition for purposes of this study. We say, in general, that satellite navigation is a process of identifying the space and time coordinates of satellite data products (in this case meteorological satellites). Note that this characterization departs somewhat from the classical usage of navigation which implies the definition and maneuvering of the position of ships, aircraft, satellites, etc. A more exact definition is given in Chapter 2. A fundamental component of any satellite navigation system is a model of the satellite's orbital properties. This investigation is primarily concerned with the mathematical and physical nature of near earth meteorological satellite orbits and thus meteorological satellite navigation requirements. The study also considers the basic nature of coordinate systems and the various measures of time.

There are two very general orbital application areas insofar as meteorological satellites are concerned. The first and more traditional application of orbital analysis is the process of tracking the position and motion of satellites, by the space agencies, so as to provide ephemeris and antenna pointing information to ground readout stations and operations command facilities. Considering that in this process, the actual characteristics of an orbital plane are defined, this can be referred to as a navigation process. However, for our purposes, we shall consider this process as an "orbital tracking" problem.

The second application is the analytic treatment of orbital motion in a model designed for processing the meteorological data, generated by spacecraft instrumentation. In this case, there are very different computational and operational restraints than in the case of orbit tracking. Primarily we are concerned with developing efficient and quick computational routines that retain a relatively high degree of orbital position accuracy, but are not bogged down with the multiplicity of external forces that orbit tracking models must consider.

The practical outcome of the study is a set of orbital computer models, which are adaptable in a very general fashion, to a variety of analytic near-earth satellite navigation systems. The usability of these models is insured because they are based on the conventional orbital elements available from the primary meteorological satellite agencies, i.e. the National Environmental Satellite Service (NESS), the National Aeronautical Space Administration (NASA), the European Space Agency (ESA), and the National Space Development Agency (NASDA) of Japan. The reader may refer to Appendix A for an explanation.

Meteorological satellites, whether they are of the experimental or operational type, are classified as either geosynchronous (≃ 24 hour period) or polar low orbiter (≃ 100 minute period) by the above agencies. The low orbiters may be placed in either sun-synchronous or non-sun-synchronous orbit. All of these satellites are in nearly circular orbit, and in general, are at altitudes at which atmospheric drag is not a significant factor over the prediction time scale under consideration (≃ 1-2 weeks). This investigation will be addressed to these types of orbits.

Chapter 2.0 considers some basic concepts which are crucial to an understanding of the satellite navigation problem. Chapter 3.0 provides a set of definitions and an explanation of the various measures of time. A discussion of station coordinates (latitude) is given in Chapter 4.0 along with some fundamental geometric definitions. Chapter 5.0 represents the major portion of the analysis, that is, a discussion of the two body orbit problem and a method to calculate orbital position vectors given a set of "Classical Orbital Elements". Chapter 6.0 considers the time varying properties of an orbit and goes on to look at the resultant effects of the aspherical gravitational potential of the earth on the orbital characteristics of a satellite. The topic of the orbital revisit problem is considered in Chapter 7.0. Finally, appendices are included which provide a set of computer models which can be used to calculate orbital position vectors and the various orbital periods which are discussed in the chapter on perturbation theory.

A principle reference used in this analysis is the very fine compendium on Orbit Mechanics by Pedro Ramon Escobal (1965), hereafter EB. This work stands alone as an aid to solving orbital mechanics problems faced by satellite workers and scientists. Other very helpful references used in this study were The Handbook on Practical Navigation by Bowditch (1962) and a translation of a Russian text on orbit determination by Dubyago (1961). The latter work provides a very interesting historical sketch of the development of orbital mechanics and man's understanding of the motion of celestial bodies.

2.0 BASIC CONCEPTS

2.1 Orbit Mechanics and Satellite Navigation

The following definitions are essential to an understanding of the ensuing analysis:

Orbital Mechanics: A branch of celestial mechanics concerned with orbital motions of celestial bodies or artificial spacecraft.

Celestial Mechanics: The calculation of motions of celestial bodies under the action of their mutual gravitational attractions.

Astrodynamics: The practical application of celestial mechanics, astroballistics, propulsion theory, and allied fields to the problem of planning and directing the trajectories of space vehicles.

Navigation (General): The process of directing the movement of a craft so that it will reach its intended destination: subprocesses are position fixing, dead reckoning, pilotage, and homing.

Navigation (Satellite): The process of determining a set of unique transformations between the coordinates of satellite data points in a satellite frame of reference and their associated terrestrial or planetary coordinates. (This definition should be contrasted with "Satellite Image Alignment", which is a non-analytic, mostly subjective process in which the two or more images to be aligned often have different aspect ratio characteristics.)

The major areas of Orbital Mechanics are:

1. Satellite Orbit Injection
   a. Thrust (Ballistic, Propulsion) forces
   b. Drag forces
   c. Lift forces

2. Determination of Orbital Elements

   a. Position vector, velocity vector, and initial time $(\vec{r}, \dot{\vec{r}}, t_o)$

   b. Two position vectors and times $(\vec{r}_1, t_1, \vec{r}_2, t_2)$

   c. Three pairs of azimuth-elevation angles and times $[(\Phi_1, H_1, t_1), (\Phi_2, H_2, t_2), (\Phi_3, H_3, t_3)]$

   d. Slant-range, range-rate, and time observations $[(d_1, \dot{d}_1, t_1), (d_2, \dot{d}_2, t_2)\ldots]$

   e. Mixed observations (angles, ranges, range-rates, times)

3. Orbital Properties and Tracks

   a. Orbital elements

   b. Velocities and periods

   c. Position vectors

   d. Direct and retrograde orbits

   e. Equator crossing data

   f. Orbital revisit frequencies

4. Orbital Analytics (Keplermanship)

   a. Nodal passages

   b. Satellite rise and set times

   c. Line of sight periods and eclipses

   d. Orbital architecture

The ensuing analysis will be primarily concerned with the topics outlined in parts 3 and 4. Since meteorological satellite navigation methods are generally not affected by how satellites are placed in orbit nor how the various space agencies track these satellites so as to produce orbital elements (other than the associated errors), we will put aside any further discussion of parts 1 and 2, and instead concentrate on the material outlined in parts 3 and 4.

2.2 Satellite Navigation Modeling

Satellite navigation modeling can be considered to be a five part problem:

1. The time dependent determination of the spacecraft orbital position in an inertial coordinate system.

2. The time dependent determination of the spacecraft orientation (attitude) in an inertial coordinate system.

3. The specification or determination (time dependent) of the optical paths of the imaging or sounding instrument with respect to the spacecraft.

4. The integration of the above static and dynamic aspects of the spacecraft into a model which can provide measurement pointing vectors in the inertial frame of reference.

5. The transformation of the inertial pointing vectors to pointing vectors in the preferred (non-inertial) coordinate system.

The first requirement of an analytic navigation technique is a model which can solve for satellite position at any specified time. In fact, the determination of spacecraft orientation is absolutely dependent on knowledge of satellite position if ground based or star based attitude determination techniques are applied. A discussion of this topic can be found in Smith and Phillips (1972) and is presently being extended by Phillips (1979). With the knowledge of spacecraft position and orientation, the dynamics of the actual on-board instrumentation can then be considered. Finally, upon integration of these three dynamic aspects of an orbiting satellite into an appropriate model, pointing vectors can be obtained which fix the relationship between an instrument field-of-view and a terrestrial coordinate (latitude, longitude, height).

2.3 Satellite Orientation

It is important to distinguish between the effect of varying satellite position and varying satellite orientation on the apparent earth scene. First of all it is instructive to define the various terms associated with satellite orientation:

Attitude: Orientation of the principal axis of a spacecraft, e.g. the spin axis, with respect to the principal axis (spin axis) of the earth, usually given in terms of declination and right ascension with respect to a celestial frame of reference.

Precession: The angular velocity of the axis of spin of a spinning rigid body, which arises as a result of steady uneven external torques acting on the body.

Nutation: A high frequency spiral, bobbing, or jittering motion of a spinning rigid body, about a mean principal axis, due to asymmetric weight distribution or short period torque modulation.

Wobble: An irregular vacillation of a body about its mean principal axis due to non-solid body characteristics.

Figure 2.1 has been provided to illustrate these definitions.

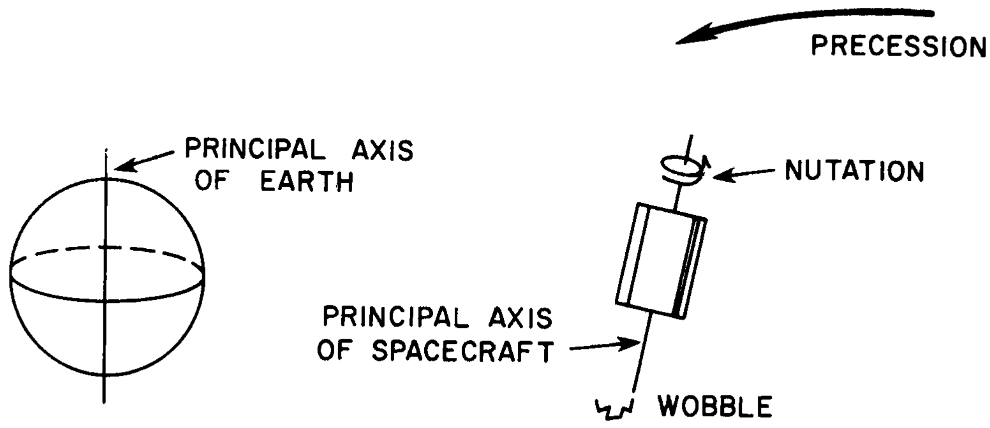


Figure 2.1 Dynamics of Satellite Orientation

Variation in the orientation of a meteorological satellite can lead to both translations and rotations of earth fields with respect to a fixed satellite field-of-view. These apparent motions are superimposed on real motions due to variation in the orbital position. A requirement of any satellite navigation model is the inclusion of procedures to separate the apparent motions from the real motions which are essentially independent processes. Therefore, this investigation will be devoted to the determination of orbital position as these calculations generally preface the determination of the remaining navigational parameters.

2.4 Applications of a Satellite Navigation Model

Finally, an important question concerning satellite navigation is: "What does a navigation model provide?" Essentially, it provides the following three capabilities:

1. The capability of placing grid and/or geographic-topographic annotation information in or on the data. This process should be called a "Gridding" process.

2. A means to specify the terrestrial or planetary coordinate of a given data point coordinate, or conversely, to specify the data point coordinate corresponding to a given terrestrial or planetary coordinate. This process should be called a "Navigational Interrogation" process.

3. A framework for transforming the raw satellite imagery into alternate cartographic (map) projections. The actual process of reorganizing the raw data into a new projection should be called a "Mapping" process.

Note the actual navigation process only involves specifying, calculating, or determining the appropriate parameters inherent to the navigation model and utilizing them to calculate coordinate transformations.

3.0 TIME

3.1 Basic Systems of Time

Any navigational process, by its very nature, involves various systems of time. Therefore, we need the following definitions:

Mean Solar Time (MST): Time that has the mean solar second as its unit and is based on the mean sun's motion. One mean solar second is 1/86,400 of a mean solar day. One solar day is 24 hours of mean solar time.

Greenwich Mean Time (GMT): Mean solar time at the meridian of Greenwich, England. Also referred to as Universal Time (UT0), Zulu Time, Z-Time, or Greenwich Civil Time:

$$GMT = MST + n \tag{3.1}$$

where n is the number of time zones to the west of the Greenwich meridian as shown in Figure 3.1. There are also higher order systems of Universal Time (UT1, UT2) which are corrected for variations in the earth's rotational rate due to secular, irregular, periodic seasonal and periodic tidal terms and polar motion due to solar and lunar gravitational effects on the earth's equatorial bulge. These corrections are not significant for the time periods we are considering.

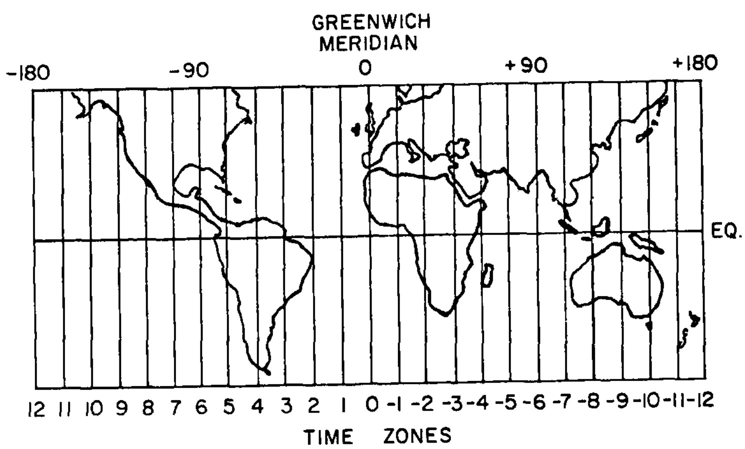


Figure 3.1 Time Zones

Ephemeris Time (ET): A uniform measure of time defined by laws of dynamics and determined in principle from the orbital motions of the planets, particularly of the earth. One ephemeris second (ISU:1960) is 1/31556925.9747 of a tropical year defined by the mean motion of the sun in longitude at the epoch 1900, January 0, 12 hours (12:00 GMT, Dec. 31, 1899). An ephemeris day is 86,400 ephemeris seconds. The earth's rotation suffers periodic and secular variations in rotation so that ephemeris time is defined by:

$$ET = GMT + \Delta t \tag{3.2}$$

where Δt is an annual increment tabulated in the American Ephemeris and Nautical Almanac. For instance, using values from the American Ephemeris and Nautical Almanac (1978), Table 3.1 is generated:

Table 3.1: Ephemeris Time Correction Increments

Year	Δt
1956.5	31.52
1957.5	31.92
1958.5	32.45
1959.5	32.91
1960.5	33.39
1961.5	33.80
1962.5	34.23
1963.5	34.73
1964.5	35.40
1965.5	36.14

Note that Δt can not be calculated in advance. It is determined from observed and predicted positions of the moon.

It is also worth noting that the change in the time increment from year to year is fairly insignificant. The result of this characteristic of ephemeris time, is that short term orbital predictions ($\simeq$ 5 years) can effectively ignore ephemeris corrections. Although this may simplify operational satellite orbit prediction, incremental correction must be included when considering long term orbital calculations such as historical earth-sun configurations. Table 3.2 represents a listing of incremental corrections from the American Nautical and Ephemeris Almanac (1978).

Atomic Time (AT): A measure of time based on the oscillations of the U.S. Cesium Frequency Standard (National Bureau of Standards, Boulder, Colorado). The standard is based on the U.S. Naval Observatory's suggested value of 9,192,631,770 oscillations per second of the cesium atom - isotope 133. The reference epoch has been defined as January 1, 1958 $0^h0^m0^s$ GMT. The standard time scale to which U.S. orbital tracking stations are synchronized is the Universal Time Coordinated (UTC) system. This system is derived from an atomic time scale. Prior to 1972 the UTC system operated at a frequency offset from the AT system. Since January 1, 1972 the UTC system is derived from a rubidium atomic frequency standard. The new measurements used to convert to UTC come from various global stations and are thus referred to as Station Time (ST).

Tropical Year: Period of one revolution of the earth measured between two vernal equinoxes. Equal to 365.24219879 mean solar days or 365 days, 5 hours, 48 minutes, 46 seconds or 31,556,925.9747 ephemeris seconds. Also referred to as an Astronomical Year, Equinoctial Year, Natural Year or Solar Year.

Anomalistic Year: Period of one revolution of the earth measured between perhelion to perhelion (see Figure 3.2). Equal to 365.259641204

Table 3.2: Ephemeris Time Correction Table (From the 1978 American Ephemeris and Nautical Almanac)

Date (0^h UT)	ΔT(A)	ΔUT1
1956	s	s
Jan. 1	+31.34	−0.08
Jan. 4	31.34	− .08
Jan. 4	31.34	− .02
Apr. 1	31.43	− .04
July 1	31.52	− .07
Oct. 1	31.56	− .01
1957		
Jan. 1	+31.67	−0.04
Apr. 1	31.79	− .06
July 1	31.92	− .07
Oct. 1	32.00	− .02
1958		
Jan. 1	+32.17	−0.04
Apr. 1	32.32	− .05
July 1	32.45	− .06
Oct. 1	32.52	− .01
1959		
Jan. 1	+32.67	−0.03
Apr. 1	32.80	− .03
July 1	32.91	− .06
Oct. 1	33.00	.00
1960		
Jan. 1	+33.15	−0.01
Apr. 1	33.28	− .03
July 1	33.39	− .02
Oct. 1	33.45	+ .03
1961		
Jan. 1	+33.58	+0.02
Apr. 1	33.70	+ .02
July 1	33.80	+ .04
July 31	33.81	+ .06
Aug. 1	33.81	+ .01
Oct. 1	33.86	+ .04
1962		
Jan. 1	+33.99	+0.04
Apr. 1	34.12	+ .01
July 1	34.23	.00
Oct. 1	34.31	+ .02
1963		
Jan. 1	+34.47	−0.03
Apr. 1	34.58	− .05
July 1	34.73	− .09
Oct. 1	34.83	− .09
Oct. 31	34.90	− .12
Nov. 1	34.90	− .02
1964		
Jan. 1	+35.03	−0.08
Mar. 31	+35.22	−0.15

Date (0^h UT)	ΔT(A)	ΔUT1
1964	s	s
Apr. 1	+35.22	−0.05
July 1	35.40	− .11
Aug. 31	35.47	− .11
Sept. 1	35.47	− .01
Oct. 1	35.52	− .02
Dec. 31	35.73	− .11
1965		
Jan. 1	+35.73	−0.01
Feb. 28	35.86	− .06
Mar. 1	35.86	+ .04
Apr. 1	35.94	.00
June 30	36.14	− .08
July 1	36.14	+ .02
Aug. 31	36.24	− .01
Sept. 1	36.24	+ .09
Oct. 1	36.31	+ .06
1966		
Jan. 1	+36.54	−0.05
Apr. 1	36.76	− .03
July 1	36.99	− .02
Oct. 1	37.18	+ .02
1967		
Jan. 1	+37.43	+0.01
Apr. 1	37.65	+ .02
July 1	37.87	+ .04
Oct. 1	38.04	+ .10
1968		
Jan. 1	+38.29	+0.09
Jan. 31	38.37	+ .09
Feb. 1	38.37	− .01
Apr. 1	38.52	.00
July 1	38.75	+ .01
Oct. 1	38.95	+ .04
1969		
Jan. 1	+39.20	+0.03
Apr. 1	39.45	+ .02
July 1	39.70	+ .01
Oct. 1	39.91	+ .03
1970		
Jan. 1	+40.18	0.00
Apr. 1	40.45	− .03
July 1	40.70	− .05
Oct. 1	40.89	− .01
1971		
Jan. 1	+41.16	−0.04
Apr. 1	41.41	− .05
July 1	41.68	− .08
Oct. 1	41.92	− .09
Dec. 31	+42.22	−0.15

Date (0^h UT)	ΔT(A)	ΔUT1
1972	s	s
Jan. 1	+42.22	−0.04
Apr. 1	42.52	− .34
June 30	42.82	− .64
July 1	42.82	+ .36
Oct. 1	43.07	+ .11
Dec. 31	43.37	− .19
1973		
Jan. 1	+43.37	+0.81
Apr. 1	43.67	+ .51
July 1	43.96	+ .22
Oct. 1	44.19	− .01
Dec. 31	44.48	− .30
1974		
Jan. 1	+44.48	+0.70
Apr. 1	44.73	+ .45
July 1	44.99	+ .19
Oct. 1	45.20	− .02
Dec. 31	45.47	− .29
1975		
Jan. 1	+45.47	+0.71
Apr. 1	45.73	+ .45
July 1	45.98	+ .20
Oct. 1	46.18	.00
Dec. 31	46.45	− .27
1976		
Jan. 1	+46.45	+0.73
Apr. 1	(46.7)	(+ .5)
July 1	(47.0)	(+ .2)
Oct. 1	(47.2)	(.0)
1977		
Jan. 1	(+47.4)	
Apr. 1	(47.7)	
July 1	(47.9)	
Oct. 1	(48.1)	
1978		
Jan. 1	(+48.4)	
Apr. 1	(48.6)	
July 1	(48.8)	
Oct. 1	(49.1)	
1979		
Jan. 1	(+49.3)	

The quantity $\Delta T(A) = 32^s\!.18 + TAI - UT1$ provides a first approximation to $\Delta T = ET - UT$, the reduction from Universal to Ephemeris Time. TAI is the scale of International Atomic Time formally introduced on 1972 January 1, but extrapolated to previous dates; UT1 is the observed Universal Time, corrected for polar motion. The correction $\Delta UT1 = UT1 - UTC$ is given for use in connection with broadcast time signals, which are now UTC in most countries. Coded values of $\Delta UT1$ are now given in the primary time signal emissions, and may be as much as $\pm 0^s\!.8$. Discontinuities in UTC can occur at 0^h UT on the first day of a month (exception: 1956 Jan. 4, discontinuity at 19^h UT). Special entries are given for the two dates bracketing any discontinuity greater than $0^s\!.02$. Values within parentheses are either provisional (two decimals) or extrapolated (one decimal). Additional information is given in the explanation concerning time scales (page 527) and concerning the use of ΔT with ephemerides (pages 539–541).

Table 3.2 Continued

CORRECTIONS

The American Ephemeris, 1970–1978

The corrections tabulated below should be *added* to $A_E+180°$ and $A_S+180°$ in the Ephemeris for Physical Observations of Jupiter for the years 1970–1978. These corrections should also be *subtracted* from the Longitude of Central Meridian (System I and System II).

	°
1970	+0.03
1971	+0.02
1972	+0.02
1973	+0.01
1974	0.00
1975	−0.01
1976	−0.02
1977	−0.03
1978	−0.03

The American Ephemeris, 1972–1980

All the *negative* values of the Astrometric Declination of the four principal minor planets, Ceres, Pallas, Juno, Vesta, for the years 1972–1980 require a correction of $-0''.1$.

For example, on page 281 of this volume:

1978 Aug. 16 *for* $-31°15'52''.4$ *read* $-31°15'52''.5$

The American Ephemeris, 1972–1977

The mean motion for the Earth in the table of mean elements at the top of page 216 is referred to a moving equinox while the mean motions for Mercury, Venus and Mars are referred to a fixed equinox. For consistency, the Earth's mean motion should also have been referred to a fixed equinox; in which case its value should have been 0.985609.

CIVIL CALENDAR

New Year's Day	Sun.	Jan.	1	Labor Day	Mon.	Sept.	4
Lincoln's Birthday	Sun.	Feb.	12	Columbus Day	Mon.	Oct.	9
Washington's Birthday . .	Mon.	Feb.	20	Veterans Day	Sat.	Nov.	11
Memorial Day	Mon.	May	29	General Election Day . . .	Tue.	Nov.	7
Independence Day	Tue.	July	4	Thanksgiving Day	Thu.	Nov.	23

mean solar days or 365 days, 6 hours, 13 minutes, 53 seconds. Keep in mind that the perhelion is continually precessing.

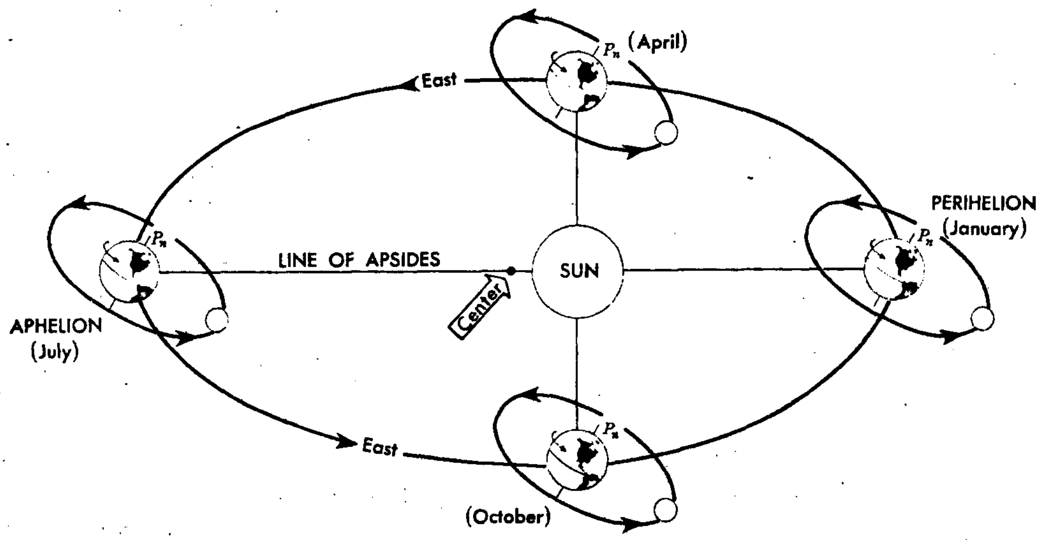


Figure 3.2 Nodal Passages of the Earth's Orbit
(From Bowditch, 1962)

Julian Day: The number of each day, counted consecutively since the beginning of the present Julian period on January 1, 4713 B.C. The Julian Day begins at noon, 12 hours later then the corresponding civil day (see Table 3.3).

Julian Calendar: A calendar replaced by the Gregorian Calendar. The Julian year was 365.25 days, the fraction allowed for the extra day every fourth year (leap year). There are 12 months, each 30 or 31 days except for February which has 28 days or in leap year 29. "Thirty days hath September, April, June, and November. All the rest have 31, excepting February, which has 28, although in leap years 29."

Table 3.3: Julian Day Number (From EB, 1965)

Days Elapsed at Greenwich Noon, A.D. 1950–2000

YEAR	JAN. 0	FEB. 0	MAR. 0	APR. 0	MAY 0	JUNE 0	JULY 0	AUG. 0	SEP. 0	OCT. 0	NOV. 0	DEC. 0
1950	243 3282	3313	3341	3372	3402	3433	3463	3494	3525	3555	3586	3616
1951	3647	3678	3706	3737	3767	3798	3828	3859	3890	3920	3951	3981
1952	4012	4043	4072	4103	4133	4164	4194	4225	4256	4286	4317	4347
1953	4378	4409	4437	4468	4498	4529	4559	4590	4621	4651	4682	4712
1954	4743	4774	4802	4833	4863	4894	4924	4955	4986	5016	5047	5077
1955	243 5108	5139	5167	5198	5228	5259	5289	5320	5351	5381	5412	5442
1956	5473	5504	5533	5564	5594	5625	5655	5686	5717	5747	5778	5808
1957	5839	5870	5898	5929	5959	5990	6020	6051	6082	6112	6143	6173
1958	6204	6235	6263	6294	6324	6355	6385	6416	6447	6477	6508	6538
1959	6569	6600	6628	6659	6689	6720	6750	6781	6812	6842	6873	6903
1960	243 6934	6965	6994	7025	7055	7086	7116	7147	7178	7208	7239	7269
1961	7300	7331	7359	7390	7420	7451	7481	7512	7543	7573	7604	7634
1962	7665	7696	7724	7755	7785	7816	7846	7877	7908	7938	7969	7999
1963	8030	8061	8089	8120	8150	8181	8211	8242	8273	8303	8334	8364
1964	8395	8426	8455	8486	8516	8547	8577	8608	8639	8669	8700	8730
1965	243 8761	8792	8820	8851	8881	8912	8942	8973	9004	9034	9065	9095
1966	9126	9157	9185	9216	9246	9277	9307	9338	9369	9399	9430	9460
1967	9491	9522	9550	9581	9611	9642	9672	9703	9734	9764	9795	9825
1968	9856	9887	9916	9947	9977	*0008	*0038	*0069	*0100	*0130	*0161	*0191
1969	244 0222	0253	0281	0312	0342	0373	0403	0434	0465	0495	0526	0556
1970	244 0587	0618	0646	0677	0707	0738	0768	0799	0830	0860	0891	0921
1971	0952	0983	1011	1042	1072	1103	1133	1164	1195	1225	1256	1286
1972	1317	1348	1377	1408	1438	1469	1499	1530	1561	1591	1622	1652
1973	1683	1714	1742	1773	1803	1834	1864	1895	1926	1956	1987	2017
1974	2048	2079	2107	2138	2168	2199	2229	2260	2291	2321	2352	2382
1975	244 2413	2444	2472	2503	2533	2564	2594	2625	2656	2686	2717	2747
1976	2778	2809	2838	2869	2899	2930	2960	2991	3022	3052	3083	3113
1977	3144	3175	3203	3234	3264	3295	3325	3356	3387	3417	3448	3478
1978	3509	3540	3568	3599	3629	3660	3690	3721	3752	3782	3813	3843
1979	3874	3905	3933	3964	3994	4025	4055	4086	4117	4147	4178	4208
1980	244 4239	4270	4299	4330	4360	4391	4421	4452	4483	4513	4544	4574
1981	4605	4636	4664	4695	4725	4756	4786	4817	4848	4878	4909	4939
1982	4970	5001	5029	5060	5090	5121	5151	5182	5213	5243	5274	5304
1983	5335	5366	5394	5425	5455	5486	5516	5547	5578	5608	5639	5669
1984	5700	5731	5760	5791	5821	5852	5882	5913	5944	5974	6005	6035
1985	244 6066	6097	6125	6156	6186	6217	6247	6278	6309	6339	6370	6400
1986	6431	6462	6490	6521	6551	6582	6612	6643	6674	6704	6735	6765
1987	6796	6827	6855	6886	6916	6947	6977	7008	7039	7069	7100	7130
1988	7161	7192	7221	7252	7282	7313	7343	7374	7405	7435	7466	7496
1989	7527	7558	7586	7617	7647	7678	7708	7739	7770	7800	7831	7861
1990	244 7892	7923	7951	7982	8012	8043	8073	8104	8135	8165	8196	8226
1991	8257	8288	8316	8347	8377	8408	8438	8469	8500	8530	8561	8591
1992	8622	8653	8682	8713	8743	8774	8804	8835	8866	8896	8927	8957
1993	8988	9019	9047	9078	9108	9139	9169	9200	9231	9261	9292	9322
1994	9353	9384	9412	9443	9473	9504	9534	9565	9596	9626	9657	9687
1995	244 9718	9749	9777	9808	9838	9869	9899	9930	9961	9991	*0022	*0052
1996	245 0083	0114	0143	0174	0204	0235	0265	0296	0327	0357	0388	0418
1997	0449	0480	0508	0539	0569	0600	0630	0661	0692	0722	0753	0783
1998	0814	0845	0873	0904	0934	0965	0995	1026	1057	1087	1118	1148
1999	1179	1210	1238	1269	1299	1330	1360	1391	1422	1452	1483	1513
2000	254 1544	1575	1604	1635	1665	1696	1726	1757	1788	1818	1849	1879

Gregorian Calendar: The calendar used for civil purposes throughout the world, replacing the Julian calendar and closely adjusted to the tropical year.

Note that it is common practice among satellite data users to refer to the Julian day or date of a data set in terms of the day number of the corresponding year (1-365 or 1-366). This is not inconsistent with the classical definition since the initial day of the sequence is arbitrary.

3.2 The Annual Cycle and Zodiac

We must also consider the definition of sidereal time, but before doing so, a brief discussion of the annual cycle and the zodiac is in order. As the earth progresses through its annual cycle, there are four solar passages which are used to distinguish the seasons and divide the earth into its so called climate zones. There are two equator crossing (equinoxes) and two maximum excursion passages (solstices) of the sun with respect to the earth (see Figure 3.3). These are:

1. March or Spring Equinox
2. June or Summer Solstice
3. September or Autumnal Equinox
4. December or Winter Solstice

It is commonplace to refer to the summer and winter solstice latitudes as the tropic of cancer and the tropic of capricorn, respectively.

To an observer on the earth the sun appears to achieve a maximum latitudinal excursion of $+23^{o}27'$ or $-23^{o}27'$ at the solstices. The zone between these two parallels is often referred to as the torrid zone. The apparent motion of the sun, of course, is due to the inclination of the earth's orbit about the sun. The apparent track of the sun is along a plane which is called the ecliptic. When the sun reaches a solstice position, the opposite hemisphere is having its winter in which the limits of the circumpolar sun are approximately $23^{o}27'$ from the pole.

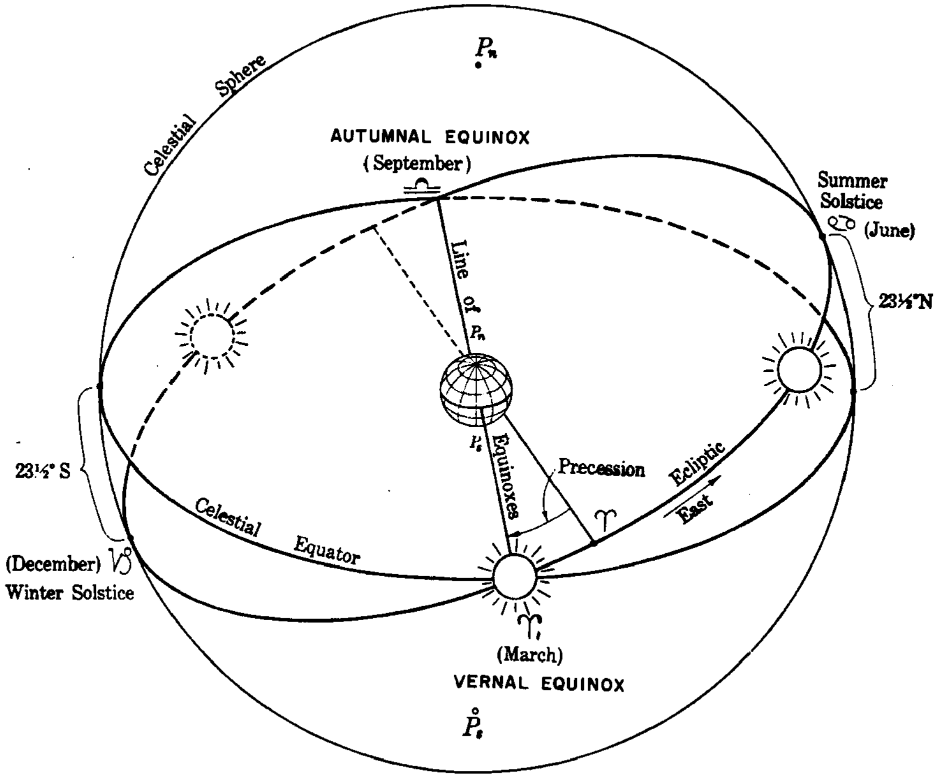


Figure 3.3 Solar Passages (From Bowditch, 1962)

These two polar circles define the boundaries between the temperate zones and the frigid zones, that is, the so-called arctic circle and antarctic circle parallels (see Figure 3.4).

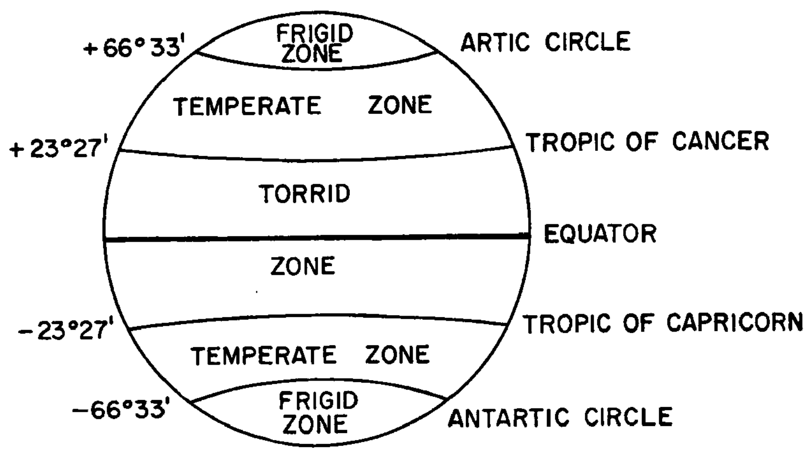


Figure 3.4 Climate Zones

The names used to describe the boundaries of the torrid zone were given some 2000 years ago when the sun was entering the constellations Cancer and Capricorn at the time of the solstices. By the same token the spring and autumnal equinoxes were taking place at the time the sun was entering the constellations Aires and Libra. Thus, it is appropriate to refer to the solstices and the equinoxes as zodiacal passages. What is the zodiac?

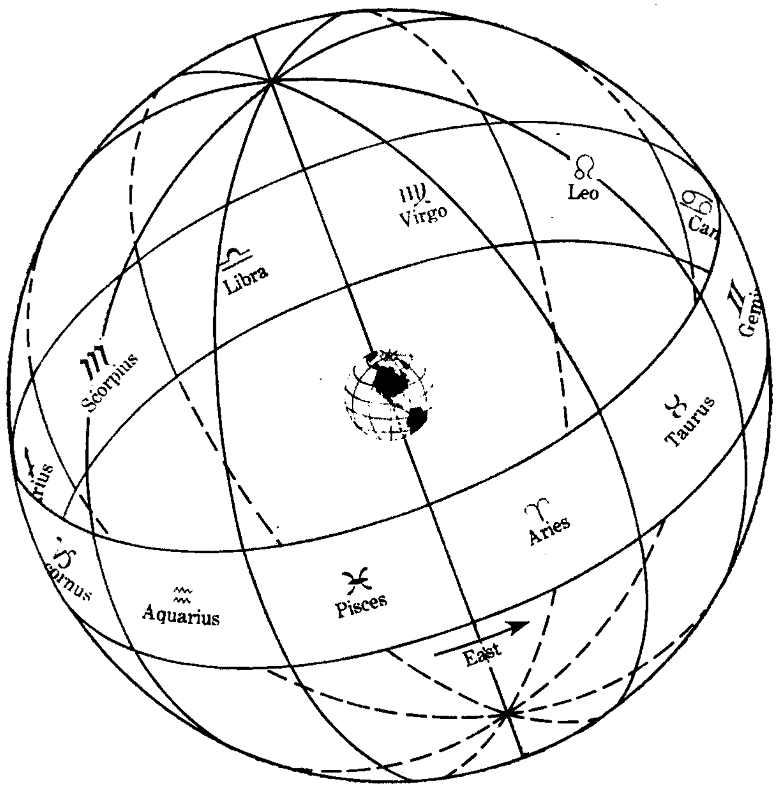


Figure 3.5 The Zodiac (From Bowditch, 1962)

Strictly, the zodiac is the circular band of sky extending 8° on each side of the ecliptic (see Figure 3.5). The navigational planets and the moon are within these limits. The zodiac is divided into 12 sections of 30° each, each section being given the name and symbol (sign) of the constellation within it. The sun remains in each section for approximately one month. Due to the precession of the equinoxes, the sun no longer enters the aforementioned constellations at the seasonal passages. However, astronomers still list the sun as entering these constellations; this is their principal astronomical significance. The pseudo-science of astrology assigns additional significance, not recognized by all scientists to the position of the sun and planets among the zodiacal signs (see Bowditch, 1962).

Since the precession of the equinoxes plays an important role in celestial position fixing, we shall define it:

<u>Precession of the Equinoxes</u>: A slow conical motion of the earth's axis (like the spinning of a top) about the vertical to the plane of the ecliptic, having a period of about 26,000 years (25,781 years) caused by the perturbative attractions of the sun, moon, and other planets on the equatorial protuberence (bulge) of the earth. It results in a gradual westward motion of the equinoxes (50.27 arc-seconds per year). Because of the precession, the zodiacal configuration with respect to the sun at its seasonal passages, has shifted approximately one section or constellation westward.

At the time of the definition of the zodiac, the sun was entering the constellation Aires at the time of the Spring Equinox. This solar position is of major importance to the sidereal reference system of time. The celestial meridian corresponding to the sun position at the time of a spring or vernal (from the Greek for spring) equinox defines the reference meridian for sidereal time. The expression "vernal equinox" and associated expressions, are applied to both "times" and "points" of occurrence of various phenomena. The vernal equinox is also called the "first point of Aries" (γ) or the "rams horns", although strictly speaking we should now call it the "first point of Pisces" due to the precession of the equinoxes.

3.3 Sidereal Time

We can now provide a set of definitions which describe the sidereal time system:

Sidereal Time: Time that is based on the position of the stars. A sidereal period is the length of time required for one revolution of a celestial body about its primary axis, with respect to the stars. Thus, a sidereal year is one revolution of the earth around the sun with respect to the fixed celestial reference.

Now there are 365.24219879 mean solar days in a tropical year. Due to the earth's revolution about the sun and the respective orientation of the sun and a fixed celestial reference (star reckoning), a sidereal day is actually shorter in time than a solar day. In fact, it is easy to show that there is exactly one more sidereal day in an annual period (vernal equinox to vernal equinox) than there are mean solar days (see Figure 3.6). Thus:

$$\begin{aligned} 1 \text{ mean solar time unit} &= 1.002737909 \text{ sidereal time units} \\ &= 366.24219879/365.24219879 \end{aligned}$$

Therefore, a sidereal day is 3'56" shorter than a solar day.

Sidereal Year: A sidereal year (i.e. the period of revolution of the earth relative to the stars) is 365.2563662 mean solar days (365 days, 6 hours, 9 minutes, 10 seconds) due to the precession of the equinoxes (50.27" per year).

$$365.2563662 = \frac{360^{\circ}0'50.27''}{360^{\circ}} \cdot 365.24219879$$

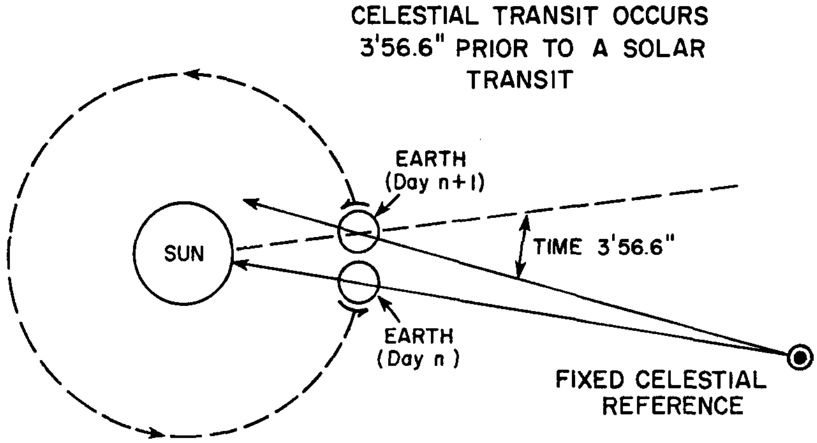


Figure 3.6 Difference between a solar and sidereal year (Not exact scale).

Hour Angles: Angular distance west of a celestial meridian or hour circle of a body (e.g. the sun) measured through 360° (see Figure 3.7). There are three conventionally defined hour angles:

1. Local Hour Angle (LHA): Angular distance west of the Local celestial meridian.

2. Greenwich Hour Angle (GHA): Angular distance west of the Greenwich celestial meridian.

3. Sidereal Hour Angle (SHA): Angular distance west of the Vernal Equinox celestial meridian (γ).

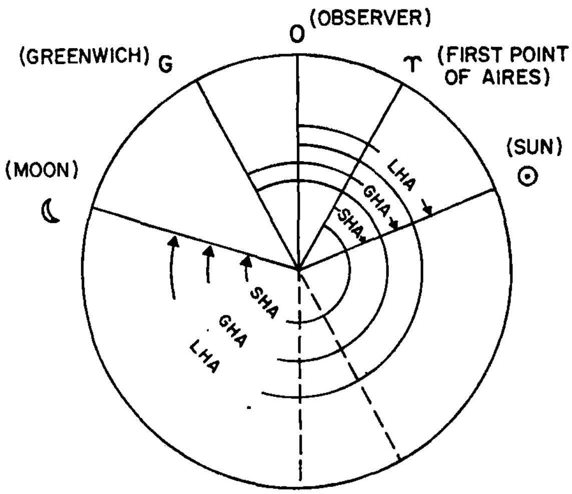


Figure 3.7 Hour Angles

4.0 GEOMETRICAL CONSIDERATIONS

4.1 Definitions of Latitude (Station Coordinates)

Since the earth is not a perfect sphere, there are a selection of coordinates to choose from. Most systems are based on the assumption that the earth can be represented by an oblate spheriod; that is, a geometrical shape in which sections parallel to the equator are perfect circles and meridians are ellipses (see Figure 4.1).

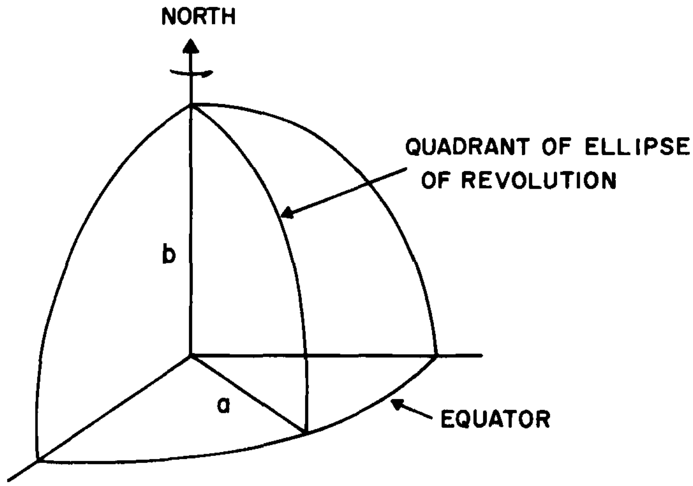


Figure 4.1 Model of the earth (From EB, 1965)

We define an oblate spheroid in terms of two radial axes (a, b) where:

a ≡ semi-major axis

b ≡ semi-minor axis

We can now define the flattening (f) parameter which is related to the eccentricity of the ellipsoid of revolution. We also define the eccentricity (e), a parameter which will be considered in the discussion of orbital calculations and conic sections. The flattening (f) and eccentricity (e) are given by:

$$f = (a-b)/a \tag{4.1}$$

$$= 0 \text{ for a perfect sphere}$$

$$e = \sqrt{a^2 - b^2}/a \tag{4.2}$$

$$= 0 \text{ for a spheroid or a circular orbit}$$

Also:

$$e = \sqrt{2f - f^2} \tag{4.3}$$

$$f = 1 - \sqrt{1 - e^2}$$

Note that in the limit as $b \to 0$ then $e \to 0$ and $f \to 0$. Values of these parameters for the earth are given by:

$$\begin{aligned} a &= 6378.214 \text{ km} \\ b &= 6356.829 \text{ km} \\ e &= 8.1820157 \cdot 10^{-2} \\ f &= 3.35289 \cdot 10^{-3} \end{aligned} \tag{4.4}$$

Note that:

$$b = a \cdot (1-f) \tag{4.5}$$

We can also define a mean earth radius (c) by a weighted average:

$$\begin{aligned} c &= (2a + b)/3 \\ &= 6371.086 \text{ km} \end{aligned} \tag{4.6}$$

Using our adopted model of the geometric shape, we can define the two conventional measures of latitude. Following the approach given in Chapter 2 of EB and using Figure 4.2 as a guide we first consider geocentric latitude:

Geocentric Latitude: The acute angle (ϕ) wrt the equatorial plane determined by a line connecting the geometric center of the ellipsoid and a point on its surface.

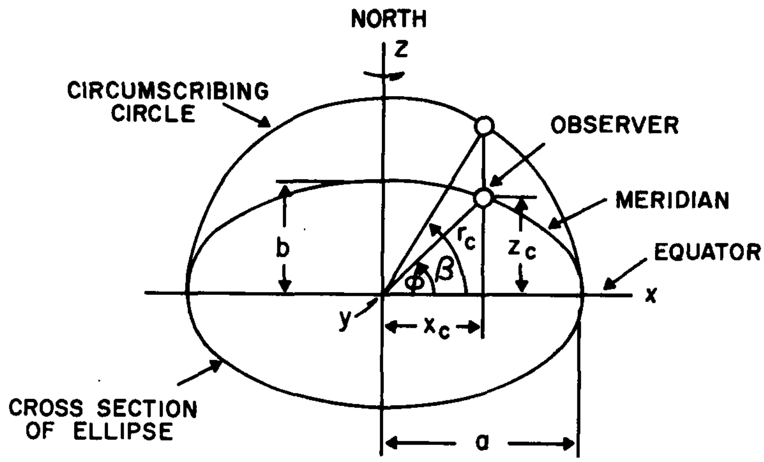


Figure 4.2 Ellipsoid of revolution defining geocentric latitude (Based on a figure from EB, 1965)

It is convenient to define the rectangular components (x_c, z_c), as we shall see later. It is also helpful to provide a derivation of x_c and z_c in terms of a, e and ϕ. To do so, we first define the reduced latitude β:

$\beta \equiv$ the acute angle wrt the equatorial plane determined by a line connecting the geometric center of the ellipsoid and a point on a circumscribing circle (see Figure 4.2). We will use the circumscribing circle later in the discussion of the eccentric anomaly.

Since:

$$x_c = r_c \cos\phi = a \cdot \cos\beta \qquad (4.7)$$

$$z_c = r_c \sin\phi = a\sqrt{1-e^2}\,\sin\beta \qquad (4.8)$$

therefore:

$$r_c = \sqrt{x_c{}^2 + z_c{}^2} = a\,\sqrt{1 - e^2\sin^2\beta} \qquad (4.9)$$

and:

$$\sin\phi = \frac{z_c}{r_c} = \frac{\sqrt{1 - e^2}\ \sin\beta}{\sqrt{1 - e^2\ \sin^2\beta}} \tag{4.10}$$

$$\cos\phi = \frac{x_c}{r_c} = \frac{\cos\beta}{\sqrt{1 - e^2\sin^2\beta}} \tag{4.11}$$

We square (4.10) and (4.11) and after multiplying by $\sqrt{1 - e^2}$:

$$\sin^2\phi = \frac{(1 - e^2)\sin^2\beta}{1 - e^2\sin^2\beta} \tag{4.12}$$

$$(1 - e^2)\cos^2\phi = \frac{(1 - e^2)\cos^2\beta}{1 - e^2\sin^2\beta} \tag{4.13}$$

now add (4.12) and (4.13) and after some manipulation:

$$\sqrt{1 - e^2\sin^2\beta} = \frac{\sqrt{1 - e^2}}{\sqrt{1 - e^2\cos^2\phi}} \tag{4.14}$$

We now combine (4.10) and (4.14) to solve for $\sin\beta$:

$$\sin\beta = \frac{\sin\phi}{\sqrt{1 - e^2\cos^2\phi}} \tag{4.15}$$

similarly for (4.11) and (4.14):

$$\cos\beta = \frac{\sqrt{1 - e^2}\ \cos\phi}{\sqrt{1 - e^2\ \cos^2\phi}} \tag{4.16}$$

Combining (4.16) and (4.7) with (4.15) and (4.8):

$$x_c = \frac{a\sqrt{1 - e^2}\ \cos\phi}{\sqrt{1 - e^2\cos^2\phi}} \tag{4.17}$$

$$z_c = \frac{a\sqrt{1 - e^2}\ \sin\phi}{\sqrt{1 - e^2\cos^2\phi}} \tag{4.18}$$

Next, we define geodetic latitude, again following EB:

<u>Geodetic Latitude</u>: The acute (ϕ') wrt the equatorial plane determined by a line normal to the tangent place of a point on the surface of the ellipsoid and intersecting the equatorial plane. Geodetic latitude is often referred to as geographic latitude (see Figure 4.3).

Recalling Eqns. (4.7) and (4.8):

$$x_c = a \cos\beta \tag{4.7}$$

$$z_c = a \sqrt{1 - e^2} \sin\beta \tag{4.8}$$

we can now differentiate:

$$-dx_c = a \sin\beta (d\beta) \tag{4.19}$$

$$dz_c = a \sqrt{1 - e^2} \cos\beta (d\beta) \tag{4.20}$$

Now note:

$$ds = \sqrt{(-dx_c)^2 + (dz)^2} = a \sqrt{1 - e^2\cos^2\beta} (d\beta) \tag{4.21}$$

and finally:

$$\sin\phi' = \frac{-dx_c}{ds} = \frac{\sin\beta}{\sqrt{1 - e^2\cos^2\beta}} \tag{4.22}$$

$$\cos\phi' = \frac{dz_c}{ds} = \frac{\sqrt{1 - e^2} \cos\beta}{\sqrt{1 - e^2\cos^2\beta}} \tag{4.23}$$

Finally, using Equations (4.10, 4.11) and (4.22, 4.23), it is easy to show that:

$$\phi' = \tan^{-1}[\tan\phi/(1-f)^2]$$

$$\phi = \tan^{-1}[\tan\phi' \cdot (1-f)^2] \tag{4.24}$$

This provides a convenient transformation between the station coordinate systems.

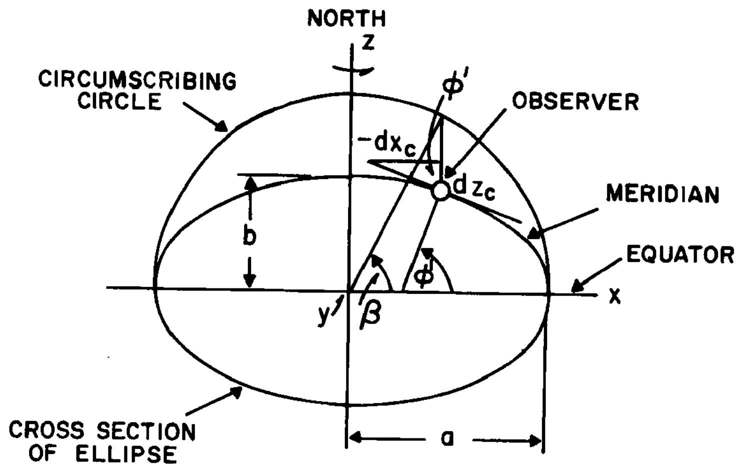


Figure 4.3 Ellipsoid of revolution defining geodetic latitude (Based on a figure from EB, 1965)

A third definition of latitude is often used, particularly in the process of surveying, that is astronomical latitude:

Astronomical Latitude: The acute angle (ϕ'') wrt the equatorial plane formed by the intersection of a gravity ray with the equatorial plane. This latitude is a function of the local gravitational field (direction of a plumb-bob), and is thus affected by local terrain. Tabulation of station errors is required to convert to geodetic latitude. Note that most maps are in either geodetic or astronomical latitude whereas navigational analysis will usually use a geocentric system.

4.2 Cartesian - Spherical Coordinate Transformations

It is necessary to define transformations between a spherical frame of reference and a cartesian frame of reference. For satellite navigation purposes, two systems are convenient:

1. Declination-Right Ascension-Radial System (δ,ρ,r) where we have chosen declination to be defined in the same sense as co-latitude:

$$x = r \cdot \sin(\delta) \cdot \cos(\rho)$$
$$y = r \cdot \sin(\delta) \cdot \sin(\rho) \qquad (4.25)$$
$$z = r \cdot \cos(\delta)$$

$$\delta = \cos^{-1}[z/\sqrt{x^2 + y^2 + z^2}]$$
$$\rho = \tan^{-1}[y/x] \qquad (4.26)$$
$$r = \sqrt{x^2 + y^2 + z^2}$$

2. Latitude-Longitude-Radial System (ϕ,λ,r):

$$x = r \cdot \cos(\phi) \cdot \cos(\lambda)$$
$$y = r \cdot \cos(\phi) \cdot \sin(\lambda) \qquad (4.27)$$
$$z = r \cdot \sin(\phi)$$

$$\phi = \sin^{-1}[z/\sqrt{x^2 + y^2 + z^2}]$$
$$\lambda = \tan^{-1}[y/x] \qquad (4.28)$$
$$r = \sqrt{x^2 + y^2 + z^2}$$

4.3 Satellite - Solar Geometry

A standard requirement for satellite data analysis is the definition of the angular configuration of a satellite and the sun with respect to a terrestrial position (ϕ,λ,r). In order to specify the three usual angles (zenith, nadir, azimuth), we first define the following polar coordinates:

$(\phi_\odot, \lambda_\odot, r_\odot) \equiv$ solar position

$(\phi_s, \lambda_s, r_s) \equiv$ satellite position

$(\phi, \lambda, r) \equiv$ reference point

Converting these three positions to their terrestrial position vectors:

$\vec{V}_\odot \equiv$ solar vector in earth coordinates (from 4.27)

$\vec{V}_s \equiv$ satellite vector in earth coordinates (from 4.27)

$\vec{V}_p \equiv$ reference point in earth coordinates (from 4.27)

We can define the solar and satellite zenith $(\Theta_\odot, \Theta_s)$, nadir $(\eta_\odot, \eta_s)$, and azimuth $(\Phi_\odot, \Phi_s)$ angles and relative zenith (Θ_r) and azimuth (Φ_r) angles:

$$\begin{aligned} \text{Solar zenith} &\equiv \Theta_\odot = \cos^{-1}[\vec{V}_p \cdot (\vec{V}_\odot - \vec{V}_p)] \\ \text{Solar nadir} &\equiv \eta_\odot = \cos^{-1}[-\vec{V}_\odot \cdot (\vec{V}_p - \vec{V}_\odot)] \end{aligned} \tag{4.29}$$

$$\begin{aligned} \text{Satellite zenith} &\equiv \Theta_s = \cos^{-1}[\vec{V}_p \cdot (\vec{V}_s - \vec{V}_p)] \\ \text{Satellite nadir} &\equiv \eta_s = \cos^{-1}[-\vec{V}_s \cdot (\vec{V}_p - \vec{V}_s)] \end{aligned} \tag{4.30}$$

$$\text{Relative zenith} \equiv \Theta_r = \cos^{-1}[(\vec{V}_\odot - \vec{V}_p) \cdot (\vec{V}_s - V_p)] \tag{4.31}$$

Figure 4.4 illustrates the zenith and nadir angle definitions.

In order to define the azimuth angles we first define a pointing vector $(\vec{V}_{90})$ which is subtented 90° from $\vec{V}_p$ in the same hemisphere as $\vec{V}_p$ and in the plane defined by the center of the earth, the north pole,

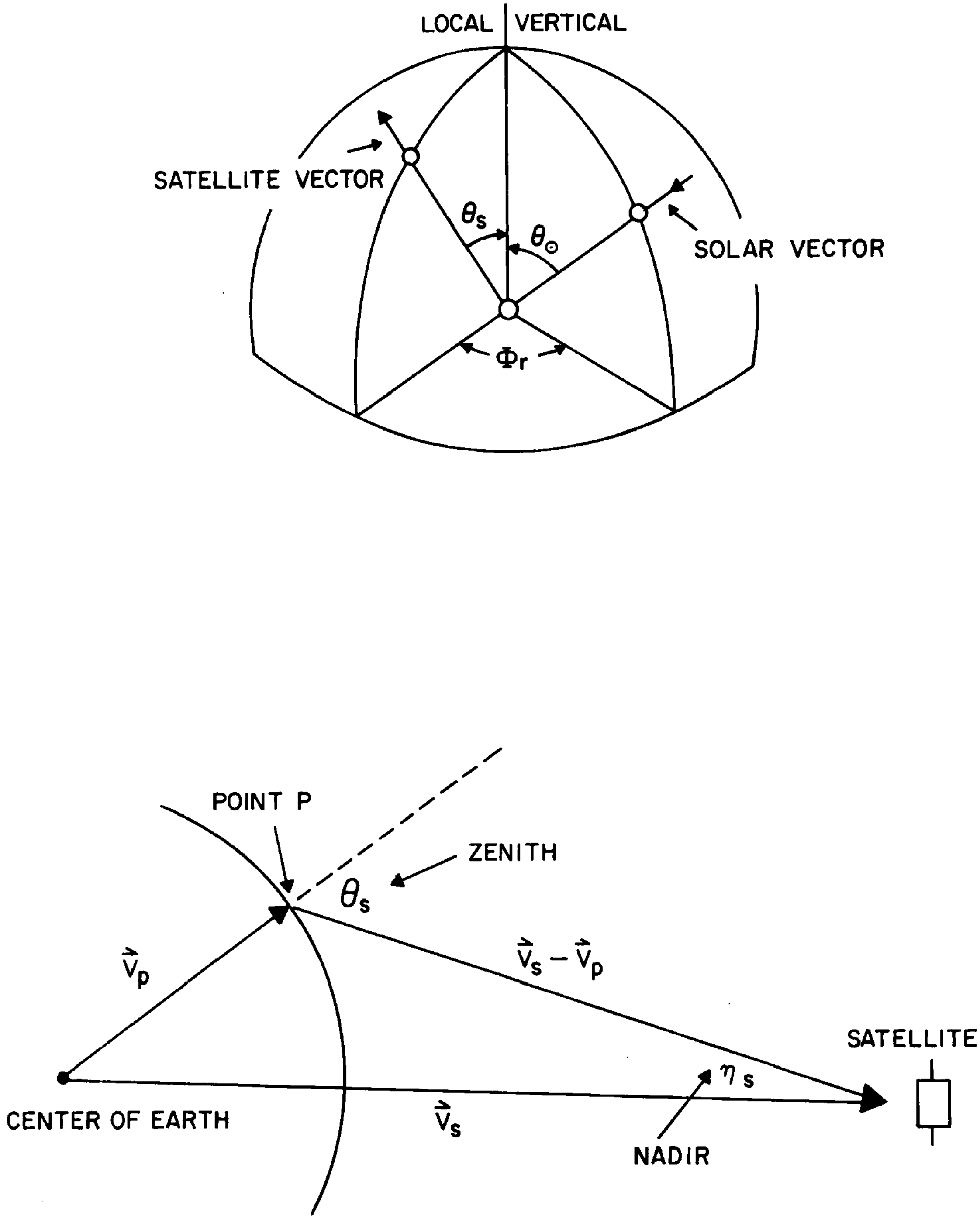


Figure 4.4 Definition of zenith and nadir angles.

and the endpoint of $\vec{V}_p$. Let:

$$\vec{S}_\odot = (\vec{V}_\odot - \vec{V}_p) / \| \vec{V}_\odot - \vec{V}_p \| \tag{4.32}$$

Furthermore, we define:

$$\begin{aligned} \vec{X}_\odot &= \vec{V}_{90} / \| \vec{V}_{90} \| \\ \vec{Z}_\odot &= \vec{V}_p / \| \vec{V}_p \| \\ \vec{Y}_\odot &= \vec{X}_\odot \text{ X } \vec{Z}_\odot \end{aligned} \tag{4.33}$$

$$\begin{aligned} \Phi_1 &= \cos^{-1}[(\vec{Z}_\odot \text{ X } \vec{S}_\odot \text{ X } \vec{Z}_\odot)\cdot\vec{X}_\odot] \\ \Phi_2 &= \cos^{-1}[(\vec{Z}_\odot \text{ X } \vec{S}_\odot \text{ X } \vec{Z}_\odot)\cdot\vec{Y}_\odot] \end{aligned} \tag{4.34}$$

The solar zenith is then given by:

$$\Phi_\odot \begin{cases} = \Phi_1 & \text{for } \Phi_2 \le 90 \\ = 360-\Phi_1 & \text{for } \Phi_2 > 90 \end{cases}$$

The satellite azimuth (Φ_s) is defined in the same way. Finally, we have the relative azimuth:

$$\Phi_r = \text{MOD}(|\Phi_\odot - \Phi_s|, 180) \tag{4.35}$$

See Figure 4.5 for an illustration.

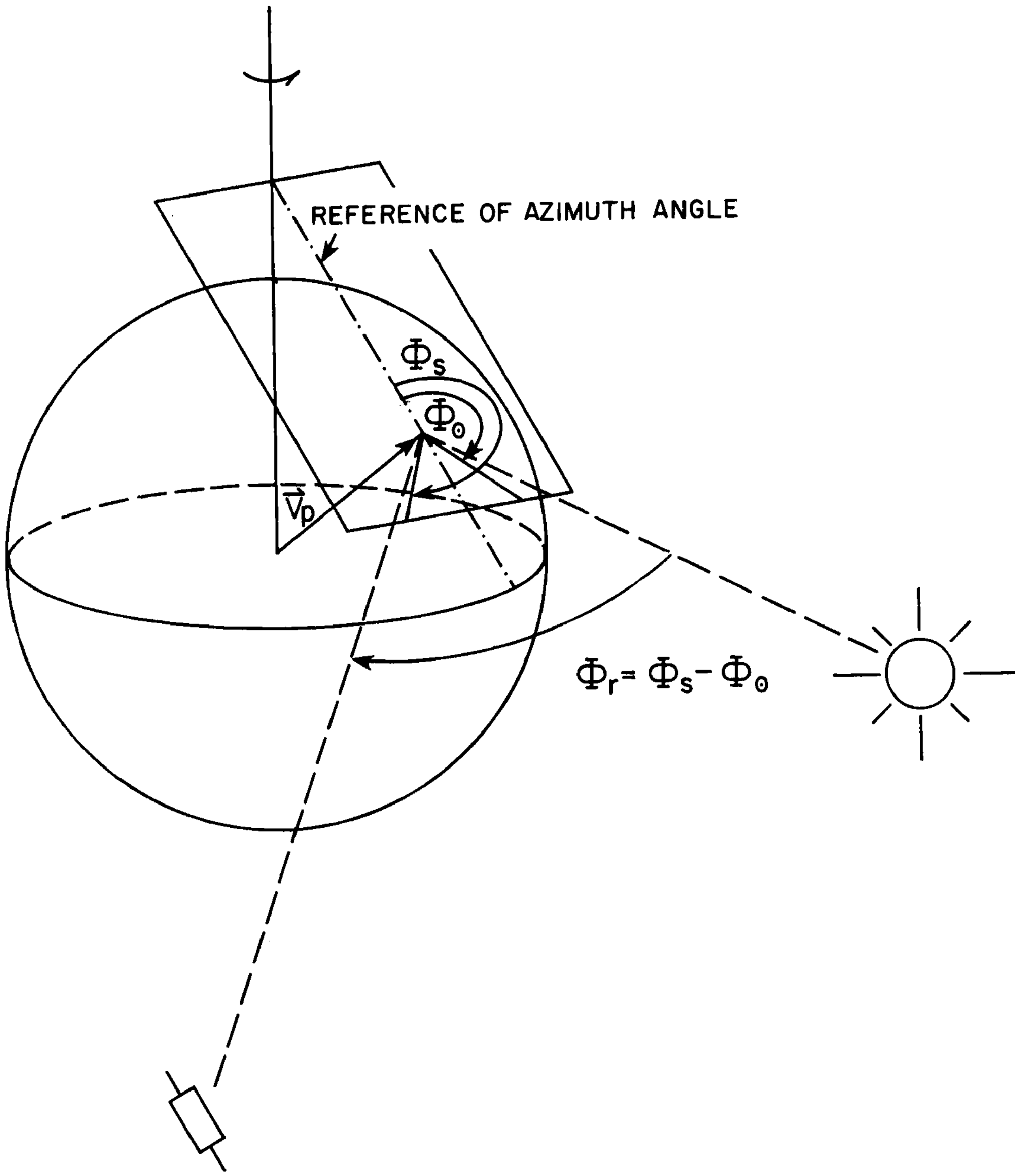


Figure 4.5 Definition of azimuth angles.

5.0 THE TWO BODY PROBLEM

5.1 The Inverse Square Force Field Law

We continue the analysis by considering the two body problem, ignoring all of the perturbative influences (i.e., thrust, drag, lift, radiation pressure, proton bombardment or solar wind, assymetrical electromagnetic forces, auxillary bodies and any aspherical gravitational potential of either body), that is we consider only the mutual attractions of a body A with a body B and the resultant motions. Furthermore, we assume that the motion under consideration is that of a satellite or planetary body B (secondary body of mass m_2) with respect to a central body A (primary body of mass m_1).

For closed solutions we will utilize the inverse square force field law:

$$\frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} = -\frac{K^2\mu\vec{r}}{r^3} \tag{5.1}$$

First, we determine the origin of the above equation. Essentially, Equation 5.1 embodies the laws of Kepler and Newton. To review:

<u>Kepler's Laws</u> (Empirical-aided by astronomical observations)

I. Within the domain of the solar system all planets describe elliptical paths with the sun at one focus.

II. The radius vector from the sun to a planet generates equal areas in equal times.

III. The squares of the periods of revolution of the planets about the sun are proportional to the cubes of their mean distances from the sun.

<u>Newton's Laws of Motion</u>

I. Every body will continue in its state of rest or of uniform motion in a straight line except insofar as it is compelled to change that state by an impressed force.

II. Rate of change of momentum (mv) is proportional to the impressed force and takes place in the line in which the force acts.

$$F = ma = m(dv/dt)$$

III. Action and reaction are equal and opposite.

Newton's Law of Universal Gravitation

Any two bodies in the universe attract one another with a force (F_{12}) which is directly proportional to the product of their masses (m_1,m_2) and inversely proportional to the square of the distance (r_{12}) between them:

$$\begin{aligned} F_{12} &= Gm_1m_2/r_{12}^2 \\ &= K^2m_2/4_{12}^2 \end{aligned} \tag{5.2}$$

where:

$K^2 = Gm_1$

$G \equiv$ Universal Gravitational Constant

$= 6.373 \cdot 10^{-8}$ dyne$\cdot$cm$^2\cdot$gm^{-2}

$m_1 \equiv$ larger mass (e.g. the earth)

$m_2 \equiv$ smaller mass (e.g. a satellite)

We can derive the inverse square force field law from Newton's second law and his law of universal gravitation. Adopting the notation in Chapter 2 of EB, the Universal Law of Gravitation states:

$$F_{12} = \frac{Gm_1m_2}{r_{12}^2} \tag{5.3}$$

Now consider an arbitrary inertial reference frame shown in Figure 5.1. The force in the x direction F_{1x} is:

$$F_{1x} = F_{12}\cos\Theta = F_{12}\cdot(x_2 - x_1)/r_{12} \tag{5.4}$$

therefore:

$$F_{1x} = \frac{Gm_1m_2}{r_{12}^2} \cdot \frac{x_2 - x_1}{r_{12}} \tag{5.5}$$

and finally:

$$F_{1x} = \frac{Gm_1m_2}{r_{12}^3}(x_2 - x_1) \tag{5.6}$$

$\equiv$ force on body 1

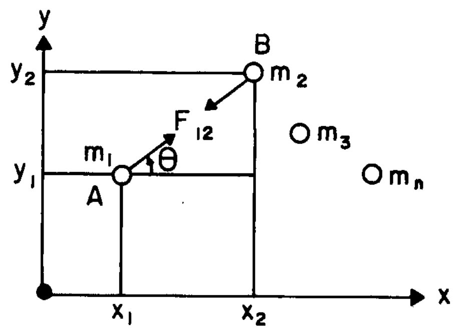


Figure 5.1 Arbitrary inertial coordinate reference frame

Newton's second law states that the unbalanced force on a body in the x direction is given by:

$$F_{1x} = m_1 \frac{d^2x_1}{dt^2} \tag{5.7}$$

therefore:

$$m_1 \frac{d^2x_1}{dt^2} = Gm_1m_2 \frac{(x_2 - x_1)}{r_{12}^3} \tag{5.8}$$

Now repeating the analysis for the y and z components we find:

$$m_1 \frac{d^2\vec{r}_1}{dt^2} = Gm_1m_2 \frac{(\vec{r}_2 - \vec{r}_1)}{r_{12}^3} \tag{5.9}$$

or:

$$\frac{d^2\vec{r}_1}{dt^2} = K^2 \frac{m_2}{m_1} (\vec{r}_2 - \vec{r}_1)/r_{12}^3 \tag{5.10}$$

Now transform to a <u>relative</u> inertial coordinate system as shown in Figure 5.2. From above:

$$m_1 \frac{d^2\vec{r}_1}{dt^2} = Gm_1m_2 \frac{\vec{r}_{12}}{r_{12}^3} \tag{5.11}$$

where:

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 \tag{5.12}$$

Now considering only the x component:

$$x_{12} = x_2 - x_1 \tag{5.13}$$

we note that:

$$\frac{d^2x_{12}}{dt^2} = \frac{d^2x_2}{dt^2} - \frac{d^2x_1}{dt^2} \tag{5.14}$$

which is the desired expression for the acceleration of body 2 with respect to body 1.

From our arbitrary inertial analysis:

$$m_1 \frac{d^2x_1}{dt^2} = Gm_1m_2 \frac{x_{12}}{r_{12}^3} \tag{5.15}$$

$$m_2 \frac{d^2x_2}{dt^2} = Gm_2m_1 \frac{x_{21}}{r_{21}^3} \tag{5.16}$$

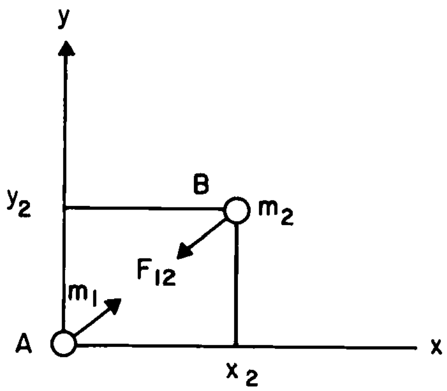


Figure 5.2 Relative inertial coordinate reference frame.

Now since $r_{12} = r_{21}$, and cancelling masses, then:

$$\frac{d^2x_1}{dt^2} = Gm_2 \frac{x_{12}}{r_{13}^3} = Gm_2 \frac{(x_2-x_1)}{r_{12}^3} \tag{5.17}$$

$$\frac{d^2x_2}{dt^2} = Gm_1 \frac{x_{21}}{r_{12}^3} = Gm_1 \frac{(x_1-x_2)}{r_{12}^3} \tag{5.18}$$

and subtracting the two equations yields:

$$\frac{d^2x_2}{dt^2} - \frac{d^2x_1}{dt^2} = \frac{d^2x_{12}}{dt^2} = Gm_1 \frac{(x_1-x_2)}{r_{12}^3} - Gm_2 \frac{(x_2-x_1)}{r_{12}^3} \tag{5.19}$$

$$\frac{d^2x_{12}}{dt^2} = -G(m_1 + m_2) \frac{(x_2 - x_1)}{r_{12}^3} \tag{5.20}$$

Now repeating the analyses for the y and z components we find:

$$\frac{d^2\vec{r}_{12}}{dt^2} = - Gm_1 \frac{(m_1 + m_2)}{m_1} \frac{\vec{r}_{12}}{r_{12}^3} \tag{5.21}$$

and finally:

$$\frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} = -K^2\mu\frac{\vec{r}}{r^3} \tag{5.22}$$

where:

$$\mu = (m_1 + m_2)/m_1$$

$\equiv$ normalized mass sum

We generally apply (5.22) to a system where the primary mass (m_1) is much greater than the secondary mass (m_2), yielding μ approximately 1.0.

Often in the study of orbital mechanics, an n-body system arises in which the desired origin of the coordinate system is the mass center or barycenter; that is, motion is relative to the barycenter and not any single central body (see Figure 5.3). We refer to such a reference system as a <u>Barycentric Coordinate System</u> (see a review in Chapter 2 of EB). The utility of this frame of reference arises in the event that the trajectory of a space vehicle would undergo <u>less disturbed</u> motion if referred to a barycenter. Since we are primarily concerned with near earth satellites we will forego an examination of the barycentric coordinate system. It is useful to examine the governing equation, however:

$$\frac{d^2\vec{r}_{B2}}{dt^2} = - G\left[\sum_{i=1}^{n} m_i\right]\frac{\vec{r}_{B2}}{r_{B2}^3} + G\sum_{\substack{i=1\\i\neq 2}}^{n} m_i\vec{r}_{i2}\left(\frac{1}{r_{B2}^3} - \frac{1}{r_{i2}^3}\right) \tag{5.23}$$

where:

n = 1 (the primary mass of the system)

n = 2 (the space vehicle under consideration)

and B represents the barycenter.

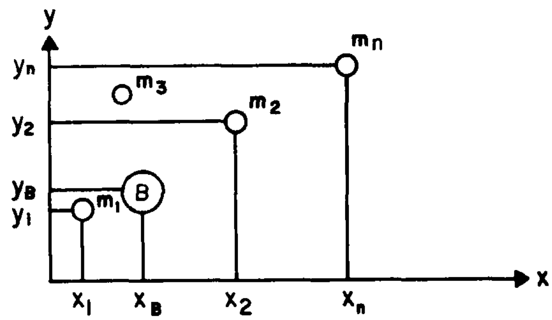


Figure 5.3 Barycentric coordinate reference frame. (Based on a figure from EB, 1965)

5.2 Coordinate Systems and Coordinates

We first define the celestial sphere:

<u>Celestial Sphere</u>: An imaginary sphere of indefinitely large radius, having the earth as the origin and the funadmental plane being an infinite extension of the Earth's equatorial plane (see Figure 5.4). To define the celestial sphere we first extend a line along the fundamental plane to a point fixed by the vernal equinox (γ), which is the reference meridian, and let that be the x-axis. The z-axis is given by the earth's spin axis or principal axis. An orthogonal coordinate system is finally established by defining the y-axis as the cross product of the z and x axes (see Figure 5.5).

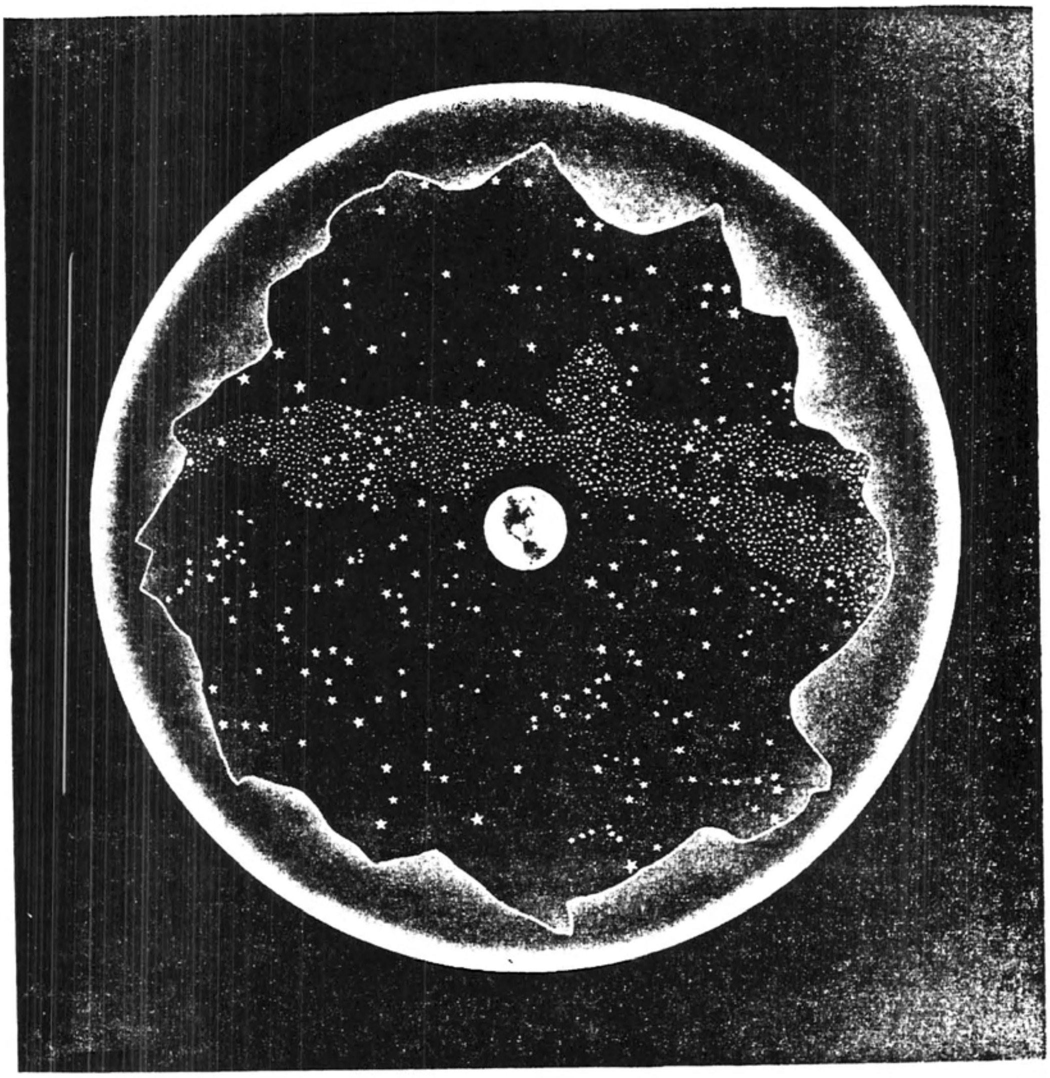


Figure 5.4 The celestial sphere (From Bowditch, 1962)

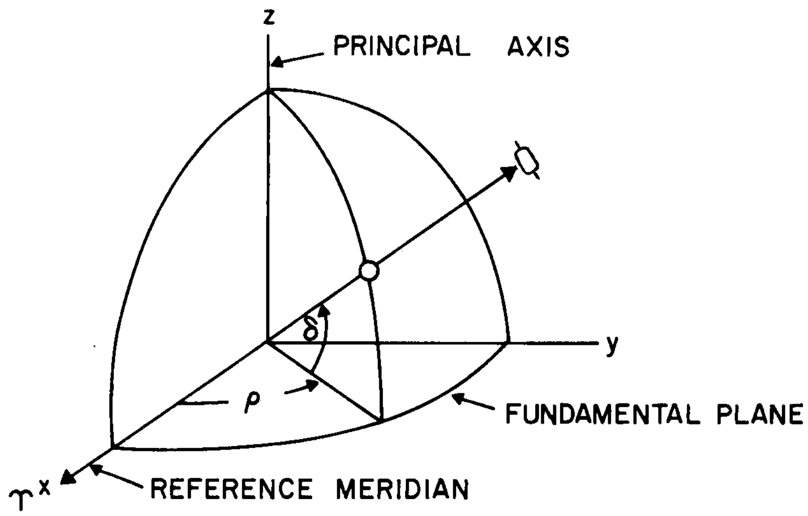


Figure 5.5 The right ascension - declination inertial coordinate system.

This celestial reference frame is often termed a right ascension-declination inertial coordinate system, in which declination (δ) is analogous to latitude (ϕ) (or as the case may be - colatitude), and right ascension (ρ) is analogous to longitude (λ) or hour angle (HA). Note that we refer to the equatorial plane as the fundamental plane, the z-axis as the principal axis, the the vernal equinox as the reference meridian. Also note that the celestial coordinate system is not truly an inertial system since it utilizes the terrestrial spin axis as the principal axis. Since the earth's spin axis precesses (giving rise to the westward precession of the equinoxes) we are left with a non-inertial reference frame if we consider very long time periods. There is also a lunar influence on the earth's spin axis which causes a nutation having a periodicity of approximately 18.5

years. Superimposed on these motions is the so-called Chandler Wobble, which has a period of approximately 14 months and is due to the non-solid nature of the earth itself. For our purposes, the non-inertial variation in the terrestrial spin axis is ignored.

It should be noted that we can define our coordinate system in any way we choose, however, simplicity and convenience are the watchwords. In designing coordinate systems for the various orbiting bodies or vehicles contained in the solar system, the same basic principles that are used for the earth centered (geocentric) celestial coordinate system are applied. Examples of various coordinate systems adopted for orbital analysis are referred to as follows (see EB):

Reference Body	Coordinate System
Earth	Geocentric
Sun	Heliocentric
Moon	Selenographic
Mars	Arcocentric
Satellite	Orbit Plane

It should also be pointed out that there are a choice of coordinates to be used once the coordinate system is defined. Again, the choice is arbitrary, however, the chosen coordinate parameters should have a natural relationship between the observer and the observed depending on whether measurement, calculation, or description is the nature of the problem on hand. Again, there are a variety of choices:

1. Declination (δ) - right ascension (ρ) - radial distance (r)
2. Declination (δ) - hour angle (HA) - radial distance (r)

3. Latitude (ϕ) - longitude (λ) - height (h)

4. Elevation (H) - azimuth (ϕ) - slant range (d)

5. Zenith (Θ) - azimuth (ϕ) - altitude (h)

6. Cartesian (x,y,z)

The solution of the governing equation (5.22) given in an earth-relative celestial coordinate system will yield three constants after the first integration (of the three component equations), and three constants after the second. Since (5.22) is an acceleration form of a linear, second order, ordinary differential equation, the first set of constants are initial velocity terms $(\mathring{x},\mathring{y},\dot{z})$ and the second set of constants are initial position terms (x_o,y_o,z_o). Thus, if we are given a position vector and a velocity vector at an epoch time t_o (six orbital elements and an epoch), we have a means to solve the governing equation.

Usually, this set of initial elements is not available since observations of the secondary body B are made from a rotating primary body A (that is a coordinate system that is different from that in which the analysis will be performed). That is why elevation-azimuth angle observations or range-range rate signals must first be transformed to a set of convenient orbital elements in the preferred coordinate system. Since this problem comes under the more general problem of orbital determination we will not consider it any further.

5.3 Selection of Units

Simplicity and computational efficiency can be achieved with the proper selection of units, based on the particular orbital problem. The proper choice of physical units for length, mass, and time is primarily determined by the dimensionality of the primary body A. We shall discuss two systems of units; the Heliocentric (solar origin) and Geocentric (terrestrial origin) systems.

1. Heliocentric Units

Length: Astronomical Unit (A.U.)

The mean distance between the sun and a fictitious planet, subjected to no perturbations, whose mass and sidereal period are the values adopted by Gauss in his determination of $K_{\odot}$ (we will discuss $K_{\odot}$ later).

1 A.U. = $1.496 \cdot 10^{8}$ km ($\simeq$ 93,000,000 miles) per A.U.

Mass: Mass of Sun ($m_{\odot}$)

$m_{\odot} = 1.9888822 \cdot 10^{33}$ gm per solar mass (s.m.)

Now if we use our previous definition:

$$G(m_{\odot} + m_p) = K^2\mu \tag{5.24}$$

where:

$m_{\odot} \equiv$ mass of sun

$m_p \equiv$ mass of planet

$K^2 = Gm_{\odot}$

$\mu = (m_{\odot} + m_p)/m_{\odot}$

we can define normalized mass factors for the nine planets. Note that the mass of a planet in the heliocentric system would also include the mass of its moons. Table 5.1 provides normalized mass factors for the nine planets.

Table 5.1: Solar System Normalized Mass Factors

Planet	Normalized Mass Factor (μ)
Mercury	1.0000001
Venus	1.0000024
Earth-Moon	1.0000030
Mars	1.0000003
Jupiter	1.0009547
Saturn	1.0002857
Uranus	1.0000438
Neptune	1.0000512
Pluto	1.0000028

2. Geocentric Units

Length: Earth equatorial radius (e.r.)

1 e.r. = 6378.214 km ($\approx$ 3960 miles) per e.r.

Mass: Mass of earth (m_e)

$m_e = 5.9733726 \cdot 10^{27}$ gm per earth mass (e.m.)

Note the mass of the moon (m_m):

$m_m = 7.3473218 \cdot 10^{25}$ gm per moon mass (m_m)

must be considered as part of the planetary mass when considering the earth orbit in a heliocentric system, but is ignored when considering a satellite in a geocentric system.

5.4 Velocity and Period

We need to define the velocity and period of an orbiting body. Consider first the circular orbit of a satellite at height h (mass m_s) above the earth (radius R_e). Therefore, the geocentric radius r is given by:

$$r = R_e + h \tag{5.25}$$

and:

$$m_s \ddot{\vec{r}} = - m_s \cdot K^2 \mu \vec{r}/r^3 \tag{5.26}$$

However, the magnitude of $m_s \ddot{\vec{r}}$ is a centrifugal force $-m_s \cdot V^2/r$ where V is the circular velocity at orbital altitude. Therefore in scalar form:

$$m_s \frac{V^2}{r} = \frac{m_s \cdot K^2 \cdot \mu}{r^2} \tag{5.27}$$

$$V^2 = \frac{K^2 \cdot \mu}{R_e + h} \tag{5.28}$$

$$V = \sqrt{K^2 \mu/(R_e + h)} \tag{5.29}$$

$$V = K \sqrt{\mu/(R_e + h)} \tag{5.30}$$

Therefore V is the required orbit velocity for a circular orbit at height h.

Since the circular orbital track would be a distance of $2\pi(R_e + h)$, for a single revolution, the orbital period (P) would be $2\pi \cdot (R_e + h)/V$, or:

$$P = \frac{2\pi \cdot (R_e + h)^{3/2}}{K \sqrt{\mu}} \tag{5.31}$$

Note that as the height of a satellite increases, the velocity required to maintain it in circular orbit decreases. See Figure 5.6 for an illustration. Note, however, from a propulsion point of view, more energy is expended in lifting a satellite against gravity to reach a higher orbit, than is gained in the reduction or the forward speed required for orbit injection.

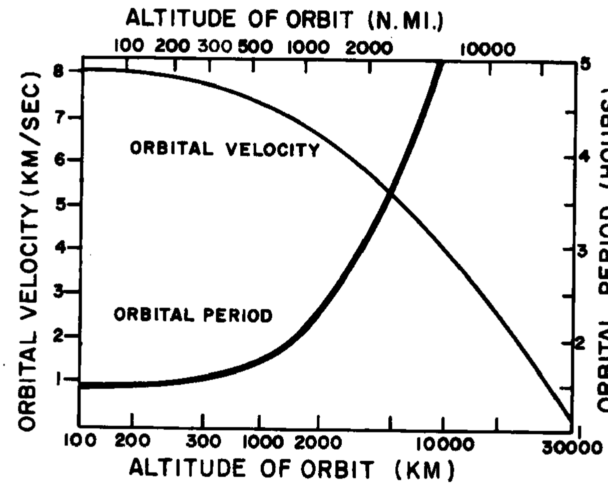


Figure 5.6 Velocity and period of a satellite in circular orbit as a function of altitude (From Widger, 1966)

If we solve $P = 2\pi.(R_e + h)^{3/2}/(K\mu^{1/2})$ for h using a period P of 24 hours, we have solved for the required height of a geosynchronous satellite; that is, an orbital configuration in which the period is that of a single rotation of the earth. The required height for a geosynchronous satellite in a circular orbit is thus approximately 35,863 km (42241.214 km from geocentric origin).

Now since we know the orbital period P, we can determine the ground speed (V_{gs}) of a circular orbit, i.e., the velocity at radius R_e. Since the path of one revolution is $2\pi \cdot R_e$, then

$$V_{gs} = 2\pi \cdot R_e / P$$

$$= \frac{R_e}{R_e + h} \circ K \circ \sqrt{\frac{\mu}{R_e + h}} \tag{5.32}$$

and applying equation (5.29):

$$V_{gs} = \frac{R_e}{(R_e + h)} \cdot V \tag{5.33}$$

Table 5.2 tabulates various orbital characteristics as a function of satellite altitude.

Table 5.2: Orbital Characteristics as a Function of Altitude - R_e = 6370 km or 3435 N. miles (From Widger, 1966).

Orbit Altitude Km	Orbit Altitude N. Miles	$R_e + h$ Km	$R_e + h$ N. Miles	$\left(\frac{R_e+h}{R_e}\right)$	$\left(\frac{R_e}{R_e+h}\right)$	Orbital Velocity km/hr	Orbital Velocity knots	Ground Velocity (Non-Rotating Earth) km/hr	Ground Velocity (Non-Rotating Earth) knots	Orbital Period hours	Orbital Period min.	Westward Displace. Per Orbit Deg. Long.
150	81	6520	3516	1.024	.9770	28111	15245	27464	14894	1.458	87.48	21.87
185	100	6555	3535	1.029	.9717	28080	15203	27285	14773	1.468	88.08	22.02
200	108	6570	3543	1.031	.9695	28004	15188	27150	14725	1.476	88.56	22.14
250	135	6620	3570	1.039	.9622	27901	15130	26846	14558	1.492	89.52	22.38
278	150	6648	3585	1.044	.9582	27839	15099	26675	14468	1.502	90.12	22.53
300	162	6670	3597	1.047	.9550	27795	15074	26544	14396	1.509	90.54	22.64
350	189	6720	3624	1.055	.9478	27690	15017	26245	14233	1.526	91.56	22.89
371	200	6741	3635	1.058	.9450	27649	14994	26128	14169	1.533	91.98	23.00
400	216	6770	3651	1.063	.9408	27589	14962	25956	14076	1.543	92.58	23.15
450	243	6820	3678	1.071	.9339	27488	14905	25671	13920	1.560	93.60	23.40
463	250	6833	3685	1.073	.9322	27462	14893	25600	13883	1.565	93.90	23.48
500	270	6870	3705	1.079	.9271	27386	14851	25390	13768	1.578	94.68	23.67
550	297	6920	3732	1.086	.9204	27287	14798	25115	13620	1.595	95.70	23.93
556	300	6926	3735	1.087	.9197	27277	14793	25087	13605	1.597	95.82	23.96
600	324	6970	3759	1.094	.9138	27189	14745	24845	13474	1.612	96.72	24.18
649	350	7019	3785	1.102	.9075	27095	14694	24589	13335	1.629	97.74	24.44
650	351	7020	3786	1.102	.9073	27092	14692	24581	13330	1.629	97.74	24.44
700	378	7070	3813	1.110	.9009	26995	14640	24320	13189	1.647	98.82	24.71
741	400	7111	3835	1.116	.8957	26919	14597	24111	13075	1.661	99.66	24.92
750	405	7120	3840	1.118	.8945	26902	14588	24064	13049	1.664	99.84	24.96
800	432	7170	3867	1.126	.8883	26807	14536	23813	12912	1.682	100.92	25.23
834	450	7214	3885	1.131	.8842	26725	14503	23630	12824	1.697	101.82	25.46
850	459	7220	3894	1.134	.8821	26715	14487	23565	12779	1.699	101.94	25.49
900	486	7270	3921	1.141	.8761	26624	14436	23325	12647	1.717	103.02	25.76
927	500	7297	3935	1.146	.8729	26575	14411	23197	12579	1.727	103.62	25.91
950	513	7320	3948	1.149	.8701	26531	14388	23085	12519	1.735	104.10	26.03
1000	540	7370	3975	1.157	.8642	26441	14338	22850	12391	1.753	105.18	26.30
1019	550	7389	3985	1.160	.8620	26408	14320	22764	12344	1.760	105.60	26.40
1050	567	7420	4002	1.165	.8583	26352	14290	22618	12265	1.771	106.26	26.57
1100	594	7470	4029	1.173	.8526	26264	14243	22393	12144	1.788	107.26	26.82
1112	600	7482	4035	1.175	.8513	26243	14232	22341	12116	1.793	107.58	26.90
1150	621	7520	4056	1.181	.8469	26179	14194	22171	12021	1.806	108.36	27.09
1200	648	7570	4083	1.189	.8413	26089	14147	21949	11902	1.825	109.50	27.38
1205	650	7575	4085	1.189	.8409	26083	14145	21933	11895	1.826	109.56	27.39
1250	674	7620	4109	1.196	.8360	26005	14103	21740	11790	1.842	110.52	27.63
1297	700	7667	4135	1.204	.8307	25925	14059	21536	11679	1.860	111.60	27.90
1300	701	7670	4136	1.204	.8305	25919	14057	21526	11674	1.861	111.66	27.92
1350	728	7720	4163	1.212	.8251	25834	14011	21316	11560	1.879	112.74	28.19
1390	750	7760	4185	1.218	.8208	25769	13974	21151	11470	1.894	113.64	28.41
1400	755	7770	4190	1.220	.8198	25752	13966	21111	11449	1.897	113.82	28.46
1450	782	7820	4217	1.228	.8146	25670	13921	20911	11340	1.915	114.90	28.73
1483	800	7853	4235	1.233	.8111	25615	13891	20776	11267	1.928	115.68	28.92
1500	809	7870	4244	1.236	.8094	25589	13876	20712	11231	1.934	116.04	29.01
1550	836	7920	4271	1.243	.8043	25508	13833	20516	11126	1.952	117.12	29.28
1575	850	7945	4285	1.247	.8016	25468	13810	20415	11070	1.961	117.66	29.42
1600	863	7970	4298	1.251	.7992	25428	13789	20322	11020	1.971	118.26	29.57
1650	890	8020	4325	1.259	.7942	25349	13747	20132	10918	1.989	119.34	29.84
1668	900	8038	4335	1.262	.7924	25321	13730	20064	10880	1.996	119.76	29.94
1700	917	8070	4352	1.267	.7893	25267	13703	19943	10816	2.008	120.48	30.12
1750	944	8120	4379	1.275	.7844	25191	13662	19760	10716	2.027	121.62	30.41
1761	950	8131	4385	1.277	.7834	25175	13651	19722	10694	2.031	121.86	30.47
1800	971	8170	4406	1.283	.7796	25113	13619	19578	10617	2.046	122.76	30.69
1850	998	8220	4433	1.291	.7749	25039	13578	19403	10522	2.064	123.84	30.96
1853	1000	8223	4435	1.291	.7745	25033	13574	19388	10513	2.066	123.96	30.99
35815	19326	42185	22761	6.622	.1510	11052	5992	—	—	24.000	1440.00	—

5.5 Elliptic Orbits

In the consideration of elliptic orbits governed by our principle equation, the radius r, of the second body from the primary body, can be given by:

$$r = p/(1 + e \cdot \cos\nu) \qquad (5.34)$$

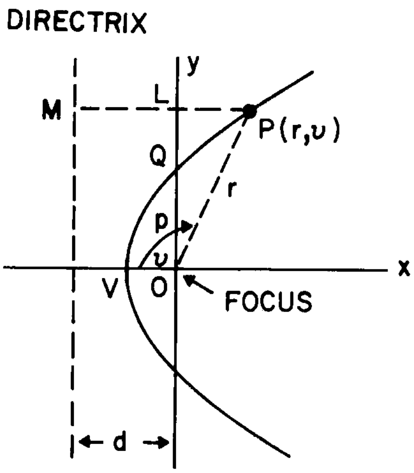
which is simply the equation describing conic sections (see Figure 5.7), where:

$e \equiv$ eccentricity

$\nu \equiv$ true anomaly

$p \equiv$ semi-parameter of conic

$\quad = ed$



If a point P moves so that its distance from a fixed point (called the focus) divided by its distance from a fixed line (called the directrix) is a constant e (called the eccentricity), then the curve described by P is called a conic (so-called because such curves can be obtained by intersecting a plane and a cone at different angles). If the focus is chosen at origin O the equation of a conic in polar coordinates (r,ν) is, if OQ = p and LM = d:

$$r = \frac{p}{1 + e\cos\nu} = \frac{ed}{1 + e\cos\nu}$$

Figure 5.7 Conic sections (Based on a figure from Spiegal, 1968).

Thus we see that if $p \neq 0$, then:

$0 < e < 1$ the conic is an ellipse

$e = 1$ the conic is a parabola

$1 < e < \infty$ the conic is a hyperbola

In the following discussions the term semi-major axis (a) will be used. It is defined as half the maximum diameter of the conic. Note that (see Dubyago, 1961):

$$a = 0 \quad \text{for parabolic motion}$$

$$0 < a < \infty \quad \text{for elliptic or circular motion}$$

$$-\infty < a < 0 \quad \text{for hyperbolic motion}$$

For an ellipse, a and p are related through e by $p=ed=a(1-e^2)$.

As an aside, it is interesting to note that for any arbitrary position of a vehicle, within the influence of the terrestrial gravitational field, there is a given escape velocity (V_{esc}). The magnitude of the initial velocity vector $\dot{\vec{r}}$ determines the type of path, that is:

elliptic if $\|\dot{\vec{r}}\| < V_{esc}$

parabolic if $\|\dot{\vec{r}}\| = V_{esc}$

hyperbolic if $\|\dot{\vec{r}}\| > V_{esc}$

The escape velocity from a celestial body is given by:

$$V_{esc} = (2gR)^{1/2} \tag{5.35}$$

where:

$g \equiv$ gravitational constant of body

$R \equiv$ radius of body

For the earth and moon, the escape velocities of a missile launched from the surface are:

Body	V_{esc}
Earth	$\simeq 11 \text{ km} \cdot \text{sec}^{-1}$
Moon	$\simeq 2.5 \text{ km} \cdot \text{sec}^{-1}$

Contrast the above to the velocity of an air parcel at the earth's surface (no wind):

$$V_{par} = \Omega R_e = 7.292 \cdot 10^{-5} \cdot 6371 \text{ km} \cdot \text{sec}^{-1}$$
$$\doteq 0.46 \text{ km} \cdot \text{sec}^{-1} \tag{5.36}$$

where Ω is the earth's angular velocity and R_e is the earth radius.

The equation for an ellipse, in polar coordinates with the origin at a focus, is given by:

$$r = a(1 - e^2)/(1 + e \cdot \cos\nu) = p/(1 + e \cdot \cos\nu) \tag{5.37}$$

Noting that $p \neq 0$, $0 < e < 1$, and $0 < a < \infty$ for the planets, constitutes a proof of Kepler's First Law.

A proof of Kepler's Second Law requires an integration of the area swept out by the radius vector $\vec{r}$. This results in the definition of the orbital period P in the relative inertial coordinate system which we have established. The period is then given by:

$$P = \frac{2\pi}{K\sqrt{\mu}} a^{3/2} \tag{5.38}$$

which corresponds to equation (5.31). A proof of equation (5.38) is given in Chapter 3 of EB.

This is the appropriate form in a relative inertial coordinate system. Note that for circular orbits:

$$V = K\sqrt{\mu/a} \tag{5.39}$$

which corresponds to equation (5.30). For elliptic orbits V is not constant. We will derive the velocity for elliptic orbits in Chapter 6.

Now since the period P of a body is:

$$P = \frac{2\pi}{K\sqrt{\mu}}\, a^{3/2} \tag{5.40}$$

we can square both sides to get Kepler's Third Law:

$$P^2 = \frac{4\pi^2}{K^2\mu}\, a^3 \tag{5.41}$$

The squares of the periods of revolution of the planets about the Sun are proportional to the cubes of their mean distances from the Sun.

It is interesting that Kepler derived his laws empirically, involving many years of laborious data reduction. His 3rd law did not include the mass factor μ since the accuracy in his data simply did not allow the detection of the secondary mass effect (see EB).

5.6 The Gaussian Constant

We can now define the Gaussian constant K_Θ, noting that:

$$P^2 = \frac{(2\pi)^2}{\mu K^2}\, a^3 \tag{5.42}$$

and choosing a heliocentric system of characteristic units. It is a simple matter to compute the numerical value of K^2 or the Gaussian constant:

$$K_\Theta = \sqrt{K^2} \tag{5.43}$$

thus:

$$K_\Theta = \frac{2\pi}{P\sqrt{\mu}}\, a^{3/2} \tag{5.44}$$

Now since the period of the Earth is 365.256365741 mean solar days (celestial period), and if the semi-major axis of the earth's orbit is taken to be 1 A.U. and $\sqrt{\mu}$ = 1.0000015, then K_Θ = 0.017202099 A.U.$^{3/2}$·day^{-1}.

This was the procedure Gauss used to determine $K_\odot$ in his 1809 publication "Theoria Motus Corporum Coelestium In Sectionibus Conicis Solem Ambientium", i.e., Theory of the Motion of Heavenly Bodies Revolving Round the Sun in Conic Sections (see EB). Similar procedures are used to obtain the gravitational constants of the other planets. Table 5.3 provides gravitational constant data for the planets.

Table 5.3: Gravitational Constants of the Major Planets (From Escobal, 1965)

Planet	Semimajor Axis (km)	Gravitational Constants (K_p) (A.U.$^{3/2}$/Mean Solar Day)
Mercury	2,424	6.960×10^{-6}
Venus	6,100	2.691×10^{-5}
Earth	6,378.15	2.99948×10^{-5}
Mars	3,412	9.786×10^{-6}
Jupiter	71,420	5.3153×10^{-4}
Saturn	60,440	2.908×10^{-4}
Uranus	24,860	1.136×10^{-4}
Neptune	26,500	1.240×10^{-4}
Pluto	4,000	2.700×10^{-5}

Note that for Table 5.3, 1 A.U. = 149,599,000 km and K_p is related to $K_\odot$ by $K_p = K_\odot \sqrt{m_p/m_\odot}$. Also note that in the geocentric system, the present value of K_e (earth gravitational constant) is 0.07436574 e.r.$^{3/2}$ $\cdot$ min^{-1}.

5.7 Modified Time Variable

It is often convenient in the treatment of orbital problems to transform the time dimension to the so-called modified time variable (τ). The transformation involves a gravitational constant (e.g., K_Θ or K_e) and an epoch time t_o. In Heliocentric units:

$$\tau = K_\Theta(t-t_o) \tag{5.45}$$

whereas in Geocentric units:

$$\tau = K_e(t-t_o) \tag{5.46}$$

The advantage of using this quantity can be seen if we recast the governing equation in terms of τ. Since:

$$d^2\tau = K^2 d^2 t \tag{5.47}$$

then:

$$\frac{d^2\vec{r}}{dt^2} = -K^2\mu\vec{r}/r^3 \tag{5.48}$$

transforms to:

$$\frac{d^2\vec{r}}{d\tau^2} = -\mu\vec{r}/r^3 \tag{5.49}$$

and K^2 does not appear.

Use of characteristic units, leads to a new unit of velocity (V_{csu}), the circular satellite unit velocity (see Chapter 3 of EB):

$$V_{csu} = K\sqrt{\frac{\mu}{a}} \tag{5.50}$$

In the Heliocentric System:

$$V_{csu} = K_\Theta \sqrt{\frac{1}{1\ \text{A.U.}}} = 0.017202099\ \frac{\text{A.U.}^{3/2}}{\text{day}} \sqrt{\frac{1}{1\ \text{A.U.}}} \tag{5.51}$$

$$V_{csu} = 0.017202099 \ \frac{\text{A.U.}}{\text{day}} \cdot 1.496 \cdot 10^{11} \frac{\text{m}}{\text{A.U.}} \cdot \frac{1 \text{ day}}{86400 \text{ sec}} \tag{5.52}$$

$$= 29{,}785 \text{ m/sec}$$

In the Geocentric System:

$$V_{csu} = K_e \sqrt{\frac{1}{1 \text{ e.r.}}} = 0.07436574 \ \frac{\text{e.r.}^{3/2}}{\text{min}} \sqrt{\frac{1}{1 \text{ e.r.}}} \tag{5.53}$$

$$V_{csu} = 0.07436574 \ \frac{\text{e.r.}}{\text{min}} \cdot 6.378214 \cdot 10^{6} \frac{\text{m}}{\text{e.r.}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

$$= 7{,}905 \text{ m/sec} \tag{5.54}$$

5.8 Classical Orbital Elements

Let us first establish an elliptic frame of reference in which we consider coordinates along x_ω, y_ω axes in a plane containing the orbit (see Figure 5.8).

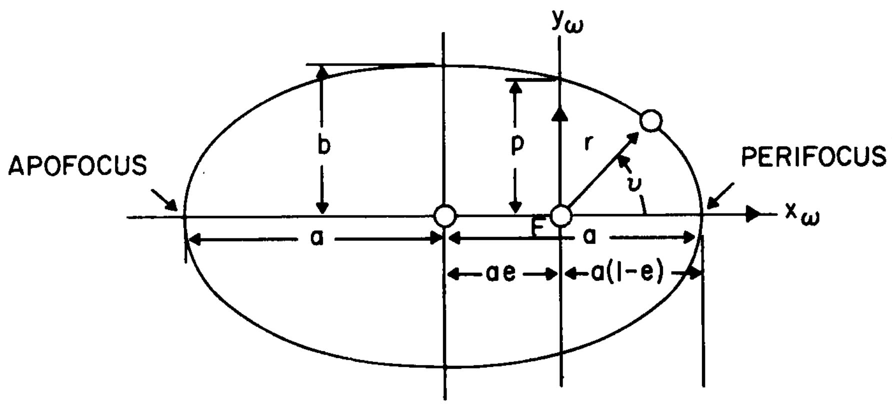


Figure 5.8 Elliptic frame of reference (Based on a figure from EB, 1965)

We have already defined:

$e \equiv$ eccentricity

$= \sqrt{a^2 - b^2}/a$

$a \equiv$ semi-major axis

$b \equiv$ semi-minor axis

$p \equiv$ semi-parameter of conic

$= a(1-e^2)$

$\nu \equiv$ true anomaly

In addition, the positions where $dr/d\tau$ are zero are called apsis (plural for apse). Elliptical orbits possess two points where the above condition is satisfied, i.e., the minimum radius position (perifocus) and the maximum radius position (apofocus). In discussing the sun in its ecliptic, we refer to the apsis as perihelion and aphelion (see Figure 5.9).

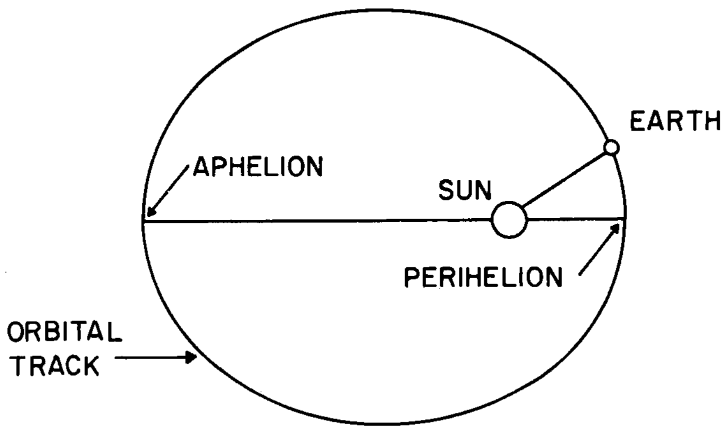


Figure 5.9 Perihelion and aphelion of earth in solar orbit (Not exact scale)

A complete set of orbital elements sufficient to describe an orbit are the "Classical Orbital Elements". They are as follows:

1. Epoch Time (t_o): Julian day and GMT time for which the following elements are defined.

2. Semi-major Axis (a): Half the distance between the two apsis of perifocus and apofocus.

3. Eccentricity (e): Degree of ellipticity of the orbit.

4. Inclination (i): Angle between the orbit plane and the equatorial plane of the primary body.

5. Mean Anomaly (M_o): Angle in orbital plane with respect to the center of a mean circular orbit, having a period equivalent to the anomalistic period, from perifocus to the satellite position (anomalistic period is discussed in Chapter 6).

6. Right Ascension of Ascending Node (Ω_o): Angle in orbital plane between vernal equinox (reference meridian) and northward equator crossing.

7. Argument of Perigee (ω_o): Angle in orbit plane from ascending node to perifocus.

The above set of elements satisfies the requirement of defining six constants and an epoch time noted in Section 5.2. Note that if the epoch time were to correspond to perifocus, the mean anomaly would be zero and thus would be an unnecessary parameter. This is generally not the case with either NASA, NESS, ESA, or JMS orbital element transmissions. Of the 7 parameters, the three angular quantities (M_o, Ω_o, ω_o) are subscripted similar to t_o indicating that they are time dependent quantities. The time dependence of a two body orbit will be discussed in Chapter 6. The European Space Agency has used true anomaly rather than mean anomaly in their orbital transmissions for the Meteosat and GOES-1 satellites. This presents no difficulty as will be seen in the following section. Appendix A provides examples of orbital parameter transmissions for various U.S., European, and Japanese satellites.

5.9 Calculation of Celestial Pointing Vector

First we recall the essential angles:

i ≡ Orbital inclination

Ω_o ≡ Right ascension of ascending node (note that $℧_o$ is defined as the right ascension of descending node)

ω_o ≡ Argument of perigee

Following the approach given in Chapter 3 of EB, the angles i, Ω_o, ω_o (the "Classical Orientation Angles") are used to define the orbit plane in celestial space, defined by an orthogonal (I, J, K) coordinate system (as shown in Figure 5.10).

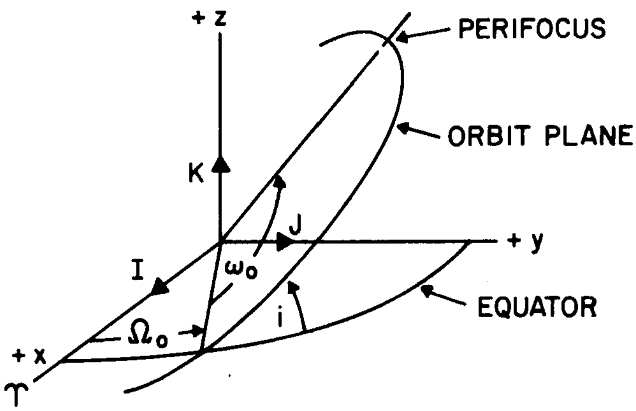


Figure 5.10 The Classical Orientation Angles and the Orthogonal I, J, K Coordinate System (Based on a figure from EB, 1965)

Note that:

$0 \leq i < \pi$

$0 \leq \Omega_o < 2\pi$

$0 \leq \omega_o < 2\pi$

From Figure 5.10 it is convenient to define retrograde and direct orbits:

1. Retrograde: Orbits whose motion is in the direction of y to x.
2. Direct or Prograde: Orbits whose motion is in the direction of x to y.

Compare the above with the classic definition of a retrograde orbit:

> Motion in an orbit opposite to the usual orbital direction of celestial bodies within a given system; i.e., a satellite motion, in a direction opposite to the motion of the primary body.

Since the use of angles is cumbersome, we transform to a set of orthogonal vectors (P, Q, W) in a cartesian reference frame (see Figure 5.11):

> P is a vector pointing toward perifocus
>
> Q is in the orbit plane and advanced 90° from P
>
> W is the normal to the orbit plane

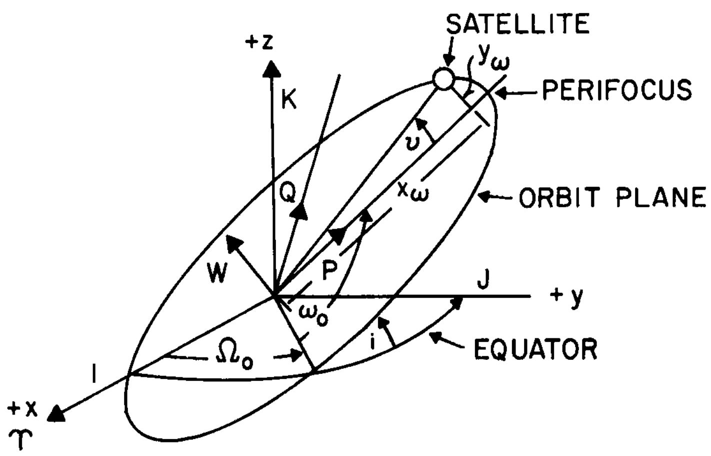


Figure 5.11 The P, Q, W orthogonal reference frame (Based on a figure from EB, 1965)

The set of orthogonal vectors (U, V, W) can also be defined (see Figure 5.12). These vectors will not be used in our analysis, however, they are useful vectors for additional analytical study (see EB for an explanation):

U is the vector always pointing at the satellite in the plane of the orbit

V is the vector advanced from U, in the sense of increasing true anomaly, by a right angle

W is the normal to the orbital plane and is given by U x V

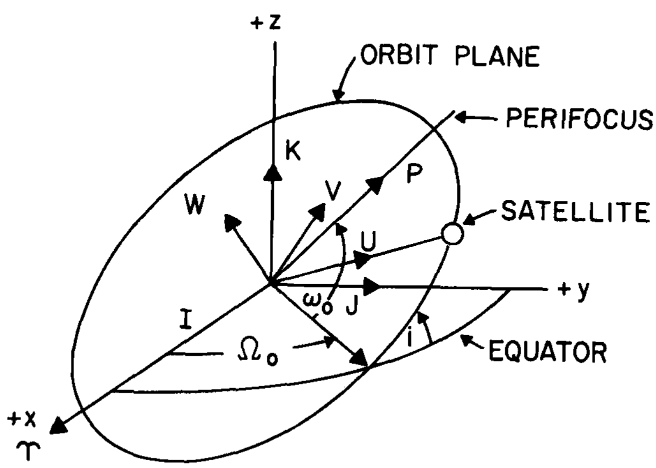


Figure 5.12 The U, V, W orthogonal reference frame (Based on a figure from EB, 1965)

Note that if the satellite is at its perifocal position, the (P, Q, W) orthogonal set is equivalent to the (U, V, W) orthogonal set.

Since (i, Ω_o, ω_o) are the Euler angles of a coordinate rotation, we can develop a transformation between the (I, J, K) system and the

(P, Q, W) system. The direction cosines of this transformation are thus:

$$P_x = \cos\omega_o \cdot \cos\Omega_o - \sin\omega_o \cdot \sin\Omega_o \cdot \cos i$$
$$P_y = \cos\omega_o \cdot \sin\Omega_o + \sin\omega_o \cdot \cos\Omega_o \cdot \cos i \qquad (5.55)$$
$$P_z = \sin\omega_o \cdot \sin i$$

$$Q_x = -\sin\omega_o \cdot \cos\Omega_o - \cos\omega_o \cdot \sin\Omega_o \cdot \cos i$$
$$Q_y = -\sin\omega_o \cdot \sin\Omega_o + \cos\omega_o \cdot \cos\Omega_o - \cos i \qquad (5.56)$$
$$Q_z = \cos\omega_o \cdot \sin i$$

$$W_x = \sin\Omega_o \cdot \sin i$$
$$W_y = -\cos\Omega_o \cdot \sin i \qquad (5.57)$$
$$W_z = \cos i$$

Therefore we have:

$$\begin{bmatrix} P \\ Q \\ W \end{bmatrix} = \begin{bmatrix} P_x P_y P_z \\ Q_x Q_y Q_z \\ W_x W_y W_z \end{bmatrix} \cdot \begin{bmatrix} I \\ J \\ K \end{bmatrix} \qquad (5.58)$$

where (P, Q, W) is wrt the orbit plane frame of reference and (I, J, K) is wrt the celestial frame of reference. Note that the (P, Q, W) system utilizes $(x_\omega, y_\omega, z_\omega)$ coordinates (see Figure 5.10) whereas the (I, J, K) system utilizes (x, y, z) coordinates (see Figure 5.11).

Now if (P, Q, W) are mutually orthogonal and we define the transformation matrix B, where:

$$B = \begin{bmatrix} P_x & P_y & P_z \\ Q_x & Q_y & Q_z \\ W_x & W_y & W_z \end{bmatrix} \tag{5.59}$$

then:

$$\begin{bmatrix} x_\omega \\ y_\omega \\ z_\omega \end{bmatrix} = B \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{5.60}$$

and since:

$$B^{-1} = B^T \tag{5.61}$$

therefore:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^T \begin{bmatrix} x_\omega \\ y_\omega \\ z_\omega \end{bmatrix} \tag{5.62}$$

where:

$$B^T = \begin{bmatrix} P_x & Q_x & W_x \\ P_y & Q_y & W_y \\ P_z & Q_z & W_z \end{bmatrix} \tag{5.63}$$

so that:

$$\begin{aligned} x &= x_\omega P_x + y_\omega Q_x + z_\omega W_x \\ y &= x_\omega P_y + y_\omega Q_y + z_\omega W_y \\ z &= x_\omega P_z + y_\omega Q_z + z_\omega W_z \end{aligned} \tag{5.64}$$

Now since the satellite always remains in the P,Q orbital plane, then z_ω is always zero. Therefore:

$$x = x_\omega P_x + y_\omega Q_x$$
$$y = x_\omega P_y + y_\omega Q_y \qquad (5.65)$$
$$z = x_\omega P_z + y_\omega Q_z$$

implying that if we can determine (x_ω, y_ω), we can solve for a celestial position vector. Note that if we remain in the orbital plane coordinate system as long as possible, we will have an easier time than working in a 3-dimensional system.

In order to determine orbit plane coordinates we need to derive Kepler's Equation which relates geometry or position in the orbit plane to time. We will restrict the analysis to an elliptical formulation, ignoring the parabolic and hyperbolic formulations. We first need a new definition, i.e., the eccentric anomaly (see Figure 5.13).

Eccentric Anomaly (E): The angle measured in the orbital plane from the P axis to a line through the origin and another point defined by the projection of the moving vehicle in the y_ω direction upon a circumscribing circle. Note that this angle is analogous to the angle β (reduced latitude) which was defined in Chapter 4 during the discussion of station coordinates.

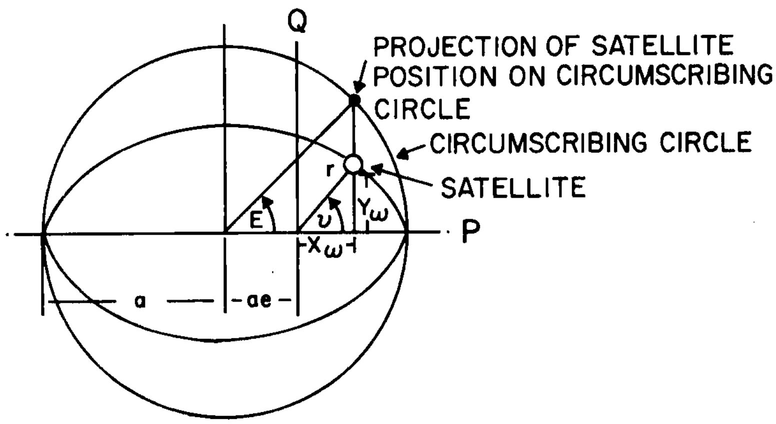


Figure 5.13 Definition of eccentric anomaly
Based on a figure from EB, 1965)

Recalling the definition of true anomaly (also shown in Figure 5.13):

True Anomaly (ν): Angle in the orbital plane with respect to a focus of the ellipse from the perifocal position to the satellite position.

and with the aid of the previous figure:

$$x_\omega = r \cos \nu$$
$$y_\omega = r \sin \nu \tag{5.66}$$

$$x_\omega = a \cdot \cos E - a \cdot e \tag{5.67}$$

Now since:

$$r = p/(1 + e \cdot \cos\nu) \tag{5.68}$$

then:

$$r = p/(1 + e \cdot x_\omega/r) \tag{5.69}$$

or:

$$p = r + e \cdot x_\omega \tag{5.70}$$

But we know:

$$p = a(1 - e^2) \tag{5.71}$$

therefore from equation (5.67):

$$x_\omega = a(\cos E - e) \tag{5.72}$$

we have:

$$r + e \cdot a(\cos E - e) = a(1 - e^2) \tag{5.73}$$

$$r = a(1 - e^2 - e\cos E + e^2) \tag{5.74}$$

$$r = a(1 - e\cos E) \tag{5.75}$$

Now since:

$$r^2 = x_\omega{}^2 + y_\omega{}^2 \tag{5.76}$$

by manipulation:

$$y_\omega = a(\sin E \cdot \sqrt{1-e^2}) \tag{5.77}$$

and thus equations (5.72) and (5.77) give us orbital plane coordinates in terms of Classical Orbital Elements and the eccentric anomaly.

We can now develop the relationship between E and ν. Noting that:

$$\begin{aligned} a(\cos E - e) &= r\cos\nu \\ &= \frac{a(1 - e^2)}{1 + e\cos\nu} \cdot \cos\nu \end{aligned} \tag{5.78}$$

and with suitable manipulation:

$$\cos\nu = \frac{\cos E - e}{1 - e\cos E} \tag{5.79}$$

Also:

$$\begin{aligned} a\sin E\sqrt{1-e^2} &= r\sin\nu \\ &= \frac{a(1 - e^2)}{1 + e\cos\nu} \sin\nu \end{aligned}$$

Now using equation (5.79) to define $\cos\nu$ and with suitable manipulation:

$$\sin\nu = \frac{\sin E\sqrt{1-e^2}}{1 - e\cos E} \tag{5.80}$$

Equations (5.79) and (5.80) thus provide a transform pair between E and ν. If we invert the expressions, we have a transform pair between ν and E. It is easy to show that:

$$\begin{aligned} \cos E &= \frac{\cos\nu + e}{1 + e\cos\nu} \\ \sin E &= \frac{\sqrt{1-e^2} \cdot \sin\nu}{1 + e\cos\nu} \end{aligned} \tag{5.81}$$

Now we will go through a brief derivation of Kepler's equation. First we note:

$$\dot{x}_\omega = -\, a\, \dot{E}\, \sin E$$
$$\dot{y}_\omega = a\, \dot{E}\, \sqrt{1-e^2}\, \cos E \qquad (5.82)$$

Next we require some identities that are basic properties of orbits. From equation (5.49):

$$\frac{d\vec{\dot{r}}}{d\tau} = \ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} \qquad (5.83)$$

therefore:

$$\vec{r} \times \ddot{\vec{r}} = \frac{-\mu}{r^3}\, \vec{r} \times \vec{r} = 0 \qquad (5.84)$$

Now since:

$$\frac{d}{d\tau}(\vec{r} \times \dot{\vec{r}}) = \vec{r} \times \ddot{\vec{r}} + \dot{\vec{r}} \times \dot{\vec{r}} \qquad (5.85)$$

therefore:

$$\frac{d}{d\tau}(\vec{r} \times \dot{\vec{r}}) = 0 \qquad (5.86)$$

and:

$$\vec{r} \times \dot{\vec{r}} = \vec{h} \equiv \text{a vector constant} \qquad (5.87)$$

$$(\vec{r} \times \dot{\vec{r}}) \cdot \vec{h} = h^2 \equiv \text{a scalar constant} \qquad (5.88)$$

A proof in Chapter 3 of EB shows that:

$$r = \frac{h^2/\mu}{1 + e\cos\nu} \qquad (5.89)$$

and therefore:

$$\mu p = \mu \cdot a(1 - e^2) = h^2 = (\vec{r} \times \dot{\vec{r}}) \circ (\vec{r} \circ \dot{\vec{r}}) \qquad (5.90)$$

Now expanding the right hand side of equation (5.90):

$$\mu \cdot a(1 - e^2) = \begin{bmatrix} i & j & k \\ x_\omega & y_\omega & 0 \\ \dot{x}_\omega & \dot{y}_\omega & 0 \end{bmatrix} \circ \begin{bmatrix} i & j & k \\ x_\omega & y_\omega & 0 \\ \dot{x}_\omega & \dot{y}_\omega & 0 \end{bmatrix} \tag{5.91}$$

results in the following:

$$\mu \circ a(1 - e^2) = (x_\omega \dot{y}_\omega - y_\omega \dot{x}_\omega)^2 \tag{5.92}$$

From the definitions of x_ω, y_ω, $\dot{x}_\omega$, $\dot{y}_\omega$ it is easy to show that:

$$\frac{\sqrt{\mu}}{a^{3/2}} = (1 - e \cos E)\dot{E} \tag{5.93}$$

Now if we integrate equation (5.87) from $\tau' = 0$ to $\tau' = \tau$:

$$\frac{\sqrt{\mu}}{a^{3/2}} \int_o^\tau d\tau' = \int_o^{E_\tau} (1 - e \cos E)\, dE' \tag{5.94}$$

we find:

$$\frac{\sqrt{\mu}}{a^{3/2}} \tau = E_\tau - e \sin E_\tau \tag{5.95}$$

We now recall the definition of the modified time variable:

$$\tau = K(t - t_o) \tag{5.96}$$

where we understand that from the integration limits, the initial time t_o corresponds to the point on the orbit where $E = 0$. We shall call this time T, the time of perifocal passage. Substituting for τ, such that $E \equiv E_t$, we have Kepler's equation:

$$\frac{\sqrt{\mu}}{a^{3/2}} K(t - T) = E - e \sin E \tag{5.97}$$

Now we call $\sqrt{\mu}K/a^{3/2}$ the mean motion n, where:

$$n = \frac{\sqrt{\mu}}{a^{3/2}} K \tag{5.98}$$

and it is now apparent that we have a formulation for the mean anomaly (M):

$$M = n(t - T) \tag{5.99}$$

Note that M is one of the Classic Orbit Elements:

Mean Anomaly (M): Angle in orbital plane with respect to the center of a mean circular orbit, having a period equivalent to the anomalistic period, from perifocus to the satellite position. We shall defer our discussion of anomalistic period until we discuss perturbation theory in Chapter 6.

We now see what the mean motion has to do with the period. Recalling equation (5.40):

$$P = \frac{2\pi}{K\sqrt{\mu}} a^{3/2} \tag{5.100}$$

Therefore the mean motion constant (n) and the period (P) are simply reciprocal quantities:

$$n = \frac{2\pi}{P} \qquad P = \frac{2\pi}{n} \tag{5.101}$$

It is important to note why the recovery of an accurate value of the semi-major axis (a) from raw orbit tracking data is so important. Since the period is directly proportional to $a^{3/2}$, any error in recovering the semi-major axis translates to a cumulative error in position due to an incorrect period. Figure 5.14 provides a graph for both a low orbiting satellite and a geosynchronous satellite indicating the period error corresponding to errors in specifying the semi-major axis.

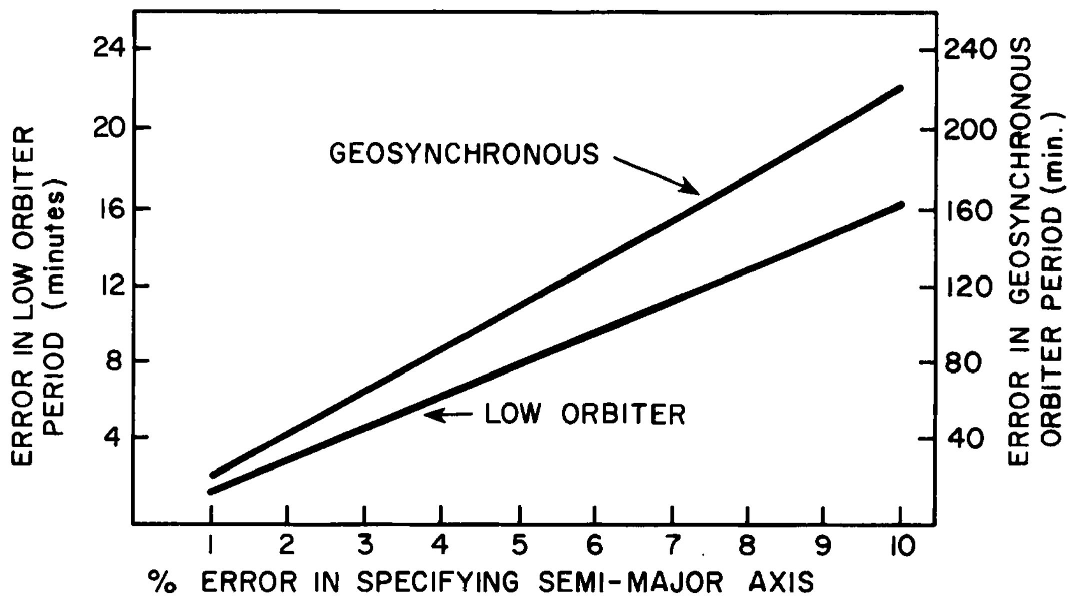


Figure 5.14 Error in determining satellite period corresponding to error in recovering the semi-major axis

From equations (5.97) and (5.99) we have a relationship between M and E:

$$M = E - e \sin E \tag{5.102}$$

however, we want E in terms of M. Since equation (5.102) is a transcendental equation we can transform it. First equation 5.102 is differentiated:

$$dM = (1 - e \cos E) dE \tag{5.103}$$

Next we rearrange and integrate from the position of perigee at which $E_o = M_o = 0$, to an arbitrary position in the orbit corresponding to (E_t, M_t):

$$\int_o^{E_t} dE = E_t = \int_o^{M_t} \frac{dM}{1 - e \cdot \cos E} \tag{5.104}$$

We can now express the term under the integral of equation 5.104 as a Fourier expansion:

$$E_t = \int_o^{M_t} \left\{ \frac{a_o}{2} + \sum_{m=1}^{\infty} \left(a_m \cdot \cos \frac{m\pi M}{\ell} + b_m \sin \frac{m\pi M}{\ell} \right) \right\} dM \tag{5.105}$$

where 2ℓ is the period of the function and:

$$a_m = \frac{1}{\ell} \int_o^{2\ell} (1 - e \cos E)^{-1} \cos \left(\frac{m\pi M}{\ell} \right) dM$$

$$b_m = \frac{1}{\ell} \int_o^{2\ell} (1 - e \cos E)^{-1} \sin \left(\frac{m\pi M}{\ell} \right) dM \tag{5.106a}$$

$$a_o = \frac{1}{\ell} \int_o^{2\ell} (1 - e \cos E)^{-1} dM$$

Now substituting for dM from equation (5.103) and noting that $2\ell = 2\pi$:

$$a_o = \frac{1}{\pi} \int_o^{2\pi} dE = 2$$

$$a_m = \frac{1}{\pi} \int_o^{2\pi} \cos(m \cdot M) dE \tag{5.106b}$$

and all $b_m = 0$ since we are integrating an even function. Now using our definition of M from equation (5.102):

$$a_m = \frac{1}{\pi} \int_0^{2\pi} \cos \left\{ m(E - e \sin E) \right\} dE$$

Now using an integral representation property of Bessel functions (see Abramowitz and Stegun, 1972):

$$a_m = 2 J_m(me) \tag{5.107}$$

where J_m is a Bessel function of the first kind of order m and argument me:

$$J_m(me) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{me}{2}\right)^{2k+m}}{k!(k+m)!} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{me}{2}\right)^{2k+m}}{k!\Gamma(k+m+1)} \tag{5.108}$$

We can now rewrite equation (5.105) as:

$$E_t = \int_0^{M_t} \left\{ 1 + \sum_{m=1}^{\infty} 2 J_m(me) \cos(mM) \right\} dM \tag{5.109}$$

and integrating, we can finally express the eccentric anomaly E, explicitly in terms of M and e with a Fourier-Bessel series:

$$E = M + 2 \sum_{m=1}^{\infty} \frac{1}{m} J_m(me) \sin(mM) \tag{5.110}$$

where E and M represent the eccentric and mean anomaly at an arbitrary time t.

The above expression remains cumbersome for computer calculations. However, the series term can be expanded in powers of e. Noting that $e < 1.0$, we can truncate at some power of e, say 5:

$$J_1(1\cdot e) = \frac{(-1)^o\left(\frac{e}{2}\right)^1}{0!\circ 1!} - \frac{(-1)^1\left(\frac{e}{2}\right)^3}{1!\circ 2!} + \frac{(-1)^2\left(\frac{e}{2}\right)^5}{2!\circ 3!} + \ldots$$

$$= \frac{e}{2} - \frac{e^3}{16} + \frac{e^5}{384} + \ldots$$

$$J_2(2\circ e) = \frac{e^2}{2} - \frac{e^4}{6} + \ldots \qquad (5.111)$$

$$J_3(3\cdot e) = \frac{9}{16} e^3 - \frac{81}{256} e^5 + \ldots$$

$$J_4(4\cdot e) = \frac{2}{3} e^4 + \ldots$$

$$J_5(5\cdot e) = \frac{625}{768} e^5 + \ldots$$

Now if we collect terms in similar powers of e:

$$\begin{aligned} E = M &+ \frac{2}{1}\cdot\frac{e}{2}\circ \sin(M) \\ &+ \frac{2}{2}\circ\frac{e^2}{2}\circ \sin(2M) \\ &+ \frac{2}{3}\cdot\frac{9}{16}\circ e^3\cdot \sin(3M) - \frac{2}{1}\circ\frac{1}{16}\circ e^3\cdot \sin(M) \qquad (5.112) \\ &+ \frac{2}{4}\circ\frac{2}{3}\cdot e^4\circ \sin(4M) - \frac{2}{2}\circ\frac{1}{6}\cdot e^4\cdot \sin(2M) \\ &+ \frac{2}{5}\circ\frac{625}{768}\cdot e^5\circ \sin(5M) - \frac{2}{3}\circ\frac{81}{256}\circ e^5\circ \sin(3M) \\ &+ \frac{2}{1}\cdot\frac{1}{384}\cdot e^5\cdot \sin(M) \end{aligned}$$

Simplifying:

$$\begin{aligned} E = M + \sin(M)\circ e &+ \frac{\sin(2M)}{2}\circ e^2 + \frac{1}{8}[3\circ \sin(3M) - \sin(M)]e^3 \\ &+ \frac{1}{6}[2\circ \sin(4M) - \sin(2M)]\circ e^4 \qquad (5.113) \\ &+ \frac{1}{384}[125\cdot \sin(5M) - 81\circ \sin(3M) + 2 . \sin(M)]\cdot e^5 \end{aligned}$$

We now note that all the coefficients of the expansion are less than one, thus insuring that the truncation in powers of e only ignores increasingly smaller terms. Now we can apply the trigonometric multiple angle relationships:

$$\begin{aligned} \sin(2M) &= 2\sin(M)\cos(M) \\ \sin(3M) &= 3\sin(M) - 4\sin^3(M) \\ \sin(4M) &= 4\sin(M)\cos(M) - 8\sin^3(M)\cos(M) \\ \sin(5M) &= 5\sin(M) - 20\sin^3(M) + 16\sin^5(M) \end{aligned} \tag{5.114}$$

Substituting and simplifying we arrive at our final equation for E in explicit terms; an expression which involves only a single sin and cos calculation insofar as computational requirements are concerned:

$$\begin{aligned} E = M &+ \sin(M) \circ e + \sin(M)\cos(M)e^2 \\ &+ [\sin(M) - (3/2)\sin^3(M)]e^3 \\ &+ [\sin(M)\cos(M) - (8/3)\sin^3(M)\cos(M)]e^4 \\ &+ [\sin(M) - (17/3)\sin^3(M) + (125/24)\sin^5(M)]e^5 \end{aligned} \tag{5.115}$$

Note that if we consider only the first power term (for example, in the event e is very small), then:

$$E \overset{\circ}{=} M + e\circ\sin(M) \tag{5.116}$$

To illustrate the error in ignoring the higher order terms we examine the eccentric anomaly of the sun with respect to the earth under various orders of expansion. Table 5.4 provides the results. Appendix D provides a computer solution for an apparent solar orbit which considers the above expansion.

Table 5.4 Eccentric Anomaly of Sun wrt Earth Under Various Orders of Expansion (Eccentricity of solar orbit is .081820157)

Mean Anomaly	Eccentric Anomaly				
	e^1	e^2	e^3	e^4	e^5
0	0.000000	0.000000	0.000000	0.000000	0.000000
15	15.021177	15.022850	15.022978	15.022987	15.022988
30	30.040910	30.043809	30.043980	30.043987	30.043986
45	45.057856	45.061203	45.061300	45.061292	45.061291
60	60.070858	60.073757	60.073698	60.073678	60.073677
75	75.079032	75.080706	75.080494	75.080478	75.080479
90	90.081820	90.081820	90.081546	90.081546	90.081548
105	105.079032	105.077359	105.077147	105.077164	105.077165
120	120.070858	120.067960	120.067900	120.067920	120.067919
135	135.057856	135.054508	135.054605	135.054613	135.054611
150	150.040910	150.038011	150.038182	150.038176	150.038176
165	165.021177	165.019503	165.019631	165.019621	165.019622
180	180.000000	180.000000	180.000000	180.000000	180.000000

The stage is now set for the calculation of a celestial pointing vector. We first transform the epoch from t_o to the time of perifocal passage (T). Since:

$$M_o = n(t_o - T) \tag{5.117}$$

therefore:

$$T = t_o - M_o/n \tag{5.118}$$

Thus we can now solve for M at any arbitrary time t:

$$M = n(t - T) \tag{5.119}$$

and then solve for E:

$$E = M + e \sin(M) + \ldots \tag{5.120}$$

We now solve for x_ω and y_ω and note that z_ω is always 0:

$$x_\omega = a(\cos E - e)$$
$$y_\omega = a(\sin E \cdot \sqrt{1-e^2}) \qquad (5.121)$$
$$z_\omega = 0$$

Now transform to a celestial pointing vector:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^T \begin{bmatrix} x_\omega \\ y_\omega \\ 0 \end{bmatrix} \qquad (5.122)$$

where B^T is the transpose of the celestial frame-orbital plane transformation matrix. This completes the desired solution.

It is useful to summarize the relationships between M, ν, and E:

$$M = E - e \cdot \sin E$$
$$E \doteq M + e \cdot \sin M + \ldots \qquad (5.123)$$

$$\cos \nu = (\cos E - e)/(1 - e \cos E)$$
$$\sin \nu = \sqrt{1 - e^2} \cdot \sin E/(1 - e \cos E) \qquad (5.124)$$

$$\cos E = (\cos\nu + e)/(1 + e \cos\nu)$$
$$\sin E = \sqrt{1 - e^2} \cdot \sin\nu/(1 + e \cos\nu) \qquad (5.125)$$

Now recall that ESA uses True Anomaly (ν_o) rather than Mean Anomaly (M_o) in their orbital element transmissions. Thus before we can apply equation (5.118), we first transform ν_o to an initial eccentric anomaly E_o:

$$E_o = \cos^{-1}[(\cos \nu_o + e)/(1 + e \cos \nu_o)] \qquad (5.126)$$

The initial mean anomaly can now be solved:

$$M_o = E_o + e \cdot \sin E_o \qquad (5.127)$$

5.10 Rotation to Terrestrial Coordinates

Finally, we transform to our rotating frame of reference (i.e., the earth). This is accomplished by noting that the observer's meridian is rotating with an angular velocity equal to $\dot{\rho}$, that is the sidereal rate of change. Thus the observer's right ascension can be given by:

$$\rho = \rho_o + \dot{\rho}(t - t_e) \qquad (5.128)$$

in which we have defined:

$$\begin{aligned} \rho_o &= SHA \\ \dot{\rho} &= (2\pi/P_d)\cdot S \end{aligned} \qquad (5.129)$$

where P_d is the daily period (24 hours), t_e is a sidereal epoch, and SHA is the sidereal hour angle at the epoch t_e. We can choose SHA = 0, i.e. a time when the Greenwich meridian is in conjunction with the vernal equinox. To do so, the "Universal and Sidereal Time" table from the American Ephemeris and Nautical Almanac can be used. Table 5.5 provides an example from the 1978 version for January, in which can be seen that on January 1, at 17 16 00 GMT the vernal equinox and the Greenwich meridian are aligned. S simply converts solar mean time to sidereal time, where:

$$S = 366.25/365.25 \qquad (5.130)$$

Thus, by rotating the (x, y, z) vector through an angle ρ, we finally achieve our desired earth reference vector (x_e, y_e, z_e):

$$\begin{aligned} x_e &= \cos(\rho)\cdot x + \sin(\rho)\cdot y \\ y_e &= -\sin(\rho)\cdot x + \cos(\rho)\cdot y \\ z_e &= z \end{aligned} \qquad (5.131)$$

Now, using the transformation between cartesian and spherical coordinates, we can solve for the sub-satellite point (ϕ_{sp}, λ_{sp}) in geocentric coordinates and the satellite height (h). First, we solve for latitude and longitude (ϕ,λ) and the radius coordinate (r) in a spherical reference frame:

$$\phi = \sin^{-1}[z_e/\sqrt{x_e^2 + y_e^2 + z_e^2}]$$

$$\lambda = \tan^{-1}[y_e/x_e] \qquad (5.132)$$

$$r = \sqrt{x_e^2 + y_e^2 + z_e^2}$$

Finally we transform to geocentric coordinates (ϕ_{sp}, λ_{sp}) and height (h):

$$\phi_{sp} = \cos^{-1}[\cos\phi/\sqrt{1-e^2\sin^2\phi}]$$

$$\lambda_{sp} = \lambda \qquad (5.133)$$

$$h_{sp} = r - R_e$$

where R_e is the earth radius at latitude ϕ_{sp} and e is the eccentricity of the earth itself.

Computer codes adopted to the above methodology are given in Appendices B and D. Appendix B considers an earth-satellite configuration whereas Appendix D considers an earth-sun configuration. Appendix C consists of a numerical routine used to determine an earth satellite equator crossing period which will be discussed in Chapter 6. Appendix E gives two approximate solutions for determining solar position; these routines can be compared to the solution given in Appendix D. Appendix F represents a set of library routines applicable to the aforementioned orbital codes, and finally Appendix G provides a solution for determining the required inclination for a sun-synchronous orbit (this problem is discussed in Chapter 6).

Table 5.5: Universal and Sidereal Time Table for January, 1978 (From the American Ephemeris and Nautical Almanac, 1978)

Date		Julian Date	G. SIDEREAL TIME (G.H.A. of the Equinox) Apparent	Mean	Equation of Equinoxes at 0^h U.T.	G.S.D. 0^h G.S.T.	UNIVERSAL TIME (Greenwich Transit of the Equinox) Date	Apparent	Mean
0^h U.T.		244	h m s	s	s	245		d h m s	s
Jan.	**0**	3508·5	6 37 13·506	13·280	+0·226	**0200·0**	**Jan.**	0 17 19 55·662	55·886
	1	3509·5	6 41 10·059	09·835	·223	**0201·0**		1 17 15 59·756	59·976
	2	3510·5	6 45 06·611	06·391	·220	**0202·0**		2 17 12 03·849	04·067
	3	3511·5	6 49 03·164	02·946	·218	**0203·0**		3 17 08 07·941	08·157
	4	3512·5	6 52 59·718	59·501	·217	**0204·0**		4 17 04 12·031	12·248
	5	3513·5	6 56 56·275	56·057	+0·218	**0205·0**		5 17 00 16·118	16·339
	6	3514·5	7 00 52·835	52·612	·222	**0206·0**		6 16 56 20·203	20·429
	7	3515·5	7 04 49·397	49·167	·229	**0207·0**		7 16 52 24·285	24·520
	8	3516·5	7 08 45·960	45·723	·238	**0208·0**		8 16 48 28·367	28·610
	9	3517·5	7 12 42·524	42·278	·246	**0209·0**		9 16 44 32·451	32·701
	10	3518·5	7 16 39·086	38·834	+0·252	**0210·0**		10 16 40 36·537	36·791
	11	3519·5	7 20 35·644	35·389	·255	**0211·0**		11 16 36 40·626	40·882
	12	3520·5	7 24 32·200	31·944	·256	**0212·0**		12 16 32 44·718	44·972
	13	3521·5	7 28 28·753	28·500	·254	**0213·0**		13 16 28 48·812	49·063
	14	3522·5	7 32 25·305	25·055	·250	**0214·0**		14 16 24 52·906	53·153
	15	3523·5	7 36 21·857	21·610	+0·247	**0215·0**		15 16 20 57·000	57·244
	16	3524·5	7 40 18·409	18·166	·244	**0216·0**		16 16 17 01·093	01·334
	17	3525·5	7 44 14·963	14·721	·242	**0217·0**		17 16 13 05·183	05·425
	18	3526·5	7 48 11·519	11·276	·242	**0218·0**		18 16 09 09·273	09·515
	19	3527·5	7 52 08·076	07·832	·244	**0219·0**		19 16 05 13·361	13·606
	20	3528·5	7 56 04·634	04·387	+0·247	**0220·0**		20 16 01 17·448	17·697
	21	3529·5	8 00 01·193	00·943	·250	**0221·0**		21 15 57 21·535	21·787
	22	3530·5	8 03 57·752	57·498	·254	**0222·0**		22 15 53 25·622	25·878
	23	3531·5	8 07 54·310	54·053	·257	**0223·0**		23 15 49 29·710	29·968
	24	3532·5	8 11 50·868	50·609	·260	**0224·0**		24 15 45 33·799	34·059
	25	3533·5	8 15 47·424	47·164	+0·260	**0225·0**		25 15 41 37·890	38·149
	26	3534·5	8 19 43·979	43·719	·259	**0226·0**		26 15 37 41·983	42·240
	27	3535·5	8 23 40·531	40·275	·256	**0227·0**		27 15 33 46·077	46·330
	28	3536·5	8 27 37·082	36·830	·252	**0228·0**		28 15 29 50·172	50·421
	29	3537·5	8 31 33·633	33·385	·247	**0229·0**		29 15 25 54·268	54·511
	30	3538·5	8 35 30·183	29·941	+0·243	**0230·0**		30 15 21 58·362	58·602
	31	3539·5	8 39 26·735	26·496	·239	**0231·0**		31 15 18 02·456	02·692
Feb.	**1**	3540·5	8 43 23·289	23·052	·237	**0232·0**	**Feb.**	1 15 14 06·546	06·783
	2	3541·5	8 47 19·845	19·607	·238	**0233·0**		2 15 10 10·635	10·873
	3	3542·5	8 51 16·403	16·162	·241	**0234·0**		3 15 06 14·721	14·964
	4	3543·5	8 55 12·964	12·718	+0·246	**0235·0**		4 15 02 18·806	19·055
	5	3544·5	8 59 09·524	09·273	·251	**0236·0**		5 14 58 22·891	23·145
	6	3545·5	9 03 06·084	05·828	·256	**0237·0**		6 14 54 26·979	27·236
	7	3546·5	9 07 02·642	02·384	·258	**0238·0**		7 14 50 31·069	31·326
	8	3547·5	9 10 59·197	58·939	·257	**0239·0**		8 14 46 35·162	35·417
	9	3548·5	9 14 55·748	55·495	+0·254	**0240·0**		9 14 42 39·257	39·507
	10	3549·5	9 18 52·298	52·050	·248	**0241·0**		10 14 38 43·354	43·598
	11	3550·5	9 22 48·847	48·605	·242	**0242·0**		11 14 34 47·450	47·688
	12	3551·5	9 26 45·397	45·161	·236	**0243·0**		12 14 30 51·546	51·779
	13	3552·5	9 30 41·948	41·716	·232	**0244·0**		13 14 26 55·640	55·869
	14	3553·5	9 34 38·500	38·271	+0·229	**0245·0**		14 14 22 59·733	59·960
	15	3554·5	9 38 35·054	34·827	+0·227	**0246·0**		15 14 19 03·824	04·050

6.0 PERTURBATION THEORY

6.1 Concept of Gravitational Potential

We will now consider the deviation of an orbit from the ideal, two body, inverse square-force field law. In order to do so, we must distinguish the concepts of empirically correcting orbit calculations due to a non-perfect two body system, and the actual prediction of orbit positions based on a physical model which accounts for forces that perturb a body from perfect two body motion. The first technique has received a good deal of study under the general heading of "Differential Correction". A discussion of this topic is given in Chapter 9 of EB, by Dubyago (1961), and by Capellari et al. (1976). The method consists of bringing a predicted orbit position into agreement with a set of actual orbit measurements in such a way so as to adjust a set of constant orbital elements to satisfy a new local time period. Thus the methodology does not necessarily consider the physical reasons why an orbit is perturbed.

The general area of "Perturbation Theory" consists of developing a set of reasonable, time dependent quantities which arise due to various perturbation forces, which in turn lead to time dependent expressions for the orbital elements themselves. This theory, although not necessarily adaptable to analytic techniques, has a physical basis in fact. Since the satellite navigation problem is not really compatible with the required procedures used in Differential Correction techniques, we shall address the following discussion to perturbation techniques.

We first need to consider the governing equation in terms of the concept of potential. Following the approach of Kozai (1959) and EB and using a spherical coordinate system defined by the earth's equatorial plane, we define a potential (V):

$$V + \frac{\mu K^2}{r} \tag{6.1}$$

where:

$$r = \sqrt{x^2 + y^2 + z^2} \tag{6.2}$$

and (x, y, z) are the cartesian components of a radius vector $\vec{r}$ extended from the earth center to an arbitrary satellite position. Taking partial derivatives with respect to x, y and z yields:

$$\begin{aligned} \frac{\delta V}{\delta x} &= -\frac{\mu K^2}{r^2}\frac{\delta r}{\delta x} \\ \frac{\delta V}{\delta y} &= -\frac{\mu K^2}{r^2}\frac{\delta r}{\delta y} \\ \frac{\delta V}{\delta z} &= -\frac{\mu K^2}{r^2}\frac{\delta r}{\delta z} \end{aligned} \tag{6.3}$$

and since:

$$\frac{\delta r}{\delta x} = \frac{x}{r}\ ;\ \frac{\delta r}{\delta y} = \frac{y}{r}\ ;\ \frac{\delta r}{\delta z} = \frac{z}{r} \tag{6.4}$$

then:

$$\frac{d^2x}{dt^2} = \frac{\delta V}{\delta x}\ ;\ \frac{d^2y}{dt^2} = \frac{\delta V}{\delta y}\ ;\ \frac{d^2z}{dt^2} = \frac{\delta V}{\delta z} \tag{6.5}$$

or simply:

$$\frac{d^2\vec{r}}{dt^2} = \nabla V \text{ (grad V)} \tag{6.6}$$

Equation 6.6 thus states that the acceleration of a body is due to the gradient of what we shall call a potential V.

If we generalize the problem, it is easily seen that V can be expressed as a summation of normalized point masses (m_i):

$$V = \sum_{i=1}^{n} \frac{m_i K^2}{r_i} \tag{6.7}$$

Now if we consider the earth as a series of concentric (circular) masses about its center, we see that if we assume an oblate spheroid (bulging equator), we are considering a non-symmetric force field as shown in Figure 6.1. Makemson et al. (1961) have provided a spherical harmonics expansion of the aspherical potential V_e of the earth:

$$\begin{aligned} V_e = \frac{K^2 m_e}{r} \Bigg[1 &+ \frac{J_2}{2r^2} (1 - 3 \sin^2\delta) \\ &+ \frac{J_3}{2r^3} (3 - 5 \sin^2\delta)\sin\delta \\ &- \frac{J_4}{8r^4} (3 - 30 \sin^2\delta + 35 \sin^4\delta) \\ &- \frac{J_5}{8r^5} (15 - 70 \sin^2\delta + 63 \sin^4\delta)\sin\delta \\ &+ \frac{J_6}{16r^6} (5 - 105 \sin^2\delta + 315 \sin^4\delta - 231 \sin^6\delta) \\ &+ \epsilon \Bigg] \end{aligned} \tag{6.8}$$

where:

m_e = mass of earth in earth mass units = 1

K = the terrestrial gravitational constant

$\delta = \sin^{-1}(z/r)$

r = distance from the earth center to a spacecraft in e.r. units

and the J_i's are the spherical harmonic coefficients of the earth's gravitational potential. Equation (6.8) has been normalized such that $J_1 = 1$. The term ϵ simply expresses the error due to ignoring higher order terms. The lower order coefficients have been tabulated by Makemson et al. (1961) and are given in Table 6.1.

Table 6.1: Harmonic Coefficients of the Earth's Gravitational Potential

$$J_2 = +1082.28 \cdot 10^{-6}$$
$$J_3 = -2.30 \cdot 10^{-6}$$
$$J_4 = -2.12 \cdot 10^{-6}$$
$$J_5 = -0.20 \cdot 10^{-6}$$
$$J_6 = +1.00 \cdot 10^{-6}$$

Equation (6.8) is actually a simplification of the gravitational potential of the earth. When considering the departures from symmetry, there are two kinds of spherical harmonics: zonal harmonics (departures due to the ellipticity of the meridians), and tesseral harmonics (departures due to the ellipticity in latitudinal cross sections). Only zonal harmonics are considered in the above expansion. This is a standard model adopted in general perturbation techniques (see Escobal (1968) for a discussion of higher order models).

Since we can express the governing equation as:

$$\frac{d^2\vec{r}}{dt^2} = \nabla V_e \qquad (6.9)$$

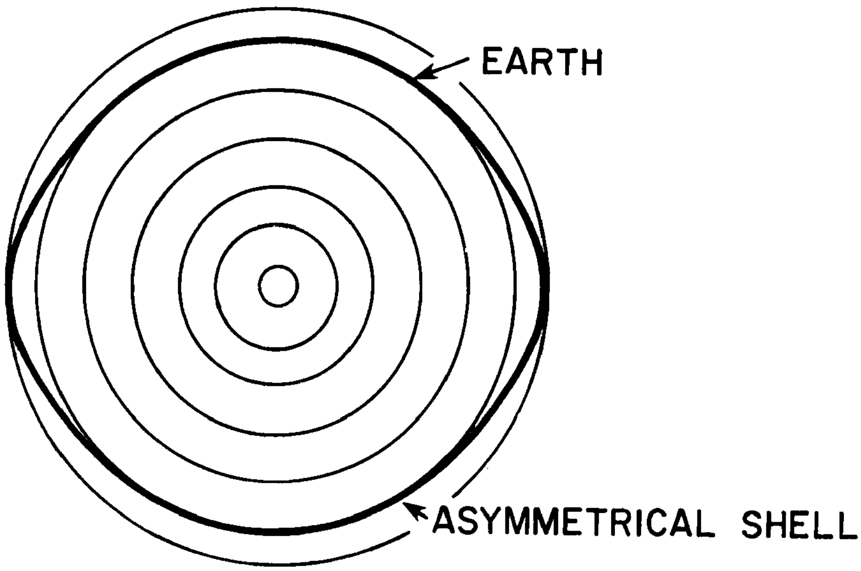


Figure 6.1 Depiction of the earth as a sequence of concentric mass shells

by differentiating equation (6.8) with respect to x, y, z and using equation (6.4) we have the equations of motion of a satellite with respect to an oblate spheroidal central body (expressed to order J_3):

$$\frac{d^2x}{dt^2} = \frac{\delta V_e}{\delta x} = \frac{-K^2 m_e x}{r^3}\left[1 + \frac{3}{2}\frac{J_2}{r^2}(1 - 5\sin^2\delta) + \frac{5}{2}\frac{J_3}{r^3}(3 - 7\sin^2\delta)\sin\delta + \ldots\right]$$

$$\frac{d^2y}{dt^2} = \frac{\delta V_e}{\delta y} = -\frac{-K^2 m_e y}{r^3}\left[1 + \frac{3}{2}\frac{J_2}{r^2}(1 - 5\sin^2\delta) + \frac{5}{2}\frac{J_3}{r^3}(3 - 7\sin^2\delta)\sin\delta + \ldots\right] \quad (6.10)$$

$$\frac{d^2z}{dt^2} = \frac{\delta V_e}{\delta z} = \frac{-K^2 m_e z}{r^3}\left[1 + \frac{3}{2}\frac{J_2}{r^2}(3 - 5\sin^2\delta) + \frac{5}{2}\frac{J_3}{r^3}(6 - 7\sin^2\delta)\sin\delta + \ldots\right] + \frac{K^2 m_e}{r^2}\left[\frac{3}{2}\frac{J_3}{r^3} + \ldots\right]$$

This lays the foundation for considering the motion of a satellite with respect to an oblate spheriodal central body and under the influence of additional perturbative effects.

6.2 Perturbative Forces and the Time Dependence of Orbital Elements

A satellite, under the influence of a perfect inverse square force field law, would have a set of constant orbital elements:

$$[a, e, i, M_o, \Omega_o, \omega_o]$$

devoid of any time dependence. However, due to perturbative forces, the orbital elements are acted upon leading to shifts or oscillations. There are a number of effects which can be considered as perturbative forces:

1. Aspherical gravitational potential
2. Auxillary bodies (e.g. sun, moon, planets)
3. Atmospheric drag
4. Atmospheric lift
5. Thrust
6. Radiation Pressure (shortwave and longwave radiation)
7. Galactic particle bombardment, e.g. protons (i.e. solar wind)
8. Electromagnetic field asymmetry

The most important of these effects on earth satellites is due to the first factor; the aspherical gravitational potential of the earth itself. Atmospheric drag becomes significant for the lower orbit satellites (heights less than 850 km).

The aim of general perturbation theory is to develop closed expressions for the time dependence of the orbital elements. It has been shown that perturbations possess different characteristics (see Chapter 10 of EB and Dubyago (1961) for a review):

1. Secular variations
2. Long term periodic variations
3. Short term periodic variations

In working with meteorological satellite orbits, we are primarily concerned with non-oscillatory secular perturbations which cause ever increasing or decreasing changes of particular orbital elements away from their values at an epoch t_o as shown in Figure 6.2.

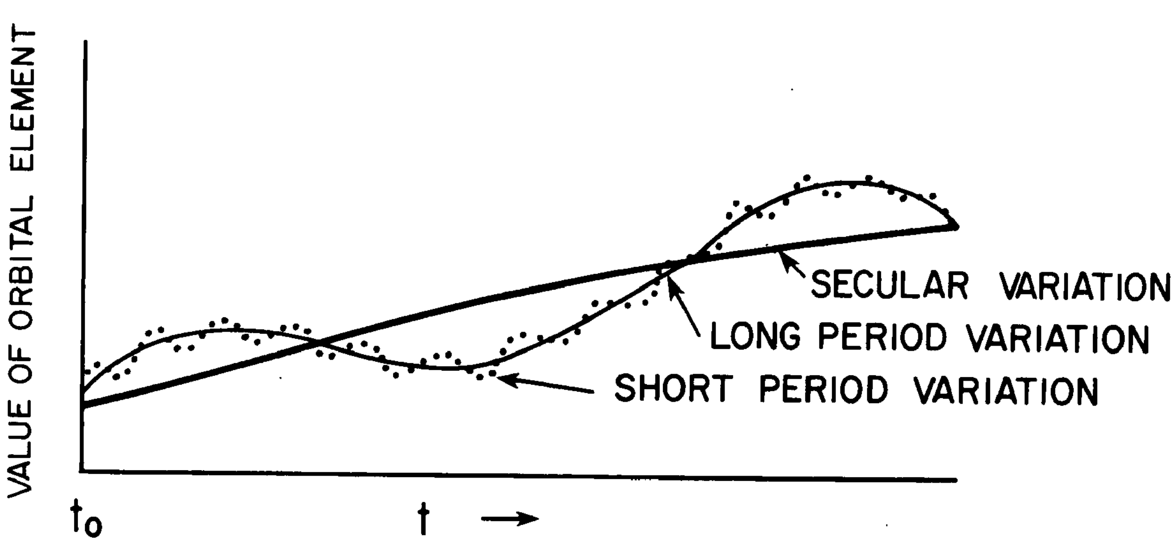


Figure 6.2 Three principle types of orbital perturbations

The aspherical gravitational potential of the earth primarily effects M, Ω, and ω (where we understand that M, Ω, and ω without subscripts are no longer constant). The other elements (a, e, i) undergo minor periodic variations about their mean values due to the aspherical gravitational potential, but in terms of meteorological satellite orbits, are not considered significant. In general, long period variations are caused by the continuous variance of ω whereas short period variations are caused by linear combinations of variations in M and ω.

The general form of the equation of motion in a relative inertial coordinate system is given by:

$$\frac{d^2\vec{r}_{12}}{dt^2} = -K^2\mu\,\frac{\vec{r}_{12}}{r_{12}^3} + K^2\sum_{j=3}^{m}\frac{m_j}{m_1}\left(\frac{\vec{r}_{2j}}{r_{2j}^3} - \frac{\vec{r}_{1j}}{r_{1j}^3}\right) + \left[\Sigma a_2 - \Sigma a_1\right] \qquad (6.11)$$

where subscript 1 indicates the earth, subscript 2 indicates the satellite, the summation over m represents accelerations due to all auxillary bodies of mass m_j (moon, sun, planets), and the bracketed term represents the difference in accelerations of the satellite and the earth created by non-vacuum properties of the surrounding environment (i.e., drag, lift, thrust, radiation pressure, protons, electromagnetic fields). If we tabulate the accelerations due to the non-vacuum properties:

1. Drag (D^P): $D^P = \frac{1}{2} C_D \rho_a A\, V_r^{\,2}$

2. Lift (L^P): $L^P = \frac{1}{2} C_L \rho_a A\, V_r^{\,2}$

3. Thrust (T^P): $T^P = T(t)/\left\{m_o - \int_{t_o}^{t} \frac{dm}{dt}(t)dt\right\}$

4. Radiation Pressure (RP^P): $RP^P = S \cdot W/c$

5. Particle Flux ($PF^P(\measuredangle)$): $PF^P(\measuredangle) = \frac{1}{2}\, C_p \rho_p A \cdot V_r^{\,2}$

6. Electromagnetic Effects (EM^P): $EM^P = F_\varepsilon + F_m$

where:

C_D ≡ empirical drag coefficient (dimensionless)

C_L ≡ empirical lift coefficient (dimensionless)

ρ ≡ density term for atmosphere (ρ_a) or particles (ρ_p)

A ≡ cross section of satellite

V_r ≡ relative motion of satellite with respect to residual atmosphere.

$T(t)$ ≡ time dependent thrust function

m_o ≡ vehicle mass at time of initial thrust

$\int_{t_o}^{t} \frac{dm}{dt}(t)dt$ ≡ integral of vehicle mass flow rate

$S \equiv$ sensitivity coefficient of satellite (includes the effect of the radiative characteristics of its exposed surfaces and its cross-sectional area and has units of area)

$W \equiv$ total irradiance at satellite

$c \equiv$ velocity of light

$C_p \equiv$ empirical particle flux coefficient (dimensionless - the tilted arrow for the particle flux term $PF^P(\swarrow)$ indicates that it is dominated by a point source of solar protons)

$(F_\varepsilon + F_m) \equiv$ unbalanced electromagnetic forces

and note that the first term on the right hand side of equation (6.11) is given by equation (6.10), we can thus express the force field law, specifically for a satellite with respect to an oblate spheroidal earth, in a non-vacuum medium, and affected by the auxillary bodies of the solar system.

In terms of meteorological satellites we are generally considering nearly circular, free flying orbits with altitudes greater than 800 km. In addition, updated orbital parameters from the satellite agencies can be expected at a frequency of no greater than two weeks. Given these boundary conditions, most of the above perturbation terms can be ignored. The major perturbation effect, of course, is the non-sphericity of the earth and the resultant effect on the gravitational potential field.

The minor terms insofar as meteorological satellites are concerned, are the lunar effect, atmospheric drag, and solar radiation pressure. In general the minor terms need not be included in orbit propagations that take place over a one to two week period, if we consider the allowable error bars associated with satellite navigation requirements. That is to say, ignoring the effect of the minor perturbations does not lead to position or ephemeris errors significantly greater than the resolution of the data fields under analysis.

It is important to note that the space agencies responsible for tracking satellites often include the minor terms in retrieving orbital elements. This is due to the fact that generalized orbit retrieval packages have been developed for the extensive variety of operational and experimental satellites, and missiles rather than retrieval packages individually tailored to specific types of satellites. The primary difficulty with treating the minor terms in a satellite navigation model is that the required mathematics does not lend itself to streamlined analytic calculations, a principle requirement for processing the vast amounts of data produced by most meteorological satellite instruments. This is the principle reason for retaining only the major perturbation effect (asymmetric gravitational potential) which can be handled in a direct analytic fashion.

Following EB, if we consider the potential of an aspherical earth (V_e) with respect to the potential of a perfectly spherical earth (V_p), where:

$$V_p = K^2(m_e + m_s)/r \tag{6.12}$$

$$V_e = \frac{K^2 m_e}{r}\left[1 + \frac{J_2}{2r^2}\,(1-3\,\sin^2\delta) + \frac{J_3}{2r^3}\,(3-5\,\sin^2\delta)\sin\delta + \ldots\right] \tag{6.13}$$

then the difference in these two potentials can be said to define a perturbative function (R):

$$R = V_e - V_p \tag{6.14}$$

We can then say the potential V_p gives rise to perfect two body motion whereas the difference function R leads to perturbations about that motion. Using the definition for r and δ:

$$\begin{aligned} r &= a(1 - e^2)/(1 + e \cos\nu) \\ \sin\delta &= \sin i \cdot \sin(\nu + \omega) = z/r \end{aligned} \tag{6.15}$$

we can develop an explicit expression for the perturbative function. The following equation is then an expansion of R to order J_4:

$$\begin{aligned} R = K^2 m_e \Bigg[& \frac{3}{2}\frac{J_2}{a^3}\left(\frac{a}{r}\right)^3 \left\{\frac{1}{3} - \frac{1}{2}\sin^2 i + \frac{1}{2}\sin^2 i \cdot \cos 2(\nu + \omega)\right\} \\ & - \frac{J_3}{a^4}\left(\frac{a}{r}\right)^4 \left\{\left(\frac{15}{8}\sin^2 i - \frac{3}{2}\right)\sin(\nu + \omega) \right. \\ & \left. - \frac{5}{8}\sin^2 i \cdot \sin 3(\nu + \omega)\right\} \sin i \\ & - \frac{35}{8}\frac{J_4}{a^5}\left(\frac{a}{r}\right)^5 \left\{\frac{3}{35} - \frac{3}{7}\sin^2 i + \frac{3}{8}\sin^4 i \right. \\ & + \sin^2 i\left(\frac{3}{7} - \frac{1}{2}\sin^2 i\right)\cos 2(\nu + \omega) \\ & \left. + \frac{1}{8}\sin^4 i \cdot \cos 4(\nu + \omega)\right\} \Bigg] \end{aligned} \tag{6.16}$$

Brouwer and Clemence (1961) and Sterne (1960) have provided the analysis necessary to relate time derivatives of the orbital elements to derivatives in R. These expressions as given by EB are as follows:

$$\frac{da}{dt} = \frac{2}{na}\frac{\delta R}{\delta M}$$

$$\frac{de}{dt} = \frac{(1-e^2)}{na^2 e}\frac{\delta R}{\delta M} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\delta R}{\delta \omega}$$

$$\frac{di}{dt} = \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\delta R}{\delta \omega}$$

$$\frac{dM}{dt} = n - \frac{(1-e^2)}{na^2 e}\frac{\delta R}{\delta e} - \frac{2}{na}\frac{\delta R}{\delta a} \tag{6.17}$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2\sqrt{1-e^2}\,\sin i}\frac{\delta R}{\delta i}$$

$$\frac{d\omega}{dt} = \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\delta R}{\delta i} + \frac{\sqrt{1-e^2}}{na^2 e}\frac{\delta R}{\delta e}$$

It is now possible to partition the resultant derivatives into secular components, long period oscillatory components, and short period oscillatory components.

If we ignore the oscillatory components (in a, e, and i) we can then develop secular perturbation expressions for any selected order of the gravitational potential expansion. It is this process, for satellite applications, which eliminates the time dependence in a, e, and i while including it in M, Ω, and ω. Next note that the time dependence of an arbitrary orbital element (χ) can be expressed as a Taylor series expansion:

$$\chi = \chi_o + \dot{\chi}(t - t_o) + \ddot{\chi}(t - t_o)^2/2! + \ldots \tag{6.18}$$

where χ_o is the initial value at an epoch t_o, and $\dot{\chi}$, $\ddot{\chi}$, ..., are time derivatives. Now, if we ignore all but first order time derivatives and consider only the first order variations of the aspherical gravitational potential (due to J_2), we can express the time dependence of M, Ω, and ω in simple finite difference form with adequate accuracy:

$$\begin{aligned} M &= M_o + \dot{M}(t - t_o) \\ \Omega &= \Omega_o + \dot{\Omega}(t - t_o) \\ \omega &= \omega_o + \dot{\omega}(t - t_o) \end{aligned} \tag{6.19}$$

where $\dot{M} = \overline{n}$ is defined as the Anomalistic Mean Motion and $\dot{\Omega}$, $\dot{\omega}$ are the first derivatives of Ω and ω. These expressions, derived in Chapter 10 of EB, are given by:

$$\dot{M} = \overline{n} = n \left[1 + \frac{3}{2} J_2 \frac{\sqrt{1-e^2}}{p^2} \left(1 - \frac{3}{2} \sin^2 i\right)\right] \tag{6.20}$$

$$\dot{\Omega} = -\left(\frac{3}{2} \frac{J_2}{p^2} \cos i\right) \overline{n} \tag{6.21}$$

$$\dot{\omega} = \left(\frac{3}{2} \frac{J_2}{p^2} \left[2 - \frac{5}{2} \sin^2 i\right]\right) \overline{n} \tag{6.22}$$

which are all functions of a, e, and i. It is important to note that as long as the latter 3 parameters remain nearly constant with time, it is not necessary to apply implicit numerical techniques to the solutions of equations (6.20), (6.21), and (6.22). However, a principal effect of atmospheric drag on low orbit satellites is to modify the values of a, e and i as a function of the eccentric anomaly. This is due to the fact that the essential effect of drag is to de-energize a satellite orbit and thus reduce the dimension (semi-major axis) of the orbit ellipse. In addition, if the initial orbit is highly non-circular, the variation in the drag effect due to the elliptic path leads to modification of the orbit inclination. If a low-flying satellite (small period or high eccentricity) were being considered, time dependent expressions for the semi-major axis, eccentricity, and inclination should be included. EB provides a set of expressions for drag induced derivatives of a, e, and i in Chapter 10 of his text, however, to include these expressions in an orbital solution would require a multiple step iterative approach to the calculation of the six derivative quantities. According to Fuchs (1980), with respect to the satellite navigation problem, drag induced perturbations need not be considered for meteorological satellites until orbital altitudes start falling below 850 km.

With equation (6.20) we can define the Anomalistic Period ($\overline{P}$):

$$\overline{P} = 2\pi/\overline{n} \text{ (perifocus to \underline{varying} perifocus)} \qquad (6.23)$$

Contrast this with the non-perturbative or mean period P:

$$P = 2\pi/n \text{ (perifocus to \underline{non-varying} perifocus)} \qquad (6.24)$$

Expanding to second order variations in potential results in terms of J_2 and J_4, where the Anomalistic Mean Motion $\bar{\bar{n}}$ is given by: (see EB):

$$
\begin{aligned}
\bar{\bar{n}} = n \Bigg[1 &+ \frac{3}{2} J_2 \frac{\sqrt{1-e^2}}{p^2} \left(1 - \frac{3}{2} \sin^2 i\right) \\
&+ \frac{3}{128} J_2^{\,2} \frac{\sqrt{1-e^2}}{p^4} \Big(16\sqrt{1-e^2} + 25(1-e^2) - 15 \\
&+ [30 - 96\sqrt{1-e^2} - 90(1-e^2)] \cos^2 i \\
&+ [105 + 144\sqrt{1-e^2} + 25(1-e^2)] \cos^4 i\Big) \\
&- \frac{45}{128} J_4 \frac{\sqrt{1-e^2}}{p^4} e^2 (3 - 30 \cos^2 i + 35 \cos^4 i) \Bigg]
\end{aligned}
\tag{6.25}
$$

and the Anomalistic Period (P) and the Mean Anomaly (M) are given by:

$$
\begin{aligned}
\bar{\bar{P}} &= 2\pi/\bar{\bar{n}} \\
M &= M_o + \bar{\bar{n}}(t - t_o)
\end{aligned}
\tag{6.26}
$$

The first derivative terms $\dot{M}$, $\dot{\Omega}$, and $\dot{\omega}$ are given by:

$$
\dot{M} = \bar{\bar{n}} \tag{6.27}
$$

$$
\begin{aligned}
\dot{\Omega} = -\Bigg\{ \frac{3}{2} \frac{J_2}{p^2} \bar{\bar{n}} \cos i \Bigg[1 &+ \frac{3}{2} \frac{J_2}{p^2} \Bigg\{ \frac{3}{2} + \frac{e^2}{6} - 2\sqrt{1-e^2} \\
&- \left(\frac{5}{3} - \frac{5}{24} e^2 - 3\sqrt{1-e^2}\right) \sin^2 i \Bigg\} \Bigg] \\
&+ \frac{35}{8} \frac{J_4}{p^4} n \left(1 + \frac{3}{2} e^2\right) \left(\frac{12 - 21 \sin^2 i}{14}\right) \cos i \Bigg\}
\end{aligned}
\tag{6.28}
$$

$$\dot{\omega} = \left\{ \frac{3}{2} \frac{J_2}{p^2} \bar{\bar{n}} \left(2 - \frac{5}{2} \sin^2 i\right) \left[1 + \frac{3}{2} \frac{J_2}{p^2} \left\{2 + \frac{e^2}{2} - 2\sqrt{1-e^2} - \left(\frac{43}{24} - \frac{e^2}{48} - 3\sqrt{1-e^2}\right) \sin^2 i\right\}\right] - \frac{45}{36} \frac{J_2}{p^4} e^2 \, n \cos^4 i - \frac{35}{8} \frac{J_4}{p^4} n \left[\frac{12}{7} - \frac{93}{14} \sin^2 i + \frac{21}{4} \sin^4 i + e^2 \left\{\frac{27}{14} - \frac{189}{28} \sin^2 i + \frac{81}{16} \sin^4 i\right\}\right]\right\} \tag{6.29}$$

Note that the sign of the expression for $d\Omega/dt$ (see Equation (6.21)) indicates why orbits must retrograde to achieve a sun synchronous configuration (eastward precession of ascending node). Since $d\Omega/dt$ must be positive and the expression is of the form −[positive constant] $\cdot \cos i$, then the cosine of i must be negative. This requires $i > 90$.

It is worth comparing the first derivative terms ($\dot{M}$, $\dot{\Omega}$, $\mathring{\omega}$) for the first and second order expansions for both short period polar orbiting satellites and longer period geosynchronous satellites. Using typical orbital data we can generate Table 6.2 from the computer routine given in Appendix B.

Table 6.2: Comparison of First Derivative Terms for First and Second Order Expansions (deg/day)

	First Order		Second Order	
	Polar	Geosynchronous	Polar	Geosynchronous
n	4985.237053	357.564532	4985.237053	357.564532
$\mathring{M}$	4982.408922	357.577648	4982.410662	357.577648
$\mathring{\Omega}$	.990040	-.013117	.993605	-.013115
$\mathring{\omega}$	-2.666695	.026234	-2.664593	.026237

6.3 Longitudinal Drift of a Geosynchronous Satellite

We can now show that a geosynchronous satellite has a Ω term, even if the inclination and eccentricity are zero. Setting $i = 0$ and using a first order expansion:

$$\dot{\Omega} = \frac{d\Omega}{dt} = \left(-\frac{3}{2}\frac{J_2}{p^2}\right) n \left[1 + \frac{3}{2} J_2 \frac{\sqrt{1-e^2}}{p^2}\right] \tag{6.30}$$

Now since $n = K/a^{3/2}$ and $p = a(1 - e^2)$, and if we set $e=0$, and letting:

$$J_2 = 1082.28 \cdot 10^{-6}$$

$$K = 0.07436574 \text{ e.r.}^{3/2}/\text{min}$$

$$a = 6.6229 \text{ e.r.}$$

then:

$$\frac{d\Omega}{dt} = -\left(\frac{3}{2}\frac{J_2}{a^2}\right)\frac{K}{a^{3/2}}\left[1 + \frac{3}{2} J_2 \frac{1}{a^2}\right] \tag{6.31}$$

$$= -0.01332^{\circ}\ \text{day}^{-1} \text{ westward drift}$$

This gives rise to the so-called figure 8 orbit track of a geosynchronous satellite as shown in Figure 6.3.

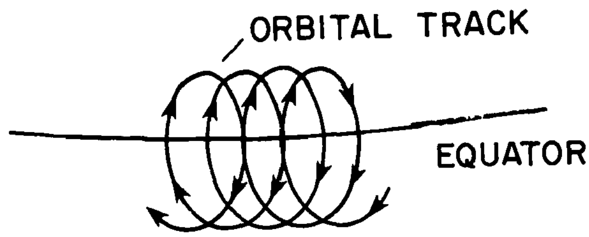


Figure 6.3 Figure 8 orbital track of a geosynchronous satellite

6.4 Calculations Required for a Perturbed Orbit

To calculate an orbital position vector, now that M, Ω, ω are no longer constant requires 2 more steps than the analysis given in Chapter 5. Recalling that prior to orbit calculations we determined the time of perifocal passage (T):

$$T = t_o - M_o/n \tag{6.32}$$

we must now update Ω and ω to time T since they are no longer constant parameters; we shall call these new initial terms ω_T and Ω_T:

$$\begin{aligned} \omega_T &= \omega_o + \mathring{\omega}(T - t_o) \\ \Omega_T &= \Omega_o + \mathring{\Omega}(T - t_o) \end{aligned} \tag{6.33}$$

Finally, instead of considering the transformation matrix B (see Equation 5.59) as constant, we must calculate ω and Ω at the specified time t:

$$\begin{aligned} \omega &= \omega_T + \mathring{\omega}(t - T) \\ \Omega &= \Omega_T + \mathring{\Omega}(t - T) \end{aligned} \tag{6.34}$$

and then use these values to calculate the direction cosines for the transformation matrix B:

$$B = \begin{bmatrix} P_x(t), & P_y(t), & P_z(t) \\ Q_x(t), & Q_y(t), & Q_z(t) \\ W_x(t), & W_y(t), & W_z(t) \end{bmatrix} \tag{6.35}$$

where:

$$\begin{aligned} P_x(t) &= \cos\omega \cdot \cos\Omega - \sin\omega \cdot \sin\Omega \circ \cos i \\ P_y(t) &= \cos\omega \circ \sin\Omega + \sin\omega \cdot \cos\Omega \cdot \cos i \\ P_z(t) &= \sin\omega \circ \sin i \\ Q_x(t) &= -\sin\omega \circ \cos\Omega - \cos\omega \cdot \sin\Omega \circ \cos i \\ Q_y(t) &= -\sin\omega \circ \sin\Omega + \cos\omega \circ \cos\Omega \cdot \cos i \\ Q_z(t) &= \cos\omega \circ \sin i \\ W_x(t) &= \sin\Omega \circ \sin i \\ W_y(t) &= -\cos\Omega \cdot \sin i \\ W_z(t) &= \cos i \end{aligned} \tag{6.36}$$

This requirement slightly alters the run-time on a computer as shown in Table 6.3.

Table 6.3: Difference in Computational Time Between Non-Perturbed and Perturbed Orbit Calculations (times are given in relative units (RU) for a CDC-7600: 1 RU ≡ .25 milliseconds of CPU time)

No. of Vector Calculations	Non-Perturbed	Perturbed
1	1.00	1.08
10	9.20	10.00
50	44.00	50.00
100	88.00	100.00

6.5 Equator Crossing Period

There is another satellite period to be considered assuming varying orbital elements. This is the so-called synodic, nodal, or equator crossing period, which is very useful to operational tracking stations. The equator crossing period is most easily defined if we first let:

$$\begin{aligned} \nu^{+} &= 360 - \omega_T \\ \nu^{-} &= 180 - \omega_T \end{aligned} \tag{6.37}$$

and use the relationships between E and ν:

$$\begin{aligned} \cos E &= \frac{\cos\nu + e}{1 + e\cos\nu} \\ \sin E &= \frac{1-e^2 \sin\nu}{1 + e\cos\nu} \end{aligned} \tag{6.38}$$

yielding two solutions E^{+} and E^{-}. By defining ν^{+} and ν^{-} according to Equation (6.37) we have placed the satellite at its equatorial crossing nodes. We can now solve for M^{+} and M^{-}:

$$M^{+,-} = E^{+,-} - e\sin E^{+,-} \tag{6.39}$$

and since $M = \bar{n}(t - T)$, we can solve for the times of equator crossings:

$$
\begin{aligned}
t^{+}_{eqcs} &= \frac{M^{+}}{\bar{n}} + T \\
t^{-}_{eqcs} &= \frac{M^{-}}{\bar{n}} + T
\end{aligned}
\tag{6.40}
$$

where + indicates a northward excursion and - indicates a southward excursion. Finally, the equator crossing period ($\tilde{P}$) is given by:

$$
\tilde{P} = 2 \cdot \left| t^{+}_{eqcs} - t^{-}_{eqcs} \right| \tag{6.41}
$$

The difficulty with the above approach is that over a half period, ω is varying, so that application of Equation (6.37) is only approximate. A rather simple solution to this problem is a numerical iterative approach in which two adjacent equator crossing nodes are found to a specified degree of accuracy. Appendix C provides a listing of a routine which will isolate a pair of equator crossings for a perturbed orbit. By applying the computer codes given in Appendices B and C, Table 6.4 is generated. This table compares the differences between the mean period, anomalistic period, and synodic period for both operational polar orbiter and geosynchronous satellites. Typical orbit data have been used in the calculations.

Table 6.4: Comparison of Three Satellite Periods (minutes)

	Polar	Geosynchronous
Mean	103.987	1440.108
Anomalistic (first order)	104.046	1440.055
Synodic	104.102	1339.935

Finally, to illustrate the application of a perturbed model, Figures 6.4 and 6.5 are provided. These figures portray typical orbital paths of both a geosynchronous satellite (GOES-3) and a polar orbiting satellite (TIROS-N).

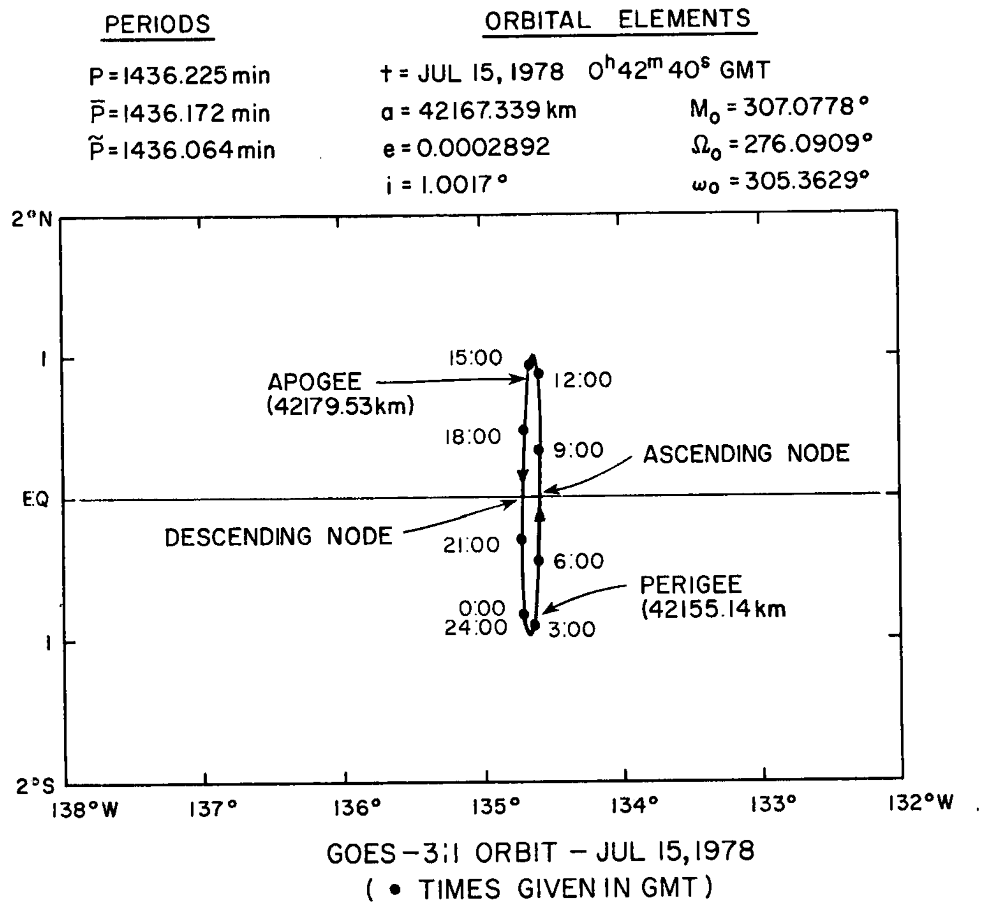


Figure 6.4 Typical orbital path of a geosynchronous satellite (GOES-3)

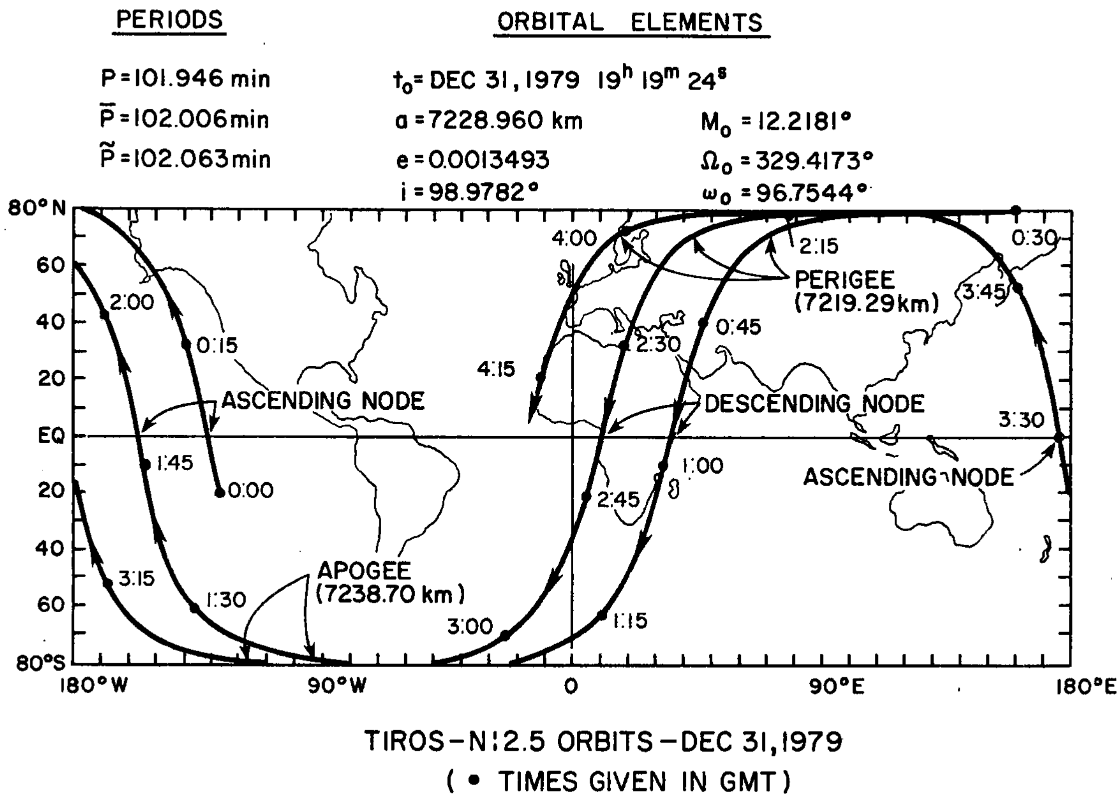


Figure 6.5 Typical orbital path of a polar orbiting satellite (TIROS-N)

6.6 Required Inclination for a Sun-Synchronous Orbit

Another problem which we can address, is the determination of the required inclination angle for sun synchronous orbits for a given orbital period (P). This is simply a matter of requiring $\dot{\Omega}$ to be 360 degrees per mean solar year. Now since:

$$\frac{d\Omega}{dt} = -\left(\frac{3}{2}\frac{J_2}{p^2}\cos i\right)\bar{n} \tag{6.42}$$

$$\bar{n} = n\left[1 + \frac{3}{2}J_2\frac{\sqrt{1-e^2}}{p^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right] = 2\pi/\bar{P} \tag{6.43}$$

$$p = a(1 - e^2) \tag{6.44}$$

$$n = \frac{\sqrt{\mu}}{a^{3/2}}K = 2\pi/P \tag{6.45}$$

We simply require that i satisfies:

$$\frac{360 \text{ degrees}}{365.24219879 \text{ days}} = -\frac{3}{2}\left(\frac{J_2}{p^2}\cos i\right) n\left[1 + \frac{3}{2}J_2\frac{\sqrt{1-e^2}}{p^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right]$$

or: (6.46)

$$.985647336 \text{ deg}\cdot\text{day}^1 = -\frac{3}{2}\left(\frac{1082.28\cdot 10^{-6}}{p^2}\cos i\right) n \left[1 + \frac{3}{2}1082.28\cdot 10^{-6}\frac{\sqrt{1-e^2}}{p^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right]$$

Note that a is assumed to be in cannonical units:

$$a(\text{e.r.}) = a(\text{km})/R_e(\text{km})$$

This equation is easily solved numerically. Since the right hand side of equation (6.46) is monotonically increasing as i goes from 90° to 180°, we can use a Newton's method approach in the interval ($90^\circ \leq i \leq 180^\circ$) to isolate, to a specified tolerance, a solution matching the left hand side. By applying this procedure, Table 6.5 has been generated which gives the required satellite height and inclination for a sun synchronous orbit, given the satellite period. A circular orbit (e=0) is assumed. A listing of a computer routine is given in Appendix G.

Table 6.5: Required Orbital Inclination for a Sun Synchronous Satellite Given a Satellite Period (e=0)

Period (minutes)	Height (km)	Inclination (Deg)
90	274.36	96.5893
100	758.44	98.4366
110	1226.62	100.5585
120	1680.80	102.9718

6.7 Velocity of a Satellite in a Secularly Perturbed Elliptic Orbit

A final problem we might want to solve is the determination of the velocity V of a satellite in an elliptic orbit at time t. Since we know:

$$x_\omega = a(\cos E - e)$$
$$y_\omega = a\sqrt{1 - e^2}\,\sin E \qquad (6.47)$$

and thus:

$$\dot{x}_\omega = -a\,\dot{E}\,\sin E$$
$$\dot{y}_\omega = a\,\dot{E}\sqrt{1-e^2}\,\cos E \tag{6.48}$$

and since V is simply:

$$V = \sqrt{\dot{x}_w^{\,2} + \dot{y}_w^{\,2}} \tag{6.49}$$

then:

$$V = a\,\dot{E}\sqrt{\sin^2 E + (1-e^2)\cdot\cos^2 E} \tag{6.50}$$

Note immediately that for a circular orbit where e=0:

$$V = a\,\dot{E}\sqrt{\sin^2 E + \cos^2 E} \tag{6.51}$$

$$= a\,\dot{E}$$

and since if e=0 then E=M, thus:

$$V = a\dot{M} \tag{6.52}$$

Now since:

$$\dot{M} = n\left[1 + \frac{3}{2}J_2\,\frac{\sqrt{1-e^2}}{p^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right] \tag{6.53}$$

and if we ignore the perturbation term then $\dot{M} = n$, and we have a velocity expression for a circular, non-perturbed orbit:

$$V = an \tag{6.54}$$

Now note that since:

$$n = \sqrt{\mu}\,K/a^{3/2} \tag{6.55}$$

then:

$$V = \frac{a\sqrt{\mu}\,K}{a^{3/2}} = K\sqrt{\frac{\mu}{a}} \tag{6.56}$$

which is similar to Equation 5.30, an expression that is independent of time.

In the case e = 0, we consider the perturbative effects:

$$V = a\,\dot{E}\sqrt{\sin^2 E + (1 - e^2)\cdot\cos^2 E} \tag{6.57}$$

using:

$$\begin{aligned} E &= M + e\cdot\sin M \\ \dot{E} &= \dot{M}(1 + e\cdot\cos M) \end{aligned} \tag{6.58}$$

where:

$$\dot{M} = \bar{n} = n\left[1 + \frac{3}{2}J_2\,\frac{\sqrt{1-e^2}}{p^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right] \tag{6.59}$$

and:

$$\begin{aligned} M &= n(t - T) \\ n &= \sqrt{\mu}\,K/a^{3/2} \\ p &= a(1 - e^2) \\ J_2 &= 1082.28 \cdot 10^{-6} \end{aligned} \tag{6.60}$$

Thus we have solved for V as a function of time, knowing only the orbital elements.

7.0 THE ORBITAL REVISIT PROBLEM

7.1 Sun Synchronous Orbits

Does a satellite pass over the same point on each orbit if it is sun synchronous? It would, only if the equator crossing separation is an integer factor of 360°. For example:

1. Assume a 60 minute period. After 1 orbit period, the earth would rotate 15° underneath the satellite. This would continue 24 times until the satellite was back to exactly the same point that it started.

2. Assume a 120 minute period. In this instance, there would be a 30° equator crossing separation. Therefore since 360/30 = 12 is an exact integer, the satellite would return to the same point.

Tables 7.1 and 7.2 are useful.

Table 7.1: Orbit Crossing Separations up to 90°

Period	Longitudinal Separation			Integer Number of Orbits
20 min	x 15°/60 min =	5°	which divides 360°	72 times
40 min	"	10°	"	36 times
60 min	"	15°	"	24 times
80 min	"	20°	"	18 times
120 min	"	30°	"	12 times
160 min	"	40°	"	9 times
180 min	"	45°	"	8 times
240 min	"	60°	"	6 times
360 min	"	90°	"	4 times

Table 7.2: Complete Table for Orbit Crossing Separations with 1 to 6 Hour Periods.

Period		Longitudinal Separation		Integer Number of Orbits
60.0 min	x 15^{o}/60 min =	15.0^{o}	which divides 360^{o}	24 times
62.60870 min	"	15.65217^{o}	"	23 times
65.45455 min	"	16.34364^{o}	"	22 times
68.57143 min	"	17.14286^{o}	"	21 times
72.0 min	"	18.0^{o}	"	20 times
75.78947 min	"	18.94737^{o}	"	19 times
80.0 min	"	20.0^{o}	"	18 times
84.70588 min	"	21.17647^{o}	"	17 times
90.0 min	"	22.5^{o}	"	16 times
96.0 min	"	24.0^{o}	"	15 times
102.85714 min	"	25.71429^{o}	"	14 times
110.76923 min	"	27.69231^{o}	"	13 times
120.0 min	"	30.0^{o}	"	12 times
130.90909 min	"	32.72727^{o}	"	11 times
144.0 min	"	36.0^{o}	"	10 times
160.0 min	"	40.0^{o}	"	9 times
180.0 min	"	45.0^{o}	"	8 times
205.71429 min	"	51.42857^{o}	"	7 times
240.0 min	"	60.0^{o}	"	6 times
288.0 min	"	72.0^{o}	"	5 times
360.0 min	"	90.0^{o}	"	4 times

3. Now consider a period which results in a longitudinal separation which does not divide 360° an integer number of times, such as 100 minutes. Then 100 x 0.25 = 25 degree longitudinal crossing, which divides 360° exactly 14.4 times. If we let the first crossing occur at 0° longitude (Greenwich Meridian), Table 7.3 gives the equatorial crossing sequence.

Table 7.3: Equator Crossings for a Non-Integer Separation Factor

	Orbit Number	Equatorial Crossing Longitude
	0	0°
	1	25°W
	2	50°W
CYCLE 1	3	75°W
	⋮	⋮
	13	325°W (35°E)
	14	350°W (10°E)
	15	15°W
	16	40°W
CYCLE 2	⋮	⋮
	27	315°W (45°E)
	28	340°W (20°E)
	29	5°W
CYCLE 3	⋮	⋮
	42	330°W (30°E)
	43	355°W (5°E)
	44	20°W
CYCLE 4	⋮	⋮
	57	345°W (15°E)
	58	10°W
CYCLE 5	⋮	⋮
	72	0°

Note that it takes 5 complete orbital cycles or 72 orbital periods until the pattern repeats. It is easy to see why this gets more complicated if the period is something like 101.358 minutes. Basically, to determine how many cycles are required to repeat the sequence, the smallest integer (I) must be found such that:

$$I \times P(\text{period}) = \text{another integer}$$

Thus, in order to find I:

1. Calculate the orbits per cycle (N):

   $N = 360/(0.25P)$ where the period(P) is in minutes.

2. Now N is given by:

   $N = n_1n_2n_3n_4\ldots$

   Take the decimal portion and divide it by a power of 10 corresponding to the number of places in the decimal portion at a preferred decimal accuracy.

3. Simplify that fraction to its least common denominator (LCD).

4. The LCD is the smallest integer I. Example:

   Assume an orbit of 110 minutes. How many cycles and orbits must pass before the orbit pattern repeats itself?

   $N = 360/(0.25 \cdot 110) = 13.09090909\ldots$

   Let us make our calculation accurate to 4 decimal places, thus:

   $N = 13.0909$

   Take the decimal portion 0909 and divide it by 10,000, yielding 909/10,000. Since any power of ten (10^9) can be given as the multiples of its prime factors, i.e., $10^9 = 5^9 \cdot 2^9$, then the numerator 909 would have to be divisible by 5 or 2 to have a lower least common denominator. Thus, in this case, 10,000 is the LCD because 909 is not divisible by 2 or 5. Therefore, it would take 10,000 cycles or 130,909 orbits for the orbit pattern to repeat itself to within 4 decimal place accuracy.

Also note that even though the orbit pattern of a sun-synchronous satellite does not repeat every cycle, this does not make it any less sun-synchronous. It simply pseudo-randomizes the equator crossings. Actually, there is a predictable phase pattern to the equator crossing changes although it can be considered as a randomizing process.

7.2 Multiple Satellite System: Mixed Sun-Synchronous and Non-Sun-Synchronous Orbits

In order to achieve uniform spatial and temporal sampling, future satellite systems will include various sun-synchronous and non-sun-synchronous satellites. The basic problem is to design an orbit configuration which will yield an optimal revisit frequency over all parts of the globe. Since the topic of diurnal variability has become such an important consideration in radiation budget studies, future satellite systems cannot afford to provide only twice a day coverage of the globe. The most successful technique which has been used to design the orbit architecture for a multiple satellite system is the computer simulation of multiple satellite orbits. By "flying satellites" in a computer, the revisit frequencies for a global spatial grid can be computed for a variety of orbital parameters. Campbell and Vonder Haar (1978) used this approach for the specification of the optimal orbit inclination for a system of polar-orbiting satellites designed to measure the earth's radiation budget. Circular orbits were used in their analysis.

It should be recognized that when considering polar orbiting satellites, an analysis of the revisit problem must include not only the orbital period but also the scanning pattern of the satellite instrument. As the satellite height increases, the period increases and thus the longitudinal separation of equator crossings increases. A fixed nadir viewing instrument would miss global strips (swaths) to the east and west of the orbital track as the satellite height is increased. If a satellite instrument is designed to scan across the orbital track, the longitudinal separation can be increased up to the point at which the atmospheric path length would have to be considered.

Essentially, the solution of the orbital revisit problem should be an attempt to sample the three dimensional volume: latitude, longitude and local time. Polar orbiters with inclinations near 90° would sample all latitudes and longitudes in a time period of approximately one month. However, only a very narrow local time interval would be sampled because of the slow precession rates. Satellites with lower inclination orbits such as 30°, would precess rapidly (about 5° per day) for an 800 Km altitude orbit, sampling 12 hours in a month. Computer simulations indicate that a set of satellites at 80° and 50°, and 80°, 60°, and 50° inclinations would provide nearly optimum sampling for two and three low orbit satellite systems, respectively (see Campbell and Vonder Haar, 1978). The geosynchronous satellites are examples of satellite platforms which provide fixed spatial and angular sampling but can provide high temporal sampling.

Another factor which must be included in the analysis is the quantity which is being measured. For observations of emitted flux, observations at any time of day generally provide good results. However, when considering albedo measurements, observations at night are useless and observations near sunrise or sunset (local times 600 and 1800) are very difficult to analyze because of the high solar grazing angles. Any variation of the observed field must also be considered in the orbital design. For radiation budget purposes, a set of 80^o, 50^o and sun-synchronous satellites is better than an 80^o-60^o-50^o set. The sun-synchronous orbit should be located at some local time between 900 and 1500 so as to provide uniform quality albedo estimates. The drifting orbiters are able to measure the diurnal variations. There are, of course, additional requirements for which orbits at other times of the day might be more useful. For example, in order to observe the earth's surface, an orbit at 8:00 am local time might be best since there are generally fewer clouds to obscure the ground.

8.0 CONCLUSIONS

This investigation has been directed toward the study of the orbit properties of near earth meteorological satellites, and in particular, the application of the results to the satellite navigation problem. Beginning with some basic definitions of time and coordinate systems, the basic foundation for the solution of the two body Keplerian orbit was outlined. This solution was adapted to the conventional orbital element parameters available from the meteorological satellite agencies so as to develop computer models for calculating orbital position vectors as a function of time. This is a fundamental requirement for any analytic satellite navigation model.

The invariant two body solution was then extended to a perturbed solution in which the time variant nature of an orbit was considered. Using a formulation called the perturbation function, derived from a harmonic expansion of the earth's gravitational potential, a set of closed form time derivatives of particular orbital elements were examined. From these definitions, it was possible to examine various orbital characteristics of near earth satellites.

Next, a discussion of the orbit revisit problem was provided as a means to highlight the significance of exact computer solutions to the orbital properties of meteorological satellites. Finally, a set of computer codes for calculating orbital position vectors and various orbital period quantities is provided in the appendices. The input to these routines is based on the "Classical Orbital Elements" available from the operational satellite agencies. A brief description of the source of these elements is provided in Appendix A.

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APPENDIX A

EXAMPLES OF NESS, NASA, ESA, AND NASDA ORBITAL ELEMENT TRANSMISSIONS

APPENDIX A

EXAMPLES OF NESS, NASA, ESA, AND NASDA ORBITAL ELEMENT TRANSMISSIONS

Classical Orbital Elements for meteorological satellites are, in general, provided by the operational satellite agencies, i.e., NESS, NASA, ESA, and NASDA. Although actual satellite tracking data may be provided by other agencies such as the North American Air Defense Command (NORAD), the reduction of this data to the conventional elements is under the management of the operational space agencies. Before providing examples of orbital element transmissions for various satellites from these agencies, a brief explanation of the format is required. As discussed in Chapters 5 and 6, the standard elements include:

1. Epoch Time (t_o)
2. Semi-major Axis (a)
3. Eccentricity (e)
4. Inclination (i)
5. Mean Anomaly or True Anomaly (M_o or ν_o)
6. Right Ascension of Ascending Node (Ω_o)
7. Argument of Perigee (ω_o)

In the discussion of Chapters 5 and 6, these elements were referred to as "Classical Orbital Elements" although in actuality, the space agencies refer to the above set of elements by other names. The three basic categories of orbital elements that appear on standard orbital transmission documents are as follows:

1. Keplerian Elements
2. Osculating Elements
3. Brouwer Mean Elements

There are no differences in the definitions of the classical elements insofar as the above categories are concerned, however, there are differences in the time varying properties of orbital elements with respect to the three categories. Referring to the Orbital Elements as Keplerian, implies that pure unperturbed two body motion is under consideration. Referring to the Classical Elements as Brouwer Mean Elements implies that time derivatives are involved with respect to various elements and that the elements themselves are based on Brouwer theory (see Brouwer and Clemence, 1961) or Brouwer-Lyddane theory (see Cappellari et al., 1976). Keplerian or Brouwer Mean elements are the standard products of the operational space agencies. The model developed in Chapters 5 and 6 incorporates the basic physics considered in Brouwer or Brouwer-Lyddane theory but uses a different formulation, see Kozai (1959) or EB (1965).

Referring to a set of elements as Osculating Elements can lead to some confusion. We say, in general, that an orbit osculates (kisses) an instantaneous position and velocity vector. In this sense, various sets of elements compatible with the various orders of perturbation theory could propogate an orbit which kisses or osculates a pre-defined position-velocity constraint which is known to define an orbit. When the space agencies label a set of orbital elements as osculating, they are indicating that the elements used in a Keplerian theory will osculate a position-velocity constraint which could have been based on two-body theory or perhaps a perturbation theory applied to raw tracking data used to generate the ephemeris constraint. Therefore osculating elements can be considered as Keplerian elements, although the elements themselves may represent a fit to ephemeris data based on any number of perturbation models.

The above points may seem academic in terms of reducing tracking station data to a set of orbital elements, however, the distinction is very important. It is instructive to discuss this statement by example. We will consider the approaches used by NESS and NASA in their generation of orbital elements for TIROS-N, GOES, and Nimbus-7 satellites. TIROS-N, which is a NESS operated polar orbiting satellite, is radar tracked by NORAD. In addition, NORAD reduces approximately a week of tracking data to a set of orbital elements which are compatible with the NORAD perturbation model (the model itself is classified). Perturbation factors included in this model include zonal and meridional asymmetries in the earth's gravitational potential, lunar forces, atmospheric drag, and solar radiation pressure. The retrieved orbit elements are then used to propogate approximately 3 weeks of ephemeris data which are transmitted to NESS, who in turn, retrieves either Keplerian Elements or Brouwer Mean Elements based on unperturbed two body theory or Brouwer-Lyddane theory. The orbit retrieval package is based on sub-systems of the NASA Goddard Trajectory Determination System (GTDS) which is a large computer package designed for a vast array of NASA orbital problems, and is developed and maintained by the NASA Goddard Space Flight Center. Therefore, NESS can provide either unperturbed or perturbed model elements, but it must be recognized that these elements represent fits to model produced ephemeris data, not raw tracking data (see Ellickson, 1980).

The retrieval of GOES-East and GOES-West orbital elements takes place at both NESS and NASA. The NESS produced elements are based on approximately one week of tri-lateration (3 station) ranging data generated by the 5 NOAA operated tracking stations (Wallops Island, VA;

Seattle, WA; Honolulu, HI; Santiago, Chile; Ascension Island). The type of model used to fit the ranging data is based on unperturbed two-body motion, so by definition, the NESS produced orbital elements for GOES are Keplerian. NASA, on the other hand, bases its orbit retrievals on range and range-rate data available from its own global network of tracking stations. Unlike NESS, NASA uses the GTDS perturbation model to retrieve orbital elements which are then used to propogate an ephemeris stream. These model data are finally fit by a Keplerian model to produce a set of elements which osculate a position-velocity vector pair which best characterizes a two body orbit. NASA then transmits these elements under the heading of Osculating Elements, although it is understood that they are Keplerian Elements. NASA uses a very similar procedure for producing Nimbus-7 orbital elements, however, the elements derived from the model ephemeris stream are Brouwer Mean Elements based on Brouwer-Lyddane theory.

The following ten cases are examples of various orbital transmission documents from four operational space agencies (NASA, NESS, ESA, JMS) for the following seven different satellites:

1. GOES-2 (Eastern Geosynchronous)
2. GOES-3 (Western Geosynchronous)
3. GOES-1 (Indian Ocean Geosynchronous, also called GOES-A)
4. METEOSAT (European Geosynchronous)
5. GMS (Japanese Geosynchronous)
6. TIROS-N (NESS Polar Orbiter)
7. NIMBUS-G (NASA Polar Orbiter)

CASE 1: GOES-2: NASA Transmission

TWX022...8 710-828-9716

DE GWWW 040
02/0818Z
FM MISSION AND DATA OPERATIONS NASA GSFC GREENBELT MD
TO GSRM/NAVSPASUR DAHLGREN VA
GSRM/NORAD COC CHEYENNE MTN COMPLEX CO/DOFSO ATTN CHIEF ANALYST
GSRM/WILHELM F STERNWARTE BERLIN W GERMANY ATTN ZIMMER
LSRM/RAE FARNBOROUGH ENGLAND ATTN KING-HELE SPACE DEPT
GSTS/JOE JOHNS CODE 933/WILLINGHAM CODE 572/PETRUZZO CODE 581
GSTS/MARSH CODE 490
CPOB/PROKOPCHAK
GSTS/UNIV OF WISCONSIN COLLECT TWX 910-286-2771
GTOS/SOCC/MCINTOSH/SHARTS
GSRM/RUWOMSA/MILLSTONE HILL WESTFORD MA ATTN SRIDHARAN
GSRM/RUWJHTA/WHITE SANDS MISSILE RANGE NM ATTN MEYERS
GSRM/AFCRL HANSCOM AFB BEDFORD MA ATTN SUYA/HUSSEY
GSRM/RUWTGPA/SEL BOULDER CO ATTN SCHOEDER/NBS BOULDER CO ATTN W HANSON
GSTS/COMPUT

THE FOLLOWING ARE THE OSCULATING ORBITAL ELEMENTS
FOR SATELLITE 1977 48A GOES-2
COMPUTED AND ISSUED BY THE GODDARD SPACE FLIGHT CENTER.
EPOCH 79 Y 02 M 23 D 00 H 00 M 0.000 S UT.

SEMI-MAJOR AXIS	42432.7798	KILOMETERS
ECCENTRICITY	.006227	
INCLINATION	0.0271	DEGREES
MEAN ANOMALY	309.9886	DEGREES
ARGUMENT OF PERIGEE	331.4553	DEGREES
MOTION , PLUS	0.0262	DEG. PER DAY
R.A. OF ASCEND. NODE	148.3225	DEGREES
MOTION MINUS	0.0131	DEG. PER DAY
ANOMALISTIC PERIOD	1449.81255	MINUTES
HEIGHT OF PERIGEE	35790.43	KILOMETERS
HEIGHT OF APOGEE	36318.85	KILOMETERS
VELOCITY AT PERIGEE	11103.	KM. PER HR
VELOCITY AT APOGEE	10965.	KM. PER HR.
GEOC. LAT. OF PERIGEE MINUS	0.013	DEGREES
INERTIAL COORDINATES REFERENCE TRUE OF DATE		
X	14996.5485	KILOMETERS
Y	39513.8631	KILOMETERS
Z	-19.6313	KILOMETERS
X DOT	-2.8821	KM. PER SEC.
Y DOT	1.0761	KM. PER SEC.
Z DOT	0.0003	KM. PER SEC.

02/0819Z MAR GWWW

Case 2: GOES-2: NESS Transmission

TWX019...710-828-9716

RE GTOS 007
01/1630Z
FM SOCC/MCINTOSH
TO GMOC
GPHY
GPOB/M PROKOPCHAK
GSRM/RNWRMOA/NORAD COC CHEYENNE MTN COMPLEX CO/DOFO CHIEF ANALYST
GSRM/RUWTGPA/J SCHROEDER, SEL BOULDER/W HANSON, NBS
GSRM/RUWOMSA/MILLSTONE HILL WESTFORD MA ATTN SRIDHARAN
GSRM/RUEOFFA/AFGL HANSCOM AFB MA LYS/B MEYERS, SUA/ROBINSON
GSTS/B RICHARDSON WILLINGHAM CODE 572
GSTS/R MARSH CODE 490
GSTS/PHIL PEASE CODE 933
GSTS/UNIV OF WISC SPACE SCI AND ENGRNG CENTER TWX 910-286-2771
GSTS/COLO ST UNIV DEPT OF ATMOSPHERIC SCIENCE TWX 910-930-9008
CSU LIBRARIES

/SUS DUPE/

PREDICTED POST MANEUVER

ORBITAL ELEMENTS FOR GOES-2

EPOCH 79Y 02M 28DAT 04H 28MIN 24SEC UT

SEMI-MAJOR AXIS (KM) 42164.189

ECCENTRICITY 0.000156

INCLINATION (DEG) 0.059

R. A. OF ASC. NODE (DEG) 144.047

ARGUMENT OF PERIGEE (DEG) 138.064

MEAN ANOMALY (DEG) 202.303

LONGITUDE (DEG WEST) 100.0

Case 3: GOES-3: NASA Transmission

TWX005...710-828-9716

DE GWWW 037
10/2056Z
FM MISSION AND DATA OPERATIONS NASA GSFC GREENBELT MD
TO GSRM/NAVSPASUR DAHLGREN VA
GSRM/NORAD COC CHEYENNE MTN COMPLEX CO/DOFSO ATTN CHIEF ANALYST
GSRM/WILHELM F STERNWARTE BERLIN W GERMANY ATTN ZIMMER
LSRM/RAE FARNBOROUGH ENGLAND ATTN KING-HELE SPACE DEPT
GSTS/R NARSH CODE 490/E WILLINGHAM CODE 572/B RICHARDSON CODE 572
GSTS/C. PETRUZZO CODE 581/J. JOHNS CODE 933
GSTS/UNIV OF WISCONSIN COLLECT TWX 910-286-2771
GPOE/M. PROKOPCHAK
GTOS/SOCC/MCINTOSH
GSRM/RUWOMSA/MILLSTONE HILL WESTFORD MA ATTN SRIDHARAN
GSRM/RUWJHTA/WHITE SANDS MISSILE RANGE NM ATTN MEYERS
GSRM/AFGL HANSCOMB AFB BEDFORD MASS ATTN,SUA/ROBINSON,LY/MYERS
GSRM/RUWGTPA/BOULDER CO ATTN SEL/SCHROEDER,NES/HANSON
GSRM/RUWJHTA/WHITE SANDS MISSLE RANGE/XPD ATTN CLAP 26ADS
GSTS/COMPUT

THE FOLLOWING ARE THE OSCULATING ORBITAL ELEMENTS
FOR SATELLITE 1978 62A GOES-3
COMPUTED AND ISSUED BY THE GODDARD SPACE FLIGHT CENTER.
EPOCH 78 Y 07 M 09 D 18 H 20 M 0.000 S UT.

SEMI-MAJOR AXIS		42237.1011	KILOMETERS
ECCENTRICITY		.001572	
INCLINATION		1.0121	DEGREES
MEAN ANOMALY		352.7205	DEGREES
ARGUMENT OF PERIGEE		162.6745	DEGREES
MOTION	PLUS	0.0267	DEG. PER DAY
R.A. OF ASCEND. NODE		275.9454	DEGREES
MOTION	MINUS	0.0133	DEG. PER DAY
ANOMALISTIC PERIOD		1439.78823	MINUTES
PERIOD DOT			MIN. PER DAY
HEIGHT OF PERIGEE		35792.55	KILOMETERS
HEIGHT OF APOGEE		35925.32	KILOMETERS
VELOCITY AT PERIGEE		11077.	KM. PER HR.
VELOCITY AT APOGEE		11042.	KM. PER HR.
GEOC. LAT. OF PERIGEE	PLUS	0.301	DEGREES

10/2056Z JUL GWWW

Case 4: GOES-3: NESS Transmission

ORBITAL ELEMENTS FOR GOES 3 ,SATID 7806201
EPOCH 78Y 07M 15D AT 00HR 42MIN 40SEC UT
SEMI-MAJOR AXIS- 42167.339 KM
ECCENTRICITY 0.0002892
INCLINATION 1.00173 DEG
R. A. OF ASC. NODE 276.0909 DEG
ARGUMENT OF PERIGEE 305.3629 DEG
MEAN ANOMALY 307.0778 DEG
LONGITUDE 134.6859 DEG W
ATTITUDE - SPIN VECTOR
   R. A. - 14.383 DEG
   DECLIN- -88.707 DEG
SPIN PERIOD/RATE- 0.60000 SEC / 100.0000 RPM

17/1817Z JUL 78 GTOS

Case 5: GOES-1: NASA Transmission

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TWX005...710-828-9716

DE GWWW 027'E
30/1611Z
FM MISSION AND DATA OPERATIONS NASA GSFC GREENBELT MD
TO GSRM/NAVSPASUR DAHLGREN VA
GSRM/NORAD COC CHEYENNE MTN COMPLEX CO/DOFSO ATTN CHIEF ANALYST
GSRM/WILHELM F STERNWARTE BERLIN W GERMANY ATTN ZIMMER
GSRM/AFGL HANSCOMB AFB BEDFORD MASS ATTN SUA/ROBINSON,LYS/B. MEYERS
GSRM/RUWTGPA/NBS BOULDER CO ATTN HANSON
LSRM/RAE FARNBOROUGH ENGLAND ATTN KING-HELE SPACE DEPT
GPOB/PROKOPCHAK
GSTS/BRYANT CODE 581/WIRTH CODE 490/WILLINGHAM CODE 572
GSTS/UNIV OF WISCONSIN COLLECT TWX NR. 910-286-2771
GTOS/SOCC/L RANNE
GSRM/RUWOMSA/MILLSTONE HILL WESTFORD MA ATTN SRIDHARAN
GSRM/RUWJHTA/B. MEYERS WHITE SANDS MISSILE RANGE, N. MEX
GSRM/RUWJHTA/OLAP 26ADS WHITE SANDS MISSILE RANGE NM/XPD
GSTS/JOE JOHNS CODE 933
LESR/PALLASCHICE, K. AUBECK
GSTS/COMPUT

THE FOLLOWING ARE THE OSCULATING ORBITAL ELEMENTS
FOR SATELLITE 1975 100A GOES-A    COMPUTED AN
COMPUTED AND ISSUED BY THE GODDARD SPACE FLIGHT CENTER.
EPOCH 78 Y 10 M 27 D  0 H  0 M  0.0   S UT.
SEMI-MAJOR AXIS              42113.5688  KILOMETERS
ECCENTRICITY                   .000820
INCLINATION                     0.1106  DEGREES
MEAN ANOMALY                   81.6045  DEGREES
ARG. OF PERIFOCUS              20.7101  DEGREES
     MOTION       PLUS          0.0269  DEG. PER DAY
R.A. OF ASCEND. NODE          274.7950  DEGREES
     MOTION       MINUS         0.0135  DEG. PER DAY
ANOMALISTIC PERIOD           1433.42979  MINUTES
PERIOD DOT                               MIN. PER DAY
HT. OF PERIFOCUS              35700.891  KILOMETERS
HT. OF APOFOCUS               35769.967  KILOMETERS
VEL. AT PERIFOCUS               11085.  KM. PER HR.
VEL. AT APOFOCUS                11066.  KM. PER HR.
GEOC. LAT OF PERIFOCUS PLUS      0.037  DEGREES

30/1611Z OCT GWWW
```

Case 6: GOES-1: ESA Transmission (During the FIRst GARP Global Experiment - FGGE)

FM LESP/OPB ATT, ESCC

TO GAQD
LPFN/G LAEMMEL, DFVLR OBERPFAFFENHOFEN
LPFN/NOC OPS, DFVLR
LPFN/ORB COMP, DFVLR
LPFN/G RATTEI, DFVLR OBERPFAFFENHOFEN
GTOS/F KAHWAJY, NOAA-NESS
GTOS/P EYCLESHEIMER, NOAA-NESS
GCFN/NOCC
DLD/ T O HAIG, UNIVERSITY OF WISCONSIN TLX 910-286-2771
DLD/ SITBON, LMD ECOLE POLYTECHNIQUE PARIS -TLX 691596
DLD/R LASBLEIZ, CMS LANNION -TLX 950256
DLD/G FERRAND, EOPO TOULOUSE -TLX 520862

INFO DLD/MM AUBECK, GARDNER, LAUE, MUENCH, PALLASCHKE, ROTH,
NETWORK, SCHEDULING, SPACON, ESOC
DLD/A LUKASIEWICZ, REDU
DLD/P ESTARIA, VILSPA -TLX 42555

ORBITAL PARAMETERS FOR GOES-A (7510001) RUN NUMBER 22

DERIVED ELEMENTS	HEIGHT OF PERIGEE (KM)	=	35769.563065
	HEIGHT OF APOGEE (KM)	=	35812.069978
	SEMI MAJOR AXIS (KM)	=	42168.960521
	ECCENTRICITY	=	.000504
	INCLINATION (DEG)	=	.171442
	ASCENDING NODE (DEG)	=	77.228633
	ARG. OF PERIGEE (DEG)	=	125.944991
	TRUE ANOMALY (DEG)	=	3.044481
STATE VECTOR	X - COMPONENT (KM)	=	-37811.384898
	Y - COMPONENT (KM)	=	-18620.453813
	Z - COMPONENT (KM)	=	98.024500
	X - COMPONENT (KM/SEC)	=	1.358878
	Y - COMPONENT (KM/SEC)	=	-2.759605
	Z - COMPONENT (KM/SEC)	=	-.005791

EPOCH (UT) 79 YR 2 MO 19 DA 0 HO 0 MI .000 SE
ORBIT NUMBER 217.3583

Case 7: METEOSAT: ESA Transmission

```
NR23    RR    ESOC    DARMSTADT    ALLEMAGNE    APR 18/1504Z

FM LESR/ORB ATT, ESOC

TO  GAQD
    LPFN/G LAEMMEL, DFVLR OBERPFAFFENHOFEN
    LPFN/NOC OPS, DFVLR
    LPFN/ORB COMP, DFVLR
    GSTS/G MARECHEK, CODE 572.3 GSFC
    GSTS/R SCLAFFORD, CODE 861.2 GSFC
    GCEN/NOCC
    GTOS/NOAA
    GOPS/OPERATIONS CENTRE BRANCH CODE 512 GSFC

INFO DLD/MM BERLIN, KUMMER, MUENCH, PALLASCHKE, ROBSON, ROTH,
        SOOP, WALES, NETWORK, SCHEDULING, ESOC
    DLD/A LUKASIEWICZ, REDU
    DLD/MR P SIBTON, LMD ECOLE POLYTECHNIQUE TLX 691596

ORBITAL PARAMETERS FOR METEOSAT          RUN NUMBER   33


DERIVED ELEMENTS     HEIGHT OF PERIGEE (KM)  =    35768.439692
                     HEIGHT OF APOGEE (KM)   =    35806.748998
                     SEMI MAJOR AXIS (KM)    = H  42165.738345
                     ECCENTRICITY            =         .000454
                     INCLINATION (DEG)       =         .191114
                     ASCENDING NODE (DEG)    =      189.854027
                     ARG. OF PERIGEE (DEG)   =      253.674435
                     TRUE ANOMALY (DEG)      =      120.281037

STATE VECTOR         X - COMPONENT (KM)      =  -38585.968653
                     Y - COMPONENT (KM)      =  -17026.022147
                     Z - COMPONENT (KM)      =      33.927094
                     X - COMPONENT (KM/SEC)  =       1.239819
                     Y - COMPONENT (KM/SEC)  =      -2.812761
                     Z - COMPONENT (KM/SEC)  =        .009952

EPOCH (UT)           78 YR   4 MO   17 DA   0 HO   0 MI    .000 SE
ORBIT NUMBER           145.0388

18/1505Z APR 78 LESR
```

Case 8: GMS: NASDA produced elements transcribed onto GMS data tapes and decoded by the McIDAS system at the University of Wisconsin's Space Science and Engineering Center.

```
***     N L S S   M C I D A S

                          1278337        3 DEC 1978
 ETIMY=  781203   ETIMH=         0  SEMIMA= 4216600   ECCEN=      106
ORBINC=      56   MEANA=  283585  PERHEL=   89874  ASNODE=  198114
```

Case 9: TIROS-N: NESS Transmission

TIROS-N NAVIGATION SYSTEM POLAR SPACECRAFT EPHEMERIS ACCESS ROUTINE INITIALIZATION REPORT AT JAN 02, 1980 VER 3.0 PAGE 1

EPOCH OF CURRENT CYCLE IS 79/12/31 19 19 23.664 SPACECRAFT ID IS ****** DEFAULT NUMBER OF INTERPOLATION POINTS IS 10

START TIME OF DATA GRADES				END TIME OF DATA GRADES				INTERVAL OF DATA	LENGTH OF DATA
GRADES	DATE	DAY	SECONDS	GRADES	DATE	DAY	SECONDS		
FINE	12/30/79	79.364	0.	FINE	1/11/80	80. 11	0.	100 SECONDS	12 CYCLES
MEDIUM	1/ 5/80	80. 5.	600.	MEDIUM	3/17/80	80. 77	0.	10 MINUTES	72 CYCLES
COURSE	3/10/80	80. 70	3600.	COARSE	6/14/80	80.166	0.	1 HOURS	96 CYCLES

ELEMENTS AT CURRENT CYCLE's EPOCH

KEPLERIAN		INERTIAL TOD		BROUWER MEAN	
SEMI-MAJOR AXIS	7221.8962554074	X	-2568.2800593576	SEMI-MAJOR AXIS	7228.9597759711
ECCENTRICITY	0.0012051329	Y	280.5696240752	ECCENTRICITY	0.0013492807
INCLINATION	98.9826322459	Z	6737.4203664218	INCLINATION	98.9782134269
RT ASC OF ASC NODE	329.4207821364	XDOT	-5.3608748958	RT ASC OF ASC NODE	329.4172856807
ARG OF PERIGEE	63.5514823988	YDOT	3.9020314858	ARG OF PERIGEE	96.7543541300
MEAN ANOMALY	45.3887663021	ZDOT	-2.3898005021	MEAN ANOMALY	12.2180973526

0002 PSCEAR - FOR INTERPOLATION PURPOSES 10 POINTS WILL BE USED INSTEAD OF THE INPUT VALUE 0

TIME. BROUWER ELEMENTS 800102. 0.

7228.96 0.001349 98.98 329.42 96.75 12.22

ORBITAL PERIOD IN SECONDS 6123.89

Case 10: NIMBUS-G: NASA Transmission

```
NASA GSFC
TWA016...710-828-9716
DE GWWW 025'E
09/01012
FM MISSION AND DATA OPERATIONS NASA GSFC GREENBELT MD
TO GSRM/NAVSPASUR DAHLGREN VA
LESR/CMS LANNION FRANCE ATTN R. LASBLEIZ TELEX 22301
GSRM/NORAD COC CHEYENNE MTN COMPLEX CO/DOFSO ATTN CHIEF ANALYST
GSRM/WILHELM F STERNWARTE BERLIN W GERMANY ATTN ZIMMER
LSRM/RAE FARNBOROUGH ENGLAND ATTN KING-HELE SPACE DEPT
GSTS/DR THOMAS VON DER HAAR DEPT ATMOSPHERIC SCIENCES
COLORADO ST UNIV CO TWX 910-930-9008
GSTS/D. WRIGHT CODE 912
GSTS/COMPUT

THE FOLLOWING ARE THE BROUWER MEAN ORBITAL ELEMENTS
FOR SATELLITE 1978  98A NIMBUS-G
COMPUTED AND ISSUED BY THE GODDARD SPACE FLIGHT CENTER.
EPOCH 78 Y 11 M 03 D 00 H 00 M  0.000 S UT.
SEMI-MAJOR AXIS                7325.1057  KILOMETERS
ECCENTRICITY                    .000843
INCLINATION                     99.2905  DEGREES
MEAN ANOMALY                   129.2702  DEGREES
ARGUMENT OF PERIGEE            229.0408  DEGREES
     MOTION       MINUS          2.6686  DEG. PER DAY
R.A. OF ASCEND. NODE           219.3325  DEGREES
     MOTION       PLUS           0.9908  DEG. PER DAY
ANOMALISTIC PERIOD             103.98734  MINUTES
HEIGHT OF PERIGEE               940.79  KILOMETERS
HEIGHT OF APOGEE                953.14  KILOMETERS
VELOCITY AT PERIGEE            26579.   KM. PER HR
VELOCITY AT APOGEE             26534.   KM. PER HR.
GEOC. LAT. OF PERIGEE  MINUS    48.183  DEGREES


INERTIAL COORDINATES  REFERENCE TRUE OF DATE

X                           -5648.0572  KILOMETERS
Y                           -4674.5658  KILOMETERS
Z                            -218.9159  KILOMETERS
X DOT                          -0.9239  KM. PER SEC.
Y DOT                           0.7799  KM. PER SEC.
Z DOT                           7.2715  KM. PER SEC.
```

APPENDIX B

COMPUTER SOLUTION FOR AN EARTH SATELLITE ORBIT (PERTURBED TWO BODY)

APPENDIX B

COMPUTER SOLUTION FOR AN EARTH SATELLITE ORBIT (PERTURBED TWO BODY)

```
      SUBROUTINE SATPOS(IYRDAY,SATTIM,ICOOR,XSAT,YSAT,ZSAT,SATLAT,SATLON
     *,SATHGT)
C
C     DETERMINE A SATELLITE POSITION VECTOR ACCORDING TO A KEPLERIAN ORBIT
C
C     ERIC A. SMITH
C     DEPARTMENT OF ATMOSPHERIC SCIENCE
C     COLORADO STATE UNIVERSITY/FOOTHILLS CAMPUS
C     FORT COLLINS, COLORADO 80523
C     TEL 303-491-8533
C
C     REFERENCES.
C
C     BOWDITCH,NATHANIEL,1962.
C     AMERICAN PRACTICAL NAVIGATOR - AN EPITOMIE OF NAVIGATION.
C     U.S. NAVY HYDROGRAPHIC OFFICE,H.O. PUB. NO. 9.
C     UNITED STATES GOVERNMENT PRINTING OFFICE,1524 PP.
C
C     ESCOBAL,PEDRO RAMON,1965.
C     METHODS OF ORBIT DETERMINATION.
C     JOHN WILEY AND SONS,INC.,NEW YORK/LONDON/SYDNEY,463 PP.
C
C     INPUT PARAMETERS
C
C     IYRDAY = YEAR ( YYDDD IN JULIAN DAY )
C     SATTIM = TIME ( HOURS IN GMT )
C     ICOOR = 0 FOR TERRESTRIAL COORDINATES
C           = 1 FOR CELESTIAL COORDINATES
C
C     OUTPUT PARAMETERS
C
C     XSAT = X COMPONENT OF SATELLITE POSITION VECTOR ( KM )
C     YSAT = Y COMPONENT OF SATELLITE POSITION VECTOR ( KM )
C     ZSAT = Z COMPONENT OF SATELLITE POSITION VECTOR ( KM )
C     SATLAT = SATELLITE LATITUDE ( DEGREES )
C     SATLON = SATELLITE LONGITUDE ( DEGREES )
C     SATHGT = SATELLITE HEIGHT ( KM )
C
C     LATITUDE IS GIVEN IN TERMS OF SPHERICAL COORDINATES
C     USE THE FOLLOWING TRANSFORMATION TO CONVERT TO GEOCENTRIC LATITUDE
C
C               S=RDPDG*SATLAT
C               SATLAT=ACOS(COS(S)/SQRT(1.0-(E*SIN(S))**2))/RDPDG
C
      REAL J2,J4,INC,MMC,MANOML
C
C     CONTROL KEYS AND BROUWER MEAN ORBITAL ELEMENTS
C
C     IOSAT = SATELLITE TYPE
C             SET POSITIVE FOR INITIALIZING NEW SATELLITE TYPE
C             SET NEGATIVE FOR RETAINING OLD SATELLITE TYPE WITH NEW ORBIT PARMS
C                IOSAT IS THEN SET POSITIVE
C     IMORT = 0 FOR ORBIT ANOMALY GIVEN AS MEAN ANOMALY ( E.G. NASA )
C           = 1 FOR ORBIT ANOMALY GIVEN AS TRUE ANOMALY ( E.G.  ESA )
C     IOSEC = 0 FOR   ZERO ORDER SECULAR PERTURBATION THEORY
C           = 1 FOR  FIRST ORDER SECULAR PERTURBATION THEORY
C           = 2 FOR SECOND ORDER SECULAR PERTURBATION THEORY
C     IEDATE = EPOCH DATE ( YYMMDD IN CALENDER FORM )
C            = DATE FOR WHICH FOLLOWING ORBITAL PARAMETERS ARE VALID
C     IETIME = EPOCH TIME ( HHMMSS IN GMT )
C            = TIME FOR WHICH FOLLOWING ORBITAL PARAMETERS ARE VALID
C     SEMIMA = SEMI-MAJOR AXIS ( KM )
C            = HALF THE DISTANCE BETWEEN TWO APSES OF APO-FOCUS AND PERI-FOCUS
C     OECCEN = ECCENTRICITY OF EARTH ORBIT ( UNITLESS )
C            = DEGREE OF ELLIPTICITY OF ORBIT
C     ORBINC = ORBIT INCLINATION ( DEGREES )
C            = ANGLE BETWEEN THE ORBIT AND EQUATORIAL PLANES
C     OANOML = ORBIT ANOMALY AT EPOCH TIME ( DEGREES )
C            = ANGLE IN ORBITAL PLANE BETWEEN PERI-FOCUS AND SATELLITE POSITION
C              GIVEN AS EITHER A MEAN ANOMALY OR A TRUE ANOMALY
```

```
C      PERIGE = ARGUMENT OF PERIGEE AT EPOCH TIME ( DEGREES )
C             = ANGLE IN ORBIT PLANE FROM ASCENDING NODE TO PERI-FOCUS
C      ASNODE = RIGHT ASCENSION OF ASCENDING NODE AT EPOCH TIME ( DEGREES )
C             = ANGLE IN EQUATORIAL PLANE BETWEEN VERNAL EQUINOX(PRINCIPLE AXIS)
C               AND NORTHWARD EQUATOR CROSSING
C      PERIOD = PERIOD ( MINUTES )
C             = STATEMENT OF KEPLERS THIRD LAW
C               THIS PARAMETER IS CALCULATED IN SATPOS
C      APEROD = ANOMALISTIC PERIOD ( MINUTES )
C             = TIME BETWEEN THE PASSAGE FROM ONE PERI-FOCUS TO THE NEXT
C               THIS PARAMETER IS CALCULATED IN SATPOS
C      EPEROD = NODAL PERIOD ( MINUTES )
C             = TIME BETWEEN THE PASSAGE FROM ONE EQUATOR CROSSING TO THE NEXT
C               THIS PARAMETER IS CALCULATED IN EQCROS
C
       COMMON/ORBCOM/IOSAT,INORT,IOSEC,IEDATE,IETIME,SEMIMA,OECCEN,ORBINC
      *,OANOML,PERIGE,ASNODE,PERIOD,APEROD,EPEROD
C
C      DEFINITIONS
C
C      MEAN ANOMALY(M)        . ANGLE IN ORBITAL PLANE WITH RESPECT TO THE CENTER
C                               OF A MEAN CIRCULAR ORBIT(HAVING A PERIOD EQUIVALENT
C                               TO THE ANOMALISTIC PERIOD)FROM PERI-FOCUS TO THE
C                               SATELLITE POSITION.
C      TRUE ANOMALY(M)        . ANGLE IN ORBITAL PLANE WITH RESPECT TO A FOCUS OF
C                               THE ELLIPTIC FROM PERI-FOCUS TO THE SATELLITE
C                               POSITION.
C      ECCENTRIC ANOMALY(E)   . ANGLE IN ORBITAL PLANE WITH RESPECT TO THE CENTER
C                               OF A CIRCLE CIRCUMSCRIBING THE ELLIPSE OF MOTION
C                               FROM PERIFOCUS TO THE SATELLITE POSITION.
C
C      ORBITAL CONSTANTS
C
C      PI = VALUE OF PI
C      SOLYR = NUMBER OF DAYS IN SOLAR YEAR ( DAYS )
C      SIDYR = NUMBER OF DAYS IN SIDEREAL YEAR ( DAYS )
C      RE = EQUATORIAL EARTH RADIUS ( KM )
C      GRACON = TERRESTRIAL GRAVITATIONAL CONSTANT ( KE=SQRT(G*ME*60**2/RE**3 )
C               WHERE KE = TERRESTRIAL GRAV CON ( 0.07436574 EM**.5*ER**1.5/MIN )
C                     G = UNIVERSAL GRAV CONSTANT ( 6.673E-8 DYNE*CM**2*GM**-2 )
C                     ME = MASS OF EARTH ( 5.9733726E27 GM PER EM )
C                     RE = RADIUS OF EARTH ( 6.378214E8 CM PER ER )
C      F = FLATTENING   OF THE EARTH ( F=(A-B)/A            , F=1-SQRT(1-E**2)   )
C      E = ECCENTRICITY OF THE EARTH ( E=SQRT(A**2-B**2)/A , E=SQRT(2*F-F**2)   )
C          WHERE F = FLATTENING OF EARTH ( 3.35289E-3 )
C                E = ECCENTRICITY OF EARTH ( 8.1820157E-2 )
C                A = SEMI-MAJOR EARTH AXIS - EQUATORIAL ( 6378.214 KM )
C                B = SEMI-MINOR EARTH AXIS - POLAR      ( 6356.829 KM )
C                  = A*(1-F)
C                C = MEAN EARTH RADIUS ( 6371.086 KM )
C                  = (2*A+B)/3
C      J2 = SECOND HARMONIC COEF OF EARTHS ASPHERICAL GRAVITATIONAL POTENTIAL
C      J4 = FOURTH HARMONIC COEF OF EARTHS ASPHERICAL GRAVITATIONAL POTENTIAL
C      IRFDAY =  YYDDD WHEN CELESTIAL COOR SYS COINCIDES WITH EARTH COOR SYS
C                 I.E. TRANSIT OF FIRST POINT OF AIRES WITH GREENWICH MERIDIAN
C      IRFHMS = HHMMSS WHEN CELESTIAL COOR SYS COINCIDES WITH EARTH COOR SYS
C                 I.E. TRANSIT OF FIRST POINT OF AIRES WITH GREENWICH MERIDIAN
C      CHRANG = CELESTIAL HOUR ANGLE - ZERO AT TRANSIT TIME ( DEGREES )
C      PREVEQ = PERIOD OF THE PRECESION OF THE VERNAL EQUINOX ( YEARS )
C      OBCLIP = OBLIQUITY OF THE ECLIPTIC ( DEGREES )
C      NUMIT = MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR CALC ECCENTRIC ANOMALY
C      EPSILN = CONVERGENCE CRITERION USED FOR CALC ECCENTRIC ANOMALY
C      LYRDAY = PREVIOUS VALUE OF IYRDAY
C      LOSAT = PREVIOUS VALUE OF IOSAT
C
       DATA PI/3.14159265358979/
       DATA SOLYR,SIDYR/365.24219879,366.24219879/
       DATA RE/6378.214/
       DATA GRACON/0.07436574/
       DATA F/3.35289E-3/
```

```
      DATA E/8.182015?E-2/
      DATA J2,J4/+1082.28E-6,-2.12E-6/
      DATA IRFDAY,IRFHMS/78001,171600/
      DATA CHRANG/0.0/
      DATA PREVEQ/25781.0/
      DATA OBCLIP/23.45/
      DATA NUMIT,EPSILN/20,1.0E-8/
      DATA LYRDAY,LOSAT/-1,-1/
      DATA INIT/0/
C
C     INITIALIZE CONSTANTS
C
      IF(INIT.NE.0)GO TO 1
      INIT=1
      RDPDG=PI/180.0
      TWOPI=2.0*PI
      SOLSID=SIDYR/SOLYR
      RHMS=FTIME(IRFHMS)
      CHA=RDPDG*CHRANG
C
C     ROTATION RATE OF THE VERNAL EQUINOX IN TERMS OF SIDEREAL TIME
C
      VEQ=TWOPI*SOLSID/(PREVEQ*SOLYR*1440.0)
C
C     TERRESTRIAL ROTATION RATE IN TERMS OF SIDEREAL TIME
C
      ROT=TWOPI*SOLSID/1440.0
C
C     TEST TO SEE IF DAY OR SATELLITE HAS CHANGED NECESSITATING PARM UPDATE
C
    1 IF(IYRDAY.EQ.LYRDAY.AND.IOSAT.EQ.LOSAT.AND.IOSAT.GT.0)GO TO 9
      IOSAT=IABS(IOSAT)
      LYRDAY=IYRDAY
      LOSAT=IOSAT
C
C     CONVERT EPOCH TO JULIAN DAY-TIME
C
C     IEPDAY = YEAR-DAY OF EPOCH ( YYDDD IN JULIAN DAY )
C     IEPHMS = HOUR-MINUTE-SECOND OF EPOCH ( HHMMSS IN GMT )
C
      IEPDAY=MDCON(1,IEDATE)
      IEPHMS=IETIME
C
C     DEFINE MEAN ANOMALY
C
C     EXPLICIT RELATIONSHIPS BETWEEN V,E, AND M ARE GIVEN BY THE FOLLOWING
C
C        COS(V)=(COS(E)-I)/(1-I*COS(E))
C        SIN(V)=SQRT(1-I**2)*SIN(E)/(1-I*COS(E))
C        COS(E)=(COS(V)+I)/(1+I*COS(V))
C        SIN(E)=SQRT(1-I**2)*SIN(V)/(1+I*COS(V))
C        M=E-I*SIN(E)
C
      IF(IMORT.EQ.0)MANOML=OANOML
      IF(IMORT.NE.0)CTA=COS(RDPDG*OANOML)
      IF(IMORT.NE.0)EANOML=ACOS((CTA+OECCEN)/(1.0+OECCEN*CTA))
      IF(IMORT.NE.0)MANOML=(EANOML-OECCEN*SIN(EANOML))/RDPDG
      MANOML=AMOD(MANOML,360.0)
      IF(MANOML.LT.0.0)MANOML=360.0+MANOML
C
C     DEFINE ECCENTRICITY FACTOR AND ORBITAL SEMI-PARAMETER
C
      EFACTR=SQRT(1.0-OECCEN**2)
      OSPARM=(SEMIMA/RE)*EFACTR**2
C
C     CALCULATE INCLINATION SIN AND COS TERMS
C
      INC=RDPDG*ORBINC
      SI=SIN(INC)
      CI=COS(INC)
```

```
C
C     MEAN MOTION CONSTANT
C
      MMC=GRACON*(RE/SEMIMA)**1.5
C
C     CALCULATE ORBITAL PERIOD
C
      PERIOD=TWOPI/MMC
C
C     CALCULATE ANOMALISTIC MEAN MOTION CONSTANT AND DERIVITIVES BASED
C     ON SELECTED ORDER OF SECULAR PERTURBATION THEORY
C
      IF(IOSEC.EQ.0)GO TO 2
      IF(IOSEC.EQ.1)GO TO 3
      GO TO 4
C
C     ZERO ORDER
C
    2 AMMC=MMC
      DPER=0.0
      DASN=0.0
      GO TO 5
C
C     FIRST ORDER
C
    3 AMMC=MMC*(1.0+(1.5*J2*EFACTR/OSPARM**2)*(1.0-1.5*SI**2))
      DPER=+(1.5*J2*(2.0-2.5*SI**2)/OSPARM**2)*AMMC/RDPDG
      DASN=-(1.5*J2*CI/OSPARM**2)*AMMC/RDPDG
      GO TO 5
C
C     SECOND ORDER
C
    4 AMMC=MMC*(1.0+(1.5*J2*EFACTR/OSPARM**2)*(1.0-1.5*SI**2)+(0.0234375
     **J2**2*EFACTR/OSPARM**4)*(16.0*EFACTR+25.0*EFACTR**2-15.0+(30.0-96
     *.0*EFACTR-90.0*EFACTR**2)*CI**2+(105.0+144.0*EFACTR+25.0*EFACTR**2
     *)*CI**4)-(0.3515625*J4*EFACTR*OECCEN**2/OSPARM**4)*(3.0-30.0*CI**2
     *+35.0*CI**4))
      DPER=+((1.5*J2*AMMC/OSPARM**2)*(2.0-2.5*SI**2)*(1.0+(1.5*J2/OSPARM
     ***2)*(2.0+OECCEN**2/2.0-2.0*EFACTR-(1.791666667-OECCEN**2/48.0-3.0
     **EFACTR)*SI**2))-1.25*J2**2*OECCEN**2*MMC*CI**4/OSPARM**4-(4.3750*
     *J4*MMC/OSPARM**4)*(1.714285714-6.642857143*SI**2+5.25*SI**4+OECCEN
     ***2*(1.928571429-6.75*SI**2+5.0625*SI**4)))/RDPDG
      DASN=-((1.5*J2*AMMC*CI/OSPARM**2)*(1.0+(1.5*J2/OSPARM**2)*(1.5+OEC
     *CEN**2/6.0-2.0*EFACTR-(1.666666667-0.208333333*OECCEN**2-3.0*EFACT
     *R)*SI**2))+(4.375*J4*MMC/OSPARM**4)*(1.0+1.5*OECCEN**2)*(0.8571428
     *57-1.5*SI**2)*CI)/RDPDG
C
C     CALCULATE ANOMALISTIC PERIOD
C
    5 APEROD=TWOPI/AMMC
C
C     DETERMINE TIME OF PERI-FOCAL PASSAGE
C
C     IPFDAY = YEAR-DAY OF PERIFOCUS ( YYDDD IN JULIAN DAY )
C     IPFHMS = HOUR-MINUTE-SECOND OF PERIFOCUS ( HHMMSS IN GMT )
C
      JYEAR=INIDIV(IEPDAY,1000)
      JDAY=MOD(IEPDAY,1000)
      EHMS=FTIME(IEPHMS)
      TIME=EHMS-RDPDG*MANOML/(60.0*AMMC)
      IF(TIME.GE.0.0)IS=+1
      IF(TIME.LT.0.0)IS=-1
      IT=ABS(TIME)/24.0+1.0
      IDAY=IS*IT
      IF(IDAY.GT.0)IDAY=IDAY-1
      PHMS=TIME-IDAY*24.0
      IF(IDAY.EQ.0)GO TO 8
      JDAY=JDAY+IDAY
      IF(JDAY.LT.1)GO TO 6
      JTOT=NUMYR(JYEAR)
```

```
      IF(JDAY.GT.JTOT)GO TO 7
      GO TO 8
6     JYEAR=JYEAR-1
      JDAY=NUMYR(JYEAR)+JDAY
      GO TO 8
7     JYEAR=JYEAR+1
      JDAY=JDAY-JTOT
8     IPFDAY=1000*JYEAR+JDAY
      IPFHMS=ITIME(PHMS)
      PHMS=FTIME(IPFHMS)
C
C     ADJUST PERIGEE AND ASCENDING NODE TO TIME OF PERI-FOCAL PASSAGE
C
      DIFTIM=TIMDIF(IFPDAY,EHMS,IPFDAY,PHMS)
      PERPFP=PERIGE+DPER*DIFTIM
      PERPFP=AMOD(PERPFP,360.0)
      IF(PERPFP.LT.0.0)PERPFP=360.0+PERPFP
      ASNPFP=ASNODE+DASN*DIFTIM
      ASNPFP=AMOD(ASNPFP,360.0)
      IF(ASNPFP.LT.0.0)ASNPFP=360.0+ASNPFP
      KEY=1
C
C     CALCULATE DELTA-TIME ( FROM TIME OF PERI-FOCUS TO SPECIFIED TIME )
C
9     DIFTIM=TIMDIF(IPFDAY,PHMS,IYRDAY,SATTIM)
      IF(IUSEC.EQ.0.AND.KEY.EQ.0)GO TO 10
      KEY=0
C
C     CALCULATE TIME DEPENDENT VALUES OF PERIGEE AND ASCENDING NODE
C
      PER=RDPDG*(PERPFP+DPER*DIFTIM)
      ASN=RDPDG*(ASNPFP+DASN*DIFTIM)
C
C     CALCULATE PERIGEE AND ASCENDING NODE SIN AND COS TERMS
C
      SP=SIN(PER)
      CP=COS(PER)
      SA=SIN(ASN)
      CA=COS(ASN)
C
C     CALCULATE THE (P,Q,W) ORTHOGONAL ORIENTATION VECTORS
C
C     P POINTS TOWARD PERI-FOCUS
C     Q IS IN THE ORBIT PLANE ADVANCED FROM P BY A RIGHT ANGLE IN THE DIRECTION
C     OF INCREASING TRUE ANOMALY
C     W COMPLETES A RIGHT HANDED COORDINATE SYSTEM
C
      PX=+CP*CA-SP*SA*CI
      PY=+CP*SA+SP*CA*CI
      PZ=+SP*SI
      QX=-SP*CA-CP*SA*CI
      QY=-SP*SA+CP*CA*CI
      QZ=+CP*SI
C     WX=+SA*SI
C     WY=-CA*SI
C     WZ=+CI
C
C     DEFINE MEAN ANOMALY(M) AT SPECIFIED TIME
C
10    MANOML=AMOD(AMMC*DIFTIM,TWOPI)
C
C     CALCULATE ECCENTRIC ANOMALY(E) AT SPECIFIED TIME
C
C     THE SOLUTION IS GIVEN BY A SIMPLIFIED NUMERICAL (NEWTONS) METHOD
C     AN EXPLICIT RELATIONSHIP INVOLVES A BESSEL FUNCTION OF THE FIRST KIND J(N)
C
C        E = M+2*SUM(N=1,INFINITY)(J(N)(N*I)*SIN(N*M))
C
      EOLD=MANOML
      DO 11 I=1,NUMIT
```

```
      EANOML=MANOML+OECCEN*SIN(EOLD)
      IF(ABS(EANOML-EOLD).LT.EPSILN)GO TO 12
   11 EOLD=EANOML
C
C     EXPRESSION FOR MAGNITUDE OF SATELLITE RADIUS VECTOR ( R )
C
C        R = RE*OSPARM/(1.0+OECCEN*COS(EANOML))
C
C     GENERATE A POSITION VECTOR WITH RESPECT TO THE FOCUS AND IN THE ORBITAL
C     PLANE.  NOTE THAT THE Z COORDINATE IS BY DEFINITION ZERO.
C
   12 XOMEGA=SEMIMA*(COS(EANOML)-OECCEN)
      YOMEGA=SEMIMA*(SIN(EANOML)*EFACTR)
      ZOMEGA=0
C
C     TRANSFORMATION TO A CELESTIAL POINTING VECTOR BY UTILIZATION OF THE
C     TRANSPOSE OF THE (P,Q,W) ORTHOGONAL TRANSFORMATION MATRIX.  NOTE THAT
C     THE THIRD ROW CONTAINING W IS NOT REQUIRED BECAUSE ZOMEGA IS ZERO.
C
      XSAT=XOMEGA*PX+YOMEGA*QX
      YSAT=XOMEGA*PY+YOMEGA*QY
      ZSAT=XOMEGA*PZ+YOMEGA*QZ
      IF(ICOOR.NE.0)GO TO 13
C
C     DETERMINE TRANSFORMATION MATRIX FOR ROTATION TO TERRESTRIAL COORDINATES
C
      DIFTIM=TIMDIF(IRFDAY,RHMS,IYRDAY,SATTIM)
      RAS=CRA+DIFTIM*(ROT-VEQ)
      RAS=AMOD(RAS,TWOPI)
      SRA=SIN(RAS)
      CRA=COS(RAS)
      XS=XSAT
      YS=YSAT
      ZS=ZSAT
C
C     ROTATION TO TERRESTRIAL POINTING VECTOR
C
      XSAT=+CRA*XS+SRA*YS
      YSAT=-SRA*XS+CRA*YS
      ZSAT=+ZS
C
C     CONVERT TO SPHERICAL COORDINATES
C
   13 SS=XSAT*XSAT+YSAT*YSAT
      SATLAT=ATAN2(ZSAT,SQRT(SS))/RDPDG
      SATLON=ATAN2(YSAT,XSAT)/RDPDG
      SATHGT=SQRT(SS+ZSAT*ZSAT)
      RETURN
      END
```

APPENDIX C

COMPUTER SOLUTION FOR FINDING A SYNODIC PERIOD

APPENDIX C

COMPUTER SOLUTION FOR FINDING A SYNODIC PERIOD

```
      SUBROUTINE EQCRUS(NUM,IORBIT,IGDAY,GHMS)
C
C     EMPIRICAL DETERMINATION OF EQUATOR CROSSINGS AND NODAL PERIOD
C
C     ERIC A. SMITH
C     DEPARTMENT OF ATMOSPHERIC SCIENCE
C     COLORADO STATE UNIVERSITY/FOOTHILLS CAMPUS
C     FORT COLLINS, COLORADO 80523
C     TEL 303-491-8533
C
C     INPUT PARAMETERS
C
C     NUM = NUMBER OF ORBITS FOR WHICH TO CALCULATE EQUATOR CROSSING PARAMETERS
C           IF NUM IS SET TO ZERO NO EQUATOR CROSSING INFORMATION IS PRINTED
C     IORBIT = ORBIT NUMBER OF INITIAL GUESS EQUATOR CROSSING
C     IGDAY = YEAR-DAY OF INITIAL GUESS ( YYDDD )
C     GHMS = GMT TIME OF INITIAL GUESS ( HOURS )
C
      COMMON/ORBCOM/IOSAT,IMORT,IOSEC,IEDATE,IETIME,SEMIMA,OECCEN,ORBINC
     *,OANOML,PERIGE,ASNODE,PERIOD,APEROD,EPEROD
      DATA CRIT/0.00001/
      IPASS=1
      IYRDAY=IGDAY
      SATTIM=GHMS
      NEG=0
      XINC=0.02
 1    CALL SATPOS(IYRDAY,SATTIM,0,XSAT,YSAT,ZSAT,SATLAT,SATLON,SATHGT)
      IF(ABS(SATLAT).LT.CRIT)GO TO 5
      IF(SATLAT.GT.0.0)GO TO 3
      NEG=1
      IODAY=IYRDAY
      XOTIM=SATTIM
 2    SATTIM=SATTIM+XINC
      IF(SATTIM.GE.24.0)IYRDAY=IYRDAY+1
      IF(SATTIM.GE.24.0)SATTIM=SATTIM-24.0
      GO TO 1
 3    IF(NEG.NE.0)GO TO 4
      SATTIM=SATTIM-XINC
      IF(SATTIM.LT.0.0)IYRDAY=IYRDAY-1
      IF(SATTIM.LT.0.0)SATTIM=SATTIM+24.0
      GO TO 1
 4    XINC=XINC/10.0
      IYRDAY=IODAY
      SATTIM=XOTIM
      GO TO 2
 5    IF(IPASS.EQ.2)GO TO 6
      IPASS=2
      IEDAY=IYRDAY
      EHMS=SATTIM
      IYRDAY=IEDAY
      SATTIM=EHMS
      SATTIM=SATTIM+APEROD/60.0
      IF(SATTIM.GE.24.0)IYRDAY=IYRDAY+1
      IF(SATTIM.GE.24.0)SATTIM=SATTIM-24.0
      NEG=0
      XINC=0.02
      GO TO 1
 6    IFDAY=IYRDAY
      FHMS=SATTIM
      EPEROD=TIMDIF(IEDAY,EHMS,IFDAY,FHMS)
      IF(NUM.LT.1)RETURN
      WRITE(6,100)PERIOD,APEROD,EPEROD
100   FORMAT(*0              PERIOD = *,F15.6,/,* ANOMALISTIC PERIOD = *,F
     *15.6,/,*        NODAL PERIOD = *,F15.6,/)
      WRITE(6,101)
101   FORMAT( *  ORBIT    DATE  YYDDD  HHMMSS   LATITUDE  LONGITUDE  SAT
     *HEIGHT*,/)
      DELT=EPEROD/60.0
      IORB=IORBIT
      IYRDAY=IEDAY
```

```
      SATTIM=EHMS
      DO 8 I=1,NUM
      IF(I.EQ.1)GO TO 7
      IORB=IORB+1
      SATTIM=SATTIM+DELT
      IF(SATTIM.GE.24.0)IYRDAY=IYRDAY+1
      IF(SATTIM.GE.24.0)SATTIM=SATTIM-24.0
7     IDATE=MDCON(2,IYRDAY)
      IHMS=ITIME(SATTIM)
      CALL SATPOS(IYRDAY,SATTIM,0,XSAT,YSAT,ZSAT,SATLAT,SATLON,SATHGT)
      WRITE(6,102)IORB,IDATE,IYRDAY,IHMS,SATLAT,SATLON,SATHGT
102   FORMAT(1X,I6,2X,I6,2X,I5,2X,I6,3(2X,F9.3))
8     CONTINUE
      RETURN
      END
```

APPENDIX D

COMPUTER SOLUTION FOR A SOLAR ORBIT
(PERTURBED TWO BODY)

APPENDIX D

COMPUTER SOLUTION FOR A SOLAR ORBIT (PERTURBED TWO BODY)

```
      SUBROUTINE SOLAR3(IYRDAY,SOLTIM,ICOOR,XSUN,YSUN,ZSUN,SUNLAT,SUNLON
     *,SUNHGT)
C
C     COMPUTE SUN POSITION VECTOR ACCORDING TO A KEPLERIAN ORBIT
C
C     ERIC A. SMITH
C     DEPARTMENT OF ATMOSPHERIC SCIENCE
C     COLORADO STATE UNIVERSITY/FOOTHILLS CAMPUS
C     FORT COLLINS, COLORADO 80523
C     TEL 303-491-8533
C
C     REFERENCES.
C
C     BOWDITCH,NATHANIEL,1962.
C     AMERICAN PRACTICAL NAVIGATOR - AN EPITOMIE OF NAVIGATION.
C     U.S. NAVY HYDROGRAPHIC OFFICE,H.O. PUB. NO. 9.
C     UNITED STATES GOVERNMENT PRINTING OFFICE,1524 PP.
C
C     ESCOBAL,PEDRO RAMON,1965.
C     METHODS OF ORBIT DETERMINATION.
C     JOHN WILEY AND SONS,INC.,NEW YORK/LONDON/SYDNEY,463 PP.
C
C     THE AMERICAN EPHEMERIS AND NAUTICAL ALMANAC,1978.
C     ISSUED BY THE NAUTICAL ALMANAC OFFICE
C               UNITED STATES NAVAL OBSERVATORY
C                            AND
C               HER MAJESTYS NAUTICAL ALMANAC OFFICE
C               ROYAL GREENWICH OBSERVATORY
C     U.S. GOVERNMENT PRINTING OFFICE,WASHINGTON DC,573 PP.
C
C     INPUT PARAMETERS
C
C     IYRDAY = YEAR ( YYDDD IN JULIAN DAY )
C     SOLTIM = TIME ( HOURS IN GMT )
C     ICOOR  = 0 FOR TERRESTRIAL COORDINATES
C            = 1 FOR CELESTIAL COORDINATES
C
C     OUTPUT PARAMETERS
C
C     XSUN   = X COMPONENT OF SUN POSITION VECTOR ( KM )
C     YSUN   = Y COMPONENT OF SUN POSITION VECTOR ( KM )
C     ZSUN   = Z COMPONENT OF SUN POSITION VECTOR ( KM )
C     SUNLAT = SUN LATITUDE ( DEGREES )
C     SUNLON = SUN LONGITUDE ( DEGREES )
C     SUNHGT = SUN HEIGHT ( KM )
C
C     LATITUDE IS GIVEN IN TERMS OF SPHERICAL COORDINATES
C     USE THE FOLLOWING TRANSFORMATION TO CONVERT TO GEOCENTRIC LATITUDE
C
C              S=RDPDG*SATLAT
C              SATLAT=ACOS(COS(S)/SQRT(1.0-(E*SIN(S))**2))/RDPDG
C
      REAL MMC,MANOML,INC
C
C     BROUWER MEAN ORBITAL ELEMENTS
C
C     IEYDAY = EPOCH DAY ( YYDDD IN JULIAN DAY )
C     IEPHMS = EPOCH TIME ( HHMMSS IN GMT )
C     SEMIMA = SEMI-MAJOR AXIS ( KM )
C     OECCEN = ECCENTRICITY OF SOLAR ORBIT ( UNITLESS )
C     ORBINC = ORBIT INCLINATION ( DEGREES )
C     PERHEL = ARGUMENT OF PERIHELION AT EPOCH TIME ( DEGREES )
C     ASNODE = RIGHT ASCENSION OF ASCENDING NODE AT EPOCH TIME ( DEGREES )
C
      DATA IEYDAY,IEPHMS/78001,230000/
      DATA SEMIMA/149596138.2/
      DATA OECCEN/0.016751/
      DATA ORBINC/23.452/
      DATA PERHEL/281.221/
      DATA ASNODE/0.0/
```

```
C
C     ORBITAL CONSTANTS
C
C     PI = VALUE OF PI
C     SOLYR = NUMBER OF DAYS IN SOLAR YEAR ( DAYS )
C     SIDYR = NUMBER OF DAYS IN SIDEREAL YEAR ( DAYS )
C     RS = ASTRONOMICAL UNIT ( KM )
C     GRACON = GAUSSIAN GRAVITATIONAL CONSTANT ( KS=SQRT(G*MS*86400**2/RS**3 )
C              WHERE KS = GAUSSIAN GRAV CON ( 0.017202099 SM**.5*AU**1.5/DAY )
C                     G = UNIVERSAL GRAV CONSTANT ( 6.673E-8 DYNE*CM**2*GM**-2 )
C                    MS = MASS OF SUN ( 1.9888822E33 GM PER SM )
C                    RS = ASTRONOMICAL UNIT ( 1.496E13 CM PER AU )
C                          NOTE.  MEAN EARTH-SUN DISTANCE IS 1.00000003*AU
C                                 SEMI-MAJOR AXIS IS 0.999974186*AU
C     SQRTMU = SECOND BODY MASS CORRECTION FACTOR ( SQRTMU=SQRT(1+(ME+MM)/MS )
C              WHERE SQRTMU = MASS CORRECTION FACTOR ( 1.00000152 SM**.5 )
C                    ME = MASS OF EARTH ( 5.9733726E27 GM )
C                    MM = MASS OF MOON  ( 7.3473218E25 GM )
C                    MS = MASS OF SUN   ( 1.9888822E33 GM )
C     F = FLATTENING   OF THE EARTH ( F=(A-B)/A            , F=1-SQRT(1-E**2)   )
C     E = ECCENTRICITY OF THE EARTH ( E=SQRT(A**2-B**2)/A , E=SQRT(2*F-F**2)   )
C         WHERE F = FLATTENING OF EARTH ( 3.35289E-3 )
C               E = ECCENTRICITY OF EARTH ( 8.1820157E-2 )
C               A = SEMI-MAJOR EARTH AXIS - EQUATORIAL ( 6378.214 KM )
C               B = SEMI-MINOR EARTH AXIS - POLAR      ( 6356.829 KM )
C                 = A*(1-F)
C               C = MEAN EARTH RADIUS ( 6371.086 KM )
C                 = (2*A+B)/3
C     IRFDAY =  YYDDD WHEN CELESTIAL COOR SYS COINCIDES WITH EARTH COOR SYS
C               I.E. TRANSIT OF FIRST POINT OF AIRES WITH GREENWICH MERIDIAN
C     IRFHMS = HHMMSS WHEN CELESTIAL COOR SYS COINCIDES WITH EARTH COOR SYS
C               I.E. TRANSIT OF FIRST POINT OF AIRES WITH GREENWICH MERIDIAN
C     CHRANG = CELESTIAL HOUR ANGLE - ZERO AT TRANSIT TIME ( DEGREES )
      DATA PREVEQ/25781.0/
C     OBCLIP = OBLIQUITY OF THE ECLIPTIC ( DEGREES )
C     KEY = 0 FOR COMPUTING ECCENTRIC ANOMALY WITH ITERATIVE METHOD
C             SIMPLIFIED NEWTONS METHOD
C         = 1 FOR COMPUTING ECCENTRIC ANOMALY WITH EXPLICIT METHOD
C             FOURIER-BESSEL SERIES
C         = 2 FOR COMPUTING ECCENTRIC ANOMALY WITH 2ND ORDER EXPANSION OF
C             FOURIER-BESSEL SERIES
C         = 3 FOR COMPUTING ECCENTRIC ANOMALY WITH 3RD ORDER EXPANSION OF
C             FOURIER-BESSEL SERIES
C         = 4 FOR COMPUTING ECCENTRIC ANOMALY WITH 4TH ORDER EXPANSION OF
C             FOURIER-BESSEL SERIES
C     NUMIT = MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR CALC ECCENTRIC ANOMALY
C     EPSILN = CONVERGENCE CRITERION USED FOR CALC ECCENTRIC ANOMALY
C
      DATA PI/3.14159265358979/
      DATA SOLYR,SIDYR/365.24219879,366.24219879/
      DATA RS/149600000.0/
      DATA GRACON/0.017202099/
      DATA SQRTMU/1.00000152/
      DATA F/3.35289E-3/
      DATA E/8.1820157E-2/
      DATA IRFDAY,IRFHMS/78001,171600/
      DATA CHRANG/0.0/
      DATA PREVEQ/25791.0/
      DATA OBCLIP/23.45/
      DATA KEY/2/
      DATA NUMIT,EPSILN/20,1.0E-8/
      DATA INIT/0/
C
C     INITIALIZE CONSTANTS
C
      IF(INIT.NE.0)GO TO 1
      INIT=1
      RDPDG=PI/180.0
      TWOPI=2.0*PI
      SOLSID=SIDYR/SOLYR
```

```
      EHMS=FTIME(IEPHMS)
      RHMS=FTIME(IRFHMS)
      CHA=RDPDG*CHRANG
C
C     ROTATION RATE OF THE VERNAL EQUINOX IN TERMS OF SIDEREAL TIME
C
      VEQ=TWOPI*SOLSID/(PREVEQ*SOLYR*1440.0)
C
C     TERRESTRIAL ROTATION RATE IN TERMS OF SIDEREAL TIME
C
      ROT=TWOPI*SOLSID/1440.0
C
C     DEFINE ECCENTRICITY FACTOR
C
      EFACTR=SQRT(1.0-OECCEN**2)
C
C     MEAN MOTION CONSTANT
C
      MMC=SQRTMU*GRACON/1440.0*(RS/SEMIMA)**1.5
C
C     CALCULATE THE (P,Q,W) ORTHOGONAL ORIENTATION VECTORS
C
      INC=RDPDG*ORBINC
      PER=RDPDG*PERHEL
      ASN=RDPDG*ASNODE
      SI=SIN(INC)
      CI=COS(INC)
      SP=SIN(PER)
      CP=COS(PER)
      SA=SIN(ASN)
      CA=COS(ASN)
      PX=+CP*CA-SP*SA*CI
      PY=+CP*SA+SP*CA*CI
      PZ=+SP*SI
      QX=-SP*CA-CP*SA*CI
      QY=-SP*SA+CP*CA*CI
      QZ=+CP*SI
      WX=+SA*SI
      WY=-CA*SI
      WZ=+CI
C
C
C     DEFINE MEAN ANOMALY(M) AT SPECIFIED TIME
C
C
    1 DIFTIM=TIMDIF(IEYDAY,EHMS,IYRDAY,SOLTIM)
      MANOML=AMOD(MMC*DIFTIM,TWOPI)
C
C     CALCULATE ECCENTRIC ANOMALY(E) AT SPECIFIED TIME
C
      IF(KEY.GE.1.AND.KEY.LE.4)GO TO 3
C
C     ITERATIVE METHOD - SIMPLIFIED NEWTONS METHOD
C
      EOLD=MANOML
      DO 2 N=1,NUMIT
      EANOML=MANOML+OECCEN*SIN(EOLD)
      IF(ABS(EANOML-EOLD).LT.EPSILN)GO TO 9
    2 EOLD=EANOML
      GO TO 9
    3 GO TO(4,6,7,8),KEY
C
C     EXPLICIT METHOD - FOURIER-BESSEL SERIES
C
    4 EANOML=MANOML
      EOLD=EANOML
      DO 5 N=1,NUMIT
      X=N*OECCEN
      Y=N*MANOML
      EANOML=EANOML+2*BESFK(N,X,EPSILN)*SIN(Y)/N
      IF(ABS(EANOML-EOLD).LT.EPSILN)GO TO 9
    5 EOLD=EANOML
```

```
      GO TO 9
C
C     2ND ORDER EXPANSION
C
    6 SM=SIN(MANOML)
      CM=COS(MANOML)
      EANOML=MANOML+SM*OECCEN+SM*CM*OECCEN*OECCEN
      GO TO 9
C
C     3RD ORDER EXPANSION
C
    7 SM=SIN(MANOML)
      CM=COS(MANOML)
      E1=OECCEN
      E2=OECCEN*E1
      E3=OECCEN*E2
      EANOML=MANOML+SM*E1+SM*CM*E2+(SM-1.5*SM*SM*SM)*E3
      GO TO 9
C
C     4TH ORDER EXPANSION
C
    8 SM=SIN(MANOML)
      CM=COS(MANOML)
      SMCM=SM*CM
      S3M=SM*SM*SM
      E1=OECCEN
      E2=OECCEN*E1
      E3=OECCEN*E2
      E4=OECCEN*E3
      EANOML=MANOML+SM*E1+SMCM*E2+(SM-1.5*S3M)*E3+(SMCM-8*S3M*CM/3)*E4
C
C     GENERATE A POSITION VECTOR WITH RESPECT TO THE FOCUS AND IN THE ORBITAL
C     PLANE.  NOTE THAT THE Z COORDINATE IS BY DEFINITION ZERO.
C
    9 XOMEGA=SEMIMA*(COS(EANOML)-OECCEN)
      YOMEGA=SEMIMA*(SIN(EANOML)*EFACTR)
      ZOMEGA=0
C
C     TRANSFORMATION TO A CELESTIAL POINTING VECTOR BY UTILIZATION OF THE
C     TRANSPOSE OF THE (P,Q,W) ORTHOGONAL TRANSFORMATION MATRIX.  NOTE THAT
C     THE THIRD ROW CONTAINING W IS NOT REQUIRED BECAUSE ZOMEGA IS ZERO.
C
      XSUN=XOMEGA*PX+YOMEGA*QX
      YSUN=XOMEGA*PY+YOMEGA*QY
      ZSUN=XOMEGA*PZ+YOMEGA*QZ
      IF(ICOOR.NE.0)GO TO 10
C
C     DETERMINE TRANSFORMATION MATRIX FOR ROTATION TO TERRESTRIAL COORDINATES
C
      DIFTIM=TIMDIF(IRFDAY,RHMS,IYRDAY,SOLTIM)
      RAS=CHA+DIFTIM*(ROT-VEQ)
      RAS=AMOD(RAS,TWOPI)
      SRA=SIN(RAS)
      CRA=COS(RAS)
      XS=XSUN
      YS=YSUN
      ZS=ZSUN
C
C     ROTATION TO TERRESTRIAL POINTING VECTOR
C
      XSUN=+CRA*XS+SRA*YS
      YSUN=-SRA*XS+CRA*YS
      ZSUN=+ZS
C
C     CONVERT TO SPHERICAL COORDINATES
C
   10 SS=XSUN*XSUN+YSUN*YSUN
      SUNLAT=ATAN2(ZSUN,SQRT(SS))/RDPDG
      SUNLON=ATAN2(YSUN,XSUN)/RDPDG
      SUNHGT=SQRT(SS+ZSUN*ZSUN)
```

```
      RETURN
      END
      FUNCTION NFAC(N)
C
C     CALCULATES N FACTORIAL
C
      NFAC=1
      IF(N.LE.0)RETURN
      DO 1 I=1,N
 1    NFAC=NFAC*I
      RETURN
      END
      FUNCTION BESFK(N,X,EPS)
C
C     CALCULATES BESSEL FUNCTION OF FIRST KIND OF ORDER N USING ARGUMENT
C     X TO A PRECISION TOLERENCE OF EPS
C
      REAL NUMER
      FAC=X**N/(2.0**N*NFAC(N))
      BESFK=FAC
      XSQ=X*X
      TWN=2*N
      ISN=(+1)
      INT=0
      NUMER=1
      DENOM=1
 1    BOLD=BESFK
      ISN=(-1)*ISN
      INT=INT+2
      NUMER=NUMER*XSQ
      DENOM=DENOM*INT*(TWN+INT)
      BESFK=BESFK+ISN*FAC*NUMER/DENOM
      IF(ABS(BESFK-BOLD).GE.EPS)GO TO 1
      RETURN
      END
```

APPENDIX E

COMPUTER SOLUTIONS FOR A SOLAR ORBIT (APPROXIMATE AND NON-LINEAR REGRESSION)

APPENDIX E

COMPUTER SOLUTIONS FOR A SOLAR ORBIT (APPROXIMATE AND NON-LINEAR REGRESSION)

```
      SUBROUTINE SOLAR1(IYRDAY,SOLTIM,ICOOR,XSUN,YSUN,ZSUN,SUNLAT,SUNLON
     *,SUNHGT)
C
C     COMPUTE SUN POSITION VECTOR ACCORDING TO EMPIRICAL FORMULAE
C
C     ERIC A. SMITH
C     DEPARTMENT OF ATMOSPHERIC SCIENCE
C     COLORADO STATE UNIVERSITY/FOOTHILLS CAMPUS
C     FORT COLLINS, COLORADO 80523
C     TEL 303-491-8533
C
C     INPUT PARAMETERS
C
C     IYRDAY = YEAR ( YYDDD IN JULIAN DAY )
C     SOLTIM = TIME ( HOURS IN GMT )
C     ICOOR = 0 FOR TERRESTRIAL COORDINATES
C           = 1 FOR CELESTIAL COORDINATES ( NOT AVAILABLE )
C
C     OUTPUT PARAMETERS
C
C     XSUN = X COMPONENT OF SUN POSITION VECTOR ( KM )
C     YSUN = Y COMPONENT OF SUN POSITION VECTOR ( KM )
C     ZSUN = Z COMPONENT OF SUN POSITION VECTOR ( KM )
C     SUNLAT = SUN LATITUDE ( DEGREES )
C     SUNLON = SUN LONGITUDE ( DEGREES )
C     SUNHGT = SUN HEIGHT ( KM )
C
      DATA PI/3.14159265/
      DATA AVEHGT/149600000.0/
      RDPDG=PI/180.0
      IYEAR=INTDIV(IYRDAY,1000)
      IDAY=MOD(IYRDAY,1000)
      TOT=NUMYR(IYEAR)
      DAY=IDAY-1+SOLTIM/24.0
      THET1=2*PI*DAY/TOT
      THET2=2*THET1
      THET3=3*THET1
      C1=COS(THET1)
      S1=SIN(THET1)
      C2=COS(THET2)
      S2=SIN(THET2)
      C3=COS(THET3)
      S3=SIN(THET3)
      SUNDEC=0.006918-0.399912*C1+0.070257*S1-0.006758*C2+0.000907*S2
     * -0.002697*C3+0.00148*S3
      DISCOR=1.00011+0.034221*C1+0.00128*S1+0.000719*C2+0.000077*S2
      SUNLAT=SUNDEC/RDPDG
      SUNLON=-15*(SOLTIM-12.0)
      SUNHGT=AVEHGT/SQRT(DISCOR)
      XLAT=RDPDG*SUNLAT
      XLON=RDPDG*SUNLON
      XSUN=SUNHGT*COS(XLAT)*COS(XLON)
      YSUN=SUNHGT*COS(XLAT)*SIN(XLON)
      ZSUN=SUNHGT*SIN(XLAT)
      RETURN
      END
```

```
      SUBROUTINE SOLAR2(IYRDAY,SOLTIM,ICOOR,XSUN,YSUN,ZSUN,SUNLAT,SUNLON
     *,SUNHGT)
C
C     COMPUTE SUN POSITION VECTOR ACCORDING TO NON-LINEAR REGRESSION
C
C     ERIC A. SMITH
C     DEPARTMENT OF ATMOSPHERIC SCIENCE
C     COLORADO STATE UNIVERSITY/FOOTHILLS CAMPUS
C     FORT COLLINS, COLORADO 80523
C     TEL 303-491-8533
C
C     MODIFICATION OF A ROUTINE SUPPLIED BY THE NATIONAL ENVIRONMENTAL
C     SATELLITE SERVICE ( NESS )
C
C     INPUT PARAMETERS
C
C     IYRDAY = YEAR ( YYDDD IN JULIAN DAY )
C     SOLTIM = TIME ( HOURS IN GMT )
C     ICOOR = 0 FOR TERRESTRIAL COORDINATES
C           = 1 FOR CELESTIAL COORDINATES
C
C     OUTPUT PARAMETERS
C
C     XSUN = X COMPONENT OF SUN POSITION VECTOR ( KM )
C     YSUN = Y COMPONENT OF SUN POSITION VECTOR ( KM )
C     ZSUN = Z COMPONENT OF SUN POSITION VECTOR ( KM )
C     SUNLAT = SUN LATITUDE ( DEGREES )
C     SUNLON = SUN LONGITUDE ( DEGREES )
C     SUNHGT = SUN HEIGHT ( KM )
C
      REAL LP
C
C     ORBITAL CONSTANTS
C
C     PI = VALUE OF PI
C     SOLYR = NUMBER OF DAYS IN SOLAR YEAR ( DAYS )
C     SIDYR = NUMBER OF DAYS IN SIDEREAL YEAR ( DAYS )
C     IRYDAY =  YYDDD WHEN CELESTIAL COOR SYS COINCIDES WITH EARTH COOR SYS
C               I.E. TRANSIT OF FIRST POINT OF AIRES WITH GREENWICH MERIDIAN
C     IRFHMS = HHMMSS WHEN CELESTIAL COOR SYS COINCIDES WITH EARTH COOR SYS
C               I.E. TRANSIT OF FIRST POINT OF AIRES WITH GREENWICH MERIDIAN
C     CHRANG = CELESTIAL HOUR ANGLE - ZERO AT TRANSIT TIME ( DEGREES )
C     PREVEQ = PERIOD OF THE PRECESION OF THE VERNAL EQUINOX ( YEARS )
C     OBCLIP = OBLIQUITY OF THE ECLIPTIC ( DEGREES )
C     (IEYDAY,IEPHMS) = EPOCH TIME BASE FOR REGRESSION
C     (C0-C9,E1-E7) = REGRESSION CONSTANTS
C
      DATA PI/3.14159265358979/
      DATA SOLYR,SIDYR/365.24219879,366.24219879/
      DATA IRYDAY,IRFHMS/78001,171600/
      DATA CHRANG/0.0/
      DATA PREVEQ/25781.0/
      DATA OBCLIP/23.45/
      DATA IEYDAY,IEPHMS/58261,0/
      DATA C1/0.17664978490940000E3/
      DATA C2/0.98564734495500007E0/
      DATA C3/0.22695698212594030E-12/
      DATA C4/0.25443661034350000E3/
      DATA C5/0.98560026220020310E0/
      DATA C6/0.11243740398899790E-12/
      DATA C7/0.22794176405045000E3/
      DATA C8/0.52953865653818660E-1/
      DATA C9/0.15574662626624150E-11/
      DATA E1/0.33502000000000000E-1/
      DATA E2/0.23444780594227700E2/
      DATA E3/0.35645296220244890E-6/
      DATA E4/0.25583333333333330E-2/
      DATA E5/0.15338888888888880E-3/
      DATA E6/0.14960000000000000E9/
      DATA E7/0.72741200000000000E-2/
```

```
      DATA INIT/0/
C
C     INITIALIZE CONSTANTS
C
      IF(INIT.NE.0)GO TO 1
      INIT=1
      RDPDG=PI/180.0
      TWOPI=2.0*PI
      SOLSID=SIDYR/SOLYR
      EHMS=FTIME(IEPHMS)
      RHMS=FTIME(IRFHMS)
      CHA=RDPDG*CHRANG
C
C     ROTATION RATE OF THE VERNAL EQUINOX IN TERMS OF SIDEREAL TIME
C
      VEQ=TWOPI*SOLSID/(PREVEQ*SOLYR*1440.0)
C
C     TERRESTRIAL ROTATION RATE IN TERMS OF SIDEREAL TIME
C
      ROT=TWOPI*SOLSID/1440.0
C
C     CALCULATE TIME DIFFERENCE IN DAYS
C
    1 DIFTIM=TIMDIF(IEYDAY,EHMS,IYRDAY,SOLTIM)
      D=DIFTIM/1440.0
C
C     CALCULATE REGRESSION
C
      DSQ=D*D
      LP=C1+C2*D+C3*DSQ
      ALP=C4+C5*D-C6*DSQ
      OMEGA=C7-C8*D+C9*DSQ
      LP=RDPDG*AMOD(LP,360.0)
      ALP=RDPDG*AMOD(ALP,360.0)
      OMEGA=RDPDG*AMOD(OMEGA,360.0)
      XSOL=LP+E1*SIN(ALP)
      XEPS=RDPDG*(E2-E3*D+E4*COS(OMEGA)+E5*COS(2*LP))
      RSUN=E6*10.0**(-E7*COS(ALP))
C
C     COMPUTE A CELESTIAL POSITION VECTOR
C
      XSUN=RSUN*COS(XSOL)
      YSUN=RSUN*SIN(XSOL)*COS(XEPS)
      ZSUN=RSUN*SIN(XSOL)*SIN(XEPS)
      IF(ICOOR.NE.0)GO TO 2
C
C     DETERMINE TRANSFORMATION MATRIX FOR ROTATION TO TERRESTRIAL COORDINATES
C
      DIFTIM=TIMDIF(IRYDAY,RHMS,IYRDAY,SOLTIM)
      RAS=CHA+DIFTIM*(ROT-VEQ)
      RAS=AMOD(RAS,TWOPI)
      SRA=SIN(RAS)
      CRA=COS(RAS)
      XS=XSUN
      YS=YSUN
      ZS=ZSUN
C
C     ROTATION TO TERRESTRIAL POINTING VECTOR
C
      XSUN=+CRA*XS+SRA*YS
      YSUN=-SRA*XS+CRA*YS
      ZSUN=+ZS
C
C     CONVERT TO SPHERICAL COORDINATES
C
    2 SS=XSUN*XSUN+YSUN*YSUN
      SUNLAT=ATAN2(ZSUN,SQRT(SS))/RDPDG
      SUNLON=ATAN2(YSUN,XSUN)/RDPDG
      SUNHGT=SQRT(SS+ZSUN*ZSUN)
      RETURN
      END
```

APPENDIX F

LIBRARY ROUTINES FOR ORBITAL SOFTWARE

APPENDIX F

LIBRARY ROUTINES FOR ORBIT SOFTWARE

```
      FUNCTION FLALO(M)
C
C     PACKED INTEGER ( SIGN DDD MM SS ) LATITUDE-LONGITUDE TO FLOATING POINT
C
C     INPUT PARAMETERS
C
C     M = PACKED INTEGER ( SIGN DDD MM SS ) LATITUDE-LONGITUDE
C
      IF(M.LT.0)GO TO 1
      N=M
      X=1.0
      GO TO 2
 1    N=-M
      X=-1.0
 2    FLALO=FLOAT(INTDIV(N,10000))+FLOAT(MOD(INTDIV(N,100),100))/60.0+FL
     *OAT(MOD(N,100))/3600.0
      FLALO=X*FLALO
      RETURN
      END
      FUNCTION FTIME(M)
C
C     PACKED INTEGER ( SIGN HH MM SS ) TIME TO FLOATING POINT
C
C     INPUT PARAMETERS
C
C     M = PACKED INTEGER ( SIGN HH MM SS ) TIME
C
      IF(M.LT.0)GO TO 1
      N=M
      X=1.0
      GO TO 2
 1    N=-M
      X=-1.0
 2    FTIME=FLOAT(INTDIV(N,10000))+FLOAT(MOD(INTDIV(N,100),100))/60.0+FL
     *OAT(MOD(N,100))/3600.0
      FTIME=X*FTIME
      RETURN
      END
      FUNCTION GCIRC(XLAT1,XLON1,XLAT2,XLON2)
C
C     GREAT CIRCLE ARC DISTANCE IN KILOMETERS
C
C     INPUT PARAMETERS
C
C     XLAT1 = LATITUDE OF FIRST POINT ( DEGREES )
C     XLON1 = LONGITUDE OF FIRST POINT ( DEGREES )
C     XLAT2 = LATITUDE OF SECOND POINT ( DEGREES )
C     XLON2 = LONGITUDE OF SECOND POINT ( DEGREES )
C
      DATA PI/3.14159265/
      DATA XKMPDG/111.12/
      DATA XMIN/2.0/
      RDPDG=PI/180.0
      COSLAT=COS(RDPDG*XLATAV(XLAT1,XLAT2))
      YLAT=XLATSB(XLAT2,XLAT1)
      YLON=XLONSB(XLON2,XLON1)
      X=XKMPDG*YLAT
      Y=XKMPDG*YLON*COSLAT
      GCIRC=SQRT(X*X+Y*Y)
      IF(GCIRC.LT.XMIN)RETURN
      YLAT1=RDPDG*XLAT1
      YLAT2=RDPDG*XLAT2
      YLON=RDPDG*YLON
      GCIRC=XKMPDG*ACOS(SIN(YLAT1)*SIN(YLAT2)+COS(YLAT1)*COS(YLAT2)*
     * COS(YLON))/RDPDG
      RETURN
      END
      FUNCTION GEOLAT(IDIR,XLAT)
C
C     GEODETIC-GEOCENTRIC LATITUDE CONVERSION
```

```
C
C      INPUT PARAMETERS
C
C      IDIR = 1 FOR GEODETIC TO GEOCENTRIC
C           = 2 FOR GEOCENTRIC TO GEODETIC
C      XLAT = LATITUDE ( DEGREES )
C
       DATA PI/3.14159265/
       DATA RE,RP/6378.388,6356.912/
       IF(IDIR.LT.1.OR.IDIR.GT.2)RETURN
       RDPDG=PI/180.0
       F=(RE-RP)/RE
       FAC=(1.0-F)**2
       YLAT=RDPDG*XLAT
       GO TO(1,2),IDIR
 1     GEOLAT=ATAN(TAN(YLAT)*FAC)/RDPDG
       RETURN
 2     GEOLAT=ATAN(TAN(YLAT)/FAC)/RDPDG
       RETURN
       END
       FUNCTION ILALO(X)
C
C      FLOATING POINT LATITUDE-LONGITUDE TO PACKED INTEGER ( SIGN DDD MM SS )
C
C      INPUT PARAMETERS
C
C      X = FLOATING POINT LATITUDE OR LONGITUDE
C
       IF(X.LT.0.0)GO TO 1
       Y=X
       I=1
       GO TO 2
 1     Y=-X
       I=-1
 2     J=3600.0*Y+0.5
       ILALO=10000*INTDIV(J,3600)+100*MOD(INTDIV(J,60),60)+MOD(J,60)
       ILALO=I*ILALO
       RETURN
       END
       FUNCTION INTDIV(I,J)
C
C      INTEGER DIVIDE WITHOUT ROUNDOFF PROBLEMS
C
C      INPUT PARAMETERS
C
C      I = NUMERATOR
C      J = DENOMINATOR
C
       DATA CON/1.0E-11/
       K=1
       IF(I*J.LT.0)K=-1
       X=IABS(J)
       INTDIV=IABS(I)/X+CON
       INTDIV=K*INTDIV
       RETURN
       END
       FUNCTION IROUND(X)
C
C      ROUNDS A FLOATING POINT NUMBER
C
C      INPUT PARAMETERS
C
C      X = FLOATING POINT NUMBER TO CONVERT
C
       IF(X)1,2,3
 1     IROUND=X-0.5
       RETURN
 2     IROUND=0
       RETURN
 3     IROUND=X+0.5
```

```
      RETURN
      END
      FUNCTION ITIME(X)
C
C     FLOATING POINT TIME TO PACKED INTEGER ( SIGN HH MM SS )
C
C     INPUT PARAMETERS
C
C     X = FLOATING POINT TIME
C
      IF(X.LT.0.0)GO TO 1
      Y=X
      I=1
      GO TO 2
 1    Y=-X
      I=-1
 2    J=3600.0*Y+0.5
      ITIME=10000*INTDIV(J,3600)+100*MOD(INTDIV(J,60),60)+MOD(J,60)
      ITIME=I*ITIME
      RETURN
      END
      SUBROUTINE JULDAY(IYRDAY,ITIT)
C
C     CONVERT JULIAN DAY TO ALPHA HEADING
C
C     INPUT PARAMETERS
C
C     IYRDAY = YEAR-DAY ( YYDDD )
C     ITIT = 20 CHARACTER TITLE
C
      DIMENSION ITIT(1),MONTHS(2,12)
      DATA MONTHS/2HJA,2HN ,2HFE,2HB ,2HMA,2HR ,2HAP,2HR ,2HMA,2HY ,2HJU
     *,2HN ,2HJU,2HL ,2HAU,2HG ,2HSE,2HP ,2HOC,2HT ,2HNO,2HV ,2HDE,2HC /
      IDATE=MDCON(2,IYRDAY)
      IY=INTDIV(IYRDAY,1000)
      JDAY=MOD(IYRDAY,1000)
      IM=MOD(INTDIV(IDATE,100),100)
      ID=MOD(IDATE,100)
      ENCODE(20,100,ITIT)MONTHS(1,IM),MONTHS(2,IM),ID,IY,JDAY
 100  FORMAT(2A2,I2,4H, 19,I2,1H(,I3,4H)    )
      RETURN
      END
      FUNCTION MDCON(IDIR,IDATE)
C
C     CONVERSION BETWEEN YYMMDD ( YEAR-MONTH-DAY ) AND YYDDD ( YEAR-JULIAN DAY )
C
C     INPUT PARAMETERS
C
C     IDIR = 1 FOR YYMMDD TO YYDDD
C          = 2 FOR YYDDD TO YYMMDD
C     IDATE = DATE
C
      DIMENSION NUM(12)
      DATA NUM/31,59,90,120,151,181,212,243,273,304,334,365/
      IF(IDIR.LT.1.OR.IDIR.GT.2)RETURN
      GO TO(1,2),IDIR
 1    IY=INTDIV(IDATE,10000)
      IM=MOD(INTDIV(IDATE,100),100)
      ID=MOD(IDATE,100)
      IF(IM.LT.1)IM=1
      IF(IM.GT.12)IM=12
      LEAP=MOD(IY,4)
      ITOT=0
      IF((IM-1).NE.0)ITOT=NUM(IM-1)
      IF(LEAP.EQ.0.AND.IM.GT.2)ITOT=ITOT+1
      IJD=ITOT+ID
      MDCON=1000*IY+IJD
      RETURN
 2    IY=INTDIV(IDATE,1000)
      IJD=MOD(IDATE,1000)
```

```
      LEAP=MOD(IY,4)
      MAX=365
      IF(LEAP.EQ.0)MAX=366
      IF(IJD.LT.1)IJD=1
      IF(IJD.GT.MAX)IJD=MAX
      ITOT=0
      DO 3 I=1,12
      IF(I.EQ.1)NDAY=NUM(I)
      IF(I.NE.1)NDAY=NUM(I)-NUM(I-1)
      IF(LEAP.EQ.0.AND.I.EQ.2)NDAY=NDAY+1
      ITOT=ITOT+NDAY
      IF(IJD.GT.ITOT)GO TO 3
      IM=I
      ID=IJD-ITOT+NDAY
      MDCON=10000*IY+100*IM+ID
      GO TO 4
    3 CONTINUE
      MDCON=10000*IY+100*12+31
    4 RETURN
      END
      FUNCTION NUMDY(IYD1,IYD2)
C
C     TIME DIFFERENCE IN DAYS ( SECOND MINUS FIRST )
C
C     INPUT PARAMETERS
C
C     IYD1 = FIRST YEAR-DAY ( YYDDD )
C     IYD2 = SECOND YEAR-DAY ( YYDDD )
C
      IY1=INTDIV(IYD1,1000)
      ID1=MOD(IYD1,1000)
      IY2=INTDIV(IYD2,1000)
      ID2=MOD(IYD2,1000)
      I=1
      IF(IY1.GT.IY2)I=-1
      IF(IY1.EQ.IY2.AND.ID1.GT.ID2)I=-1
      IF(I.LT.0)GO TO 1
      JY1=IY1
      JD1=ID1
      JY2=IY2
      JD2=ID2
      GO TO 2
    1 JY1=IY2
      JD1=ID2
      JY2=IY1
      JD2=ID1
    2 NUMDY=0
    3 IF(JY1.GE.JY2)GO TO 4
      NUMDY=NUMDY+NUMYR(JY1)-JD1+1
      JY1=JY1+1
      JD1=1
      GO TO 3
    4 NUMDY=NUMDY+JD2-JD1
      NUMDY=I*NUMDY
      RETURN
      END
      FUNCTION NUMYR(IYEAR)
C
C     NUMBER OF DAYS IN A YEAR
C
C     INPUT PARAMETERS
C
C     IYEAR = YEAR
C
      NUMYR=365
      LEAP=MOD(IYEAR,4)
      IF(LEAP.EQ.0)NUMYR=366
      RETURN
      END
      FUNCTION TIMDIF(IYD1,TIME1,IYD2,TIME2)
```

```
C
C      TIME DIFFERENCE IN MINUTES ( SECOND MINUS FIRST )
C
C      INPUT PARAMETERS
C
C      IYD1 = FIRST YEAR-DAY ( YYDDD )
C      TIME1 = FIRST TIME IN HOURS
C      IYD2 = SECOND YEAR-DAY ( YYDDD )
C      TIME2 = SECOND TIME IN HOURS
C
       TIMDIF=1440.0*NUMDY(IYD1,IYD2)+60.0*(TIME2-TIME1)
       RETURN
       END
       FUNCTION XLATAV(XLAT1,XLAT2)
C
C      AVERAGES TWO LATITUDE VALUES
C      LATITUDE RUNS FROM +90.0 NORTH TO -90.0 SOUTH
C
C      INPUT PARAMETERS
C
C      XLAT1 = FIRST LATITUDE
C      XLAT2 = SECOND LATITUDE
C
       XLATAV=(XLAT1+XLAT2)/2.0
       RETURN
       END
       FUNCTION XLATSB(XLAT1,XLAT2)
C
C      SUBTRACTS TWO LATITUDE VALUES
C      LATITUDE RUNS FROM +90.0 NORTH TO -90.0 SOUTH
C
C      INPUT PARAMETERS
C
C      XLAT1 = MINUEND
C      XLAT2 = SUBTRAHEND
C
       XLATSB=XLAT1-XLAT2
       RETURN
       END
       FUNCTION XLONAV(IDIR,XLON1,XLON2)
C
C      AVERAGES TWO LONGITUDE VALUES
C      LONGITUDE RUNS FROM +180.0 EAST TO -180.0 WEST
C
C      INPUT PARAMETERS
C
C      IDIR = 1 TO COMPUTE AVERAGE LONGITUDE ASSUMING SHORTEST VECTOR BETWEEN TWO
C               MERIDIANS
C           = 2 TO COMPUTE AVERAGE LONGITUDE ASSUMING VECTOR EXTENDING FROM XLON1
C               TO XLON2 IN THE WEST TO EAST DIRECTION
C      XLON1 = FIRST LONGITUDE
C      XLON2 = SECOND LONGITUDE
C
       IF(IDIR.LT.1.OR.IDIR.GT.2)RETURN
       GO TO(1,4),IDIR
 1     IF(ABS(XLON1-XLON2).GT.180.0)GO TO 3
 2     XLONAV=(XLON1+XLON2)/2.0
       RETURN
 3     XLONAV=(XLON1+XLON2+360.0)/2.0
       IF(XLONAV.GT.180.0)XLONAV=XLONAV-360.0
       RETURN
 4     IF(XLON1.GT.XLON2)GO TO 3
       GO TO 2
       END
       FUNCTION XLONSB(XLON1,XLON2)
C
C      SUBTRACTS TWO LONGITUDE VALUES
C      LONGITUDE RUNS FROM +180.0 EAST TO -180.0 WEST
C
C      INPUT PARAMETERS
```

```
C
C     XLON1 = MINUEND
C     XLON2 = SUBTRAHEND
C
      XLONSB=XLON1-XLON2
      IF(ABS(XLONSB).GT.180.0)GO TO 1
      RETURN
 1    IF(XLONSB.GT.0.0)GO TO 2
      XLONSB=XLONSB+360.0
      RETURN
 2    XLONSB=XLONSB-360.0
      RETURN
      END
```

APPENDIX G

COMPUTER ROUTINE FOR DETERMINING THE INCLINATION REQUIRED FOR A SUN-SYNCHRONOUS ORBIT

APPENDIX G

COMPUTER ROUTINE FOR DETERMINING THE INCLINATION REQUIRED FOR A SUN-SYNCHRONOUS ORBIT

```
      PROGRAM SUNSYNCH
      REAL KE,MMC,J2,LHS
      DATA PI/3.14159265/
      DATA RE/6378.214/
      DATA KE/0.07436574/
      DATA J2/1082.28E-6/
      DATA E/0/
      DATA NUM,PINIT,PINT/4,90.0,10.0/
      DATA MAX,CRIT/500,1.0E-5/
      WRITE(6,100)
100   FORMAT(*1    PERIOD       HEIGHT         INCLINATION*,//)
      TWOPI=2*PI
      RDPDG=PI/180.0
      EXP=2.0/3.0
      LHS=360.0/365.24219879
      P=PINIT-PINT
      DO 3 I=1,NUM
      P=P+PINT
      A=(KE*P/TWOPI)**EXP
      H=RE*A-RE
      MMC=360.0/P*1440
      SP=A*(1.0-E**2)
      N=0
      XINC1=90
      XINC2=180
1     N=N+1
      IF(N.GT.MAX)GO TO 2
      XINC=(XINC1+XINC2)/2.0
      RH=RHS(XINC,J2,SP,MMC,E)
      IF(ABS(RH-LHS).LT.CRIT)GO TO 2
      IF(RH.GE.LHS)XINC2=XINC
      IF(RH.LT.LHS)XINC1=XINC
      GO TO 1
2     WRITE(6,101)P,H,XINC
101   FORMAT(1X,F10.2,2X,F10.4,2X,F15.6)
3     CONTINUE
      WRITE(6,102)
102   FORMAT(1H1)
      STOP
      END
      FUNCTION RHS(XINC,J2,SP,MMC,E)
      REAL J2,MMC
      DATA RDPDG/0.017453293/
      XI=RDPDG*XINC
      RHS=-1.5*(J2*COS(XI)/SP**2)*MMC*(1.0+(1.5*J2*SQRT(1.0-E**2)/SP**2)
     * *(1.0-1.5*SIN(XI)**2))
      RETURN
      END
```

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7. Author(s) Eric A. Smith			8. Performing Organization Rept. No. 321
9. Performing Organization Name and Address Department of Atmospheric Science Colorado State University Fort Collins, Colorado 80523			10. Project/Task/Work Unit No. 11. Contract/Grant No. NSF ATM-7807148 NSF ATM-7820375 ONR N00014-79-C-0793
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15. Supplementary Notes

16. Abstracts

An analysis is carried out which considers the relationship of orbit mechanics to the satellite navigation problem, in particular, meteorological satellites. A preliminary discussion is provided which characterizes the distinction between "classical navigation" and "satellite navigation" which is a process of determining the space time coordinates of data fields provided by sensing instruments on meteorological satellites. Since it is the latter process under consideration, the investigation is orientated toward practical applications of orbit mechanics to aid the development of analytic solutions of satellite orbits.

Using the invariant two body Keplerian orbit as the basis of discussion, an analytic approach used to model the orbital characteristics of near earth satellites is given. First the basic concepts involved with satellite navigation and orbit mechanics are defined. In addition, the various measures of time and coordinate geometry are reviewed. The two body problem is then examined beginning with the fundamental governing equations, ie.e the inverse square force field law. After a discussion of the mathematical and physical nature of this equation, the Classical Orbital Elements used to define an elliptic orbit are described. The mathematical analysis of a procedure used to calculate celestial position vectors of a satellite is then outlined. It is shown that a transformation of Kepler's time equation (for an elliptic orbit) to an expansion in powers of ecdentricity removes the need for numerical approximation.

The Keplerian solution is then extended to a perturbed solution, which considers first order time derivatives of the elements defining the orbital plane. Using a formulation called the gravitational perturbation function, the form of a time variant perturbed two body orbit is examined. Various characteristics of a perturbed orbit are analyzed including definitions of the three conventional orbital periods, the nature of a sun-synchronous satellite, and the velocity of a non-circular orbit.

Finally, a discussion of the orbital revisit problem is provided to highlight the need to develop efficient, relatively exact, analytic solutions of meteorological satellite orbits. As an example, the architectural design of a satellite system to measure the global radiation budget without deficiencies in the space time sampling procedure is shown to be a simulation problem based on "computer flown" satellites. A set of computer models are provided in the appendices.

17. Key Words and Document Analysis

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