MANNING'S N VALUES FOR FLOODPLAINS WITH SHRUBS AND WOODY VEGETATION

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ABSTRACT

An improved methodology has been developed for the determination of Manning's n and other hydraulic roughness values for shrubs and woody vegetation. This method involves the measurement of horizontal plant density, stem diameter, and the height and width of the leaf mass of a typical plant. Recent investigation has shown that the plant stiffness modulus may be predicted with good accuracy by using stem diameter and plant height in a non-linear relationship. New relationships have been developed for the calculation of Mannings n values for both submerged and partially submerged vegetation. These relationships for flow through vegetated channels still require a trial and error solution when both depth and velocity are unknown, but simplify the solution technique significantly. A stage-discharge table can now be directly constructed for flood elevation studies with out trial and error solutions. A simple example of the calculation of Manning's n values using the method is presented so that the practitioner can follow the method and apply it in the field.

INTRODUCTION

Previous research has indicated that hydraulic roughness in vegetated channels can be related to the frontal area of plants, the height of the plant, the stem stiffness, stem diameter, and the horizontal density of the plants (i.e. number of plants per unit area) among other factors. (Freeman, et. al., 1996) The purpose of this paper is to present data developed subsequent to the 1996 paper and current methodology that can be used in the estimation of hydraulic roughness in vegetated channels. The developments described in this paper will assist engineers

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and scientists in the determination of hydraulic roughness values for channels that are vegetated for either aesthetic or habitat values.

ESTIMATION OF MANNING'S N VALUE

Prediction equations for the estimation of hydraulic roughness and Manning's n value for the vegetation present in vegetated channels (equations 1 and 5) have been developed and reported previously in Freeman, et. al., 1996 and Rahmeyer and Werth, 1996. The equations have been further modified to allow calculation of C_D, V*/V as well as Manning's n value for the bed and channel combined as shown in Equations 2 through 4 and 6 - 8 (Rahmeyer, 1998).

The equations were developed for two types of flows, submerged (plants fully under water) and partially submerged flows (plants protruding above the water surface). The divide between the two types of flow is when flow depth is approximately 80% of the height of the plant. When flow reaches approximately 80% of the height, the plant bends sufficiently to become submerged in the flow.

Submerged Flows

For submerged flows (Y_o>0.8 Plant Height) the original prediction equation for only the vegetative portion of the total roughness is (See Figure 1 for plant variable definitions):

$$n_{veg} = 0.039\ K_n \left(\frac{E_s A_s}{\rho V^2 A}\right)^{0.141} \left(\frac{H}{Y_O}\right)^{0.175} (M\ A)^{0.191} \left(\frac{\nu}{V\ R_h}\right)^{0.0155} \tag{1}$$

For submerged conditions (Y_o > 0.8 H) the equations to predict the coefficient of drag of the plants and the total hydraulic roughness and Manning's n values are as follows:

$$C_D = 0.202 \left(\frac{E_s A_s}{\rho V^2 A_i}\right)^{0.247} \left(\frac{H}{Y_O}\right)^{0.328} \left(\frac{1}{(M\ A_i)}\right)^{0.631} \left(\frac{\nu}{V\ R_h}\right)^{0.156} \tag{2}$$

$$\frac{V*}{V} = 0.183 \left(\frac{E_s A_s}{\rho A_i}\right)^{0.183} \left(\frac{H}{Y_O}\right)^{0.243} (M\ A_i)^{0.273} \left(\frac{\nu}{R_h}\right)^{0.115} \left(\frac{1}{V*}\right)^{0.481} \tag{3a}$$

Equation 3a can be solved for V resulting in Equation 3b:

$$V = 5.468 \left(\frac{\rho A_i}{E_s A_S}\right)^{0.183} \left(\frac{Y_o}{H}\right)^{0.243} \left(\frac{1}{MA_i}\right)^{0.273} \left(\frac{R_h}{\nu}\right)^{0.115} (V^*)^{1.481} \quad \text{(3b)}$$

This equation (3b) can be used in Manning's equation to solve for n. If we use the definition of $V^*=(gR_hS)^{1/2}$ the solution is direct for n if the depth of flow is known. This represents a major improvement over previous methodology.

$$n = 0.183\, K_n \left(\frac{E_s A_s}{\rho A_i}\right)^{0.183} \left(\frac{H}{Y_o}\right)^{0.243} (MA_i)^{0.273} \left(\frac{\nu}{R_h}\right)^{0.115} \left(\frac{1}{V^*}\right)^{1.481} R_h^{2/3}\, S^{1/2} \quad (4)$$

The variables in the above equations are defined as:

A_i	Frontal area of an individual plant blocking flow, ft^2. (H'xW)
A_S	Total cross-sectional area of the stem(s) of an individual plant measured at H/4 from plant base, ft^2.
E_s	Modulus of Plant Stiffness, lbs/ft^2.
g	Acceleration of gravity (32.2 ft/sec^2)
H	Average undeflected plant height, ft.
H'	Undeflected height of the leaf mass of a plant, ft.
K_n	Units conversion for Manning's Equation = 1.49 $ft^{1/3}/sec$ (1.0 $m^{1/3}/sec$ in metric units).
M	Relative plant density, number of plants per ft^2.
n_{veg}	Manning's resistance coefficient for vegetation and channel bed.
R_h	Hydraulic radius (R=Channel Area/Channel Perimeter), ft. For a wide channel R_h is taken equal to Y_o.
V	Mean channel velocity, ft/sec.
Y_O	Flow depth, ft.
W	Width of "average" plant, ft.
ν	Fluid dynamic viscosity, ft^2/s.
ρ	Fluid density, $slugs/ft^3$

The Manning's n value calculated in Equations 3 and 4 is the total roughness while the value from Equation 1 is only the roughness due to the vegetation and does not include the bed roughness. Equations 3 and 4 (as well as 7 and 8 later) were developed from the same data as equations 1 and 5 but were based on total roughness rather than just the roughness of the vegetation. The equations presented here can now be easily implemented into a computer routine for use with models such as HEC-2, HEC-RAS or other hydraulic modeling packages.

The variable definitions for plant dimensions and water depth are shown in Figure 1. The plant widths and plant heights used in the development of the equations represented the approximate averages of the plants evaluated. The average widths and/or heights used in calculations should be reduced if large voids exist in the leaf mass, for round or oval leaf mass shapes, and/or for the lack of leaves during winter flows. This reduction in frontal area will account for the reduced plant blockage area resulting from these factors. To calculate the frontal area of the plants blocking flow, the average plant width, W, is multiplied by the average height of the leaf mass, H'.

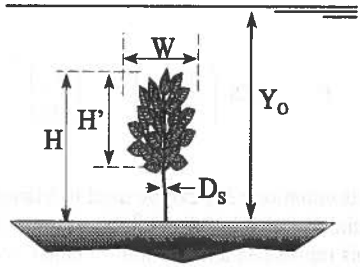

Figure 1 Plant Dimensions for Submerged Flow (From Rahmeyer, 1998)

Partially Submerged Flows

Partially submerged flow is defined as flow at a depth of less than 80% of the plant height (H). The equation for the vegetation portion of Manning's n value for partially submerged ($Y_o < 0.8$ H) flow is:

$$n_{veg} = 2.2x10^{-6}\, K_n \left(\frac{E_s A_s}{\rho V^2 A_*}\right)^{0.242} (MA_*)^{0.0623} \left(\frac{\nu}{VR_h}\right)^{0.662} \qquad (5)$$

The equations for CD, and V*/V for partially submerged flow are as follows:

$$C_D = 3.624e{-}09 \left(\frac{E_s A_s}{\rho V^2 A_i *}\right)^{0.448} \left(\frac{1}{(M A_i *)}\right)^{0.882} \left(\frac{\nu}{V R_h}\right)^{1.061} \qquad (6)$$

$$\frac{V^*}{V} = 9.159x10^{-5} \left(\frac{E_s A_S}{\rho A_i^*}\right)^{0.207} (MA_i^*)^{0.0547} \left(\frac{R_h}{\nu}\right)^{0.490} (V^*)^{0.0761} \qquad (7a)$$

Equation 7a can be reduced to solve for V as shown in Equation 7b, and using the definition of $V^* = (gR_hS)^{1/2}$ equation 7b can be solved for velocity directly if depth of flow is known for the calculation of A_i^*.

$$V = 1.092x10^4 \left(\frac{\rho A_i^*}{E_s A_S}\right)^{0.207} \left(\frac{1}{MA_i^*}\right)^{0.0547} \left(\frac{\nu}{R_h}\right)^{0.490} (V^*)^{0.924} \qquad (7b)$$

Equation 7b can then be used in Manning's equation to give an equation for the calculation of n directly (when depth is known) for partially submerged vegetation as shown in Equation 8.

$$n = 9.159x10^{-05} K_n \left(\frac{E_s A_s}{\rho A_i^*}\right)^{0.207} (MA_i^*)^{0.0547} \left(\frac{R_h}{\nu}\right)^{0.490} \left(\frac{1}{V^*}\right)^{0.924} R_h^{2/3} S^{1/2} \qquad (8)$$

A_i^* is defined as the undeflected frontal area of the plant that is partially submerged. It is calculated by:

$$A_i^* = \left[Y_O - (H - H')\right] W \qquad (9)$$

with the variables required being shown in Figure 1. Again it should be noted that the plant widths and plant heights used in the development of the equations represented the approximate averages of the plants evaluated. The average widths and/or heights used in calculations should be reduced if large voids exist in the leaf mass, for round or oval leaf mass shapes, and/or for the lack of leaves during winter flows. This reduction in frontal area will account for the reduced plant blockage area resulting from these factors. To calculate the frontal area of the plants blocking flow for partially submerged conditions, the average plant width, W, is multiplied by the average height of the submerged leaf mass [Yo-(H-H')].

All equations are dimensionless (with the exception of Equations 4 and 8 which contain K_n) and units can be converted to metric values without change to the equations (K_n becomes 1.0 $m^{1/3}$/sec). Since the depth of flow is also an unknown in most flow problems, an iterative solution will still be required for partially submerged plants. The solution should converge rapidly, however.

Determination of Plant Stiffness

One of the major drawbacks to using this method has been lack of a method to estimate E_s for the various plant types. Very little data existed for plants other than those tested by Rahmeyer, et al., and new plants had to be checked in the field to determine the value to use for E_s. Subsequent work has yielded a method to facilitate field measurements and a method to estimate the stiffness without the need for extensive field measurements.

Variables used to calculate plant stiffness values (H, D_s, and F_{45}) are measured as shown in Figure 2. The values obtained are used in Equation 10 to obtain the stiffness values for the plant. The plant stiffness is calculated by measuring the force (applied at ½ of the plant height, H/2) necessary to bend the plant to a 45° angle and measuring the stem diameter at one fourth of the height (H/4). The plant stiffness modulus is defined as:

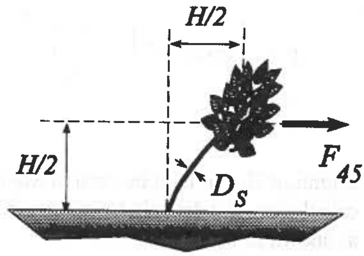

Figure 2 Plant Dimensions for Determining Stiffness

$$E_s = \frac{F_{45}H^2}{3I} = \left(\frac{64}{\Pi D_s^4}\right)\frac{F_{45}\ H^2}{3} = 6.791\left(\frac{F_{45}H^2}{D_s^4}\right) \tag{10}$$

The calculation of E_s has been problematic requiring large amounts of field effort and time to evaluate the results of the field analysis. In an effort to reduce the amount of work required to estimate E_s, work was done that developed a good relationship between the stem area ($\pi D_s^2/4$) and the force required to bend the stem to 45°. This research produced a linear relationship between force (F_{45}) and stem area for an individual stem. (Freeman, 1997)

Subsequent to the field work which produced the linear relationship between force and stem area, Equation 11 has been developed which relates the plant height and stem diameter to E_s directly without the necessity of calculating F_{45}. Equation 11 was developed based only on data from the plants tested in the original flume work at Utah State University and early field work by Freeman. Figure 3 compares the predicted values of Equation 11 with those obtained by subsequent field measurements by Freeman (1997). This equation relates two of the three variables involved in the determination of the stiffness modulus to the observed modulus with good results. The prediction equation for E_s is:

$$E_s(psf) = 160{,}000\left(\frac{H}{D_s}\right) + 454\left(\frac{H}{D_s}\right)^2 + 37.8\left(\frac{H}{D_s}\right)^3 \tag{11}$$

Equation 11 may be adjusted to give a slightly better fit to the observed data now that the additional data is available, but the observed data show good overall agreement when compared to the predicted values. The fact that Equation 11 does a generally good job of estimating E_s indicates that this method will be accurate enough to give guidance to engineers and scientists that must have an estimate of hydraulic roughness for project comparisons when field E_s data is not available.

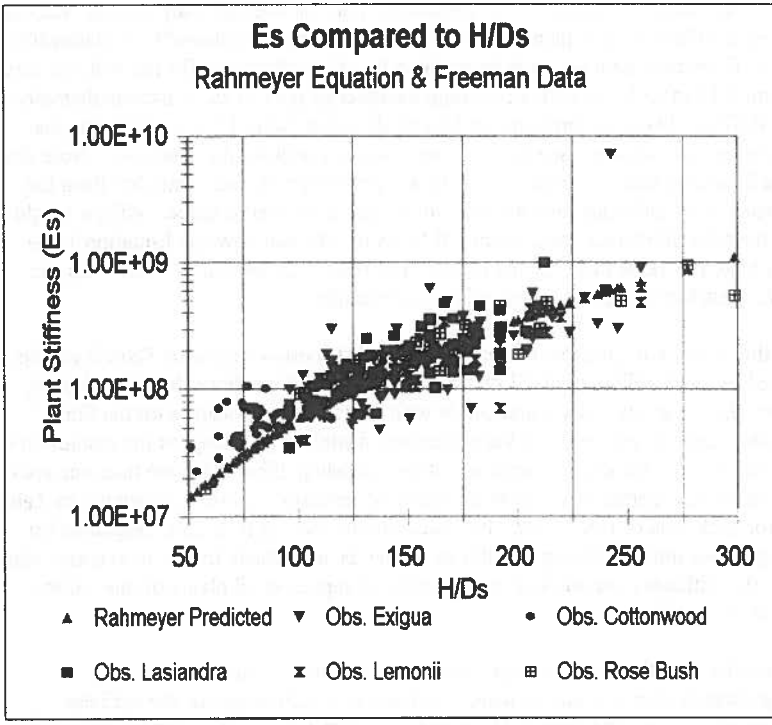

Figure 3. Comparison of field data with E_s prediction equation.

This equation requires estimation of only a stem diameter and a plant height -- both of which are quickly obtained in the field or estimated from growth studies. It appears that the equation may slightly under predict the value of E_s for low H/D_s values but equation 11 was not developed using any data from cottonwood trees which represent most of the small values of the H/D_s data shown in Figure 3. The data from the flume experiments used by Rahmeyer in developing the prediction equation are not shown in the figure.

EXAMPLE PROBLEM

Consider a simple flood plain 80 feet (ft) wide and covered with a stand of Salix exigua (sand bar) and Salix lasiandra (Pacific) willows. The flow is 1200 cfs and the flood plain slope is 0.0020. What is the hydraulic resistance (n value) of the flood plain and the depth and velocity of flow. (Here we assume no stream channel through area -- i.e. flow through an isolated flood plain for simplicity. If a channel is present one would use one of the standard methods of channel compositing to obtain the effective n value for the entire channel.)

From the field evaluation it was determined that the average plant density was 0.04 exigua willows/ft^2 (16 plants/400 ft^2) and 0.02 lasiandra willows/ft^2 (8 plants/400 ft^2). From field data we have determined that stem diameters for the willows vary from 0.12 in to 2.0 in with an average diameter of 0.87 in and a median diameter of 0.67 in. These are broken into bins as shown in Table 1 for the Salix exigua. Only one size class is used for the Salix lasiandra willows for simplicity. Note that the bins have been selected arbitrarily, are probably reduced in number from the results of a field study, and are only for exigua to conserve space. Willow height in the area of interest varies from 3 ft to 18 ft. We can now use Equation 11 to estimate the value of E_s for the plants. The results are shown in Table 1 for the size class bins and plants involved in this example.

If the stands are relatively uniform, a single distribution similar to Table 2 can be used for each willow type. If the characteristics of the willow stand (i.e. height, stem diameter, etc) vary significantly with respect to the location on the flood plain, it may be necessary to keep the areas distinct, depending on the resolution of the model and the desired accuracy of the modeling effort. If more than one area must be considered, it would be advisable to develop a distribution similar to Table 2 for each area or river reach. For the determination of hydraulic roughness for large areas under differing conditions, it may be acceptable to use an average value for the stiffness modulus and use the value to represent all plants of one variety over all conditions.

It can be seen from this example that the plant stiffness modulus can vary significantly over a group of plants. If there is a wide range for the stiffness modulus, it would be wise to incorporate the distribution of E_s and height into the calculation of hydraulic roughness presented below.

Once the values for E_s for each plant type have been obtained, the values must be composited together to give a usable value of n for the flood plain. This is especially true for one-dimensional modeling with such models as HEC-2. Even if using multi-dimensional models such as FESWMS or RMA-2 areas must also be composited to obtain a Manning's n value or roughness value for rather large areas to facilitate modeling.

The compositing process involves accounting for the relative number of each plant type found on the flood plain. This involves the combination of plant characteristic variables into a composite value for use in the above equations for predicting roughness values. This is done using the following equations:

$$A_{ave} = \sum\left[A_i \frac{M_i}{M_{total}}\right] \qquad E_{ave} = \sum\left[E_i \frac{M_i}{M_{total}}\right] \qquad A_{S_{ave}} = \sum\left[A_{Si} \frac{M_i}{M_{total}}\right]$$

Table 1. Example of Determination of Plant Tension and Stiffness Modulus from Field Data.

Determining Tension and Stiffness Modulus for Salix exigua Willow Stand			
Size Class Median (in)	Number of Stems in Sample	Willow Stand Average Height (ft)	Stiffness Modulus E_s[1] (from Eq. 11) (psf)
1.85	2	16.5	6.9 E+07
1.05	7	12	1.28 E+08
0.55	16	7	1.7 E+08
Mean = 0.78 Median = 0.55		Mean = 9.1 Median = 7	Mean 1.51 E+08 Med. 1.72 E+08
Salix Lasiandra			
0.65		6.0	7.47 E+07

[1] Note: Stem Diameters must be converted to ft before calculation of E_s in Eq. 11.

Table 2. Plant Data for Example Problem

Item	Exigua	Lasiandra	Composite[1]
Average Plant Height (H)	9	6	8
Average Leaf Mass Width (W)	2	3	2.3
Average Leaf Mass Height (H')	9	6	8
Average Stem Diameter (D_s)	0.78	0.65	0.74
Number of Stems per Plant (#)	4	8	-
Calculated Stem Area ($A_s=\pi D_s^2/4$ * #)	0.133	0.184	0.0150
Calculated Frontal Area (A)	18.0	18.0	18.0
Calculated Stiffness Modulus (E_s)	1.72e+08	7.47e+07	1.39e+08
Area of Plot for Plant Count-400 ft^2	400	400	-
Average # Plants in Measured Plots	16	8	-
Calculated Plant Density (M_i=#/Area)	0.04	0.02	0.06[2]
Relative Plant Density	0.667	0.333	1.0

1. Methodology Described Above.
2. Total Plant Density is Sum of Individual Plant Densities

$$H_{ave} = \sum\left[H_i \frac{M_i}{M_{total}}\right] \quad A_{*ave} = \sum\left[A_{*i} \frac{M_i}{M_{total}}\right] \quad H'_{ave} = \sum\left[H'_i \frac{M_i}{M_{total}}\right]$$

The total plant density is the sum of individual plant densities. In this example we have two plants with individual densities of 0.04 (exigua) and 0.02 (lasiandra) plants/m^2, density the total density (M_{total}) is 0.04 + 0.02 = 0.06 and M_i/M_{total} is equal to 0.667 for the willows and 0.333 for the dogwoods. The plant density and other composited values are shown in the last column of Table 2 for the reach in consideration for this example.

The resulting "composited" or averaged values from the above equations are used in either equations 2, 3 and 4 or 6, 7 and 8 depending whether the plants are submerged or partially submerged.

The solution of the prediction equations is an iterative solution where a depth of flow is assumed and the velocity which results from the solution is compared to the velocity calculated from the continuity equation (Q=AV), the value for depth adjusted and the equation recalculated until the velocity from Equations 3b or 7b and the calculated depth (from the continuity equation, Q=VA) are equal. These calculations are presented in Table 3.

The resulting value for Manning's n in this example is 0.137 and the flow is in the submerged regime (8.475 ft > 8.0 ft composite height). The depth could have been as shallow as 6.4 feet (80% of 8 ft) and still be considered in the submerged

Table 3. Trial and Error Solution for Example Problem.

SUBMERGED FLOW CALCULATIONS					
Variable	Value	Trial 1	Trial 2	Trial 3	Trial 4
Assumed Depth	8	8.4	8.5	8.45	8.475
Rh (assume rect chan)	6.67	6.94	7.01	6.98	6.99
Ai = HW (S)	18.4	19.32	19.55	19.435	19.49
(Ea*As/rho*Ai)^0.183	7.45	7.39	7.37	7.38	7.37
(H/Yo)^0.243	1	0.99	0.985	0.987	0.986
(M*Ai)^0.273	1.03	1.04	1.045	1.043	1.044
(nu/Rh)^0.115	0.27	0.23	0.226	0.226	0.226
(1/V*)^1.481	1.87	1.82	1.80	1.808	1.805
n = (Equation 4)	0.14	0.140	0.138	0.138	0.138
V = (Equation 3b)	1.683	1.756	1.774	1.765	1.769
Vcon = Q/A	1.875	1.785	1.764	1.775	1.767

flow regime. Obviously the higher willows in the flood plain will be in the partially submerged regime at a flow depth of 6.4 feet, but the majority of the stand should be submerged if the measurements and compositing are properly done. If the depth had been calculated using Equation 8 for unsubmerged flow the resulting depth (10.5 ft) would be higher than the composite plant (8.0 ft) and the composite vegetation would obviously submerged, indicating the need to use Equations 3b and 4 for submerged flow.

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