DISSERTATION

WEEKLY CONTROL OF ALPINE SEASONAL RESERVOIR

Submitted by

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In partial fulfillment of the requirements

for the Degree of Doctor of Philosophy

Colorado State University

Fort Collins, Colorado

Spring, 1977

ABSTRACT OF DISSERTATION

WEEKLY CONTROL OF ALPINE SEASONAL RESERVOIR

In the present research a technique was developed to maximize the returns from the operation of a seasonal alpine reservoir for the production of electrical energy. The emphasis rests on a comprehensive approach to the problem, and the following fields were considered: hydrology, power economics, operation research and decision theory.

Flow forecast can markedly improve the efficiency of reservoir operation. The hydrology of the concerned alpine watershed was thoroughly analyzed, in order to develop a runoff forecast model. Linear relations were established between the flow to be forecasted and precipitation, amount of water stored in the watershed, sum of degree days and annual change in glaciated area. This procedure performed well for runoff forecasts for periods exceeding three months. For periods of shorter duration, however, forecast based on the antecedent flow conditions performed best.

Looked after are the successive weekly releases which maximize the returns resulting from the operation over a specified period of a reservoir of a given size. The return function depends on the price of the produced energy. To take into account implicitly the variations with time of the demand, the energy price was assumed to vary during the week exponentially with the weekly amount of release. Furthermore a monthly price variation was superimposed on the weekly price variation.

The solution technique to determine the optimal releases strategy was first developed for the deterministic case. It is based on the solution of the system of equations given by the

Kuhn-Tucker conditions. As the direct solution of this system of equations was complicated, the following alternative approach was devised. The operation period was divided into two parts, the drawdown phase, and the refill phase. For each of these phases, the system of equations given by the Kuhn-Tucker conditions was solved by successive approximations, the physical constraints being first ignored, and then introduced in a stepwise way. Finally the two phases were linked together and the optimality of the solution checked. The advance knowledge of the approximate nature of the optimal releases sequence allowed to reduce to a minimum the guesswork and the number of iterations necessary to arrive at the optimum.

Stochastic reservoir operation was solved by introducing the notion of expected future return of storage, developed by Masse. A relation could then be established between this variable and some relevant hydrologic variables. With this approach, it is possible to take into account the magnitude of the forecasted inflow in the decision process.

The application of the developed technique to a seasonal reservoir fed by an alpine watershed showed that the method was both feasible and attractive. By taking into account the properties of the optimal solution, it was possible to reduce substantially the amount of computation.

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ACKNOWLEDGMENTS

To do a doctorate research work in Switzerland, while most of the committee members are in the United States is not an easy task. The work could finally be brought to an end only because all the persons involved fully cooperated.

Special thanks are due to Dr. Hubert J. Morel-Seytoux, the advisor, whose help, criticism and suggestions were determinant in the accomplishment of this work. Furthermore, the author appreciates the time he could spend with Professor Morel-Seytoux during his trips to Europe.

The writer expresses sincere appreciation to Professor Themistocles Dracos, Head of the Institute of Hydromechanics and Water Resources Management, Federal Institute of Technology, Zürich. Professor Dracos provided for the financial support for this research, which is gratefully acknowledged, and gave helpful suggestions during the course of the investigations.

The writer would also like to thank Dr. Maurice Bryson, Associate Professor of the Statistics Department, Dr. Neil S. Grigg, Associate Professor and Dr. Thomas G. Sanders, Assistant Professor, Department of Civil Engineering, who served on the graduate committee, for their valuable criticism and comments. Finally the author extends his thanks to Dr. Vujica Yevjevich, who was chairman at the writer's preliminary examination, and Dr. Petar Todorovic, who served on the committee for the same examination.

It is impossible to list all the persons who contributed to this research work. However, the author would like to mention Mr. Pierre Guillot and Robert Grard, Electricite de France, Grenoble, who initiated the author to Masse's work. The author is grateful to Electro Watt, Consultant Engineers, Zürich, who granted the author a leave of absence during the critical phase of this study, supplied useful information on Swiss energy problems, and gave, when necessary, moral support.

The author would also like to thank his wife, Ana-Maria, his family and friends, for their patience, understanding and encouragement during the fulfillment of this work.

TABLE OF CONTENTS

Chapter		Page

Chapter		Page

Chapter		Page

LIST OF TABLES

Table		Page

LIST OF FIGURES

Figure		Page

Figure		Page
5.11	Relation between Future Marginal Return of Storage and Inflow, Given by Least Squares Fit: Initial Date, July 1; Initial Storage, 23.963 Millions M^3	237
5.12	Future Marginal Return of Storage, Distribution Curve: Initial Date, April 1, Initial Storage, 7.569 Millions M^3	249
5.13	Relation between Future Marginal Return of Storage and Summer Inflow, Given by Least Squares Fit: Initial Date, April 1; Initial Storage, 7.569 Millions M^3	250
5.14	Relation between Future Marginal Return of Storage and Inflow During April, Given by Distribution Curve: Initial Date, April 1; Initial Storage, 7.569 Millions M^3	251
5.15	Relation between Future Marginal Return of Storage and Inflow During April, Given by Least Squares Fit: Initial Date, April 1; Initial Storage, 7.569 Millions M^3	252
5.16	Future Marginal Return of Storage, Distribution Curve: Initial Date, April 1; Initial Storage 4.569 Millions M^3	253
5.17	Relation between Future Marginal Return of Storage and Summer Inflow, Given by Least Squares Fit: Initial Date, April 1; Initial Storage, 4.569 Millions M^3	254
5.18	Relation between Future Marginal Return of Storage and Inflow During April, Given by Distribution Curve: Initial Date, April 1; Initial Storage, 4.569 Millions M^3	255
5.19	Relation between Future Marginal Return of Storage and Inflow During April, Given by Least Squares Fit: Initial Date, April 1; Initial Storage, 4.569 Millions M^3	256
5.20	Future Marginal Return of Storage, Distribution Curve, Initial Date, April 1; Initial Storage, 1.569 Millions M^3	257

Figure		Page

Figure | Page

LIST OF SYMBOLS

Symbol	Description
P	Annual precipitation index
$P_{i,j}$	Cumulated precipitation index from i^{th} month to j^{th} month
T	Mean daily temperature in degree Celsius
T_d	Sum of positive degree-days over a given period
D	Average rate of snowmelt, mm per day
Q	Annual runoff, in millions m^3
$Q_{i,j}$	Cumulated runoff from i^{th} month to j^{th} month, in million m^3
Q_i	Runoff during month i
q_i	Runoff during i^{th} week of the hydrologic year
A	Catchment area, in Km^2
$A_{g\ell}$	Area of the catchment covered with glaciers
Y	Dependent variable in regression equation
X_i	i^{th} independent variable in regression equation
a	Intercept in regression equation
b_i	i^{th} regression coefficient
r	Correlation coefficient
R	Multiple correlation coefficient
R^2	Coefficient of multiple determination
$\hat{Y}$	Value of the dependent variable computed on the basis of the regression equation

Symbol	Description
ε	Residual
a_o	Intercept for the forecast regression equation
S	Storage capacity of reservoir, in millions m^3
s_i	Reservoir content at the beginning of the i^{th} week, in millions m^3
h	Head on the turbines in m
e	Energy rate function in KWh/m^3
x_i	Release in the i^{th} week of the year in millions m^3
$Pr(x)$	Price of one KWh as a function of the weekly release, in cents
Pr_{monthj}	Average monthly price of one KWh
z	Dummy variable
$B(i,s_i,x_i)$	Total return from a weekly release
V	Total return from reservoir operation over a given period
$\alpha_i(\bar{s}_i)$	Maximum marginal return of a release for a given week and reservoir content
n	Number of decision variables
m	Number of constraint equations
$g_j(\ldots)$	j^{th} constraint equation
$L(\ldots)$	Lagrangian form
k	Index of the week when reservoir operation is started
λ_j	Lagrange multiplier for water balance equation

Symbol	Description
β_j	Lagrange multiplier for release constraint equation
γ_j	Lagrange multiplier for storage constraint equation
ℓn	Natural logarithm
ℓ	Index of the week when reservoir is empty
λ_ℓ^d	Marginal return of the release during the drawdown phase, if the reservoir is empty in the ℓ^{th} week
λ_ℓ^f	Marginal return of the release during the refill phase, if the reservoir is empty in the ℓ^{th} week
$\lambda_{\ell,i}^d$	Marginal return of the release of the i^{th} week, if the reservoir is empty in the ℓ^{th} week
$\lambda_{\ell,i,j}^d$	Marginal return of the releases from the i^{th} to the j^{th} week, if the reservoir is empty in the ℓ^{th} week
p_i	Probability of occurrence of i^{th} event
$E(X)$	Expected value of the random variable X
$Var(X)$	Variance of the random variable X
$\sigma(X)$	Standard deviation of the random variable X
$P(X)$	Probability distribution of the sample data of X
$F(X)$	Probability distribution fitted to the sample data of X
$N(\bar{x},\sigma)$	Normal probability distribution with mean $\bar{x}$, and standard deviation σ
λ_{k,s_k}^d	Future marginal return for a given storage on a given date, drawdown phase

Symbol	Description
λ^f_{k,s_k}	Future marginal return for a given storage on a given date, refill phase
a_d, b_d	Constants of the equation relating inflow to the logarithm of the future marginal return of storage, drawdown phase
a_f, b_f	Constants of the equation relating inflow to the logarithm of the future marginal return of storage, refill phase
$\lambda^f_{k,s_k,min}$	Limiting value of the future marginal return of storage, refill phase
$\lambda^f_{k,s_k,max}$	Limiting value of the future marginal return of storage, refill phase
$Q^d_{k,\ell-1}$	Cumulated inflow from the k^{th} to $(\ell-1)^{th}$ week, drawdown phase
$Q^f_{k,52}$	Cumulated inflow from the k^{th} to the end of the water year refill phase
$Q^f_{k,52,limit}$	Minimum inflow for which the reservoir gets empty

Chapter 1

INTRODUCTION

The Second World Congress on Water Resources was held in New Delhi in December 1975. A rapid glance through the addresses and articles of invited personalities and a quick look through the delivered papers indicate how vital for mankind an adequate supply of water is and how involved and difficult the management of this resource can be.

Although water is used for different purposes, the problems to be solved remain more or less the same. It consists in meeting the human needs for water in an optimal way. This problem is not trivial because both the availability of water and the demand for water vary throughout the year. Furthermore these variations are most often not in phase and partially random. Hence the present research will restrict itself to the problem of optimal use of water for energy production, but the methodology followed remains quite general and applies to many water resources problems.

At the Second World Congress on Water Resources, about 600 papers by experts from 43 countries were received for consideration, and many other congresses were held in 1975 on this topic. This shows how active research is but also raises the question of the utility of starting a new research work in this field.

A thorough literature review in the field of water resources reveals that the related research went through different stages and concerned itself with different areas. These activities occurred either concurrently or one after the other, but rarely they happened one with the other. Seldom the information and experience gained from one stage or field were taken into account to solve problems of the next stages or of the nearby fields. Hence there is a need today for a global approach to water resources problems and for an integration of the knowledge existing today. And this is the purpose of the present research.

The first researchers directed their efforts towards the determination of design criteria for reservoirs built for well specified purposes. Only recently they have started to look into the problem of the operation of these reservoirs. Furthermore, they based at the beginning their computations on the assumption that the inflows sequences are known in advance. Today, one begins to give a greater attention to the stochastic nature of the inflows to the reservoirs. These different aspects or stages of the basic problem are closely related to each other. However researchers rarely took this fact into account.

Water resources problems touch many areas. One can, however, distinguish two main fields. The first one deals with the availability of water, and the second one, with the determination of the returns produced by the use of water. These last years, hydrology and operation research have experienced tremendous developments and very sophisticated methods have been produced. But seldom advanced techniques from the two fields can be used concurrently with great efficiency; rarely the pieces of the puzzle fit together.

If one considers the field of operation research for itself, the same situation appears. Sophisticated mathematical tools exist but they cannot be used in practical cases because they lead to cumbersome computations, or because the very nature of the problem under consideration precludes their application. On the other side, real world problems are often much constrained and present special features which allow rather fast to obtain at least an approximate solution of the problem.

These facts induced the author to start a research work where all the aspects of a water resources problem would be considered. The aim is then to shape each relevant element in such a way that it fits optimally in the puzzle represented by the problem to be solved.

The problem studied here deals with the optimal control of an alpine seasonal reservoir for the production of electrical energy. The dimensions of the reservoir, the characteristics of the power scheme and the inflow pattern are given. The task is to determine the successive releases from the reservoir in such a way that the annual returns resulting from the reservoir operation are maximized.

The solution of real-world optimization problems quite often requires computations-stages, where trial and error approaches must be used. To avoid these stages, we shall try to simplify the complicated relations which relate inflows to optimal releases, in such a way that they can be solved algebraically. This can be achieved by introducing in these relations the characteristics of the optimal solution. This procedure will be applied for deterministic reservoir operation as well as for stochastic reservoir operation.

Before starting this research work, the author had many discussions with reservoir operators working for Swiss public utilities. These discussions indicated that there was a real need to study this problem in a systematic way. Also the influence of the randomness of the inflow on reservoir operation should be considered, as well as the potentialities of runoff forecasts.

This study is also challenging because it gives the opportunity to integrate the different fields concerned: meteorology, glaciology, hydrology, hydroelectric power technology, economics, statistics and operation research. Also special efforts will be devoted to the identification of those elements which give some indications about the nature of the optimal solution.

The very nature of the problem under consideration supplies the structure of the outline. The second chapter deals with the hydrology of the contributing watershed, and the third one defines the objective function. In the fourth chapter, the methodology for reservoir operation is derived, assuming that the complete inflows sequence is known in advance, while the fifth chapter shows the extension of the derived methodology to the cases of stochastic reservoir operation with and without flow forecast.

Most of the reservoirs recently built in Switzerland lie at an elevation exceeding 1600 m (5250 ft) which implies that they are all located in the Alps. The hydrology of these catchments is intricate as rain, snow, ice and temperature are involved in the runoff producing phenomena. The lack of reliable data further complicated the hydrologic analyses. Nevertheless runoff forecast models for durations equal or exceeding three months could be established. For this purpose, linear regression equations were determined which relate the runoff to be forecasted with the antecedent precipitation and with the antecedent runoff. Unfortunately, due to the relative large amount of summer rain, the reliability of these forecasts is low. Better results were obtained for the forecast of runoff for periods of shorter duration, using the Markov property of the flows.

The power market in Switzerland has features of its own. Striking is the existence of many independent utilities which follow their own pricing policy. Yet it was all the same possible to establish a realistic function which relates the unit energy price to the weekly release from the reservoir. This function, an exponential curve, which takes into account the hourly, weekly and monthly variations in the energy price, leads to a nonlinear objective function.

Deterministic reservoir operation was studied first. Besides the fact that the annual water balance equation must be satisfied, two other types of physical constraints may apply: in any week, no water can be released if the reservoir is empty, and no water can be stored if the reservoir is full. Hence the problem consists in optimizing a nonlinear objective function subject to linear constraints. The system of equations given by the Kuhn-Tucker conditions was too complicated to be solved directly. Consequently, the following alternative approach was devised. The annual operation of the reservoir was divided into two partial operation periods, the drawdown phase and the refill phase. For each of these phases, the system of equations given by the Kuhn-Tucker conditions was derived, ignoring the physical constraints of the system. This way of doing led to simple functional relations between the optimal release of any given week and the inflow recorded from the beginning of the same week to the end of the considered phase. In a second step, the first ignored constraints were progressively introduced in each operation phase and the releases strategy, corrected if necessary. Finally, the two phases were linked together and the optimality of the computed solution checked. Computational experience confirmed that this procedure leads lastly to the desired solution.

The notion of future marginal return of storage developed by Masse enabled an easy transition from deterministic to stochastic reservoir operation. For, according to Masse, the instantaneous marginal return of the optimal weekly release must be equal to the expected future marginal return of the storage. A systematic analysis of the computer runs performed in the deterministic case indicated that, for a given date and reservoir content, a simple relation exists between the inflow recorded from the given date to the end of the considered phase, and the corresponding future marginal return of storage. The determination of this relation permitted to compute the future marginal return of storage for any magnitude of the relevant inflow, and consequently the expected value looked after. With the derived relation, it was also possible to incorporate the runoff forecast into the decision process.

The last chapter summarizes the research work, deals with the evaluation of the derived solution technique and shows the possible fields of application.

The results of this research indicate quite clearly that complex looking problems can be solved with simple mathematical tools, provided that one tries to understand the basic underlying mechanisms. In these last years, too much emphasis has been given to theoretical and computer oriented solutions. One should again in the future, stress and consider the importance of the physical nature of the problem.

Chapter 2

HYDROLOGY

The optimization technique to be used to control a reservoir depends greatly on the hydrologic characteristics of the related drainage area. Hydrology determines magnitude and succession of the reservoir inflows. It comes in at the planning stage, for the selection of the optimal reservoir capacity, as well as during the operation period, for the determination of the optimal releases sequence. Consequently it must be studied first.

To put the problem into its natural background, Chapter 2 begins with a short description of the physical geography of Switzerland. This part is followed by the presentation of the criteria used to select the test watershed. Then the physical elements involved in the runoff process are analyzed separately. The chapter ends with the establishment of water balance and prediction equations for the test watershed.

2.1 Physical Geography of Switzerland

2.1.1 Topography. Switzerland covers 41,300 sq km (16,000 sq mi), that is a little more than one-sixth of the area of Colorado and can be divided into three main regions (Fig. 2.1). Two of them, the Jura and the Alps are mountainous, the third one, the so-called Midlands, consists of alluvial plains and hills. The Jura forms the border with France in the western and the northern part of the country; it covers one-sixth of the territory with peaks between 1,000 and 1,700 m (3,300 and 5,000 ft respectively). The Alps divide Switzerland into two parts; they extend from west to

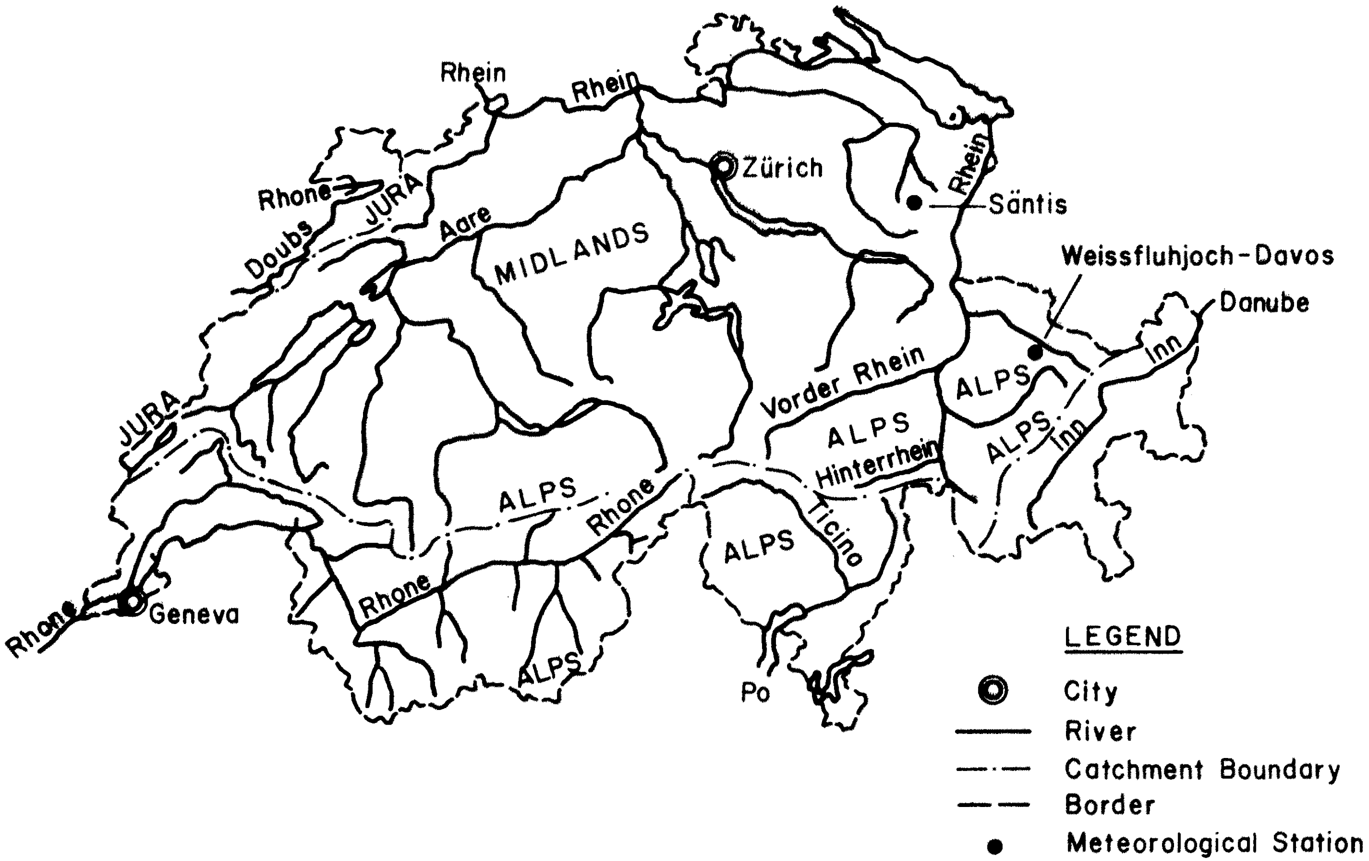


Fig. 2.1 Switzerland and its Rivers

east over about half of the country. They show quite a few peaks exceeding 4,000 m (13,000 ft). The Midlands lie in between, on the northern side of the Alps, at an elevation between 300 and 600 m (1,000 and 2,000 ft respectively).

2.1.2 Climate. The vast mass of the Alps and the related variations in altitude and in exposure account for the unusual diversity of the climate. On the whole it is continental. The average annual temperature stays around 8.5° Celsius (47°F) at 500 m (1,640 ft); at 2,500 m (8,200 ft) it drops to -1.8° Celsius (29°F). In summer some regions enjoy a very mild climate but the perennial snow is encountered between 2,500 and 2,800 m (8,200 and 9,200 ft respectively) and about 1,500 sq km (600 sq mi) are covered by glaciers.

The average annual precipitation for the whole country amounts to about 1,500 mm (59 in.); its areal distributions are much affected by the presence of the Jura and the Alps. However throughout the year, the precipitation is roughly evenly distributed. The minimum monthly values are recorded in winter, the maximum ones in summer.

2.1.3 Hydrography. The waters in Switzerland flow to four main river systems. The most important one is the Rhine River Basin which drains 68 percent of the area, then comes the Rhone River Basin with 18 percent, followed by the Pô River Basin with 9 percent and finally the Danube River Basin with 5 percent (Fig. 2.1). The average annual runoff depth equals 1,030 mm (41 in.). This leads to a runoff coefficient of 0.70.

The annual flow pattern is related, among others, to the annual precipitation cycle and to the snow and icemelt phenomena.

Parde (1955), recognizing this fact, has classified river regimes with appreciable snow cover influence on the basis of a coefficient which represents the percentage of the warm season flow contributed by meltwater. For Switzerland he arrived at the five following subdivisions: "Pluvio-nival," "nivo-pluvial," "transition to nival," "pure nival," to "nivo glacial," and "glacial" regime. In the "pluvio-nival" regime the computed coefficient ranges between 6 and 14 percent, in the "glacial" regime it exceeds 51 percent. As most storage reservoirs in Switzerland are located at elevations exceeding 1,500 m (4,900 ft) only the last three regimes are of interest.

2.2 Description of the Selected Watershed

2.2.1 Selection criteria. The watershed selected to test the optimization method must satisfy some criteria. First its hydrology must be similar to that of the basins where most of the seasonal storage reservoirs were erected. In Switzerland these basins belong either to the subalpine or to the alpine range. Alpine here means situated above the tree line. Second, the basin must be homogeneous in order to allow reliable runoff forecasts. Third, and most important, a sufficient amount of runoff, rainfall, temperature, and topographical data must be available.

This criterion caused the greatest problems as the flows of many rivers have been modified during the last decades by the construction of diversions. All the drainage basins of the existing reservoirs had to be eliminated, mainly because the available runoff series were too short or the virgin flows were difficult to reestablish.

Consequently a basin had to be selected without an existing reservoir: Hinterrhein at Hinterrhein.

Concerning the third criterion, the following remarks are of interest. The gauging station of Hinterrhein has been in operation since 1945 and the records are rated as good. Besides the precipitation gauges located in the main and in the nearby valleys a few totalizers were established about forty years ago at high elevation. Also snow courses have been taken regularly since the early sixties. Finally some topographical data on the glaciers of the watershed have been collected since the end of the nineteenth century. Hence from the point of view of availability of data the situation seems to be quite good.

2.2.2 Physical characteristics of the Hinterrhein watershed at Hinterrhein. One usually considers the Hinterrhein River as the actual origin of the Rhine River. The catchment of the selected gauging station covers 53.7 sq km (20.66 sq mi) of which 17.3 percent were covered by glaciers in 1962. Its drainage area is comprised between latitude north 46° 28' and 46° 34' and longitude east 9° 02' and 9° 13' (Fig. 2.2).

The highest point, the Rheinwaldhorn, exceeds 3,400 m (11,150 ft) while the gauging station lies at 1,583 m (5,194 ft) close to the northern entrance of the San Bernardino Tunnel which joins the northern with the southern part of Switzerland. According to the hypsographic curve (Fig. 2.3) the average elevation amounts to 2,380 m (7,808 ft) and more than 70 percent of the catchment lie below 2,600 m (8,530 ft).

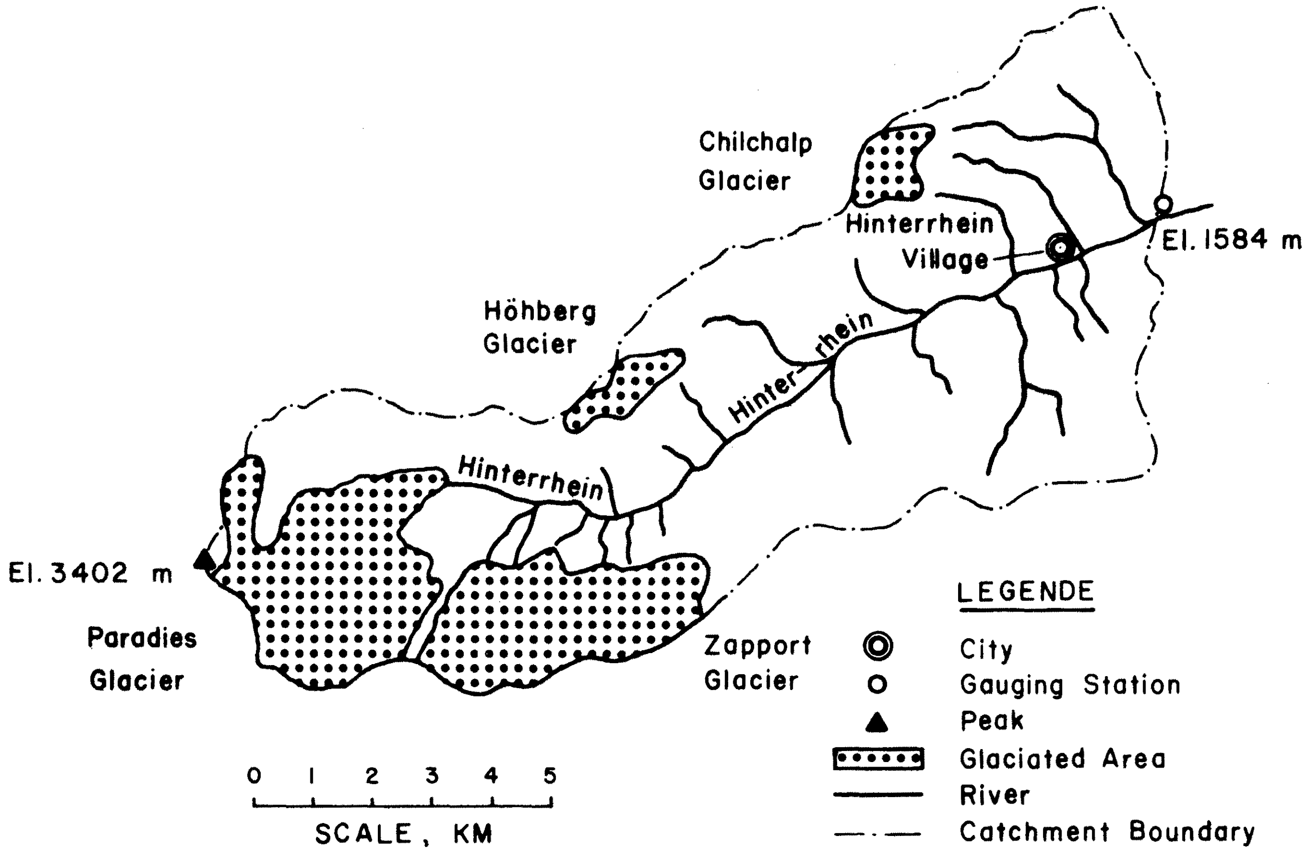


Fig. 2.2 Catchment of Hinterrhein at Hinterrhein
(after National Topographie Map)

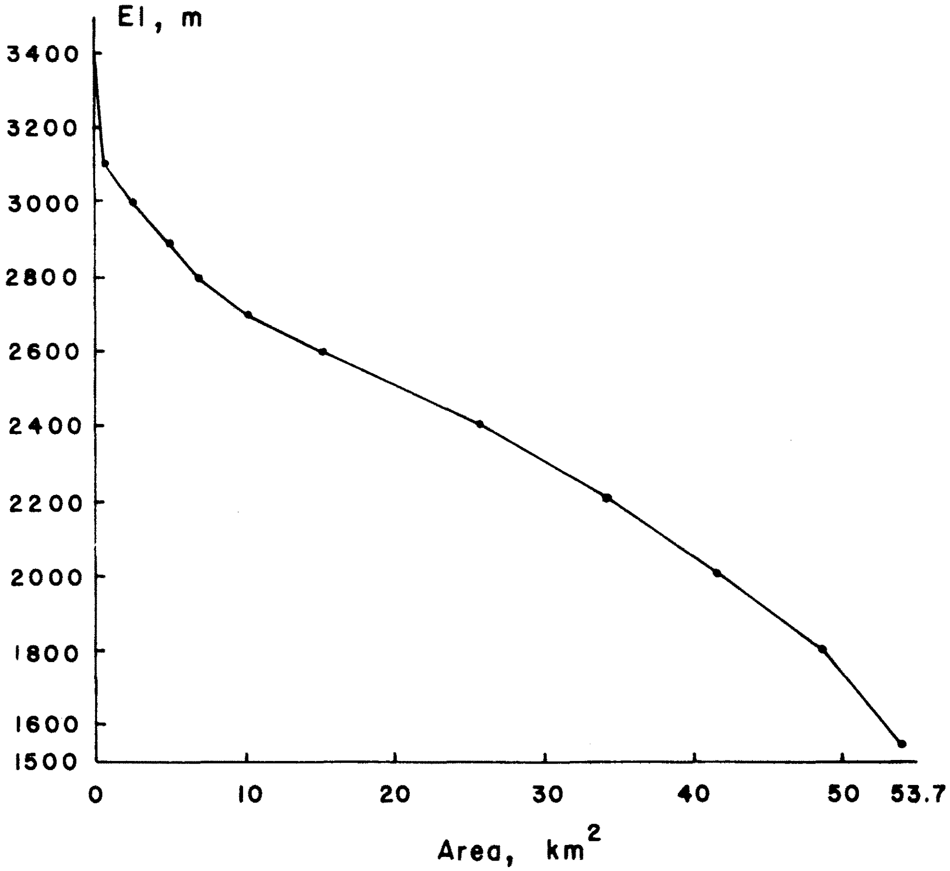


Fig 2.3 Hypsographic Curve

The river itself originates at about 2,400 m (7,874 ft) at the lower end of the Paradies Glacier. It then flows with a gross stream channel slope of 4.7 percent (Widmoser, 1974) in a steep and narrow valley. Towards the lower end of the drainage basin the channel slope decreases and the valley widens.

The vegetation is of the alpine type. Although small woods can be found near Hinterrhein Village most of the catchment is covered with grass, isolated trees, and bushes. As the elevation increases the trees and later on the bushes progressively disappear. At the highest elevations even grass becomes sparse; there the slopes are very steep and rocks prevail.

Quite a few glaciers exist in the Hinterrhein catchment. The two most important ones are the Paradies and the Zapport Glacier. The Paradies Glacier, the actual source of the Rhine, occupies the upper end of the valley; the Zapport Glacier extends over the highest reaches of the right side and on the left side there are only a couple of glaciers of smaller extent. In 1962 these glaciers covered a little more than 9 sq km (3.46 sq mi). Their average elevation amount-d to about 2,850 m (9,350 ft).

2.2.3 <u>River regime</u>. It is typically alpine or according to Pardé's classification "nivo-glacial." The precipitation is evenly distributed throughout the year but the runoff is mainly concentrated during the summer months. This shows the importance of temperature. Table 2.1 gives the location of the 0° Celsius isotherm and the lower limit of the snow cover during the year for regions situated on the northern side of the Alps (Lugeon, 1928).

Table 2.1

Average Monthly Elevation of Isotherm Zero and of Lower Limit of Snow Cover

Month	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
Isotherm Zero	2350	1300	-	-	700	1150	1900	2550	3100	3700	3650	3350
Lower Limit Snow Cover	1650	1300	650	600	600	750	1000	1400	1950	2450	2700	2350

Elevations are given in m

From October to March precipitation is stored as snow and ice and the resulting runoff is small. In April the isotherm 0° Celsius reaches the lower area of the watershed and snowmelt is started; it reaches its maximum rate in June. Glaciermelt starts later and is responsible for the high flows registered in July, August and even in September. Finally the summer rains also contribute to the runoff. Hence, the total summer flows result from the superposition of three different phenomena.

The maximum annual flow equals 1.7 times the minimum annual flow for the period with available records. The seasonal distribution of flows is even more remarkable. Winter runoff barely reaches 14 percent of the annual total. The minimum occurs in February. From that moment on runoff increases until July or August when the daily peaks can be quite high. Then starts the recession which ends in February.

2.3 Precipitation

2.3.1 General pattern. Cyclonic type precipitation brings the greatest amount of moisture into the area under study. Most often these air masses originate either from the Atlantic Ocean or from the Mediterranean Sea. However the resulting precipitation within the catchment itself is small because the Hinterrhein Valley lies across the most frequent wind directions and because it is sheltered from the same winds by high mountain ranges. As cyclonic type precipitation covers in general large areas, good correlations should exist between the records of nearby stations.

With increasing elevation a larger percentage of precipitation falls as snow. Lliboutry (1965) indicates that at 1,500 m (4,900 ft), 40 percent of the annual precipitation falls as snow, at 2,000 m (6,500 ft), 60 percent, at 2,500 m (8,200 ft), 75 percent, at 3,000 m (9,800 ft), 90 percent and finally at 3,500 m (11,500 ft), 96 percent.

This is important for two reasons. First it is well-known that the reliability of a precipitation gauge decreases as the amount of catch from snow increases; generally such gauges tend to underestimate the real precipitation. Hence here, records coming from stations located at high elevation should be considered with care. Second, the percentage of precipitation fallen as snow influences greatly the magnitude of the terms appearing in the energy balance equation.

Many researchers (see bibliographies given by Kubat, 1972 and Havlik, 1969) have studied in the last thirty years the relations existing between elevation and precipitation. Some of them even tried to prove the existence of an altitude above which precipitation should decrease. The lack of gauges at high elevations complicated the solution of the problem. However today the idea of an elevation with maximum rain has been abandoned and it is generally admitted that precipitation increases roughly linearly and constantly with elevation. The process of rain formation and the available records justify this conclusion. For orographic effect plays a most important role in cyclonic type precipitation. On his side Havlik (1969) studied thoroughly the existing data of rain gauges of the Alps. He showed that the variations in mean annual rainfall between stations are directly related to the corresponding elevation changes. Yet Havlik does not exclude the existence of local conditions which might infirm his general law.

The data series of the recording gauge of Hinterrhein Village shows an annual average of 1,732 mm precipitation (68 in.) for the period 1901-1940. It also indicates that summer is the most frequent wettest season followed by fall and spring. Winter is the driest one. The relevant figures appear in Table 2.2.

Unfortunately the highest recording rain gauge of the valley is located at the lower end of the catchment under study which shows the importance of the above analysis of the effects of elevation on rainfall.

2.3.2 Analysis of data. The Hinterrhein Valley is particularly well equipped with recording rain gauges: in the catchment itself, Hinterrhein Village; further downstream, Splügen and Andeer; and in two nearby valleys, Vals, Inner-Ferrera and Avers am Bach. These six stations provide for a good regional coverage. Furthermore six non-recording gauges were installed at high elevation. Location and characteristics of these stations can be found in Table 2.3 and in Fig. 2.4.

Of the six selected recording precipitation gauges only two, Splügen and Hinterrhein, present gaps in their records. The missing values were determined by linear regressions with the four remaining stations with complete series. The obtained correlation coefficient for annual precipitation exceeds in both cases 0.92 for a sample equal or greater than eighteen. The major computed statistical parameters (mean, variance and coefficient of variation) appear in Table 2.4.

Table 2.2

Average Monthly and Quarterly Precipitation of Hinterrhein

A. Monthly Averages
1901 - 1940

Month	Oct	Nov	Dec	Jan	Feb	Mar	April	May	June	July	Aug	Sept
Precipitation	201	151	98	68	77	131	141	161	160	188	185	171

Precipitation is given in mm

B. Quarterly Averages
1901 - 1940

Period	Oct to Dec	Jan to March	April to June	July to Sept	Total
Precipitation	450	276	462	544	1732

Precipitation is given in mm

Table 2.3

List of Selected Precipitation Gauges

A. Recording Gauges
1945 - 1974

Station Name	Elevation m	Years With Incomplete Records
Vals	1290	-
Hinterrhein	1619	1949, 52, 53, 1962-68
Splügen	1460	1968, 69
Avers	1960	
Innerferrera	1475	
Andeer	980	

B. Totalizers
1945 - 1973

Station Name	Elevation m	Years With Missing Records
Gemskanzel-Rheinwaldfirn	2916	1945, 47, 52, 53, 54
Aelpetlistock	2393	-
Muotaulta-Annarosa	2800	1964
Crapet Prassignola	2650	1945, 59
Piz Curvèr	2810	-
Piz Scalotta	2965	1966, 67

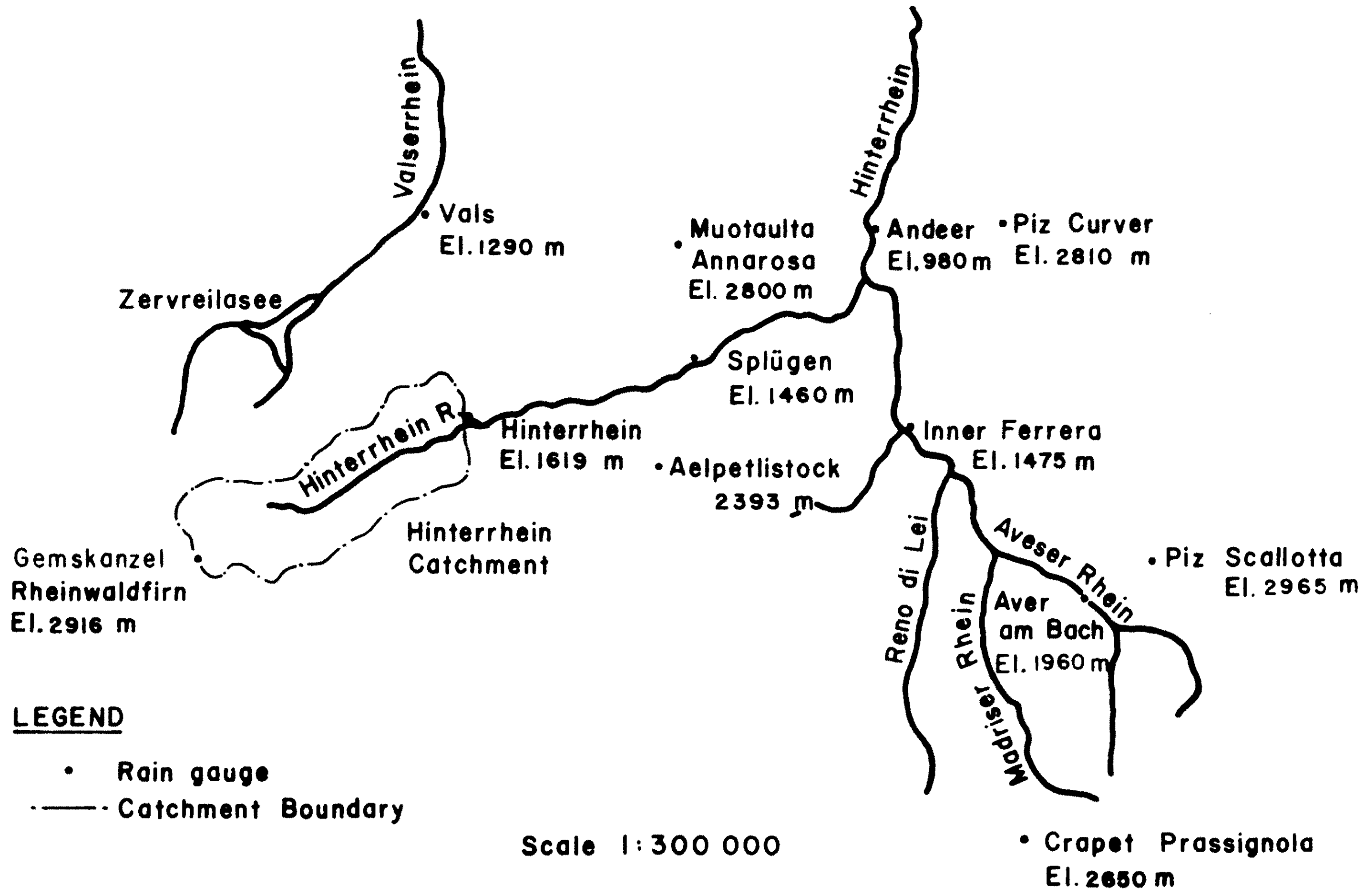


Fig. 2.4 Location of Selected Raingauges

Table 2.4

Annual Precipitation: Main Statistics

A. Recording Gauges
1945 - 1974

Station Name	Elevation m	Mean mm	Stand. Dev. mm	Coeff. of Var.
Andeer	980	931	159	0.171
Vals	1290	1050	191	0.182
Splügen	1460	1239	226	0.182
Innerferrera	1475	1199	219	0.183
Hinterrhein	1619	1610	311	0.193
Avers	1960	1031	165	0.160

B. Totalizers
1945 - 1973

Station Name	Elevation m	Mean mm	Stand. Dev. mm	Coeff. of Var.
Aelpetlistock	2393	1695	341	0.201
Crapet	2650	1476	252	0.171
Muotaulta-Annarosa	2800	1395	227	0.163
Gemskanzel-Rheinwaldfirn	2916	1417	328	0.231
Piz Curvèr	2810	1068	219	0.205
Piz Scalotta	2970	1355	243	0.179

The performed analyses call for the subsequent comments. The available records are representative and reliable. The high correlation coefficients existing between the records of the selected rain gauges confirm that the areal precipitation distribution is homogeneous. Also the influence of elevation on the annual precipitation becomes apparent. However the available data show the complexity of the problem (Table 2.4). The trend is quite obvious for Andeer, Splügen, and Hinterrhein but it is not at all evident for Andeer, Inner-Ferrera and Vals. Hence, it seems difficult to extrapolate from these data what is really happening in the higher ranges of the watershed.

The nonrecording gauges might perhaps give a hint in this respect. For four of them the long-term annual average lies around 1,400 mm (55 in.) while for the other two it is markedly different. This difference may result from local factors and it was decided to eliminate the gauges of Aelpetlistock and Piz Curver (Table 2.4). Although the four remaining stations behave identically on the average, substantial differences were registered in single years so that the annual correlation coefficients between stations dropped to 0.74.

The low recorded water depths are striking. Even if one assumes that they are off by about 20 percent the real value would then lie around 1,800 mm. This would imply a very small increase from the 1,732 mm of Hinterrhein. As a summary, the information coming from nonrecording gauges is not very reliable and cannot be used here. It supplies, however, qualitative indications.

2.3.3 Precipitation index. What procedures should be used to determine the areal precipitation depth of the catchment? Hinterrhein Village, the highest recording station in the valley, lies at the lower end of the catchment. According to the trend prevailing in the Alps the areal precipitation should be greater than the value recorded at Hinterrhein Village, but by how much?

In 1949 Uttinger published an isohyetal map for Switzerland. It gives for the upper ranges of the catchment under study values exceeding 2,500 mm a year. The data registered after the publication of this map, especially those of Gemskanzel, contradict these figures. Hence this map is of no use here. Furthermore the existence of gaps in its record series does not allow the selection of Hinterrhein Village as a bench station for the estimation of areal rainfall.

The solution consists of abandoning the idea of evaluating the areal rainfall which was replaced with another variable--the precipitation index. By definition it is the arithmetic mean of the values recorded at the six selected rain gauges. The number of considered stations is large enough to eliminate local influences and to set forth regional trends. Unfortuantely this approach implies that in the subsequent analyses no water balance equation can be established for the concerned watershed.

According to Tables 2.5 and 2.6 the annual precipitation index varies between 715 and 1,584 mm with an average of 1,163 mm. The coefficient of variation is of the same order of magnitude for the different periods except for the summer quarter which experiences the greatest variations. Figure 2.5 shows the annual precipitation index as a function of time and the related five-year moving average scheme. No major trend could be detected. Finally, the normal distribution fits well the recorded annual, semiannual and quarterly

Table 2.5

Precipitation Index: Main Statistics

1945 - 1974

Period	Oct to March	April to June	July to Sept	April to Sept	Oct to Sept
Mean	406	342	416	757	1163
Stand. Dev.	121	60	157	178	196
Coeff. Var.	0.298	0.176	0.378	0.236	0.169
Min.	204	194	181	375	715
Max.	666	484	807	1096	1584

Precipitation is given in mm

Table 2.6

Precipitation Index: Quarterly and Annual Values

1946 - 1974

Year	Oct to March mm	Apr to June mm	July to Sept mm	April to Sept mm	Oct to Sept mm	Annual 5 Years Mov. Aver. mm
1946	459	484	439	922	1381	-
47	308	312	391	704	1011	-
48	340	339	415	754	1094	1075
49	204	307	204	511	715	1085
1950	424	371	378	751	1175	1132
51	666	353	410	764	1430	1119
52	431	321	496	817	1248	1241
53	374	300	353	653	1027	1201
54	321	423	579	1002	1323	1164
55	451	298	229	527	978	1123
56	274	317	655	972	1246	1169
57	244	358	437	795	1039	1081
58	379	353	525	879	1258	1202
59	510	194	181	375	885	1189
1960	488	286	807	1096	1584	1178
61	654	319	207	526	1180	1184
62	416	298	269	568	984	1225
63	311	413	561	975	1286	1177
64	536	315	241	555	1090	1173
65	339	263	744	1006	1345	1279
66	312	378	470	848	1160	1304
67	612	393	512	905	1516	1312
68	528	416	464	879	1407	1266
69	403	432	296	729	1132	1242
1970	476	289	349	638	1114	1123
71	433	332	276	608	1041	1054
72	289	354	282	634	923	1046
73	220	391	447	840	1060	-
1974	362	295	435	732	1094	-

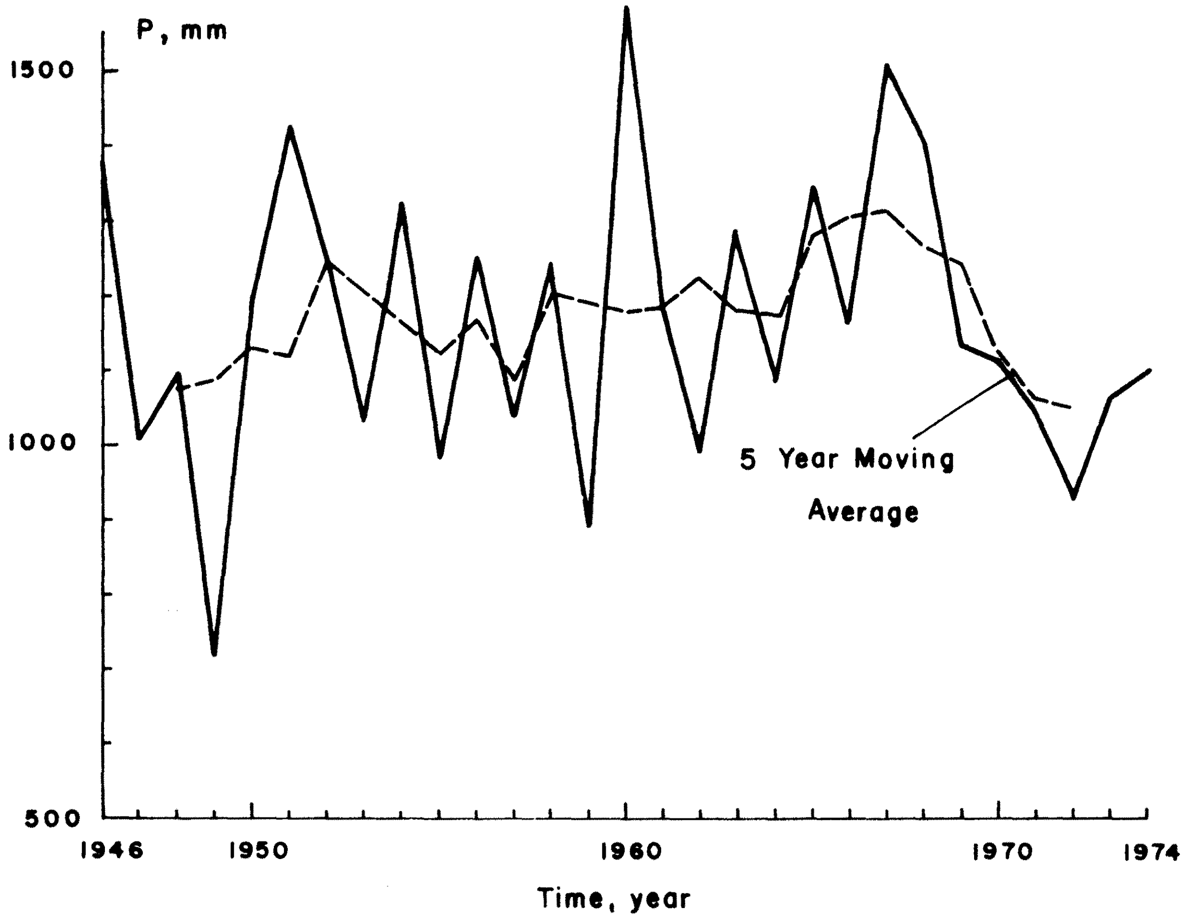


Fig. 2.5 <u>Precipitation Index: Time Series Plot of Annual Values</u>

values (Figs. 2.6, 2.7 and 2.8). Statistically these variables can also be considered as independent from one another (Table 2.7).

As a conclusion, a fair amount of reliable information on precipitation exists in the Hinterrhein Valley. The selected six rain gauges provide a good areal coverage and allow the computation of a representative precipitation index.

2.4 Temperature

2.4.1 Methodology.

The elevation range of the Hinterrhein watershed is such that snowmelt and glaciermelt play an important role in the water cycle. Many different and complex processes influence the production rate of meltwater, the most important one being heat transfer. Absorbed solar radiation, net longwave radiation exchange between the snowpack and its environment, convective heat transfer from the air, latent heat of vaporization released by the condensate, conduction of heat from underground and heat content of rainwater are some of the heat sources which must be taken into account.

Although not all the factors just mentioned are of equal importance even the determination of only some of these parameters would require a prohibitive amount of instruments. If it seems still possible to measure many variables in a small size experimental watershed, this endeavor becomes impossible for a watershed like the one under consideration. Fortunately temperature is a good index of the heat transfer processes associated with snowmelt (U.S. Army Corps of Engineers, 1956). This variable is easy to measure so that this kind of data should be widely available. It explains why temperature

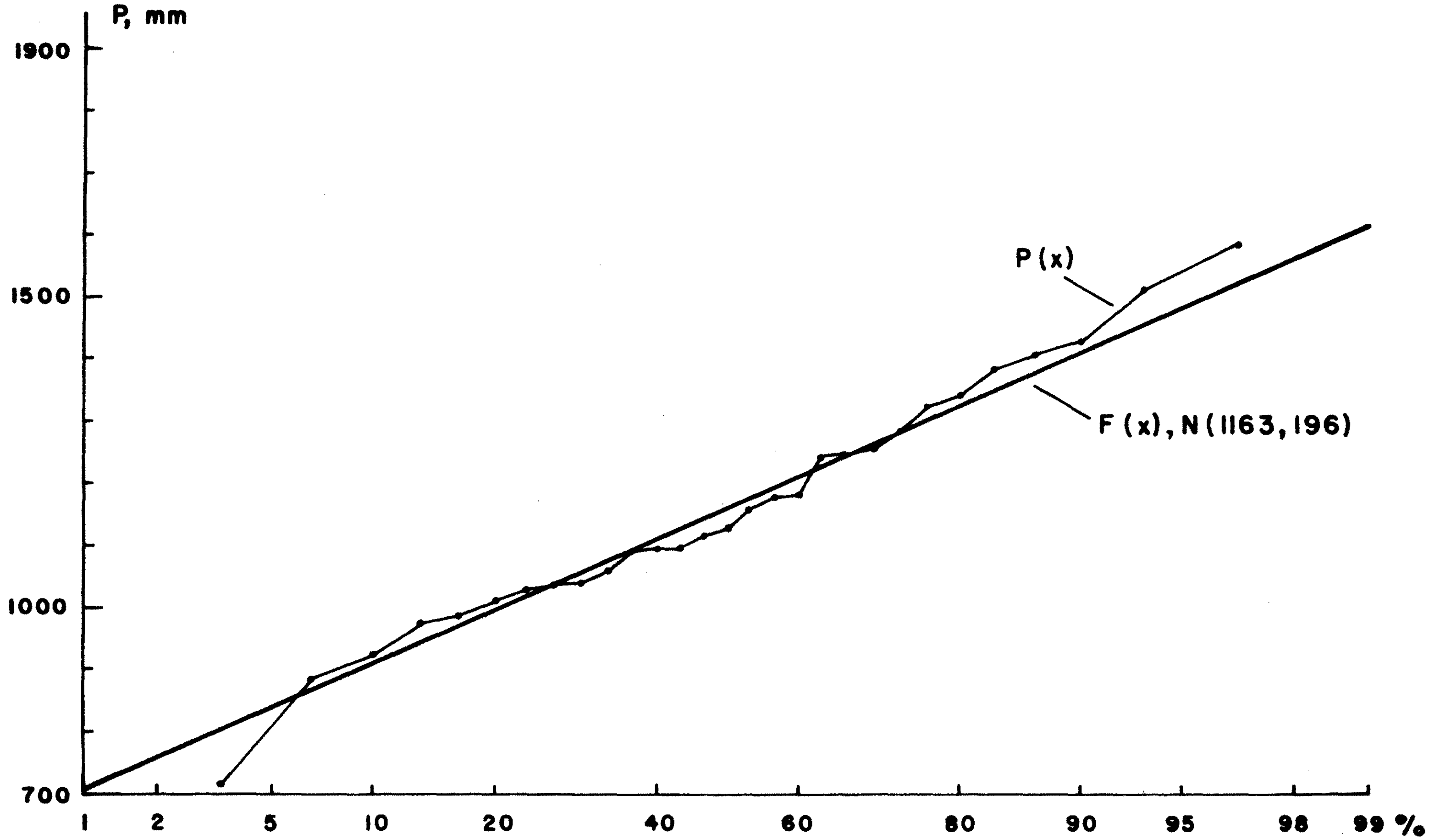


Fig. 2.6 Precipitation Index: Probability Distribution of Annual Values

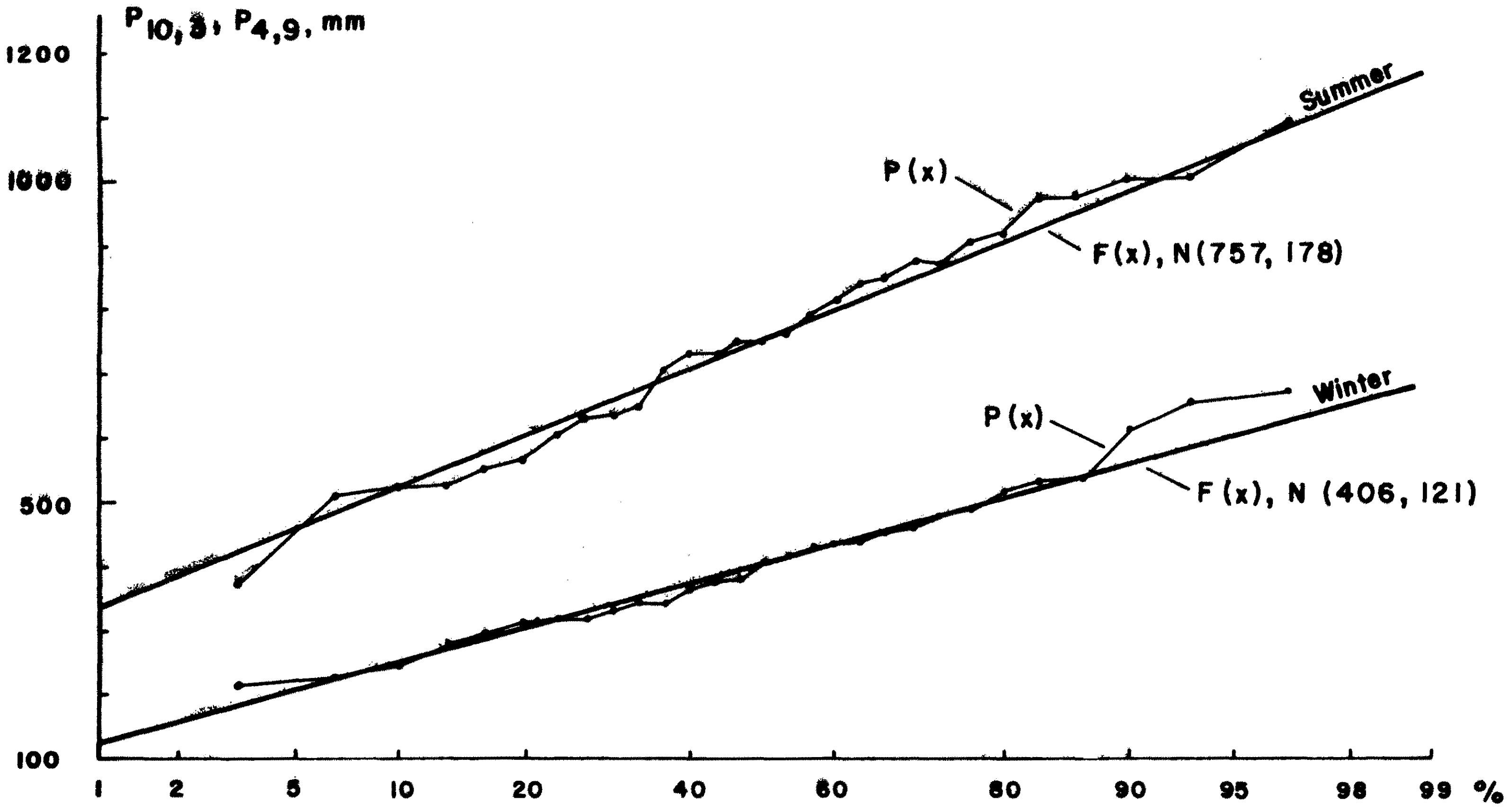


Fig. 2.7 Precipitation Index: Probability Distribution of Seasonal Values

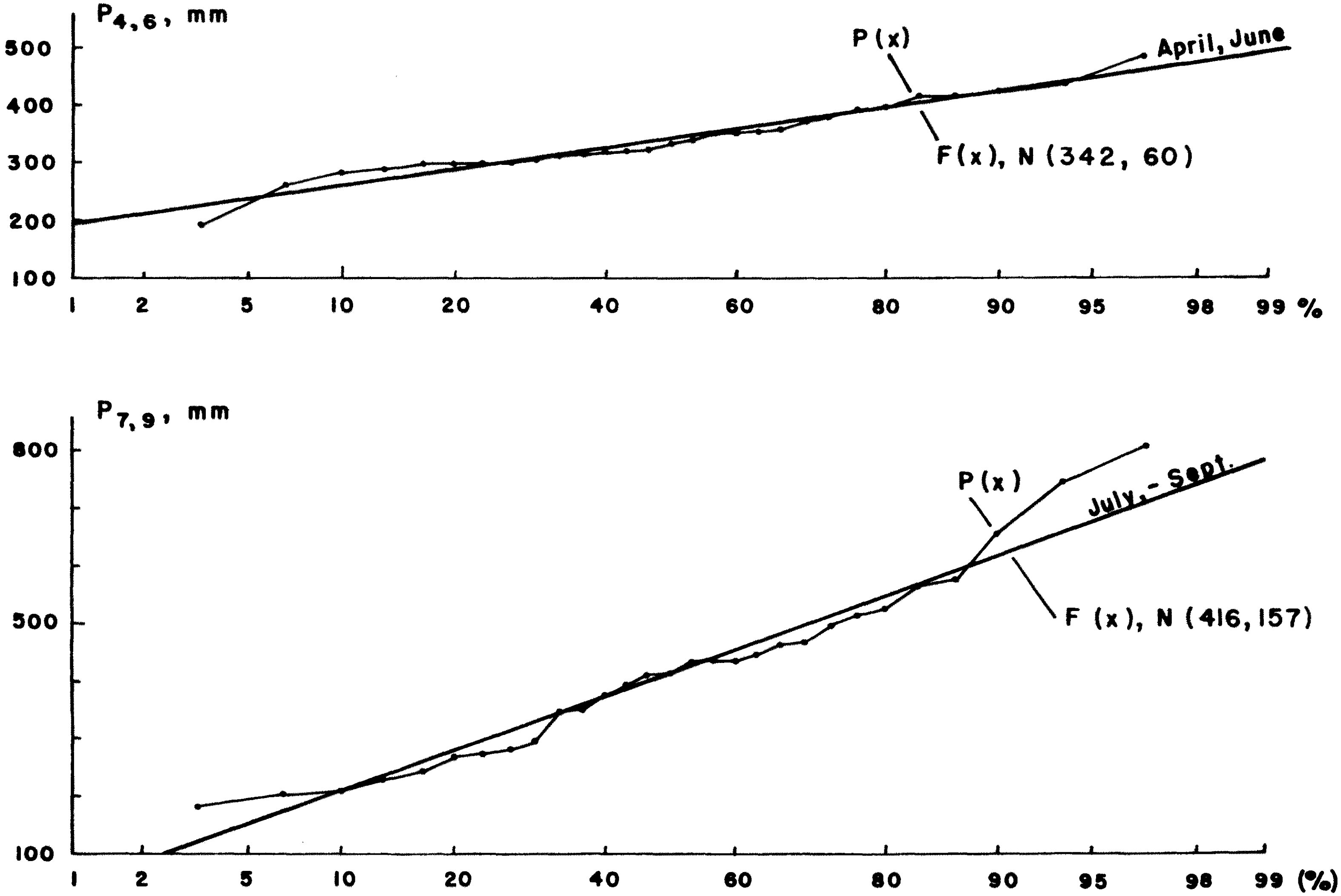


Fig. 2.8 Precipitation Index: Probability Distribution of Quarterly Values

Table 2.7

Correlation Coefficients between Annual, Seasonal and Quarterly Values

1945 - 1974

	Annual Corr. Coeff.	Pairs of Periods		
		(Apr to Sept - (Oct to March)	(Apr to June) - (Oct to March)	(July to Sept) - (Apr to June)
Precipitation	- 0.104	- 0.180	- 0.070	0.171
Number of Degree Days	- 0.015	-	-	0.391
Runoff	0.009	- 0.183	- 0.016	0.271

indexes are the most widely used methods of computing snow and glaciermelt.

The degree-day method has been retained quite often as an index and it will also be our approach here. Because snow and glaciermelt result directly from the heat transfer from the air in excess of 0° Celsius, the degree-day method postulates that the daily production rate of snowmelt is proportional to the number of degree-days above freezing.

2.4.2 Selection of the bench station. At high elevations only a few sites have been recording temperature for more than twenty years. Säntis (elevation 2,500 m) and Weissfluhjoch-Davos (elevation 2,680 m) are two of the most important ones in Switzerland. The record series from Säntis presents the disadvantage that some modifications were introduced in 1960 into the measuring technique which caused a systematic shift in the records. Furthermore Säntis is located far away from the watershed under study. Consequently the station of Weissfluhjoch-Davos was retained. It lies within 60 km of Hinterrhein catchment so that the data of this station are also valid for this area. Anyway according to Hoinkes (1968) temperature prevailing at elevations exceeding 2,000 m is nearly the same all over the Alps and is not much affected by local conditions.

2.4.3 Analysis of data. The temperature readings occur three times a day, namely at 7:30, 13:00, and 21:30. According to Zingg (1951) the mean daily temperature computed as:

$$T = \frac{T_{7:30} + T_{13:00} + 2.T_{21:30}}{4} \qquad (2\text{-}1)$$

gives the best correlation with the snowmelt mechanism. For Weissfluhjoch-Davos the average sum of positive degrees for the period 1951-60 (Zingg, 1961) appears in Table 2.8.

Hence at 2,680 m melting starts only in May. It reaches its full strength in July and August. Local snowmelting can still occur in October but often the glaciers are already frozen by mid September.

The main statistical parameters of the sum of positive degrees appear in Table 2.9 while the basic data appear in Table 2.10. The missing values were obtained by extrapolation from the records of Säntis. Figure 2.9 shows the time series of the annual sum of positive degree days and the corresponding 5 years moving average scheme. No strong trend could be detected. The normal distribution provides a good fit to the different data series which can be considered as statistically independent from one another (Figs. 2.10, 2.11, and 2.12, Table 2.7).

Of interest is not only the sum of positive degrees at one point but over the whole range covered by the watershed. Usually one assumes that temperature varies linearly with elevation at a rate of 0.65° Celsius per 100 m (Zingg, 1951). Because of the presence of negative values the sum of positive degrees does not vary exactly linearly with elevation. However, the linearity holds approximately for a small range like between about 2,500 and 2,800 m (Zingg, 1951).

2.5 Glacier

2.5.1 Scope of the work. Glaciers play an important role in the hydrology of alpine rivers. Kasser (1955) indicates that for the Rhône at Porte du Scex (catchment 5,220 sq km, glaciated area

Table 2.8

Average Number of Degree Days of Weissfluhjoch-Davos

1951 - 1960

Period	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Total
Number of Degree Days	45	6	1	-	2	1	3	39	95	168	166	122	648

in degrees Celsius

Table 2.9

Number of Degree Days: Main Statistics

1945 - 1974

Period	April to June	July to Sept	Sept	April to Sept	Oct to Sept
Mean	138	451	119	589	655
Stand. Dev.	40.5	78.2	42.5	101.2	100.5
Coeff. Var.	0.29	0.17	0.36	0.17	0.15
Min.	65	306	35	442	472
Max.	271	578	205	849	887

in degrees Celsius

Table 2.10

Number of Degree Days: Monthly, Quarterly and Annual Values

Hydr. Year	April to June °C	July to Sept °C	Sept °C	April to Sept °C	Oct to Sept °C	5 Year Mov. Aver. °C
1946	144	450	125.3	594	608	-
47	271	578	152.9	849	887	-
48	127	405	129.8	532	613	752
49	157	544	181.9	701	787	766
1950	217	548	90.4	765	867	724
51	111	495	145.5	606	676	732
52	155	484	34.5	639	676	687
53	140	496	140.0	636	654	621
54	125	348	136.3	472	560	598
55	107	354	90.6	461	539	573
56	75	446	157.9	521	563	601
57	122	371	87.0	493	548	629
58	157	558	162.3	714	795	624
59	149	494	147.3	643	698	640
1960	151	306	54.6	457	515	665
61	132	496	205.2	628	642	643
62	111	493	124.2	604	675	650
63	129	485	123.0	614	687	641
64	193	484	133.5	676	729	638
65	137	309	50.3	446	472	639
66	153	384	156.1	536	626	619
67	118	485	100.8	603	683	606
68	142	316	68.6	458	587	649
59	128	451	132.4	579	660	666
1970	111	467	143.4	578	690	638
71	112	521	90.0	633	711	668
72	92	350	37.9	442	542	650
73	171	512	147.7	683	737	-
74	65	459	114.4	524	569	-

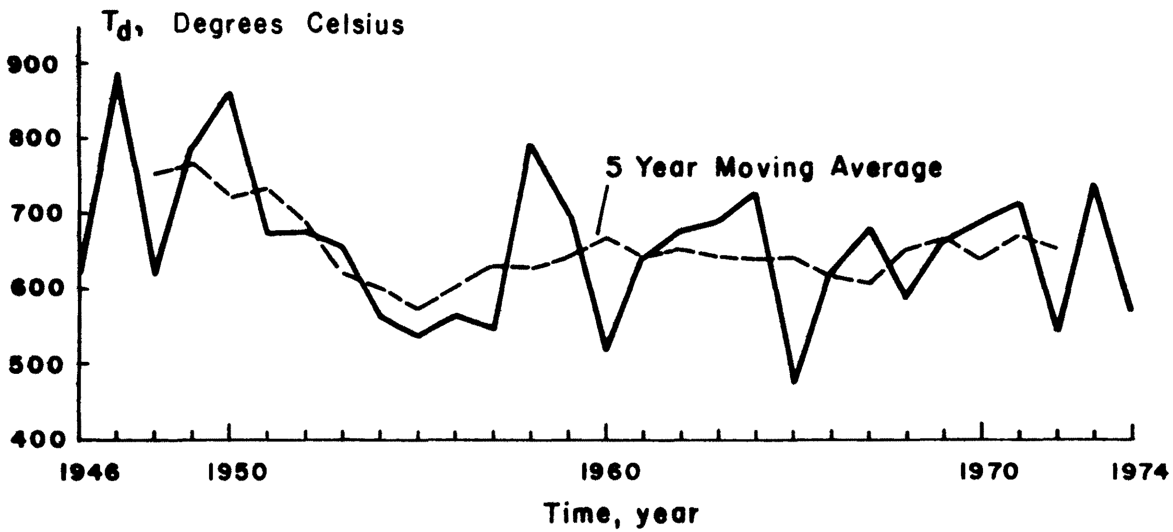


Fig. 2.9 Number of Degree Days: Time Series Plot of Annual Values

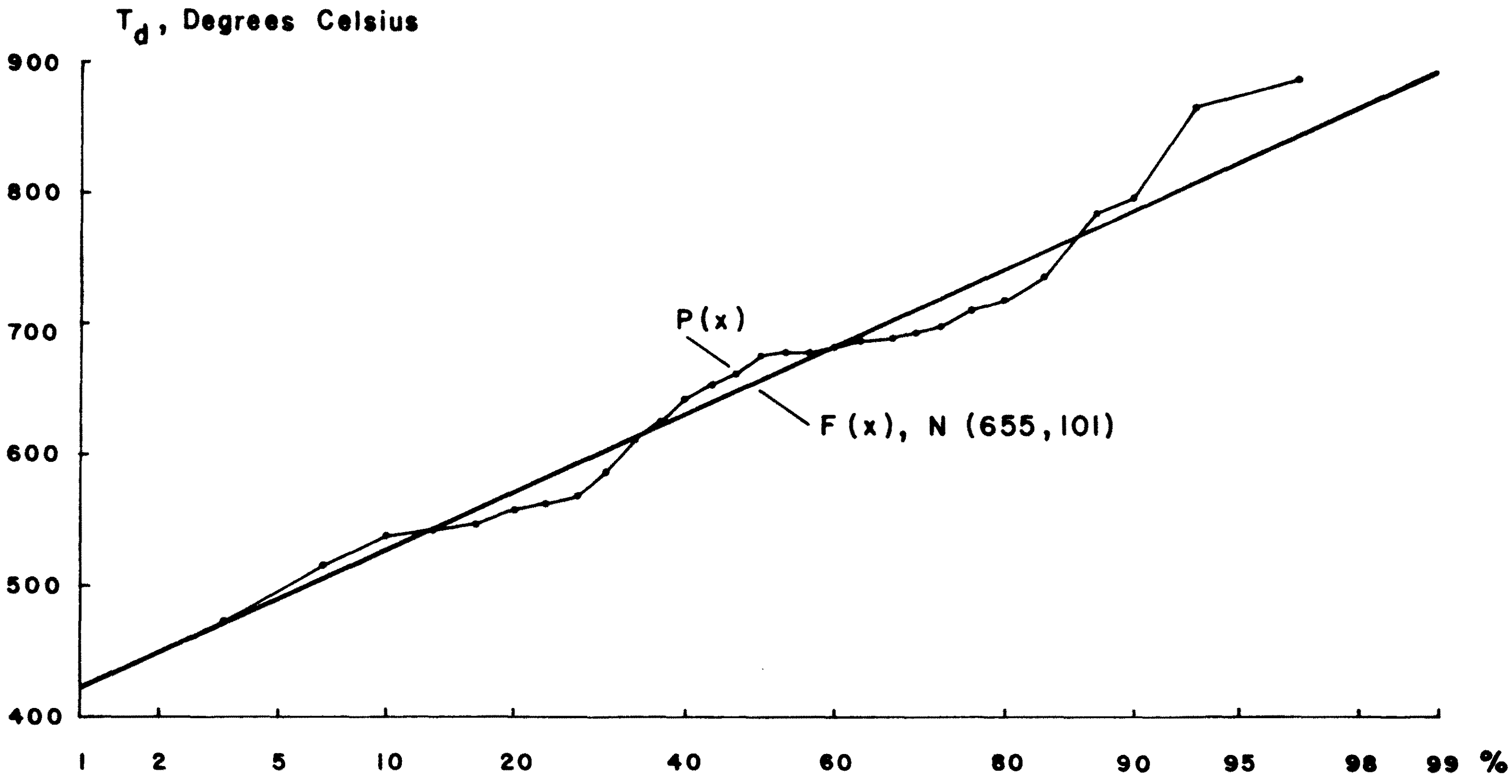


Fig. 2.10 Number of Degree Days: Probability Distribution of Annual Values

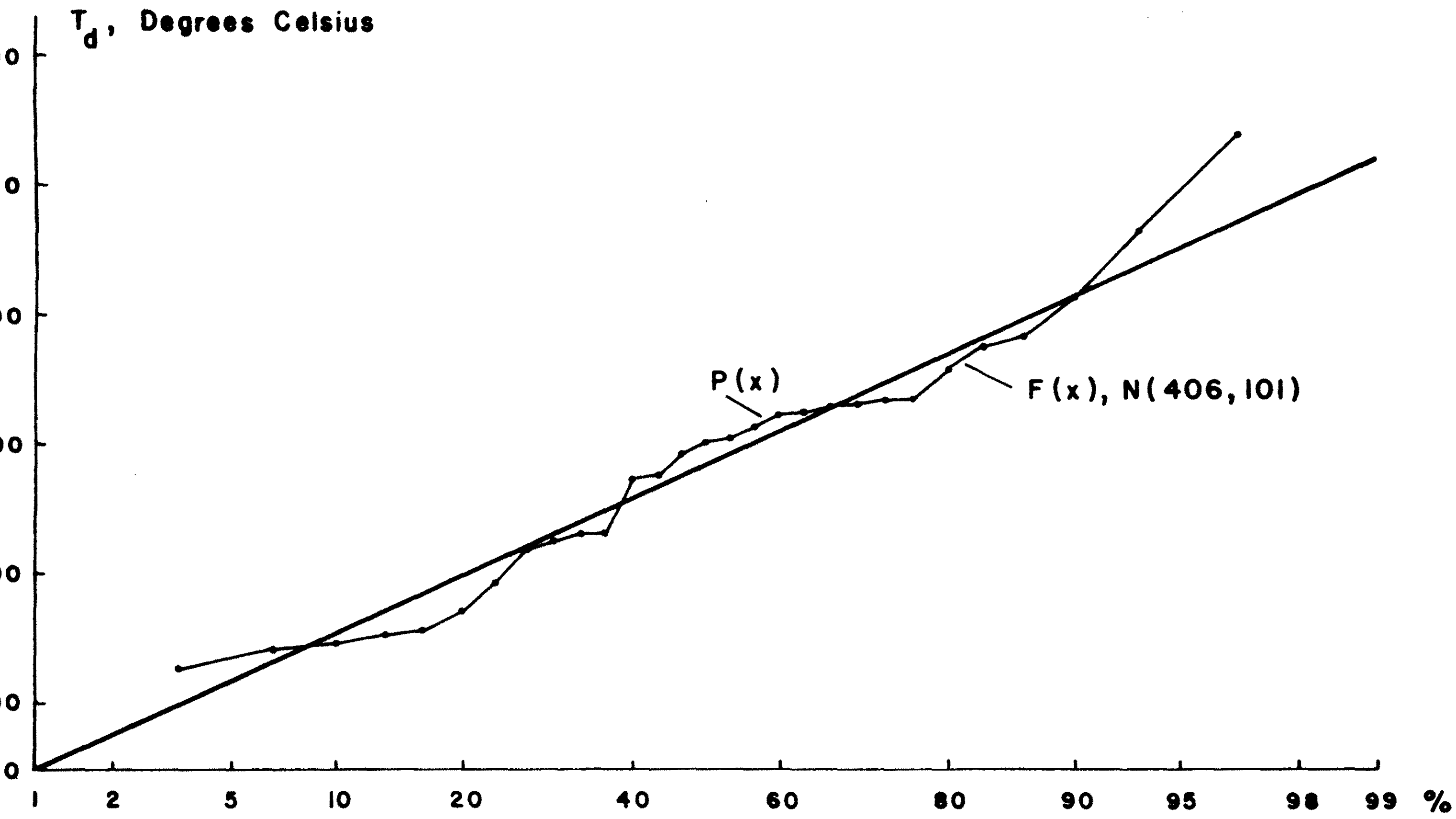


Fig. 2.11 Number of Degree Days: Probability Distribution of Summer Values

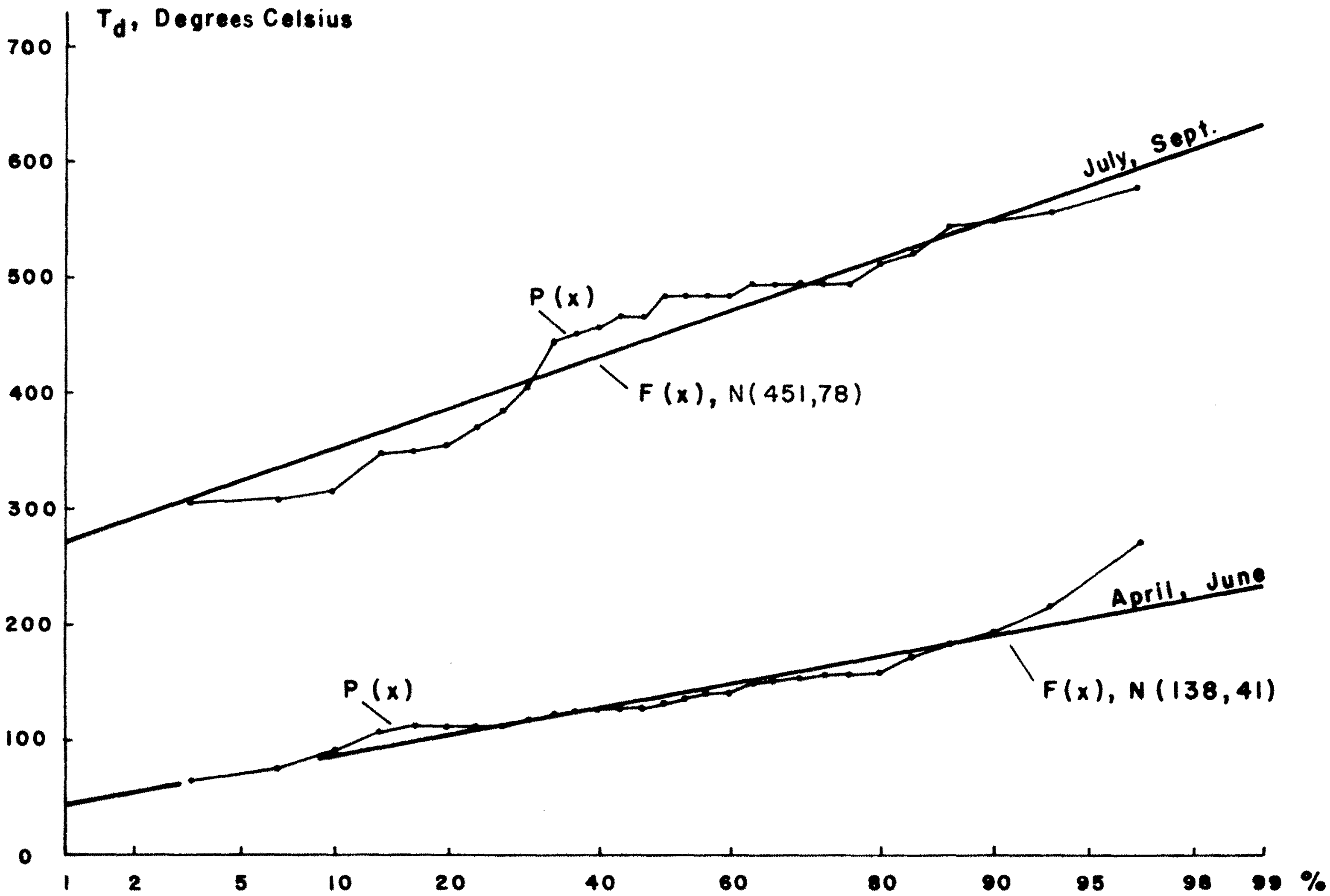


Fig. 2.12 <u>Number of Degree Days: Probability Distribution of Quarterly Values</u>

16 percent), the contributions of the glaciers amount to 11 percent of the summer runoff. It is consequently absolutely necessary to acquaint ourselves with their properties and characteristics. The section starts with a brief description of glaciology in Switzerland, continues with a qualitative and quantitative analyses of glacier activities, and ends with the application of the gained knowledge to the catchment of Hinterrhein.

2.5.2 Glaciology in Switzerland. The glaciated areas of the Alps are remnants from the Würm glaciation. It occurred thousands of years ago and covered most of Switzerland. Since then the ice has been steadily retreating with some smaller advances excepted.

The earliest indications concerning the extension of the glaciated area in Switzerland go back to 1876. At that date this area was evaluated at 1,895 sq km (733 sq mi) of which 663 sq km (255 sq mi) belonged to the Rhine catchment (Mercanton, 1958). The surveying done for the new national map (1934) supplied the following new values: 1,581 sq km (610 sq mi) and 548 sq km (211 sq mi) respectively. For Switzerland the retreat within 58 years amounts to 14.8 percent of the total area. For the Rhine catchment the corresponding figure is a little higher: 17.4 percent.

The average elevation of the lower end of all the glacier tongues retreated by about 90 m between 1876 and 1934. In the thirties it comprised between 1,450 and 1,950 m (4,750 ft and 6,400 ft respectively). The most frequently recorded values lay around 2,500 m (8,200 ft).

Systematic surveys were started two decades ago. Today length and elevation variations as well as aerial photographs are taken each year for nearly all the glaciers. Furthermore mass balances (see next section) which require detailed studies of area and elevation changes are computed regularly for some of them. Finally experimental watersheds were established on a glacier.

2.5.3 Qualitative analysis of glacier activities. In the Handbook of Applied Hydrology (1964) glacier is defined as "a body of ice originating on land by the recrystallization of snow or other forms of solid precipitation and showing evidence of past or present flow." The author continues in the following way, "... however many glaciologists hold to a more stringent definition. In order to qualify as a glacier an ice mass must have: 1) an area where snow or ice usually accumulates in excess of melting and 2) another area where the wastage of snow or ice usually exceeds the accumulation, and there must be 3) a slow transfer of mass by creep from the first region to the second." This lengthy definition shows clearly that many complex factors interact. Especially, as compared to snowmelt, one more parameter comes into play: the mass flow.

Schematically one can explain the formation of glaciers in the following way. The last glaciation in the Alps was initiated some thousands of years ago by a gradual decrease in temperature. As a consequence the heat available in summer was no longer sufficient to melt everywhere the snow fallen during the preceding winter season. Snow began to accumulate in layers roughly parallel to the prevailing topography. With the increase of snow depth two phenomena occurred: first the snow of the bottom layers was transformed into ice through compaction and second the whole mass became unstable and started to move down into the valley. This ice mass flow continued until it reached areas where the available summer heat was sufficient to melt away all the incoming ice flow.

According to this explanation, snowfall exceeds snowmelt in the upper portion of the glacier, in the lower portion the opposite is true. One calls the upper zone the accumulation zone and the lower one the ablation zone. They are separated by the firn limit where accumulation equals ablation. The firn limit is also the highest level to which the winter snow cover retreats during summer and as such changes from year to year.

All the glaciers of the Alps are temperate glaciers: the ice temperature is everywhere at the melting point except for the top layers which more or less follow the thermal cycle of the air. Hence glaciers are extremely sensitive to climatic changes. They grow or retreat according to the changing energy and moisture inputs. To quantify these movements one computes their mass balance. It is defined as the net quantity of water gain or loss occurring in a glacier over a specified duration. If the mass balance is nil the glacier is said to be in equilibrium. In the Alps the mass balance has been mostly negative for the last hundred years.

Meltwater does not come equally from all the zones. The highest areas do not contribute at all, even in summer, as the snow melted during the day freezes again at night. If the highest contributing zones lie just above the firn limit the greatest quantity of water is in fact supplied by the glacier tongue which is completely thawed in summer.

A complicated drainage system exists on and in the glacier. As ice is impervious meltwater flows first on the surface in small channels. These channels end into pits through which the water reaches the bottom and the glacier stream. Each year, in spring, this complex intra-glacial network must be rebuilt; it takes time, which explains why extensive glaciermelt starts so late in the season. Also a cold spell in September can completely stop the meltwater flow although later on temperature is again above the freezing point.

2.5.4 Quantitative analysis of glacier activities. The international hydrologic decade gave a new impulse to the alpine glaciology. The researchers mainly addressed themselves to the collection of new data and to the scientific analysis of the glacial phenomena. Of special interest for the present study we shall present the results of three research teams. In Switzerland, Zingg, Kasser and Lang; in Austria, Hoinkes and Lang.

To get familiar with the magnitude of the variables involved some data about the most important parameters are given hereafter. They come from the best surveyed glaciers of Switzerland and Austria. The first information deals with area changes (Table 2.11).

For the Aletschglacier the mass balance was 17 times negative and 11 times positive from 1945-46 to 1972-73. For the Hintereisferner glacier it was 12 times negative and 5 times positive for the period 1952-53/1967-68. Finally Table 2.12 gives the variation of the firn line elevation for some Swiss glaciers.

The just given numbers confirm that the mechanisms involved are complex. Each glacier behaves in its own way. It may even behave differently from year to year.

Table 2.11

Area Changes of Glaciers

A. Glacier of Aletsch

Date	Area km^2	Percentage Decrease in Area
1933	135.27	
		5.9
1962	127.27	
		3.6
1971	122.64	

B. Glacier of Hintereisferner

Date	Area km^2	Percentage Decrease in Area
1953	10.24	
		11.8
1968	9.03	

Table 2.12

Annual Elevation Variations of the Firn Limits of Some Glaciers

Glacier:	Gries	Limmern	Silvretta	Hintereisferner
1959 - 60	-	2720	-	2880
60 - 61	-	2650	2740	2940
61 - 62	3130	2840	3150	3080
62 - 63	2840	2750	2900	3010
63 - 64	2900	2950	3160	3180
64 - 65	2770	2510	2490	2770
65 - 66	2780	2420	2510	2850
66 - 67	2800	2860	2715	2920
67 - 68	2710	2530	2645	2850
68 - 69	2740	2740	2800	2960
69 - 70	3040	2820	2730	-
70 - 71	3030	2930	2880	-
71 - 72	2680	2750	2800	-
72 - 73	3070	2900	2980	-

Elevations are given in m

Fortunately the laws governing some parts of the glacial mechanisms are simpler. Hence, according to Zingg (1951), the following relation exists between produced meltwater and positive degree-days:

$$D = 4.5 \cdot T_d \tag{2-2}$$

where D is the depth of melted water in mms and T_d the sum of positive degree-days.

This relation was established on an experimental watershed, situated close to the station of Weissfluhjoch-Davos at 2,540 m. Obviously the proportionality factors depend on the prevailing local conditions and may be different for ice and for snow.

Lang (1967) analyzed the relation existing between glacier runoff and related meteorological factors. The basic data came from an experimental watershed on the Aletsch Glacier (elev. 2,200 m, area 4,480 sq m) and from the corresponding total drainage basin, global radiation turned out to be the best indicator for glacier runoff. Temperature, however, gave the best correlation with the flows of the complete drainage basin.

Finally Kasser (1955) made extensive studies with the catchment of the Rhone River at Porte du Scex (5,220 sq km, glaciation 16 percent). Kasser used the formula developed by Zingg to evaluate the contribution from the glaciated area. He obtained a correlation coefficient of 0.80 for a regression between annual runoff, related precipitation and glaciermelt. He also mentioned that the change in glaciated area had a non-negligible affect on the total amount of seasonal runoff.

2.5.5 Glaciers of the Hinterrhein catchment. Unfortunately not much information is avaiable. The Paradies and the Zapport Glacier form most of the glaciated area. In Table 2.13 figure the length and elevation variations of the tongue of the Paradies Glacier from 1876 up to today. In 1876, the glaciers covered 17.60 sq km (6.8 sq mi), by 1933, this area amounted to 11.88 sq km (4.58 sq mi) and in 1962 only 9.29 sq km (3.58 sq mi) were still glaciated. These figures correspond to a reduction of the glaciated area of 32.5 percent for the first period, and of 21.7 percent for the second period.

Compared to the other Swiss glaciers, this decrease in area is exceptionally high. Furthermore, according to Table 2.13, the Paradies Glacier has nearly constantly retreated since 1933. No information exists about mass balance and firn limit. The firn limit should lie between 2,800 (9,200 ft) and 3,000 m (9,850 ft).

The last column of Table 2.13 supplies the computed glaciated areas. The areal changes were taken proportional to the length variations. It is a crude approximation, but with the available information, no other alternative exists. This is especially true because the data collected on other glaciers cannot be transferred to the Hinterrhein catchment.

2.6 Runoff

2.6.1 Annual, seasonal, quarterly and monthly flows. Runoff is the final result of the action of temperature on snow and on glaciers, and of the watershed on rain. In this part however, as far as possible, the runoff phenomenon is considered for itself. The relationships existing between the different elementary cycles will be studied later on.

Table 2.13

Glaciated Area of Hinterrhein: Main Characteristics

Date	Elev. Lower End of Glacier m	Annual Horizontal Retreat m	Cumulated Horizontal Retreat m	Measured Glaciated Area km^2	Computed Glaciated Area km^2
1876	2213			17.600	
1933	2308	-	-	11.880	
1945	-	227	227		11.125
46	-	14			11.079
47	-	46			10.926
48	-	62			10.720
49	-	4			10.706
1950	-	82			10.434
51	-				10.366
52	-	61.5			10.297
53	-				10.229
54	-	20			10.163
55	-	53.5			9.985
56	2355	40			9.852
57	2357	33.5			9.741
58	2358	16			9.687
59	2359	31			9.584
1960	-	37.5			9.522
61	2360.5				9.460
62	2363	51	779	9.290	9.290
63	2361	48.5			9.129
64	-	48			9.049
65	2363				8.969
66	2365	25			8.886
67	2365	2			8.879
68	2365	-11.5			8.918
69	2356	10			8.884
1970	2358	14			8.838
71	2362	43.7			8.693
72	2362	0.0			8.693
73	2362	9.0			8.663
1974	2362	10.6	978.3		8.627

The hydrologic year starts on October 1st, and ends on September 30th of the following year. The Hinterrhein River carries on the average 106 million cubic meters (3,750 million ft^3) yearly (Fig. 2.13). Of this amount, 12 percent are recorded in the winter, and 88 percent in the summer semester (Table 2.14); the months of July, August and September yield themselves 51 percent of the annual flow. Actually the flow regime is typical of an alpine river. The discharges are low from December to March; April and May show a substantial increase while the highest values are registered from June to August. September can bring occasionally high flows and October and November are transition months. The Table 2.16 visualizes this fact.

The normal distribution provides a good fit to annual and semiannual, and quarterly flows (Table 2.15, Fig. 2.14, 2.15 and 2.16). For the periods just mentioned, the flows are all statistically independent from one another (Table 2.7). Only the successive monthly flows show significant correlation coefficients (Table 2.17). Their variations with time present an interesting pattern. The correlation coefficient is minimum in November and in May, and maximum in January and in September. Between these extreme points, it increases and decreases regularly. Noteworthy is the negative correlation coefficient between the monthly flows of May and June.

2.6.2 Weekly flows. To obtain a better insight into the annual flow cycle and its mechanisms, it is absolutely necessary to consider weekly discharges. Actually this time step is ideal; it eliminates the random components of the time series, while it still preserves the basic trends.

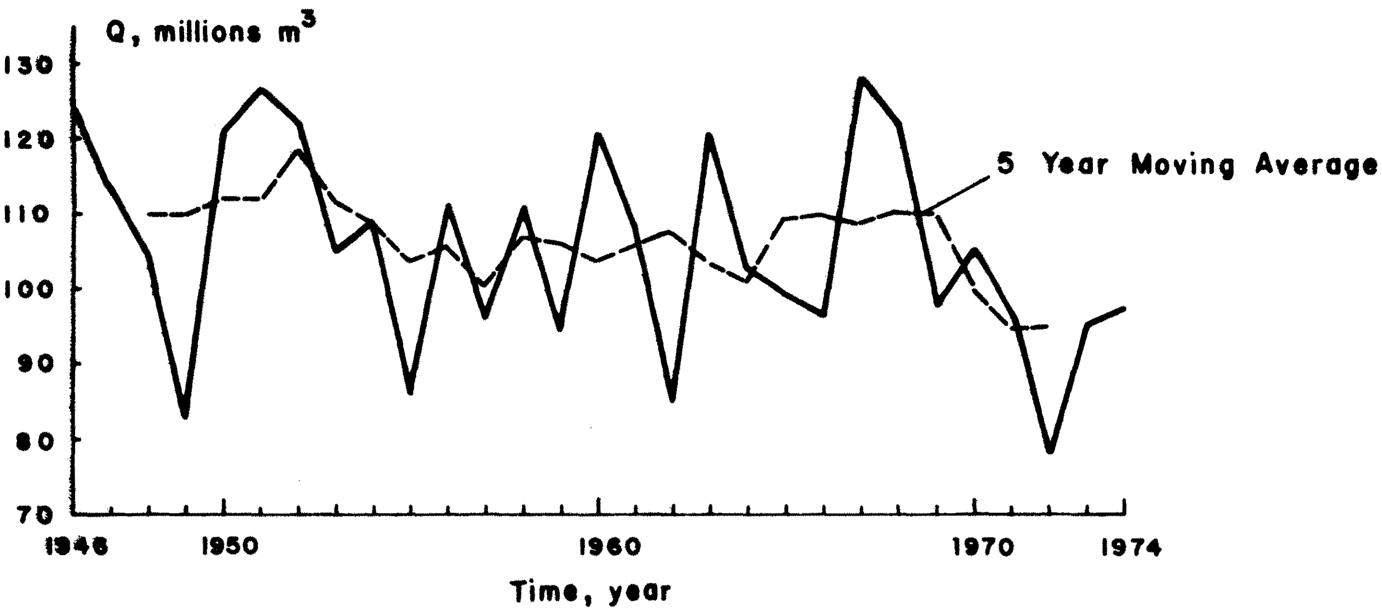


Fig. 2.13 Runoff: Time Series Plot of Annual Values

Table 2.14

Runoff: Main Statistical Parameters

1945 - 1974

	Winter Flows	April-June	July-Sept	April-Sept	Annual Flows
Average	12.796	39.159	54.077	93.236	106.032
Stand. Dev.	4.813	5.952	11.420	14.231	14.303
Coeff. Var.	0.376	0.152	0.211	0.153	0.135
Min.	6.246	50.609	77.683	70.932	77.891
Max.	23.467	26.179	38.995	117.868	128.081

All the numbers are given in millions m^3

Table 2.15

Runoff: Quarterly, Seasonal and Annual Values

Year	Oct to March 10^6m^3	April to June 10^6m^3	July to Sept 10^6m^3	April to Sept 10^6m^3	Annual 10^6m^3	5 Year Mov. Aver. 10^6m^3
1946	12.377	43.475	69.777	113.252	125.629	-
47	7.789	48.532	57.094	105.626	113.415	-
48	9.781	39.790	54.802	94.592	104.373	109.49
49	9.250	31.729	41.733	73.462	82.712	109.79
1950	12.812	50.609	57.912	108.521	121.333	111.56
51	9.246	47.589	70.279	117.868	127.114	111.58
52	15.380	47.107	59.804	106.911	122.291	118.88
53	16.406	34.468	53.567	88.035	104.441	111.90
54	23.467	37.746	58.011	95.757	119.224	109.09
55	7.801	39.456	39.182	78.638	86.439	103.88
56	9.171	26.179	77.683	103.862	113.033	105.25
57	11.195	39.049	46.020	85.069	96.264	100.08
58	11.049	41.390	58.838	100.228	111.277	107.01
59	22.218	32.166	38.995	71.161	93.379	106.03
1960	9.404	45.041	66.673	111.714	121.118	103.83
61	21.086	40.754	46.255	87.009	108.095	105.78
62	14.326	29.973	40.959	70.932	85.258	107.54
63	6.246	40.688	74.116	114.804	121.050	103.22
64	17.728	45.094	39.348	84.442	102.170	100.91
65	10.783	34.518	54.246	88.764	99.547	109.48
66	11.961	40.780	43.789	84.569	96.530	109.61
67	18.401	38.697	71.163	109.860	128.081	108.74
68	17.928	45.592	58.224	103.816	121.744	109.90
69	16.530	36.825	44.447	81.272	97.802	109.92
1970	8.657	38.931	57.776	96.707	105.364	99.88
71	14.176	35.176	47.254	82.430	96.606	94.58
72	6.286	32.366	39.239	71.605	77.891	94.52
73	6.854	37.633	50.764	88.397	95.251	-
74	12.769	34.334	50.380	84.714	97.483	-

Table 2.16

Runoff: Average Monthly Values

1945 - 1974

Period	Oct	Nov	Dec	Jan	Feb	March	April	May	June	July	Aug	Sept
Runoff	1.94	1.10	0.60	0.42	0.32	0.38	1.31	4.84	8.78	8.56	7.21	4.61

Numbers are given in m^3/s

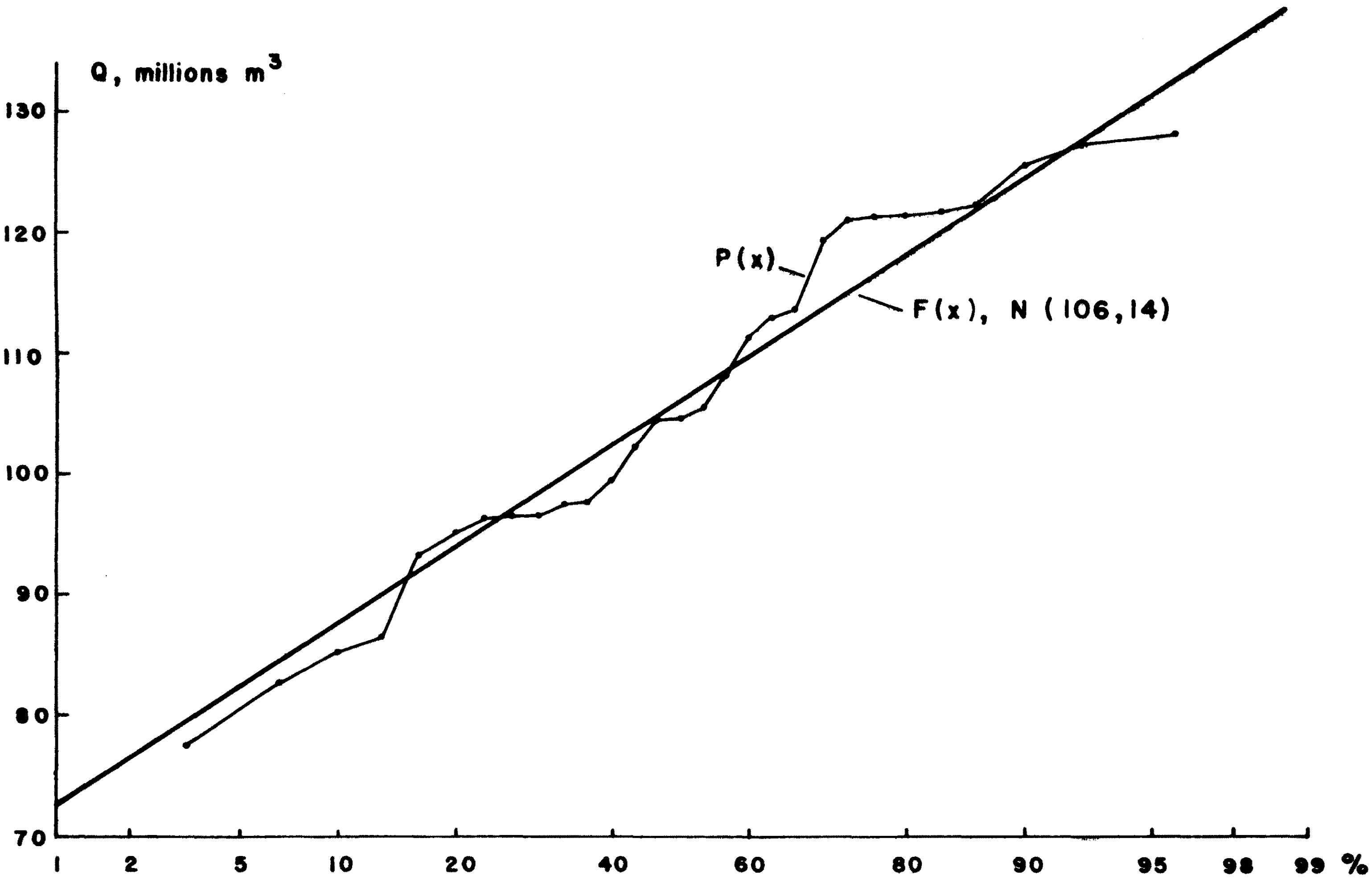


Fig. 2.14 Runoff: Probability Distribution of Annual Values

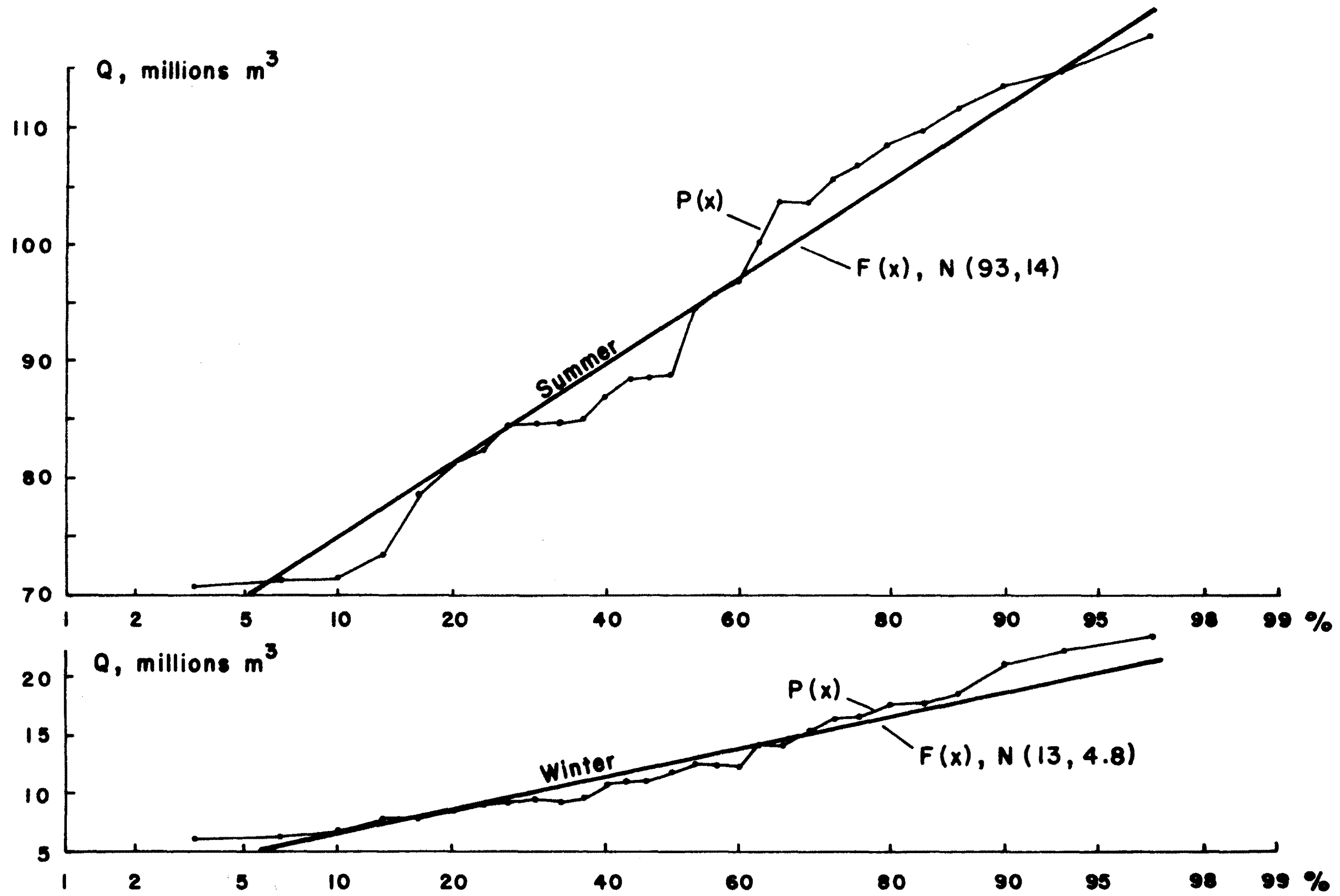


Fig. 2.15 Runoff: Probability Distributions of Summer and Winter Values

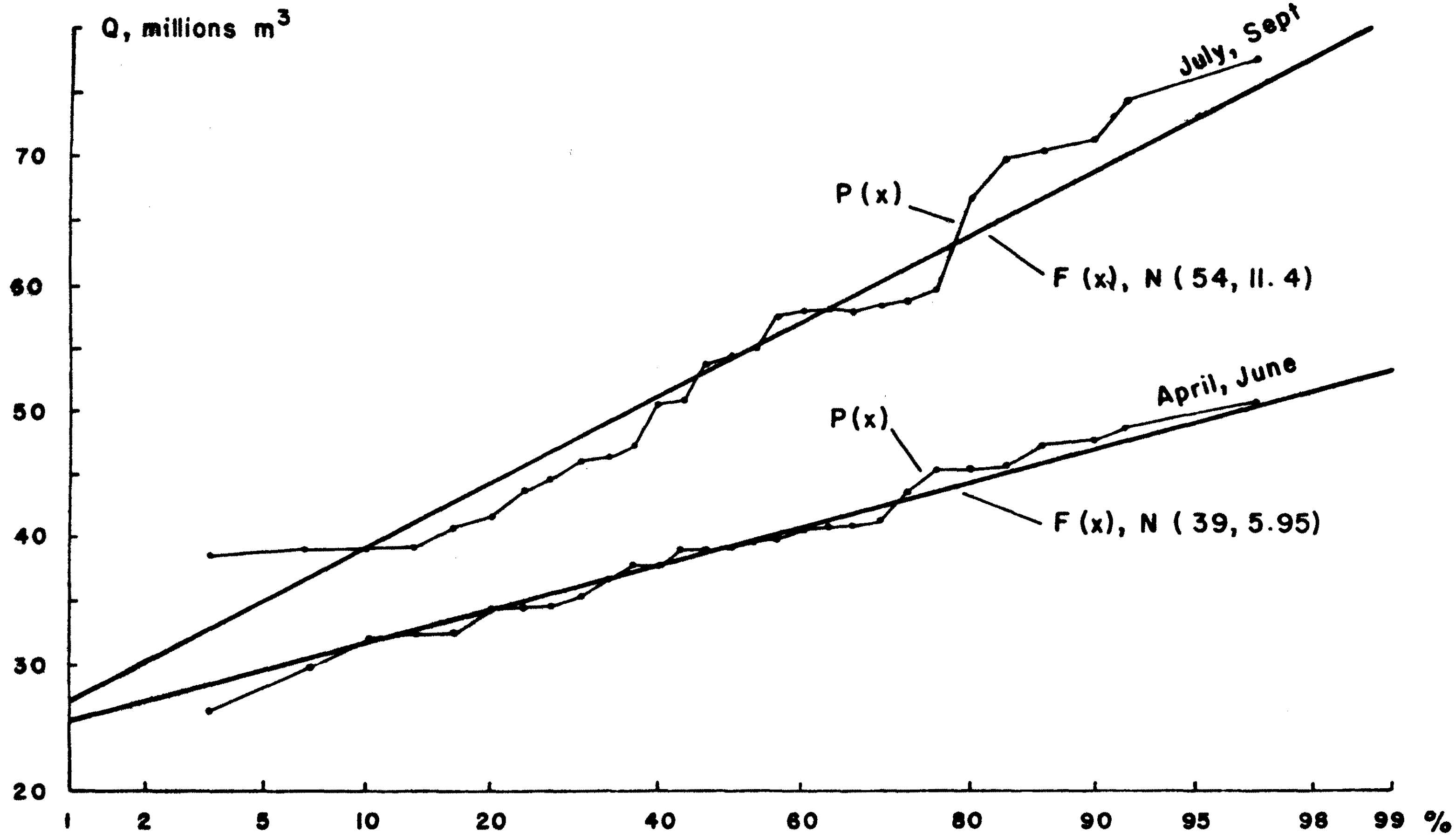


Fig. 2. 16 Runoff: Probability Distribution of Quarterly Values

Table 2.17

Runoff: Correlation Coefficient between Successive Monthly Flows

1945 - 1974

Pair of Months	Correlation Coefficient
Oct - Sept	0.381
Nov - Oct	0.172
Dec - Nov	0.619
Jan - Dec	0.754
Feb - Jan	0.720
Mar - Feb	0.469
April - Mar	0.296
May - April	0.107
June - May	-0.144
July - June	0.190
Aug - July	0.394
Sept - Aug	0.526

Table 2.18 and 2.19 contain the main statistical parameters of this variable. The mean value reaches its minimum at the end of February, and its maximum between the end of June and beginning of July. The mean value decreases and increases regularly during most of the year, except for the period of June to August where it stays relatively constant.

The links existing between the successive weekly flows change within the year. Tables 2.18 and 2.19 display the computed lag one autocorrelation coefficient of the weekly flows and the corresponding five year moving average scheme. The annual variation of this variable is related to the annual temperature cycle. For the autocorrelation coefficient increases from mid-October on and reaches its maximum in the second half of winter, when temperature is minimum. At that date, discharge depends mainly on the antecedent runoff. Then the autocorrelation coefficient starts decreasing and reaches its lowest value, when temperature is highest. In summer, runoff results mainly from glaciermelt, which is directly controlled by the prevailing air temperature. As temperature fluctuates greatly from day to day, the resulting runoff undergoes the same changes. On the whole, the autocorrelation is low. Remarkable is the apparent contradiction between the variations of the monthly and of the weekly correlation coefficients.

The selection of the appropriate statistical distribution for weekly flows caused some problems. When the hydrometeorological conditions are homogeneous like in winter and in summer, the normal distribution provides a good fit. In early fall and in spring, however, the available data samples are not always homogeneous. In fall,

Table 2.18

Runoff: Main Statistics of Weekly Flows

Winter Semester

Week Index	Mean $10^6 m^3$	Stand. Dev. $10^6 m^3$	Coeff. Var.	Min. $10^6 m^3$	Max. $10^6 m^3$	Corr. Coeff.	Mov. Aver. Corr. Coeff.
1	1.687	1.481	0.878	0.533	7.245	0.70	-
2	1.170	0.937	0.801	0.430	4.805	0.25	-
3	1.012	1.063	1.050	0.366	5.251	0.74	0.51
4	0.943	0.793	0.841	0.307	3.436	0.41	0.49
5	0.943	0.922	0.978	0.222	4.062	0.45	0.58
6	0.758	0.627	0.827	0.187	3.141	0.59	0.60
7	0.681	0.453	0.665	0.253	2.157	0.73	0.71
8	0.560	0.339	0.605	0.229	2.053	0.81	0.80
9	0.459	0.143	0.312	0.213	0.782	0.95	0.87
10	0.417	0.117	0.281	0.196	0.619	0.92	0.91
11	0.385	0.117	0.304	0.179	0.621	0.92	0.94
12	0.334	0.093	0.278	0.158	0.562	0.96	0.94
13	0.307	0.087	0.283	0.145	0.504	0.94	0.94
14	0.293	0.087	0.297	0.143	0.444	0.95	0.95
15	0.276	0.077	0.279	0.146	0.410	0.94	0.94
16	0.257	0.071	0.276	0.143	0.407	0.95	0.93
17	0.237	0.069	0.291	0.131	0.407	0.92	0.94
18	0.219	0.065	0.297	0.127	0.384	0.91	0.95
19	0.203	0.066	0.325	0.118	0.378	0.97	9.93
20	0.192	0.071	0.370	0.096	0.369	0.98	0.93
21	0.191	0.076	0.398	0.073	0.391	0.87	0.93
22	0.187	0.066	0.353	0.074	0.333	0.93	0.89
23	0.190	0.067	0.353	0.089	0.322	0.90	0.86
24	0.204	0.084	0.412	0.101	0.410	0.78	0.83
25	0.241	0.105	0.436	0.107	0.532	0.81	0.78
26	0.310	0.123	0.397	0.115	0.644	0.75	0.72

Table 2.19

Runoff: Main Statistics of Weekly Flows

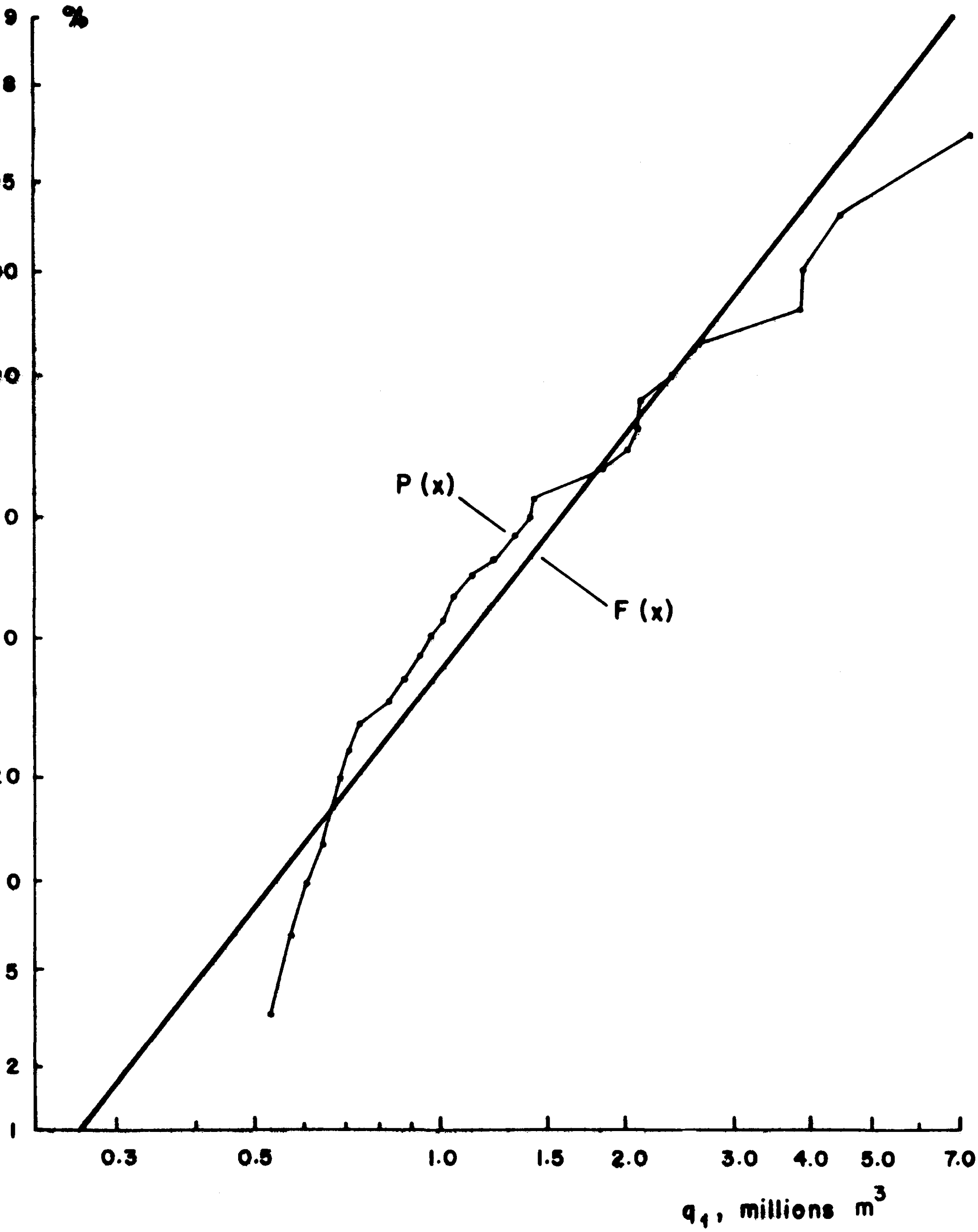
Summer Semester

Week Index	Mean $10^6 m^3$	Stand. Dev. $10^6 m^3$	Coeff. Var.	Min. $10^6 m^3$	Max. $10^6 m^3$	Corr. Coeff.	Mov. Aver. Corr. Coeff.
27	0.466	0.244	0.524	0.126	1.042	0.75	0.72
28	0.598	0.406	0.679	0.157	2.189	0.67	0.71
29	0.880	0.491	0.558	0.240	2.059	0.59	0.66
30	1.107	0.623	0.563	0.307	2.676	0.72	0.63
31	1.623	1.105	0.681	0.473	5.187	0.58	0.56
32	2.417	1.106	0.458	0.785	4.534	0.61	0.53
33	3.167	1.563	0.494	0.880	6.000	0.30	0.41
34	3.152	1.388	0.440	1.426	7.395	0.42	0.38
35	4.170	1.740	0.417	1.790	9.253	0.15	0.32
36	4.530	1.540	0.340	1.460	7.243	0.43	0.39
37	4.870	1.785	0.367	2.253	8.558	0.28	0.36
38	6.046	1.828	0.302	2.256	10.558	0.67	0.42
39	6.113	2.105	0.344	2.400	11.474	0.27	0.41
40	5.532	1.843	0.333	1.880	9.167	0.45	0.41
41	5.476	1.857	0.339	2.002	8.700	0.40	0.31
42	5.485	1.857	0.339	3.203	12.438	0.27	0.35
43	4.615	1.169	0.253	2.233	7.542	0.16	0.37
44	4.308	1.197	0.278	2.592	6.985	0.46	0.33
45	5.093	2.091	0.411	2.034	11.631	0.57	0.30
46	4.451	1.840	0.413	1.896	10.368	0.21	0.35
47	4.120	1.782	0.433	1.648	9.510	0.10	0.36
48	3.645	2.189	0.601	1.658	13.669	0.43	0.30
49	3.768	2.073	0.550	1.389	9.231	0.51	0.27
50	2.991	1.792	0.599	1.146	9.701	0.23	0.31
51	2.628	1.930	0.734	0.953	11.006	0.10	0.31
52	1.888	1.223	0.648	0.768	7.012	0.30	-
						0.43	-

for instance, many realizations of a sample belong to the recession part of the hydrograph, but a few may correspond to the high glacier-melt flows. On the other hand in spring, most of the recorded values are quite low because snowmelt has not yet started, but for some others, snowmelt is already going on in its full strength. This situation shows the drawbacks of dating the flows according to the calendar year. A method as proposed by Laufer (1972), which consists of dating the flows according to the time to peak would surely eliminate some of the difficulties. Here, the best procedure, at least theoretically, consists of fitting two distribution curves for the data: one for the lower values, and another one for the higher values. Practically, however, so few data fall into the second category, that it is not possible to fit a curve to them. Hence a single distribution curve was fitted to the data, namely the lognormal. On the whole it works quite satisfactorily (Fig. 2.17).

2.6.3 Recession curve A typical hydrograph consists of a rising limb, a crest segment and a falling limb or recession. The prevailing meteorology during snowmelt determines mainly the character of the rising limb. The recession curve, on the other side, results from the withdrawal of water from storage within the basin, and hence markedly influenced by the basin characteristics.

Eight years were randomly selected. Figure 2.18 presents the corresponding recession curves during winter, on a weekly basis. Four of them show an identical shape with about equal rate of flow decreases; three others have amazing shapes. Furthermore the initial and final date of the recession curve vary from year to year.



Remark: The data follow a lognormal distribution.

Fig. 2. 17a **Runoff: Probability Distribution of Weekly Flows**

- Oct 1, Oct 7

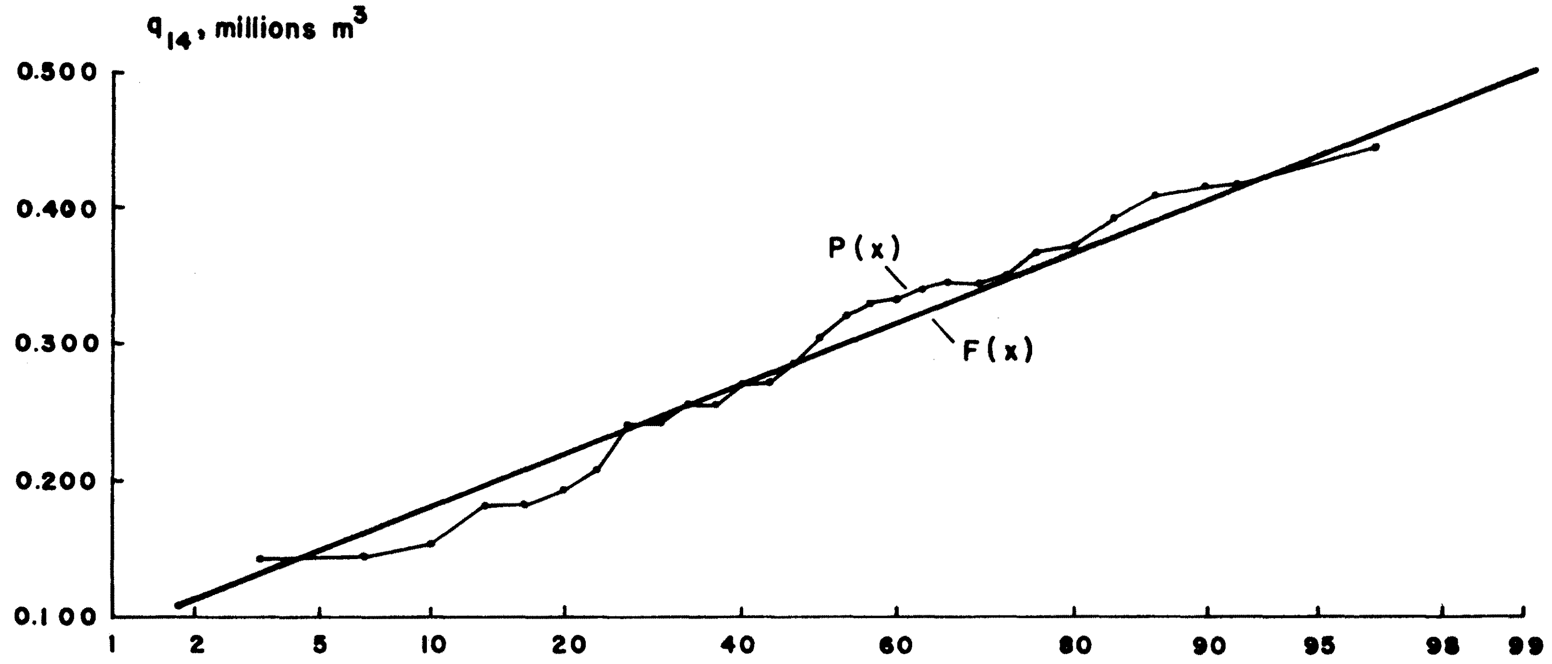
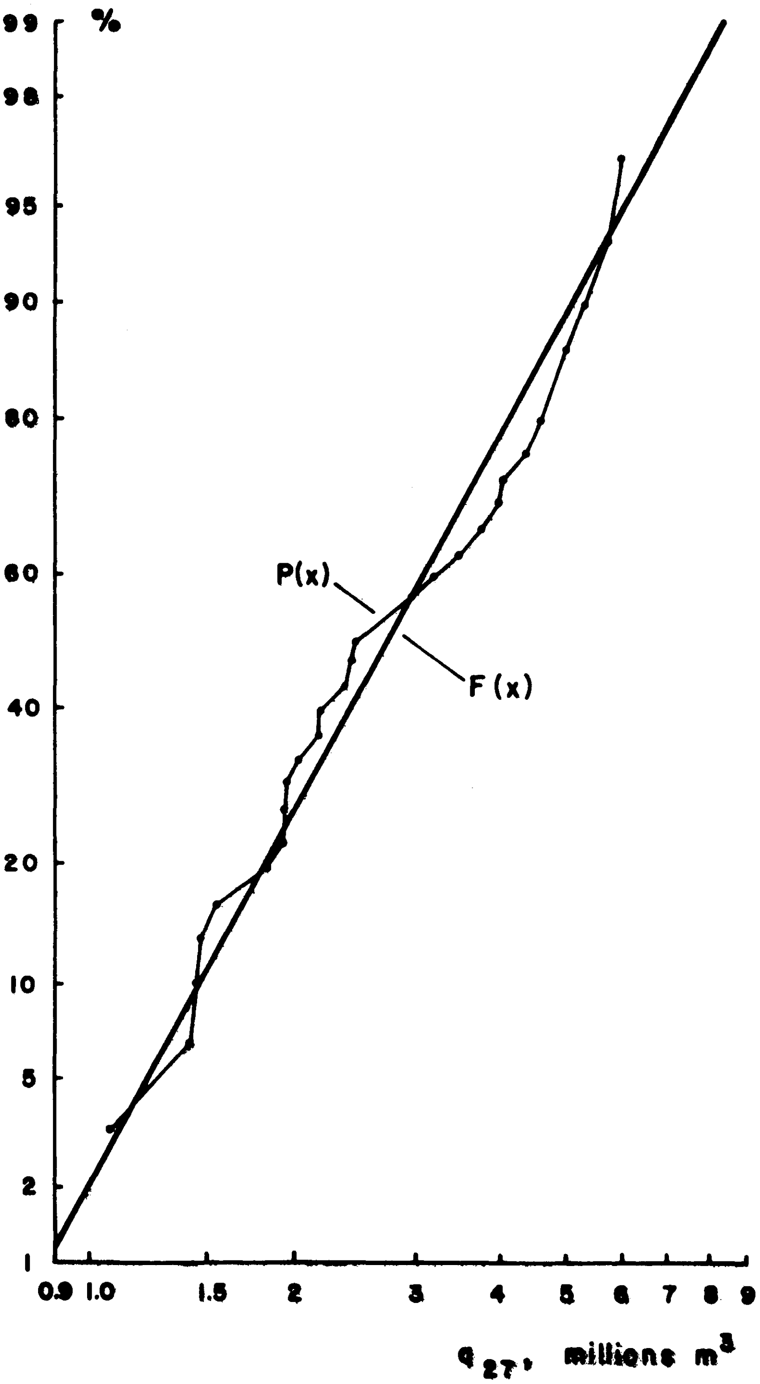


Fig. 2.17 b Runoff: Probability Distribution of Weekly Flows - Dec 31, Jan 6



Remark: Data Follow a Lognormal Distribution

Fig. 2.17c Runoff: Probability Distribution of Weekly Flows - April 1, April 7

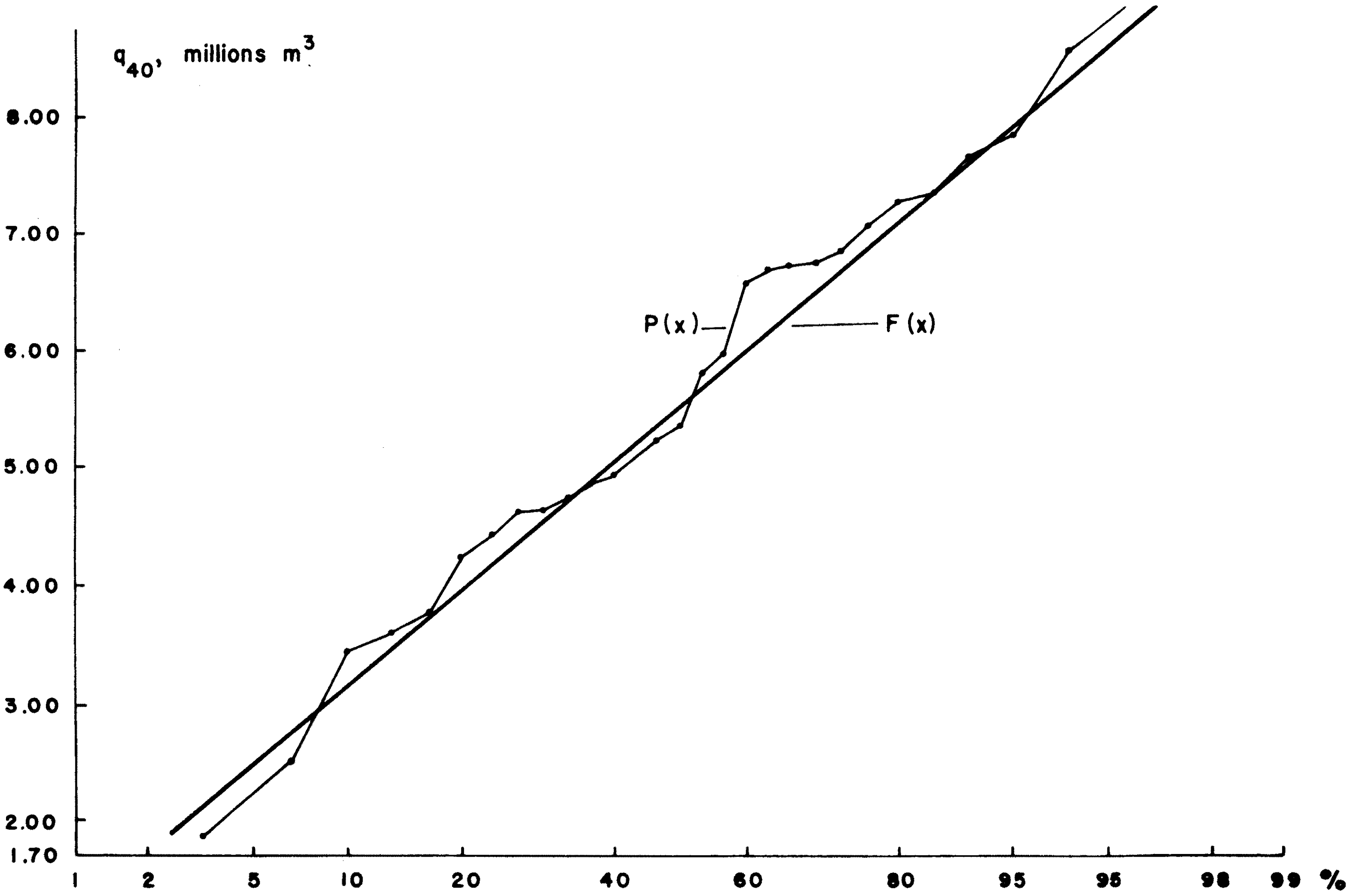


Fig. 2.17 d **Runoff: Probability Distribution of Weekly Flows –July 1, July 7.**

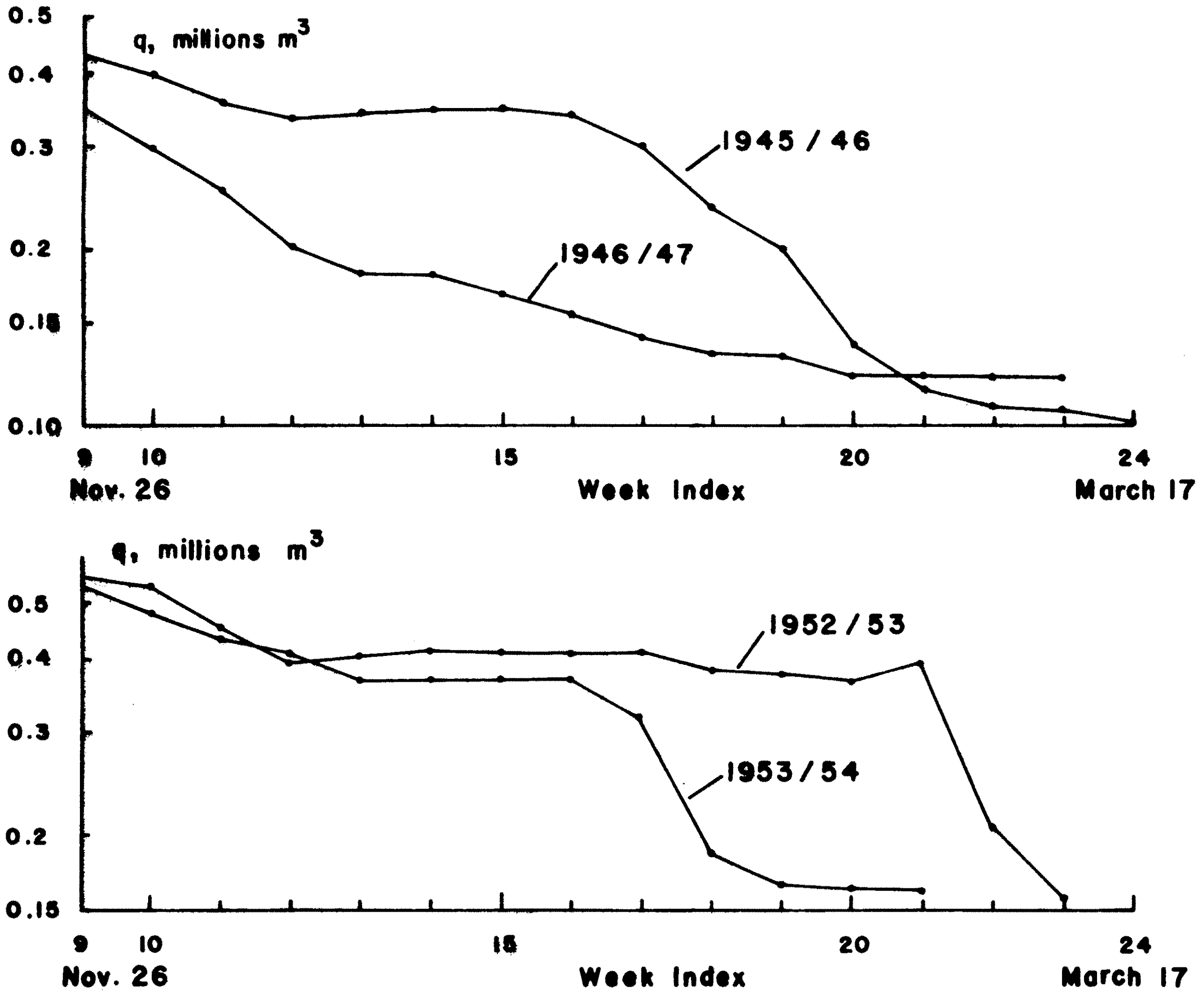


Fig. 2.18 a <u>Runoff: Recession Curve – 1945/46, 1946/47, 1952/53, 1953/54</u>

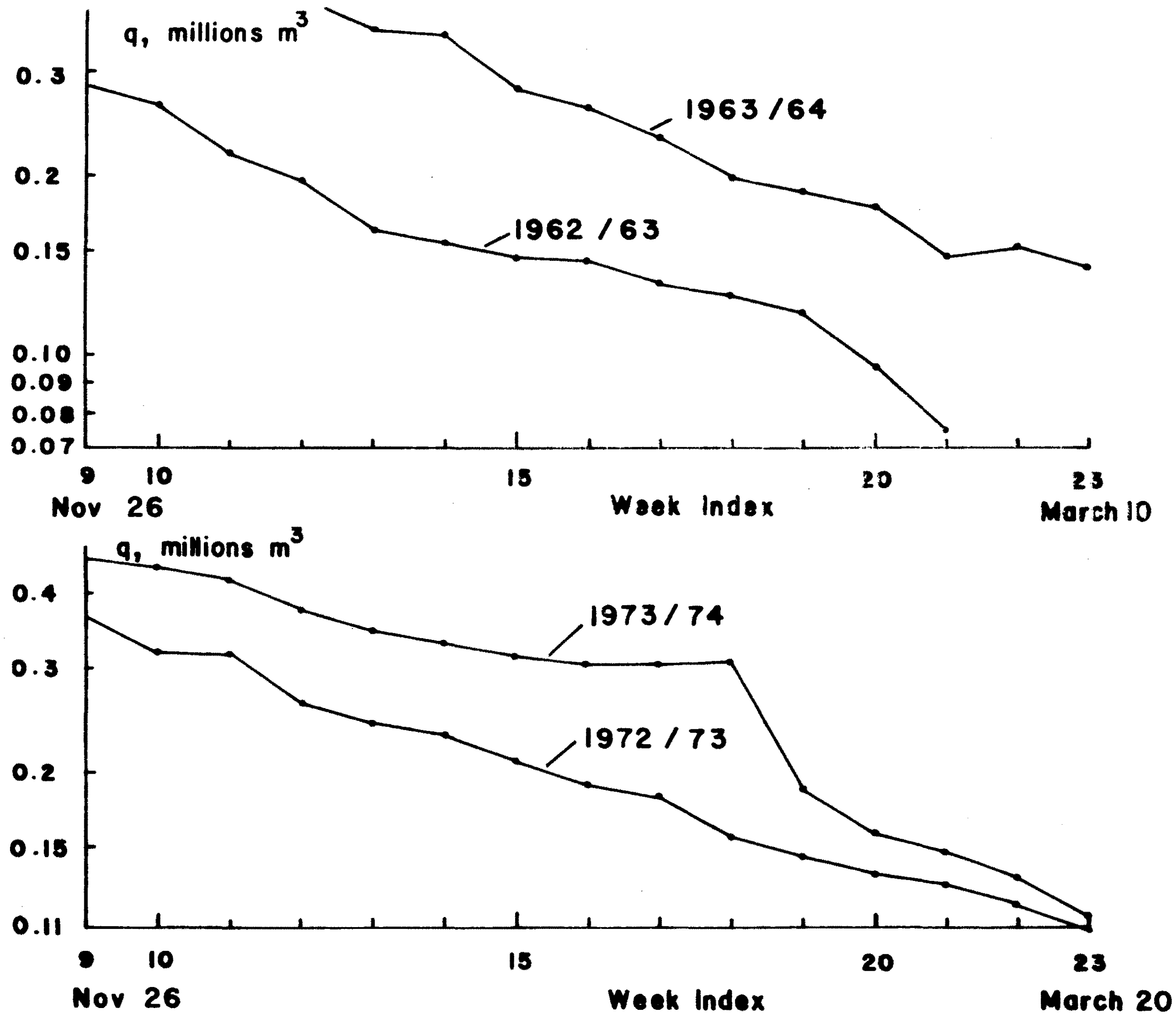


Fig. 2.18 b Runoff: Recession Curves - 1962/63, 1963/64, 1972/73, 1973/74

Hence, it seems difficult to establish reliable runoff forecast during winter, on the basis of this procedure. The amount of work implied would be considerable, and the resulting gain in information, small.

2.6.4 Date of minimum and maximum weekly flows. The minimum weekly flow occurs between mid-February and mid-March. Its magnitude ranges between 0.073 (2.58 million ft^3) and 0.301 million cubic meters (10.6 million ft^3). No relation seems to exist between the date of occurrence of this minimum and its corresponding magnitude.

The maximum weekly flow takes place between the middle of June and the middle of July. Its value oscillates between 7.0 and 8.0 million of cubic meters (24.8 and 28.2 million ft^3 respectively). Isolated peaks were registered in late August or September. Generally these peaks were of higher magnitude, from 9.0 to 11.0 million of cubic meters (31.8 and 39 million ft^3).

2.7 Water Balance Equations

2.7.1 Water cycle. Before establishing the relations which exist between the different variables involved in the runoff process, let us recall briefly the water cycle of an alpine watershed. If one considers the catchment as a closed system, precipitation is then its input, and runoff, its main output. However temperature completely controls the output rate.

In winter and until late into spring, the input occurs mainly as snow. After its deposition on the ground, snow undergoes different physical transformations, called ripening, which cause among others an increase in its density. In fact snowmelt cannot start before ripening has taken place.

Generally this happens in April, when the 0° Celsius isotherm has reached the lower end of the catchment. The area where snow is melting extends as temperature further increases. However snowmelt can be momentarily stopped by the occurrence of sudden cold spells. Nevertheless more than enough energy is available on the average is melt away continuously the snow of the lower ranges of the catchment. During that period, the input plays a minor role.

Although the 0° Celsius isotherm reaches the lower end of the glacier tongue already in May, glacier starts contributing appreciably to runoff only in summer. For the months of May and June are used to rebuild the glacier drainage system which was destroyed during the preceding winter. In summer, the input plays progressively a greater role, as the input is immediately melted, so that it reaches the channel system fast. Hence runoff is closely related to temperature.

What happens in September is quite complex. A series of cold days at the beginning of the month, stops glaciermelt definitively for the season, although high temperatures may be recorded later on. On the other hand, if it is constantly warm, the ablation may become quite extensive; as the drainage system already exists. Consequently not only the total energy input is of importance but also its distribution within the month. The same applies for precipitation. The elevation range of the watershed is such that no appreciable interannual storage of precipitation exists, except when it has snowed in the second half of September. In these situations the fallen snow reaches the river only in the following year.

Temperature starts decreasing in fall. The 0° Celsius isotherm comes down into the valley and the watershed is progressively frozen. Runoff diminishes parallel to this decrease in temperature. Only in isolated years a sudden warm spell caused a short increase in runoff. Winter flows result essentially from glacier ice melted through friction and earth heat.

Losses occur in two forms: infiltration and evapotranspiration. Here infiltration is not important and it is assumed that the annual change in underground storage is negligible.

The importance of evapotranspiration is also difficult to evaluate. Experimental data exist for lower elevated regions. These values were extrapolated for alpine watersheds, taking into account the lower annual average temperature and the smaller vegetation cover. On the other side, values computed from water balance equations appear in literature. Kasser (1965), for example, computed an average annual evapotranspiration of 350 mm for the alpine watershed of Mattmark (65 km^2, glaciation 39 percent, average elevation 2,850 m). Other authors found values around 200 mm. As of today it is not possible to determine exactly the value of this variable. Hence it will appear only implicitly in the subsequent computations.

2.7.2 Selected hydrologic model. Many methods exist to relate runoff with the variables involved in its formation. The nature of the phenomena, the purpose of the study and the available data makes one approach more attractive than other ones. Here a simple and robust technique is required.

The multiple linear-regression technique was selected after examination of different possibilities. In hydrology this statistical tool has been extensively used mainly because of its simplicity, its power and flexibility. It consists in investigating the relationship existing between a dependent variable, in our case, runoff, and a group of independent variables, in our case, precipitation, snowmelt and glaciermelt. The general equation of a multiple linear regression involving p independent variables is

$$Y = a + b_1 X_1 + b_2 X_2 + \ldots + b_p X_p \tag{2-3}$$

in which a is the intercept and $b_1, b_2, \ldots, b_p$, the regression coefficients. Furthermore, Y represents the runoff during a specified period, or the output of the system, and $X_1, X_2, \ldots, X_p$, the input. The period of analysis is arbitrary, but it should respect the physical nature of the phenomena. The comparison of inputs and outputs over a hydrologic year is the best choice, shorter periods, however, can also lead to good results.

As only some of the variables involved in the runoff process were retained, it was not possible to establish a deterministic relation between inputs and outputs. Hence the standard least squares method was used to estimate the intercept and the regression coefficients. The R, the correlation coefficient, measures the goodness of the selected model. Finally R^2, called the coefficient of determination, gives the percentage of the total variance of the output explained by the inputs.

Another way to test the goodness of the selected model is to compute $\hat{Y}$, the estimated output, on the basis of the derived equation, and to compare it with Y, the effective output. The difference of these two values appears often in the literature as ε, and is called residual or lack of fit:

$$\varepsilon = Y - \hat{Y} \tag{2-4}$$

The sum of the residuals from the available data sample must always be equal to zero. Residuals, however may show interesting statistical properties, which often give some clues to the nature of the studied phenomena. Generally the model can be considered as good if the residuals are randomly distributed.

In the subsequent paragraphs, regression equations were established on an annual, semiannual and quarterly basis.

2.7.3 Annual water balance. In the simplest model annual runoff, the output, was regressed against annual precipitation index, the input. The equation reads as follows:

$$Y = a + b_1 X_1 \tag{2-5}$$

where

Y = annual runoff, in millimeters

X_1 = annual precipitation index in millimeters

The computed correlation coefficient amounts to 0.83 (Table 2.20, Fig. 2.19, Fig. 2.20). This procedure implies that the influences of temperature and of glaciermelt are negligible, and that all the precipitation fallen during the hydrologic year appears as runoff in the same year.

The last assumption should be correct on the average. For about 72 percent of the catchment lies below 2,600 m. At this

Table 2.20

Water Balance Equation

Y mm	X_1 mm	X_2	a	b_1	b_2	Stand Error Percent	Corr. Coeff.
$Q_{10,9}$	$P_{10,9}$	-	666.61	1.125	-	7.5	0.83
$Q_{10,9}$	$P_{10,9}$	$\frac{4.5 \cdot A_{G\ell} \cdot T_{d,4,9}}{A}$	- 85.97	1.288	1.172	4.4	0.94
$Q_{4,9}$	$P_{10,9} - 2/3 \cdot Q_{10,3}$	$\frac{4.5 \cdot A_{G\ell} \cdot T_{d,4,9}}{A}$	- 94.03	1.265	1.166	4.8	0.95
$Q_{4,6}$	$P_{10,6} - 2/3 \cdot Q_{10,3}$	$T_{d,4,6}$	+155.51	0.563	1.759	7.6	0.86
$Q_{7,9}$	$P_{7,9}$	$\frac{4.5 \cdot A_{G\ell} \cdot T_{d,7,9}}{A}$	+ 87.80	1.157	1.194	12.3	0.81
$Q_{7,9}$	$P_{10,9} - 2/3 \cdot Q_{10,6}$	$\frac{4.5 \cdot A_{G\ell} \cdot T_{d,7,9}}{A}$	-150.63	1.198	1.463	8.0	0.92

Runoff and precipitation in millimeters

$Q_{i,j}$ = Runoff from i^{th} month to j^{th} month of calendar year

Standard Form of Equation: $Y = a + b_1 X_1 + b_2 X_2$

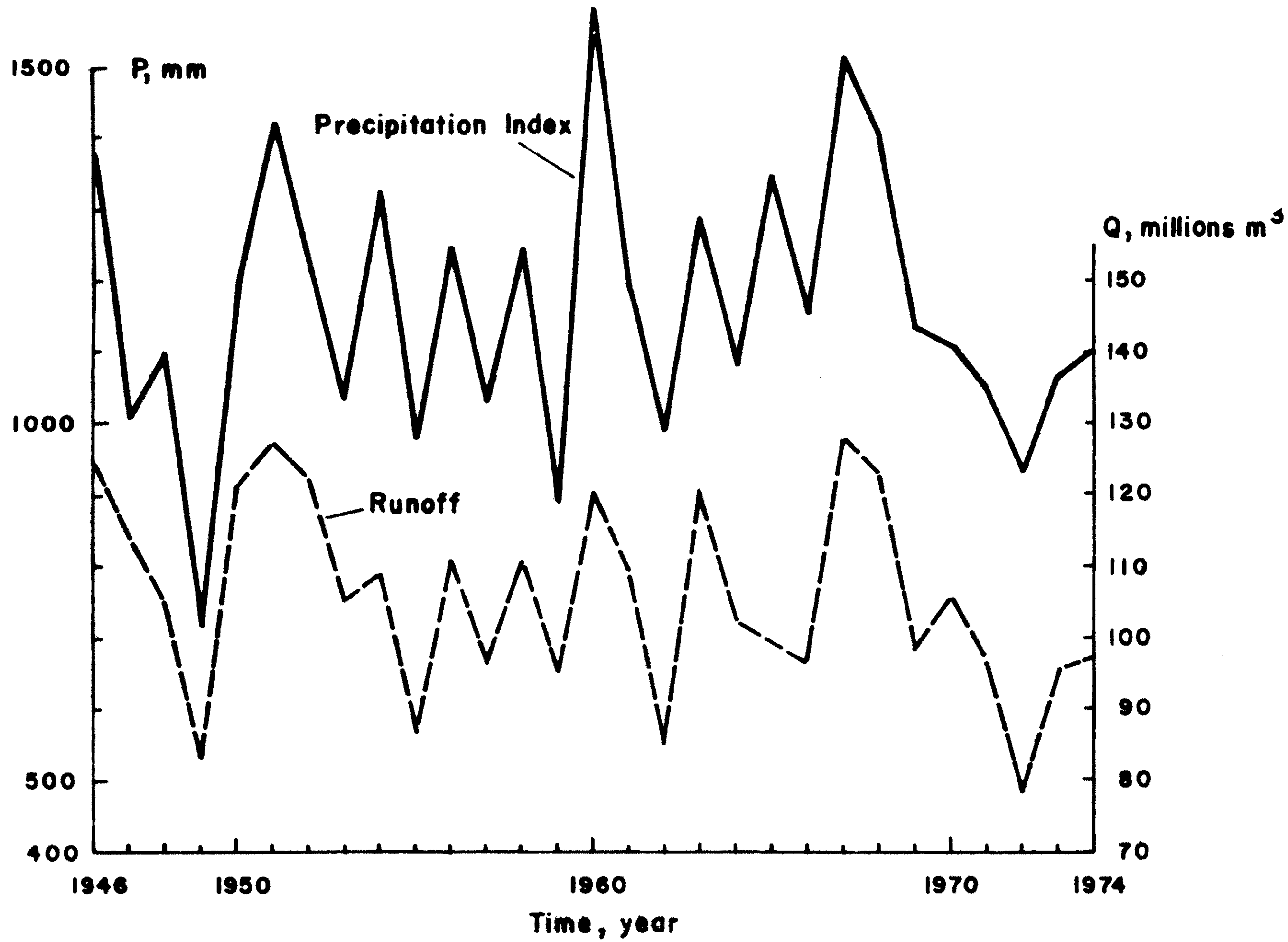


Fig. 2.19 Correspondence between Annual Precipitation Index and Annual Runoff

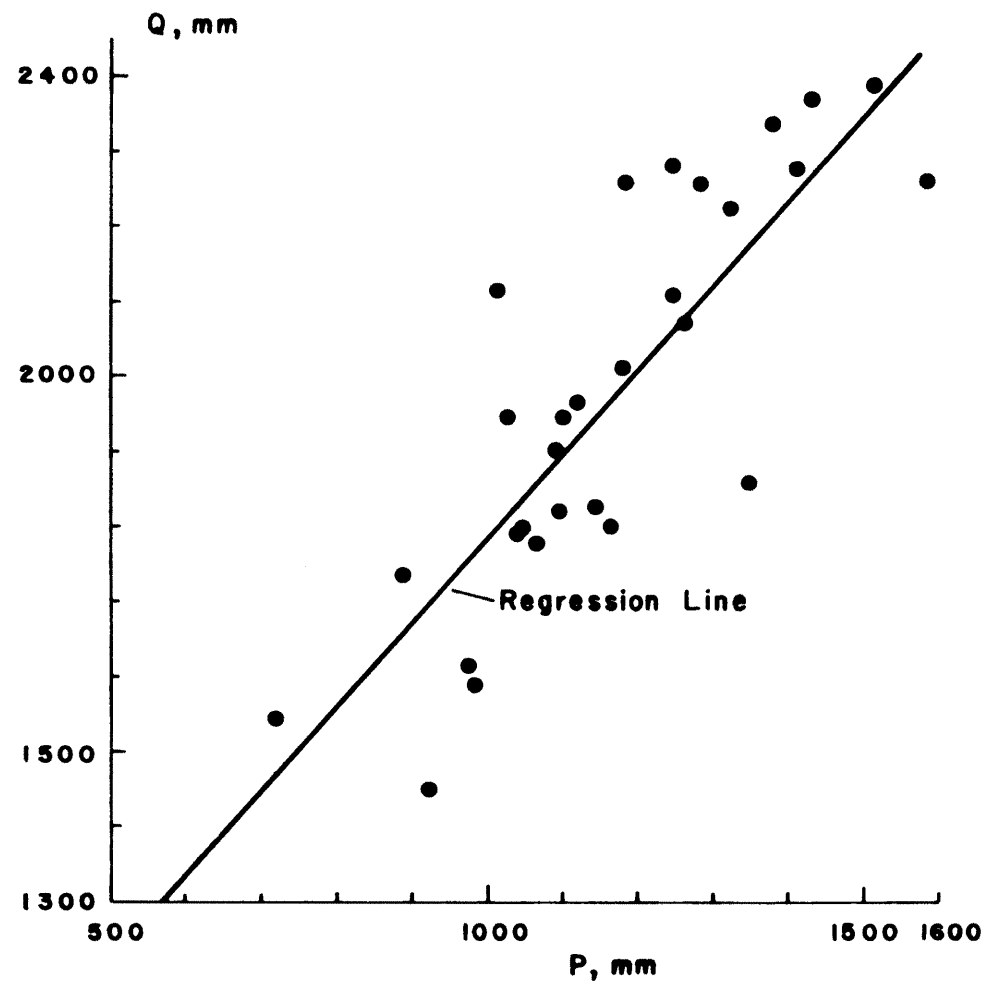


Fig. 2.20 Regression between Annual Precipitation Index and Annual Runoff

elevation, snow melts completely until middle of July, and appears again on the ground only in the middle of October. Hence perennial snow plays a minor role in this watershed. Only snow fallen in late September may sometimes not have time to melt completely until the end of September.

To test the first assumption, the residuals were plotted in different ways. A systematic trend appeared when the residuals were plotted as a function of time. The values of the earlier years were systematically positive, the ones corresponding to the recent years, systematically negative (Fig. 2.21). This may have been caused by the decrease in glaciated area.

In the final model, an additional independent variable which represents the contribution to runoff of the glaciers was introduced into the equation. It is defined as follows:

$$X_2 = 4.5 \cdot \frac{A_g \ell}{A} \cdot T_d \qquad (2\text{-}6)$$

where

X_2 = annual amount of meltwater produced by the glaciers in millimeters

4.5 = degree day factor

$A_g \ell$ = glaciated area of the catchment

A = catchment area

T_d = number of degree days from April to September

Three points should be mentioned concerning this expression. The glaciated area changes each year, according to the figures given in Table 2.8. The exact value of the degree day factor is not known; the selected number corresponds to the most frequently encountered

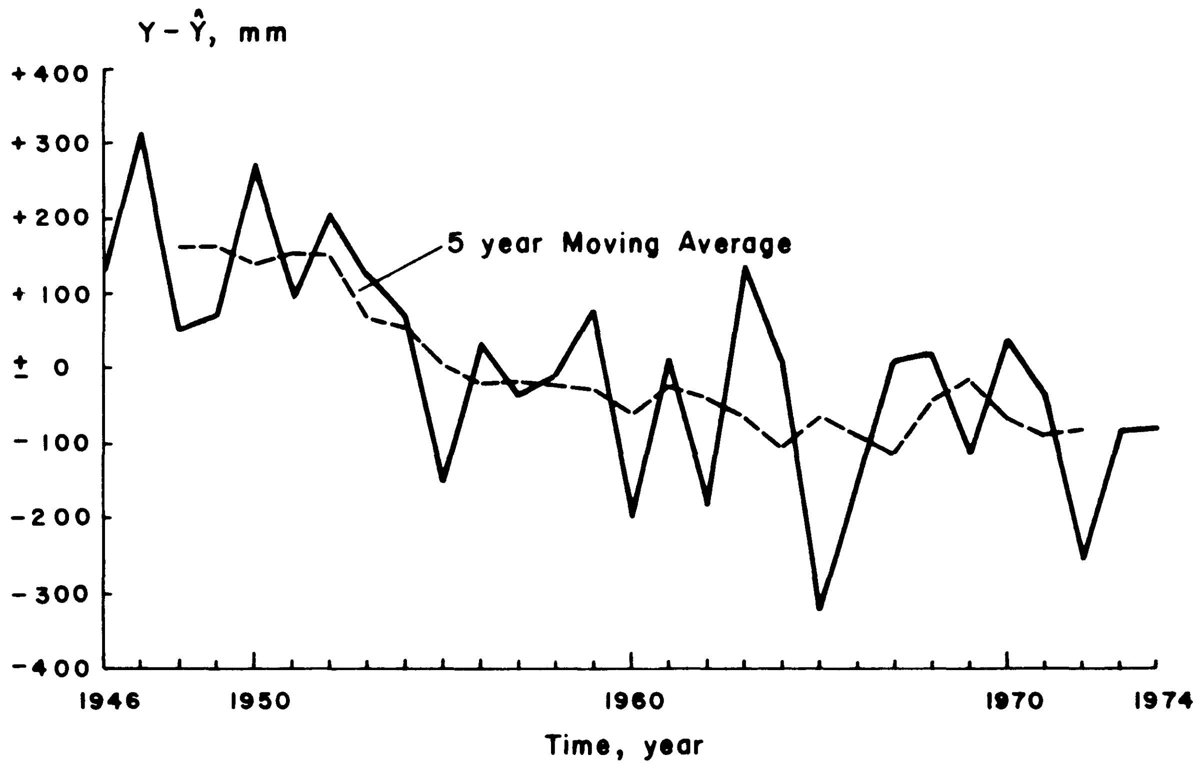


Fig. 2.21 Precipitation-Runoff Model, Plot of Residuals for Annual Values

value in the literature. And finally it was assumed that the whole glacier was contributing to snowmelt.

The regression equation reads as:

$$Y = a + b_1 X_1 + b_2 X_2 \tag{2-7}$$

The computed correlation coefficient amounts to 0.94; a net improvement from the previous case (Table 2.20). Figure 2.22 shows the residuals plotted as a function of time. The previously noticed trend no longer exists. However the plot of the five year moving average scheme revealed another one of smaller amplitude and period.

To explain this new pattern, the years with greatest negative and positive residuals were analyzed separately. The recorded greatest negative residuals corresponded essentially to two categories of years. In the first category, large amounts of snow had fallen at the very end of September, which could not melt till the end of the hydrologic year. In the second category, a long series of cold days had happened at the beginning of September, which definitively stopped the glaciermelt. On the other hand, the highest positive residuals coincided with the years where the temperature recorded in September was continuously high. This led to an excessive glaciermelt.

As a summary, precipitation index and the sum of degree days explain well the recorded variations in runoff. The obtained precision is sufficient and the computed residuals, quite low. With this approach, however, it is not possible at all to model the complex phenomena happening in September. This would require a lot more data, and is beyond the scope of this study.

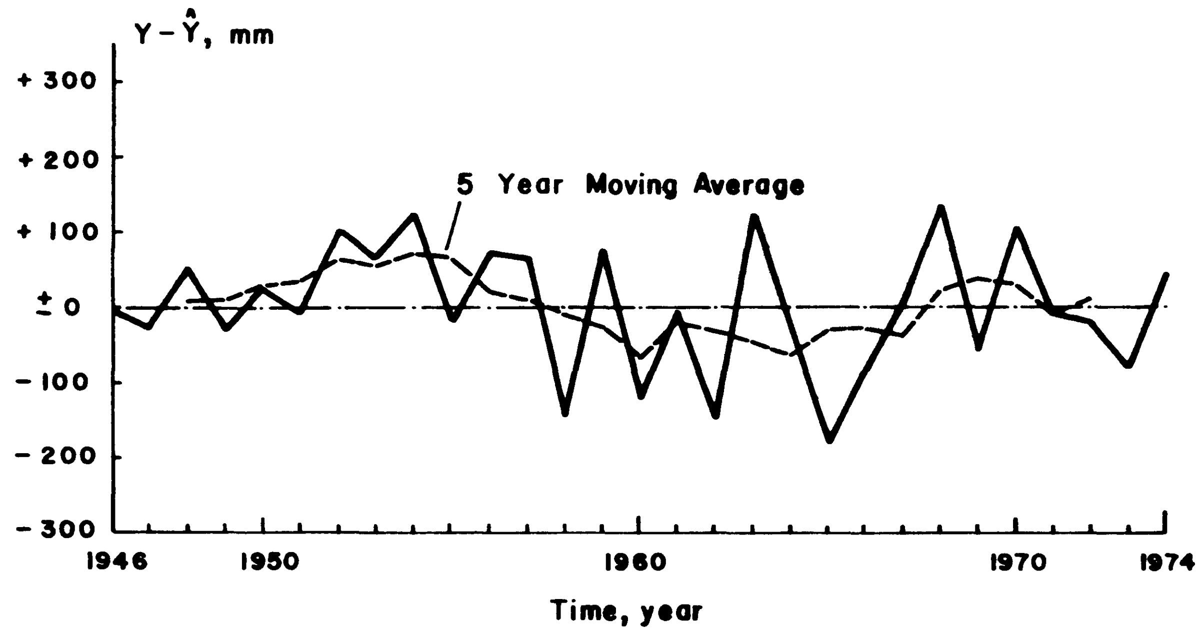


Fig. 2. 22 <u>Precipitation – Glaciermelt – Runoff Model, Plot of Residuals</u>

<u>Annual Values</u>

2.7.4 Seasonal and quarterly water balances. The analyses of the preceding section indicated that reliable relations exist between annual precipitations, glaciermelt and runoff. The same approach with minor modifications was used to establish relations on a seasonal and quarterly basis.

Seasonal runoff is defined as the flow taking place between April 1 and the end of September. The main difficulty lies in the determination of the amount of water stored as snow in the watershed on April 1. Theoretically it is the difference between winter precipitation and winter runoff. However the exact areal winter precipitation depth is not known. It must amount approximately to 1.5 times the computed precipitation index. Hence, to avoid the occurrence of negative storage values, winter flows were first divided by 1.5 and then subtracted from winter precipitation. Note that the least squares method is rather insensitive to variations in the value of this factor.

The regression equation reads as follows:

$$Y = a + b_1 X_1 + b_2 X_2 \qquad (2\text{-}8)$$

where:

Y = runoff from April to September in millimeters

X_1 = precipitation index from April to September increased by the amount of water stored as snow in the catchment as of April 1 (Table 2.20)

X_2 = contribution from glaciermelt (Table 2.20)

The computed correlation coefficient amounts to 0.95 (Table 2.20). The plot of the residuals versus time is similar to the one corresponding to the annual balance equation (Fig. 2.23).

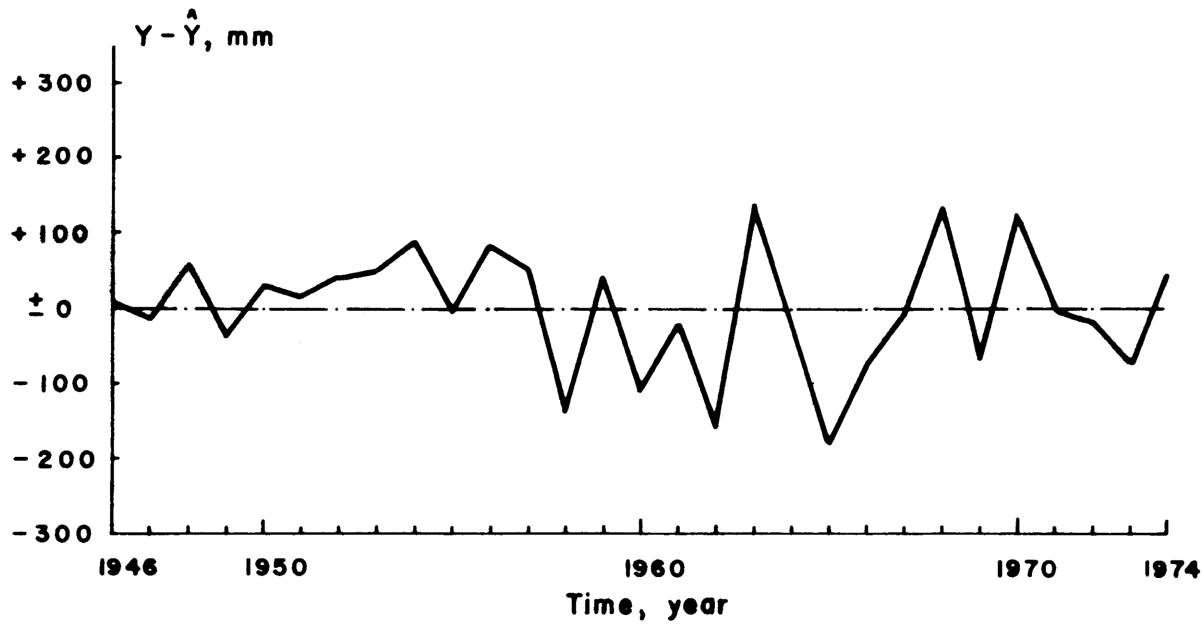


Fig. 2.23 Precipitation – Glaciermelt – Runoff Model, Plot of Residuals for Summer Values

From April to June, runoff results mainly from snowmelt. Consequently the definition of the second independent variable was modified, as it is not known which part of the watershed really contributes to snowmelt:

$$X_2 = 4.5 \cdot T_d \tag{2-9}$$

where

T_d = number of degree days from April to June

And the equation reads as follows:

$$Y = a + b_1 X_1 + b_2 X_2 \tag{2-10}$$

with X_1 = water stored as snow in the catchment as of April 1, increased by the precipitation index from April to June (Table 2.20)

X_2 = contribution from snowmelt

The correlation coefficient is equal to 0.86 (Table 2.20). The end of June is an arbitrary cutting-point, hence the lower correlation coefficient. On the average, snowmelt is still going on, and glaciermelt has already started. Surprisingly, the plot of residuals versus time no longer exhibits the trend noted in the other cases (Fig. 2.24). This seems to prove that the trend is strongly connected with the phenomena occurring in the second half of summer.

The last computed regression deals with the flows of July to September. In a first attempt, runoff was related to the precipitation and the sum of degree days of the period considered. A low correlation coefficient resulted from this procedure. This confirms that at the beginning of July, not all the snow has melted, and

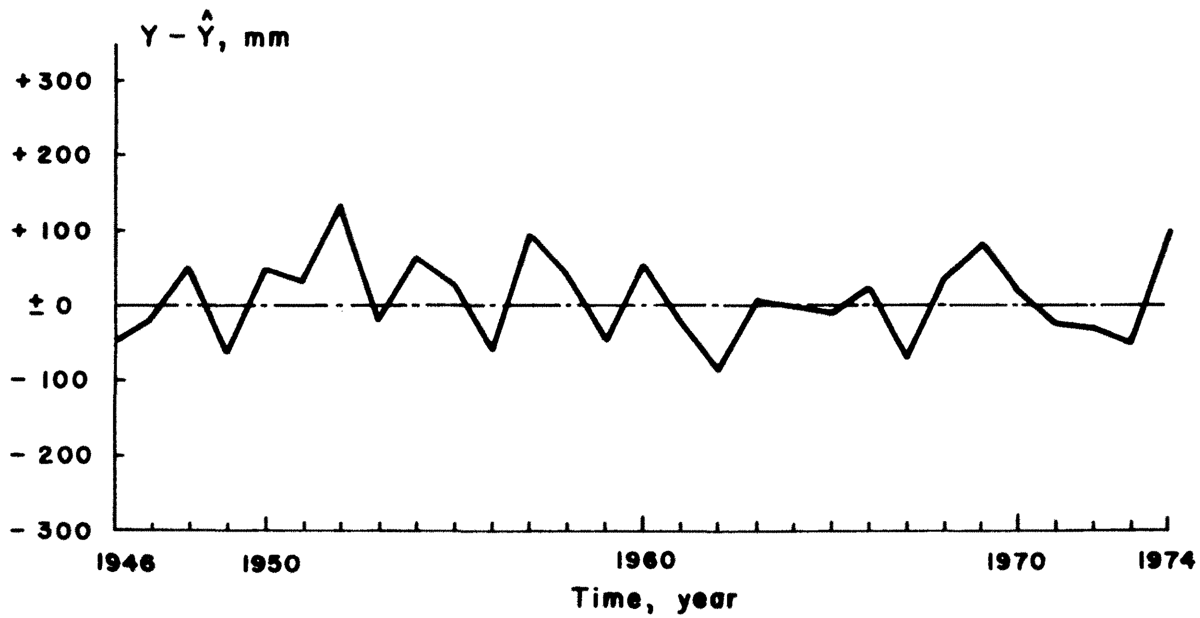


Fig. 2.24 Precipitation - Glaciermelt - Runoff Model, Plot of Residuals for Quarterly Values - April, June

seasonal storage plays still an important role. Hence the following terms were considered:

$$Y = a + b_1 X_1 + b_2 X_2 \tag{2-11}$$

where:

Y = runoff from July to September

X_1 = water stored in the catchment as of July 1 increased by the precipitation index from July to September (Table 2.20)

X_2 = contribution from glaciermelt (Table 2.20)

The computed correlation coefficient amounts to 0.92 and the plot of the residuals versus time shows again the trend noticed for the annual flow (Fig. 2.25, Table 2.20).

2.7.5 Discussion of results. Considering the nature of the problem and the small amount of data available, the performance of the selected model is very good. Difficulties appeared only with the modeling of the runoff phenomena in September.

On an annual basis, precipitation explains 69 percent of the runoff variance. Hence it plays the most important role, followed by temperature and change in glaciated area. For the period of April to June, however, temperature comes first, while precipitation has a minor influence and glacial contributions are not significant at all. Most of the runoff comes from storage. The situation changes again in the summer quarter. Precipitation and glaciermelt become determinant, as at that date of the year, most of the storage has been exhausted. The weekly and monthly autocorrelation

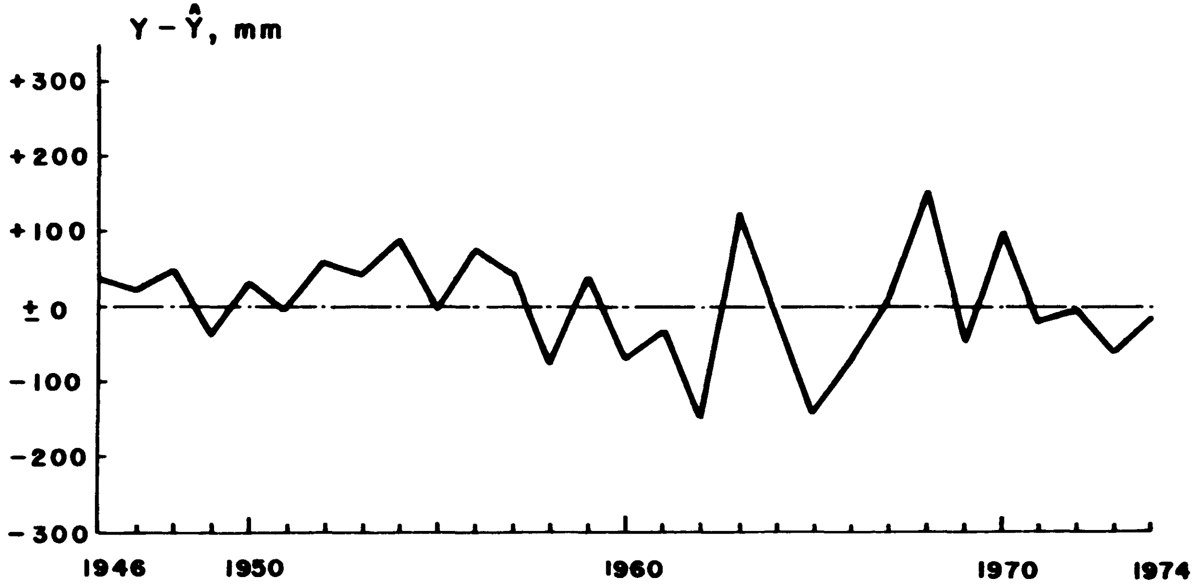


Fig. 2.25 Precipitation - Glaciermelt - Runoff, Plot of Residuals for Quarterly Values.-July, Sept

coefficients vary within the the year according to a pattern which confirms the preceding remarks.

In spring, the weekly autocorrelation coefficient decreases with time because the relative contribution of base flow to runoff diminishes. Also temperature becomes more and more a critical parameter, as snowmelt reaches higher elevations where the prevailing climate is much cooler. The opposite trend recorded for monthly values result essentially from the inadequacy of the calendar year divisions to model some hydrologic phenomena. In March and during the first half of April, discharge is still low; it starts increasing in the second half of April. The magnitude of this increase changes from year to year. In fact May is the first month where snowmelt has been going on continuously. Hence the highest correlation coefficient is obtained between the months May and June. Its negative sign indicates that during these two months, runoff originates essentially from storage. Depending on the rate of snowmelt in May, more or less stored water is available for the runoff in June.

In summer, temperature controls the contribution of glaciermelt. The weekly variations in temperature during this period may be quite important, hence the low autocorrelation coefficients. On a monthly basis however, some of these variations are smoothed out. Furthermore the increase with time in the monthly autocorrelation coefficients reflects the inertia of the glaciermelt process.

2.8 Runoff Forecast Models

2.8.1 Preliminary remarks. It seems reasonable to assume that the knowledge in advance of the reservoir inflows can increase the efficiency of its operation. In fact, depending on the period of year, seasonal, quarterly, monthly and weekly flow forecasts may be needed. This will be shown in the next chapters. For the time being it is sufficient to know that different types of forecast are required.

The approach followed to forecast runoff changes from case to case. A method adequate for seasonal flow forecast may completely fail for weekly flow forecast, and vice-versa. Hence to be complete, many situations should be studied, which is beyond the scope of this reasearch. However, here, only the most typical cases were retained and analyzed.

2.8.2 Runoff forecasts for a duration of three months and more. One needs this type of forecast for the summer semester. The water balance equations established in Section 2.7 supply all the required information. According to Eq. 2-3, the general equations reads as:

$$Y = a + b_1 X_1 + b_2 X_2 + b_2 X_2 + b_3 X_3 + \ldots b_p X_p \qquad (2\text{-}12)$$

where Y represents now the runoff during the period of forecast, starting from the date the forecast is done, like 3,4,..., or 6 months, and $X_1, X_2, \ldots X_p$ different physical variables related to the runoff process, like precipitation and temperature. At the date the forecast is done, some of the variables of Eq. 2-12 are known, some are not. The unknown variables appear with their expected value, that is as a constant. The constant terms can be regrouped. If, for example, one assumes that X_3 to X_p are not known at the date the forecast is done, the modified equation reads as follows:

$$Y = a_o + b_1 X_1 + b_2 X_2 \tag{2-13}$$

with

$$a_o = a + b_3 \bar{X}_3 + \ldots + b_p \bar{X}_p \tag{2-14}$$

As in Section 2.7, the reliability of the forecast model is obtained by computing $\hat{Y}$, the estimated output, on the basis of the derived equation, and to compare it with Y, the real output. The difference between these two values is called ε, the residual or lack of fit. But the residuals play also another important role. They supply all the information which is needed to generate synthetic data. For this purpose one must know the standard deviation and the statistical distribution followed by the residuals. The two examples presented hereafter illustrate the just developed theory.

The first example deals with the runoff forecast for the period April 1 to September 30. If the annual precipitation is divided up into a winter and a summer component, the water balance equation becomes:

$$Y = a + b_1 X_1 + b_2 X_2 + b_3 X_3 \tag{2-15}$$

where

Y = runoff from April 1 to September 30

X_1 = winter storage, or winter precipitation index minus a fraction of winter runoff

X_2 = summer precipitation index

X_3 = number of degree days from April 1 to September 30

However as of April 1, neither summer precipitation, nor number of degree days are known, so that these two variables appear in the regression equation with their expected value:

$$Y = a + b_2 \bar{X}_2 + b_3 X_3 + b_1 X_1 \tag{2-16}$$

The constant terms are grouped under a_o so that the final equation is

$$Y = a_o + b_1 X_1 \tag{2-17}$$

The results appear in Table 2.21. The correlation coefficient amounts to 0.46, which is rather low. According to Figure 2.26 the residuals follow a normal distribution.

The second example deals with the forecast of the runoff for the period July 1 to September 30. Here again, the precipitation index is divided into two components, the first one representing the precipitation from October to June 30, and the second one, the precipitation from July 1 to September 30. The modified water balance equation is then:

$$Y = a + b_1 X_1 + b_2 X_2 + b_3 X_3 \tag{2-18}$$

where

Y = runoff from July 1 to September 30

X_1 = water stored as precipitation in the watershed, or precipitation index from October to June 30 minus a fraction of the runoff during the same period

X_2 = precipitation index from July 1 to September 30

X_3 = number of degree days for July 1 to September 30

Table 2.21

Runoff Forecast Equations

Y mm	X_1 mm	X_2 mm	a_o	b_1	b_2	Corr. Coeff.
$Q_{4,9}$	$P_{10,3}{}^{-2/3} \cdot Q_{10,3}$	-	1434.13	1.224	-	0.46
$Q_{4,6}$	$P_{10,3}{}^{-2/3} \cdot Q_{10,3}$	-	594.71	0.546	-	0.42
$Q_{7,9}$	$P_{10,6}{}^{-2/3} \cdot Q_{10,6}$	-	902.21	1.028	-	0.38
$Q_{7,9}$	Q_6	Q_5	617.51	9.851	12.719	0.30
$Q_{8,9}$	Q_7	Q_6	172.07	15.270	2.253	0.45
Q_9	Q_8	Q_7	3.166	6.760	3.650	0.56
Q_4	q_{26}	q_{25}	27.877	188.212	-123.054	0.59
$\ell n q_1$	$\ell n q_{52}$	$\ell n q_{51}$	- 8.287	10.615	9.907	0.74
$\ell n q_{27}$	$\ell n q_{26}$	$\ell n q_{25}$	4.320	23.613	- 5.698	0.84

Runoff and precipitation in millimeters

$Q_{i,j}$ = runoff from i^{th} to j^{th} month of the calendar year

q_i = weekly runoff during i^{th} week of the water year

Standard Form of Equation:

$$Y = a_o + b_1 X_1 + b_2 X_2$$

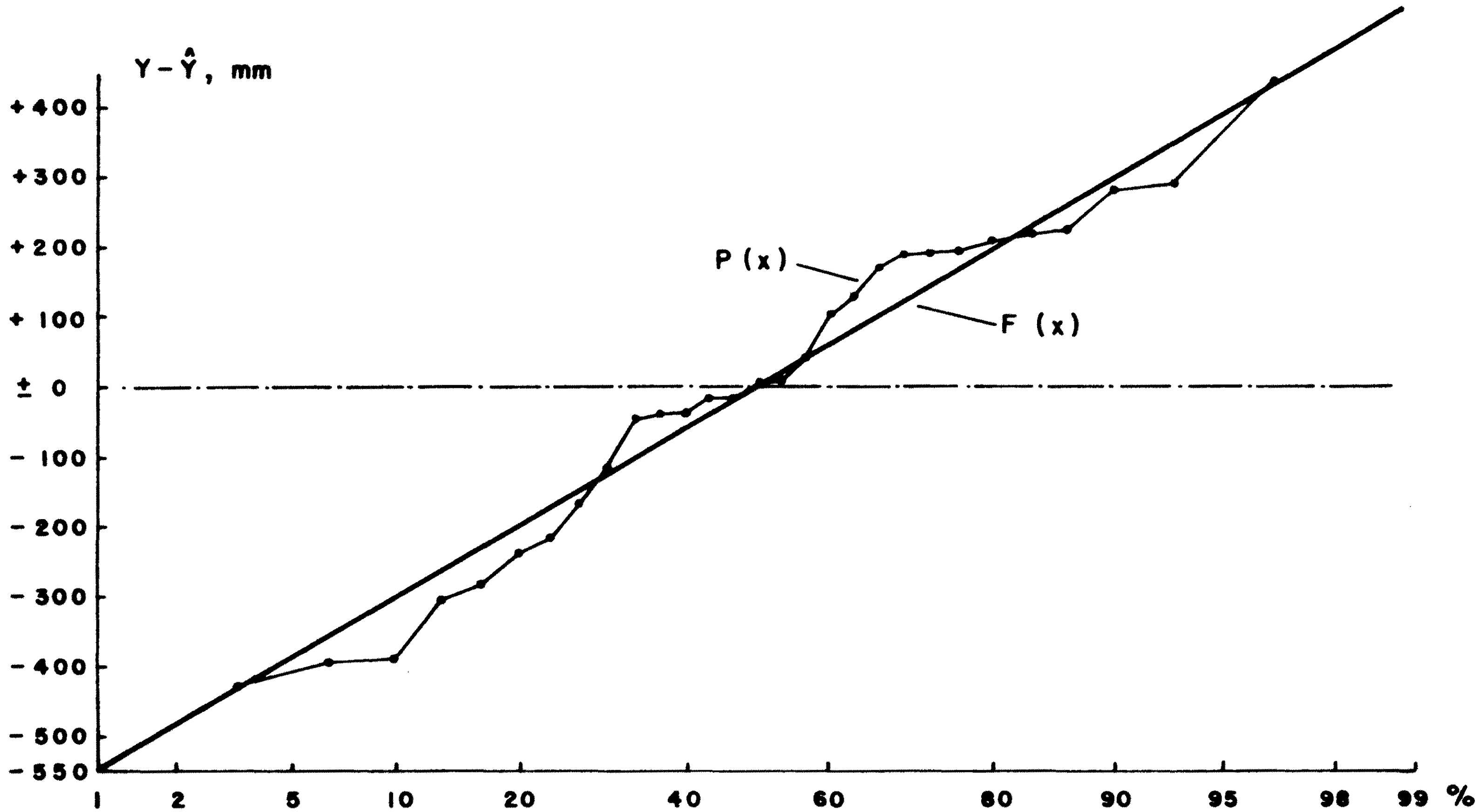


Fig. 2.26 <u>Semi - annual Flow Prediction: Residual for Period April, September</u>

As of July 1, the summer quarter precipitation and the corresponding number of degree days are not known. These two variables enter the equation with their expected value, so that we have now:

$$Y = a_o + b_1 X_1 \tag{2-19}$$

The correlation coefficient amounts to only 0.38 (Table 2.21) which shows the limits of this approach. The residuals are normally distributed (Fig. 2.27).

2.8.3 Runoff forecasts for periods of one, two and three months. The approach based on the water balance equation performed poorly for quarterly runoff forecasts. Different facts led to this poor result. First, the selected period, July 1 to September 30 does not correspond to a well-defined phase of the runoff cycle. This implies that the identification of the state of the watershed as of July 1 is complex. In fact it requires a large amount of data coming directly from the watershed itself; and these data are not available. Second, at that period of the year, the amount of water stored as snow in the catchment is small.

The approach followed here takes into account that linear relations exist between the successively recorded monthly flows. The general equation which expresses these relations reads as

$$Y = a + b_1 X_1 + b_2 X_2 \tag{2-20}$$

where

Y = monthly, bimonthly, or quarterly flow to be forecasted

X_1 = runoff during the month preceding the date of the forecast

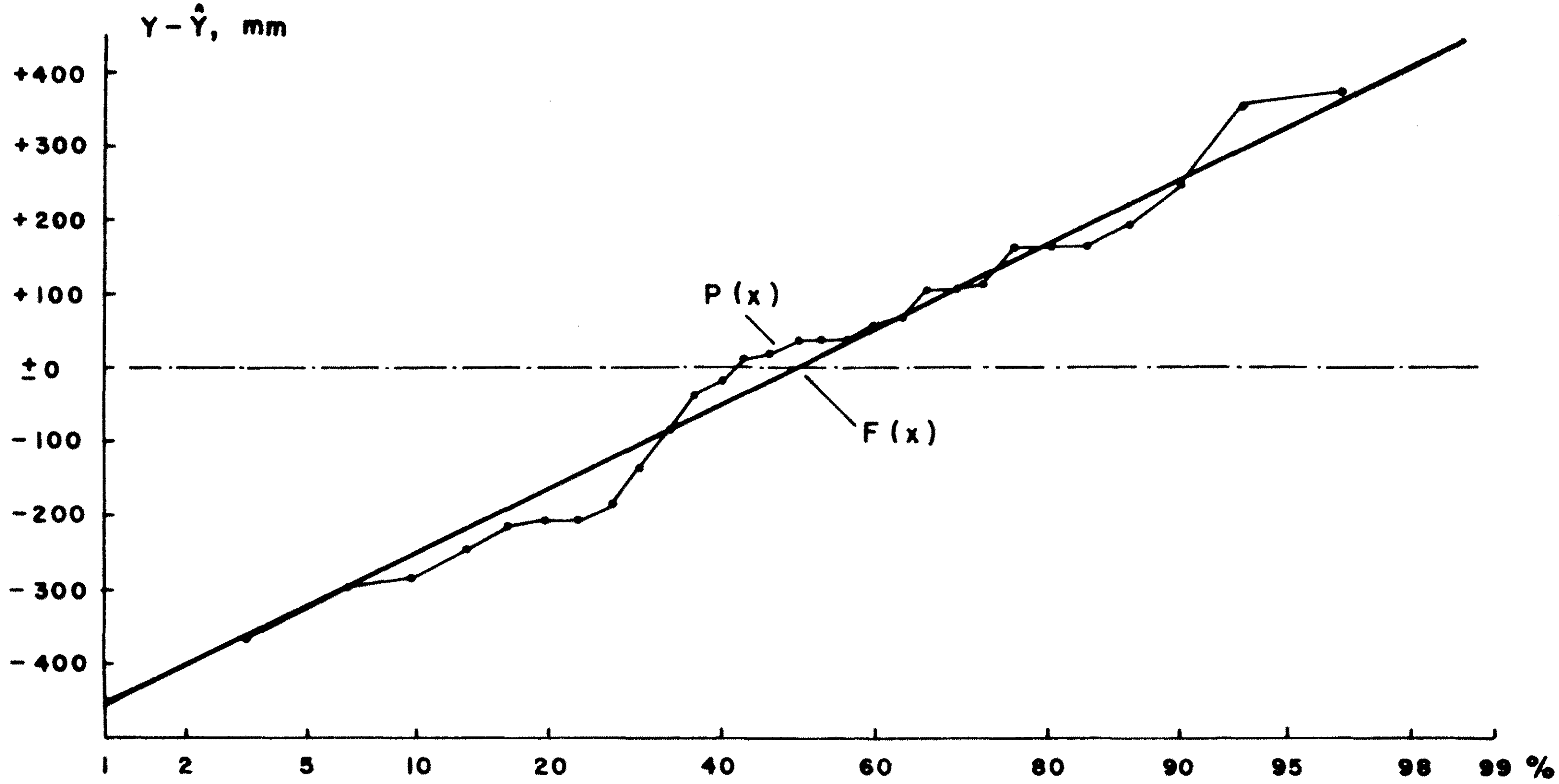


Fig. 2.27 **Quarterly Flow Prediction: Residuals for Period July, September**

X_2 = runoff during the month preceding the month which corresponds to the variable X_1.

The terms, a, b_1 and b_2 are determined by the least squares method as for the water balance equations. Finally residuals can also be computed as this relation is established on a statistical basis.

With this method, forecast for the runoff from July 1 to September 30, from August 1 to September 30, and for September were established. From Table 2.21 one can see that the correlation coefficients vary between 0.30 and 0.56. The residuals follow a normal distribution.

The forecast for the runoff of April represents a special case. It is based on the flows occurring during the last two weeks of March (Table 2.21).

2.8.4 Runoff forecast for weekly flows. The performance of the method based on monthly flows improved as the length of the forecast period decreased. It seems then logical to extend this approach to weekly flows forecast. The general equation is:

$$Y = a + b_1 X_1 + b_2 X_2 \tag{2-21}$$

where

Y = natural logarithm of the weekly flow to be forecast

X_1 = natural logarithm of the runoff during the week preceding the date to the forecast

X_2 = natural logarithm of the runoff during the week preceding the week corresponding to variable X_1.

In this case the logarithms are used because the weekly runoff is lognormally distributed. This approach was applied for the forecast of weekly runoff in April and in October. In both cases, the residuals are normally distributed, and the correlation coefficient, quite high (Fig. 2.28, 2.29, Table 2.21).

2.9 Final Remarks

For the basin under study, precipitation, snow and glaciermelt form the inputs, while temperature completely controls runoff, the output. Runoff is concentrated in summer when temperature is highest. The variations in discharges during September, which is partly responsible for the interannual runoff fluctuations, result also from temperature.

Water balance equations relating runoff to precipitation, snow and glaciermelt led to good results. This fact confirms the validity of the assumption of linearity between the effects of the retained variables. Furthermore the water balance equations allowed the establishment of flow forecast equations for the summer season. However for short periods, the flow forecast equations should be based on the Markov property of the runoff process. This approach worked quite well for the forecast weekly flows.

The information gained in this chapter on runoff phenomena will be used to select and develop the reservoir operation model.

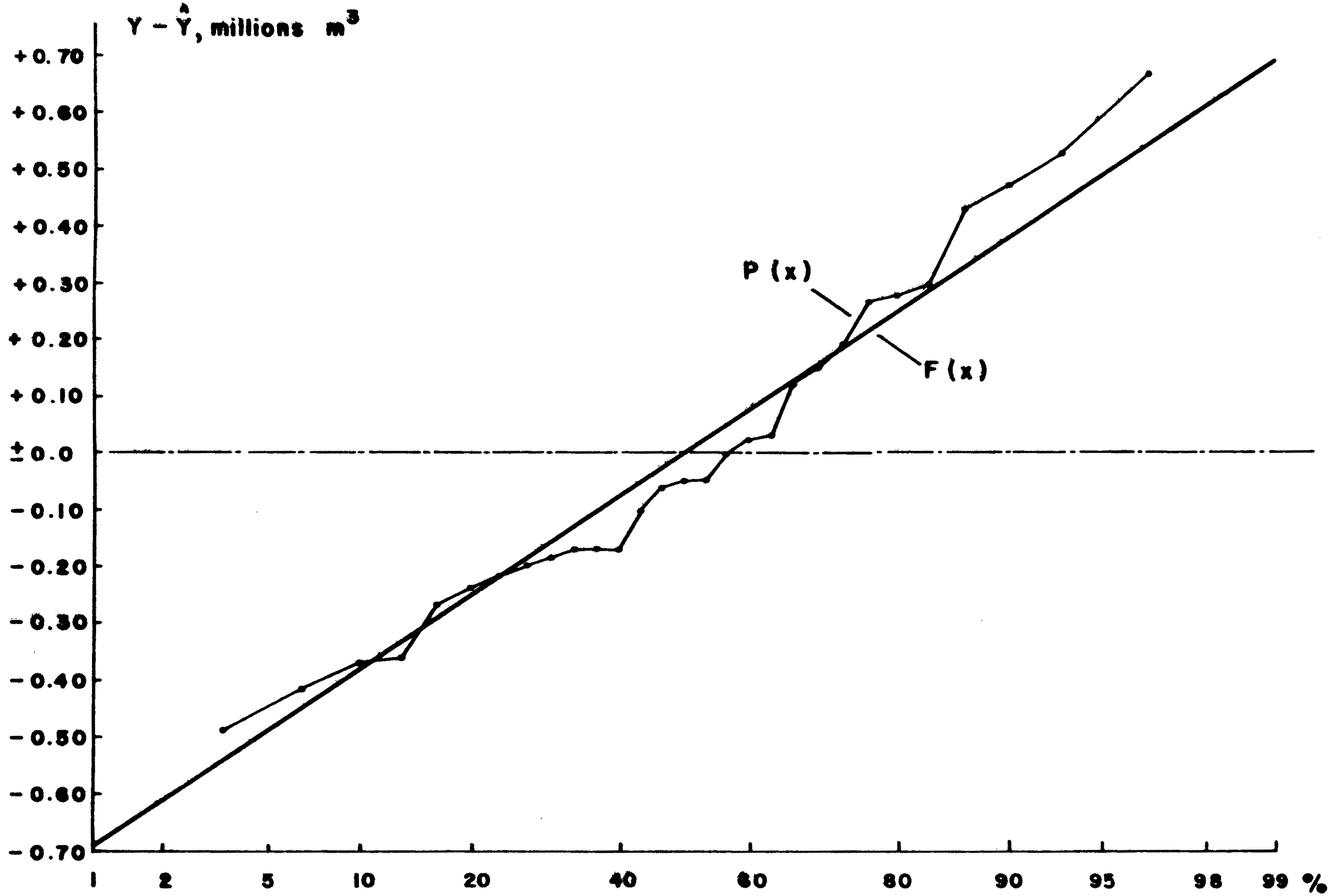


Fig. 2.28 Weekly Flow Prediction: Residuals for April 1, April 7

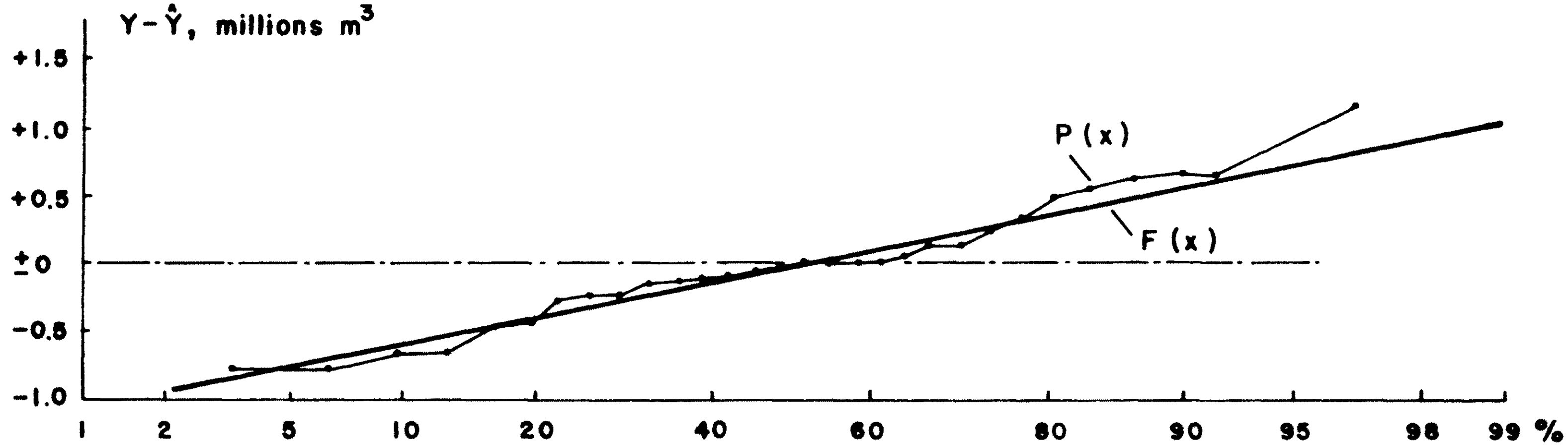


Fig. 2.29 Weekly Flow Prediction, Residuals for Oct 1, Oct 7

Chapter 3

ECONOMICS OF POWER PRODUCTION

This chapter deals with the specification of the technical and economical characteristics of the power plant selected for the case study. It begins with the analysis of the Swiss power demand curve. After a brief description of the existing Swiss electrical network, a typical storage scheme is selected. The chapter ends with the determination of the energy price function and of the related return function.

3.1 Analysis of the Demand Curve

Electric energy plays an important role in Switzerland. In the water year 1973-74, public and private utilities supplied 34,095 GWh, which represents about 17 percent of the total gross annual primary energy consumption.

The existing statistics on energy consumption classify the users into seven categories. Category 1 encompasses domestic users and the service sector. It is a heterogeneous sector which contains the following groups: households, schools and hospitals, office buildings, stores and warehouses, factories with less than 20 employees, and small farms. The other categories are more homogeneous. Hence category 2 groups the general industries, while the electrical, thermal and chemical industries can be found in category 3. The railways appear under category 4, the electrical boilers, under category 5, the pumped storage schemes under category 6 and the losses under category 7.

According to Table 3.1, the households and the services sector represent the greatest users with 47 percent of the total annual consumption. They are followed by the general industries, with 19 percent, and by the electrical, thermal and chemical industries, with 14 percent of the annual total consumption.

To analyze the monthly variations of the energy consumption, it is sufficient to consider two groups. The first one corresponds to the first category, and the second one to all the other categories. In the first group, energy is mainly used for lighting, for heating, and for small electric appliances. Accordingly, the highest demand occurs in winter, from December to January, and the lowest demand, in summer from July to August. The differences between the minimum and the maximum monthly consumption amounts to roughly 30 percent of the average annual consumption. For the second group, energy is used to produce mechanical energy and the electric consumption is related to the industrial activity of the country. The annual low is reached during the summer vacations, and the maximum in winter. The overall variations, however are small and stay within a bandwidth of 20 percent of the average annual value. The aggregate variation in total energy consumption follows the same pattern during the year. On the whole it is higher in winter, and smaller in summer. If 100 is taken as the annual average, the monthly consumption varies between 91 and 108.

The industrial activity is not constant during a month, not even during a week or a day. On Saturdays and Sundays, factories are closed, and during the day, there are working hours. Hence the

Table 3.1

Electrical Energy Consumption in Switzerland for the Hydrologic Year 1973/74. Data taken from the Bulletin SEV/VSE, 66, 3, 8 February 1975

Category	Group	Consumption in GWh	Consumption in %
1	Household + Services	15,998	47
2	General Industries	6,662	19
3	Electr., Chemical, Thermal Industries	4,655	14
4	Railways	2,001	6
5	Electric Boilers	55	-
6	Pumped Storage Schemes	1,613	5
7	Losses	3,111	9
	Total	34,095	100

energy demand can be characterized by a curve with annual, monthly, weekly and daily cyclic components. This fact plays an important role in the production of the energy itself as well as in the determination of the prices at which the produced energy is sold.

3.2 Electricity Production System

Different types of power stations are in operation to produce most efficiently the energy required by the consumers: run of river, storage, pumped storage, thermal and nuclear power plants. Table 3.2 shows their relative importance within the Swiss network. Note that pumped storage developments do not figure separately. Hence 78 percent of the produced annual energy comes from hydropower. Of these 78 percent, 49 percent is supplied by run of river plants, and 29 percent by storage schemes.

Table 3.3 shows the production pattern on a seasonal basis. The figures for the run of river plants call for no special comments. As expected, they produce less power in winter than in summer. Concerning the thermal and nuclear power plants, one should remember that, as there is an excess of power supply in summer, all the maintenance operations on these schemes are scheduled, whenever possible for that period. The values obtained for the storage schemes are a little surprising, at least at the first glance. In fact they result from the limited storage capacity of the reservoirs. Water has to be released in summer to avoid spillage.

Monthly production figures for the storage schemes appear on Table 3.4. They are high in winter and during snowmelt, and low in

Table 3.2

Annual Electricity Production of Swiss Power Plants for the Hydrologic Year 1973/74. Data Taken from Bulletin SEV/VSE, 66, 11, 7 June 1975.

Type of Plant	Production in GWh	in %
Run of River	18,291	49
Storage	10,631	29
Thermal & Nuclear	8,326	22
Total	37,248	100

Table 3.3

Semiannual Electricity Production of Swiss Power Plants for the Hydrologic Year 1973/74. Data taken from Bulletin ASE/UCS 66, 3, 8 February 1975

	Production					
	Winter		Summer		Total	
Type of Plant	in GWh	in %	in GWh	in %	in GWh	in %
Run of River	7,573	41	10,718	59	18,291	100
Storage	5,530	52	5,101	48	10,631	100
Nuclear + Thermal	4,801	58	3,525	42	8,326	100
Total	17,904	48	19,344	52	37.248	100

Table 3.4

Monthly Electricity Production of Swiss Storage Schemes for the Hydrologic Year 1973/74. Data taken from Bulletin SEV/VSE 66, 9, 3 May 1975.

Month	Production in GWh	in %
October	383	4
November	876	8
December	835	8
January	1159	11
February	1118	10
March	1162	11
April	744	7
May	328	3
June to September	4026	38
Total	10631	100

May and October, just before, respectively just after, the snowmelt process.

The demand curve is satisfied in the following way. Thermal and nuclear stations operate as far as possible at the installed capacity throughout the year; they supply part of the base load. Run of river plants supply the remaining part of the base load; however their production rate depends on the available inflows which undergo strong seasonal variations. Pumped storage schemes cover the peak loads. The storage plants have a far more complex role, which is furthermore constantly modified as the configuration of the production system and its relation to the demand curve change. First they transfer energy from summer to winter, as they store water in summer when demand is low, and release it in winter when demand is high. Second they are used as a network regulator and as a reserve in case of breakdowns.

3.3 Electrical Distribution System

About 1,200 utilities supply Switzerland with electrical energy. Nearly each of them has its own sources of energy, its distribution network, its pricing policy and its consumers. They constitute a heterogeneous group. However the ten most important utilities supply 70 percent of the energy and their networks are interconnected. Hence similitudes exist all the same between them and some general patterns of organizations can be identified.

Today there are two large groups of producers and distributors: purely private undertakings and state owned undertakings. Both groups get their energy from three different sources. First, they may own completely some power stations which they are free to operate as they want. Second, many recent hydro-schemes were built in partnership as these schemes required for their construction great capital outlays which could not be provided by a single utility. Many combinations have resulted from both state and private organizations participating in the same schemes. Most often, all the partners have the same rights and obligations in operating the power plant, in proportion to their financial contribution. However they are not completely free to produce energy how and whenever they want. They have to take into account the wishes of the other partners of the pool. Third, utilities can buy energy directly on the power market. For this purpose, they may sign contracts among themselves to buy or to sell fixed quantities of energy at fixed prices for a given period. These contracts are on a long, middle or short-term basis. For very short terms, energy is sold, according to the laws of supply and demand.

From these considerations, it results that the determination of precise and unique energy prices is difficult, if not impossible. The existence of many independent utilities, competing against each other, further complicates this problem as they are reluctant to give information on this topic.

3.4 Selection of the Power Scheme

As no existing power plant could be found with enough available hydrological and meteorological data, a fictitious one had to be defined which represents a more or less typical case. The material contained in the preceding section supplies the basic information.

However some further points have to be discussed, before we can describe the selected power scheme.

In the Swiss Alps, there are numerous small valleys located at high altitude and separated by mountain chains. This configuration led to the construction of regional schemes. Numerous diversions and tunnels collect the water in different valleys and over a wide area and bring it to the reservoir. The power producing system itself is much simpler: penstocks or pressure shafts connected to power plants in one or two stages and with no important intermediate pond. The total gross head ranges from 200 to 1,000 m. Values between 400 and 600 m are the most frquent ones for the Rhine Valley (Eidg. Verkehrs-und Energiewirtschafts-Departement, 1973).

The storage capacity of the reservoir is also an important factor to consider. The average ratio between the reservoir capacity and the related mean annual inflow was computed for the thirty-one main alpine schemes with seasonal reservoir (see preceding reference). On the average, the storage capacity equals half the mean annual inflow. But for the more recent developments, the storage capacity is slightly greater, about 70 percent of the inflows. On the other hand, Varlet (1966) tried to define mathematically the capacity of the reservoir which allows for a complete regulation of the annual flow. He showed that for watersheds lying above 1,800 m, with more than two-thirds of the annual runoff taking place within the three summer months, the storage volume should be equal to 40 percent of the annual inflow. If one excludes the summer months from the regulation, the reservoir capacity must amount to 60 percent of the annual inflows. Hence the results of Varlet's computations coincide well with the situation prevailing in Switzerland.

The last parameter to consider is the average annual duration of operation in hours. These figures vary within a great range for the existing storage system (Härry 1957, Leuthold). Most often, however, they oscillate between 2,500 and over 3,000 hours.

The layout of the selected power scheme appears in Fig. 3.1. It consists of a reservoir with a capacity of 72 million cubic meters, a penstock and a power station. The maximum head amounts to 486 m when the reservoir is full, the minimum head, to 418 m, when the reservoir is empty. Fig. 3.2 shows the reservoir content versus elevation curve. It was adapted from an existing scheme. On Fig. 3.3 appears the gross energy rate function, which gives the gross quantity of energy produced by the release of one unit of water through the turbines for a specified reservoir content. As the losses resulting from power production are assumed to be constant, the net quantity of energy produced is obtained by multiplying the values of Fig. 3.3 by 0.85. The most important characteristics of the scheme are summed up in Table 3.5.

3.5 Selection of the Energy Prices

Two sources were considered to determine the energy prices to use in this study: the energy demand and supply curves and the available literature on pricing policy.

As explained earlier, the demand for energy changes over the year, during the week and during the day. The same remark applies for the energy supply. However the variations of these two variables

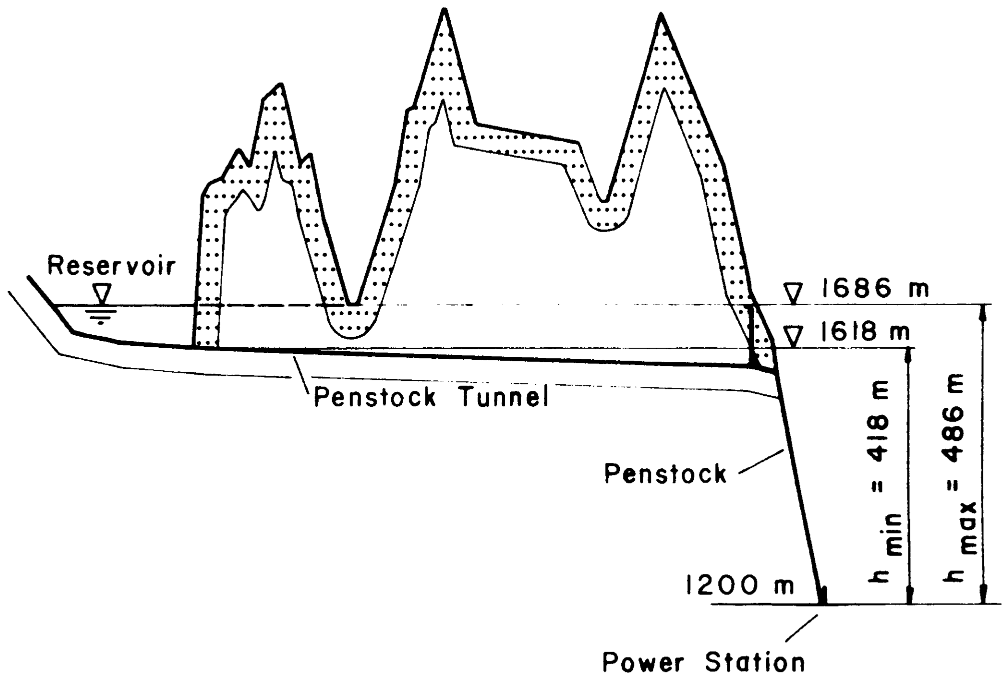


Fig. 3.1 <u>Selected Power Scheme</u>

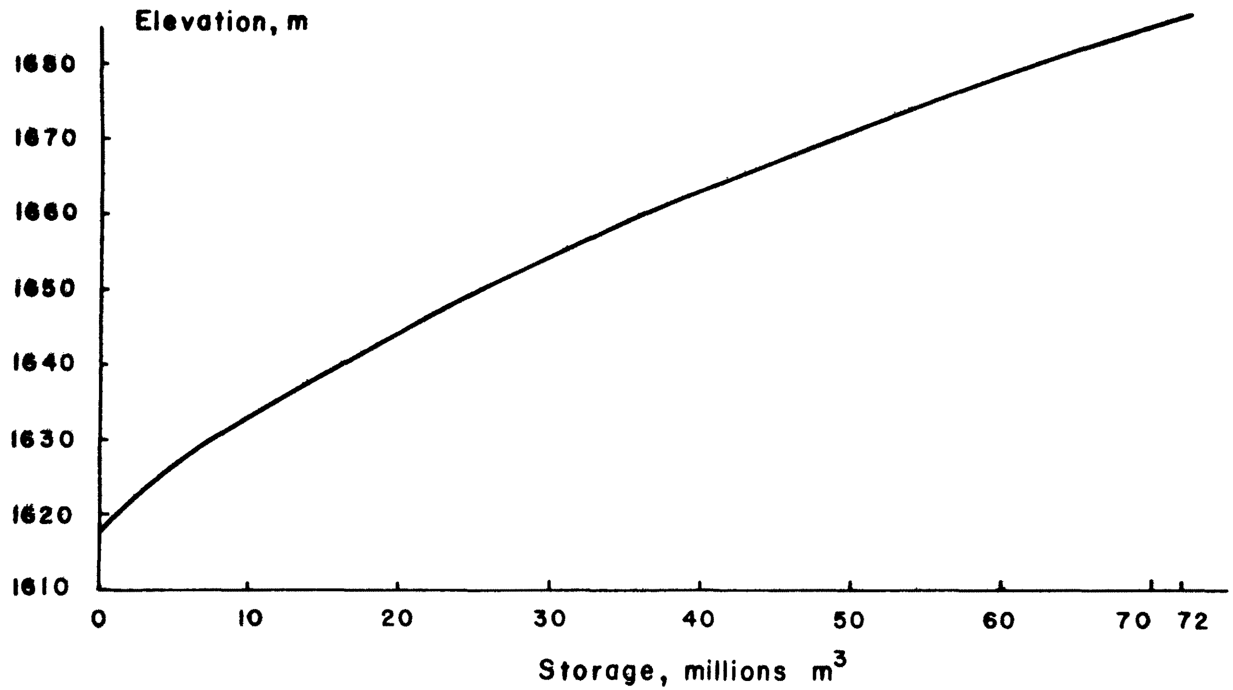


Fig. 3.2 Reservoir Content Versus Elevation Curve

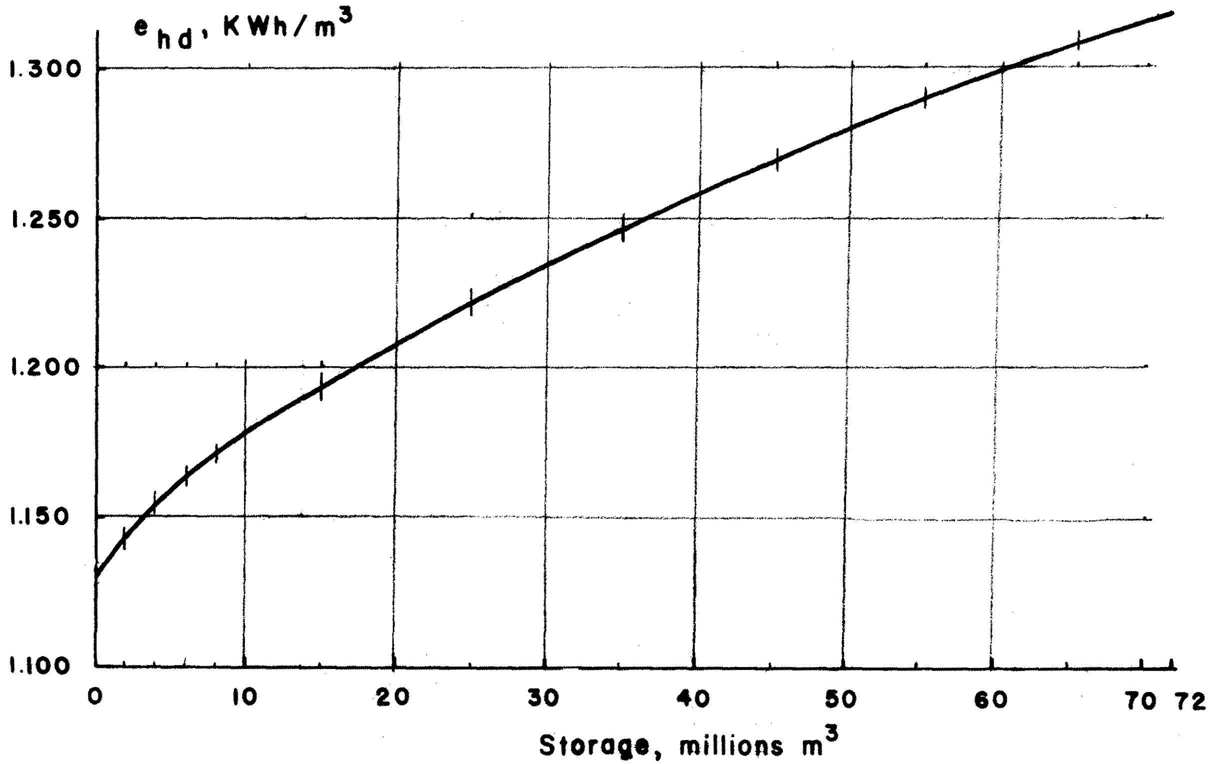


Fig. 3.3 **Energy Rate Function**

Table 3.5

Characteristics of Selected Power Scheme
Data were Kindly Supplied by Elektrizitätswerke der Stadt Zürich

Variable	Symbol	Magnitude	Units
Catchment Area	A	53.7	km^2
Average Annual Inflow	Q	106.10^6	m^3
Storage Capacity	S	72.10^6	m^3
Design Release	-	10	m^3/s
Maximum Head	h_{max}	486	m
Minimum Head	h_{min}	418	m
Max. of Energy Rate Function	$e_{hd}(s_{max})$	1.321	KWh/m^3
Min. of Energy Rate Function	$e_{hd}(s_{min})$	1.136	KWh/m^3
Average Annual Duration of Operation	-	3000	Hours

are not necessarily in phase so that, at least theoretically, there exist periods where the demand is in excess and vice-versa, periods where the supply is in excess. This situation must necessarily have an effect on the prices. Hence, according to the law of supply and demand, energy should be priced higher in winter than in summer. Furthermore, during the week, it should be cheaper on Saturday and Sunday than during the working days. Finally prices should also vary during the day. This is the qualitative information obtained from the analysis of the energy demand and supply curves.

The existing literature was screened and the contents of the available articles were compared among each other (Devantéry, 1950; Härry, 1957; Galli, 1965; Frankhauser, 1972 etc.). On the whole, they showed a good concordance. Although some small differences appeared, general trends could be set forth, which confirmed the conclusions drawn in the preceding paragraph. So, for example, many authors propose the use of 3 classes of price during the working days, and only 2 for the weekend. Based on these analyses, the following approach was selected.

As the week is the smallest time step considered in this study, the prices were compiled on a weekly basis. During the 5 working days, peak prices are charged during 3.5 hours per day, off-peak prices, during 7.5 hours per day, the night prices, during 13 hours per day. On Saturdays and Sundays, only 2 categories apply: off-peak prices during 8 hours per day and night prices during 16 hours per day. After combining the equivalent prices on a weekly basis and after ordering them according to their magnitude, Fig. 3.4 was obtained.

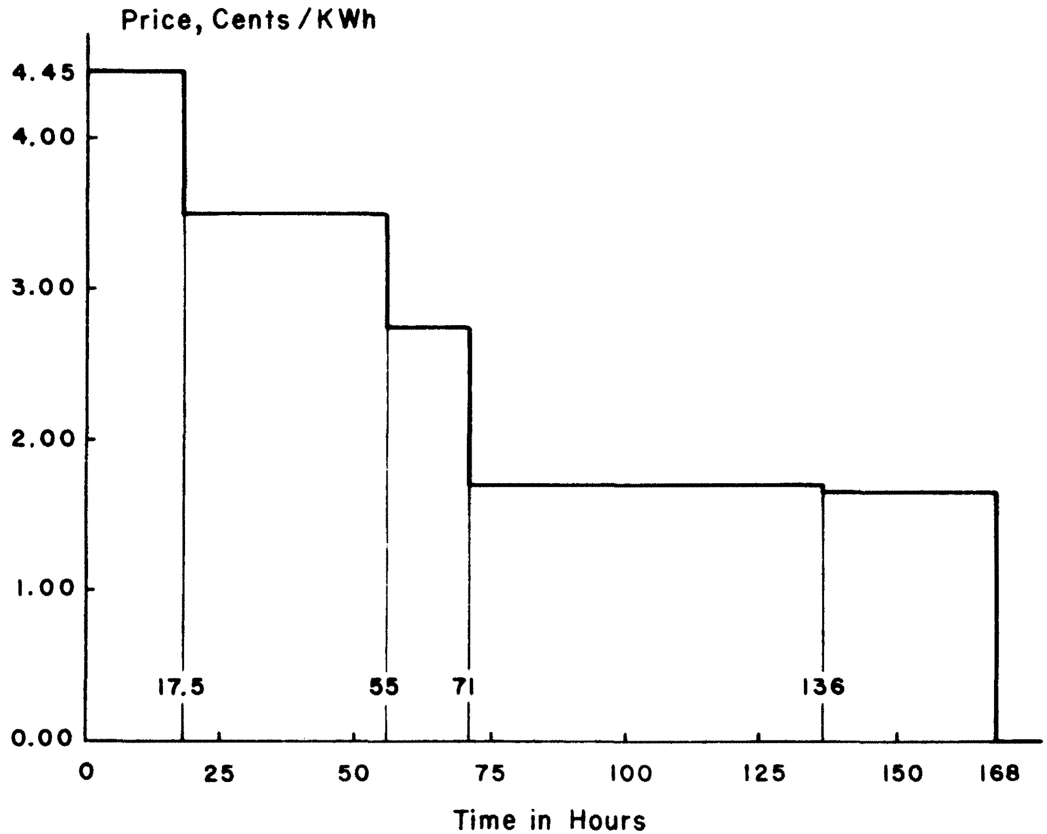


Fig. 3.4 Price of Electricity as a Function of Weekly Hours of Operation

Together with Table 3.6, it gives the full information on the selected prices. A quick computation indicates that the average weekly price for one KWh amounts to 2.5 cents.

On more variation comes into play: the monthly one. Table 3.7 shows the monthly pattern of energy price changes. They were selected in such a way that the average annual price amounts to 2.5 cents per KWh. Furthermore the average winter price is twice the summer price, ratio often used in practice.

3.6 Return Function

The formulation of the return function depends on the selection of the time grid. This quantity can be any value between one hour and one year, but the orientation of the present studies reduces a great deal the choice.

The reservoir operation requires a grid width smaller than one month in order to obtain reliable and precise results. On the other hand, time steps equal to or shorter than one day cannot be considered in this study, as nonhydrologic factors like availability of turbines, cost of starting or stopping them, load of the distribution system, etc., begin to play an important role. Gathering information on these problems is beyond the scope of this research. It appears then that a time step of a week is a good choice. The results are precise, and the electro-technical problems are not yet determinant. Consequently a time step of one week will be retained for all further studies.

To determine the return function, two more assumptions are necessary. First, one assumes that water is drawn from the reservoir at a constant rate (10 m^3/s), which implies that there exists a

Table 3.6

Selected Energy Prices

	Category	Price Cents/KWh	Duration in Hours/Day	Duration in Hours/Week
Weekdays	Peak	4.45	3.5	17.5
(5 days)	Off-peak	3.50	7.5	37.5
	Night	1.70	13.0	65.0
Weekend	Off-peak	2.75	8.0	16.0
(2 days)	Night	1.65	16.0	32.0
Weekly Averages	-	2.5	-	168

Table 3.7

Average Monthly Prices of Electricity
(Adapted from H. Frankhauser, 1972)

Month	Price in Cents/KWh
October	2.84
November	3.28
December	3.66
January	3.77
February	3.66
March	3.32
Winter	3.42
April	2.30
May	1.56
June	1.11
July	1.11
August	1.48
September	1.95
Summer	1.58
Year	2.50

unique relation between weekly hours of operation of the turbines and the total volume of water released in the corresponding period. Second, we assume that the distribution of the releases during the week is performed in an optimal way. This means that water is first released during the periods where the energy is most expensive. Hence, as the amount of water released during a week increases, the corresponding return per KWh produced diminishes (law of diminishing returns). These two assumptions allowed to establish a relation between weekly hours of operation of the power plant and weekly releases, and consequently between weekly releases and energy prices (Fig. 3.5).

We decided to fit a continuous curve to the price diagram of Fig. 3.5, to make the computation easier. The curve to be selected had to lead to an average weekly price of 2.5 cents per KWh and to provide a good approximation to the existing diagram. The exponential curve of Fig. 3.6 performed best. This choice is further justified by the fact that quite often time decaying phenomena of the type appearing here (law of diminishing returns) are modeled by exponential functions. Hence,

$$Pr\ (x) = 4.73 \cdot e^{-\frac{x}{4.2}} \tag{3-1}$$

where

Pr (x) = Marginal price in cents of one KWh corresponding to a specified total weekly release

x = total weekly release in million m^3

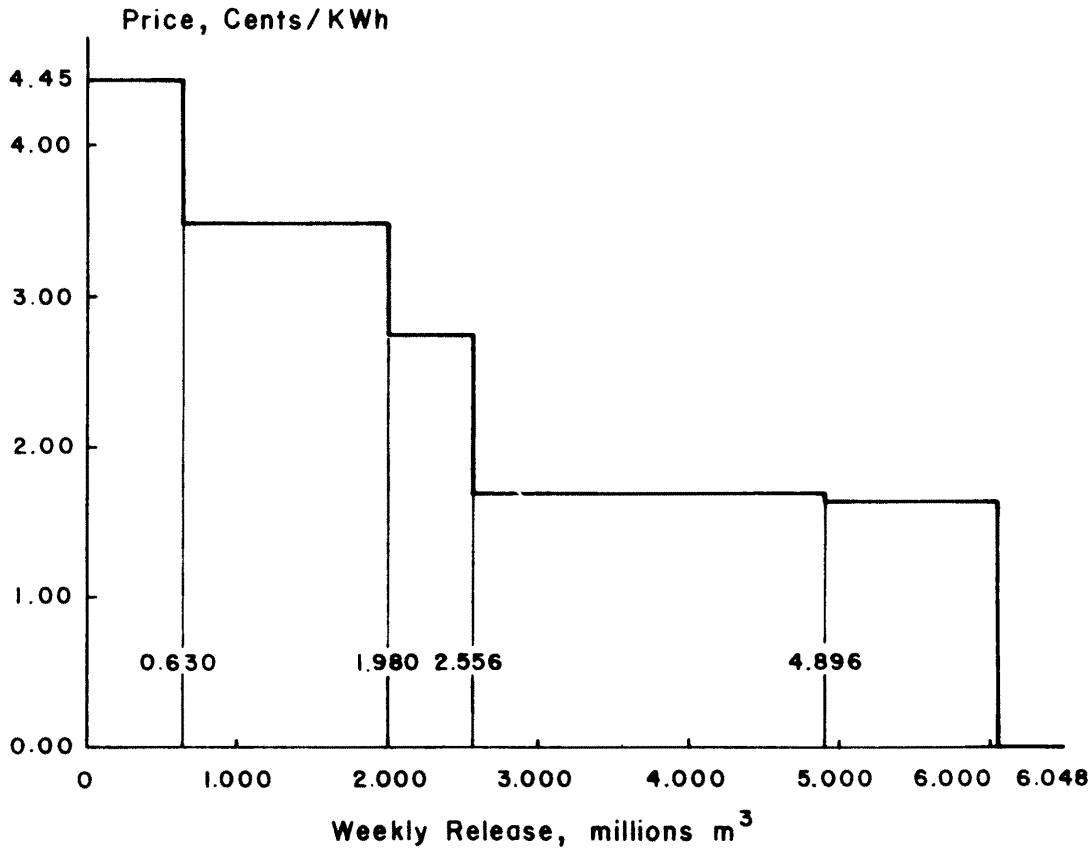


Fig. 3.5 Price of Electricity as a Function of Weekly Release

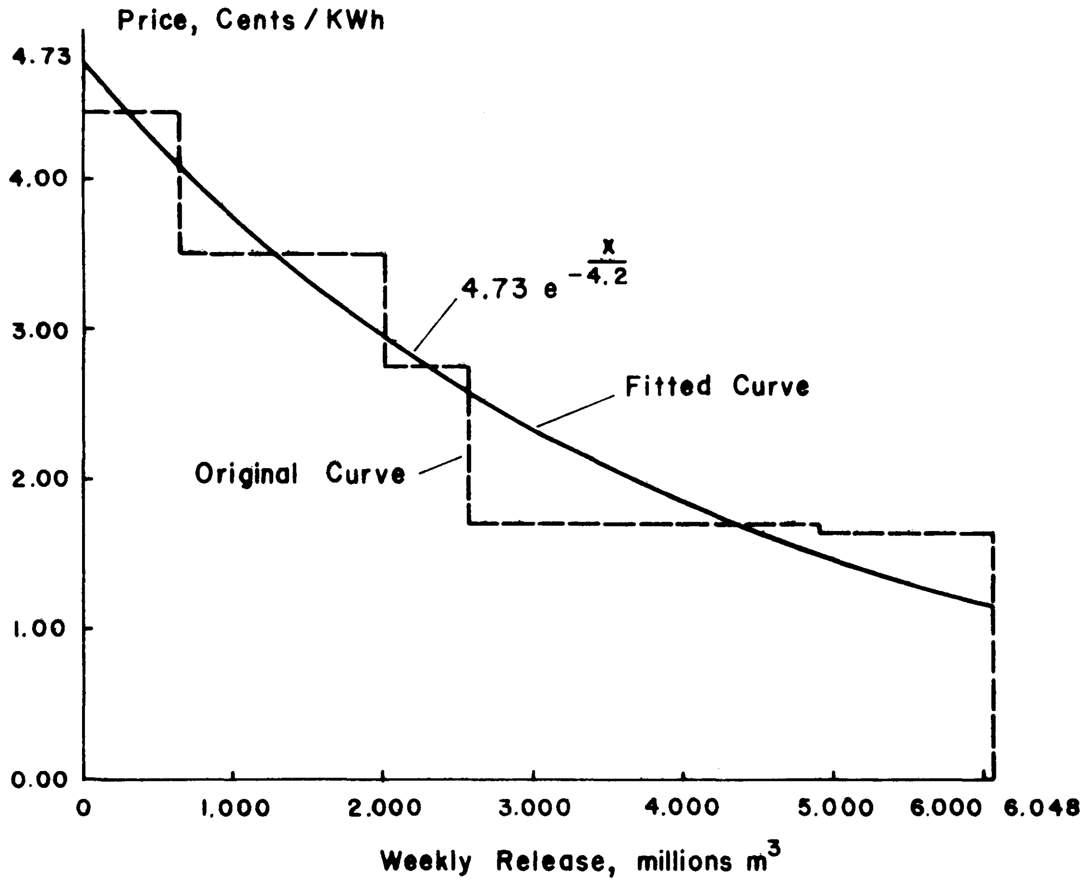


Fig. 3.6 Price of Electricity, Fitted Exponential Curve

However the prices change throughout the year, so that the complete relation reads as follows:

$$Pr_i\ (x_i) = \frac{4.73}{2.50} \cdot Pr_{month\ j} \cdot e^{-\frac{x_i}{4.2}} \tag{3-2}$$

where

$Pr_i\ (x_i)$ = Marginal price in cents of one KWh corresponding to a specified total release in the i^{th} week

$Pr_{month\ j}$ = Average monthly price, according to Table 3.7

x_i = Total release in million m^3 during the i^{th} week

i = running index for the weeks

j = running index for the months

The return from a release during a specified week is

$$B\ (i,\ s_i,\ x_i) =$$

$$10^6 \cdot 0.85 \cdot \frac{4.73}{2.50} \cdot Pr_{month\ j} \cdot \int_0^{x_i} e_{hd}\ (s_i) \cdot e^{-\frac{z}{4.2}}\ dz \tag{3-3}$$

where

$B(i,\ s_i,\ x_i)$ = return resulting from a given release in the i^{th} week and for a specified reservoir content, in cents

$e_{hd}(s_i)$ = energy rate function, in KWh/m^3

s_i = storage content in the i^{th} week, in million m^3

z = dummy variable for the amount of water released during the i^{th} week in million m^3

The factor 0.85 is introduced to convert gross into net energy, and the factor 10^6, to be systematic with the units.

This expression can be simplified. The variation in head during a week is small as compared to the total head. One does not make a great error if it is taken as a constant equal to the average of the initial and final head of the considered period. Hence Eq. 3-3 reads as

$$B\ (i,\ s_i,\ x_i) =$$

$$10^6 \cdot 1.608 \cdot Pr_{\text{month } j} \cdot e_{hd}(\bar{s}_i) \int_0^{x_i} e^{-\frac{z}{4.2}}\, dz \qquad (3\text{-}4)$$

where

$e_{hd}(\bar{s}_i)$ = Average value of the energy rate function during the i^{th} week, in KWh/m^3

To alleviate the writing a new expression, $\alpha\ (s_i)$, is introduced, so that

$$B\ (i,\ x_i,\ s_i) =$$

$$10^6 \cdot \alpha_i\ (\bar{s}_i) \int_0^{x_i} e^{-\frac{z}{4.2}}\, dz \qquad (3\text{-}5)$$

where

$$\alpha_i(\bar{s}_i) = 1.608 \cdot Pr_{\text{month } j} \cdot e_{hd}\ (\bar{s}_i)$$

If one operates the reservoir over a complete year, the total annual return V amounts to

$$\sum_{i=1}^{52} B\ (i,\ x_i,\ s_i) \qquad (3\text{-}6)$$

Now all the variables necessary for the computations are defined and the computations can be started.

3.7 Concluding Remarks

The Swiss production and distribution network for electrical energy has a pattern of its own. As far as possible it was tried to respect its characteristics. However, this does not mean that the methodology to be developed in the subsequent chapters is not general; on the contrary. For, the selected method rests on quite general principles; the particular nature of the return function introduces here and there only some computational simplifications.

Chapter 4

DETERMINISTIC RESERVOIR OPERATION

Deterministic reservoir operation is the topic of this chapter. It starts with the review of the existing theorems on nonlinear optimization and with the formulation of the problem to be solved. Then, on the basis of the results from preliminary computations, a new solution technique is proposed. The chapter ends with the application of this method to numerous cases, in order to evaluate its performance.

4.1 Introduction

Required is the construction of an optimal storage policy. To this end, the model was taken as simple as possible but still including the main features of the problem. It comprises a single storage reservoir with a hydroelectric plant, a sequence of inflows and a set of price functions for the produced electric energy. The water year is broken up into 52 weeks, or time intervals, starting from October 1 and ending on September 30 of the following year. At the beginning of each week a decision must be made about storage use in that interval, taking into account the current reservoir level. It is done in such a way that the returns from the energy produced during the rest of the year's operation get maximum. In this chapter, the sequence of weekly inflows for the year under study is supposed to be known in advance.

The new method, which will be quite general, will be tested on a particular case. Chapter 3 supplied the characteristics of the selected reservoir and power plant, as well as the benefit functions associated with the weekly releases. The discharges of the Hinterrhein River recorded at Hinterrhein from 1945 to 1974 constituted the inflows to the model.

The achievements of the computer industry about one decade ago made possible the application of many new and quite involved mathematical techniques. The whole operation research field experienced then a tremendous development, which was also stimulated by the theoretical studies of researchers such as Dantzig (1963), Bellman (1957, 1962), and Kuhn and Tucker (1950).

Two directions were mainly followed: a purely theoretical one and a trial and error oriented one. The purely theoretical approach found relatively few application fields as in real world problems it led quite often to unmanageable situations, despite the great power of the computer. On the other hand, the trial and error oriented approach flourished all over the world. It tries to reach the optimal solution by proceeding by successive approximations. However, more and more, the physical nature of the problem got forgotten. The most explicit example of this trend is supplied by dynamic programming (Warren Hall, 1966) where the optimal solution is obtained by an enlightened enumeration of all the possible solutions.

Today it is time to go back and to introduce again into the solution technique the physical nature of the problem under study. Operation research should be considered as for what it is: a tool among others.

In this respect, French researchers followed an interesting path. Varlet (1923), Boulinier (1943), Giguet (1945), and Massé (1946) derived reservoir operating rules directly based on the physical nature of the problem. They arrived, in simple cases, at usable results. For more complicated cases, however, they had again to rely on trial and error methods. This was especially true for real world problems (see Appendix A).

The purpose here is to show that a more efficient combination of the theoretical and of the experimental approaches is possible. The theoretical path will be followed as far as possible, in order to reduce to a minimum the guesswork

4.2 Kuhn-Tucker Conditions

The optimization problem consists in maximizing a nonlinear objective function, the variables of which are subject to linear equality and inequality constraints. It can be formulated as:

$$\text{Max. } V\ (x_1, x_2, \ldots, x_n)$$

subject to (4-1)

$$g_j\ (x_1, x_2, \ldots, x_n) \geq 0 \quad \text{for} \quad j = 1, m,$$

where $x_1, x_2, \ldots, x_n$ are the n unknown decision variables, and $g_1\ (x_1, x_2, \ldots, x_n)$, $g_2\ (x_1, x_2, \ldots, x_n)$ and $g_m\ (x_1, x_2, \ldots, x_n)$ the m constraints.

Stark (1972) suggested to classify the methods for solving nonlinear optimization problems into two broad categories: the classicial techniques and the search techniques. Classical optimization seeks optimal solutions by solving systems of equations, while search techniques proceed iteratively by successive approximations. Here we shall apply the first approach.

Assume for a while that the objective function $V(x_1,x_2,\dots,x_n)$ is unconstrained, that means that the n variables can vary without any restriction. For the problem

$$\text{Max. } V(x_1,x_2,\dots,x_n) \tag{4-2}$$

calculus yields easily the necessary conditions for an optimal solution:

$$\frac{\partial V}{\partial x_i}\bigg|_{x_i^*} = 0, \text{ for } i = 1,2,\dots,n, \tag{4-3}$$

where $\big|_{x_i^*}$ implies evaluation at the optimal solution. Hence the first derivative must vanish. Furthermore if the objective function is known to be strictly concave, the solution is also a global optimum.

Unconstrained problems occur infrequently in the real world. An objective function with variables subject to a series of equality constraints represents the first generalization of the unconstrained case:

$$\begin{aligned} &\text{Max. } V(x_1,x_2,\dots,x_n) \\ &\text{s.t. } g_j(x_1,x_2,\dots x_n) = 0,\ j = 1,\dots,m \end{aligned} \tag{4-4}$$

In this case, one can form a so-called Lagrangian expression with the property that any values of the variables which maximize the original objective function subject to its equality constraints will also maximize the value of the Lagrangian function. The Lagrangian form is obtained by multiplying each equality constraint $g_j(x_1,x_2,\dots,x_n)$ by its own so-called Lagrange multiplier λ_j, an artificial variable, and by adding the resulting product to the original objective function. Hence the new optimization problem becomes:

$$\text{Max. } L\ (x_1,x_2,\ldots,x_n;\ \lambda_1,\lambda_2,\ldots,\lambda_m)$$

where

$$L\ (x_1,x_2,\ldots,x_n;\ \lambda_1,\lambda_2,\ldots,\lambda_m) = V\ (x_1,x_2,\ldots,x_n) + \sum_{j=1}^{m} \lambda_j \cdot g_j(x_1,x_2,\ldots,x_n) \tag{4-5}$$

The constrained problem has been changed into an unconstrained one, at the cost of m additional variables, the Lagrange multipliers. Again, at the optimum, the first derivatives must vanish:

$$\begin{aligned} \frac{\partial L}{\partial x_i} &= \frac{\partial V}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \cdot \frac{\partial g_j}{\partial x_i}\bigg|_{x_i^*} = 0, \quad i = 1,2,\ldots,n \\ \frac{\partial L}{\partial \lambda_j} &= g_j\ (x_1,x_2,\ldots,x_n) = 0 \quad j = 1,2,\ldots,m \end{aligned} \tag{4-6}$$

Solving this system of equations yields the desired solution. Note that the second set of conditions are automatically satisfied.

H. W. Kuhn and A. W. Tucker (1950) made the most important contributions in the field of nonlinear optimization. They extended the Lagrange multipliers technique to the situations where the decision variables are subject to inequality constraints. According to them, the necessary conditions for an optimum to the problem formulated at the beginning of this section (Eq. 4-1) are:

$$1.\quad \text{If } x_i^* > 0,\ \frac{\partial V}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \cdot \frac{\partial g_j}{\partial x_i}\bigg|_{x_i^*} = 0;\ i = 1,2,\ldots,n$$

$$2.\quad \text{If } x_i^* = 0,\ \frac{\partial V}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \cdot \frac{\partial g_j}{\partial x_i}\bigg|_{x_i^*} \leq 0;\ i = 1,2,\ldots,n \qquad (4\text{-}7)$$

$$3.\quad \text{If } \lambda_j > 0,\ g_j\ (x_1^*, x_2^*, \ldots x_n^*) = 0;\ j = 1,2,\ldots,m$$

$$4.\quad \text{If } \lambda_j = 0,\ g_j\ (x_1^*, x^*, \ldots, x_n^*) \geq 0;\ j = 1,2,\ldots,m$$

$$5.\quad x_i^* \geq 0;\ i = 1,2,\ldots,n$$

$$6.\quad \lambda_j \geq 0;\ j = 1,2,\ldots,m$$

If the objective function is concave, these conditions are sufficient. As compared to the classical Lagrange multiplier approach, there are two new conditions (Eqs. 4-2 and 4-4), which take into account the possibility that the maximum may occur at a boundary rather than at an interior point. Suppose for example that we are at an interior point. Then, as x_i can take on any value, $\partial V/\partial x_i$ must vanish in order to have an optimum (Eq. 4-1). On the other hand, assume that the maximum is at a corner, for example $x_i = 0$. The partial derivative $\partial V/\partial x_i$ can either be zero, negative or positive. Of the 3 possibilities only the last one is not possible, as an increase in x_i would lead to an increase in V, the return function. Hence the partial derivative must be equal or smaller than zero, as indicated by Eq. 4-2.

Equation 4-3 stipulates that a given constraint equation is tight, and that the corresponding Lagrange multiplier is greater than zero. If the constraint equation is not tight, the associated Lagrange multiplier vanishes. Finally Eqs. 4-5 and 4-6 express the nonnegativity of the variable x_i and of the Lagrange multipliers λ_j.

Theoretically the formulation of the Kuhn-Tucker conditions is a great contribution to the solution of nonlinear models. It is especially useful to test whether an available solution is optimal. However the search of solution according to this method may become quite tedious. The system of equations may be of higher dimension, and the associated equations, nonlinear. Often, more than one solution set exists and the optimal solution must be determined by successive eliminations of the unfeasible ones. Quite a formidable task if the number of variables is large.

4.3 Problem Formulation

The optimization problem is first brought into its canonical form. The associated Kuhn-Tucker conditions are derived in the second part. The section ends with some considerations on the complexity of the determinations of the optimal solution.

The objective function is given by the sum of the returns resulting from the weekly water releases during the period of operation. Hence,

$$V\ (x_k, x_{k+1}, \ldots, x_{52}) = \sum_{i=k}^{52} B_i\ (i,\ x_i,\ s_i).$$

where

$$B_i\ (i,\ x_i,\ s_i) = 10^6 \cdot \alpha_i(\bar{s}_i) \cdot \int_0^{x_i} e^{-z/4.2}\, dz \qquad (4\text{-}8)$$

The index i refers to the i^{th} week of the water year. According to the above notation, the reservoir operation starts in the k^{th}

week of the water year. The term x_i represents the water release from the reservoir during the i^{th} week, and $\bar{s}_i$, the corresponding average reservoir content. The term $\alpha_i(\bar{s}_i)$ is an aggregate factor. For a given week and reservoir content, it is obtained by multiplying the energy produced by a unit release of water by the associated maximum energy price (Table 3.6 and Table 3.7).

During the reservoir operation, there are some physical boundaries which must be observed. Water releases cannot exceed the inflows when the reservoir is empty, and the releases must be at least equal to the inflows when the reservoir is full. Furthermore for the period of operation, the mass balance equation must be satisfied. These remarks lead to three types of constraints.

First, there is an equality constraint. It expresses that for the period of reservoir operation, the sum of the total inflow and of the initial storage is equal to the sum of the total release and of the final storage:

$$s_k + \sum_{i=k}^{52} q_i = \sum_{i=k}^{52} x_i + s_{53} \tag{4-9}$$

where s_k stands for the reservoir content at the beginning of the k^{th} week, or initial storage. Similarly s_{53} stands for the final storage. The corresponding standard equation form is,

$$s_k - s_{53} + \sum_{i=k}^{52} q_i - \sum_{i=k}^{52} x_i = 0 \tag{4-10}$$

The second type of constraint equations stipulates that in any week, the release must be smaller or at most equal to the sum of the storage available at the beginning of the week and of the inflow during the same period. We shall call these constraints, release constraints:

$$x_j \le s_k + \sum_{i=k}^{j} q_i - \sum_{i=k}^{j-1} x_i,\ j = k,51 \tag{4-11}$$

or in standard form:

$$s_k + \sum_{i=k}^{j} q_i - \sum_{i=k}^{j} x_i \ge 0,\ j = k,51 \tag{4-12}$$

The third type of constraints indicates that the storage in the reservoir cannot exceed the reservoir capacity (storage constraints):

$$s_k + \sum_{i=k}^{j} q_i - \sum_{i=k}^{j} x_i \le S,\ j = k,51 \tag{4-13}$$

where S represents the reservoir capacity. Bringing all the terms on the right-hand side yields the standard form:

$$S - s_k - \sum_{i=k}^{j} q_i + \sum_{i=k}^{j} x_i \ge 0,\ j = k,51 \tag{4-14}$$

Hence we are confronted with a nonlinear optimization problem, subject to equality and inequality constraints:

$$\text{Max } V(x_k, x_{k+1}, \ldots, x_{52}) = 10^6 \cdot \sum_{i=k}^{52} \alpha_i(\bar{s}_i) \cdot \int_0^{x_i} e^{-z/4.2}\, dz$$

$$\text{s.t. } s_k - s_{53} + \sum_{i=k}^{52} q_i - \sum_{i=k}^{52} x_i = 0$$

$$s_k + \sum_{i=k}^{j} q_i - \sum_{i=k}^{j} x_i \ge 0,\ j = k,51 \tag{4-15}$$

$$S - s_k - \sum_{i=k}^{j} q_i + \sum_{i=k}^{j} x_i \ge 0,\ j = k,51$$

$$x_i \ge 0,\ i = k,52$$

The procedure described in the preceding section can be applied. The Lagrangian form is then:

$$L(x_k, x_{k+1}, \ldots, x_{52}, \lambda, \beta, \gamma) =$$

$$10^6 \cdot \sum_{i=k}^{52} \alpha_i(\bar{s}_i) \cdot \int_0^{x_i} e^{-z/4.2}\, dz$$

$$+ \lambda(s_k - s_{53} + \sum_{i=k}^{52} q_i - \sum_{i=k}^{52} x_i) \qquad (4\text{-}16)$$

$$+ \sum_{j=k}^{51} \beta_j \cdot (s_k + \sum_{i=k}^{j} q_i - \sum_{i=k}^{j} x_i)$$

$$+ \sum_{j=k}^{51} \gamma_j \cdot (S - s_k - \sum_{i=k}^{j} q_i + \sum_{i=k}^{j} x_i)$$

In this formulation, λ is the Lagrange multiplier for the mass balance equation, β_j, for the release constraints, and γ_j, for the storage constraints. The index k refers to the week where the reservoir operation is started. The associated Kuhn-Tucker conditions are:

1. $\alpha_i(\bar{s}_i) \cdot e^{-x_i/4.2} - \lambda - \sum_{j=i}^{51} \beta_j + \sum_{j=i}^{51} \gamma_j = 0 \quad i = k,52;\ x_i \neq 0.$

2. $\alpha_i(\bar{s}_i) \cdot e^{-x_i/4.2} - \lambda - \sum_{j=i}^{51} \beta_j + \sum_{j=i}^{51} \gamma_j \leq 0 \quad i = k,52;\ x_i = 0$

3'. $s_k - s_{53} + \sum_{i=k}^{52} q_i - \sum_{i=k}^{52} x_i = 0$

3''. $s_k + \sum_{i=k}^{j} q_i - \sum_{i=k}^{j} x_i = 0 \quad j = k,51,\ \beta_j > 0$ (4-17)

3'' $S - s_k - \sum_{i=k}^{j} q_i + \sum_{i=k}^{j} x_i = 0 \quad j = k,51, \; \gamma_j > 0$

4'. $s_k + \sum_{i=k}^{j} q_i - \sum_{i=k}^{j} x_i \geq 0 \quad j = k,51, \; \beta_j = 0$

4''. $S - s_k - \sum_{i=k}^{j} q_i + \sum_{i=k}^{j} x_i \geq 0 \quad j = k,51, \; \gamma_j = 0$

5. $x_i \geq 0 \quad i = k,52$

6. $\lambda, \beta_j, \gamma_j \geq 0 \quad j = k,51$

To simplify the writing, the asterisk relating to the optimal value of x_i has been omitted. Equation 1 applies when the optimal value of a given x_i is an interior point. In that case, the partial derivative of the return function versus the given variable must vanish. If, on the contrary, the optimum is on a boundary, Eq. 2 holds. For tight constraints, Eq. 3 must be satisfied, for loose constraints, Eq. 4 holds.

For $k=1$, there are 155 unknowns, even 156 if one considers that the final storage is not necessarily known in advance. The resolution of this formidable looking system of equations is further complicated by the presence of nonlinear terms. Yet the solution can be found algebraically, but it is time consuming and not efficient. Hence our task now consists in trying to simplify the basic system of equations, in order to determine the variables in an easier way.

4.4 Preliminary Developments

4.4.1 Scope and purpose of the preliminary studies. The relations derived under Section 4.3 are too cumbersome to be solved directly. They must be modified and simplified in such a way that they become easily tractable. To assess the possibilities of simplification of the basic equations, and to familiarize ourselves with the problem, numerous preliminary computations were performed. They consisted in determining more or less intuitively optimal release strategies for different types of reservoir, and for varying initial and final storages. For all the cases studied, the operation was done for a complete water year. The value of the index k (see Section 4.3) is thus set equal to 1. This section summarizes the results of these analyses.

It starts with the annual operation of a reservoir with infinite storage capacity. Then, progressively, the simplifying assumptions are eliminated until the case under study corresponds to the one we must solve. No quantitative information will be given hereafter. The emphasis rests on the physical understanding of the reservoir control problem. Proofs and derivations of equation will be restricted to a minimum. This section should supply the physical and intuitive background on which the solution technique will be built. The related mathematical derivations will appear in a later section.

4.4.2 Reservoir of infinite capacity with known final storage. The simplest possible case consists of a reservoir which can store and release any amount of water. Furthermore, we assume that the reservoir is full at the beginning and at the end of the water year. The problem reduces then to the optimal redistribution over the year, of the inflows. Mathematically the problem becomes:

$$\text{Max. } V\,(x_1, x_2, \ldots, x_{52}) = \sum_{i=1}^{52} 10^6 \cdot \int_0^{x_i} \alpha_i(\bar{s}_i) \cdot e^{-z/4.2}\, dz$$

$$\text{s.t.} \quad s_1 - s_{53} + \sum_{i=1}^{52} q_i - \sum_{i=1}^{52} x_i = 0 \qquad (4\text{-}18)$$

The release and storage constraints have disappeared and only the annual mass balance equality constraint remains. The related Kuhn-Tucker conditions are:

$$1. \quad \alpha_i(\bar{s}_i) \cdot e^{-x_i/4.2} - \lambda = 0 \quad i = 1{,}52 \quad \text{if} \quad x_i \neq 0$$

$$2. \quad \alpha_i(\bar{s}_i) \cdot e^{-x_i/4.2} - \lambda \leq 0 \quad i = 1{,}52 \quad \text{if} \quad x_i = 0 \qquad (4\text{-}19)$$

$$3. \quad s_1 - s_{53} + \sum_{i=1}^{52} q_i - \sum_{i=1}^{52} x_i = 0$$

Equation 1 expresses that the partial derivative of the Lagrangian form vanishes for those variables which, at the optimal, constitute an interior point. For those variables which at the optimal are at a corner, the partial derivatives must be smaller or equal to zero (Eq. 2). The annual water balance equation is given by Eq. 3. As by assumption initial and final content are identical, the above conditions become:

$$1. \quad \alpha_i(\bar{s}_i) \cdot e^{-x_i/4.2} = \lambda \quad i = 1{,}52 \quad x_i \neq 0$$

$$3. \quad \sum_{i=1}^{52} x_i = \sum_{i=1}^{52} q_i \qquad (4\text{-}20)$$

Hence the Kuhn-Tucker conditions reduce to simple and easily tractable equations, quite a change from the cumbersome system of equations of Section 4.4.3! Furthermore the derived equations attach interesting properties to the optimal strategy.

First, as indicated by Eq. 1 of (4-20), the Lagrange multiplier λ represents the marginal value of the returns from the weekly releases. Second, in fact a corollary of the first property, the marginal value of the return of the different releases are constant throughout the year. Or, in other words, a release strategy is optimal, when the marginal returns of the releases are equal. Third the Lagrange multiplier λ has a further important meaning. It can be considered as the marginal cost of honoring a constraint, cost meaning here loss in the value of the objective function. Hence in this case it represents the total derivative of the maximum value of the objective function with respect to a relaxation in the water balance equation.

The determination of the optimal sequence of releases presents no difficulty, as the system of equations (4-20) can be solved algebraically. The introduction of the natural logarithms into the first equation of (4-20) yields:

$$\frac{-x_i}{4.2} = \ln \lambda - \ln \alpha_i(\bar{s}_i) \quad i = 1,52 \tag{4-21}$$

which substituted into the second equation of (4-20), leads to:

$$4.2 \cdot \sum_{i=1}^{52} \ln \alpha_i(\bar{s}_i) - 4.2 \cdot 52 \cdot \ln \lambda = \sum_{i=1}^{52} q_i$$

or

$$\ln \lambda = \frac{4.2 \sum_{i=1}^{52} \ln \alpha_i(\bar{s}_i) - \sum_{i=1}^{52} q_i}{4.2 \cdot 52} \tag{4-22}$$

from which the values of the x_i are easily computed.

The terms $\alpha_i(\bar{s}_i)$ of Eq. 4-22 depend on the successive weekly reservoir contents reached during the year. However this parameter is not known in advance. We have to proceed by iteration. An initial reservoir content curve for the whole period of operation is selected, which allows to determine the optimal Lagrange multiplier and the associated releases. With the obtained releases, a new reservoir content curve is computed which is then compared to the preceding one. If both curves are not too different, the optimum is reached. On the contrary, the computation is repeated with the new curve, as initial reservoir content curve. The procedure is repeated until both initial and final curves are identical.

The computations showed interesting properties of the optimal reservoir content curve. As the energy price is higher in winter than in summer, the releases concentrated mainly in winter, while the reservoir was filled in summer (Fig. 4.1). This implies that for a real reservoir, the release constraints would have been violated in late winter and in spring. Also, for the cases studied, the optimal reservoir content curves in winter, and the date when the minimum contents were recorded, are much less dependent on the inflows than the corresponding summer reservoir content curves. Finally, initial and final storage influence the release strategy only locally.

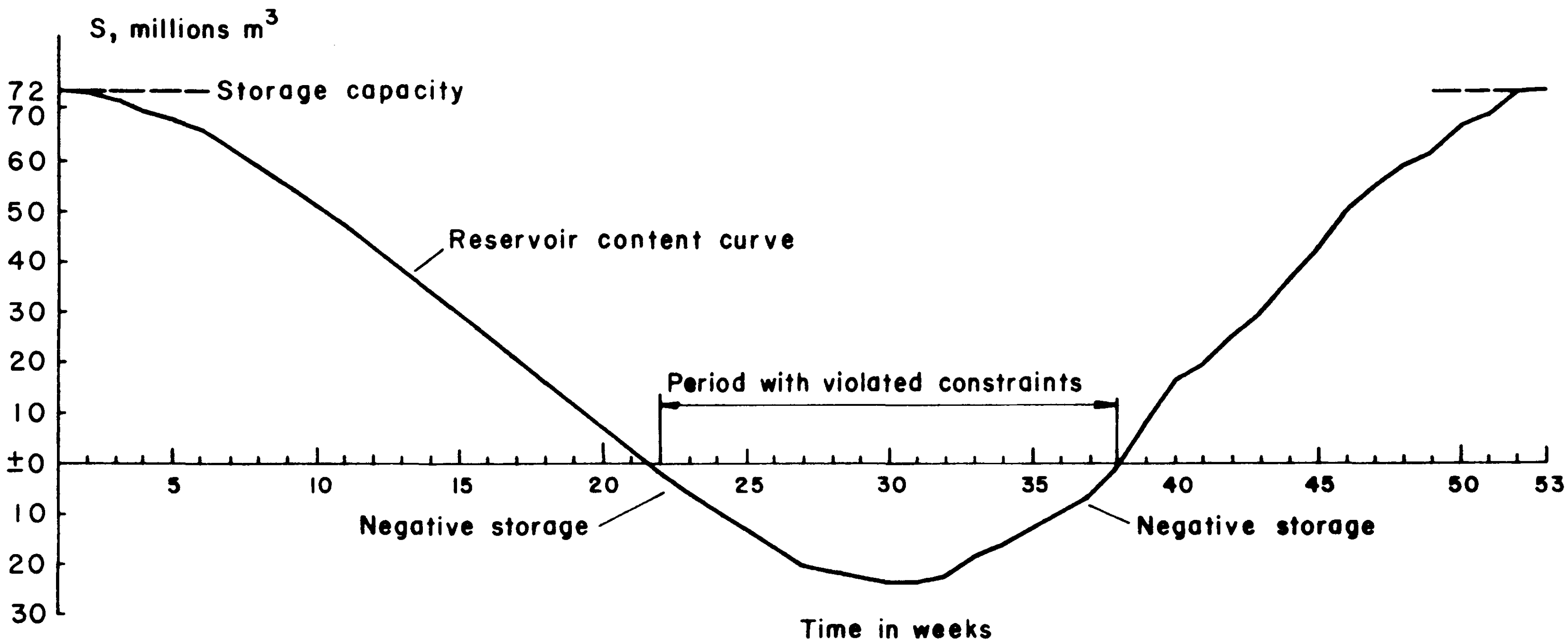


Fig. 4.1 **Reservoir of Infinite Capacity, Optimal Trajectory**

Hence the selection of a reservoir with infinite capacity allowed to simplify drastically the Kuhn-Tucker conditions, and to set forth some interesting properties of the optimal solution. In the next subsections, the problem will be progressively complicated, and we shall follow what happens to the just derived properties.

4.4.3 Reservoir of finite capacity under simplified conditions. The behavior of a real reservoir is analyzed here. However the weekly inflows are assumed to vary smoothly during the year, and in such a way, that only one release constraint becomes tight during the annual operation. This tight constraint corresponds to the week in which the reservoir gets empty. Furthermore we suppose that the reservoir gets empty at the end of the 28th week, and that it is again full at the end of the water year.

Compared to the previous case, one more constraint comes into play. It expresses that the reservoir gets empty at the end of the 28th week. Hence the optimization problem is:

$$\text{Max. } V\ (x_1,x_2,\ldots,x_{52}) = \sum_{i=1}^{52} 10^6 \cdot \int_0^{x_i} \alpha_i(\bar{s}_i)\cdot e^{-z/4.2}\, dz$$

$$\text{s.t. } s_1 - s_{53} + \sum_{i=1}^{52} q_i - \sum_{i=1}^{52} x_i = 0 \qquad (4\text{-}23)$$

$$s_1 + \sum_{i=1}^{28} q_i - \sum_{i=1}^{28} x_i = 0$$

The associated Kuhn-Tucker conditions become:

$$
\begin{aligned}
&1\ \text{a)} \quad \alpha_i\, e^{-x_i/4.2} - \lambda - \beta_{28} = 0 \quad i \le 28 \quad x_i \neq 0 \\
&\text{b)} \quad \alpha_i\, e^{-x_i/4.2} - \lambda = 0 \quad i > 28 \quad x_i \neq 0 \\
&2\ \text{a)} \quad \alpha_i \cdot e^{-x_i/4.2} - \lambda - \beta_{28} \le 0 \quad i \le 28 \quad x_i = 0 \\
&\text{b)} \quad \alpha_i \cdot e^{-x_i/4.2} - \lambda \le 0 \quad i > 28 \quad x_i = 0 \\
&3\ \text{a)} \quad s_1 - s_{53} + \sum_{i=1}^{52} q_i - \sum_{i=1}^{52} x_i = 0 \\
&\text{b)} \quad s_1 + \sum_{i=1}^{28} q_i - \sum_{i=1}^{28} x_i = 0
\end{aligned}
\tag{4-24}
$$

The just established relation looks similar to those of subsection 4.4.1; two supplementary conditions have appeared, however, which result from the additional release constraint equation. The relations given under (1) can be rewritten as follows:

$$
\begin{aligned}
&\alpha_i \cdot e^{-x_i/4.2} = \lambda + \beta_{28} \quad i \le 28 \ x_i > 0 \\
&\alpha_i \cdot e^{-x_i/4.2} = \lambda \quad i \ge 28 \quad x_i > 0
\end{aligned}
\tag{4-25}
$$

Hence it seems that the annual reservoir operation can be divided into two periods, each one being characterized by a different but constant value of the marginal returns of the release.

In the cases studied, the sequences of weekly releases follow a well-defined pattern. In winter, they are greater than the corresponding inflows, in summer, smaller. Furthermore, the reservoir content diminishes regularly starting from October until the middle of April; from; from that date on, it increases again regularly. All these remarks suggest to break down the operation of the reservoir into two periods. The first period, called by definition the drawdown phase, lasts from October 1 until the reservoir content is minimum. The second one, called the refill phase, begins in the week the reservoir content is minimum, and lasts until the end of September (Fig. 4.2).

According to the equations of (4-25), the marginal value of the releases stays constant during the drawdown phase and is equal to the sum of the two Lagrange multipliers λ, and β_j. The same property holds during the refill phase. But the marginal value of the release is equal to λ, and is consequently smaller than that of the drawdown phase as λ and β_j are both positive. Hence the properties derived in subsection 4.4.1 are still valid; but they apply only to part of the annual operation.

The date the reservoir gets empty, or the date of emptiness, plays an important role, as it allows to break down the main problem into two subproblems. At the beginning of this subsection, we assumed that this date was known in advance. However, sensitivity analyses on this parameter showed that this variable is very stable. In fact, as a first approximation, it can be considered as a constant. The exact determination will be given later on. For the time being, we shall continue to assume that we know its exact value in advance.

4.4.4 <u>Reservoir of finite capacity with known final conditions.</u> The optimization problem is considered here in its entire generality except for the assumption that the final condition is known. It is the existence of tight constraints which makes the problem under

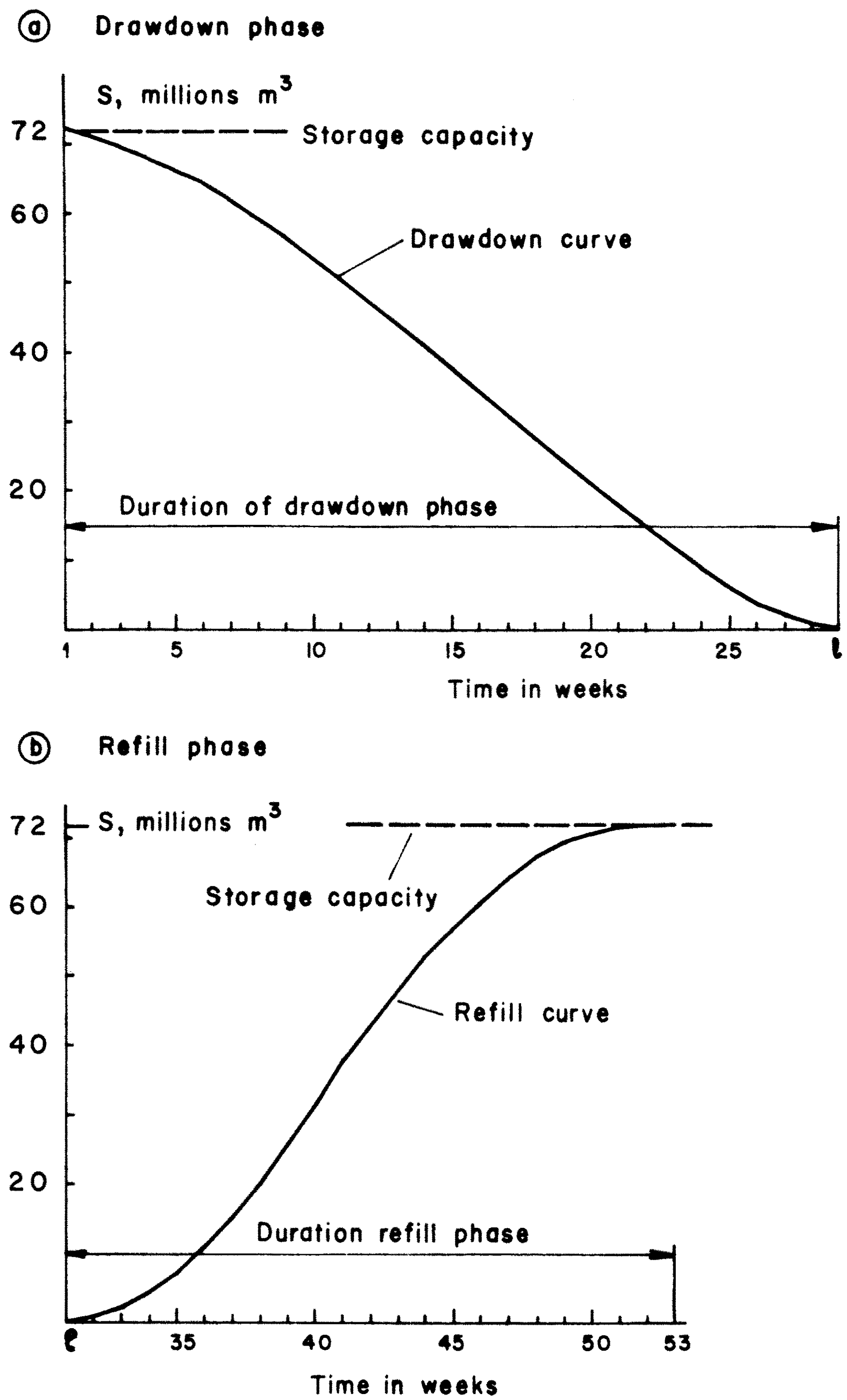


Fig. 4.2 Division of the Operation Period into Drawdown and Refill Phases

study intricate. The advance knowledge of which ones are tight would greatly diminish the amount of work. Yet the optimal solution possesses some further properties which help to reduce the computational burden.

Assume for a while that the optimal releases strategy for a complete year has been determined, and that it was found that the reservoir is full at the beginning of the 1st, 3rd, 50th and 53rd week, and empty at the beginning of the 31st and 33rd week. The optimization problem is:

$$\text{Max. } V\,(x_1, x_2, \ldots, x_{52}) = \sum_{i=1}^{52} 10^6 \cdot \int_0^{x_i} \alpha_i(\bar{s}_i) \cdot e^{-z/4.2}\, dz$$

$$\begin{aligned}
\text{s.t. } & s_1 - s_{53} + \sum_{i=1}^{52} q_i - \sum_{i=1}^{52} x_i = 0 \\
& s_1 + \sum_{i=1}^{30} q_i - \sum_{i=1}^{30} x_i = 0 \\
& s_1 + \sum_{i=1}^{32} q_i - \sum_{i=1}^{32} x_i = 0 \qquad\qquad (4\text{-}26) \\
& S - s_1 - \sum_{i=1}^{2} q_i + \sum_{i=1}^{2} x_i = 0 \\
& S - s_1 - \sum_{i=1}^{49} q_i + \sum_{i=1}^{49} x_i = 0
\end{aligned}$$

The first constraint expresses the annual water balance constraint. The two next ones are release constraints for the two weeks when the reservoir is empty. And the last two equations represent storage constraints for the weeks when the reservoir is full. The associated Kuhn-Tucker conditions follow hereafter:

$$\alpha_1 \cdot e^{-x_1/4.2} - \lambda + \gamma_{49} - \beta_{32} - \beta_{30} + \gamma_2 = 0$$

$$\alpha_2 \cdot e^{-x_2/4.2} - \lambda + \gamma_{49} - \beta_{32} - \beta_{30} + \gamma_2 = 0$$

$$\alpha_3 \cdot e^{-x_3/4.2} - \lambda + \gamma_{49} - \beta_{32} - \beta_{30} = 0$$

$$\cdots \qquad \cdots = 0$$

$$\alpha_{30} \cdot e^{-x_{30}/4.2} - \lambda + \gamma_{49} - \beta_{32} - \beta_{30} = 0$$

$$\alpha_{31} \cdot e^{-x_{31}/4.2} - \lambda + \gamma_{49} - \beta_{32} = 0$$

$$\alpha_{32} \cdot e^{-x_{32}/4.2} - \lambda + \gamma_{49} - \beta_{32} = 0 \tag{4-27}$$

$$\alpha_{33} \cdot e^{-x_{33}/4.2} - \lambda + \gamma_{49} = 0$$

$$\cdots \qquad = 0$$

$$\alpha_{49} \cdot e^{-x_{49}/4.2} - \lambda + \gamma_{49} = 0$$

$$\alpha_{50} \cdot e^{-x_{50}/4.2} - \lambda = 0$$

$$\cdots \qquad = 0$$

$$\alpha_{52} \cdot e^{-x_{52}/4.2} - \lambda = 0$$

The just written equations express that the partial derivatives of the Lagrangian form must vanish for the optimal solution. However these equations apply only if the associated release is different from zero.

In the negative, the concerned relation becomes an inequality. If, for example, the release of the 34th week equals zero, then we have:

$$\alpha_{34} \cdot e^{-x_{34}/4.2} - \lambda + \gamma_{49} \leq 0 \tag{4-28}$$

Note that we did not repeat here the relations associated with the partial derivatives of the objective function with respect to the Lagrange multipliers. In fact, they are identical to the constraint equations given under (4-26).

A closer look at these relations suggests that the marginal value of the releases varies during the year according to a definite pattern. Remembering that the Lagrange multipliers are always greater or equal to zero, it appears that the marginal return of the releases is maximum during the weeks of the drawdown phase when the reservoir is neither empty nor full. On the other hand, the marginal values of the releases is minimum during the weeks of the refill phase when the reservoir is neither full nor empty. For the complete year, the following pattern exists. The marginal value of the weekly releases increases from October 1 on until it reaches its maximum value. It stays then constant as long as the reservoir is not empty. After the date of emptiness, however, it decreases until it hits its lowest value. It remains at this value until the reservoir is full, and the cycle is repeated (Fig. 4.3). Although established on the basis of a particular example, this property is quite general. It stipulates that for an optimal solution, no release can be modified without either violating a constraint, or diminishing the total return. The just developed property is important to determine whether a strategy is optimal or not.

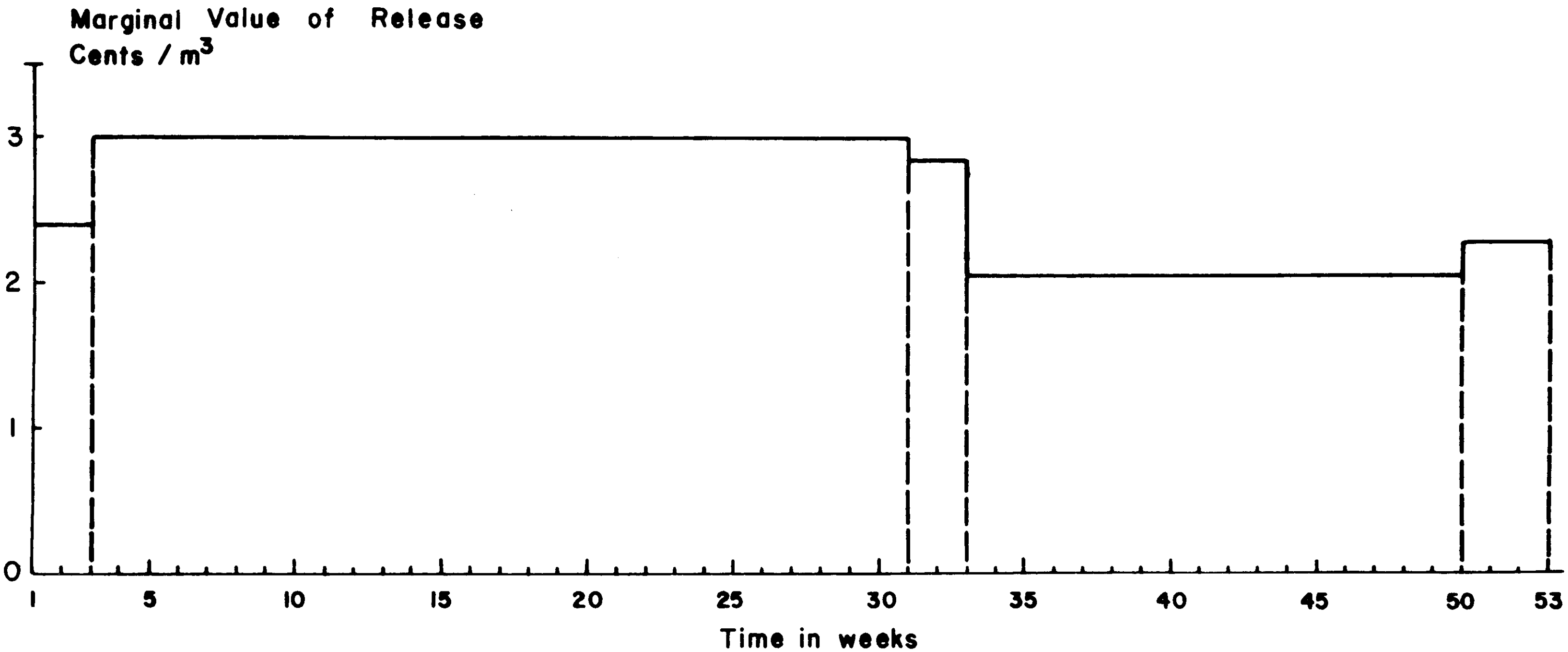


Fig. 4.3 **Marginal Values of Releases: Typical Pattern of Variation**

What happened to the basic properties of the optimal strategy described in subsection 4.4.2? They still hold, if we restrict their application to those parts of the drawdown and refill phases, for which the reservoir is neither full or empty.

The main purpose of seasonal storage schemes is to store water in summer in order to release it in winter. This fact implies that the reservoir content changes within the year roughly according to the same pattern. The highest level is reached at the end of summer, the minimum one, in the middle of spring; and in between, it decreases or increases more or less regularly depending on the inflows sequence. Accordingly, storage constraint equations can be tight most probably only in September or October, and the release constraint equation, in April and May. Furthermore computational experience shows that these constraints do not modify markedly the overall shape of the optimal reservoir content curve. Hence it should be possible to obtain a solution very close to the optimum by simply ignoring all the constraint equations, except the water balance equation and the one associated with the date the reservoir gets the first time empty. The optimal solution is then computed by successive improvement of the nearly optimal solution.

4.4.5 Final storage. The last important point to study concerns the determination of the reservoir content at the end of the operation period. In most of the preceding examples, the reservoir was assumed to be full at the end of the water year. Is that necessarily true?

The selected reservoir belongs to the category of so-called seasonal reservoir, which means that water is transferred only from one season to the other. There is no carry-over storage. The regulation takes place within the year and the total annual releases are equal to the total annual inflows. Hence if the reservoir is full at the start of the operation, it will again be full at the end of the operation.

However, it does not always work that way. If during reservoir operation, a wet winter follows a dry summer it seems reasonable to give up the idea of filling completely the reservoir by September 30. Furthermore, we know that the marginal value of the weekly releases vary within the year according to a definite pattern. Especially this variable takes on its minimum value during the refill phase. As a consequence, the marginal value of the release of the last week of the water year must be smaller, or at most equal to that of the first week of the following year. In other words, the final storage depends on what is happening in the following year.

The capacity of the existing Swiss seasonal storage reservoir is so small as compared to the average annual inflows, that for an optimal strategy the reservoir is very often full at the end of the year. This fact suggests the following methodology. In a first step, the reservoir is taken as full on September 30, and the optimal strategy is determined accordingly. In a second step, the final reservoir content is modified until the marginal value of the release of the last week of the year is smaller or equal to that of the first week in the new year.

Complete information on this topic, especially on the characteristics of the following year, will be given later. For the present time, it suffices to remember that the final storage depends on what is happening in the following year, and that is determined by successive approximations.

4.5 Solution Technique

4.5.1 Outline of the methodology. The objective is to construct an optimal release policy for the selected power scheme. The duration of operation corresponds to the water year and the releases are determined on a weekly basis. According to the preceding discussions, the solution of the system of equations resulting from the Kuhn-Tucker conditions yields the optimal release policy. However this system of equations is too complicated to be solved directly. Hence, based on the derivations of Section 4.4.4, the following solution technique is proposed.

At the beginning of the computations we temporarily assume that the reservoir is full at the end of the water year. Furthermore, an initial reservoir content curve is selected which represents the storage level reached at the beginning of each week of the year. These assumptions allow us to compute the initial energy rate function and to determine the week when the reservoir gets empty.

The water year is then divided into two parts. The period preceding the week in which the reservoir is empty corresponds to the drawdown phase. The optimal strategy is determined separately for each phase. To this end, they are first computed assuming that no constraints are binding. In a second step, the constraints are progressively introduced and, if necessary, the initial strategy is modified until no more constraints are violated.

The next step consists of assessing the final storage. For this purpose, the marginal return from the release of the last week of the operation period is compared to the average marginal return from the release of the first week of the water year. The average marginal return represents the marginal return which is obtained on the average from the release of the first week of the water year. If the value of the refill phase is higher, the final storage must be decreased, if it is lower the final storage must be increased until, for an identical storage, both values are identical.

Once these operations have been performed, drawdown and refill phase are linked together. Two things can happen. Either the conditions of optimality for the complete year are still satisfied, or not. In the first case, we proceed to the following step, in the latter case, the date of emptiness must be modified and the release strategy, must be determined again according to this new date.

Finally, the reservoir content curve resulting from the optimal release strategy is computed and compared to the initial one. If the two curves are markedly different, computations are repeated, using the resulting storage curve as initial curve. This procedure is applied until both curves are roughly identical.

The different steps are described in more detail in the following subsections.

4.5.2 Date of emptiness. The drawdown phase ends at the end of the week for which the reservoir ends up empty for the first time. This date must be known before the optimization procedure is started, to render possible the partition of the reservoir operation period, into drawdown and refill phases. Fortunately a mathematical expression can be derived to compute this date. Furthermore this parameter does not vary greatly from case to case.

Quite generally, an optimal release strategy is such that any feasible modification in the magnitude of the computed releases leads to a decrease in the total returns. Hence, if we assume that the reservoir gets empty in the week preceding, or following the date of emptiness corresponding to the optimal strategy, the total benefit must also decrease, so that the optimal strategy yields for the drawdown phase, the highest possible value for the marginal returns of the releases.

For the calculation of the date of emptiness, one considers only the drawdown phase. The assumptions that the reservoir is empty at the beginning of the ℓ^{th} week, that the correct energy rate function is available, and that no constraints are tight, lead to the following optimization problem:

$$\text{Max. } V\,(x_1,x_2,\ldots x_{\ell-1}) = \sum_{i=1}^{\ell-1} 10^6 \cdot \alpha_i(\bar{s}_i) \cdot \int_0^{x_i} e^{-z/4.2}\,dz$$

$$\text{s.t.}\quad s_1 + \sum_{i=1}^{\ell-1} q_i - \sum_{i=1}^{\ell-1} x_i = 0 \qquad (4\text{-}29)$$

The equality constraint postulates that the reservoir is empty at the beginning of the ℓ^{th} week. According to Section 4.4.3, the associated Kuhn-Tucker conditions are:

$$1. \quad \alpha_i(\bar{s}_i)\cdot e^{-x_i/4.2} - \lambda_\ell^d = 0 \quad x_i \neq 0 \quad i = 1, \ell - 1$$

$$2. \quad \alpha_i(\bar{s}_i)\cdot e^{-x_i/4.2} - \lambda_\ell^d \leq 0 \quad x_i = 0 \quad i = 1, \ell - 1 \qquad (4\text{-}30)$$

$$3. \quad s_i + \sum_{i=1}^{\ell-1} q_i - \sum_{i=1}^{\ell-1} x_i = 0$$

The ℓ of the Lagrange multiplier λ_ℓ^d which represents also the marginal value of the releases indicates that the reservoir is empty at the beginning of the ℓ^{th} week, while d refers to the drawdown cycle.

The introduction of the natural logarithms into the first equation gives:

$$x_i = 4.2\ (\ln \alpha_i - \ln \lambda_\ell^d) \quad x_i \neq 0 \quad i = 1, \ell - 1 \qquad (4\text{-}31)$$

With this expression, the releases x_i are eliminated from the third equation of (4-30) so that:

$$\ell n\ \lambda_\ell^d = \frac{1}{4.2(\ell - 1)} \cdot \left[4.2 \sum_{i=1}^{\ell-1} \ell n\ \alpha_i - s_i - \sum_{i=1}^{\ell-1} q_i\right],\ x_i \neq 0 \qquad (4\text{-}32)$$

The obtained solution is feasible provided that none of the constraint equations are violated. In a first approximation, the influence of the storage constraints on the date of emptiness is neglected, so that only the control of the release constraints remains. As the monthly energy price decreases markedly in spring, the release constraint for the last week of the drawdown phase is the most stringent one. Hence the computed solution is feasible, if for the week under consideration, the release exceeds the inflow.

On the contrary, water would have been released in the previous weeks, which was not yet in the reservoir.

The same computations are done assuming that the reservoir gets empty one week later. If the corresponding Lagrange multiplier $\lambda^d_{\ell+1}$ is smaller than λ^d_ℓ, then the reservoir gets empty in the ℓ^{th} week for the optimal strategy. In the negative, the date of emptiness is delayed by one more week, and the same computations are repeated until the following condition is satisfied:

$$\lambda^d_\ell \geq \lambda^d_{\ell+1} \tag{4-33}$$

The fulfillment of this condition expresses that the highest possible value for the Lagrange multiplier has been obtained.

The derived expression for the determination of the date of emptiness of the reservoir holds only if the initial assumptions are satisfied. It may well happen that the existence of tight storage constraints, and the real storage content curve lead to a different value of this parameter. However computational experience showed that this seldom happened.

At this stage of the computations, the water year is divided into drawdown and refill phases, according to the information given by this section.

4.5.3 Drawdown phase. Required is an optimal release strategy for the drawdown phase. The initial storage and the weekly inflows sequence are known, whereas the duration of the drawdown phase was computed in the preceding subsection. Making the assumption that no constraints are tight reduces the problem to:

$$\text{Max. } V\,(x_1, x_2, \ldots x_{\ell-1}) = \sum_{i=1}^{\ell-1} 10^6 \cdot \alpha_i(\bar{s}_i) \cdot \int_0^{x_i} e^{-z/4.2}\,dz$$

$$\text{s.t. } s_1 + \sum_{i=1}^{\ell-1} q_i - \sum_{i=1}^{\ell-1} x_i = 0 \qquad (4\text{-}34)$$

According to Section 4.4.3, the relevant Kuhn-Tucker conditions are:

$$1.\quad \alpha_i \cdot e^{-x_i/4.2} - \lambda_\ell^d = 0 \quad x_i \neq 0 \quad i = 1,\ \ell-1$$

$$2.\quad \alpha_i \cdot e^{-x_i/4.2} - \lambda_\ell^d \leq 0 \quad x_i = 0 \quad i = 1,\ \ell-1 \qquad (4\text{-}35)$$

$$3.\quad s_i + \sum_{i=1}^{\ell-1} q_i - \sum_{i=1}^{\ell-1} x_i = 0$$

These equations are identical to those derived under subsection 4.5.2. Hence, after different modifications of the basic relations (see subsection 4.5.2), we have:

$$x_i = 4.2 \cdot [\ln \alpha_i(\bar{s}_i) - \ln \lambda_\ell^d]$$

$$s_1 + \sum_{i=1}^{\ell-1} q_i - \sum_{i=1}^{\ell-1} x_i = 0 \qquad (4\text{-}36)$$

The expression for $\ln \lambda_\ell^d$ obtained in subsection 4.5.2 is introduced into the first equation, so that:

$$x_i = \frac{1}{\ell-1}\left[s_1 + \sum_{i=1}^{\ell-1} q_i + 4.2 \cdot (\ell-1) \cdot \ln \alpha_i - 4.2 \cdot \sum_{i=1}^{\ell-1} \ln \alpha_i\right]$$

$$i = 1,\ \ell-1 \qquad (4\text{-}37)$$

This formula determines all the releases for the drawdown phase.

Such a simple solution results directly from the assumption that no constraints are tight. Quite a change from the cumbersome system of equations given in Section 4.4.3. Of course the just computed releases are not necessarily feasible and they will be corrected later on, if necessary. However it is more efficient to proceed by successive approximations, than to try to solve directly the original system of equations.

4.5.4 Refill phase . After introduction of the correct initial and final conditions into the mass balance equation, the releases for the refill phase are computed in the same way as those of the drawdown phase. Here the reservoir is empty at the start of the operation and full at the end of the water year. The optimization problem consists then in:

$$\text{Max. } V\,(x_\ell, x_{\ell+1}, \ldots, x_{52}) = \sum_{i=\ell}^{52} 10^6 \cdot \alpha_i(\bar{s}_i) \cdot \int_0^{x_i} e^{-z/4.2}\, dz$$

$$\text{s.t.} \; - s_{53} + \sum_{i=\ell}^{52} q_i - \sum_{i=\ell}^{52} x_i = 0 \tag{4-38}$$

The associated Kuhn-Tucker conditions (see Section 4.4.3) are:

$$1. \quad \alpha_i(\bar{s}_i) \cdot e^{-x_i/4.2} - \lambda_\ell^f = 0 \quad x_i \neq 0 \quad i = \ell, 52$$

$$2. \quad \alpha_i(\bar{s}_i) \cdot e^{-x_i/4.2} - \lambda_\ell^f \leq 0 \quad x_i = 0 \quad i = \ell, 52 \tag{4-39}$$

$$3. \quad -s_{53} + \sum_{i=\ell}^{52} q_i - \sum_{i=\ell}^{52} x_i = 0$$

Applying the same transformations to relation 1 and 3 yields:

$$\ell n\, \lambda_\ell^f = \frac{1}{4.2[52-(\ell-1)]} \cdot \left[4.2 \cdot \sum_{i=\ell}^{52} \ell n\, \alpha_i + s_{53} - \sum_{i=\ell}^{52} q_i \right] \tag{4-40}$$

and

$$x_i = \frac{1}{4.2(52-(\ell-1)]} \cdot$$

$$\left[-s_{53} + \sum_{i=\ell}^{52} q_i + 4.2[52-(\ell-1)]\ell n\, \alpha_i - 4.2 \sum_{i=\ell}^{52} \ell n\, \alpha_i \right] \quad i = \ell,52$$

The remarks done at the end of subsection 4.5.3 apply here also. Furthermore, one should remember that the derived releases result from the assumption that the reservoir is full at the end of the operation. This restriction will be eliminated in one of the following steps.

4.5.5 Introduction of the constraints - equations. At this stage of the computations, there exist two optimal strategies, one for the drawdown, and one for the refill phase. The next step consists in checking whether the release strategies violate any constraint and, if necessary, in correcting it in such a way that the incriminated constraint is no longer violated. The methodology followed to correct the releases is identical for both operation phases.

a) Drawdown phase, storage constraint. Figure 4.4 shows the correction procedure. The constraints equations are satisfied step by step, starting from the beginning of the operation period. In the selected example, the release strategy violates the storage constraint in the second and in the third week.

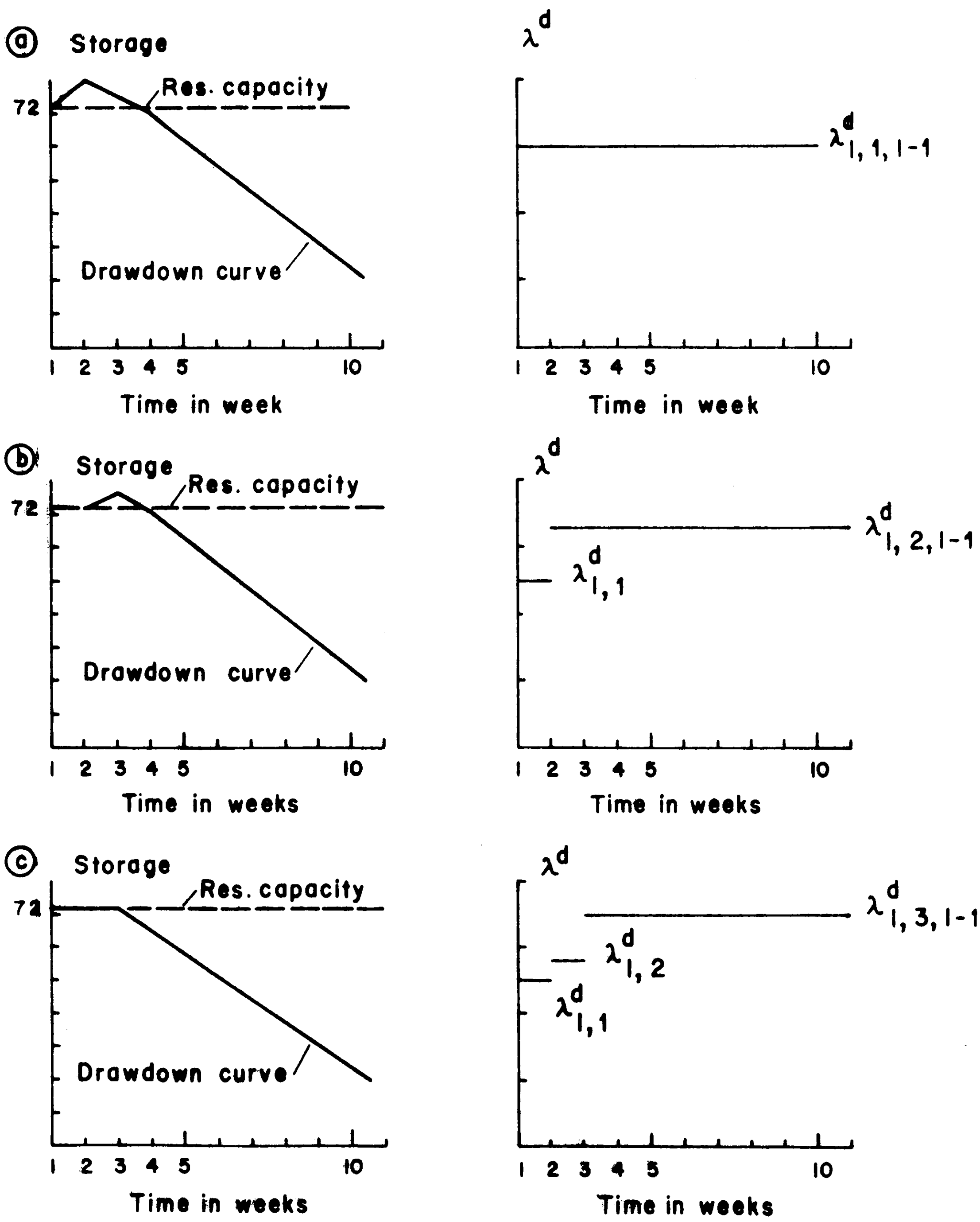


Fig. 4.4 Drawdown Phase: Satisfaction of Storage Constraints

We begin by increasing the release of the first week so that the reservoir is just full in the second week. The procedure of subsection 4.5.3 is applied to compute the releases for the remaining weeks, starting the operation in the second week with a full reservoir. Diagram b shows the newly determined reservoir content curve.

As the storage constraint for the third week is still violated, the same procedure is repeated but this time for the second week. We continue in the same way until no more constraints are violated.

The release strategy available from the third week on is optimal. For, as the marginal value of the release for the period (1,3) is lower than that of the following periods, water should be transferred from the period (1,3) to the period $(3,\ell)$; but this is not possible without violating a constraint. Hence the strategy from the third week on is optimal.

The optimality conditions are also applied to the releases of the first subperiod. If necessary they are modified.

b) Refill phase, release constraint. The releases are successively corrected until no more constraints are violated (see Fig. 4.5). Here again, the strategy is optimal for the period $(\ell+2,52)$ as no water can be transferred from the region of low marginal value, to the region of high marginal value.

c) Refill phase, storage constraint. The same methodology holds and Fig. 4.6 supplies the necessary information.

4.5.6 <u>Final storage</u>. We are now in a position to relax the restriction on the final storage. To this end, the year under study is linked to another year, but which one?

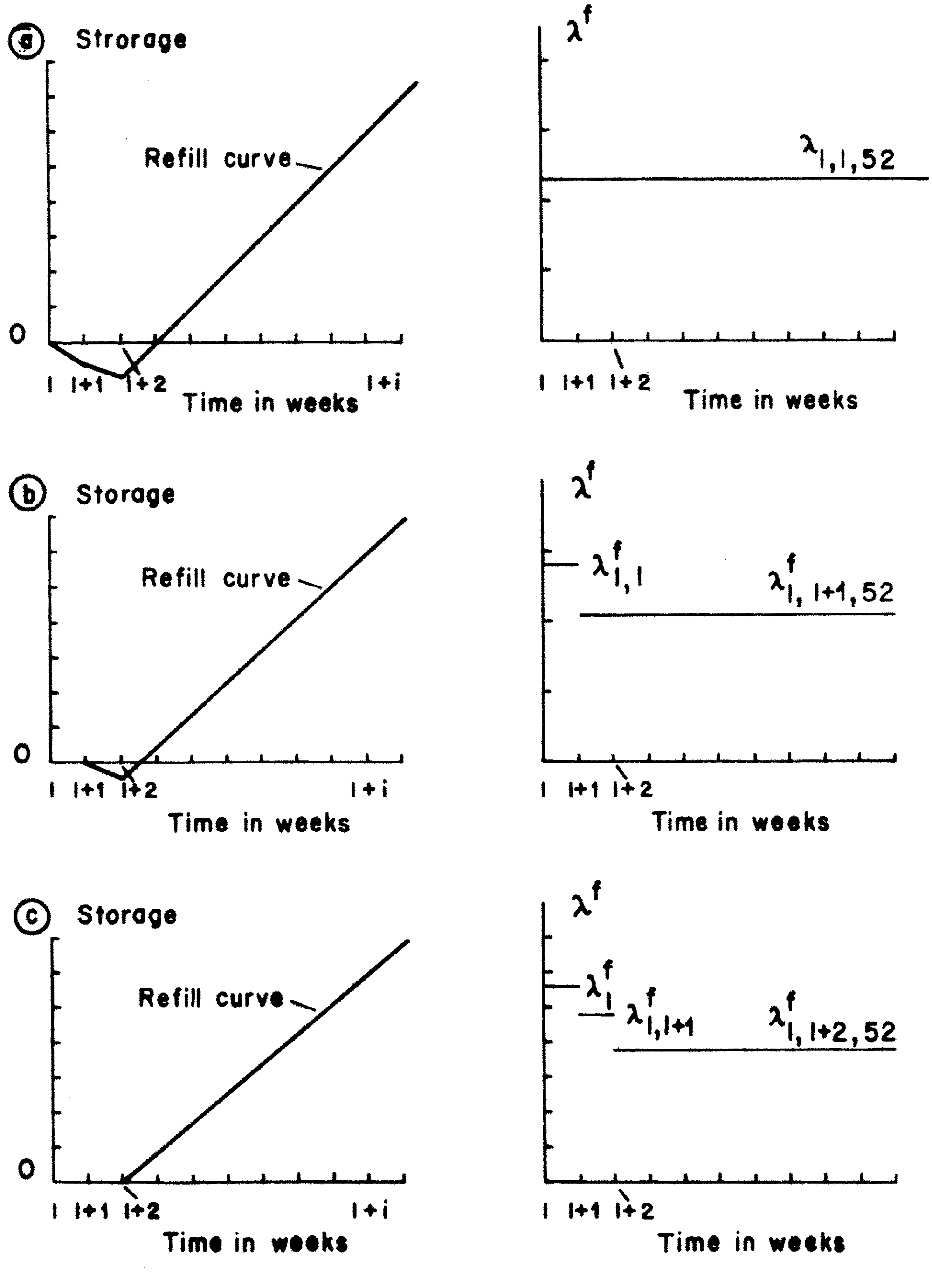


Fig. 4.5 <u>Refill Phase: Satisfaction of Release Constraints</u>

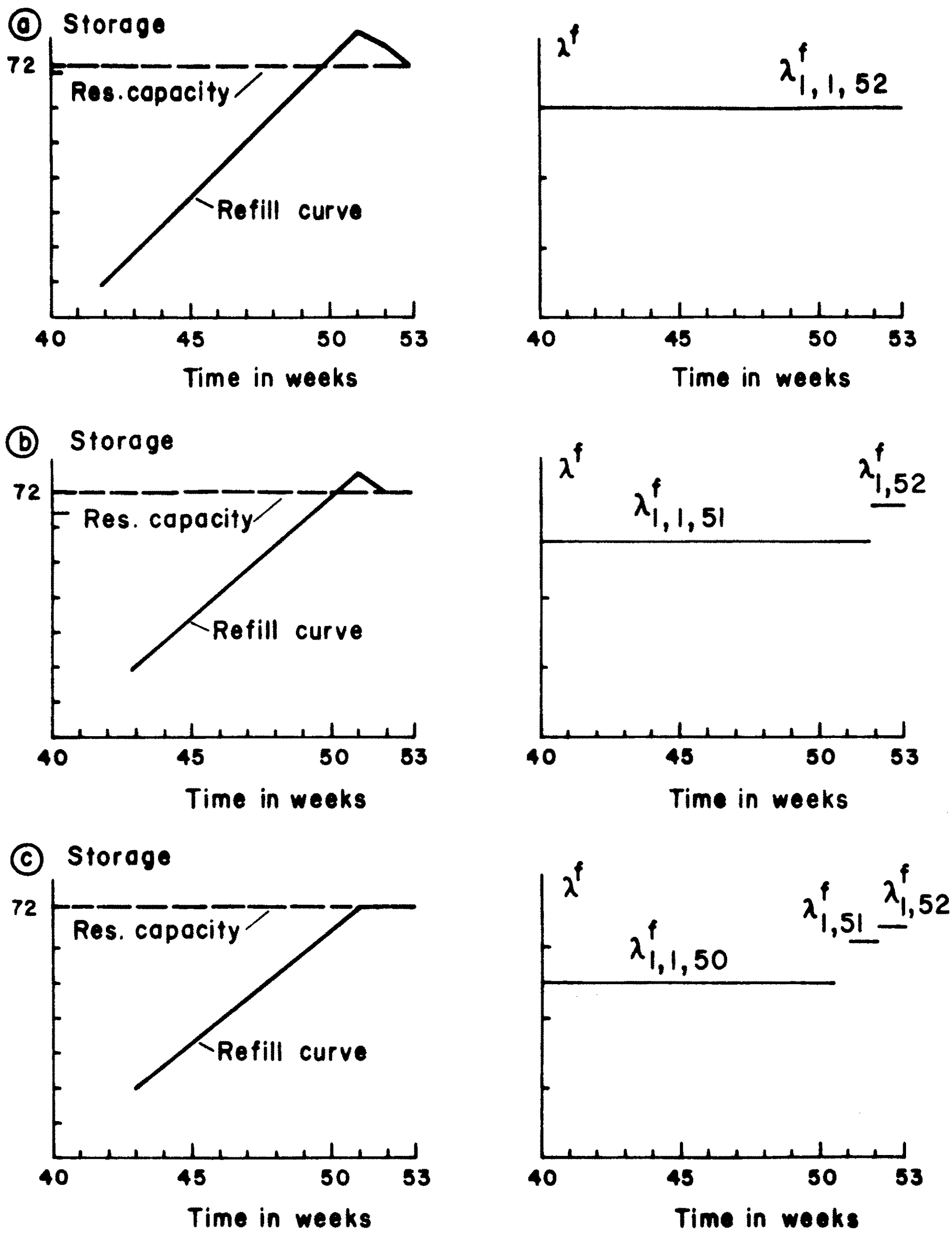


Fig. 4.6 Refill Phase: Satisfaction of Storage Constraints

The hydrology of the selected watershed does not allow to forecast in early summer the inflows of the forthcoming winter and summer. This fact justifies the selection of a year which corresponds to an average hydrologic condition. Actually the inflows of the second year should be such that the marginal returns of the associated releases are equivalent to the average of the marginal returns obtained from the reservoir operation over many years. To use a term which will be defined in Chapter 5, the marginal value of the releases of the second year, must correspond to the expected marginal value. Computations showed that a year with weekly inflows equal to the corresponding average of the recorded values satisfies the above condition. Consequently the year under study will be linked to a so-called average year.

The procedure becomes now obvious. The marginal return of the last week of operation is compared to that of the first week in the following year, and for a full reservoir. If the first one is smaller than the latter one, the reservoir should remain full. In the contrary, the final storage should be decreased until both marginal returns become equal.

The conditions of optimality are satisfied. For a reservoir content smaller than the reservoir capacity, both marginal values are equal, and no transfer of water is necessary. For a full reservoir, water should be transferred from the period with low marginal returns to that of high marginal returns, but this is not possible without violating the constraints (Fig. 4.7).

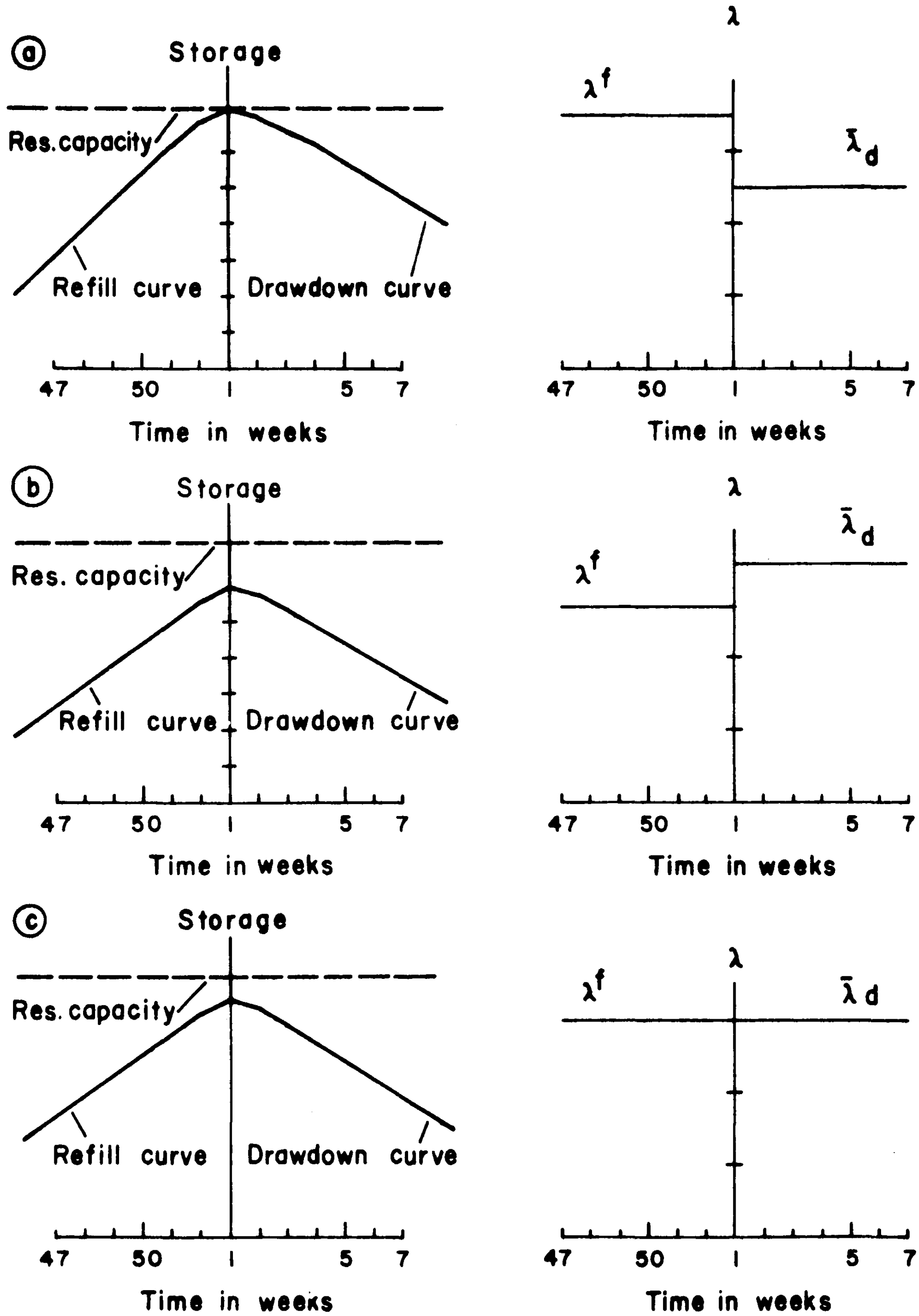


Fig. 4.7 Optimal Final Storage Capacity

4.5.7 Linkage of the drawdown to the refill phase. After the separate determination of the optimal sequence of releases, the drawdown phase is linked to the refill phase. The computed release strategies must satisfy one condition to be optimal: the marginal value of the release of the last week of the drawdown phase must be greater, or at least equal to that of the following week. In this situation, it is no longer possible to transfer water from the region of low marginal value to that of high marginal value:

$$\lambda^{d}_{\ell-1} > \lambda^{f}_{\ell} \tag{4-41}$$

If this condition is not satisfied, then either the reservoir gets empty in a later week, or it does not get empty at all. The latter situation arises, if the magnitude of the summer inflows is small as compared to that of the winter inflows.

4.5.8 Optimal strategy. Finally the computed reservoir content curve is compared to the initial one. If the two curves are not too different from each other, and if the marginal values of the releases vary according to the required patterns (see Section 4.4), the problem is solved. However, in all the cases studied, a second iteration was necessary. In the second iteration, the new initial reservoir content curve corresponds to the curve obtained at the end of the first run.

4.5.9 Concluding remarks. The methodology just described seems to be lengthy and complicated. This wrong impression results from two reasons. First, the methodology was developed here in such a way that the described steps correspond to the routines of the computer program. Second, all the special cases which might come up have been mentioned. In reality, however, they seldom occur, or at least not at the same time.

Computational experience showed that in most cases, the first guess of the date of emptiness was the correct one. Furthermore, the reservoir nearly always got empty and the storage constraints rarely got tight during the drawdown phase. These facts which are directly related to the hydrologic, constructive and economical characteristics of the hydropower plant, reduced substantially the amount of computations and made the methodology quite attractive.

4.6 Computer Program and Case Study

4.6.1 Introductory remarks. The computer program follows exactly the methodology developed in the preceding sections. It contains a main program and nine subroutines. On the whole, the division into subroutines is identical to the division of Section 4.5 into subsections.

To illustrate the computational procedures, a simple example is worked through. The selected inflows sequence corresponds to the mean values for the period of 1945 to 1974, and the reservoir is taken as full at the beginning of the water year. It is a simple case but nevertheless it shows the essential features of the program.

4.6.2 Preparatory routines. Under this heading are grouped the routines which perform computations not directly related to the optimization procedure. Routine LOF1 reads and prints out, if desired, the input data. Among them are the weekly inflows sequence, the characteristics of the hydroelectric scheme, the set of energy prices and some indications concerning the initial reservoir content curve.

On the basis of this information, LOF2 establishes the initial reservoir content curve and LOF3 computes accordingly the energy rate function. The relevant data appear in Tables 4.1 and 4.2.

4.6.3 Routine LOF4. It determines the date of emptiness according to the formula given by Eq. 4-32. Assuming that the reservoir is full at the start of the operation and empty at the beginning of the 31st week, this formula leads to:

$$\lambda_{31}^{\ell} = \frac{1}{4.2 \cdot 30} (4.2 \cdot 55.924 - 72.000 - 15.707) \tag{4-42}$$

The inflows and the reservoir content are given in million m^3. The term 55.924 represents the sum of the $\ell n\ \alpha_i(\bar{s}_i)$, 72.000, the initial reservoir content, and 15.707, the inflows during the drawdown phase. The same computations were repeated for a date of emptiness occurring in the 30th and the 32nd week. The results follow hereafter:

$$\lambda_{30}^{\ell} = 3.214,\ \lambda_{31}^{\ell} = 3.216,\ \lambda_{32}^{\ell} = 3.177 \text{ cents/m}^3 \tag{4-43}$$

Hence the highest marginal returns from the releases are obtained when the reservoir is empty at the beginning of the 31st week. Furthermore for a marginal value of 3.177, the reservoir content is negative in the 31st week.

4.6.4 Routine LOF5. This routine is identical for the drawdown and refill phase. It determines the optimal sequence of releases according to Eq. 4-37 for the case that neither storage, nor release constraints are tight. The release for the first week of the year is obtained as follows:

Table 4.1

Annual Reservoir Operation with Average Inflows
Drawdown Phase, First Iteration

Week Index	LOFL q_i 10^6 m^3	LOF1 Pr_j Cents/KWh	LOF2 s_i 10^6 m^3	LOF3 e_{hd} KWh/m^3	LOF3 α_i Cts·KWh/m^3	LOF4 λ Cts/m^3	LOF5 x_i 10^6 m^3	LOF7 x_i 10^6 m^3	LOF7 s_i 10^6 m^3
1	1.687	2.84	70.800	1.319	6.025	3.216	2.637	2.637	71.525
5	0.943	3.09	61.200	1.302	6.470	"	2.937	2.937	65.350
9	0.459	3.39	51.600	1.283	6.995	"	3.264	3.264	55.480
13	0.307	3.66	42.000	1.262	7.430	3.216	3.517	3.517	42.952
17	0.237	3.77	32.400	1.240	7.516	"	3.565	3.565	29.718
21	0.191	3.66	22.800	1.216	7.152	"	3.357	3.357	16.816
25	0.241	3.32	13.200	1.188	6.340	"	2.851	2.852	5.571
29	0.880	2.30	3.600	1.154	4.263	3.216	1.184	1.184	0.188
31			0.000						0.000

$$\lambda = \frac{1}{4.2\cdot 30} \cdot [4.2 \cdot 55.924 - 72 - 15.707]$$

Table 4.2

Annual Reservoir Operation with Average Inflows
Refill Phase, First Iteration

Week Index	LOF1 q_i $10^6\ m^3$	LOF1 Pr_j Cents/KWh	LOF2 s_i $10^6\ m^3$	LOF3 e_{hd} KWh/m^3	LOF3 α_i $Cts\cdot KWh/m^3$	LOF4 λ Cts/m^3	LOF5 x_i $10^6\ m^3$	LOF6 x_i Cts/m^3	LOF7 x_i $10^6\ m^3$	LOF7 s_i $10^6\ m^3$
31	1.623	1.77	1.636	1.145	3.255	2.402	1.276	2.385	1.306	0.158
33	3.167	1.56	8.182	1.171	2.938	2.402	0.846	"	0.876	3.053
37	4.870	1.11	21.274	1.211	2.162	"	0.000	"	0.000	16.998
41	5.476	1.11	34.366	1.245	2.221	2.402	0.000	2.385	0.000	39.852
45	5.093	1.48	47.458	1.274	3.033	"	0.980	"	1.010	58.534
49	3.768	1.95	60.550	1.301	4.079	"	2.225	2.385	2.255	70.191
52	1.888	1.95	70.364	1.319	4.135	2.402	2.281	2.638	1.888	72.000
53			72.000							72.000

$$\lambda = \frac{1}{4.2\cdot 13}(4.2 \cdot 15.157 + 72.000 - 88.209)$$

$$x_1 = \frac{1}{30}[72.000 + 15.707 + 4.2 \cdot 30 \cdot 1.796 - 4.2 \cdot 55.924]$$

and

$$x_1 = 2.637 \cdot 10^6 \text{ m}^3 \tag{4-44}$$

For the refill phase, a slight modification must be introduced into the general procedure. Equation 4-40 holds only for weeks with releases greater than zero. It may happen however that no water should be released during some weeks. In these cases, the concerned weeks just drop out of the equation. Hence in the selected example, there are only 14 weeks with releases greater than zero (Table 4.2).

4.6.5 Routine LOF6. The constraints are introduced in this routine stepwise and chronologically. For this purpose, one divides the phase under study into two parts. A first subperiod, where the constraints are tight, and a second one, where they are loose. The constraints are introduced successively until the reservoir content curve stays completely within the limits of the reservoir, and in such a way that the strategy for the second period is always optimal (see Section 4.5.5 and Fig. 4.4, 4.5, and 4.6).

It so happens that no constraints are violated during the drawdown phase. For the refill phase, the reservoir content exceeds its capacity at the beginning of the 52nd week. Hence the reservoir content is set to the reservoir capacity at the beginning, assuming that the reservoir operation ends at the beginning of the 52nd week. The recomputed value of the marginal return is then:

$$\lambda^f_{31,52} = \frac{1}{4.2 \cdot 13}(-4.2 \cdot 15.157 - 72 + 88.209) = 2.385 \text{ Cents/m}^3 \tag{4-45}$$

The related reservoir content curve does not violate anymore any constraint, and we can proceed to the next step.

4.6.6 Routine LOF7. This routine checks separately for the two phases whether the computed strategies are optimal. If it is not the case, the releases are modified until the Kuhn-Tucker conditions are satisfied. The releases had to be recomputed most often for the subperiods with tight constraints. In the application problem, the releases as obtained from LOF6 were optimal, so that no corrections were required.

4.6.7 Routine LOF8. It links the refill phase to the drawdown phase of the following year, to determine the optimal storage on September 30th. To this end, the marginal return of the last week of September is compared to that of the first week of October, for identical content and according to the procedure developed in Section 4.5. Here marginal value $\lambda_{31,52}$, 2.638, is smaller than the marginal return $\bar{\lambda}^d$, which in this example is equivalent to $\lambda^d_{31,}$ or 3.216. Hence the reservoir must be full at the end of the water year, quite a logical conclusion. For the reservoir in question was designed for seasonal regulation and consequently should get full when average hydrologic conditions prevail. In the contrary there would be a waste of storage.

4.6.8 Main program and routine LOF9. Drawdown and refill phase are coupled together in the main program. If the computed $\lambda^d_{31,31}$, is greater than $\lambda^f_{31,32}$, the date of emptiness has been selected correctly and the problem is solved. This is what happened in the present case study as 3.216 is greater than 2.385. In the opposite, the computation should have been started again from the beginning, with a later date of emptiness. Routine LOF9 has been written to take care of the case when the reservoir does not get empty.

In the last step, the resulting reservoir content curve is compared to the initial one. If the difference between them is too important, then a new iteration must be started using the resulting reservoir content curve as initial one. Two iterations are enough to reach an adequate precision, as indicated by the computed example (Table 4.3 and 4.4 and Fig. 4.8). Figure 4.9 shows the optimal strategies for different initial contents, and Table 4.5 the associated marginal returns of the releases.

4.7 Results and Evaluation

4.7.1 Purpose and scope of the applications. The just described solution technique was applied to different cases. These applications confirmed the soundness of the assumptions on which the solution technique is based. Furthermore they set forth some properties of the optimal solution which helped to speed up the computations and gave some indications concerning the path to follow for the reservoir operation under uncertainty.

The system consisted of the hydro-power plant and of the reservoir described in Chapter 3, while the gauging station of Hinterrhein supplied the required inflows.

The reservoir was operated first for the 29 years of available records, starting on October 1 with a full reservoir. Then, to assess the influence of the initial condition, we performed two additional series of runs with each a different initial storage. Finally we also varied the duration of operation, which ranged from a complete year to three months. The list of runs appear on

Table 4.3

Annual Reservoir Operation with Average Inflows
Drawdown Phase, Second Iteration

Week Index	LOF1 q_i $10^6\ m^3$	LOF1 Pr_j Cents/KWh	LOF2 s_i $10^6\ m^3$	LOF3 e_{hd} KWh/m^3	LOF3 α_i $Cts \cdot KWh/m^3$	LOF4 λ Cts/m^3	LOF5 x_i $10^6\ m^3$	LOF7 x_i $10^6\ m^3$	LOF7 s_i $10^6\ m^3$
1	1.687	2.84	71.525	1.320	6.030	3.198	2.663	2.663	71.512
5	0.943	3.09	65.350	1.310	6.507	"	2.983	2.983	65.184
9	0.459	3.39	55.480	1.291	7.037	"	3.312	3.312	55.118
13	0.307	3.66	42.952	1.264	7.442	3.198	3.547	3.547	42.429
17	0.237	3.77	29.718	1.234	7.476	"	3.566	3.566	29.133
21	0.191	3.66	16.816	1.198	7.053	"	3.321	3.321	16.298
25	0.241	3.32	5.571	1.161	6.199	"	2.779	2.779	5.266
29	0.880	2.30	0.188	1.136	4.205	3.198	1.149	1.149	0.173
31			0.000						0.000

Table 4.4

Annual Reservoir Operation with Average Inflows
Refill Phase, Second Iteration

Week Index	LOF1 q_i 10^6 m^3	LOF1 Pr_j Cents/KWh	LOF2 s_i 10^6 m^3	LOF3 e_{hd} KWh/m^3	LOF3 α_i Cts·KWh/m^3	LOF4 λ Cts/m^3	LOF5 x_i 10^6 m^3	LOF6 x_i Cts/m^3	LOF7 x_i 10^6 m^3	LOF7 s_i 10^6 m^3
31	1.623	1.77	0.158	1.136	3.235	2.409	1.239	2.392	1.269	0.177
33	3.167	1.56	3.053	1.151	2.885	"	0.757	"	0.787	3.211
37	4.870	1.11	16.988	1.199	2.140	"	0.000	"	0.000	17.369
41	5.476	1.11	39.852	1.258	2.245	2.409	0.000	2.392	0.000	40.233
45	5.093	1.48	58.534	1.297	3.087	"	1.042	"	1.072	58.824
49	3.768	1.95	70.191	1.318	4.134	"	2.268	2.392	2.298	70.256
52	1.88	1.95	72.000	1.321	4.143	2.409	2.277	2.643	1.888	72.000
53			72.000							72.000

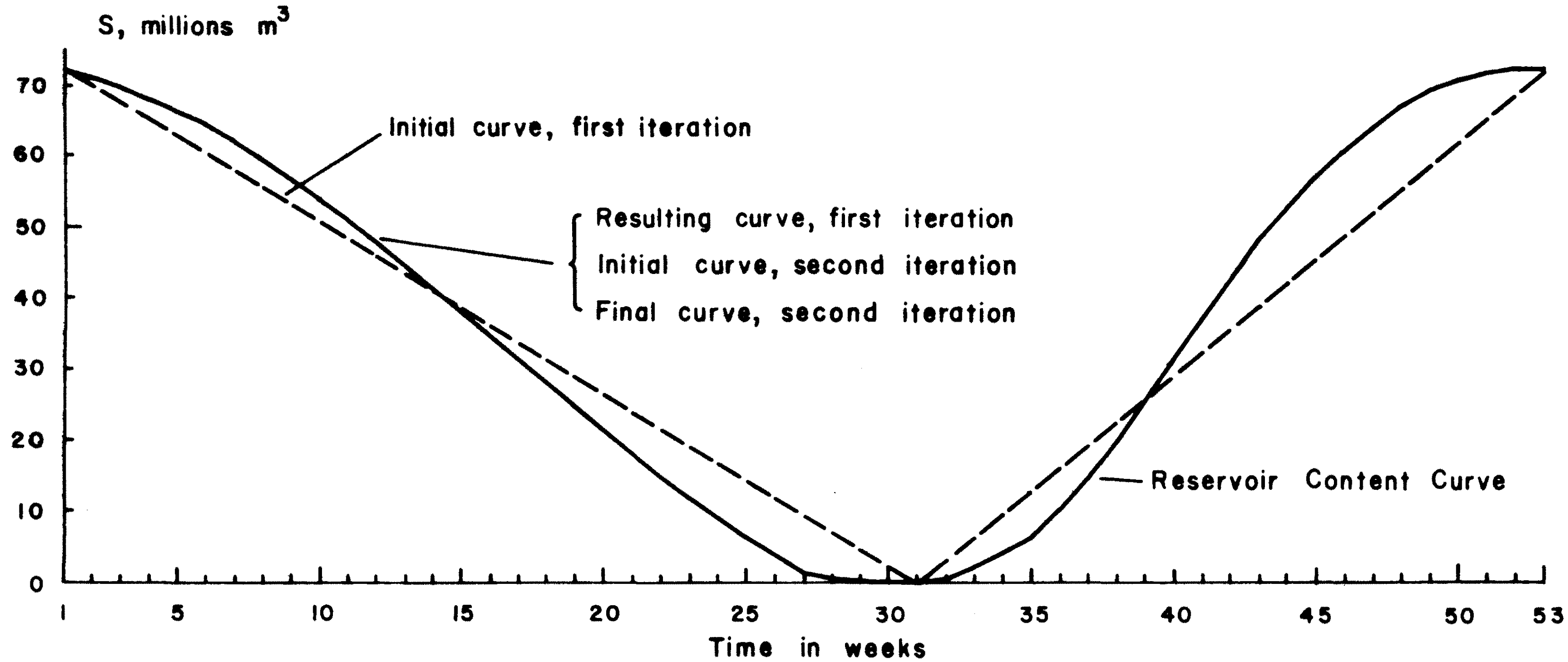


Fig. 4.8 Reservoir Operation with Average Inflows: Optimal Trajectory

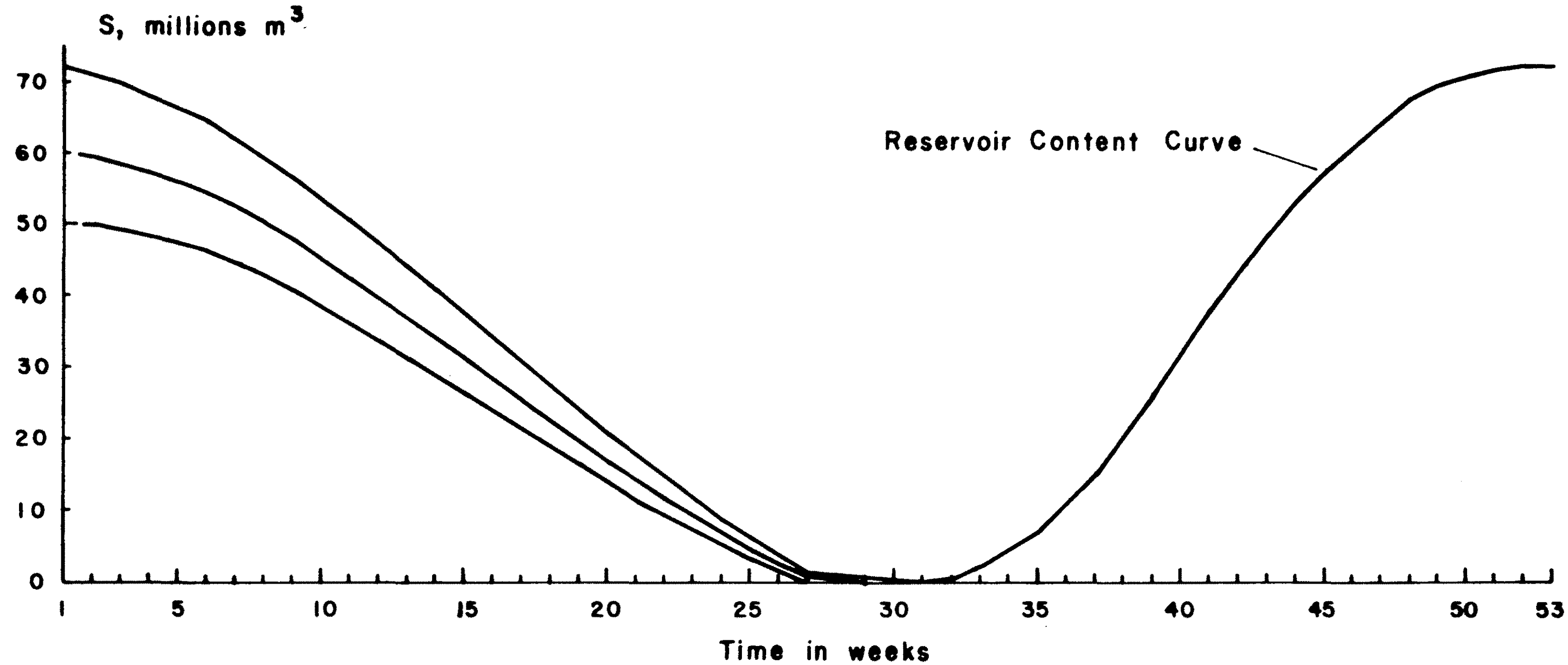


Fig. 4.9 Reservoir Operation for Average Inflows: Optimal Trajectory for Different Initial Conditions

Table 4.5

Annual Reservoir Operation with Average Inflows
Marginal Value of Releases during Drawdown Phase
and Date of Emptiness

Initial Storage 10^6 m^3	Marginal Return Cents/m^3	Date of Emptiness Week Index
72	3.198	31
70	3.245	30
68	3.293	30
66	3.341	30
64	3.391	30
62	3.442	29
60	3.495	29
58	3.548	29
56	3.603	29
54	3.658	28
52	3.716	28
50	3.775	27

Table 4.6. The selection of the reservoir operation durations and of the initial contents were done in such a way that the applications covered all the cases with which a reservoir operator has to deal in real life.

The present section begins with the analysis of the results from the annual reservoir operation with maximum initial storage. Then follows the discussion of the influence of the initial condition on the optimal strategy. The section ends with the presentation of the effects of the duration of operation.

4.7.2 Date of emptiness. Starting with maximum storage, the reservoir content dropped to zero at least once during the annual control in 25 out of 29 years. In the remaining four years, the reservoir never got empty and the marginal value of the releases stayed constant throughout the year.

These special cases happened when for instance a dry summer followed a wet winter. Consequently the marginal returns of the drawdown releases were unusually low, and those of the refill phase, unusually high, so that both could become equal without violating any constraint. Accordingly the ratio:

$$\frac{\text{Sum of refill releases}}{\text{Sum of drawdown releases}} \qquad (4\text{-}46)$$

is a first rough indicator of whether a reservoir gets empty or not. The smaller its value, the higher the probability that it gets empty.

However this is only a gross approximation, as not only the total amounts of releases play a role but also their distribution within their respective phases. If high inflows occur at the beginning of winter, or at the end of summer, while the reservoir

Table 4.6

List of Computed Cases

Start of operation

	1 October	31 December	1 April	1 July
Initial Content	72.000	48.809	7.569	39.963
Initial Content	66.000	40.809	1.569	31.963
Initial Content	60.000	32.809	0.000	23.963

Initial content in 10^6 m^3

is still full, respectively already full, they cannot be stored and must be released immediately. Hence they do not have any bearing on the determination of the marginal value of the releases of their respective phase. Accordingly, the following ratio is a better indicator of whether the reservoir gets empty or not:

$$\frac{\text{Sum of refill releases with constant marginal returns}}{\text{Sum of drawdown releases with constant marginal returns}} \tag{4-47}$$

Two remarks result from this relation: the importance of the initial condition and the fact that both winter and summer inflows must be known to tell with certainty whether a reservoir gets empty during the annual operation.

The approximate equation derived in Section 4.4 (Eq. 4-32) to determine the date of emptiness proved to be powerful. It failed to give the correct answer in a couple of cases only. Also this date did not vary greatly from year to year. The reservoir was empty 15 times on April 29, 4 times on April 15, and 2 times on April 8. This is a small scatter.

The date the reservoir gets the first time empty depends on the relative magnitude of the weekly inflows in April. The inflows during the week preceding the date of emptiness were always much smaller than those of the following week.

Quite often, the refill cycle did not start immediately after the date of emptiness, and the reservoir remained empty during a couple of weeks. In two cases even, the actual refill phase began only in the middle of May.

4.7.3 Final storage. The final storage amounted to the maximum reservoir capacity in 18 out of 29 years. Furthermore for 7 of the remaining 11 years, it exceeded 99 percent of the reservoir capacity. The lowest obtained level corresponded to a little more than 65 million m^3.

The maximum storage was reached quite often before the end of the water year. September 15 and 22 were the most frequently recorded dates, whereas September 1 was the earliest one.

4.7.4 Drawdown phase. The computed drawdown curves did not differ greatly from each other. They were on the whole smooth and similar in their shape. These characteristics result from the secondary role played by the inflows which represent only a small fraction of the total quantity of water released during the drawdown phase. Actually one can divide the drawdown curve roughly into three parts (Fig. 4.10).

The first one lasts from the beginning of October until about mid November. It corresponds to the period where the greatest variations in the drawdown curve are recorded. The reservoir is nearly full and the inflows during October can in some cases markedly influence the release strategy. If there is no more storage space available the incoming water must be immediately released. Furthermore, the inflows themselves undergo great variations in this time of the year, as depending on the prevailing temperature, glaciermelt is either completely stopped or still going on. It follows from these remarks that the reservoir may still be full in mid-November.

The second part lasts from mid-November until mid-March. A nearly constant rate of reservoir content decrease with time characterizes this part of the curve. This rate of decrease is greater for years with large storage in November, than for years with low

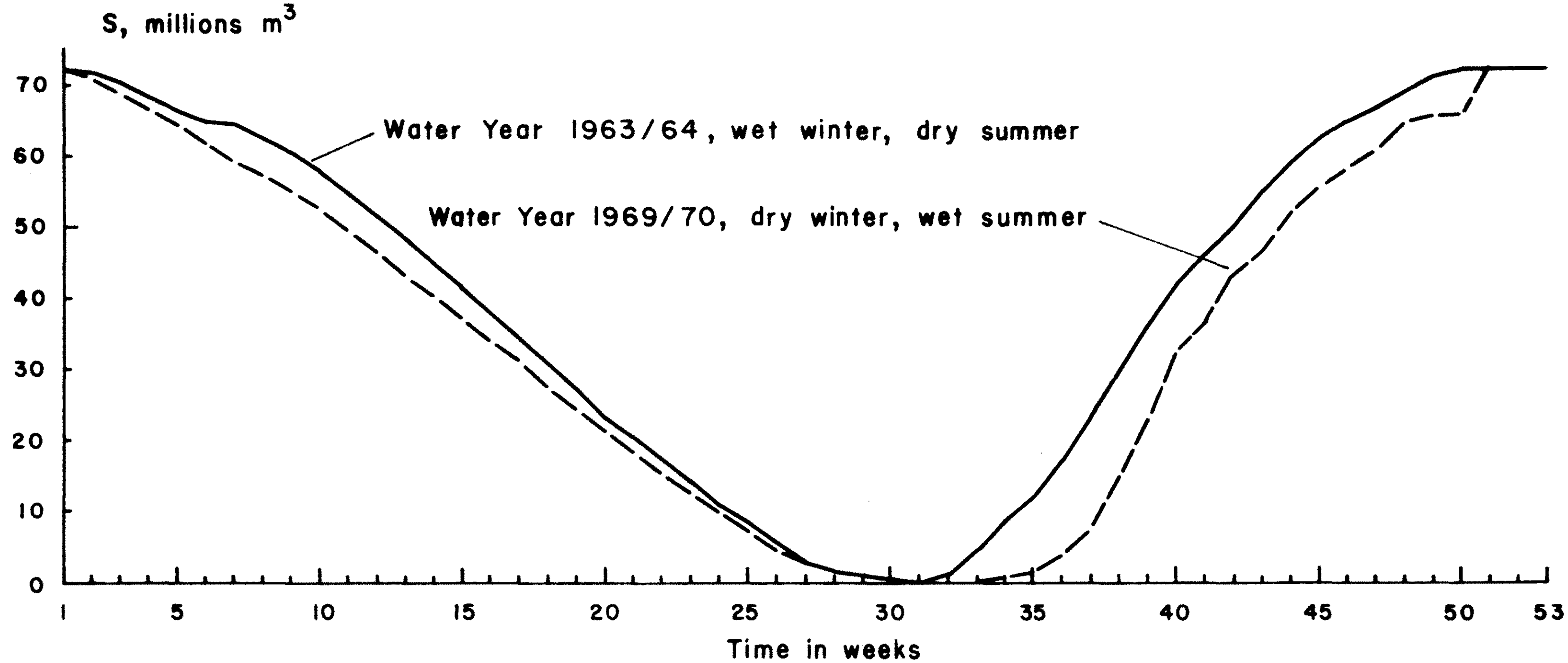


Fig. 4.10 Typical Drawdown and Refill Curves

storage in November, so that as time passes, the reservoir content curves from the different years get closer to each other.

The third part covers the last four to six weeks of the drawdown phase. The rate of decrease of the reservoir content changes completely and becomes such that the tangent to the reservoir content curve becomes horizontal on the date on which the reservoir is empty. The strategy in these weeks is markedly influenced by the associated inflows sequence.

The analysis of the release sequences confirmed these remarks (Table 4.7). The greatest variations in the releases were recorded during the first weeks of operation. However, they were much smaller from mid-November on. The range of the releases obtained for a given week and for 29 years amounted to about 25 percent of the corresponding average value. The related standard deviation was equal to 5 percent of the average release. Towards the end of the drawdown phase, the variations in the releases obtained for the different years increased again but stayed smaller than the ones recorded in October. Hence even within the drawdown phase, the determination of the releases obey to different rules.

In 24 out of 29 cases, storage constraints were never tight during the drawdown phase. Hence most often the marginal value of the returns stayed constant during that phase. However, in the most general case, the value of this parameter first increases until the last binding constraint has been reached; from that moment on it stays constant until the reservoir is empty.

4.7.5 Refill phase. The situation here differs completely from the preceding case. Inflows play an important role as the

Table 4.7

Annual Reservoir Operation with Maximum Initial Storage
Summary of the Results

Week Index	Date	Releases Aver.	σ	Min.	Max.	Reservoir Content Aver.	σ	Min.	Max.	Return Aver.	σ	Min.	Max.
		10^6 m^3				10^6 m^3				10^6 m^3			
1	1 Oct	2.968	0.957	2.406	61048	72.000	-	72.000	72.000				
14	31 Dec	3.590	0.163	3.358	3.938	41.013	2.105	37.963	45.815				
27	1 Apr	1.147	0.151	0.932	1.325	2.172	1.342	0.403	6.156	4.691	0.330	4.068	5.214
40	1 July	0.251	0.377	0.000	1.346	31.721	4.481	16.805	41.012				
52	22 Sept	1.593	0.609	0.848	2.972	-	-	-	-				

Statistics are based on 29 years of records

*The date is that of the first day of the week operation starts with an empty reservoir. Hence the shape of the reservoir content curve becomes irregular and great variations may exist from one year to another. However one can again divide the refill phase into three subperiods, of which the first and the last one are not always present (Fig. 4.10).

The first subperiod corresponds to the time when the reservoir is empty. It may last up to four weeks. Two opposite facts cause this situation. First, in a wet summer, there is no hurry in starting to fill immediately the reservoir, as more than enough water is available in June, July and August to do so. Second, the reservoir stays also empty for a longer period when the inflows after the date of emptiness are quite small; in this case the incoming water is immediately released. These remarks imply that the releases of the first subperiod depend on the total summer inflows as well as on the weekly inflows.

The second subperiod covers the weeks during which the reservoir is neither full nor empty. Its duration changes from year to year. The releases are mostly small and the shape of the refill curve depends directly on the inflows sequence, so that sudden jumps may appear in it, depending on the prevailing hydrology. However initial and final conditions play a minor role.

The third subperiod extends over the last four weeks of reservoir operation. Then the reservoir stays close to its maximum, so that the magnitude of the inflows determines the releases. The water coming from high inflows must be immediately released, as no storage space is available.

The releases of the refill phase undergo greater variations than those of the drawdown phase. Yet the same general trend exists. For the available sample, the variations in the release associated to a given week are smallest from mid-June to mid-August; they increase as one moves towards the beginning and the end of the operation period (Table 4.7).

The marginal value of the returns follow a typical pattern of variation during the refill phase. It decreases just after the date of emptiness until it reaches its minimum, which corresponds to the actual refill phase. It increases again as soon as the maximum storage capacity has been hit.

4.7.6 Initial condition. The initial storage affects the form of the optimal release strategy. To set forth these influences, the reservoir was first operated for the complete year and for the available inflows sequences but with three different initial conditions. The performed computations showed the importance of the date of emptiness. Usually the release strategy after this date was independent from the initial condition, so that the date of emptiness is really a cutting-point for reservoir operation. Furthermore, the differences between the drawdown curves corresponding to the selected initial conditions decreased with time. They were maximum at the start of the operation, and nil when the reservoir was empty. Accordingly, the releases differed most at the beginning of the drawdown phase. Finally for identical inflows sequence, the reservoir got empty earlier with lower initial storage (Fig. 4.9).

The reservoir was also operated for periods shorter than a year. In these cases, the preceding conclusions must be modified a little. For if the initial storage was extremely large, the reservoir no longer got empty and the influence of the initial condition was felt over the entire period of operation. On the other hand, the reservoir did not get full in late September if the operation was started after April with a low initial storage (Fig. 4.11 and Table 4.8).

4.7.7 Returns. The program was run for four durations of operation with each three initial conditions. The average returns resulting from the 12 studied cases were tabulated and compared among each other (Table 4.9). The standard deviation of the average revenue was smallest for the annual operation, where it amounted to 7-8 percent of the corresponding mean value. In fact the absolute value of the standard deviation was about the same for an operation over 52, 39 and 26 weeks. The greatest variations were observed for the operation over 13 weeks (Fig. 4.12, 4.13, 4.14 and 4.15).

4.8 Conclusions

A general system of equations to solve the problem of annual reservoir operation was established at the beginning of the Chapter. The nature of this control problem allowed to simplify the basic system of equations. Especially the year was broken down into two parts, the drawdown and the refill phase, which could then be optimized separately. Once the solutions for the two periods were found, the two phases were coupled together, and the Kuhn-Tucker conditions checked. The so obtained release strategy is the solution to the initial problem.

The computer program developed according to this methodology was tested on the available 29 years of records. The tests were successful. The simplifying assumptions which led to quite simple mathematical expressions for the initial solution, supplied release strategies which in most cases lay close to the optimal one. Also enlightened guess of

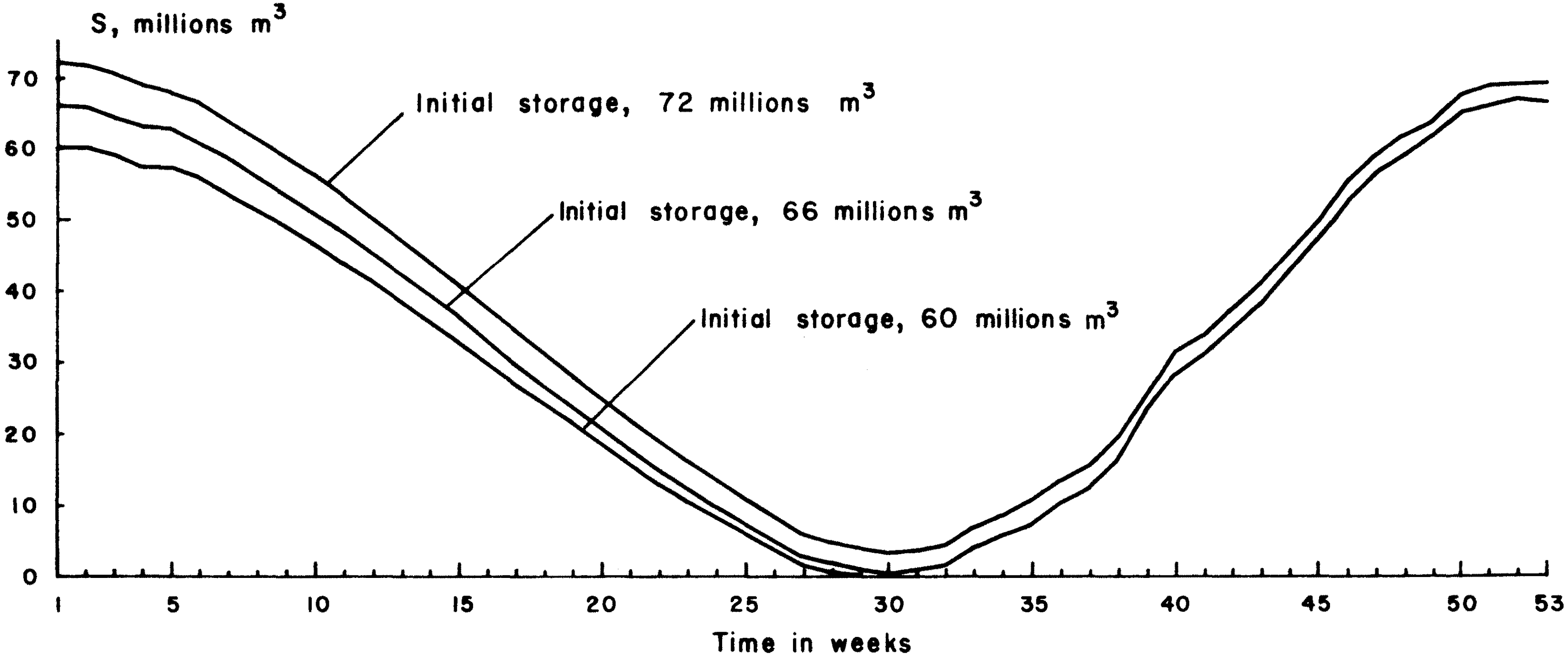


Fig. 4.11 <u>Influences of Initial Conditions on Optimal Trajectory</u>

Table 4.8

Average Releases Obtained for Selected Weeks

Duration of Operation Weeks	Start of Operation	Initial Storage 10^6 m^3	Average Releases 1 Oct	31 Dec 10^6 m^3	1 Apr	1 July	22 Sept
52	1 October	72.000	2.928	3.590	1.147	0.251	1.593
52	"	66.000	2.446	3.399	0.968	"	1.590
52	1 October	60.000	2.237	3.195	0.787	0.251	1.583
49	31 December	48.809	-	4.036	1.567	0.251	1.612
49	"	40.809	-	3.583	1.139	"	1.592
49	31 December	32.809	-	3.071	0.683	0.251	1.584
26	1 April	7.569	-	-	2.222	0.251	1.614
26	"	1.569	-	-	1.077	"	1.586
26	1 April	0.000	-	-	0.440	0.251	1.584
13	1 July	39.963	-	-	-	0.780	1.718
13	"	31.963	-	-	-	0.326	1.518
13	1 July	23.963	-	-	-	0.119	1.390

Average is based on 29 years of records

Table 4.9

Computed Average Returns

Duration of Operation Weeks	Start of Operation	Initial Storage 10^6 m^3	Returns Aver. 10^6 Fr	δ 10^6 Fr
52	1 October	72.000	4.691	0.330
"	"	66.000	4.481	0.328
52	1 October	60.000	4.258	0.329
39	31 December	48.809	2.999	0.336
"	"	40.809	2.736	0.323
39	31 December	32.809	2.448	0.306
26	1 April	7.569	0.828	0.324
"	"	1.569	0.690	0.305
26	1 April	0.000	0.637	0.303
13	1 July	39.963	0.571	0.224
"	"	31.963	0.414	0.236
13	1 July	23.963	0.294	0.200

Statistics are based on 29 years of records

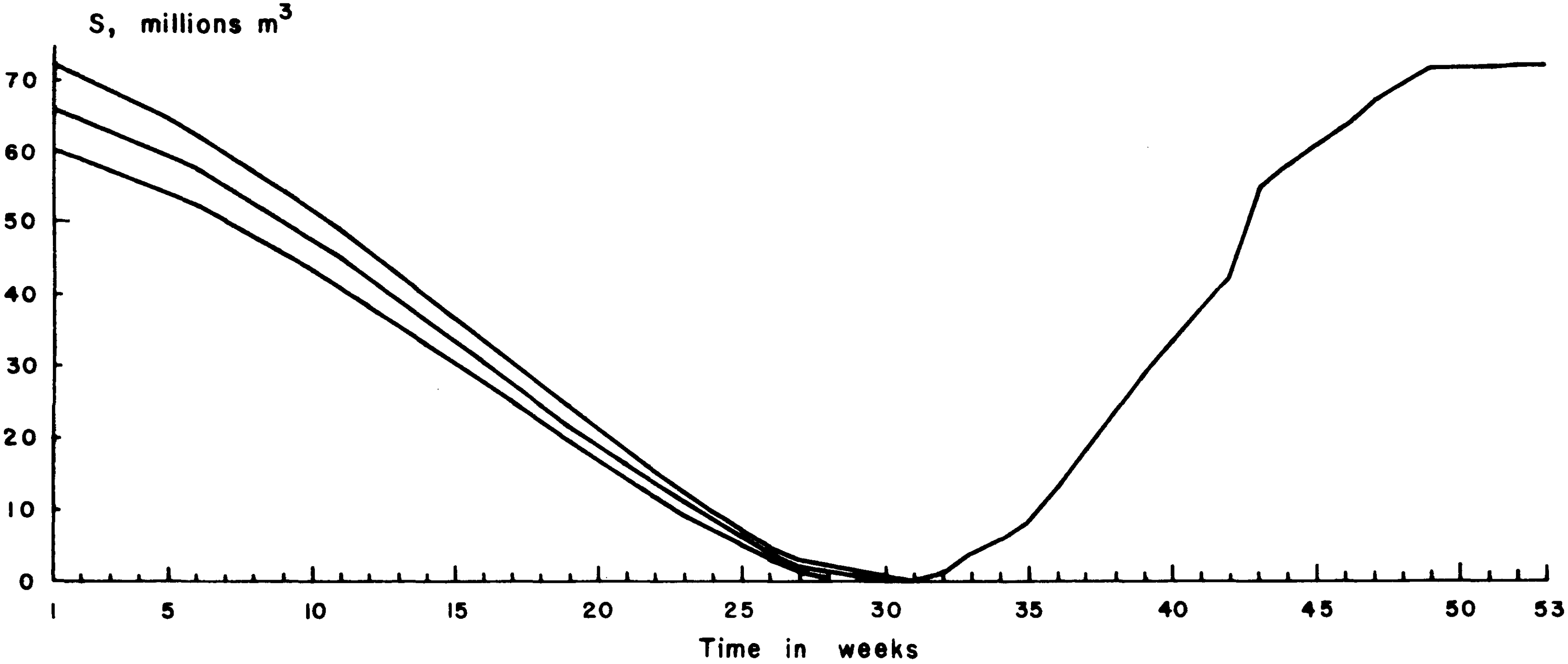


Fig. 4.12 Water Year 1972/73. Reservoir Operation over 52 Weeks

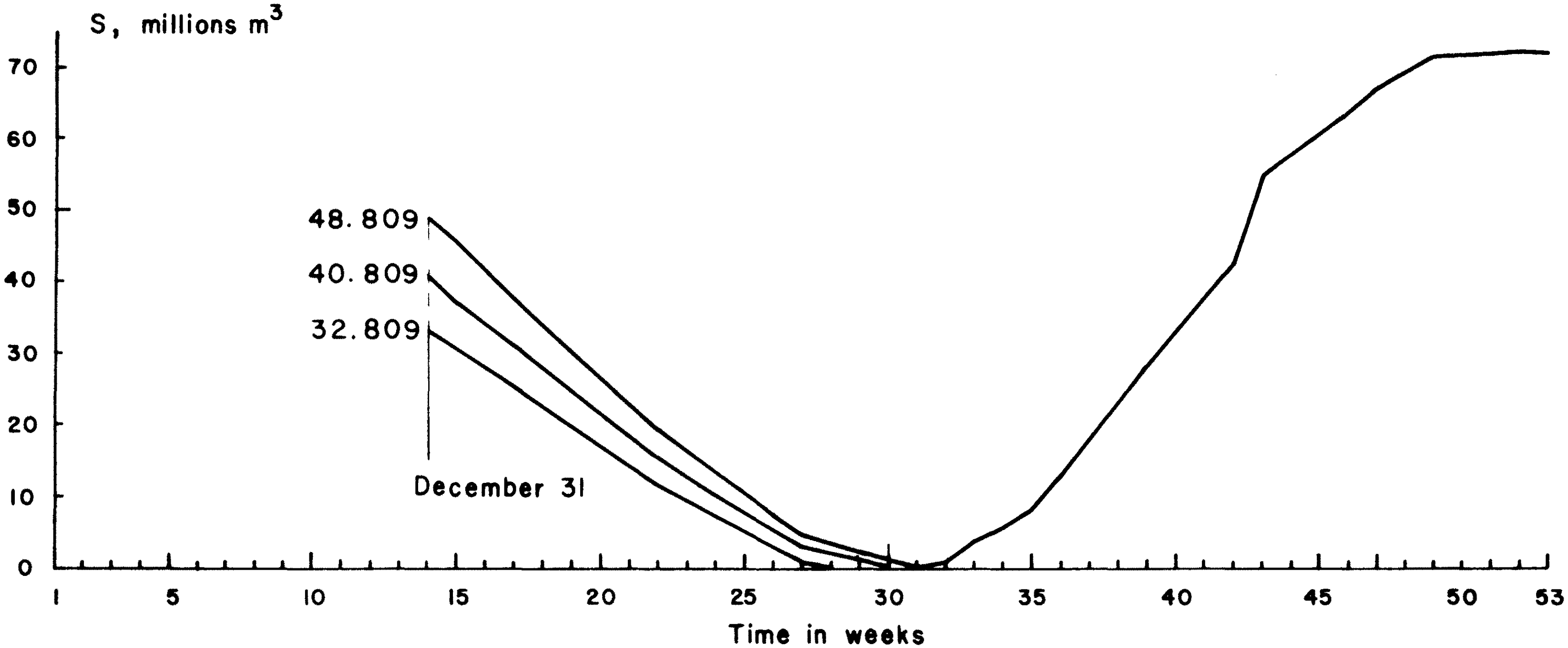


Fig. 4.13 Water Year 1972/73. Reservoir Operation over 39 Weeks

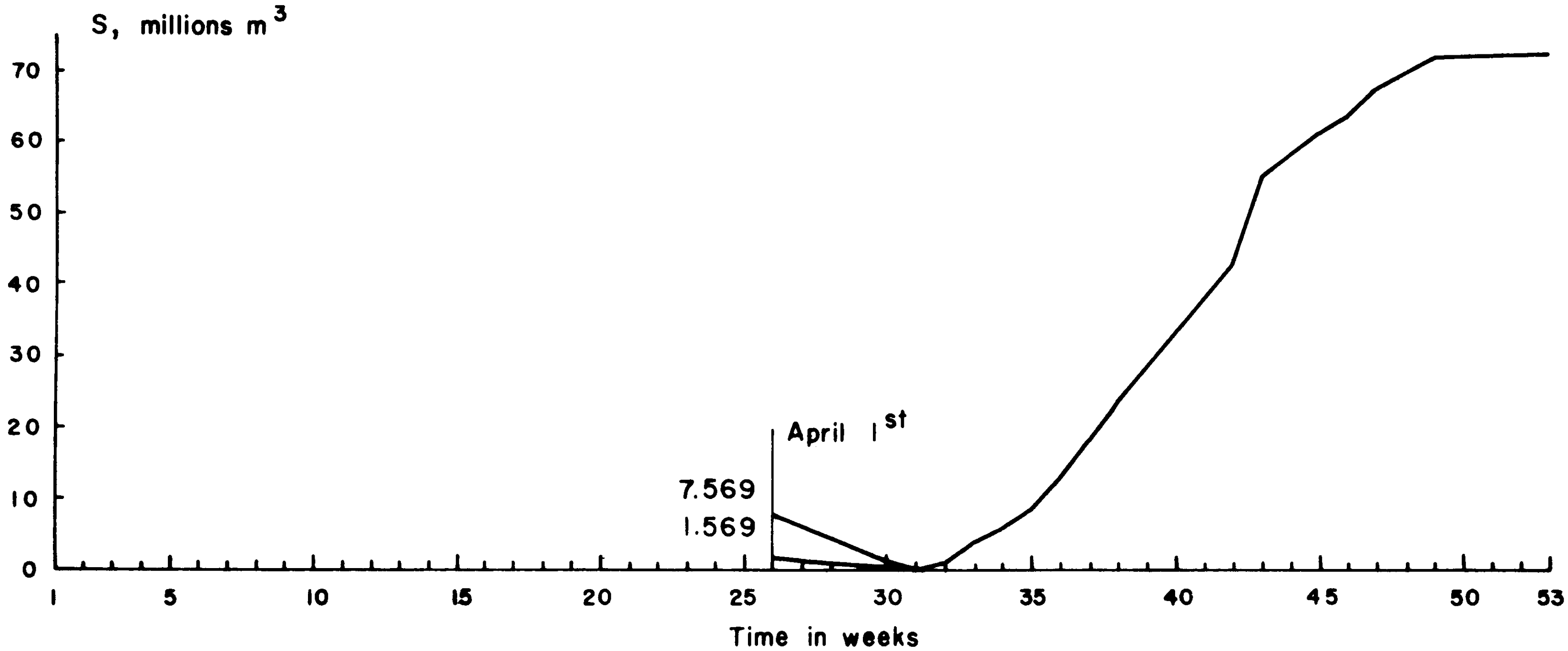


Fig. 4.14 <u>Water Year 1972/73. Reservoir Operation over 26 Weeks</u>

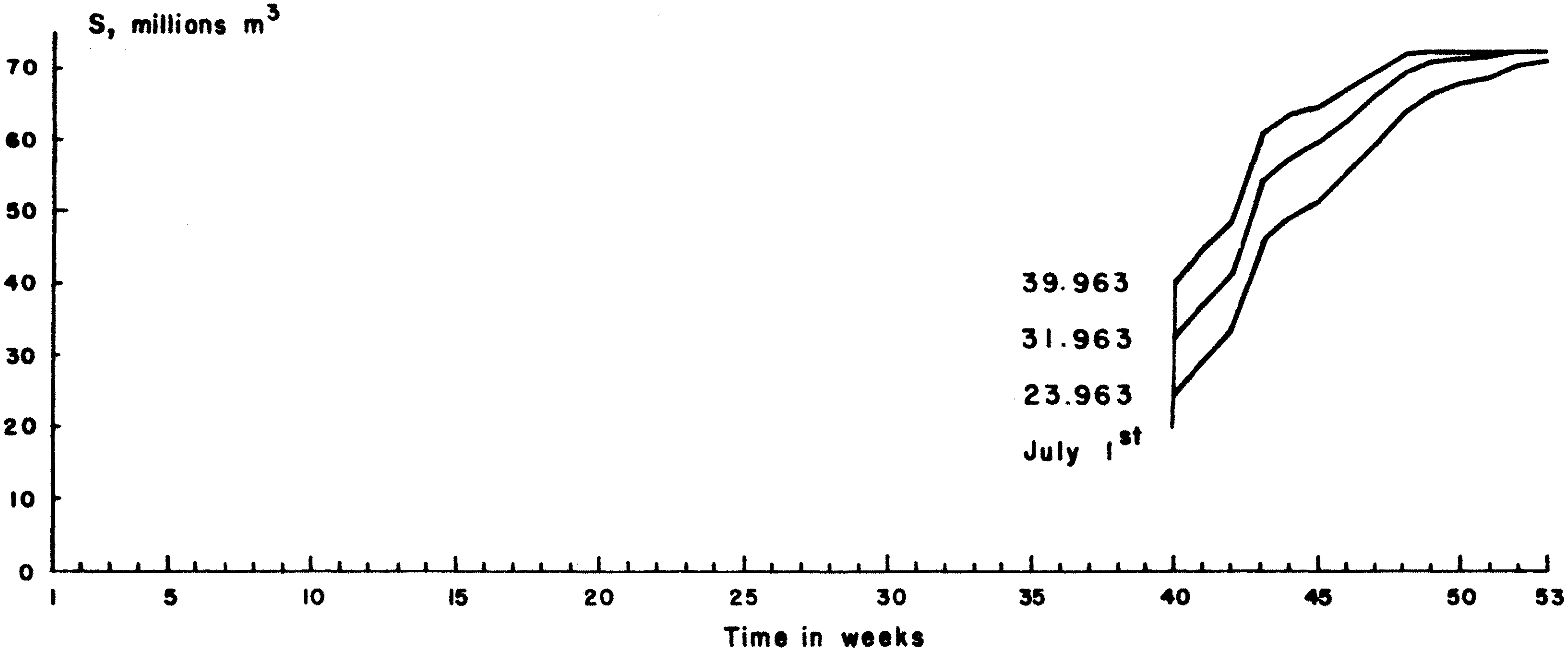


Fig. 4.15 Water Year 1972/73. Reservoir Operation over 13 Weeks

of the initial reservoir content curve reduced further the amount of computations. Finally the performed runs conveyed a host of interesting information.

The properties of the optimal trajectories change within the year. During the drawdown phase, the inflows play a secondary role. Hence the drawdown curves from the available inflows sequences lie close to each other in their middle part; they differ only at their ends. During the refill phase, however, the inflows play an important role and substantial differences between the reservoir content curves corresponding to the available records exist. They are generally greatest at the beginning and at the end of the refill phase.

For reservoir operation under uncertainty, it is important to know the relations existing between the optimal releases and some parameters. These relations are not simple and furthermore their nature changes over the year. The initial storage influences markedly the strategy in the drawdown phase, whereas the total inflows during the summer semester are determinant for the refill phase. However from September to October, and from the middle of March, to the middle of May, the magnitude of the weekly inflows plays an important role.

With the gained experience, we are now in a good position to tackle the problem of stochastic reservoir operation.

Chapter 5

STOCHASTIC RESERVOIR OPERATION

Up to now, we assumed that the complete sequence of future weekly inflows was known at the moment the release strategy was determined. Though unrealistic, this assumption allowed to establish interesting properties of the optimal solution. The situation in the real world, however, is different. While operating an existing reservoir, we do not know the future inflow; at most we can guess more or less successfully what it will be. Hence a new parameter, the uncertainty about the magnitude of future inflow comes into play.

The notions of risk and uncertainty are introduced first. Then follows a short review of the techniques employed in the water resources field to cope with this problem, and the description of the method retained in this study. The chapter ends with the application to the cases with and without flow prediction.

5.1 Decision Theory

5.1.1 Risk and uncertainty. Massé (1946a) and Maas et al. (1962) adequately treated this topic. What follows is largely inspired from their work.

In some problems, the exact consequences of an action cannot be told in advance. What will finally happen, depends in fact on data not available at the time the decision was taken. In these situations we are faced with uncertainty.

An alternative, however, must be chosen, its consequences being known or not. A course of actions must be selected. The decision maker then takes a risk, as he does not know with certainty what the consequences of his decision will be. The risk may be small or great; it may imply losses of foregone benefits.

Decision theory developed models to help persons confronted with these problems. One point, however, should be kept in mind. The decision maker must first define his own attitude towards risk, regardless of the complexity and reliability of the existing models. The models show only the consequences of different decisions with the associated risks and then the person concerned selects a course of action depending on his attitude towards risk. Hence a subjective element will always be present in problems dealing with uncertainty.

5.1.2 Traditional approaches. In water resources problems, the state of nature markedly influences the outcome of an adopted course of action. Although the state of nature is not known at the time the alternative is selected, one can beforehand establish a list of all the possible states of nature along with their likelihood, and determine the consequences each one would have on any course of action. The final step consists then in selecting that alternative with the most desirable outcome.

Different selection criteria exist. The so-called traditional ones do not take into account the likelihood of the state of nature. Two of them are shortly presented hereafter.

In the approach based on the principle of maximum returns, one determines first for each alternative the minimum returns it guarantees whatever the state of nature. One selects in a second step that alternative with the highest minimum return. This approach is too pessimistic as it considers for each decision only the worst possible consequence. Furthermore irrelevant considerations may affect the outcome of the selection.

In the approach based on the minimax risk principle, the alternative with the smallest risk involved is preferred. Maas et al. (1962) define in this case "the risk of any combination of a decision and a state of nature by the excess of the maximum return attainable in that state of nature over the return that actually results from a given decision in that state of nature." Here again there are some weaknesses, the most important one being that the alternatives in presence affect the decision.

5.1.3 Expected value approach. Often the likelihood of an event, which is measured by the probability of its occurrence, is known and therefore it is quite natural to introduce it into the decision process.

The first moment, or expected value plays an important role in decision theory. It is defined as:

$$E\ (X) = \sum_i p_i \cdot x_i \tag{5-1}$$

where x_i is the i^{th} possible outcome of an event and p_i, the probability of its occurrence. The summation extends over the complete set of possible outcomes. The expected value is a good measure of central tendency. Yet, it suffers from an important drawback, as it gives no information about the spread of the outcomes of an event.

Hence, even though two decisions give rise to the same expected value, one may be definitely preferred, if the standard deviation of its outcomes is smaller. Furthermore, many persons are ready to settle for a decision with a smaller expected value provided that the spread of the related outcomes is reduced. This attitude, which is currently followed in the field of insurance, allows to reduce the severity of the greatest possible losses. Many techniques were devised to quantify this attitude: certainty equivalent, gamblers indifference map, risk discounting, and so on. But the expected value approach seems still to be the most widely used method in the water resources field. Why?

The expected value approach leads to reasonable decisions, whenever the actuarial risk situation prevails. This situation arises whenever one has to take concurrently many decisions which are similar in their nature and independent from one another, and when one is more interested in the overall consequences of one's decisions. Actually this is the case in the field of water resources, as will be shown later.

5.1.4 Expected value approach and reservoir operation. We must deal with two kinds of uncertainty while operating a reservoir. The input into the system, and the required output from the system are random. Here, however, the demand is considered as deterministic so that only the hydrologic uncertainty of the inflow remains.

The climate in Switzerland is such that the interannual variations in runoff are small. The ratio between the maximum and the minimum recorded annual flow does not exceed 1.7. Hence the spread of the inflows is small and we are in a favorable situation to apply the expected value approach.

Seasonal reservoirs may play different roles within a power producing system (See Chapter 3). Despite the existences of these different roles, the expected value method still applies. For the value of the sold energy, that is its price, implicitly reflects the prevailing role played by the reservoir. Energy is expensive when the offer is small and the risk of shortage high. Also a cautious operator will value energy during a period of possible shortage much higher than what an operator would do, who likes to take risks. Furthermore a reservoir operator is always free to select the decision which he finds most appropriate. In this situation, the present method would just help him in the evaluation of the different alternatives.

Other arguments speak for the expected value approach. Power utilities often own many reservoirs, in which case they are in an actuarial risk situation. On the other hand, a company may own only one reservoir but then is interested in maximizing the returns over the entire life span of the reservoir. Here again, actuarial risk situation prevails.

Hence in this study, we shall cope with uncertainty by using the expected value approach. This method presents many advantages, we are however aware of its limits.

5.2 Operation Research and Uncertainty

5.2.1 Literature review. The mere decision to select the criterion of the expected value to weigh the merits of alternatives does not solve the problem itself. Many papers (Takeuchi, 1972) have been published these last years in the area of stochastic optimization. Two facts may have caused this abundance of articles. Either the problem is difficult and requires to be solved by the joint efforts of many researchers, or no general method exists and each case asks for a special treatment. In reality, the truth lies in between. Hence only the methodologies relevant for the case under study will be reviewed here. Roefs (1968) prepared a good summary of the existing procedures, which Croley (1972) took over and completed.

According to Roefs, two basic methods exist to solve stochastic optimization problems: Implicit Stochastic Optimization (ISO) and Explicit Stochastic Optimization (ESO). Croley himself has added a third one, which is in fact a combination of ISO and of ESO: the Alternative Stochastic Optimization (ASO) technique.

Monte Carlo Dynamic Programming introduced by Young (1967) belongs to the first category. To optimize the operation of a reservoir under uncertainty, Young applies first a deterministic optimization technique to each of the many available inflows sequences. The related optimal sets of releases are recorded. In a second step, the computed releases are related to some variables like storage, or inflows, that have an influence on the release strategy and that describe the state of the system. Multiple linear regression analysis is most often used in the second step. Finally, the established relations supply the information required to operate the reservoir.

In the second technique, one introduces the probability distribution of the inputs directly into the optimization procedure. Stochastic Linear Programming developed by Manne (1960) characterizes well this approach. Manne looks for that set of probabilities which maximize the expected total benefit of reservoir operation. The solution of ESO consists of a table of optimal decisions indexed on the reservoir content and on the amount of previous inflow.

Finally Croley proposes a combination of both methods. First, as in ISO, the returns of the reservoir are optimized successively for various input samples, and then related release strategies are recorded. Second, this time as in ESO, one evaluates the distribution of the decisions corresponding to the first stage of the operation period. Then an appropriate decision, corresponding either to the mean, mode or median of the obtained statistical distribution, is selected, which applied to the system brings it to the beginning of the second stage. One repeats the same procedure for the second and all the following stages. Finally relations are established between decision and relevant state variables as in ISO. To reduce the burden of computations, the system is operated, in each case, only over a reduced period instead of the complete one.

5.2.2 Evaluation of the reviewed methods. As the ISO technique relies heavily on simulation, the problem does not have to be solvable by analytical techniques. So input time series with long persistence can be handled without problems. The application of ISO may require a lot of computations. However the real difficulties and drawbacks of this method appear in the last step, when relations are established between decision variables and relevant parameters describing the state of the system.

What are the relevant state variables? What type of relations select? Multiple linear models were quite often used. But these models suffer from important drawbacks. The reliability of the established relation decreases sharply for values of the state variables which are much greater or smaller than their means. Furthermore they ignore completely the physical characteristics of the problem under study.

The ESO method has some definite advantages over the ISO method. As the probability of the input is introduced directly into the computation scheme, it is no longer necessary to calculate problematic multivariate regression equations. Unfortunately this procedure leads to unmanageable difficulties as soon as the system, or the hydrology of the associated inputs becomes complicated. When the successive inputs into the system are highly correlated, tremendous computer storage is required. The same remark holds if the inputs cover a wide range.

Croley combined both methods in an optimal way and he arrived at a better procedure. A high amount of computations is the cost for this improvement. To cope with this drawback Croley applied the deterministic optimization technique to the system for only a part of the operation period. The duration of the reduced operation period is selected in such a way that the resulting benefits are not too different from those obtained from an operation over the complete time span. This artifice may reduce substantially the amount of computations in some cases. Here however it brings small gains. Finally Croley could not avoid the last controversial step of ISO, where decision variables are related to state variables.

For our purpose the following points from these techniques will be retained. As proposed by Young, we shall operate the reservoir for several samples of inflows sequences, and we shall consider in each run, as done by Croley, only the decision corresponding to the first stage of the operation period. But we want to eliminate completely the step of the ISO technique dealing with multiple linear regressions and try to introduce into the solution technique, the physical properties of the optimal solution. To do this, some additional notions are needed.

5.2.3 Massé's principles of optimality. Massé (1946a) showed that in deterministic reservoir operation, the marginal instantaneous return of the current release equals the marginal future return of the current storage, also called marginal value of the stored water. The following examples illustrate this principle.

Assume first that the marginal instantaneous benefit of the current release is smaller than the marginal future return of the current storage. The associated strategy cannot be optimal, as a reduction in the current release leads to an increase of the benefit resulting from reservoir operation. The spared water, when released in a later period, will yield a higher marginal instantaneous benefit because, by hypothesis, the related marginal return is higher. Hence this shifting of water brings some gains.

Assume on the other hand that the marginal instantaneous benefit of the current release is greater than the marginal future return of the current storage. Here again the associated strategy cannot be optimal, as an increase in the current release leads to an increase of the benefit resulting from reservoir operation. If more water is released in the current period and if less water is stored for future use, the overall consequence of this shifting of water is an increase of the total benefits.

Hence the optimum is reached when no transfer of water from one period of time to another one can bring any gain, or when both marginal values are equal. An important property is related to this principle.

Let us define the trajectory of the water levels, as the curve representing the successive levels reached by the water surface in the reservoir during the operation period. Quite naturally, the optimal trajectory is that reservoir content curve, which results from the application to the system of a sequence of optimal decisions.

Then the following property holds. The optimal trajectory is such that the marginal future return of the storage levels reached during the operation of the reservoir is constant. This property however holds only when the reservoir is neither full nor empty.

Massé (1946b) also generalized the principle of optimality for the case of stochastic optimization. The term marginal future benefit is replaced by the term expected marginal future benefit of storage. Hence (Morel-Seytoux, 1974) "at the optimum the marginal instantaneous benefit of the release equals the marginal future expected benefit of the storage."

Hence in stochastic optimization, the major problem becomes the determination of the expected future benefit associated with a given initial storage s_i. We shall deal with this problem in the next subsection.

5.2.4 Expected marginal future benefit of storage. Different variables affect the expected future benefit of storage. Obviously the maximum reservoir storage capacity, the price of the energy and the inflow plays a major role. However if these variables are held constant, the expected future benefit of storage is a function of the date of the year and of the considered reservoir content. Date and reservoir content will be taken as parameters in the subsequent studies. Hence the final aim consists in establishing a two-dimensional grid which represents the expected future benefit of storage as a function of time and of reservoir content.

Two paths stay open to achieve this goal. One can proceed either on a theoretical, or on an experimental basis. Massé followed the theoretical approach and obtained closed form solutions for simple reservoir systems and inflow sequences. However the problem becomes soon unmanageable when the inflows sequences present strong persistence. Consequently here, the variable looked after will be determined experimentally.

The expected future benefit of a given storage and for a given date is obtained as follows. The selected storage is taken as initial storage and the reservoir is operated in an optimal way starting from the selected date until the end of the water year. The obtained marginal benefit of storage is recorded. This procedure is repeated for the available, recorded or simulated inflows sequences. Finally the expected value of the so obtained sample of marginal benefits is taken. One proceeds in the same way for all the other grid points.

5.2.5 Evaluation of the proposed method. At this point, all the elements necessary to apply the proposed stochastic optimization technique have been defined separately, so that it is time now to put the puzzle together.

We face the following situation. We must be able to decide at the beginning of any week of the water year how much water should be released during the coming week, so that the returns from the energy produced during the rest of the year's operation get maximum. The content of the reservoir at the date of the decision, and the antecedent flows sequence are known, while the magnitude of the flow following the date of the decision is not known.

The stochastic optimization problem is solved in two steps. First, the marginal value of the releases and the expected future benefit of storage are determined separately. Table 3.7 and Figs. 3.3 and 3.6 supply all the information necessary to evaluate the marginal return of the releases as a function of the amount of water released during one week. On the other hand, as indicated in the preceding subsection, the expected future marginal return of storage must be calculated for 52 dates and for different storage contents. Once these operations have been performed, we can proceed to the second step. It consists in selecting the required weekly release in such a way that Massé's principle of optimality is satisfied. What have we achieved?

First the computer program developed in Chapter 4 can be used to evaluate the expected future benefit of storage. As the technique proposed in Chapter 4 is very efficient, this property is of great importance. Furthermore the information gained from the runs performed according to the list of Table 4.6 are still relevant for the present problem. Second the future marginal return of storage is an attractive variable to work with. It has a concrete physical meaning so that we always know what we are doing. Third, we systematically work with benefits. The transfer back into the decision domain occurs in the last step only. This way of doing is consistent and saves time. Fourth, as will be shown later, this method can be easily modified to take into account runoff forecasts. Finally, up to now no multiple linear regression techniques were used.

But all these advantages are compensated by a tremendous amount of computations. If 15 different initial storages are retained per date, then we must perform 15 times 52 weeks x 29 years runs, which amounts to more than 23,000 runs. This is simply not possible.

Hence the purpose of the forthcoming sections is to develop shortcut techniques to reduce the computational burden. A special effort will be done to incorporate into the solution technique the physical properties of the optimal solution.

5.3 Stochastic Reservoir Operation without Flow Forecast

5.3.1 Preliminary remarks. Why at all deal with this case? Why not immediately introduce the runoff prediction model into the computations scheme? Strong arguments plead for the present approach. Before tackling complicated problems, it is absolutely necessary to master the simple ones. Furthermore, for some parts of the year it is not possible to establish reliable runoff forecasts. Finally it may be interesting to compare the merits of both methods.

In the stochastic case, a computed optimal decision is the best one only with a given probability. It does not always lead to the maximum return. Most often, when the inflows sequence which was unknown at the moment the decision was taken becomes known, it will then be possible to determine an alternative decision which yields higher return. But this fact does not question the validity of the approach followed here, which by definition leads on the average to the best results. Consequently its merits can and must be evaluated only after many decisions of the same kind have been taken.

Also one cannot compute an optimal release strategy for the complete reservoir operation period when the magnitude of the future inflow is not known. The nature of the optimal decision changes constantly as time goes on and more information becomes available. Hence the optimal weekly releases must be determined successively from one week to another, taking into account each time all the information which is available.

5.3.2 Outline of the methodology. As indicated in Section 5.2, the purpose of the present chapter is to compute the expected future marginal return of storage for different dates of the year and for different initial reservoir contents. Table 5.1 gives the list of all the performed computer runs. According to this Table, some of the initial dates considered here correspond to those selected to test the computer program. These dates are well spread over the year and should encompass all the special cases which might be encountered while determining the expected future benefit of storage. The number and the nature of the chosen initial reservoir contents are such that they should enable us to set forth the special features of the problem.

Hence the reservoir was operated for each of the selected initial storages from the initial date to the end of the water year. This procedure was repeated for the available 29 inflow sequences. Finally we calculated the expected value of the so obtained sample of future marginal return of storage.

In the solution technique developed in Chapter 4, the reservoir operation period was divided into drawdown and refill phase. This procedure allowed to compute separately the optimal sequence of releases for both phases. In addition, a linear relation could be established between the natural logarithm of the marginal value of the release and the sum of the releases during the concerned phase, and a term related to the storage levels corresponding to the optimal reservoir content curve. This relation will be extensively used in the subsequent paragraphs, as it allows to reduce substantially the amount of computations required to estimate the expected future marginal return of storage.

Table 5.1

Stochastic Reservoir Operation, List of Computed Cases

Initial Date	Initial Storage x 10^6 m^3				
October 1	72.000	69.000	66.000	63.000	60.000
December 31	48.809	44.809	40.809	36.809	32.809
April 1	7.569	4.569	1.569	0.000	-
July 1	39.963	35.963	31.963	27.963	23.963

The presentation of this topic follows the outline of Chapter 4. Hence we shall deal first with the drawdown phase and second with the refill phase. Then we shall consider the so called special cases which arise when the reservoir is full or nearly empty. The numerical applications are grouped together and appear after the theoretical derivations.

5.3.3 Drawdown phase. Assuming that the reservoir operation is started on the k^{th} week with an initial storage s_k, we obtained in Section 4.5.2 (Eq. 4-32), the following equation:

$$\ell n \ \lambda^d_{\ell,k} = \frac{1}{4.2\cdot(\ell-k)} \cdot \left[4.2 \cdot \sum_{i=k}^{\ell-1} \ell n\alpha_i(\bar{s}_i) - s_k - \sum_{i=k}^{\ell-1} q_i\right] \tag{5-2}$$

where $\lambda^d_{\ell,k}$ represents the marginal value of the release of the k^{th} week, provided that the reservoir gets empty at the beginning of the ℓ^{th} week, $(\ell-k)$, the number of weeks during the drawdown phase, $\bar{s}_i$, the average reservoir content during the i^{th} week, $\alpha_i(\bar{s}_i)$, the product of the energy equivalent for the average storage $\bar{s}_i$ of 1 m^3 of water and of the prevailing energy price (see Section 3.6, Eq. 3-5), and the last term, the total inflow during the drawdown phase. This relation holds as long as no storage constraints are active during the drawdown phase, and only if the reservoir is empty at the beginning of the ℓ^{th} week.

In fact $\lambda^d_{\ell,k}$ is equal to the future marginal return of storage which we are looking for. From now on, we shall refer to this variable as λ^d_{ℓ,s_k}, where k denotes the day under consideration, and s_k, the corresponding initial storage; d indicates that we are in the drawdown phase.

To determine the expected future marginal benefit of storage, it is then sufficient to evaluate successively for each of the available inflows sequences the value of $\lambda^{d}_{\ell,k}$ and to take the expected value of the so obtained data sample. However the same result can also be obtained analytically in a much simpler way. For this purpose, let us rewrite Eq. 5-2 as:

$$\ell n\ \lambda^{d}_{k,s_k} = a_d + b_d \cdot Q_{k,\ell-1} \tag{5-3}$$

where

$$a_d = \frac{\sum_{i=k}^{\ell-1} \ell n\alpha_i\ (\bar{s}_i)}{(\ell-k)} - \frac{s_k}{4.2\ (\ell-k)}$$

and

$$b_d = \frac{-1}{4.2(\ell-k)}$$

The term $Q_{k,\ell-1}$ represents the cumulative inflow recorded during the drawdown phase.

Now if we assume for a while that the date the reservoir is empty and the optimal drawdown curve are independent of the inflows sequence, then a_d and b_d become a constant and the natural logarithm of the future marginal return of storage varies linearly with the inflow $Q_{k,\ell-1}$. Furthermore, as $Q_{k,\ell-1}$, which represents the inflow integrated over several weeks, is approximately normally distributed, the logarithms of the future marginal return of storage are also normally distributed. Finally, according to the reproductive properties of the normal distribution, we have:

$$E\left[\ell n\ \lambda^{d}_{k,s_k}\right] = a_d + b_d \cdot E\left[Q_{k,\ell-1}\right] \tag{5-4}$$

and

$$\text{Var}\left[\ell n\ \lambda^{d}_{k,s_k}\right] = b_d^2 \cdot \text{Var}\left[Q_{k,\ell-1}\right] \tag{5-5}$$

where E [...] and Var [...] mean the expected value and the variance of the expression in the brackets, respectively.

Consequently it should be possible with the help of the just derived relation, to compute analytically the expected future marginal benefit of storage, without operating the reservoir for all the existing inflows sequences. This fact would represent a tremendous gain in computations time.

But the derivation of this important property was made possible only by the existence of restrictive assumptions. We want to show now that these assumptions are not as restrictive as they seem to be at the first glance.

First, concerning the date on which the reservoir gets the first time empty, the analysis performed in Chapter 4 indicated that this parameter can be considered as a constant, at least in the first approximation. Hence one can expect that this assumption has a negligible influence on the determination of the expected marginal value of storage.

Second, we assumed that the sum of the natural logarithm of the terms $\alpha_i(\bar{s}_i)$ was a constant. The optimal drawdown curve depends, at least theoretically, on the inflows sequence. The calculations done in Chapter 4 however showed that the variations from one inflow sequence to another in the shape of the optimal drawdown curve were relatively small. As the logarithm of the terms $\alpha_i(\bar{s}_i)$ enters relation 5-3, the variations under consideration are further reduced.

So, for example, the value of this parameter changes by 10 percent if for a given date one passes from an empty to a full reservoir. This proves that the different reservoir trajectories do not affect much the final value of the sum of the natural logarithm of the term $\alpha_i(\bar{s}_i)$. Hence this second assumption is also not restrictive.

More severe though are the limitations resulting from the assumptions that no storage constraints are binding, and that the reservoir always gets empty. These cases will be treated separately later, so that we can ignore them now.

Consequently for most of the situations prevailing during the drawdown phase, the expected future marginal benefit of storage can be computed directly with the help of a simple relation. The most important terms appearing in this relation are the expected value of the relevant inflows sequence and a variable related to the optimal drawdown curve corresponding to the sequence of average inflows.

5.3.4 <u>Refill phase</u>. An expression relating marginal value of releases with some parameters of the optimal reservoir content curve was also developed for the refill phase (Section 4.5.4, Eq. 4-40). It reads as follows:

$$\ln \lambda^f_{k,s_k} = \frac{1}{4.2(53-k)} \cdot \left[4.2 \sum_{i=k}^{52} \ln \alpha_i(\bar{s}_i) + s_{53} - \sum_{i=k}^{52} q_i \right] \tag{5-6}$$

where λ^f_{k,s_k} represents the future marginal benefit of storage for the week k and initial storage s_k, s_{53}, the final storage, $\bar{s}_i$, the average reservoir content in the i^{th} week, $\alpha_i(\bar{s}_i)$, the product of the energy equivalent for the average storage $\bar{s}_i$ of 1 m^3 of water, and of the prevailing energy price (see Eq. 3-5), and q_i, the inflow during the i^{th} week. This equation is rewritten as:

$$\ell n \; \lambda^{f}_{k,s_k} = a_f + b_f \cdot Q_{k,52} \tag{5-7}$$

where

$$a_f = \frac{\sum_{i=k}^{52} \ell n\alpha_i(\bar{s}_i)}{53-k} + \frac{s_{53}}{4.2(53-k)}$$

and

$$b_f = \frac{-1}{4.2(53-k)}$$

The term f refers to the refill phase while $Q_{k,52}$ stands for the inflow recorded during the refill phase. In words, the natural logarithm of the future return of storage varies linearly with the inflow recorded during the refill phase.

But this linear relation holds only if the following conditions are satisfied: full reservoir at the end of the operation period, no active storage or release constraints, and optimal reservoir content curves not too different from each other.

According to the information supplied by the computer runs of the deterministic case, these conditions are satisfied when average flow condition prevails. However, if the inflow recorded during the refill phase is low, the reservoir may not get full at the end of the water year, while, if the inflow recorded during the refill phase is high, the reservoir gets full before the end of the water year. Furthermore the optimal refill curves corresponding to different inflow sequences may differ widely among each other as they depend directly on the inflows sequences, which themselves vary greatly.

These facts limit the application of the just derived formula more seriously than was the case in the drawdown phase. We expect nevertheless that it provides a good approximation for the expected future marginal benefit of storage:

$$E\left[\ln \lambda^f_{k,s_k}\right] = a_f + b_f \cdot E\left[Q_{k,52}\right] \tag{5-8}$$

The future marginal benefits of storage follow a log-normal distribution in the drawdown phase, and the same property should exist in the refill phase. As the linear relation which allowed to derive this property holds only for average inflows sequences, the future marginal benefits of storage follow a log-normal distribution in their central range. What happens at the extremes?

Let us gradually diminish the magnitude of the inflow during the refill phase and see what happens. As the inflow decreases, the amount of water released decreases also, while the marginal future benefit of storage increases. Soon the situation will come up where the reservoir will not get full at the end of the water year. From that moment on, the rate of increase of λ^f_{k,s_k} decreases. Actually an upper boundary for this parameter exists. Theoretically it cannot exceed the product of the maximum price of a KWh times the energy equivalent for the relevant reservoir content of one m^3 of water. Practically it is related to the minimum possible reservoir content reached at the end of the water year.

On the other hand, let us now gradually increase the magnitude of the inflow during the refill phase. As the inflow increases the amount of water released increases also, while the marginal future benefit of storage decreases. Soon, the reservoir gets full earlier than the end of the water year, and the rate of increase of λ^f_{k,s_k} decreases. Here a lower boundary for this parameter exists, which theoretically corresponds to the maximum quantity of water which can be released through the power plant.

Hence the underlying distribution curve of the future marginal return of storage must exhibit these properties. The four parameters log-normal distribution curve fulfills these conditions. This distribution curve is bounded on both sides and, as suggested by its name, is defined by four parameters: Mean, standard deviation, upper and lower boundaries. In the case under study, the following expression is normally distributed:

$$\ell n \left[\frac{\lambda^f_{k,s_k} - \lambda^f_{k,s_k,\min}}{\lambda^f_{k,s_k,\max} - \lambda^f_{k,s_k}} \right] \tag{5-9}$$

where $\lambda^f_{k,s_k,\min}$ and $\lambda^f_{k,s_k,\max}$ are the lower, and upper boundary, respectively.

Generally the use of this type of curves requires a great amount of work. Especially the determination of the upper and lower boundary is tedious. Here though, the problem is simpler as the physical nature of the problem allows a good first guess of the value of the boundaries. The minimum reservoir content at the end of the year defines the upper boundary, and the largest weekly release in summer, the lower boundary. The situation is further simplified by the fact that the expected future marginal return of storage is rather insensitive to the exact determination of these boundaries.

Upper and lower boundary depend on the considered date and initial storage. Their effects and relative importance vary. If the initial reservoir content is relatively low, essentially the upper boundary comes into play. Inversely if the initial reservoir content is relatively high, the opposite is true. Also there must exist a content for which both boundaries are equally unimportant.

The knowledge that the future marginal benefits of storage resulting from different inflow sequences follow a well defined distribution curve is here important for two reasons. First it allows to check the validity of Eq. 5-7, as the expected future marginal return of storage can be computed in two different ways: on the basis of Eq. 5-8 and through the relevant distribution curve. Second it permits to establish a relation between inflow and associated marginal future benefit of storage. To each probability level, correspond a specified amount of inflow during the period of reservoir operation, and a value for the future marginal return of storage, so that the two variables are related on a statistical basis. Hence once the probability distributions of both variables have been determined, one can derive one variable from the other and vice-versa.

Hence, in the refill phase as in the drawdown phase, a simple relation could be established between inflow and future marginal return of storage. The computations involved to arrive at this result were though more complicated.

5.3.5 Mixed strategy situation. Relations 5-3 and 5-7 hold each separately either for the drawdown, or for the refill phase. Yet situations may arise where it is not known in advance whether the reservoir stays in the refill, or in the drawdown phase. So, for instance, with April 1 as an initial date, the reservoir never gets empty during the operation period for some inflows sequences and the equation for the refill phase applies. On the other hand, for other inflows sequences, it gets empty and the equation for the drawdown phase applies. For cases similar to the one just described, we shall talk of mixed strategy situations as two types of release strategy must be considered to determine the optimal release.

If the reservoir gets empty, the following relation (see Section 5.3.3, Eq. 5-3) holds:

$$\ell n\lambda^{d}_{k,s_k} = \frac{\sum_{i=k}^{\ell-1} \ell n\alpha_i(\bar{s}_i)}{\ell-k} - \frac{s_k}{4.2(\ell-k)} - \frac{1}{4.2(\ell-k)} \cdot Q_{k,\ell-1} \qquad (5\text{-}10)$$

where ℓ stands for the assumed date of emptiness, and $Q_{k,\ell-1}$, for the inflow recorded from date k until the date the reservoir is empty.

If the reservoir does not get empty, the subsequent relation (see Section 5.3.4, Eq. 5-7) applies:

$$\ell n\lambda^{f}_{k,s_k} = \frac{\sum_{i=k}^{52} \ell n\alpha_i(\bar{s}_i)}{53-k} + \frac{s_{53}-s_k}{4.2(53-k)} - \frac{1}{4.2(53-k)} \cdot Q_{k,52} \qquad (5\text{-}11)$$

where $Q_{k,52}$ stands for the inflow recorded from date k until the end of the water year.

The future marginal return of storage is computed according to both formulas and the highest obtained value is retained, as the return must be maximum. Hence, if the computed value of λ^{d}_{k,s_k} exceeds the one obtained for λ^{f}_{k,s_k}, the reservoir stays in the drawdown phase for the given inflows sequence, and vice-versa.

The expected future marginal return of storage results from the combination of these two strategies. Depending on the statistical characteristics of the inflows sequences $Q_{k,\ell-1}$ and $Q_{k,52}$, one strategy will apply more or less often than the other one. Consequently the following expression can be written for the expected value of the logarithm of the future marginal return of storage:

$$E\left[\ell n\lambda_{k,s_k}\right] = p\cdot E\left[\ell n\lambda^{d}_{k,s_k}\right] + q\cdot E\left[\ell n\lambda^{f}_{k,s_k}\right] \tag{5-12}$$

with

$$p + q = 1$$

where p stands for the probability that the formula for the drawdown phase holds, and q, for the probability that the formula for the refill phase holds. The last steps of the computations consist then in the determination of the values of p and of the two expected future marginal returns of storage.

To evaluate p, consider the limiting case where both formulas lead to the same value of the future marginal return of storage. This happens when for the optimal strategy, the reservoir just gets empty, and when no release constraint affects the magnitude of the releases. As both future marginal returns are equal, the two equations can be combined and, solving for $Q_{k,52}$, one obtains:

$$\begin{aligned} Q^{f}_{k,52} = 4.2\cdot(53-k)\cdot &\left[+ \frac{\sum_{i=k}^{52} \ell n\alpha_i(\bar{s}^{f}_{i})}{(53-k)} - \frac{\sum_{i=k}^{\ell-1} \ell n\alpha_i(\bar{s}^{d}_{i})}{(\ell-k)} \right. \\ &\left. + \frac{s^{f}_{53} - s_k}{4.2\cdot(53-k)} + \frac{s_k}{4.2(\ell-k)} \right] + \frac{(53-k)}{(\ell-k)}\cdot Q^{d}_{k,\ell-1} \end{aligned} \tag{5-13}$$

The terms indexed with a small f are related to the refill phase, and those indexed with a small d, to the drawdown phase.

This equation indicates that for each value of $Q^{d}_{k,\ell-1}$, corresponds a value of $Q^{f}_{k,52}$ called by definition $Q^{f}_{k,52,\mathrm{limit}}$, such that the reservoir gets empty if $Q^f_{k,52}$ exceeds $Q^f_{k,52,\text{limit}}$. Furthermore, the probability that the reservoir gets empty is obtained by integrating out the inflow $Q^d_{k,\ell-1}$:

$$p = \int_0^\infty \text{Prob}\left[Q^f_{k,52} \geq Q^f_{k,52,\text{limit}}(Q^d_{k,\ell-1})\right] \cdot f\,(Q^d_{k,\ell-1}) \cdot d(Q^d_{k,\ell-1}) \tag{5-14}$$

The term $f\,(\cdot)$ represents the probability distribution of the inflow $Q^d_{k,\ell-1}$. The computations are straightforward but may become quite tedious.

Computational experience indicated that for the cases under consideration, the future marginal returns of storage are log-normally distributed. Hence, at least theoretically Eq. 5-10 and 5-11 can be used to evaluate the expected values of λ^d_{k,s_k} and of λ^f_{k,s_k}, provided that the correct values for $\bar{Q}^d_{k,\ell-1}$ and for $\bar{Q}^f_{k,52}$ are introduced into the respective equation. Practically however, the determination of $\bar{Q}^d_{k,\ell-1}$ and of $\bar{Q}^f_{k,52}$ may become as tedious as the computations of p.

Consequently in a mixed strategy situation it is simpler to determine the expected future marginal return of storage on the basis of the results obtained from the fictive reservoir operation for different inflows sequences.

5.3.6 Full or nearly full reservoir. The methodology developed in Section 5.3.3 for the drawdown phase holds only if no storage constraints are binding. The procedure to follow when such a situation arises, appears hereafter.

In September or October, the reservoir is usually full or nearly full. Sometimes, high inflows are recorded during this period and water must be released from the reservoir, regardless of what the optimal strategy would be, just to avoid overtopping of the reservoir. Although these situations do not happen often, they must be considered, as they affect the magnitude of the expected future marginal benefit of storage.

Basically, we are again faced with a mixed strategy situation, as either the general theory as developed under Section 5.3.3 applies, or the one to be derived hereafter. If the general theory holds, the following relation leads to the optimal strategy:

$$\ell n\lambda^{d}_{k,s_k} = \frac{1}{4.2(\ell-k)}\left[4.2\sum_{k}^{\ell-1}\ell n\alpha_i(\bar{s}_i)-s_k-\sum_{i=k}^{\ell-1}q_i\right] \tag{5-15}$$

If the inflow during the k^{th} week is high, the reservoir capacity may be exceeded at the end of that week. To avoid this, the following water balance equation must be satisfied:

$$s_k + q_k - x_k \leq S \tag{5-16}$$

which expresses the release x_k must be selected in such a way that the reservoir content at the end of the considered week is smaller or at most equal to the reservoir capacity. At the limit, we have:

$$x_k = q_k - S + s_k \tag{5-17}$$

To determine the corresponding marginal return of the release, one multiplies the right hand side of Eq. 3-1 by $\alpha_k(\bar{s}_k)$ and replace x_k in the same equation by the right hand side of expression 5-17 so that:

$$\ell n\lambda^{d}_{k,s_k} = \ell n\alpha_k(\bar{s}_k) + \frac{S - s_k}{4.2} - \frac{q_k}{4.2} \tag{5-18}$$

which is in fact the second relation looked after.

As long as the value for the marginal future return of storage given by Eq. 5-18 is greater than that of Eq. 5-15, the general theory holds, in the opposite Eq. 5-18 applies.

Note that theoretically a storage constraint may become binding later, like in week k+2, or k+3, etc. This situation however happened extremely seldom and did not affect greatly the value of the expected future marginal return of storage. Hence it can be ignored.

From this stage of the computations on, the procedure is identical to the one developed in the preceding subsection. It will therefore not be repeated.

5.3.7 Empty reservoir. The last point to deal with concerns the week of the year, when the reservoir is empty. While studying the results of deterministic reservoir operation, we noticed that quite often the reservoir remained empty for more than one week. This period corresponds again to a mixed strategy situation as either the reservoir is empty a further week, or it stays in the drawdown phase or in the refill phase. If the reservoir stays in the drawdown phase, relation 5-2 holds:

$$\ell n\lambda^{d}_{k,s_k} = \frac{1}{4.2(\ell-k)}\left[4.2\sum_{i=k}^{\ell-1}\ell n\alpha_i(\bar{s}_i) - Q_{k,\ell-1}\right] \tag{5-19}$$

where s_k does not appear, as its value is equal to zero.

If the reservoir is in the refill phase, relation 5-6 holds:

$$\ell n\lambda^{f}_{k,s_k} = \frac{1}{4.2(53-k)}\left[4.2\sum_{i=k}^{52}\ell n\alpha_i(\bar{s}_i) + s_{53} - Q_{k,52}\right] \tag{5-20}$$

If the reservoir stays empty one more week, the weekly release is equal to the weekly inflow, which, according to Eq. 4-21, leads to the following expression for the future marginal return of storage:

$$\ell n\lambda_{k,s_k} = \ell n\alpha_k(\bar{s}_k) - \frac{q_k}{4.2} \qquad (5\text{-}21)$$

The future marginal return of storage is computed according to the three formulas, and the highest of the obtained values is retained. This way of doing makes impossible the violation of the release constraint. The further steps are identical to those described in Section 5.3.5.

We have now reviewed the most important cases which confront the reservoir operator. To substantiate the theoretical derivations, we shall present in the forthcoming subsection, some numerical examples.

5.3.8 Applications. The purpose here is to show that the results of the numerical applications confirm the developed theory and that the derived methodology is both feasible and attractive. These examples should also indicate the order of magnitude of the variables involved. The reservoir was operated for four initial dates and for five initial contents per selected date. These cases cover the whole range of situations which may happen during the operation of a reservoir. They appear in the same order as in the theoretical derivations.

a) Drawdown phase. The situation existing on December 31 and, to a smaller extent, on October 1, is typical for that case. Tables 5.2, 5.3, 5.5 and 5.6 display the computed future marginal returns of storage while Table 5.4 and 5.7 supply the corresponding expected values. Typical distributions curve of this variable appear on Figs. 5.1 and 5.2.

The spread of the computed future marginal returns of storage is quite small, if the cases corresponding to the two highest initial storages of October 1 are excluded. This fact confirms that during the drawdown phase, the release strategy is primarily influenced by the reservoir content prevailing on the decision date, whereas the associated inflows sequence plays a secondary role. Normal and log-normal distribution curves provide an equally good fit to the computed future marginal returns of storage.

b) Refill phase. This case is well characterized by the situation existing when reservoir operation is started on July 1. Tables 5.8 and 5.9 show the computed future marginal returns of storage and Table 5.10, the expected values obtained according to different methods. Figs. 5.3, 5.6 and 5.9 display the fitted probability distributions while the relations between marginal future returns of storage and associated inflows appear on Figs. 5.4, 5.5, 5.7, 5.8, 5.10 and 5.11.

Compared to the drawdown phase, the spread of the calculated marginal values is much greater. The four parameters log-normal distribution provides a good fit for nearly the complete range of available data. The influence of the upper and lower boundary becomes evident as normal, log-normal and four parameters log-normal distributions lead to different expected future returns of storage. However these differences decrease as the initial storage increases, which indicates that the influence of the boundaries on the expected

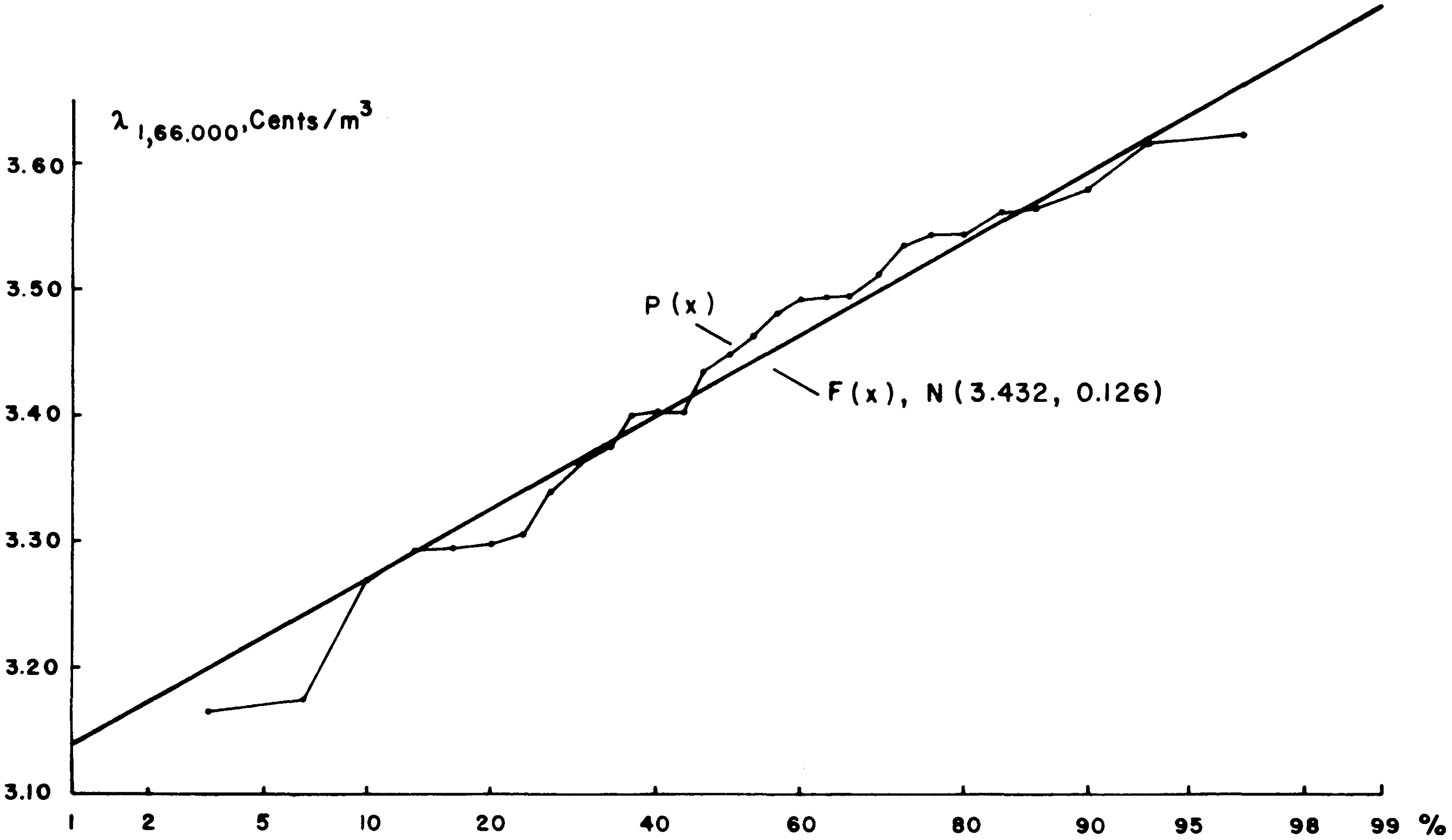


Fig. 5.1 **Future Marginal Return of Storage; Distribution Curve: Initial Date, October 1; Initial Storage, 66 Millions M^3**

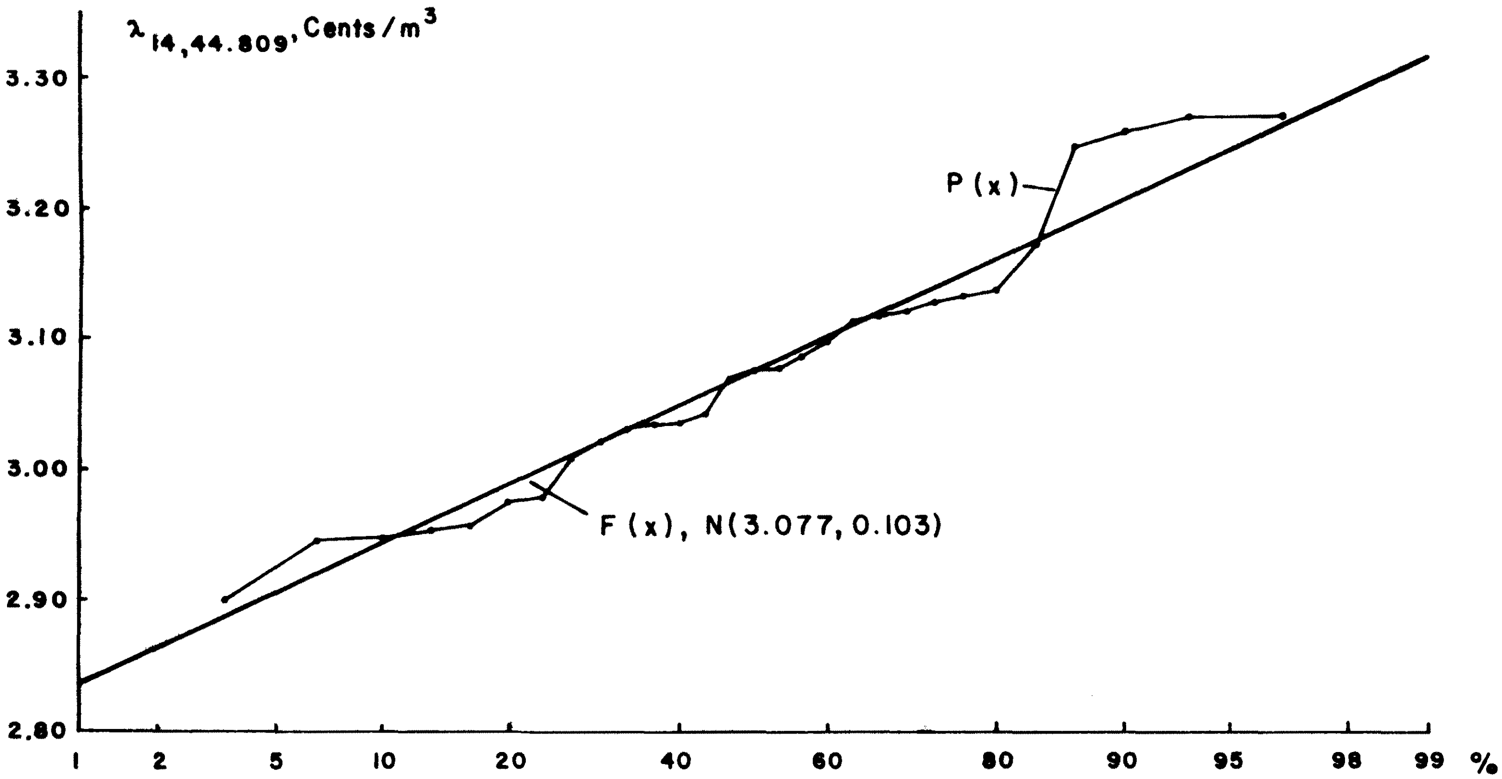


Fig. 5.2 Future Marginal Return of Storage, Distribution Curve:

Initial Date, December 31; Initial Storage, 44.809 Millions M^3

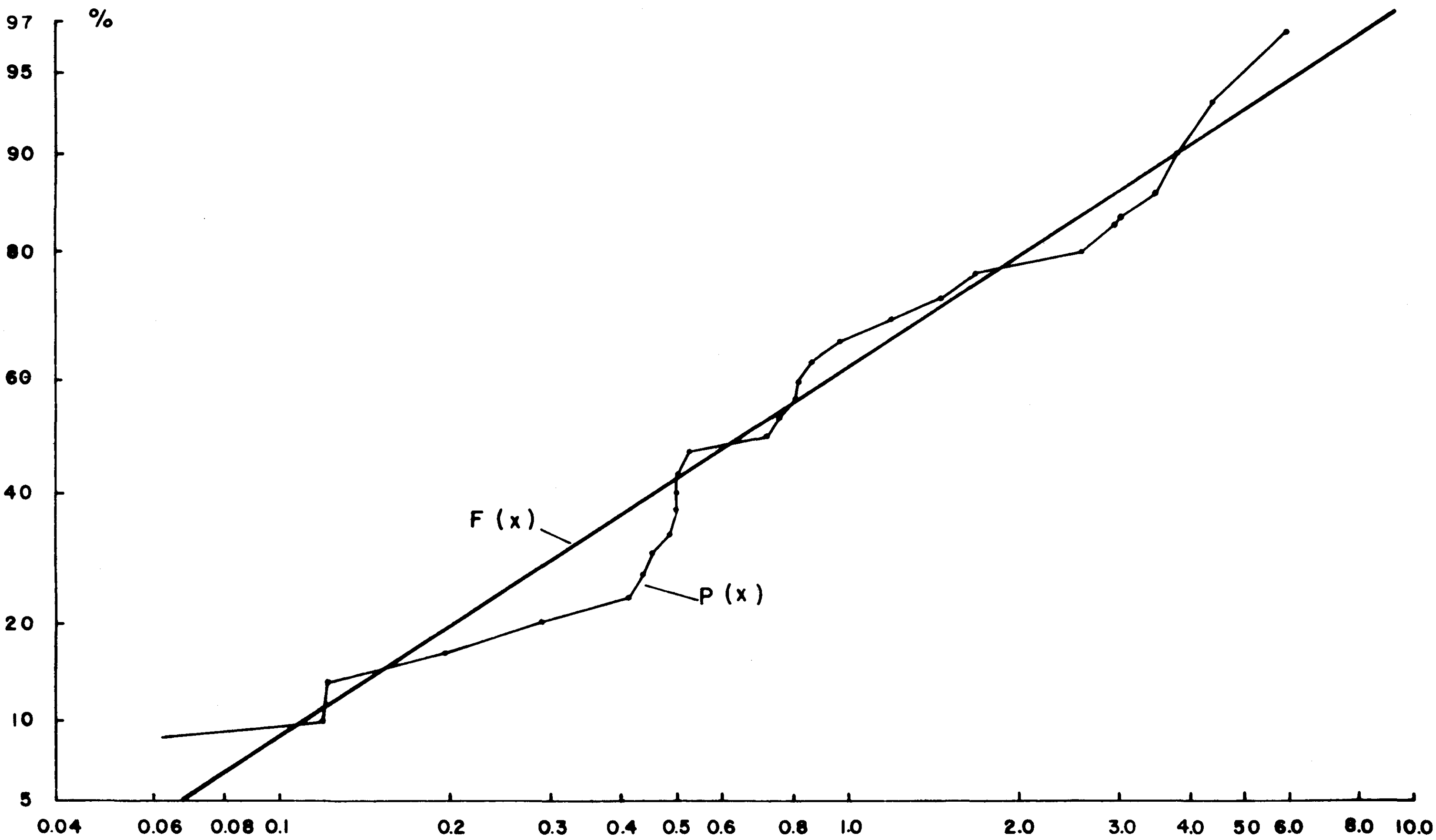


Fig. 5.3

Future Marginal Return of Storage Distribution Curve: Initial Date, July 1; Initial Storage, 39.963 Millions M^3

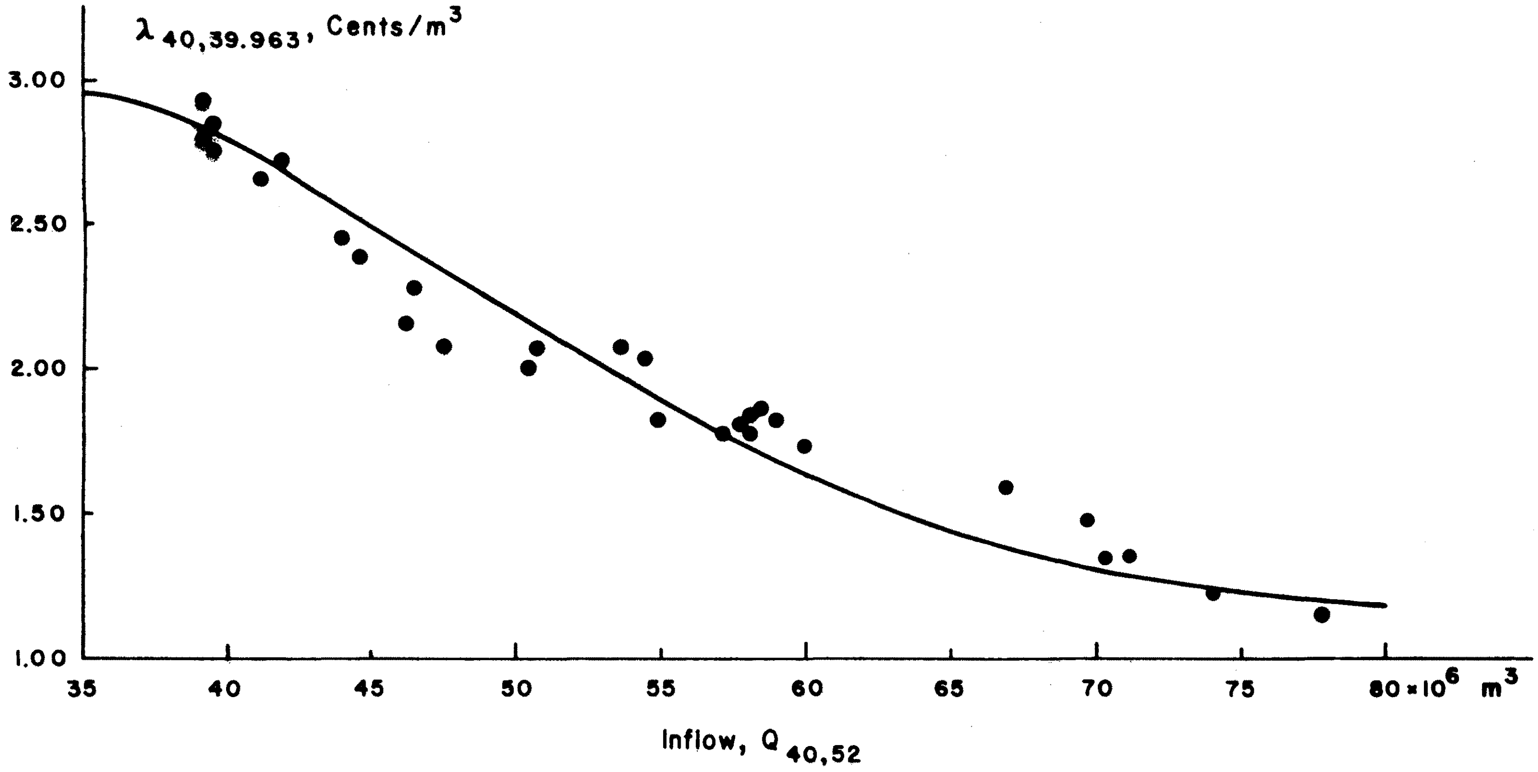


Fig. 5.4 Relation between Future Marginal Return of Storage and Inflow, Given by Distribution Curves; Initial Date, July 1; Initial Storage, 39.963 Millions M^3

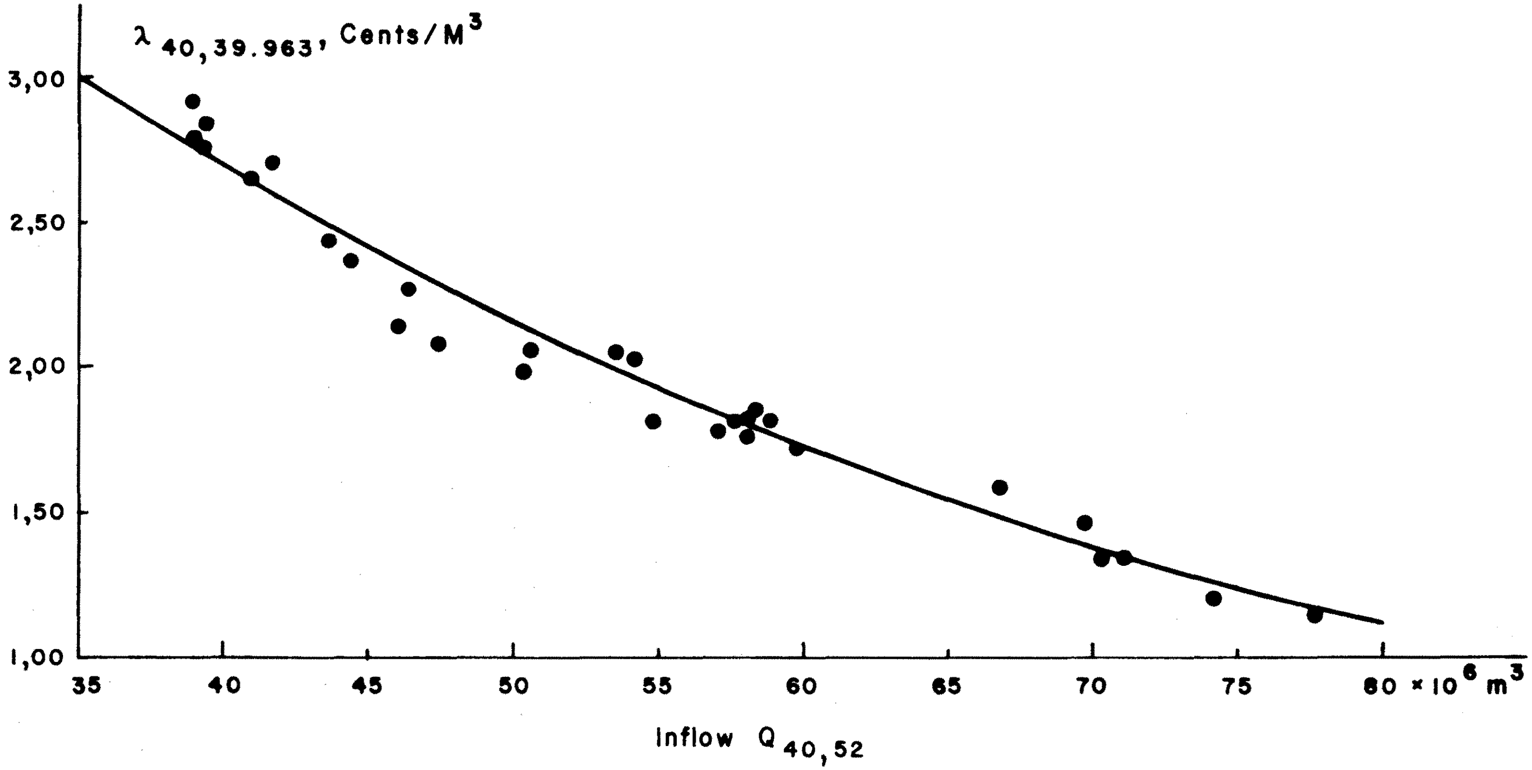
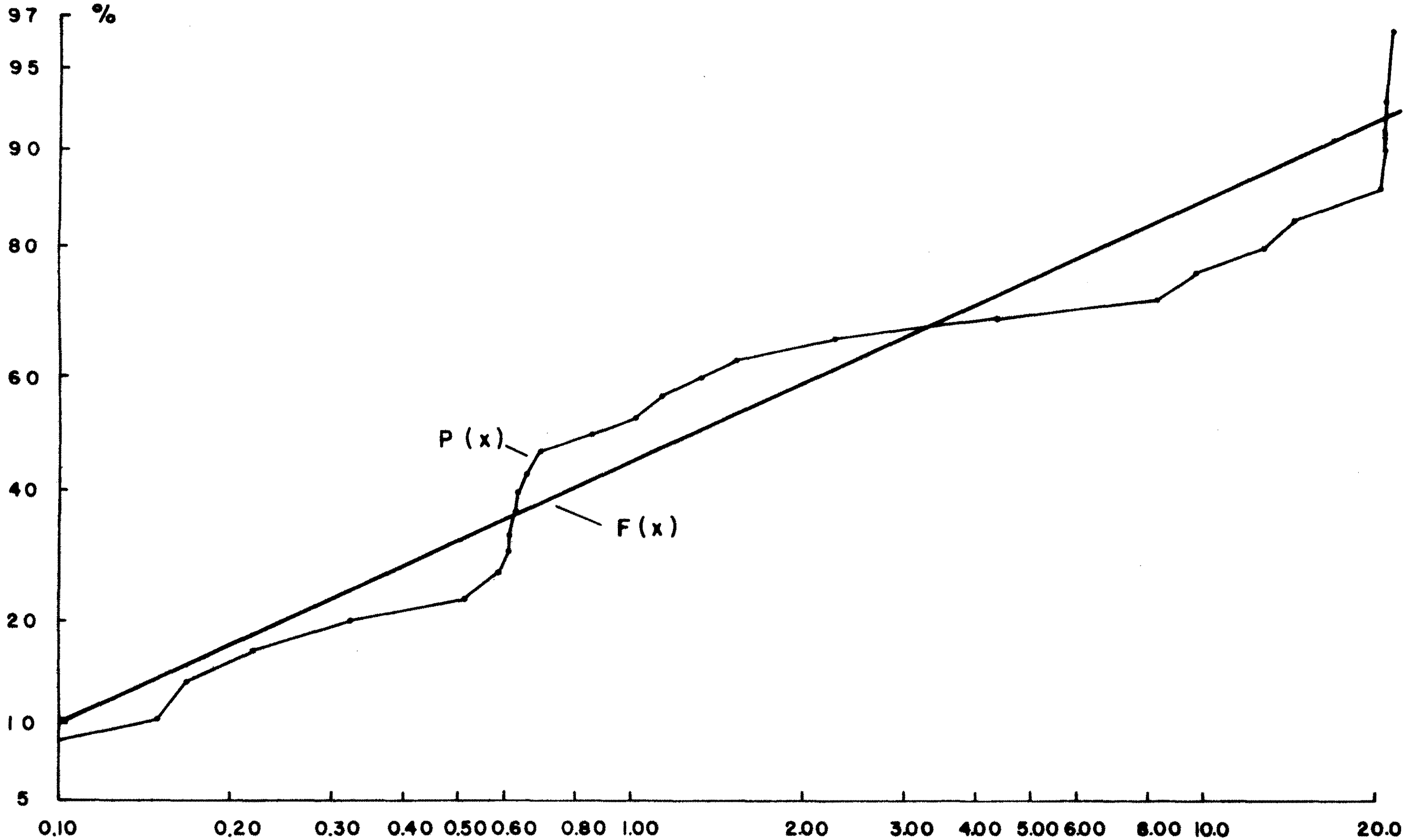


Fig. 5.5 Relation between Future Marginal Return of Storage and Inflow, Given by Least Squares Fit; Initial Date, July 1; Initial Storage 39.963 Millions M^3



$$\frac{\lambda_{40,31.963} - 1.360}{3.400 - \lambda_{40,31.963}}$$

Fig. 5.6

Future Marginal Return of Storage; Distribution Curve: Initial Date, July 1; Initial Storage, 31,963 Millions M^3

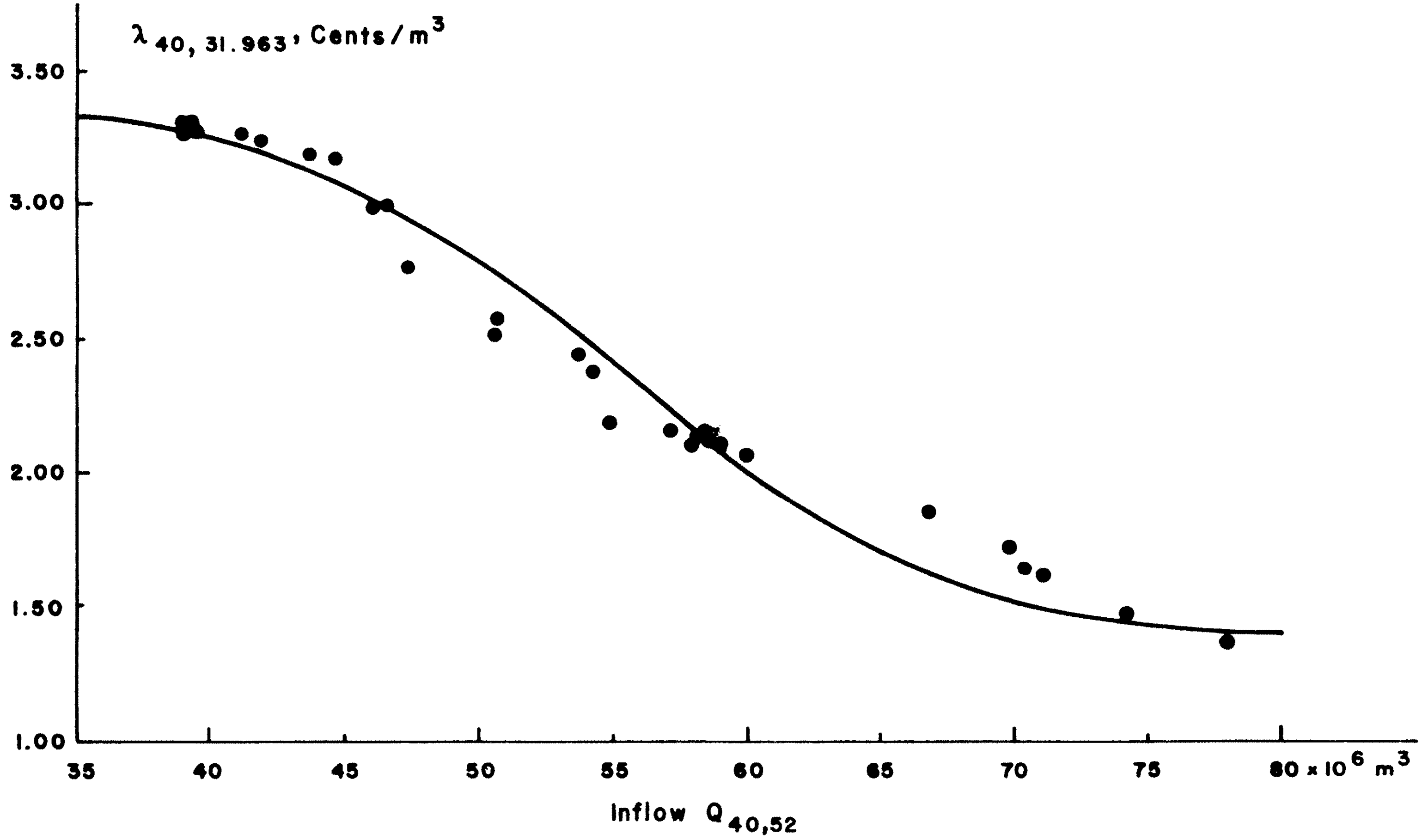


Fig. 5.7 Relation between Future Marginal Return of Storage and Inflow, Given by Distribution Curves; Initial Date, July 1; Initial Storage, 31,963 Millions M^3

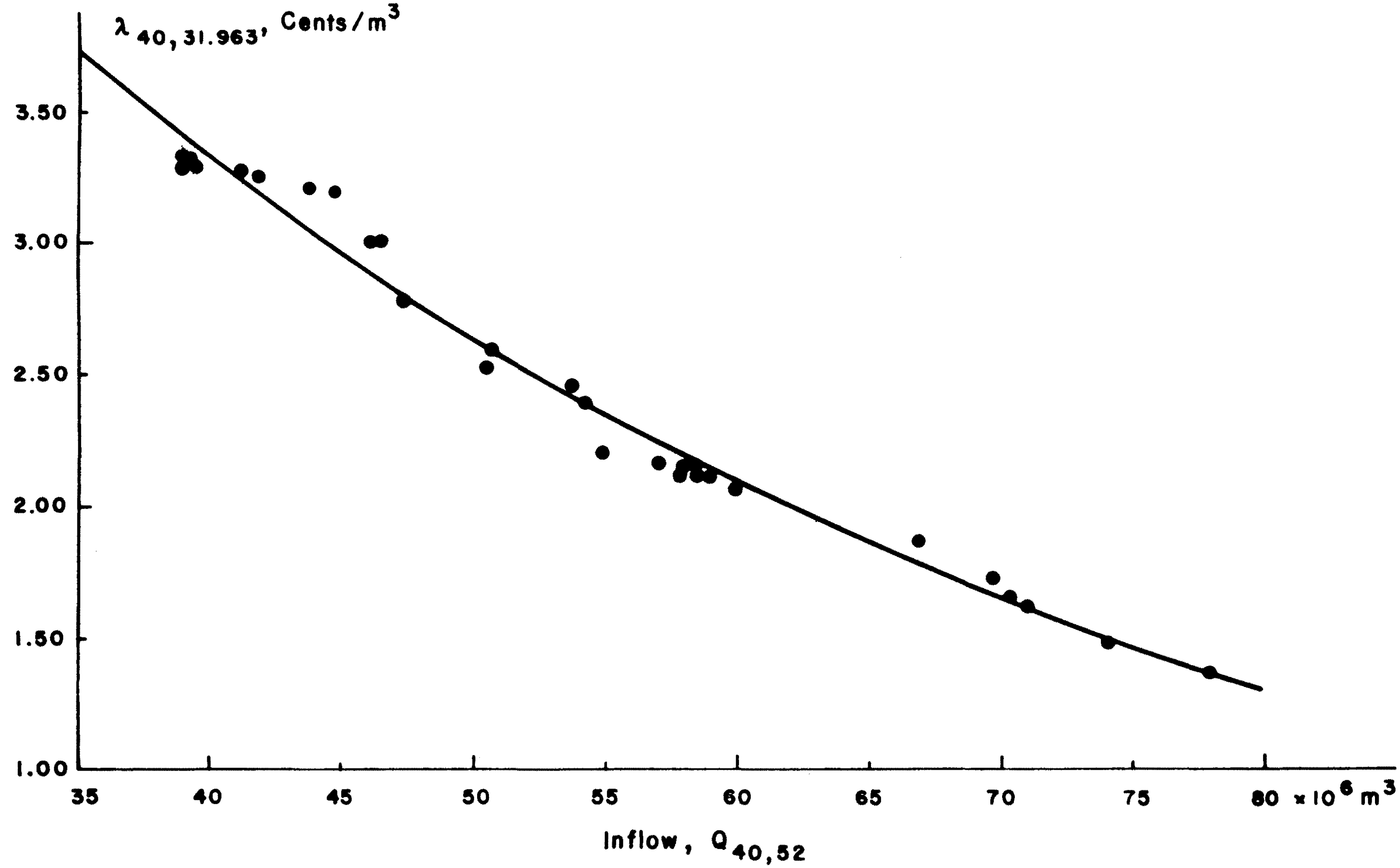


Fig. 5.8 <u>Relation between Future Marginal Return of Storage and Inflow, Given by Least Squares Fit; Initial Date, July 1; Initial Storage, 31.963 Millions M^3</u>

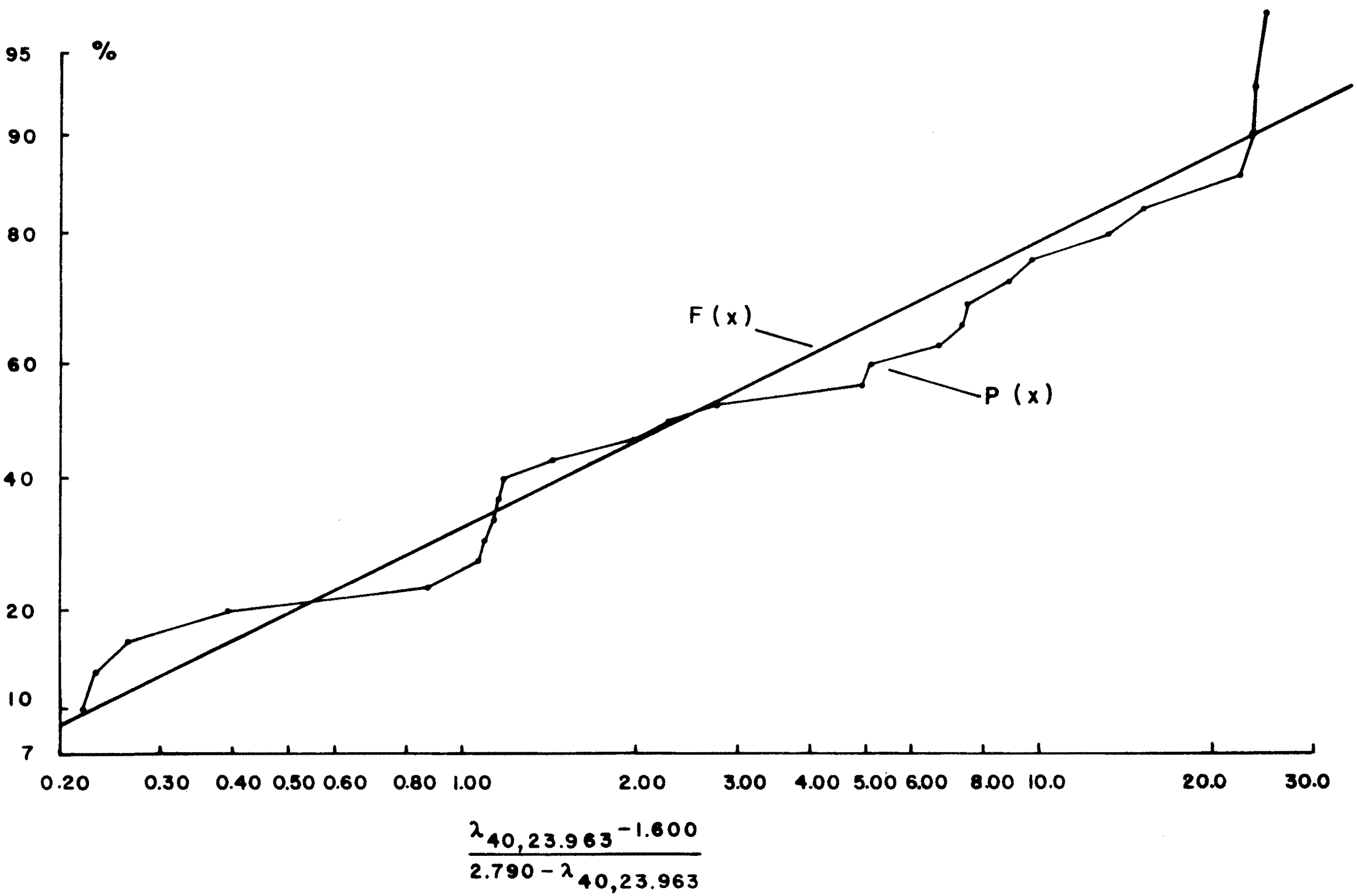


Fig. 5.9 Future Marginal Return of Storage: Distribution Curve: Initial Date, July 1; Initial Storage, 23.963 Millions M^3

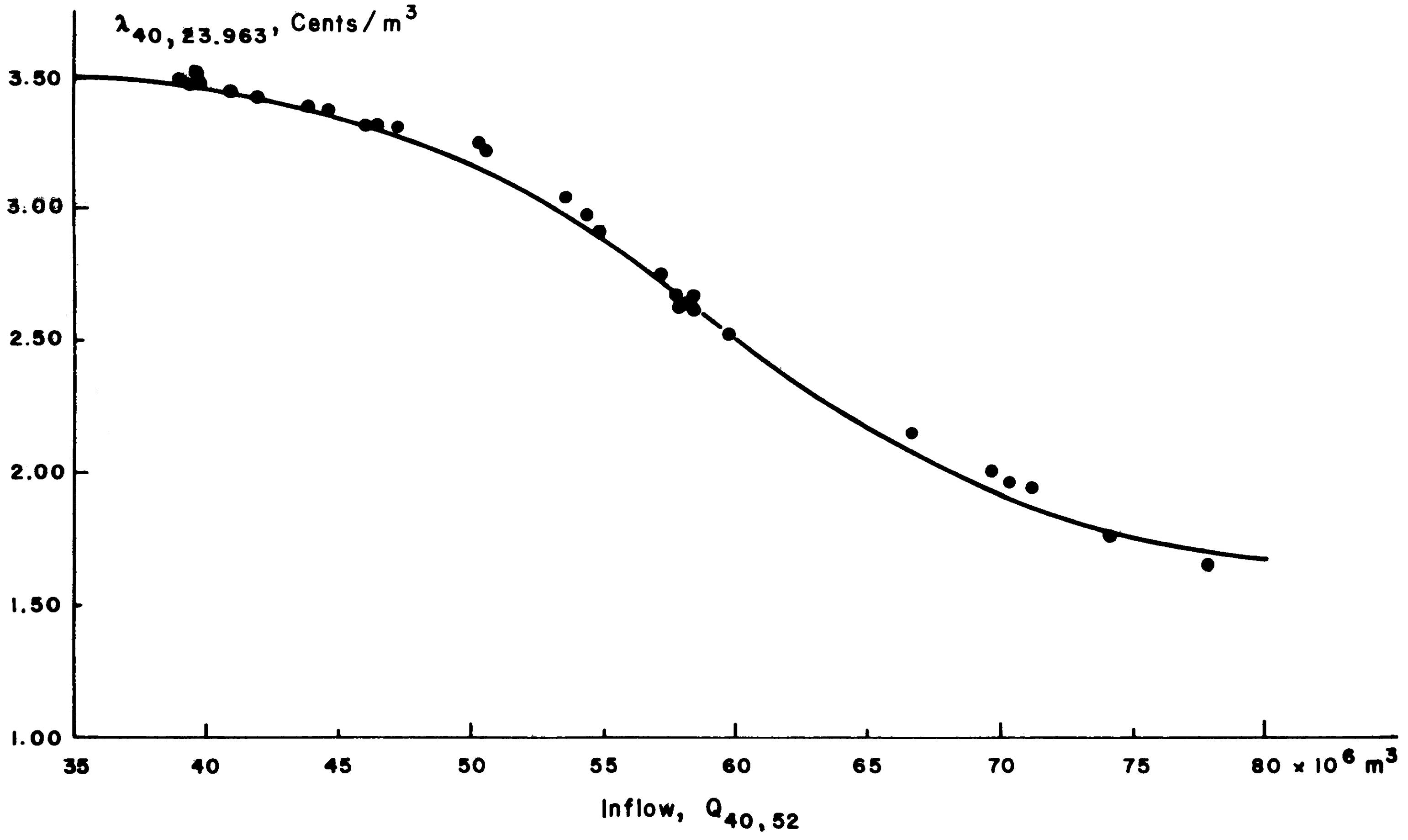


Fig. 5.10 <u>Relation between Future Marginal Return of Storage and Inflow, Given by Distribution Curves; Initial Date, July 1; Initial Storage, 23.963 Millions M^3</u>

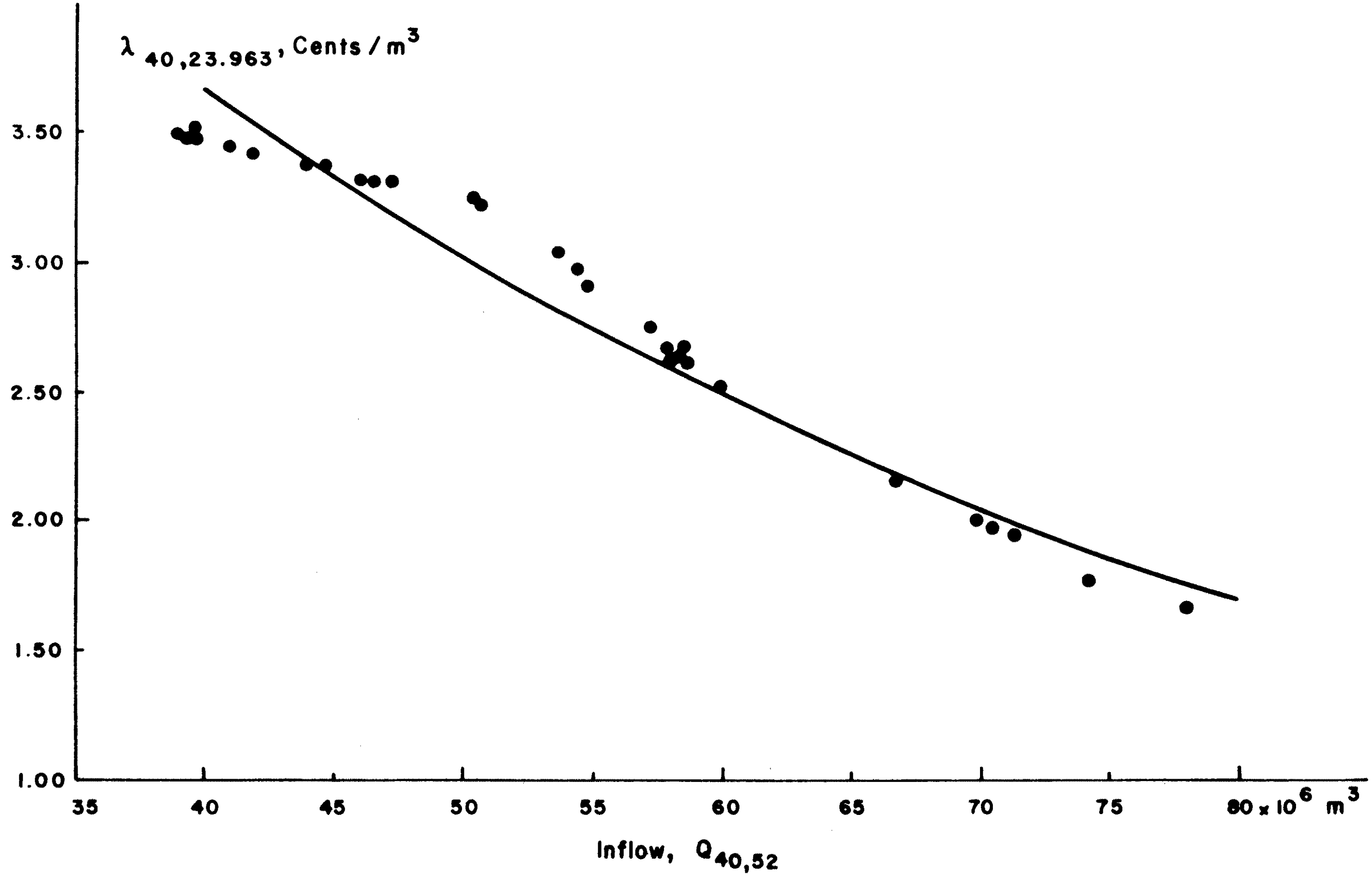


Fig. 5.11 Relation between Future Marginal Return of Storage and Inflow, Given by Least Squares Fit; Initial Data, July 1; Initial Storage 23 963 Millions M^3

Table 5.2

Future Marginal Return of Storage, Values Computed for October 1

Initial Storage	76.000 $\times 10^6$ m^3 Cents/m^3	69.000 $\times 10^6$ m^3 Cents/m^3	66.000 $\times 10^6$ m^3 Cents/m^3	63.000 $\times 10^6$ m^3 Cents/m^3	60.000 $\times 10^6$ m^3 Cents/m^3
1946	3.160	3.242	3.320	3.400	3.483
47	3.308	3.385	3.463	3.545	3.631
48	3.250	3.325	3.402	3.481	3.561
49	3.304	3.354	3.432	3.512	3.593
1950	3.223	3.292	3.362	3.434	3.507
51	3.311	3.384	3.460	3.536	3.614
52	3.117	3.190	3.264	3.340	3.417
53	3.088	3.157	3.227	3.299	3.371
54	2.876	3.043	3.109	3.175	3.244
55	3.346	3.417	3.491	3.564	3.643
56	3.328	3.399	3.472	3.545	3.621
57	3.248	3.318	3.389	3.463	3.540
58	3.282	3.352	3.424	3.497	3.573
59	1.420	2.052	2.927	3.292	3.253
1960	3.256	3.332	3.412	3.495	3.579
61	2.407	3.016	3.089	3.164	3.241
62	3.286	3.314	3.342	3.402	3.478
63	3.379	3.455	3.535	3.617	3.701
64	3.093	3.159	3.225	3.294	3.371
65	3.277	3.347	3.419	3.492	3.566
66	2.079	3.305	3.377	3.449	3.526
67	3.100	3.167	3.235	3.304	3.376
68	2.429	3.126	3.196	3.270	3.346
69	3.155	3.223	3.293	3.363	3.435
1970	3.338	3.410	3.485	3.561	3.641
71	3.151	3.224	3.299	3.376	3.454
72	3.357	3.417	3.492	3.580	3.670
73	3.398	3.471	3.546	3.621	3.698
1974	3.181	3.249	3.325	3.402	3.481

Table 5.3

Future Marginal Return of Storage, Main Statistics Computed for October 1

Initial Storage	72.000 $\times 10^6$ m^3	69.000 $\times 10^6$ m^3	66.000 $\times 10^6$ m^3	63.000 $\times 10^6$ m^3	60.000 $\times 10^6$ m^3
Average	3.067	3.246	3.345	3.432	3.504
Stand. dev.	0.440	0.258	0.144	0.125	0.134
Coeff. var.	0.143	0.079	0.043	0.036	0.038
Min.	1.420	2.052	2.927	3.164	3.241
Max.	3.398	3.471	3.546	3.621	3.701

All the numbers are given in Cents/m^3

Table 5.4

Future Marginal Return of Storage, Expected Values Computed for October 1, according to Different Methods

Distribution Curve	n	Storage 72.000 x 10^6 m^3	n	Storage 69.000 x 10^6 m^3	n	Storage 66.000 x 10^6 m^3	n	Storage 63.000 x 10^6 m^3	n	Storage 60.000 x 10^6 m^3
Normal	29	3.067	29	3.246	29	3.345	29	3.432	29	3.504
Log-Normal	29	3.025	29	3.233	29	3.342	29	3.428	29	3.501
Log-Normal*	24	3.237	28	3.286	28	3.358	29	3.428	29	3.501
Log-Normal**	5	2.186	1	2.052	1	2,927	-	-	-	-
Marg. Ret. Stor. for Aver. Infl.	-	3.198	-	3.269	-	3.341	-	3.416	-	3.495

All the numbers are given in Cents/m^3

* For the cases where reservoir content is smaller than 72.000 x 10^6 m^3 on October 8

**For the cases where reservoir content is equal to 72.000 x 10^6 m^3 on October 8

Table 5.5

Future Marginal Return of Storage,
Values Computed for December 31

Initial Storage	48.809 $\times 10^6$ m^3 Cents/m^3	44.809 $\times 10^6$ m^3 Cents/m^3	40.809 $\times 10^6$ m^3 Cents/m^3	36.809 $\times 10^6$ m^3 Cents/m^3	32.809 $\times 10^6$ m^3 Cents/m^3
1946	2.772	2.941	3.131	3.336	3.563
47	2.866	3.035	3.219	3.415	3.631
48	2.789	2.947	3.116	3.304	3.504
49	3.202	3.249	3.296	3.419	3.626
1950	2.883	3.036	3.196	3.364	3.541
51	2.932	3.087	3.251	3.429	3.621
52	2.815	2.980	3.161	3.353	3.556
53	2.827	2.976	3.141	3.318	3.524
54	2.878	3.031	3.191	3.359	3.535
55	3.012	3.122	3.287	3.460	3.646
56	2.961	3.118	3.283	3.456	3.637
57	2.889	3.042	3.203	3.371	3.559
58	2.942	3.098	3.262	3.433	3.617
59	3.214	3.261	3.309	3.358	3.520
1960	2.871	3.023	3.182	3.369	3.589
61	2.775	2.900	3.090	3.293	3.514
62	3.225	3.272	3.321	3.420	3.612
63	2.973	3.130	3.300	3.489	3.700
64	2.916	3.071	3.232	3.418	3.625
65	2.959	3.116	3.280	3.453	3.634
66	2.924	3.079	3.241	3.412	3.602
67	2.921	3.076	3.239	3.409	3.587
68	2.794	2.953	3.120	3.304	3.504
69	2.982	3.140	3.306	3.480	3.664
1970	2.976	3.133	3.299	3.475	3.670
71	2.853	2.957	3.136	3.326	3.527
72	3.226	3.273	3.322	3.387	3.593
73	3.016	3.176	3.344	3.520	3.704
74	2.859	3.010	3.168	3.347	3.550

Table 5.6

Future Marginal Return of Storage, Main Statistics Computed for December 31

Initial Storage	48.809 $\times 10^6$ m^3	44.809 $\times 10^6$ m^3	40.809 $\times 10^6$ m^3	36.809 $\times 10^6$ m^3	32.809 $\times 10^6$ m^3
Average	2.940	3.077	3.228	3.396	3.592
Stand. dev.	0.132	0.103	0.074	0.061	0.058
Coeff. var.	0.045	0.033	0.023	0.018	0.016
Min.	2.772	2.900	3.090	3.293	3.504
Max.	3.226	3.273	3.344	3.520	3.704

All the numbers are given in Cents/m^3

Table 5.7

Future Marginal Return of Storage, Expected Values Computed for December 31, according to Different Methods

Distribution Curve	Storage 48.809 x 10^6 m^3	Storage 44.809 x 10^6 m^3	Storage 40.809 x 10^6 m^3	Storage 36.809 x 10^6 m^3	Storage 32.809 x 10^6 m^3
Normal	2.940	3.077	3.228	3.396	3.592
Log-Normal	2.937	3.075	3.228	3.395	3.591

All the numbers are given in Cents/m^3

Table 5.8

Future Marginal Return of Storage, Value Computed for July 1, in Cents/m^3

Initial Storage	39.963 10^6 m^3	35.963 10^6 m^3	31.963 10^6 m^3	27.963 10^6 m^3	23.963 10^6 m^3
1946	1.469	1.596	1.726	1.862	2.009
47	1.768	1.959	2.170	2.482	2.754
48	1.828	2.004	2.196	2.518	2.906
49	2.710	3.040	3.252	3.335	3.442
1950	1.782	1.944	2.113	2.325	2.626
51	1.346	1.491	1.652	1.813	1.970
52	1.737	1.888	2.052	2.232	2.516
53	2.073	2.224	2.441	2.706	3.036
54	1.824	1.982	2.138	2.346	2.635
55	2.806	3.225	3.306	3.391	3.481
56	1.146	1.256	1.377	1.509	1.654
57	2.158	2.490	3.022	3.248	3.331
58	1.830	1.976	2.132	2.331	2.615
59	2.938	3.228	3.309	3.394	3.484
1960	1.595	1.722	1.858	2.005	2.153
61	2.279	2.653	3.000	3.243	3.325
62	2.655	3.069	3.269	3.352	3.440
63	1.216	1.333	1.462	1.602	1.756
64	2.854	3.221	3.303	3.387	3.476
65	2.038	2.186	2.388	2.646	2.964
66	2.456	2.821	3.212	3.293	3.377
67	1.352	1.482	1.625	1.782	1.953
68	1.850	1.997	2.155	2.361	2.651
69	2.383	2.732	3.182	3.280	3.363
1970	1.814	1.971	2.143	2.368	2.659
71	2.099	2.371	2.779	3.223	3.305
72	2.775	3.224	3.306	3.389	3.480
73	2.067	2.312	2.591	2.936	3.232
1974	2.007	2.201	2.516	2.943	3.240

Table 5.9

Future Marginal Return of Storage, Main Statistics Computed for July 1

Initial Storage	39.963 x 10^6 m^3	35.963 x 10^6 m^3	31.963 x 10^6 m^3	27.963 x 10^6 m^3	23.963 x 10^6 m^3
Average	2.029	2.262	2.447	2.666	2.856
Stand. dev.	0.507	0.605	0.626	0.619	0.584
Coeff. var.	0.250	0.267	0.256	0.232	0.204
Min.	1.146	1.256	1.377	1.509	1.654
Max.	2.938	3.228	3.306	3.394	3.481

All the numbers are given in Cents/m^3

Table 5.10

Future Marginal Return of Storage, Expected Values
Computed for July 1, according to Different Methods

Initial Storage	39.963 10^6 m^3	31.963 10^6 m^3	23.963 10^6 m^3
Normal distribution	2.029	2.447	2.856
Log-Normal distr.	1.967	2.368	2.790
Four par. log-norm. distr.	1.951	2.498	2.959
Upper boundary	3.250	3.400	3.560
Upper boundary	1.120	1.360	1.600
Linear relation*	2.028	2.414	3.001
Least squares**	1.998	2.441	2.848

All the numbers are given in Cents/m^3

* Computed according to the formula $E\left[\ln\lambda^f_{40,s_{40}}\right] = a_f+b_f\cdot \bar{Q}_{40,52}$

**Obtained from the linear regression between

$\ln\lambda_{40,s_{40}}$ and $Q_{40,52}$

value of storage depends on the initial content. It suggests also that the upper boundary plays a more important role than the lower boundary. Finally the expected future marginal return of storage derived from Eq. 5-7 lies remarkably close to the corresponding value given by the four parameters log-normal distribution. This is a very important result as it allows to reduce greatly the amount of computations required to determine the expected future marginal returns of storage.

The relation established between inflow and associated future marginal return of storage on the basis of their respective distribution curves provides a fair fit to the "experimental" data. The fit is best for the lowest initial storage. On the whole, the reliability of the determined relation is roughly constant for the whole range of data, including the tails. Some of the recorded discrepancies between "experimental" and theoretical results can be explained by the fact that the original inflow sample is not exactly normally distributed. Also the four parameters log-normal distribution is not flexible enough to follow all the changes in curvature of the "experimental" distribution curve.

A linear regression equation was computed between the natural logarithm of the future marginal returns of storage and the related inflows, to assess the merits of the just derived theoretical curve. As expected, the "least squares" curve fails completely to reproduce the situation prevailing for extreme data. It provides however a good fit for the central range of the experimental data, and when the boundaries play a secondary role.

c) Mixed strategy situation. To illustrate this case, we shall consider the situation existing when the reservoir operation is started on April 1. Table 5.11 and 5.12 contain the computed future marginal returns of storage while Table 5.13 supplies the corresponding expected values. Figs. 5.12, 5.16 and 5.20 show the fitted distribution curves, two for each initial storage, and Figs. 5.13, 5.14, 5.15, 5.17, 5.18, 5.19, 5.21, 5.22, 5.23, the relation existing between the future marginal return of storage and the corresponding relevant hydrologic variable.

The sample of marginal values of storage was divided into two groups, depending on whether or not the reservoir got empty for the associated optimal trajectory. The log-normal distribution provided a good fit to each group of data. This happened mainly because upper and lower boundary play here a secondary role. In fact the mixed strategy situation prevents the occurrence of extreme cases. If for example the inflow recorded from the start of the reservoir operation until the end of the water year is low in a given year, the reservoir does not get empty. If it would all the same have been empty, the corresponding future marginal value of storage would have been much lower. The expected values derived from Eq. 5-10 and 5-11 lie quite close to those obtained from the distribution curves.

Generally a definite relation exists between future marginal return of storage and the inflow recorded from the start of the reservoir operation until the end of the water year. However this relation holds only for the smaller values of the considered inflow. As indicated by the relevant figures, above a certain inflow level, the future marginal returns of storage seem to be randomly distributed. These points correspond to the case where the reservoir gets empty.

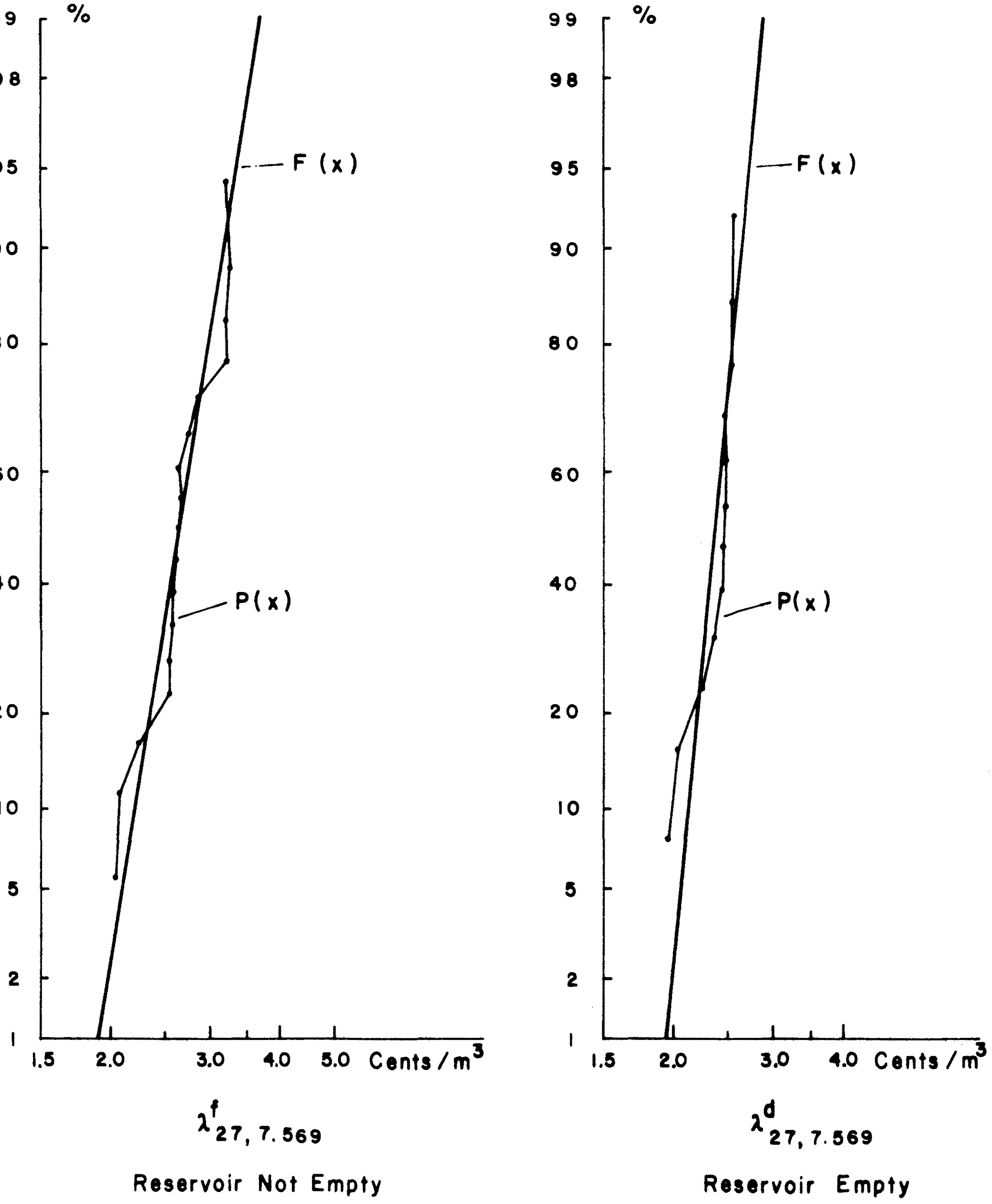


Fig. 5.12 Future Marginal Return of Storage Distribution Curves; Initial Date, April 1; Initial Storage, 7.569 Millions^3

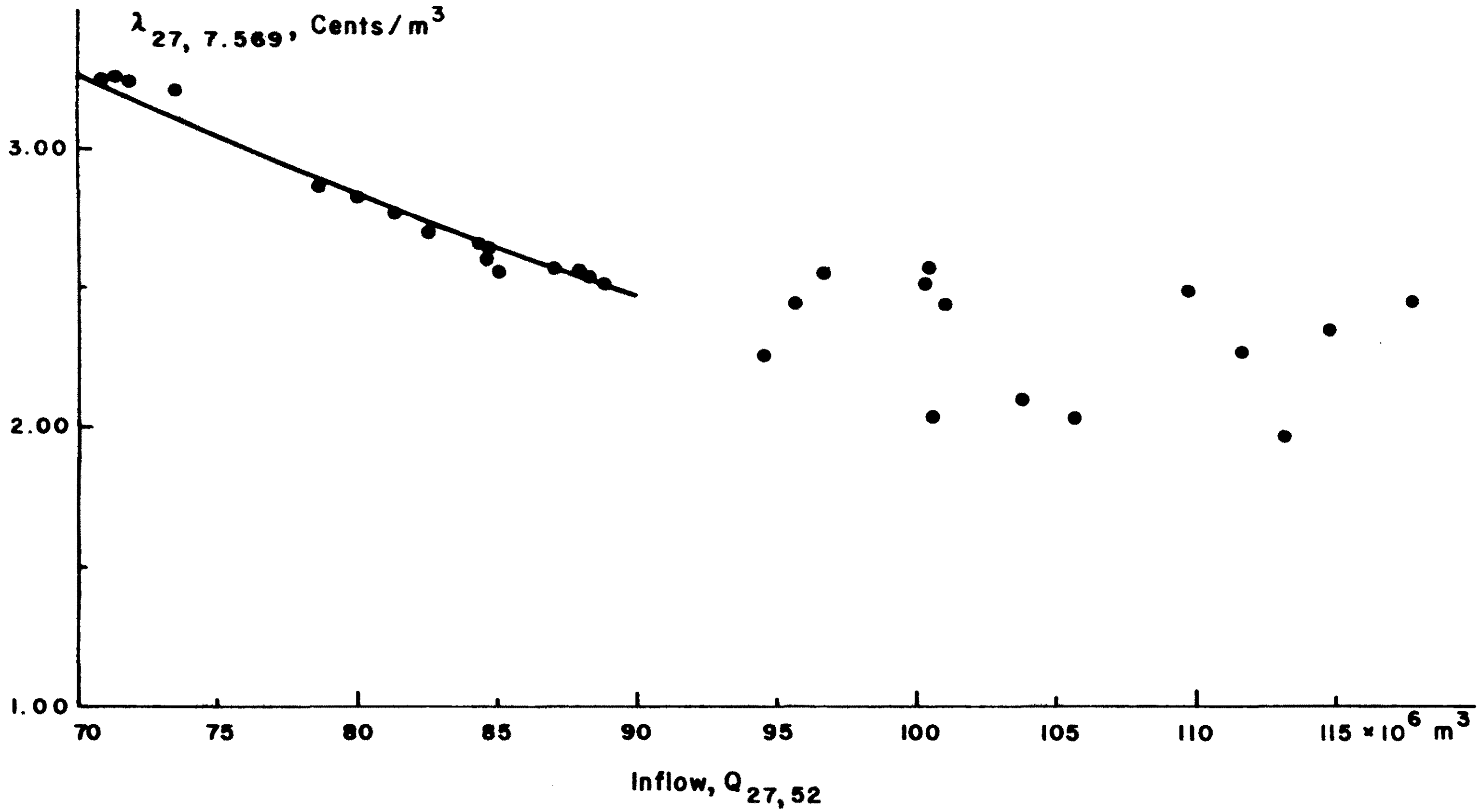


Fig. 5.13 Relation between Future Marginal Return of Storage and Summer Inflow, Given by Least Squares Fit; Initial Date, April 1; Initial Storage 7.569 Millions M^3

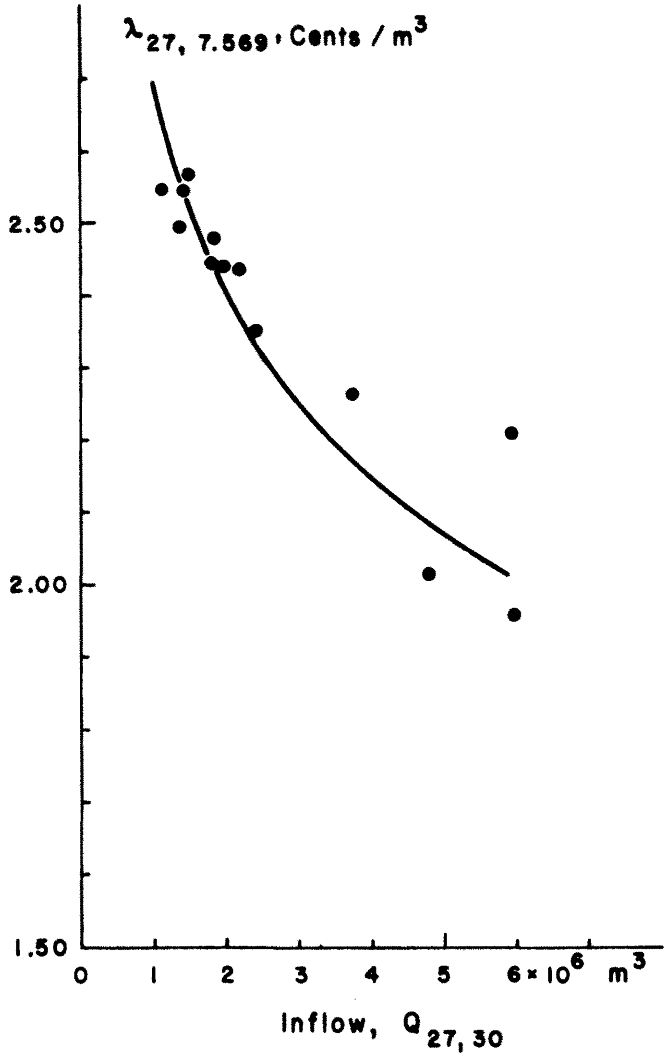


Fig. 5.14 Relation between Future Marginal Return of Storage and Inflow during April, Given by Distribution Curves; Initial Date, April 1; Initial Storage, 7,569 Millions M^3

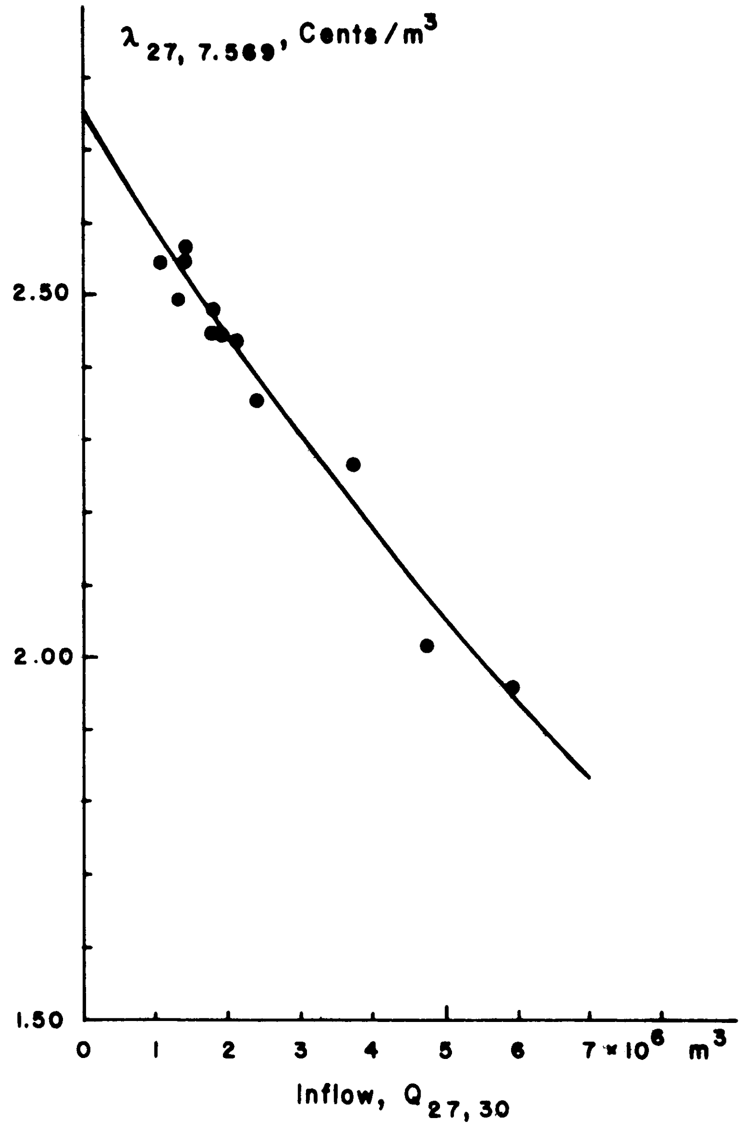


Fig. 5.15 Relation between Future Marginal Return of Storage and Inflow during April, Given by Least Squares Fit; Initial Date, April 1; Initial Storage, 7.569 Millions M^3

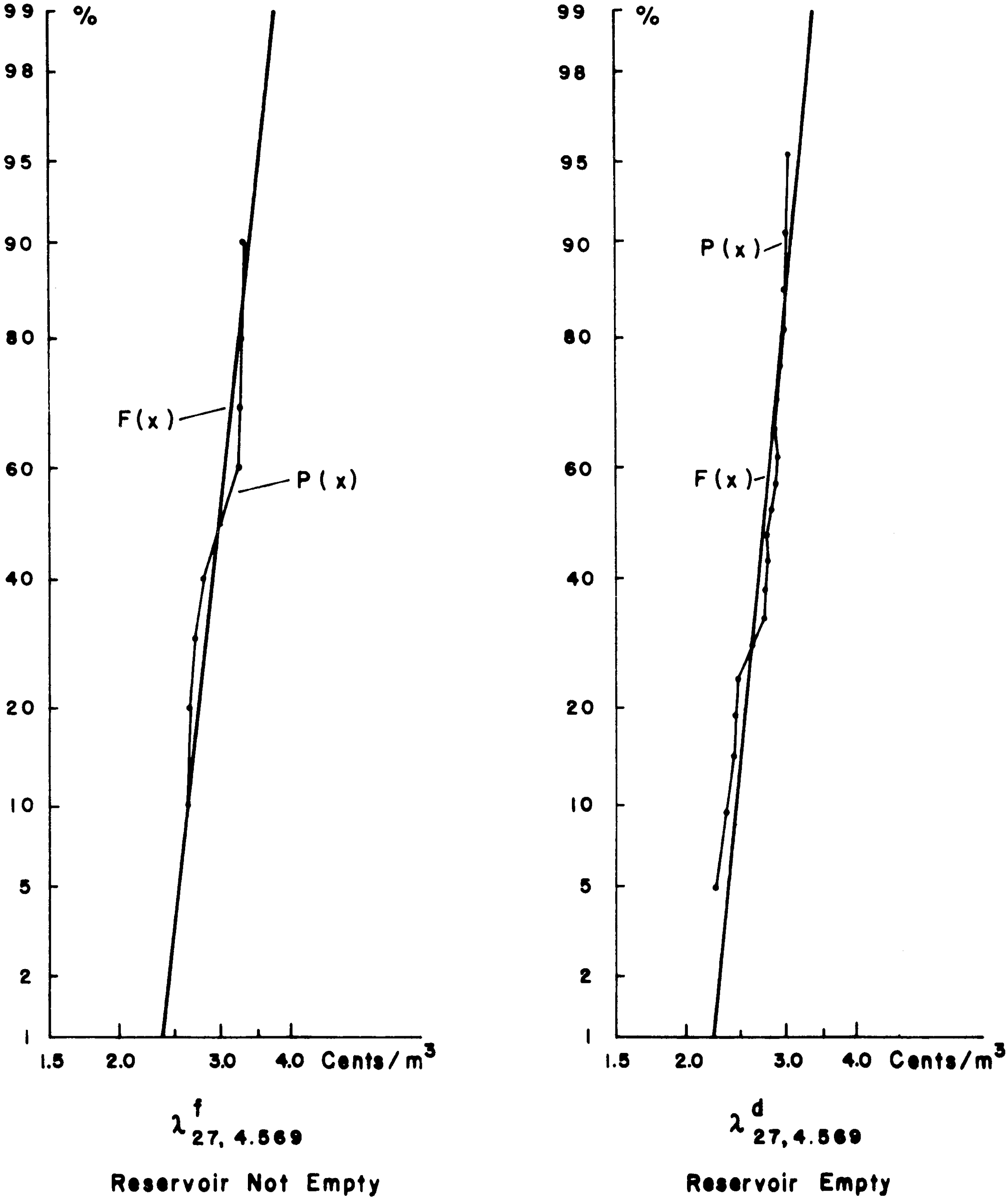


Fig. 5.16 Future Marginal Return of Storage, Distribution Curves; Initial Date, April 1; Initial Storage, 4,569 Millions M^3

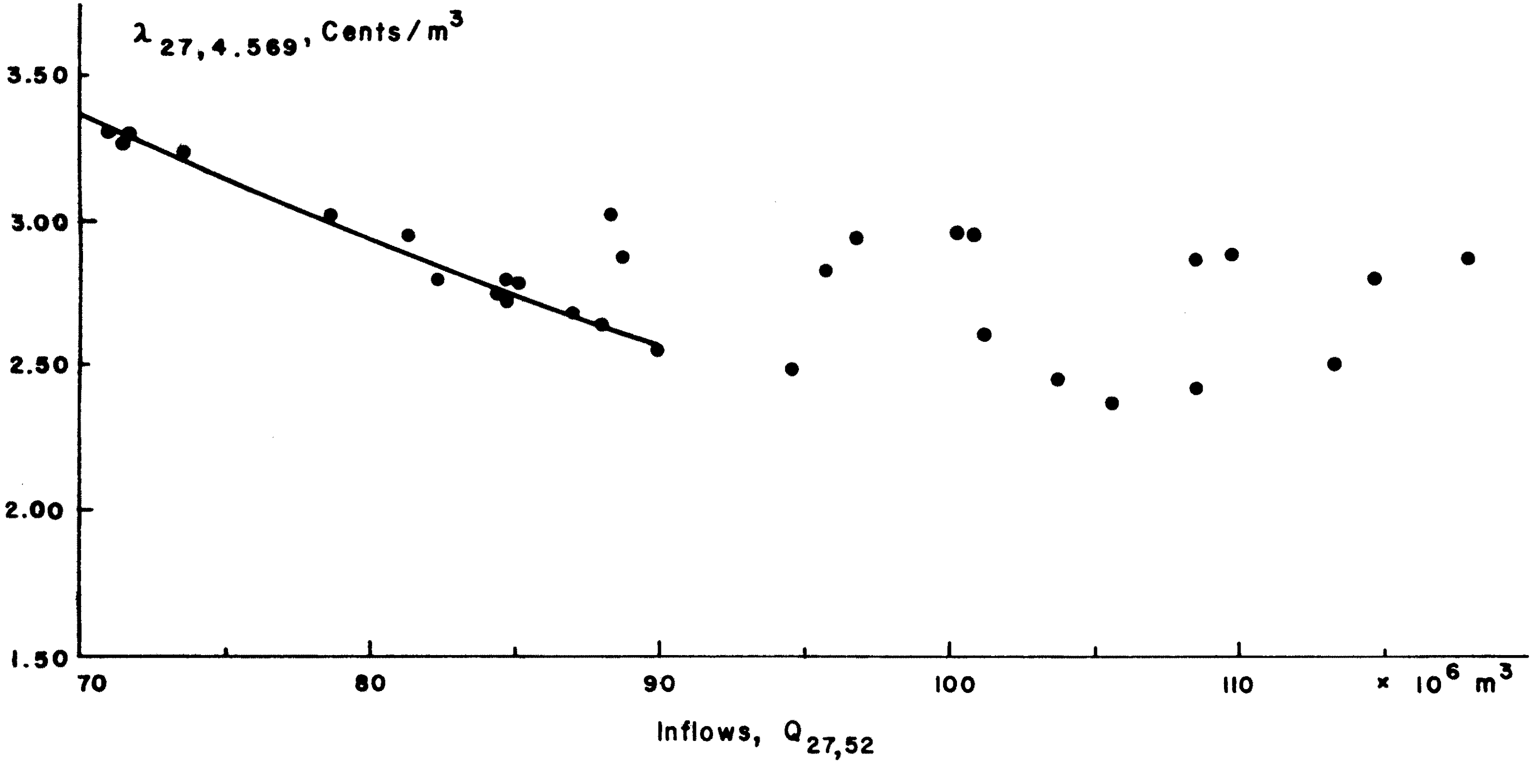


Fig. 5.17 Relation between Future Marginal Return of Storage and Summer Inflow, Given by Least Squares Fit; Initial Date, April 1; Initial Storage, 4,569 Millions M^3

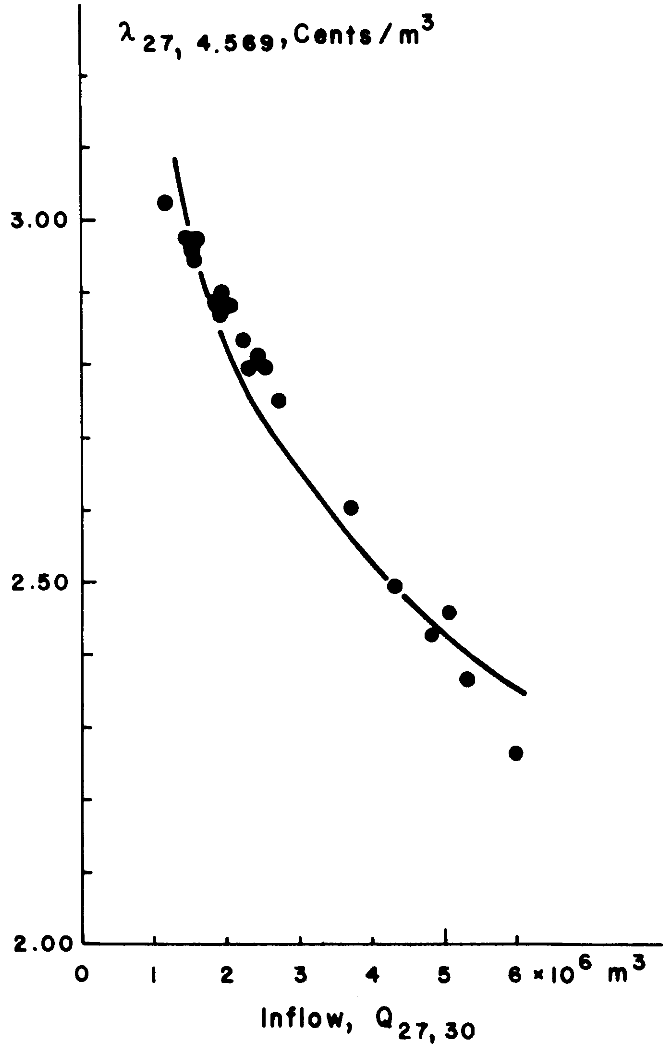


Fig. 5.18 Relation between Future Marginal Return of Storage and Inflow during April, Given by Distribution Curves; Initial Date, April 1; Initial Storage, 4.569 Millions M^3

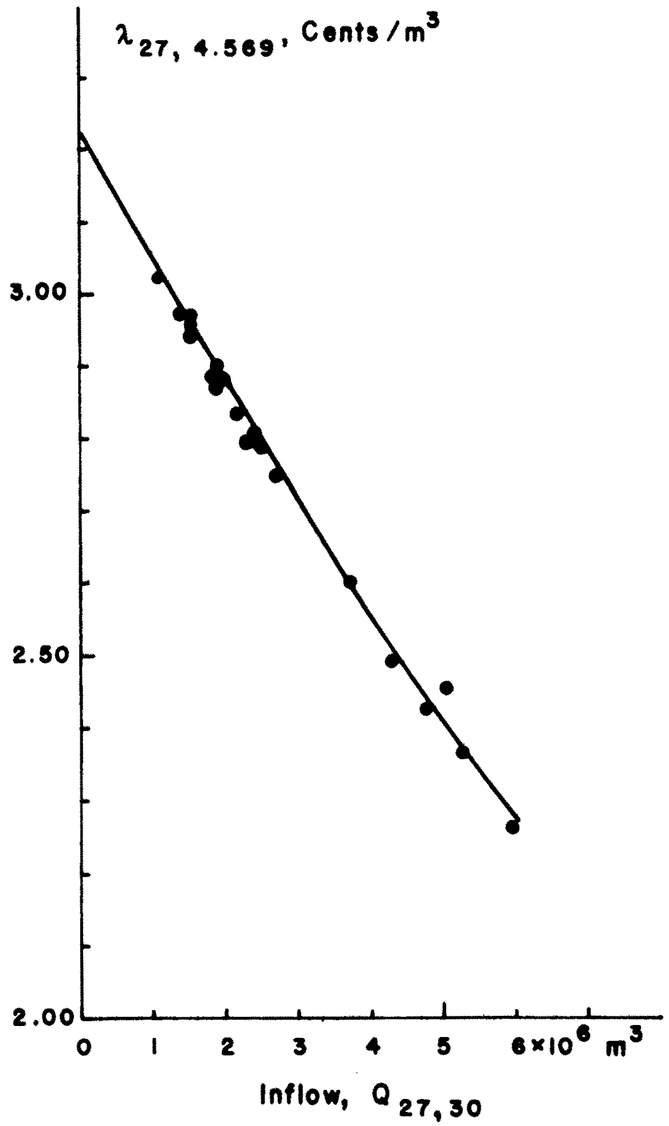


Fig. 5.19 Relation between Future Marginal Return of Storage and Inflow during April, Given by Least Squares Fit; Initial Date, April 1; Initial Storage, 4.569 Millions M^3

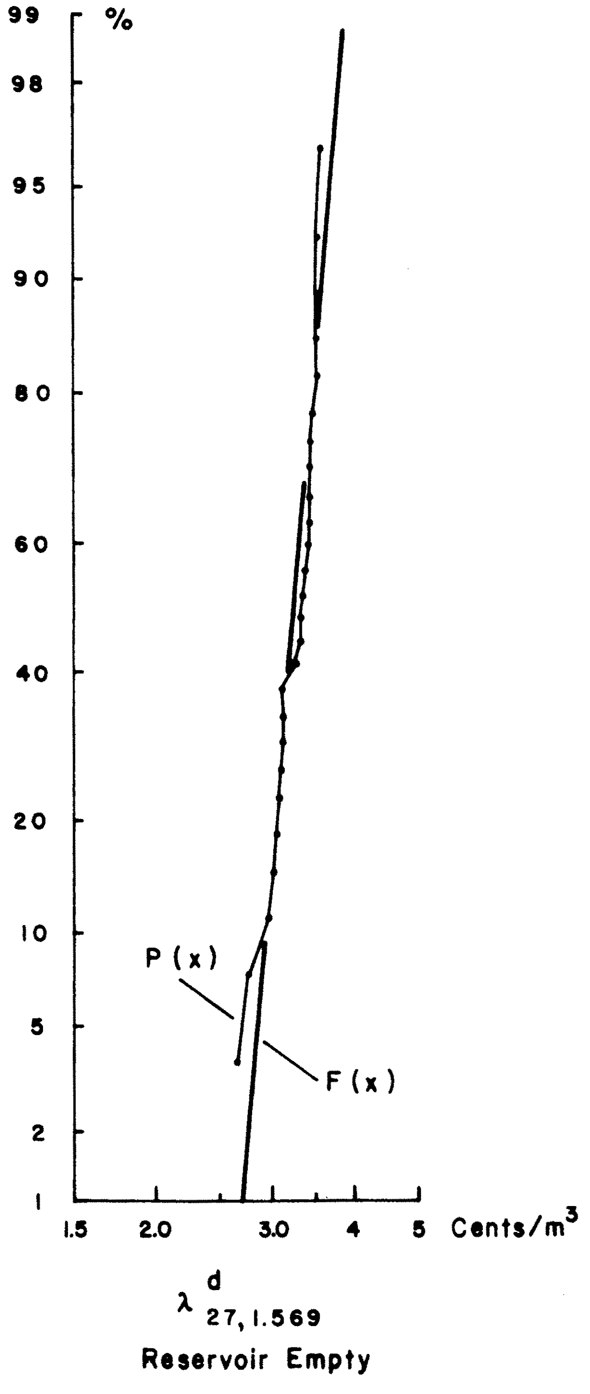


Fig. 5.20 Future Marginal Return of Storage, Distribution Curve; Initial Date, April 1; Initial Storage, 1.569 Millions M^3

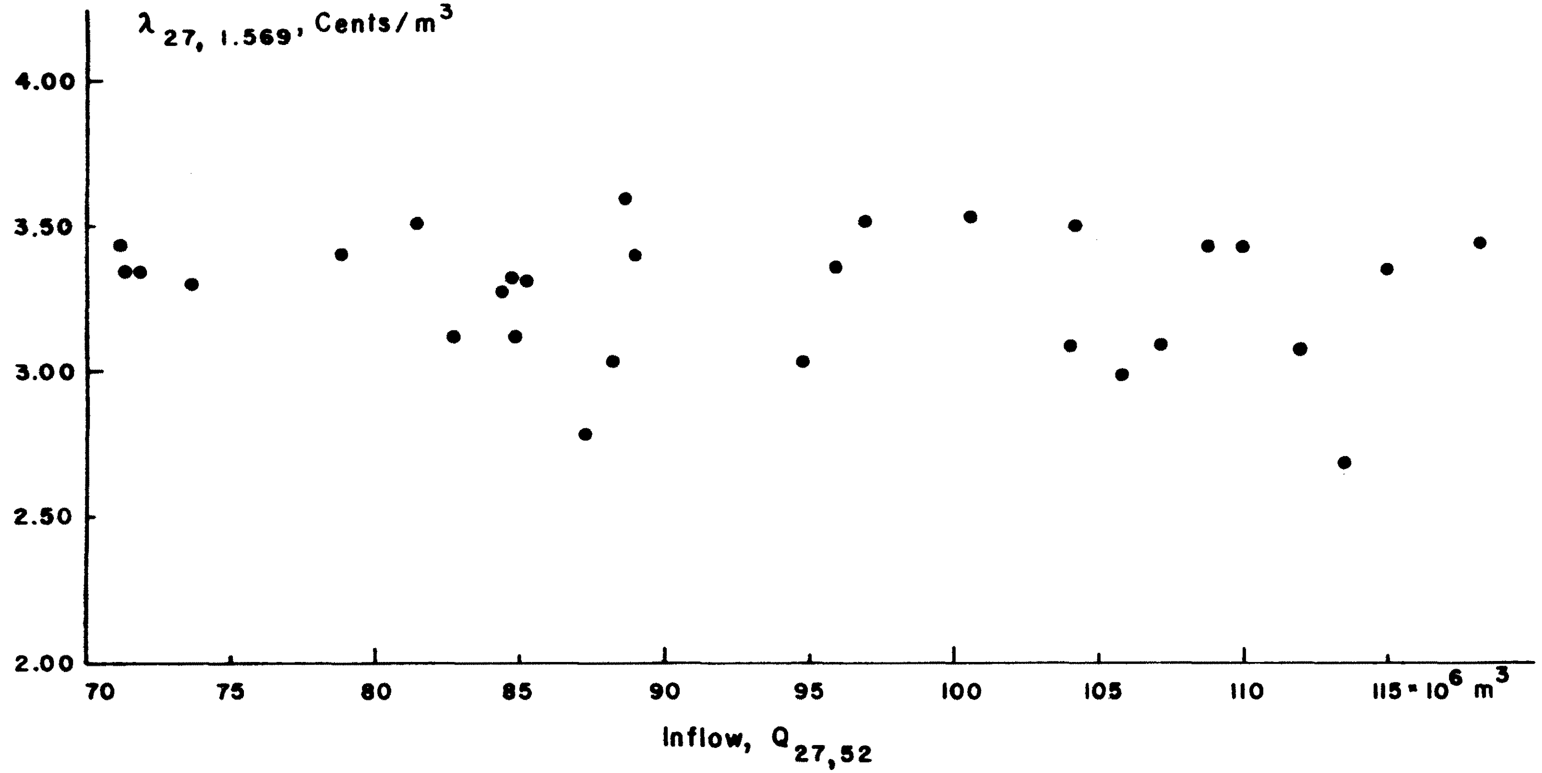


Fig. 5.21 Relation between Future Marginal Return of Storage and Summer Inflow, Initial Date, April 1; Initial Storage, 1,569 Millions M^3

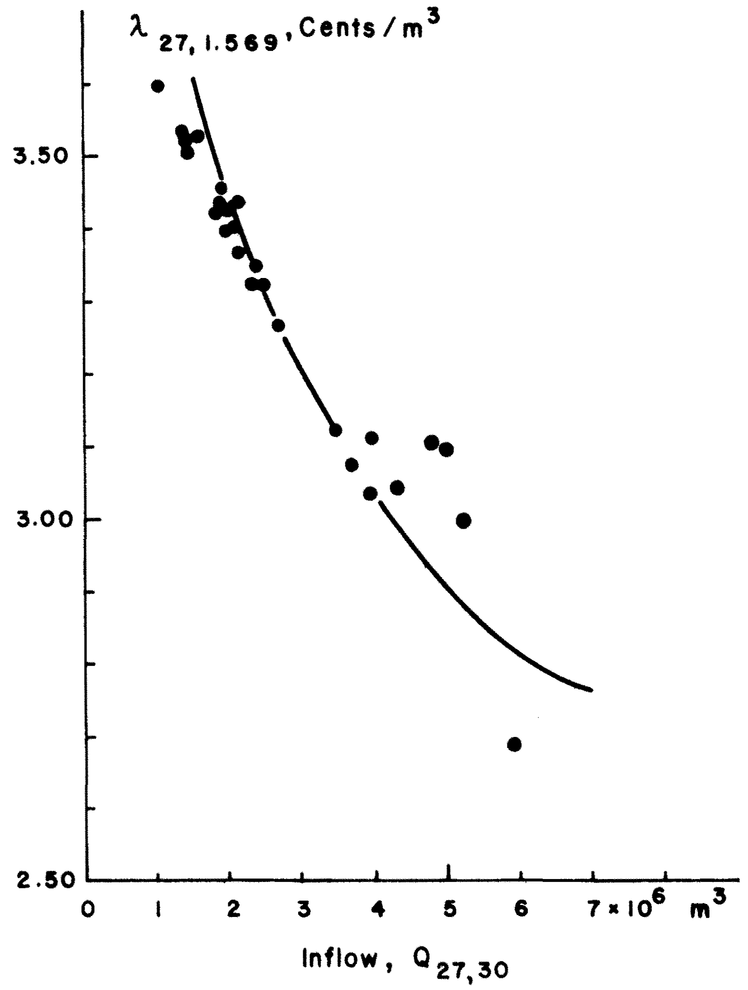


Fig.5.22 Relation between Future Marginal Return of Storage and Inflow during April, Given by Distribution Curves; Initial Date, April 1; Initial Storage, 1,569 Millions M^3

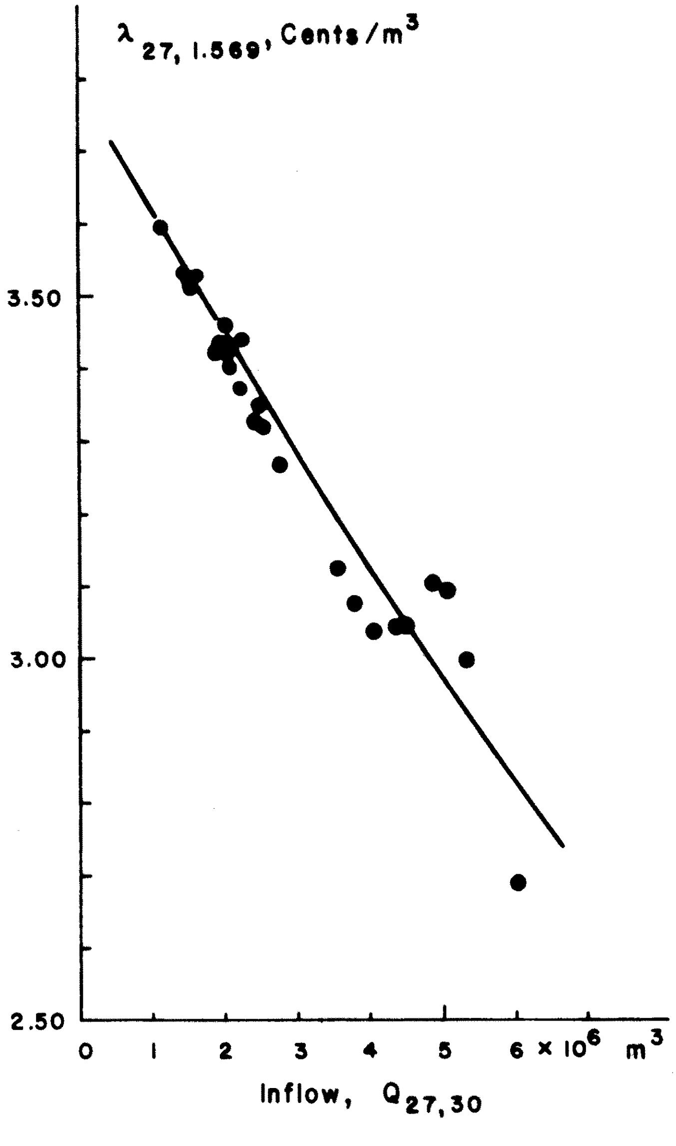


Fig. 5.23 Relation between Future Marginal Return of Storage and Inflow during April; Given by Least Squares Fit; Initial Date, April 1; Initial Storage, 1.569 Millions M^3

Table 5.11

Future Marginal Return of Storage, Values Computed for April 1

Initial Storage	7.569 $\times 10^6$ m^3 Cents/m^3	4.569 $\times 10^6$ m^3 Cents/m^3	1.569 $\times 10^6$ m^3 Cents/m^3	0.000 $\times 10^6$ m^3 Cents/m^3
1946	1.957	2.266	2.693	3.460
47	2.028	2.362	2.999	3.714
48	2.250	2.492	3.045	3.638
49	3.209	3.258	3.309	3.746
1950	2.441	2.879	3.421	3.828
51	2.445	2.880	3.459	3.977
52	2.040	2.425	3.101	3.811
53	2.549	2.640	3.036	3.427
54	2.442	2.838	3.373	3.691
55	2.868	3.026	3.404	3.862
56	2.569	2.961	3.519	3.936
57	2.560	2.794	3.320	3.723
58	2.499	2.970	3.530	3.982
59	3.248	3.299	3.353	3.659
1960	2.267	2.600	3.075	3.690
61	2.571	2.682	2.792	3.429
62	3.251	3.302	3.439	3.995
63	2.352	2.796	3.357	4.077
64	2.652	2.750	3.268	3.960
65	2.509	2.878	3.420	3.771
66	2.642	2.799	3.326	3.834
67	2.481	2.894	3.439	3.931
68	2.090	2.454	3.095	3.667
69	2.769	2.958	3.516	4.016
1970	2.540	2.946	3.526	4.045
71	2.685	2.814	3.120	3.934
72	3.241	3.293	3.346	3.482
73	2.547	3.027	3.598	3.948
1974	2.603	2.718	3.121	3.633

Table 5.12

Future Marginal Return of Storage, Main Statistics Computed for April 1

Initial Storage	7.569 x 10^6 m^3	4.569 x 10^6 m^3	1.569 x 10^6 m^3	0.000 x 10^6 m^3
Average	2.562	2.828	3.276	3.788
Stand. dev.	0.350	0.273	0.227	0.190
Coeff. Var.	0.137	0.097	0.069	0.050
Min.	1.957	2.266	2.693	3.427
Max.	3.251	3.302	3.598	4.077

All the numbers are given in Cents/m^3

Table 5.13

Future Marginal Return of Storage, Expected Values Computed for April 1, according to Different Methods

Distribution Curve	n	Storage 7.569 $10^6\ m^3$	n	Storage 4.569 $10^6\ m^3$	n	Storage 1.569 $10^6\ m^3$	n	Storage 0.000 $10^6\ m^3$
Normal	29	2.562	29	2.828	29	3.276	29	3.788
Log Normal	29	2.540	29	2.815	29	3.268	27	
Log Normal[a]	12	2.374	20	2.739	25	3.280	-	-
Log Normal[b]	17	2.664	9	2.991	4	3.191	-	-
Log Normal[c]	12	2.377						
Log Normal[d]	17	2.670						

All the numbers are given in Cents/m^3

a) For the cases where the reservoir gets empty

b) For the cases where the reservoir does not get empty

c) Computed according to the Formula $E\left[\ln\lambda^{d}_{27,7.569}\right] = a_d - b_d\ \bar{Q}'_{27,30}$

d) Computed according to the Formula $E\left[\ln\lambda^{f}_{27,7.569}\right] = a_f - b_f\ \bar{Q}'_{27,52}$

Accordingly, these data points should be plotted against the inflow recorded from the start of the operation until the date the reservoir is empty, and then a new trend appears.

As the boundaries play a secondary role, the fit provided by the linear regression approach is nearly equivalent to the one provided by the relation based on the respective distribution curves of the involved variables. Yet both relations are only valid in the central portion of the available data.

The probability that for a given initial storage, the reservoir gets empty, was also evaluated. Obviously, the higher the initial content, the lower the probability that the reservoir gets empty.

d) Full or nearly full reservoir. This situation arises for example on October 1, when the initial storage exceeds 69 million m^3. As indicated in Section 5.3.6, two types of decision may apply. Accordingly the available sample of future marginal returns of storage is divided into two groups. In the first group belong all the cases for which no storage constraints are active, in the second one, all the cases for which the reservoir is still full at the end of the first week. Fig. 5.24 shows the fitted log-normal distribution.

e) Empty reservoir. The last case to deal with concerns what happens when the reservoir is empty. If April 1 is selected as initial date, most probably the reservoir will be again empty at the beginning of the following week, as on the average, the reservoir remains empty until the end of April. Fig. 5.26 shows that the releases are independent from the corresponding inflows recorded between April 1 and the end of the water year. However if we relate the release with the corresponding weekly inflow a definite trend

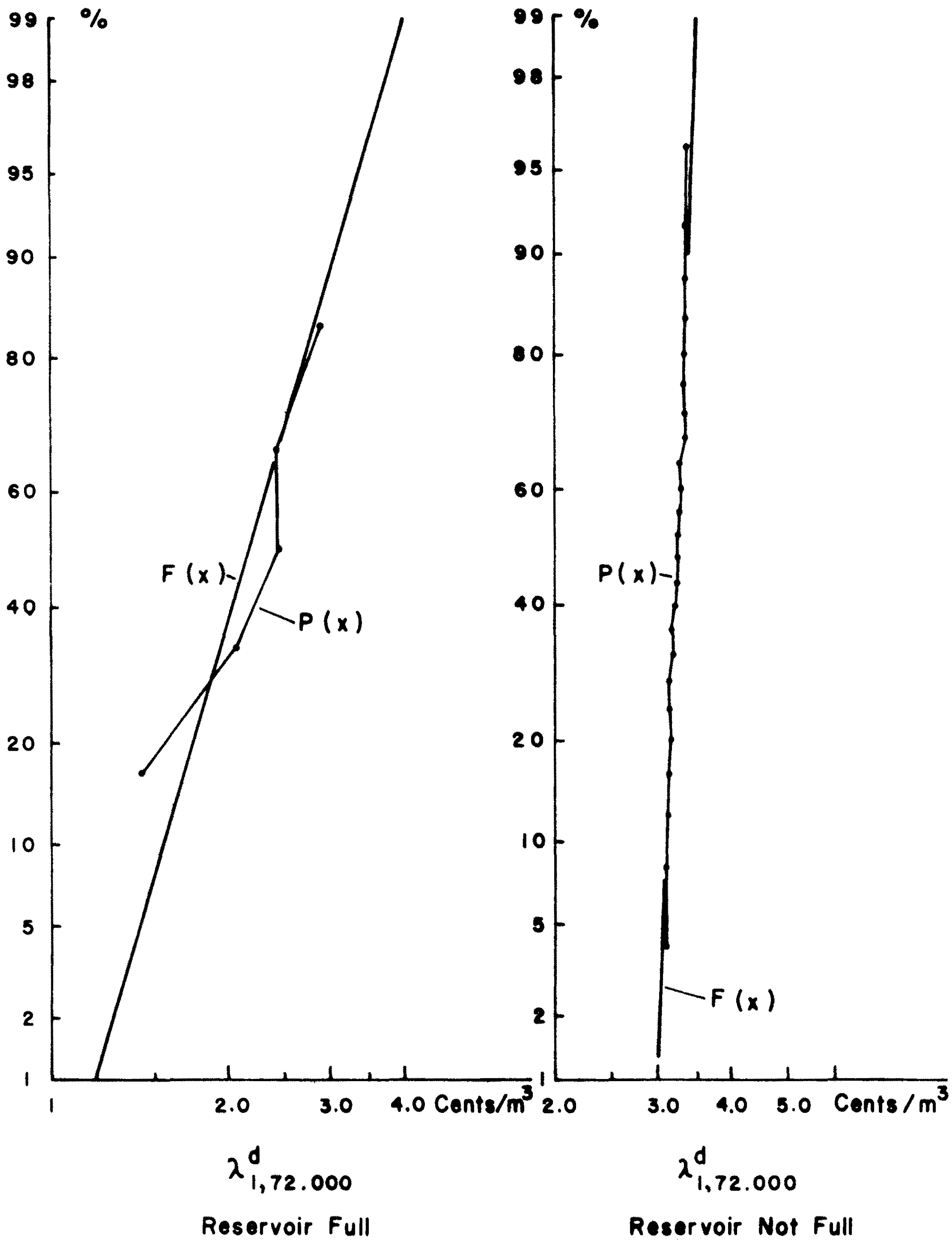
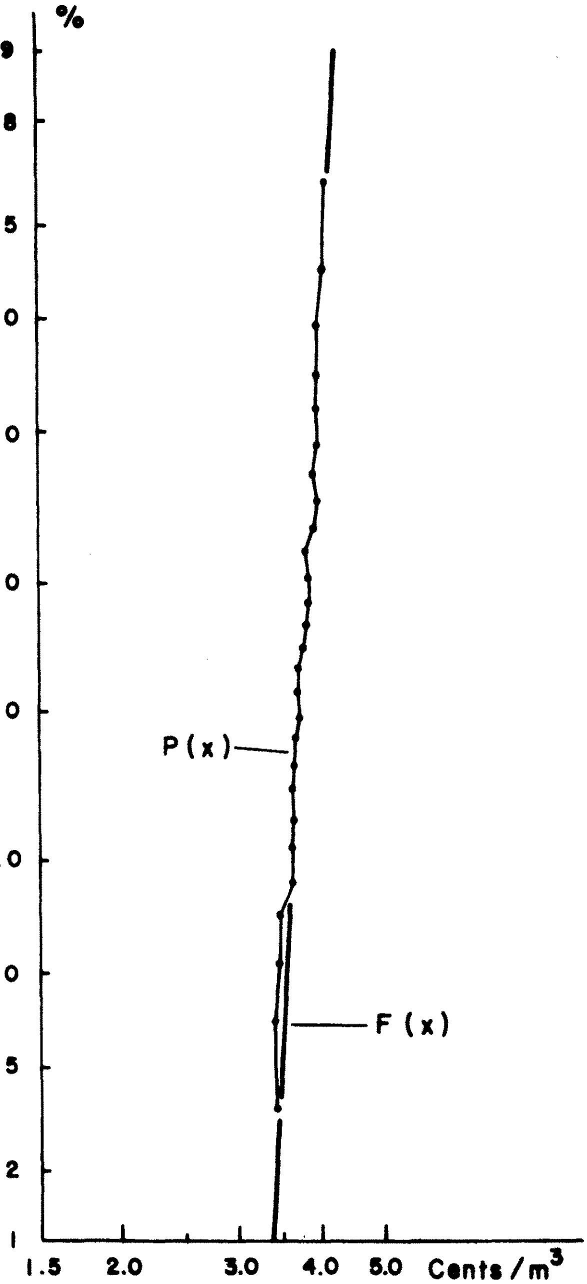


Fig. 5.24 Future Marginal Return of Storage, Distribution Curves; Initial Date, October 1, Initial Storage, 72.000 Millions M^3



$\lambda^{d}_{27,\ 0.000}$

Reservoir Empty

Fig. 5.25 Future Marginal Return of Storage, Distribution Curve; Initial Date, April 1, Initial Storage 0.000 Millions M^3

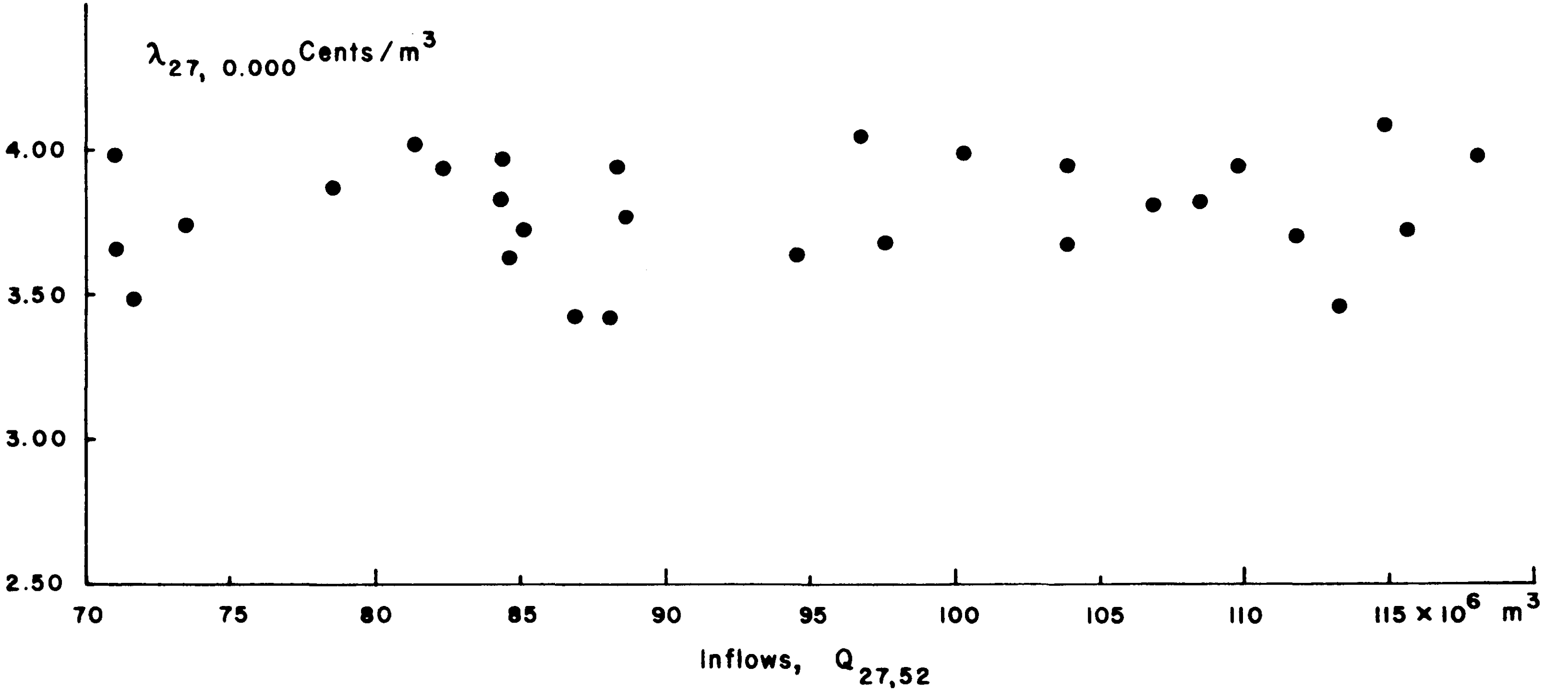


Fig. 5.26 Relation between Future Marginal Return of Storage and Summer Inflow, Initial Date, April 1; Initial Storage, 0.000 Millions M^3

appears (Fig. 5.27). The reservoir was never in the refill phase and eight times in the drawdown phase. The log-normal distribution provided a good fit to the computed future marginal returns of storage (Fig. 5.25).

5.3.9 Summary and outlook. Reservoir operation under uncertain future requires the determination of the expected future marginal return of storage. The computations performed to this end showed tha* the factors which have an influence on this variable change during the year. The water year can be roughly divided into three main periods. From October to the end of February, the drawdown phase prevails, while from May to September, the refill phase prevails. In May and April, the reservoir is in mixed strategy situation as either the drawdown, or the refill phase may apply. Finally special cases arise in September or October when the reservoir is nearly full, and in April, when the reservoir is empty.

Two linear expressions relating inflow during the relevant phase and natural logarithm of the associated future marginal return of storage could be established. The first relation holds for the drawdown phase, the second one, for the refill phase. Furthermore, for these two phases it is possible with the help of these relations to estimate directly the expected future marginal return of storage. The calculation of this expected value requires only the knowledge of the optimum reservoir content curve corresponding to the sequence of average inflows. This property allows to compute the expected future marginal return of storage much faster than what was anticipated at the beginning of this section. Unfortunately the situation is not simple when mixed strategy situation prevails, or when the reservoir is nearly full, full or empty.

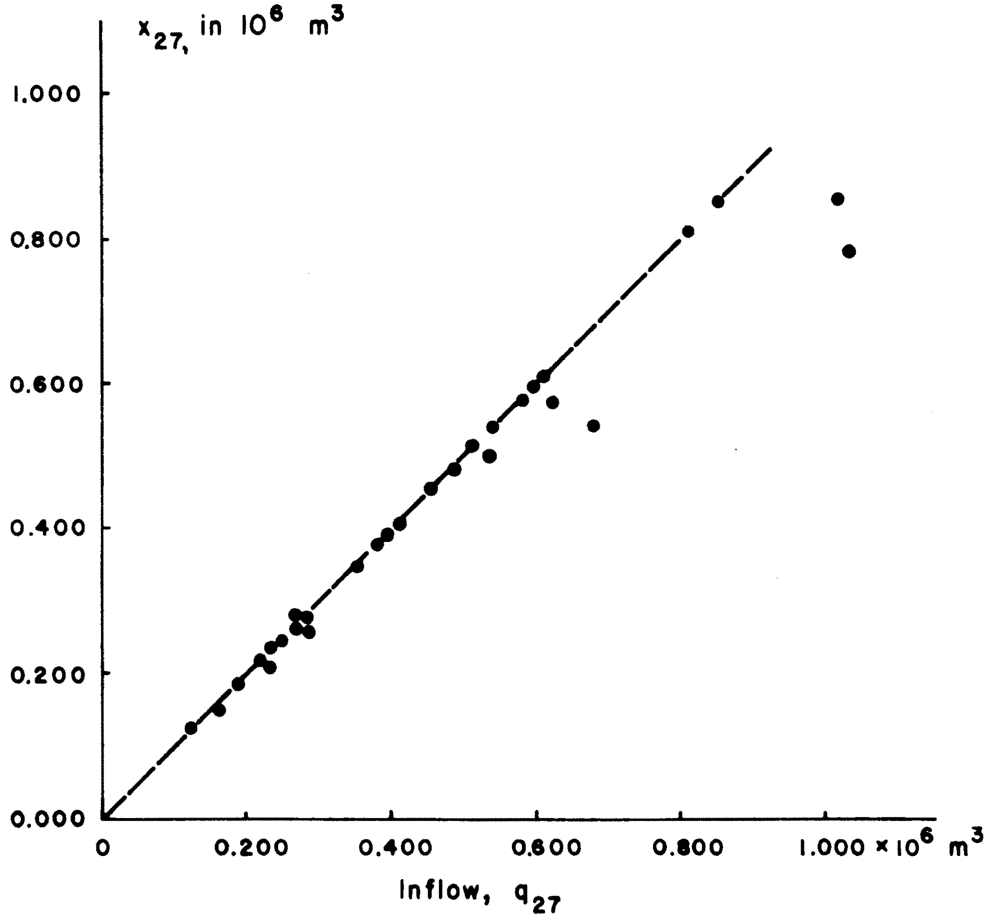


Fig. 5.27 Relation between Weekly Inflow and Weekly Release, Initial Date, April 1; Initial Storage, 0.000 Millions M^3

The final aim of this reserach is to introduce runoff forecast into the decision process. Yet before forecasting the magnitude of a variable, one must assess which variables are relevant for the determination of a release. The just performed analyses supply important information on this topic.

In winter, for a given date and initial storage, the inflow from that date until the end of April is relevant. However computations showed that the spread of the future marginal returns of storage corresponding to different inflows sequences is very small, so that for practical purposes, one can consider that the optimal release strategy depends only on the initial date and on the initial storage.

In late spring and summer, the inflow from the date under consideration until the end of the water year are relevant whereas the distribution of these inflows within this period plays a secondary role. Also a reliable relation exists between this variable and the corresponding future marginal return of storage.

The most complicated situation arises in March or April when anyone of the three following variables can be important: inflow from the date the reservoir operation is started until the date the reservoir is empty, or inflow recorded from the same initial date until the end of the water year, or inflow recorded during the week following the decision day.

Finally the inflow during the week following the decision day can also become relevant in September or October when the reservoir is nearly full.

Hence, depending on the date considered, different types of forecast must be done. The steps involved in the stochastic optimization with flow forecast are described in the next section.

5.4 Stochastic Reservoir Operation with Flow Forecast

5.4.1 Principles of hydrologic forecasting. As already mentioned, the parameters relevant for optimal reservoir operation change within the year. In late fall and winter, the inflow $Q_{k,\ell}$ is determinant; in early spring, either $Q_{k,\ell}$, or $Q_{k,52}$, or q_k; in late spring, $Q_{k,52}$, and in early fall q_k, where k stands for the initial date of reservoir operation, and ℓ, for the week on which the reservoir gets empty.

To obtain the most efficient strategy, these parameters should be known at the date the release strategy is selected. The complexity of the runoff process makes impossible the supply of such an information. The hydrologic models developed in Chapter 2 allow however the determination of the approximate value of the relevant parameters, along with an estimation of the reliability of the supplied information.

The established linear runoff forecast models relate either weekly and monthly inflow with the inflow of the two preceding weeks, or monthly inflow with the inflow of the two preceding months, or quarterly and semi-annual inflow with meteorological variables (see Table 2.21). As these forecast equations contain variables which are known at that date, they can be rewritten in the following way:

$$Y = \bar{Y} + \varepsilon_t \tag{5-22}$$

where Y stands for the hydrologic variable to be forecasted. The influence of the variables known at the date of the forecast appear in $\bar{Y}$, whereas the influence of the other terms appear in ε_t. Hence the forecast equation is made of a so-called deterministic component $\bar{Y}$, and of a stochastic component, ε_t. The stochastic component

ε_t follows in all the cases considered here a normal distribution with mean equal to zero and well specified standard deviation σ_t:

$$\varepsilon \sim N(0,\sigma_t) \tag{5-23}$$

Actually the standard deviation σ_t expresses the reliability of the forecast. The greater its value, the smaller the reliability of the forecast, and vice-versa. In any case, it should be smaller than that of the recorded data sample of the variable to be predicted. Hence the forecasted variable still remains a random variable following the same type of statistical distribution as the original variable, only its expected value and standard deviation are different. And this remark is the key to the solution of the present problem.

5.4.2 Outline of the methodology. Twenty nine years of records exist at the selected gauging station. In fact one can consider these records as a sample belonging to a special forecast, namely to the one where forecasted mean and variance are equal to those of the recorded data. Hence the methodology derived in Section four still applies; only the appropriate statistical parameters must be introduced into the relevant equations. Here, mean and variance are given by the forecast equation, in Section four, they came from the available records. But the major difference results from the fact that no inflow data sample corresponding to the forecasted mean and variance anymore exists. And the following question arises: is the available information sufficient to derive the optimal release strategy? Yes the existing information is sufficient, except when the reservoir is in mixed strategy situation. In these cases additional information is needed and it is obtained by making extensive use of simulation.

The purpose of simulation is, among others, to establish experimentally some properties of a phenomenon which cannot be derived analytically because of the complexity of the problem. In the present situation, it consists of generating artificial samples of the forecasted hydrologic variable. Once the synthetic data have been generated, the problem is identical to that studied in Section three. That is the future marginal return corresponding to each of the generated inflow data can be computed, the appropriate distribution curve, fitted, the expected value of future marginal return, evaluated, and the corresponding release determined. The computation steps appear in details in the forthcoming subsections.

5.4.3 Drawdown phase. The inflow during the period extending from the date the forecast is done until the date the reservoir gets empty becomes the relevant parameter during the drawdown phase. However, as during the greatest part of the drawdown phase the magnitude of the inflow is small as compared to the prevailing storage, the inflow plays a secondary role and release depends practically solely on the initial storage. Only in early spring, when the reservoir content is low does the inflow play an important role. This corresponds in fact to the period when the reservoir is in mixed strategy situation. The forthcoming derivations apply to this case.

The equation to forecast the relevant hydrologic parameter reads as:

$$Y = a_o + b_1X_1 + b_2X_2 \tag{5-24}$$

where Y stands for $Q_{k,\ell}$, X_1, for the inflow during the week (k-1), and X_2, for the inflow during the week (k-2). Associated with this equation is ε_t, the residual, which is normally distributed, with mean zero and standard deviation σ_t. Equation 5-24 supplies the expected value, and σ_t, the standard deviation of the forecasted $Q_{k,\ell}$, so that:

$$Y = \bar{Y} + \varepsilon_t$$

with (5-25)

$$\bar{Y} = a_o + b_1X_1 + b_2X_2$$

The required sample of $Q_{k,\ell}$ is obtained with the help of a random number generator or from a table of random normally distributed numbers (Rand Corporation, 1955). Then the relation established in Section 5.3 (Fig. 5.18, Fig. 5.22) supply the future marginal return of storage corresponding to each of the generated inflow data $Q_{k,\ell}$. As no boundaries are relevant here, the future expected marginal return of storage is easily computed.

5.4.4 Refill phase. The total inflow recorded during the period extending from the initial date to the end of the water year plays the major role here. Depending on the length of the forecast period, different types of runoff models were used. For periods exceeding two months, the following type of relation was retained:

$$Y = a_o + b_1X_1 \tag{5-26}$$

where Y stands for $Q_{k,52}$ or the inflow to be forecasted, X_1, for the amount of water stored in the watershed at the date of the forecast, and a_o, the constant term.

If the length of the forecast period is smaller than two months, the Markov property of the runoff is used, and the typical equation reads as

$$Y = a_o + b_1X_1 + b_2X_2 \tag{5-27}$$

where Y stands for $Q_{k,52}$, for example one month, and X_1 and X_2, for the monthly flow recorded during the two months preceding the date of the forecast.

However, both equations reduce to

$$Y = \bar{Y} + \varepsilon_t \tag{5-28}$$

Here again $\bar{Y}$ represents the influence of the deterministic component of the model and is equivalent to the expected value of the forecasted variable, whereas ε_t is a normally distributed variable with zero mean and standard deviation σ_t, which corresponds to the standard deviation of the predicted variable. A sample of inflow data $Q_{k,52}$ with the required statistical properties is easily generated on the basis of the available information.

The subsequent steps are identical to those of the drawdown phase namely: determination of the future marginal return of storage corresponding to each generated value of $Q_{k,52}$, and computations of the expected value. Note that the upper and lower boundaries of the future marginal return of storage depend only on the initial content, which simplifies greatly the fitting of the four parameter log-normal distribution.

However a shortcut exists, when the reservoir is in the refill phase. In these situations, the relation drawn on Figs. 5.4, 5.7 and 5.10 supply directly the marginal return looked after and it is no longer necessary to generate synthetic values for the inflow $Q_{k,52}$. It suffices to look on these graphs for the future marginal return of storage corresponding to the expected value of the forecasted inflow. The so obtained number is the value we are looking for.

In mixed strategy situation though, simulation is necessary. The fact that the future marginal return of storage is log-normally distributed introduces some simplifications into the computations.

5.4.5 Mixed strategy situation. As indicated earlier, two different operation rules may apply at the beginning of spring, depending on whether or not the reservoir gets empty during the water year. Hence two samples of inflow data are generated concurrently, one representing the inflow from the initial date until the end of the water year, and the other one, the inflow from the initial date until the date on which the reservoir is empty. The two future marginal returns on storage corresponding to each pair of generated inflow data are computed and the highest obtained value is retained. This operation is repeated for the whole sample of generated data. Finally the weighted expected value is evaluated. The details of the just described computations appear in subsections 5.4.3 and 5.4.4.

5.4.6 Full or nearly full reservoir. At the beginning of fall the inflow during the week following the initial date k becomes determinant, if it exceeds a specified threshold. Here again, the easiest way consists in generating weekly inflow data. The relevant forecast equation is:

$$Y = a_o + b_1X_1 + b_2X_2 \tag{5-29}$$

with Y standing for the natural logarithm of the inflow during the week k, X_1, for the natural logarithm of the inflow during the week $(k-1)$ and X_2, for the natural logarithm of the inflow during the week $(k-2)$.

This equation must now be reduced to its standard form:

$$Y = \bar{Y} + \varepsilon_t \tag{5-30}$$

which allows the generation of the required weekly inflow data sample.

From this point on, the methodology follows the steps outlined in subsection 5.3.6. The reservoir is in a mixed strategy situation. If the generated inflow is smaller than the threshold, the release is a constant, independent from the weekly inflow, if it is greater, the release is such that the reservoir just becomes full at the end of the following week. The future marginal return of storage results directly from these considerations and finally the weighted expected value is evaluated.

5.4.7 Empty reservoir. In April, when the reservoir is empty, three situations can occur: either the reservoir stays in the drawdown phase, or in the refill phase, or it stays empty one more week. Accordingly different parameters may become relevant: Q_k, $Q_{k,52}$ or q_k. It is again a mixed strategy situation and three samples of inflow must be generated. The relevant forecast equations have already been described in a section of Chapter 2 and in subsection 5.4.3, 5.4.4 and 5.4.6. Once the inflow data have been generated, the future marginal returns of storage are determined by a group of three and the highest value is retained. This operation is repeated for the whole sample and finally the weighted expected value is evaluated.

5.4.8 Applications. Some typical cases will be worked through in this subsection to show the steps involved in the determination of the optimal releases. The water year 1947-48 was selected to perform these applications. It represents average hydrologic conditions.

The chosen initial date and initial storage correspond to those of the examples of subsection 5.3.8.

a) Drawdown phase. The computations carried through in the preceding section indicated that the optimal releases could be considered as independent from the inflows sequence. Hence it is not worthwhile to establish runoff forecasts for this phase. An exception to this statement constitutes what happens in April, when the reservoir is nearly empty. This case will be analyzed later.

b) Refill phase. This case is well characterized by the situation existing when reservoir operation is started on July 1. The relevant parameter is the inflow recorded during the months of July, August and September. According to Table 2.21, the relevant forecast equation reads as:

$$Q_{July,Sept} = 902.21 + 1.028(P_{Oct,June} - 0.667\ Q_{Oct,June}) + \varepsilon_t \qquad (5\text{-}31)$$

As $P_{Oct,June}$ amounts to 679 mm, and $Q_{Oct,June}$, to 923 mm, one obtains:

$$Q_{July,Sept} = 1025.454 + \varepsilon_t \text{ in mm} \qquad (5\text{-}32)$$

with

$$\varepsilon_t \sim N(0,\ 192.865) \qquad (5\text{-}33)$$

This equation leads to an expected inflow of 55.067 million m^3 which is not too different from 54.077 million m^3, the sample average. This fact results directly from the low correlation coefficient existing between the dependent and the independent variables. Table 5.14 shows the generated sample of inflows $Q_{40,52}$.

Depending on the initial storage, Fig. 5.4, 5.7 or 5.10 is used to compute the future marginal returns of storage. Fig. 5.28, 5.29

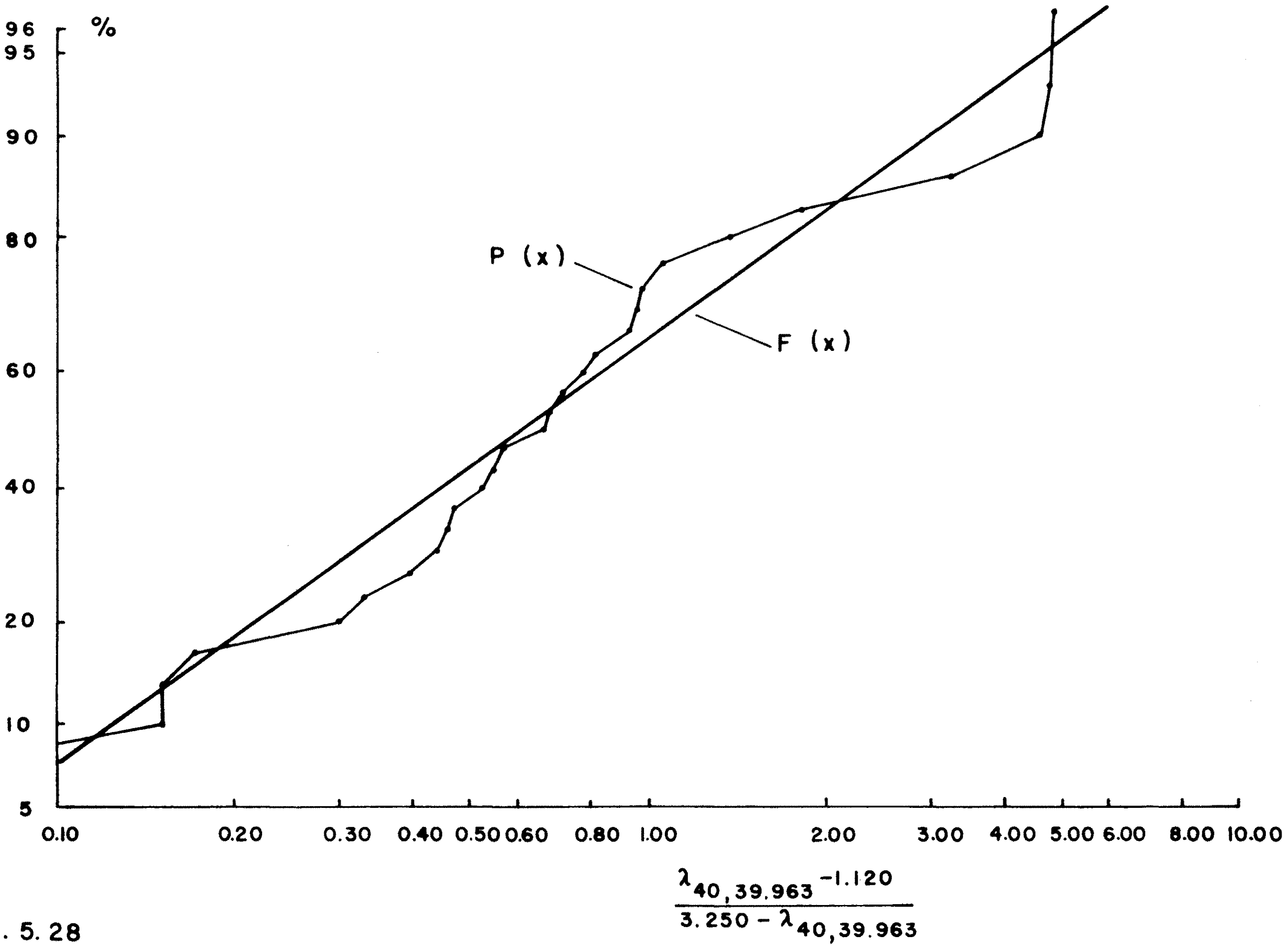
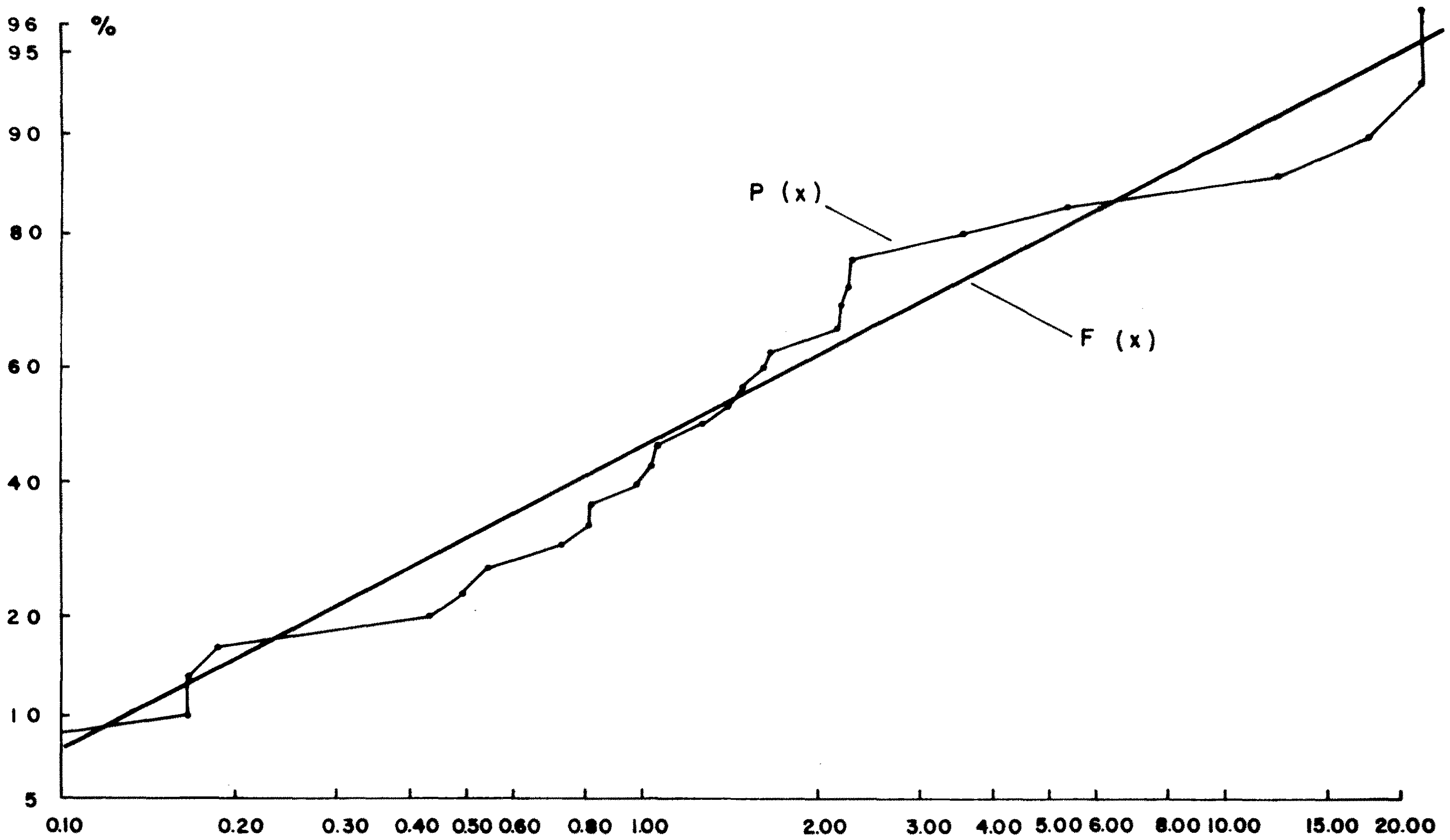


Fig. 5. 28

Future Marginal Return of Storage, Distribution Curve: 1948, Initial Date, July 1; Initial Storage, 39.963 Millions M^3



$$\frac{\lambda_{40,\,31.963} - 1.360}{3.400 - \lambda_{40,\,31.963}}$$

Fig. 5.29

Future Marginal Return of Storage: Distribution Curve: 1948, Initial Date, July 1, Initial Storage, 31.963 Millions M^3

and 5.30 display the fitted distribution curves, and Table 5.14 and 5.18 the obtained expected future marginal return of storage. Note that in this case, simulation is not necessary and that the same result could have been obtained by determining the future marginal return of storage corresponding to the expected value of the forecasted inflow $Q_{40,52}$.

c) Mixed strategy situation. Reservoir operation when started on April 1 illustrates well this case. Two parameters can be determinant: either $Q_{27,52}$, the inflow from April to September, or $Q_{27,30}$, the inflow during April. The following forecast equation applies in the first case (see Table 2.21):

$$Q_{April,Sept} = 1434.13 + 1.224\ (P_{Oct,March} - 0.667\ Q_{Oct,March}) + \varepsilon_t \tag{5-33}$$

As $P_{Oct,March}$ amounts to 340 mm and $Q_{Oct,March}$ to 182 mm, we obtain

$$Q_{April,Sept} = 1762.557 + \varepsilon_t \text{ in mm}$$

with (5-34)

$$\varepsilon_t \sim N(0,\ 230.076)$$

In the second case, the forecast equation reads as:

$$Q_{April} = 1.497 - 6.608\ q_{26} + 10.107\ q_{25} \tag{5-35}$$

knowing that q_{26} amounted to 0.532 million m^3 and q_{25}, to 0.644 million m^3 one obtains:

$$Q_{April} = 4.490 + \varepsilon_t \quad (\text{in } 10^6\ m^3)$$

with (5-36)

$$\varepsilon_t \sim N(0,\ 1.147)$$

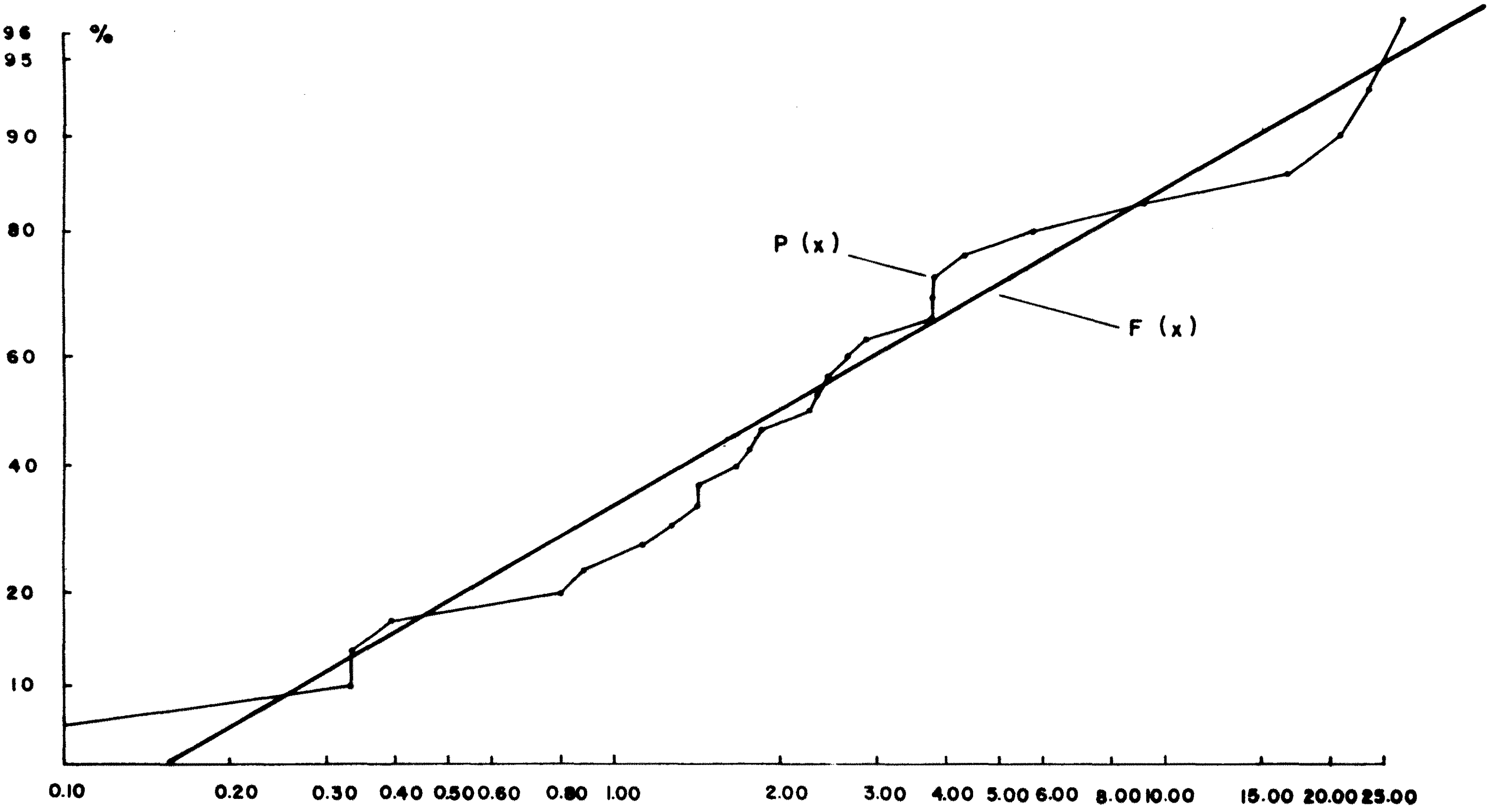


Fig. 5.30

Future Marginal Return of Storage; Distribution Curve: Year 1948, Initial Date, July 1; Initial Storage, 23 963 Millions M^3

Table 5.14

Future Marginal Return of Storage, Values Computed for 1948, July 1

	$Q_{July,Sept}$ x 10^6 m^3	$\lambda^f_{40,39.963}$	$\lambda^f_{40,31.963}$ Cents/m^3	$\lambda^f_{40,23.963}$
1	59.677	1.65	2.03	2.52
2	37.089	2.88	3.31	3.48
3	37.508	2.87	3.29	3.47
4	65.300	1.43	1.68	2.15
5	45.066	2.49	3.08	3.37
6	58.134	1.72	2.08	2.64
7	53.008	2.01	2.58	2.99
8	56.827	1.79	2.27	2.75
9	57.384	1.77	2.22	2.70
10	60.296	1.61	1.98	2.47
11	52.337	2.05	2.62	3.03
12	36.723	2.89	3.31	3.49
13	55.702	1.85	2.37	2.83
14	40.559	2.75	3.25	3.45
15	52.014	2.07	2.63	3.05
16	50.471	2.15	2.76	3.15
17	81.080	1.17	1.39	1.67
18	66.155	1.40	1.65	2.09
19	53.758	1.97	2.50	2.96
20	47.289	2.35	2.95	3.27
21	56.696	1.80	2.27	2.75
22	53.278	1.98	2.55	2.97
23	76.416	1.22	1.43	1.73
24	50.384	2.16	2.77	3.15
25	49.486	2.22	2.83	3.19
26	66.163	1.40	1.65	2.09
27	55.414	1.87	2.39	2.85
28	50.436	2.17	2.76	3.15
29	55.074	1.89	2.40	2.87

Table 5-15 displays the generated inflow samples and the corresponding future marginal return of storage. Depending on the intiial storage, Figure 5.14, 5.18 or 5.22 was used to compute the future marginal return of storage if the reservoir was in the draw-down phase, and Fig. 5.13, 5.17, or 5.20 if the reservoir was in the refill phase. The results are summed up in Table 5.18.

d) Full or nearly full reservoir. October 1 is here the selected initial date while the initial storage equals 72 million m^3. The relevant parameter is q_1, the inflow during the week following the start of the reservoir operation. The following prediction equation applies (see Table 2.21):

$$\ell nq_1 = -0.445 + 0.570\ \ell nq_{52} + 0.532\ \ell nq_{51} + \varepsilon_t \tag{5-37}$$

As for q_{52} and q_{51}, an inflow of 2.386 and 2.196 million m^3 respectively was recorded, the prediction equation reduces to

$$\ell nq_1 = 0.469 + \varepsilon_t \qquad (\text{in } 10^6\ m^3)$$

with (5-38)

$$\varepsilon_t \sim N(0,0460)$$

Table 5.17 displays the generated samples of inflows and the computed future marginal return of storage. As usually the reservoir stays empty until the end of April, and due to the statistical properties of the generated hydrologic variables, the refill phase was never started here. Table 5.18 summarizes the computations results.

Table 5.15

Future Marginal Return of Storage Value Computed for April 1

	$Q_{Apr,Sept}$ x 10^6 m^3	$Q_{Apr.}$ x 10^6 m^3	$\lambda^f_{27,7.569}$ Cents/m^3	$\lambda^d_{27,7.569}$	$\lambda^f_{27,4.569}$	$\lambda^d_{27,4.569}$
1	92.648	3.149	2.37	-	-	2.62
2	74.659	5.400	3.05	-	3.15	-
3	99.320	4.934	2.16	-	-	2.43
4	93.945	3.977	2.33	-	-	2.52
5	111.403	4.836	-	2.09	-	2.44
6	83.307	5.706	2.71	-	2.81	-
7	94.798	2.909	2.31	-	-	2.65
8	83.394	4.150	2.71	-	2.81	-
9	109.933	4.681	-	2.10	-	2.45
10	77.550	2.895	2.93	-	3.03	-
11	82.801	5.971	2.73	-	2.83	-
12	103.681	4.652	-	2.10	-	2.45
13	103.508	1.964	-	2.40	-	2.84
14	74.473	4.136	3.07	-	3.16	-
15	108.413	3.931	-	2.15	-	2.53
16	108.870	3.935	-	2.15	-	2.53
17	70.693	3.977	3.23	-	3.35	-
18	99.406	3.240	-	2.23	-	2.61
19	81.269	3.912	2.78	-	2.88	-
20	90.782	5.386	2.44	-	2.52	-
21	102.136	4.757	-	2.08	-	2.44
22	96.144	6.477	2.27	-	2.35	-
23	106.028	4.312	-	2.13	-	2.50
24	76.450	5.199	2.97	-	3.08	-
25	84.135	3.425	2.67	-	2.77	-
26	97.244	3.112	2.23	-	-	2.63
27	110.291	4.181	-	2.14	-	2.50
28	87.557	4.067	2.58	-	2.65	-
29	87.632	6.655	2.58	-	2.65	-

Table 5.16

Future Marginal Return of Storage, Values Computed for 1948, April 1, Empty Reservoir

	$Q_{April,Sept}$ x 10^6 m^3	Q_{April} x 10^6 m^3	q_{27} x 10^6 m^3	$\lambda^d_{27,0.000}$ Cents/m^3	$\lambda^{d\,*}_{27,0.000}$ Cents/m^3
1	92.648	3.149	0.818	3.485	-
2	74.659	5.400	0.684	-	3.570
3	99.320	4.934	0.634	-	3.612
4	93.945	3.977	0.810	-	3.464
5	111.403	4.836	0.803	-	3.470
6	83.307	5.706	0.521	-	3.711
7	94.798	2.909	1.055	3.535	-
8	83.394	4.150	2.249	3.284	-
9	109.933	4.681	.954	-	3.347
10	77.550	2.895	1.027	3.538	-
11	82.081	5.971	1.076	-	3.252
12	103.681	4.652	0.844	-	3.436
13	103.508	1.964	0.777	3.740	-
14	74.473	4.136	1.074	3.286	-
15	108.413	3.931	0.630	-	3.616
16	108.870	3.935	0.753	-	3.511
17	70.693	3.977	1.343	3.318	-
18	99.406	3.240	0.721	-	3.538
19	81.269	3.912	1.218	3.331	-
20	90.782	5.386	1.159	-	3.188
21	102.136	4.757	0.964	-	3.339
22	96.144	6.477	0.981	-	3.326
23	106.028	4.312	1.266	3.252	-
24	76.450	5.199	0.693	-	3.562
25	84.135	3.425	1.012	3.429	-
26	97.244	3.112	1.109	3.493	-
27	110.291	4.181	0.633	-	3.613
28	87.557	4.067	1.112	3.300	-
29	87.632	6.655	0.736	-	3.526

*The values of this column correspond to the case where q_{27} is determinant.

Table 5.17

Future Marginal Return of Storage
Values Computed for 1948, October 1

	q_1 x 10^6 m^3	$\lambda^d_{1,72.000}$	Cents/m^3 $\lambda^d_{1,72.000}$
1	1.932	3.237	
2	1.638	3.237	
3	2.933	-	2.999
4	2.427	3.237	
5	1.038	3.237	
6	2.314	"	
7	2.587	"	
8	0.866	"	
9	1.206	3.237	
10	2.800	-	3.098
11	1.313	3.237	
12	0.911	"	
13	1.058	"	
14	1.595	3.237	
15	2.824	-	3.080
16	0.770	3.237	
17	1.531	"	
18	0.925	"	
19	2.446	"	
20	1.791	"	
21	0.765	"	
22	0.985	"	
23	2.091	"	
24	1.336	3.237	
25	2.830	-	3.076
26	1.734	3.237	
27	1.706	3.237	
28	4.045	-	2.303
29	1.625	3.237	

Table 5.18

Comparison of the Future Marginal Return of Storage Computed according to Different Methods, Year 1948

Initial Date	Initial Content $\times 10^6$ m^3	Marginal Value of Storage (Cents/m^3): Deterministic Case	Stochastic without Forecast	Stochastic with Forecast
Oct. 1	72.000	3.250	3.025	3.181
April 1	7.569	2.250	2.540	2.460
"	4.569	2.492	2.815	2.689
April 1	0.000	3.628	3.268	3.446
July 1	39.963	1.828	1.951	1.921
"	31.963	2.196	2.498	2.452
July 1	23.963	2.906	2.959	2.900

5.4.9 Summary. The methodology developed in the course of this chapter is so flexible that it allowed without major problems the introduction of the runoff forecasts into the optimization scheme. Hence the outline of this section follows exactly that of the preceding section.

Only the dimension of the problem increased by one, so that the problem became three-dimensional: date, storage and forecasted inflow. As a consequence, computational difficulties appeared which required the introduction of the simulation technique. This technique had to be applied whenever the reservoir is in mixed strategy situation. However, for all the other cases, the introduction of the runoff forecast presented no difficulties, the more so because the relations established in Section 5.3 between relevant hydrologic parameters and future marginal return of storage could be used directly.

The last question to answer concerns the performance of this approach. This question cannot be answered definitely here. Many more computation would be required and the outcome of these analyses depends also on the hydrology of the watershed under consideration. We shall now just mention some trends which appeared while the reservoir was operated with the runoff of the selected watershed. Runoff forecasts led to an increase in the efficiency of reservoir operation mainly in mixed strategy situations. This fact resulted from the good reliability of the related runoff forecasts. However the gains due to runoff forecasts were smaller in the refill phase, as the reliability of the hydrologic forecast model is quite low. Finally runoff forecasts bring no substantial gains in the drawdown phase.

5.5 Final Remarks

The method developed to solve the problem of deterministic reservoir operation allowed to derive an efficient and reliable procedure to operate the reservoir under uncertainty. This procedure makes full use of the basic properties of the optimal trajectory. For the determination of the optimal releases, the water year was divided into different periods, for which relations were established between optimal releases and relevant hydrologic parameters. The great advantage of the derived relations is that they are reliable for the complete range of the relevant hydrologic variables. Furthermore they permit the introduction of the runoff forecasts without major difficulties.

Different factors made this positive result possible. Some of them are common to nearly all the reservoir operation problems, some result from the particular nature of the problem studied here. Common to all the reservoir operation problems are the fact that the reservoir stays successively in the drawdown and in the refill phase, and that in these periods, different parameters are relevant. On the other hand, however, the hydrology, the energy rate function and the price of the produced energy are specific to the problem under study. Despite of the existence of these special features, we claim that the methodology is quite general. The complete evaluation of the generality of the derived methodology appears in the next chapter.

Chapter 6

SUMMARY, RECOMMENDATIONS AND CONCLUSIONS

The results of the computations presented in the preceding chapters show that the aims of the present research have been fulfilled. A thorough analysis of the hydrologic characteristics of the reservoir inflows and of the return function, and the study of the reservoir operation mechanisms led to the development of an original solution technique.

The solution technique for deterministic reservoir operation results directly from the hydrologic, technical and economical characteristics of the system under consideration. As the date when the reservoir gets empty the first time does not change much from one sequence of weekly inflows to another, the annual reservoir operation can be broken down into two parts. The first part, the drawdown phase, covers the period preceding the date of first emptiness of the reservoir; the second part, the refill phase, covers the period following the date of first emptiness of the reservoir. To further reduce the amount of computations, the physical constraints of the system are first ignored. The optimal sequence of releases is then determined separately for both phases by solving the system of equations given by the Kuhn Tucker conditions. In a second step, the physical constraints are reintroduced into the computations and the release strategy is modified if necessary. Finally both phases are linked together and, if required, the release strategy is again modified to fit this new situation. However, because the date of first emptiness of the reservoir is nearly independent from the magnitude of the inflow, and because the physical constraints of the system affect the nature of the optimal release strategy only during a few weeks of the year, the solution obtained in the first stage of the computations is very close to the optimal one. Hence most often only a few iterations are necessary to arrive at the final optimal solution.

For stochastic reservoir operation without flow forecast the notion of future marginal return of the storage is introduced. A release during a given week is optimal if its instantaneous marginal return is equal to the expected future marginal return of the storage. The analyses of the cases studied in the deterministic situation revealed that depending on the date and reservoir content considered, the future marginal return of the storage is related to a different but well specified parameter. Hence during the drawdown phase, the relevant parameter is the inflow recorded between the selected date and the date of first emptiness of the reservoir; during the refill phase, it is the inflow recorded between the selected date and the end of the water year. Furthermore, the natural logarithms of the future marginal return of the storage follow a normal distribution in the drawdown phase, and a four parameter normal distribution in the refill phase. The determination of the expected values looked after presents no special difficulties. A complication arises however in April and May, when the reservoir is nearly empty. In these cases, the reservoir can either be in the drawdown or in the refill phase. One says then that the reservoir is in a mixed strategy situation. The natural logarithms of the future marginal return of storage are distributed according to two different normal distributions, depending on whether they belong to the drawdown, or to the refill phase. The same kind of situation prevails in September or October, when the reservoir is nearly full.

The introduction of the runoff forecast into the decision process involved no major difficulties. For the refill phase, the existing relation between future marginal return of storage and relevant inflow variable supplies directly for any forecast magnitude of inflow, the corresponding future marginal return of the storage. For the drawdown phase, the same type of approach applies. However, as the inflow plays only a minor role as compared to that of the initial storage, the runoff forecast brings only small gains. In a mixed strategy situation, it is necessary to generate an artificial sample for each of the two relevant inflow variables, to determine the corresponding future marginal return of the storage, to select by pairs the highest obtained value, and then to compute the expected value of the so obtained sample of future marginal returns of the storage. The same procedure must be followed when the reservoir is nearly full, that is in September and October.

The advantages of the derived method are as follows. By taking into account the hydrologic, technical and economical aspects of the problem, it was possible to simplify drastically the system of equations given by the Kuhn Tucker conditions, and to solve the original problem by a successive approximations technique. However it is not a trial and error approach, as the optimal solution is reached by a systematic correction of the initial unfeasible strategy. Also the use of the notion of future marginal return of the storage allows to pass easily from deterministic to stochastic reservoir operation without flow forecast, and from stochastic reservoir operation without flow forecast to stochastic reservoir operation with flow forecast. Furthermore, the functional relations established between future marginal return of the storage and relevant inflow variable exhibit a reliability which stays constant for the complete range covered by the relevant hydrologic parameter.

The methodology developed here is complete and makes possible an optimal operation of the reservoir. However, due to the limited amount of time available, some points could not be studied in the desired depth. Hence the availability or the search for additional meteorological and hydrologic data should lead to more reliable forecasts, especially for periods of short duration. On the other hand, concerning reservoir operation, the following point deserves closer consideration: the relation existing in the refill phase between the total inflow and the future marginal return of storage. Also a more refined theoretical development may lead in mixed strategy situation to a solution technique which does not require the use of simulation in stochastic reservoir operation with flow forecast. Of interest would be the evaluation of the gains brought about by the introduction of the runoff forecasts into the optimization procedure. Finally adaptive optimization procedure could also be considered.

A point not mentioned up to now deals with the comparison of the release strategy obtained with the present solution technique with those established by some public and private Swiss utilities. This task was finally abandoned because of the great amount of work required to obtain meaningful results. First no dam exists in the watershed under study, so that the problem of correlation existing between the runoff of different watersheds immediately arises. Second, meetings with representatives of different utilities showed that not one, but many methodologies exist in Switzerland. Which one should then be used for comparison purposes? One, or all of them? Third, breakdown, unavailability of turbines, fluctuations of energy prices, changes in the structure of the power network markedly influence the release strategy, so that different release strategies cannot be directly compared. Hence meaningful comparisons involving not too much work are only possible for average conditions. But this is of little interest, as reservoir operation under average conditions presents no major difficulty.

As already mentioned, the derived solution technique is tailored to the power system under consideration, so that the legitimate question of the generality of the derived methodology arises. The different points involved will be reviewed one after the other in the following paragraphs.

The hydrology of alpine watersheds is such that 61 percent of the annual runoff takes place within 3 months, while the contributions of the six winter months amount to only 12 percent of the annual discharge. This situation is extremely favorable for dividing the reservoir operation into drawdown and refill phase. However one should keep in mind that this feature is not peculiar to alpine watersheds. Many rivers of the world present this feature, although it may not always be so marked. Furthermore this very characteristic makes the construction of a reservoir interesting; if the natural flows of a river were already regulated, or nearly regulated, the construction of a reservoir would bring no great gain. Hence one can assume that for most rivers which feed reservoirs, one shall encounter runoff conditions similar to those of alpine watersheds.

The energy price function comes into play in two places of the optimization procedure. First, the difference in prices existing between winter and summer increases the tendency, already given by the hydrology, of the reservoir to be empty every year nearly at the same date.

Smaller differences between winter and summer energy prices can imply a greater variation in the date of emptiness. This fact however does not question the validity of the derived method; at most it may complicate it a little. On the other hand, one should remember that the existence of differences in winter and summer energy prices makes the construction of a reservoir attractive. Second, with the selected energy price function, an exponential curve, interesting mathematical properties could be established for the optimal strategy. Of course, the energy price function is not necessarily an exponential curve; yet, even if this is not the case, it should be possible to derive interesting mathematical relations between optimal releases and some hydrologic variables. The computations may be more involved, but the procedure remains the same.

The solution technique holds for any reservoir capacity, provided that the reservoir was built for seasonal regulation. In the selected example, the minimum and maximum possible head differ by only 17 percent. This property reduces the number of iterations required to reach the optimal solution. Hence one should expect slower convergence to the optimum for run of the river schemes, but the general methodology is still valid.

The time step retained in this research amounts to one week. Obviously it can be decreased or increased without major difficulties.

As pointed out in the introduction, the present research restricted itself to the problem of reservoir operation for the production of electrical energy. Clearly the same methodology applies for reservoir operation for irrigation or flood control. The only prerequisite is that an adequate objective function exists.

Finally the notion of expected future marginal return of storage allows quite easily the extension of the present method to the cases where more than one reservoir exists in the concerned system.

Consequently the methodology derived in this research is quite general, as it lends itself to many different applications without major modifications. Furthermore the results of this study confirm that a thorough understanding of the processes involved in reservoir operation problems is quite rewarding. It allows tailoring the solution technique to the problem at hand and hence a high computational efficiency is reached.

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Note:

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LIST OF ABBREVIATIONS

ASE	Association Suisse des Electriciens
SEV	Schweiz. Elektrotechnischer Verein
UCS	Union des Centrales Suisses d'Electricité
VSE	Verband Schweiz. Elektrizitätswerke

APPENDIX A

The solution technique developed in Chapter 4 is partly based on ideas contained in three French papers dealing with reservoir operation problems. Hereafter, these ideas will be shortly presented and discussed.

The oldest of these papers was published in 1923 by Varlet. It deals with the regulation of the flows of a river, on which a reservoir has been built. The aim is to release water from the reservoir in such a way that the resulting river flow is regulated as much as possible. To achieve this goal, Varlet, as Rippl did (1883), proposed a graphical procedure based on the extensive use of mass curves. His method consists in plotting twice the cumulated inflows to the reservoir (Y-axis) as a function of time (X-axis). The origin of the first mass curve corresponds to the origin of the axes, whereas the origin of the second mass curve is shifted downward on the Y-axis by a quantity equal to the reservoir capacity (Fig. A.1).

Any state of the reservoir, characterized by a date and the storage level reached at that date can be represented by a point in the XY plane. Furthermore the drawn mass curves are such that any state, which can possibly be reached by the reservoir during its operation must correspond to a point located within the area limited by the two mass curves. For when the reservoir is empty, the representative point lies on the upper curve, and when the reservoir is full, it lies on the lower curve. Finally, the succession of levels hit by the reservoir during its operation is represented by a continuous curve located within the enclosed area.

Hence if initial and final reservoir contents are known in advance, the reservoir control problem boils down to the determination of the

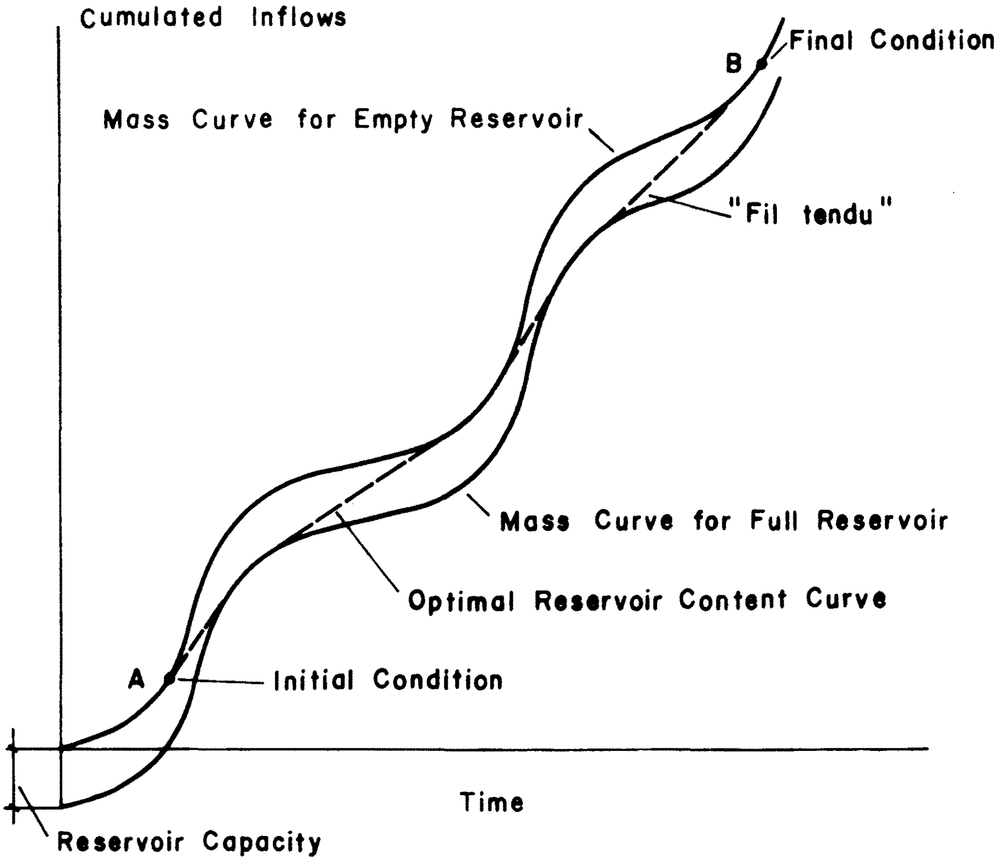


Fig. A.1 Optimal Reservoir Content Curve according to Varlet optimal curve which joins the two extreme points. For this purpose, Varlet developed the method of the "fil tendu." This method assimilates the reservoir content curve to a thread and, according to Varlet, the optimal reservoir content curve is obtained by stretching the thread between the two extreme points.

The approach described above leads to two important properties of the optimal solution. First, as shown by Figure A.1, the optimal reservoir content curve is made of straight parts separated by curved ones. In other words, periods with constant releases alternate with periods with varying releases. Second, as a corollary to the first property, it appears that it is not always possible to regulate completely the flows of a river with a given reservoir.

One can use Varlet's approach to optimize the energy production of a hydro-power scheme, provided that the following additional assumptions are stipulated. The utility concerned is only interested in the production of firm energy, which it sells at a constant price. Also the head on the turbines does not vary with time. These assumptions markedly reduce the field of applications of Varlet's method. Nevertheless the idea of the "fil tendu" conveys an intuitive idea of what the optimal solution is. It also indicates that during the period of operation, the reservoir passes through different phases and finally it shows the role played by the initial and final conditions.

Boulinier took over and developed the ideas set forth by Varlet. In an interesting article published in 1943, he applies this method to the control of the level of Lake Geneva. The level of Lake Geneva, according to an international treaty, can fluctuate only within a range of about 2 meters. Furthermore, downstream of the gates which control the lake level, there exists a run of the river power scheme, the energy production of which should be maximized. Hence the problem consists of maintaining the lake level within the range defined by the international treaty and in maximizing the energy output of the power scheme. It is in fact a multi-purpose optimization problem.

Boulinier solved this problem in a stepwise procedure. He determined first for each day of the year the lake level at which the gates should be opened, in order to make sure that the maximum level would not be exceeded at a later date. This succession of levels specifies along with the minimum level the range within which Lake Geneva can be controlled for power production purposes only.

In the second step, Boulinier optimized the firm energy output of the hydro-power plant. Using Varlet's method, he computed for a given sequence of inflows a curve representing, as a function of time, the minimum lake levels which guarantee a given firm release. He repeated the same procedure for different annual inflow sequences and determined the envelop curve of the obtained minimum levels. Finally, these computations were performed for different amounts of firm releases. The operation rule consists then in maintaining the lake level always above the appropriate minimum levels envelop curve (Fig. A.2).

However the strategy just described applies only when the lake level stays close to the critical levels. If enough water is available, the present strategy is of no use. As a third step, Boulinier introduced a new condition into the problem. The power plant has a limited capacity and spills occur each time the release exceeds this value. Consequently, as a new condition, Boulinier stipulated that the amounts of spills must be minimized. To satisfy this condition, Boulinier in much the same way

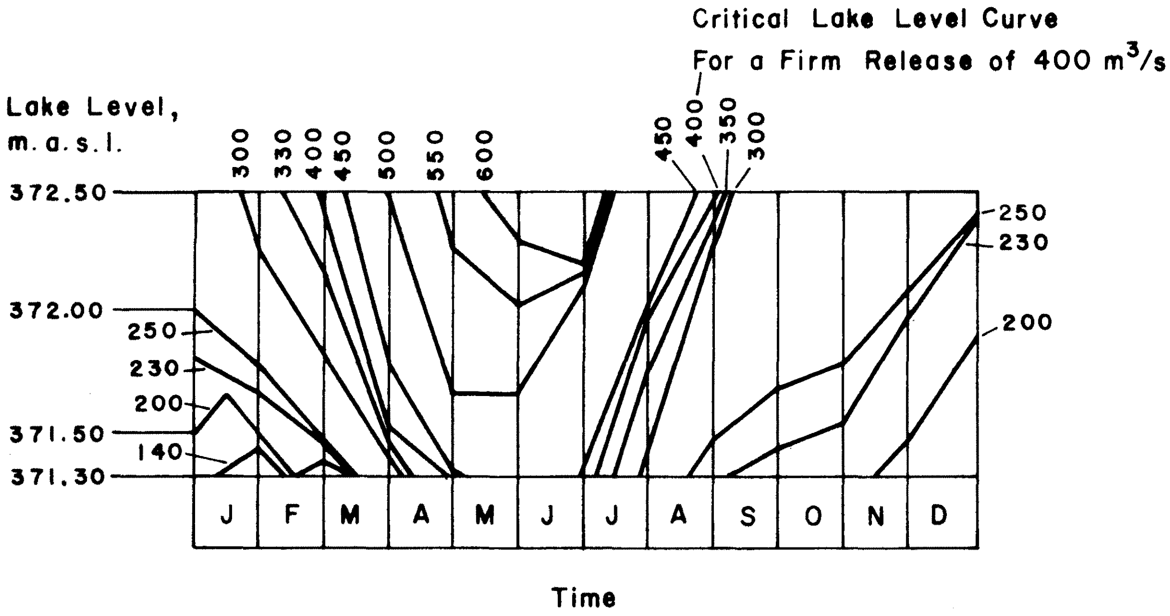


Fig. A.2 Lake of Geneva: Critical Lake Level Curve for Different Firm Releases (after Boulinier) as before, determined the envelop curve of the maximum lake levels, above which the release should be set equal to the power station capacity.

At this stage of the computations, two limiting lake level curves exist which indicate the release strategy to follow in extreme situations. Unfortunately both curves are of no use for situations corresponding to neither of the extreme cases. Boulinier, though, was not very explicit about what to do in these cases.

The present article is instructive in many respects. To begin with, it represents an application of Varlet's approach and shows quite clearly its limitations. Although Boulinier indicated ways of introducing head and energy price variations into the computations, this path does not seem promising to solve real world problems.

Yet this method is most important to understand the mechanisms of reservoir operation. The nature and the characteristics of the optimal solution is clearly displayed, as well as the effect of the initial and final conditions. Furthermore the present procedure proved that the reservoir, during the period of operation, goes through different phases. Phases with constant releases alternate with phases with varying releases. There exists also periods, where the optimal strategy is independent from initial and final conditions.

Giguet (1945), conscious of the limitations of the preceding approach, started from a completely different point. Right at the beginning of his article, he introduced a new variable u, called the marginal utility of the release. It represents the return produced by the release of an amount dz of water. Hence the instantaneous return produced by an instantaneous release x of water during a period of time dt amounts to

$$\int_{0}^{x} u \, dz \tag{A-1}$$

and the total return resulting from reservoir operation for a duration t_1 equals

$$U = \int_{0}^{t_1} \int_{0}^{x(t)} udzdt \tag{A-2}$$

Hence the problem to be solved consists in determining the sequence of releases $x(t)$, in such a way that the value of U is maximized. To stay as general as possible, Giguet assumed that the marginal utility of the release was a function of time, release, reservoir content, and prevailing meteorology.

According to Giguet, reservoir control for energy production reduces to transferring water from those periods where the marginal utility of the release is low to those where the marginal utility is high. However this transfer of water is not always possible. This fact led Giguet to divide the reservoir operation period into different phases. Whenever the water transfer is possible without any restriction, Giguet said that the reservoir is in a "régime équilibré," while when this is not the case, he said that the reservoir is in a "régime bloqué." For the situation where the reservoir is in a "régime bloqué," he introduced some further subdivisions, depending on whether the reservoir is full or empty at that time.

After these introductory considerations, Giguet derived a series of rules and theorems which stipulate the conditions which must be satisfied in order for a release strategy to be optimal. Among others, he proved that for an optimal solution, the marginal utility of the releases varies with time according to a well specified pattern.

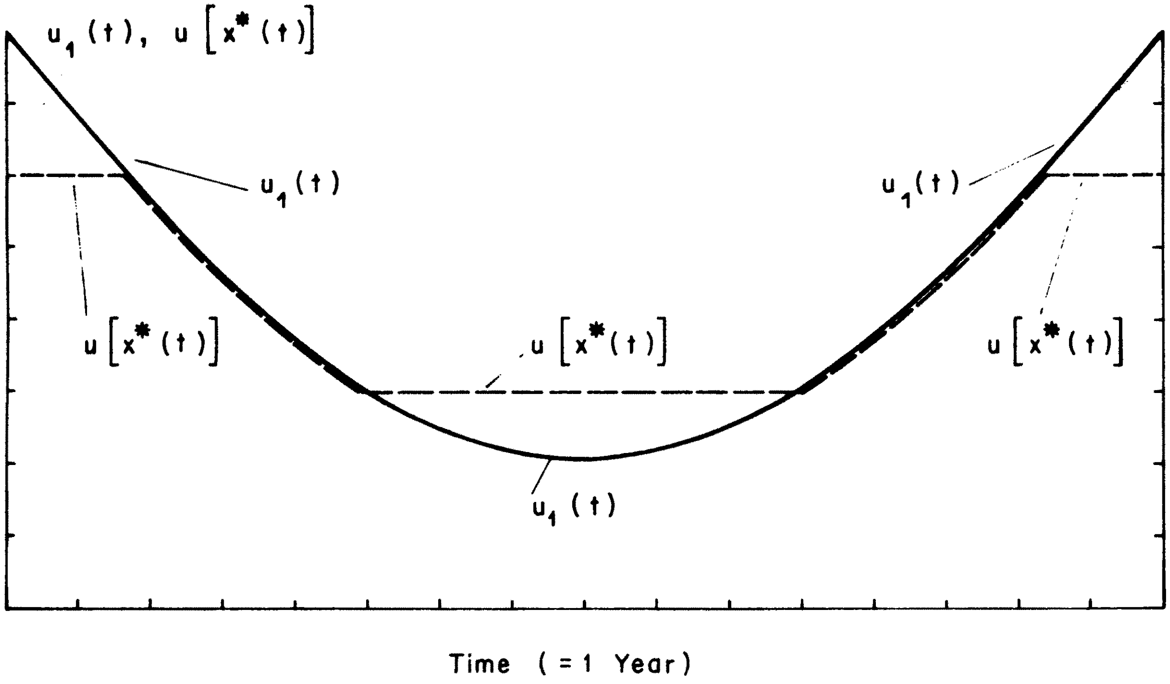
Furthermore Giguet introduced two special marginal utility functions. The first one, u_o, represents the marginal utility when the release is equal to zero, the second one, u_1, the marginal utility when the release is set equal to the reservoir inflow.

Finally he derived the optimal release strategy on an annual basis for the case where u_1 is a sinusoidal function. As shown by Figure A.3, the optimal release strategy is made of branches where the marginal return of the releases is either decreasing or increasing, separated by periods where the marginal return of the releases is constant.

Compared to the previous methodology, there is a tremendous improvement. Most important, the present method can accommodate cases where the marginal return of the release is not a constant. Note that Varlet's approach leads to the same results whenever the marginal return of the release is a constant.

The drawbacks of Giguet's approach are that the determination of the optimal solution requires a certain amount of trial and error, and that it is a deterministic method. Finally no attention has been given to the role and the influence of the initial and final conditions on reservoir operation rules.

The three reviewed papers contain original ideas on the solution of reservoir operation problems. The first two provide an intuitive idea of the properties and nature of the optimal reservoir content curve. The last one, more theoretical and more general, sets forth interesting properties of the marginal return of the successive releases. The three papers, combined together, give a good understanding of the reservoir operation problem.



$u_1(t)$ = marginal utility when release is equal to zero as a function of time

$u[x^*(t)]$ = marginal utility of optimal release as a function of time

Fig. A.3 Typical Pattern of Annual Variation of the Marginal Utility of the Optimal Releases Sequence (after Giguet)

However, if they show well the properties of the optimal solution, they fail, though, to indicate a systematic and efficient way to get at it. Hence the purpose of the present research was to derive from the available information a methodology which leads as directly as possible to the optimal solution.