DISSERTATION

ENACTED RESPONSIVENESS AND RESPONSIVENESS AS A DISPOSITION: LEVERAGING AND VALUING STUDENT THINKING

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ABSTRACT

ENACTED RESPONSIVENESS AND RESPONSIVENESS AS A DISPOSITION: LEVERAGING AND VALUING STUDENT THINKING

Over the past few decades there has been increased attention on instructional practices that incorporate and build on student thinking. To effectively implement these practices, it is essential for an instructor to demonstrate responsiveness to student thinking. Although important, responsiveness is not well-understood at the post-secondary level. In this three-paper dissertation, I first use a thematic analysis to analyze twenty-nine articles that discuss constructs related to responsiveness to student thinking from within the K-16 science and mathematics education literature. Results from this analysis shed light on a distinction between responsiveness as a disposition and enacted responsiveness, which is articulated in the definition for responsiveness to student thinking that I propose. To better understand instructional practices that interact with and impact instructor responsiveness, in the second paper I analyze results from an instructional practices survey that was distributed to college calculus instructors at twelve institutions. Results from quantitative analyses highlight categorizations of instructional practice that relate to responsive practices, indicating that responsiveness can occur in both student-oriented and instructor-oriented classes. In the third paper, I investigate instructor responsiveness to student thinking as a disposition (that guides action) and responsiveness to student thinking as an action (the enacted evidence of the underlying disposition), drawing on interview data from eight college calculus instructors. A thematic analysis of the task-based interviews indicated that instructors who exhibited a responsive disposition to their students' thinking enact this through eliciting student thinking, reflecting on student thinking, and responding to student thinking. Further, these instructors view themselves as decision-makers, and thus feel empowered to act on their responsive disposition. The results from this dissertation have implications for researchers interested in teacher growth and professional development providers.

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Chapter 1

Introduction

This dissertation work was initially motivated by discussions with other graduate student instructors about their lack of preparation for teaching at the college level, in particular, their expressed lack of preparedness to engage in instructional practices other than a traditional lecture. Upon entering graduate school, I felt confident in my ability to be in front of a class of undergraduate students to teach and facilitate the learning of mathematics (using a variety of student-centered activities) because of my secondary teaching experiences and professional development. For my dissertation work, I began looking at the knowledge instructors draw upon while teaching that is distinct from their content knowledge, with the goal of understanding ways to support new graduate student instructors. This developed into an interest in considering other intangible aspects that enable effective instruction. In particular, it developed into an interest in investigating instructor responsiveness to student thinking. Since responsiveness to student thinking is not clearly defined, and consequently, not well-understood, I began by developing a definition rooted in the literature, and then sought to understand what responsiveness to student thinking looked like in practice. This dissertation study aims to answer the research questions: (1) What is responsiveness to student thinking? and (2) How do college calculus instructors exhibit responsiveness to student thinking?

1.1 Study Design

The focus of this study is on investigating the responsiveness to student thinking demonstrated by college Calculus 1 instructors. I focus on Calculus 1 because of the high impact nature of this course, as it is a requirement for most students pursuing degrees in science, technology, engineering, or mathematics. Additionally, Calculus 1 is taught by a variety of instructors with varying experiences, ranging from novice graduate teaching assistants to experienced teaching faculty or research mathematicians. This variation allows for increased breadth in the demonstrations of responsiveness to student thinking, which in turn allows for a better understanding of the construct of responsiveness. Further, by focusing on a variety of instructors at the post-secondary level, I aim to better understand how responsiveness is demonstrated at this level, beyond the context of in-the-moment instruction in inquiry-based classes that has been the primary focus of studies considering responsive instruction.

This study draws on a mixed method design combining both quantitative and qualitative data and analyses in order to gain a better understanding of instructor responsiveness than either method could provide on its own (Creswell & Plano Clark, 2017). The quantitative data allows for a broader scope, considering instructional and responsive practices across several instructors at multiple institutions across the United States. The qualitative data, on the other hand, allows for more in-depth analyses focusing on the responsiveness of instructors from one institution. For the first phase of this study, quantitative and qualitative data were collected and analyzed separately using a concurrent triangulation study design (Creswell & Plano Clark, 2017). With this design, quantitative data and qualitative data were collected simultaneously, and then each set of data was analyzed separately using the appropriate methods. Then the results were brought together for interpretation and to triangulate findings. For the third interview with participants, the quantitative and qualitative data analyses informed the interview protocol, drawing on an explanatory design (Creswell & Plano Clark, 2017).

1.2 Study Background: Progress through Calculus

The data collection for this study is supported by the National Science Foundation, under the auspices of the Mathematical Association of America *Progress through Calculus* (PtC) project (NSF DUE #1430540). PtC is a research study looking at the precalculus, Calculus I (differential and integral calculus), and Calculus II course sequence (referred to as P2C2) and related support programs at masters and doctorate granting institutions. Phase 1 of this study involved a census survey that was distributed to all institutions offering a graduate degree (masters or PhD) in mathematics (N=330). This survey had an overall response rate of 67.6%. The project team used the survey results, which included details about the structures and programs in place for the P2C2 course sequence, and graduation statistics (using IPEDS data, IES NCES, 2018) to identify twelve institutions as case study sites for Phase II of the project. The institutions selected for these in-depth case studies represented a variety of institution sizes, locations, and selectivity. Additionally, these institutions' census survey results indicated that they had various interesting structures and systems in place to support students in the P2C2 sequence, such as the use of student-centered pedagogies, student support services, and robust professional development of graduate teaching assistants.

The PtC project team visited the case study sites at least two times over the 2017-2018 and 2018-2019 academic years. In addition to collecting qualitative data through interviews and observations, we distributed surveys to all P2C2 instructors and their students, in addition to sending a survey to all mathematics department members (Apkarian et al., 2019). In this dissertation study, I use the survey results from the Calculus 1 instructors of record who were teaching during the Fall 2017 semester.

I selected one of the twelve PtC case study institutions for additional qualitative data collection for my dissertation study. This institution was selected because of the role of both graduate teaching assistants and teaching faculty in the calculus courses, and because of its standing as a highly selective institution. I made this choice so that I could begin with the assumption that the calculus instructors (both graduate teaching assistants and teaching faculty) had a strong mathematical background with a solid understanding of the content that I would be using for discussion in task-based interviews. This is important to note since there is evidence that a lack of content knowledge can be a hindrance to instructor interpreting and responding to student work (Musgrave & Carlson, 2017; Pascoe & Stockero, 2017).

1.3 Overview of Papers

My dissertation comes in the form of three papers. The first is a comprehensive analysis of the literature related to responsiveness to student thinking, resulting in a definition for responsiveness to student thinking, with sub-definitions of responsiveness as a disposition and as enacted responsiveness. In the second paper, I examine enacted responsiveness by analyzing survey data from instructors from all twelve PtC institutions. In the third paper, I examine responsiveness as a disposition by analyzing interview data from eight instructors at the one highly selective institution.

1.3.1 Paper 1: What is Responsiveness to Student Thinking?

The goal of this paper is to answer the following research questions:

> (1) What terms/constructs are researchers using to discuss instructors' responsiveness to student thinking?
>
> (2) How can responsiveness to student thinking be defined so that it encapsulates these existing and related terms/constructs and provides researchers with a common language to discuss responsiveness to student thinking moving forward?

In order to answer these questions, and research question (1) in particular, I first needed to identify relevant terms, constructs, and descriptions that are used by researchers (or teacher/researchers) to discuss responsive instruction or responsiveness to student thinking. Thus, I conducted a strategic literature search to arrive at a representative corpus of data.

This first paper provides an overview of a selection of existing literature surrounding instructor responsiveness to student thinking, expanding on the themes of responsive instruction in science and mathematics articulated by Robertson, Atkins, Levin, and Richards (2016). I drew on relevant literature cited in Robertson, Scherr, and Hammer's (2016) book *Responsive Teaching in Science and Mathematics*, using snowball sampling to select additional related articles. In addition to this corpus of data, I incorporated articles considering responsive instruction at the post-secondary level from colleague recommendations or from my own experience in the field. It is important to note again that in this selection of literature, it was not my goal to overview every construct that could be considered related to responsiveness to student thinking, but rather to include a representative sample that could be leveraged to propose a definition for responsiveness that encompasses a variety of conceptualizations of related constructs.

In this paper I aimed to distill the literature for each construct to a short summary describing the construct that remained true to the authors' original conceptualization. To answer research question (2), I then conducted a thematic analysis on these descriptive summaries of the constructs, highlighting similarities and differences. Then, leveraging the results from a thematic analysis, I propose a definition for responsiveness to student thinking that consists of two dimensions – responsiveness as a disposition and the enactment of responsiveness (Gehrtz, 2019). These definitions are:

> *Responsiveness as a disposition* is attending to student thinking, mathematical meanings, and needs (including both cognitive and affective needs), and an awareness of the need to provide an appropriate response based on student understandings or contributions. A responsive disposition relies on an instructor's valuing of student thinking.
>
> *Enacted responsiveness* is responding to student thinking by focusing on the substance of student contributions (including those that are incorrect or incomplete) when determining the direction of the lesson, through leveraging and building on student contributions, and by making connections between student ideas and what is accepted as disciplinary conventions.

1.3.2 Paper 2: College Calculus Instructors' Instructional Practices and Responsiveness to Student Thinking

Since the first paper in this dissertation illuminated the distinction between an underlying disposition of responsiveness and an enacted responsiveness, the goals of this second paper are to categorize the instructional practices of college Calculus 1 instructors in order to better understand enacted responsiveness, answering the research questions: (1) In what ways can college calculus instruction be categorized to indicate responsiveness to student thinking? and (2) How is responsive instruction related to other instructor characteristics (e.g., years of experience, involvement in decisions regarding instructional approach or course content)? In order to answer these research questions, an instructional practices survey was distributed to all P2C2 instructors from twelve

PtC case study institutions. This survey included forty-one Likert-scale type questions along with background and demographic questions. The survey data sheds light on instructional practices that are common (or not common), including five practices identified as demonstrating responsiveness to student thinking, among these instructors at the twelve case study institutions.

In order to answer research question (1), I conducted an exploratory factor analysis on survey responses from the ninety-four Calculus 1 instructors that participated to characterize instructors' practices, requesting four factors. These four factors index instructional practices themes of *student-thinking centered instructional practices*, *student-to-student interactions in class*, *equitable instructional practices*, and *instructor-centered instructional practices*.

In order to answer research question (2), I first needed to identify aspects of responsive instruction, in addition to various instructor characteristics. Since the survey included instructional practice items that I identified as demonstrating responsive instruction, and included background and demographic questions, I used the results from the survey and factor analysis to further investigate the relationship between instructional practices and responsive instruction. There is evidence in the literature that suggests instructional approach (e.g., Kuster, Johnson, Keene, and Andrews-Larson, 2017), experience (e.g., Jacobs, Lamb, & Philipp, 2010; Stahnke, Schueler, & Roesken-Winter, 2016), professional development (e.g., Borko, 2004), and involvement in the decision-making for a course (e.g., Schoenfeld, 2011) could impact an instructor's enactment of responsiveness. Thus, I used various statistical analyses to investigate the relationship between the factors characterizing instruction and instructor characteristics (such as their rank, years of experience, decision-making role) from the survey data.

Results highlight the relationship between the factors and the class time spent on instructor-centered and student-centered activities, as well as the limited relationship between various instructor characteristics and instructional practices. Consequently, results highlight that there are additional aspects that enable (or constrain) the enactment of responsiveness that goes beyond instructor characteristics, again pointing to the importance of considering responsiveness as a disposition.

1.3.3 Paper 3: Responsiveness as a Disposition and Its Impact on Instruction

In my work to define responsiveness to student thinking in the first paper of this dissertation, it became clear to me that responsiveness as a disposition is critical in the enactment of responsiveness. Additionally, the second paper looking at Calculus I instructors' practices pointed to the fact that an instructor's enactment of responsiveness goes beyond their experience and role as a decision-maker, supporting the hypothesis that one's disposition might play a critical role in instructional decisions related to responsiveness to student thinking. Thus, the goal of this third paper is to study responsiveness as a disposition, aiming to answer the research question: How do college calculus instructors exhibit responsiveness to student thinking? Specifically, by focusing on both responsiveness as a disposition (that guides action) and responsiveness as an action (the enacted evidence of the underlying disposition).

In this paper, I focus on instructors' practices that shed light on their underlying responsive disposition because dispositions are difficult to classify and understand on their own. Throughout this dissertation, I draw on Thornton's (2006) definition of *dispositions in action*:

> Dispositions are habits of mind including both cognitive and affective attributes that filter one's knowledge, skills, and beliefs and impact the action one takes in classroom or professional setting. They are manifested within relationships as meaning-making occurs with others and they are evidenced through interactions in the form of discourse. (Thornton, 2006, p. 62)

I use this definition because it considers not only the underlying disposition, but also the impact that the disposition has on action. This allows me to leverage the enactment of responsiveness as a way to gain insight into responsiveness as a disposition. Thornton (2006) expresses a similar sentiment in her work stating that this conceptualization of "'dispositions in action' … move[s] beyond reflection, self-assessment and perceptions to examine how dispositions are manifested within the classroom and how they impact pedagogy and ultimately the learning process" (Thornton, 2006, p. 56).

To answer my research question, I conducted a thematic analysis on sixteen hours of interview data that were collected over the course of one academic year, interviewing four Calculus 1 recitation leaders and four Calculus 1 instructors of record from one of the PtC case-study sites. The thematic analysis uncovered practices that shed light on instructors' underlying responsive dispositions and documented how differences in an instructor's role as a decision-maker is related to their enactment of responsiveness.

1.4 Discussion

The overarching research goal of this dissertation is to investigate instructor responsiveness to student thinking, aiming to better understand what this looks like as both an underlying and enacted disposition. The work presented in this dissertation addresses this research goal by defining responsiveness to student thinking, leveraging various researchers' constructs that are in the realm of responsive instruction. I then use this definition and survey results from 94 Calculus I instructors to consider how responsiveness to student thinking is present in various instructional practices that occur both during in-the-moment instruction and during other practices, such as in planning and assessing, shedding light on the enactment of responsiveness. I then examine how a responsiveness to student thinking demonstrates a responsiveness disposition, drawing on interview data from eight Calculus 1 instructors.

Chapter 2

Paper 1: What is Responsiveness to Student Thinking?

There are a number of terms that have been used in the literature to indicate a responsiveness to student thinking, such as teacher follow-up, sensitivity to students, and uptake. Additionally, the terms 'responsiveness' and 'responsive teaching' are used regularly, without clearly defining what exactly is meant or encompassed by these terms. Education researchers would benefit from a clearly articulated definition that encapsulates what responsiveness is so that we can more effectively build off one another's work. In this paper, I use thematic analysis to analyze twenty-nine articles that draw on constructs indicating or related to responsiveness to student thinking from within the science and mathematics education literature. Results from this analysis shed light on a distinction between responsiveness as a disposition and as enacted responsiveness, which is articulated in my proposed definition of responsiveness.

Instructional practices that are responsive to students' thinking have been shown to be beneficial for student learning, enhancing students' conceptual understandings (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989), to promote student agency and voice (Coffey, Hammer, Levin, & Grant, 2011), and to promote equitable participation (Empson, 2003; Robertson, Scherr, & Hammer, 2016). Despite these benefits, there is not a consensus about what responsiveness to student thinking entails, or an agreed upon definition of what is encompassed by responsiveness (e.g., Lineback, 2015; Empson, 2014). Terms such as teacher follow-up, uptake, sensitivity to students, mathematical caring relations, among many others seem to fall under the umbrella of responsiveness, and so the field would benefit from a common definition and term that refers to these student-thinking centered interactions that leverage student contributions to inform instruction.

In the book *Responsive Teaching in Science and Mathematics* (Robertson, Scherr, & Hammer, 2016), Robertson, Atkins, Levin, and Richards summarize the conceptualization, and enactment of responsive teaching as involving: (a) *a foregrounding of the substance of students' ideas*, instantiating "intellectual empathy" by listening to students with the goal of understanding their thinking rather than to simply evaluate or correct it, (b) *recognizing the disciplinary connections within students' ideas*, identifying "disciplinary progenitors" (Harrer, Flood, & Wittmann, 2013) or "seeds of science" (Hammer & van Zee, 2006), and (c) *taking up and pursuing the substance of student thinking*, where the classroom direction emerges and is moved along by the students' contributions (Robertson, Atkins, Levin, & Richards, 2016, p. 1). Although Robertson, Atkins, Levin, and Richards (2016) have provided a summary of themes in literature (largely at the K-12 level in science) involving responsive instruction, there is still a need to provide a more explicit definition of the construct responsiveness. Currently there is a lack of precision in the terms researchers use to describe responsiveness and responsive teacher moves (like "take up"), and there are a number of instances where researchers use "responsive" or "responsiveness" without further articulating what they mean, indicating that there is a shared understanding of these terms despite the fact that they are used to describe a variety of forms and teacher actions. Proposing an explicit definition will allow the education research community to stop creating new terms that encapsulate responsiveness to student thinking and responsive instruction, and researchers will be able to more effectively build on one another's work, furthering our understanding of responsiveness and ways to support instructors in developing responsiveness to student thinking.

In this paper, I provide a brief summary of research related to responsiveness to student thinking and responsive instructional practices, with the goal of developing a definition for responsiveness rooted in these various terms, definitions, and characteristics. Specifically, my research questions are (1) What terms/constructs are researchers using to discuss instructors' responsiveness to student thinking? and (2) How can responsiveness to student thinking be defined so that it encapsulates these existing and related terms/constructs and provides researchers with a common language to discuss responsiveness to student thinking moving forward?

2.1 Selection of Literature Related to Responsiveness

In order to arrive at a collection of literature related to instructor responsiveness to student thinking, I first situated my search for constructs and terms in Robertson, Scherr, and Hammer's (2016) book *Responsive Teaching in Science and Mathematics*. A few of the constructs were explicitly discussed throughout the chapters, and were included in the corpus of data for this analysis of responsiveness. Additionally, I looked through the sources cited by relevant chapters, selecting additional articles if the title appeared to indicate a construct or term related to responsiveness or responsive instruction. This resulted in a collection of twenty articles and one book that aligned with the goals of my analysis. After reading the abstracts and skimming over the relevant parts of the articles, I selected nine articles and one book that met at least one of the following criteria (henceforth referred to as the selection criteria): (1) the authors defined a term or construct within the realm of instructor responsiveness, or (2) the authors explicitly discussed teacher activity that was responsive to students or their thinking (beyond evaluative responses). I then used snowball sampling, adding six additional relevant articles that were described or cited in this initial collection of ten sources. All articles in this initial collection were situated at the K-12 level. To this corpus of data, I added ten articles that I knew were relevant to this analysis based on my experience in the field and through conversations with colleagues. These ten articles also met the selection criteria, and included eight studies at the post-secondary level and two at the K-12 level.

In addition to the corpus of articles selected using snowball sampling, I also conducted a search of abstracts using key words "responsive*" and "math*" [1], excluding "cultural*" since Culturally Responsive Teaching is beyond the scope of the analysis for this paper, in the following databases: Academic Search Premier, PsycINFO, ERIC, and Psychology and Behavioral Sciences. I limited the search to peer reviewed journals in English, and the following Subject/Thesaurus Terms: mathematics education, academic achievement, teaching methods, learning, mathematics, decision making, mathematics teachers, education, science education, and teaching. This resulted in

[1] I use the keyword "math*" in the database search, and do not include the keyword "science", because Robertson, Scherr, and Hammer's (2016) book primarily focuses on literature regarding responsive teaching in science.

84 articles, with ten articles with titles that appeared relevant to the analysis presented here. After reading each abstract and/or paper, three were excluded from the analysis because they described the "Responsive Classroom approach" which is a social and emotional learning intervention that is focused on students' emotional well-being alone without attending to students' mathematical thinking (e.g., Ottmar, Rimm-Kaufman, Berry, & Larsen, 2013), two were excluded because they used the term responsive in relation to the skill of responding in Jacobs, Lamb, and Philipp's (2010) construct of professional noticing which is already included in the corpus of data, and one was excluded because it referred to students' responsiveness to their own learning (Owens, Perry, Conroy, Georghegan, & Howe, 1998). See Table 2.1 for a list of articles included in this analysis of the definitions of responsiveness/responsive instruction, organized by the method of selection.

Table 2.1: Corpus of Data, Selection Method, and Primary Selection Criteria (1 or 2) Satisfied.

	Meets Selection Criteria 1	Meets Selection Criteria 2
Referenced in Robertson, Scherr, & Hammer (2016)	Hackenberg (2005, 2010) Jacobs, Lamb, & Philipp (2010) Lineback (2015) Pierson (2008) Sherin, Jacobs, & Philipp (2011) van Es (2011)	Brodie (2011) O'Connor & Michaels (1993) Hammer, Goldberg, & Fargason (2012) van Zee & Minstrell (1997)
Identified from articles referenced in Robertson, Scherr, & Hammer (2016)	Bishop, Hardison, & Przybyla-Kuchek (2016) Mason (2011)	Collins (1982) Saxe, Gearhard, & Seltzer (1999) Fennema et al. (1996) Franke, Carpenter, Levi, & Fennema (2001)
Recommended by experts in the field/From my own experience as a researcher in the field	Ader & Carlson (2019) Empson (2014) Jaworski (2002) Potari & Jaworski (2002) Teuscher, Moore, & Carlson (2015)	Franke, Turrou, & Webb (2011) Johnson (2013) Kuster, Johnson, Keene, & Andrews-Larson (2017) Kuster, Johnson, Rupnow, & Garrison-Wilhelm (2018) Laursen & Rasmussen (2019)
Database Search	Adhami (2001)	Abdulhamid & Venkat (2018) Kim (2017) Koole & Elbers (2014)

The goal of this analysis is to identify and understand some of the terms and constructs researchers use to describe responsiveness and responsive instruction, and to work towards a definition that clearly articulates what is encapsulated by these terms. Consequently, I exclude literature related to aspects that support (or constrain) instructors in their ability to be responsive to students' thinking, such as their mathematical knowledge for teaching (e.g., Ball, Thames, & Phelps, 2008) and generative listening (the act of listening to students' contributions with the intent to make meaning out of it rather than simply evaluate it; e.g. Yackel, Stephan, Rasmussen, & Underwood, 2003). Although these are important features to consider in order to more fully understand responsive instruction, consideration of what enables responsiveness is outside of the scope of this paper.

2.2 Overview of Literature Related to Responsiveness

The following provides a brief overview of several of the constructs related to responsiveness to student thinking from this corpus of data. I begin by defining and describing the constructs identified as related to responsiveness (satisfying the first criterion for selection), drawing on the definitions given by the authors and extracting meaning for these terms based on how the author describes them. I then overview teacher activity that is related to responsive instruction (satisfying the second criterion for selection). It is important to note that although this selection of terms/constructs is not exhaustive, it is representative of the myriad of ways that researchers discuss responsiveness to student thinking, drawing on research from both the K-12 and post-secondary levels, as well as examples from science and mathematics.

2.2.1 Constructs Related to Responsiveness to Student Thinking

The constructs and terms discussed in this section are those that satisfied Criteria 1 of the selection criteria, in that the various authors defined or described a specific term that fell under the umbrella of instructor responsiveness to student thinking.

Responsiveness

Pierson (2008) uses the term *responsiveness* to describe classroom discourse that is reflected in teacher follow-up moves, specifically the extent to which the teacher takes up[2] student ideas (validating them) and focuses on student thinking (rather than emphasizing teacher thinking) during in-the-moment instruction and teacher-student interactions. Additionally, she asserts that responsiveness can be thought of "as a sensitivity or genuineness in conversation: an attempt to understand what another is thinking displayed in how a conversational partner builds, questions, probes, clarifies, or takes up that which another has said," articulating that responsiveness necessarily includes an eliciting of student thinking (Pierson, 2008, p. 25). Additionally, this highlights that responsiveness goes beyond simply responding by indicating it is a "sensitivity" evidenced by how an instructor engages with student contributions.

In later work, Bishop (Pierson), Hardison, and Przybyla-Kuchek (2016) refer to *responsiveness* as "a characteristic of interactions wherein students' mathematical ideas are present, valued, attended to, and taken up as the basis for instruction" (p. 1173). They highlight the importance of engaging with student ideas and responding in order to better understand their mathematical meanings, asserting that to understand in and of itself is a responsive act. In their study, they developed a framework for coding responsiveness in whole class mathematics discussions by focusing on student contributions and teacher follow-up moves. The student contributions were classified as none, minimal (contributions involving routine calculations, recalling facts, etc.), considerable (contributions indicating strategies, but without justification), and substantive (contributions involving mathematical ideas grounded with justifications, or doing other instances of mathematical argumentation). Teacher moves were then coded as demonstrating low, medium, or high levels of taking up student contributions, indicating the degree to which student contributions were used to inform instruction.

[2]Pierson (2008) does not define 'takes up', but here (and throughout the remainder of the paper) I interpret this to mean the identifying, incorporating, and leveraging of student ideas and contributions during instruction. Further, I only use the phrase 'take up' in describing various conceptualizations of responsiveness when the phrase has been explicitly used by the authors in their own descriptions of the construct.

The Redirection

Lineback (2015) expands upon Pierson's (2008) work on responsiveness, developing the notion of *the redirection*, identifying it as a new construct that is "sensitive enough to capture and characterize differences in how a teacher responds to his or her students" (Lineback, 2015, p. 421). Lineback (2015) notes that the redirection allows researchers to capture nuances in the variability of a teacher's responsiveness in both the short term, by highlighting fluctuations moment to moment, and in the long term, by how the teacher uses or changes the use of their redirections over time. She defined a *redirection* as a

> teacher's bid to shift, refocus, or redirect the attention of the class from one scientific phenomenon/question/activity to another. A redirection consists of a question or comment submitted by an instructor to his or her class that stops the ongoing flow of the students' work and attempts to shift their attention either by presenting an alternative (scientific) locus for the students' attention or by prompting the students to engage in a new (scientific) activity. (p. 430)

Thus, a key component of the redirection is the shift in either focus or activity. Additionally, a redirection is simply the teacher's "bid" to shift the direction and that it does not depend on the redirection actually being acknowledged by the students.

Inherent in a redirection is the valuing of student thinking, and creation of space for student contributions to arise. Simply acknowledging a student comment is not a redirection since it does not allow an opportunity for the student's idea to be considered by the class. Similarly, if a teacher poses a probing question to encourage a student to elaborate on their contribution, then this is also not considered a redirection since it does not shift the class activity nor allow for the student's contribution to be considered by the whole class (Lineback, 2015).

In her coding scheme, Lineback (2015) distinguishes between activity redirections that are responsive to students' idea and those that are not. Additionally, she differentiates between focus redirections that are connected to student contributions in the current discourse sequence (and therefore are a responsive redirection) and those that seek to revisit a previous contribution (indicating a delayed responsive redirection). Focus redirections that are not tied to student ideas or contributions are considered redirections that are not responsive. Lineback (2015) provides an even finer-grained characterization of classroom discourse that demonstrates a responsive redirection considering if the teacher or students' thinking is on display, the role of elaboration techniques (ranging from a mechanism to encourage the explanation of a scientific idea to getting students to compare, contrast, or extend their contributions), and how the teacher leverages student contributions (e.g., probing the class to consider an individual's contribution, interpret a contribution, or poses an individual student's question back to the class).

Lineback (2015) asserts that the redirection allows for a finer-grained understanding of instructor responsiveness, characterizing various ways instructors take up student thinking in the moment, by how they respond and allow student ideas to impact class activity. She further asserts that this allows researchers to understand variation in responsiveness in both the short term and long term.

Decentering

Decentering (Piaget, 1955) typically refers exclusively to the cognitive processes underlying making sense of and interacting with someone else's thinking (with the potential to enable responsiveness). Teuscher, Moore, and Carlson (2015) go further when they draw on the notion of *decentering* to consider a teacher's attempt to understand students' reasoning and the actions they take in order to respond while teaching. The authors describe decentering as an individual setting aside their own way of thinking in an attempt to understand what someone else understands, which relies on reflective student-teacher interactions. Drawing on Steffe and colleagues' (1983) definition, Teuscher, Moore, and Carlson (2015) describe how it is essential for a teacher to construct a sophisticated second-order model to decenter, which are "[the models] observers may construct of the subject's knowledge in order to explain their observations (i.e., their experience) of the subject's states and activities" (Steffe, von Glasersfeld, Richards, & Cobb, 1983, p. xvi). They note that decentering involves a valuing of the substance of student contributions, including contributions that are incorrect because these still serve to inform the teacher's response.

Teuscher, Moore, and Carlson (2015) point to the role of decentering in teachers' in-the-moment decisions based on student thinking. They distinguish between teachers who are "oriented to engage in decentering acts" (p. 450), and those who are not, highlighting the ways that teachers follow up and respond based on either the students' way of thinking or the teacher's way of thinking. Teachers who are not oriented to decenter often assume that students will interpret everything as they intend and will be able to "grasp everything his or her students do" (p. 450). Teachers who are oriented to engage in decentering, on the other hand, "appreciate the instructional power gained by considering and attempting to make sense of student thinking" (p. 450), and often leverage their second-order models of student thinking to inform instruction. Additionally, a teacher's capacity to decenter influences their "decisions to pose (or not pose) a question, the nature of teachers' questions, [and] the quality of their explanations" (Teuscher, Moore, & Carlson, p. 453). Decentering is necessary for a teacher to be able to "effectively [focus] on student thinking" which involves "discerning and acting on student thinking in ways that support student learning" (p. 434). If a teacher is not creating second-order models of a student's thinking, then they cannot effectively respond based on student understanding.

Ader and Carlson (2019) also leverage the construct of decentering in order to document how teachers make sense of and use student thinking during in-the-moment teacher-student interactions, attending specifically to teacher questions and discussions. Drawing on classroom video and interview data, Ader and Carlson (2019, p. 10)

> identify patterns and distinctions in the teacher's attentiveness to students' thinking, including the degree to which the teacher attempted to understand students' thinking, and the degree to which the teacher's model of the students' thinking informed her instructional decisions, explanations, and questions.

This analysis shed light on five levels documenting teachers' propensity to decenter, evidenced by the degree to which teachers' questions and comments showed interest (or not) in student thinking, appeared to make sense of student thinking (or not), and whether they were focused on student answers, student thinking, or the teacher's thinking. Teachers demonstrating the highest level of decentering appeared to make sense of student thinking, and then took into account not only the student's thinking, but also how the student might interpret their (the teacher's) follow up response to help the student advance their thinking.

Sensitivity to Students

Another construct that is aligned with responsiveness to student thinking is an instructor's *sensitivity to students*. Barbara Jaworski describes *sensitivity to students* as "the teacher's knowledge of students' thinking, attention to their needs and the ways in which the teacher interacts with individuals and guides group interactions" (2002, p. 72). Sensitivity to students includes sensitivities to both cognitive and affective needs, with cognitive sensitivities concerning "the appreciation and recognition of students' thinking which can then be developed further by appropriate challenge" (Potari & Jaworski, 2002, p. 374) and affective sensitivities involving "fostering students' personal beliefs in and valuing of their ability to do mathematics and think mathematically" (Potari & Jaworski, 2002, p. 374). There is evidence that sensitivity to students can be present in both planning and in-the-moment instruction.

Sensitivity to students is one component in the teaching triad, an analytical tool that has been used to analyze secondary mathematics teaching, along with *management of learning* and *mathematical challenge* (Potari & Jaworski, 2002). Management of learning includes the facilitation of the classroom environment, including planning tasks and establishing norms, and mathematical challenge describes the challenges (typically in the form of a task) posed to students requiring new/extended mathematical thinking. Potari and Jaworski (2002) highlight the dual nature of sensitivity to students and mathematical challenge, which indicates the important connection between attending to student needs and a response aligned with those needs. Additionally, when an instructor demonstrates sensitivity to students, they tend to respond to the student's current way of reasoning rather than funneling explanations and conversations back to their own way of thinking, demonstrating responsiveness, and indicating a propensity to decenter.

Jaworski points to previous research that has focused on whether or not sensitivity to students "supports effective challenge (or not)" (2002, p. 73), noting that mathematical challenge is considered successful when students are able to engage with mathematical ideas that progress their thinking. When sensitivity to students and mathematical challenge complement one another to promote students' progress in mathematical thinking, then these two constructs are considered to be in *harmony*. It is possible for sensitivity to suppress challenge, particularly when an instructor is hesitant to challenge students within their zone of proximal development because of how the student might perceive or respond to the challenge, indicating that responsiveness to affect can come at the expense of responsiveness to student thinking.

Mathematical Caring Relations

An additional construct that is related to responsiveness to student thinking is that of *mathematical caring relations*. Amy Hackenberg (2010) states that a mathematical caring relation "involves holding the work of orchestrating mathematical learning for students together with an orientation to monitor and respond to energetic fluctuations that may accompany student-teacher interactions." This involves responsiveness to student thinking and responsiveness to student energy, with student energy being in the same vein as sensitivities to student affect in the work of Jaworski and colleagues. Additionally, Hackenberg asserts that mathematical caring relations are inseparable from learning, in that

> mathematical caring relations occur in the context of aiming for mathematical acts of learning. Mathematics teachers may act as carers in general, but they start to act as mathematical carers when they work to harmonize themselves with and open new possibilities for students' mathematical thinking, while maintaining focus on students' feelings of depletion and stimulation that may accompany student-teacher interactions. (2005, p. 237)

Hackenberg (2010) describes an instructor's ability to decenter as central to the notion of mathematical caring relations. Hackenberg (2010) asserts that decentering is not simply knowing that a student thinks differently, but acting on this to help the student make progress in their understandings. If a follow-up task presented by the teacher is not appropriate because of the teacher's lack of decentering, then the student might be overwhelmed and understanding might be hindered. Alternatively, if the teacher suggests a problem that is too easy, then the student's learning/understanding is not enhanced. Hackenberg (2010) describes the balance between the mathematical challenge presented and the students' energy as *harmony*, indicating the instructor is monitoring and making adjustments in response to their understanding of student thinking. This also is in alignment with how Jaworski and colleagues have described harmony between student sensitivities and mathematical challenge. It is important to note that Hackenberg (2010) distinguishes between cognitive decentering that requires "intuitive responsiveness" and a "deliberate ... formulation of how to interact" (p. 240), where the latter requires additional analytical cognitive activity in order for the instructor to decide how to respond compared to the former. She also notes that decentering often requires both intuitive and deliberate analytical activity.

Professional Noticing

van Es (2011, p. 135) described *noticing* as "heightened sensitivities to particular aspects of [experts'] work, as well as techniques for analyzing, using, and inquiring into these features of their practices." Although there are many different views on what is involved in teacher noticing (Mason, 2011), most scholars agree that it consists of attending to and making sense of particular events during instructional practices (Sherin, Jacobs, & Philipp, 2011). Jacobs, Lamb, and Philipp (2010) refine this further and conceptualize the construct of *professional noticing* as "the set of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 172).

Professional noticing allows for unpacking the decision-making that occurs during in-the-moment instruction that incorporates and builds on children's understanding, rather than attending to the variety of things that a teacher might notice during instruction. A teacher's attention to their students' strategies is important because it is one way of documenting how they gain insight into student understandings. Additionally, a teacher's interpretation of student understandings sheds light on both the details of how they think about a particular student's understanding as well as if their reasoning is in line with research on students' mathematical development. For responding, Jacobs, Lamb, and Philipp (2010) point to the fact that they are interested in a teacher's intended responding, not focusing on a 'correct' response based on student contributions or artifacts, but rather whether or not the teacher's response is consistent with their interpretation of student understanding and is consistent with the research on student mathematical thinking and development.

In their investigation of teacher professional noticing, Jacobs, Lamb, and Philipp (2010) focus on in-the-moment decision-making that teachers engage with in their interactions with students, and not long term decision-making involved in planning or preparing. This situates professional noticing skills as happening nearly simultaneously, as one "integrated teaching move" (Jacobs, Lamb, & Philipp, 2010, p. 173) because the teacher needs to attend, interpret, and decide how to respond during the real time of a classroom interaction.

Responsive Teaching

Empson (2014) uses the term *responsive teaching* as teaching that takes in the evidence of student thinking, interprets student thinking, and "responds helpfully" (p. 24), as she acts as a teacher/researcher studying her own professional noticing. She details her reflections and decision-making as she works with four students on their understanding of base ten once a week for nine weeks. Empson discussed her decision-making in posing problems, considering when to push students to use a more sophisticated method and when to hold back, aiming to support student thinking rather than take over their thinking with hers. She also articulates that her extensive knowledge of student thinking informed her in-the-moment decisions, but that it did not provide a prescription of how to respond. Empson uses the word sensitivity to highlight attention to student understanding, aiming to balance this with her goals for progressing the mathematics. This aligns with Jaworski's (2002) and Hackenberg's (2010) notion of harmony between pedagogical goals and student affect or energy.

2.2.2 Teacher Activity Related to Responsive Instruction

Various literature uses the words responsive or responsiveness, but do not explicitly define these terms, indicating an assumption that there is a shared understanding of what is encompassed by these terms. In this section, I provide an overview of a selection of literature that refer to responsiveness or responsive teacher activity and explicitly describe some of the activity that these constructs entail, satisfying selection Criteria 2. I first overview research that discusses teacher activity related to responsiveness to student thinking, distinguishing between teacher activity that involves eliciting student thinking and teacher activity that involves responding to student thinking. I then discuss aspects of specific pedagogical approaches and curriculum that relate to responsiveness to student thinking.

Teacher Activity Involving Eliciting Student Thinking

Integrated assessment refers to "the extent of opportunity for students to reveal their understandings, to receive interpretations of their contributions, and to provide interpretations of others' contributions" (Saxe, Gearhart, & Seltzer, 1999, p. 11). The rating scale used for integrated assessment sheds light on a teacher's effort to gain access to student understandings by documenting how often questions are asked, and whether or not these questions actually provide insight into student understanding.

The notion of integrated assessment is in line with the work of Fennema et al. (1996) and Franke et al. (2001) on *eliciting student thinking*. Fennema et al. (1996) use a teacher's eliciting of student thinking to document levels of teacher demonstration of Cognitively Guided Instruction (CGI). In lower levels (level 1 and 2) of CGI, the teacher provides less opportunities for students to engage with the mathematics and "elicits or attends to children's thinking or uses what they share in a very limited way" (p. 412). Higher levels (levels 4-A and 4-B) of CGI indicate that the teacher is eliciting student thinking, and is also using these student contributions (along with knowledge of either general or specific student thinking) to inform instruction. Franke et al. (2001) focused on how teacher's elicited student thinking "beyond what was required to solve the particular problems posed" (p. 660), informing how the teacher facilitated the sharing of student strategies (particularly those that were incomplete or unclear).

Reflective toss is another term that documents discursive patterns during instruction related to eliciting student thinking. van Zee and Minstrell (1997) describe a reflective toss as consisting of a student contribution, followed by a teacher question that results in additional student comments. They leverage the reflective toss to document the ways a teacher's question influences and elicits subsequent student thinking. van Zee and Minstrell (1997) highlight the following emergent goals in their analysis involving the reflective toss: (1) it serves to engage all students in considering a method proposed by a student, (2) it aids in refining and clarifying aspects of a previously discussed method, and (3) allows for the evaluation of an alternative method.

Teacher Discourse Involving Student Thinking

Teacher *uptake* of a student response occurs when the teacher references a student contribution immediately preceding their comment or question, indicating a cohesion of ideas (Collins, 1982). Additionally, the construct of *follow-up* described by Brodie (2011) aligns with uptake as it focuses on how the teacher "picks up on a contribution made by a learner", explicitly referencing their idea (Brodie, 2011, p. 180). However, Brodie (2011) asserts that follow-up encompasses teacher moves beyond that which would be considered uptake.

Franke, Turrou, and Webb (2011) also use the term *follow-up* in their work, referring to it as teacher moves that result in the articulation of additional student thinking. They documented the role and importance of teacher follow-up to student thinking and mathematical explanations, concluding that it plays a critical role in shaping opportunities for students. In particular, they found that it was not enough for teachers to simply ask "How did you solve that?", but that it was important for teachers to press for explanations that were correct and complete. Additionally, the authors point to the importance of teacher follow-up being connected to what the student has articulated (which aligns with Collins's (1982) and Brodie's (2011) definitions), and creating a classroom environment with norms and expectations that every student's thinking is valued.

Koole and Elbers (2014) also acknowledge the importance of the connection between a teacher's response and students' contributions. They use conversation analysis to highlight how *scaffolding* and teacher responsiveness are interactional phenomenon, aiming to understand *local interactional responsiveness*. As the teacher provides scaffolding, they provide help that "is contingent on and responsive to the level of expertise the child is showing" (p. 58). From this perspective, the researchers are not focused on teacher intent or cognition, but rather how the teacher demonstrates their understanding of the students' thinking during the interaction in how they respond. However, Koole and Elbers (2014) note that teachers appear to draw on their experience and knowledge of the student that goes beyond the in-the-moment evidence of student thinking during the interaction, stating that “the teacher’s explanations often appear to be based on an assumption of the nature of the student’s difficulty, rather than on a diagnosis of the student’s understanding” (p. 67). For example, if a student were to say that they did not understand, and the teacher did not attempt to find out what the student’s problem was, “start[ing] the explanation on the basis of a hypothetical problem, rather than the problem the student showed him” (p. 66), then this would demonstrate an interaction that is not responsive at the content level.

O’Connor and Michaels (1993) consider teachers’ *revoicing* of student contributions in their work documenting how teachers align student ideas with conventions of the mathematical community. Specifically, in revoicing, the teacher reformulates student contributions in order to align the utterances with that which would be “more recognizable to the wider world”, linking student “experiences and inventions with the conventional knowledge categories of the wider world” (O’Connor & Michaels, 1993, p. 326). Teachers can also leverage revoicing as a way to “draw further inference” from a student’s utterance, expanding and clarifying student contributions still allowing the student to claim or disclaim ownership of the contribution. Additionally, revoicing can also be used to “lend power and authority to the student’s relatively weak voice” (ibid, p. 327), especially when a student is hesitant to participate or when the student’s contribution is not initially considered by the class.

Adhami (2001) uses the term *tactical* to refer to a type of question that is “couched in the particular pupils’ responses, and which involves negotiation of meaning, handling of misconceptions and attention to minute and idiosyncratic steps of reasoning” (p. 28), and argues that these questions can be seen as *responsive questioning*. Tactical questions are responsive questions in that they are contingent on student contributions, and have the potential to impact the direction of the class lesson. The motivations behind tactical questions extend beyond simply eliciting student participation, but also include a valuing of the substance of a student contribution, demonstrating a “genuine acceptance and engagement with pupils’ ideas in their own language” (p. 34). Adhami (2001) states that it is possible to anticipate some tactical questions, drawing on one’s experience or knowledge of common misconceptions in student thinking, but tactical questions still require a teacher to generate these in the moment, using "largely, the actual phrases offered by the youngsters" (p. 30) – which allows the students to have a sense of ownership of the development of the ideas.

Abdulhamid and Venkat (2018) use teachers' *elaboration* to students' contributions during classroom interactions as a way to examine responsive teaching, offering some "stages of implementation' towards the ideals of responsive teaching" (p. 81). In their classifications of elaboration, Abdulhamid and Venkat (2018) consider situations where a student offers an incorrect response (which they refer to as breakdown situations), situations where a student offers a correct response and the teacher views their strategy or solution as inefficient (sophistication situations), and situations where the correct response is incorporated into the discourse of the class (individuation/collectivization). The authors detail themes in teachers' elaboration and lack of elaboration, pointing to how elaboration can help move a teacher to more responsive instruction. Specifically, Abdulhamid and Venkat (2018) distinguish between elaboration in breakdown situations that are focused on the student's incorrect answer or focused on the task. In the sophistication situations, the teacher's elaboration can take the form of offering a more efficient strategy, eliciting a more efficient strategy, or interrogating the student's offer for efficiency. When elaboration is provided in the individuation/collectivization situations, Abdulhamid and Venkat (2018) point to it taking the form of a class chorus that is either confirming or interrogating individual students, a form that is confirming, interrogating, repeating, or decompressing individual student's offer to the whole class, or a form of collective reasoning where the teacher "facilitates and creates a collaborative form mathematical discourse" (Abdulhamid & Venkat, 2018, p. 91).

Although the discourse moves discussed in this section have the potential to demonstrate responsiveness to student thinking, high demonstrations of these teacher moves are not synonymous with high levels of responsiveness (Pierson, 2008). It is not enough to simply have the instructor's response take on the linguistic form of explicitly referencing the student contribution, but rather "responsiveness considers the underlying intent and function of the speech act" (Pierson, 2008, p. 29). Abdulhamid and Venkat (2018) also point to this when they state that "responsive teaching is considered in the context of classroom interaction with a view to increasing teachers' awareness of the need to provide appropriate follow-up to students' offers (answers or contributions) in ways that extend or expand possibilities for mathematics learning" (p. 81), highlighting the importance of how teachers use their follow-up moves to facilitate opportunities to expand on student contributions, creating space for them to learn.

Responsive Teacher Activity in Inquiry-Based Instruction

Johnson (2013) and colleagues (Kuster, Johnson, Rupnow, & Garrison-Wilhelm, 2018; Kuster, Johnson, Keene, & Andrews-Larson, 2017) have documented and described a variety of instructional practices that instructors enact during inquiry-oriented instruction, an instructional approach situated in the theory Realistic Mathematics Education (see e.g., Gravemeijer & Doorman, 1999) focused on students' reinvention of the mathematics (for more information on inquiry-oriented instruction see e.g., Kuster, Johnson, Keene, & Andrews-Larson, 2017; Larsen, 2013; Laursen & Rasmussen, 2019). Although they have not identified and defined a distinct construct that falls under the responsiveness umbrella, they have pointed to specific practices that demonstrate responsive teacher activity during in-the-moment inquiry instruction. Specifically, Kuster, Johnson, Rupnow, and Garrison-Wilhelm (2018) identified seven local instructional practices that support the four inquiry-oriented principles (see e.g., Kuster, Johnson, Keene, & Andrews-Larson, 2017). Namely, the fourth practice, "Teachers are responsive to student contributions, using student contributions to inform the lesson", supports the principle of *building on student contributions* and the principle of *developing a shared understanding*. This practice highlights that responsiveness to student contributions includes the use of student ideas in informing the direction of the class. Additionally, by using student contributions to inform the lesson, instructors are able to create new instructional space for students to interact with one another's strategies and understandings, thus creating the opportunity for the development of rich mathematical meanings (Kuster, Johnson, Rupnow, & Garrison-Wilhelm, 2018). Kuster, Johnson, Rupnow, and Garrison-Wilhelm (2018) developed a rubric describing various levels of the local instructional practices (including teachers being responsive to student contributions), noting that there are differing amounts of instructional space created depending on the degree to which an instructor is responsive to their students' ideas or if they tend to direct discussions back to their own thinking/contributions.

Laursen and Rasmussen (2019) draw attention to instructional practices in inquiry-based instruction that are cognitively responsive to students' mathematical thinking in that they make use of student ideas and strategically respond with talk that is "well timed and well targeted to surface and explore students' prior knowledge, to help students organize or connect important ideas, and to support students' changing views" (p. 3). Further, they note it is essential for instructors to employ "adaptive and responsive facilitation skills" as they create space for students to engage with the mathematics, extending and connecting student ideas with the accepted conventions of the mathematical community.

Additionally, Johnson (2013) has identified various teaching episodes that demonstrate *responsive mathematical activity*, aiming to better understand the relationship between student mathematical activity and instructor mathematical activity. In summary, she described teacher's mathematical activity as:

> … interpreting students' mathematical reasoning and contributions; analyzing and evaluating students' mathematical contributions, conjectures, and arguments; and identifying mathematical connections, both between multiple student contributions and between student contributions and known mathematical results,

which conveys responsiveness to student thinking (Johnson, 2013, p. 773). Johnson's (2013) summary of responsive teacher activity further highlights the importance of identifying mathematical connections between student contributions and known mathematical results, which aligns with one of the goals of teacher revoicing (O'Connor & Michaels, 1993).

Hammer, Goldberg, and Fargason (2012) also describe responsive teaching as coordinating student contributions and identifying productive connections to the discipline, stating, "the teacher works first to engage students in the pursuit, and then to support them in their pursuit in ways that afford progress toward canonical practices and ideas… a responsive approach can 'cover' a similar set of topics as in a more traditional one, but the route emerges responsively and may be idiosyncratic" (p. 55). They highlight that responsive teaching "adapt[s] and discover[s] instructional objectives responsively to student thinking" (p. 55), identifying how student ideas can be leveraged to foster productive scientific thinking.

2.2.3 Curriculum and Resources Supporting Responsive Instruction

There has been some work looking into the influence curriculum plays in supporting student-thinking centered instruction. In this section I briefly discuss resources for instruction that are designed to enable instructor responsiveness.

Cognitively Guided Instruction (CGI) is a professional development program designed to support instruction that is rooted in responsiveness to student thinking. CGI was developed through research on student thinking in mathematics, and includes models of student mathematical development. These models can be used to "interpret, transform, and reframe [instructors'] informal or spontaneous knowledge about students' mathematical thinking" (Carpenter, Fennema, & Franke, 1996, p. 3). Instructors using CGI draw on their knowledge of both general research-based student development and understanding, and specific knowledge of their own students' understanding in order to make instructional decisions, responding to student needs and building on what a student knows (Carpenter et al., 1989).

Kim (2017) has considered the impact of curriculum resources on teachers' becoming more responsive to students' mathematical thinking. She uses the terms *responsive teaching* and *student-thinking-centered teaching* almost synonymously, indicating that responsive teaching is necessarily focused on and rooted in students' mathematical thinking. She states that "key practices of student-thinking-responsive teaching[3] are understanding students' mathematical thinking and using it well in instruction" (Kim, 2017, p. 344), and that "student-thinking-responsive teaching trie[s] to establish a productive mathematical discourse community in the classroom and expects students to explain their ideas, critique or build on their classmate's idea, and justify the former in light of the latter and of subsequent discussion" (ibid, p. 344). In Kim's (2017) discussion

[3]I take 'student-thinking-responsive teaching' (Kim, 2017) to be synonymous with responsive teaching that is student-thinking-centered.

of one of the teacher's implementation of the curriculum, she highlighted opportunities for the teacher to become more responsive to student thinking, including: anticipating student strategies (in pre-assessment phase), understanding student sense-making and their articulation of their ideas (in the implementation phase), and knowing more about student strategies and improve one's own pedagogical strategies (in the post-assessment phase). Note that this indicates that there are opportunities for instructors to be responsive to their students' thinking during a variety of phases of instruction, including during planning, in-the-moment instruction, and in reflection after assessing student understanding and strategies.

Hammer, Goldberg, and Fargason (2012) describe their initial thoughts on designing a responsive curriculum that is grounded in student reasoning, rather than starting from "the established body of knowledge…that one expects students are ready to learn" (p. 55). They posit that responsive teaching cannot rely on a predetermined sequences of tasks because it is the moment-to-moment decisions, based on their interpretation and reasoning about student contributions, that makes responsive instruction. Nonetheless, Hammer, Goldberg, and Fargason (2012) are working to develop a curriculum that "supports teacher attention and responsiveness to student reasoning and participation" (p. 67), drawing on knowledge and trajectories of student thinking. Their vision for the curriculum includes "generative launching questions" as well as suggestions (i.e. a "menu of possibilities" (ibid, p. 69)) for follow-up activities aimed to support teachers in recognizing opportunities to categorize and explore student ideas, support student argumentation, and support students in their writing or representations.

In addition to his discussion on *tactical questioning*, Adhami also points to *strategic questioning* as questions that are "formulated … through analysis of task, and … on the range of attainments, or more general reasoning ability, in a class" (p. 28) as *responsive questioning*. Strategic questions were provided in the curriculum materials Adhami (2001) references (*Thinking Maths*) and served as the "spine of the lesson as a whole" (p. 28) providing a sequence of questions aiming to facilitate a progression of thinking. It is important to note, however, that Adhami (2001) recommends switching back and forth between tactical questions and strategic questions in order to balance pedagogical goals with student contributions.

2.3 Thematic Analysis

In order to develop a definition for responsiveness to student thinking that encapsulates related terms and constructs (answering the second research question), a thematic analysis was conducted on the descriptions from the literature. For a first level of coding, the data were categorized by the following themes, as indicated in the organization of the previous section: constructs related to responsiveness (satisfying selection criteria 1), teacher activity related to responsive instruction (satisfying selection criteria 2), and curriculum and resources that support responsive instruction. Teacher activity related to responsive instruction was also grouped by the subthemes of teacher activity related to eliciting student thinking, teacher discourse involving student thinking, and responsive teacher activity in inquiry-based instruction.

After this initial categorizing, I coded the data to determine the variety of forms that responsiveness to student thinking can take on based on the descriptions of responsiveness in the literature. Several researchers allude to the difference between a tendency to be responsive and the enactment of responsiveness. This distinction indicates that it might be productive to consider responsiveness as a disposition (i.e. tendency to be responsive) and responsiveness in action (i.e. enacted responsiveness). For example, Hackenberg (2010) differentiates between intuitive responsiveness and deliberate interactions. Robertson, Atkins, Levin, and Richards (2016) also point to the role of disposition in responsive instruction, stating, "We suspect, instead, that responsive teaching grows out of and is grounded in a stance toward students and their ideas rather than through any particular structure of activities" (p. 3), and Teuscher, Moore, and Carlson (2015) refer to the propensity and capacity to decenter.

While dispositions themselves can be difficult to identify and understand, researchers have studied dispositions in relation to how they impact behavior. Thornton defines *dispositions in action* as (2006, p. 62):

habits of mind including both cognitive and affective attributes that filter one's knowledge, skills, and beliefs and impact the action one takes in classroom or professional setting. They are manifested within relationships as meaning-making occurs with others and they are evidenced through interactions in the form of discourse.

Table 2.2: Codes and Sample Coded Excerpts.

	Code	Coded Excerpt
Responsiveness as a Disposition	Attention to student thinking	"…a characteristic of interactions wherein students' mathematical ideas are present, … attended to …" (Bishop, Hardison, & Przybyla-Kuchek, 2016)
	Attention to other student needs	"…maintaining a focus on students' feelings of depletion and stimulation…" (Hackenberg, 2005)
	Valuing of student thinking	Tactical questions … include a valuing of the substance of a student contributions, demonstrating a "genuine acceptance and engagement with pupils' ideas…" (Adhami, 2001)
Enacted Responsiveness	Eliciting student thinking	Elicited student thinking "beyond what was required to solve the particular problems posed" (Franke et al., 2001)
	Responding to/on the basis of student thinking	"discerning and acting on student thinking in ways that support student learning" (Teuscher, Moore, & Carlson, 2015)
	Responding to other student needs	"…an orientation to monitor and respond to energetic fluctuations…" (Hackenberg, 2010)
Other themes	Timing of responsiveness	"A redirection consists of a question or comment submitted by an instructor to his or her class that stops the ongoing flow of the students work…" (Lineback, 2015)

Drawing on Thornton's (2006) definition, the constructs, terms, and conceptualizations of responsiveness were coded as evidencing a responsive disposition, enacted responsiveness, or both. I began by highlighting all excerpts pointing to aspects that impact action (indicating responsiveness as a disposition), and all excerpts demonstrating teacher activity (indicating an enacted responsiveness). For example, the phrase "attention to minute and idiosyncratic steps of reasoning" from Adhami's (2001) conceptualization of *tactical questing* was coded as indicating responsiveness as a disposition and the phrase "eliciting student thinking" used in several constructs was coded as indicating an enacted responsiveness. The results section includes themes and a description for responsiveness as a disposition and an enacted responsiveness.

I then used open coding to determine emergent subthemes of each, also attending to additional themes (e.g. the timing of responsiveness). Table 2.2 provides a list of the emergent codes, as well as sample coded excerpts. Note that for the sample coded excerpts, I highlight segments quoted from the original sources in order to clearly document the presence of the themes in the corpus of data; however, in the thematic analysis I also coded all of the text from my descriptions/summaries of the various constructs to understand these themes further.

2.4 Themes from the Literature: What is Encompassed by Responsiveness?

2.4.1 Forms of Responsiveness

The results from this thematic analysis of the literature shed light on similarities and differences in the conceptualizations of responsiveness to student thinking. First of all, the analysis points to the variety of forms that responsiveness can take on, namely, it has been considered as discourse moves, a skill set, knowledge, orientations, characteristics of interactions, and as teacher activity. Table 2.3 categorizes the constructs according to their primary form. It is important to note that nearly all of the constructs and descriptions of responsiveness to student thinking could be considered as teacher activity since most are evidenced by some kind of action or behavior. Thus, subthemes of teacher activity are discussed subsequently and are not detailed in the table.

The variety of forms that responsiveness takes on further highlights how complex responsiveness as a construct really is. Teacher *uptake* (Collins, 1982), *follow-up* (Brodie, 2011), *revoicing*

Table 2.3: Forms of Responsiveness.

Form	Terms/Constructs
Discourse Moves	Uptake (Collins, 1982) Follow-up (Brodie, 2011) Revoicing (O'Connor & Michaels, 1993) Reflective toss (van Zee & Minstrell, 1997) Tactical questioning (Adhami, 2001)
Characteristics of Interactions	Responsiveness (Pierson, 2008; Bishop, Hardison, & Przybyla-Kuchek, 2016) Follow-up (Franke, Turrou, & Webb, 2011)
Mental Actions & Knowledge	Sensitivity to students (Jaworski, 2002; Potari & Jaworski, 2002) Decentering (Teuscher, Moore, & Carlson, 2015; Ader & Carlson, 2019)
Orientations	Mathematical caring relations (Hackenberg, 2005; 2010)
Skill Set	Professional noticing (Jacobs, Lamb, & Philipp, 2010)

(O'Connor & Michaels, 1993), *reflective toss* (van Zee & Minstrell, 1997), and *tactical questioning* (Adhami, 2001) all take the form of discourse moves in student-teacher interactions. Teacher *uptake* (Collins, 1982), *revoicing* (O'Connor & Michaels, 1993), and *tactical questioning* (Adhami, 2001) highlight the teacher's comment/question and how they use, expand, or clarify student contributions. *Reflective toss* (van Zee & Minstrell, 1997) is demonstrated by an additional comment by the student. A prominent characteristic of these constructs is the cohesion between the student contribution and teacher's response, indicated by an explicit reference to the student's contribution.

In addition to being referenced as discourse moves, responsiveness is discussed as characteristic of interactions, mental actions and knowledge, an orientation, and a skill set. Namely, Bishop, Hardison, and Przybyla-Kuchek (2016) refer to *responsiveness* as a characteristic of an interaction that is evidenced by how student ideas are "present, valued, attended to, and taken up as the basis of instruction" (p. 1173). Teuscher, Moore, and Carlson's (2015) and Ader and Carlson's (2019) use of *decentering* takes into consideration a teacher's attempt to understand student reasoning when considering how teachers respond to student contributions, shedding light on their mental actions. In Jaworski's (2002) definition of *sensitivity to students*, she identifies this as a component of a teacher's knowledge of student thinking, in addition to attention to students' needs and interactions with students, indicating an orientation. In Hackenberg's (2010) conceptualization of *mathematical caring relations*, she considers these relations to require an orientation to monitor and respond to student thinking and energy, and Jacobs, Lamb, and Philipp (2010) describe *professional noticing* as a set of interrelated skills.

Despite the variety of forms that responsiveness to student thinking might take on, one clear similarity between these descriptions is the central role of student thinking, reasoning, or contributions. This is not surprising as several of these constructs were identified as being related to responsiveness *because* of their attention to and valuing of student thinking or contributions. Further, some of the constructs additionally attend to aspects that enable responsiveness, illuminating a distinction between responsiveness as a disposition (i.e. an orientation or valuing of student thinking) and an enactment of this disposition (i.e. teacher activity responding to student contributions).

Responsiveness as a Disposition

A number of the responsiveness descriptions in the literature allude to the impact of an underlying disposition on an instructor's demonstration of responsiveness. In Hackenberg's (2005; 2010) description of *mathematical caring relations*, she discusses instructors' orientation to monitor and respond to energetic fluctuations, indicating an awareness of the need to provide appropriate follow-up to students that is rooted in an accurate interpretation of student understandings (i.e. ability to decenter). Similarly, Ader and Carlson (2019) discuss teachers' propensity to *decenter*, which aligns with one's orientation to engage in decentering acts. The construct of *elaboration* (Abdulhamid & Venkat, 2018) also highlights this awareness of the need for appropriate follow-up in the context of classroom interactions, and *professional noticing* (Jacobs, Lamb, & Philipp, 2010) specifically points to an intentionality in recognizing opportunities to connect mathematics with students' current understandings. Further, this indicates that there is an underlying attribution of value to student contributions in *professional noticing* since the instructor aims to leverage productive student thinking and understandings in making these connections. Potari and Jaworski

(2002) also point to instructors' valuing of students' ability to do mathematics and to think mathematically, in addition to highlighting instructor appreciation and recognition of student thinking. The descriptions of *tactical questioning*, *decentering*, *responsiveness*, and *redirection* also indicate a valuing of student thinking and reasoning.

These constructs all illustrate what is encompassed by a responsive disposition towards student thinking with themes pointing to the valuing or appreciation of student thinking and ability to do mathematics, as well as an awareness of the need to provide appropriate follow-up to facilitate the students' mathematical progress. This valuing and awareness of appropriate follow-up aligns with Thornton's (2006, p. 62) definition of *disposition in action* since these are the "habits of mind" or orientations that impact the actions that a teacher takes. Further, "they are manifested within relationships as meaning-making occurs" in how a teacher attends, engages, and attempts to understand student thinking, looking for opportunities to make connections to disciplinary conventions (ibid, p. 62). In addition to shedding light on responsive dispositions, these constructs also demonstrate an enactment of this disposition, "evidenced through interactions" (ibid, p. 62) or teacher activity. For example, *sensitivity to students* involves a disposition ("habits of mind", ibid, p. 54) that includes an appreciation and recognition of students' thinking and ability to do mathematics, which in turn impacts the action a teacher takes. This disposition is "manifested within relationships as meaning-making occurs" as the teacher attends to student needs, draws on their knowledge of student thinking, and recognizes opportunities for appropriate challenge (ibid, p. 62). This is then evidenced "in providing students with the chance to explore, discuss, and argue but also to encourage the building of confidence in their own ideas" (Potari & Jaworski, 2002, p. 370), indicating an enactment of this disposition.

Enacted Responsiveness

All of the constructs and descriptions overviewed in this paper highlight instructors' activity that evidences responsiveness, or in other words, their enacted responsiveness. Instructors demonstrate responsiveness in the ways that they attempt to understand student thinking and engage with student reasoning, and in how they facilitate opportunities for student contributions to arise, or draw out student thinking. Specifically, the phrase "elicit[ing] student thinking" is mentioned explicitly in descriptions of five of the constructs (*eliciting student thinking* (Fennema et al., 1996; Franke et al., 2001), *reflective toss* (van Zee & Minstrell, 1997), *tactical questioning* (Adhami, 2001), *elaboration* (Abdulhamid & Venkat, 2018), *responsiveness* (Pierson, 2008; Bishop, Hardison, & Przybyla-Kuchek, 2016)), and six (including the four articles articulating responsive teacher activity in inquiry-based instruction and the constructs of *revoicing* (O'Connor & Michaels, 1993), *uptake* (Collins, 1982), and *follow-up* (Brodie, 2011)) allude to the fact that student thinking or ideas are present, referring to student contributions, which may or may not have been elicited by the instructor. The constructs of *integrated assessment* (Saxe, Gearhart, & Seltzer, 1999), *follow-up* (Franke, Turrou, & Webb, 2011), and *redirection* (Lineback, 2015) also do not explicitly reference eliciting student thinking, but they do refer to creating space for contributions or opportunities for students to (further) reveal their understandings. Six constructs, *sensitivities to students* (Jaworski, 2002; Potari & Jaworski, 2002), *mathematical caring relations* (Hackenberg, 2005; 2010), *professional noticing* (Jacobs, Lamb, & Philipp, 2010), *decentering* (Teuscher, Moore, & Carlson, 2015; Ader & Carlson, 2019), *responsive teaching* (Empson, 2014) and *scaffolding/local interactional responsiveness* (Koole & Elbers, 2014), rely on a demonstration of student thinking, but do not articulate the form of this demonstration or how it was realized. These teacher actions illustrate how an instructor draws out student contributions or engages with student reasoning, which are critical in developing a teacher's understanding of student thinking.

Another similarity between these descriptions and definitions, highlighting an important component of enacted responsiveness, is that it requires an actual response from the instructor indicating they are attending to the students' thinking or contributions *and* are responding based on this. In particular, it is important for an instructor to respond to students in ways that are consistent with student demonstrations of understanding. Instructors can respond by leveraging, building on, or incorporating student thinking/ideas into the discourse of the class, using student contributions to inform instruction. Terms in these descriptions indicating a response are *guides*, *act*, *respond*, *taken up*, and *inform*. Additionally, instructors often respond in order to progress students' mathematical thinking, supporting the development of rich mathematical meanings, or to facilitate and connect student ideas with other student contributions or with established disciplinary conventions.

2.4.2 Timing of Responsiveness

All of the constructs and descriptions of responsiveness point to teacher responsiveness that occurs during in-the-moment instruction. For example, in Laursen and Rasmussen's (2019) discussion of responsive teaching, they comment how instructors' talk that is "well timed and well targeted" is crucial for "adaptive and responsive facilitation skills" (p. 3), pointing to in-the-moment interactions with students. Similarly, Ader and Carlson (2019) and Teuscher, Moore, and Carlson (2015) highlight decision-making that occurs during in-the-moment instruction that builds on students' understandings through *decentering*. Jacobs, Lamb, and Philipp (2010) document how teachers attend, interpret, and respond almost instantaneously in *professional noticing*, and Pierson (2008) documents *responsiveness* in the extent to which teachers' take up, respond, and focus on student thinking during class. Additionally, the constructs that are specifically related to discourse moves, such as *reflective toss* (van Zee & Minstrell, 1997), *uptake* (Collins, 1982), and *revoicing* (O'Connor & Michaels, 1993) rely on in-the-moment student-teacher interactions.

In addition to responsiveness that occurs in-the-moment, several of the constructs leave room for responsiveness to student thinking to extend to instructor reflection or planning. There is evidence that *sensitivity to students* occurs during instructors' reflecting and planning when instructors attend to students' difficulties, ways of thinking, and affective and emotional needs when designing a lesson plans and sequences of tasks for student to engage with (that are within their zone of proximal development) during instruction (Jaworski, 2002). Additionally, in Koole and Elbers' (2014) discussion of *scaffolding* and *local interactional responsiveness*, they note that teachers often draw on their experience and knowledge that extends beyond the in-the-moment demonstration of student thinking when responding to students. This highlights that previous knowledge, interactions, and reflection plays a part during in-the-moment responsiveness. This is also consistent with

Lineback's (2015) distinction between responsive redirections and delayed responsive redirections where the teacher revisits a previous student contribution. Since there is not an explicit reference to responsiveness occurring only during in-the-moment instruction in descriptions of *responsive instruction* in Johnson's (2013), Kuster, Johnson, Rupnow, and Garrison-Wilhelm's (2018), and Hammer, Goldberg, and Fargason's (2012) work, these discussions leave open the possibility for some of the instructor's work interpreting students' reasoning, identifying mathematical connections, deciding how to build on student contributions, and adapting instructional objectives to occur outside of class during an instructor's reflections or planning or preparing for class.

2.5 Discussion: Defining Responsiveness to Student Thinking

After considering these various constructs and the definitions or descriptions provided in the original sources, I propose the following definition for *responsiveness to student thinking*, highlighting the distinction between responsiveness as a disposition and enacted responsiveness:

> *Responsiveness as a disposition* is attending to student thinking, mathematical meanings, and needs (including both cognitive and affective needs), and an awareness of the need to provide an appropriate response based on student understandings or contributions. A responsive disposition relies on an instructor's valuing student thinking.
>
> *Enacted responsiveness* is responding to student thinking by focusing on the substance of student contributions (including those that are incorrect or incomplete) when determining the direction of the lesson, through leveraging and building on student contributions, and by making connections between student ideas and what is accepted as disciplinary conventions.

In order to model this connection between responsiveness as a disposition and enacted responsiveness, I draw on Blömeke, Gustafsson, and Shavelson's (2015) model of teacher competence as a continuum (see Figure 2.1). Blömeke, Gustafsson, and Shavelson (2015) proposed this model of teacher competence in order to address the dichotomy between cognitive and situated perspectives in studying teachers and their practice. This model demonstrates how the perspectives can complement one another by considering teacher competence as a continuum including "traits (cognitive, affective, motivations) that underlie the perception, interpretation and decision-making that give rise to observed behavior in a particular real-world situation" (Blömeke, Gustafsson, & Shavelson, 2015, p. 16).

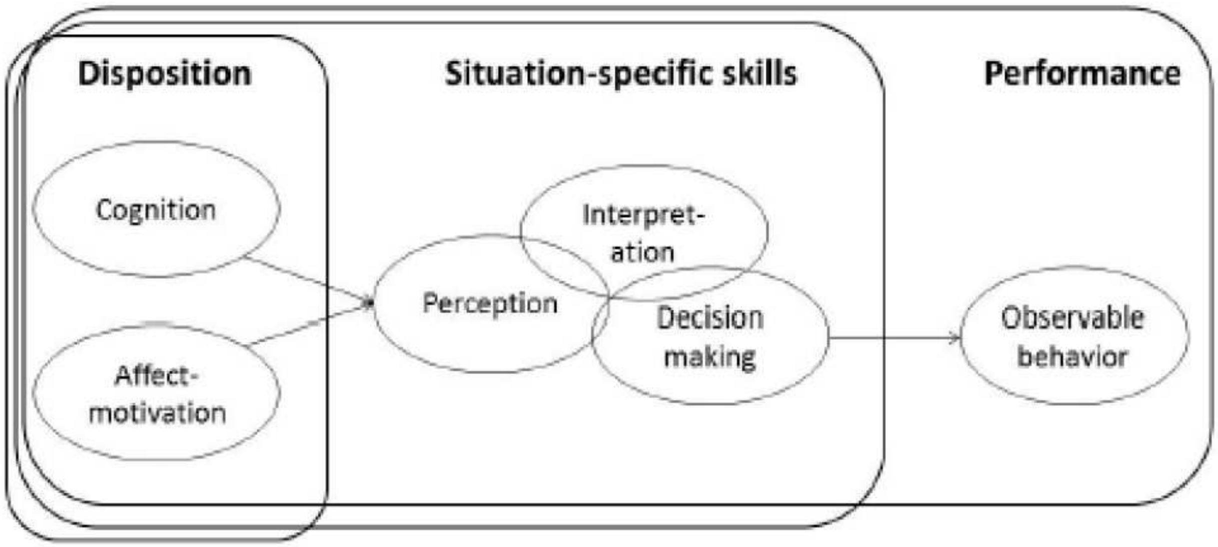


Figure 2.1: Blömeke, Gustafsson, and Shavelson's (2015) model of teacher competence

My definition for responsiveness to student thinking mirrors this continuum of teacher competence because it includes an underlying disposition of responsiveness, situation-specific skills, thought-processes, and actions that are influenced by this disposition, which then are evidenced in how a teacher responds on the basis of student thinking. Additionally, these components are specifically exhibited by various conceptualizations of responsiveness through the constructs analyzed in this paper. For example, an instructor's propensity to *decenter* is primarily an underlying disposition that influences how an instructor engages and attempts to understand their students' thinking (Teuscher, Moore, & Carlson, 2015; Ader & Carlson, 2019), *professional noticing* is described as a set of skills that include an instructors' ability to attend, interpret, and respond to students' mathematical strategies and understandings, and then the constructs that highlight teacher discourse moves (e.g., *revoicing* (O'Connor & Michaels, 1993), *reflective toss* (van Zee & Minstrell, 1997)) are tied explicitly to observable behaviors.

In Figure 2.2, I draw on this model to illustrate the relationship between responsiveness as a disposition and enacted responsiveness, highlighting how they complement one another to support a more complete understanding of *responsiveness to student thinking*. Responsiveness as a disposition is an underlying component that influences how an instructor makes meaning from their students' thinking and contributions by impacting how they create space for student ideas to emerge, in how they engage with student ideas, attend to student thinking, interpret demonstrations of student understandings, and in how they elicit, build, clarify, and reflect on student contributions. These actions, influenced by responsiveness as a disposition, are further evidenced by how an instructor responds based on student thinking, leveraging student ideas in instruction and supporting disciplinary connections rooted in the substance of student contributions. Although not explicitly documented in the diagram, it is important to note that responsiveness can be demonstrated both during in-the-moment instruction and in how an instructor uses student thinking in other instructional practices, such as in planning or assessing.

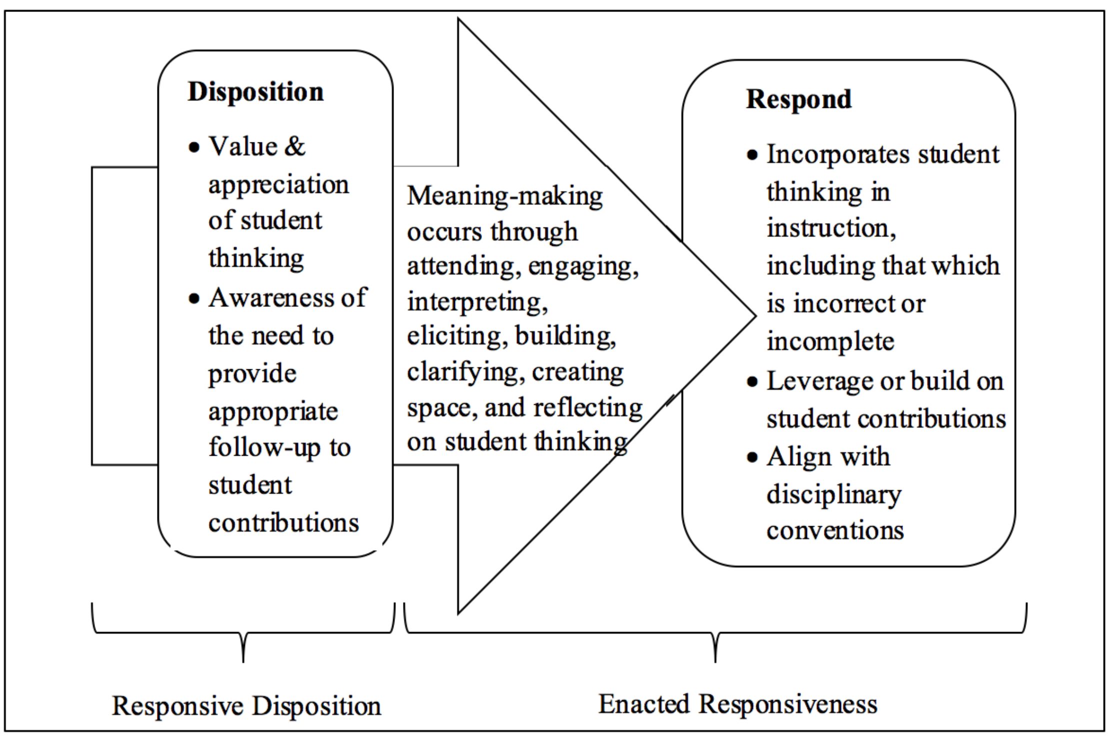


Figure 2.2: Responsiveness to student thinking: Responsiveness as a disposition and as enacted responsiveness

Although it is possible to consider responsiveness in each of these stages (as some of the constructs in the realm of responsiveness discussed in this paper do), my conceptualization of *responsiveness to student thinking* requires both a responsive disposition and enacted responsiveness. In other words, I do not consider enacted responsiveness to be *responsiveness to student thinking* without an underlying disposition that supports an instructor's ability to make meaning of the student's contributions and leverage this in some way to support instruction. For example, if an instructor only engages with students' thinking on a surface level through observable behavior, such as restating a student's contribution without attempting to understand the substance and meaning within the student's contribution, then, although this could be considered enacted responsiveness, I would not truly consider this interaction as *responsiveness to student thinking*. However, I recognize that it is possible for an instructor to be engaging with student thinking in a way that is not visible or evidenced by observable actions, indicating *responsiveness to student thinking*. Conversely, if an instructor values student thinking and is aware of the need to provide appropriate follow-up to student thinking (having a responsive disposition), but does not act on this to demonstrate an enacted responsiveness, then I would not consider this as *responsiveness to student thinking* as there is no evidence of a response.

Considering responsiveness as a disposition and as in action together allows for a more complete picture of what is encapsulated by *responsiveness to student thinking*, as well as highlights the importance of both an instructors' underlying interest and valuing of student thinking and response on the basis of student thinking. The constructs of *uptake* (Collins, 1982), *follow-up* (Brodie, 2011), and *reflective toss* (van Zee & Minstrell, 1997) are designed to document identifiable forms in classroom discourse, highlighting that a teacher responds on the basis of student ideas (in that they reference student ideas in the question/comment posed after the student contribution), but these constructs seem to be missing something that r*esponsiveness to student thinking* <u>should</u> entail – the intent or interest in the substance of student thinking that extends beyond the act of responding. Pierson (2008) takes a similar perspective in her discussion of the similarities and differences of her conception of *responsiveness* and the construct of *revoicing* (O'Connor & Michaels, 1993), and Teuscher, Moore, and Carlson (2015) highlight how *revoicing* is a “teacher-centered activity only requiring a student to put forth some statement to be revoiced” (p. 436). However, it is again important to note that it is possible for instructors engaging in activities documented by *uptake* (Collins, 1982), *follow-up* (Brodie, 2011), and *revoicing* (O’Connor & Michaels, 1993) to value student ideas and to be deeply engaged with the student thinking, indicating responsiveness, although it might not be overtly apparent in the interaction (Colestock & Sherin, 2016).

Throughout this paper, I avoid explicitly distinguishing between responsiveness to student ‘contributions’ and responsiveness to student ‘thinking’ using these terms relatively interchangeably, primarily because I did not want to make assumptions about the meanings the original authors’ intended. However, I take responsiveness to student ‘contributions’ to mean that an instructor is responding to what the student has said (which may or may not communicate their mathematical thinking), without engaging with the substance of student thinking or understanding; whereas I take responsiveness to student ‘thinking’ to include the assumption that an instructor is engaging with and attempting to understand the mathematical strategies and reasoning articulated by the student. *Responsiveness to student thinking*, as I have defined it in this paper, includes both an underlying and enacted disposition – which inherently includes an assumption that instructors demonstrating *responsiveness to student thinking* are attending and responding on the basis of (underlying) student thinking rather than simply responding to student contributions without using the contribution as a window into student conceptions.

2.6 Conclusion and Implications

I began this paper with Robertson, Atkins, Levin, and Richards’ (2016) summary of themes from a selection of literature surrounding responsive instruction. The corpus of data considered in the analysis presented in this paper included literature at post-secondary level in mathematics, which provides a broader perspective on responsiveness and is crucial for defining responsiveness that encompasses a variety of conceptualizations and enactments. Further, the thematic analysis sheds additional light on some of the nuances between the similarities and differences in conceptualizations of responsiveness, as well as reiterates some of the themes from Robertson, Atkins, Levin, and Richards' (2016) summary.

In collecting a selection of literature describing constructs and activity related to responsiveness for the thematic analysis, I sought to include a broad, yet representative sample of various conceptions, and was primarily focused on better understanding how responsiveness was exhibited in responsive instruction. All of the conceptualizations of responsiveness included descriptions of instructor activity, but the thematic analysis also illustrated an underlying aspect of responsive instruction – the presence and impact of responsiveness as a disposition. Although this underlying component of responsiveness was not explicitly discussed in conjunction with the evidence of an enacted responsiveness in all of the literature, it is a critical component of *responsiveness to student thinking*, enabling effective student-thinking centered instruction.

By distinguishing between responsiveness as a disposition and an enacted responsiveness as dimensions of *responsiveness to student thinking*, professional development providers can leverage both of these aspects to support instructors in incorporating more student-thinking centered instruction, rather than only focusing on the development of skills evidenced by an enacted responsiveness (which are often the focus of such professional development). It is important to note, however, that there is still work that needs to be done to more fully understand responsiveness as a disposition, enacted responsiveness, and ways to support instructors in their endeavors to teach responsively. Specifically, there is a need to investigate demonstrations of responsiveness that go beyond in-the-moment inquiry-based instruction, particularly at the post-secondary level. Further, since instructors are likely to enact responsiveness differently, it is important to better understand these differences and their impact on instruction and student learning (Lineback, 2015).

Chapter 3

Paper 2: College Calculus Instructors' Instructional Practices and Responsiveness to Student Thinking

In the past few decades there has been increased attention on instructional practices that incorporate and build on student thinking. To effectively implement these practices, it is essential for an instructor to demonstrate responsiveness to student thinking, by focusing on student contributions/ideas when determining the direction of the lesson, leveraging and building on student contributions, and making connections between student ideas and what is accepted as disciplinary conventions during in-the-moment instruction or during other instructional practices. Although important, responsiveness is not well-understood at the post-secondary level. To better understand instructional practices that indicate responsiveness to student thinking, and aspects that impact an instructor's responsiveness, an instructional practices survey was distributed to college calculus instructors at twelve institutions. Results from quantitative analyses highlight categorizations of instructional practices that relate to responsive practices, indicating that responsiveness can occur in both student-oriented and instructor-oriented classes. Additionally, there is evidence that responsiveness is not limited to experienced instructors, and that it is a complex construct that needs to be researched further.

3.1 A Focus on Student Thinking

There have been increased calls for more student-centered approaches to instruction at both the K-12 and post-secondary levels (e.g., Principles to Action, NCTM, 2014; IP Guide, Ludwig et al., 2018), largely motivated by the positive effects of this type of instruction on student learning (e.g. Freeman et al., 2014; Kogan & Laursen, 2014; Laursen, Hassi, Kogan, & Weston, 2014). A key component of effective student-thinking centered instruction (regardless of instructional format) is an instructor's responsiveness to student thinking, which is demonstrated by their valuing of student thinking and use of student contributions in informing the direction of class (Laursen & Rasmussen, 2019; Kuster, Rupnow, Johnson, & Garrison-Wilhelm, 2018). Elsewhere, I've conducted a review of literature related to responsiveness to student thinking in order to develop a definition that encompasses the variety of terms and constructs researchers use when describing an instructor's attention and response to student thinking and contributions (Gehrtz, 2019). I've conducted a thematic analysis and proposed the following definition for responsiveness to student thinking, distinguishing between responsiveness as a disposition and enacted responsiveness, stating that:

> *Responsiveness as a disposition* is attending to student thinking, mathematical meanings, and needs (including both cognitive and affective needs), and an awareness of the need to provide an appropriate response based on student understandings or contributions. A responsive disposition relies on an instructor's valuing of student thinking.
>
> *Enacted responsiveness* is responding to student thinking by focusing on the substance of student contributions (including those that are incorrect or incomplete) when determining the direction of the lesson, through leveraging and building on student contributions, and by making connections between student ideas and what is accepted as disciplinary conventions. (Gehrtz, 2019).

Additionally, I discussed examples of teacher activity that support the enactment of a responsive disposition, which are evidenced by how an instructor responds on the basis of student thinking (enacted responsiveness). Some activities include an instructor's attempt to understand student thinking and make meaning of student contributions through "attending, engaging, interpreting, eliciting, building, clarifying, creating space, and reflecting on student thinking" (Gehrtz, 2019). One of the goals of this study is to better understand enacted responsiveness, particularly at the post-secondary level, by considering how these activities relate to college calculus instructors' broader instructional practice.

Instruction that focuses on student thinking creates new instructional space for student contributions, providing pedagogical opportunities for the instructor to recognize and build on student thinking, fostering students' rich mathematical meanings and understandings (Larsen, Johnson, & Weber, 2013; Leatham, Peterson, Stockero, & Van Zoest, 2015; Kuster, Johnson, Keene, & Andrews-Larson, 2018). Additionally, instruction that is responsive to student thinking has been shown to provide more positive learning experiences (Thornton, 2006), promote students' conceptual understandings and learning (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989), student agency and voice (Coffey, Hammer, Levin, & Grant, 2011), and equitable participation when every student's thinking is valued (Empson, 2003; Robertson, Scherr, & Hammer, 2016). This attention to student thinking also shapes the type of things students come to see as valued in the classroom, and creates opportunities for instructors' ongoing learning about student thinking (Cobb, Stephan, McClain, & Gravemeijer, 2001; Franke, Carpenter, Levi, & Fennema, 2001; Radoff & Hammer, 2016). Although there are many benefits for students when there is a focus on instructional practices that build on their mathematical thinking, there is evidence that these practices are not well-understood and difficult to enact (Speer & Wagner, 2009; Sherin, 2002). The goals of this paper are to further consider what responsiveness looks like, how it is related to various instructional practices, and how it is developed, in order to better support instructors in their implementation of responsive instructional practices, which are evidence of their enacted responsiveness.

3.1.1 Research Goals Surrounding Responsiveness to Student Thinking

My hypothesis is that all effective instruction at the post-secondary level is enabled by an instructors' responsiveness to student thinking. Since a wide variety of instructors teach introductory calculus at this level, ranging from novice graduate students to experienced tenured faculty, I focus my study on the responsiveness of college calculus instructors. Additionally, calculus is a high-impact course as it is often the first mathematics class taken by students who are intending to pursue a degree in science, technology, engineering, or mathematics (STEM). Students' experiences in calculus have the potential to impact their STEM degree trajectory, and poor instruction is often cited as a contributor to students' decision to switch out of STEM (Bressoud & Rasmussen, 2015; Seymour & Hewitt, 1997). Thus, it is important to seek ways to support effective calculus instruction. By better understanding how calculus instructors with a variety of experiences and perspectives enact responsiveness, we can gain a more thorough understanding of the construct of responsiveness itself, and will be able to better support instructors in their implementation of responsive instruction.

In this study, I investigate college calculus instructors' enacted responsiveness to student thinking. Specifically, I investigate the following research questions: (1) In what ways can college calculus instruction be categorized to indicate responsiveness to student thinking? and (2) How is responsive instruction related to other instructor characteristics (e.g. years of experience, involvement in decisions regarding instructional approach or course content)? By answering research question 1, we can better understand instruction at the post-secondary level, and connections between various instructional practices and responsiveness to student thinking. Further, answering this research question will allow for a better understanding of what responsiveness looks like, in general, at the post-secondary level, which then can be leveraged to support instructors in their implementation of responsive practices. By answering research question 2, we can begin to understand which types of instructors are more likely to teach responsively, and will further illuminate aspects that might influence an instructor's ability to be responsive to students' thinking, which can aid in the creation of professional development targeting responsiveness to student thinking.

3.2 Influences on Instructor Responsiveness

There are a number of aspects that have the potential to impact instructor responsiveness to student thinking, such as one's instructional approach, experience and professional development, as well as the extent to which an instructor has agency in their instructional decisions. In this section I will briefly overview literature that points to how these various aspects have the potential to influence instructor responsiveness to student thinking.

3.2.1 Instructional Approach

At the post-secondary level in general, and college calculus specifically, lecture is still the primary form of instruction (Blair, Kirkman, & Maxwell, 2013; Vroom et al., 2019). Here I refer to lecture-oriented instruction as instructor-centered because lecture reflects a "dominant direction of communication [being] from the lecturer to the students" (Pritchard, 2010, p. 610), indicating that a focus is on the instructors' ideas and contributions. However, this does not exclude student-instructor or student-student interactions in class, it is just that these are secondary to lecture.

Effective instructor-centered instruction is useful for "modeling expert thinking and ways of doing mathematics" (Rasmussen & Wawro, 2017, p. 563), but there is evidence that students often do not take away the main ideas intended to be communicated by the instructor (Lew, Fukawa-Connelly, Meija-Ramos, & Weber, 2016). Thus, it is important for instructor-centered instructional practices to still be student-thinking focused, demonstrating responsiveness to student understandings and ideas, which also enables instructors to be more aware of what students are gleaning from class.

In instructor-centered classes there are fewer opportunities to be responsive to student ideas in-the-moment, and so it might seem as if these instructors demonstrate less responsiveness than more active or student-centered classes. However, at the heart of responsive instruction is an attention to student thinking and how the instructor responds on the basis of student understandings. Thus, if the instructor is gaining insight to student thinking by regularly considering work in a non-evaluative way, seeking to understand what the students might be thinking, and uses this information to inform their lectures, then they are demonstrating responsiveness to student thinking. Further, instructor-centered classes can support responsiveness to student thinking when the instructor probes students with follow-up questions, seeking explanations that provide more opportunities to respond by building on student thinking, and does not simply elicit student answers for evaluative purposes.

I refer to student-centered classes as those where the dominant activities in class involve student contributions, between pairs of students, in small groups, or to the whole class. As with instructor-centered instruction, student-centered instruction does not mean that the only activities in class are those where students are making contributions, but rather that these are the primary activities occurring. In more student-centered classrooms, there tend to be more opportunities to elicit and respond to student thinking during in-the-moment instruction. One type of student-centered instruction is that of *inquiry-oriented instruction*. With inquiry-oriented instruction, curricular materials support student reinvention of mathematical concepts, and include tasks "that build on student concepts and reasoning as the starting point from which more complex and formal reasoning develops" (Wawro, Rasmussen, Zandieh, Sweeney, & Larson, p. 321). Kuster, Johnson, Keene, and Andrews-Larson (2017) describe four principles that are central to inquiry-oriented instruction drawing on research literature from the K-12 and post-secondary levels. Two of these principles are *generating student ways of reasoning* and *building on student contributions*. In order to effectively enact these two principles, an instructor must be responsive to their students' understandings, eliciting student thinking and reasoning extending beyond simply sharing answers (Hufferd-Ackles, Fuson, & Sherin, 2004), and then responsively adjust instruction and discussion accordingly, based on the development of student contributions and ideas (Kuster et al., 2017). The third and fourth principles then build off of this work with the instructor making efforts to develop a shared understanding of the mathematics (third principle) and connecting this to a standard mathematical language and notation (fourth principle). Again, responsiveness to student thinking is crucial for the effective enactment of these principles, particularly because it is the instructor's responsibility to respond to student ideas, by first recognizing when "student mathematical thinking might compel one to take a particular path because of the opportunity it provides at that moment to build on that thinking to further student mathematical understanding" (Larsen, Johnson, & Weber, 2013, p. 118). Then, it is the instructor's responsibility to select and sequence student ideas in a way that develops a shared understanding that is consistent with that which is accepted in the field of mathematics (Kuster et al., 2017; Johnson, 2013).

In a similar vein, Laursen and Rasmussen (2019) describe four pillars of inquiry-based instruction rooted in the literature surrounding inquiry-based learning (IBL) traditions (e.g. Mahavier, 1999) and inquiry-oriented instruction traditions (e.g. Cobb et al., 1991; Freudenthal, 1991), bringing these domains together under a common vision of Inquiry-Based Mathematics Education. These four pillars include: *student engagement in meaningful mathematics, student collaboration for sense-making, instructor inquiry into student thinking,* and *equitable instructional practices*.

Rasmussen and Kwon (2007) note that instructor inquiry into student thinking requires that the instructor elicit student ideas and create a classroom community where student thinking and contributions are valued. Further, it is the instructor's responsibility to support students in refining their ideas and moving the mathematical agenda forward. Without an awareness of students' mathematical ideas and strategies, and the instructors' ability to take this information and adapt instruction as necessary, students are not provided with the opportunities to develop rich mathematical meanings that are connected to both their development of ideas and in alignment with the community of mathematicians.

3.2.2 Experience and Professional Development

Since responsiveness involves attending and responding to students' thinking, and the development of these skills take time, experience plays a role in the development of responsiveness to student thinking (Lineback, 2015). Huang and Li (2012, p. 430) found that more experienced teachers were "more likely to be aware of developing students' mathematics thinking and ability when helping students develop mathematical knowledge", further indicating that responsiveness can develop over time (Jacobs, Lamb, & Philipp, 2010; Stahnke, Schueler, & Roesken-Winter, 2016). There is evidence that expert instructors are able to better monitor and understand student contributions in more detail than novice instructors (Kaiser, Blömeke, König, Busse, Döhrmann, & Hoth, 2017), and are able to anticipate and interpret teaching situations more quickly and accurately (Santagata & Yeh, 2016, p. 5). Although experience alone may support an instructor in their responsiveness – by giving them a knowledge base and understanding of student thinking to draw on while they are engaged in teaching or planning (Silverman & Thompson, 2008; Leatham, Peterson, Stockero, & Van Zoest, 2015) – if the only way an instructor interacts with student thinking is through evaluation, it is unlikely that this experience will support a teacher in leveraging student thinking in their instruction. Additionally, there is also evidence that indicates that experienced teachers are not necessarily experts at demonstrating responsive instructional practices (Dreher & Kuntze, 2014).

Extensive research surrounding mathematics instructor professional development (PD) highlights that PD is an effective way to improve and impact instructional practices (Borko, 2004). A primary way to do this is by giving instructors opportunities to focus on student thinking in a non-evaluative way (as we most commonly do as teachers). Cognitively Guided Instruction (CGI) is one such program that is rooted in research on student mathematical thinking and development. Instructors that have participated in CGI have been able to better predict problems that their students will have difficulty solving (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989), which points to instructors being better able to anticipate student thinking. When an instructor is able to effectively anticipate student thinking, this allows for opportunities for them to respond to these anticipated understandings, either in class or outside of class in their planning practices or office hours. Additionally, instructors that begin to focus on student thinking, really trying to unpack and understand what a student understands and why they have that understandings, create opportunities to develop their own learning and understanding of student mathematical thinking and strategies (Franke, Carpenter, Levi, & Fennema, 2001).

There is evidence that instructors can learn to attend, interpret, and respond to students' mathematical strategies (referred to as *professional noticing* by Jacobs, Lamb, and Philipp (2010)) through PD or teacher education (e.g., Jacobs et al., 2010; Sherin & van Es, 2009). When instructors are able to increase their ability to notice, they move from superficially evaluating student thinking to being able to interpret more meaning, which allows for instructional decisions informed by student understanding (Richards & Robertson, 2016). Leatham, Peterson, Stockero, and Van Zoest (2015) point to the complexity of identifying class interactions and student contributions where student thinking can be leveraged, identifying this as a barrier to effectively using student thinking during instruction. Pascoe and Stockero (2017) found that graduate student instructors were able to increase their ability to notice Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOSTs, see Leatham, Peterson, Stockero, & Van Zoest, 2015) after PD that introduced the construct of professional noticing and highlighted what might be important for an instructor to notice during a lesson. By being able to notice these opportunities, teachers are better able to capitalize on them to leverage student thinking in their instruction.

Although Jacobs, Lamb, and Philipp (2010) found that instructors' ability to attend to and interpret student strategies (two components of professional noticing) grew with teaching experience and with PD, they did not find the same evidence of the instructors' ability to respond to student thinking increase with experience alone. Further, there is evidence that instructors do not move predictably through sequenced levels of responsive instruction (Robertson, Richard, Elby, Walkoe, 2016), and so although PD can support responsiveness to student thinking, it is important to consider other aspects that might impact responsiveness beyond teaching experience.

3.2.3 Role of Making Instructional Decisions

There is evidence that instructors who embrace their role as decision-makers tend to provide more responsive instructional experiences for students (Thornton, 2006). An instructor's in-the-moment decisions are shaped by the interaction of their knowledge, goals, and beliefs (Schoenfeld, 2011). Specifically, the decisions that an instructor makes are impacted greatly by their goals that are activated at any given moment during their instructional practice (Schoenfeld, 1998). If an instructor's main goal is to "cover" all the required material in a course, where something is considered "covered" if the instructor talks about it in class, then they are probably less likely to demonstrate responsiveness to student thinking, deciding to create space for student contributions or to adjust instruction based on students' understandings (Yoshinobu & Jones, 2012). On the other hand, if an instructor's goal is develop students' mathematical meanings, it is critical to gauge where students are at with their understanding, and to adjust instruction in order to support students' meanings, then they are likely to make instructional decisions that reflect responsiveness.

Since responsive instruction requires an instructor to regularly make instructional decisions based on their understanding of student thinking and mathematical strategies, it is essential for an instructor to capitalize on their role as a decision-maker to move the direction of the class forward, keeping alignment with one's pedagogical goals (Richards & Robertson, 2016). If an instructor feels constrained by the amount of material to cover or feels as if they do not have the opportunity to make decisions based on their students' understanding of the material, then this will likely impact their ability to demonstrate responsiveness. Additionally, there is evidence that pre-service teachers decision-making skills were the least developed of the professional noticing skills of attending to, interpreting, and deciding how to respond to student thinking (Jacobs, Lamb, & Philipp, 2010; Schack et al., 2013), which further indicates that one's role in making instructional decisions (regarding course content or instructional approach) could impact one's ability to be responsive to their students' thinking.

3.2.4 Additional Aspects

There are many other aspects that have the potential to influence an instructor's responsiveness to student thinking. For example, one's knowledge of the mathematical content, as well as their knowledge of how to teach that content has the potential to impact responsiveness to student thinking (see e.g., Ball, Thames, and Phelps, 2008). For example, in order for an instructor to effectively elicit and respond to student thinking throughout instruction, an instructor must be able to understand the mathematics of the student's contribution/thinking, what is (and is not) productive about the thinking, where the student's understandings likely came from and will likely lead. Further, an instructor must understand various ways to respond to this thinking and how those responses will interact with the rest of the curriculum – all of which draw on knowledge of the content and knowledge that is leveraged when teaching the content. In this study, I draw on participants' experience as a proxy for the knowledge they draw upon while teaching because deeply considering knowledge beyond experience is outside the scope of this study. However, it is an open area to investigate further once enacted responsiveness is better understood.

3.3 Methodology and Results

This study is part of a larger mixed-methods study investigating college mathematics instructor responsiveness to student thinking. The survey data collected was also a part of the NSF-funded grant *Progress through Calculus* (PtC) (NSF DUE #1430540), a study investigating multiple dimensions of calculus instruction across the United States. As part of the PtC project, a census survey was distributed to all (N=330) mathematics masters and doctoral granting institutions (with a response rate of 67.6%) (Rasmussen et al., 2019). The project team used the census survey data, which included details about the structures and programs in place for the precalculus to Calculus II sequence at these institutions, along with graduation statistics (IPEDS data; IES NCES, 2018), to identify twelve institutions for in-depth case studies. These twelve case-study institutions vary greatly in size, location, and selectivity.

3.3.1 Post-Secondary Instructional Practices Survey-Mathematics Design and Data Collection

In order to gather information about instruction at the case study sites, the PtC research team added seventeen items to the Post-Secondary Instructional Practices Survey (PIPS) (Walter, Henderson, Beach, & Williams, 2016). Seven of the additional items were related to equitable teaching practices (Montgomery County Public Schools, 2010; Tanner, 2013; Dorman, 2003), five were adapted from the Mathematics Classroom Observation Protocol for Practices (Gleason, Livers, & Zelkowski, 2015), and five were taken from surveys designed to measure pedagogical content knowledge (Jang, Guen, & Hseih, 2009; Zelkowski, Gleason, Cox, & Bismarck, 2013). These additional items were modified as necessary to reflect the structure of the PIPS items. This new expanded survey (referred to as PIPS-Mathematics, PIPS-M) included forty-one instructional practices items which instructors rated on a 5-point Likert scale (1=not at all descriptive, 2=minimally descriptive, 3=somewhat descriptive, 4=mostly descriptive, 5=very descriptive) (Apkarian et al., 2019).

From the set of forty-one items on the PIPS-M, I identified five items that reflected a responsiveness to student thinking, in that the items explicitly reflected either attention to students and their thinking or they indicated an adjustment of instruction based on students' contributions or thinking. Table 3.1 shows these *responsiveness* items (R1-R5), as well as the source of the item.

Table 3.1: PIPS-M Survey Responsiveness Items.

Responsiveness Item	Item Source
R1: I use a variety of approaches (e.g., questioning, discussion, formal/informal assessments) to gauge where my students are in their understanding of concepts	Jang, Guen, & Hsieh, 2009
R2: I use student questions & comments to determine the focus & direction of classroom lessons	PIPS: Walter et al., 2016
R3: I use student assessment results to guide the direction of my instruction during the semester	PIPS: Walter et al., 2016
R4: I adjust my teaching based upon what students currently do or do not understand	Zelkowski et al., 2012
R5: I consider students' thinking/understanding when planning lessons	Created by PtC Researchers

The PIPS-M survey was distributed to all precalculus, Calculus I, and Calculus II instructors at the twelve case-study institutions during the fall 2017 semester. For this study, I only consider results from the Calculus I instructors of record, as this is the population being considered in the larger study on instructor responsiveness. Future work will examine how instructor responsiveness varies between these courses. The Calculus I instructors who participated in this study include thirty-four graduate students, seven postdoctoral fellows, twenty-five lecturers/adjuncts, and twenty-eight faculty (total N=94) from the twelve PtC case study institutions. See Appendix B for more details on the twelve institutions and participants.

3.3.2 Factor Analysis

To answer the first research question, an exploratory factor analysis with varimax rotation was conducted on the PIPS-M items after confirming a Kaiser-Meyer-Olkin measure of sampling adequacy (KMO=.725) and a significant Bartlett's test of sphericity ($\chi^2(820) = 1635.870; p = 0.00$). Solutions for two, three, four, and five factors were considered using varimax rotation of the factor loading matrix, suppressing items with loadings less than .3. The two-factor (2F) option mostly aligns with the 2F solution described in Walter, Henderson, Beach, and Williams's (2016) factor analysis of the original twenty-four PIPS items, and accounted for 29.296% of the total variance. The first factor in the 2F model indicates student-centered practice and the second factor generally indicates instructor-centered practice. However, since the PIPS-M includes a total of forty-one items, the other models provide more detail on the instructional practices of a participant.

The three-factor (3F) option, accounting for 33.716% of total variance, and four-factor (4F) option, accounting for 37.68% of total variance, had very similar groupings. In the 4F option, the fourth factor loaded six items over .3 (with only three of these loading over .4); three of these items cross-loaded on the third factor and one cross-loaded on the second factor.

Table 3.2: Inter-Item correlations, Mean, and Standard Deviation for Fifth Factor in 5F Solution.

Item	Test questions require students to apply course concepts to unfamiliar situations	I know most of my students by name	I use a grading curve as needed to adjust student scores	M	SD
Test questions require students to apply course concepts to unfamiliar situations	1.00	.345	.075	3.00	1.207
I know most of my students by name	-	1.00	.227	4.08	1.385
I use a grading curve as needed to adjust student scores	-	-	1.00	2.60	1.446

The five-factor (5F) solution, accounting for 40.953% of the total variance, is preferred because of the detail provided by the factors. Further, the 5F solution is also supported by a qualitatively logical grouping of items, except for the three items loading over .3 on the fifth factor, accounting for 3.57% of the variance. Cronbach's alpha was computed for the items on this factor; the alpha was .447 which indicates low reliability. Table 3.2 shows the inter-item correlations, mean, and standard deviation for these items. The low correlations support the decision to remove these items.

The three items loading on factor five in the 5F solution were removed and a factor analysis with varimax rotation was conducted (after confirming a Kaiser-Meyer-Olkin measure of sampling adequacy (KMO=.760) and a significant Bartlett's test of sphericity, $\chi^2(703) = 1473.750; p = 0.00$), requesting four factors and suppressing loadings less than .3. This four factor option accounts for 39.55% of the total variance.

3.3.3 Results from Factor Analysis

The factor analysis begins to answer the first research question by illuminating a categorization of instructional practices that highlights a responsiveness (or lack of responsiveness) to student thinking (as indicated by the *responsiveness* items) since the factors are generated by grouping together items that instructors answered as similarly descriptive of their instruction. Table 3.3 includes operational definitions for each of the factors, as well as sample items loading on the factors. A table including all the PIPS-M items with their rotated factor loadings can be found in Appendix B.

After rotation, the first factor in the factor analysis accounted for 15.37% of the variance, the second factor accounted for 9.76%, the third factor for 7.45%, and the fourth factor for 6.97% of the variance. In order to assess whether the data from the variables in each of the four requested factors form reliable scales, Cronbach's alphas were computed. The alphas were .891, .859, .781, and .668 on factor one, two, three, and four respectively, which indicates that the items would form scales that have good internal consistency.

The first factor seems to index instructional practices that demonstrate a *student-thinking centered* approach to in-class activities, and had strong loadings on sixteen items. The items loading on this factor can be divided into two broad categories – items that reflect what an instructor does and items that reflect what students do in class. Instructors who rate items loading on this factor as descriptive of their instruction have students explore/discuss ideas before direct instruction, share their ideas, work on problems individually, reflect on processes used to solve problems, and make connections between ideas and concepts on assignments. These instructors use strategies to support underrepresented groups, encourage participation from a wide range of students, connect content to students' lives and future work, discuss multiple problem-solving approaches, give feedback during class, use student comments, questions, and assessments to guide the direction of

Table 3.3: Operational Definitions for the PIPS-M Factors with Sample Items Loading on the Factors.

Factor	Operational definition	Sample PIPS-M Items
1. Student-thinking centered instructional practices	In this type of instruction, the instructor provides time for students to explore and reflect, they create opportunities to gain insight into student thinking and strategies in class, and use student questions, comments, and assessment results to guide the class's focus and direction.	I provide time for students to reflect about the processes they use to solve problems R1: I use a variety of approaches (e.g., questioning, discussion, informal/formal assessments) to gauge where my students are in their understanding of concepts
2. Student-to-student interactions in class	In this type of instruction, peer-to-peer support is encouraged and students work together in pairs or groups, discussing course topics, mathematical difficulties, or ideas.	I structure class so that students work together in pairs or small groups I structure class so that students talk with one another about course topics
3. Equitable instructional practices	In this type of instruction, a wide variety of students participate in class, and the instructor adjusts teaching based on what (a wide variety of) students do and do not understand.	A wide range of students respond to my questions in class R4: I adjust my teaching based upon what students currently do or do not understand
4. Instructor-centered instructional practices	In this type of instruction, the instructor primarily explains concepts, based on their assumption of their students' knowledge, while students listen and take notes.	I structure my course with the assumption that most of the students have little useful knowledge of the topics My class sessions are structured to give students a clear/structured set of notes

instruction, and use a variety of approaches to gauge where students are at with their understandings.

This factor indexing *student-thinking centered instruction* also illuminates aspects of instructor responsiveness as three of the five responsiveness items (R1, R2, and R3) loaded on this factor. The connection between *student-thinking centered instruction* and responsive instruction is also documented in the literature as Kim (2017) uses these terms synonymously, highlighting that "understanding students' thinking and using it well in instruction" is a key component of responsive instruction (p. 344). The *student-thinking centered instruction* factor also points to this because it indicates that instructors who use student questions and comments to determine the focus and direction of classroom lessons (R1) also tend to provide opportunities for students to explore or discuss their understanding of concepts before direct instruction, and create opportunities during class to gain insight into student thinking, encouraging participation from a wide range of students. This suggests that instructors who demonstrate some level of responsive instruction create opportunities during class to inquire into student thinking by making space for student contributions (Kuster, Johnson, Keene, & Andrews-Larson, 2017). Additionally, in this student-thinking centered instruction, instructors value a variety of student contributions and understandings, and are responsive to these by adjusting the direction and focus of class accordingly.

This factor also seems to align with the third pillar of inquiry-based mathematics instruction – "Instructors inquire into student thinking" (Laursen & Rasmussen, 2019, p. 10). The items loading on this factor reflect several of the seven actions that instructors do in an inquiry-oriented classroom as indicated by Kuster, Rupnow, Johnson, and Garrison-Wilhelm (2018), including the following: Teachers elicit student reasoning and contributions (#2), teachers actively inquire into student thinking (#3), teachers are responsive to student contributions, using student contributions to inform the lesson (#4), students are engaged in one another's thinking or reasoning (#5), and teachers guide and manage the development of the mathematical agenda (#6). The connection between *student-thinking centered instruction* and these inquiry-oriented local instructional practices indicate that instructors who report these practices as descriptive of their instruction likely are focused on student-thinking, and incorporate opportunities both in and out of class to inquire into student thinking, using this to guide the direction of instruction. However, it is important to note that despite this alignment with the inquiry-oriented practices, the items on this factor are broad enough to also encompass student-thinking focused instruction that is not necessarily rooted in having students work on problems in groups, or reinventing the mathematics with little to no lecture.

The second factor seems to index instruction that promotes *student-to-student interactions*, and had strong loadings on six items. This factor includes items that indicate that instructors who structure their class so that students work in pairs or groups also tend to create opportunities for students to talk with one another about course topics or their mathematical difficulties, and engage with one another in peer-to-peer support and through constructive criticism of each other's ideas. This factor aligns with the second pillar of inquiry-based mathematics education, which states "Student collaboratively process mathematical ideas" (Laursen & Rasmussen, 2019, p. 10). It is important to point out that there are no items that load on this factor that indicate what the instructor is doing while the students work with one another, pointing to the absence of evidence of the third pillar of inquiry-based mathematics education. This indicates that it is possible to have students actively involved and interacting with one another in class without the instructor being responsive to students' thinking (Campbell, Cabrera, Michael, & Patel, 2017). This is further supported by the lack of responsiveness items loading with this factor. However, if an instructor incorporates these practices promoting student-to-student interactions in class, they could use this as an opportunity to gain insight into student thinking and leverage and respond to student ideas as they facilitate the student interactions, demonstrating a responsiveness to student thinking. Thus, while it is possible for these instructional practices to promote responsive instruction, this factor alone captures the student interactions but not the way an instructor uses these interactions to leverage to student thinking.

The third factor loaded five items that seem to align with the fourth pillar of inquiry-based mathematics education, which is "Instructors foster equity in their design and facilitation choices"

(Laursen & Rasmussen, 2019, p. 10). Items like "A wide range of students respond to my questions in class" and "A wide range of students participate in class" that load on this factor indicate there is more equitable participation from students because they highlight that it is not the same group of students responding to instructor questions and participating in the class. Further, it is interesting to note that the item "I use strategies to encourage participation from a wide range of students" does not load with this factor, indicating a difference between *encouraging* participation from a wide range of students and *actual* participation from a wide range of students. A result that highlights this point, and has implications for students' continuation through STEM, is from Ellis, Kelton, and Rasmussen's (2014) study where they found a statistically significant difference in how Persisters (students that continue from Calculus I to Calculus II) and Switchers (students who were intending to take Calculus II, but did not take it after Calculus I) experienced classroom practices such as whole-class discussion. In particular, Persisters experienced high levels of whole-class discussion, and Switchers experienced low levels, within the same class. Ellis, Kelton, and Rasmussen (2014) hypothesize that the instructors might be viewing themselves as holding a whole-class discussion when they ask a variety of students to respond, but the students who are not explicitly involved in this discussion likely recognize that it is not a *whole-class* discussion.

Additionally, although this third factor seems to evidence *equitable practices*, there are several items loading on other factors that also align with equitable instruction. For example, the twenty-seven equitable classroom practices developed by Montgomery County Public Schools' *Equity Initiatives Unit* (2010) highlight that there are opportunities to incorporate equitable teaching practices within a variety of classroom environments. Table 3.4 shows the alignment between survey items and equitable classroom practices on factors one, two, and three. It is important to note that although a number of these practices load with the factors indexing *student-thinking centered instruction* and *student-to-student interactions*, the incorporation of these practices do not necessary imply equitable experiences for students. The results from Johnson, Andrews-Larson, Keene, Melhuish, Keller, and Fortune's (2019) study further highlight the potential for inequities in classes with student-centered instruction, as they found significant differences in men and women's performance an inquiry-oriented abstract algebra classes that was not present in non-inquiry-oriented classes.

This third factor from the factor analysis also includes items that indicate that the instructor is responsive to students' thinking/contributions, adjusting teaching based on what students do and do not understand (R4). It is important to note that the inclusion of this item with this *equitable instructional practices* factor highlights that simply having a wide variety of students participate is not enough, but rather it is important to take and build off student contributions when moving the mathematical agenda forward. This, together with the item "There is a sense of community among the students in my class" also loading on this factor, particularly aligns with Hand's (2012) work surrounding equitable instruction where she describes the importance of supporting students in "taking up space" in the classroom. "Taking up space" includes both a participation in mathematical practices *and* participating in the development of the classroom community where one sees themself belonging.

The fourth factor seems to index instructional practices that demonstrate an *instructor-centered approach* to in-class activities, with seven items loading on the factor. Instructors who rated these practices as descriptive of their instruction probably spend most of the class time lecturing, while also providing some opportunities for students to ask and answer questions.

Since R5 ("I consider students' thinking/understanding when planning lessons") also loads on this factor, the analysis indicates that responsiveness to student thinking can occur in instructor-centered classes. This is particularly noteworthy since the majority of the literature discussing instructor responsiveness is situated within the K-12 level, where student-teacher interactions are traditionally more prevalent during in-the-moment instruction, and in the context of inquiry-oriented

Table 3.4: Equitable Classroom Practices Alignment with PIPS-M Items.

Factor	Item	Equitable Classroom Practices
1. Student-Thinking Centered Instructional Practices	I provide activities that connect course content to my students' lives and future work	Uses students' real life experiences to connect school learning to students' lives
	I structure class so that more than one approach to solving a problem is discussed	Uses a variety of visual aids and props to support student learning
	R1: I use a variety of approaches (e.g., questioning, discussion, formal/ informal assessments) to gauge where my students are in their understanding of concepts	Uses multiple approaches to consistently monitor students' understanding of instruction, directions, procedures, processes, questions, and content
	I understand students' previous conceptions, skills, knowledge, and interests related to a particular topic	Identifies students' current knowledge before instruction
	I provide feedback on student assignments without assigning a formal grade	Gives students effective, specific oral and written feedback that prompts improved performance
	I provide students with immediate feedback on their work during class (e.g., student response systems; short quizzes)	
	*I use strategies to encourage participation from a wide range of students	Seeks multiple perspectives
	*When calling on students in class, I use randomized response strategies (e.g., picking names from a hat)	Uses random response strategies
2. Student-to-student Interactions in Class	I structure class so that students work together in pairs or small groups	Uses cooperative learning structures
	I structure class so that students talk with one another about course topics	
	*I require students to work in predetermined or randomized groups	
	I structure class to encourage peer-to-peer support among students	
	I structure class so that students discuss their mathematical difficulties with other students	
3. Equitable Instructional Practices	I explain concepts in a variety of ways	Uses a variety of visual aids and props to support student learning
	*A wide range of students participate in class	Acknowledges all students' comments, responses, questions, and contributions
	*A wide range of students respond to my questions in class	Seeks multiple perspectives
* Indicates items on the PIPS-M that were written with the intention of measuring instructional practices that support an equitable experience for students		

classes at the post-secondary level (Gehrtz, 2019). In instructor-centered instruction, much of the responsiveness to student thinking is done in preparing and planning for the class rather than in-the-moment interactions with students during class. The inclusion of the item "I ask students to respond to questions during class time" has the potential to support responsiveness in instructor-centered classes, particularly if the instructor capitalizes on the opportunity to gain insight into student thinking, probing into how students arrived at their answers, and then using this insight to guide the lesson or in their preparation of future lessons. Additionally, if the instructor has a good understanding of common student strategies and conceptions on a particular concept, they can use this knowledge as they plan, incorporating multiple strategies and explanations into their lecture. This aligns with the research from the factor analysis as the item "I explain concepts in this class in a variety of ways" has cross-loadings on the *instructor-centered instructional practices* factor and the *equitable teaching practices* factor.

3.3.4 Instructor Characteristics and Responsive Instruction

In order to investigate other factors that may be related to the instructional practices described by these factors, a composite score was created using a weighted average (including cross-loaded items only in the factor with their highest loading). These composite scores were then centered around zero (by subtracting the mean) and normalized (by dividing by the standard deviation). Thus, instructors who rated items loading on the *student-thinking centered instruction* (Factor 1) as more descriptive of their teaching than average, for example, would have a positive score, and those that rated these items as less descriptive than average of their instruction would have a negative score for this factor.

Relationship between class activities and factors

In addition to the instructional practices Likert-type items on the PIPS-M, the survey also included multiple choice questions (with space to elaborate on one's response if necessary) regarding instructors' number of weekly contact hours with students, level of decision-making in the course, and blunt categorization of how class time is spent. The survey concluded with a series of demographic questions.

For the blunt categorization of class activity, instructors were asked to report the average percentage of class time, over the whole term, that their students spent on the following activities (with the total percentage summing to 100%): listening to the instructor lecture or solve problems, participating in whole-class discussions, working on tasks in small groups, and working on tasks individually. Pearson's r was computed for the *student-thinking centered instruction* factor and these four activities. A nonparametric correlation, Spearman's rho, was computed for each of the other three factors (as they were skewed) and the percentage of time students spent on these four activities. Recall that the factor scores were centered and normalized, so a positive/negative mean indicates whether or not the instructor is rating instructional practices for each factor (as a whole, using a weighted average) as more/less descriptive of their instruction than average.

Results from the correlations confirmed that there is a relationship between the types of activities students spend time doing in class and the instructional practices documented by the factors. Specifically, the instructional practices captured in factors one, two, and three (*student-thinking centered*, *student-to-student interactions*, and *equitable instructional practices*, respectively) have significant *negative* correlations with the percentage of class time students spend listening to the instructor lecture (see Table 3.5 for correlation coefficients and significance levels). This indicates that instructors whose practices align with these three factors spend significantly less time lecturing than those instructors who do not rate the practices loading on these three factors as descriptive of their instruction. Further, since listening to the instructor lecture or solve problems is an instructor-centric activity, it is no surprise that there is a positive correlation (although not statistically significant, $r(90) = .122, p = .247$) between this practice and the factor indexing *instructor-centered instruction*.

The direction of the correlation was positive between the *student-thinking centered instruction* factor and the amount of time students spent participating in whole-class discussions ($r(85) =$

$.709, p < .001$) and working on tasks individually ($r(85) = .597, p < .001$). This means that instructors who rate the instructional practices loading on this factor as descriptive of their instruction tend to structure class so that students have time participating in whole-class discussions and working on tasks individually. The direction of the correlation between the *student-thinking centered instruction* factor and the time students spent listening to the instructor lecture or solve problems was negative, indicating that the more descriptive this factor was of the instructor's teaching, the less time students spent listening to the instructor lecture ($r(85) = -.614, p < .001$). It is also documented in the literature that self-identified non-lecturers are less likely to report engaging in teacher-centric activities and more likely to engage in student-centric activities (Johnson, Keller, Fukawa-Connelly, 2018). Results also indicated that there is not a significant correlation between the *student-thinking centered instruction* factor and the time students spent working on tasks in small groups ($r(85) = -.009, p = .935$), indicating that group work focused classes are not the only structure that enable an instructor to focus on student-thinking and could even constrain student-thinking centered instruction depending on how the instructor uses student contributions during group work.

The correlation between the factor indexing *student-to-student interactions* and the time students spend listening to the instructor lecture or solve problems ($r(90) = -.520, p < .001$) and the time they spend working in small groups was also significant ($r(90) = .683, p < .001$). The direction of the correlation with lecture was negative and the direction of the correlation with group work was positive. This further highlights that instructors rating the items on the *student-to-student interactions* factor as more descriptive of their instruction also tend to structure class so that students are working in small groups. It is interesting to note that in addition to the items loading on this factor that explicitly align with group work (e.g., "I structure class so that students work together in pairs or small groups"), there are also items loading on this factor describing opportunities in class for students to talk to one another about course topics, mathematical difficulties, or ideas (e.g., "I structure class so that students discuss their mathematical difficulties with other students"), but this factor is still not significantly correlated with the time students spend participating in whole-class discussions ($r(90) = .129, p = .220$) or working on tasks individually ($r(90) = .028, p = .790$).

These analyses indicate that classes whose instructor rates high levels of *student-to-student interactions* include infrequent time where students listen to the instructor lecture and frequent time with students working together in groups, but no consistency on the time spent having a whole class discussion or with students working on problems individually. This factor did not include any items that were identified as indicating responsiveness to student thinking, while the *student-thinking centered* factor included three responsiveness items. Additionally, the *student-thinking centered* factor included infrequent lecture and frequent whole class discussion and time for students to work individually, with no consistent amount of time with students working in groups. Together, these analyses indicate that having students work in groups may not afford responsiveness to student thinking as much as students working individually, paired, or with spending time having whole class discussions.

Table 3.5: Correlation Coefficients (Pierson's r/Spearman's rho) Between Factors and the Class Time Students Spent on Various Activities.

Factor	Time students spend listening to the instructor lecture or solve problems	Time students spend participating in whole-class discussions	Time students spend working on tasks in small groups	Time students spend working on tasks individually
1. Student-thinking centered	$-.614$**	.709**	$-.009$	.597**
2. Student-to-student interactions	$-.520$**	.129	.683**	.028
3. Equitable instructional practices	$-.287$*	.428**	.072	.026
4. Instructor-centered	.122	.053	$-.146$	.076
*$p < .01$; **$p < .001$				

There was also a significant positive correlation between the factor indexing *equitable teaching practices* and the time students spend participating in whole-class discussions ($r(91) = .438, p < .001$). This further reiterates that the incorporation of equitable teaching practices correlates with the amount of time or opportunities students' have to participate in whole-class discussions, and is less related to the time students spend working on tasks in small groups or individually (which do not have a significant correlation with this factor). Further, the direction of the correlation of the *equitable teaching practices* factor and the time students spend listening to lecture is negative, indicating the more an instructor's practices align with this factor, the less time they spend lecturing. Table 3.5 shows the correlations between the four factors and the various activities students spent time doing during class.

Relationship between experience and factors

Years of experience as an instructor of record is one way to approximate an instructor's experience teaching. To investigate if there was a statistically significant association between years of experience as an instructor of record (question in the demographics portion of the survey) and instructional types, as indicated by the factors, a correlation was computed. Since the data for years of experience as instructor of record was skewed (skewness=2.015), the Spearman rho statistic was calculated. The results indicate that there is no significant association between the number of years as instructor of record and any of the factors. This indicates that an instructor's experience does not relate to specific instructional types, such as instructor-centered instructional practices or student-thinking centered instructional practices. Another proxy for experience is the rank of the instructor (e.g. graduate student, postdoctoral fellow, lecturer/adjunct, faculty member). In order to determine if an instructor's rank was related to their types to instructional practices, a one way ANOVA was conducted. However, there were no statistically significant differences found among the rank of instructors on *student-thinking centered instruction*, *student-to-student interactions*, *equitable instructional practices*, or *instructor-centered instruction.*

Relationship between decision-making role and factors

Instructors were asked the following questions regarding their involvement in the decision-making of the course.

Q1. How are most decisions about course content (e.g., syllabi, exams, homework, pacing, grading) made for Calculus 1? Clarify if you wish.

Q2. How are most decisions made about instructional approach (e.g., use of clickers, group work, active learning) made for Calculus 1? Clarify if you wish.

Instructors were provided with the following options, along with space to clarify their response if necessary: (1) I make most decisions; (2) I'm part of a team that makes the most decisions; (3) Someone else makes the most decisions.

To determine how an instructor's involvement with the decision-making in the course was related to their instructional practices, as indicated by the factors (continuous scale variables), a one-way ANOVA was conducted for Q1 and Q2. Results indicate that there is no significant difference between the three levels of instructor involvement in decision-making in regards to course content (Q1) on the *student-thinking centered instruction* factor (factor one), the *student-to-student interactions* factor (factor two), the *equitable instruction* factor (factor three), or the *instructor-centered instruction* factor (factor four). However, a statistically significant difference was found among the three levels of instructor involvement in decision-making about instructional approach, $F(2, 90) = 3.617, p = .031$ on at least one of the factors. A post-hoc Tukey HSD test indicated that instructors who are responsible for making the most decisions and instructors who indicated someone else makes the most decisions about instructional approach differed significantly in their implementation of practices loading on the *equitable instruction practices* factor with a high effect size ($p = .026, d = 1.24$). Those instructors responsible for making instructional decisions ($M = .0938$) were more likely to rate these equitable teaching practices as more descriptive of their instruction than those who indicated they did not make decisions about instructional approach ($M = -1.095$).

3.4 Discussion

The factor analysis sheds light on practices that instructors rate as similarly descriptive of their instruction. Further, the four factors indexing *student-thinking centered instruction*, *student-to-student interactions*, *equitable teaching practices*, and *instructor-centered instruction* shed light on instructional practices that reflect responsiveness to student thinking, answering the first research question. Responsiveness survey items loaded on three of the four factors, including factors indexing both student-centered and instructor-centered instructional practices, providing evidence that there are opportunities for instructors to be responsive to their students' thinking in either of these types of instruction. In more student-centered classrooms, instructors have more opportunities to draw out and build on student thinking in the moment, whereas in classes that are more instructor-centered, instructors still have opportunities to be responsive to their students' thinking outside of class in planning or in how they write homework or assessments. Table 3.6 highlights how each factor aligns with student- or instructor-centered instruction and opportunities to be responsive to students' thinking.

Table 3.6: Alignment with Factors and Opportunities to be Responsive.

Factor	Student-Centered Instruction	Instructor-Centered Instruction	Opportunities to be Responsive in Instruction
1. Student-thinking centered instructional practices	Yes	Yes	Yes
2. Student-to-student interactions in class	Yes	No	Yes
3. Equitable instructional practices	Yes	Yes	Yes
4. Instructor-centered instructional practices	No	Yes	Yes

It is important to point out that for each of the factors where responsiveness items loaded there were also items loading that indicated the instructor was eliciting or drawing out student thinking.

This is crucial because if an instructor does not value student contributions or does not know or recognize that students might have different mathematical understanding grounded in some productive thinking, then they cannot truly be responsive to students' mathematical strategies and thinking. On the *student-thinking centered instruction* factor, instructors use a variety of approaches to gauge students' understanding and provide opportunities for their students to share their ideas (or their group's ideas). The *equitable teaching practices* factor loads items that indicate that a wide range of students are participating and answering questions in class. For the *instructor-centered instruction* factor, students are asked to respond to questions in class. However, simply because students are responding to questions does not necessarily indicate that the instructor is gaining insight into student thinking, especially if the instructor is focused primarily on student answers rather than how they understand or arrived at those answers.

As mentioned in the results section, there were no responsiveness items loading on the factor indexing *student-to-student interactions*, but this does not mean that there are not opportunities for instructors to leverage and be responsive to students' thinking during this type of instruction. In fact, instruction that incorporates student-to-student interactions provides many opportunities for instructors to gain insight into student mathematical thinking and reasoning, but the instructor has to be intentional in using these opportunities to impact instruction and to be responsive to their students' thinking. Simply having students engage with one another is not enough.

The results from the analysis investigating the relationship between the factors and reported class activities highlight the possible role that certain class activities can play in support of responsive instruction. While the factors themselves illuminate that responsiveness to student-thinking can occur in instructor-centered environments, there were no significant correlations between this factor (*instructor-centered instruction*) and the amount of time instructors reported that their students spent listening to lecture, participating in whole-class discussions, working on tasks in small groups, or working on tasks individually, indicating that such classes do not have one clear structure. The remaining three factors are more clearly student-centered, with only two factors, the *student-thinking centered instruction* and *equitable instructional practices* factors, loading items identified as indicating responsiveness to student thinking. Instructors who rated the items loading on these two factors as descriptive of their instruction spend significantly less time lecturing and significantly more time having a whole class discussion, compared to those instructors who do not rate the instructional practices loading on these factors descriptive of their instruction. Additionally, instructors who rated the items loading on *student-thinking centered instruction* also more frequently have their students work individually compared to other instructors. Interestingly, there is a not a significant correlation between the *student-thinking centered instruction* and *equitable instructional practices* factors and the amount of time students spend working in groups, though there is a significant correlation between this activity and the *student-to-student interactions* factor. This indicates a more nuanced understanding of the roles that specific student-centered instructional practices can play in fostering a responsive instructional environment.

Although there is evidence in the literature that experience and professional development have the potential to influence an instructor's responsiveness to student thinking, the findings here indicate that instructional practices, as characterized by the factors, are not related to instructors' experience teaching as an instructor of record or their rank as a graduate student, postdoctoral fellow, lecturer, or as a faculty member. This aligns with the results from Johnson, Keller, and Fukawa-Connelly's study (2018) which found that respondent's self-identification as a "lecturer" (indicating their primary instructional practice was lecture) or a "non-lecturer" (indicating their primary instructional practice was not lecture) was independent of their experience teaching, the nature of the course, or the nature of the institution.

Similarly, there is literature that indicates that the role of an instructor as a decision-maker could impact their responsiveness to student thinking, but results from this study provide evidence that instructors' involvement in the decision-making for course content does not significantly impact their instructional practices characterized by the factors, including their responsiveness to student thinking. This could be because the factors largely highlight practices describing instruction and might be less related to the actual content of the course. Instructors' role in decision-making in regards to instructional approach does relate to the level they incorporate equitable teaching practices. This could highlight that instructors who have a role in making-decisions regarding the instructional approach of a course leverage this decision-making to create opportunities to incorporate more of these equitable teaching practices such as explaining concepts in a variety of ways, creating opportunities for a wide range of students to participate, and building a sense of community among students in class.

3.4.1 Other Impacts on Responsiveness

In summary, if an instructor's experience, rank, or their role as a decision-maker (in regards to course content) does not significantly impact their instructional practices and responsiveness to student thinking, as characterized by the factors, what does have a significant impact? There are a number of aspects that were not captured by the survey that likely play a critical role in an instructor's ability to be responsive to student thinking, including their disposition and mathematical knowledge for teaching, since an instructor's disposition is continually shaped by their experiences and interacts with their mathematical knowledge for teaching to influence what they attend to during instruction and how responsive they are to students and students' thinking (Hand, 2012; van Es & Sherin, 2002).

Disposition

I have argued that an instructor's disposition to student thinking plays an important role in how they attend and respond to student thinking, either in class or outside of class (Gehrtz, 2019). This is supported by Robertson, Scherr, and Hammer (2016) as they also hypothesize that responsive instruction goes beyond an articulated sequence of activities, and state that they suspect "that responsive teaching grows out of and is ground[ed] in a stance toward students and their ideas" (p. 3). This seems to indicate that an instructor's disposition towards students and their thinking impacts the level to which they value student contributions and are able to demonstrate responsiveness. Additionally, consider Thornton's (2006, p. 62) definition of disposition:

> Dispositions are habits of mind including both cognitive and affective attributes that filter one's knowledge, skills, and beliefs and impact the action one takes in classroom

> or professional setting. They are manifested within relationships as meaning-making occurs with others and they are evidenced through interactions in the form of discourse.

This definition further highlights the impact of dispositions on instructional decisions as it points to dispositions as a lens through which an instructor's knowledge, goals, and beliefs are filtered, going beyond an underlying system of beliefs.

Mathematical Knowledge for Teaching

There has been much research surrounding teachers' knowledge of student thinking, particularly at the K-12 level, by considering this as one component of instructors' mathematical knowledge for teaching (MKT). MKT includes both knowledge of content and the pedagogical knowledge needed to facilitate learning and effective instruction. In particular, it includes knowledge about the typical ways students think about mathematics (productively and unproductively), common student strategies, examples that are especially useful for developing students' mathematical meanings, organization of course content, and the relative difficulty of topics for students (Ball, Thames, & Phelps, 2008). An instructor's MKT influences their decisions, and impacts what they notice and attend to during various instructional practices (Jacobs, Lamb & Philip, 2010). Further, robust MKT enables an instructor to be able to infer the student mathematics, recognizing what might be productive in students' incomplete or incorrect answers, which sets the stage for an instructor to respond and build on student understanding during both in-the-moment instruction and while they are planning for instruction. When an instructor has a better understanding of student mathematical thinking, and how it develops, then they are better able to respond and to effectively guide student thinking (Borko, 2004), as there is evidence that a lack of MKT constrains an instructor's ability to respond effectively in the moment (Johnson & Larsen, 2012).

In other components of this larger study, I explore responsiveness as a disposition among eight instructors from one institution. More research is needed to better understand how MKT and disposition relate to responsiveness to student thinking.

3.5 Potential Limitations

There are some potential limitations to using a survey that relies on instructors' self-reports of what is happening in class, such as instructors overestimating the amount of time spent on student-centered activities and underestimating the amount of time spent on instructor-centered activities such as lecture (e.g., Ellis, Kelton, & Rasmussen, year; Fung & Chow, 2002; Hayward, Weston, & Laursen, 2018). However, there is not a consensus on whether or not self-report surveys are considered a limitation (Desimone, 2009). Specifically, there is evidence that when instructors are reporting on their teaching over a specific time frame, considering specific behaviors and activities, surveys can provide information consistent with observations (Desimone, 2009). Further, Hayward, Weston, and Laursen (2018) suggest that when "survey and observation instruments are well designed and properly aligned, surveys may be a trustworthy, efficient, and less costly method of measuring teaching practices" (p. 733). In particular, their results point to evidence that survey instruments designed to capture descriptions of what happens in class are largely accurate, compared to survey instruments that are more evaluative in nature. The Post-secondary Instructional Practices Survey (Walter, Henderson, Beach, & Williams, 2016), that served as a starting point for the survey distributed by the Progress through Calculus project team, was designed to "reduce the potential for eliciting socially acceptable responses" (p. 3) and included items that were nonevaluative. For example, the survey includes the item "I guide students through major topics as they listen and take notes" instead of an item that might "downplay" this approach as "just lecture" (p. 3). As the Progress through Calculus project team expanded the PIPS survey, creating the PIPS-Mathematics (PIPS-M), adding items of interest for our research purposes, we aimed to mirror the formatting and nonevaluative nature of the existing PIPS items. Consequently, we can reasonably conclude that the survey results used in my dissertation study are reasonably accurate descriptions of the participants' instruction.

The sample size is also a limitation of this study. Although a sample size of N=94 Calculus 1 instructors of record is a sufficient size for the factor analysis, the additional quantitative analyses investigating associations between various groups on the instructional practices factors may have been impacted by this small sample size. Specifically, when considering if there were differences between instructors who reported they made most of the decisions regarding course content and those who reported they were not involved in making these decisions, the sample size was reduced to groups of size 31 and 30 participants respectively. Consequently, the power $(1-\beta)$ is .48, using $\alpha = .05$ and medium effect size, .5 (Cohen, 1988), which is much less than the ideal power of .80.

3.6 Conclusion and Implications

The exploratory factor analysis sheds light on various categorizations of undergraduate mathematics (calculus, in particular) instruction, in addition to highlighting how these practices might be related to instruction that is responsive to students' thinking. Additionally, this categorization provides specific practices that can be incorporated by instructors interested in being more responsive to student thinking, which is especially relevant due to the findings that encourage the use of student-centered strategies in undergraduate instruction (e.g., Freeman, et al., 2014). For example, since three of the responsiveness items loaded on the factor indexing *student-thinking centered instruction*, incorporating other practices loading on this factor, such as providing opportunities for students to share their thinking in class, would be beneficial as the presence of student thinking is foundational for building on and responding to student thinking (Leatham, Peterson, Stockero, & Van Zoest, 2015). Additionally, although there seem to be more opportunities to build on student thinking during in-the-moment instruction, results indicate that responsiveness to student thinking can also occur in instructor-centered instruction through the incorporation of student thinking in the preparation for a lesson.

There is evidence that effectively building on student contributions and responding to student thinking is challenging (e.g., Speer & Wagner, 2009; Johnson & Larsen, 2012) and relies on the instructors' knowledge of student mathematical thinking (Johnson & Larsen, 2012), valuing of and eliciting student contributions (Kuster, Johnson, Keene, & Andrews-Larson, 2017), and ability to attend to interpret and respond to students' understandings (Jacobs, Lamb, & Philipp, 2010). However, the results of this analysis indicate that responsiveness to student thinking is not limited to experienced instructors, as one might expect, highlighting that responsiveness to student thinking is complex and more research needs to be done in order to more fully understand what enables (or constrains) it.

In future work, I will observe instructors to learn more about how their reported responsive instruction relates to their actual responsive instruction, and will further consider how an instructor's disposition is related to their responsiveness. Gresalfi and Cobb (2006) noted that developing "positive and productive dispositions … will allow us to begin to address the current well-documented gap between who actually has opportunities to be successful in the classroom" (p. 55). Although Gresalfi and Cobb were referring to cultivating students' dispositions, this also can apply to instructors' dispositions towards student thinking; when we better understand how we can foster responsiveness towards student thinking, then we can better support students by providing opportunities for more effective instruction building off of their understandings.

Chapter 4

Paper 3: Responsiveness as a Disposition and Its Impact on Instruction

There is evidence that instructors who are responsive to students' thinking tend to provide more positive learning experience for students. Additionally, effective instruction relies on an instructor's ability to respond to student thinking, which is especially relevant due to the increased attention on improving college mathematics instruction. In order to investigate instructor responsiveness to student thinking as a disposition (that guides action) and responsiveness to student thinking as an action (the enacted evidence of the underlying disposition), eight college calculus instructors were interviewed three times over the course of one academic year. A thematic analysis of the task-based interviews indicated that instructors who exhibited a responsive disposition to their students' thinking enact this through eliciting student thinking, reflecting on student thinking, and responding to student thinking. Further, these instructors view themselves as decision-makers, and thus feel empowered to act on their responsive disposition.

Effective instruction relies on an instructor's ability to attend and respond to students' mathematical understandings and strategies (Jacobs, Lamb, & Philipp, 2010). Additionally, there is evidence from the K-12 literature that an instructor's disposition towards student thinking influences instructional decisions, including how they interact and respond to students (Sherin & Russ, 2014; Thornton, 2006; Schoenfeld, 2008). Specifically, it has been shown that teachers who are more responsive to their students' thinking are generally recognized as more effective teachers who provide more positive learning experiences for their students (Thornton, 2006). This is especially relevant due to the increased attention on improving mathematics instruction, and in particular, the focus on student-thinking-centered instruction.

In my work defining *responsiveness to student thinking* based on K-16 literature surrounding responsiveness and responsive instruction, I highlight two dimensions of *responsiveness to student* *thinking* – an enacted responsiveness and responsiveness as a disposition (Gehrtz, 2019), which I define as follows:

> *Responsiveness as a disposition* is attending to student thinking, mathematical meanings, and needs (including both cognitive and affective needs), and an awareness of the need to provide an appropriate response based on student understandings or contributions. A responsive disposition relies on an instructor's valuing of student thinking.
>
> *Enacted responsiveness* is responding to student thinking by focusing on the substance of student contributions (including those that are incorrect or incomplete) when determining the direction of the lesson, through leveraging and building on student contributions, and by making connections between student ideas and what is accepted as disciplinary conventions.

Although there is evidence that suggests that this distinction between an underlying and enacted disposition exists and is productive, few of the articles that I considered in my thematic analysis called this distinction out explicitly, and even fewer articles articulated what the underlying disposition, orientation, or propensity to be responsive consisted of or how it impacted practice. For example, Hackenberg (2010, p. 240) uses the phrase "intuitive responsiveness" as a contrast to "a more deliberate, analytical formulation of how to interact" when discussing an instructors' effort to understand their students' thinking, but does not define or describe what "intuitive responsiveness" actually entails. Similarly, Robertson, Atkins, Levin, and Richards (2016, p. 3) hint at the impact of disposition on responsive instruction when they state, "We suspect, instead, that responsive teaching grows out of and is grounded in a stance toward students and their ideas rather than through any particular structure of activities", but they do not elaborate on this "stance toward students and their ideas." Thus, more work needs to be done to better understand the role of an underlying disposition on *responsiveness to student thinking*. Thornton (2006) notes the benefits of considering instructor dispositions in the context of teaching practice; consequently, the primary goal of this paper is to investigate responsiveness as a disposition, by identifying components of responsive instruction and the role of decision-making (indicating an enactment of a responsive disposition).

4.1 Research Related to Dispositions and Decision-Making

Although instructor dispositions towards teaching and students play an integral role in how instructional activities are chosen and enacted, it is not always clearly articulated what is encompassed by one's "disposition." Dispositions can refer to an instructor's beliefs, inclinations, values, attitudes, and ability, among other things (Splitter, 2010). For this paper, I draw upon Thornton's (2006) definition of *dispositions in action* that arose as a result of her work studying middle school teachers' dispositions:

> Dispositions are habits of mind including both cognitive and affective attributes that filter one's knowledge, skills, and beliefs and impact the action one takes in classroom or professional setting. They are manifested within relationships as meaning-making occurs with others and they are evidenced through interactions in the form of discourse (p. 62).

This definition highlights that dispositions are more than simply latent values or beliefs, and that dispositions interact with knowledge and influence instructional practices. Based on other's previous work and my own experiences, I hypothesize that an instructor's disposition towards student thinking influences how they interact with students during class or office hours, how they interact with student thinking in class how they approach grading and thinking about student work.

In her work on dispositions, Thornton (2006) developed a continuum of examples using classroom discourse analysis that describes teachers' orientations to student interactions ranging from a *responsive disposition* to a *technical disposition*. *Responsive dispositions* are those that are responsive to the needs and learning of students, including emotional, cultural, and development needs, and *technical dispositions* are those that involve going through the motions of teaching, but not engaging on a deeper level to probe, understand, or facilitate student learning. Thornton (2006) notes that with technical dispositions, instruction varies little from student to student and from situation to situation. This framing of teacher dispositions on a continuum lends itself to distinguishing between teachers who view themselves as in-the-moment decision-makers and those who do not. More specifically, one would expect teachers who view themselves as decision-makers to continually direct classroom interactions in order to align them with their goals for student learning as well as with their students' current thinking (exhibiting responsive dispositions). Conversely, teachers who carry out their role technically are expected to follow their prescribed lesson plans or pedagogical goals without deciding to adapt to the needs of the class or students (exhibiting technical dispositions).

One of the most developed models for considering teachers as decision-makers comes from Alan Schoenfeld who has worked to describe how knowledge, goals, and beliefs interact to shape instructional practices and decisions (Schoenfeld, 1998). His work provides evidence that an instructor's knowledge about the content, context, and pedagogy influences the types of things that they attend to during instruction and why they make certain decisions. Kim (2017) has further argued that teacher knowledge, specifically of curriculum, can support teachers in enacting student-thinking centered/responsive instruction (note that she uses these terms interchangeably). Additionally, an instructor's goals (short or long term) influence how they decide to respond in the moment. For example, if a student asks a question in class, the instructor has to decide how they want to answer (with a mini-lecture, class discussion, etc.) and how long they want to spend answering the question and when (either now or later); these will vary depending on the instructor's immediate and long-terms goals for student learning. Further, an instructor's beliefs and dispositions influence which goals they prioritize. Schoenfeld (1998) notes that certain beliefs, knowledge, or goals can be strongly activated at a particular moment during instruction (either because of planning or an interaction) and that this can influence how the instructor decides to respond.

Schoenfeld (2008) has also noted that teaching is a system that involves coherence between teacher commitments and values. He highlights that even when teachers are flexible and responsive to student thinking in their classroom, attending to multiple or conflicting goals, it is possible to model their decisions with consistency. This illuminates the connection between an instructor's underlying beliefs and the instructional decisions they are making, further highlighting that responsive dispositions can be enacted through decision-making.

Another framework that unpacks how teachers act as decision-makers is that of professional noticing (Mason, 2002; Sherin, Jacobs, Philipp, 2011), which has been used as a way to connect an instructor's knowledge and practice with their disposition towards student thinking (Hand, 2012). This framework focuses specifically on how a teacher decides to respond to students' mathematical understandings, complementing Schoenfeld's framework that models all the decisions an instructor makes while teaching. Jacobs, Lamb, and Philipp (2010) describe noticing as: attending to, interpreting, and deciding how to respond to student strategies and understandings. An instructor's disposition to student thinking has been shown to impact the types of things that they attend to during instruction, impacting how and what they respond to (Sherin & Russ, 2014). I conjecture that in order for teachers to effectively attend, interpret, and respond to their students' understandings, they must have a responsive disposition that values student contributions and allows them to capitalize on their role as decision-maker.

The frameworks discussed above focus on in-the-moment decision-making, highlighting different processes and aspects that impact instructional decisions. Schoenfeld links knowledge, goals, and beliefs with decision-making, and Jacobs, Lamb, & Philipp link attending and interpreting with how an instructor interacts with specific students' understandings. However, neither framework attends explicitly to the underlying responsive disposition that guides instructors' behavior, and how instructional decisions shed light on this underlying disposition. This study is guided by the following research question: How do college calculus instructors exhibit responsiveness to student thinking? In particular this work investigates instructor responsiveness by focusing on both responsiveness as a disposition (that guides action) and responsiveness as an action (the enacted evidence of the underlying disposition).

4.2 Research Design and Methodology

This study is part of a larger mixed-methods studying investigating college instructors' responsiveness to student thinking, distinguishing between responsive dispositions and enacted responsiveness. For this paper, I focus on the qualitative data collection and analysis component of this study. To understand responsiveness as a disposition and how it impacts college mathematics instruction, I focus my study at one university and in one content area - calculus. I chose calculus because this is a course that impacts a vast array of students, with varying interests and educational goals, and is taught by a wide variety of instructors with their own varying experience, interests, and educational goals (Bressoud, Mesa, & Rasmussen, 2015).

I conduct this work from a situated cognition learning perspective which emphasizes the importance of context in the development of understanding and knowledge (Brown, Collins, & Duguid, 1989). From this perspective, it is essential to consider the multiple facets (i.e. content, level of instruction, teacher knowledge, teacher beliefs) that are tied to and interact to give rise to various knowledge impacting teaching practice (Putnam & Borko, 2000). Specifically, in trying to research and improve teacher practice, we must attend to teachers' dispositions as a part of this surrounding context. Although interviews were conducted outside of a teaching context, instructors were asked to consider their teaching practice in addition to examining student work, which is a common and authentic practice for most instructors. Additionally, since reflection has been used as a way to assess and define dispositions (Thornton, 2006), the use of semi-structured interviews affords opportunities for instructors to be reflective on their practice, which can shed light on their underlying dispositions and motivations for instructional decisions beyond what could be learned by observing teachers in situ. Further, as Thornton (2006, p. 67) notes, "instead of coming from an *a priori* notion of what makes for the best dispositions, this [dispositions in action] approach comes from observations of teachers in practice and what transpired in the classroom," suggesting that dispositions in action can be "evidenced and documented through examining classroom discourse" (Thornton, 2006, p. 67). This supports the notion that as instructors' engage in an approximation of practice, by describing their instruction, instructional decisions, engaging with student work, and reflecting how they would respond to students, we can gain insight into the underlying disposition that supports these enactments of practice.

4.2.1 Interview Participants

For this study, eight Calculus 1 instructors from one highly selective institution were interviewed. Participants were recruited from a highly selective instruction with the assumption that these instructors would have the mathematical content knowledge necessary for interpreting student work. This choice was made because it has been documented in the literature that the lack of such content knowledge is a barrier to an instructor's ability to interpret student work and be responsive to student thinking and contributions (Musgrave & Carlson, 2017; Pascoe & Stockero, 2017). Four participants were new graduate teaching assistants (GTAs) who were leading recitation sections (Adem, Bao, Huang, and Cruz)[4], two were experienced GTAs and were instructors of record (with multiple semesters experience teaching Calculus 1 – Kevin and Daniel), and two were experienced teaching faculty (Ava and Jennifer). Of the teaching faculty, one was in her first year at this institution, but had several years of experience teaching as a graduate student at another institution (Ava). The other teaching faculty had received her PhD at this institution and had ten years of experience teaching Calculus 1 (and other courses) at this institution (Jennifer). The participants' experience teaching calculus (at the beginning of the study in Fall 2017) at this particular institution is summarized in Table 4.1. These participants were selected because of their varying levels of experience instructing and interacting with students. Additionally, this variation of roles and responsibilities related to the instruction of calculus is likely to influence their perception of their role as decision-maker, and consequently provides greater insight into responsiveness as a disposition and how this is enacted in instruction.

4.2.2 Qualitative Data Collection

A series of three interviews were conducted with each of the participants over the course of one academic year. The first interview was designed to learn about the participants' experiences teaching, career goals, and perspective on what it looks like to be a good instructor. The first interview also included one prompt where instructors were asked to interpret student work and

[4] All participants' names are pseudonyms.

Table 4.1: Summary of Participants' Calculus Teaching Experience.

Participant	Semesters Leading Recitations	Semesters as Instructor of Record
Adem*	1	0
Bao*	1	0
Huang	1	0
Cruz	1	0
Kevin	1	2
Daniel	1	8
Ava**	0	1
Jennifer***	2	20
*Had experience leading recitation at a previous institution **Had experience teaching a variety of courses as instructor of record during graduate experience ***Includes experience teaching as a graduate student and as faculty		

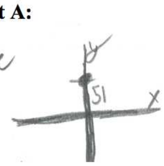
describe how they would respond if the student visited their office hours. Figure 4.1 includes an excerpt from this portion of the interview.

Indicate whether each of the following statement is TRUE or FALSE. If the statement is true, explain how you know it's true. If it is false, give a counterexample **and** explain why it is a counterexample.

If $\lim_{x\to 3^+} f(x) = 51 = \lim_{x\to 3^-} f(x)$, then $f(x)$ is continuous at $x = 3$.

Student A:

True



The graph shows that x=3 their is a Solid dot indicating that f(x) is continuous at x=3.

Questions posed during the interview:

- What is this student thinking? How would you interpret this response?
- If Student A were to come to your office hours, what sorts of things might you do to help them have a better understanding of this concept?

Figure 4.1: Excerpt from initial interview protocol

The second interview was a task-based interview adapted from one used previously to examine college instructor mathematical knowledge for teaching where instructors were asked to work through calculus prompts, interpret student work to those prompts, and then discuss how they would respond to the students' thinking in office hours (Speer & Frank, 2013). This second interview included three calculus prompts (one focusing on limit, one on derivative, and one on integral), and had a similar form to the interview excerpt presented in Figure 4.1. The third interview was designed to facilitate a discussion revolving around various responsive instructional practices, such as using student questions and comments to inform instruction. The interview protocols are included in Appendix C for reference. Table 4.2 details the qualitative data and timing it was collected.

Table 4.2: Summary of Qualitative Data Collected.

Semester	Fall 2017	Spring 2018
Data Collected	Initial Interview (via video call) Before the start of the term for new graduate students In week 9 for experienced instructors Task-Based Interview (face-to-face) Week 11 for all participants	Follow-up Interview (face-to-face) Week 13 for all participants

4.2.3 Qualitative Data Analysis

The interviews were audio-recorded and transcribed for analysis. The interview data were analyzed using thematic analysis (Braun & Clarke, 2006), by first highlighting all utterances related to a consideration of students or their thinking. These segments were then coded as either demonstrating *enacted responsiveness* or *responsiveness as a disposition*. Segments coded as *enacted responsiveness* included segments where instructors were responding specifically to students' work (e.g., "I would just go back over the definition with them"), and segments coded as *responsiveness as a disposition* were segments that demonstrated a general attending to students' needs, learning, and understanding (e.g., "I [try to] put myself into [the students'] positions, thinking about if I am first learning this concept."). I then used inductive open-coding to determine themes, paying specific attention to how responsiveness as a disposition influenced responsiveness in action. After arriving at three general categories that described how instructors' were enacting responsive dispositions (or not) in their practice, I used deductive coding to code all the interviews using these categories. During this process, I also used inductive coding to develop subcategories as necessary. In future work refining this coding process, I will deductively code all of the interviews with the subcategories, and will have a second coder in order to establish inter-rater reliability.

In the next phase of data analysis, I highlighted all utterances that pointed to instructors' decision-making and their role as a decision-maker. This coding documented themes between dispositions and decision-making that has been discussed in the literature. I then compared and contrasted how an instructor's view of themself as a decision-maker shed light on their enactment of responsiveness.

4.3 Findings

The thematic analysis revealed how an instructor elicits, reflects, and responds to student thinking and mathematical understandings (demonstrating enacted responsiveness), which serves as a proxy for understanding their underlying responsive disposition. The following section describes these themes (from inductive and deductive coding) and preliminary subthemes (from inductive coding) demonstrating the two dimensions of responsiveness to student thinking. It is important to note that this analysis does not simply seek to create a list of activities that instructors must do in order to demonstrate a responsive disposition, but rather to better understand the types of activities that seem to stem from and are enabled by a responsive disposition. After identifying such activities, I then discuss the role of decision-making in these actions, and present three vignettes that illustrate various enactments of responsiveness, underlying dispositions, and decision-making.

4.3.1 Eliciting Student Thinking

The thematic analysis of the interviews highlighted multiple underlying reasons why instructors might elicit student thinking, shedding light on their underlying disposition of responsiveness. Instructors that elicited student thinking either sought to draw out understandings they anticipated students would have (either correct or incorrect), or sought to gain insight into student thinking in order to gauge understanding. There were also instances where instructors did not elicit student thinking explicitly; these tended to be situations where either the instructor was able to interpret student thinking from the student's work or they sought to interpret the work without prompting for student thinking (e.g., "Well, I'd first have to figure out what they were getting at in answering this question."). The following excerpts demonstrate distinct motives for eliciting student thinking, and come in response to being asked "When you are planning, or teaching or grading, how do you consider student thinking?" and "What if this student came to your office hours, what sorts of things would you do to help them have a better understanding of this concept?", respectively, with the second two excerpts below being in response to the latter question.

> *Eliciting to draw out common student errors:* "I have been spending time every week coming up with five challenging problems, and I think, 'What's all the stuff they mess up on the test?' And I can put them all into [these] problems ... I said I'll work through all of these with you so they don't just blatantly do all the mistakes ... They'll kind of know that they are not sure what they're doing, ... and so I have noticed that by me kind of drawing these to the forefront ... [they] seem pretty good when [there are] similar ... stumbling blocks on the later assessments."
>
> *Eliciting to gain insight into student thinking:* "The first thing I would ask them is for them, now that they have the opportunity to take as much time as they want, try to explain to me what they were thinking."
>
> *Eliciting to guide a student through a problem:* "I would probably just ask them like what's going on throughout time - like which car is moving faster. And then based on that, which one went farther during this time."

These excerpts give insight into instructors' underlying disposition of responsiveness to student thinking. Instructors that exhibit a more responsive disposition tend to demonstrate multiple motivations for eliciting student thinking, drawing out student thinking in a variety of situations. The most common of the subcodes listed above was that of *eliciting to gain insight into student thinking*. This is likely due to the fact instructors were asked to respond to students' work several times throughout the interviews and they felt they needed more information about how the student was thinking in order to respond accordingly.

4.3.2 Reflecting on Student Thinking

Instructors who regularly reflected on their students and their students' understandings demonstrated a responsive disposition towards student thinking. These instructors tended to reference students or their thinking when discussing the motivation behind various instructional practices and decisions. The following excerpts come from one instructor's interview (faculty member Ava) - note the variety of ways that this single instructor attends to students and reflects on student experiences and thinking. These excerpts together highlight a responsiveness as a disposition to student thinking, and provide insight into why they make certain instructional decisions when enacting this disposition.

> *Reflecting on students' affect:* "I have felt that my students have a lot of anxiety just because they are trying to prepare for this test ... I am supposed to be very conscientious about how much information I share with my students, and I get that because we want the experience to be uniform. So if I am telling my students more than other instructors, then that is not fair ... I personally don't care about fairness, but I understand that fairness is a consideration ... And it's one way for me to alleviate my own anxiety, and my students' anxiety was just to tell them what I wanted them to know [referring to previous teaching experience where she had complete control over the course]... I was still able to help them to focus on the things that I thought were important."
>
> *Reflecting on student difficulties with content:* "Right now my students across the board - so students who I know came in with strong backgrounds and students who came in with maybe weaker backgrounds - are all having trouble with sigma notation and writing down Riemann sums."
>
> *Reflecting on student thinking when grading:* "Definitely when I am grading I have more time and space to think, 'Oh you've written down this thing in this weird way,' let me try to figure out where it is coming from."
>
> *Reflecting on student thinking when planning:* "I mean ideally when I plan a lesson I think about what my students will struggle with and what they will feel very natural [with], but I don't always do a good job of it. I don't always have the time and space to really think about what exactly is going to be the challenging part, and I also don't always do a good job of predicting what is going to be the challenging part."
>
> *Reflecting on specific student thinking and understanding:* "I would want for them to draw me a picture, ... because if they drew me just a single point, then I am worried that they are only thinking of this as single point instead of a single point in a continuous function. But if they are thinking of this as a single point in a continuous function, then I think that they have some understanding of what is going on with the limit."

Other instructors demonstrated a responsiveness to student thinking by reflecting on similar aspects as those described above. For example, one instructor said “I just try to put myself inside their head as best as I can”, which would be classified as *reflecting on student thinking when planning*. This segment, however, demonstrates reflection that is relatively vague and focuses on the instructor’s perspective on student thinking, whereas Ava’s reflection attends to more specific aspects of student thinking, such as things they will struggle with or find more natural. Another example of an instructor demonstrating *reflecting on student difficulties with content* is evidenced by the statement, “We try to think ... through the first time I learned this, what was tough for me. And we write that on the board and go over it. And I think a lot of the times we get it sort of correct, and some of the times we don’t.” This differs from Ava’s reflection on student difficulties because she also reflects on some of the potential underlying factors that contribute to student difficulties with content (and refers to specific difficulties), whereas this excerpt points to the recognition of potential student difficulties, and largely draws on the instructor’s own experience as a student, rather than knowledge of actual student difficulties. Instructors who demonstrated a more responsive disposition towards student thinking reflected on students regularly throughout the interview, regardless of specifically being asked to consider student thinking (like Ava). Additionally they reflected on underlying factors that contribute to student understandings, rather than simply identifying/recognizing student thinking and (mis)understandings.

4.3.3 Responding to Student Thinking

Most of the segments coded as *responding to student thinking* came from instructors’ responses to the prompt, “If this student were to come to your office hours, what sorts of things would you do to help them have a better understanding?” after examining the student’s work. Instructors’ responses demonstrate their underlying disposition of responsiveness towards student thinking. On one end of the spectrum, instructors responded to the specific student work by selecting examples or explanations tailored to the student’s understanding, enacting a responsive disposition towards student thinking. These responses took the form of supported student activity or teacher explanations, with the former including prompts from the instructor eliciting additional student contributions, and the latter involving an explanation given by the instructor. The following excerpts illustrate these types of responses:

> *Responsive to specific student thinking, leveraging additional student contributions:* "I would ... ask them to draw me a graph of a function such that its derivative changes from positive to negative at some particular point. Again we can see that maybe that was just a slip here. ... If [the student] drew another graph for me where it was decreasing and then increasing, then I would know they don't really understand what the sign of the derivative means. Then I would have to go back into this idea. I could ... <*describes specifically what they would do*>... Whereas if they drew a correct graph, then I would know they were kind of grasping for where to go with this, and then we could talk specifically [about] if you realize that was a minimum, what should you have looked at next."

> *Responsive to specific student thinking, through instructor explanation:* "I would probably draw them like a an example where you have like the 12, and then make sure that they understand like what the limit really means that you're – what's happening as you're getting close, not like what's happening at the point."

On the other end of the spectrum, there were instructors who demonstrated a lack of responsiveness to student thinking, or a technical disposition (Thornton, 2006). These instructors often responded directly to the mathematics prompt explaining how they would solve the problem instead of building off the student's demonstration of their understanding (as indicated by student work). Instructors also provided vague or general responses that were not tied to the specific student's work to which they were asked to respond.

> *Not responsive to student thinking, focused on the mathematics:* "I would just abandon [the student's] answer, and just start over with - I know this is the graph of f', what does this tell me about the slope? Or what does f' tell you about the original function?"

> *Not responsive to specific student thinking:* "Once they've conveyed to me what they were trying to say, then that helps to identify the mistakes in their thinking, the holes in their understanding, and then I would address those. Probably in this case it's some confusion between concavity and, you know, inflection points, and local min and local maxes – that sort of thing. And so I try and lay it out, you know, you have your function, the first derivative, second derivative, you know, points where its zero at each one of these – what those mean."

Instructors also discussed additional ways in which they responded to student thinking (during planning, grading, writing assessments, in-the-moment instruction, etc.), beyond the instances when explicitly asked to respond to a student during office hours after examining student work. These segments were coded as *responsiveness as a disposition* because they were not in response to specific students' work, but rather highlighted the instructor's role as a decision-maker enacting a responsive disposition. The following excerpts demonstrate the variety of ways that instructors responded to students and their understandings.

> *Responding based on insights to student thinking from grading:* "I didn't know what they would have trouble with, and [then] I saw their quizzes – I had to grade those things. Then it was pretty clear where I needed to emphasize things and where they were fine, and I [could] skip over stuff."
>
> *Responding based on insights to student thinking from office hours:*"That was something [a student] asked me and in office hours, but I thought, 'oh they're all worried [about] that, and they think everybody else gets it.'"
>
> *Responding to student thinking during planning:* "I think it is very important to go in planning a lecture, or planning a lesson, not just saying 'I'm gonna push a bunch of information at the students,' but try to guide them through having at least some rudimentary understanding of it. And in order to do that effectively, you really do have to like be able to empathize with their situation – like I'm just being told this completely new thing, where am I going to misunderstand this? And then take and put in specific notes or examples to illustrate where students are going to in misunderstand things."
>
> *Responding to student thinking while writing homework or exam problems:* "If I think [a problem is] going to send them down a completely wrong road, I either might change the problem a little bit or give them a hint, say, 'Hey you notice this thing.' But I think it's important to at least be in the mindset when you're writing down homework problems or exam questions or any of that, you have to be in the mindset of what somebody who doesn't know the stuff very well would try."
>
> *Responding by adjusting in-the-moment instruction based on student understandings:* "If I have, you know, a lesson planned that's really a series of questions, and I hope that we answer all of these questions – and then in the middle of trying to answer one of the questions, maybe, we'll get derailed and somebody will say actually, 'I have a lot of confusion about this piece.' And we need to focus on that, and we end up covering things that I didn't plan to cover at all – which I want to have that flexibility, but it would be great if I could also be entirely prepared for whatever they were going to ask me."
>
> *Respond to common student errors:* "[If] you know that 50% of people are going to make this mistake on an exam or quiz, or something – so if you know that that's a

> mistake that's commonly made, you can take and set aside a minute and say, 'You know this is what you might be thinking. This is why it's wrong.' And then if you have that sort of for the big mistakes that people commonly make, then that I mean, that helps you guide your lectures and examples you do."

These excerpts demonstrate a variety of ways instructors can respond to student thinking, which point to varying degrees of a responsive disposition and the enactment of this disposition. When an instructor primarily responds to the mathematics prompt, without engaging with the students' thinking, this indicates the enactment of a more technical (and less responsive) disposition. When an instructor responds based on the substance of the student's thinking, this indicates the enactment of a more responsive disposition. A responsive disposition can be enacted at various phases of instruction, for example when an instructor leverages the insights they gain into student thinking from grading, office hours, and in-the-moment student-teacher interactions to inform instruction.

4.3.4 The Role of Decision-Making in the Enactment of a Responsive Disposition

The thematic analysis highlights the role of decision-making on the enactment of a responsive disposition through an instructor's eliciting, reflecting, and responding to student thinking. The instructors interviewed for this study varied in the degree to which they viewed themselves as decision-makers, ranging from those who clearly articulated that they did not make decisions regarding the course content or instructional approach to those who occupied the role of course coordinator and were part of a team that made several decisions about the course. All of the participants who were instructors of record demonstrated enacting their role as decision-makers by the way they discussed planning and in-the-moment instructional decisions. These instructors particularly addressed their autonomy in regards to instructional approach. Of the participants who were involved in leading recitation sections (who were all first year graduate students at this institution), each indicated they were not involved in making decisions regarding the content or instructional approach for the course, but two demonstrated taking on the role of a decision-maker throughout the interviews.

The Cases of Adem and Cruz

Decision-making. In order to highlight the differences between those who view themselves as decision-makers and those who do not through their enactment of responsiveness, I first focus on two participants. Adem and Cruz were two of the new graduate students that were leading recitations for two variations of Calculus 1 during Fall 2017. Adem was co-leading his section with an experienced undergraduate teaching assistant and Cruz was co-leading his section with another new graduate teaching assistant. Adem self-admittedly had very little experience teaching and had very little professional development around instruction, although he worked as a teaching assistant for a recitation section during his first year of graduate school at a different institution. During the interviews (throughout the course of an academic year), Adem regularly referred to the fact that he was not writing lesson plans and that the course coordinator told him what to go over in each recitation session. Additionally, he highlighted his role as a graduate student who did not have complete control over his class and had to stick to the rigid course outline indicated in the syllabus. Adem often pointed to his lack of agency as a barrier to adjusting based on student understanding, stating:

> If [students] don't understand what's going on, then at some point you have to stop and sort of either go over things again or do something differently. And again, that could just not be feasible if someone is telling you you have to cover this and this throughout the semester. You might not have time to go back, and . . . at that point you – I guess you just tell them to read the book or something.

This excerpt illustrates that Adem does not view himself as a decision-maker, and this impacts whether or not he elicits student thinking and how he responds to students.

Cruz, on the other hand, demonstrated taking on a decision-making role despite the fact that he also had very little teaching or professional development experience. Namely, he discussed deciding what to include in the first five minutes of class when introducing the recitation activity. Cruz did not refer to the rigidity of the syllabus, but did acknowledge that the pace of the course left little opportunity to incorporate a variety of approaches, such as discussion and formal/informal assessments, in class. However, Cruz never indicated a lack of time or agency to address students' understandings, and discussed how he attempts to adjust instruction based on student questions and comments, saying:

> I try to adapt and I try to just take the questions as they go, but I don't really have a strategy. It's kind of just like all really messy and I hope that when I get more experience I'll have better strategies for it, which I think is the point, right? I think it's a learning experience not just for them [the students], but for me.

This comment indicates that he views himself as a decision-maker, taking class time to address student questions. Additionally, it points to his view that one's teaching can improve, which contrasts Adem's perspective that being a good instructor is "an inherent quality that some people have... like a sales person. I think some people are sort of naturally born sales people and some aren't." This positionality also points to their differing dispositions regarding becoming good instructors.

Eliciting Student Thinking. Adem and Cruz also differed in their demonstrations of eliciting student thinking, pointing to the role of decision-making in the enactment of a responsive disposition. Adem, who did not view himself as a decision-maker, tended to provide more vague comments about eliciting student thinking, such as "It's good to ask questions" or "You don't know whether the silence and like the occasional nod means everybody knows what's going on... so I think informal assessments would be important to gauge that sort of thing" indicating that these might be good things to include, without being clear if he actually has implemented such things in his own instruction. Additionally, Adem points to the students' responsibility to ask questions if they do not understand (when he paused during a lecture), and did not demonstrate explicitly eliciting student contributions during the interviews, even during hypothetical office hour situations. Cruz, on the other hand, takes on the role of a decision-maker and capitalizes on opportunities in office hours, in particular, to elicit student thinking stating regularly, "I would have them explain it out to me" or "I would make them do [the problem] in front of me."

Reflecting and Responding. Adem and Cruz also demonstrated differences in their reflecting and responding during the interview. Adem primarily reflected on his own experience as a student (when prompted) and on the constraints to being responsive to students' thinking and ideas, highlighting his lack of experience and lack of time due to the amount of content that needed to be covered in the course. When asked how he would to respond to a student in office hours after looking at their work on a mathematics problem, Adem tended to state that he would go over the definition with the student or gave a vague response articulating what he typically does in office hours, without attending to the specific student whose work he was asked to respond to.

Cruz reflected on the difficulty of knowing common student mistakes, stating that they were often different than he anticipated. However, he also reflected on how helpful it was to listen to the experienced recitation instructors' perspectives on student difficulties. When Cruz was asked how he would respond to a student in office hours based on his interpretation of their work, he often stated that he would give an example or described how he would review the content with the student, guiding them to the answer through a series of questions.

In summary, Adem and Cruz differed in the extent to which they demonstrated a view of themselves as decision-makers, which appeared related to the exhibited differences in how they demonstrated eliciting, reflecting, and responding to student thinking. Specifically, Adem did not discuss eliciting student thinking in his class other than mentioning that he pauses during lecture to allow them time to ask questions, but emphasized that it was the students' responsibility to voice their questions. This is particularly interesting because Adem is teaching a recitation section where the primary goal is for students to work in groups on application problems with the instructors moving between groups to answer questions. Cruz, on the other hand, demonstrated some eliciting of student thinking, though this was largely during the portion of the interview when he was asked how he would respond to a student during office hours. Adem reflected on his own experiences as a student (when prompted), and primarily responded to students by explaining the content, whereas Cruz reflected on student difficulties with the content, and responded to students in office hours by guiding them through the steps with examples and questions. Table 4.3 provides a summary of Adem's and Cruz's demonstrations of eliciting, reflecting, and responding, in addition to including a summary of Ava's, which will be discussed in the next section.

The Case of Ava

Decision-making. To further highlight the role of decision-making in the enactment of a responsive disposition, consider now the case of Ava, a new faculty member with several years of experience as instructor of record during her graduate career. Ava, like Adem, also mentioned the timing constraints regarding the amount of material that she is expected to cover in the Calculus 1 course, saying:

> It has been hard for me to get used to all the coordination, just because all of sudden I need to fit everything that I want [to do] into somebody else's structure. And there are things that I want to do that don't fit neatly into the calculus structure, and I don't have the time and space to do them.

However, she still demonstrates capitalizing on her role as a decision-maker. Ava regularly uses informal assessments, a variety of instructional approaches (i.e. think-pair-share, group work, lecture), and writes additional homework problems to supplement those assigned to all sections by the coordinator. Additionally she described how she adjusts instruction in-the-moment stating:

> I think that it is also important to have an idea what you want to cover, and make sure that you're trying to cover content, but the point in the class is for the students to get something out of it. So if they have questions, then I think that that should absolutely determine the focus of the class. Right? I want them to understand and learn, and you know, who cares about what I wanted them to know – it's about what they need help understanding.

Ava also responds to student understanding by revisiting topics that students struggle with on exams throughout the remainder of the semester. These examples provide evidence that Ava views herself as a decision-maker since she finds ways to respond to her students' understanding despite the regimented structure of the coordinated class.

Eliciting Student Thinking and Reflecting. Additionally, Ava tends to elicit and reflect on student thinking more than Adem and Cruz, further highlighting how one's view of their role as decision-maker can enable their enactment of a responsive disposition. Ava elicits student thinking to gain insight into to student thinking, saying things like:

> I definitely think it's important, as much as possible, to ask the students to verbalize... And it's nice that I have an opportunity to do that; I get to talk to them in the help

> room, and in office hours, and in class I get to ask them to say more about what they're thinking.

Further, she has created a classroom environment where students are comfortable expressing their confusions, regularly asking questions, stating that she spends most of class time answering students' questions. Ava also spent a considerable amount of time in the interview reflecting on her teaching and on her students' thinking. She reflected on a greater variety of things than Adem and Cruz, including student affect and anxieties, difficulties with content, student thinking demonstrated in their work while she grades, as well as reflecting on how she incorporates student thinking in her planning. Ava commented that she should spend more time reflecting:

> One thing that I think I should be doing that might help me be a better instructor is reflecting more on the things that I've done. I think oftentimes I have a lesson plan that went pretty well, or I am happy with how the dialog went in class that day. And what I should do is sit down and think about what went well and how I can refine it to do it better next time.

This statement highlights that Ava considers herself to be a decision-maker, and that this impacts her instruction; namely, Ava reflects on what she does in class and ways to improve her teaching.

Responding. Ava differs from both Adem and Cruz in the ways that she discussed responding to a student in office hours based on her interpretation of student work. Adem and Cruz primarily focused on the mathematics and getting the student to the correct answer, with Adem explaining the ideas and Cruz occasionally bringing in student contributions through guiding questions. Ava, however, tended to focus on the students and their understanding. She would often begin by having the student explain their strategies or reasoning, and then would provide examples or explanations that were based on what the student said. Additionally, when interpreting student work, Ava would often hypothesize about not only what the student was thinking, but also the potential root of their understanding, saying things like in response to the student work demonstrated in Figure 4.1 above:

> Oh, so this sounds like there is something going wrong with the pictures that we often draw in class where we have a line, and then we have a hole in the line and we say – okay, the limit exists but it is discontinuous.

In this statement, Ava interprets the student error to be stemming from examples that are typically used in class to discuss limits and continuity. She goes on to hypothesize that the student is probably thinking that a closed circle on a graph indicates continuous and an open circle means discontinuous.

Adem, Cruz, and Ava differ in the extent to which they identify as decision-makers (with Ava demonstrating the most and Adem demonstrating the least decision-making agency), in addition to differing in their demonstrations of responsiveness to student thinking, namely, in how they elicit, reflect, and respond to student thinking. These three instructors range from viewing it as the students' responsibility to share their thinking or communicate their questions (Adem) to actively eliciting student thinking in a variety of ways (Ava). From Ava's discussion of her teaching practices, it is evident that she has developed norms with her students and a class environment where students are comfortable asking their questions, sharing their thinking, and responding to her prompts to elaborate on their thinking. Additionally, Adem, Cruz, and Ava reflect on different aspects of student understanding, ranging from a focus on constraints to leveraging student thinking in instruction (Adem) to focusing on common student understandings (Cruz) and reflecting on students and their thinking during nearly every aspect of instruction (Ava). Finally, they differ in the extent to which they respond on the basis of student thinking, demonstrating a lack of responsiveness by focusing on the constraints to using student ideas in instruction (Adem) and demonstrating more robust responsiveness that incorporates and responds to student thinking both in the moment and throughout the course, revisiting content as necessary (Ava). Table 4.3 provides examples of the types of eliciting, reflecting, and responding demonstrated by Adem, Cruz, and Ava. These differences in eliciting, reflecting, and responding suggest that one's view of their role as a decision-maker interacts with their enactment of a responsive disposition. Ava's demonstration of responsiveness is qualitatively different than that of Adem and Cruz that goes beyond the number of instances she demonstrated eliciting, reflecting, and responding to student thinking during the interviews. In future work, I will unpack the nuances that distinguish the robustness of Ava's responsiveness to student thinking from Adem's and Cruz's responsiveness to student thinking.

Table 4.3: Summary of Adem, Cruz, and Ava's Eliciting, Reflecting, and Responding.

	Adem	Cruz	Ava
Elicit	-Students' responsibility to ask questions when he pauses in lecture -Quizzes are the biggest insight into student thinking -If students have a problem or question on the spot, says he might not be able to handle it	-Asks students in office hours to explain their work to him	-Regularly asks students to explain their thinking in class and in office hours -Uses a variety of informal assessments and strategies to gauge student understanding -Students tells her when they do not understand
Reflect	Reflected on: -His experience learning the content -How an instructor were to use a variety of approaches in class, they would open themselves up to more surprises and it would take more time to plan	Reflected on: -How it is difficult to know "common pitfalls" -The insights about student difficulties discussed by experienced teaching assistants -How he would like to prepare more next time he leads a recitation	Reflected on: -How she would like to reflect more -Student affect -Student difficulties with content -Student thinking when she was grading assignments -Student thinking/ understanding when planning for instruction
Respond	-Go over definitions -Use brief lecture to explain -Not feasible to use students' questions and comments to direct the class lesson -When asked to respond to a specific student's thinking, he describes the general way he approaches student questions including no detail regarding the specific student's understanding	-Expressed adapting and taking student questions as they come -Repeat explanations when students do not understand -When asked to respond to specific student's thinking, he explains the mathematics, occasionally including student contributions	-Adjusts instruction in-the-moment based on students' questions and comments -Revisits topics after the exam throughout the semester -When asked to respond to a specific student's thinking, she asks the student to explain their thinking and then either explains building on student contributions or guides the students through relevant examples

4.4 Potential Limitations

In this study, I set out to better understand responsiveness to student thinking among college calculus instructors. My goal for this phase of my work was not to evaluate the responsiveness demonstrated by instructors, nor to develop levels of responsiveness to student thinking. Before identifying levels of responsiveness, I wanted to identity what responsiveness to student thinking could look like among college calculus instructors. That said, it could be considered a limitation that none of the interview participants demonstrated robust responsiveness that leveraged rich theoretical models of student thinking and research on common ways of thinking. However, I would argue that this is not a limitation of this study, but rather it is representative of the population considered for this study. Since the context of my dissertation study is under the work of the Progress through Calculus grant, which focuses on instruction at masters and PhD granting institutions, the instructors teaching introductory courses at these types of institutions rotate regularly. This makes it difficult for instructors to develop robust understandings of common student thinking for one particular course, and likely has an impact on the ways that they demonstrate responsiveness to student thinking. In future work, I will consider the case of Jennifer, and other experienced instructors with many semesters of experience teaching the same course, in order to investigate if teaching the same course and the same types of students repeatedly is related to demonstrations of higher levels of responsiveness.

Although the goal of this study was not to identify levels of responsiveness to student thinking among college calculus instructors, understanding what high level responsiveness to student thinking can help put the responses among my participants into perspective. To help do that, consider the prompt and student response in Figure 4.1. An instructor demonstrating high levels of responsiveness to this student's thinking would elicit, reflect, and respond to student thinking in ways that draw out the student's conceptual understanding (rather than only procedural understanding), reflect on this understanding to develop a model of how the student understands the related ideas, and respond based on that model. This is in line with Ader and Carlson's (2017) highest level of decentering, in which instructors demonstrate taking action to understand student thinking and build on student ideas.

Related to the student thinking illustrated in Figure 4.1, an instructor exhibiting robust responsiveness might ask the student questions to elicit more about their understanding of function, limits, and continuity. For example, they could ask "Will you tell me more about why you have the point where you do on the graph?", "What is happening with the function near that point?", or "Why does the solid dot indicate $f(x)$ is continuous at $x = 3$?" and "What does $x = 3$ mean?" These questions demonstrate a valuing of student ideas and thinking, and do not assume that the student is completely incorrect with no productive ideas because they did not draw an accurate representation of $\lim_{x\to 3^+} f(x) = 51 = \lim_{x\to 3^-} f(x)$. Further, these questions can provide insight into student conceptual understanding and do not simply ask the student to state the definition of limit or continuity.

After eliciting student thinking, an instructor who demonstrates a high level of responsiveness to student thinking might reflect on the student response to construct their own model of the students' thinking. If the student responded to the first question by saying the limit was a point on the graph (rather than indicating it was a codomain value), the instructor might be able to conclude that this student is viewing the domain and range as separate locations and considering the limit as a point on the graph, implying continuity. This view of limit falls under the *proximity metaphor for the limit of a function and continuity* discussed by Oehrtman (2009). This metaphor for limit can be productive, but since the student is conflating the value of the function with the value of the limit there are additional aspects of the student's understanding that will need to be addressed. If the student responded to the third question saying that a solid dot means the function is continuous and an open circle at that point means that it is not continuous, then the instructor could conclude that the student can identify removable discontinuities. Although the student demonstrates an understanding of the meaning of an open circle on a graph, the student does not understand when a function might have a removable discontinuity and how this relates to $\lim_{x\to 3^+} f(x) = 51 = \lim_{x\to 3^-} f(x)$ in the hypothesis of the true/false statement (or what assumptions can be made from this hypothesis).

An instructor demonstrating a high level of responsiveness would take the information they elicited about student thinking, which was further developed through their reflection on the student thinking, and then would respond based on their understanding of what the student is thinking. For example, an instructor could respond to this student by providing examples or discussing limits using language such as “the function $f(x)$ becomes approximately 51 when x is approximately 3.” This might help the student to gain a better understanding of what assumptions can be made by the hypothesis of the true/false statement, in addition to better understanding what $f(x)$ might be doing near the codomain value of 51. Additionally, an instructor demonstrating responsiveness to student thinking could respond by supporting the student to have a better understanding of the difference between the function value at $x = 3$ and the limit value as x approaches 3. It is important to note that these responses demonstrate an attention to the student’s conceptual understanding of limit and continuity, and are not aimed to simply have the student arrive at the answer.

4.5 Discussion and Implications

In addition to unpacking the role of decision-making in responsiveness, this analysis has illuminated the distinction between responsiveness as a disposition and enacted responsiveness. Instructors were even aware of this distinction between an underlying disposition of responsiveness and of what it looks like to be enacted through responsive instruction. This is demonstrated in the Findings section demonstrating *responding to student thinking while writing homework or exam problems*. Here the instructor highlights the importance of being “in the mindset” of considering what students might do (demonstrating an underlying disposition of responsiveness) when he is writing exam or homework problems (enacting responsiveness through instructional decisions).

This work has also documented how instructors’ underlying disposition of responsiveness interacts with their actions as decision-makers in responding to their students’ thinking. The primary emergent themes from the thematic analysis highlight that instructors who exhibit a responsive disposition to their students’ thinking enact this through eliciting student thinking, reflecting on student thinking, and responding to student thinking. Since I regularly asked instructors how they would respond to students after showing them student work, it is not surprising that responding came up as one of the emergent codes; however, eliciting and reflecting on student thinking surfaced without explicit prompting, pointing to the importance of the presence of student thinking in responsive instruction (regardless of instructional format), which I identified as part of the underlying responsiveness dimension of the construct *responsiveness to student thinking* (Gehrtz, 2019). Additionally, considering the data more broadly, there is evidence of the extent to which an instructor values student thinking and is aware of the need to provide appropriate responses to students based on their understanding – another aspect of an underlying responsive disposition. For example, consider together all the opportunities Ava uses to elicit student thinking, the variety of ways she reflects about her instruction and engaging with her students, in addition to the ways she leverages the insight she gains from eliciting student thinking and reflecting to inform her instruction; these behaviors point to her responsive disposition that is sensitive to students and their thinking, demonstrating a genuine interest in engaging with the substance of their thinking and using it to constantly inform her instructional practices.

The analysis presented in this paper also points to the role of decision-making in supporting the enactment of a responsive disposition, with instructors who identify as decision-makers (independent of their official decision-making role in the course) creating more opportunities to meaningfully engage with student thinking through eliciting, reflecting, and responding to student ideas. However, it is likely that the underlying disposition also plays a critical role in an instructor's empowerment as a decision-maker. Thus, more work still needs to be done to investigate responsiveness as a disposition; however, this is challenging work "fraught with challenges and ethical dilemmas" (Thornton, 2006, p. 67) – which is one reason why looking at enacted dispositions provides valuable insight into underlying dispositions.

Currently, much professional development surrounding student-centered instruction focuses on the teaching practices and the logistics of facilitating such learning, often neglecting the development of the underlying dispositions that enable student-centered instruction (Thornton, 2006). Since there is evidence that dispositions can be reshaped and developed (Hand, 2012), we should strive to not only improve instruction, but to foster responsive dispositions towards student thinking. Some studies have already demonstrated that instructors' orientations to student thinking can be developed through opportunities to look at student work and engage with it on a deeper level to unpack student thinking rather than as an evaluative measure (e.g., Nickerson, Lamb, & LaRochelle, 2017). As we gain a greater understanding of these underlying dispositions and how they impact responsive instruction (in how they elicit, reflect, and respond to student thinking), we can create professional development that more specifically targets this underlying factor that impacts instructor decisions and practice. Further, this area of research has the potential to drastically impact undergraduate instruction; when we better understand how we can foster responsive dispositions and responsive instruction, we can better support students through student-thinking centered instruction.

Chapter 5

Conclusion

5.1 Summary of Dissertation Study

5.1.1 Paper 1

This dissertation paper overviews the literature related to responsiveness to student thinking, including constructs discussed at both the K-12 and post-secondary levels, and uses thematic analysis to develop a definition for responsiveness that education researchers can use moving forward. Results from the thematic analysis indicate that responsiveness takes the form of both an underlying disposition and an enacted responsiveness. The definition proposed encompasses the variety of forms of responsiveness discussed in the literature. Specifically, responsiveness to student thinking relies on an underlying disposition, which can be considered to include mental actions (such as decentering), knowledge, and orientations, and responsiveness to student thinking is evidenced through discourse moves, actions (pointing to skill sets), and characteristics of interactions. Further, several of the constructs alluded to opportunities to demonstrate responsiveness to student thinking beyond in-the-moment instruction or student-teacher interactions. The distinction between underlying and enacted responsiveness, and evidences of responsiveness during a variety of instructional practices were further investigated in the second and third paper of this dissertation.

5.1.2 Paper 2

The second paper of this dissertation focused primarily on the enactment of responsiveness and provided further evidence that responsiveness to student thinking could occur in or outside of class, particularly because responsiveness items loaded on the *instructor-centered instructional practices* factor. This result also points to opportunities to demonstrate responsiveness to student thinking in instructor-centered classes by leveraging student thinking during planning. Further, the analysis shed light on the fact that not all student-centered instruction is inherently responsive to students' thinking. Conversely, student-thinking centered instruction aligns with responsiveness and is significantly correlated with the amount of class time spent engaging in whole-class discussions, along with the time students spend working on tasks individually. Results discussed in the second paper also indicated that there was not a significant difference between Calculus 1 instructors who were involved in making decisions (either as part of a team or individually) about course content and those who were not on all of the instructional practices factors. There was, however, a difference between these groups on the factor indexing *equitable instructional practices* with decision-makers rating these practices as more descriptive of their instruction.

5.1.3 Paper 3

Results from the qualitative analysis in the third paper provide insight into responsiveness to student thinking as an underlying disposition, and how it impacts instruction. Instructors demonstrating a responsive disposition tended to discuss several motivations and instances of eliciting student thinking, either in class or in office hours. Additionally, they regularly reflected on student thinking and responded on the basis of that demonstration of student understandings, either through student-centered or instructor-centered activities/explanations. Further, instructors responded on the basis of student thinking that was demonstrated in student work during grading, student-teacher interactions during office hours, past experiences, and student contributions during class. The qualitative analysis also pointed to the role of decision-making on the enactment of responsiveness to student thinking with instructors who demonstrated decision-making actions also demonstrating more eliciting, reflecting, and responding to student thinking.

5.1.4 Synthesis of Findings from Papers 1, 2, and 3

By considering the results from these three papers together, we can further understand responsiveness to student thinking. For example, the quantitative results from the second paper indicated that an instructor's role as a decision-maker, as measured by their response to the questions:

> Q1. How are most decisions about course content (e.g., syllabi, exams, homework, pacing, grading) made for Calculus 1? Clarify if you wish.

Q2. How are most decisions made about instructional approach (e.g., use of clickers, group work, active learning) made for Calculus 1? Clarify if you wish.

were generally not correlated with their instructional practices as indicated by the factors. However, results from the qualitative analysis in the third paper shed light on how an instructor's role as a decision-maker does impact their instructional decisions, and in particular, how they elicit, reflect, and respond to student thinking. Taken together, these results shed light on the distinction between one's "official" role in making decisions for the course (regarding decisions related to either the content or the instructional approach), which was documented by the survey, and one's "unofficial" role as a decision-maker, which is highlighted by the agency one demonstrates in making instructional decisions (often having only small-scale impacts on the trajectory of a lesson or the course) based on their students' needs and understandings. In future work, I plan to further consider how one's underlying disposition impacts one's view of themself as an "unofficial" decision-maker.

The results from the three papers presented in this dissertation also illuminate how responsiveness to student thinking can be demonstrated in both instructor-centered and student-centered instructional practices, if the instruction is student-thinking centered. For example, in the qualitative analyses presented in the third paper, instructors demonstrated responding to the substance of student thinking by engaging the students in exploring the mathematics prompt or by offering an explanation tailored to the demonstration of student understanding. Additionally, the *student-thinking centered* factor included the following instructional practices that could be incorporated in either an instructor-centered (i.e. predominantly lecture) or student-centered (i.e. predominantly non-lecture) classes:

- I provide time for students to reflect about the processes they use to solve problems
- I use strategies that have been shown to support students from underrepresented groups
- I provide activities that connect course content to my students' lives and future work

- I expect students to make connections between related ideas or concepts when completing assignments
- I structure class so that more than one approach to solving a problem is discussed
- R2: I use student questions and comments to determine the focus and direction of classroom lessons
- R3: I use student assessment results to guide the direction of my instruction during the semester
- I understand students' previous conceptions, skills, knowledge, and interests related to a particular topic
- I provide feedback on student assignments without assigning a formal grade

A number of these instructional practices also could be incorporated during various other activities, such as during planning, grading, or writing assessments. This further highlights that responsiveness to student thinking is not limited to in-the-moment responsiveness. These results are supported by the qualitative analysis in the third paper when instructors indicated that they leverage their insights into student thinking gained during office hours, grading student work, and from experience teaching when planning for lessons.

5.2 Implications

Although there is still more work that needs to be done to understand responsiveness to student thinking as an underlying and enacted disposition, results from this study can be used to inform professional development supporting responsiveness. Since there is evidence that dispositions can be molded and shaped, I envision a professional development program that attends to fostering responsiveness as a disposition as well as supporting responsiveness in action. In order to support responsiveness as a disposition, I imagine including activities where instructors engage with student work, in a non-evaluative way, reflecting on the productive mathematics underlying students' incomplete and incorrect answers. Additionally, during this activity, it would be productive to incorporate relevant research on student thinking related to the topic, having instructors reflect on how they would address common student conceptions. This practice would allow instructors to be more prepared when similar student ideas are brought up in class; since they have already reflected on how they would respond, this has the potential to result in instruction that is adaptive based on student contributions. Interview data from my dissertation study highlighted that not knowing how to respond to student questions was one constraint to allowing students' questions and comments to influence the direction of the lesson, with instructors saying things like, "if you realize it's a problem there on the spot, then you might be able to handle it, but you wouldn't have thought through maybe the best way to explain it."

I also envision professional development that supports the enactment of responsiveness by incorporating a discussion surrounding questioning techniques, or other methods of eliciting student thinking since the presence of student ideas is the first step in attempting to understand what students might be understanding and responding on the basis of student understanding. Additionally, the professional development could include a discussion of how to incorporate various instructional practices that are grouped together on the *student-thinking centered instructional practices* factor from the factor analysis discussed in the second paper in this dissertation. Finally, I would envision this professional development to have an ongoing component where instructors were encouraged to regularly reflect on their students and their students' thinking, along with ways to leverage their thinking in instruction. Reflection seems to be a key component in responsiveness because when students' understandings are regularly on an instructor's mind, then they are more likely to draw on this knowledge during various instructional practices. This is particularly evident in the case of Ava, as she was constantly tying her responses to the interview prompts back to her students and how almost all of her instructional decisions were informed by reflecting on students' affect and mathematical understandings.

5.3 Conclusion

At the beginning of this dissertation work I set out to better understand ways to support novice instructors in more student-centered instruction, but through this work I have come to realize that I was actually interested in supporting instructors in instruction that is responsive to student thinking, or student-thinking centered instruction. This type of instruction is where student thinking is valued and leveraged, even when it is incomplete or incorrect. My dissertation work has shed light on how student-thinking centered instruction is responsive instruction, and can be incorporated in both instructor-centered and student-centered classrooms. Since lecture is still the most common form of instruction at the post-secondary level, this result points to opportunities to support post-secondary instructors in implementing responsiveness by nudging them towards small changes that do not require a drastic shift in pedagogical practices, but have the potential to make a big impact on student experiences and learning.

In future work, I plan to further consider the role of one's disposition on their enactment of responsiveness. In particular, I am interested to learn more about how one's disposition towards the discipline impacts instruction. This surfaced in one of my interviews with a novice graduate student who said,

> I think it's less important than in other academic areas [for an instructor to include a variety of approaches, such as discussion]. I think in math it's a little more cut-and-dry so you don't necessarily have to like open thing up to discussion in a calculus course . . . my undergrad was in political science and there it makes a lot more sense to open up like the classroom to discussion because things are more opinion based, it's less rigid.

This indicates that his view of the differences between political science and mathematics would impact the types of instructional practices he would incorporate in class. Coffey and Edwards (2016) also point to the role subject matter plays in responsive instruction in their analysis of prospective elementary teachers' responses in the context of student contributions in mathematics and science. I would also be interested in investigating the impact of professional development or course coordination on responsiveness to student thinking (as both a disposition and enacted), the relationship between responsive instruction and student learning and experiences in a class, in addition to considering how responsiveness to student thinking is evidenced during in-the-moment teacher-student interactions (in class or in office hours).

In conclusion, this dissertation contributed to the work of examining instructor responsiveness to student thinking. It has provided a theoretical contribution in the proposed definition of responsiveness to student thinking, rooted in existing literature, and provided additional insights on responsiveness as an underlying disposition and enacted responsiveness through the analysis and results from my dissertation study. Additionally, this work has provided a practical contribution through implications for practice, pointing to instructional practices that align with responsiveness to student thinking and possibilities for professional development designed to foster responsiveness as both an underlying and enacted disposition.

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Appendix A

PIPS-M Survey

Items are numbered in the appendix for reference in this document. Participants did not see the item numbers or letters. This survey was built and distributed via Qualtrics (Qualtrics, 2018). The text **your institution** *was replaced by the institution's name in the survey participants viewed.*

INTRODUCTION

Which of the following courses are you teaching *this term*? Mark all that apply.
[*Site-specific options of each of the targeted courses*]
None of the above.

COURSE-SPECIFIC LOOP

This loop is repeated for each of the courses an instructor selects in the introductory item, with that course's name piped in to certain items. For demonstration purposes, we reference **Math 101: Calculus 1.**

1. Weekly contact hours students have with you for **Math 101: Calculus 1** per section.

a.	Lecture	________
b.	Discussion/recitation	________
c.	Office hours	________
d.	Other	________

2. How are most decisions about *course content* (e.g., syllabi, exams, homework, pacing, grading) made for **Math 101: Calculus 1**? Clarify if you wish.

a.	I make most decisions.	□	________
b.	I'm part of a team that makes most decisions.	□	________
c.	Someone else makes most decisions.	□	________

3. How are most decisions about *instructional approach* (e.g., use of clickers, group work, active learning) made for **Math 101: Calculus 1**? Clarify if you wish.

a.	I make most decisions.	□	________
b.	I'm part of a team that makes most decisions.	□	________
c.	Someone else makes most decisions.	□	________

4. What percent of regular class time in **Math 101: Calculus 1**, over the whole term, did your students spend... [must total 100]

a.	Listening to the instructor lecture or solve problems	________
b.	Participating in whole-class discussions	________
c.	Working on tasks in small groups	________
d.	Working on tasks individually	________

5. Please indicate the degree to which the following statements are descriptive of your teaching in **Math 101: Calculus 1.**

Response options: (1) Not at all descriptive; (2) Minimally descriptive; (3) Somewhat descriptive; (4) Mostly descriptive; (5) Very descriptive

a.	I guide students through major topics as they listen	□□□□□
b.	I provide activities that connect course content to my students' lives and future work	□□□□□
c.	My syllabus contains the specific topics that will be covered in every class session	□□□□□
d.	I provide students with immediate feedback on their work during class (e.g., student response systems; short quizzes)	□□□□□
e.	I structure my course with the assumption that most of the students have little useful knowledge of the topics	□□□□□
f.	I use student assessment results to guide the direction of my instruction during the semester	□□□□□
g.	I ask students to respond to questions during class time	□□□□□
h.	I use student questions and comments to determine the focus and direction of classroom lessons	□□□□□
i.	In my class a variety of means (models, drawings, graphs, symbols, simulations, tables, etc.) are used to represent course topics and/or solve problems	□□□□□
j.	I structure class so that students explore or discuss their understanding of concepts before direct instruction	□□□□□
k.	My class sessions are structured to give students a clear/structured set of notes	□□□□□
l.	I structure class so that students talk with one another about course topics	□□□□□
m.	I structure class so that students constructively criticize one another's ideas	□□□□□
n.	I structure class so that students discuss their mathematical difficulties with other students	□□□□□
o.	I structure class so that students work on problems individually during class.	□□□□□
p.	I structure class so that students work together in pairs or small groups	□□□□□
q.	I structure class so that more than one approach to solving a problem is discussed	□□□□□
r.	I provide time for students to reflect about the processes they use to solve problems	□□□□□
s.	I give students frequent assignments worth a small portion of their grade	□□□□□
t.	I expect students to make connections between related ideas or concepts when completing assignments	□□□□□
u.	I provide feedback on student assignments without assigning a formal grade	□□□□□
v.	Test questions focus on important facts and definitions from the course	□□□□□
w.	Test questions require students to apply course concepts to unfamiliar situations	□□□□□
x.	Test questions contain well-defined problems with one correct solution	□□□□□
y.	I use a grading curve as needed to adjust student scores	□□□□□
z.	A wide range of students respond to my questions in class	□□□□□
aa.	I know most of my students by name	□□□□□
bb.	When calling on students in class, I use randomized response strategies (e.g., picking names from a hat)	□□□□□
cc.	I structure class to encourage peer-to-peer support among students (e.g., ask peer before you ask me, having group roles, developing a group solution to share, etc.)	□□□□□
dd.	There is a sense of community among the students in my class	□□□□□
ee.	I require students to work in predetermined or randomized groups	□□□□□
ff.	I use strategies that have been shown to support students from underrepresented groups	□□□□□
gg.	I consider students' thinking/understanding when planning lessons	□□□□□
hh.	I use a variety of approaches (e.g., questioning, discussion, formal/informal assessments) to gauge where my students are in their understanding of concepts	□□□□□
ii.	I understand students' previous conceptions, skills, knowledge, and interests related to a particular topic	□□□□□
jj.	I explain concepts in this class in a variety of ways	□□□□□
kk.	I adjust my teaching based upon what students currently do or do not understand	□□□□□
ll.	I give feedback on homework, exams, quizzes, etc.	□□□□□
mm.	I structure class so that students share their ideas (or their group's ideas) during whole class discussions	□□□□□
nn.	I use strategies to encourage participation from a wide range of students	□□□□□
oo.	A wide range of students participate in class	□□□□□

6. Generally speaking, do other **Math 101: Calculus 1** instructors use a teaching style similar to yours?

a.	Yes	◻
b.	No	◻
c.	Too varied to choose	◻
d.	I don't know	◻

7. How do you feel about the instructional approach(es) being used to teach **Math 101: Calculus 1** at **your institution**?

a.	Very unhappy	◻
b.	Somewhat unhappy	◻
c.	Neutral	◻
d.	Somewhat happy	◻
e.	Happy	◻

8. Please use the following space to explain or clarify your previous response.

____
____
____

9. Is there anything else you would like us to know about how **Math 101: Calculus 1** is taught at **your institution**?

____
____
____

DEMOGRAPHIC & INDIVIDUAL CONTEXT ITEMS

1. How many years have you been at **your institution**, in any capacity?

   ____

2. For how many years have you taught precalculus/calculus courses, at **your institution** or elsewhere?

a.	As instructor of record/primary instructor	________
b.	As a teaching assistant (TA)	________

3. What is your primary area of research, if you have one?

   ____

4. (Select all that apply) Do you consider yourself to be:

a.	Man	☐
b.	Transgender	☐
c.	Woman	☐
d.	Not listed (please specify)	☐ ________
e.	Prefer not to disclose	☐

5. (Select all that apply) Do you consider yourself to be:

a.	Alaska Native of Native American	☐
b.	Black or African American	☐
c.	Central Asian	☐
d.	East Asian	☐
e.	Hispanic or Latinx	☐
f.	Middle Eastern or North African	☐
g.	Native Hawaiian or Pacific Islander	☐
h.	Southeast Asian	☐
i.	South Asian	☐
j.	White	☐
k.	Not listed (please specify)	☐ ________
l.	Prefer not to disclose	☐

6. (Select all that apply) Do you consider yourself to be:

a.	Asexual	☐
b.	Bisexual	☐
c.	Gay	☐
d.	Straight (heterosexual)	☐
e.	Lesbian	☐
f.	Queer	☐
g.	Not listed (please specify)	☐ ________
h.	Prefer not to disclose	☐

7. What is your age, in years?

   ____

8. (Select all that apply) Do you consider yourself to be:

a.	International instructor	☐
b.	First-generation college student (i.e., neither parent nor guardian completed a Bachelor's degree)	☐
c.	First-generation higher education (i.e., first in your family to pursue an advanced degree such as PhD)	☐
d.	Person with a disability	☐
e.	English language learner (i.e., the primary language spoken in your childhood home was not English)	☐
f.	Parent or care-giver	☐
g.	Prefer not to disclose	☐

9. Are there any aspects of your identity (or who you are) that have impacted your experience at **your institution**? Please explain.

____

____

____

10. Is there anything else you would like us to know about you or your experiences at **your institution**?

____

____

____

Appendix B

Additional Information from Quantitative Data and Analysis

Table B.1 provides a breakdown of the survey participants, and institutional demographics for the twelve Progress through Calculus case study sites. Note that the response rates reports are computed using all responses (including incomplete survey responses) from precalculus, Calculus 1, and Calculus II (P2C2) instructors of record, graduate student instructors of record, and graduate/ undergraduate instructional assistants (teaching assistants or recitation leaders), and not just the responses of the Calculus I instructors of record used in the factor analysis.

Table B.1: Demographic and Sample Size Information.

Institution	N (Calculus I instructors)	Response rate (all P2C2 Instructors)	U.S. Region	Control	Carnegie Classification	Approximate Undergraduate Student Enrollment
1	4	46%	NE	Public	DRU	6,300
2	3	79%	SW	Public	MA/L	11,000
3	6	58%	SE	Public	RU/H	22,500
4	12	67%	W	Public	RU/VH	23,900
5	14	74%	SW	Public	RU/VH	49,900
6	9	38%	SE	Public	RU/H	41,000
7	12	73%	SE	Private	RU/VH	6,600
8	11	91%	SW	Public	RU/H	20,900
9	4	63%	NE	Public	RU/H	12,800
10	4	84%	MW	Private	DRU	14,800
11	2	69%	NE	Private	RU/H	3,200
12	13	64%	NE	Public	RU/H	13,600

Figure B.1 provides a scree plot from the exploratory factor analysis on the forty-one PIPS-M items. The leveling off of the eigenvalues after four factors on the scree plot supports a four-factor solution.

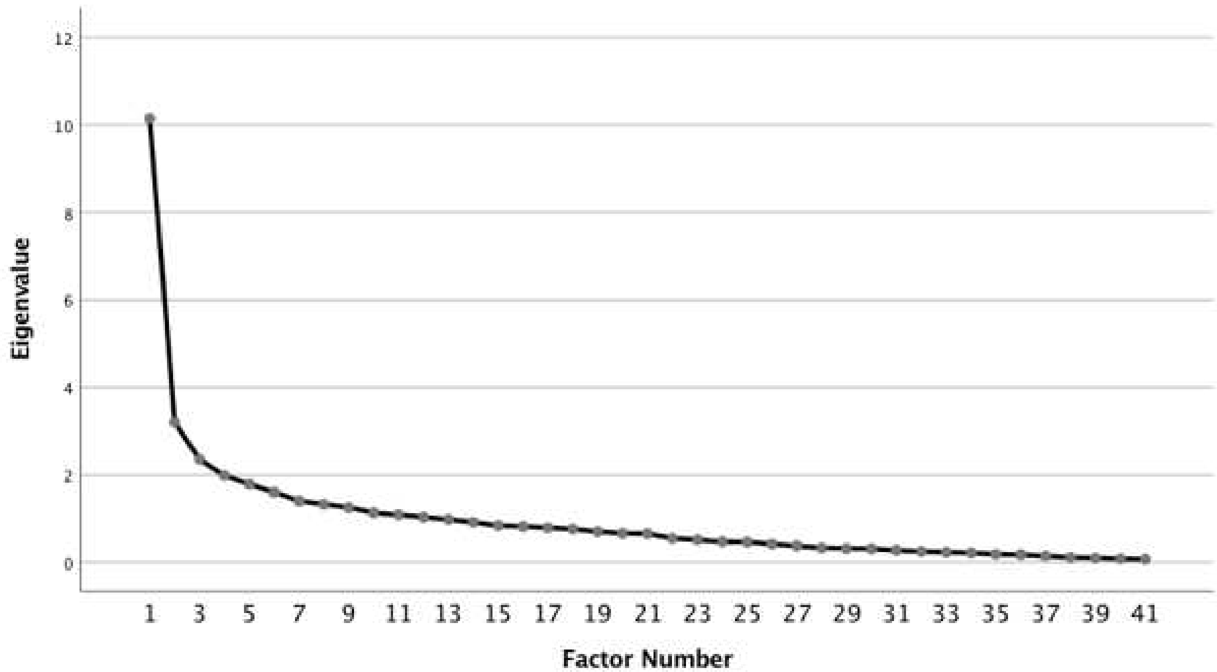


Figure B.1: Scree plot of eigenvalues in factor analysis

Table B.2 includes the PIPS-M survey items and their rotated factor loadings, suppressing loading less than .375 for readability. The table also provides a categorization of the items included with each of the four factors.

Table B.2: Factor Loadings for Rotated Factors

	Item	Factor Loadings 1	2	3	4
Student-Thinking Centered Instructional Practices	I provide time for students to reflect about the processes they use to solve problems	.681			
	I structure class so that students explore or discuss their understanding of concepts before direct instruction	.662			
	I structure class so that students work on problems individually during class	.650			
	I structure class so that students share their ideas (or their group's ideas) during whole class discussions	.611			
	I use strategies that have been shown to support students from underrepresented groups	.595			
	I provide activities that connect course content to my students' lives and future work	.567			
	I provide students with immediate feedback on their work during class (e.g., student response systems; short quizzes)	.547			
	R1: I use a variety of approaches (e.g., questioning, discussion, formal/informal assessments) to gauge where my students are in their understanding of concepts	.546			
	I expect students to make connections between related ideas or concepts when completing assignments	.528			
	I use strategies to encourage participation from a wide range of students	.477			
	When calling on students in class, I use randomized response strategies (e.g., picking names from a hat)	.469			
	I structure class so that more than one approach to solving a problem is discussed	.464			
	R2: I use student questions and comments to determine the focus and direction of classroom lessons	.459			
	R3: I use student assessment results to guide the direction of my instruction during the semester	.411			
	I understand students' previous conceptions, skills, knowledge, and interests related to a particular topic	.409			
	I provide feedback on student assignments without assigning a formal grade	.387			
Student-to-Student Interactions in Class	I structure class so that students work together in pairs or small groups		.833		
	I structure class so that students talk with one another about course topics		.775		
	I require students to work in predetermined or randomized groups		.631		
	I structure class to encourage peer-to-peer support among students (e.g., ask peer before you ask me, having group roles, developing a group solution to share)	.405	.614		
	I structure class so that students discuss their mathematical difficulties with other students	.462	.569		
	I structure class so that students constructively criticize one another's ideas		.480		
Equitable Instructional Practices	A wide range of students respond to my questions in class			.781	
	A wide range of students participate in class			.751	
	R4: I adjust my teaching based upon what students currently do or do not understand			.487	.436
	I explain concepts in this class in a variety of ways			.430	.425
	There is a sense of community among the students in my class		.376	.411	
Instructor-Centered Instructional Practices	I structure my course with the assumption that most of the students have little useful knowledge of the topics				.516
	My class sessions are structured to give students a clear/structured set of notes				.483
	I guide students through major topics as they listen				.479
	Test questions contain well-defined problems with one correct solution				.478
	Test questions focus on important facts and definitions from the course				.433
	I ask students to respond to questions during class time				.422
	R5: I consider students' thinking/understanding when planning lessons				.387

**Suppressing loadings less than .375 for readability*

Table B.3 includes the correlations between factors.

Table B.3: Spearman's rho Correlation Between Factors

Factor	1. Student-thinking centered instruction	2. Student-to-student interactions in class	3. Equitable instruction	4. Instructor-centered instruction
1. Student-thinking centered instruction	–	.230*	.504**	.134
2. Student-to-student interactions in class		–	.410**	.019
3. Equitable instruction			–	.116
4. Instructor-centered instruction				–

Appendix C

Interview Protocols

Initial Interview (via Skype - 30 minutes)

1. What will be your role as GTA this term?
   (a) Instructor of record? Recitation leader? Lab facilitator?
2. What other experiences do you have teaching or tutoring?
3. In your opinion, what strengths do you bring to the classroom as an instructor/recitation/lab leader?
4. What do you think it means to be a good instructor?
   (a) In your opinion, how does one become a good instructor?
5. Tell me about your career aspirations. What do you want to do after you graduate?
   (a) Do you think these professional goals will influence the way that you teach? How so?
   (b) This is a similar question, but am interested in the reverse relationship. How do you think your role as a TA will influence your perspective on your career goals?
6. How would you go about solving this problem?
   (a) Are there any other ways of solving this problem? Include as many different ways as possible.
7. If a calculus 1 student encountered this problem, what sorts of things might they say or do in response to the prompt?
   (a) What are some (other) incorrect things might they do?
   (b) What are some (other) correct things might they do?
8. Here is a student X's response to the prompt. What is the student thinking? How would you interpret this response?
9. If student X were to come to your office hours, what sorts of things might you do to help them get from where they are to what you think would be a better understanding?

Task: Indicate whether each of the following statements is TRUE or FALSE. If the statement is true, explain how you know it's true. If it is false, give a counterexample **and** explain why it is a counterexample.

(a) If $\lim_{x\to 3^+} f(x) = 51 = \lim_{x\to 3^-} f(x)$, then $f(x)$ is continuous at $x = 3$.

Student A:

True

The graph shows that x=3 their is a Solid dot indicating that f(x) is continuous at x=3.

Student B:

True, the two limits are equal to each other and both are equal to a point, 51.

Student C:

True

If f(x) is equivalent in both scenarios, there is no hole or jump and thus it is continous

Student D:

That is False because it doesn't have a defined point at X=3. continuity says that the limits have to exist and if they exist they have to equal the function and it doesn't here.
If they had f(3) = 51 Then it would work.

Tasked-Based Interview (2nd Interview - 60 minutes)

1. For GTAs and new Faculty: How has it been/was it transitioning to be a graduate student/professor of practice at [institution]? How do your experiences at [institution] compare to your experiences at other institutions?

2. What do you enjoy the most about being a grad student/faculty at [institution]? What part do you enjoy the least?

3. Consider first year calculus students, what is one area of calculus that your students have the most trouble with?

    (a) What strategies do you have to help students overcome these difficulties?

4. How do you consider student thinking when planning, teaching, or grading?

    The following questions apply to tasks 1-4.

5. How would you go about solving this problem?

    (a) Are there any other ways of solving this problem? Include as many different ways as possible.

6. If a calculus 1 student encountered this problem, what sorts of things might they say or do in response to the prompt?

    (a) What are some (other) incorrect things might they do?

    (b) What are some (other) correct things might they do?

7. Here is a student X's response to the prompt. What is the student thinking? How would you interpret this response?

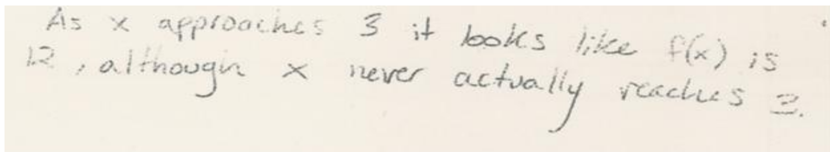
    (a) How would you describe this student's level of understanding?

8. If student X were to come to your office hours, what sorts of things might you do to help them get from where they are to what you think would be a better understanding?

Task 1: Limit Task

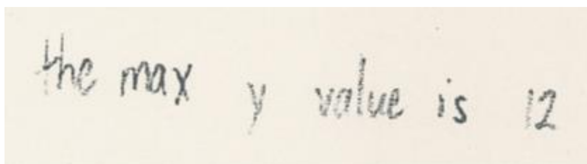
Describe what it means when we say “the limit of f(x) as x approaches 3 is 12” ($\lim_{x \to 3} f(x) = 12$).

Student A:



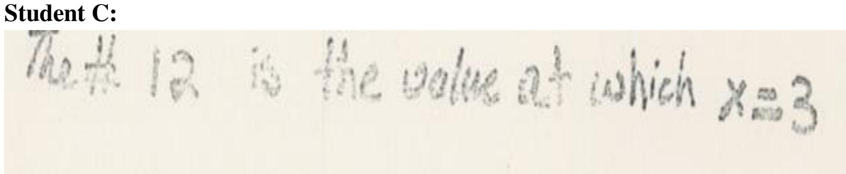
As x approaches 3 it looks like f(x) is 12, although x never actually reaches 3.

Student B:



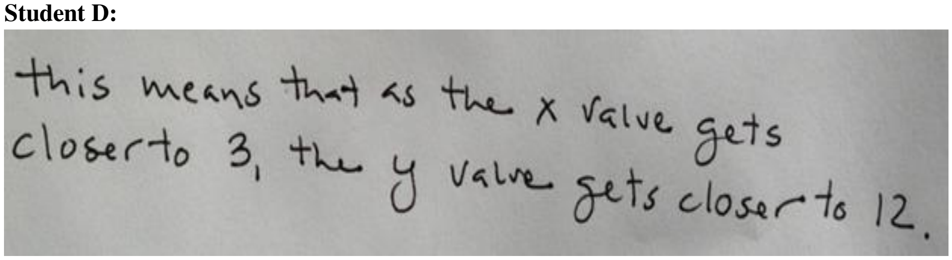
the max y value is 12

Student C:



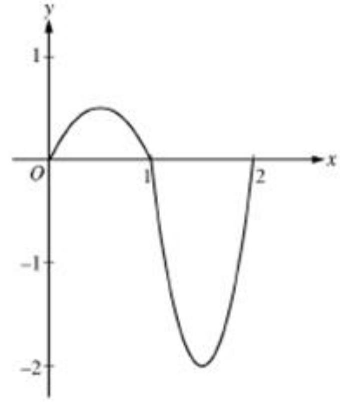
The # 12 is the value at which x = 3

Student D:



this means that as the x value gets closer to 3, the y value gets closer to 12.

Task 2: Relating the derivative to the function



Graph of f'

The figure above shows the graph of f', the derivative of a function f, for $0 \le x \le 2$. What is the value of x at which the absolute minimum of f occurs?

Student A:

The absolute minimum also occurs at $x = 1$, because we can see (concavity) behavior on either side of that point and it goes from ⊕ to ⊖ on the derivative function graph. (change in concavity)

Student B:

abs. min @ $x = 1.5$ on f. where derivative is 0 on $f'(x)$.

Student C:

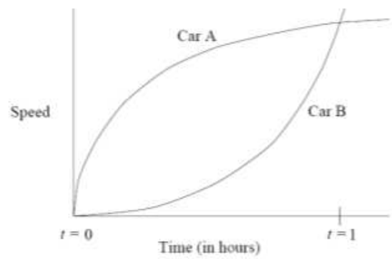
$x = 2$, because the area under f' has a net negative value

Student D:

(2,0) is the absolute minimum because the slope between [1,2] is steeper than the slope between [0,1] therefore this value is lower than the other relative min at (0,0).

Task 3: Car Task

The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are travelling in the same direction.)



State the relationship between the position of car A and car B at t = 1 hour. Provide an explanation for your answer.

Student A:

Car A and Car B probably cross each other at 1 hour because there might be 2 roads to take.

Student B:

Car A & Car B are in the same position at t=1. This is possible because car A started out with a higher increasing acceleration; car B started out accelerating slower but increased greatly as car A slowed acceleration

Student C:

Both car A&B have travelled the same distance, yet the rate ~~&~~ both cars underwent is different. initial speed/velocity × time gives you distance...?

Student D:

→ at t = 1 the cars are going the same speed but car A traveles much farther because it had a higher velocity for a longer time

Follow-Up Interview (30 minutes)

1. For new TAs: What was it like leading a lab section last semester?
    (a) If applicable: Is there anything that you would do differently looking back? Explain.
2. What are you roles and responsibilities this semester related to calculus?

    If you recall, for the last interview (and part of the very first interview) I had math problems and student work that we talked through. For those interviews I am trying to understand what instructors notice about the student work - about the mathematics, about the student errors, about how you would help or explain the problem to students. And this got me thinking about the dispositions instructors have about student thinking.
3. First of all, do you have a sense of how or when you might have learned to interpret student thinking and make sense of student work?
4. For Faculty: As coordinator, do you try to incorporate common student errors or other things about students' thinking about calculus into your meetings with instructors or TAs?
5. On the survey that you filled out last semester there were a bunch of instructional practices that were asked to rate how descriptive they were of your teaching. Now I want go over some of the statements again.

    Ask the following questions for each of the responsiveness statements:
    1: Use a variety of approaches (e.g., questioning, discussion, formal/informal assessments) to gauge where students are in their understanding of concepts
    2: Use student questions and comments to determine the focus and direction of classroom lessons
    3: Adjust teaching based upon what students currently do or do not understand
    4: Use student assessment results to guide the direction of instruction during the semester
    5: Consider students' thinking/understanding when planning lessons

    (a) Do you think that it is important to ... ? Why/why not?
    (b) Which would you like to incorporate this more into your instruction? Why?
    (c) Which is the most difficult to incorporate? Why?
    (d) Which are you satisfied the most with the degree to which you incorporate this practice?