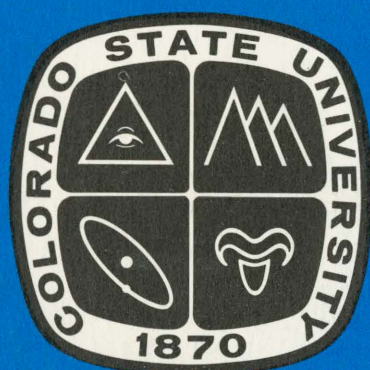


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A POLYNOMIAL REPRESENTATION OF $6.3 \mu\text{m}$ WATER VAPOR AND $4.3 \mu\text{m}$ CO_2 ATMOSPHERIC TRANSMISSIVITIES

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A POLYNOMIAL REPRESENTATION OF 6.3 μm WATER VAPOR
AND 4.3 μm CO₂ ATMOSPHERIC TRANSMISSIVITIES

By

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ABSTRACT

A POLYNOMIAL REPRESENTATION OF 6.3 μm WATER VAPOR AND 4.3 μm CO_2 ATMOSPHERIC TRANSMISSIVITIES

The research reported in this paper describes a polynomial model formulated for the depiction of 6.3 μm H_2O and 4.3 μm CO_2 transmissivities in the atmosphere. A six degree function utilizing a corrected optical path is found sufficient for expressing convolved tabulated transmittance data of 10 cm^{-1} resolution, with an RMS error of about one percent transmission. With this model transmissivities may be calculated for many spectral intervals in a relatively small amount of time with the precision required for most atmospheric problems.

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LIST OF SYMBOLS

a_v	spectral absorption coefficient
b_v	spectral optical path correction exponent
$C_n(\Delta\nu)$	spectral interval polynomial absorption coefficient
K_v	spectral absorption coefficient
p	atmospheric pressure
p_e	effective atmospheric pressure
p_0	standard atmospheric pressure
s	line intensity
T	atmospheric temperature
T_c	effective atmospheric temperature
T_0	standard atmospheric temperature
u	optical mass
u^*	effective optical path
$W(\Delta\nu)$	spectral interval transmission function
x_i	half width function
x	logarithm of effective optical path
Y	spectral interval polynomial absorption function
α	Lorentz half width
α_0	Lorentz half width at standard conditions
γ	pressure correction exponent
$\gamma_{\Delta\nu}$	spectral interval pressure correction exponent
λ	temperature correction exponent
$\lambda_{\Delta\nu}$	spectral interval temperature correction exponent
ν	frequency
$\Delta\nu$	spectral interval

LIST OF SYMBOLS

τ_{ν}	monochromatic radiation transmittance
$\tau_{\Delta\nu}$	spectral interval radiation transmittance

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I. INTRODUCTION

In order to simplify and to reduce computer time required for calculating transmittances in the 6.3 μm water vapor band and in the 4.3 μm CO_2 band calculated transmission data were fitted with a polynomial model. Coefficients for the model were determined from calculated transmittances tabulated in 10 cm^{-1} intervals for both bands at standard temperature (296°K) and pressure (1013 mb). Pressure and temperature effects were taken into account by means of a corrected optical depth. A six degree polynomial in the logarithm of corrected optical depth was sufficient for representing the tabulated transmittances in most of the 10 cm^{-1} intervals of these bands with an RMS precision of better than 1% in transmission.

II. DEVELOPMENT

For monochromatic radiation the transmissivity, τ_ν , at a frequency, ν , is given by

$$\tau_\nu = \exp(-K_\nu u) \quad (1)$$

where u is the optical mass and K_ν the absorption coefficient. For an absorption band where there are many discrete spectral absorbing lines the absorption coefficient is in reality a sum of the absorption coefficients of all spectral lines extending to the frequency ν . In other words,

$$K_\nu = \sum_i K_{\nu_i} \quad (2)$$

where K_{ν_i} is the absorption coefficient of the i^{th} line. In the atmosphere, the absorption coefficient is described with reasonable accuracy by the Lorentz line shape

$$K_{\nu_i} = \frac{s_i \alpha_i}{\pi[(\nu - \nu_i)^2 + \alpha_i^2]} \quad (3)$$

where s_i is the line intensity, α_i the Lorentz half width and ν_i is the frequency of the line center.

Combining the three equations above, one obtains

$$\tau_\nu = \exp\left[-\frac{u}{\pi} \sum_i \frac{\alpha_i s_i}{[(\nu - \nu_i)^2 + \alpha_i^2]}\right] \quad (4)$$

In order to express Eq. 4 in a simple polynomial form it must be assumed that α_i in the denominator be expressed by a characteristic value, $\tilde{\alpha}_i$, which is good for all temperature and pressure conditions

(Goody, 1964). Using this assumption and the fact that for the Lorentz shape pressure broadening may be represented by the approximation

$$\alpha_i \cong \alpha_{0_i} \left(\frac{p}{p_0}\right) \left(\frac{T}{T_0}\right)^n, \quad \text{where } n = \frac{1}{2}, \quad (5)$$

and α_{0_i} refers to the Lorentz half-width at STP, equation 4 may be simplified. Multiplying Eq. 5 by u ,

$$\alpha_i u \cong \alpha_{0_i} \left(\frac{p}{p_0}\right) \left(\frac{T}{T_0}\right)^{-\frac{1}{2}} u,$$

however, the temperature dependence is in fact more complicated and the power at (p/p_0) is not restricted to unity, so in order to establish the equality, a more general form is introduced.

$$\alpha_i u = \alpha_{0_i} \left(\frac{p}{p_0}\right)^\lambda \left(\frac{T}{T_0}\right)^\lambda u$$

$$\text{or} \quad \alpha_i u = \alpha_{0_i} u_* \quad (6)$$

$$\text{where} \quad u_* = \left(\frac{p}{p_0}\right)^\lambda \left(\frac{T}{T_0}\right)^\lambda u, \quad (7)$$

u_* is the effective optical path. Introducing the above simplification into Eq. 4 we obtain

$$\tau_v = \exp\left[-\frac{u_*}{\pi} \sum_i x_i s_i\right] \quad (8)$$

where $x_i = \alpha_{0_i} [(\nu - \nu_i)^2 + \alpha_i^2]^{-1}$. For a given spectral interval it will

be assumed that $\frac{1}{\pi} \sum_i x_i s_i$ may be represented by a constant value a_v which further reduces Eq. 8 to

$$\tau_v = \exp[-a_v u_*]. \quad (9)$$

From Goody's (1964) discussion of single line and band models, it was found that theoretically the power of the optical path need not be restricted to unity, especially in the case of overlapping lines. In a study by Roach (1961) in which laboratory data and empirical fits for infrared and near-infrared CO₂ and water vapor bands developed by Howard, Burch and Williams (1955) were utilized, the integral absorption is proportional to the square root of the optical path for weak lines. For the strong line approximation the absorption is proportional to a multiple (218 - 460) of the log u. To account for this complex interaction between overlapping weak and strong lines it becomes necessary to introduce the exponent b_v, into equation 9 yielding

$$\tau_v = \exp[-a_v u_*^{b_v}]. \quad (10)$$

Following the method of Smith (1969) and taking the logarithm of Eq. 10 twice yields the simple polynomial form

$$\ln(-\ln \tau_v) = \ln a_v + b_v \ln u_* + R_v \quad (11)$$

where R_v, the residual required to establish the equality in Eq. 11 may be interpreted as all higher order terms in ln u_{*}.

In our application a regular polynomial expansion similar to Eq. 11 is applied to convolved transmittance data tabulated in 10 cm⁻¹ intervals of the 6.3 μm water vapor band and the 4.3 μm CO₂ band. The expansion used is of the form

$$W(\Delta v) = C_0(\Delta v) + C_1(\Delta v) x + C_2(\Delta v) x^2 + \dots + C_n(\Delta v) x^n \quad (12)$$

where $W_{\Delta\nu} = \ln(-\ln \tau_{\Delta\nu})$ and $x = \ln u_*$.

Having the tabulated transmittance data for the pressures, temperatures, and optical depths given in Table I, only three points remain unsolved. The coefficients for Eq. 12 for each 10 cm^{-1} interval need to be determined and the exponents $\lambda_{\Delta\nu}$ and $\gamma_{\Delta\nu}$ need to be determined. The former was handled in the following manner.

At STP $u_* = u$, so now $x = \ln u$. Using the tabulated values of $\tau_{\Delta\nu}$ and u , the values of the coefficients in each 10 cm^{-1} interval are simultaneously derived by the method of least squares. By statistical analysis, it was found that a polynomial of no more than six degrees in $\ln u$ is sufficient to describe the relationship of τ with u . In order to achieve an optimum description over the entire range of transmittances tabulated for a given frequency interval all values of τ equal to 1 and 0 were deleted from the analysis.

In order to determine the values of $\lambda_{\Delta\nu}$, and $\gamma_{\Delta\nu}$ for each 10 cm^{-1} spectral interval, it is first necessary to have a unique value of u_* for a given u , p , and T . These values were determined in the following way. Consider a transmittance value at a given optical depth, temperature, and pressure, $\tau_{u,p,T}$. At STP there is also a value $\tau_{u_1,STP}$ where

$$\tau_{u,p,T} = \tau_{u_1,STP} \quad (13)$$

By knowing the value of u_1 where this equality holds it is possible to assign to u_* the value u_1 . Since a set of tabulated transmittance values at various temperatures, pressures and optical depths is available (Table I) it is merely a matter of plotting a family of curves with each curve representing the relationship of τ and u at a unique

PRESSURE		TEMPERATURE		OPTICAL PATH	
CO ₂	H ₂ O	CO ₂	H ₂ O	CO ₂	H ₂ O
mb	mb	°K	°K	Atm. cm.	Atm. cm.
1013	1013	296	296	.0001	.001
700	800	273	293	.0005	.01
500	600	253	273	.001	.1
100	400	233	233	.0025	1.0
				.005	
10	200	213	213	.01	2.0
				.1	4.0
				1	8.0
				10	10.0
				50	20.0
				100	40.0
				250	80.0
				500	
				750	
				1000	
				1250	
				2500	

Table I. Pressures, temperatures, and optical paths of tabulated transmittances, used as a data base for the development of the polynomial model.

combination of temperature and pressure. This is done for each spectral interval with τ graphed against $\log u$.

If a curve of τ_{STP} vs. $\log u$ is first graphed for each interval, it is then a simple matter of taking a $\tau_{u,p,T}$ and finding the value of u_1 where $\tau_{u,p,T} = \tau_{u_1,STP}$. By tabulating a set of values for u_* , u , p , T and subjecting these to a linear regression analysis using a logarithmic form of Eq. 7,

$$(\ln u_* - \ln u) = \lambda_{\Delta v} (\ln p - \ln p_0) + \gamma_{\Delta v} (\ln T - \ln T_0)$$

the best fit values of $\lambda_{\Delta v}$ and $\gamma_{\Delta v}$ may be obtained.

For some weak lines where the graphical resolution was not sufficiently precise for finding a unique value of u_* from the curve and for some strong lines where $\frac{\partial \tau}{\partial u}$ was too high, another method was employed for finding values of u_* . In these cases, it was necessary to use the values of $\tau_{u,p,T}$ in the polynomial relationship found at STP and solve for u_* . Also, in some cases (strong CO_2) it was found necessary to determine two different values for both $\lambda_{\Delta v}$ and $\gamma_{\Delta v}$ in order to achieve a higher degree of precision. The values of $\lambda_{\Delta v}$ and $\gamma_{\Delta v}$ to be used is specified as a function of the pressure.

All transmittance values used in the derivation of this model were calculated with a program (Program LBL) written by R. McClatchey (1973) which utilized laboratory line intensity data compiled by the AFCRL (TR-73-0096).

The McClatchey program made convolved transmittance computations by utilizing the line intensity data in a Lorentz shape which was modified at pressures below 100 mb by use of the Voigt shape (Goody, 1964). For the purposes of this model the McClatchey program was run using a

$.01\text{ cm}^{-1}$ monochromatic frequency increment and a $.05\text{ cm}^{-1}$ triangular slit function half width at the optical depths specified in Table I. All influences from a given line were ignored at 2.0 cm^{-1} from the line center. The convolved transmittances were calculated at each 10 cm^{-1} starting at 1250 cm^{-1} and ending at 2450 cm^{-1} for the $6.3\text{ }\mu\text{m}$ water band and starting at 2000 cm^{-1} and ending at 2630 cm^{-1} for the $4.3\text{ }\mu\text{m}$ CO_2 band. An RMS error of less than 1% transmission with respect to the tabulated transmittances for most of the 10 cm^{-1} intervals considered was achieved with use of this model.

III. RESULTS

Tables III and IV list the coefficients of the significant terms in equation 12, found for the 4.3 μm CO_2 and 6.3 μm water vapor bands. The transmittance for a spectral interval can be calculated from these coefficients by the relation

$$\tau_{\Delta\nu} = \exp[-\exp(Y)]. \quad (14)$$

In Eq. 14 $Y = \sum_{i=0}^6 c_i x^i$ where the c_i 's are given in Tables III and IV.

Also, the values for $\lambda_{\Delta\nu}$ and $\gamma_{\Delta\nu}$ are presented in Tables III and IV.

In Eq. 15 $x = \ln u_*$ where

$$u_* = u \left(\frac{p}{p_0} \right)^{\lambda_{\Delta\nu}} \left(\frac{T}{T_0} \right)^{\gamma_{\Delta\nu}}.$$

It is noted that $\tau_{\Delta\nu}$ as computed from Eq. 10 must be constrained to the interval 0 to 1 (i.e. if $\tau_{\Delta\nu} > 1.0$, $\tau_{\Delta\nu} = 1.0$; if $\tau_{\Delta\nu} < 0$, $\tau_{\Delta\nu} = 0$).

The range of applications of this model is limited to the optical depths given in Table II. Use of values outside of these ranges could lead to serious errors in results. For errors and limitations encountered within the useful range of the model see the column headed "RMS ERROR" in Tables III and IV.

TABLE II. Useful range of model

CO ₂		H ₂ O	
u _* (atm. cm.)		u _* (atm. cm.)	
min.	max.	min.	max.
0.1	2500.0	0.001	85.0
Spectral Range (cm ⁻¹)		Spectral Range (cm ⁻¹)	
2000-2630		1250-2450	

ν (cm^{-1})	C_0	C_1	$C_2 \times 10^n$	n	$C_3 \times 10^n$	n	$C_4 \times 10^n$	n	$C_5 \times 10^n$	n	$C_6 \times 10^n$	n	γ	λ	RMS ERROR % τ	LIMITING CONDITIONS
2000	-11.6688	1.003763											.143	2.828	.03	
2010	-10.59660	.980826											-.18971	1.2434	.02	
2020	-10.23181	.97099											.05822	1.599	.13	
2030	-9.114978	1.096552	-.1518652	-1	.1960704	-1	-.6664442	-2	.8262779	-3	-.3683478	-4	.0487	3.8545	.34	
2040	-7.168705	1.095537	-.9948559	-1	.3567489	-1	-.5395615	-2	.2230342	-3	.402385	-5	.1542	3.1542	.71	
2050	-6.275427	.9783538	.1135545	-1	.2697099	-2	-.2065808	-2	.148796	-3	.1271628	-5	.3187	1.0075	1.4	
2060	-6.475222	1.028614	.8108571	-2	.5654846	-3	-.1816227	-2	.2179822	-3	-.6859203	-5	A(.58387) B(.17784)	A(.096) B(.7161)	.80	p>100, use A
2070	-5.673477	1.133939	-.1940258	+0	.1037922	+0	-.2744545	-1	.3153987	-2	-.1313592	-3	A(.3529) B(.1054)	A(.92664) B(1.10403)	.90	p>100, use A
2080	-8.111579	1.157891	-.495307	-1	.1333789	-1	-.2615897	-2	.2724533	-3	-.1252151	-4	.14188	1.489	.62	
2090	-12.20607	7.384675	-3.507023	+0	1.000936	+0	-.1559653	+0	.124387	-1	-.3974606	-3	.2066	4.1503	.38	
2100	-12.71590	4.681139	-1.380183	+0	.2674267	+0	-.2785658	-1	.1461297	-2	-.3029923	-4	.2066	4.6716	.28	
2110	-12.71690	5.542827	-2.259306	+0	.5952606	+0	-.8685228	-1	.6642176	-2	-.2086656	-3	-.07842	4.641	.13	
2120	-13.81551	6.017355	-2.441485	+0	.5987598	+0	-.7895454	-1	.5402158	-3	-.1522199	-3	.1317	5.4743	.08	
2130	-13.81551	6.255362	-3.150488	+0	.9934008	+0	-.167781	+0	.1433565	-1	-.4876130	-3	.0498	4.941	.23	
2140	-10.52849	1.007707											-.14331	6.661	.07	
2150	-11.33792	1.008598											-.01083	8.268	.02	
2160	-1399.352	1341.359	-.5376869	+3	.1146184	+3	-.1368848	+2	.8681828	+0	-.2284334	-1	-.7439	6.42	.10	$u_* < 50, \tau = 1.0$
2170	-12.65558	1.043676											-.1146	11.525	.02	
2180	-11.69599	1.046987											-.1724	12.51	.03	
2190	-13.81551	9.762707	-.7563113	+1	.280655	+1	-.5135127	+0	.4593461	-1	-.1608862	-2	-.09631	8.8833	.22	
2200	-13.81551	7.166384	-.3337185	+1	.9615654	+0	-.1523022	+0	.1250231	-1	-.4161007	-3	-.17131	10.69	.30	
2210	-7.561423	1.522972	-.2849925	+0	.8698373	-1	-.1445866	-1	.1181778	-2	-.3862541	-4	-.1103	10.69	.48	
2220	-5.549922	1.072481	-.234514	-1	.3193229	-2	-.1008166	-2	.8659035	-4	-.1975633	-5	-.1002	8.863	.92	
2230	-3.814379	1.003372	-.1975505	-1	-.568079	-2	-.4909031	-3	.2685157	-3	-.167033	-4	A(.4586) B(.1155)	A(6.41564) B(7.4264)	A(.55) B(.22)	p<100, use B
2240	-3.107889	.9154184	-.1679270	-1	.5472486	-2	-.1554092	-2	-.1565793	-3	.3913460	-4	A(.59988) B(.26684)	A(5.1173) B(5.9502)	A(1.2) B(.10)	p<100, use B
2250	-1.613904	.8004722	-.5333285	-1	.4027510	-2	.2235731	-2	-.5172158	-3	.3365971	-4	A(.7714) B(.50557)	A(2.4621) B(3.5022)	A(.10) B(1.5)	p<500, use B
2260	-1.259589	.7056034	-.2345379	-1	.1082191	-1	-.1654415	-2	.1560126	-3	-.6104281	-5	A(.85625) B(.65224)	A(.8373) B(1.732)	A(.10) B(1.8)	p<100, use B
2270	-1.01856	.6806692	-.3924527	-1	.1697213	-1	-.1704094	-2	.2270495	-4	.4112372	-5	A(.9222) B(.64872)	A(.02476) B(1.1855)	A(.42) B(1.7)	p<100, use B
2280	-.9282895	.8521525	-.8997461	-1	.3436132	-3	.6250376	-2	-.40819	-4	-.1543246	-3	A(.84955) B(.4408)	A(3.5082) B(4.6314)	A(.74) B(2.1)	p<500, use B $u_* > 100, \tau = 0$

TABLE III. Spectral intervals, empirical absorption coefficients, pressure and temperature correction exponents for 4.3 μm CO_2 band

ν (cm^{-1})	C_0	C_1	$C_2 \times 10^n$	n	$C_3 \times 10^n$	n	$C_4 \times 10^n$	n	$C_5 \times 10^n$	n	$C_6 \times 10^n$	n	γ	λ	RMS ERROR % τ	LIMITING CONDITIONS
2290	-.0412770	.7314233	-.4014692	-1	.2736055	-2	.1104330	-2	.8144719	-4			A(1.118) B(.71103)	A(2.83) B(2.10945)	A(1.0) B(1.5)	$p \leq 500$, use B
2300	.535833	.6869181	-.1546908	-1	.1309148	-1	.2987873	-2	.208231	-3			.6608	4.846	1.7	$u_* > 500$, $\tau=0$
2310	1.211583	.7908794	.8524421	-1	.2108139	-1	.1262320	-2					.77992	3.8	.80	$u_* > 250$, $\tau=0$
2320	1.733303	.689087											.88132	1.745	.20	
2330	2.076448	3.749197	.2986294	+1	.1047945	+1	.1582643	+0	.8752171	-2			.11546	7.4264	2.2	$u_* > 10$, $\tau=0$
2340	1.794512	.682285											.8668	.9795	.60	
2350	1.962694	.647478											.91325	.39985	.30	
2360	2.512322	.8656896	.2928097	-1	-.6161232	-2	-.1318683	-2	.4538172	-4	.9256632	-5	.94112	.39911	1.3	
2370	1.379881	.628314											.8985	3.3574	.10	
2380	-.4562727	.5675137	-.470478	-1	.1704993	-2	.8098327	-3	-.3352258	-5	-.4235605	-5	.54315	9.5172	1.9	
2390 A	-3.707507	.2427206	-.1141463	+0	.6857687	-1	-.8447277	-3	-.135696	-2			.5624	13.72	.50	$u_* < 75$, A
2390 B	-12.343	5.932585	-.1202592	-1	.8683469	-1	.6296423	-2	-.1346239	-2	.5886439	-4	.5624	13.72	.50	$u_* > 75$, B
2400	-13.12236	5.075257	-.2396008	+1	.7234902	+0	-.1189038	+0	.101102	-1	-.3484698	-3	-.82865	7.0381	.27	
2410	-12.71690	4.990170	-.2285069	+1	.6736437	+0	-.1081057	+0	.897723	-2	-.3027885	-3	-.2262	6.481	.26	
2420	-13.12236	4.909700	-.2123409	+1	.5976136	+0	-.9229550	-1	.7439495	-2	-.2454690	-3	-.0827	5.0843	.17	
2430	-12.71690	4.547448	-.1888515	+1	.5182736	+0	-.7767888	-1	.6065730	-2	-.1942519	-3	-.3053	5.83	.37	
2440	-12.42922	5.193784	-.2398289	+1	.6992543	+0	-.1100077	+0	.8904603	-2	-.2919119	-3	-.39423	7.386	.39	
2450	-10.38771	.975357											-.0336	10.81	.05	
2460	-15.21179	1.356926											-1.4484	8.669	.08	
2470	-14.85062	1.326848											-.5502	3.449	.08	
2480	-13.05816	1.112558											-.2006	.6326	.03	
2490	-13.22684	1.078971											-.1637	-.8178	.02	
2500	-14.51671	1.204648											-.1801	-1.276	.03	
2510	-12.56425	1.044403											-.06395	-.35745	.02	
2520	-12.9239	1.021165											.13034	.6791	.02	
2530	-16.80484	1.387741											.01828	.3281	.03	
2540	-22.02705	1.902337											-.3517	-.6391	.01	
2550	-72.51609	8.103366											-.3	-.6	.01	
2560																$\tau=0$
2570	-16.29282	1.202111											-.0083	4.012	.01	
2580	-12.86049	.997688											-.0715	2.815	.01	
2590	-130.3871	137.1886	-.6374150	+2	.1553281	+2	-.2074061	+1	.1436343	+0	-.4025499	-2	-.03	-.6	.12	$u_* < 1$, $\tau=1$
2600	-11.76677	1.047577											-.0298	-.4854	.05	
2610	-13.0082	1.094928											-.037	-.93084	.04	
2620	-11.45619	1.023624											-.0546	-.4328	.02	
2630	-11.75953	1.051602											-.2802	.5072	.10	

TABLE III. Spectral intervals, empirical absorption coefficients, pressure and temperature correction exponents for $4.3 \mu\text{m CO}_2$ band

ν (cm^{-1})	C_0	C_1	$C_2 \times 10^n$	n	$C_3 \times 10^n$	n	$C_4 \times 10^n$	n	$C_5 \times 10^n$	n	$C_6 \times 10^n$	n	γ	λ	RMS ERROR % τ	LIMITING CONDITIONS
1250	-7.523403	1.115514	.1113683	-1	-.2154055	-1	.6134898	-3	.1726722	-2	-.2617828	-3	-.4503	6.0075	.03	p<400, use B
1260	-4.824289	.9890130	-.1422394	-1	-.6519051	-2	-.5702513	-3	.2361331	-3	-.1195842	-4	.59426	4.5726	.20	
1270	-5.178666	.9867838	-.2159568	-1	-.6638722	-2	-.5444032	-3	.1164342	-3	.1207201	-4	-.09456	5.07052	.22	
1280	-5.026492	1.002985	-.1643425	-1	-.6007118	-2	-.3533454	-3	.1399816	-3	-.4964413	-5	.12452	6.1897	.30	
1290	-5.745833	1.014653	.4944907	-2	-.1114754	-1	-.2382948	-2	.1142783	-2	-.1207630	-3	-.82514	9.4782	.34	
1300	-5.622221	1.005112	-.1878542	-1	-.8458818	-2	-.4193196	-3	.1981468	-3	.8308224	-6	.2038	8.36281	.18	
1310	-3.193766	.9289146	-.4826521	-1	-.1242498	-1	.2059330	-2	.4595628	-3	-.8379227	-4	.1015	4.7395	.53	
1320	-4.698016	1.041980	-.3649320	-1	-.2554895	-1	.2253854	-2	.1028684	-2	-.1294214	-3	.53872	4.6854	.70	
1330	-2.912854	.8127313	-.6518750	-1	-.6306190	-2	.1138910	-2	.1526660	-3	-.1337210	-4	-.0887	4.64741	.82	
1340	-3.351250	.8148053	-.6906277	-1	-.7632003	-2	.1037707	-2	.2230663	-3	.4518774	-5	-.05722	4.2935	.32	
1350	-4.391205	.9498369	-.5494290	-1	-.1299426	-1	.3346050	-2	.4362039	-3	-.1014758	-3	.5745	3.8631	.21	
1360	-2.229603	.6931086	-.6881879	-1	.2115602	-2	.1808088	-2	-.6650180	-4	-.3018200	-4	.34869	4.1334	.58	
1370	-2.463575	.7358997	-.5828874	-1	.3514741	-2	.1548396	-2	-.1376282	-3	-.3141942	-4	.14223	4.08272	.65	
1380	-2.639212	.6203174	-.2637395	-1	.2197629	-2	-.5927768	-3	-.4062073	-5	.1277608	-4	.47125	2.2673	.40	
1390	-1.681148	.6247951	-.5077404	-1	.5755714	-2	.9346739	-3	-.1858333	-3	-.2559655	-4	.66982	3.23854	.78	
1400	-2.999052	.6920855	-.5447039	-1	.9934879	-2	.2019340	-2	-.3498640	-3	-.5412370	-4	.36362	2.58617	.41	
1410	-1.794604	.5275334	-.4083221	-1	.7521672	-2	.3835267	-3	-.1719390	-3	-.1390166	-4	.31636	3.02354	.53	
1420	-2.411910	.7863917	-.6300680	-1	-.1425434	-2	.1409973	-2	.2610143	-4	-.1411986	-4	.6546	1.9344	.45	
1430	-1.623185	.5446810	-.3702338	-1	.9847981	-2	.5500564	-3	-.2679472	-3	-.2494176	-4	.48111	1.8088	.45	
1440	-2.801317	.6235168	-.5039367	-1	.1057550	-1	.1486979	-2	-.3618938	-3	-.4780380	-4	.3347	2.0345	.38	
1450	-1.075371	.5502116	-.4699074	-1	-.2591721	-2	-.5223393	-3	.1950970	-3	.3260155	-4	.64316	1.2469	.73	
1460	-2.296299	.5693489	-.2695699	-1	.8661900	-2	.8623992	-5	-.2284938	-3	-.1378330	-4	.96093	-.13851	.52	
1470	-1.238994	.5584310	-.5173156	-1	.5185382	-2	.5977594	-3	-.1437413	-3	-.1496735	-4	.61541	1.0118	.70	
1480	-1.955311	.7198861	-.6069813	-1	-.1188440	-2	.9302169	-3	.2488852	-4	-.7936170	-5	.4833	1.7415	.77	
1490	-1.126785	.5791793	-.4117228	-1	.4700010	-2	.3657488	-4	-.1700426	-3	-.1170901	-4	.60451	.345	.44	
1500	-.8496409	.5134084	-.5178445	-1	.4117965	-2	.5581583	-3	-.2687433	-4	-.2017896	-5	.60347	.829	.70	
1510	-1.061634	.6514814	-.6055387	-1	.8611481	-2	.2156782	-2	-.2137083	-3	-.4182012	-4	.45922	1.97663	.77	
1520	-.6215797	.634411	-.3210257	-1	.3501981	-2	.7086512	-4	-.5318281	-4	-.2234009	-6	A(.79435) B(.665)	A(.73634) B(.794)	A(.26) B(1.2)	
1530	-1.065227	.5373775	-.4270132	-1	.5655739	-2	.4986884	-3	-.9163901	-4	-.7784704	-5	.5032	.5434	.84	p<400, use B
1540	-.8185037	.5816737	-.4637327	-1	-.3066333	-2	-.6161646	-3	.1763353	-3	.3123889	-4	.7903	.6918	.78	
1550	-.7386507	.5274873	-.4464589	-1	.5755210	-2	.6031077	-3	-.7042349	-4	-.6105420	-5	.7202	.7446	.68	
1560	-1.282116	.5047078	-.4180638	-1	.7841022	-2	.5779814	-3	-.1681248	-3	-.1548365	-4	.8442	-.9043	.51	
1570	-1.312035	.5108337	-.2747305	-1	.1807238	-2	-.1015745	-2	-.9279670	-5	.1790218	-4	.7413	-1.26	.40	
1580	-6.086696	1.012970	.1309415	-2	-.1187388	-2	-.2197165	-2	.7092823	-3	-.7866758	-4	-.05265	4.624	.13	
1590	-3.337431	.9190066	-.4150070	-1	-.7744309	-2	.1088785	-2	.1936099	-3	-.3114893	-4	.2057	1.5826	.52	

TABLE IV. Spectral intervals, empirical absorption coefficients, pressure and temperature correction exponents for 6.3 μm water vapor band

ν (cm^{-1})	C_0	C_1	$C_2 \times 10^n$	n	$C_3 \times 10^n$	n	$C_4 \times 10^n$	n	$C_5 \times 10^n$	n	$C_6 \times 10^n$	n	γ	λ	RMS ERROR %	LIMITING CONDITIONS
1600	-2.952878	.9267226	-.4091990	-1	-.1104738	-1	.1987223	-3	.3009132	-3	.3986262	-5	.27297	.74609	.88	
1610	-1.726232	.5215215	-.2254797	-1	.3514981	-3	-.1309393	-2	.4263525	-4	.2608712	-4	.8016	-.1.445	.66	
1620	-1.679269	.6384774	-.4221796	-1	.6566940	-2	.8353135	-3	-.1574958	-3	-.1877166	-4	.65285	-.9089	.31	
1630	-1.389478	.4411941	-.5442028	-1	.6497407	-2	.1030066	-2	-.1739352	-3	-.3504294	-4	.8164	-.1.492	.32	
1640	-1.292707	.5740062	-.4802520	-1	-.1142807	-3	-.6781228	-4	.6912524	-4	.1344076	-4	.7658	-.596	.31	
1650	-.9646004	.3828036	-.4000790	-1	.1089477	-1	.4053161	-3	-.2311628	-3	-.1713651	-4	.7323	-.5406	.59	
1660	-1.657941	.5957592	-.5003442	-1	.2740492	-2	.2733652	-3	-.8137568	-4	-.5866427	-5	.5051	-.02776	.95	
1670	-1.253811	.5384481	-.3147184	-1	.1224117	-1	.5523984	-3	-.3430930	-3	-.3098300	-4	.7656	-.4662	.34	
1680	-.8518642	.5656042	-.3548937	-1	.5372149	-2	.4739055	-3	-.1067042	-3	-.1113892	-4	.62573	.90404	.44	
1690	-1.251196	.5767304	-.1804385	-1	-.1176239	-2	-.1330040	-2	.8542801	-4	.2892565	-4	.77413	-.3152	.28	
1700	-.8360292	.4848361	-.3917358	-1	.1290840	-1	.8208375	-3	-.3665030	-3	-.3689185	-4	.6182	-.3125	.49	
1710	-.9493980	.5865651	-.3716811	-1	-.1073398	-2	-.4478715	-3	.1144351	-3	.2209961	-4	.7707	.5824	.81	
1720	-3.328748	.8811315	-.3457315	-1	-.8622309	-3	.5258166	-3	.4643425	-5	-.6972957	-5	.4119	1.9526	.39	
1730	-1.095224	.5614041	-.2203160	-1	.2522702	-2	-.6930630	-3	.4659614	-4	.2089049	-4	.76754	.6889	.46	
1740	-1.229221	.7056694	-.4421891	-1	.1001604	-2	.5664177	-3	.9333973	-5	-.2806864	-5	.4683	.6627	.62	
1750	-1.549372	.5831574	-.3241347	-1	.7834497	-2	.2802490	-3	-.2384433	-3	-.2082444	-4	.6186	2.012	.46	
1760	-1.879264	.5417106	-.3100323	-1	.8714060	-2	.8164442	-4	-.2156956	-3	-.1315559	-4	.7629	1.6255	.26	
1770	-1.482360	.5907030	-.3406397	-1	.2351340	-2	-.1509738	-3	.2190444	-4	.1103890	-4	.7199	-.1086	.70	
1780	-2.362442	.6364812	-.4487051	-1	.1091047	-1	.1408917	-2	-.3496269	-3	-.4500972	-4	.46725	1.9506	.35	
1790	-1.688572	.6406677	-.2803156	-1	.3976474	-2	-.8273006	-4	-.7233971	-4	-.8172406	-6	.6905	1.3432	.18	
1800	-2.183601	.6655345	-.5731159	-1	.7029463	-2	.1726994	-2	-.2334795	-3	-.4152875	-4	.4128	2.028	.55	
1810	-2.818403	.7133048	-.6344222	-1	.3416692	-2	.1817341	-2	-.1311337	-3	-.3702215	-4	.322	2.046	.41	
1820	-2.467579	.6422959	-.5620410	-1	.9378511	-2	.1857259	-2	-.2994529	-3	-.4726200	-4	.69335	1.565	.10	
1830	-2.200603	.6375558	-.6813344	-1	.2692202	-2	.1456957	-2	-.3562699	-4	-.1567368	-4	.4785	.91043	.56	
1840	-1.949537	.6090040	-.2841388	-1	.4127135	-2	-.4277478	-3	-.1099117	-3	.6514168	-6	.713	2.0	.46	
1850	-5.015885	.9835242	-.3023411	-1	-.1082766	-1	.1173906	-2	.3675893	-3	-.5673779	-4	.70352	3.2664	.10	
1860	-2.749139	.7970375	-.5875241	-1	-.3821069	-2	.8168971	-3	.6601706	-4	-.8525242	-5	.4716	1.66	.69	
1870	-2.857052	.5598702	-.5746383	-1	.6180143	-2	.8809197	-3	-.1971874	-3	-.2422129	-4	.3787	1.4173	.48	
1880	-4.581218	.9830508	-.2662520	-1	-.7536398	-2	.5140282	-3	.1991532	-3	-.3304451	-4	1.63723	1.492	.33	
1890	-2.824140	.6962483	-.6511873	-1	.3735509	-2	.1690701	-2	-.1238234	-3	-.2890593	-4	.383	1.738	.46	
1900	-4.255107	.9371587	-.4482704	-1	-.1020297	-1	.1857785	-2	.2830929	-3	-.6268593	-4	.2583	2.31	.31	
1910	-2.857568	.6359083	-.4167038	-1	.4074594	-2	.4418033	-3	-.1006299	-3	-.1120382	-4	.44275	1.543	.47	
1920	-3.326163	.8227689	-.6338880	-1	-.6129284	-2	.1063581	-2	.1756312	-3	-.5479515	-6	.2285	3.63	.22	
1930	-6.026984	1.053557	-.3155929	-1	-.1362850	-1	.2596936	-2	.3786073	-3	-.1034550	-3	.00991	4.88184	.03	
1940	-2.617471	.7300794	-.8290940	-1	-.4432245	-2	.3517908	-2	-.1891294	-3	-.8002774	-4	.29353	2.63	.80	
1950	-5.534998	1.025649	-.2195776	-1	-.1279462	-1	.1758367	-2	.4320189	-3	-.8766187	-4	.00778	3.57665	.08	

TABLE IV. Spectral intervals, empirical absorption coefficients, pressure and temperature correction exponents for 6.3 μm water vapor band

ν (cm^{-1})	C_0	C_1	$C_2 \times 10^n$	n	$C_3 \times 10^n$	n	$C_4 \times 10^n$	n	$C_5 \times 10^n$	n	$C_6 \times 10^n$	n	γ	λ	RMS ERROR % τ	LIMITING CONDITIONS
1960	-3.751299	.8565832	-.5124706	-1	-.5712889	-2	.6208276	-3	.1382761	-3	.1584703	-5	.57	3.2352	.26	
1970	-7.798852	1.073767	.1796094	-1	-.1603197	-1	-.7319673	-3	.1474496	-2	-.2031580	-3	-.02623	1.22953	.03	
1980	-4.930807	.9342666	-.5431239	-1	-.1566033	-1	.2188849	-2	.4415825	-3	-.6789466	-4	.01297	4.40115	.04	
1990	-3.738759	.7910507	-.5532003	-1	-.1938514	-2	.2052361	-2	.4297634	-4	-.6082696	-4	.46406	4.5315	.64	
2000	-6.002226	.9814609	.1174670	-1	-.5961920	-2	-.3472491	-2	.9026121	-3	-.7271331	-4	-.14754	2.8127	.08	
2010	-4.327603	.8784282	-.5462483	-1	-.7393181	-2	.2339617	-2	.2147462	-3	-.7044762	-4	-.3466	5.9185	.42	
2020	-5.700011	.9839282	.1269819	-1	-.6426326	-2	-.2998061	-2	.1030953	-2	-.1045786	-3	-.14493	4.35104	.04	
2030	-8.439707	1.534201	-.2630327	+0	.3532721	-1							-.406	5.79634	.03	
2040	-5.074994	.9706586	-.1924880	-1	-.8288782	-2	-.8757833	-3	.2806208	-3	-.8101925	-6	-.05696	4.739	.05	
2050	-11.53295	2.319683	-.2990429	+0	.4979471	-2	.3509445	-2	-.2408299	-4			.568	3.948	.02	
2060	-4.984385	.9412570	-.1478779	-1	-.1321623	-1	-.1608954	-2	.1473672	-2	-.1779824	-3	.43966	5.4	.22	
2070	-7.469541	1.050980	-.1896076	-1	.1999134	-2	-.1845006	-2	.7103605	-3	-.9540667	-4	.2524	3.309	.09	
2080	-7.418272	1.001302	-.3179280	-2	.1303392	-2	-.2793750	-2	.8373476	-3	-.9467817	-4	.1925	7.007	.02	
2090	-6.203654	.9912012	-.1169829	-1	-.5223104	-2	-.2923888	-2	.7686243	-3	-.4216393	-4	.42512	5.5074	.03	
2100	-7.989391	1.094532	-.1777856	+0	.1132367	+0	-.2903229	-1	.2544588	-2			.13652	2.8407	.01	
2110	-12.62268	1.395346											.095	3.16	.10	
2120	-8.743065	1.276693	-.2774983	+0	.1474123	+0	-.3471644	-1	.2912374	-2			.0564	3.474	.03	
2130	-7.297670	1.144933	-.2155349	+0	.1292404	+0	-.3424728	-1	.3062523	-2			-.154	5.58	.01	
2140	-7.783127	1.335991	-.2871638	+0	.9348258	+0	-.1035395	-1					.07932	7.0453	.02	
2150	-8.808495	1.180073	-.1378444	+0	.7391174	-1	-.1929461	-1	.1764266	-2			.1096	4.7934	.01	
2160	-8.464965	1.201994	-.1828124	+0	.8391135	-1	-.1791054	-1	.1325444	-2			.09056	8.9081	.02	
2170	-9.955573	1.380030	-.285800	+0	.9548358	-1	-.1073846	-1					.04037	3.1193	.01	
2180	-9.100236	1.470646	-.3288353	+0	.1203543	+0	-.1476783	-1					-.07143	7.958	.05	
2190	-12.20607	1.406288											.3428	3.9	.02	
2200	-9.316707	1.477720	-.3449222	+0	.1037735	+0	-.1077963	-1					-.0663	10.261	.10	
2210	-10.13867	1.187353											-.0052	6.669	.03	
2220	-11.77252	1.374533											-.0455	5.347	.03	
2230	-10.20201	1.054434											.1873	8.805	.01	
2240	-10.34802	1.126794											-.1608	8.709	.02	
2250	-9.895746	1.126414											.15464	5.6372	.03	
2260	-11.69888	1.268287											.21604	3.605	.02	
2270	-35.29655	6.315727											1.931	-11.11	.05	
2280	-11.85668	1.165333											3.134	9.74	.01	
2290	-10.07241	.9740010											-.3355	11.776	.02	
2300	-16.45504	1.904020											.52044	-.5015	.02	

TABLE IV. Spectral Intervals, empirical absorption coefficients, pressure and temperature correction exponents for 6.3 μm water vapor band

ν (cm^{-1})	C_0	C_1	$C_2 \times 10^n$	n	$C_3 \times 10^n$	n	$C_4 \times 10^n$	n	$C_5 \times 10^n$	n	$C_6 \times 10^n$	n	γ	λ	RMS ERROR % τ	LIMITING CONDITIONS
2310	-11.56798	1.025496											.1247	8.264	.01	$\tau=0$
2320	-12.93035	1.302551											.1157	7.858	.01	
2330	-20.23708	2.788853											.12472	1.0172	.01	
2340	-19.83727	2.615219											-.1956	5.793	.01	
2350	-14.29588	1.378627											-.1956	5.793	.01	
2360	-18.21139	2.113971											.07984	2.3382	.01	
2370	-19.20545	2.340820											-.104	2.8583	.01	
2380	-12.89249	.9001730											-.665	11.49	.01	
2390	-26.26830	3.858676											-.1177	1.70365	.01	
2400	-23.76723	3.321964											-.03321	2.2152	.01	
2410	-13.13293	.7369810											.116	7.8575	.01	
2420	-55.65472	10.44066											-.06245	.76724	.01	
2430	-109.1294	22.43474											-.10	.767	.01	
2440																
2450	-62.21981	11.82282											-.06245	.76724	.01	

TABLE IV. Spectral Intervals, Empirical absorption coefficients, pressure and temperature correction exponents for 6.3 μm water vapor band

IV. APPLICATION TO THE ATMOSPHERE

The transmittance model described above is applicable to homogeneous temperature and pressure conditions. To apply this model to atmospheric conditions (i.e. varying temperature and pressure along ray path) it is necessary to introduce an equivalent temperature and pressure of the nonhomogeneous medium such that the transmittance of the nonhomogeneous medium is approximately equal to that of a homogeneous medium at equivalent temperature and pressure. An equivalent temperature and pressure may be defined as the mean values for the medium:

$$T_e(u) = \frac{\int_0^u T \, du}{\int_0^u du}, \quad p_e(u) = \frac{\int_0^u p \, du}{\int_0^u du}$$

The transmittance in each spectral interval where there is absorption by both the CO_2 and H_2O in the atmosphere may be approximated by

$$\tau_{\Delta\nu}(u) = \tau_{\Delta\nu}(\text{H}_2\text{O}) \times \tau_{\Delta\nu}(\text{CO}_2).$$

V. CONCLUSION

The polynomial model reported here enables the rapid calculation of atmospheric transmission function in the $4.3\text{ }\mu\text{m}$ CO_2 and the $6.3\text{ }\mu\text{m}$ H_2O absorption bands with a precision sufficient for most atmospheric problems. A relatively small amount of data is required for the calculation of these functions which encompass two important spectral regions in the atmosphere. High spectral and vertical resolution may be realized with the use of this model in some applications.

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