

Effectiveness of a Vertical Barrier against Intrusion of Flood Plain Infiltrated Water into an Aquifer

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Abstract. In this paper the process of unsaturated aquifer recharge from a river channel and its flood plain laterally confined with fully penetrating barriers is investigated. The same basic methodology described previously by the authors [Morel Seytoux et al., 1988] for the cases of recharging areas in a homogeneous medium is applied here in case of heterogeneities due to damming. An approximate solution is obtained by matching two one-dimensional flows, a vertical and a horizontal one. The formulation leads to an integro-differential equation, which can be solved numerically. The results show the effectiveness of the barrier and its dependency upon project parameters.

1. Introduction

“Groundwater barriers” are often called different structures, like subsurface dams and storage dams. They can be made of artificial material, such as concrete, or of more or less impermeable natural material like sand or silt. Damming ground water for conservation purposes is certainly not a new concept. Groundwater dams were constructed on Sardinia in Roman times and damming of ground water was practised by ancient civilisations in North Africa. In the last century they have been developed and applied in many parts of the world, notably in arid region as a method of overcoming water storage. More recently groundwater barriers have been considered for river management purposes. In the last few years their use for aquifer protection in river restoration projects has given this method renewed attention. The use of groundwater barriers is of increasing interest since they allow the storage of the infiltration water below the river bed and drastically reduce the lateral spreading of the percolating water. Thus the possibility of interaction between the surface water and the lateral aquifer outside the barriers is definitely reduced.

2. Aim of the research

Aim of the present study is to develop a conceptual model and a mathematical code for the development of the water table rise under stationary and unsaturated recharge conditions below the flood plain of a river in case the lateral spreading of the mound is limited by the presence of a barrier made of soil of known characteristics. The scope of practical interest for engineering projects is to define the effectiveness of the barrier and its dependency upon project parameters.

3. Description of the problem

The aquifer is assumed to be homogeneous, anisotropic, unconfined and has a finite depth. The geometry of the problem is shown in Fig.1. In the case of low and medium water levels, the river flows within its current bed, which shows a smaller permeability due to the clogging process. In this case the

recharge rate below the river channel is negligible and the water table level is horizontal. The groundwater level is initially the same at both sides of the barrier.

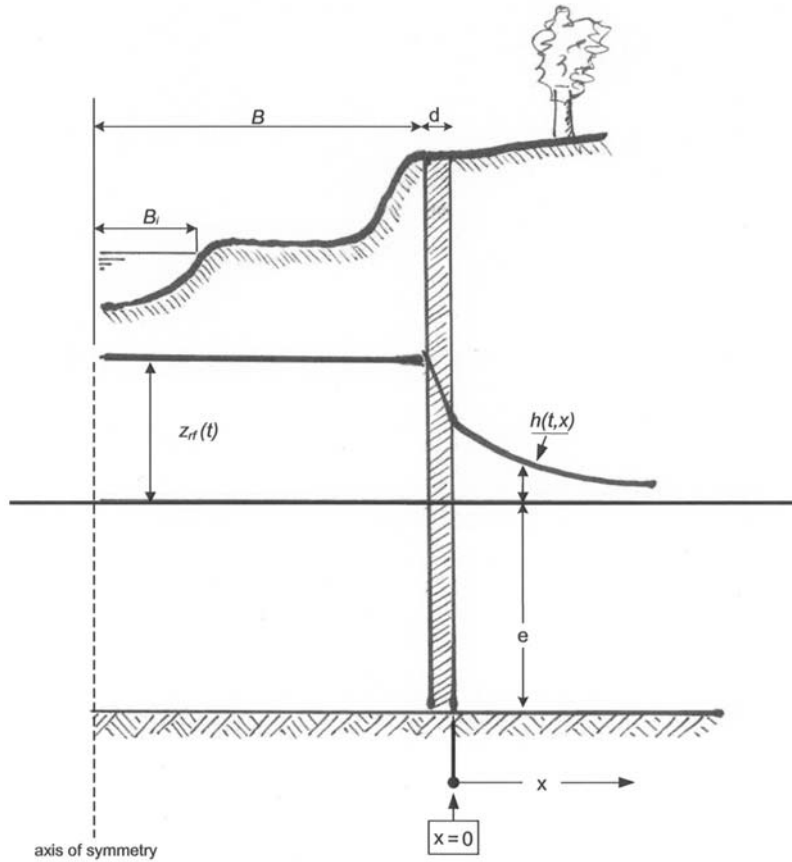


Figure 1. The geometry of the problem

In the case of high water levels, the river channel fills up its flood plain, which has a higher permeability than the clogged layer in the current bed. Thus it is realistic to assume that the recharge rate below the flood plain is significant, but still not big enough to ensure saturated conditions. The unsaturated descending front reaches the water table and at this time the reflected front rises filling the empty pore space in the unsaturated aquifer below the flood plain. In this case the rising mound is almost flat and can be approximated as an upward horizontal front. The hydraulic head drop within the barrier is the difference between the level of the reflected front and the water table level on the external side of the barrier. The water table level laterally keeps decreasing and at infinity reaches conditions at rest.

4. Basic approach

Clearly the flow phenomenon following the arrival of the percolation flux at the initial location of the water table is two-dimensional in nature. Following a general methodology previously applied in stream-aquifer interaction modelling and aquifer recharge [Morel-Seytoux H.J. and C. Miracapillo, 1988] the two-dimensional nature of the flow process is approximated by the matching (linking and coupling) of a dimensional vertical flow process and an horizontal one (Fig.2). In this manner the complexity due to the higher dimensions is avoided. The justification for the approximation is that, except in the barrier, where a relevant

hydraulic head drop happens along a short flow path, in the rest of the aquifer the vertical velocity is negligible and Dupuit assumptions still hold. The major difficulty in the procedure is the development of the matching condition which leads ultimately to an integral equation formulation. A discretized form of the integral equation can be solved easily to provide numerical answers to a specific problem.

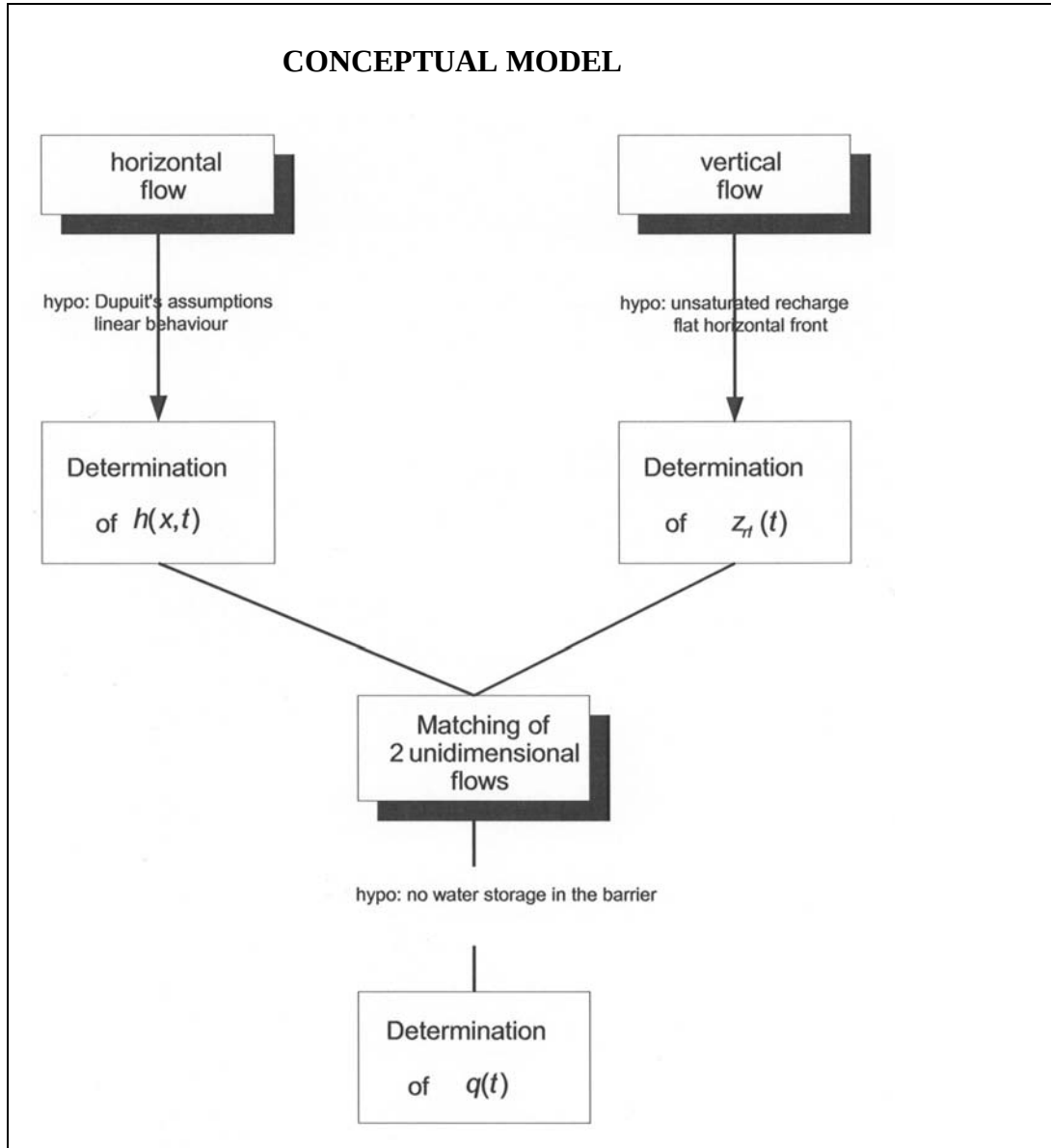


Figure 2. Conceptual Model

5. Mathematical formulation of the problem

The mathematical formulation presented in this paper is not the most general because it is based on several assumptions which are related to the geometry and to the physics of the problem, namely:

- the width of the river channel is much smaller than the width of the flood plain
- the thickness of the barrier is much less than the width of the river bed
- the barriers are located at the external limits of the flood plain

- the geometry of the problem has an axis of symmetry in the centre of the river bed
- the bottom of the aquifer is horizontal
- the initial water table profile is horizontal everywhere (condition at rest)
- the infiltration rate below the flood plain is less than the vertical hydraulic conductivity
- due to the clogged layer at the bottom of the river bed, the infiltration rate below the river channel is negligible
- the descending flux below the flood plain is unsaturated
- infiltration in the unsaturated zone occurs under steady conditions

In this situation it is realistic to assume that when the unsaturated front hits the water table, the infiltration flux will split into two parts, one reflected (filling upward the residual pore space still available after the passage of the descending unsaturated front) and one transmitted outside the barrier (into the aquifer not overlain by the recharge area).

In Fig.1 the position of the water table at time t is indicated. For reasons of symmetry only one half of the system is considered and displayed. The profile is approximated between the two barriers by a plateau at a distance z_{rf} above the initial water table level and by a profile $h(x, t)$ laterally outside the barriers. The origin for the x axis is the external side of the barrier. Within the barrier the water table is approximated by a straight line connecting the aquifer elevations at the two sides, namely z_{rf} and $h(0, t)$. Thus the hydraulic head drop in the barrier

is $z_{rf}(t) - h(0, t)$.

In the following mathematical formulation the change of the water content in the barrier due to the development of the mound is not considered. It is assumed that this quantity is negligible compared to the change of water storage in the neighbouring part of the aquifer.

5.1. Determination of the mound height $h(x, t)$

The evolution of the mound $h(x, t)$ height is governed by the linear form of the one-dimensional Boussinesq equation:

$$\frac{\partial h}{\partial t} = \frac{K_H e}{\phi} \frac{\partial^2 h}{\partial x^2} = \frac{T}{\phi} \frac{\partial^2 h}{\partial x^2} = \kappa \frac{\partial^2 h}{\partial x^2} \quad (1)$$

where t is time, K_H is fully saturated hydraulic conductivity in the horizontal direction, e is saturated thickness, ϕ is effective porosity, x is abscissa in the horizontal direction, T is transmissivity (or, more precisely, horizontal transmissivity) and κ is aquifer diffusivity. In the particular case when the lateral flux at the boundary is constant and of value 1 and for an initially flat water table, the solution of the boundary value problem for eq. (1) is well known [e.g. Carslaw, H.S. and Y.C. Jaeger, 1959] and is:

$$K_{h,q}(x, t) = \frac{2(\kappa t)^{1/2}}{T} \left\{ \frac{\exp(-x^2 / 4\kappa t)}{\pi^{1/2}} - \frac{x}{(4\kappa t)^{1/2}} \operatorname{erfc} \left[\frac{x}{(4\kappa t)^{1/2}} \right] \right\} \quad (2)$$

In eq. (2) $K_{h,q}(x,t)$ is the mound height response to a unit step excitation of lateral flux (also known as the unit step response of height due to excitation of boundary flux). When the boundary flux varies with time it is known from linear system theory [e.g. Dooge, 1973; Morel-Seytoux, 1979; Morel-Seytoux H.J. et al., 1988] that the response $h(x,t)$ is related to the boundary flux $q(t)$ (a discharge per unit length of strip, thus of dimension area per time) by the convolution equation:

$$h(x,t) = q(0)K_{h,q}(x,t) + \int_0^t K_{h,q}(x;t-\tau) \frac{\partial q(\tau)}{\partial \tau} d\tau \quad (3)$$

where $q(0)$ is the value of $q(t)$ at time zero. In particular at $x = 0$ the value of h is:

$$h(0,t) = h(t) = 2\sqrt{\frac{t}{\pi\phi T}}q(0) + \int_0^t 2\sqrt{\frac{t-\tau}{\pi\phi T}} \frac{\partial q(\tau)}{\partial \tau} d\tau \quad (4)$$

because the value of $K_{h,q}(x,t)$ for $x = 0$, as deduced from eq. (2), is:

$$K_{h,q}(0,t) = 2\sqrt{\frac{t}{\pi\phi T}}$$

5.2. Determination of the position of the reflected front

It is known from the theory of multiphase flow in porous media [e.g. Morel-Seytoux, 1969, 1973, 1987] that the velocity of propagation, V_f of a front of water contents is given by the expression:

$$V_f = \frac{v_w^+ - v_w^-}{\theta^+ - \theta^-} \quad (5)$$

where v_w^+, θ^+ and v_w^-, θ^- are the respective values of water velocities (Darcy velocities) and water contents on both sides of the front.

Application of eq. (5) to the problem at hand yields the result:

$$\frac{dz_{rf}}{dt} = \frac{r(B - B_i) - q(t)}{(\tilde{\theta} - \theta_o)(B - B_i) + (\tilde{\theta} - \theta_i)B_i} \quad (6)$$

where θ_o is the uniform and steady water content in the unsaturated zone under the flood plain (above the rising water table), $\tilde{\theta}$ is the water content at saturation and θ_i is the initial water content under the river channel.

In this case the water storage in the barrier and its change during the rise of the water table are neglected. This is justified by the fact that the water storage under the flood plain equal to

$$(\tilde{\theta} - \theta_i)z_{rf}B_i + (\tilde{\theta} - \theta_o)z_{rf}(B - B_i)$$

is much bigger than the water storage in the barrier, which is equal to

$$\frac{1}{2}(\theta_d - \theta_{di})(z_{rf} - h)d$$

where θ_d and θ_{di} are respectively the water content at saturation and the initial water content in the barrier and d is the thickness of the barrier.

5.3. Determination of the transmitted flux (matching equation)

Using the flow net approach [e.g., Morel Seytoux and Miracapillo 1988; Morel-Seytoux et al., 1990] the recharge rate $q(t)$ can be expressed by Darcy's law as:

$$q = \bar{A} \frac{\Delta H}{\frac{\Delta L_V}{K_V} + \frac{\Delta L_H}{K_H}} \quad (7)$$

where $\bar{A}$ is the average cross-sectional area of the flow tubes, ΔH is the piezometric head drop, K_V and K_H are the values of the horizontal and vertical hydraulic conductivities, ΔL_V and ΔL_H are the average lengths of the flow tubes in the horizontal and vertical directions which carry water away from the infiltration area to the lateral boundary ($x = 0$) across the aquifer underneath the flood plain and across the barrier.

Thus
$$\Delta L_V = \Delta L_{Vo} + \Delta L_{Vd}$$

and
$$\Delta L_H = \Delta L_{Ho} + \Delta L_{Hd}$$

where the subscript o and d refer to the aquifer and to the barrier. The average length of the flow lines is obtained by the arithmetic mean of the boundary flow lines, namely:

$$\Delta L_{Ho} = 0.5B \quad \Delta L_{Hd} = d$$

$$\Delta L_{Vo} = 0.5(z_{rf}(t) + e)$$

$$\Delta L_{Vd} = 0.5(z_{rf}(t) - h(o, t))$$

Similarly, the average area of the flow per unit length is

$$\bar{A} = 0.5[B + (e + h(o, t))]$$

$\Delta H(t)$ in eq. (8) is the hydraulic head drop from the recharge surface along the flow pathline to the boundary at $x=0$.

Since the water table mound under the flooding plane is approximated with a horizontal “plateau”, the piezometer head can only drop within the barrier from the value of $z_{rf}(t)$ to the value of $h(0, t)$.

Thus
$$\Delta H(t) = z_{rf}(t) - h(0, t)$$

Substitution of the above written equations in eq.(7) gives

$$q(t) = K_{eq}(t) [z_{rf}(t) - h(0, t)] \quad (8)$$

where K_{eq} in the next general case is

$$K_{eq} = K_{Ho} \frac{\alpha \delta (B + e + h)}{\alpha (z_{rf} - h + 2d) + \delta (z_{rf} + e) + \alpha \delta B} \quad (9)$$

where $\alpha = K_{Vo} / K_{Ho}$ and $\delta = K_d / K_{Ho}$.

If z_{rf} remains during the rise of the water table much smaller than e (and consequently h too),

$$K_{eq} = K_{Ho} \frac{\alpha \delta (B + e)}{\alpha (z_{rf} - h + 2d) + \delta e + \alpha \delta B}$$

and if $(z_{rf} - h)$ is much smaller than d (for instance for little values of t) the expression for K_{eq} simplifies to:

$$K_{eq} = K_{Ho} \frac{\alpha \delta (B + e)}{\alpha 2d + \delta e + \alpha \delta B}$$

and in this case the dependence of K_{eq} on the time is eliminated.

5.4. Coupling of the various components

The horizontal flow $q(t)$, which spreads laterally out of the barrier is given by eq.(8), with a constant K_{eq} .

The expression for $z_{rf}(t)$ can be obtained from integration of eq. (6), with the result:

$$z_{rf}(t) \cong \frac{r(B - B_i)t}{(\tilde{\theta} - \theta_o)(B - B_i) + (\tilde{\theta} - \theta_i)B_i} - \frac{1}{(\tilde{\theta} - \theta_o)(B - B_i) + (\tilde{\theta} - \theta_i)B_i} \int_0^t q(\tau) d\tau \quad (10)$$

The water table height at $x=0$, $h(0)$, is obtained from eq.(4) in case $q(0) = 0$ since $z_{rf}(0) = h(0) = 0$, yielding:

$$h(t) = \int_0^t 2\sqrt{\frac{t-\tau}{\pi\phi T}} \frac{\partial q(\tau)}{\partial \tau} d\tau \quad (11)$$

Substitution of the expressions for z_{rf} and h from eqs.(10) and (11) into eq.(8) leads to an integro-differential equation for the unknown $q(t)$, specifically:

$$\begin{aligned} & \left[(\tilde{\theta} - \theta_o)(B - B_i) + (\tilde{\theta} - \theta_i)B_i \right] q(t) + K_{eq} \int_0^t q(\tau) d\tau + \\ & + \left[(\tilde{\theta} - \theta_o)(B - B_i) + (\tilde{\theta} - \theta_i)B_i \right] \frac{2K_{eq}}{\sqrt{\pi T \phi}} \int_0^t \sqrt{t-\tau} \frac{\partial q(\tau)}{\partial \tau} d\tau \\ & = K_{eq} r(B - B_i)t \end{aligned}$$

which can be rewritten

$$\begin{aligned} & B_{eq} q(t) + K_{eq} \int_0^t q(\tau) d\tau + \frac{2K_{eq} B_{eq}}{\sqrt{\pi T \phi}} \int_0^t \sqrt{t-\tau} \frac{\partial q(\tau)}{\partial \tau} d\tau \\ & = K_{eq} r(B - B_i)t \end{aligned} \quad (12)$$

where

$$B_{eq} = (\tilde{\theta} - \theta_o)(B - B_i) + (\tilde{\theta} - \theta_i)B_i.$$

5.5. Solution of the equation

Equation (12) is a linear integral equation which can be solved in principle using for example the Laplace transform technique (e.g. Abdulrazzak and Morel-Seytoux, 1983). However in this case complex integration is required and successive integrals of special functions would appear. It is easier to solve eq. (12) numerically by discretizing it in the form:

$$B_{eq} q(n) + K_{eq} \left[(1-\lambda)q(0) + \sum_{v=1}^n q(v) + \lambda q(n) \right] + \quad (13)$$

$$\frac{2K_{eq} B_{eq}}{\sqrt{\pi T \phi}} \sum_{v=1}^n [q(v) - q(v-1)] \lambda(n-v+1) =$$

$$= K_{eq}(B - B_i)rn$$

for $n = 1, 2, 3, \dots, N$

where $q(n)$ is the value of $q(t)$ at discrete integer values of a selected period of time and N is the total time horizon of interest. The parameter λ is analogous to the time weight used in finite differences schemes. A value of 1 indicates a fully implicit scheme, a value of 0.5 indicates a Crank-Nicolson scheme and a value of zero indicates an explicit scheme. For stability reasons it is sometimes necessary to use a value of λ in the range $0.5 < \lambda \leq 1$.

In this case, where time is measured from the moment when the descending wetting front hits the water table, $q(0)$ is zero. In eq. (13) a particular “integrated discrete kernel” is defined as [e.g. Morel Seytoux and Miracapillo, 1988]:

$$\Delta(m) = \int_0^1 \sqrt{m - \tau} d\tau = \frac{2}{3} [m^{3/2} - (m-1)^{3/2}]$$

The discretization leads to an explicit linear system of algebraic equations in the unknowns $q(1), q(2), q(n)$ the ordinates of $q(n)$ at discrete intervals of time.

The most general expression for linear system of algebraic equations is

$$Aq = B$$

where q and B are vectors and A is the matrix of coefficients.

In this case A is a lower triangular matrix and the solution of the linear equations system can be obtained by forward substitution.

The single terms are

$$b_i = K_{eq}(B - B_i)ri$$

$$a_{ii} = B_{eq} + K_{eq} + \frac{2K_{eq}B_{eq}}{\sqrt{\pi T\phi}} D_1$$

$$a_{ij} = K_{eq} + \frac{2K_{eq}B_{eq}}{\sqrt{\pi T\phi}} [D_{i-j+1} - D_{i-j}]$$

where

$$D_i = \left[\frac{2}{3} i^{3/2} - (i-1)^{3/2} \right]$$

6. Results

The model evaluates the effectiveness of a vertical barrier to prevent contaminated infiltrated water from entering the clean surrounding aquifer. The vertical barrier reduces the lateral recharge rate into the aquifer. The reduction

depends on the characteristics of the barrier, namely its thickness and its permeability. The mathematical code is implemented and some numerical results are shown. The evolution of the lateral recharge rate outside the barrier (not below the flood plain) is shown for different values of the thickness of the barrier (Fig.3) and for different values of the hydraulic conductivity of the barrier (Fig.4). Input parameters are: width of the flood plain=40m, width of the current bed=10m, water content of saturation=25%, initial water content=5%, water content of the descending front=20%, vertical hydraulic conductivity of the aquifer=0.01m/hour, horizontal hydraulic conductivity of the aquifer=0.05m/hours, recharge rate =0.0025m/hour, thickness of the aquifer=20m, number of time steps=60.

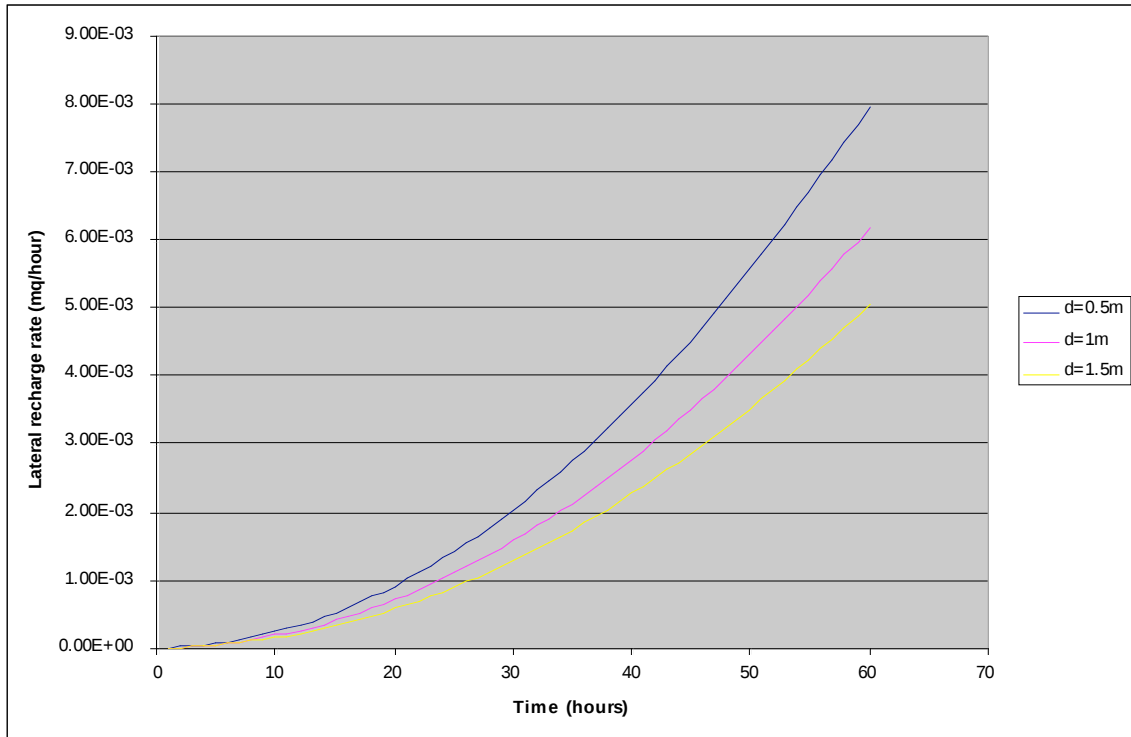


Figure 3. Evolution of the lateral recharge rate into the aquifer outside the barriers for a given value of the hydraulic conductivity of the barrier ($K_d=0.001\text{m/hour}$) and different values of the thickness of the barrier ($d=0.5\text{m}$, 1m , 1.5m).

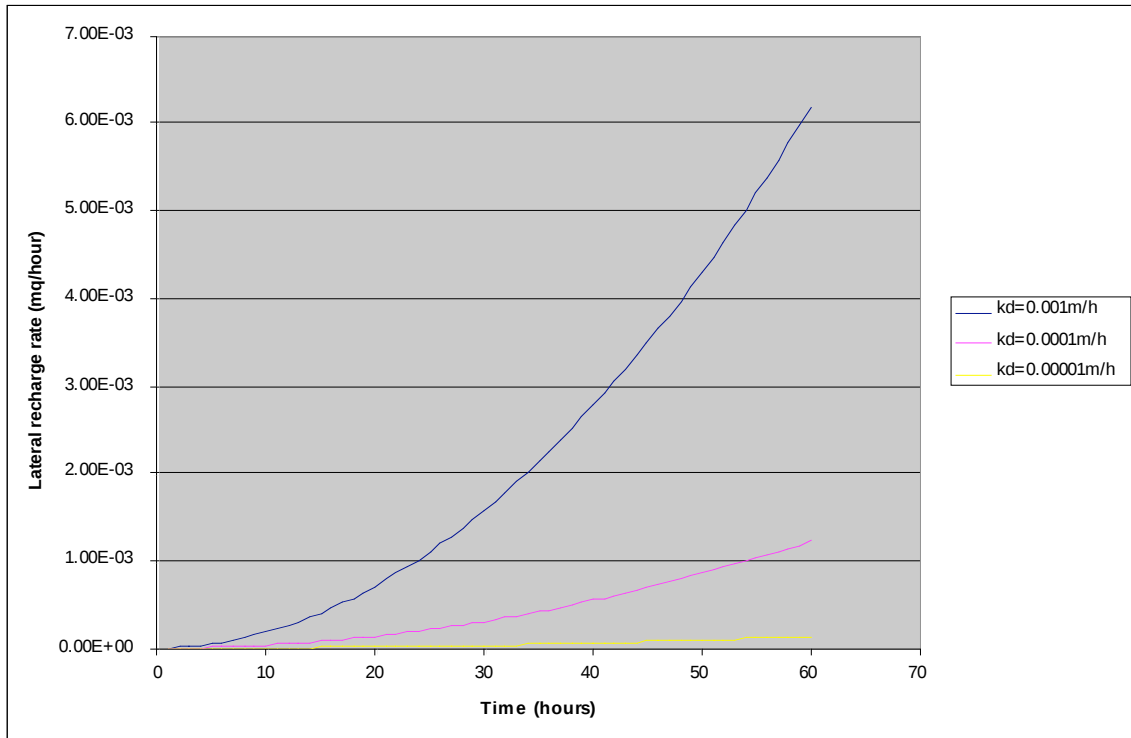


Figure 4. Evolution of the lateral recharge rate into the aquifer outside the barriers for a given value of the thickness of the barrier ($d=1\text{m}$) and different values of the hydraulic conductivity of the barrier ($K_d=0.0.001\text{m/hours}$, 0.0001m/hours , 0.00001m/hours).

7. Conclusion

This study is based on a methodology previously applied and already tested in different cases, namely:

- saturated aquifer recharge from a river channel in an homogeneous aquifer (Abdulrazzak, M.J. and H.J. Morel-Seytoux, 1983)
- unsaturated aquifer recharge in an homogeneous anisotropic aquifer (Morel-Seytoux H.J. et al., 1990)
- unsaturated aquifer recharge from a circular spreading basin in an homogeneous anisotropic aquifer (Morel-Seytoux, H.J. and C. Miracapillo, 1988)

In this study the previous methodology is extended to the case of a flood plain laterally confined with fully penetrating barriers.

The procedure based on the matching of unidirectional flows, a vertical and a horizontal one, allows the determination of integrated characteristics, such as the overall discharge through the barriers and the water stored under the flood plain between the barriers. These integrated quantities are important because they enable to determine the effectiveness of the barriers. This knowledge can be very useful, for instance, in the planning and design of measures for groundwater protection purposes.

This procedure has also additional advantages. Firstly, it accounts for different values for the specific yield in the aquifer below the river channel, below the flooding plane and away from it. In fact the fillable pore space depends on the water stored in the unsaturated zone above the rising water table. Secondly, it can also account in a simply way for anisotropy between the horizontal and the vertical

directions. This is a useful feature since significant anisotropy is the rule rather than the exception.

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