HYDROGRAPH SYNTHESIS FOR UNGAGED WATERSHEDS

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1. INTRODUCTION

1.1 The Problem

Flood estimation for ungaged catchments has long been a problem for engineers concerned with the design of spillways, diversion works, bridges, culverts, and flood control works. The great majority of such hydraulic structures is constructed on small watersheds, and since small streams have not, in the past, been gaged to nearly the same relative extent as large streams, most of the designs have to be prepared without the benefit of streamflow records.

Various techniques have been used to provide design flood estimates under such circumstances. These include flood formulae, enveloping curves of floods, regional frequency studies, and the rational method, all of which provide an estimate of the peak discharge of the design flood, and various methods of hydrograph synthesis, which provide an estimate of the complete hydrograph of the design flood. A complete hydrograph, as opposed to a peak discharge estimate, is required in all circumstances where storage is involved, such as dam spillways, and culverts where pondage is permissible, and also in cases where the time distribution of runoff is important such as flood control works.

It is the purpose of this paper to review briefly the various procedures for hydrograph synthesis with emphasis on recent developments, and as part of this review, to describe a new procedure for synthesizing the hydrograph of surface runoff by routing rainfall-excess through catchment storage.

1.2 Components of Streamflow

Hydrologists recognize that water falling as rain and subsequently appearing as streamflow can follow various paths from the point where it reaches the catchment surface to the point where it leaves the catchment. Since the storage delay imposed on the water varies with the path it takes, it has been found convenient in hydrograph synthesis to regard a streamflow hydrograph as consisting of various components, each having followed a different path to the outlet. Four components are usually recognized, as follows:

(i) Channel precipitation - water that fell directly on the water surfaces of the stream system,

(ii) Surface runoff - water that ran off overland into the stream channels,

(iii) Interflow - water that infiltrated into the ground and then seeped out again onto the surface or into the stream channels without having reached the ground-water reservoir,

(iv) Ground-water flow - water that has seeped into the stream channels from the ground-water reservoir.

These four components, and their relation to the other parts of the runoff cycle, are illustrated in Figure 1.

Channel precipitation is normally a small portion of the total streamflow, and is usually treated in combination with surface runoff. The term "surface runoff" will, therefore, be taken to include channel precipitation in the remainder of the paper. Little is known about the properties of interflow, and the nature of this type of flow is not well defined from a fluid mechanics point of view. As a result, it is usually treated either as part of the surface runoff or as part of the ground-water flow.

Since ground-water flow is normally a small component of flood flows, most hydrograph synthesis procedures have been restricted to the hydrograph of surface runoff, with allowance for ground-water flow in flood estimation being made in an arbitrary manner. Recent trends in this field, however, are towards treating ground-water flow as well as surface runoff in a rational way, or towards synthesizing the complete hydrograph without separation into its components.

1.3 Procedures for Hydrograph Synthesis

Several different approaches to hydrograph synthesis have been made in the past, and these can be classified as follows:

(i) Modified rational (or time-area) method,
(ii) Synthetic unitgraphs,
(iii) Runoff routing,
(iv) Runoff cycle models,
(v) Correlation studies.

Development of these various methods will be reviewed in sections 2 to 5 below.

In Section 4, dealing with runoff routing methods, some emphasis is given to recent work by the author on the development of a computational model for synthesis of the surface runoff hydrograph from rainfall-excess, and means for evaluating the parameters of the model from streamflow and catchment data. This procedure does not provide a means of flood estimation for ungaged catchments since it requires streamflow data for evaluation of the model parameters, but it does provide a means of representing the effect of a catchment on a rainfall-excess input, which is a necessary step in development of a flood estimation method for ungaged catchments.

Section 6 summarizes the development of hydrograph synthesis procedures and indicates the trends of current research in this field.

2. MODIFIED RATIONAL METHOD

This procedure, sometimes called the time-area method, is a development of the rational method first proposed by Ross (1921) and elaborated by Hawken (1921). It derives the surface runoff hydrograph by applying the rainfall-excess hyetograph to a time-area diagram of the catchment in accordance with the principle of superposition, just as the hyetograph is applied to a unitgraph in more modern procedures. The time-area diagram is a histogram showing the areas contributing runoff to the outlet of the catchment within various time periods after the commencement of rainfall-excess. It can be prepared in various ways, but essentially involves determination of the travel time of water from various points on the catchment to the outlet, and measurement of the areas falling within various class intervals of travel time.

This procedure is little used nowadays because of two limitations to its validity. The first, a severe one, is that the time-area diagram takes account of only the translational effects of catchment storage, and neglects the attenuation or storage effects. Translation time can be looked upon as a rough approximation to the effects of flood wave celerity, and the storage effect concerns the attenuation of the wave. Rational method translation time, however, is based upon a concept of "drops of water" travelling over the catchment surface, rather than of wave motion and storage effects. In other words, it is based on an erroneous concept of the runoff process. The second limitation is that the theory is a linear one, which restricts its generality.

3. SYNTHETIC UNITGRAPHS

3.1 General

In the unitgraph method (Sherman 1932a; Johnstone and Cross 1949, pp. 134-159), the time-area diagram of the modified rational method is replaced by a unitgraph, which is the surface runoff hydrograph that would result from unit rainfall-excess occurring uniformly in unit time. Since the combined effects of rainfall factors and loss factors are specified or standardized in this definition, it follows that the unitgraph represents the effect on hydrograph shape of the physiographic factors only. In this respect it is similar to the time-area diagram, but, being derived from actual records of rainfall and streamflow, the unitgraph reflects in its shape the integrated effect of all the physiographic factors and is, therefore, preferable to the time-area diagram.

Rainfall-excess hyetographs are applied to unitgraphs in exactly the same way as they are to time-area diagrams to determine surface runoff hydrographs. The method is therefore, only approximate when applied to non-linear catchments, for which discharge is not a linear function of storage (Johnstone and Cross 1949) or for which discharge is not a linear function of cross sectional area of flow in the channels (Dooge 1959). The term "storage" in this context implies the total channel and detention storage on the catchment, though this is usually not stated in the literature. With storage thus defined, it is obviously almost a practical impossibility to measure the storage in order to check whether it is a linear function of discharge, but fortunately there is a single indirect criterion of linearity that replaces both of the above conditions. This is that the lag of the catchment should be constant, and independent of flood magnitude.

In cases where the rainfall and streamflow data necessary for unitgraph derivation are unavailable, it is necessary to synthesize the unitgraph by measuring certain physiographic characteristics of the catchment and applying empirical relationships between these characteristics and unitgraph parameters. The empirical relationships are determined previously by analysis of gaged catchments on a regional basis.

3.2 Miscellaneous Methods

Several approaches to the solution of this problem have been used. Sherman (1932b) proposed a procedure for modifying the unitgraph of a catchment assumed to be hydraulically similar to the catchment for which the unitgraph is being synthesized. This procedure, therefore, accounts only for differences in catchment area, and neglects the effect on the hydrograph of any other factors that may be different on the gaged and the ungaged catchments.

Bernard (1935) prepared a graph giving daily distribution graph ordinates (a modified form of the unitgraph) as a function of a "watershed factor" that combines the effects of watershed shape, the condition, shape, and slope of the main stream, and the shape of the rainfall intensity-duration relation. These curves are not applicable to small catchments, since they produce only a daily distribution graph and not a short period unitgraph, but they may be of value for large catchments in the region for which they were derived.

McCarthy in 1938 developed a procedure that has been described by Johnstone and Cross (1949, pp. 217-221). This consists of reducing derived unitgraphs to what they would be for a hydraulically similar catchment of 10 sq. m., and empirically relating model peak to a model slope factor and stream pattern factor. Other unitgraph parameters were then related to the unitgraph peak.

Morgan and Hullinghorst (1939) carried out least squares correlations of parameters of the instantaneous S-curve (from which the unitgraph can be derived) with catchment area, mean travel distance and mean height of watershed above the outlet.

Several procedures make use of dimensionless hydrographs to define the shape of the unitgraph or the flood hydrograph. Commons (1942) presented such a graph which could be made dimensional when the peak discharge and some time parameter were known. The Soil Conservation Service, (U. S. Dept. of Agr., 1957) use a similar dimensionless graph together with a relation between peak discharge and period of rise, and various methods for the determination of period of rise. Bender and Roberson (1961) also present a dimensionless unitgraph for the Willamette River basin, Oregon, which they use, however, in deriving unitgraphs from complex storms on gaged catchments rather than in synthesizing unitgraphs from catchment characteristics. Finally, Morgan and Johnson (1962) describe a procedure due to Mitchell in which dimensionless

S-curves are used together with a relation between lag and catchment area to synthesize unitgraphs for Illinois streams.

3.3 Snyder's Method and its Development

All of the unitgraph synthesis procedures so far described appear to have been developed quite independently of each other, and have little in common except for the general concept of relating unitgraph characteristics to catchment characteristics. In 1938, however, Snyder (1938) presented a synthetic unitgraph method that has formed the basis for a large number of subsequent procedures. The original procedure utilizes an empirical relationship between lag (defined as time from the center of mass of rainfall-excess to the peak of the resulting hydrograph) and a shape parameter. Time of rise is determined from the lag, and the unitgraph peak discharge, base length, and daily distribution graph ordinates are also determined from empirical relationships with lag. Thus the complete unitgraph can be prepared.

Further development of Snyder's method by the U. S. Corps of Engineers is shown in the Corps Engineering Manual (U. S. Dept. of Army, 1948, pp. 10-15). Values of the coefficients of the empirical equations of the method are presented for various parts of the United States. Variations from the original procedure are that the base length equation and distribution graph ordinates are not used, but in their place are given a means of estimating the width of the unitgraph at ordinates of 50 percent and 75 percent of the peak, and a procedure for shaping the recession limb by means of the S-curve.

Taylor and Schwarz (1952) developed a procedure generally similar to Snyder's, but different equations relating unitgraph parameters to catchment characteristics were used, and an additional catchment characteristic (the equivalent mean slope of the main stream) and the unit period of the unitgraph were included in the correlations. This procedure should be an improvement over Snyder's since it includes the effect of an additional catchment characteristic, but the complexity of the empirical equations makes re-derivation of the coefficients and exponents for other regions difficult.

Laden, Reilly, and Minotte (1940) also presented a procedure similar in some respects to Snyder's method. They developed mass curves of distribution graphs as a function of Snyder's lag for Upper Ohio River catchments. Also, they gave empirical relations for unitgraph base length in terms of lag, and peak discharge as a function of base length and a shape parameter.

The U. S. Bureau of Reclamation (U. S. Dept. of Interior) use a procedure with some superficial resemblance to Snyder's method. Lag (defined as time period between center of mass of rainfall-excess and the time when half the total volume of runoff has been discharged) is related empirically to a function of Snyder's shape parameter and stream slope. The shape of a synthetic unitgraph is determined, after computation of the lag, from dimensionless mass curves of derived unitgraphs in the region concerned.

Recently, a unitgraph procedure for sewered areas has been presented (Eagleson, 1962). This utilizes empirical relations for lag in terms of mean length and velocity of flow, peak flow in terms of lag, and base length and unitgraph width in terms of peak flow.

3.4 Unitgraph Synthesis by Runoff Routing

All of the unitgraph synthesis procedures described above in Sections 3.2 and 3.3 rely entirely on empirical relations between characteristics of the unitgraph and characteristics of the catchment. In recent years there has been an increasing tendency to synthesize hydrographs and unitgraphs by routing rainfall or rainfall-excess through storage intended to represent the catchment storage. A catchment storage model is therefore set up (a mathematical model, computational model, or some kind of physical analog), and the parameters of the model (such as storage delay time) are determined by analysis of actual rainfall and streamflow data. These model parameters are then empirically related to catchment characteristics on a regional basis. These procedures therefore, differ from previously described synthetic unitgraph procedures in that they attempt to treat the effects of catchment storage in a rational rather than an empirical manner, but they still depend on empirical relations to determine the characteristics of the storage.

Once the catchment model is defined in accordance with some particular concept of the runoff process, and its parameters evaluated from the empirical relations, a unitgraph can be synthesized by routing an input consisting of one inch of rainfall-excess occurring instantaneously all over the catchment through the storage model. The resulting output will be the instantaneous unitgraph if the storage model is a linear one. In general, the same procedure can be used for any input and with a linear or non-linear storage model, and the output will be the corresponding hydrograph. This procedure can be called runoff routing since it consists of routing the surface runoff through storage. Since the procedures for unitgraph synthesis and hydrograph synthesis are essentially the same, the development of both will be described together under the heading of "Runoff Routing" in Section 4.

4. RUNOFF ROUTING

4.1 General

The basis of runoff routing procedures has been explained in Section 3.4 above. Use of such a procedure for hydrograph synthesis on ungaged catchments involves:

(i) Setting up a catchment or storage model,

(ii) evaluating the parameters of the model for gaged catchments, and relating these to measurable catchment characteristics,

(iii) computing the model parameters for the ungaged catchment from these relations, and routing any desired rainfall-excess pattern through the model.

A rather detailed literature survey of runoff routing procedures has been given elsewhere (Laurenson 1962b), but it is appropriate here to review briefly these procedures. Items (i), (ii) and (iii) above will be dealt with in Sections 4.2, 4.3, and 4.4 respectively.

4.2 Catchment or Storage Models

4.2.1 Literature Survey

Various catchment models have been assumed for the purpose of computing hydrographs, most being characterized by the separation of translation effects from storage effects. As has been pointed out by Dooge (1959), the modified rational method of Ross (1921) described in Section 2 is a form of runoff routing in which translation is taken into account but attenuation is neglected. The assumed catchment model is illustrated diagrammatically in Figure 2(a).

An advance on this concept was introduced by Zoch (1934, 1936, 1937) in a series of papers presenting a mathematical means of computing hydrographs. Zoch assumed that elements of rainfall-excess on different parts of the catchment had different translation times to the outlet, but all passed through the same concentrated linear storage. The same model was adopted by Clark (1945) and is shown diagrammatically in Figure 2(b). It is convenient to regard the concentrated storage as being located at the outlet of the catchment.

A catchment model somewhat similar to the Zoch-Clark model was used by Parsons (1944). He prepared storage-discharge relations for both surface and sub-surface runoff by integrating the areas under the recession curves of these components, and relating the storage to the discharge some constant period earlier. Hydrographs were synthesized by lagging the rainfall-excess and interflow by the appropriate amount, and routing through the appropriate storage. The model therefore, consists of two concentrated (but presumably non-linear) storages in parallel located at the outlet of the catchment.

Nash (1957, 1960) developed an equation for the instantaneous unitgraph based on a view of the catchment as consisting of a series of equal, linear, concentrated storages, each discharging directly into the next, as indicated diagrammatically in Figure 2(c). All runoff is assumed to pass through all the storages.

Application of an instantaneous unit input to Nash's model results in an equation for the instantaneous unitgraph having the form of the two parameter Gamma function:

$$u\ (0,t) = \frac{V\ (t/K)^{n-1} e^{-t/K}}{K\ n\ !} \tag{1}$$

where u (0,t) is the instantaneous unitgraph ordinate at time t , V is the volume of the unitgraph, and K is the storage delay time of each of the n linear concentrated storages. The factorial of n is replaced by the Gamma function $\Gamma(n)$ in application since the method of determining n results in non-integral values in general.

An instantaneous unitgraph equation of the same form as Nash's was earlier developed by Edson (1951) under different assumptions. He assumed, as did Zoch and Clark, that rainfall-excess elements from different parts of the catchment were transposed by different amounts, and then routed through a single concentrated linear storage. However, unlike Zoch's general equation and Clark's method, a restriction was placed on the form of the time-area diagram in that its integral (the time area concentration curve) was specified as being of parabolic form. These assumptions as to the form of the catchment and its storage are represented by Figure 2(d).

The catchment models described so far form a continuous line of development, the modified rational method model providing for translation but not attenuation effects of catchment storage, while the other models provide for both effects. Zoch, Clark, Parsons and Edson, however, allow for translation by lagging the inflows and routing through a single concentrated storage, while Nash does not lag the inflows but routes all inflow through a succession of concentrated storages, thus obtaining a translational effect. Next to be described are two models that appear to derive nothing from the previous work, but which nevertheless recognize the distributed nature of catchment storage (thus recognizing both translation and attenuation effects), and also allow for the further fact that catchment storage is, in general, a non-linear function of discharge, so that storage delay time varies with discharge.

Appleby (1954) recognizes that catchment storage is distributed along the stream channels and on the ground surface, and idealizes this by assumptions that imply a catchment model as shown in Figure 2(e).

In a discussion of Nash's (1960) paper, Appleby (1961) states that, for the particular case of linear storage (constant K), the unitgraph equation derived from his model reduces to an equation of the same form as that developed by Nash, i.e., the differential quotient of the incomplete Gamma function. Thus, three different catchment models, Nash's (Figure 2(c)), Edson's (Figure 2(d)), and Appleby's (Figure 2(e)), all result in the same two-parameter equation of the instantaneous unitgraph.

A model with distinct similarities to Appleby's has been used by Dawson (1958, 1960), who represents the catchment as a plane, rectangular surface. Hydraulic computation was applied to this idealized catchment to determine the hydrograph from uniform rainfall, and the computational method was then developed into a novel semi-graphical non-linear routing procedure for application to varying intensities on natural catchments.

Sugawara and Maruyama (1956) propose several different catchment models to represent the runoff process, with provision for infiltration, permanent losses, ground-water flow, interflow, and surface runoff. In all of these, storage effects are assumed linear and concentrated, and translation effects are not separated from attenuation effects. Different models were found to be applicable to different catchments. Equations of the instantaneous unitgraph (dealing with surface runoff only) were developed for two catchment models. The first was the same as that later used by Nash (Figure 2(c)), but with only two storages. Sugawara and Maruyama's second model for unitgraph derivation can be illustrated as in Figure 2(f). This is a more general model as it provides for rainfall-excess from different parts of the catchment to be routed through different amounts of storage.

Two recent American contributions to the literature on runoff routing describe methods for synthesizing hydrographs on large catchments, primarily for flood forecasting. They introduce a new concept in that the computational model for any catchment can be built up by combining a number of standard elements in correct relation to each other to represent the actual catchment. This feature distinguishes these two methods from all of those previously described.

The first of these was presented by Rockwood (1958), who provided elements of "catchment storage," "lake storage," and "channel storage," and combined these as necessary to represent the catchment and stream system. These components are illustrated in Figure 2(g). The second was by Harder (1962), who described a non-linear electronic analog runoff routing computer consisting of a number of "river resistor units," current generators, and "channel storage units," which can be put together as required to represent a catchment.

Dooge (1959) developed two equations for the instantaneous unitgraph based on two slightly different storage models. The first is a very general model consisting of a series of unequal linear, concentrated storages irregularly distributed along the stream channels, and separated by linear channels (velocity independent of stage), and is illustrated in Figure 2(h). The second model, shown in Figure 2(i) is slightly less general in that rainfall-excess from all points on a given isochrone is assumed to pass through the same number of reservoirs, and all the reservoirs are assumed equal.

4.2.2 General Model

The present author, (Laurenson 1962b), after experimenting with several different models, adopted one illustrated in Figure 3.

In this model, the catchment is divided into about ten sub-areas bounded by isochrones (or lines of equal travel time to the outlet). The rainfall-excess hyetograph of the topmost sub-area is routed through a single, non-linear, concentrated storage having a storage-outflow relation of the form:

$$S = K(q) \cdot q \tag{2}$$

where S is volume of storage, q is rate of outflow, and K, a function of q, is the storage delay time. The output from this storage is combined with the rainfall-excess hyetograph from the second sub-area, and the combined flow routed through a second non-linear, concentrated storage. This process is repeated until all sub-areas have been accounted for, at which point the output from the storage represents the catchment hydrograph.

This is a very general catchment model since it takes rational account of:

(i) time variations in rainfall-excess,
(ii) areal variations in rainfall-excess,
(iii) the fact that rainfall-excess from different parts of the catchment passes through different amounts of storage,
(iv) non-linear storage effects, and
(v) the fact that catchment storage is distributed rather than concentrated. (This is true since a series of concentrated storages produces the translation typical of distributed storage as well as the attenuation typical of concentrated storage.)

4.3 Evaluation of Model Parameters

4.3.1 Author's Model

Section 4.2 above describes the catchment storage models that have been proposed, either explicitly or implicitly by various authors. It is now appropriate to examine the techniques used by these authors and others to evaluate the parameters of the models in order that they can be used for the synthesis of hydrographs. For the sake of continuity, the catchment model developed by the author will be treated first.

Evaluation of the parameters of this model consists of delineating the isochrones in such a way that they are separated by equal increments of travel time, and determining the functional relationships $K = K(q)$ (See Eqn: 2) for the various storages.

The configuration of the isochrones is determined from a contour map of the catchment assuming that travel time through any reach is directly proportional to length of flow path, and inversely proportional to the square root of slope. These assumptions fix the shape and relative positions of the isochrones, but the values of travel time cannot be assigned to the several isochrones until travel time has been evaluated for any one point on the catchment.

Evaluation of travel time for one point on the catchment is approached through the determination of catchment lag (using the original definition of time from center of mass of rainfall-excess to center of mass of surface runoff). It can be demonstrated that lag (so defined) is an "average" travel time for the catchment, and is equal to the travel time of a point on the catchment corresponding to the centroid of the time-area diagram (plot of area between isochrones against travel time). Furthermore, travel time for any point can logically be interpreted as the storage delay time for rainfall-excess occurring at that point and passing through catchment storage to the outlet. Thus, if lag is determined from rainfall and streamflow records, the storage delay time of points corresponding to the centroid of the time-area diagram is known, and travel time or storage delay time for all isochrones can be computed.

For the South Creek Experimental Catchment of The University of New South Wales, lag was found to vary with mean discharge of the flood from which it was determined according to the empirical relation:

$$t_m = 64\ q_m^{-0.27} \tag{3}$$

where t_m is lag in hours, q_m is mean discharge in c.f.s., and the coefficient of correlation between the logarithms of lag and mean discharge was 0.90. The variation of lag with discharge indicates non-linearity of the catchment response.

As explained above, catchment lag can be identified with storage delay time of the point corresponding to the centroid of the time area diagram (K_m), and, further, mean discharge can reasonably be identified with discharge (q) in this context, so Eqn. 3 was rewritten:

$$K_m = 64\ q^{-0.27} \tag{4}$$

It was then assumed that a delay time-outflow relation of this general form applied to every point on the catchment, but that the coefficient (64 in Eqn. 4) for any given point was proportional to the relative delay time indicated by the isochrone through that point. Since the relative delay time of the point corresponding to the centroid of the time-area diagram is 0.66 on this catchment, and since the catchment storage is represented by ten storages, the upper nine each having a delay time of one tenth the maximum delay time for the catchment, and the last having a delay time of half this, it follows that the delay time for each of the upper nine storages is given by:

$$K_1 = \frac{0.1}{0.66} . 64\ q^{-0.27} \tag{5}$$

and that of the lowest storage is half of this.

Now, since the configuration of the isochrones and the functional relations for storage delay time have been determined, it is possible, for any given rainfall-excess, to determine the inflows to each storage and to route the runoff through the system.

4.3.2 Other Models

The parameters of the Clark model are the base length of the time-area diagram and the delay time of the single, linear, concentrated storage. The base length of the time-area diagram, or maximum translation time of the catchment, is taken by Clark (1945)as the time between the end of rainfall-excess and the point of contraflexure on the recession limb of the hydrograph (assumed virtually constant for different floods). The end of rainfall-excess is the time at which the last contribution to the flood wave enters the catchment, while the point of contraflexure on the recession limb of the hydrograph represents the arrival at the gaging station of the last inflow to channel storage, since the exponential part of the recession represents withdrawal of water from channel storage with no inflow. Consequently, the interval between these two times is taken as the maximum translation time of the catchment exclusive of the translation due to reservoir-type storage. After the base length has been determined, the time-area diagram is proportioned by assuming that travel time for any point on the catchment is proportional to its distance from the outlet.

On the assumption that the recession limb of the hydrograph represents withdrawal of water from storage with no inflow, Clark showed that the storage delay time is given by

$$K = - \frac{q}{(1 - x)\, dq/dt} \qquad (6)$$

For reservoir-type storage, assumed by Clark, the weighting factor x equals zero. Although K was shown to increase with decreasing discharge, Clark assumed in application that it was constant, and specified that it should be evaluated at the point where it has its minimum value, i.e., at the point of contraflexure.

Clark's catchment model has been adopted by a number of authors (Johnstone, Eaton, and O'Kelly) in developing synthetic unitgraph procedures for particular regions. Each of these three has, however, introduced slight modifications of the original Clark procedure, and they all differ in the means of determining the numerical and graphical parameters of the model for a particular catchment.

Johnstone (unpublished data 1948, Johnstone and Cross 1949, pp. 234-5) used the surface runoff hydrograph instead of the total hydrograph to determine the parameters, and related them empirically to catchment characteristics. Maximum translation time was related to length and slope of main stream and a branching factor, and storage delay time was related to width of catchment and overland slope. Also, his proportioning of the time-area diagram took account of the effects of slope variations on travel time. In a study of seven Tasmanian streams, Eaton (1954) developed equations for base length of time-area diagram in terms of length of main stream, catchment area, and a branching factor, and for storage delay time in terms of the same three factors and width of catchment. He proportioned the time-area diagram both by Clark's method and by Horton's virtual channel inflow graph (Horton 1941).

O'Kelly (1955) graphically related base length of time-area diagram and storage delay time to median overland slope, and used a virtual channel inflow graph in the form of an isosceles triangle in place of a true time-area diagram.

The catchment models developed by Nash and Edson both imply a two-parameter unitgraph equation of the form of Eqn. 1. Three independent methods of determining the two parameters for a particular catchment have been reported (by Nash, Edson, and Gray respectively), and these are described below.

Nash (1960) used the method of moments, and derived the relation between the unitgraph parameters (n and K) and the first and second moments of the unitgraph about the origin. These moments were then correlated empirically with catchment area, overland slope, and length of main stream. A unitgraph can be synthesized for an ungaged catchment by measuring these catchment characterisitcs, computing the unitgraph moments from the empirical relations, thus determining the parameters n and K , and substituting these in Eqn. 1 to determine the unitgraph.

Edson (1951), instead of relating the unitgraph parameters to the moments of the unitgraph, developed their relations to the peak and time to peak of the unitgraph. He did not, however, take the further step of correlating peak and time to peak with catchment characteristics, which would be necessary for a synthetic unitgraph procedure.

A third method of fitting the Nash-Edson equation to an actual unitgraph, the method of maximum likelihood, was used by Gray (1961), who also obtained empirical relations between the parameters of the equation and the length and slope of the main stream.

Determination of the parameters of the models of Appleby and Rockwood is by trial and error reproduction of known hydrographs, varying the parameters until a satisfactory reproduction is obtained. The author is not aware of the means used to determine the parameters of Harder's or of Sugawara and Maruyama's models, but presumes these also are determined by trial and error. Before these models could be used for hydrograph synthesis on ungaged catchments, therefore, some means of computing the parameters from catchment characteristics would have to be developed.

Use of Dooge's models requires determination of two graphical and three numerical parameters. These parameters could be determined by various methods developed prior to Dooge's work if one or two simplifying assumptions are made, but no published work as yet describes the application of Dooge's equation to unitgraph synthesis.

Use of the equations developed by Zoch for synthesis of hydrographs or unitgraphs would require determination of the area-distance curve (plot of width of catchment against distance from outlet), the velocity of flow, the delay time of the storage, and the maximum length of flow. Zoch does not deal with the difficult problems of determining velocity and delay time for actual catchments, and no published example of the application of his equations to a real catchment is known.

4.4 Computation of Synthetic Hydrographs

4.4.1 General Methods

The previous two sections have dealt respectively with the models proposed to represent catchment storage effects, and evaluation of the parameters of these models. This section deals briefly with the methods that have been used to compute hydrographs once the model has been defined and its parameters evaluated. These include mathematical, numerical and analog methods.

Strictly mathematical procedures can be used to determine the output from a concentrated linear storage resulting from a concentrated input. Those authors that have produced an equation for the hydrograph or unitgraph (Zoch 1934, 1936, 1937, Nash 1957, 1960, Edson 1951, Sugawara and Maruyama 1956, and Dooge 1959) have performed this mathematical routing, and application of their methods once the parameters have been determined merely involves numerical evaluation of the right hand sides of their equations.

Use of the Clark model simply involves a single routing through concentrated linear storage, and the Muskingum method of routing (a "coefficient" method) has been used by Clark and the others who adopted his model.

Where a large number of routings is involved, particularly non-linear routings, as in Rockwood's (1958) and Harder's (1962) "built-up" models, the use of a computer is almost essential. Rockwood used a digital computer in his case and, as has been noted, Harder used an electronic analog. Multiple, non-linear routing on a digital computer for hydrograph synthesis has also been used by the present author (Laurenson 1962b), using non-linear coefficient method routing equations derived by Laurenson (1962a).

A specially constructed linear heat flow analog was used by Appleby (1954) as a hydrograph computer, but his tests were not designed to verify the analog.

4.4.2 Testing of Author's Model

The catchment storage model developed by the author and described in Section 4.2.2, with parameters evaluated as described in Section 4.3.1, was tested by using it to reproduce the hydrographs for thirteen periods, containing 22 individual peaks, on South Creek Experimental Catchment.

Computations were performed on an electronic computer. Input consisted of rainfall hyetographs for the recording rain gages (pluviometers), average areal rainfall for each sub-area, the loss rate for each burst of the storm, and catchment storage data. The computations performed by the program, which are illustrated diagrammatically on Figure 4, consisted of computing a hyetograph of rainfall-excess for the topmost sub-area, converting this to a "hydrograph" of rainfall-excess, routing it through the non-linear storage, adding the outflow to the rainfall-excess "hydrograph" of the second sub-area, and so on.

Some of the results are shown in Figure 5, where (a) shows one of the best reproductions of actual surface runoff hydrographs, (b) an average result, and (c) the worst reproduction. On the whole, the results were encouraging, but little success was had with the smaller rises with peaks less than about 20 c.f.s./sq. mi. This failure with small rises has also been experienced by other hydrograph synthesis procedures. On the other hand, significant floods on the catchment were reproduced reasonably well.

5. OTHER NEW APPROACHES TO HYDROGRAPH SYNTHESIS

5.1 Runoff Cycle Models

Most of the hydrograph synthesis procedures described thus far have been concerned only with rainfall-excess and the resulting surface runoff. The few that take account of ground-water flow assume that the volumes of surface and sub-surface runoff are known. None of them has been concerned with the problem of determining what part of the total precipitation actually appears as streamflow. As opposed to this limitation to part of the runoff cycle, much current thinking is in the direction of synthesizing streamflow hydrographs from rainfall (as distinct from rainfall-excess) by setting up models of the complete runoff cycle. An overall model of the runoff cycle, such as that shown in Figure 1, must first be postulated, and then detailed models of all components of the overall model developed.

Little published work on this type of model is available, but Linsley and Crawford (1960) and Crawford and Linsley (1962) have done considerable work in this field. Their overall catchment model is illustrated in Figure 6 (taken from their 1962 report). Computational models of all components of the overall model have been developed, and combined in a digital computer program, which accepts hourly or daily rainfall as an input (as well as parameters of the model), and produces estimates of hourly or daily evapotranspiration and streamflow.

The models of the individual components of the overall model were based on concepts of how the various processes occur on natural catchments. For example, the routing of surface runoff and interflow through storage was achieved by translation to the outlet and routing through a single, concentrated, linear storage. Thus, these two aspects of the procedure utilize Clark's model as shown in Figure 2(b). This emphasizes the novelty of this hydrograph synthesis procedure since the problem treated by all the other procedures described in this paper forms only one small part of the problem treated by Crawford and Linsley.

5.2 Correlation Studies

A further approach to hydrograph synthesis for ungaged catchments has been developed by Reich (1962). This assumes that flood hydrographs can be adequately represented by the three parameter Gamma function. Parameters of this function are then correlated directly with storm and catchment factors, the significant factors being determined in a stepwise multiple correlation study. In application, the appropriate catchment factors can be measured, the storm factors selected, and the empirical relations used to compute the parameters of the Gamma function, which can then be evaluated to represent the hydrograph.

This procedure differs from those described in Section 4 which utilize the two-parameter Gamma function in that it synthesizes a hydrograph directly instead of a unitgraph, thus avoiding use of the linear, superposition principle. The shape of the Gamma function is such that the method is only applicable to simple storms and not to complex storms that would produce unusually-shaped or multi-peaked hydrographs.

5.3 Functional Series Representation of Surface Runoff Hydrograph

Amorocho and Orlob (1961) (see also Amorocho 1963) have introduced a procedure in which the hydrograph is represented by an infinite series, the first term of which is the convolution integral (representing unitgraph theory), succeeding terms being generalizations of the convolution integral of successively higher order. The first term contains an impulse response function (the unitgraph for a linear catchment), and each succeeding term has a corresponding generalized impulse response function of increasing degree of non-linearity. It appears that only a few terms of the infinite series are necessary for the representation of hydrographs.

The several impulse response functions for any particular catchment are determined by analysis of hyetographs and hydrographs, but so far this has only been done for a small artificial catchment with an impervious surface, and with a step function rainfall. Problems of analysis for continuously variable inputs are yet to be overcome and, further, the method is presently applicable only to surface runoff hydrographs, and not to combined surface and subsurface runoff.

In a sense, this method is a generalization of the unitgraph method that takes account of the non-linear response of catchments. For use on ungaged catchments, the various impulse response functions would have to be related to or derived from catchment characteristics, just as unitgraphs are now synthesized from catchment characteristics.

6. CONCLUSION

It can be seen from the above review that the modified rational or time-area method, while itself obsolete because it does not recognize the runoff process as a storage process, nevertheless contained features still widely used. These are the time-area diagram itself, which integrates the effects of catchment shape and the factors affecting translation time, and the principle of superposition, which is the basis of unitgraph theory.

Introduction of the unitgraph concept was followed by the development of synthetic unitgraph procedures, which relate characteristics of the unitgraph to catchment characteristics on a regional basis. Use of synthetic unitgraphs is now the most widely used means of synthesizing hydrographs for ungaged catchments. Early synthetic unitgraph procedures, and many modern ones, are based entirely on correlation of unitgraph parameters with catchment factors, but, following the work of Clark (1945), a line of approach was developed in which the unitgraph is synthesized by routing rainfall-excess through catchment storage.

This approach is a special case of runoff routing, and led to more general forms of runoff routing procedure not restricted by the unitgraph requirements of temporal and areal uniformity of rainfall-excess, and linearity of catchment response. This field of runoff routing is still in an early stage of development, but is certain to be developed to a much greater extent, as it provides a very general approach to the synthesis of hydrographs.

For complete generality, a runoff routing procedure should possess the five features listed in Section 4.2.2, and incorporated in the author's procedure. Results of this procedure indicate that the storage model and the means of evaluating its parameters are reasonable but that some improvement is necessary for small rises.

Future research on runoff routing is likely to be directed towards obtaining a better understanding of the physics of the runoff process. Up to date, catchment models have been postulated on the basis of the investigator's understanding of this process, and have been tested by their ability to reproduce measured hydrographs from measured rainfall. The best of the models are still rather artificial, and further improvement will require measurement of the movement of water on the catchment between the time it falls as rain and the time it appears as streamflow at the outlet of the catchment. Such detailed measurements would permit the development of a catchment model representing the runoff process more realistically, and therefore, more reliably, than current models do. Theoretical studies are likely to investigate the role of wave motion in both sheet and channel flow during flood runoff, a topic largely neglected to date.

A number of other approaches to hydrograph synthesis for ungaged catchments is still in the early stages of development. Perhaps the most significant of these is the use of computational models of the entire runoff cycle. Runoff routing procedures might well serve as one of the components of such overall models. Another approach is direct correlation of hydrograph parameters with rainfall and catchment factors, which can be compared with correlation of unitgraph parameters with catchment factors under standardized rainfall and loss conditions as in synthetic unitgraph procedures. Finally, the use of functional series to represent the hydrograph instead of computing it as the output from some logical system may have application in this field when the techniques are further developed.

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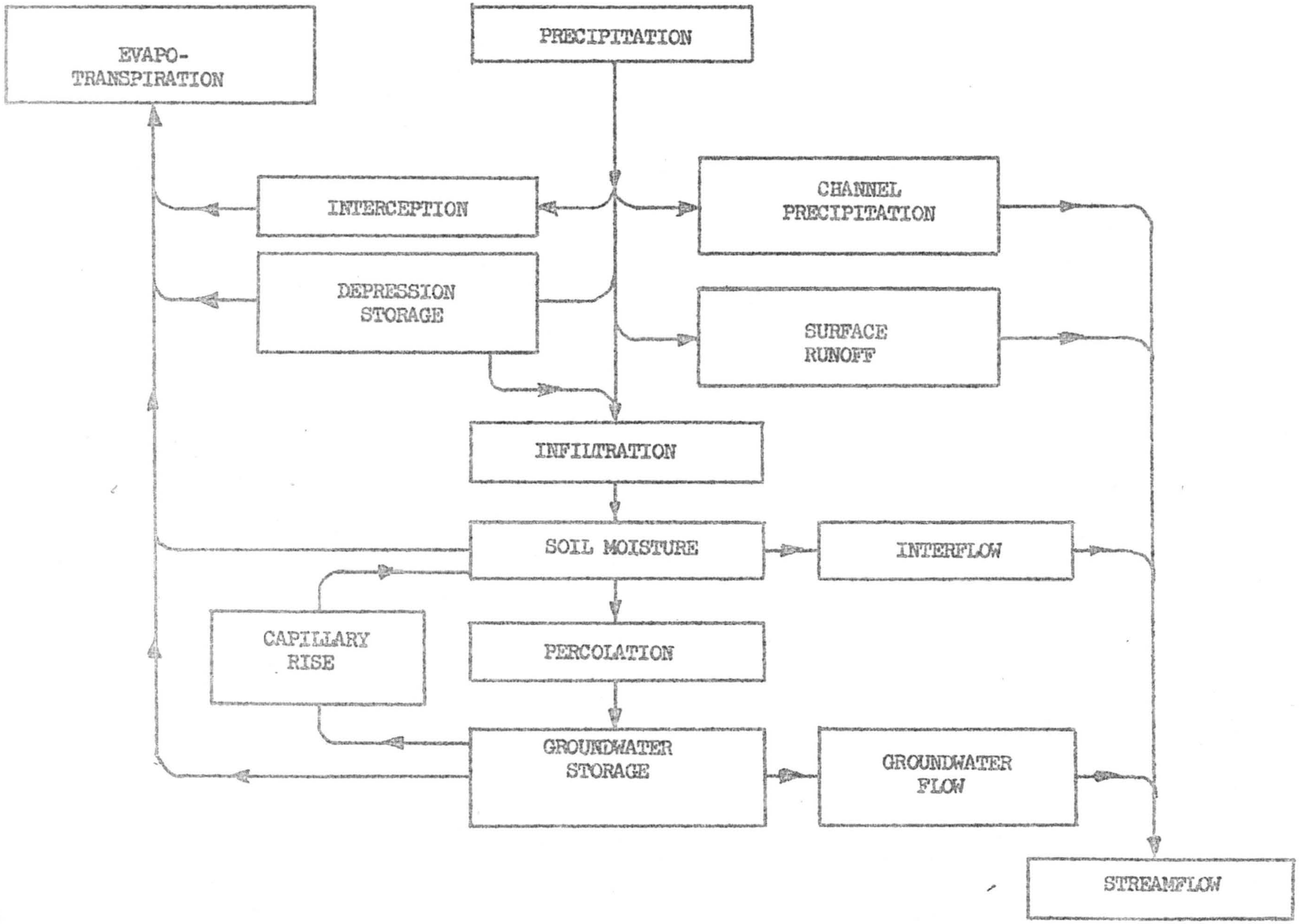
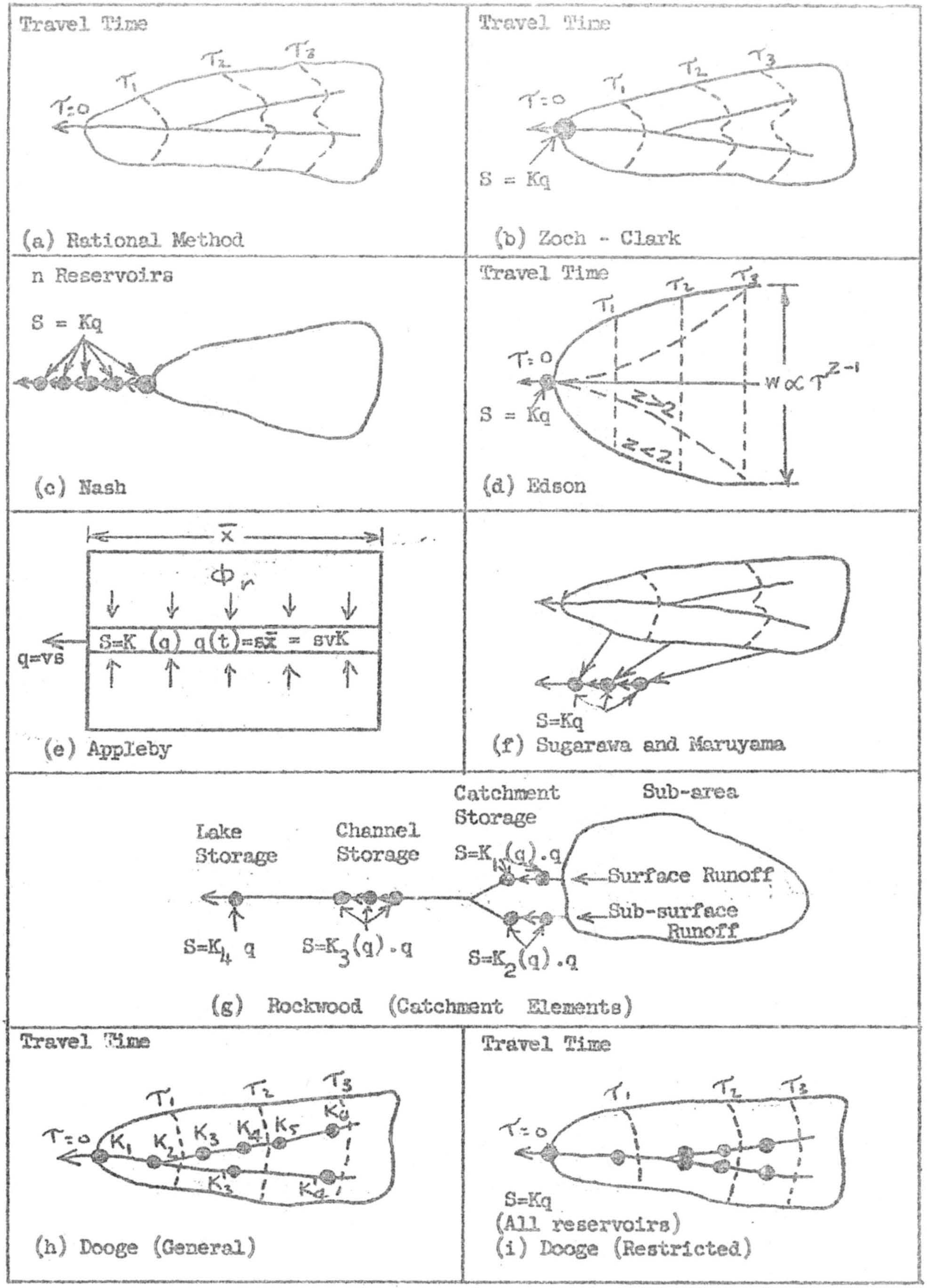


FIG. 1 - The Runoff Cycle



(a) Rational Method

(b) Zoch - Clark

(c) Nash

(d) Edson

(e) Appleby

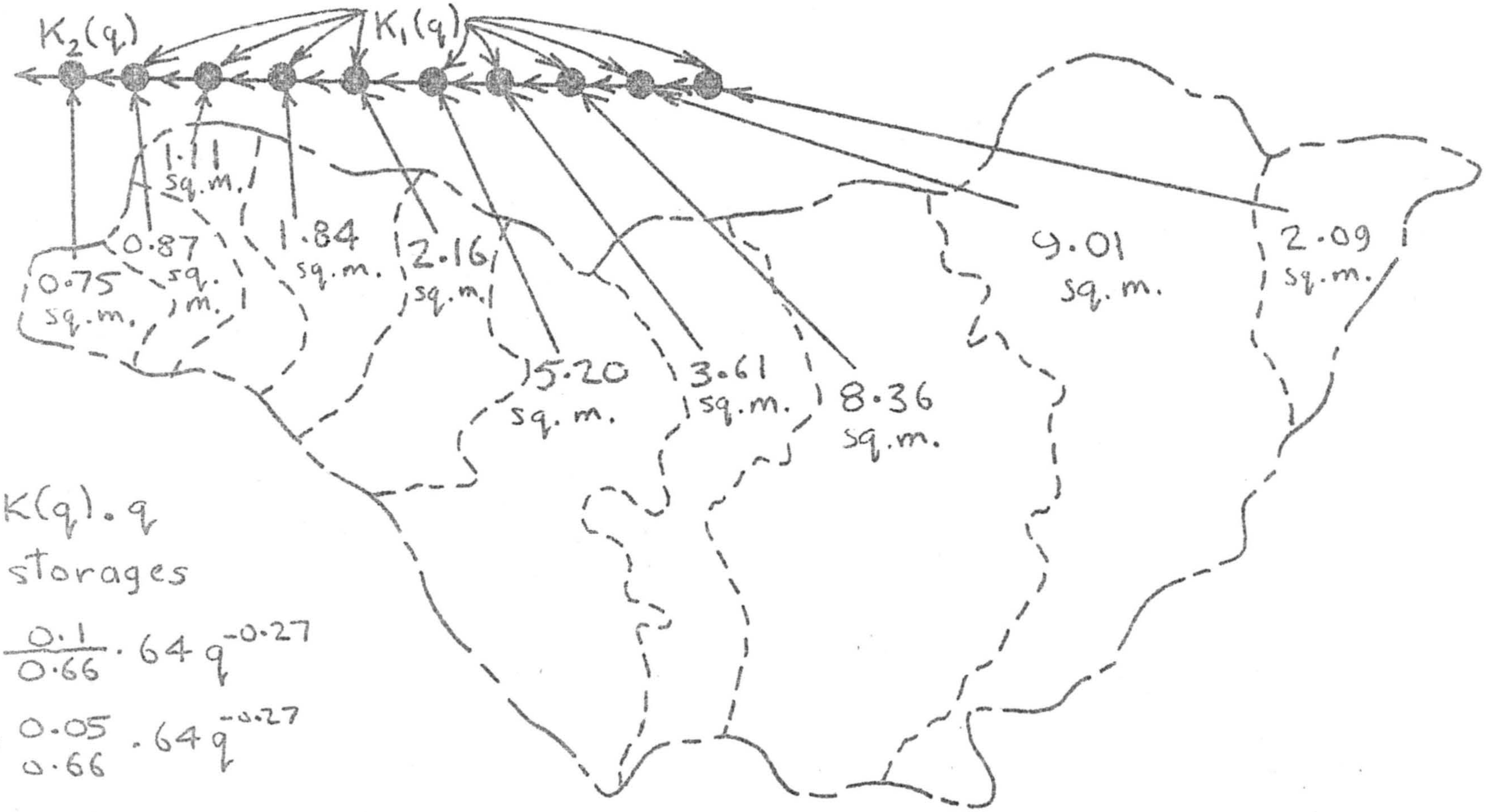
(f) Sugarawa and Maruyama

(g) Rockwood (Catchment Elements)

(h) Dooge (General)

(i) Dooge (Restricted)

FIG. 2 - Catchment Storage Models



$S = K(q) . q$

All storages

$K_1 = \frac{0.1}{0.66} . 64 q^{-0.27}$

$K_2 = \frac{0.05}{0.66} . 64 q^{-0.27}$

FIG. 3 – General catchment storage model.

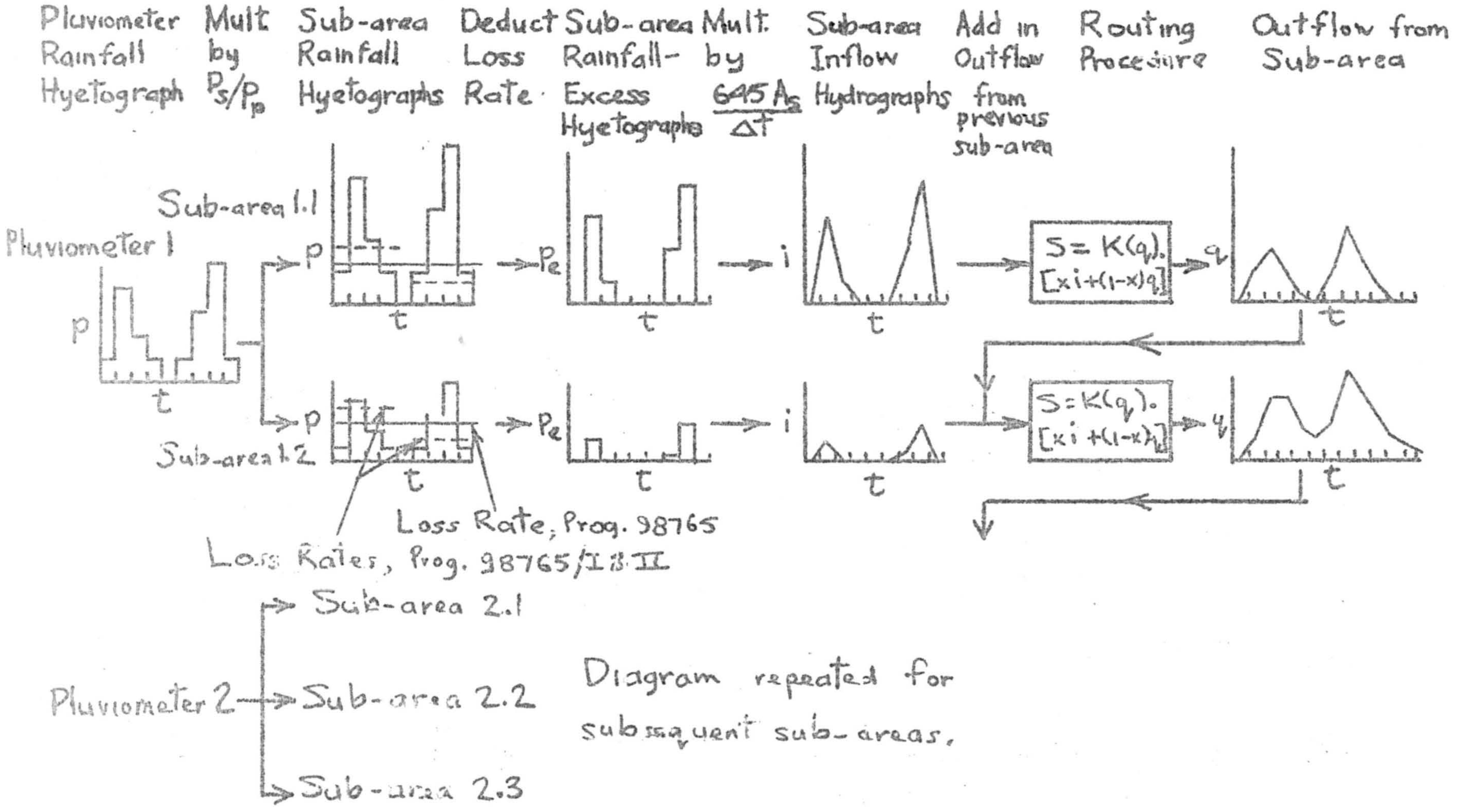


FIG. 4 - Computations for Hydrograph Synthesis

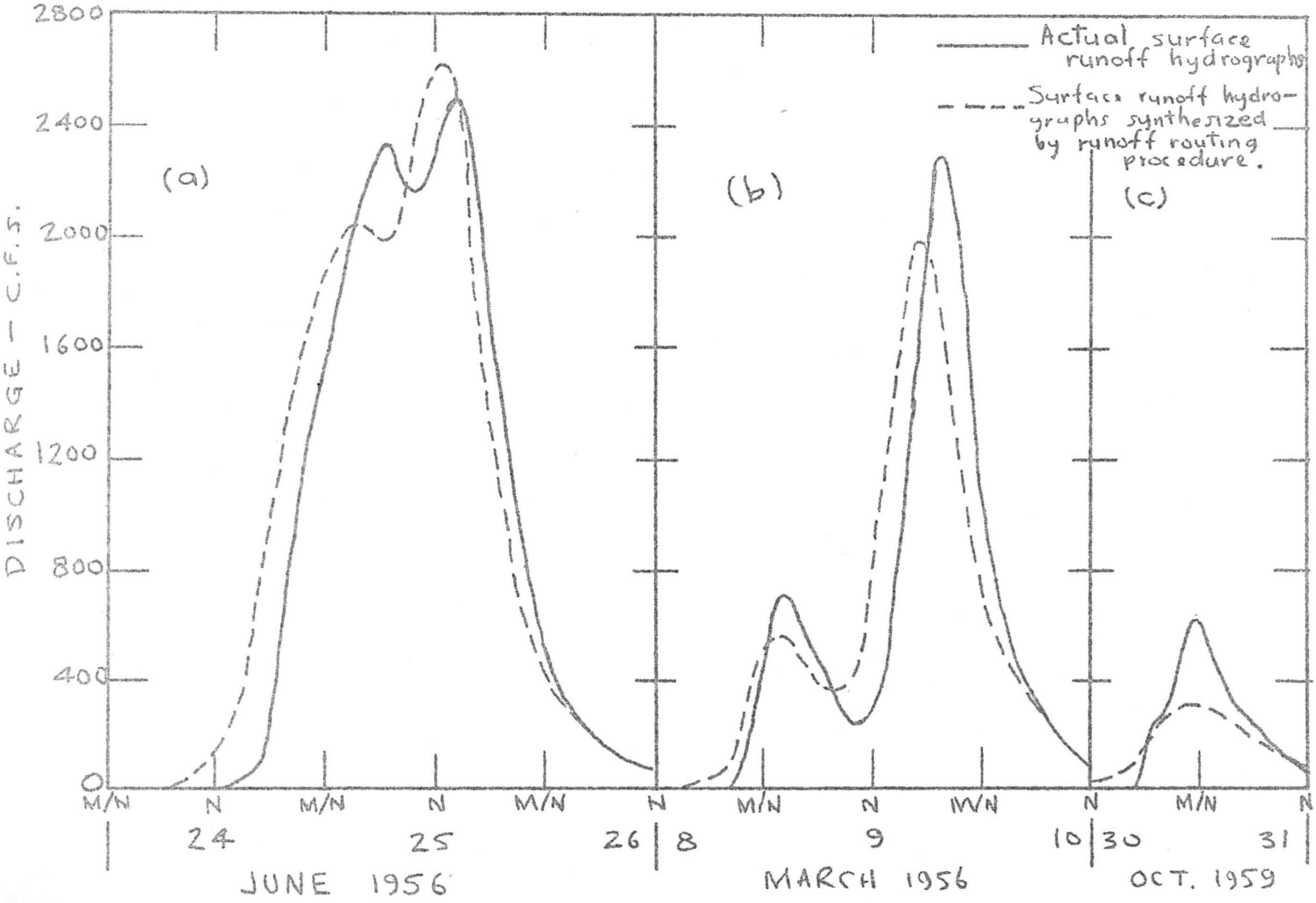


FIG.5— Runoff routing results: (a) best, (b) typical, (c) worst, result.
South Creek Experimental Catchment.

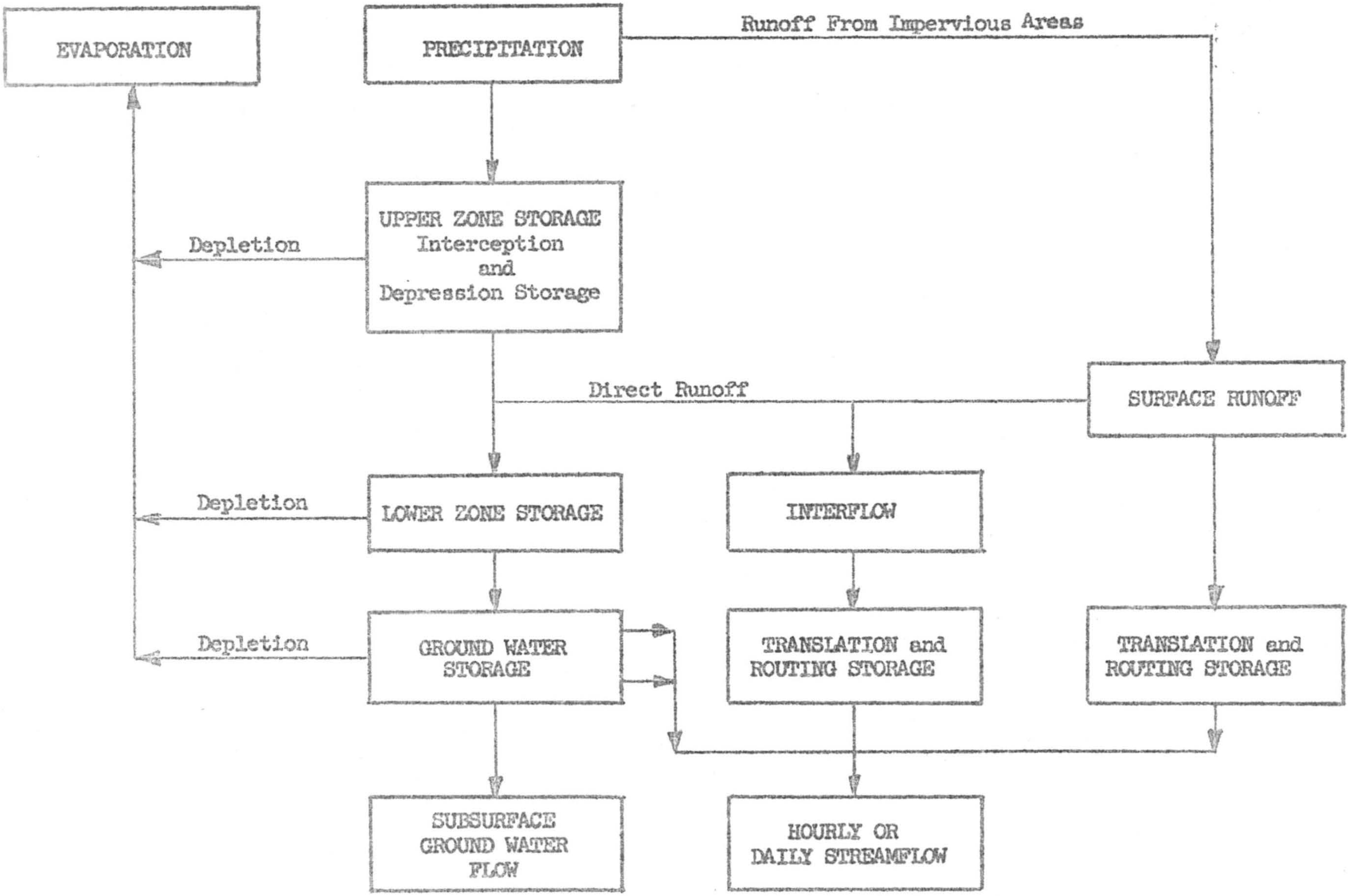


FIG.6 – Runoff cycle model used by Linsley and Crawford.