REVIEW OF RECENT TRENDS IN THE ANALYSIS OF THE DEVELOPMENT OF RIVER NETWORKS

By

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I. INTRODUCTION

An attempt has been made in the Engineering Research Center at Colorado State University to initiate a basic study of the development of river networks. An approach by selecting and investigating a large river basin has been planned. For this purpose, the Colorado River Basin has been selected, and some initial work has already been done. This study should result in some generalizations about the laws that governed the development of this river network. Later on, the results may be applied to other river basins.

Meanwhile, the objectives of this report are:

a) To review the literature concerning the development of river networks;

b) To outline the concepts, methods, and hypotheses which are currently used in explaining the processes of river network development;

c) To present in a summary form the main conclusions reached on this subject up to date;

d) To single out the recent trends in analysis of the development of river networks; and

e) To suggest new approaches to be used in the intended study of the Colorado River network.

This report represents the preliminary investigation and will be the basis for further study of the problem of river networks.

II. GENERAL THEORIES ON DEVELOPMENT OF RIVER NETWORKS

In general, rivers are the most important and powerful agents in shaping the earth's surface. After the most significant large scale modifications of the original upland, accomplished during the Pleistocene epoch, analogous processes of landscape development have continued on a smaller scale into the present time. Neglecting wind, ice, and wave action, which can be locally very significant, running water is most responsible for landscape development. However, factors affecting the river network development are so numerous that the influence of each factor cannot be determined in a rational or purely mathematical way. Therefore, a number of general concepts have been developed in order to explain and describe the evolution of land form [128]; they are:

1. Davis' cycle theory;
2. Penck's differential theory;
3. The equilibrium theory; and
4. Crickway's principle of unequal activity.

These four theories are briefly described below.

1. Davis' cycle theory

Beginning in 1899, Davis developed his concept of an erosional or geomorphological cycle which had its beginning immediately after the completion of an uplifted area, such as a mountain range created by an endogenetic geodynamic process. Then weathering, erosion, and detrition began to act on the uplifted area and gradually proceeded to reduce it to a base level. This completed the cycle. However, a new cycle starts whenever a new endogenetic diastrophism occurs. According to Davis, three stages can be recognized in the geomorphological cycle: youth, maturity, and old age. The youth of a cycle is characterized by small number of valleys strongly V-shaped, with a lot of falls and rapids. In its maturity, the drainage system becomes more integrated, the waterfalls and rapids evident in its youth have disappeared, and most of the rivers are in a dynamic-equilibrium condition. The extent of the relief reaches the maximum that is possible. In old age, the valleys become very broad, most of the relief has disappeared due to the continental planation. At this stage, the level of the drainage basin approaches the base level of erosion. Finally, the end of the cycle is reached when all relief has been reduced to the base level, leading to a gently undulating plain called by Davis, "peneplane." Davis' theory has been widely accepted.

2. Penck's differential theory

During the 1920's, Penck presented a different interpretation of the landscape development. According to his differential theory (a special contribution to the theory of slope development), uplift and planation are concurrent phenomena and there is no cycle; instead, slope development is a differential process in which several stages may be identified: a) Waxing development in which the uplift is faster than the denudation (leading to convex slope profiles); b) Stationary development in which uplift and denudation proceed at an equal rate (leading to straight slopes and parallel slope recession); and c) Waving development in which the denudation rate exceeds the rate of uplift (leading to concave slopes). Objections against Penck's interpretation of slope development are directed toward his interpretation of particular slopes rather than against his consideration of slopes as outcomes of a differential process.

3. The equilibrium theory

The equilibrium theory regards the present appearance of a landscape as the outcome of a dynamic equilibrium of the forces in action. Accordingly, a landscape will preserve its character only if the forces acting on it stay the same; if the forces change, some slopes will waste away while others will be created.

4. Crickway's principle of unequal activity

The principle of more or less equal activity of exogenetic agents in landscape development, applied in the three previous theories, was rejected by Crickway in 1959-1960. According to Crickway, the activity of exogenetic agents is unequal. Thus, endogenetic movements may lift parts of the earth's crust at a slow or rapid rate while denudation is also taking place -- the anagenetic stage of the landscape development. Once the endogenetic movements cease, denudation will continue -- the catagenetic stage of landscape development. Since denudation is mostly achieved by rivers cutting away at the bottom of slopes, each valley develops in correspondence with the intensity of lateral river action.

Even though the Davis cycle theory is, to date, probably the most widely accepted one, none of the above four theories is either fully accepted or completely rejected.

III. BASIC CONCEPTS OF QUANTITATIVE ANALYSIS

Various ideas, concepts, and methods were used in the past in studying the problem of river networks. All of them may be grouped, in general, into two categories:

1) Qualitative analysis, and

2) Quantitative analysis.

Until about 1945, the geomorphologists were concerned primarily with the history of evolution of land forms and operated almost entirely on a descriptive basis. After Horton in 1945, a strong progress has been made in quantitative analysis of river network development, the category to which the main emphasis is given in this report.

Under the growing realization that the classical descriptive analysis of river network development has very limited value in practical engineering applications, the quantitative analysis has been extensively developed recently. Some of the basic concepts used in quantitative analysis are outlined here.

1. Closed and open system

According to the closed system of analysis, the drainage basin is considered closed as in thermodynamics, so that energy cannot be added or taken out of the drainage basin under consideration. The system is thus simplified but, unfortunately, it is not adequate for a geomorphological analysis. More realistic is the open system, according to which the geomorphic processes operate in open systems such as drainage basins. In geomorphology, one is dealing almost entirely with open systems, in which the energy can be added in some places (precipitation, heat, etc.) or taken out (heat losses by convection, conduction, and radiation, evapotranspiration, use of water, etc.) [29].

2. Events of rare frequency

According to this concept [93], individual catastropic events such as heavy rainstorms, floods, mudflows, and heavy landslides of rare frequencies, having very high power, are responsible for some of the most significant land forms. Large boulders found in mountain streams, for instance, could not be transported downstream without the large power of catastropic events of rare frequency. The larger the size of material, the larger is the discharge necessary to provide the force required for moving the material [173].

3. Concept of entropy

The development of the landscape involves not only the total available energy but its distribution as well, a factor described as entropy, adopted from the comparable concept in thermodynamics.

Thus, the entropy principle, somewhat similar to that implied in the second law of thermodynamics in relation to thermal energy, is introduced in the analysis of river network development. The concept of entropy is expressed in terms of the probability of various states. Entropy treats of the distribution of energy; that is, the distribution of energy in a river system tends toward the most probable state. According to this concept, the most probable condition exists when energy in a river system is as uniformly distributed as can be permitted by physical constraints. From this general consideration, the theoretical solutions for the hydraulic geometry, including longitudinal river profile, may be obtained, and it will be found that they agree closely with field observations [92].

4. Dimensional and inspectional analysis

The concept of the dimensional and inspectional analysis permits quantitative geomorphic studies to be placed on a sound geometrical and mechanical basis. Land-form elements of fluvially dissected land masses are analyzed according to dimensions. Combinations of dimensional elements produce dimensionless numbers which provide descriptive indices of the terrain, irrespective of scale. These indices are very useful for the comparison of landscapes of different drainage basins. Complete similarity of land forms in two regions exists when all corresponding linear dimensions are in the same scale ratio and when all corresponding dimensionless numbers are the same [155].

Dimensional analysis is a somewhat faster and simpler operational tool than is inspectional analysis, and therefore has been widely used in engineering research. However, it is based on random selection and combination of dimensional elements, while inspectional analysis is based upon the application of basic physical laws such as those of mass, momentum, and energy conservation. Hence, the concept of inspectional analysis is more reliable than that of dimensional analysis.

5. Relationships of drainage basin variables

The concept of the relationship of drainage basin variables takes into account large numbers of drainage basin and network properties expressed as quantities, factors, and parameters interrelated in a large variety of ways, in order to obtain some descriptive measures of land form development. Since this concept has been intensively used in many geomorphological studies, with large number of variables, all the appropriate drainage basin properties will be briefly analyzed and described in the next chapter [74, 150].

IV. CHARACTERISTIC PARAMETERS AND PROPERTIES USED IN QUANTITATIVE ANALYSIS

In the last two decades, strong effort has been made in the direction of quantification of land form description in general and of river network in particular. With the consideration of the drainage basin as a geomorphic unit in which running water and associated mass gravity movements are the most important agents of form development, it will become obvious that there are a variety of local and secondary influences affecting its development. Hence, the geomorphic character of a drainage basin may be measured and quantitatively described in a variety of ways. For this purpose, many properties, quantities, and parameters were introduced in quantitative analyses of drainage basins. These properties and quantities may be grouped into the following classes:

1. Properties measured or counted from channel networks or drainage basins;

2. Areal properties of drainage basins; and

3. Relief properties of drainage basins.

The majority of these properties will be listed here and briefly described. Moreover, the most common names of parameters or properties, their proposed symbols, their dimensions, and their descriptions or definitions will be presented in summarized form.

1. <u>Properties measured or counted from a channel network or drainage basin</u>

1.1 <u>Stream order or basin order</u> [u] is the measure of the position of a stream or basin in the hierarchy of tributaries and represents the highest order stream or basin. The smallest, most elementary unbranched stream is designated as the order 1; the two first order streams combine to form a stream of the order 2; etc. The order of the stream increases as it goes downstream, so that the order of the main stream is the highest one.

1.2 <u>Number of streams or basins of order u</u> [N_u] represents the total number of all streams counted as the stream segments of a given order u which are present in a drainage basin.

1.3 <u>Bifurcation ratio</u> [R_b] is the ratio of the number of streams of lower stream order to the number of streams of next higher stream order:

$$R_b = \frac{N_u}{N_{u+1}}$$

1.4 <u>Stream-entrance angle</u> [ξ, degrees] represents the angle between the centerlines of the tributary and the main stream.

1.5 <u>Stream azimuth</u> [α, degrees] is defined as the angle between the vertical axis directed toward the north and the principal drainage line, measured in degrees in the clockwise direction.

1.6 <u>Stream length</u> [L_u, miles] is the total length of all streams of a given order in a drainage basin under consideration, measured as the stream segments of the same order.

1.7 <u>Mean segment length of order u</u> [$\bar{L}_u$, miles] represents by definition the sum of all stream lengths of order u, measured as stream segments, divided by the number of streams of the same order:

$$\bar{L}_u = \frac{L_u}{N_u} \quad .$$

1.8 <u>Total length of order u</u> [ΣL_u, miles] is the sum of the total stream lengths of all stream orders within a drainage basin under consideration. Total length of order u is a cumulative for a given order and includes the stream lengths of all lower order streams.

1.9 <u>Stream length ratio</u> [R_L] represents the ratio of the mean segment length of order u to the mean segment length of the first lower stream order

$$R_L = \frac{L_u}{L_{u-1}} \quad .$$

1.10 <u>Length of overland flow</u> [L_o, feet] is the length of the flow of water over the ground before it becomes concentrated in definite stream channels. To a large degree, the length of overland flow is synonymous with the length of sheet flow as it is quite commonly used.

1.11 <u>Drainage development ratio</u> [R_{Lb}], describing the degree of drainage development in a given drainage basin, is the ratio of the stream length ratio to the bifurcation ratio:

$$R_{Lb} = \frac{R_L}{R_b} \quad .$$

1.12 <u>Basin perimeter</u> [P, miles] is presented by the length measured along the border or divide of the drainage basin of a given order, as projected onto the horizontal plane of the map.

1.13 <u>Basin length</u> [L_b] represents the length measured as the longest dimension of the drainage basin of a given order, parallel to the principal drainage line.

1.14 <u>Inflection angle of contour lines</u> [Ψ, degrees] is represented by the angle which a contour line makes with itself where it depicts a channel.

2. <u>Areal properties of a drainage basin</u>

2.1 <u>Basin area of order u</u> [A_u, square miles] is the total area of a stream of a given order at a specified location, measured in a horizontal plane which is enclosed by a drainage divide.

2.2 <u>Basin area ratio</u> [R_a] is defined as the ratio of the basin area of any order u to the basin area of the first lower order:

$$R_a = \frac{A_u}{A_{u-1}} \quad .$$

2.3 <u>Interbasin area</u> [A_i, square miles] represents the area between adjacent drainage basins, which has not developed a drainage channel, but drains directly into a higher order channel.

2.4 Drainage density [D, miles per square mile] is expressed as the ratio of the total length of all streams in the drainage basin under consideration and the total area of the same drainage basin:

$$D = \frac{\Sigma L_u}{A} \quad .$$

2.5 Constant of channel maintenance [C, square miles per mile] is the inverse of the drainage density and it represents the magnitude of drainage area required to maintain one unit length of the channel. Hence, it is defined as the ratio of the total drainage area to the total length of all streams for a given basin order:

$$C = \frac{A_u}{\Sigma L_u} = \frac{1}{D} \quad .$$

2.6 Stream frequency [F, number per square mile] represents the ratio of the total number of streams, counted as the stream segments of a given order which are present in a drainage basin, to the total basin area of the same order:

$$F = \frac{N_u}{A_u} \quad .$$

2.7 Texture ratio [T, number per mile] is defined as the ratio of the number of crenulations on that contour line which has the maximum number of crenulations within the drainage basin to the length of the perimenter of the drainage basin:

$$T = \frac{N_c}{P} \quad .$$

2.8 Basin circularity [R_e] is defined as the ratio of the total area of drainage basin to the area of circle of the same perimeter (A_{cp}):

$$R_c = \frac{A}{A_{cp}} \quad .$$

2.9 <u>Basin elongation ratio</u> [R_e] is the ratio between the diameter of a circle (D_c) with the same area as the drainage basin, and the basin length measured as the maximum length of the drainage basin parallel to the principal drainage line (L_b):

$$R_e = \frac{D_c}{L_b} .$$

3. Relief properties of drainage basin

3.1 <u>Stream channel slope</u> [Θ_c, feet per mile; degrees; percents] is defined as the angle between the bottom line of the channel and the horizontal line passing through the mouth of the stream channel.

3.2 <u>Stream channel segment slope of order u</u> [Θ_u, degrees] represents the average stream channel slope of all stream channel segments of the same stream order u.

3.3 <u>Stream channel slope ratio</u> [R_s] is defined as the ratio of the stream channel segment slope of the order u to the stream channel segment slope of the next higher order stream:

$$R_s = \frac{\Theta_u}{\Theta_{u+1}} .$$

3.4 <u>Ground slope orthogonal to contour</u> [Θ_g, degrees] is defined as the mean angle between the horizontal surface and the ground surface orthogonal to contour lines of the watershed as a whole.

3.5 <u>Valley-side slope</u> [Θ_{max}, degrees] represents the maximum angle between the horizontal line and the valley side, measured at intervals along the valley walls on the steepest parts of the contour orthogonals running from divides to adjacent stream channels.

3.6 <u>Ratio of channel slope to ground slope</u> [R_{cg}] is defined by the ratio of the stream channel slope to the ground slope orthogonal to contour

$$R_{cg} = \frac{\Theta_c}{\Theta_g} .$$

3.7 <u>Dihedral angle between valley sides</u> [ξ, degrees] is defined as the angle between the two valley sides, measured in the same cross-section.

3.8 <u>Basin relief</u> [H, feet] is the maximum relief in the drainage basin defined as the difference of elevation between summit and valley floor at the exit section of the drainage basin, or the elevation difference between the lowest and highest points of a basin.

3.9 <u>Relief ratio</u> [R_h] is defined as the ratio between the basin relief and the basin length:

$$R_h = \frac{H}{L_b} \quad .$$

3.10 <u>Relative relief</u> [R_{hp}] is defined as the ratio of the basin relief to the basin perimeter

$$R_{hp} = \frac{H}{P} \quad .$$

3.11 <u>Relative basin height</u> (in hipsometry) [y] is the ratio of the height of a given contour (h), measured above the mouth of considered stream, to the basin relief

$$y = \frac{h}{H} \quad .$$

3.12 <u>Relative basin area</u> (in hipsometry) [x] is the ratio between the partial area (a) of the drainage basin above a given contour and the total basin area of the drainage basin under consideration

$$x = \frac{a}{A} \quad .$$

3.13 <u>Hypsometric integral</u> [I] represents the area under the dimensionless hypsometric curve (relative basin height vs. relative basin area)

$$I = \int_{0.0}^{1.0} x \, dy \quad .$$

3.14 <u>Volume of landmass</u> [V, cubic feet; cubic miles] represents the volume of earth material contained between the ground surface, the horizontal surface passing through the mouth of the drainage basin, and ther vertical surface passing through the drainage divide:

$$V = \int_{\text{bottom elev.}}^{\text{summit elev.}} a \, dh$$

3.15 <u>Curvature of slope profile</u> [K, radians per foot] is defined as the ratio of the angle of slope profile curvature measured in radians to the length of the slope profile curvature.

V. RESULTS OF PREVIOUS WORK

As a result of previous field and laboratory works, and various studies undertaken in the past, some laws, significant regularities, and relationships were established in describing the development of river networks. These are described below.

For example, the composition of the river network of a drainage basin can now be expressed quantitatively in terms of stream order, drainage density, bifurcation ratio, and stream-length ratio. Two fundamental laws connect the numbers and lengths of streams of different orders in a drainage basin: the law of stream numbers and the law of stream lengths. The first law expresses the relation of the number of streams of a given order to the stream order in terms of an inverse geometric series, of which the bifurcation ratio is the base. The second law expresses the average length of streams of a given order in terms of stream order, average length of streams of the first order, and the stream-length ratio. This law takes a form of a direct geometric series.

Furthermore, the erosive force and the rate of erosion (the quantity of material actually removed from the soil surface per unit of time and area) at a distance from the watershed line is directly proportional to the runoff intensity, the distance, a function of the slope angle, and a proportionality factor (representing the quantity of material which can be torn loose and eroded per unit time and surface area, with unit runoff intensity, slope, and terrain) [74].

The drainage density, surface water runoff, and the movement of ground water are parts of a single hydrologic system controlled by and related to the transmissibility of the bedrock and its overlying soil mantle [23].

For a given drainage area, the channel slope is directly proportional to a power function of the size of rock fragments on the bed, and for a given size of bed material the channel slope is inversely proportional to a power function of the drainage area. Also, the length of stream increases directly as a power of the drainage area [66].

The heights of the valley benches are significantly related to the depth of flow corresponding to the mean annual flood, and to the slope of the stream channel [78].

The hydraulic geometry -- depth, width, velocity, and suspended load -- is related to the river discharge as a power function at a given cross section. Similar relations with discharge exist among the cross sections along the river, under the condition that discharge at all sections is equal in frequency of occurrence. Then, the functions differ only in numerical values of coefficients and exponents [88].

Stream order is related to stream length, number of streams, size of drainage area, and discharge by simple exponential function. The relation of discharge to width, depth, velocity, slope, and other hydraulic factors can be approximated by a simple power function.

Suspended-load measurements made during various stages of a few individual floods have provided a close approximation to the suspended load rating curve obtained from periodic measurements taken at sediment stations over a period of years [89].

The most probable longitudinal stream profile approaches the condition in which the downstream rate of production of entropy per unit mass is constant [92].

Observed relationships between channel length, drainage-basin area, and stream-order number are dependent on the constant of channel maintenance. The texture ratio, maximum slope angle, stream gradients, drainage-basin shape, annual sediment loss per unit area, are related to the relief ratio, which is a valuable means of comparing geomorphic characteristics. Hypsometric curves reveal that the point of maximum erosion within a drainage basin migrates upchannel and that the mass-distribution curve of any basin has a similar evolution to that of the longitudinal stream profile [130].

The weighted mean per cent of silt-clay in the channel and on the banks of stable alluvial stream channels may be used as a parameter descriptive of the shape of stream channels. As the percentage of silt and clay increases, the shape of stream channels can be described as a width-to-depth ratio by a simple mathematical equation. As the weighted mean per cent of silt-to-clay increases downstream along a given river, the depth increases more rapidly and the width less rapidly with discharge, than if the weighted means were constant, while the width-to-depth ratio decreases. Conversely, as the weighted mean decreases downstream, the depth increases less rapidly and the width more rapidly with discharge than if the weighted means were constant, while the width-to-depth ratio increases. Unstable channels may be recognized by changes in their width-to-depth ratio. In general, aggrading channels have a higher width-to-depth ratio, whereas degrading channels have a lower one than indicated by weighted mean per cent silt-to-clay [133].

Mean annual sediment yield from small semiarid drainage basins is related to a ratio of basin relief to length. Mean annual runoff from small semiarid drainage basins is related to drainage density. Mean annual sediment yield per unit area decreases with increase in drainage area [136].

Statistical analysis of slope angle of valley walls has shown a high positive correlation between the valley wall slopes and adjacent channel gradients, indicating a high degree of adjustment among component parts of a drainage system.

Drainage basin height, slope steepness, stream channel gradients, and drainage density show in general a good but negative correlation with the integral of the hypsometric curve. Mature basins of low relief, gentle slopes, gentle stream gradients, and low drainage density tend to have relatively high integrals. Areas of strong relief, steep slopes, steep stream gradients, and high drainage density tend to give relatively low integrals in the average drainage basin of the third or fourth order [147].

VI. SUGGESTIONS FOR FUTURE STUDY

A remarkable increase in the application of analytical and experimental techniques to quantitative analysis of geomorphic problems has been made during the last two decades. These investigations have taken two principal directions:

1) Description of the river network quantitatively and more precisely through the use of mathematical statistics and of probability theory, and through the use of other analytical techniques;

2) Application of physical and chemical principles to field and laboratory studies of geomorphic processes.

The first of these two principal tendencies in the analysis of river networks is the subject of these suggestions for future study. In the light of previous results, on one hand, and the possibility of compilation of some basic data, on the other, the following concepts to be incorporated into the future study of river network development are recommended:

1. The potential energy concept;

2. A successive small-large watershed approach; and

3. The stochastic approach.

These concepts are briefly explained below.

1. Potential energy concept

The running water and associate mass gravity movements -- the most important agents of landform development -- represent at the same time the main components of the total energy involved in the geomorphic work in a drainage basin. Hence, it is desirable to concentrate effort on the study of these two energy components. Their magnitudes and time and space distribution over a drainage basin certainly would be a reasonable measure of the disposable geomorphic work in a basin under study.

The running water could be introduced either through precipitation, surface runoff, or river flows, by expressing its influence either in total or specific form (per unit area, or per unit area and unit time, etc.).

The gravitational attraction could be taken into account either through the ground steepness as ground slope orthogonal to contour measured as angle in degrees, or by applying the hypsometric integral and volume of landmass.

2. Successive small-large watershed approach

Many regularities and relationships concerning river network development have been established in the past. Some of them are valid in general, but many of them are valid only for particular streams or groups of streams of particular stream order. On the other hand, the general tendency in landscape analysis has been mainly directed toward smaller geomorphic units, i.e., smaller drainage basins, and hence, lower stream orders.

For the regularities and relationships to be generalized, they should be extended toward higher stream orders. It is quite probable that some of them are valid up to a certain stream order and beyond that invalid or valid only with restrictions or modifications. This should be established by determination of the relationship of drainage basin variables from lower to higher stream orders.

3. Stochastic approach

The geomorphic processes are responsible for the formation of watersheds and river networks and for their alteration with space and time. Each of these processes, considered separately in a differential form at a given unit space and unit time, is of deterministic nature. However, their combined action over an area and a period of time is so complicated and unpredictable that the total mechanism is of a stochastic rather than a deterministic behavior. There are so many causes at work that the influence of each cannot be readily identified. Therefore, the statistical and probability methods, as modern tools, must be applied in order to adequately describe these stochastic phenomena.

A considerable number of physiographic, climatic, and other data should be involved in a future study in order to develop reliable stochastic models. To facilitate the future analysis of river network development and to make possible the handling of large amounts of data, general statistical models should be designed. These models should be common for all stream orders as far as is possible. The data, if possible, should be expressed in dimensionless form (in terms of sample means, for example). Finally, sample statistics should be computed, population parameters estimated, and comparison made between streams of different order, based on these parameters.

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APPENDIX

ABSTRACTS OF SOME OF THE IMPORTANT REFERENCES

There have been many studies conducted in the United States and overseas which are related to the qualitative and quantitative analysis of river network development; but for the sake of brevity, only some of the important ones are reviewed here.

1. Carlston, C. W., Drainage density and streamflow. U.S. Geol. Surv. Prof. Paper 422-C, 1963.

The drainage density, surface water runoff, and the movement of ground water are parts of a single hydrologic system controlled by the transmissibility of the bedrock and its overlying soil mantle. A mathematical model of such a system, developed by Jacob, has been adapted to show that transmissibility is related to ground water recharge, to drainage density, and to the height of the water table at the water table divide.

It was concluded by the author that the terrain transmissibility controls the amount of precipitation which passes through the underground system of a given area. Moreover, the surface water component increases with decreasing transmissibility. The close relation of drainage density to mean annual flood per unit area indicates that the drainage network is adjusted to the mean flood runoff. Among the 15 basins in which flood runoff was correlated with drainage density, there are large and significant differences in relief, in valleyside and stream slopes and in amounts and intensitites of precipitation. However, these factors have no discernible effect on the relation of the magnitude of the floods to drainage density. Transmissibility of the terrain appears to be the dominant factor in controlling the scale of drainage density and the magnitude of the mean annual flood for basins up to 75 to 100 square miles in area.

The author provides a quantitative physical model for the origin of one of the most important elements of land form characteristics, drainage density.

2. Chorley, R. J., Geomorphology and general systems theory. U.S. Geol. Surv. Prof. Paper 500-B, 1962.

Fluvial geomorphic phenomena are examined by the author within the two model systems: closed (with fixed boundaries) and open system (the previous one being included in the latter one as its special case).

The main characteristics of the closed system are: a progressive increase in entropy (the degree to which energy becomes unable to perform work), the irreversible character of operation, the importance of the initial system conditions, the absence of intermediate equilibrium states, and the historical bias.

The open system characteristic of a tendency toward a steady state by self-regulation is equated with the geomorphic concepts of grade and dynamic equilibrium; and despite continued relief reduction, it is suggested that certain features and landscape geometry, as well as certain phases of landscape development, can be viewed profitably as partially or completely time-independent adjustments.

Seven advantages of treating landforms within an open system framework are suggested by the author: 1) the focusing of attention on the possible relationships between form and process; 2) the recognition of the multivariate character of most geomorphic phenomena; 3) a more liberal view of changes of form through time; 4) the liberalizing of attitudes toward the aims and methods of geomorphology; 5) the consideration of the whole landscape assemblage; 6) the study of areas where a previous erosional history is lacking; 7) the introduction to geography via geomorphology.

3. Fahnestock, R. K., Morphology and hydrology of a glacial stream - White River, Mount Rainier, Washington. U. S. Geol. Surv. Prof. Paper 422-A, 1963.

This paper is a study of the process by which a valley train is formed by a proglacial stream. In particular, the White River Valley was investigated, where five square miles of the 7.5 square mile drainage basin are presently covered by active ice.

Measurements of channel characteristics, such as channel width, mean depth, and mean velocities were made, and relations between variables were expressed mathematically.

Channels with steep slopes in coarse noncohesive materials were narrower, slightly shallower, and had much higher flow velocities than in cohesive materials.

The slope of the valley train was related to particle size and discharge. A systematic decrease of particles in median diameters of the valley train deposits along the stream was found downstream from the source areas. Discharges were essentially constant throughout the reach under consideration, since there were no major tributaries. Discharges were capable of transporting almost all sizes of material present and thus of modifiying the form of the valley train.

In regard to the change in pattern, a marked change from a meandering to a braided pattern took place with the onset on the high summer flows, but the pattern returned to meanders with the low flows of fall. Braiding took place most actively at large loads and discharges.

The regimen of the glacier has long-term effects in providing debris to the stream. The short-term effects of weather and runoff determine the current hydraulic characteristics, rate of deposition, and erosion, and channel pattern.

4. Hack, J. T., Studies of longitudinal stream profiles in Virginia and Maryland. U. S. Geol. Surv. Prof. Paper 294-B, pp. 45-97, 1957.

More than 100 localities on streams in seven areas of Virginia and Maryland were examined. Drainage areas ranged between 0.12 and 375 square miles. For each locality the following measurements were taken: stream length, drainage area, channel slope, channel cross section, and size of stream bed material.

The slope of a stream at a point on the channel was found to be approximately proportional to the 0.6 power of a ratio obtained by dividing the median size of the material in the stream by the drainage area of the stream at the same point. This means that for a given size of bed material the channel slope is inversely proportional to a power function of the drainage area. Also, a very uniform relation between stream length and drainage area exists, such that length increases directly as the 0.6 power of the drainage area.

The longitudinal profile of streams studied may be expressed by two simple equations, one with constant and the other with varying particle size. It was concluded by the author that stream profiles are nicely adjusted to carry away the products of erosion of their basins at rates determined by the initial relief, time, and geology of the basins.

The size of bed material (with very large variation, the size has an important effect on stream slopes) changes at any place and is determined partly by its distance from the source, partly by the initial size of the material, and partly by the relative resistance of the material to abrasion and breakage.

5. Hack, J. T., Interpretation of erosional topography in humid temperate regions. Am. Jour. Sci., v. 258-A, pp. 80-97, 1960.

Two basic concepts in geomorphology are discussed in this study. According to the author, the theory of the geomorphic cycle of erosion has dominated the science of geomorphology and strongly influenced the theoretical skeleton of geology as a whole. However, some of the principal assumptions in the theory are unrealistic. The concept of the graded stream and of lateral planation, although based on reality, are misapplied in an evolutionary development.

A more reasonable basis for the interpretation of topographic forms in an erosionally graded landscape is provided by the concept of dynamic equilibrium. According to this, every slope and every channel in an erosional system is adjusted to every other. When the topography is in equilibrium and erosional energy remains the same, all elements of the topography are downwasting at the same rate. Differences in relief and form may be explained in terms of spatial relations rather than in terms of an evolutionary development through time. However, it is recognized that erosional energy changes in space as well as time, and that topographic forms evolve as energy changes.

According to the theory of the geographic cycle, large areas of erosionally graded topography in humid regions have been considered to be "maturely dissected peneplains", while according to the equilibrium theory, this topography is what would be expected as the result of long continued erosion (its explanation does not necessarily involve changes in base level).

6. Horton, R. E., Erosional development of streams and their drainage basins: Hydrophysical approach to quantitative morphology. Geol. Soc. Am. Bull., V. 56, pp. 275-370, 1945.

The main subjects presented in this paper are: quantitative physiographic factors, infiltration theory of surface runoff, surface erosion by overland flow, origin and development of stream systems and their valleys by aqueous erosion, and drainage basin topography.

Two sets of tools -- which permit an attack along quantitative lines on the problems of the development of landforms, particularly drainage basins and their stream nets, -- are described by the author. These are measuring and operating tools.

Stream development and drainage basin topography are considered wholly from the viewpoint of the operation of hydrophysical processes. The composition of the stream system of a drainage basin can then be expressed quantitatively in terms of stream order, drainage density, bifurcation ratio, and streamlength ratio.

Two fundamental laws, the law of stream numbers and the law of stream lengths, connect the numbers and lengths of streams of different orders in a drainage basin, in terms of an inverse geometric series.

The critical length of overland flow required to produce sufficient runoff volume to initiate erosion on a given terrain is the most important single factor involved in erosion phenomena. The erosive force and the rate at which the erosion can take place at a distance from the watershed line is directly proportional to runoff intensity, the distance, a function of a slope angle, and some proportionality factor.

7. Kilpatrick, F. A. and Barnes, H. H., Channel geometry of Piedmont streams as related to frequency of floods. U. S. Geol. Surv. Prof. Paper 422-E, 1964.

For this paper, the relation of the height of valley benches to the height of water surfaces corresponding to floods of selected recurrence intervals was investigated at 34 sites in the Piedmont province, which lies in the southeastern part of the United States. The field investigation included surveys of the geometry of the channel and the measurement of water surface profiles during flood periods. The discharge-probability relation was established from records at gaging stations.

The heights of the valley benches were found to be significantly related to the depth of flow corresponding to the mean annual flood, and to the slope of the stream channel.

The slope of the channel appears to be a major factor in determining the character of the benches. The mean annual flood is contained within the channel on streams of steep slope, but may inundate the highest bench to a considerable depth on streams of mild slope.

The depth of flow corresponding to the mean annual flood has been related to bankful depth and the slope of water surface. It has been concluded that the elevation and slope of valley benches are significantly related to the frequency of flood discharge.

8. Langbein, W. G., Geometry of river channels. Am. Soc. Civil Eng., V. 90, HY2, pp. 301-312, 1964.

Rivers construct their own geometries, which vary in form, with the mean form described by "hydraulic geometry". However, hydraulic principles, as developed in firm boundary channels, appear insufficient to explain the form and profile of river channels because in them there is simultaneous adjustment of width, depth, velocity, slope, and consequently, sediment transportability. In accommodating a change in stream power, a river channel tends toward equal distribution among velocity, depth, width, and slope.

Two basic principles, the mean form which fulfills the necessary hydraulic laws and equal distribution in adjusting change in stream power, are explained by the use of three examples. First, the response to changes in flow over a sand bed between the fixed wall of a circulating flume at constant slope, with one degree of freedom - to adjust its roughness; second, accommodation of river channel, at a given cross section, to changing discharge, with one degree of freedom; and third, the river in a humid region with liberty to adjust its profile, velocities, depths and widths to accommodate the downstream increase in discharge, with three degrees of freedom. That is to say, three more relations or equations are needed to solve for variables.

Each of these three examples appears to satisfy the postulate of this paper -- that the adjustment of hydraulic geometry is toward equable accommodation of changes in stream power.

9. Leopold, L. B. and Maddock, T., The hydraulic geometry of stream channels and some physiographic implications. U. S. Geol. Surv. Prof. Paper 252, 56 p., 1953.

Hydraulic geometry -- depth, width, velocity, and suspended load of stream channels -- has been measured quantitatively and found to vary with discharge as simple power functions at a given cross section: width, $w = aQ^b$; depth, $d = cQ^f$; velocity, $v = kQ^m$; suspended sediment load, $L = pQ^j$. Similar variations in relation to discharge exist among the cross sections along the length of a river under the condition that discharge at all points is equal in frequency of occurrence. Then, functions differ only in numerical values of coefficients and exponents. These relationships at a given channel cross section and downstream when plotted on graphs are greatly similar even for river systems very different in physiographic setting. These hydraulic characteristics of stream channels slightly change progressively downstream.

The hydraulic geometry of river channels is presented here for several river systems. It is shown that similar equations apply both to rivers and to stable ("regime") irrigation canals which neither scour nor aggrade their beds. The analogy demonstrates that the average river channel-system tends to develop in a way to produce an approximate equilibrium between the channel and the water and the sediment it must transport. This approximate equilibrium appears to exist even in headward ungraded tributaries and in a given cross-section for all discharges up to the bankful stage.

10. Leopold, L. B. and Miller, J. P., Ephemeral streams - Hydraulic factors and their relation to the drainage net. U. S. Geol. Surv. Prof. Paper 282-A, 36 p., 1956.

Ephemeral streams in an arid region near Santa Fe, New Mexico, are the subject of this paper. Channels in this area are described and methods of investigation outlined; in addition, the measurements of hydraulic variables, including channel slope, discharge and sediment load, are presented.

As a result of this investigation, it was concluded that stream order is related to stream length, number of streams, drainage area, and discharge by simple exponential function. The relation of discharge to width, depth, velocity slope, and other hydraulic factors can be approximated by simple power function.

According to this data, suspended-load measurements made during various stages of a few individual floods provide a close approximation to the suspended load rating curve obtained from periodic measurements taken at a sediment station over a period of years.

In the ephemeral streams studied, velocity increases downstream at a faster rate than in perennial rivers(these are associated with an increase in suspended-load concentration downstream of ephemeral channel, because of infiltration capacity).

The tendancy for stream channels to maintain a quasi-equilibrium with imposed discharge and sediment load is shown to be characteristic of ephemeral channels in the headwaters of the drainage basin.

11. Leopold, L. B. and Langbein, W. B., The concept of entropy in landscape evolution. U. S. Geol. Surv. Prof. Paper 500-A, 20 p., 1962.

The concept of entropy is expressed in terms of probability of various states. Entropy treats the distribution of energy. The principle is introduced by the authors that the most probable condition exists when energy in a river system is as uniformly distributed as may be permitted by physical constraints. From these general considerations, equations for the longitudinal profiles of rivers are derived that are mathematically comparable to those observed in the field. The most probable river profile approaches the condition in which the downstream rate of production of entropy per unit mass is constant.

Hydraulic equations are insufficient to determine velocity, depths, and slopes of rivers that are themselves authors of their own hydraulic geometries. According to the authors, solution becomes possible by introducing the concept that the distribution of energy tends toward the most probable. This solution leads to a theoretical definition of the hydraulic geometry of river channels that agrees closely with field observations.

The most probable state for certain physical systems can also be illustrated by random-walk models. Average longitudinal profiles and drainage networks were so derived and these have the properties implied by the theory. The drainage networks derived from random walks have some of the principle properties demonstrated by Horton analysis; specifically, the logarithms of stream length and stream numbers are proportional to stream order.

12. Leopold, L. B., Wolman, M. G. and Miller, J. P., Fluvial processes in geomorphology. W. H. Freeman and Company, San Francisco, 522 p., 1964.

The origin and development of land forms have as a central theme the work of rivers, streams, and hillslope processes, since running water is the most important force that fashions the landscape. Without minimizing the importance of historical geomorphology, this book emphasizes geomorphic processes rather than the history of land forms.

The main subjects are: geomorphology and the field problem; climate and denudational processes; weathering; the drainage basin as a geomorphic unit; water and sediment in channels; channel form and process; hillslope characteristics and processes; geochronology; drainage pattern evolution; channel changes with time; and evolution of hillslopes.

Since remarkable advances in the application of analytical and experimental techniques to the study of running water and its effects have been made in the past decade, this book concentrates on these developments. Running water acts as the main agent in shaping the land surface, either by carving away relief features or by depositing masses of erosional debris. All the main processes in landscape development are described qualitatively and explained physically; also, many of them are presented quantitatively, a presentation which is very useful for quantitative analysis of river network development, as well as for the other type of land forms. This book represents both the descriptive, or qualitative, and the quantitative way of approach to problems in geomorphology.

13. Mackin, J. H., Concept of the graded river. Geol. Soc. Am. Bull.,v. 59, pp. 463-512, 1948.

The author modifies and extends the theory of grade originally set forth by Gilbert and Davis, the validity of which has been questioned.

Grade is a condition of equilibrium in streams as agents of transportation. A graded stream is one in which, over a period of years, slope is delicately adjusted to provide, with available discharge and the prevailing channel characteristics, just the velocity required for transportation of all of the load supplied from above. A graded stream responds to a change in conditions in accordance with LeChatelier's general law: "... if a stress is brought to bear on a system in equilibrium, a reaction occurs, displacing the equilibrium in a direction that tends to absorb the effect of the stress". Readjustment is effected primarily by appropriate modification of slope by upbuilding or downcutting, and only to a minor extent or not at all by concomitant changes in channel characteristics. In general, slope usually decreases in down valley direction, but because the discharge, channel characteristics, and load do not vary systematically along the stream, the graded profile is not a simple mathematical curve. Corrosive power and bed rock resistance to corrasion determine the slope of the ungraded profile, but have no direct influence on the graded profile. Mainly because of a difference in rate of down valley decrease of size of load, the aggrading profile differs in form from the graded profile; the aggraded profile is, and the graded profile is not, asymptotic with respect to a horizontal line passing through base level. It is critical in any stream profile analysis to recognize the difference in slope-controlling factors in parts of the overall profile that are graded, ungraded and aggraded.

14. Melton, A. M., An analysis of the relations among elements of climate, surface properties, and geomorphology. Off. Naval Res. Proj. N.R. 389-042, Tech. Rept. 11, Dept. Geol., Columbia Univ., 102 ., 1958.

Over 80 basins in Arizona, Colorado, New Mexico, and Utah were inspected in the field; 22 were subjected to detailed field investigations. Morphometric properties of all basins were measured from recent 1:24000 topographic maps or special drainage maps. Climate was measured by the Thornthwaite precipitation-effectiveness (P-E) index and the intensity of average five-year, one-hour rains.

As a result of this study the following conclusions were obtained. Drainage density averaged highest in basins having gullied slopes, intermediate in unmodified basins, and lowest in basins with trenched main channels. The texture of topography decreases exponentially with valley-side slope and questionably varies inversely with relative relief. High correlations exist among channel frequency, drainage density, and frequency and density of first order channels. Average channel-segment length of any order varies inversely with the total channel length for areas of the same size but differing drainage densities. Average first-order channel length is a constant proportion of average channel length of all orders, and is inversely proportional to total channel length. Valley slopes increase with infiltration capacity and P-E index, but vary inversely with wet soil strength and runoff-intensity-frequency. Greater runoff produces lower valley-side slopes. Greater infiltration permits interflow, reducing the ratio of slope runoff to channel flow. Drainage density varies with the percentage of bare area and runoff frequency-intensity, but it varies inversely with P-E index and infiltration capacity. The primary controls of topographic texture are lithology and climate, which act through the agency of secondary surface properties.

15. Rzhanitsyn, N. A., Morphological and hydrological regularities of the structure of the river net. (Translated from Russian by D. B. Krimgold). Gidrometeoizdat, Leningrad, 380 p., 1960.

This book deals with the questions related to the determination of generalized characteristics of river networks and with the laws governing their changes along the length of rivers in relation to the local peculiarities of the channel.

A river network is the result of a complex natural physiographical process. The shapes and structure of the river channel depend on physiographical conditions which determine the amounts and rates at which water reaches the earth surface (climatic factors); on conditions of runoff of this water (hydrologic factors); and on the resistance of the earth surface to erosion (geomorphological factors).

The river channels which form the river network are not homogeneous: they vary in dimensions, flow, and other characteristics from tributary to tributary and along the length of the same river.

The complicated interrelationships in a complex natural physiographic process of river network development, require a two-sided study: a) the determination of sets of averaged characteristics of the river channel system in order to reveal the general regularities of changes in each characteristic along the river and according to the size of the river; b) the determination of local morphometric characteristics of the peculiarities of each river.

The main subjects of this book are: the structure of the river net and its principal regularities; generalized characteristics of streams of a river system (in relation to stream order); morphometric characteristics and indices of channel forms; stability of the channel and rate of the channel process; characteristics of the river channel and of the stream; modeling of natural stream channels in eroding models; the possibilities of utilizing small rivers as models in the study of the channel process of large natural rivers; and some peculiarities of the natural formation of the longitudinal profile of rivers.

16. Scheidegger, A. E., Theoretical geomorphology. Prentice Hall, Inc., Englewood Cliffs, N. J., 333 p., 1961.

Mathematical treatments of the surface features of the earth, along with a coherent treatment of the exogenetic aspects of theoretical geology are presented in this book. In addition, a brief description of the physiographic facts of geomorphology and a review of some of the basic physics necessary for the understanding of the subsequent exposition are included.

The main subjects are then presented; they include the mechanics of slope formation, river bed processes, the dynamics of river valley formation, the theory of subaquatic effects, niveal effects, and some special features. Among the special processes which are dealt with are: thermal effects, dust movement, glacial scouring, climatic effects, and sideways erosion.

17. Schumm, S. A., Evolution of drainage systems and slopes in badlands at Perth Amboy, New Jersey. Geol. Soc. Am. Bull., v. 67, pp. 597-646, 1956.

A fifth-order drainage system in a small badlands area at Perth Amboy, New Jersey, was selected by the author for detailed study of geomorphic processes and landforms. The composition of this drainage network conforms to Horton's laws.

Within an area of homogeneous lithology and simple structure, the drainage network develops in direct relation to a fixed value for the minimum area required for channel maintenance. Observed relationships between channel length, drainage-basin area, and stream-order number are dependent on this constant of channel maintenance, which is in turn dependent on relative relief, lithology, and climate of the area.

Other drainage network characteristics such as texture, maximum slope angles, stream gradients, drainage-basin shape, annual sediment loss per unit area, infiltration rate, drainage pattern, and even the morphologic evolution of the area appear related to relative relief expressed as a relief ratio, which is a valuable means of comparing geomorphic characteristics. Relief ratio and stream gradients attain a constant value when approximately 25 percent of the mass of the basin has been eroded, while the basin shape becomes essentially constant at 40 percent of mass removed (in accordance with Strahler's hypothesis of time-independent forms of the steady state).

Hypsometric curves reveal that the point of maximum erosion within a drainage basin migrates upchannel and that the mass-distribution curve of any basin has an evolution similar to that of the longitudinal stream profile.

Quantitative rather than qualitative analysis predominates in this study.

18. Schumm, S. A., The Shape of alluvial channels in relation to sediment type. U. S. Geol. Surv. Prof. Paper 352-B, pp. 17-30, 1960.

The weighted mean percent silt-clay in the channel and on the banks of stable alluvial stream channels is used as a parameter descriptive of the physical characteristics of sediment. As the percentage of silt and clay in banks and channel increases, the shape of stream channels can be described by a width-depth ratio by simple mathematical equation. Neither the mean annual discharge nor the mean annual flood significantly affects this relation.

Downstream changes in width and depth of stream channels are greatly influenced by sediment type. As weighted mean percent silt-clay increases downstream along a given river, the depth increases more rapidly and the width less rapidly with discharge than if the weighted mean were constant while the width-depth ratio decreases. Conversely, as the weighted mean decreases downstream the depth increases less rapidly and the width more rapidly with discharge than if the weighted mean were constant, while the width-depth ratio increases.

Unstable channels may be recognized by changes in width-depth ratio. In general, aggrading channels have a higher width-depth ratio than indicated by the weighted mean percent silt-clay; whereas degrading channels have a lower width-depth ratio than indicated by weighted mean percent silt-clay.

The use of percentage of silt and clay in sediment or of some similar parameter, as an indication of the physical properties of alluvium, seems to open several profitable lines of research into fluvial morphology.

19. Schumm, S. A. and Hadley, R. F., Progress in the application of landform analysis in studies of semiarid erosion. U. S. Geol. Surv. Circular 437, 13p., 1961.

The application of quantitative landform analysis to the study of erosion in 59 semiarid drainage basins in Western United States, ranging from 0.1 to 18.2 square miles is covered in this report. As a result of the analysis of topographic and hydrologic data in these semiarid areas, the authors obtained the following relations: a) the mean annual sediment yield from small drainage basins is related to a ratio of basin relief to length; b) the mean annual runoff from small drainage basins is related to drainage density; c) the mean annual sediment yield per unit area decreases with the increase in drainage area; d) the form of some convex hill slopes is related to superficial creep; e) the assymetry of drainage basins, including differences in hillslope erosion and drainage density, is related to micro-climatic variations on slopes of diverse exposure; f) the cutting of discontinuous gullies is closely related to steepening by deposition of the semiarid valley floor; g) the aggradation in ephemeral streams seems to be most prevalent in reaches where the ratio of contributing drainage area to channel length is relatively small; and h) the stream-channel shape, expressed as a width-depth ratio, is related to the percentage of silt-clay in bed and bank alluvium.

These relations were detected by measurements of terrain characteristics, which indicate the importance of quantitative terrain analysis in studies of erosion.

20. Schumm, S. A., Effect of sediment characteristics on erosion and deposition in ephemeral-stream channels. U. S. Geol. Surv. Prof. Paper, 352-C, pp. 31-70, 1961.

This study concerns five small ephemeral-stream channels in five semi-arid valleys that are being actively aggraded or eroded.

The importance of physical properties of sediment in determining stream-channel shape and the differences in the mechanics of errosion and deposition between areas are emphasized. In a drainage channel composed of fine-grained, highly cohesive sediment, deposition occurs on the sides of the channel as well as on the channel floor. As a result, there is a reduction in the channel width-depth ratio across an aggrading reach. Bank caving yields only small amounts of sediment and cave blocks are often nuclei for deposition along channel sides. Vegetation seems to aid deposition.

In contrast, channels containing only small amounts of silt-clay are aggraded from bottom to top. No plastering of fine sediments on the banks occurs. Less vegetation grows on these poorly cohesive, highly mobile sediments. Commonly, a break in the longitudinal profile is quickly removed by channel degradation. Accordingly, a bank caving seems to supply more sediment to the stream load.

The aggradation in the studied area is apparently a result of high sediment yields in the headwater parts of the drainage basins. Deposition, occurring where the rate of increase of drainage area per mile of channel is low, causes marked changes in the ephemeral-stream channels.

The relation between channel shape and silt-clay was suggested as a criterion of channel stability: aggrading channels plot well above the width-depth silt-clay regression line; whereas, degrading channels plot below the line.

21. Schumm, S. A., and Lichty, R. W., Channel widening and flood-plain construction along Cimarron River in Southwestern Kansas. U. S. Geol. Surv. Prof. Paper 352-D, pp. 71-88, 1963.

The channel of the Cimarron River in Southwestern Kansas has changed significantly during historic time. The channel changes seem typical of sandy rivers in semiarid regions. In particular, channel widening instead of degradation and channel narrowing instead of aggradation are characteristic.

The causes of natural phenomena affecting destruction and rebuilding of the Cimarron River flood plain are analyzed. According to the authors, the period of channel widening was characterized by below-average precipitation and by floods of high peak discharge, whereas the period of flood-plain construction was characterized by above-average precipitation and floods of low peak discharge. The influence of these conditions on vegetation growth is the key to the behavior of rivers. Wet years and low water allowed a vigorous growth of perennial vegetation, which stabilized the existing deposits and promoted additional deposition. The stabilization of the new flood plain by vegetation was so effective that the floods after it did not cause great changes in the valley.

In contrast to floods observed in humid regions, floods in semiarid and arid environments may be tremendously destructive to the channel and flood plain. This destruction by floods may be a characteristic of erosion in semiarid regions, where climatic fluctuations are common, and the streams are ephemeral or carry very low flows during long periods. Often these streams cannot adjust as readily as perennial streams to a change in stream regimen or a climatic fluctuation.

22. Strahler, A. N., Equilibrium theory of erosional slopes approached by frequency distribution analysis, Part I. Am. Jour. Sci. v. 248, pp. 673-696, 1950.

The slope angle of valley walls was treated by the author statistically as a measure of erosional process in the quantitative analysis of erosional landforms. Field sampling of slope angles in several mature regions differing widely in lithology, relief, vegetation, climate, and soils has yielded data suited to frequency distribution analysis. Means, standard deviations, skewness, and normal distribution fitness have provided a quantitative basis for slope description and test of significance.

Analysis has indicated a wide range of means, but ones which are symmetrical and with low dispersions. This result has been taken as evidence of prevailing conditions of form-equilibrium accompanying a steady state in an open system of erosion and transportation. High positive correlation between the valley-wall slopes and adjacent channel gradients indicates a high degree of adjustment among component parts of a drainage system.

Tests for the significance of differences in slope means of three studied localities showed that: a) factors other than lithology exert major control over observed differences in slope angles; b) directional exposure has no significant effect upon slope angles (established by measured azimuths); and c) slopes left to weathering, sheet wash, and creep without stream corrasion at the base have reclined in angle.

23. Strahler, A. N., Dynamic basis of geomorphology. Bulletin of the Geol. Soc. Am. Bull., v. 63, pp. 923-937, 1952.

For quantitative research in geomorphology, it is proposed that geomorphic processes be treated as gravitational or molecular shear stresses which act upon elastic, plastic, or fluid earth materials to produce the characteristic varieties of strain, or failure, that constitute weathering, erosion, transportation, and deposition. Gravitational stresses include all mass movements, all fluvial and glacial processes, and wave- and tide-induced currents and wind. Molecular stresses are induced by temperature changes, crystallization and melting, absorption and desiccation, or osmosis; they act at random or they act in unrelated directions with respect to gravity.

A formulation of mathematical models in future quantitative studies in geomorphology is proposed. Two basic models are possible: statistical model of experimental and sample field data and simple mathematical model based on precise statement of fundamental truths (empirical and rational models).

24. Strahler, A. N., Quantitative geomorphology of erosional landscapes. C-R. 19th Intern. Geol. Cong. Algeirs, 1952, Sec. 13, pt. 3, pp. 341-354, 1954.

The processes of erosion and transportation may be viewed as the action of gravitational and molecular shear stresses acting upon earth materials having properties of elastic solids or viscous fluids. In order to establish the quantitative relationships between process and form, a first step should be made consisting of the systematic measurement of form elements comprising the topography produced under a given combination of processes.

Landscapes developed by the combined activities of weathering, mass movements, sheet erosion, and channel erosion are subject to morphometric analysis in terms of: a) the linear aspect of the channel systems, b) the areal aspect of the drainage basins and c) the relief or gradient aspect of the surfaces. Studies recently completed or in progress have produced a fund of numerical data from selected areas over a wide range of geologic, climatic, and vegetative characteristics, with a view to determining the magnitude of the controlling factors. Particular consideration was given to stream order and length ratios, stream azimuths, drainage densities, frequency distributions of maximum slope angles, ratios of channel gradients to valley-side slopes, distributions of down-slope gravitational acceleration over the ground surface, and hypsometric functions of drainage basins.

25. Strahler, A. N., Quantitative analysis of watershed geomorphology. Am. Geophys. Union Trans., v. 38, pp. 913-924, 1957.

In quantitative analysis of watershed geomorphology, two general classes of descriptive parameters are used: a) linear scale measurements and b) dimensionless numbers.

Linear scale measurements, whereby geometrically analogous units of topography can be compared as to size, include: length of stream channel of given order, drainage density, constant of channel maintenance, basin perimeter and relief, and surface and cross-sectional areas of basins as length products. For two geometrically similar drainage basins, all corresponding length dimensions should be in fixed ration.

Dimensionless numbers include: Stream order numbers, stream length and bifurcation rations, junction angles, maximum valley-slide slopes, mean slopes of watershed surfaces, channel gradients, relief rations, and hypsometric curve properties. If two drainage basins are geometrically similar, all corresponding dimensionless numbers should be identical, even though a considerable size difference may exist. Dimension-properties can be correlated with hydrologic and sediment yield data expresed per unit area, in order to be independent of total area of watershed.

26. Strahler, A. N., Dimensional analysis applied to fluvially eroded landforms. Geol. Soc. Am. Bull., v. 69, pp. 279-300, 1958.

Dimensional analysis becomes increasingly useful in empirical quantitative studies in geomorphology by offering a systematic means of describing and comparing the form elements of the landscape. By its application through the Pi Theorem, dimensional analysis facilitates the rational formulation of mathematical relationships among variables.

The use of the Pi Theorem to reduce variables has facilitated the development of a general theory of fluvial erosion in terms of steady state systems in which geometry is adjusted to erosion intensity. This theory links together various observed phenomena of erosion and sedimentation into a unified process and will enable scientists to make more complete predictions of form changes to be anticipated when watershed treatment is altered.

Dimensional analysis permits quantitative geomorphic studies to be placed on a sound geometrical and mechanical basis. The form elements of fluvially dissected land masses were analyzed in this paper according to dimension. Stream length, relief, length of overland flow, and basin perimeter have the dimension of length (L). Drainage density, texture ration, and curvature of profile have an inverse length dimension. Areal measures and volumes have the dimensions of length squared and length cubed, respectively. Dimensionless parameters include stream-order number, stream azimuth, ground-slope angle, and channel gradient. Combinations of dimensional elements produce dimensionless numbers, such as stream-length ration, basin circularity ratio, ruggedness number, and hypsometric integral, all of which provide descriptive indices of the terrain, irrespective of scale.

A very exhaustive list of dimensions and properties of fluvially eroded landforms is described and presented.

27. Wolman, M. G., and Miller, J. P., Magnitude and frequency of forces in geomorphic processes. Jour. Geol. 68, pp. 54-74, 1960.

The relative amounts of work done on the landscape and the formation of specific features of the landscape are measures of forces involved in geomorphic processes. These forces, caused by many natural events such as floods, rainfall, wind, etc., acting during time, perform the work, the amount of which is dependent on the frequency and magnitude of natural events at their maximum. In particular, the frequency at which this maximum occurs provides a measure of the level at which the largest portion of the total work is accomplished.

The analysis of records of sediment transported by rivers indicates that the largest protion of the total load is carried by moderate flows (close to bankful stage) which occur once or twice yearly. As the variability of flow increases and hence, as the size of the river basin decreases, a larger percentage of the total load is carried by less frequent flows (in many smaller basins 90 percent of the sediment is removed by storm discharges which occur at least once every 5 years).

The transport of sand and dust by wind in general follows the same laws; i.e., the greatest bulk of sediment is transported by more moderate events.

For many processes, the rate of movement of material can be expressed as a power function of some stress, such as shear stress. Where stresses generated by frequent events are incompetent to transport available materials, less frequent ones of greater magnitude are obviously required.

In general, the effectiveness of processes which control many landforms depends upon their distribution in time as well as their magnitude.