Civil Engineering Department
Colorado Agricultural and Mechanical College
Fort Collins, Colorado

TEMPERATURE DISTRIBUTION IN THE BOUNDARY LAYER OF AN AIRPLANE WING WITH A LINE SOURCE OF HEAT AT THE STAGNATION EDGE

Part 1. Symmetric Wing in Symmetric Flow

by

Chia-Shun Yih, Jack E. Cermak, and Richard T. Shen

Prepared for the
Office of Naval Research
Navy Department
Washington, D. C.

Under ONR Contract No. Nonr-544(00)

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Abstract

Given a symmetric airplane wing in a symmetric flow with an insulated surface and a line source of heat at the stagnation edge, it is proposed to calculate the temperature distribution in the boundary layer, the effect of free convection being neglected. With the velocity distribution given by previous writers in the form of a power series of the curvilinear abscissa, a similar expansion is assumed for the temperature distribution and the resulting ordinary differential equations for the functions occurring in the coefficients of the powers are solved numerically. Since these functions do not depend on the form of the symmetric wing section, they are universal functions that can be applied to symmetric wing sections of any form in flow without yaw, for calculating the temperature distribution in the boundary layer. The results will also be found to be useful when an unsymmetric wing or a symmetric wing with yaw is considered.

1. Introduction

As a first step toward the investigation of the possibility of preventing icing on airplane wings by a line source of heat and mass, one considers a symmetric wing in symmetric flow with an insulated surface and a line source of heat at the stagnation edge, and seeks to find the temperature distribution in the boundary layer. If the effect of free convection is neglected, the velocity distribution in the boundary layer is already given by previous writers in the form of a power series of the curvilinear abscissa -- Goldstein (1:151). A similar series can be assumed for the temperature distribution. When this series is substituted in the boundary-layer equation of diffusion, ordinary differential equations for the functions occurring in the coefficients of the powers will be obtained. These equations, which do not depend on the form of the wing section, will be solved numerically. The solutions provide universal functions which are not only sufficient for calculating the temperature distribution in the boundary layer of a symmetric wing of any form in symmetric flow, but also applicable to unsymmetric wings of any form or to any symmetric wing in flow with yaw. Calculations for the unsymmetric and yaw cases are not included in this report.

2. Velocity Distribution in the Boundary Layer

In a plane perpendicular to the length of the wing, the trace of the stagnation edge will be taken to be the origin, x_1 will be measured along the boundary of the wing, and y_1 will be measured in a direction normal to that of x_1. Denoting by u_1 and v_1 the velocity components in the x_1- and y_1- directions, respectively, by U_0 the velocity of approach, and by U_1 the potential velocity outside of the boundary layer in the direction of x_1, one has the following boundary-layer equation of motion:

$$U_1 \frac{\partial u_1}{\partial x_1} + v_1 \frac{\partial u_1}{\partial y_1} = U_1 \frac{dU_1}{dx_1} + \nu \frac{\partial^2 u_1}{\partial y_1^2} \tag{1}$$

U_1 being a function of x_1 only, and ν being the kinematic viscosity.

In order to eliminate ν explicitly and to deal only with dimensionless quantities, one makes the following substitutions:

$$x = \frac{x_1}{D}, \quad y = \sqrt{\frac{U_o}{\nu D}}\, y_1, \quad u = \frac{u_1}{U_o}, \quad v = \sqrt{\frac{D}{U_o \nu}}\, v_1, \quad U = \frac{U_1}{U_o} \tag{2}$$

where D is a reference length of the wing section, and transforms (1) into

$$u u_x + v u_y = U U' + u_{yy} \tag{3}$$

where $U' = \frac{dU}{dx}$.

The equation of continuity can be written in the dimensionless form

$$u_x + v_y = 0 \tag{4}$$

which permits the use of a dimensionless steam-function ψ such that

$$u = \psi_y , \qquad v = -\psi_x$$

Thus (3) becomes

$$\psi_y \psi_{xy} - \psi_x \psi_{yy} = U U' + \psi_{yyy} \tag{5}$$

With the dimensionless potential velocity given

$$U = a_1 x + a_3 x^3 + a_5 x^5 + \cdots \tag{6}$$

where $a_1, a_3, a_5, \ldots$ are dimensionless quantities depending only on the form of the wing section, and where U is an odd function of x due to the definitions of U is an odd function of x due to the definitions of U and x and due to symmetry, one may assume

$$\psi = \psi_1 x + \psi_3 x^3 + \psi_5 x^5 + \psi_7 x^7 + \cdots \tag{7}$$

where

$$\psi_1 = f_1 \sqrt{a_1}, \quad \psi_3 = \frac{4a_3}{\sqrt{a_1}} f_3, \quad \psi_5 = \frac{6a_5}{\sqrt{a_1}}\left(g_5 + \frac{a_3^2}{a_1 a_5} h_5\right), \quad \psi_7 = \frac{8a_7}{\sqrt{a_1}}\left(g_7 + \frac{a_3 a_5}{a_1 a_7} h_7 + \frac{a_3^3}{a_1^2 a_7} k_7\right) \tag{8}$$

where f, g, h, etc. are functions of the new variable

$$\eta = y\sqrt{a_1} \tag{9}$$

From (7) and (8) one has

$$u = \psi_y = a_1 f_1' x + 4 a_3 f_3' x^3 + 6 a_5 \left(g_5' + \frac{a_3^2}{a_1 a_5} h_5' \right) x^5 + \cdots \tag{10}$$

$$v = -\psi_x = -\sqrt{a_1} \left[f_1 + \frac{12 a_3}{a_1} f_3 x^2 + \frac{30 a_5}{a_1} \left(g_5 + \frac{a_3^2}{a_1 a_5} h_5 \right) x^4 + \cdots \right] \tag{11}$$

A series of equations in f, g, h etc. are furnished by (5) whose boundary conditions are

$$f(0) = g(0) = h(0) = \cdots = 0 \quad (\text{for all subscripts}) \tag{12}$$

$$f_1'(\infty) = 1, \quad f_3'(\infty) = \frac{1}{4} \quad \text{etc.} \tag{13}$$

The solutions for f_1, f_3, g_5 and h_5 are given by Goldstein (1:151) and Schlichting (2:122).

3. Temperature Distribution in the Boundary Layer

As the fluid passes the heat source, it will carry away some heat and cause the temperature in the thermal boundary layer to rise, while the temperature outside of the layer remains practically the same as the ambient temperature T_0. The amount of heat carried into the boundary layer is a measure of the strength of the line source. Denoting this strength by $2H$, and the temperature at any point by T, one has

$$H = \rho c_p \int_0^\infty u_1 (T - T_0)\, dy_1 \tag{14}$$

where ρ is the density and c_p is the specific heat at constant pressure of the fluid. Since the wing has an insulated surface, H should be a constant independent of x_1. Using the dimensionless parameters

$$\theta = \frac{T - T_o}{T_o}, \quad u = \frac{u_1}{U_o} \tag{15}$$

and the new variable given in (9), (14) can be written as

$$H = \rho c_p U_o T_o \sqrt{\frac{\nu D}{U_o a_1}} \int_0^\infty \theta u \, d\eta$$

or

$$\frac{H \sqrt{a_1}}{\rho c_p T_o \sqrt{\nu D U_o}} = \int_0^\infty \theta u \, d\eta \tag{16}$$

Neglecting the heat generated by compressibility and by viscous shear, the boundary-layer equation of heat diffusion can be written

$$u \theta_x + v \theta_y = \frac{1}{\sigma} \theta_{yy} \tag{17}$$

where $\sigma = \frac{\nu}{\alpha}$ is the Prandtl number, α being the thermal diffusivity. Remembering (10) and (16), one assumes

$$\theta = \frac{H}{\rho c_p T_o U_o D} \sqrt{\frac{R}{a_1}} \frac{1}{\sqrt{x^2}} (\theta_o + \theta_2 x^2 + \theta_4 x^4 + \cdots) \tag{18}$$

where

$$\theta_o = F_o(\eta), \; \theta_2 = \frac{4 a_3}{a_1} F_2(\eta), \; \theta_4 = \frac{6 a_5}{a_1} \left[G_4(\eta) + \frac{a_3^2}{a_1 a_5} H_4(\eta) \right], \cdots \tag{19}$$

and $R = \frac{U_o D}{\nu}$. Substituting (18) into (17), and equating the coefficients of equal powers of x on both sides, one has the following series of ordinary differential equations:

$$\frac{1}{\sigma} F_o'' = -(f_1' F_o + f_1 F_o') \tag{20}$$

$$\frac{1}{\sigma} F_2'' = f_1' F_2 - f_1 F_2' - (f_3' F_o + 3 f_3 F_o') \tag{21}$$

$$\frac{1}{\sigma} G_4'' = 3 f_1' G_4 - f_1 G_4' - (g_5' F_o + 5 g_5 F_o') \tag{22}$$

$$\frac{1}{\sigma} H_4'' = 3 f_1' H_4 - f_1 H_4' - (h_5' F_o + 5 h_5 F_o' - \tfrac{8}{3} f_3' F_2 + 8 f_3 F_2') \tag{23}$$

where the primes denote differentiation with respect to η. The boundary conditions at the insulated surface of the wing (where $\eta = 0$) are

$$F_0'(0) = F_2'(0) = G_4'(0) = H_4'(0) = \cdots = 0 \tag{24}$$

while those at points outside of the thermal boundary layer can be written

$$F_0(\infty) = F_2(\infty) = G_4(\infty) = H_4(\infty) = \cdots = 0 \tag{25}$$

The integral condition imposed by (16) can be decomposed after a straight-forward calculation with (10) and (18), into the following integral conditions:

$$\int_0^\infty F_0 f_1' \, d\eta = 1 \tag{26}$$

$$\int_0^\infty (f_3' F_0 + f_1' F_2) \, d\eta = 0 \tag{27}$$

$$\int_0^\infty (g_5' F_0 + f_1' G_4) \, d\eta = 0 \tag{28}$$

$$\int_0^\infty (3 f_1' H_4 + 3 h_5' F_0 + 8 f_3' F_2) \, d\eta = 0 \tag{29}$$

It will now be shown that by virtue of (21) to (25) the conditions (27) to (29) are always satisfied. For instance by virtue of (21) one can write

$$2(f_3' F_0 + f_1' F_2) = \frac{1}{\sigma} F_2'' + (f_1' F_2 + f_1 F_2') + 3(f_3' F_0 + f_3 F_0')$$

integration of which yields

$$2\int_0^\infty (f_3' F_0 + f_1' F_2) \, d\eta = \left[\frac{1}{\sigma} F_2' + f_1 F_2 + 3 f_3 F_0\right]_0^\infty = 0$$

by (24), (25) and (12), so that (27) is satisfied. Writing from (22) and (23)

$$4(g_5' F_0 + f_1 G_4) = \frac{1}{\sigma} G_4'' + (f_1 G_4' + f_1' G_4) + 5(g_5 F_0' + g_5' F_0)$$

$$\frac{4}{3}(3f_1' H_4 + 3h_5' F_0 + 8f_3' F_2) = \frac{1}{\sigma} H_4'' + (f_1 H_4' + f_1' H_4) + 5(h_5 F_0' + h_5' F_0) + 8(f_3 F_2' + f_3' F_2)$$

(28) and (29) can be similarly demonstrated upon integration. In fact, all the integral conditions subsequent to (29) are always satisfied if the differential equations and the boundary conditions (24) and (25) are satisfied. Thus, only (26) remains, which takes the place of (16).

Integrating (20), one obtains

$$\frac{1}{\sigma} F_0' = - f_1 F_0 \tag{30}$$

the constant of integration being zero since $F_0'(0) = f_1(0) = 0$. A second integration gives

$$F_0 = K e^{-\sigma \int_0^\eta f_1 d\eta} \tag{31}$$

where K is determined from (26) to be

$$K = \left[\int_0^\infty e^{-\sigma \int_0^\eta f_1 d\eta} f_1' d\eta \right]^{-1} \tag{32}$$

Since the functions f_1, f_3, g_5, h_5 as well as their derivatives are tabulated by Goldstein (1:151) and Schlichting (2:122), one can calculate F_0 numerically. The results are given in Table 1 for $\sigma = 0.73$.

In solving (21) numerically one takes a trial value for $F_2(0)$, and utilising the boundary condition $F_2'(0) = 0$, proceeds to calculate the function F_2. If the condition at infinity is not satisfied, another trial value for $F_2(0)$ is taken, until the condition $F_2(\infty) = 0$ is satisfied. This method of solution also applies to (22) and (23). Values for g_5 and h_5 and their first two derivatives for intermediate values of η not entered in Table III of Schlichting (2:122) were obtained by interpolation according to the method given by Sokolnikoff (3:551). The results for F_2, G_4, and H_4 are given respectively in Tables 2, 3, and 4. The functions F_0, F_2, G_4 and H_4 are represented graphically in Figure 1.

4. A Particular Example

In order to illustrate the application of the foregoing results, temperature distributions within the boundary layer of a right circular cylinder are calculated. With the longitudinal axis of the cylinder oriented perpendicular to the direction of the undisturbed velocity field, radial temperature distributions are computed for each 10° of central angle -- the angle being measured from the stagnation line -- from 10° to and including 80°.

The dimensionless quantity $\beta = \dfrac{\theta \rho c_p T_0 U_0 D}{H\sqrt{R}}$ is calculated as a function of η for each of the values of x chosen according to the foregoing paragraph. By (18), β is found to be

$$\beta = \frac{1}{\sqrt{a_1 x^2}}\left(\theta_0 + \theta_2 x^2 + \theta_4 x^4 + \cdots\cdots\right) \tag{33}$$

and is computed for various values of η at a particular value of x by use of (19) and Tables 1 through 4. The constants

a_1 , a_3 , and a_5 are taken to be those experimentally determined by Hiemenz -- see Goldstein (1:150) -- under the following conditions:

$$U_o = 19.2 \text{ cm/sec}$$

$$D = 9.74 \text{ cm}$$

$$R = 1.85 \times 10^4 ;$$

therefore, the values for β given in Table 5 are applicable for values of R nearly equal to or equal to $\underline{1.85 \times 10^4}$.

Figure 2 shows graphically how β varies with η for the various values of x . Having obtained values for β , the temperature distribution may be calculated for particular ambient conditions and fluid properties, heat source strength, and cylinder diameter by means of (15) and (18)

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η	F_0	F_0'	η	F_0	F_0'
0.0	0.7269	-0.0000	3.5	0.0335	-0.0697
0.1	0.7267	-0.0032	3.6	0.0271	-0.0583
0.2	0.7260	-0.0123	3.7	0.0217	-0.0484
0.3	0.7241	-0.0270	3.8	0.0173	-0.0399
0.4	0.7205	-0.0463	3.9	0.0137	-0.0326
0.5	0.7147	-0.0697	4.0	0.0108	-0.0264
0.6	0.7064	-0.0963	4.1	0.0084	-0.0212
0.7	0.6953	-0.1252	4.2	0.0065	-0.0169
0.8	0.6814	-0.1554	4.3	0.0050	-0.0134
0.9	0.6642	-0.1860	4.4	0.0038	-0.0105
1.0	0.6442	-0.2159	4.5	0.0029	-0.0081
1.1	0.6211	-0.2443	4.6	0.0022	-0.0063
1.2	0.5954	-0.2703	4.7	0.0016	-0.0048
1.3	0.5672	-0.2932	4.8	0.0012	-0.0037
1.4	0.5369	-0.3122	4.9	0.0009	-0.0028
1.5	0.5049	-0.3270	5.0	0.0006	-0.0021
1.6	0.4716	-0.3373	5.1	0.0005	-0.0015
1.7	0.4375	-0.3430	5.2	0.0003	-0.0011
1.8	0.4032	-0.3440	5.3	0.0002	-0.0008
1.9	0.3689	-0.3406	5.4	0.0002	-0.0006
2.0	0.3352	-0.3333	5.5	0.0001	-0.0004
2.1	0.3024	-0.3222	5.6	0.0001	-0.0003
2.2	0.2709	-0.3080	5.7	0.0001	-0.0002
2.3	0.2409	-0.2912	5.8	0.0000	-0.0002
2.4	0.2127	-0.2725	5.9		-0.0001
2.5	0.1864	-0.2523	6.0		-0.0001
2.6	0.1622	-0.2314	6.1		-0.0001
2.7	0.1401	-0.2101	6.2		-0.0000
2.8	0.1202	-0.1889	6.3		
2.9	0.1023	-0.1683	6.4		
3.0	0.0865	-0.1486	6.5		
3.1	0.0726	-0.1300	6.6		
3.2	0.0605	-0.1127	6.7		
3.3	0.0500	-0.0968	6.8		
3.4	0.0411	-0.0825	6.9		

Table 1 -- Values of F_0 and F_0'.

η	F_2	F_2'	F_2''	η	F_2	F_2'	F_2''
0.0	-0.0528524	-0.0000	-0.0000	3.5	-0.0558	0.0804	-0.0763
0.1	-0.0529	-0.0000	-0.0404	3.6	-0.0478	0.0727	-0.0779
0.2	-0.0529	-0.0040	-0.0746	3.7	-0.0405	0.0649	-0.0770
0.3	-0.0533	-0.0115	-0.1022	3.8	-0.0340	0.0572	-0.0742
0.4	-0.0544	-0.0217	-0.1222	3.9	-0.0283	0.0498	-0.0698
0.5	-0.0566	-0.0339	-0.1339	4.0	-0.0233	0.0428	-0.0643
0.6	-0.0600	-0.0473	-0.1367	4.1	-0.0190	0.0364	-0.0582
0.7	-0.0647	-0.0610	-0.1304	4.2	-0.0154	0.0306	-0.0518
0.8	-0.0708	-0.0740	-0.1154	4.3	-0.0123	0.0254	-0.0453
0.9	-0.0782	-0.0856	-0.0925	4.4	-0.0098	0.0209	-0.0391
1.0	-0.0868	-0.0948	-0.0628	4.5	-0.0077	0.0170	-0.0332
1.1	-0.0962	-0.1011	-0.0281	4.6	-0.0060	0.0136	-0.0279
1.2	-0.1064	-0.1039	0.0097	4.7	-0.0046	0.0108	-0.0231
1.3	-0.1167	-0.1030	0.0484	4.8	-0.0035	0.0085	-0.0188
1.4	-0.1270	-0.0981	0.0859	4.9	-0.0027	0.0067	-0.0152
1.5	-0.1369	-0.0895	0.1201	5.0	-0.0020	0.0051	-0.0121
1.6	-0.1458	-0.0775	0.1504	5.1	-0.0015	0.0039	-0.0096
1.7	-0.1536	-0.0625	0.1724	5.2	-0.0011	0.0030	-0.0074
1.8	-0.1598	-0.0452	0.1881	5.3	-0.0008	0.0022	-0.0057
1.9	-0.1643	-0.0264	0.1959	5.4	-0.0006	0.0016	-0.0044
2.0	-0.1670	-0.0068	0.1962	5.5	-0.0004	0.0012	-0.0033
2.1	-0.1677	0.0128	0.1891	5.6	-0.0003	0.0009	-0.0025
2.2	-0.1664	0.0317	0.1755	5.7	-0.0002	0.0006	-0.0018
2.3	-0.1632	0.0492	0.1565	5.8	-0.0001	0.0005	-0.0013
2.4	-0.1583	0.0649	0.1333	5.9	-0.0001	0.0003	-0.0010
2.5	-0.1518	0.0782	0.1074	6.0	-0.0001	0.0002	-0.0007
2.6	-0.1440	0.0890	0.0801	6.1	-0.0000	0.0002	-0.0005
2.7	-0.1351	0.0970	0.0526	6.2		0.0001	-0.0003
2.8	-0.1254	0.1022	0.0263	6.3		0.0001	-0.0002
2.9	-0.1152	0.1049	0.0018	6.4		0.0000	-0.0002
3.0	-0.1047	0.1050	-0.0197	6.5			-0.0001
3.1	-0.0942	0.1031	-0.0381	6.6			-0.0001
3.2	-0.0839	0.0993	-0.0529	6.7			-0.0001
3.3	-0.0739	0.0940	-0.0642	6.8			-0.0000
3.4	-0.0645	0.0875	-0.0719	6.9			

Table 2 -- Values of F_2, F_2', and F_2''.

η	G_4	G_4'	G_4''	η	G_4	G_4'	G_4''
0.0	-0.0283942	-0.0000	0	3.5	-0.0426	0.0616	-0.0595
0.1	-0.0284	-0.0000	-0.0382	3.6	-0.0364	0.0556	-0.0603
0.2	-0.0284	-0.0038	-0.0703	3.7	-0.0309	0.0496	-0.0594
0.3	-0.0288	-0.0109	-0.0953	3.8	-0.0259	0.0437	-0.0569
0.4	-0.0299	-0.0204	-0.1125	3.9	-0.0216	0.0380	-0.0534
0.5	-0.0319	-0.0316	-0.1214	4.0	-0.0178	0.0326	-0.0491
0.6	-0.0351	-0.0438	-0.1218	4.1	-0.0145	0.0277	-0.0444
0.7	-0.0394	-0.0560	-0.1139	4.2	-0.0117	0.0233	-0.0394
0.8	-0.0450	-0.0673	-0.0984	4.3	-0.0094	0.0193	-0.0344
0.9	-0.0518	-0.0772	-0.0760	4.4	-0.0075	0.0159	-0.0297
1.0	-0.0595	-0.0848	-0.0484	4.5	-0.0059	0.0129	-0.0252
1.1	-0.0680	-0.0896	-0.0171	4.6	-0.0046	0.0104	-0.0212
1.2	-0.0769	-0.0913	0.0158	4.7	-0.0036	0.0083	-0.0175
1.3	-0.0861	-0.0897	0.0488	4.8	-0.0027	0.0065	-0.0143
1.4	-0.0950	-0.0849	0.0799	4.9	-0.0021	0.0051	-0.0116
1.5	-0.1035	-0.0769	0.1075	5.0	-0.0016	0.0040	-0.0092
1.6	-0.1112	-0.0661	0.1303	5.1	-0.0012	0.0030	-0.0073
1.7	-0.1178	-0.0531	0.1474	5.2	-0.0009	0.0023	-0.0057
1.8	-0.1231	-0.0383	0.1582	5.3	-0.0006	0.0017	-0.0044
1.9	-0.1270	-0.0225	0.1625	5.4	-0.0005	0.0013	-0.0034
2.0	-0.1292	-0.0063	0.1606	5.5	-0.0003	0.0009	-0.0026
2.1	-0.1298	0.0098	0.1528	5.6	-0.0002	0.0007	-0.0019
2.2	-0.1289	0.0251	0.1401	5.7	-0.0002	0.0005	-0.0015
2.3	-0.1264	0.0391	0.1234	5.8	-0.0001	0.0004	-0.0010
2.4	-0.1225	0.0514	0.1038	5.9	-0.0001	0.0003	-0.0007
2.5	-0.1173	0.0618	0.0824	6.0	-0.0001	0.0002	-0.0005
2.6	-0.1111	0.0700	0.0603	6.1	-0.0000	0.0001	-0.0004
2.7	-0.1041	0.0761	0.0384	6.2		0.0001	-0.0003
2.8	-0.0965	0.0799	0.0177	6.3		0.0001	-0.0002
2.9	-0.0885	0.0817	-0.0012	6.4		0.0000	-0.0001
3.0	-0.0804	0.0815	-0.0178	6.5			-0.0001
3.1	-0.0722	0.0798	-0.0317	6.6			-0.0001
3.2	-0.0642	0.0766	-0.0427	6.7			-0.0000
3.3	-0.0566	0.0723	-0.0510	6.8			
3.4	-0.0493	0.0672	-0.0564	6.9			

Table 3 -- Values of G_4, G_4', and G_4''.

η	H_4	H_4'	H_4''	η	H_4	H_4'	H_4''
0.0	0.017823	-0.0000	-0.0000	3.5	0.1109	-0.0995	-0.0197
0.1	0.0178	-0.0000	-0.0074	3.6	0.1010	-0.1015	0.0066
0.2	0.0178	-0.0007	-0.0111	3.7	0.0908	-0.1008	0.0292
0.3	0.0178	-0.0018	-0.0108	3.8	0.0807	-0.0979	0.0477
0.4	0.0176	-0.0029	-0.0058	3.9	0.0709	-0.0931	0.0619
0.5	0.0173	-0.0035	0.0042	4.0	0.0616	-0.0869	0.0719
0.6	0.0169	-0.0031	0.0191	4.1	0.0529	-0.0797	0.0779
0.7	0.0166	-0.0012	0.0382	4.2	0.0450	-0.0719	0.0804
0.8	0.0165	0.0027	0.0602	4.3	0.0378	-0.0639	0.0798
0.9	0.0168	0.0087	0.0832	4.4	0.0314	-0.0559	0.0768
1.0	0.0176	0.0170	0.1051	4.5	0.0258	-0.0482	0.0720
1.1	0.0193	0.0275	0.1238	4.6	0.0210	-0.0410	0.0659
1.2	0.0221	0.0399	0.1368	4.7	0.0169	-0.0344	0.0591
1.3	0.0261	0.0536	0.1424	4.8	0.0134	-0.0285	0.0519
1.4	0.0314	0.0678	0.1395	4.9	0.0106	-0.0233	0.0448
1.5	0.0382	0.0818	0.1273	5.0	0.0082	-0.0189	0.0381
1.6	0.0464	0.0945	0.1068	5.1	0.0064	-0.0151	0.0318
1.7	0.0558	0.1052	0.0762	5.2	0.0048	-0.0119	0.0262
1.8	0.0664	0.1128	0.0402	5.3	0.0037	-0.0092	0.0212
1.9	0.0776	0.1168	-0.0003	5.4	0.0027	-0.0071	0.0170
2.0	0.0893	0.1168	-0.0429	5.5	0.0020	-0.0054	0.0134
2.1	0.1010	0.1125	-0.0850	5.6	0.0015	-0.0041	0.0104
2.2	0.1122	0.1040	-0.1240	5.7	0.0011	-0.0030	0.0080
2.3	0.1226	0.0916	-0.1578	5.8	0.0008	-0.0022	0.0061
2.4	0.1318	0.0758	-0.1845	5.9	0.0005	-0.0016	0.0046
2.5	0.1394	0.0574	-0.2030	6.0	0.0004	-0.0012	0.0034
2.6	0.1451	0.0371	-0.2126	6.1	0.0003	-0.0008	0.0025
2.7	0.1488	0.0158	-0.2131	6.2	0.0002	-0.0006	0.0018
2.8	0.1504	-0.0055	-0.2049	6.3	0.0001	-0.0004	0.0013
2.9	0.1498	-0.0260	-0.1891	6.4	0.0001	-0.0003	0.0009
3.0	0.1472	-0.0449	-0.1670	6.5	0.0001	-0.0002	0.0006
3.1	0.1428	-0.0616	-0.1402	6.6	0.0000	-0.0001	0.0004
3.2	0.1366	-0.0756	-0.1104	6.7		-0.0001	0.0003
3.3	0.1290	-0.0867	-0.0794	6.8		-0.0001	0.0002
3.4	0.1204	-0.0946	-0.0487	6.9		-0.0000	0.0001

Table 4 -- Values of H_4, H_4', and H_4''.

η	10°	20°	30°	40°	50°	60°	70°	80°
0.0	4.383	2.200	1.477	1.120	0.909	0.772	0.679	0.613
0.1	4.382	2.200	1.477	1.120	0.909	0.772	0.679	0.613
0.2	4.378	2.200	1.475	1.119	0.908	0.772	0.678	0.612
0.3	4.366	2.192	1.472	1.116	0.906	0.770	0.677	0.611
0.4	4.344	2.182	1.465	1.111	0.902	0.767	0.675	0.610
0.5	4.310	2.165	1.454	1.103	0.897	0.763	0.672	0.608
0.6	4.260	2.141	1.439	1.092	0.889	0.758	0.668	0.606
0.7	4.194	2.107	1.418	1.078	0.879	0.750	0.664	0.604
0.8	4.111	2.067	1.392	1.060	0.866	0.741	0.658	0.601
0.9	4.008	2.017	1.361	1.038	0.850	0.730	0.651	0.598
1.0	3.888	1.959	1.324	1.012	0.831	0.717	0.643	0.594
1.1	3.750	1.891	1.281	0.982	0.810	0.702	0.633	0.589
1.2	3.597	1.816	1.233	0.948	0.786	0.685	0.622	0.584
1.3	3.428	1.734	1.179	0.911	0.759	0.666	0.610	0.577
1.4	3.247	1.645	1.122	0.871	0.729	0.645	0.596	0.570
1.5	3.055	1.551	1.062	0.827	0.697	0.622	0.580	0.561
1.6	2.856	1.453	0.999	0.782	0.664	0.597	0.562	0.550
1.7	2.651	1.352	0.932	0.735	0.629	0.571	0.543	0.538
1.8	2.445	1.250	0.866	0.687	0.593	0.543	0.523	0.524
1.9	2.239	1.148	0.799	0.638	0.555	0.514	0.501	0.508
2.0	2.037	1.047	0.733	0.589	0.517	0.485	0.478	0.490
2.1	1.839	0.948	0.667	0.541	0.479	0.454	0.453	0.471
2.2	1.649	0.853	0.603	0.493	0.442	0.423	0.428	0.450
2.3	1.468	0.763	0.543	0.447	0.405	0.392	0.401	0.427
2.4	1.298	0.676	0.485	0.403	0.368	0.362	0.374	0.404
2.5	1.139	0.596	0.430	0.359	0.333	0.331	0.347	0.379
2.6	0.993	0.522	0.378	0.320	0.300	0.302	0.320	0.353
2.7	0.859	0.454	0.331	0.283	0.268	0.273	0.293	0.327
2.8	0.738	0.391	0.288	0.248	0.238	0.245	0.267	0.301
2.9	0.629	0.335	0.248	0.217	0.210	0.219	0.241	0.274
3.0	0.533	0.285	0.213	0.188	0.184	0.194	0.216	0.249
3.1	0.448	0.241	0.182	0.161	0.160	0.171	0.193	0.224
3.2	0.374	0.202	0.154	0.138	0.138	0.150	0.170	0.200
3.3	0.310	0.168	0.128	0.117	0.119	0.130	0.150	0.177
3.4	0.254	0.138	0.107	0.099	0.101	0.112	0.130	0.155

Table 5 -- Values of β .

η	10°	20°	30°	40°	50°	60°	70°	80°
3.5	0.208	0.114	0.088	0.082	0.086	0.096	0.113	0.122
3.6	0.168	0.092	0.073	0.068	0.071	0.082	0.097	0.117
3.7	0.135	0.075	0.059	0.056	0.060	0.069	0.082	0.100
3.8	0.108	0.060	0.048	0.046	0.050	0.058	0.069	0.085
3.9	0.086	0.048	0.039	0.038	0.041	0.051	0.058	0.072
4.0	0.068	0.038	0.031	0.030	0.033	0.039	0.048	0.060
4.1	0.053	0.030	0.024	0.024	0.027	0.032	0.040	0.050
4.2	0.041	0.023	0.019	0.019	0.022	0.026	0.032	0.041
4.3	0.032	0.018	0.015	0.015	0.017	0.021	0.026	0.033
4.4	0.024	0.014	0.012	0.012	0.014	0.017	0.021	0.027
4.5	0.018	0.011	0.009	0.009	0.011	0.013	0.017	0.021
4.6	0.014	0.008	0.007	0.007	0.008	0.010	0.013	0.016
4.7	0.010	0.006	0.005	0.005	0.006	0.008	0.010	0.013
4.8	0.008	0.005	0.004	0.004	0.005	0.006	0.008	0.010
4.9	0.006	0.003	0.003	0.003	0.004	0.005	0.006	0.008
5.0	0.004	0.002	0.002	0.002	0.003	0.004	0.005	0.006
5.1	0.003	0.002	0.002	0.002	0.002	0.003	0.004	0.005
5.2	0.002	0.001	0.001	0.001	0.002	0.002	0.003	0.003
5.3	0.002	0.001	0.001	0.001	0.001	0.001	0.002	0.003
5.4	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002
5.5	0.001	0.000	0.000	0.000	0.001	0.001	0.001	0.001
5.6	0.001				0.000	0.001	0.001	0.001
5.7	0.000					0.000	0.000	0.001
5.8								0.000

Table 5 — Values of β (continued).

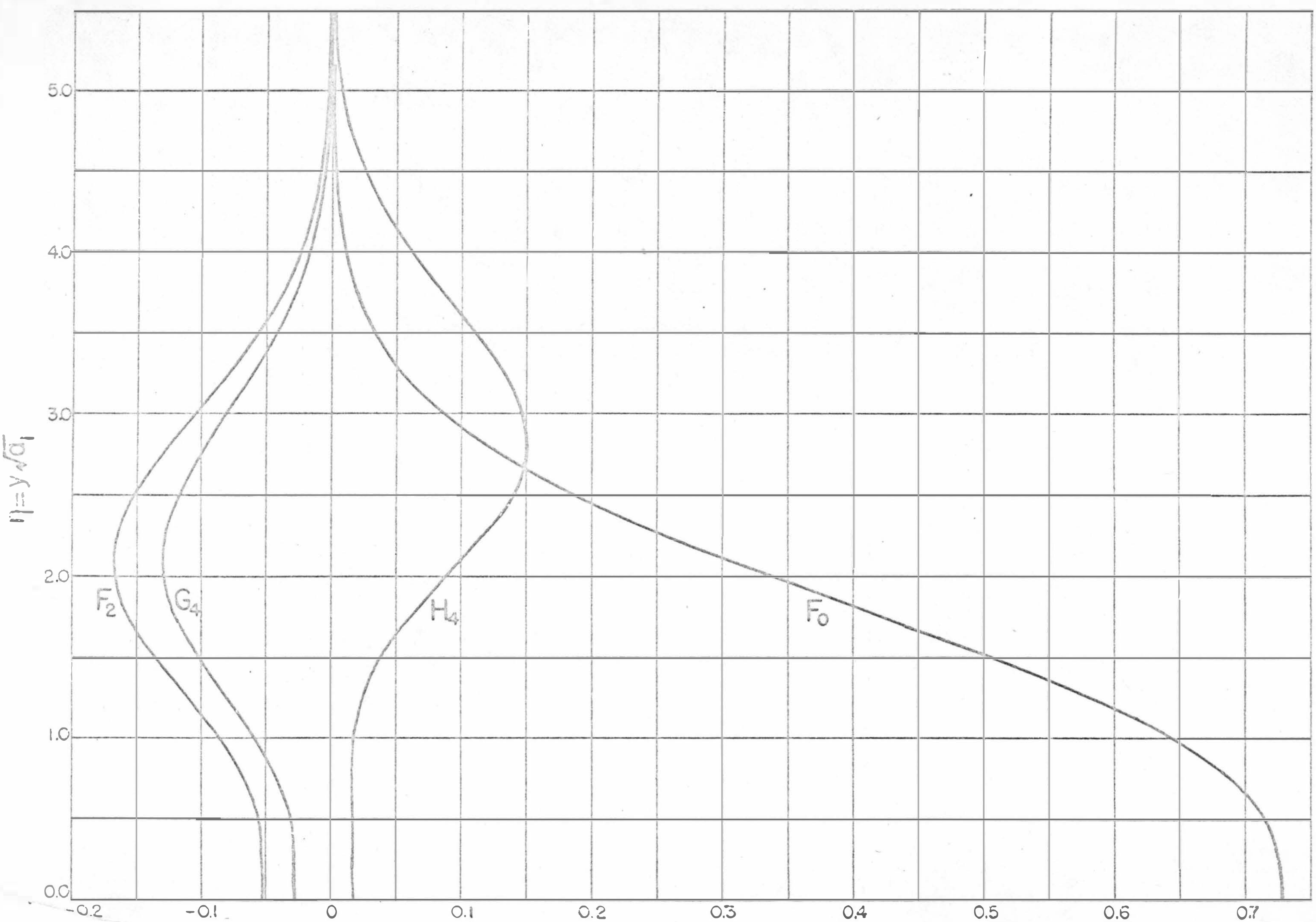

Figure 1 -- Graphs of F_0, F_2, G_4, and H_4.

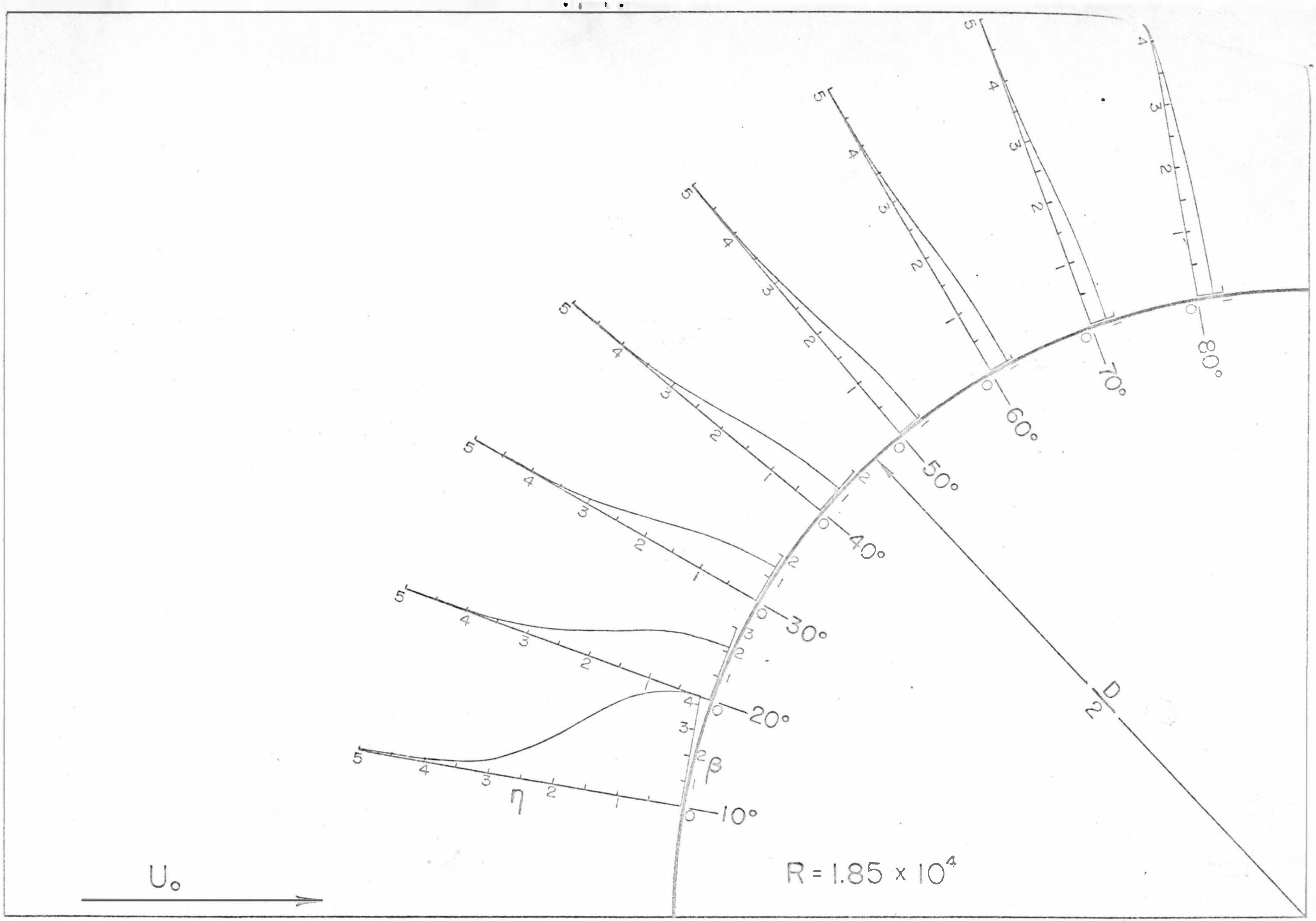

Figure 2 -- Graphs of β for a right circular cylinder in symmetric flow.