U. S. DEPARTMENT OF AGRICULTURE
AGRICULTURAL RESEARCH SERVICE
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NORTHERN PLAINS BRANCH

Progress Report No. 1

FLOW IN A SLOPING AQUIFER AS AFFECTED BY HYDRAULIC PROPERTIES OF POROUS MEDIA

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R. H. Brooks

Non-funded Contributing Project
of the
Western Regional Research Committee, Project W-51,
Drainage Design for Irrigation Agriculture

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Bouwer (1) has discussed in considerable detail the importance of unsaturated flow above the water table for shallow parallel drains. Unsaturated flow occurring on mild or steep slopes is equally important in regard to interceptor drains. This paper is a discussion of unsaturated flow of water which occurs on a steep slope parallel to the water table.

The importance of flow above the water table tends to increase as the thickness of the zone available for flow above the water table decreases. This zone may be modified by the hydraulic properties of the porous media as well as boundaries, i.e., impermeable layers, water sources, sinks, etc.

The purpose of this report is to show how the properties of porous media affect the magnitude of the flow above the water table and the position of the water table for given boundary conditions.

DESCRIPTION OF MODEL

Consider an aquifer of thickness, t , and of unit width, infinite in extent and dipping at an angle, θ , with respect to the horizontal. The aquifer consists of two homogeneous sands which abruptly join each other in a plane perpendicular to the boundaries of the aquifer as shown in figure 1. The upper boundary of the aquifer is exposed to the atmosphere but protected from evaporation while the lower boundary is assumed to be impermeable. A steady source of water is maintained at

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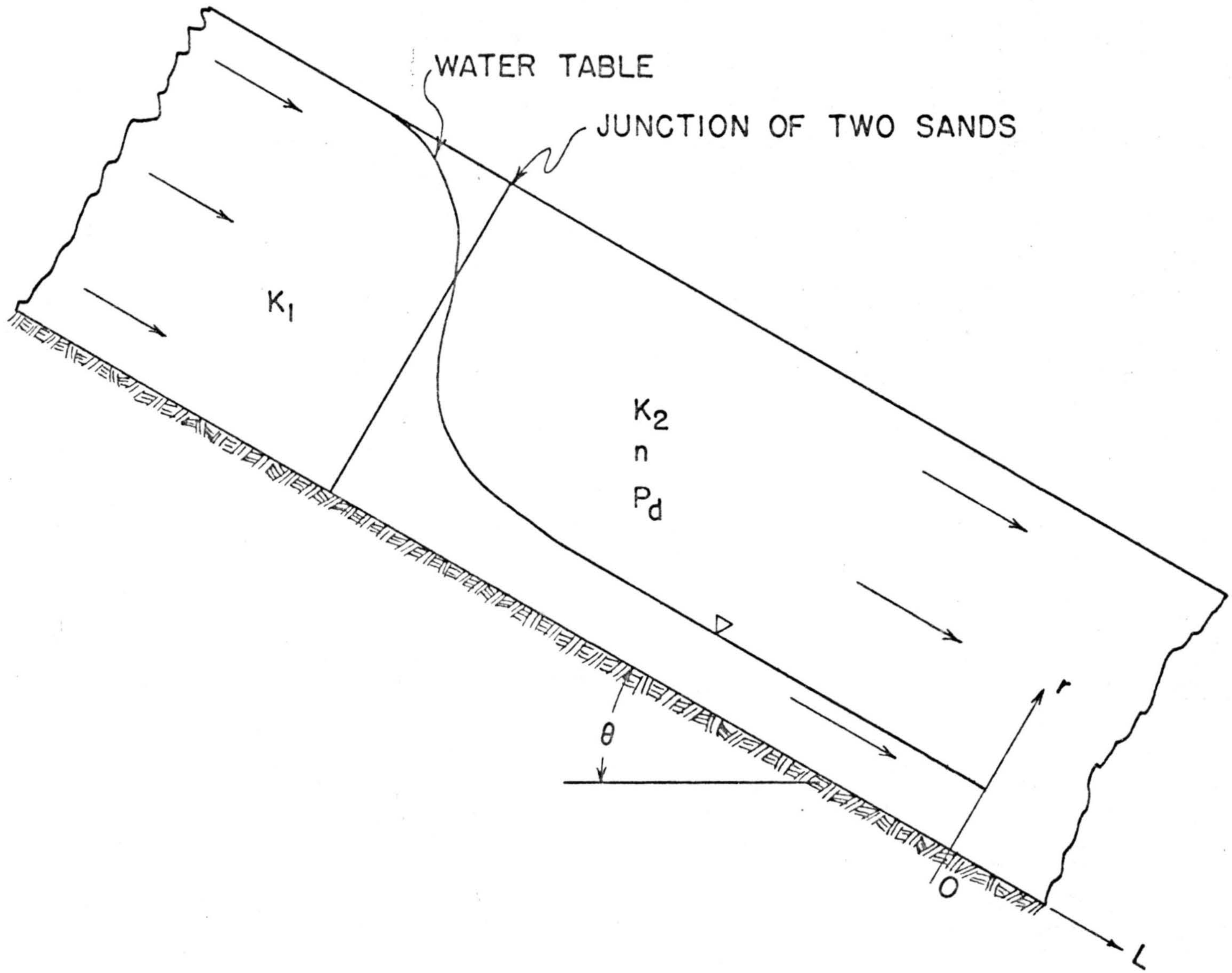

Figure 1 - Model of Sloping Aquifer and Accompanying Boundary Conditions.

a distance above and beyond the influence of the sand junction such that the upstream sand remains saturated in this region and the flow is one dimensional.

If the permeability of the sand downstream from the junction is greater than that of the upstream sand, there will be a transition or lowering of the water table in the vicinity of the junction of the two sands. The amount of lowering of the water table will depend upon the permeability and other hydraulic properties of the downstream sand. After the water table makes the transition across the junction of the two sands, it becomes parallel to the boundaries of the aquifer a short distance downstream from the junction. It is of interest to develop an equation which will predict the position of the water table downstream from the junction where the flow is parallel to the boundaries of the aquifer.

THEORY

The equation of Scott and Corey (1961) can be applied to the air-water system described above provided their assumptions are maintained. Their equation describing the change in capillary pressure P_c with respect to any direction r for a two phase system is given as

$$\frac{dP_c}{dr} = \left[\frac{q\mu}{K_w} + \rho g \sin\theta \right]_w - \left[\frac{q\mu}{K_{nw}} + \rho g \sin\theta \right]_{nw} \quad (1)$$

in which w and nw refer to the two fluid phases, i.e., wetting and non-wetting; K is the permeability, q is the component of volume flow per unit area in the direction r, μ is the fluid viscosity, ρ is the fluid density, g is the force per unit mass due to gravity, and θ is the angle of r with respect to the horizontal. The assumptions given by Scott and Corey for equation (1) are as follows:

1. The system is at steady state.
2. Thermal equilibrium exists.
3. The flow rate, q, does not vary in space for either fluid phase.
4. The properties of the matrix do not vary in space or in direction.

Since air is the non-wetting phase which is assumed to be static for the model under consideration, equation (1) becomes

$$\frac{dP_c}{dr} = \rho g \sin \theta + \frac{q\mu}{K_w} \quad . \tag{2}$$

In the analysis to follow, r, and L will be designated as fixed directions perpendicular and parallel respectively to the boundaries of the aquifer and θ will be the angle the aquifer dips with respect to the horizontal. The analysis will be based upon steady one-dimensional flow in the immediate vicinity of the designated coordinates as shown in figure 1. Since $\frac{dP_c}{dL} = 0$ along any flow path, L, equation (2) becomes

$$q = - \frac{K_w \rho g \sin \theta}{\mu} \quad . \tag{3}$$

For steady conditions, continuity requires that the total flow, Q, must be constant, i.e.,

$$Q = \int_0^t q \, dr = C_1 \tag{4}$$

where t refers to the thickness of the aquifer. Substituting equation (3) into (4) gives

$$- \frac{\rho g \sin \theta}{\mu} \int_0^t K_w dr = C_1 \quad . \tag{5}$$

As the terms preceding the integral are assumed to be constant, equation (5) may be rewritten as

$$\int_0^t K_w dr = C_2 \; . \tag{6}$$

Since C_2 is a constant everywhere in the system where the assumptions apply, then

$$\int_0^t K_{w_1} dr = \int_0^t K_{w_2} dr \; . \tag{7}$$

If the first integral is written for the upstream sand where the entire thickness of the aquifer is saturated, and the permeability of the downstream sand is such that the water table lies within the boundaries of the aquifer; then the right hand side of equation (7) can be written as the sum of two integrals, and becomes

$$\int_0^t K_{w_1} dr = \int_0^{r_{P_d}} K_{w_2} dr + \int_{r_{P_d}}^t K_{w_2} dr \; . \tag{8}$$

The limit r_{P_d} is the perpendicular distance above the lower aquifer boundary where the capillary pressure is equal to P_d. The zone between the lower boundary of the aquifer and r_{P_d} is assumed to be saturated.

In order to determine the relationship between permeability and the distance r above the boundary of the aquifer, it is necessary to reconsider equation (2) and to have a relationship between permeability and capillary pressure. It is assumed that permeability can be related to capillary pressure by the following equations:

$$K_{rw} = \left(\frac{P_d}{P_c}\right)^{\eta} , \; P_c \geq P_d$$

and (9)

$$K_{rw} = 1.0 \; , \quad P_c \leq P_d \; .$$

Relative permeability K_{rw} is the ratio of the saturated permeability K to the permeability K_w, P_c is capillary pressure and the parameters P_d and η are defined in figure 2 which is a typical experimental curve. In the direction r, q is assumed to be zero. Therefore, equation (2) can be integrated and becomes

$$P_c = (r - r_o)\,\rho g \cos\theta \tag{10}$$

where r_o is the position of the water table, i.e., where $P_c = 0$. By combining equation (9) and (10), the functional relationship between K_{rw} and r is found to be:

$$K_{rw} = \left(\frac{P_d}{(r-r_o)\,\rho g \cos\theta}\right)^{\eta} . \tag{11}$$

Substituting equation (11) into equation (8) and recalling the definition of relative permeability gives

$$K_1 \int_o^t dr = K_2 \int_o^{r_{P_d}} dr + K_2 \int_{r_{P_d}}^t \left(\frac{P_d}{\rho g \cos\theta}\right)^{\eta} \frac{dr}{(r-r_o)^{\eta}} \tag{12}$$

in which r_o and r_{P_d} are unknown quantities. Solving equation (12) and rearranging into dimensionless form produces the desired solution to the problem which is

$$\frac{K_1}{K_2} = \left(\frac{P_d/\rho g t}{\cos\theta}\right)\left(\frac{\eta}{\eta-1}\right) + \frac{r_o}{t} - \left(\frac{P_d/\rho g t}{\cos\theta}\right)^{\eta} \frac{1}{(\eta-1)\,(1-r_o/t)^{\eta-1}} \tag{13}$$

where $\frac{r_o}{t}$ is restricted to values less than or equal to $1 - \frac{P_d/\rho g t}{\cos\theta}$.

The parameters η and P_d in the above equation refer to the sand of permeability K_2.

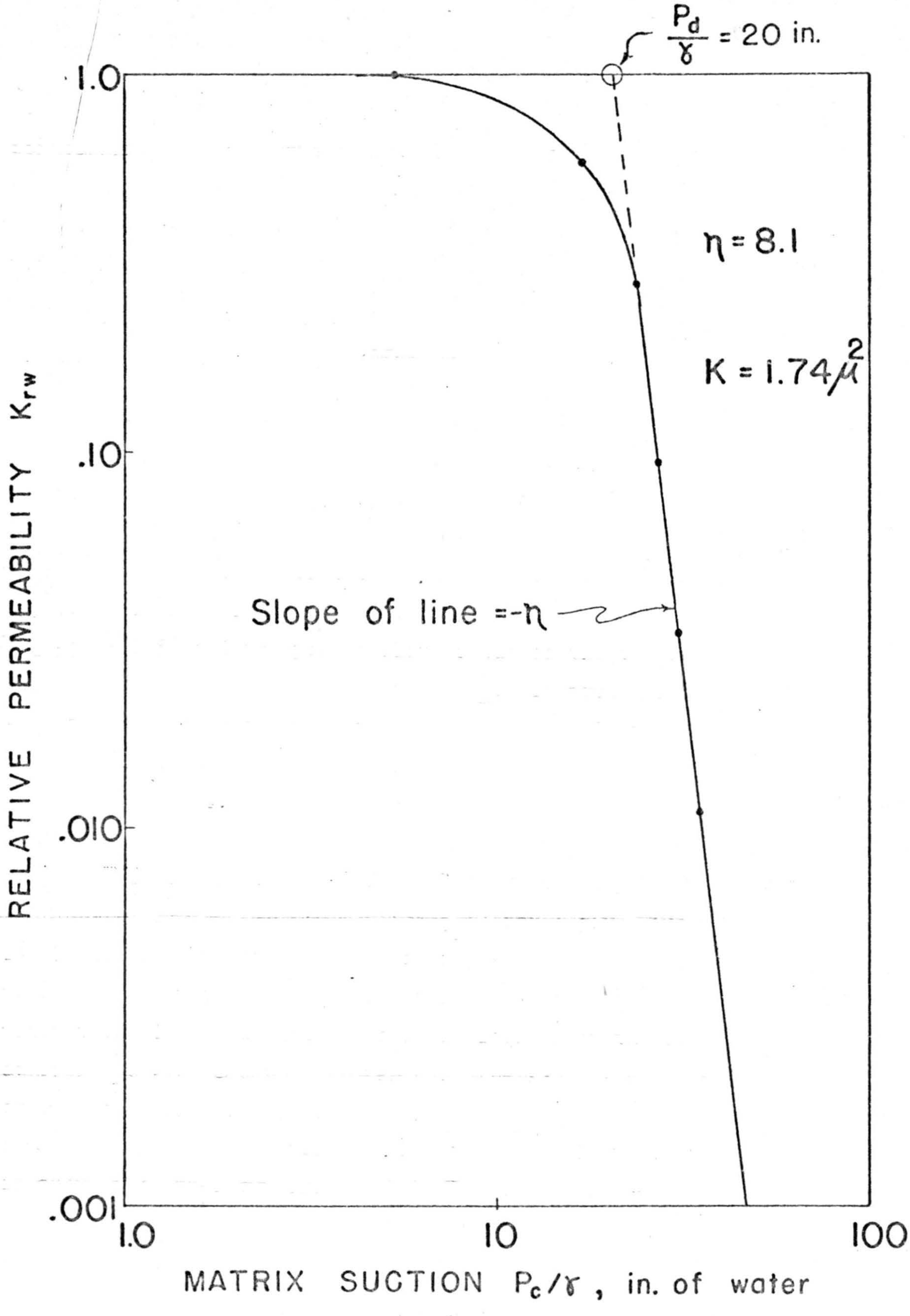

Figure 2 - Relative permeability K_{rw} as a function of matrix suction P_c/γ for a prepared sand.

Equation (13) is shown graphically in figure 3. The relative position of the water table is plotted as a function of the ratio of the permeabilities of the two sands, K_1/K_2, for various values of P_d, and η and an aquifer dip of 30°. Three families of curves have been plotted using the variables P_d and η. For each P_d value, the respective curves are for various values of η.

DISCUSSION

It is convenient to define $P_d/\rho gt$ as the relative thickness of the capillary fringe, i.e., the relative thickness of the zone between the water table and where the saturation takes a sharp reduction in value. As the relative thickness of the capillary fringe $P_d/\rho gt$ becomes small, η has little effect upon the position of the water table. If the capillary fringe, for example, is one-half the thickness of the aquifer ($P_d/\rho gt = 0.5$) and the permeability ratio of the two sands is 0.6, the relative position of the water table $\frac{r_o}{t}$ is found to range in value from -0.3 to 0.02 depending upon the value of η. On the other hand, for a relative thickness of the capillary fringe $P_d/\rho gt$ of 0.05 and for the same permeability values, the relative position of the water table $\frac{r_o}{t}$ is found to range from 0.49 to 0.54.

The ratio of water flowing above the capillary fringe to the total water flowing in the aquifer q_c/Q for a given water table position is plotted as a function of η in figure 4. The curves are for a given ratio of sand permeabilities and for three water table positions. If the water table in the downstream sand is located at the lower boundary of the aquifer $\frac{r_o}{t} = 0$ and the η value of the sand is 2.0, then 50 percent of the total flow in the aquifer is above the capillary fringe. On the other hand, if the η value is 10, then only 22 percent of the total flow is above the capillary fringe while 78 percent is within the fringe.

The model described herein is perhaps only of academic interest, and yet the results indicate that if the flow in the region above the water

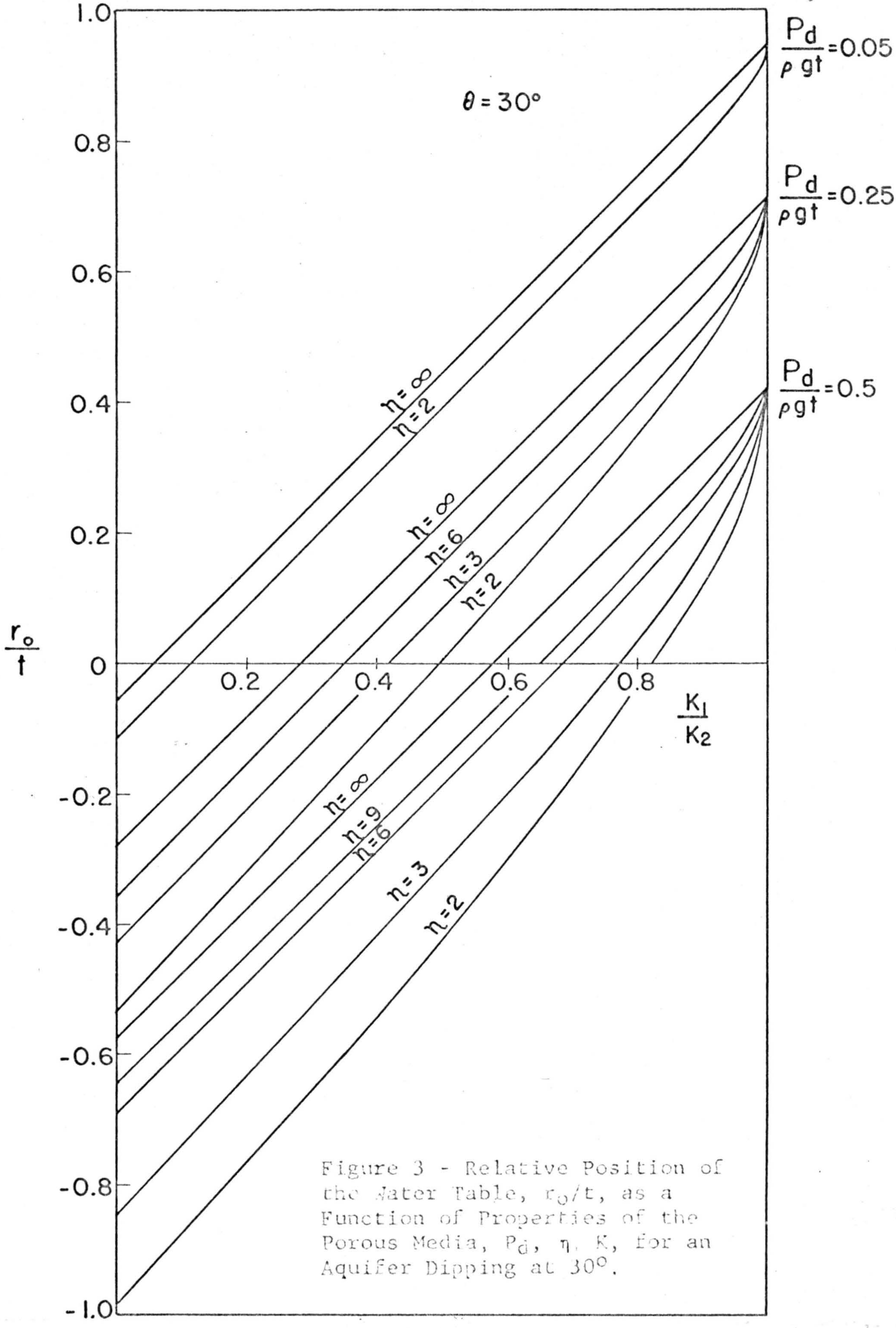

Figure 3 - Relative Position of the Water Table, r_0/t, as a Function of Properties of the Porous Media, P_d, η, K, for an Aquifer Dipping at 30°.

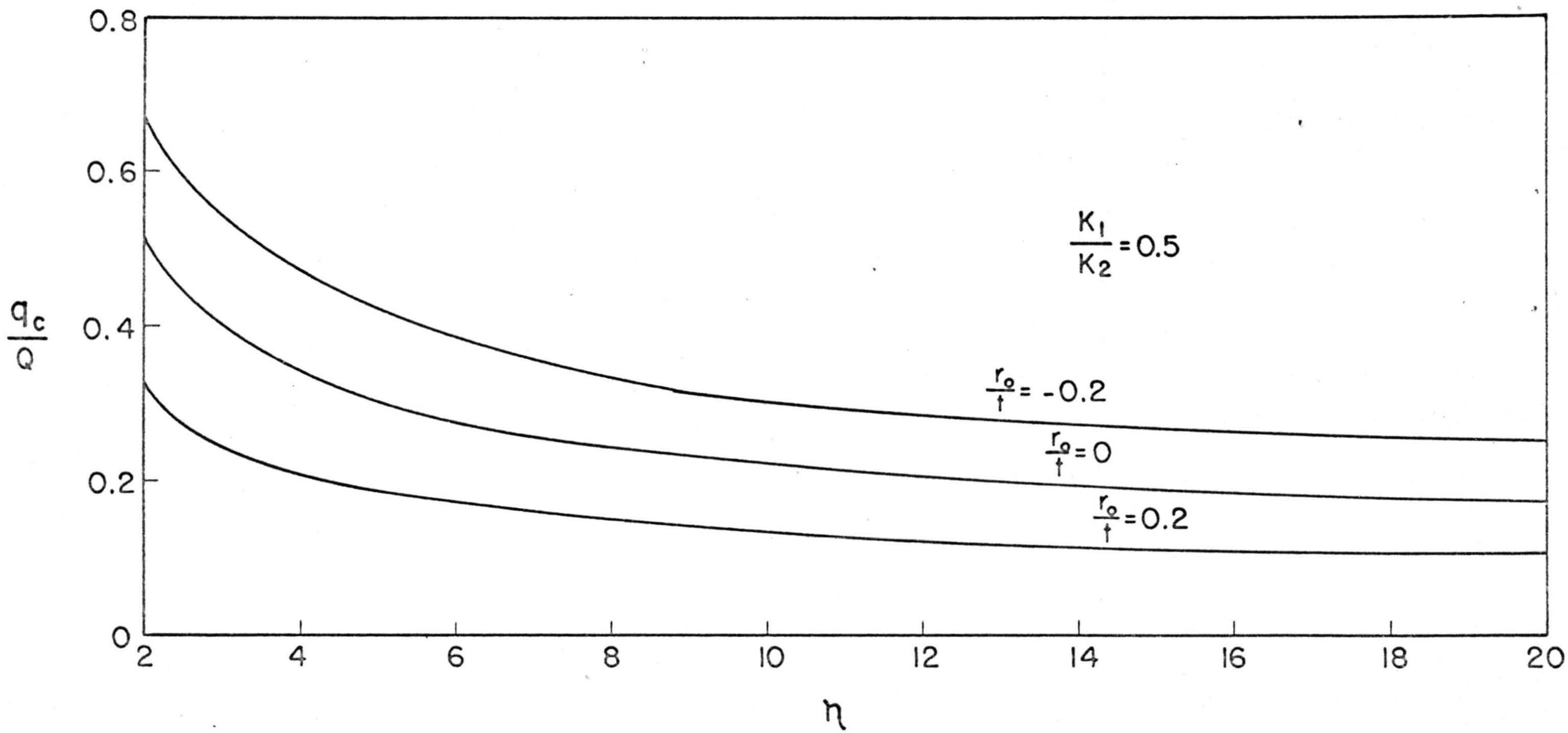

Figure 4 - Ratio of flow above capillary fringe to total flow in the aquifer, q_c/Q, as a function of η for various water table positions, r_o/t, assuming $K_1/K_2 = 0.5$.

table is ignored considerable error may be involved in the design of a drainage system. Simple laboratory techniques for determining the soil parameters which describe the permeability relationship with the capillary pressure and saturation will be helpful in considering unsaturated flow problems.

Future research on this project will involve verification of the proposed theory by using a large scaled model which will contain porous media having an extremely non-uniform pore size distribution. Difficulty has been experienced in obtaining a porous medium suitable for model work.

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