DIRECT SOLUTION FOR APRON ELEVATION

By

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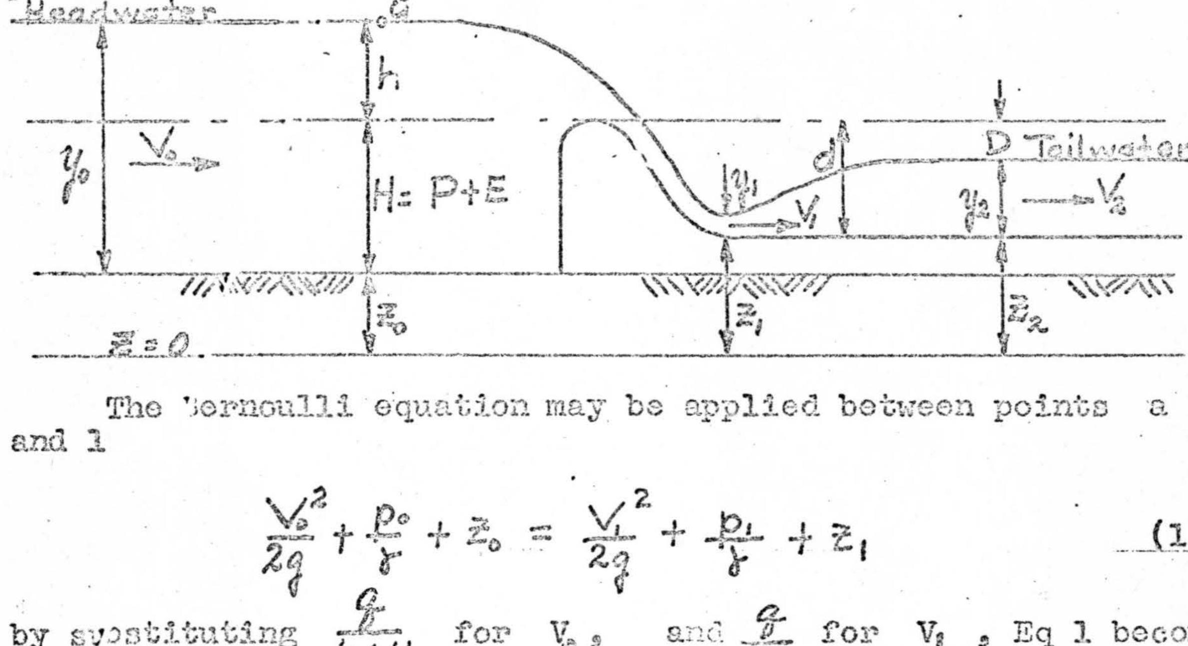
The hydraulic jump is recognized as a very efficient means of dissipating energy in a stilling pool. To utilize this phenomenon, however, it is necessary that the downstream depth of the jump be not greater than the tailwater or the jump will be swept downstream. Therefore, the required apron elevation must be determined for all discharges. The standard solution of this apron elevation has been a trial and error process requiring a considerable amount of time. By the use of dimensional analysis it is possible to put the pertinent equations in dimensionless form and to solve them once and for all in a systematic manner so that, subsequently, any factor pertaining to design may be determined directly from the solution.

The notation to be used is given in the figure. Because the final relationship is dimensionless, it is not necessary to assign units except to say that any system may be used provided it is used consistently.



The Bernoulli equation may be applied between points a and 1

$$\frac{V_0^2}{2g} + \frac{p_0}{\gamma} + z_0 = \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 \qquad (1)$$

by substituting $\frac{q}{h+H}$ for V_0, and $\frac{q}{y_1}$ for V_1, Eq 1 becomes

$$\frac{q^2}{2g(h+H)^2} + h + H = \frac{q^2}{2gy_1^2} + y_1 + H - d \qquad (2)$$

This relationship may be rearranged and expressed in terms of more significant dimensionless parameters.

$$\frac{d}{H} = \frac{q^2}{2gH^3}\left[\frac{H}{y_1} - \left(\frac{H}{h+H}\right)^2\right] + \frac{y_1}{H} - \frac{h}{H} \qquad (3)$$

or

$$\pi_1 = \frac{1}{2}\,\pi_4^2\left[\frac{1}{Y^2} - \frac{1}{(\pi_3+1)^2}\right] + Y - \pi_3 \qquad (4)$$

where $\pi_2 = D/H$, $\pi_3 = h/H$, $\pi_4 = q^2/gH^3$ and $Y = y_1/H$

Eq. 4 states the condition of flow on the apron upstream from the jump for any apron elevation, dam height, headwater elevation, and discharge per unit width of spillway. However, it does not relate the downstream depth with the tailwater depth--this expression depending upon the hydraulic jump equation.

$$y_2 = \frac{y_1}{2}\left(\sqrt{1+\frac{8q^2}{g y_1^3}} - 1\right) = d - D \tag{5}$$

which may be rearranged in dimensionless form as

$$\Pi_1 = \frac{Y}{2}\left(\sqrt{1+8\frac{\Pi_4^2}{Y^3}} - 1\right) + \Pi_2 \tag{6}$$

where $\Pi_2 = \frac{D}{H}$.

Solving Eqs. 4 and 6 simultaneously results in

$$\frac{Y}{2}\left(\sqrt{1+8\frac{\Pi_4^2}{Y^3}} - 1\right) + \Pi_2 = \Pi_1 = \frac{\Pi_4^2}{2Y^2} + Y - \Pi_3 - \frac{\Pi_4^2}{2(\Pi_3+1)^2} \tag{7}$$

which is a cubic equation that is best solved by trial and error. However, the equation may be evaluated systematically and the results tabulated as shown in the accompanying table. From this table it is possible to construct curves which may be used for direct solution of the required apron elevation for any combination of the other variables.

It should be pointed out that this solution does not take into consideration the drag on the flow due to boundary resistance or the drag and break-up on the surface of the flow due to internal turbulence and the surrounding air. These influences may be appreciable--especially in the case of high spillways.

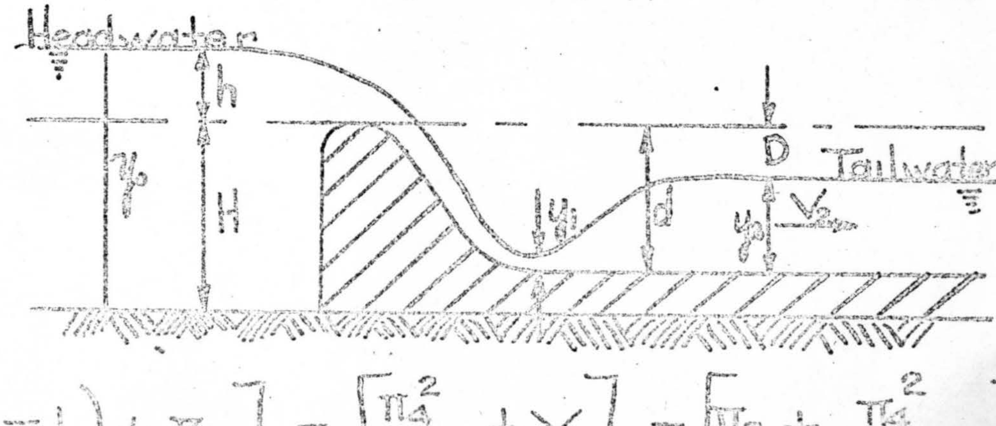
VALUES OF " π_1 FOR DETERMINATION OF APRON ELEVATION . "

π_2 → / π_3 ↓	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
			$\pi_4 = 0.2$						
0	---	0.78	1.07	1.32	1.57	1.81	2.04	2.27	2.50
0.2	0.57	0.87	1.12	1.37	1.61	1.84	2.07	2.30	2.52
0.4	0.66	0.92	1.16	1.41	1.64	1.87	2.10	2.32	2.54
0.6	0.72	0.96	1.20	1.44	1.67	1.90	2.13	2.34	2.56
0.8	0.76	1.00	1.24	1.47	1.70	1.93	2.15	2.36	2.58
1.0	0.80	1.04	1.27	1.50	1.72	1.95	2.17	2.38	2.60
			$\pi_4 = 0.4$						
0	---	1.09	1.39	1.67	1.93	2.17	2.42	2.66	2.89
0.2	0.86	1.18	1.46	1.72	1.97	2.21	2.46	2.69	2.92
0.4	0.98	1.25	1.52	1.77	2.01	2.25	2.49	2.72	2.95
0.6	1.05	1.31	1.57	1.81	2.05	2.29	2.52	2.75	2.98
0.8	1.11	1.36	1.61	1.85	2.09	2.32	2.55	2.78	3.01
1.0	1.16	1.41	1.65	1.88	2.12	2.36	2.58	2.81	3.03
			$\pi_4 = 0.6$						
0	---	1.37	1.68	1.97	2.23	2.49	2.74	2.98	3.22
0.2	1.12	1.46	1.74	2.01	2.28	2.52	2.77	3.01	3.25
0.4	1.24	1.53	1.80	2.06	2.32	2.56	2.80	3.04	3.28
0.6	1.32	1.59	1.86	2.11	2.36	2.60	2.84	3.08	3.31
0.8	1.39	1.65	1.91	2.16	2.39	2.64	2.88	3.11	3.34
1.0	1.45	1.70	1.95	2.20	2.42	2.67	2.91	3.14	3.37
			$\pi_4 = 0.8$						
0	---	1.66	1.96	2.24	2.52	2.78	3.04	3.28	3.52
0.2	1.38	1.72	2.00	2.28	2.56	2.81	3.06	3.30	3.54
0.4	1.48	1.78	2.05	2.33	2.60	2.84	3.09	3.33	3.57
0.6	1.56	1.84	2.11	2.38	2.64	2.88	3.13	3.36	3.60
0.8	1.63	1.90	2.17	2.43	2.68	2.92	3.16	3.39	3.63
1.0	1.70	1.96	2.22	2.47	2.72	2.96	3.18	3.42	3.66
			$\pi_4 = 1.0$						
0	---	---	2.20	2.52	2.79	3.05	3.30	3.55	3.80
0.2	---	1.99	2.24	2.54	2.80	3.06	3.31	3.56	3.81
0.4	1.70	2.03	2.30	2.57	2.83	3.09	3.34	3.59	3.83
0.6	1.77	2.08	2.36	2.62	2.88	3.12	3.38	3.62	3.87
0.8	1.85	2.14	2.41	2.67	2.92	3.17	3.41	3.65	3.90
1.0	1.93	2.20	2.46	2.71	2.96	3.20	3.44	3.68	3.93

$\pi_1 = \frac{d}{H}$ $\qquad \pi_3 = \frac{h}{H}$

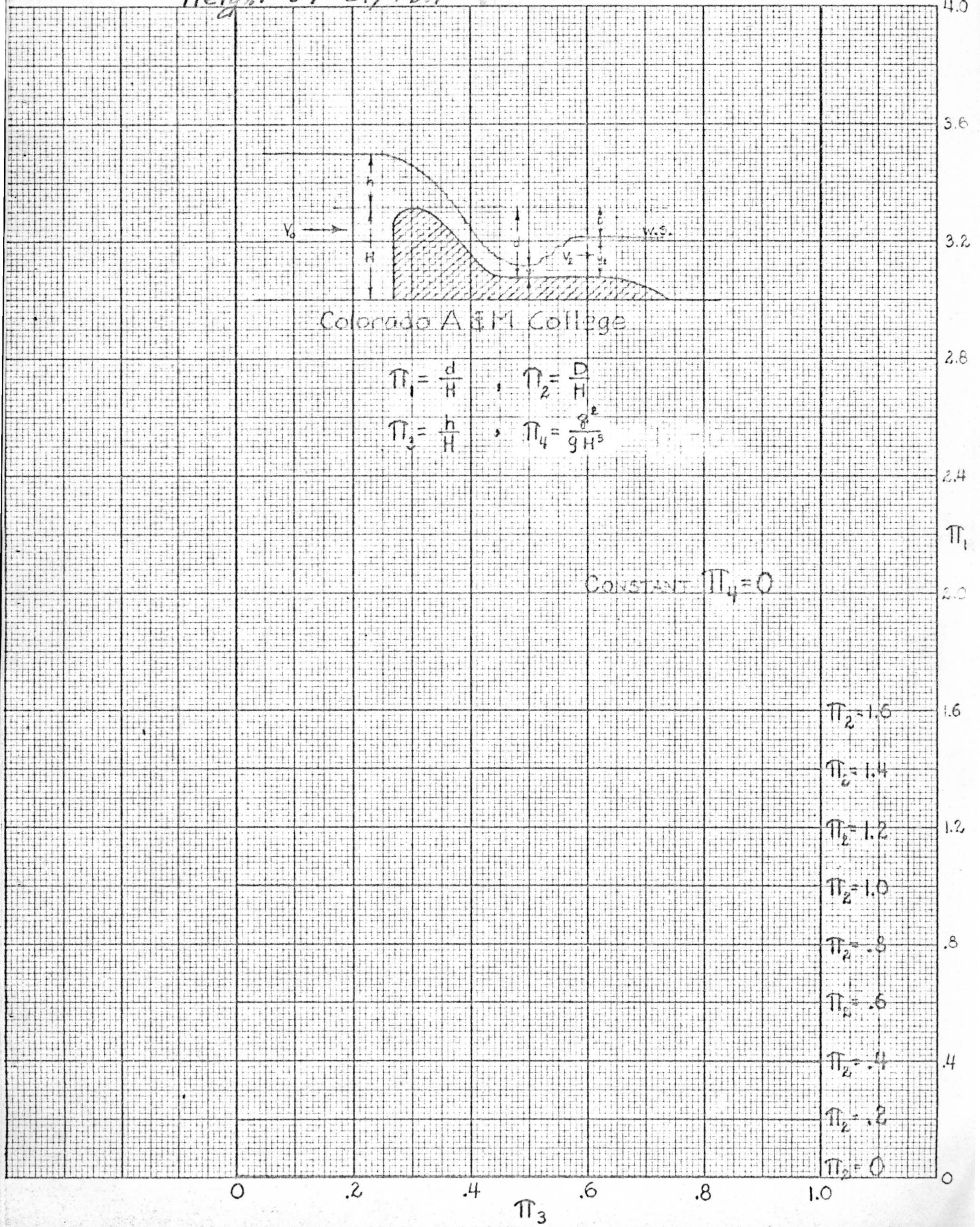
$\pi_2 = \frac{D}{H}$ $\qquad \pi_4 = \frac{q^2}{g H^3}$

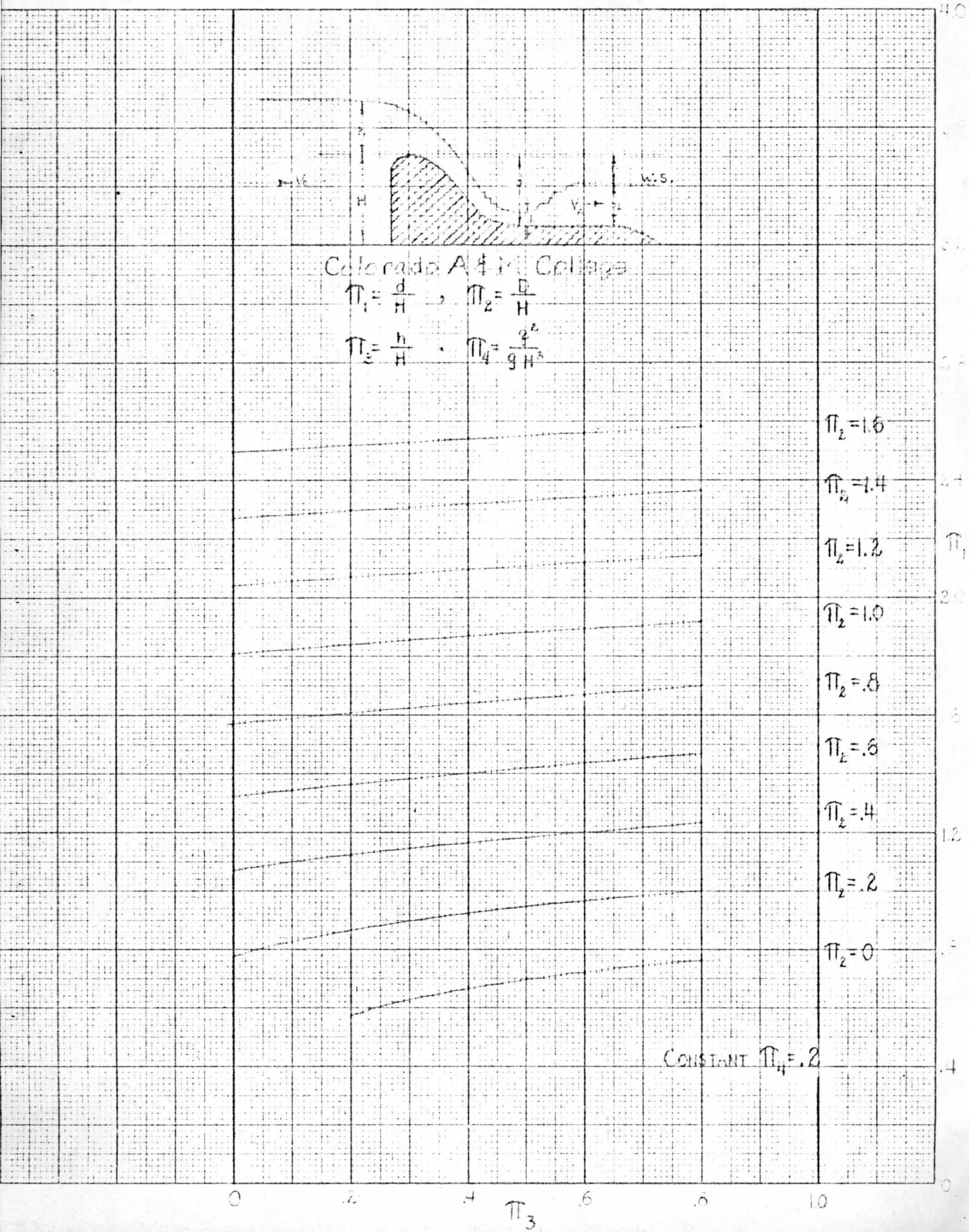
$q = V_o y_o$ $\qquad Y = \frac{y_1}{H}$



$$\pi_1 = \left[\frac{Y}{2}\left(\sqrt{1+\frac{8\pi_4^2}{Y^3}} - 1\right) + \pi_2\right] = \left[\frac{\pi_4^2}{2Y^2} + Y\right] - \left[\pi_3 + \frac{\pi_4^2}{2(\pi_3+1)^2}\right]$$

Height of Apron—





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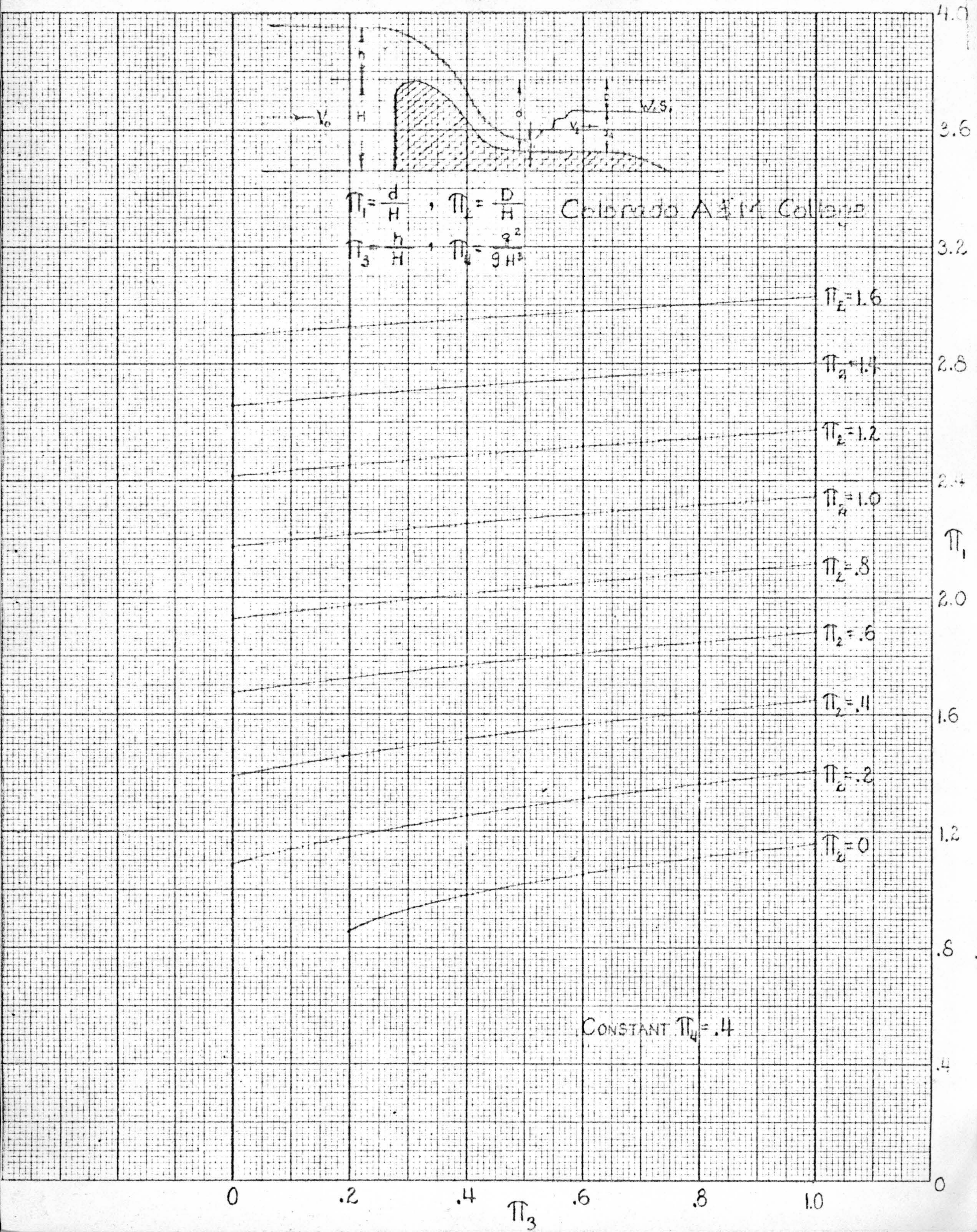
$\Pi_1 = \frac{d}{H}$, $\Pi_2 = \frac{D}{H}$
$\Pi_3 = \frac{h}{H}$, $\Pi_4 = \frac{q^2}{g H^3}$
W.S.
H
$\Pi_2 = 1.6$
$\Pi_2 = 1.4$
$\Pi_2 = 1.2$
$\Pi_2 = 1.0$
$\Pi_2 = .8$
$\Pi_2 = .6$
$\Pi_2 = .4$
$\Pi_2 = .2$
$\Pi_2 = 0$
CONSTANT $\Pi_4 = .2$
Π_1
Π_3
0
.2
.4
.6
.8
1.0
4.0
2.0
1.2
.4



V_o
h
H
d
D
V_2
d_2
W.S.
$\Pi_1 = \frac{d}{H}$, $\Pi_2 = \frac{D}{H}$
$\Pi_3 = \frac{h}{H}$, $\Pi_4 = \frac{q^2}{gH^3}$
Colorado A & M College
$\Pi_2 = 1.6$
$\Pi_2 = 1.4$
$\Pi_2 = 1.2$
$\Pi_2 = 1.0$
$\Pi_2 = .8$
$\Pi_2 = .6$
$\Pi_2 = .4$
$\Pi_2 = .2$
$\Pi_2 = 0$
CONSTANT $\Pi_4 = .4$
4.0
3.6
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2.8
2.4
2.0
1.6
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0
Π_1
0
.2
.4
.6
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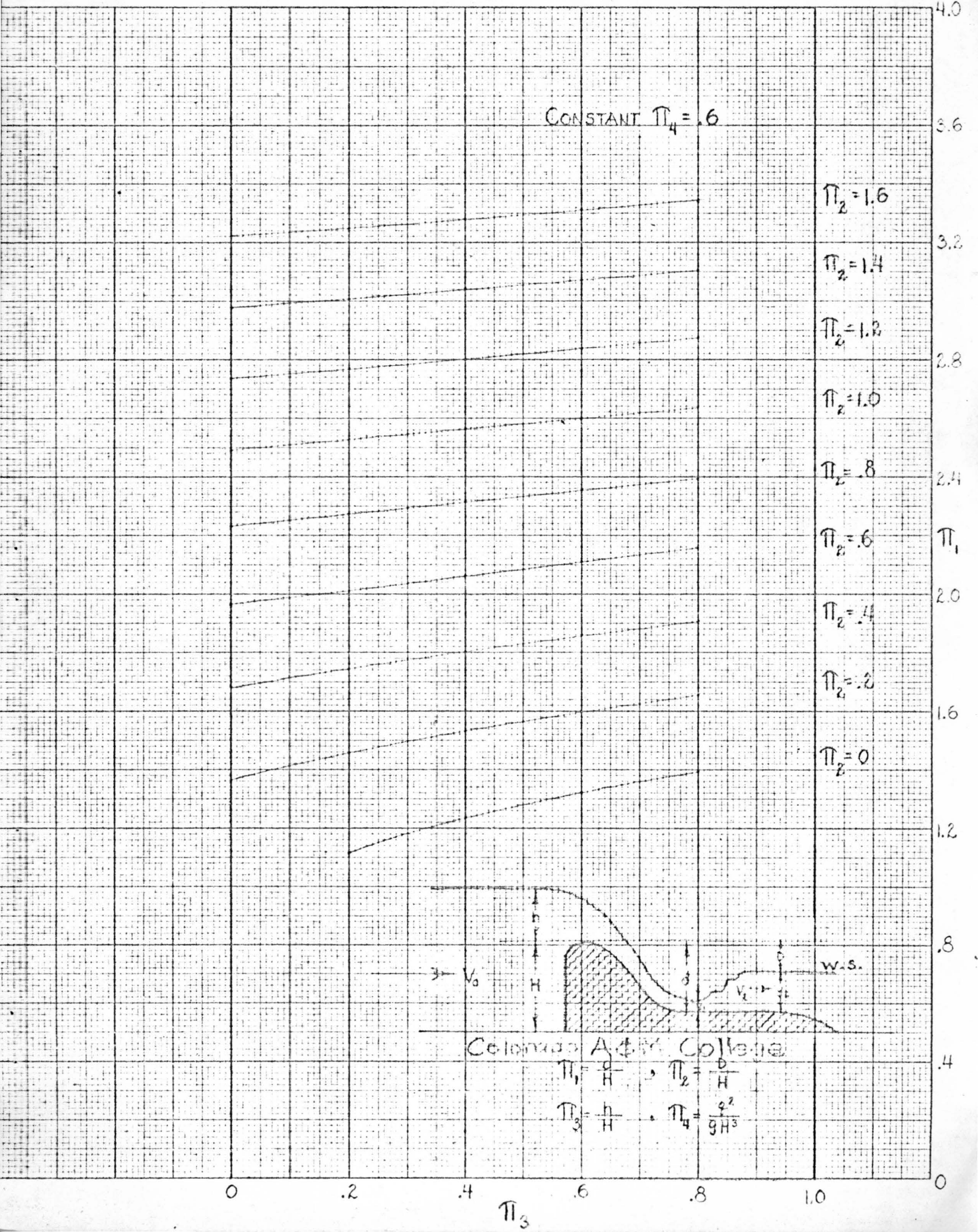
1.0
Π_3



CONSTANT Π4 = .6
Π2 = 1.6
Π2 = 1.4
Π2 = 1.2
Π2 = 1.0
Π2 = .8
Π2 = .6
Π2 = .4
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Π2 = 0
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Π3
V0
H
h
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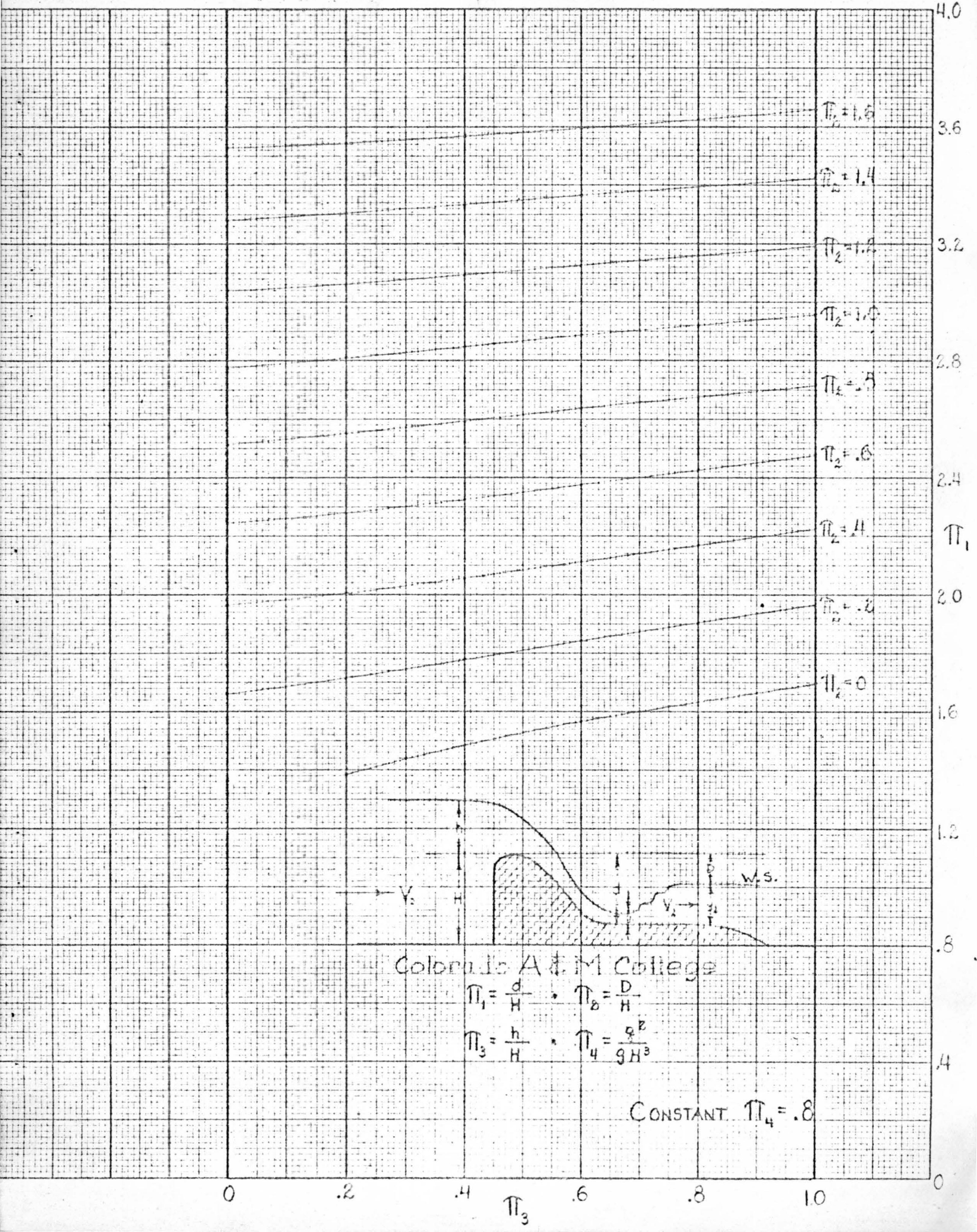
V1
W.S.
Colorado A&M College
Π1 = d/H , Π2 = D/H
Π3 = h/H , Π4 = q²/gH³



$\Pi_2 = 1.6$
$\Pi_2 = 1.4$
$\Pi_2 = 1.2$
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Π_1
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Π_3
V_1
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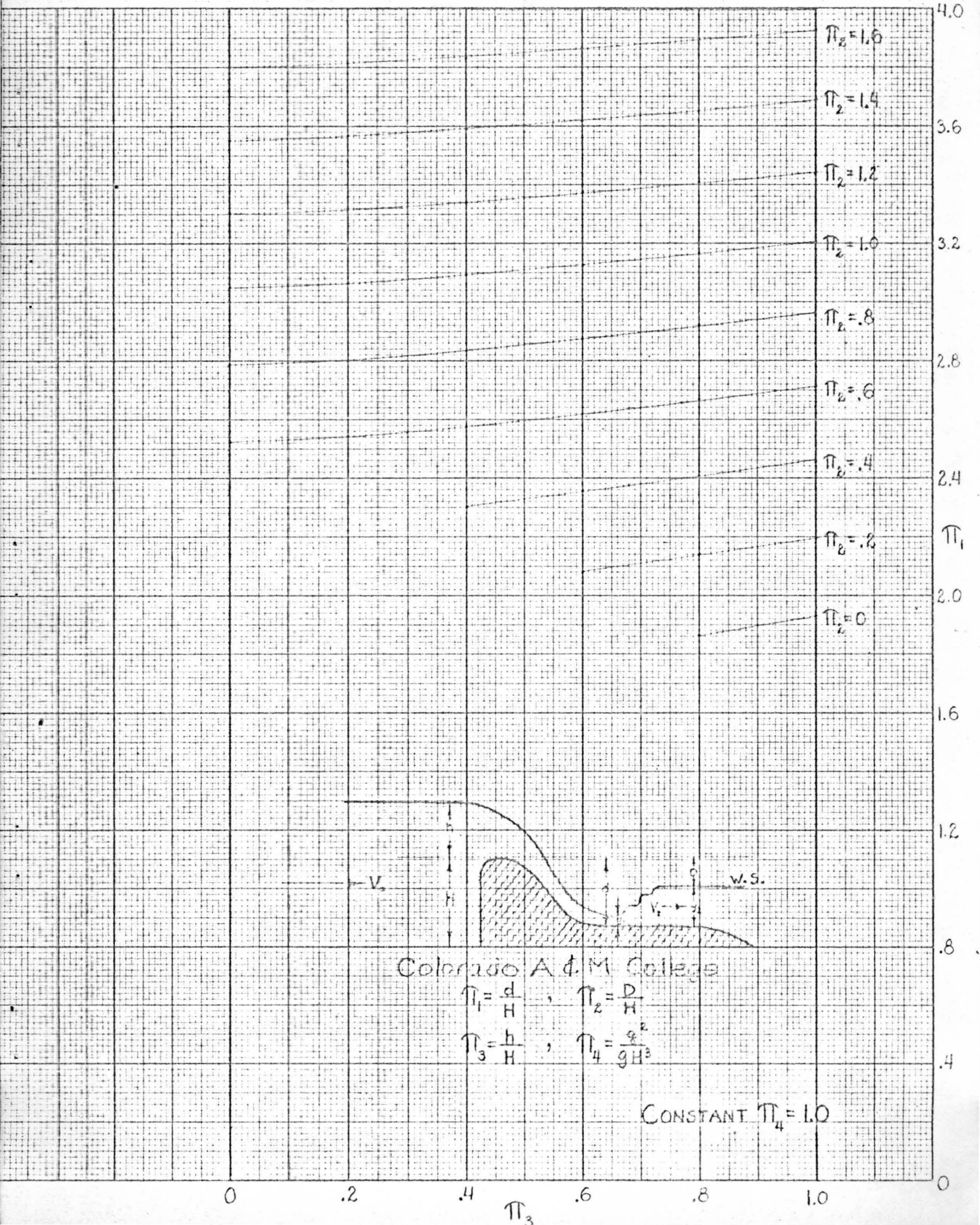
V_2
W.S.
Colorado A & M College
$\Pi_1 = \frac{d}{H}$ · $\Pi_2 = \frac{D}{H}$
$\Pi_3 = \frac{h}{H}$ · $\Pi_4 = \frac{q^2}{gH^3}$
CONSTANT $\Pi_4 = .8$



$\Pi_2 = 1.6$
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$\Pi_2 = 1.0$
$\Pi_2 = .8$
$\Pi_2 = .6$
$\Pi_2 = .4$
$\Pi_2 = .2$
$\Pi_2 = 0$
4.0
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Π_1
2.0
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0
0
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.6
.8
1.0
Π_3
W.S.
Colorado A & M College
$\Pi_1 = \frac{d}{H}$, $\Pi_2 = \frac{D}{H}$
$\Pi_3 = \frac{h}{H}$, $\Pi_4 = \frac{q^2}{gH^3}$
CONSTANT $\Pi_4 = 1.0$