DISSERTATION

MULTI-CRITERIA RELIABILITY-BASED OPTIMIZATION FOR EVALUATION AND REHABILITATION OF CONCRETE BRIDGE STRUCTURES

Submitted by

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WE HERBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER OUR SUPERVISION BY SAHARAT BUDDHAWANNA ENTITLED MULTI-CRITERIA RELIABILITY-BASED OPTIMIZATION FOR EVALUATION AND REHABILITATION OF CONCRETE BRIDGE STRUCTURES BE ACCEPTED AS FULFILLING IN PARTIAL REQUIREMENTS FOR THE DEGREE OF DOCTOR PHILOSOPHY.

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ABSTRACT OF DISSERTATION

MULTI-CRITERIA RELIABILITY-BASED OPTIMIZATION FOR EVALUATION AND REHABILITATION OF CONCRETE BRIDGE STRUCTURES

The American Society of Civil Engineers (*ASCE*) reported that approximately one-third of U.S. bridges are either in need of serious repair or functionally obsolete. Even as repairs are completed, the problem persists with more and more bridges being added to the list each year. In order to determine if repair is needed for a specific bridge, a condition evaluation is typically conducted. The selection of conditional evaluation method and the subsequent repair method has, to date, been based on the preference of the engineer and/or contractor and at times the equipment available.

The work presented in this dissertation represents a first effort to provide quantitative decision support for selection of the most economical evaluation and repair method. The two most commonly used condition evaluation techniques are non-destructive evaluation (*NDE)* and what is termed herein as semi-destructive evaluation (*SDE)*. There are several repair and rehabilitation strategies readily available for bridge girders such as increasing the structural cross-section (*CI*), attaching steel plates (*SP*), external bonded fiber reinforced polymer (*FRP*), or external prestressing (*EP*). The procedure presented herein used these above techniques for illustration and combines elements of structural reliability theory, optimization, and structural analysis. Included is the optimization (minimization) of a cost function which accounts for the approximate level of damage

present in the structure, a decision diagram to ultimately select the method based on the lowest expected cost, and preference factors in the decision procedure that account for the availability of material, labor, and equipment.

Three illustrative examples for prestressed concrete bridge girders are presented. It is envisioned that, while the procedure still relies on some basic assumptions, it can be used eventually to develop a basic set of rules for the most common combinations of bridges and sites.

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DEDICATION

To my grandmother, parents, and family

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LIST OF SYMBOLS

NDE	Non-destructive evaluation
SDE	Semi-destructive evaluation
NDT	Non-destructive test
CRS	Conventional repair strategy
CI	Cross-section increasing repair strategy
EP	External prestressing repair strategy
FRP	External fiber reinforced polymer attaching repair strategy
SP	Steel plate attaching repair strategy
RBF	Reliability-based framework
UPV	Ultra sonic pulse velocity method
PUNDIT	Portable ultrasonic non-destructive digital indicating tester
RN	Rebound number (Schmidt hammer or rebound hammer) method
PO	Pullout test method
SHM	Structural health monitoring
R	Reaction forces
β	Reliability index
β_g	Guiding reliability index
β_{target}	Target reliability index
$\Delta \beta_E$	Level of structural damage for structural evaluation
$\Delta \beta_R$	Reliability index improvement for structural repair
E	Elastic modulus
ρ	Density
ν	Poisson's ratio
POD	Probability of detection
δ	First regression parameter

θ	Second regression parameter
(c)	Crack size
PDF	Probability density function
$f_I(x)$	Initial (or prior) joint *PDF*
$f_U(x)$	Update (or posterior) joint *PDF*
$PND_Y(x)$	Probability that the cracks are not detected in the first *Y* inspections
$POD[c(x,N_i)]$	Probability of detecting for a crack of length *c*
$c(x,N_i)$	Crack length at the i^{th} inspection time
N_i	Terms of the random variable set *x*
Ψ_{fxY}	Domain of random variables that leads to $N_i \leq N_S \leq N$
M_i	Symbol for the statement "the component M_i does not fail"
X'	Failure condition
P	Probability
$f(X)$	Objective function
x_i^l	Lower bound of design variables constraints
x_i^u	Upper bound of design variables constraints
β_j	Reliability index for component or structural system *j*
β_j^0	Target reliability index of component or structural system *j*
X_d	Vector of design variables
W_i	Weight of an element *i*
β_i^l	Lower bound required safety level reliability index for the i^{th} limit state
β_i^u	Upper bound desired range reliability index for the i^{th} limit state
Cost	Life cycle cost associated with material and manufacturing
P_f	Probability of failure
$(P_f)_{target}$	Pre-specified level of safety
M_C	Material and manufacturing cost per pound
W	Fatigue
N_f	Service life in number of operations

N_i	Number of inspections
I_C	Inspection cost for all panels including repair and replacement costs
N_c	Total number of constraints
C_T	Total cost as the summation of the initial cost the expected failure cost
C_I	Initial cost, which is a function of design variables
C_F	Expected failure cost
C_D	Planning and design cost
C_P	Product cost
C_C	Construction cost
C_Q	Quality-assurance and quality control costs
C_M	Preventive and corrective maintenance costs
C_m	Total materials cost
C_{cb}	Cost of concrete in the beam
C_{sb}	Cost of reinforcing steel
C_{pb}	Cost of prestressed steel
C_{fb}	Cost of formwork
C_{sbv}	Cost of shear steel
C_{fib}	Cost of fiber in the concrete
L_b	Span length of the beam
w	Unit weights
A	Cross-sectional area
c	Unit cost

R	Structural resistance or capacity of a structure such as axial force, torsion, moment, or shear
Q	Applied load or demand on the structure such as combined dead load, live load, snow load, etc.
FOSM	First order second moment
FORM	First order reliability method
I-FORM	Inverse-first order reliability method
SOSM	Second order second moment
SOTM	Second order third moment
SORM	Second order reliability method
MCS	Monte Carlo simulation
PMF	Probability mass function
S	Safety margin
R'	Normalize values of structural resistances
μ_R	Mean values of structural resistances
σ_R	Standard deviation of structural resistances
Q'	Normalize values of structural loads
μ_Q	Mean values of structural loads
σ_Q	Standard deviation of structural loads
Φ	Standard cumulative normal distribution function
M	Lagrange function
μ	Lagrange multiplier
β_F	First order reliability index

β_{FOSM}	First order second moment reliability index
β_{FORM}	First order reliability method reliability index
β_{SOSM}	Second order second moment reliability index
β_{SOTM}	Second order third moment reliability index
α_S	Third dimensionless central moment
X	k-dimensional vector called the design vector
$g_i(X)$	Inequality constraints
$h_j(X)$	Equality constraints
$\mathrm{y}th$	Differential of f at X^*
X^*	Relative maximum or minimum point
$(Pr)_{Ri}$	Probability weighting for repair strategies

CHAPTER 1

INTRODUCTION

1.1 Background

The infrastructure in the U.S. and much of the world is in a state of disrepair as a result of deterioration, damage left un-repaired due to a lack of funds. Because funds are so limited and the need for rehabilitation and repair so significant, methods that optimize this repair process must be utilized. "Structures, like people, never get younger. Structures, like people, can maintain their good health with age, if properly cared for, examined, and treated when needed." (Ratay, 2005)

Many concrete highway bridges in the United States are deteriorating at an alarming rate. An estimated 40 % of the bridges in the United States are classified as lacking and in need of repair, rehabilitation, or replacement. In their 2005 Infrastructure Report Card the American Society of Civil Engineers (*ASCE*) reported that America's Infrastructure gets an overall grade of D but also estimated that $1.6 trillion is needed over a five-year period to bring the nation's infrastructure back to just "good" condition. Furthermore, the $1.6 trillion it is a point-in-time estimate and does not account for future population growth. In addition, many of these bridges are classified as deficient for modern-day traffic loading, having been designed to older design codes; many simply don't have the structural capacity because of poor maintenance (Naaman and Breen, 1990).

The fundamental causes of concrete bridge component deterioration are (1) the corrosion of reinforcing steel and prestressing strands due to the presence of soluble chlorides from deicing chemicals or marine exposure and (2) decay of the concrete material from a severe environment and/or chemical reactions to deicers.

Structural analysis and design are the typical background of engineers performing structural condition assessment or evaluations, but experience in field inspection of deteriorating structures is crucial. Engineers analyze and design structures for strength, stability, and deformation, but the most common problems are serviceability issues.

An in-service structure has resisted the combined effects of use, live loads, and often harsh environmental conditions over its lifetime. However, buildings, bridges, parking structures, stadiums, and all other structures do deteriorate with time as the result of repeated loadings, exposure to the elements, aging of materials, wear and tear from normal use, inadequate maintenance, and other factors.

Evaluating the condition of structures is area subset of professional engineering practice within the field of structural engineering. The requirement for maintenance, repair, and rehabilitation of buildings, bridges, and other systems within the deteriorating infrastructure is an increasingly active business. In any performance evaluation, there are two important fundamental categories to identify defects and deterioration: Firstly, defects may be introduced through poor design, manufacturing, fabrication, or construction before a structure begins its service life; Or by inappropriate operation and maintenance during a structures service lifetime. Deterioration is the gradual adverse loss of desired material properties. Eventual deterioration is normal for most construction materials as a result of aging and the weathering processes, and is typically addressed through strategic maintenance, repair, or replacement in order to avoid overstress problems which could result in failure. Defects may influence the rate of deterioration or may initiate premature deterioration for materials; hence the two are often involved in a cause and effect relationship.

Structural condition evaluation techniques consist of visual observation, measuring, photographing, probing and sampling, field and laboratory testing, engineering analyses, record keeping, documentation, and report preparation. It is quite different than forensic investigation of structures, which is the determination of the causes and modes of failure (where failure is not only collapse but also the unacceptable difference between intended and actual performance (Wierzbicki and Jones, 1989)).

While accuracy and sophistication of the field measurements, testing, and analysis are important, the reliability of the condition evaluation lies in the interpretation of the data and the judgment used in applying them to generate accurate conclusions and recommendations. However, if the recommendations made are significantly more costly than needed, then this is funding poorly spent. Thus, optimization techniques that provide decision-support to best apply these funds for both evaluation and repair are needed.

Structural reliability is a method that provides a measure of the safety reserve in a system and has been used for the evaluation of structural performance including the calibration of the *LRFD* code for steel and bridges (Nowak, 2000). It also allows one to address multiple criteria using a composite limit state function.

Structural health monitoring (*SHM*) is the procedure of establishing some knowledge of the structural condition of a structure periodically or continuously usually to provide targeted periodic maintenance. Ultimately it is envisioned that *SHM* will be able to determine the existence, locations, and degree of damage in a structure. *SHM* has tremendous potential for applications in monitoring of civil structures such as bridges and buildings (Vanik and et al., 2000).

There are numerous techniques for structural condition evaluation that have been developed over the last several decades such as loading tests, following the American Society Testing Material standard *ASTM E*455-04 (*ASTM E*455, 2004). This method uses water or sand for loading of the structure and simply measures the structures performance based on quantities such as deformations, stresses, and strains. There is also a fully destructive test which involves destruction of some portion of the structure by cutting or core drilling. Both of these methods are rather expensive and extremely time consuming and disruptive. In this dissertation non-destructive evaluation and semi-destructive evaluation techniques are hereafter referred to as *NDE* and *SDE* methods, respectively. When performing structural condition evaluations, both *NDE* and *SDE* are not only very convenient but are also much less time consuming, disruptive, and expensive than fully destructive testing.

The most common *NDE* techniques for civil engineering structures are Visual Inspection (*VT*), Penetrate Inspection (*PT*), Magnetic Particle Inspection (*MT*), Eddy Current (*EC*), Radiographic Inspection (*RT*), Ultrasonic Inspection (*UT*) and Acoustic Emission (*AE*) (Zheng and Ellingwood 1999). There are also numerous *SDE* techniques that are used in structural condition evaluation such as the Standard Pullout Test Method (*ASTM C* 900, 1987), and the Standard Test Method for Determining Residual Stresses by the Hole-Drilling Strain-Gage Method (*ASTM E*837-01e1, 2001), etc.

NDE and *SDE* methods are used for evaluation of structures and material strength, and identification of damage. They have been used in lieu of loading and other destructive tests that are costly, time consuming, and result in substantial structural damage. *NDE* and *SDE* methods have the obvious advantage over the loading and destructive tests in that they are inexpensive and leave the structure fully intact. However, they have the disadvantage that significant uncertainties are introduced and thus the ability of *NDE/SDE* methods to detect and measure structural damage results in added risk if damage goes undetected and added cost if non-existent damage is identified. It is proposed in this dissertation to introduce this uncertainty into the decision-making process through the application of existing data on *NDE/SDE* strength (and damage) detection accuracy.

1.2 Objective

The objective of this dissertation is to optimize the structural evaluation procedure and repair procedure based on reliability theory prior to initiating inspection, thus allowing one to best use limited funds and address the infrastructure deterioration efficiently and least expensively. This methodology includes the introduction of several new concepts, terms, and objective functions.

The key objectives are:

1. Develop a theory and methodology for optimal structural condition evaluation by *NDE* and/or *SDE* method and optimal structural repair and rehabilitation simultaneously.
2. Determine and calculate the structural deficiency, defined as the target reliability index (β_{target}) minus the initial estimated guiding reliability index (β_g), $\Delta\beta$, and

use this within the optimization procedure for simultaneously selecting an evaluation and repair procedure.

3. Provide quantitative decision-support for structural evaluation method selection, structural repair, and the combination thereof.

Material and/or Structural Type	**Target β**	**Design Life**	**Annual Prob. of Failure**
Wood Members in Flexure (*AF&PA*, 1996)	2.4	50 years	1.646×10^{-4}
Steel (braced compact beams in flexure, tension members at yield) members (*AISC*, 1998)	2.6	50 years	9.343×10^{-5}
Steel (braced compact beams in flexure, tension members at yield) connection (*AISC*, 1998)	4.0	50 years	6.337×10^{-7}
AASHTO, design (calibrated for girders) (Moses, 2001)	3.5	75 years	3.105×10^{-6}
AASHTO, evaluation (calibrated for girders) (Moses, 2001)	2.5	5 years	1.245×10^{-3}
RC Beams in shear and flexure (Szerszen and Nowak, 2003)	3.5	50 years	4.653×10^{-6}
RC Slabs (cast in place) (Szerszen and Nowak, 2003)	2.5	50 years	1.246×10^{-4}
RC Columns (Szerszen and Nowak, 2003)	4.0	50 years	6.337×10^{-7}

Table 1.1: Target Reliability Indices and Corresponding Annual Probabilities of Failure (excerpted from Atadero, 2006).

CHAPTER 2

LITERATURE REVIEW

2.1 Structural Condition Evaluation and Structural Monitoring Methods

There are many methods for structural condition evaluation and structural monitoring in the structural (civil) engineering which, as mentioned in chapter 1, include non-destructive evaluation (*NDE*) and semi-destructive evaluation (*SDE*).

2.1.1 Non-Destructive Evaluation (*NDE*) Methods

There are many types of non-destructive evaluation such as the ultrasonic pulse velocity (*UPV*) method, the Schmidt hammer or rebound hammer (*RN*) method, and the Windsor probe method.

Ultrasonic Pulse Velocity (UPV) Test (ASTM C 597-83)

The concept for the *UPV* method was developed by Jones and Gatfield (1962) at the Road Research Laboratory between 1945 and 1949. Leslie and Cheesman (1949) are also recognized as pioneers of *UPV*. Early *UPV* equipment was combined with a cathode-ray oscilloscope that was used for measuring the transit time (traveling time of the ultrasonic wave in a material) between two points. This equipment is very useful in the laboratory, but is somewhat difficult to use in the field. The ultrasonic pulse wave velocity is dependent on the media material density, strength, and elastic properties. The pulse velocity is equal to the path length divided by transit time

$$Pulse\ Velocity = Path\ Length\ /\ Transit\ Time \qquad (2.1)$$

(Szilard, 1982)

For accuracy, each specimen should be tested at least three times. Pulse velocity will not be influenced by the shape of the specimen, but will be influenced by the distance measured between the probes (path length). The path length should be more than the wavelength of the pulse vibrations. Thus, as one can see, there are significant sources of uncertainty which reduce the probability of detection and effect the decision-making of when and how much to repair a structure.

Ultrasonic pulse velocity equipment or a *PUNDIT* field kit shown in Figure 2.1 and consists of the *PUNDIT*, two transducers, two transducer leads, bag case, and adapter.

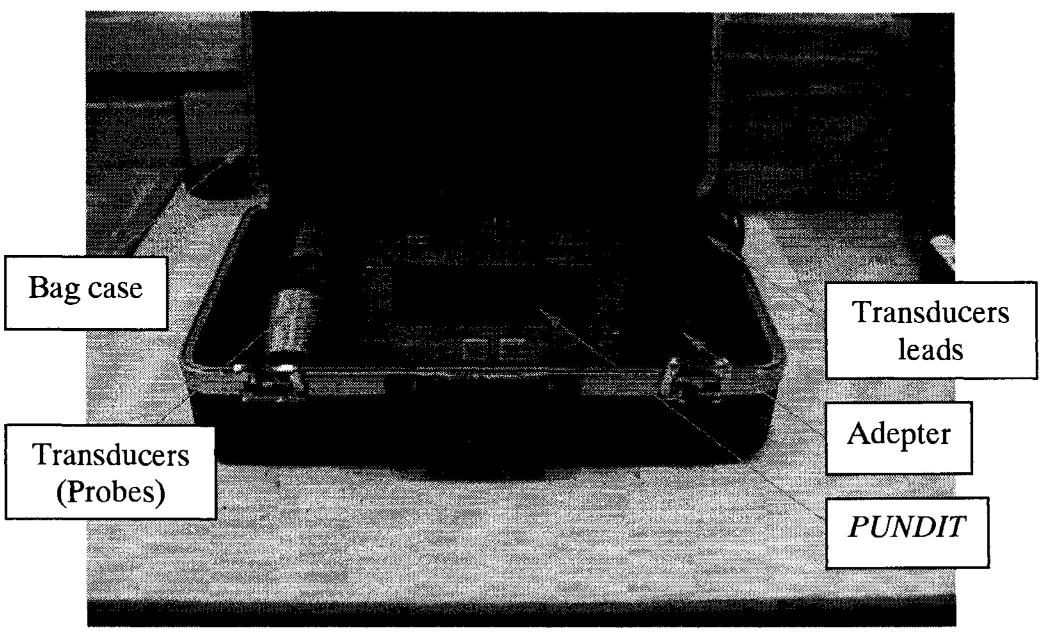


Figure 2.1: Ultrasonic Pulse Velocity Equipment (Buddhawanna, 2003).

Figure 2.2 shows a concrete test cylinder being tested via the *UPV* method in a laboratory. Again, it should be noted that human error and operator knowledge also play a role in the uncertainties associated with *NDE*.

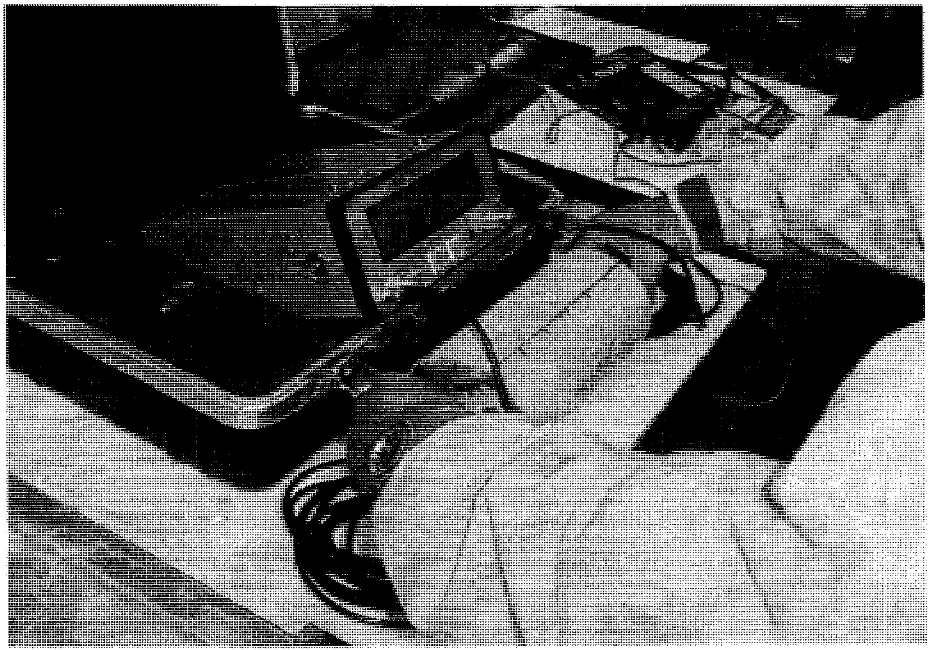


Figure 2.2: Ultrasonic Pulse Velocity Testing on Concrete Specimen (Buddhawanna, 2003).

Rebound Hammer (Schmidt Hammer, Rebound Number (RN) Method, ASTM C 805-85)

The rebound hammer equipment, shown in Figure 2.3, is a non-destructive test method for testing the quality of hardened concrete and other engineering materials such as steel, composites, etc. The rebound hammer is applied at the surface of specimens or structures (Buddhawanna, 2003).

A rebound hammer is applied at the surface and measures hardness by measuring the rebound number (*RN*) that are reported from a rebound hammer. Higher rebound numbers correlate to higher material strengths. Rebound hammer parts are shown in Figure 2.3 and 2.4 and are comprised of Rebound Hammer that is consisted by hammer body (the hollow cylindrical metal that has a diameter of about 6 cm.), plunger (the solid cylindrical metal that has a diameter about 1 cm.), hammer mass (the solid mass metal that is in the rebound hammer body), indicator, latch, monitor, connection lead, and bag case (Buddhawanna, 2003).

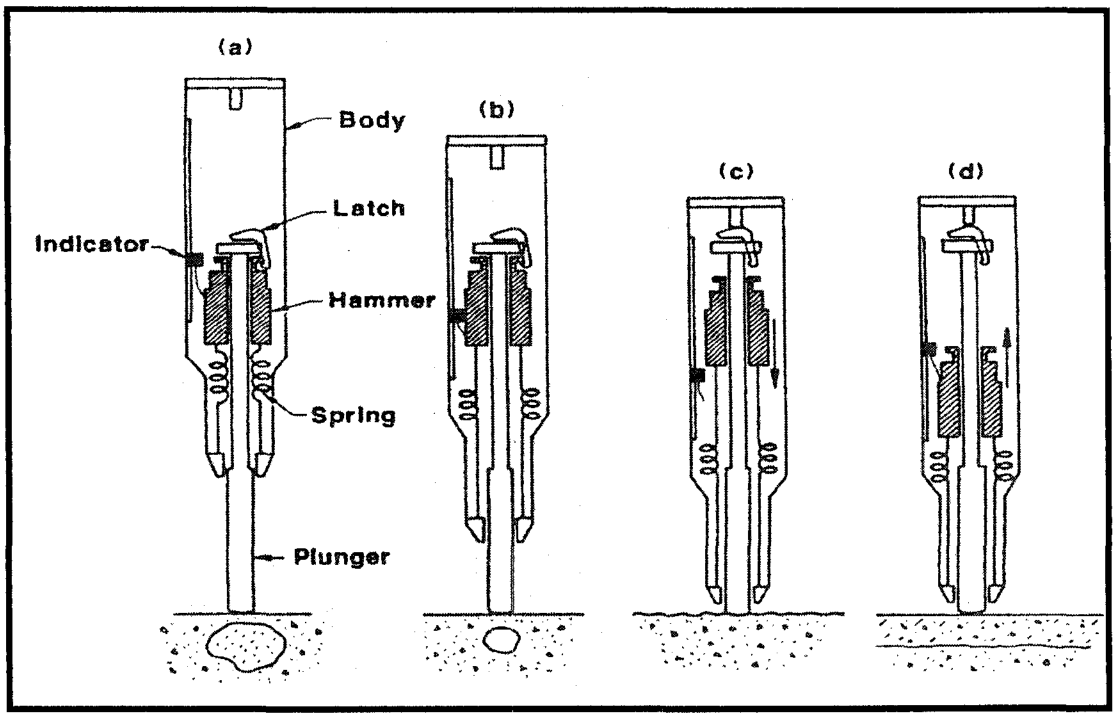


Figure 2.3: Diagram of Schmidt Hammer (Malhotra et al., 1991).

Figure 2.4: Concrete Rebound Hammer Equipment (Buddhawanna, 2003).

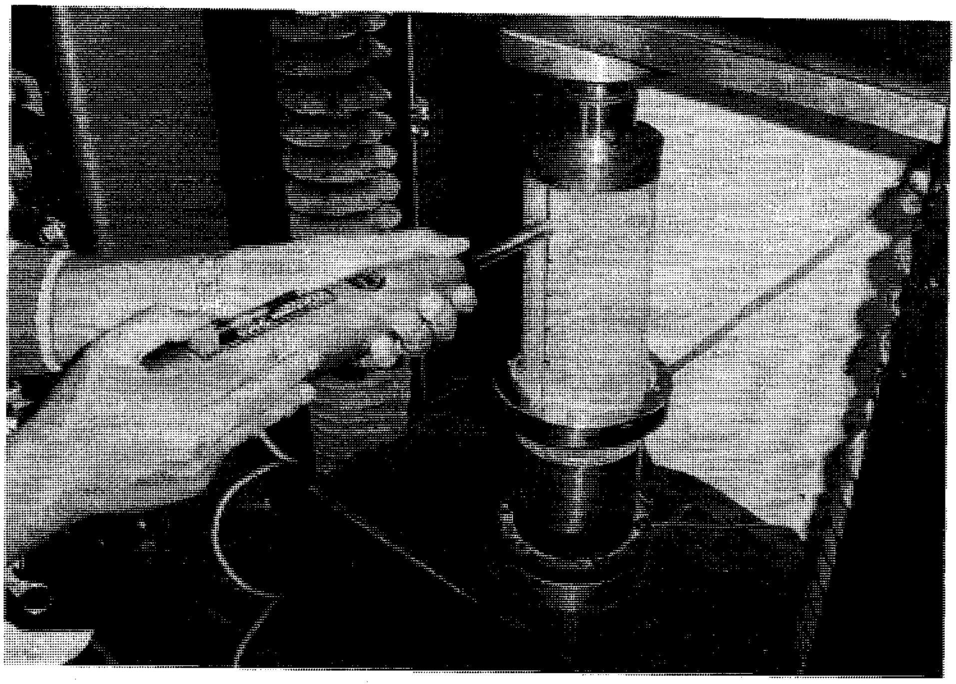


Figure 2.5: Rebound Hammer Testing on Concrete (Buddhawanna, 2003).

Windsor Probe (Probe Penetration Resistance Method (ASTM C 803)

The Windsor Probe, a non-destructive probe penetration resistance method, is shown in Figure 2.6. Penetration resistance methods are based on the determination of the depth of the penetration of probes (steel rods or pins) into the concrete. The depth of the probes correlates with the hardness of the material and can be related to the material strength (Malhotra and Carette, 1995).

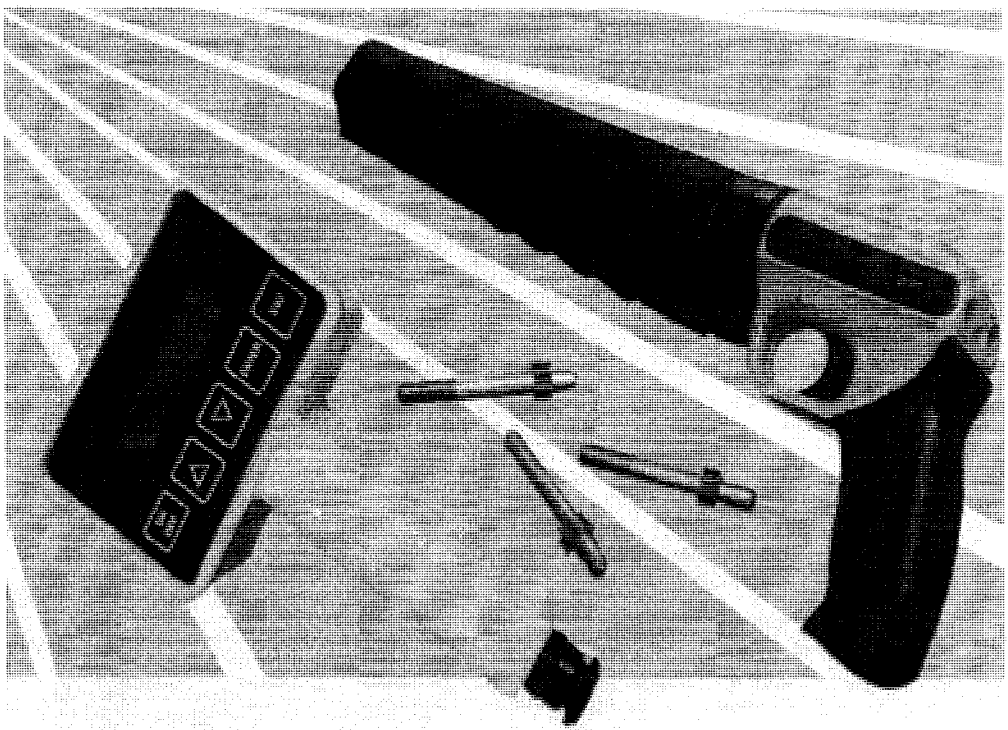


Figure 2.6: Windsor Probe Equipment (Qualitest International Inc., 2003).

2.1.2 Semi-Destructive Evaluation (*SDE*) Methods

There are many kinds of semi-destructive evaluation for assessing potentially damaged structures. These include the pullout (*PO*) method and determination of residual stresses by hole-drilling strain-gage method.

Standard Pullout (PO) Test Method (ASTM C 900-87)

The pullout test method is a Semi-Destructive Evaluation (*SDE*) method. The approach here is to insert expansion bolts into the structural material, pull them out which provides a measure of the punching shear capacity as shown in Figure 2.7 and 2.8. The punching shear is then correlated with the material strength. The correlation of an *SDE* with material strength is typically better than *NDE* with material strength, thus more accuracy is provided for decision-making but some level of additional damage to the structure is imparted. This tradeoff has not been quantified before as in this dissertation. For practical applications such as concrete strength testing using the pull-out test for, e.g. a fire damaged supporting wall, as shown in Figure 2.8, an evaluator/engineer must prepare the concrete testing surface and drill the testing hole using the pull-out test preparation kit which is shown in Figure 2.9. Then the pull-out expansion bolt is inserted into the test hole. Next, an evaluator/engineer will applies the pull-out test pull machine, as shown in Figure 2.10, on the pull-out testing expansion bolt and simply measures the pull-out force or punching shear force which has a direct correlation with the concrete compressive strength.

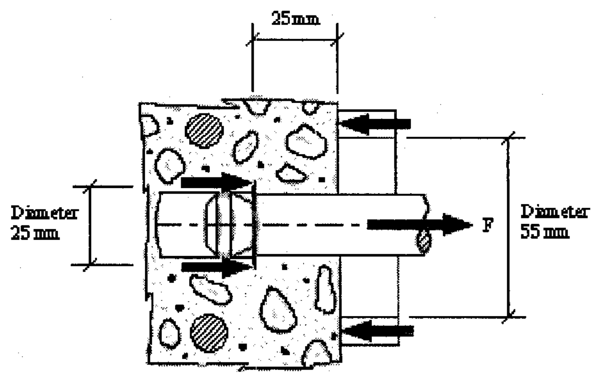


Figure 2.7: Pull-out Test (Germann Instruments, 2006).

Figure 2.8: Testing for Strength with Pull-out Test of Fire Damaged Supporting Wall (Germann Instruments, 2006).

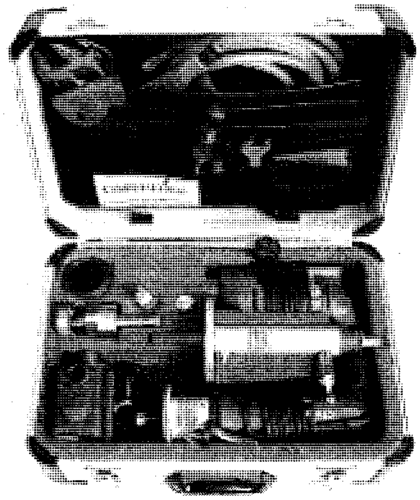


Figure 2.9: Pull-out Test Preparation Kit (Germann Instruments, 2006).

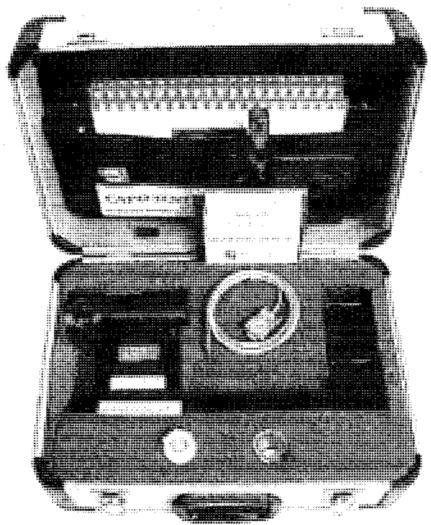


Figure 2.10: Pull-out Test Pull Machine Kit (Germann Instruments, 2006).

2.2 Reliability Analysis with Non-Destructive Evaluation (*NDE*) and Non-Destructive Testing (*NDT*)

The reliability of an evaluation technique is usually described in terms of its detection ability. The ability of a technique to detect accurately mainly depends on the crack size, because probability of detection (*POD*) typically increases as crack size increases. An extensive investigation is presented by Lewis (1999) and includes more than 20,000 inspections conducted on more than 170 cracks by 107 different inspectors. Berens and Honey (2002) performed a statistical evaluation by employing regression analysis to fit seven differential functional forms for the *POD* curve to this large data et. Based on their work, the best mathematical model for the *POD* is

$$POD(c) = \frac{\delta c^{\theta}}{1+\delta c^{\theta}} \tag{2.3}$$

where δ and θ are regression parameters that depend on the type of *NDE* technique used, and "*c*" is the crack size. This model is used by Palmberg et al. (2000) in probabilistic damage tolerance calculations for a panel with δ equal to, for example, 0.0032 $mm^{-\theta}$ and θ set at 3.5 (Acar et al., 2005).

According to the method presented by Harkness et al. (1999), failure is defined as the inability of the method to detect cracks. Therefore, the joint probability density function (*PDF*) of the random variables describing the problem should be updated as failure of the method is realized. The updated joint *PDF* of the random variables can be found by applying Bayes Theorem and expressed as

$$f_I(x) = PND_Y(x) f_U(x) \tag{2.4}$$

where $f_I(x)$ and $f_U(x)$ are the initial (or prior) and updated (or posterior) joint *PDF* respectively, $PND_Y(x)$ is the probability that the cracks are not detected in the first *Y* inspections. Since the probability of failure for detection in this case, PND_Y, is just the product of probabilities that a crack can not detected in the first *Y* inspections, it can be expressed simply as

$$PND_Y(x) = \prod_{i=1}^{Y} \{1 - POD[c(x, N_i)]\} \quad (2.5)$$

where $POD[c(x,N_i)]$ is the probability of detecting for a crack of length c, and $c(x,N_i)$ is the crack length at the i^{th} inspection time, N_i is defined in terms of the random variable set x.

After updating the distribution of random variables, the probability of failure since the last inspection can be calculated as

$$P_{fxY}(N_S) = \int_{\Psi fxY} PND_Y(x) f_X(x) dx \quad (2.6)$$

where Ψ_{fxY} is the domain of random variables that leads to $N_i \leq N_S \leq N$ (Acar et al., 2005).

In the case of *NDT*, it is related to the detection of discontinuities within the material. Depending on field conditions and the requirement for realistic test specimens, the calibration process is often quite expensive or even impossible. To prepare for an efficient and effective approach, it was proposed to divide the *NDT* system into its components and assess the reliability of each component in a way that is appropriate for the nature of each component's information (Mueller et al., 2001).

In the reliability analysis of systems, module components are investigated where components have known failure probabilities (Barlow et al., 1975; Willie, 1979). The system failure is modeled as a logical function of component failures. The probability of system failure can then be calculated with the help of elementary probability theory. For this procedure, additional assumptions about independence or knowledge of failure probabilities of certain subsystems are necessary.

As shown in Figure 2.11, system X does not fail if one of the three alternative paths (M_1-M_2-M_5, M_1-M_3-M_5 or M_4-M_5) has no failing component. This can be expressed mathematically as

$$X = X(M_1, M_2, M_3, M_4, M_5) = \{[M_1 \cap (M_2 \cup M_3)] \cup M_4\} \cap M_5 \tag{2.7}$$

where M_i is used as the symbol for the statement "the component M_i does not fail". Equation (2.7) can be rewritten according to the rules of set algebra as

$$X = (M_1 \cap M_2 \cap M_5) \cup (M_1 \cap M_3 \cap M_5) \cup (M_4 \cap M_5) \tag{2.8}$$

System failure is given by the sets complement as

$$X' = (M_1 \cap M_2 \cap M_5)' \cap (M_1 \cap M_3 \cap M_5)' \cap (M_4 \cap M_5)' \tag{2.9}$$

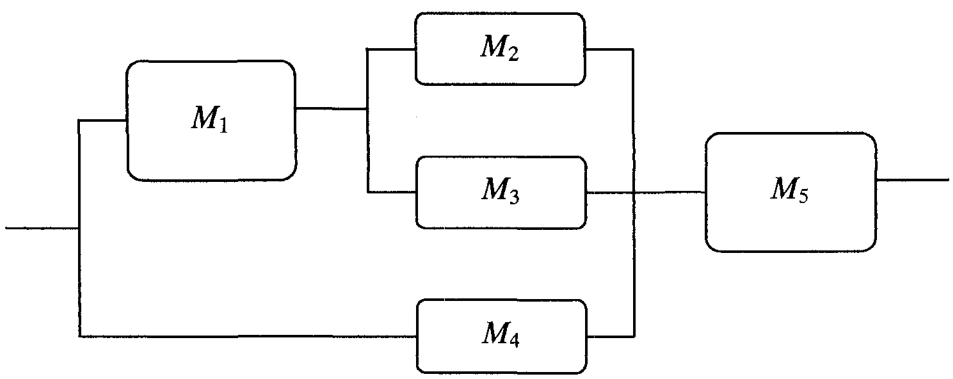


Figure 2.11: Illustration of a System of Components.

where X' is the failure condition that means all three paths are blocked by at least one failure component. M_i' occurs when M_i fails. The probability for event M_i is written as $P(M_i') = 1 - p_i$. Assuming that all components fail independent of one another (for example, component K_3 does not work more reliably than usual just because K_2 fails); the probability for the system (S) to work correctly can be calculated as:

$$
\begin{aligned}
P(X) &= P(\{[M_1 \cap (M_2 \cup M_3)] \cup M_4\} \cap M_5) \\
&= P\{[M_1 \cap (M_2 \cup M_3)] \cup M_4\} P(M_5) \\
&= \{P[M_1 \cap (M_2 \cup M_3)] + P(M_4) - P[M_1 \cap (M_2 \cup M_3) \cap M_4]\} p_5 \\
&= [P(M_1)P(M_2 \cup M_3) + p_4 - P(M_1)P(M_2 \cup M_3)P(M_4)] p_5 \\
&= \{p_1 [P(M_2) + P(M_3) - P(M_2 \cap M_3)](1 - p_4) + p_4\} p_5 \\
&= [p_1 (p_2 + p_3 - p_2 p_3)(1 - p_4) + p_4] p_5
\end{aligned}
\tag{2.10}
$$

The failure probability for X is $P(X') = 1 - P(X)$. If subsystems X_i, X_j do not fail independently of each other, the result of $P(X_i \cap X_j) = P(X_i)P(X_j)$ used in the above equation is not true. The above equation becomes more complex because additional assumptions about combined probabilities are necessary. The key for this method of system reliability assessment, which was developed during nuclear power plants risk studies in the 1960's, is the fault tree analysis and failure mode-and-effect analysis (Mueller et al., 2001).

2.3 Structural Repair, Rehabilitation, and Maintenance

Most structural systems need appropriate rehabilitation and maintenance at some point during their lifetime in order to consistently remain in satisfactory operation. Engineers typically use experience to decide how to perform an evaluation and subsequently how to repair or replace, and/or shut-down a deteriorating structural system (Kong and Frangopol, 2002). Usually, the construction cost, the inspection cost, the repair cost, the maintenance cost, the user cost, and the failure cost are essential for the repair and maintenance cost analysis of deteriorating structures (Chang and Shinozuka, 1996; Ang and De Leon, 1997; Frangopol et al., 1997 and 2000; Ang et al., 1998; Das, 1998 and 2000; Maunsell Ltd. and Transportation Research Laboratory, 1998 and 1999; Enright and Frangopol, 1999; Wallbank et al., 1999). The scheduling of inspections, evaluations, repair, maintenance and the duration of these actions depend on various parameters such as space, time, labor, material, and various sources of error. These are termed influencing factors in this research. The influencing factors are represented by probability distributions due to their high variability. In general, the probability distributions of the times of application of influencing factors are calculated based on a relative time scale. For example, time is measured from a relative reference point, such as the time of the previous maintenance. These relative times can be converted to the absolute time such as age of the structure to be used directly during the decision making processes for maintenance priorities and repair or rehabilitation strategy.

In an effort to optimize structure maintenance, repair, and rehabilitation to achieve the most cost-effective approach for an existing group of deteriorating structures, structural engineers have developed several management methods. Most of those methods are based on subjective condition assessment and empirical models to predict future condition (Aktan et al., 1996). In these methods, gathered data concentrates on structural members more than the overall system and subjective scales dependent of expert judgment are used to assess condition. Because the process of condition evaluation should depend on quantitative data more than quantitative information, the development of management methods that does not solely rely on subjective data is essential. A reliability-based method is capable of providing objective quantitative decision-making support (Kong and Frangopol, 2003).

The main purpose of evaluation and rehabilitation cost analysis is finding the optimal rehabilitation scenario for an individual structure or a group of similar, i.e. the same type, of deteriorating structures. The variability of rehabilitation cost is related not only to the type of influence factors but also to other factors such as the condition state of the structure at the time of the repair application, important level of structural members, and reliability state of the structure. In general, structures in better condition require less repair or rehabilitation cost than structures in poor condition to reach the same reliability level. Also, the same type of repair on two structures having the same reliability level is expected to be more expensive for the structure whose reliability level after repair is higher. However, current state-of-the-art structural management systems use fixed cost data based only on the type of repair without considering the level of the repair and their effects on structural system reliability. The condition of the structural system often depends on the history of its repair. Therefore, it is important to consider the relationship between cost of repair and corresponding structural reliability (Kong and Frangopol, 2004).

2.4 Repair and Rehabilitation Strategies

Many reinforced or prestressed concrete facilities such as highway bridges, buildings, etc. in the United States are beyond their design service life (*ASCE*, 2005). Because of the age of these bridges, the condition of their reinforcing steel and concrete may be questionable. Additionally, a significant number of brides were designed to resist loads that were lighter and often lower in traffic volume than they are experiencing today. This further underscores the need for effective condition evaluation and economical repair procedures for the nation's bridge. Unfortunately, there is no direct single solution that is a simple and straight forward method for optimal and suitable evaluation and repair. Furthermore, some procedures for repair of deteriorating/damaged structures are quite complex because many structures must be partially or fully utilized during repair.

There are many widely acceptable methods for structural repair which include (1) Conventional Repair Strategies (*CRS*), (2) External Bonded Fiber Reinforced Polymer (*FRP*), and (3) External Prestressing (*EP*).

Conventional Repair Strategy (CRS)

This is the first method that was developed by engineers for repair and rehabilitation of deteriorating concrete beam bridges. The purpose of this method is to directly increase the strength of the damaged structure by adding external reinforcing bars (upper and lower bars) for *RC* structures or increasing the prestressing stand area for *PC* structures. Strengthening the concrete material by epoxy grouting cracks or other defects or adding additional structural members is also considered to fall into the conventional repair category. The conventional strengthening methods included in this study are:

1. Bonded steel element: Strengthening deteriorating *RC/PC* structural members by the bonded steel plate method was developed in the 1960s in Western Europe. In this method, steel elements are glued to the concrete surface by a two-component epoxy

adhesive or can be mechanically anchored using expansion bolts to create a composite system and improve shear or flexural strength. The steel elements can be steel plates, channels, angles, or built-up members.

2. Section enlargement: In the section enlargement strategy, additional reinforced concrete members are bonded to an existing *RC/PC* structural member using the concrete cast-in-place strategy. By the section enlargement method, the columns, beams, slabs, and walls can be enlarged to increase their load resisting capacity or stiffness. A typical enlargement is approximately 2 to 3 inches for slabs and 3 to 5 inches for beams and columns.

3. Span shortening: The span shortening strategy is accomplished by adding some supports underneath existing members. The span shortening is typically accomplished using structural steel members or cast-in-place reinforced concrete members that are simple to install on a deteriorating structure. The connections between an existing member and new supports can be designed easily using bolts and adhesive anchors.

External Bonded Fiber Reinforced Polymer (FRP)

The external bonded fiber reinforced polymer composite offers tremendous potential for repair and rehabilitation of existing bridge girders as well as the construction of new bridges. *FRP* materials are robust and durable materials with high strength-to-weight ratios that serve well for many repair scenarios. These qualities make *FRP* a well-suited material to use in bridge repair and rehabilitation (Azizinamini et al., 2003).

Rehabilitation and strengthening of *RC* and other structures using *FRP* composites has become quite popular for structural work recently. This is, at least in part, due to (1) the need for significant rehabilitation for many bridges, and (2) the significant advantages *FRP* composites posses by combining good corrosion resistance with ease of handling, i.e. lightweight. The decrease in material costs for *FRP* composites has also influenced their popularity (Basham, 1994).

External Prestressing (EP)

In response to the demand for a simple and efficient process for strengthening, rehabilitation, or repairing a deteriorating existing concrete bridge, a simple method known as external prestressing is used quite routinely. The external prestressing development has been one of the major trends in construction, maintenance, repair, and rehabilitation of infrastructure during the last few decades along with the increasing use of high strength concrete and the development of concrete cable-stayed bridges. External prestressing is meant to imply the use of un-bonded prestressing tendons outside the concrete section of a *PC* beam. External prestressing strategies have been increasingly considered in the construction of new concrete structures as well, but are still more prevalent in the rehabilitation of existing bridges. This is because the method has is inexpensive and can be completed rapidly compared with many other techniques. For these reasons it is a primary method for the rehabilitation and strengthening of existing bridge beams (Naaman and Breen, 1990).

2.5 Reliability-Based Design of Structures

The primary objectives of engineering design and rehabilitation are to ensure the performance of a structural system or member. In this regard, an engineer must analyze a bridge under conditions of uncertainty while targeting structural performance that is statistically possible. In general, a probabilistic (reliability-based) analysis will be necessary in the development of a modern rehabilitation, and provides the basis for current design philosophies, i.e. load and resistance factor design (*LRFD*) (Ellingwood et al, 1982).

In structural condition assessment/evaluation procedures, the reliability-based design method is used in the early stages of the process for calculation and estimation of an unacceptable structural resistance (R). By approximation the deteriorating structural reliability index or choosing an initial temporary guiding reliability index, an evaluator or engineer can estimate what the reliability index is in the damaged or deteriorated state using a measured structural load such as dead load, live load, impact load, etc. This analysis takes into account all the random variables for both the load and the resistance.

The basic concept of reliability-based design is to use quantitative information about the load and resistance of a structure to determine the probability of failure. In addition, with the proposed formulation, designers can make the decision based on an acceptable level of risk, instead of a deterministic safety factor. The development of reliability-based design codes such as Load and Resistance Factor Design (*LRFD*) is based on the choice of the levels of reliability index (Mahadevan and Haldar, 1989).

Al-Harthy and Frangopol (1994) included uncertainties in loads, material properties, geometric properties, prestressed force levels, in order to predict structural behavior at initial, final, and ultimate stages of prestressed concrete (*PC*) girder construction. For a representative sample of *PC* beams, it was shown that the reliability levels related to the current American Concrete Institute (*ACI*) practice are non-uniform over various ranges of live load levels, span lengths, and limit states. This was also observed by Nowak (1999) and van de Lindt et al. (2005).

Choosing reliability indices given by *ACI* 318-89 (*ACI*, 1989) for *PC* girders as the target reliability indices, reliability constraints with respect to stresses at initial, final, and ultimate limit states are all satisfied simultaneously. The algorithm of the feasible directions method used to approach at the solution (Al-Harty and Frangopol, 1994).

Reliability-based structural optimization methods have been used to calculate the reliability-based design of *PC* girders. The problem is generally formulated as an optimization problem subject to reliability constraints. Reliability-based optimization concepts can be found in Frangopol (1985) and Frangopol and Moses (1994), among others. In general, the structural reliability-based optimization problem is formulated as follows:

Find X such that it minimizes $f(X)$ subject to the constraint $\beta_i(X) \geq \beta_j^0$; j = 1, 2,..., m where $x_i^l \leq x_i \leq x_i^u$; $i = 1, 2, \ldots, n$, $f(X)$ is the objective function; x is the vector of design variables; x_i^l and x_i^u are the lower and upper bound of design variables constraints; β_j is the reliability index for component or structural system j; and β_j^0 is the target reliability index of the component or structural system j.

If a structure has been evaluated and found to be structurally deficient, the selected rehabilitation procedure must suit not only the type of structure and damage/deterioration but also the construction site itself. Then the analyst must choose an appropriate target reliability index and design a suitable rehabilitation strategy considering the current condition, constraints, and desired condition of the structure. However, all of this must be accomplished within the very uncertain framework of *NDE* and *SDE* results. Thus, probabilistic design such as reliability-based design combined with optimization serves to provide the best solution to this complex problem.

2.6 Cost Optimization Using Reliability Theory

The application of reliability theory within an optimization framework is not new in civil engineering. In deterministic optimization, a structure is optimized only for a given pre-determined set of loading but in reliability-based design, the loads and the structural strengths are modeled as random variables and the probability of exceeding the structural capacity as a result of the applied loading is determined. The major problem in reliability-based optimization is the calculation of the probability of failure, which often cannot be determined due to insufficient statistical data (Sarma and Adeli, 2000).

Many objective functions have been proposed in structural engineering optimization such as minimization of the total cost, minimization of the total weight of the structure, and minimization of the total utility. A simple and convenient objective function is the minimization of the total weight of the structure, which can be expressed simply as

$$\text{Minimize} \qquad f(X_d) = \sum_{i=1}^{n} W_i \tag{2.13}$$

where X_d = the vector of design variables; W_i = the weight of an element i; and n = the total number of elements.

Constraints are applied that are related to the structural reliability of the structure. In terms of reliability indices, these constraints can be expressed as:

$$\beta_i^l \le \beta_i \le \beta_i^u, \qquad i = 1, 2, \ldots, m \tag{2.14}$$

where the lower bound β_i^l is the required safety level for the i^{th} limit state, and the upper bound β_i^u is the desired range of the reliability index (Mahadevan, 1989).

The most general form of a reliability-based cost optimization problem may be expressed as follows:

$$\begin{aligned} &\text{Minimize } Cost \\ &\text{Subject to} \quad P_f \le (P_f)_{target} \end{aligned} \tag{2.15}$$

where $Cost$ is the life cycle cost associated with material and manufacturing, fuel consumption, and inspection. P_f is the probability of failure and $(P_f)_{target}$ is the pre-specified target failure probability. Using the values given by Kale (2000) the life cycle cost is expressed as:

$$Cost = M_C W + F_C W N_f + N_i I_C \tag{2.16}$$

where M_C is the material and manufacturing cost per pound, W is the fatigue, N_f is the service life in number of operations, N_i is the number of inspections, and I_C is the inspection cost including repair and replacement costs.

In the optimization problem state, the design variables are the structural strength random variables that change the probability of failure (Acar et al., 2005). The reliability term in cost optimization is considered either directly or indirectly. In the direct way, the reliability factor is included directly in the objective function (Moses, 1977).

Moses (1977) presents the total cost (C_T) as the summation of the initial cost (C_I) and the expected failure cost (C_F) multiplied by the probability of failure (P_f). Mathematically, this is expressed as

$$C_T = C_I + P_f C_F \tag{2.17}$$

subjected to the design constraints

$$g_i(x) = \text{Structural Resistance (Supply)} - \text{Load (Demand)} \geq 0,\ i = 1,2,\ldots, N_c \tag{2.18}$$

where N_c = total number of constraints. The expected failure cost includes the cost associated with failure of the structure, such as the replacement cost, damage to property, casualties, business interruption, litigation costs, etc.

Another method known as the indirect method simply uses an objective function with the initial cost. The reliability term (failure probability) is considered indirectly in the form of a constraint, i.e. must not exceed some failure probability threshold, such as:

$$P_f \leq P_{f_{allowable}} \tag{2.19}$$

where $P_{f_{allowable}}$ is the allowable probability of failure.

Using this approach a deterministic optimization procedure can be converted into a reliability-based optimization procedure by adding one or more additional probability constraints. Both the direct and indirect methods are used by Moses (1977) for minimum cost design of reinforced concrete beams and highway girders subject to fatigue loading in which he applies a technique known as the sequential unconstrained minimization technique (*SUMT*) (Fiacco and McCormick, 1968). The probability of failure is determined from a reliability index or safety index, which is in turn computed from the basic statistics of the strength and load parameters (Frangopol and Moses, 1994). The expected failure costs are advance in advance somewhat arbitrarily.

Surahman and Rojiani (1983) present reliability-based optimization of four to ten story reinforced concrete building frames by including the reliability term in the cost function. By varying the probabilities of failure between 0.000001 and 0.01 and assuming different values for the expected failure cost, they attempt to arrive at an optimum probability of failure. Srividya and Ranganathan (1995) discuss the reliability-based cost optimization of single-story single-bay *RC* frames based on different combinations of live load and wind load within the Indian design code. They perform elastoplastic analysis and include both component and system level probabilities of failure in the form of constraints. They make relatively broad assumptions for values of failure probability. Lin and Frangopol (1996) present reliability-based cost minimization for the design of simply-supported *RC* T-girders for highway bridges using the 1994 specifications of the American Association of State Highway and Transportation Officials, *AASHTO* (1994). The initial cost is only the material cost of concrete and steel, with other costs associated with construction not explicitly considered. They point out that only 4% of structural optimization studies have involved concrete and composite structures (note that this is as of 11 years ago).

Koskisto and Ellingwood (1997) present minimum life-cycle cost optimization of prefabricated concrete structures using reliability theory. They model the total life-cycle cost as

$$C_L = C_D + C_P + C_C + C_Q + C_M + P_f C_F \qquad (2.20)$$

where C_D = planning and design cost,

C_P = product cost,

C_C = construction cost,

C_Q = quality-assurance and quality control costs,

and C_M = preventive and corrective maintenance costs.

They performed an example using a hollow core slab in which they assumed the design cost as 2.5% of the product cost, where the product cost is the sum of material and labor costs. They also assume the labor cost is approximately 43% of the material costs (C_M), and the construction cost (C_C) is set equal to 0.01 times the product of the span length and the thickness of the slab. In their example, they neglect the quality-assurance and quality control costs (C_Q), presumably due to a lack of statistics on this quantity.

Development of models for reliability-based optimization apparently was initiated by Erbatur et al. (1992) who formulated the optimization as the minimization of total cost. The term "total cost" means the initial cost (sum of all costs associated with erection and operation or the structure during its projected design life without failure) and the expected cost of failure (sum of all costs associated with the probability of failure; e.g., expected cost of disruption of normal use, expected cost associated with the loss of human life). This total cost criterion governs nearly all of the work that flourished during the 1950's and is recorded in the papers by Jones (1985), Fereig et al. (1996) among others.

The general cost function for reinforced, fiber, or prestressed concrete beams can be explained as the following equation:

$$C_m = C_{cb} + C_{sb} + C_{pb} + C_{fb} + C_{sbv} + C_{fib} \tag{2.21}$$

where C_m = total materials cost,

C_{cb} = cost of concrete in the beam,

C_{sb} = cost of reinforcing steel,

C_{pb} = cost of prestressed steel,

C_{fb} = cost of the formwork,

C_{sbv} = cost of shear steel,

and C_{fib} = cost of fiber in the concrete.

For a pretensioned beam, Equation (2.21) can be written as:

$$C_m = w_c L_b (A_{cb} - A_{sb} - A'_{sb} - A_{pb}) c_c + w_s L_b (A_{sb} + A'_{sb}) c_s + w_p L_b A_{pb} c_p + L_b p_{fb} c_f + C_{sbv} + C_{fib} \qquad (2.22)$$

where A_{cb} = cross-sectional area of concrete beam, A_{pb} = area of prestressing steel of concrete beam, A'_{sb} = area of compressive reinforcing steel of concrete beam, A_{sb} = area of tensile reinforcing steel of concrete beam, c_c = unit per weight cost of concrete, c_f = unit per weight cost of formwork, c_p = unit per weight cost of prestressing steel, c_s = unit per weight cost of reinforced steel, L_b = Span length of beam, p_{fb} = cross-sectional perimeter of form in beam, w_c = unit weight of concrete, w_p = unit weight of prestressing steel , and w_s = unit weight of steel (Sarma and Adeli, 1998).

CHAPTER 3

THEORY AND MATHEMATICAL APPROACH

3.1 Background

Because there is significant uncertainty as well as limitation in the accuracy of evaluation methods and repair strategies, reliability theory and optimization techniques are combined herein. In practical applications, each repair scenario is significantly different, of course, and may require different condition evaluation approaches to determine what the condition is, as well as what types of repair are needed. Each condition evaluation method has a different level of accuracy and a different cost associated with it. It follows logically that the uncertainties inherent in the techniques as well as the associated costs should be included in any decision-making process. It is proposed here to combine probabilistic methods, namely reliability constraints, with multi-objective optimization to identify (1) the most suitable, i.e. least costly, condition evaluation technique; and (2) the most suitable repair procedure.

3.2 Structural Reliability and Reliability-Based Design

During the last few decades significant achievements have been made in development of efficient techniques to evaluate the structural reliability of components and systems. A structural system reliability index, β can be determined by using any one of these efficient techniques.

A limit state function can be explained as the point at which the structural resistance is exceeded by the loading. Mathematically, the limit state function, g(**X**), can be expressed as

$$g(\mathbf{X}) = R\text{-}Q \tag{3.1}$$

where R represents the structure resistance (capacity), Q represents the structure load effect (demand), and **X** is the vector of structural strength random variables (Ang and Tang, 1984). If both R and Q are continuous random variables, they have a joint probability density function (*PDF*) and are typically modeled as having a normal distribution, log-normal distribution, Gamma distribution, Extreme Type I (Gumbel distribution), Extreme Type II (Freschet), Extreme Type III (Weibull distribution), or Poisson distribution (Nowak and Collins, 2000).

The limit state functions show the margin between the structural resistance and loads (including both external and internal loads). As long as the limit state function remains positive the structure is said to be in the safe region, but when it becomes negative the structure is said to be in the failure region. This is typically expressed as

$$g(R,Q) = R - Q \qquad \text{or} \qquad g(R,Q) = R/Q - 1 \tag{3.5}$$

The probability of failure (P_f) is the probability of the limit state function being less than zero. This is expressed as

$$P_f = P((R - Q) < 0 = P(g < 0) \tag{3.6}$$

which leads to

$$P_f = \int_{-\infty}^{+\infty} F_R(q_i) f_Q(q_i) dq_i \tag{3.7}$$

The probability of survival can be expressed as

$$P_S = 1 - P_f \tag{3.8}$$

A quantity known as the safety margin is the margin between the structural resistance and the loading on the structure. It can be expressed as

$$S = R - Q \tag{3.9}$$

where S, R, and Q are random variables. The probability of failure is then

$$P_f = \int_{-\infty}^{0} f_S(s)ds = F_S(0) \tag{3.10}$$

(Ang and Tang, 1984).

There are several methods and techniques which are typically used for calculation of the structural reliability index, β, such as the First order second moment (*FOSM*) method, the First order reliability method (*FORM*), the second order reliability method (*SORM*), second order second moment (*SOSM*), second order third moment (*SOTM*) (Zhao et al., 2002), and simulation techniques such as Monte Carlo simulation (*MCS*). In most civil engineering analyses the first order techniques or *MCS* is adequate for the accuracy needed. Consider a simple nonlinear limit state function which can be represented by two random variables, as shown in Figure 3.1. In this case, the reliability index is the minimum distance from the origin to the failure surface. The tangent line would be a hyper plane if the limit state function, $g(x)$, was expressed as a function of four or more random variables which is often the case.

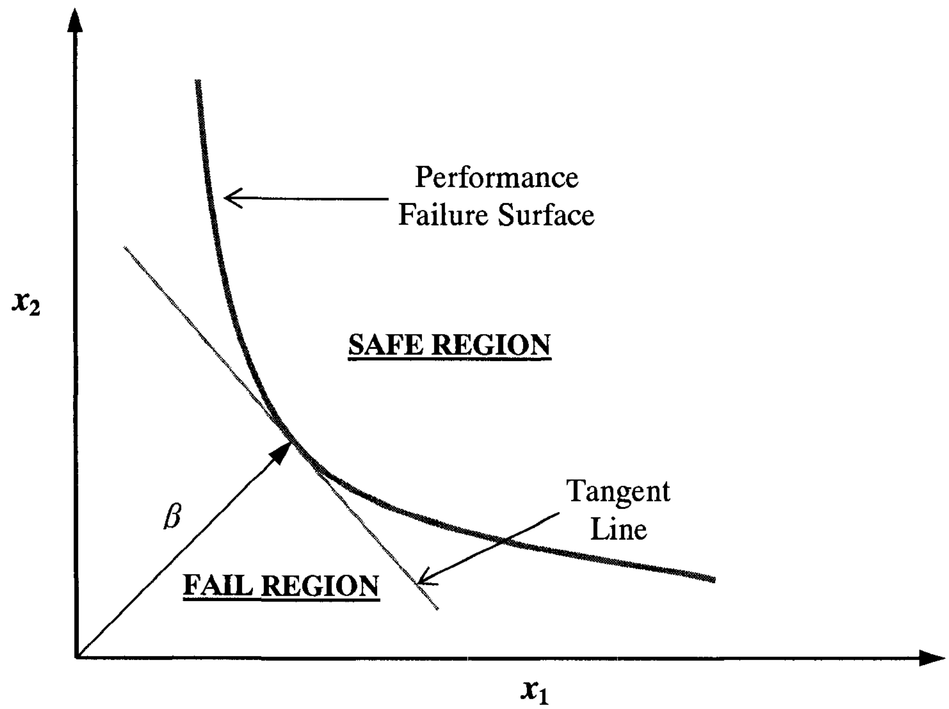


Figure 3.1: Structural Reliability Index (β) for Nonlinear Limit State Function Case.

As mentioned, the general definition of the reliability index is defined as the shortest distance from the origin point to the limit state surface (g = 0) in the standardized normal space, known as U space. In order to normalize the resistance, R, and the load effect, Q, termed R' and Q' here, one can simply write

$$R' = \frac{R - \mu_R}{\sigma_R} \tag{3.11}$$

$$Q' = \frac{Q - \mu_Q}{\sigma_Q} \tag{3.12}$$

Substitution Equation (3.11) and (3.12) into the limit state function equation (Equation 3.1) setting it equal to zero gives

$$\sigma_X X' - \sigma_Y Y' + \mu_X - \mu_Y = 0 \tag{3.13}$$

From Equation (3.13), the shortest distance (*d*) from the origin to the failure surface termed the reliability index, β, is

$$d = \beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{3.14}$$

We can then express the reliability index in term of the probability of failure as

$$P_f = \Phi(-\beta) \tag{3.15}$$

where Φ is the value of the standard normal distribution function and the probability of survival is then,

$$P_S = \Phi(\beta) \tag{3.16}$$

The first order second moment (*FOSM*) reliability index can be calculated beginning with Equation (3.1) and noting that we can express the limit state function as

$$g(X) = g(X_1, X_2, ..., X_n) \tag{3.17}$$

where $X = (X_1, X_2, ..., X_n)$ is a vector of basic state or design variables of the structural system and the function *g*(*X*) is the performance or limit state of the structural system. Then, by definition of failure probability, the probability of the limit state function g(*X*) being less than zero is

$$P_f = \int_{[g(X)<0]} \cdots \int f_{X_1,...,X_n}(x_1, x_2, ..., x_n) dx_1 dx_2 ... dx_n \tag{3.18}$$

where *f* represents the probability density function of each variable *X*. From Equation (3.11) and (3.12), we have

$$X'_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}, \qquad i = 1, 2, \ldots, n \tag{3.19}$$

and equation of the failure surface is then

$$g(\sigma_{X_1} X'_1 + \mu_{X_1}, \ldots, \sigma_{X_n} X'_n + \mu_{X_n}) = 0 \tag{3.20}$$

The distance from the origin point to the failure surface can be expressed as

$$D = \sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n} = ([X]'^t [X]')^{1/2} \tag{3.21}$$

Applying an optimization technique, the point on the failure surface having the minimum distance to the origin can be determined. The optimization here consists of minimizing the function D subject to the constraint $g(X) = 0$.

Minimize D subject to $g(X) = 0$

Applying Lagrange multipliers the optimization is easily performed. The Lagrange function (M) is

$$\begin{aligned} M &= D + \mu g(X) \\ M &= ([X]'^t [X]')^{1/2} + \mu g(X) \end{aligned} \tag{3.22}$$

The partial derivates of the Lagrange function with respect to X'_i and μ that are

$$\frac{\partial M}{\partial X'_i} = \frac{X'_i}{\sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n}} + \mu \frac{\partial g}{\partial X'_i} = 0, \quad i = 1, 2, \ldots, n \tag{3.23}$$

$$\frac{\partial M}{\partial \mu} = g(X_1, X_2, \ldots, X_n) = 0 \tag{3.24}$$

The result of these equations is the minimum distance from the origin to the failure surface and is termed the first order second moment reliability index, β_{FOSM}. This is expressed concisely as

$$\beta_{FOSM} = \frac{-\sum_i x'_i{}^* \left(\frac{\partial g}{\partial X'_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X'_i}\right)_*}} \tag{3.25}$$

In structural engineering when the limit state function is linear, it can be express as

$$g(X) = C_0 + \sum_i C_i X_i = 0 \tag{3.26}$$

Setting $g(X)$ equal to zero, the *FOSM* reliability index becomes

$$\beta_{FOSM} = \frac{C_0 + \sum_i C_i \mu_{X_i}}{\sqrt{\sum_i \left(C_i \sigma_{X_i}\right)^2}} \tag{3.27}$$

(Ang and Tang, 1984).

In the case of a nonlinear limit state function an approximate, but often relatively accurate, solution can be found by approximating the nonlinear function using a Taylor series expansion, as

$$g(X_1, X_2, \ldots, X_n) \approx g(x_1{}^*, x_2{}^*, \ldots, x_n{}^*) + \sum_{i=1}^{n} (X_i - x_i{}^*) \frac{\partial g}{\partial X_i}\Big|_{evaluate\_at\_(X_1{}^*, X_2{}^*, \ldots, X_n{}^*)}$$

or $$g(X_1, X_2, \ldots, X_n) \approx g(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}) + \sum_{i=1}^{n} (X_i - \mu_{X_i}) \frac{\partial g}{\partial X_i}\Big|_{evaluate\_at\_mean\_values} \tag{3.28}$$

From Equation (3.28), *FOSM* reliability index is

$$\beta_{FOSM} = \frac{g\left(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}\right)}{\sqrt{\sum_{i=1}^{n} \left(C_i \sigma_{X_i}\right)^2}} \quad \text{where} \quad C_i = \frac{\partial g}{\partial X_i}\Big|_{evaluate\_at\_mean\_values} \tag{3.29}$$

The reliability index in Equation (3.29) is called the *first-order second-moment mean value reliability index* (Ellingwood et al., 1982).

In the case of nonlinear limit state functions; there are no unique distances from the origin to the failure surface for the standard normalized space cases. Thus, solving this problem, one has to approximate the shortest distance from the origin to the failure surface first. The tangent plane at this point, denoted by "*" on the failure surface is $[x]'^* = (x'_1{}^*, x'_2{}^*, ..., x'_n{}^*)$ and is used to approximate the failure surface by linearizing it at the "design" point. Because the actual failure surface is nonlinear and can be either concave or convex with respect to the origin, the design point on the failure surface can be determined by setting the partial derivative of the limit state function equal to zero with respect to the normalize values of random variables that equal zero. So, the relevant tangent plane at $[x]'^* = (x'_1{}^*, x'_2{}^*, ..., x'_n{}^*)$ is

$$\sum_{i=1}^{n}(X'_i - x'_i)\left(\frac{\partial g}{\partial X'_i}\right)_* = 0 \tag{3.30}$$

where the partial derivatives $(\partial g/\partial X'_i)_*$ are determined at the point $(x'_1{}^*, x'_2{}^*, ..., x'_n{}^*)$.

The extreme point (in this case it is the design point) that has the shortest distance from the origin to the failure surface ($g = 0$) can be calculated using Lagrange multipliers. Finally, the component of X'^* can be express in the scalar from as

$$x'_i{}^* = -\alpha_i{}^* \beta; \qquad i = 1, 2, \ldots, n \tag{3.31}$$

in which

$$\alpha_i{}^* = \frac{\left(\frac{\partial g}{\partial X'_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X'_i}\right)_*^2}} \tag{3.32}$$

These values are, of course, the direction cosines with respect to the axes x'_i. When the partial derivatives are calculated at $(x'_1*, x'_2*, ..., x'_n*)$,

$$x_i* = \sigma_{X_i} x'_i* + \mu_{X_i} = \mu_{X_i} - \alpha_i * \sigma_{X_i} \beta \tag{3.33}$$

The solution of Equation (3.33) is where $g(x_1*, x_2*, ..., x_n*) = 0$ and then one can solve for β. The β is termed the first order reliability index, β_{FORM}.

The iterative algorithm for performing a *FORM* reliability index calculation is:

1) Assume the initial values of x_i* $\quad i = 1, 2, ..., n$ and normalize initial values are:

   $$x_i'* = \frac{x_i * - \mu_{x_i}}{\sigma_{x_i}}$$

2) Calculate the partial derivative of the performance function with respect to X'_i that are $(\partial g / \partial X_i')_*$ and α_i* at x_i*

3) Set $x_i* = \mu_{X_i} - \alpha_i * \sigma_{X_i} \beta$

4) Substitute x_i* in the third step in $g(x_1*, x_2*, ..., x_n*) = 0$ and solve the β_{FORM}

5) Using the β_{FORM} in the forth step, re-calculate $x'_i* = -\alpha_i \beta$

6) Repeat the second through fifth steps until the β_{FORM} convergence

(Ang and Tang, 1984).

Monte Carlo simulation (*MCS*) technique is one of the most popular simulation methods for computing failure probabilities and the structural reliability index. The strategy of the Monte Carlo simulation technique is the generation of random values mapped to probability distributions to generate values of the random variables that appear in the limit state equation. This is done a sufficient number of times and the number of failures divided by the total number of trials gives the failure probability.

This technique is very effective and can be applied to almost any type of limit state function, but the accuracy of this technique also depends on the number of simulations. In addition, random variable variance reduction techniques such as importance sampling and additional failure region knowledge can be reducing the computational effort significantly.

For a general limit state equation that is $g(X) = 0$, the probability of failure (P_f) is calculated as

$$P_f = \frac{n_f\left(g(X) < 0\right)}{n} \tag{3.34}$$

where n is the number of the simulation running times and $n_f\left(g(X) < 0\right)$ is the number of times that the limit state is less than zero $\left(g(X) < 0\right)$.

3.3 Decision Making Under Uncertain Conditions

3.3.1 Introduction

Decision making in engineering and business must often be made under uncertain conditions, i.e. where only part of the information is available. Often, rigorous statistical analysis is not possible and variables must be assumed to be a function of other components and variables within the problem, in order to facilitate a solution and provide some level of decision support. Because the usual sample data is typically not available in many decisions that must be made, the approach taken is different that traditional "confidence level" methods such as hypothesis testing.

For engineering problems, such decision-making involves must to involve natural phenomena, and uncertainties in materials, construction, and loads. For structural condition evaluation uncertainties consist of human error and limitations in the ability of the equipment to detect accurately.

3.3.2 Probability of Detection (*POD*) for Decision Making Process Diagram

Probability of detection (*POD*) in this study expresses the probability of detecting the strength of the construction materials such as concrete, steel, etc. The probability of detection curves for any *NDE* or *SDE* method can be created by utilizing the *NDE* and *SDE* testing results and trying to find the relationship between the *POD* and the testing data statistical properties.

A probability of detection curve for a particular *NDE* method can be obtained by varying the material strength of the testing specimens. For this study, the researcher tries to vary about the concrete strength by using the concrete maturity concept that vary the casting and curing temperatures and change the types of concrete as the normal and high performance concretes. Then, the researcher has tested the concrete specimens by using both the *NDE* and destructive methods, analyzes the testing data from both *NDE* and destructive methods, calculated the probability of failure (P_f) and probability of safety or probability of survival (P_S) of testing that are explained in Section 3.1.1, and tries to figure out the probability of detection (*POD*) (Zheng and Ellingwood, 1999) for solving the uncertainty and limitation of the *NDE* and *SDE* ability problems.

3.3.3 Decision Making Process Diagram for Structural Condition Evaluation Method(s) and Repair and Rehabilitation Strategy

Decision makers who must make a decision under uncertainty of testing conditions and ability will often looking for some additional information/data before finally making decision an action/testing. Some problems and cases sometimes take a lot of time and elaborate for pursuing the correcting and trusting results and data that approach to the appropriate decision. The practical tests and experiments are useful sources of information for decision making in engineering and science problems. There is no particular advantage in using a decision making process when there is a single decision point. But a decision process is very useful when there is more than one decision point or when there are two are more points of uncertainty. Figures 3.2 shows the decision making process diagram for the expanded decision making procedure in which there are several choices.

The decision tree diagram represents each event assembly by a branch. Each action is also represented by a branch. The decision making process represents the logical progression of actions and events that might occur. The sequence proceeds chronologically from left to right, with the earlier branching point appearing first. Because the design action must be taken before its operating characteristics are determined, notice that each action leads to a separate event fork, each of which consist of the same or/and similar processes of decision making.

The first decision that must be addressed is whether to conduct an evaluation or leave the structure based on the visual inspection. This decision step is not shown in Figure 3.2 but was shown earlier in the flowchart in Figure 3.5. If it is decided that an evaluation should be performed then the decision maker is faced with uncertainties associated with each method. The upper part of the decision making process diagram shown in Figure 3.2 (upper part of decision making point "a") presents a basic decision approach for the evaluation technique, E_1, and of the subsequent repair methods. The point "a" is then connected with the decision making point "b" to account for the ability of the method to detect. After making this decision, an evaluator will head to the decision making point "d" and then must make a decision about the optimal repair strategies that are shown as the decision making points "h", "i", and "j".

However, for the repair strategies, weighting factors (often referred to as preference factors in decision analysis) are provided as a probability ($(Pr)_{Ri}$) for each type of repair method. These allow the evaluator to account for the availability of materials locally, the remoteness of the site, and any preference from contractors which might affect cost. These will be discussed in detail in Section 4.6.2, 4.7.2, and 4.8.2. In this dissertation, the decision making process diagram is applied in order to provide decision support to determine the most economical condition evaluation method and repair strategy considering location, material availability, evaluation method accuracy, and uncertainties.

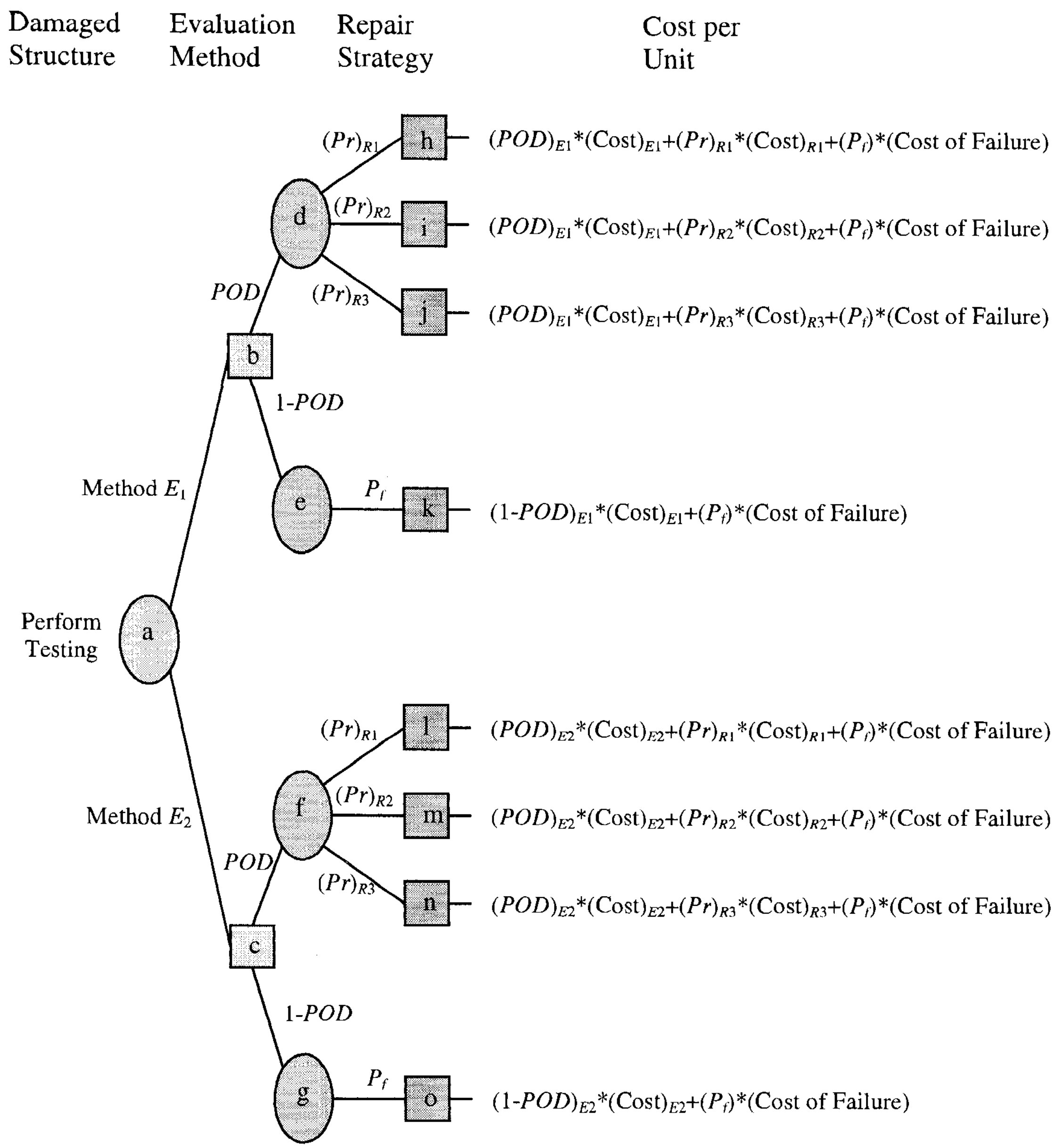


Figure 3.2: General Decision Making Process Diagram for Structural Condition Evaluation and Repair.

3.4 Engineering Optimization for Cost Minimization

The classical methods of optimization are very useful in determining the optimal solution for differentiable and continuous functions. These methods apply differential calculus in order to determine the optimal points. Because some structural optimization problems involve objective functions that are not continuous and/or differentiable these classical optimization techniques have limited practical applications. There are numerous other optimization algorithms to select from that are capable of addressing problems with a single-variable function, a multivariable function with no constraints, or a multivariable function with equality and inequality constraints. Most structural optimization problems are minimizing problems for multivariable functions (mass or cost functions) with equality and inequality constraints (Rao, 1996).

An optimization problem can be explained as the need to

Find $X = \{x_1, x_2, \ldots, x_k\}$ which minimizes $f(X)$

subject to the constraints

$$g_i(X) \leq 0, \qquad i = 1, 2, \ldots, m \qquad (3.35)$$

$$h_j(X) = 0, \qquad j = 1, 2, \ldots, n$$

where X is a k-dimensional vector called the design vector, $f(X)$ is the objective function, and $g_i(X)$ and $h_j(X)$ are inequality and equality constraints, respectively. The number of variables k and the number of constraints m or n do not need to be related. The problem in Equation (3.35) is called a constrained optimization problem. Some optimization problems do not have any constraints, for example

Find $X = \{x_1, x_2, \ldots, x_k\}$ which minimizes $f(X)$ (3.36)

This type of problem is known as an unconstrained optimization problem (Belegundu and Chandrupatla, 1999) and is (obviously) much easier to solve.

3.4.1 Multivariable Optimization with no Constraints

The necessary and sufficient conditions for maximizing and minimizing an unconstrained function with several variables begin with a Taylor series expansion of a multivariable function:

Consider the y^{th} Differential of f. If all partial derivatives of the function f ($y \geq 1$) are continuous and exist at the point X*, then

$$d^r f(X^*) = \underbrace{\sum_{i=1}^{n} \sum_{j=1}^{n} \cdots \sum_{k=1}^{n}}_{y\ldots summations} g_i g_j \ldots g_k \frac{\partial^y f(X^*)}{\partial x_i \partial x_j \ldots \partial x_k} \tag{3.37}$$

(Rao, 1996 and Onwubiko, 2000)

is called the y^{th} differential of f at X^*. For example, if $r = 2$ and $n = 3$, then

$$d^2 f(X^*) = d^2 f(x_1^*, x_2^*, x_3^*) = \sum_{i=1}^{3} \sum_{j=1}^{3} g_i g_j \frac{\partial^2 f(X^*)}{\partial x_i \partial x_j}$$

$$= g_1^2 \frac{\partial^2 f}{\partial x_1^2}(X^*) + g_2^2 \frac{\partial^2 f}{\partial x_2^2}(X^*) + g_3^2 \frac{\partial^2 f}{\partial x_3^2}(X^*) + \\ 2g_1 g_2 \frac{\partial^2 f}{\partial x_1 \partial x_2}(X^*) + 2g_2 g_3 \frac{\partial^2 f}{\partial x_2 \partial x_3}(X^*) + 2g_1 g_3 \frac{\partial^2 f}{\partial x_1 \partial x_3}(X^*) \tag{3.38}$$

The Taylor's series expansion of the function $f(X)$ at point X^* is then

$$f(X) = f(X^*) + df(X^*) + \frac{1}{2!}d^2 f(X^*) + \frac{1}{3!}d^3 f(X^*)$$

$$+ \ldots + \frac{1}{N!}d^N f(X^*) + Q_N(X^*, g) \qquad (3.39)$$

where

$$Q_N(X^*, g) = \frac{1}{(N+1)!}d^{N+1} f(X^* + \alpha g) \qquad (3.40)$$

and $0 < \alpha < 1$ and $g = X - X^*$.

A necessary condition for a maximum or minimum point is if *f*(*X*) has a maximum or minimum point at $X = X^*$ and the first partial derivatives of *f*(*X*) exists at X^*, that is

$$\frac{\partial f}{\partial x_1}(X^*) = \frac{\partial f}{\partial x_2}(X^*) = \ldots = \frac{\partial f}{\partial x_n}(X^*) = 0 \qquad (3.41)$$

A sufficient condition for a stationary point X^* to be a maximum or minimum point is that the matrix of second derivatives (Hessian matrix) of *f*(*X*) at X^* must be a positive value when X^* is a relative minimum point and must be a negative value when it is a maximum point. Mathematically, this is expressed as

$$f(X^* + g) = f(X^*) + \sum_{i=1}^{n} g_i \frac{\partial f}{\partial x_i}(X^*) + \frac{1}{2!}\sum_{i=1}^{n}\sum_{j=1}^{n} g_i g_j \frac{\partial^2 f}{\partial x_i \partial x_j}\Big|_{X = X^* + \alpha h},$$

$$0 < \alpha < 1 \qquad (3.42)$$

Because X^* is a stationary point, the necessary conditions are then

$$\frac{\partial f}{\partial x_i} = 0, \qquad i = 1, 2, \ldots, n \qquad (3.43)$$

Equation (3.42) can be reduced to

$$f(X^* + g) - f(X^*) = \frac{1}{2!}\sum_{i=1}^{n}\sum_{j=1}^{n} g_i g_j \frac{\partial^2 f}{\partial x_i \partial x_j}\Big|_{X = X^* + \alpha h},\ 0 < \alpha < 1 \qquad (3.44)$$

Then, it can be seen that the sign of $f(X^*+g)-f(X^*)$ is the same as $\sum_{i=1}^{n}\sum_{j=1}^{n} g_i g_j \frac{\partial^2 f}{\partial x_i \partial x_j}\Big|_{X=X^*+\alpha h}$.

Since the second partial derivative of $\partial^2 f(X)/\partial x_i \partial x_j$ is continuous at X^*, $\frac{\partial^2 f}{\partial x_i \partial x_j}\Big|_{X=X^*+\alpha g}$ will have the same sign as $(\partial^2 f/\partial x_i \partial x_j)|X = X^*$ for all small values of g. Thus $f(X^*+g)-f(X^*)$ will be positive, and X^* will be minimum, if

$$H = \sum_{i=1}^{n}\sum_{j=1}^{n} g_i g_j \frac{\partial^2 f}{\partial x_i \partial x_j}\Big|_{X=X^*} \tag{3.45}$$

is positive. This quantity H is a quadratic form and can be written as

$$H = [g]^T [J][g]|_{X=X^*} \tag{3.46}$$

where

$$[J]|_{X=X^*} = \left[\frac{\partial^2 f}{\partial x_i \partial x_j}\Big|_{X=X^*}\right] \tag{3.47}$$

is the second partial derivative or the Hessian matrix of $f(X)$.

In case of a saddle point, if the point (x^*, y^*) is a minimum point then

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \tag{3.48}$$

(Onwubiko, 2000).

3.4.2 Multivariable Optimization with Equality Constraints

Consider the optimization of continuous functions subjected to equality constraints:

$$\text{Minimize} \quad f = f(X)$$

$$\text{subject to} \quad h_i(X) = 0, \qquad j = 1, 2, \ldots, n \tag{3.49}$$

where

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{Bmatrix} \tag{3.50}$$

When n is less than or equal to m, if n is greater than m the problem will become over defined, and it will not have a solution. There are several methods for solving this problem such as the direct substitution, constrained variation, and Lagrange multipliers methods.

For the general problem necessary conditions, the constraint equations are $h_i(X) = 0, i = 1,2,...,n,$ and variables are $x_j, j = 1,2,...,m$. For the partial derivatives of $h_i(X)$ with respect to dx_j, there are m equations in n variables. Thus there will be in all n equations with m variables. By using the coefficients of the independent variables vanish in the equation that df equals 0, the necessary conditions for the constrained optimum of the given function are obtained. The necessary conditions for optimal solution that will be given by the Jacobian:

$$J\left(\frac{f,h_1,h_2,\ldots,h_n}{x_k,x_1,x_2,x_3,\ldots,x_n}\right)=\begin{vmatrix} \frac{\partial f}{\partial x_a} & \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} & \cdots & \frac{\partial f}{\partial x_n} \\ \frac{\partial h_1}{\partial x_a} & \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_a} & \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_2}{\partial x_n} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial h_n}{\partial x_a} & \frac{\partial h_n}{\partial x_1} & \frac{\partial h_n}{\partial x_2} & \cdots & \frac{\partial h_n}{\partial x_n} \end{vmatrix}=0 \tag{3.51}$$

where $a = n+1, n+2, \ldots, m.$

3.4.3 Lagrange Multipliers Method (Illustrated by a Two Variable-One Constraint Problem)

One of the simplest problems for engineering optimization is optimization problems that consist of two variables and one constraint, for example the problem which seeks to

Minimize $\quad f = f(x_1, x_2)$

Subject to $\quad h(x_1, x_2) = 0 \qquad (3.52)$

The necessary condition for an extreme point in Equation (3.52) is

$$\left.\left(\frac{\partial f}{\partial x_1}-\frac{\partial f/\partial x_2}{\partial h/\partial x_2}\frac{\partial h}{\partial x_1}\right)\right|_{(x_1^*,x_2^*)}=0 \tag{3.53}$$

If μ is the Lagrange Multiplier, it can be defined as

$$\mu=-\left.\left(\frac{\partial f/\partial x_2}{\partial h/\partial x_2}\right)\right|_{(x_1^*,x_2^*)}=0 \tag{3.54}$$

$$\left.\left(\frac{\partial f}{\partial x_1}+\mu\frac{\partial h}{\partial x_1}\right)\right|_{(x_1^*,x_2^*)}=0 \tag{3.55}$$

$$\left(\frac{\partial f}{\partial x_2} + \mu \frac{\partial h}{\partial x_2}\right)\Bigg|_{(x_1^*, x_2^*)} = 0 \tag{3.56}$$

Furthermore, the constraint for satisfying at the extreme point is

$$h(x_1, x_2)\Big|_{(x_1^*, x_2^*)} = 0 \tag{3.57}$$

So, Equations (3.55), (3.56), and (3.57) are the necessary conditions for a minimum point (x_1^*, x_2^*).

From the necessary conditions of Equations (3.55), (3.56), and (3.57), one can construct the function Lagrange function, M, as

$$M(x\ , x_2, \mu) = f(x_1, x_2) + \mu h(x_1, x_2) \tag{3.58}$$

From Equation (3.58), one can see that there are three variables: x_1, x_2, and μ. Therefore, the necessary conditions for a solution at the minimum point are.

$$\begin{aligned} \frac{\partial M}{\partial x_1}(x_1, x_2, \mu) &= \frac{\partial f}{\partial x_1}(x_1, x_2) + \mu \frac{\partial h}{\partial x_1}(x_1, x_2) = 0 \\ \frac{\partial M}{\partial x_2}(x_1, x_2, \mu) &= \frac{\partial f}{\partial x_2}(x_1, x_2) + \mu \frac{\partial h}{\partial x_2}(x_1, x_2) = 0 \\ \frac{\partial M}{\partial \mu}(x_1, x_2, \mu) &= h(x_1, x_2) = 0 \end{aligned} \tag{3.59}$$

The method of Lagrange Multipliers is applied throughout this dissertation work in order to identify the optimal solution for multiple variables subject to one (or more, see below) constraints. For the function f to have a stationary value, which means a minimum, maximum or saddle point, it is required that

$$df = (\partial f/\partial x_1)dx_1 + \ldots + (\partial f/\partial x_n)dx_n = 0 \tag{3.60}$$

while the constraint equation gives

$$dh = (\partial h/\partial x_1)dx_1 + \ldots + (\partial h/\partial x_n)dx_n = 0 \tag{3.61}$$

The three simultaneous equations of Equation (3.59) are then solved.

3.4.4 Application to General Problems (*m* Variables with *n* Constraints)

The most general optimization problems, which are present in this dissertation, are those with multiple variables and multiple constraints. Mathematically, this is expressed simply as

$$\begin{aligned} &\text{Minimize} \quad f = f(X) \\ &\text{Subject to} \quad h_i(X) = 0, \qquad i = 1, 2, \ldots, n \end{aligned} \tag{3.62}$$

The Lagrange function (M) is then

$$M(x_1, x_2, \ldots, x_m, \mu_1, \mu_2, \ldots, \mu_n) = f(X) + \mu_1 h_1(X) + \mu_2 h_2(X) + \ldots + \mu_n h_n(X) \tag{3.63}$$

and the necessary conditions are

$$\frac{\partial M}{\partial x_j} = \frac{\partial f}{\partial x_j} + \sum_{i=1}^{n} \mu_i \frac{\partial h_i}{\partial x_j} = 0, \quad j = 1, 2, \ldots, m \tag{3.64}$$

$$\frac{\partial M}{\partial \mu_i} = h_i(X) = 0, \qquad i = 1, 2, \ldots, n \tag{3.65}$$

Equations (3.64) and (3.65) represent a system of $m+n$ equations with $m+n$ unknowns. The solution of Equations (3.64) and (3.65) can be expressed generally as

$$X^* = \begin{Bmatrix} x_1^* \\ x_2^* \\ \cdot \\ \cdot \\ \cdot \\ x_m^* \end{Bmatrix} \quad \text{and} \quad \mu^* = \begin{Bmatrix} \mu_1^* \\ \mu_2^* \\ \cdot \\ \cdot \\ \cdot \\ \mu_n^* \end{Bmatrix} \tag{3.66}$$

where the vector X^* corresponds to the constrained minimum of the function f and the vector μ^* provides the Lagrange Multiplier minimum values (Onwubiko, 2000).

3.5 Mathematical Approach

3.5.1 Introduction

The number of objective functions depends on the total number of criteria that are used to evaluate the structural condition and the subsequent repair/rehabilitation. All objective functions are minimized by including multiple uncertainties, discussed in this chapter. Optimization for the selection of an evaluation technique has not been studied by researchers to date, as discussed in Chapter 1 of this dissertation. In addition, optimizing condition evaluation and the repair procedure at the same time has not been studied either. The latter of these would serve to help with resource allocation at preliminary stages of planning for the deteriorating infrastructures condition evaluation and performance repair as the prior explanations that have been explained in the early couple paragraphs in Chapter 1.

Initially, the equations for an objective function that describe cost, but are somehow related to the current damaged or deteriorated state of the structure must be developed. The system must also be subject to various constraints that are the result of a need for condition improvement. This desired condition improvement is quantified by the structures' structural reliability index, which recall from earlier is a measure of the safety reserve in the system.

In practical applications, it would be necessary to take into account many complex aspects that are typically neglected in optimization, e.g. the effect of internal forces, the difference in labor costs and other costs associated with manufacturing, the added cost of fabricating joints, and possibly second-order effects (buckling) (Jirasek and Bazant, 2002). The majority of these are neglected in this study since the objective is to demonstrate the methodology for optimally selecting a structural condition evaluation approach.

3.5.2 Cost Functions

Cost functions (i.e. evaluation and repair cost functions) can be expressed as the total cost of the structural condition evaluation and repair. It consists of two main terms: the condition evaluation costs and the repair costs.

The cost of the condition evaluation can expressed as

$$C_E = C_{OE} + p_E [\Delta\beta_E]^{q_E} \tag{3.67}$$

where $\Delta\beta_E$ is the level of structural damage, C_{OE} is the fixed portion of the evaluation cost which does not depend on the $\Delta\beta_E$, p_E and q_E are cost parameters which are used to calculate the variable part of structural condition evaluation cost based on the level of structural damage ($\Delta\beta_E$).

And the cost of the repair can be expressed as

$$C_R = C_{OR} + p_R [\Delta\beta_R]^{q_R} \tag{3.68}$$

where $\Delta\beta_R$ is the structural reliability index improvement, C_{OR} is the fixed portion of the evaluation cost which does not depend on the $\Delta\beta_R$, p_R and q_R are cost parameters which are used to calculate the variable part of structural performance repair cost based on the reliability index improvement ($\Delta\beta_R$).

The cost of repair was introduced by Kong and Frangopol (2004), and the cost of condition evaluation is new in this dissertation.

The total cost of the condition evaluation and performance repair is found by simply combining Equations (3.67) and (3.68) as

$$C_T = C_E + C_R = C_{OE} + p_E[\Delta\beta_E]^{q_E} + C_{OR} + p_R[\Delta\beta_R]^{q_R} \tag{3.69}$$

where C_T = the total cost of the condition evaluation and repair; β_{target} = target reliability index which is the target performance of a deteriorating structure after the repair and rehabilitation procedure is done; β_g = initial estimated guiding reliability index of a deteriorating structure which is the initial reliability index of a deteriorating structure before it has been evaluated by the condition evaluation method(s). For the level of structural damage ($\Delta\beta_E$) and reliability index improvement ($\Delta\beta_R$), both can be calculated from the different amount between the target reliability index and initial estimated (guiding) reliability index of the deteriorating structure:

$$\Delta\beta_{E,R} = \beta_{target} - \beta_g \tag{3.70}$$

Figure 3.3 illustrates the failure planes for the target and guiding reliability indices. The increase in the length of the line from the origin to the failure plane is the $\Delta\beta$.

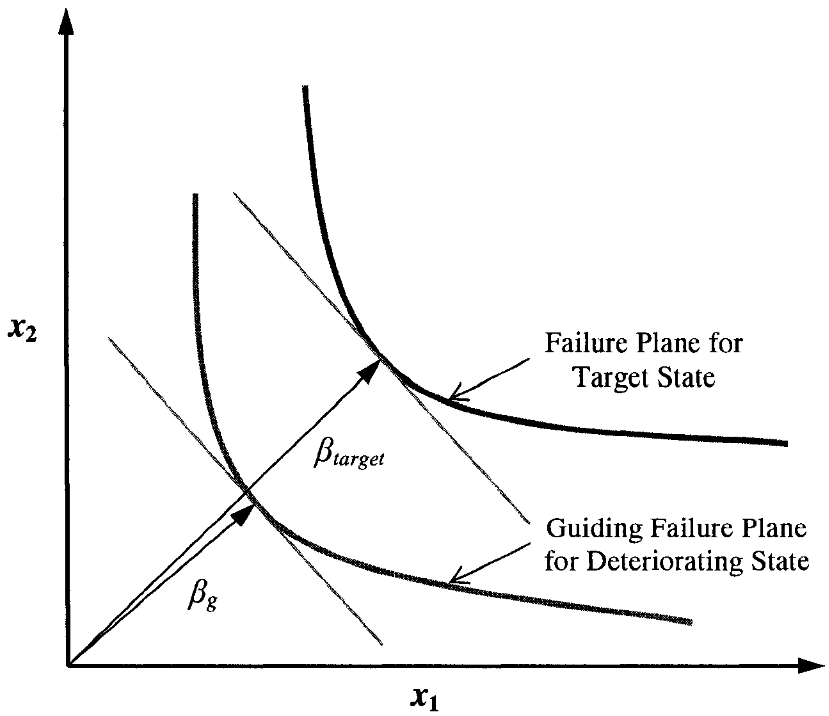


Figure 3.3: Failure Plane for Target and Deteriorating States and Target and Guiding Reliability Indices.

3.5.3 Failure Costs and Reliability Constraints

In the optimization process the minimization of the cost function is subject to the failure costs for evaluation (i.e. lack of detection) and repair strategies (structural failure). These failure costs can be expressed as the follows:

For the *NDE* methods such as *UPV* and *RN* methods

$$\left(C_{P_f}\right)_{NDE} = c_f\left(P_f\right)_{NDE} = \frac{c_f}{(1+\nu)^k}\left[f_{P_f}\left(\Delta\beta_{E,R}\right)\right]_{NDE} \tag{3.71}$$

For the *SDE* Methods such as *PO* method

$$\left(C_{P_f}\right)_{SDE} = c_f\left(P_f\right)_{SDE} = \frac{c_f}{(1+\nu)^k}\left[f_{P_f}\left(\Delta\beta_{E,R}\right)\right]_{SDE} \tag{3.72}$$

where $\left(C_{P_f}\right)_{NDE}$ or $\left(C_{P_f}\right)_{SDE}$ = the failure cost of *NDE* or *SDE* method respectively; $\left(P_f\right)_{NDE}$ or $\left(P_f\right)_{SDE}$ = the probability of failure for *NDE* or *SDE* method at the time when the failure cost is evaluated, respectively; k = the time when the failure cost is evaluated; v = discount rate of money at the time when the failure cost is evaluated; $\Delta\beta_{E,R}$ = the level of structural damage (for the condition evaluation procedure) or reliability index improvement (for the performance repair and rehabilitation procedure); c_f = the failure cost coefficient. The failure cost depends on various environmental and regional factors such as type of bridges, daily traffic volume, number of injury or death, and service losses.

3.5.4 Optimal Structural Condition Evaluation method(s) and Repair and Rehabilitation Strategy Selection

Once the optimal values and normalized cost for each condition evaluation method and repair method and other parameters affecting the analysis are determined, the most economical approach for conditional evaluation may be determined as a single method or several methods combined. Each situation would need to be handled on a case by case basis. However, the approach presented here is estimated to be robust enough to encompass approximately 70% to 90% of typical bridges in the U.S. The three major components are shown conceptually in Figure 3.4, with various overlapping sections similar to a Venn diagram. Each subject matter has influence and is influenced by one or both of the others at any given time. Further explanation is provided by the flow charts in Figures 3.5.

Figures 3.5 shows the details or flow that are developed in the dissertation. Ultimately, a set of rules that allow rapid decision making through "decision charts" could be developed if the input to the numerical model is successfully developed over time.

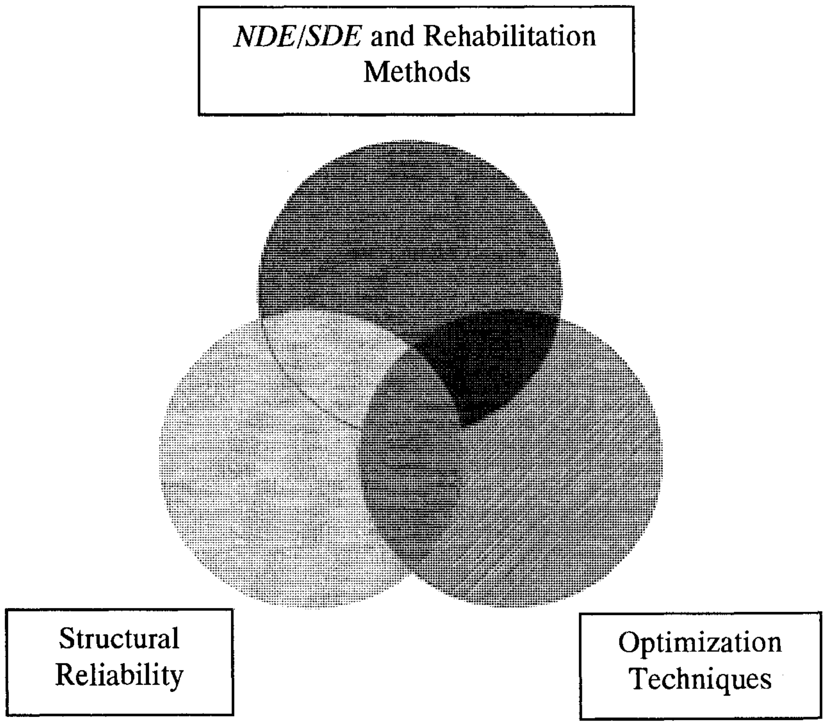


Figure 3.4: Component of Structure Repair and Rehabilitation Evaluated by *NDE* and/or *SDE* Methods.

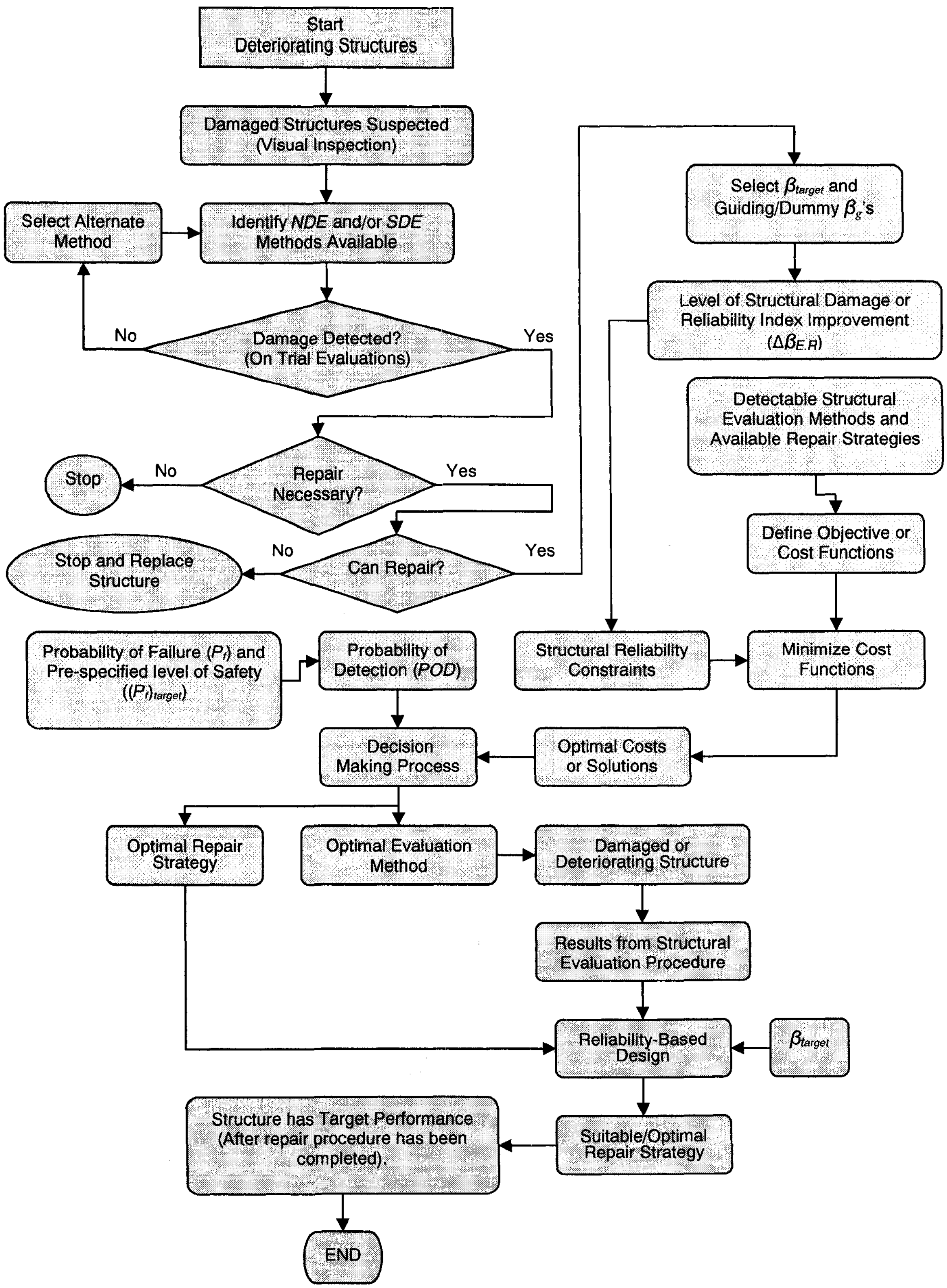


Figure 3.5: Flowchart Showing the Procedure Described in This Dissertation.

3.5.5 Summary of Procedure

Because there are numerous details associated with the work presented in chapter 1-3, a brief summary is provided here prior to presentation of the illustrative examples. This dissertation's methodology and procedure are intended to systematically determine which condition evaluation method and which repair method are most likely to provide the desired long-term results while minimizing the cost. The steps are as follows:

1) Identify and classify the deteriorating structure (low, moderate, or serious damage/deterioration) via visual inspection and based on the evaluator's experience.
2) Identify which the structural condition assessment and evaluation methods are available for possible use if a structure appears to be damaged i.e. through visual inspection, considering the type of damage observed such as material strength, wear, crack problems, etc.
3) Estimate, select, and calculate the initial guiding structural reliability index (β_g), the target structural reliability index (β_{target}), and the level of structural damage ($\Delta\beta_E$) for the structural condition evaluation procedure. For the repair method selection this would be the reliability index improvement ($\Delta\beta_R$) needed/required to bring the structure back to a healthy state. These will serve as a variable in the objective (cost) functions and structural reliability constraints for the optimization procedure. Of course, since the structure is damaged to an extent that is yet unknown, an evaluator must approximate a guiding reliability index first that depends on the level of structural damage, e.g. an educated guess.

4) The objective (cost) function for each structural condition assessment and evaluation method (*NDE/SDE*) and performance repair and rehabilitation strategy are determined and set which are dependent on the characteristic of each method and strategy.
5) Once the objective function has been minimized, the optimal values and normalized optimal values of the cost are determined.
6) A decision is made by utilizing the normalized optimal values and probabilities of detection or testing.
7) Then, the evaluation method(s) (*NDE/SDE*) are selected—perhaps one or several combined evaluation methods, which depends on the decision making process results.
8) After evaluation, the most economical repair method can be found. However, if desired by the analyst for early resource allocation decision-making, the objective functions for the combination of the evaluation method and repair method can be optimized simultaneously.
9) Finally, a damaged/deteriorated structure would be repaired or rehabilitated and would have the target reliability index and be returned to the desired state of health.

CHAPTER 4

ILLUSTRATIVE EXAMPLES

4.1 Introduction

In this study deteriorating structures such as the fire-damaged structures, aging structures, accidentally damaged structures, etc. are the structures under investigation and in need of possible further evaluation and repair following visual inspection. Then, an evaluator must decide which evaluation strategies are available and subsequently which repair methods. Finally, the, objective functions can then be formulated. The objective functions are the cost functions for the condition evaluation procedure and repair process. The functions are similar to life-cycle (cost) functions but they are not actually the same. This is because they do not involve the structure's initial cost, they only involve the costs associated with evaluation and repair once the structure is existing, i.e. in place.

The objective functions for both condition evaluation and repair methods are, in general, cost specific to each evaluation method and repair method such as time required and availability of resources. These cost functions do not account for the structural life-cycle terms, such as the cost of planning and design, products, and initial construction costs.

Recall the main idea of this study is that an evaluator will collect and formulate the information needed in the objective functions including an estimate of the current (damaged) condition of the structure. The current condition of the structure is estimated by an initial guiding reliability index (βg). Then the evaluator will move on to the optimization process in which he/she will minimize the objective function but constrained to achieve a certain predefined level of structural reliability. Once the lowest evaluation and repair cost for each combination of evaluation/repair has been obtained from the optimization it is used as input to the decision making process. The decision making process is essentially a decision tree that accounts for the cost of the evaluation, the probability that the evaluation method detects accurately, and the subsequent repair costs. Also included are the cost of a catastrophic failure and the probability of such a failure. The final decision parameter is the expected cost for each combination of evaluation/repair including the "do nothing" approach.

The illustrative examples presented in this chapter are limited to two types of evaluation: ultrasonic pulse velocity (*UPV*) and the rebound hammer or Schmidt Hammer (*RN*), and three types of repair: bonded steel elements or steel plate attaching (*SP*), fiber reinforced polymer attaching (*FRP*), and section enlargement or cross-section increasing (*CI*) for concrete bridges. The costs of the two evaluation methods per unit (evaluation) area (1 m^2) are \$75 and \$50 which also include the report preparation for *UPV* and *RN* method, respectively (Odell, 2007). The costs associated with the repair methods are \$613, \$630, and \$578 per unit (repair) area (1 m^2) for the steel plate attaching, fiber reinforced polymer attaching, and cross-section increasing strategy, respectively (Kong and Frangopol, 2004).

The examples consider an I-section prestressed concrete (*PC*) bridge girder and vary the level of structural damage ($\Delta\beta_E$) for the structural condition evaluation procedures. For the repair examples the required reliability index improvement ($\Delta\beta_R$) is varied to examine the effect of these parameters on the methods results. The $\Delta\beta_E$ and $\Delta\beta_R$ will be more than 3.5 for the serious damage problems ($\Delta\beta_{E,}$ and $\Delta\beta_R > 3.5$), between 1.5 and 3.5 for the moderate damage problems ($3.5 \geq [\Delta\beta_E$ and $\Delta\beta_R] \geq 1.5$), and less than 1.5 for the non-severe or mild damage problem ($\Delta\beta_E$ and $\Delta\beta_R < 1.5$).

In the first step, the objective functions for the structural evaluation and repair methods were formulated. Those functions are functions of the level of reliability index improvement ($\Delta\beta_R$) needed to bring the structure back to the desired condition. Then, the optimization procedure is applied to minimize the objective functions.

4.2 Objective Functions

The objective functions, hereafter referred to as a cost function, for the numerical example are the cost of the various evaluation and repair methods. They functions each have a fixed part (fixed cost) and variable part (variable cost) as discussed in section 3.2 of this dissertation. For each condition evaluation or repair method, the fixed cost and parameters for variable cost are shown in Tables 4.1 and 4.2, respectively.

Type of Condition Evaluation	Cost Function ($/m^2)	Fixed Cost C_{OE} ($/m^2)	Parameters for Variable Cost	
			p_E	q_E
Ultrasonic Pulse Velocity (*UPV*)	$C_E = C_{OE} + p_E[\Delta\beta_E]^{q_E}$	66.33	20	2.0
Schmidt Hammer (*RN*)	$C_E = C_{OE} + p_E[\Delta\beta_E]^{q_E}$	41.33	28	2.0

Table 4.1: Cost Functions Corresponding to Different Structural Condition Evaluation Methods and Associated Parameters.

Type of Repair	Cost Function ($/m^2)	Fixed Cost C_{OR} ($/m^2)*	Parameters for Variable Cost	
			p_R	q_R
Steel Plate Attaching	$C_R = C_{OR} + p_R[\Delta\beta_R]^{q_R}$	330	313	2.0
Fiber Reinforced Polymer Attaching	$C_R = C_{OR} + p_R[\Delta\beta_R]^{q_R}$	440	230	2.0
Cross-Section Increasing	$C_R = C_{OR} + p_R[\Delta\beta_R]^{q_R}$	550	178	2.0
Replacement	$C_R = C_{OR}$	1100		

*Note: Increasing the fixed cost approximately 10% from the original because the reference (Kong and Frangopol, 2004) was published in November 2004.

Table 4.2: Cost Functions Corresponding to Different Structural Repair and Rehabilitation Strategies and Associated Parameters (Kong and Frangopol, 2004).

4.3 Relationship between Concrete Strength (f_c') and Probability of Detection (*POD*)

From *NDE* test and destructive tests on the concrete specimens (Buddhawanna, 2003), the relationship between the concrete strength (f_c') and probability of detection (*POD*) for the non-destructive evaluation methods was fit to a curve in Figure 4.1. This graph shows that the accuracy of detection decreases slightly as concrete strength increases.

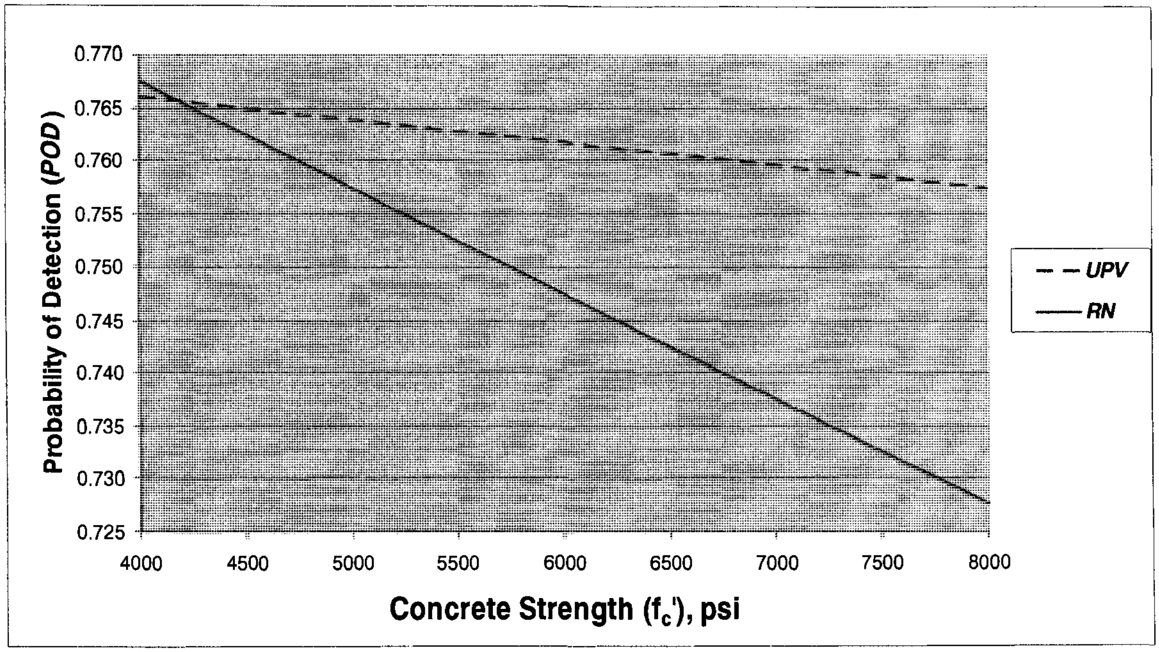


Figure 4.1: Relationship between Concrete Strength (f_c') and Probability of Detection (*POD*) for *UPV* and *RN* Methods.

For the illustrative examples, the probability of detection (*POD*) for each condition evaluation method is provided by Equations 4.1 and 4.2 for the *UPV* and *RN* method, respectively.

For *UPV* method:

$$(POD)_{UPV} = 1\text{-Ln}\ [2\text{E-06}(f_c') + 1.2591] \qquad (4.1)$$

For *RN* method:

$$(POD)_{RN} = 1\text{-Ln}\ [1\text{E-05}(f_c') + 1.2142] \qquad (4.2)$$

where $(POD)_{UPV}$ is the probability of detection for the *UPV* method, $(POD)_{RN}$ is the probability of detection for the *RN* method, and f_c' is the concrete strength.

4.4 Failure Costs and Reliability Constraints

From the existing literature, the failure costs and reliability constraints in the illustrative examples can be calculated and formulated as Equation 4.3 and 4.4 for the evaluation methods and repair methods respectively.

$$\left(C_{P_f}\right)_{Evaluation} = \frac{c_f}{(1+v)^k}\left\{\Delta\beta_E\left[t_{age}(k)\right]\right\}^2_{Evaluation} \tag{4.3}$$

$$\left(C_{P_f}\right)_{Repair} = \frac{c_f}{(1+v)^k}\left\{\Delta\beta_R\left[t_{age}(k)\right]\right\}^2_{Repair} \tag{4.4}$$

(Kong and Frangopol, 2004)

where $\left(C_{P_f}\right)_{Evaluation}$ = the failure cost of condition evaluation methods at the time when the failure cost evaluated; $\left(C_{P_f}\right)_{Repair}$ = the failure cost of performance strategies at the time when the failure cost evaluated; k = the time when the failure cost is evaluate; t_{age} = age of the structural system at the point in time k; v = discount rate of money at the time when the failure cost is evaluated; $\Delta\beta_{E,R}$ = the level of structural damage ($\Delta\beta_E$) or reliability index improvement ($\Delta\beta_R$); c_f = the failure cost coefficient. The failure cost depends on various environmental and regional factors such as type of bridges, daily traffic volume, number of injury or death, and service losses. In this study, the failure cost coefficients are approximated as \$440 and \$4,400 per unit area (increased by 10% to estimate inflation to 2008) for the evaluation and repair procedures respectively. In addition, the cost of failure can sometimes be more than \$4,400 per unit area and in some cases it is may be up to \$20,000 per unit area. The cost of failure depends on the construction site situation, importance, and area to mention a few.

In the current study, the illustrative examples are the time independent cases which means both condition evaluation and repair procedures, which are implemented in the present time [$k = 0$, $\nu = 0$, and $t_{age}(k) = t(0)$]. From this Equation (4.3) and (4.4) reduce to

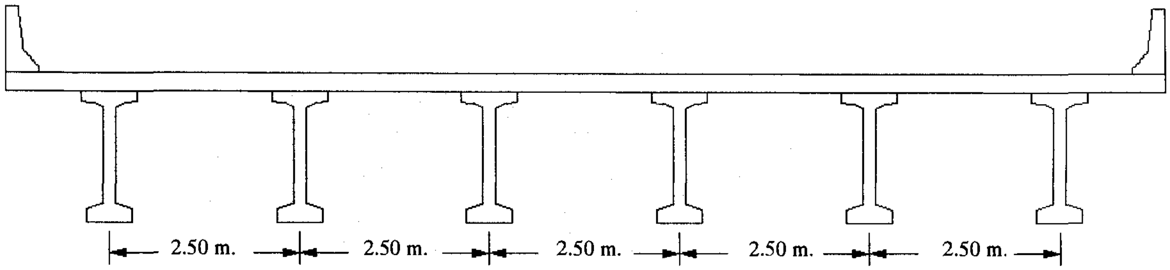
$$\left(C_{P_f}\right)_{Evaluation} = c_f \left\{\Delta\beta_E\right\}^2_{Evaluation} \tag{4.5}$$

$$\left(C_{P_f}\right)_{Repair} = c_f \left\{\Delta\beta_R\right\}^2_{Repair} \tag{4.6}$$

4.5 Illustrative Examples

4.5.1 Structural Modeling

In this dissertation, a decision making diagram process is applied for the selection of the optimal condition evaluation method and the repair method. First each optimal cost value is normalized then the problem parameters are selected such as the type of bridge girder, level of structural damage ($\Delta\beta_E$) anticipated from visual inspection, and the needed improvement in terms of the structural reliability index. The illustrative examples in this study are consisted with three examples. All of them are the bridge structures and have the structural details for the girder spacing and span lengths are shown in Figures 4.2 to 4.4. The illustrative examples are limited to a reliability index improvement ($\Delta\beta_{E,R}$) of not more than 4.5. If the $\Delta\beta_{E,R}$ is more than 4.5, the structure is assumed to have severe damage and a replacement strategy is needed.



Note: Span Length = 20.00 m.

Figure 4.2: Cross-Section of *PC* Bridge Structural Numerical Examples.

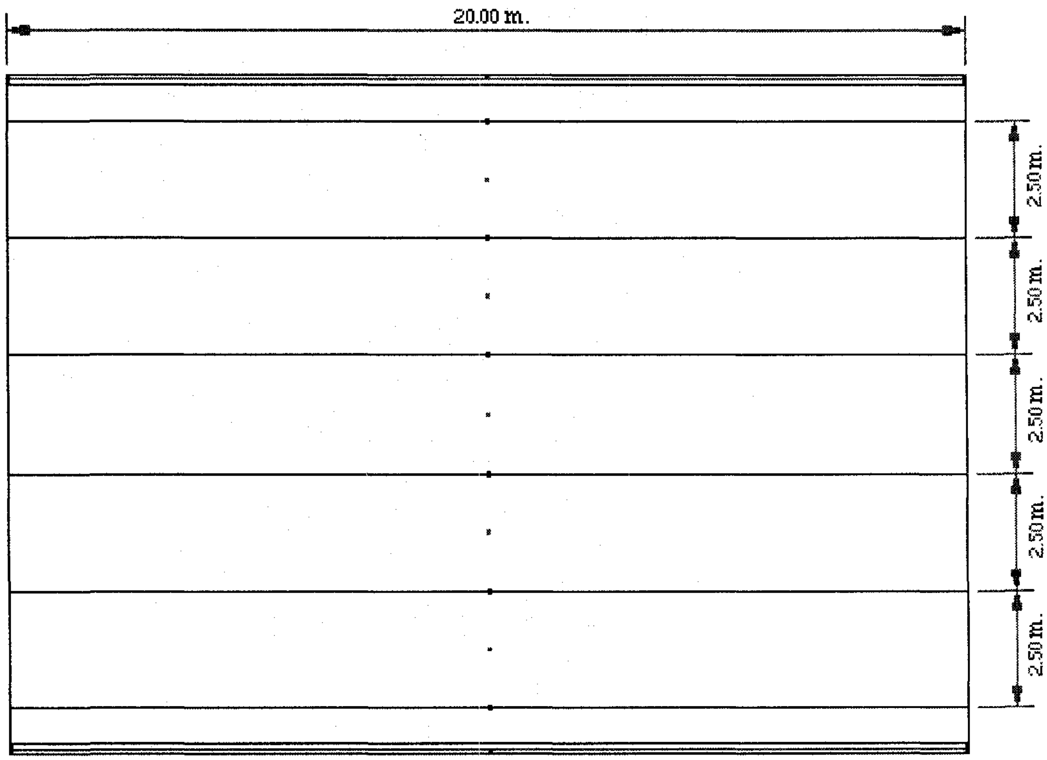


Figure 4.3: Plan of *PC* Bridge Structural Numerical Examples.

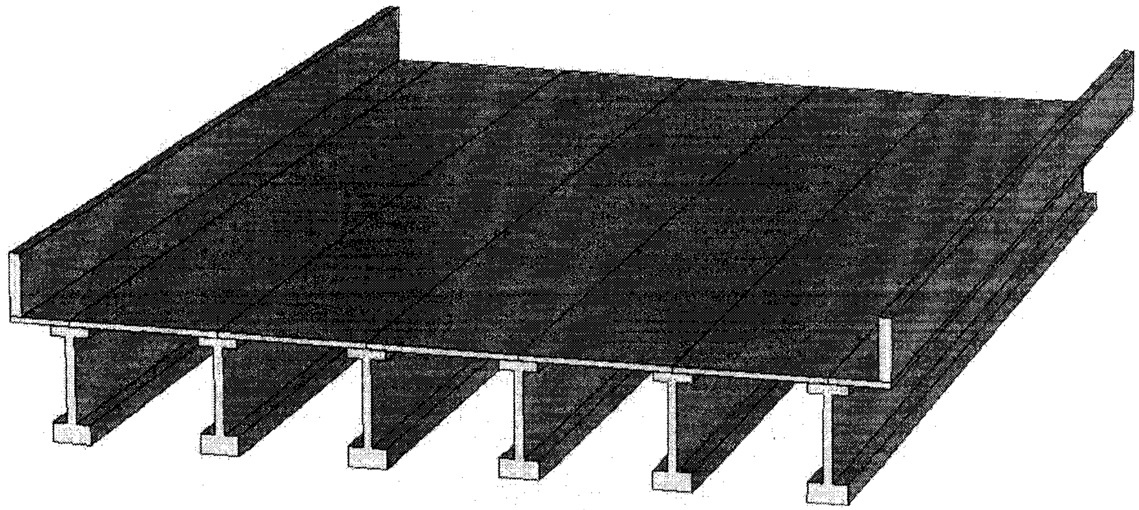


Figure 4.4: Solid Model of *PC* Bridge Girder Structural Numerical Examples.

4.5.2 Normalized Optimal Cost Values

Examples of the normalized values of the optimal costs for each procedure, which are needed within the decision process, are shown in Table 4.3. These results are calculated using a MATLAB optimization program written for this dissertation work. The MATLAB optimization programs utilize the Lagrange Multipliers method for solving the optimization problems.

Evaluation Method	Repair method	Normalized Minimum Cost
UPV	*SP*	0.612
	FRP	0.259
	CI	0.129
RN	*SP*	0.605
	FRP	0.262
	CI	0.133

Table 4.3: Example of Normalized Minimum Cost Values for combinations of Condition Evaluation Method and Repair Methods.

The optimal cost values and their normalized values from the optimization procedure for condition evaluation methods, repair methods, and their combinations (both evaluation methods and repair methods) are presented graphically in Figure 4.5, 4.6, and 4.7, respectively.

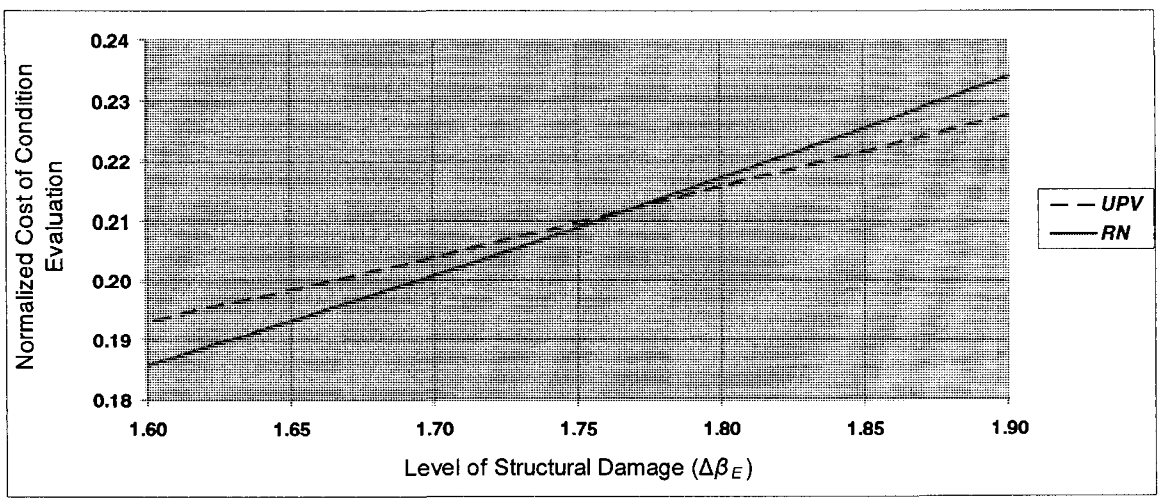


Figure 4.5: Relationship between Normalized Costs of Structural Condition Evaluation Methods and Level of Structural Damage ($\Delta\beta_E$).

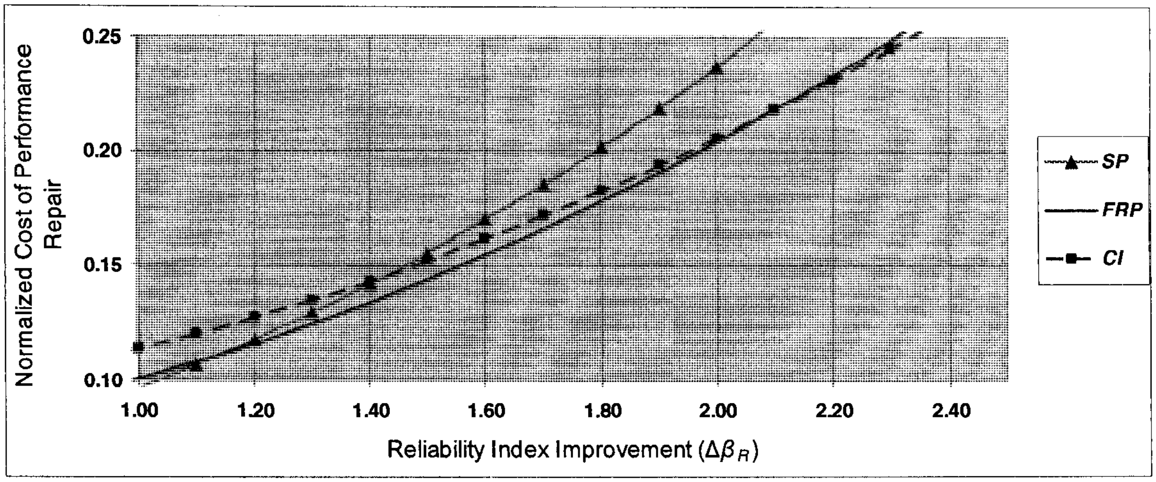


Figure 4.6: Relationship between Normalized Costs of Structural Repair Methods and Reliability Index Improvement ($\Delta\beta_R$).

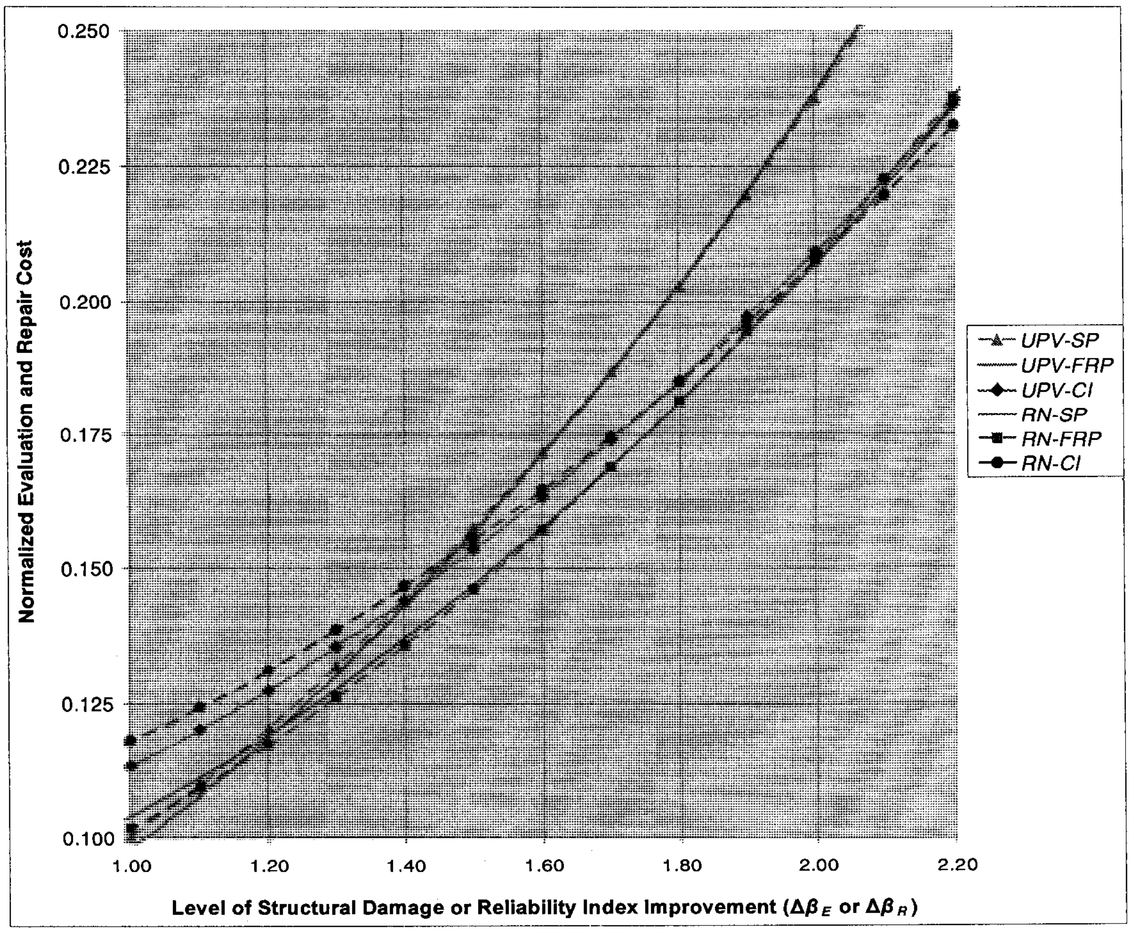


Figure 4.7: Relationship between Normalized Costs of Structural Condition Evaluation and Repair Methods and Level of Structural Damage ($\Delta\beta_E$)or Reliability Index Improvement ($\Delta\beta_R$).

4.5.3 Probability of Detection (*POD*) for *NDE*

From a previous study on *NDE* with comparison to destructive testing on concrete specimens (Buddhawanna, 2003), the probabilities of detection (*POD*) and reliability indices for both types of structural condition evaluation (*NDE*) were calculated. These *POD*'s are shown in Table 4.4 for both high performance and normal concrete strength.

	High Performance Concrete (*HPC*) $f_c' \approx 8000$ psi		Normal Concrete $f_c' \approx 4000$ psi	
	UPV	*RN*	*UPV*	*RN*
POD	0.7574	0.7276	0.7659	0.7674

Table 4.4: Probability of Detection (*POD*) for Structural Condition Evaluation Methods.

4.6 Illustrative Example 1 (Mild Structural Damage Case)

4.6.1 Structural Specification and Damage Conditions

In this illustrative example a bridge girder type known as the Colorado *CO G*68/6 (Nawy, 1999) is investigated. The target reliability index is 3.5, guiding reliability index (β_g) is 2.0 (the guiding reliability index must be approximated from visual inspection by an evaluator), and the level of structural damage ($\Delta\beta_E$) or reliability index improvement ($\Delta\beta_R$) required is 1.5. The cross-sectional dimensions are as shown in Figure 4.8 (Nawy, 1999).

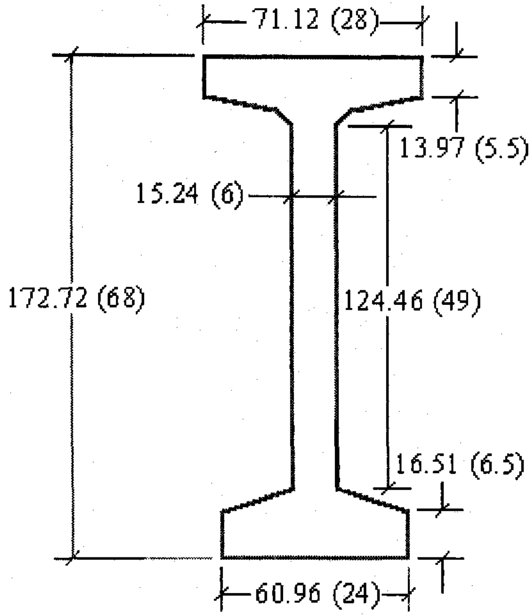


Figure 4.8: Colorado *CO G*68/6 Section Girder Cross Section [unit-cm. (in.)].

4.6.2 Repair Construction Site Condition Assumptions

In this study's illustrative examples, there are some assumed conditions for the repair/rehabilitation construction site that affect the analysis:

1) The repair/rehabilitation construction site is assumed very far from a fiber reinforced polymer factory.
2) The repair/rehabilitation construction site is far from the labor sources.
3) The repair/rehabilitation construction site is far from the equipment sources.
4) The repair/rehabilitation construction site is near the basic/normal repair material sources or construction material plants/sources (concrete, steel), metal reinforcement (rebar, prestressing tendons, etc.), structural cross-section steels

(steel plates, angles, etc.), structural connection bolts (normal bolts, expansion bolts).

From the construction site condition assumptions described above, an evaluator/engineer must be concerned about some factors that influence the condition evaluation and repair at the construction site such as:

1) Material availability factor
2) Contractor availability factor (labor availability)
3) Equipment availability factor.

In this study, the probability weightings have been assumed and estimated by the level of availability as shown in Table 4.5. In future studies a rational, perhaps, inquiry-based procedure would need to be developed and is addressed in the recommendations section of the dissertation.

	Level of Availability			Total
	Probability Weighting			
	Difficult	Moderate	Easy	
Preference Factors - Equipment - Labor - Material	0.44	0.32	0.24	1.00

Table 4.5 Estimated Probability (Cost) Weightings for Repair Strategies.

Because of the above reasons, an evaluator/engineer can approximate the probability for each type of repair method as follows:

The steel plate attaching (*SP*) repair method is a conventional repair method and it is a rather simple method for repairing a deteriorating structure. This method needs the proper size steel plates attached for re-strengthening a deteriorating structure to resist the internal forces from the structural loads cause. It requires the proper number of expansion bolts to connect between the deteriorating structural members and attached steel plates to resist the shear force that will occur in the repaired structure and to provide the rather firm connection surface between the existing structure and attached steel plate. Because of the previous reasons and repair construction conditions, an evaluator/engineer can provide a proper approximate probability of the steel plate attaching cost weighting which is assumed herein as 0.24.

The fiber reinforced polymer attaching (*FRP*) repair method needs the available source/factory of *FRP* and is a rather new repair material/strategy. The *FRP* is not a simple repair material. Furthermore, this method needs some skilled labor and equipment to attach/connect the *FRP* material with the existing structure. Sometimes it is difficult to find this kind of labor and equipment. Based on the previous reasons and repair construction conditions, an evaluator/engineer can provide a proper approximate probability of the fiber reinforced polymer attaching cost weighting as 0.44.

The cross section increasing (*CI*) repair method needs more sources of labor, equipment, and material than other repair methods. Furthermore, this method can be implemented with only some types of deteriorating structures. Based on the previous reasons and repair construction conditions, an evaluator/engineer can provide a proper approximate probability for the cross section increasing repair cost weighting herein as 0.32.

The results from this study's optimization algorithm are the normalized minimum cost values for the condition evaluation methods and repair methods. The results for the mild structural damage illustrative example are shown in Table 4.6.

From the results that are shown in Table 4.6, it shows both the calculated optimal cost values in the upper alleys/cells and normalized cost values in the lower alleys/cells for the structural condition evaluation methods, repair methods, and structural condition evaluation and repair procedure columns.

β_{target}	β_g	$\Delta\beta$	Condition Evaluation		Structural Repair		Evaluation + Repair	
			Method	Cost/Area (Normalized)	Strategy	Cost/Area (Normalized)	Strategy	Cost/Area (Normalized)
3.5	2	1.5	*UPV*	111.33 (0.183)	*SP*	1034.25 (0.155)	*UPV* +*SP*	1145.58 (0.157)
					FRP	957.5 (0.144)	*UPV* +*FRP*	1068.83 (0.147)
					CI	1011.25 (0.152)	*UPV* +*CI*	1122.58 (0.154)
			RN	104.33 (0.172)	*SP*	1034.25 (0.155)	*RN* +*SP*	1138.58 (0.156)
					FRP	957.5 (0.144)	*RN* +*FRP*	1061.83 (0.146)
					CI	1011.25 (0.152)	*RN* +*CI*	1115.58 (0.153)

Table 4.6: Optimal and Normalized Cost Values of Evaluation Methods and Repair Methods for Mild Structural Damage Case (Example 1).

4.6.3 Decision Making Step

For solving the uncertainty and limitation when utilizing of the structural condition evaluation methods, utilizing the data in Table 4.7 to solve for uncertainty and limitation of ability in *NDE* and/or *SDE* methods apply the probability of detection (*POD*).

Furthermore, it (Table 4.7) also reports all structural condition evaluation costs, repair costs, and each cost of decision making process branch that will be utilized/applied for the decision making process diagram shown in Figure 4.10.

Once the evaluator has the optimal cost values per unit area and approximated failure probability cost values per unit area the evaluator/engineer apply the data to the decision making process as shown in Figure 4.10. For illustrative purposes, it is assumed that there are two sites. For site, the deteriorating repair construction site is assumed to be located in a rather light traffic area and the bridge is assumed to be small. For this reason, the cost of catastrophic failure for this event/condition is relatively low. It is assumed to be approximately $4,400 per unit area. For the second site, the repair construction site is assumed to be located in heavy traffic conditions and a rather critical bridge structure in a large city. For this reason, the cost of failure for this event/condition is high. It is approximated as $14,000 per unit area.

The results for both sites with the aforementioned conditions are shown in Table 4.6 and 4.7, respectively. The decision making process diagram is shown in Figure 4.10 for this example.

Perform Testing	NDE / Cost/Unit	POD& / 1-POD	Repair method / Cost/Unit	Prob.	P_f	$/Unit Area 1st Site	$/Unit Area 2nd Site
Perform Testing	UPV / 111.33	0.757	SP / 1034.25	0.24	2.33E-04	$333.52	$335.76
			FRP / 957.5	0.44	2.33E-04	$506.60	$508.84
			CI / 1011.25	0.32	2.33E-04	$408.90	$411.14
		0.243			2.28E-02	$127.37	$346.25
	RN / 104.33	0.728	SP / 1034.25	0.24	2.33E-04	$325.20	$327.44
			FRP / 957.5	0.44	2.33E-04	$498.28	$500.52
			CI / 1011.25	0.32	2.33E-04	$400.58	$402.82
		0.272			2.28E-02	$128.70	$347.58

Notes: 1) In this study's illustrative examples, the target reliability index (β_{target}) is 3.5
2) The 1-*POD* is the probability of non-detection
3) The probability of failure (P_f) is 2.33E-04 for $\beta_{target} = 3.5$
4) The probability of failure (P_f) is 2.28E-02 for $\beta_g = 2$
5) The failure cost is assumed as $4,400 and $14,000 per unit area for the 1st and 2nd Site Condition respectively.

Table 4.7: *POD*, Evaluation Costs, and Repair Costs in Each Decision Making Branch for Mild Structural Damage Case.

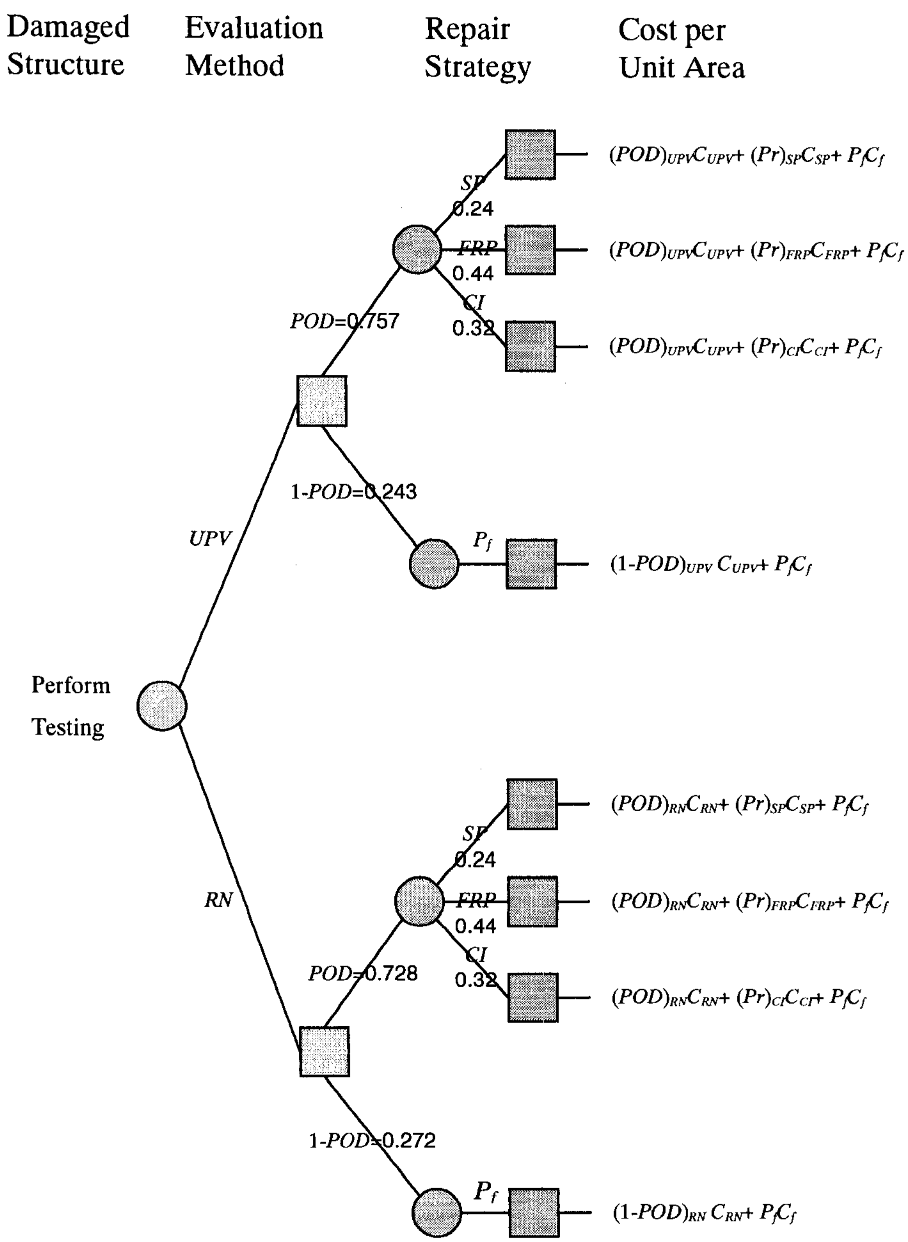
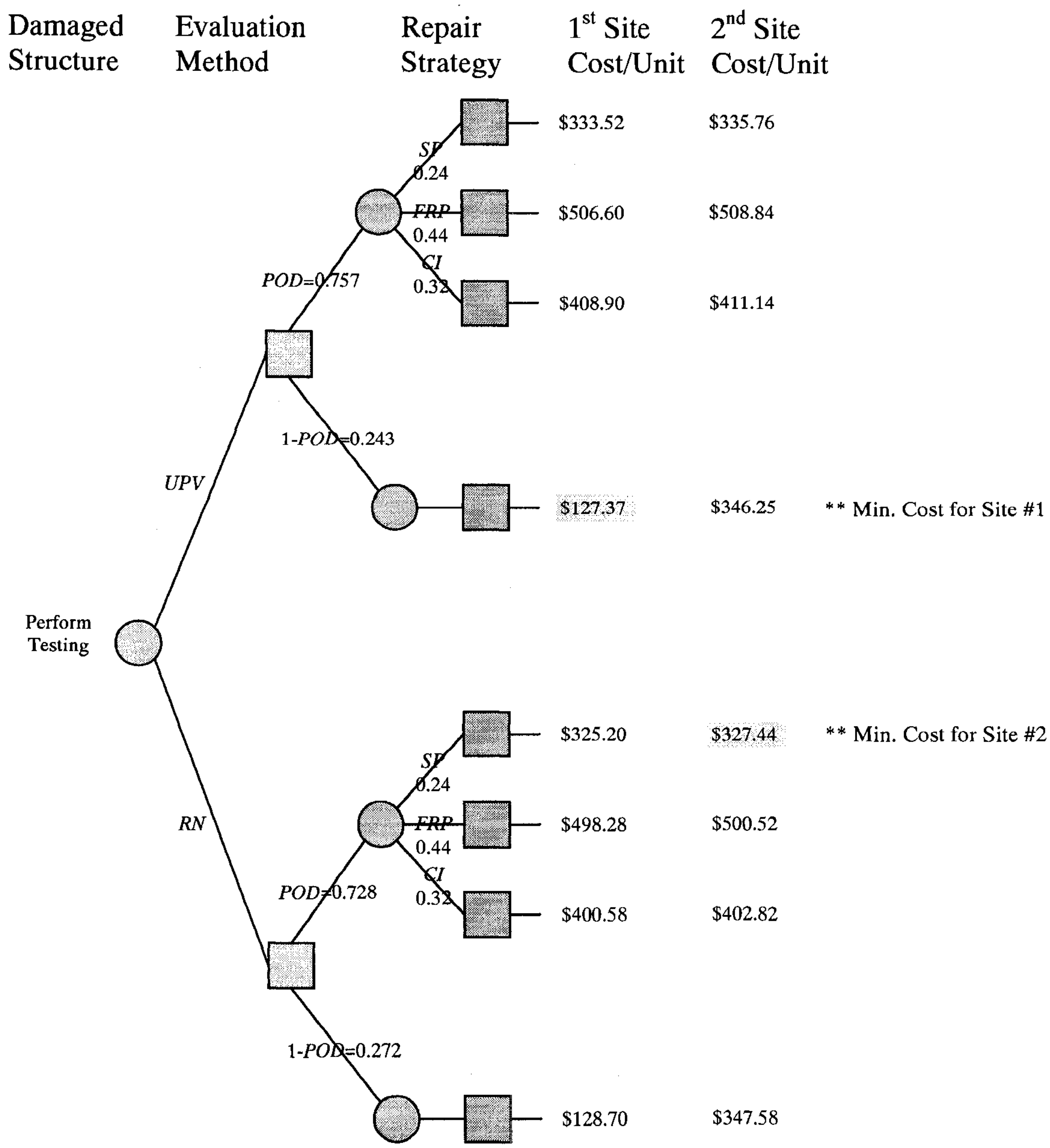


Figure 4.9: Decision Making Process Diagram for Structural Condition Evaluation and Repair.



Notes:
1) In this study's illustrative examples, the target reliability index (β_{target}) is 3.5
2) The 1-*POD* is the probability of non-detection
3) The probability of failure (P_f) is 2.33E-04 for $\beta_{target} = 3.5$
4) The probability of failure (P_f) is 2.28E-02 for $\beta_g = 2$
5) The failure cost is assumed as $4,400 and $14,000 per unit area for the 1st and 2nd Site Condition respectively.

Figure 4.10: Decision Making Process for Illustrative Example 1 (Mild Damage or $\Delta\beta_{E,R} = 1.5$).

4.6.4 Discussion

For the site condition #1 and the mild structural damage case ($\Delta\beta$ = 1.5), the most cost effective condition evaluation method was the *UPV* method and it is anticipated that repair would not be needed. This is likely because the structure is located in a low traffic area and is a small bridge. This branch is the lowest cost in this decision making diagram and was calculated as $127.37 per unit area.

For the site condition #2, consider the last column in Table 4.7 and the decision making diagram in Figure 4.10. The optimal decision making branch is the *RN* and *SP* combination which has an optimal cost value per unit area of $327.44. For this example, an evaluator would select the *RN* evaluation method and the *SP* repair method.

4.7 Illustrative Example 2 (Moderate Structural Damage Case)

4.7.1 Structural Specification and Damage Condition

In this illustrative example, a bridge girder type known as the *AASHTO* Type VI Modified (Nawy, 1999) is evaluated and repaired. The target reliability index is 3.5, guiding reliability index (β_g) is 1.0, and the level of structural damage ($\Delta\beta_E$) or reliability index improvement ($\Delta\beta_R$) required is 2.5. The cross-sectional dimensions are as shown in Figure 4.11 (Nawy, 1999).

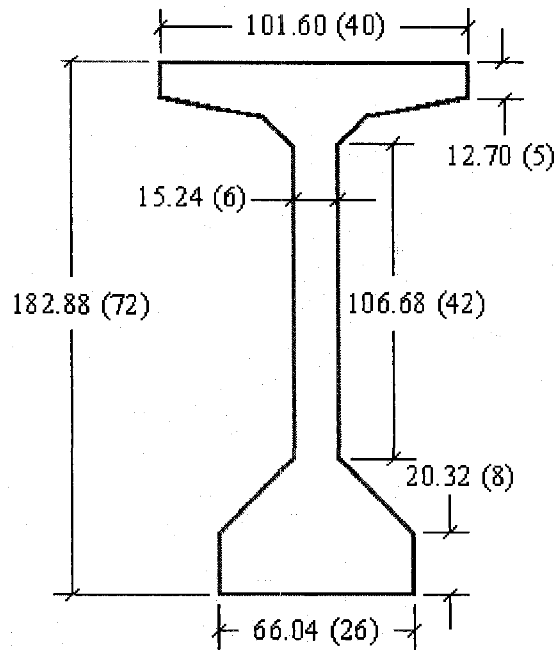


Figure 4.11: *AASHTO* Type VI Modified Section Girder Cross Section [unit-cm. (in.)].

4.7.2 Repair Construction Site Condition Assumptions

In the second illustrative example, there are some assumed conditions for the repair/rehabilitation construction site:

1) The repair/rehabilitation construction site is a moderate distance from the fiber reinforced polymer factory.
2) The repair/rehabilitation construction site is near the labor sources.
3) The repair/rehabilitation construction site is near the equipment sources.
4) The repair/rehabilitation construction site is far from the basic/normal repair material sources or construction material plants/sources (concrete, steel), metal reinforcement (rebar, prestressing tendons, etc.), structural cross-section steels

(steel plates, angles, etc.), structural connection bolts (normal bolts, expansion bolts).

As the reasons in the previous paragraphs and Table 4.5, an evaluator or engineer can approximate the probability for each type of repair method as the following.

For the steel plate attaching (*SP*) repair method, an evaluator/engineer can provide a proper approximate probability of the steel plate attaching cost weighting which is assumed herein as 0.44.

For the fiber reinforced polymer attaching (*FRP*) repair method, an evaluator/engineer can provide a proper approximate probability of the fiber reinforced polymer attaching cost weighting herein as 0.32.

For the cross section increasing (*CI*) repair method, an evaluator/engineer can provide a proper approximate probability for the cross section increasing repair cost weighting herein as 0.24.

The results for the moderate structural damage illustrative example are shown in Table 4.8. As in the first illustrative example in Section 4.6.2, the results in Table 4.8 show the calculated optimal cost values.

β_{target}	β_g	$\Delta\beta$	Condition Evaluation		Structural Repair		Evaluation + Repair	
			Method	Cost/Area (Normalized)	Strategy	Cost/Area (Normalized)	Strategy	Cost/Area (Normalized)
3.5	2	1.5	*UPV*	111.33 (0.183)	*SP*	1034.25 (0.155)	*UPV* +*SP*	1145.58 (0.157)
					FRP	957.5 (0.144)	*UPV* +*FRP*	1068.83 (0.147)
					CI	1011.25 (0.152)	*UPV* +*CI*	1122.58 (0.154)
			RN	104.33 (0.172)	*SP*	1034.25 (0.155)	*RN* +*SP*	1138.58 (0.156)
					FRP	957.5 (0.144)	*RN* +*FRP*	1061.83 (0.146)
					CI	1011.25 (0.152)	*RN* +*CI*	1115.58 (0.153)

Table 4.8: Optimal and Normalized Cost Values of Evaluation Methods and Repair Methods for Moderate Structural Damage Case (Example 2).

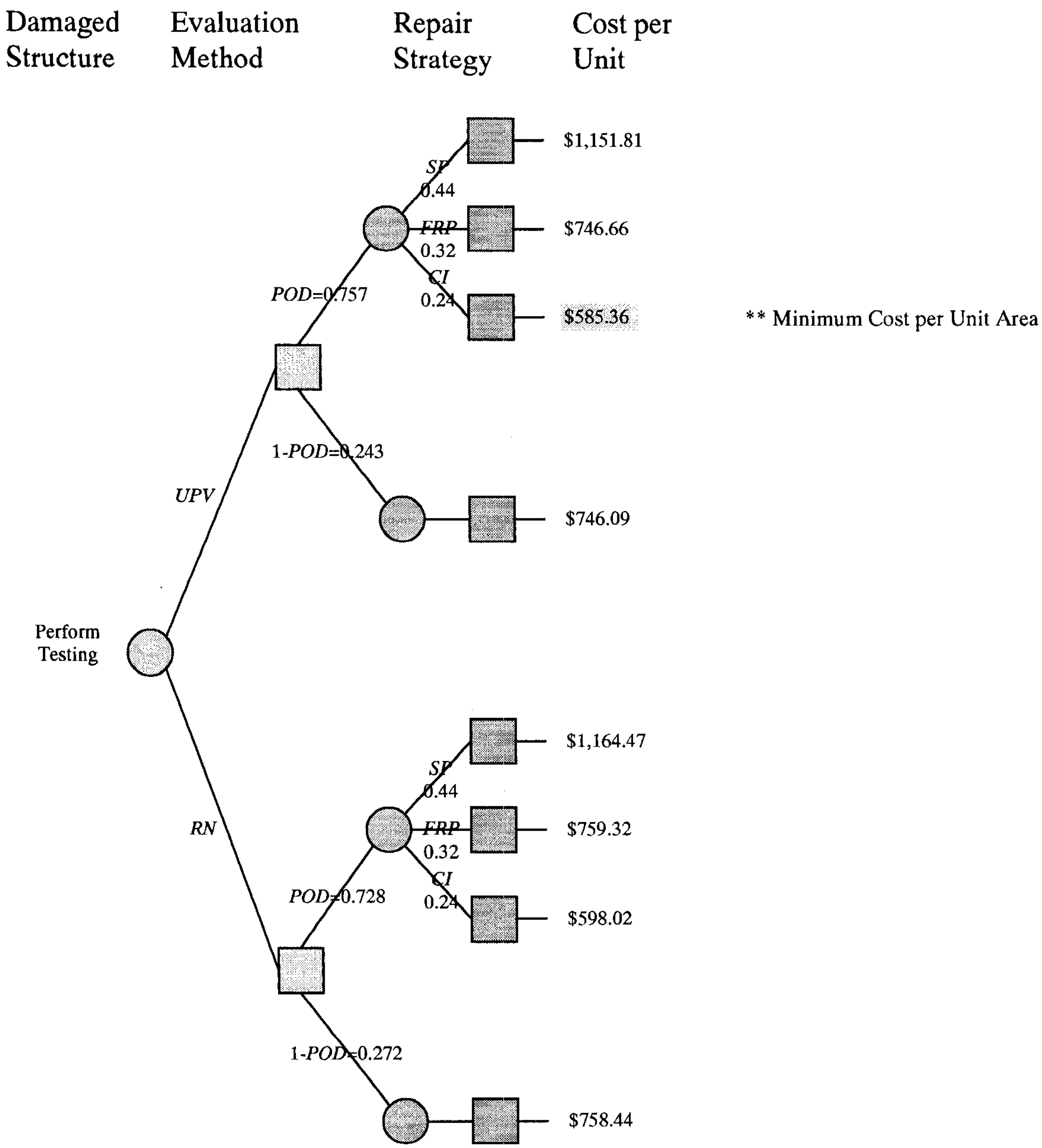
4.7.3 Decision Making Step

As in illustrative example 1, the optimization results are shown in Table 4.9 and Figure 4.12. The decision making process diagram is shown by Figure 4.12.

Perform Testing	*NDE* / Cost/Unit	*POD*& / 1-*POD*	Repair method / Cost/Unit	Prob.	P_f	Cost/Unit Area
Perform Testing	*UPV* / 191.33	0.757	*SP* / 2286.25	0.44	2.33E-04	$1,151.81
			FRP / 1877.5	0.32	2.33E-04	$746.66
			CI / 1831.25	0.24	2.33E-04	$585.36
		0.243			1.59E-01	$746.09
	RN / 216.33	0.728	*SP* / 2286.25	0.44	2.33E-04	$1,164.47
			FRP / 1877.5	0.32	2.33E-04	$759.32
			CI / 1831.25	0.24	2.33E-04	$598.02
		0.272			1.59E-01	$758.44

Notes: 1) In this study's illustrative examples, the target reliability index (β_{target}) is 3.5
2) The1-*POD* is the probability of non-detection
3) The probability of failure (P_f) is 2.33E-04 for β_{target} = 3.5
4) The probability of failure (P_f) is 1.59E-01 for β_g = 1
5) The failure cost is assumed as $4,400 per unit area.

Table 4.9: *POD*, Evaluation Costs, and Repair Costs of Evaluation methods and Repair methods in Each Decision Making Branch for Moderate Structural Damage Case.



Notes: 1) In this study's illustrative examples, the target reliability index (β_{target}) is 3.5
2) The 1-*POD* is the probability of non-detection
3) The probability of failure (P_f) is 2.33E-04 for $\beta_{target} = 3.5$
4) The probability of failure (P_f) is 1.59E-01 for $\beta_g = 1$
5) The failure cost is assumed as $4,400 per unit area.

Figure 4.12: Decision Making Process for Illustrative Example 2 (Moderate Damage or $\Delta\beta_{E,R} = 2.5$).

4.7.4 Discussion

For the second illustrative example which focused on a bridge with moderate structural damage the optimal decision making branch for this illustrative example is the *UPV* evaluation method and the *CI* repair method which has an optimal cost value per unit area of $585.36.

4.8 Illustrative Example 3 (Serious Structural Damage Case)

4.8.1 Structural Specification and Damage Condition

Similar to the previous illustrative examples, a bridge girder type known as the *PCI BT-72* (Nawy, 1999) is investigated and rehabilitated. In this illustrative example, the target reliability index is 3.5, guiding reliability index (β_g) is -1.0, and the level of structural damage ($\Delta\beta_E$) or reliability index improvement ($\Delta\beta_R$) required is 4.5. The cross-sectional dimensions are as shown in Figure 4.13 (Nawy, 1999).

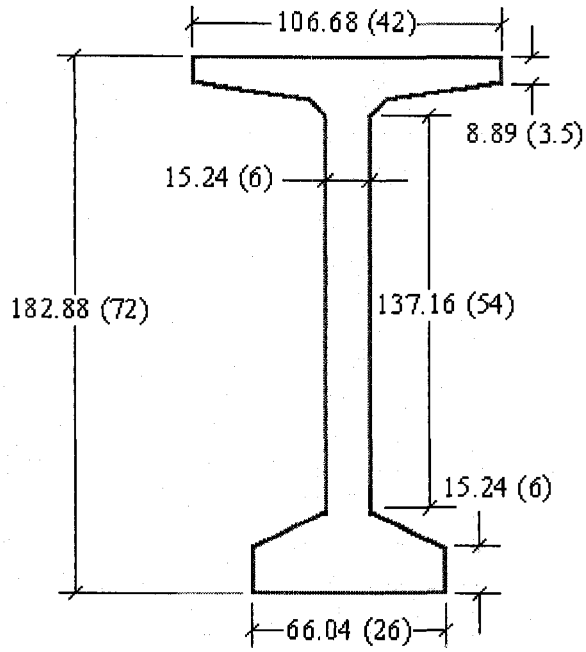


Figure 4.13: *PCI BT*-72 Section Girder Cross Section [unit-cm. (in.)].

4.8.2 Repair Construction Site Condition Assumptions

In the third illustrative example, there are some assumed conditions for the repair/rehabilitation construction site:

1) The repair/rehabilitation construction site is near the fiber reinforced polymer factory.
2) The repair/rehabilitation construction site is far from the labor sources.
3) The repair/rehabilitation construction site is far from the equipment sources.
4) The repair/rehabilitation construction site is rather far from the basic/normal repair material sources or construction material plants/sources (concrete, steel), metal reinforcement (rebar, prestressing tendons, etc.), structural cross-section

steels (steel plates, angles, etc.), structural connection bolts (normal bolts, expansion bolts).

From the construction site condition assumptions that have been described above, an evaluator/engineer must be concerned about some factors that influence the structural condition evaluation and repair construction site (same as the previous two examples). They are the material availability, contractor availability (labor availability), and equipment availability factors.

By the previous reasons and Table 4.5, an evaluator/engineer can approximate the probabilities cost weighting which are assumed herein as 0.32, 0.24, and 0.44 for the steel plate attaching (*SP*), fiber reinforced polymer attaching (*FRP*), and cross section increasing (*CI*) repair method respectively.

The results of the serious structural damage illustrative example are shown in Table 4.10 both the condition evaluation and repair method. From Table 4.10, the results are then used in the decision making process.

β_{target}	β_g	$\Delta\beta$	Condition Evaluation		Structural Repair		Evaluation + Repair	
			Method	Cost/Area (Normalized)	Strategy	Cost/Area (Normalized)	Strategy	Cost/Area (Normalized)
3.5	2	1.5	*UPV*	111.33 (0.183)	*SP*	1034.25 (0.155)	*UPV* +*SP*	1145.58 (0.157)
					FRP	957.5 (0.144)	*UPV* +*FRP*	1068.83 (0.147)
					CI	1011.25 (0.152)	*UPV* +*CI*	1122.58 (0.154)
			RN	104.33 (0.172)	*SP*	1034.25 (0.155)	*RN* +*SP*	1138.58 (0.156)
					FRP	957.5 (0.144)	*RN* +*FRP*	1061.83 (0.146)
					CI	1011.25 (0.152)	*RN* +*CI*	1115.58 (0.153)

Table 4.10: Optimal and Normalized Cost Values of Evaluation Methods and Repair Methods for Serious Structural Damage Case (Example 3).

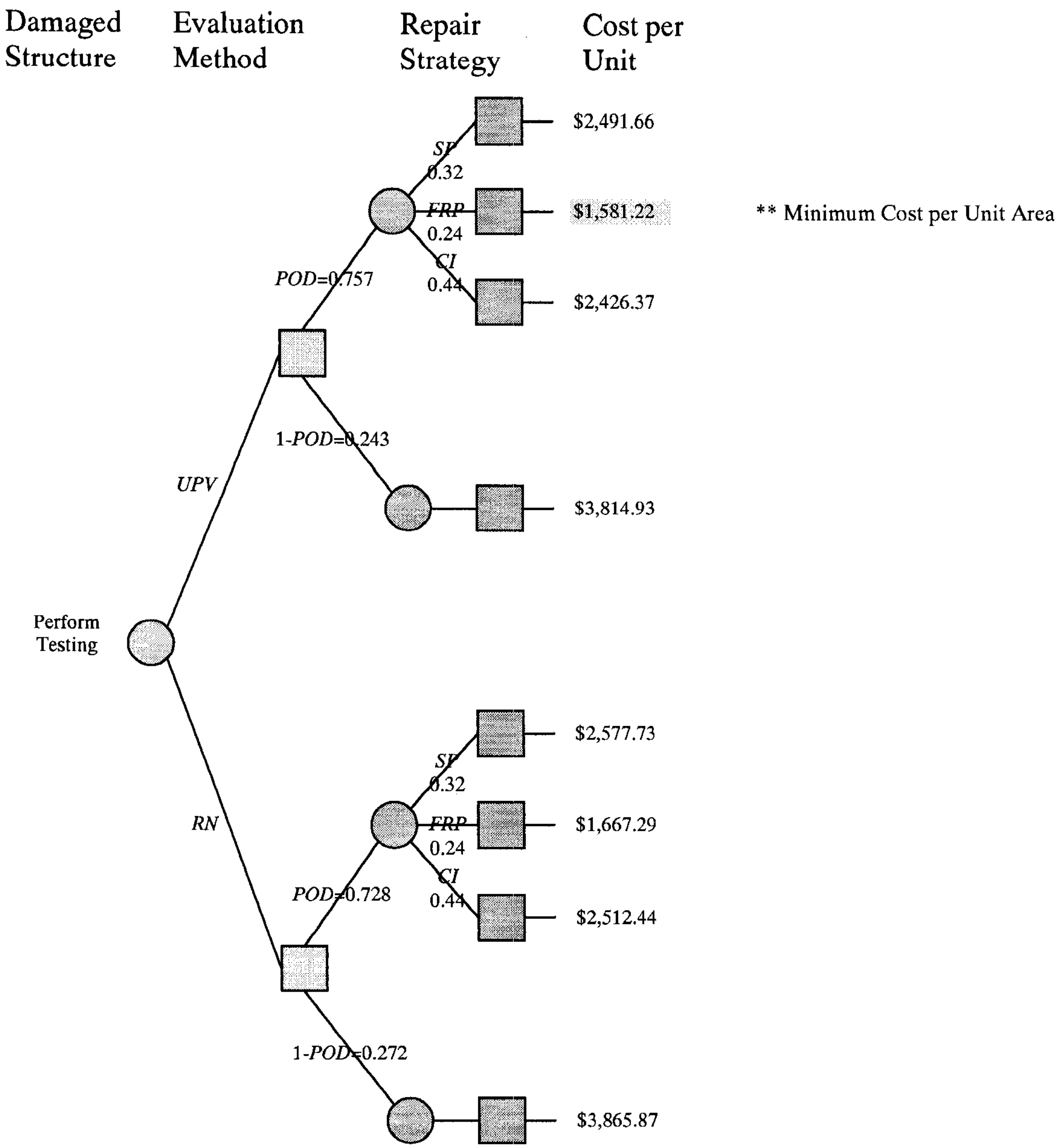
4.8.3 Decision Making Step

When an evaluator obtains the optimal cost values per unit area for this serious damage problem they are applied into the decision making process as shown in Figure 4.14.

Perform Testing	NDE / Cost/Unit	POD& / 1-POD	Repair method / Cost/Unit	Prob.	P_f	Cost/Unit Area
Perform Testing	*UPV* / 471.33	0.757	*SP* / 6668.25	0.32	2.33E-04	$2,491.66
			FRP / 5097.5	0.24	2.33E-04	$1,581.22
			CI / 4701.25	0.44	2.33E-04	$2,426.37
		0.243			8.41E-01	$3,814.93
	RN / 608.33	0.728	*SP* / 6668.25	0.32	2.33E-04	$2,577.73
			FRP / 5097.5	0.24	2.33E-04	$1,667.29
			CI / 4701.25	0.44	2.33E-04	$2,512.44
		0.272			8.41E-01	$3,865.87

Notes: 1) In this illustrative example, the target reliability index (β_{target}) is 3.5
2) The1-*POD* is the probability of non-detection
3) The probability of failure (P_f) is 2.33E-04 for β_{target} = 3.5
4) The probability of failure (P_f) is 8.41E-01 for β_g = -1
5) The failure cost is assumed as $4,400 per unit area.

Table 4.11: *POD*, Evaluation Costs, and Repair Costs of Evaluation Methods and Repair Methods in Each Decision Making Branch for Serious Structural Damage Case.



Notes: 1) In this study's illustrative examples, the target reliability index (β_{target}) is 3.5
2) The 1-*POD* is the probability of non-detection
3) The probability of failure (P_f) is 2.33E-04 for β_{target} = 3.5
4) The probability of failure (P_f) is 8.41E-01 for β_g = -1
5) The failure cost is assumed as $4,400 per unit area.

Figure 4.14: Decision Making Process for Illustrative Example 3 (Serious Damage or $\Delta\beta_{E,R}$ = 4.5).

4.8.4 Discussion

The minimum cost per unit area for the serious structural damage case illustrative example and site condition situation is $1581.22. As shown in Figure 4.14 this is for the *UPV* evaluation method and the *FRP* repair procedure.

4.9 Sensitivity Information/Analysis for Structural Condition Evaluation and Repair

Better correction and understanding of structural condition evaluation and repair of concrete bridge girders is achieved by combining structural reliability analysis and optimization techniques. This research is based on a reliability-based optimization formulation of evaluation and rehabilitation of concrete bridge girders.

As with any numerical study, it is important to understand how changes in various input parameters affect the numerical procedure, or modeling process.

4.9.1 Sensitivity Information for Condition Evaluation Method

The first investigation utilized various levels of damage (level 1 to 5) and normalized cost as the numerical and graphical formats in Table 4.12 and Figure 4.15, respectively.

Consider the uppermost line in Figure 4.15 which is for the *RN* method and the most serious structural damage condition. Note that when the levels of damage increase, the normalized costs also increase. This trend is the same for the other method (*UPV*) and other structural damage conditions (moderate and mild damage conditions).

Type or Level of Damage	Level	β_g	$\Delta\beta$	Structural Condition Evaluation Cost ($/m^2)			
				UPV	Normalized Cost	*RN*	Normalized Cost
Serious	Seri 5	-1.2	4.7	508.13	0.770	659.85	1.000
	Seri 4	-1.1	4.6	489.53	0.742	633.81	0.961
	Seri 3	-1	4.5	471.33	0.714	608.33	0.922
	Seri 2	-0.9	4.4	453.53	0.687	583.41	0.884
	Seri 1	-0.8	4.3	436.13	0.661	559.05	0.847
Moderate	Mod 5	0.8	2.7	212.13	0.321	245.45	0.372
	Mod 4	0.9	2.6	201.53	0.305	230.61	0.349
	Mod 3	1	2.5	191.33	0.290	216.33	0.328
	Mod 2	1.1	2.4	181.53	0.275	202.61	0.307
	Mod 1	1.2	2.3	172.13	0.261	189.45	0.287
Mild	Mild 5	1.8	1.7	124.13	0.188	122.25	0.185
	Mild 4	1.9	1.6	117.53	0.178	113.01	0.171
	Mild 3	2	1.5	111.33	0.169	104.33	0.158
	Mild 2	2.1	1.4	105.53	0.160	96.21	0.146
	Mild 1	2.2	1.3	100.13	0.152	88.65	0.134

Note: $\beta_{target} = 3.5$

Table 4.12: Sensitivity Information/Analysis of Structural Condition Evaluation Methods.

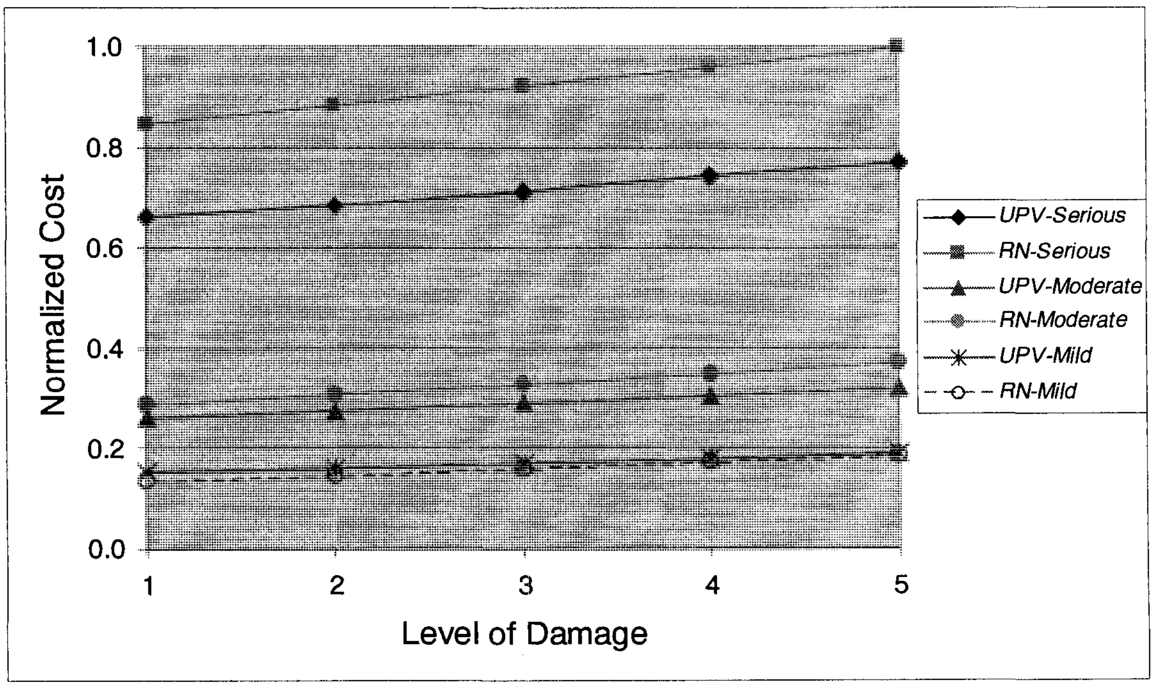


Figure 4.15: Sensitivity Information/Analysis of Structural Condition Evaluation Methods for Mild, Moderate, and Serious Cases.

4.9.2 Sensitivity Information for Repair Method

The relationship between the level of damage (levels 1 to 5 as before) and normalized cost is shown in Table 4.13 and graphically in Figures 4.16, 4.17, and 4.18 for mild, moderate, and serious damage cases, respectively.

Consider the lower line in Figure 4.16 which represents the sensitivity information for the *FRP* repair method for a girder with mild structural damage. The normalized cost increases as the damage level increases.

Type or Level of Damage	Level	β_g	$\Delta\beta$	Structural Repair Cost (\$/m^2)					
				SP	Normal. Cost	*FRP*	Normal. Cost	*CI*	Normal. Cost
Serious	Seri 5	-1.2	4.7	7244.17	1.000	5520.70	0.762	5078.45	0.701
	Seri 4	-1.1	4.6	6953.08	0.960	5306.80	0.733	4887.8	0.675
	Seri 3	-1	4.5	6668.25	0.920	5097.50	0.704	4701.25	0.649
	Seri 2	-0.9	4.4	6389.68	0.882	4892.80	0.675	4518.8	0.624
	Seri 1	-0.8	4.3	6117.37	0.844	4692.70	0.648	4340.45	0.599
Moderate	Mod 5	0.8	2.7	2611.77	0.361	2116.70	0.292	2044.45	0.282
	Mod 4	0.9	2.6	2445.88	0.338	1994.80	0.275	1935.8	0.267
	Mod 3	1	2.5	2286.25	0.316	1877.50	0.259	1831.25	0.253
	Mod 2	1.1	2.4	2132.88	0.294	1764.80	0.244	1730.8	0.239
	Mod 1	1.2	2.3	1985.77	0.274	1656.70	0.229	1634.45	0.226
Mild	Mild 5	1.8	1.7	1234.57	0.170	1104.70	0.152	1142.45	0.158
	Mild 4	1.9	1.6	1131.28	0.156	1028.80	0.142	1074.8	0.148
	Mild 3	2	1.5	1034.25	0.143	957.50	0.132	1011.25	0.140
	Mild 2	2.1	1.4	943.48	0.130	890.80	0.123	951.8	0.131
	Mild 1	2.2	1.3	858.97	0.119	828.70	0.114	896.45	0.124

Note: $\beta_{target} = 3.5$

Table 4.13: Sensitivity Information/Analysis of Structural Repair Methods.

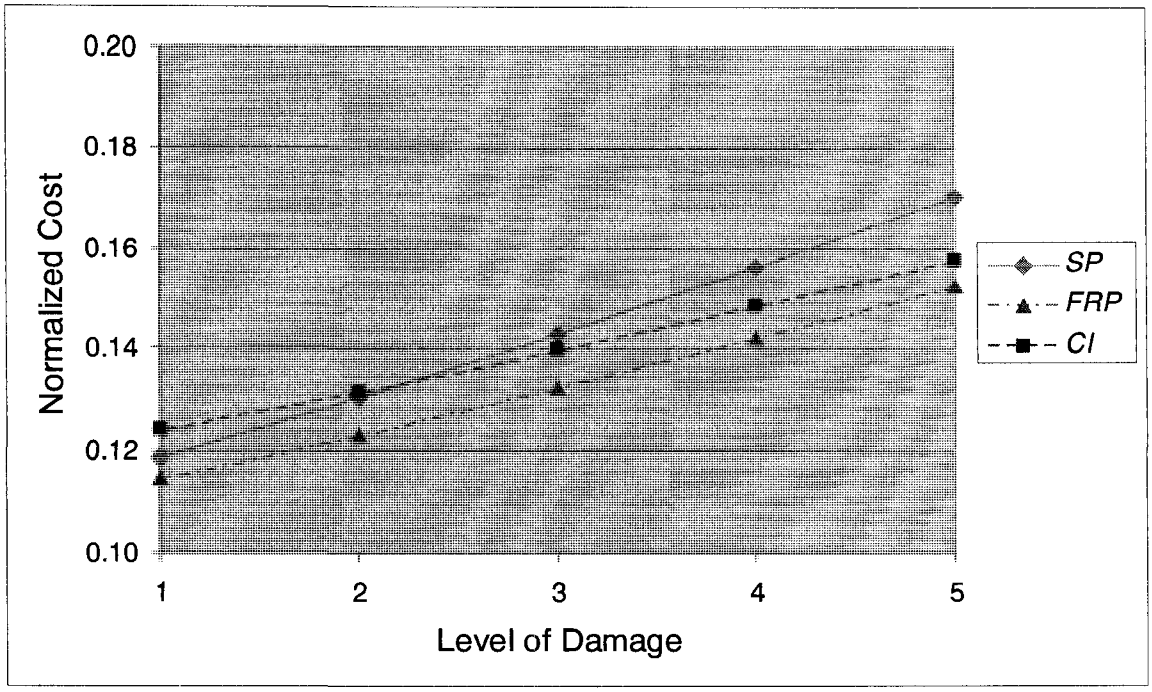


Figure 4.16: Sensitivity Information/Analysis of Structural Repair Methods for Mild Damage Case.

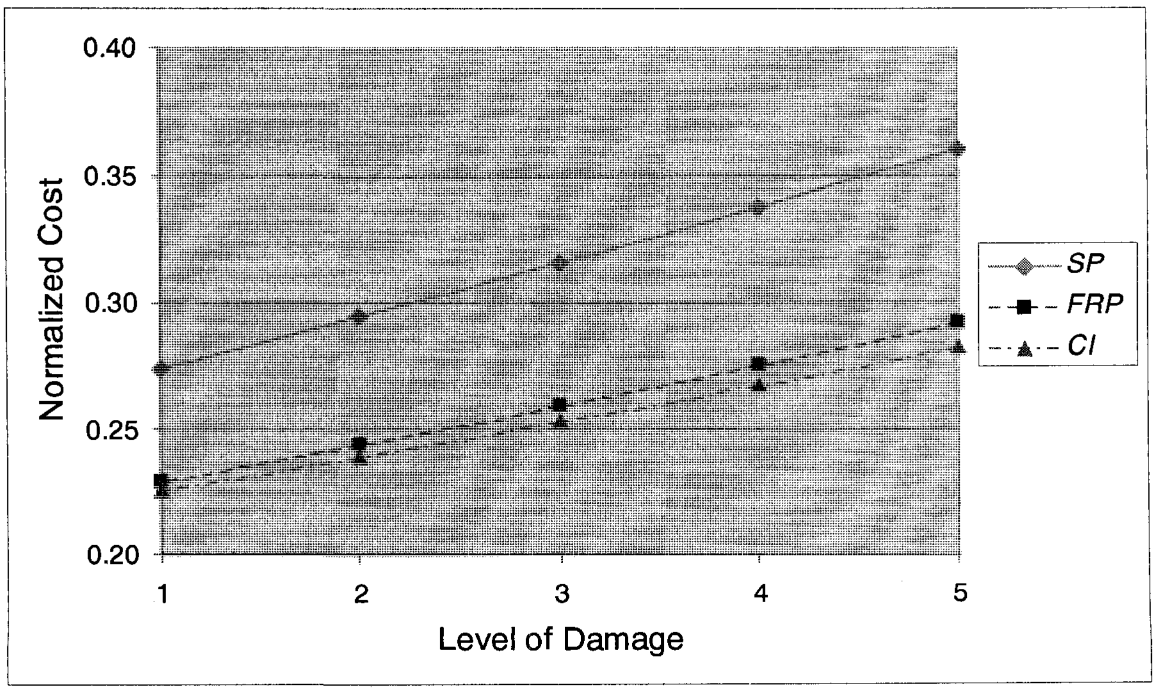


Figure 4.17: Sensitivity Information/Analysis of Structural Repair Methods for Moderate Damage Case.

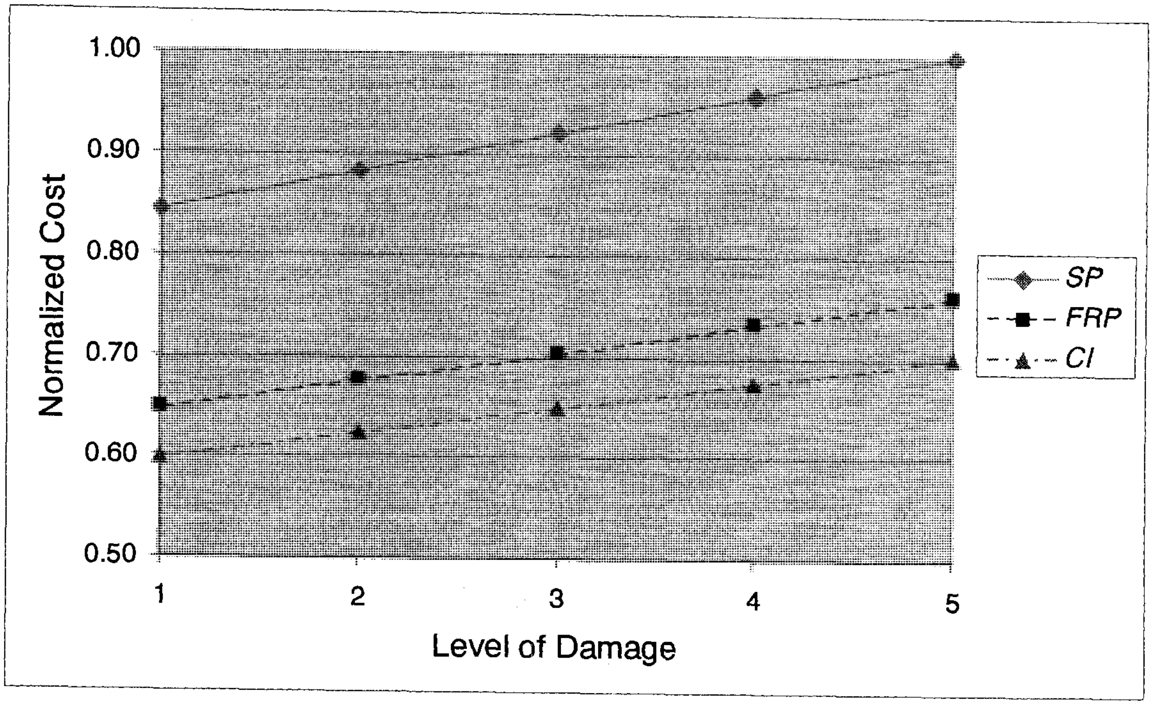


Figure 4.18: Sensitivity Information/Analysis of Structural Repair Methods for Serious Damage Case.

4.9.3 Sensitivity Information for Combined Condition Evaluation and Repair Method

The sensitivity information for condition evaluation methods and repair methods were also examined as a function of damage level (levels 1 to 5) and the results are presented in Table 4.14 and 4.15 for the *UPV* and *RN* methods, respectively. The results are also shown in graphical form in Figures 4.19, 4.20, and 4.21 for mild, moderate, and serious structural damage cases, respectively.

From Figure 4.21, consider the uppermost line which is the sensitivity information for the *RN* evaluation method combined with the *SP* repair method for a case of serious structural damage. The same positive correlation is observed.

Type or Level of Damage	Level	β_g	$\Delta\beta$	Structural Evaluation + Repair Cost ($/m^2)					
				UPV + *SP*	Normal. Cost	*UPV* +*FRP*	Normal. Cost	*UPV* +*CI*	Normal. Cost
Serious	Seri 5	-1.2	4.7	7752.30	0.981	6028.83	0.763	5586.58	0.707
	Seri 4	-1.1	4.6	7442.61	0.942	5796.33	0.733	5377.33	0.680
	Seri 3	-1	4.5	7139.58	0.903	5568.83	0.705	5172.58	0.654
	Seri 2	-0.9	4.4	6843.21	0.866	5346.33	0.676	4972.33	0.629
	Seri 1	-0.8	4.3	6553.50	0.829	5128.83	0.649	4776.58	0.604
Moderate	Mod 5	0.8	2.7	2823.90	0.357	2328.83	0.295	2256.58	0.285
	Mod 4	0.9	2.6	2647.41	0.335	2196.33	0.278	2137.33	0.270
	Mod 3	1	2.5	2477.58	0.313	2068.83	0.262	2022.58	0.256
	Mod 2	1.1	2.4	2314.41	0.293	1946.33	0.246	1912.33	0.242
	Mod 1	1.2	2.3	2157.90	0.273	1828.83	0.231	1806.58	0.229
Mild	Mild 5	1.8	1.7	1358.70	0.172	1228.83	0.155	1266.58	0.160
	Mild 4	1.9	1.6	1248.81	0.158	1146.33	0.145	1192.33	0.151
	Mild 3	2	1.5	1145.58	0.145	1068.83	0.135	1122.58	0.142
	Mild 2	2.1	1.4	1049.01	0.133	996.33	0.126	1057.33	0.134
	Mild 1	2.2	1.3	959.10	0.121	928.83	0.118	996.58	0.126

Note: $\beta_{target} = 3.5$

Table 4.14: Sensitivity Information/Analysis of *UPV* and Repair Methods.

Type or Level of Damage	Level	β_g	$\Delta\beta$	Structural Evaluation + Repair Cost ($/m^2)					
				RN + *SP*	Normal. Cost	*RN* + *FRP*	Normal. Cost	*RN* + *CI*	Normal. Cost
Serious	Seri 5	-1.2	4.7	7904.02	1.000	6180.55	0.782	5738.30	0.726
	Seri 4	-1.1	4.6	7586.89	0.960	5940.61	0.752	5521.61	0.699
	Seri 3	-1	4.5	7276.58	0.921	5705.83	0.722	5309.58	0.672
	Seri 2	-0.9	4.4	6973.09	0.882	5476.21	0.693	5102.21	0.646
	Seri 1	-0.8	4.3	6676.42	0.845	5251.75	0.664	4899.50	0.620
Moderate	Mod 5	0.8	2.7	2857.22	0.361	2362.15	0.299	2289.90	0.290
	Mod 4	0.9	2.6	2676.49	0.339	2225.41	0.282	2166.41	0.274
	Mod 3	1	2.5	2502.58	0.317	2093.83	0.265	2047.58	0.259
	Mod 2	1.1	2.4	2335.49	0.295	1967.41	0.249	1933.41	0.245
	Mod 1	1.2	2.3	2175.22	0.275	1846.15	0.234	1823.90	0.231
Mild	Mild 5	1.8	1.7	1356.82	0.172	1226.95	0.155	1264.70	0.160
	Mild 4	1.9	1.6	1244.29	0.157	1141.81	0.144	1187.81	0.150
	Mild 3	2	1.5	1138.58	0.144	1061.83	0.134	1115.58	0.141
	Mild 2	2.1	1.4	1039.69	0.132	987.01	0.125	1048.01	0.133
	Mild 1	2.2	1.3	947.62	0.120	917.35	0.116	985.10	0.125

Note: $\beta_{target} = 3.5$

Table 4.15: Sensitivity Information/Analysis of *RN* and Repair Methods.

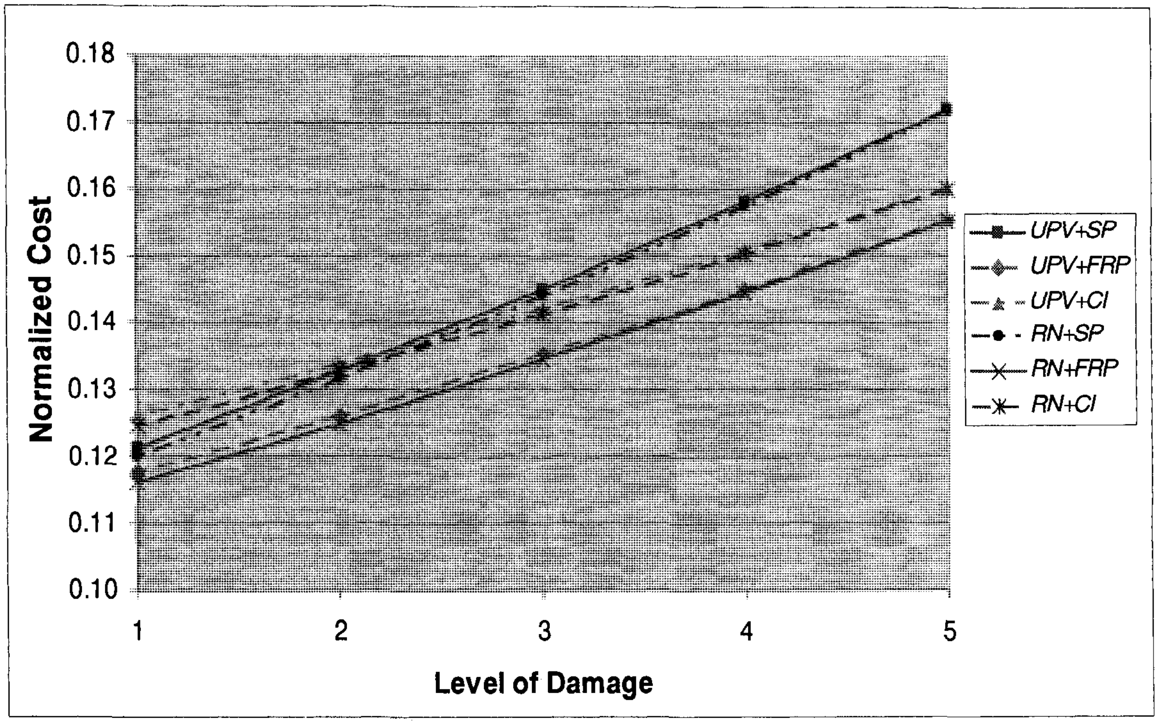


Figure 4.19: Sensitivity Information/Analysis of Structural Condition Evaluation Methods and Repair Methods for Mild Damage Case.

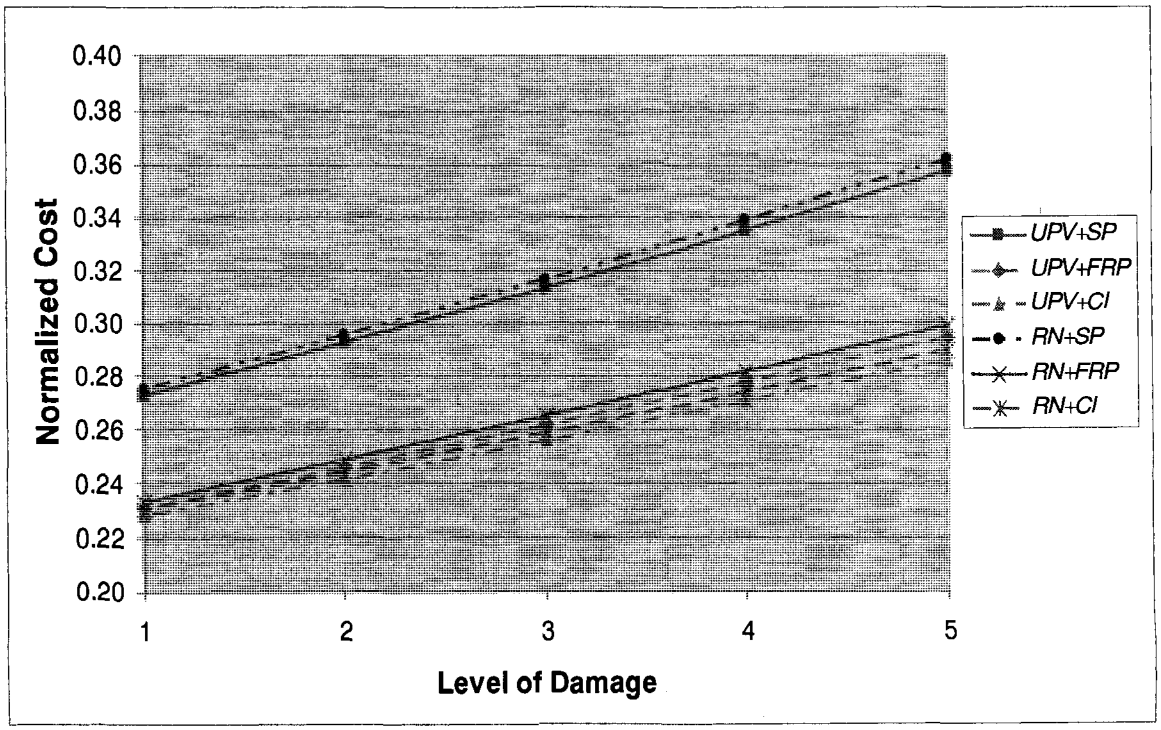


Figure 4.20: Sensitivity Information/Analysis of Structural Condition Evaluation Methods and Repair Methods for Moderate Damage Case.

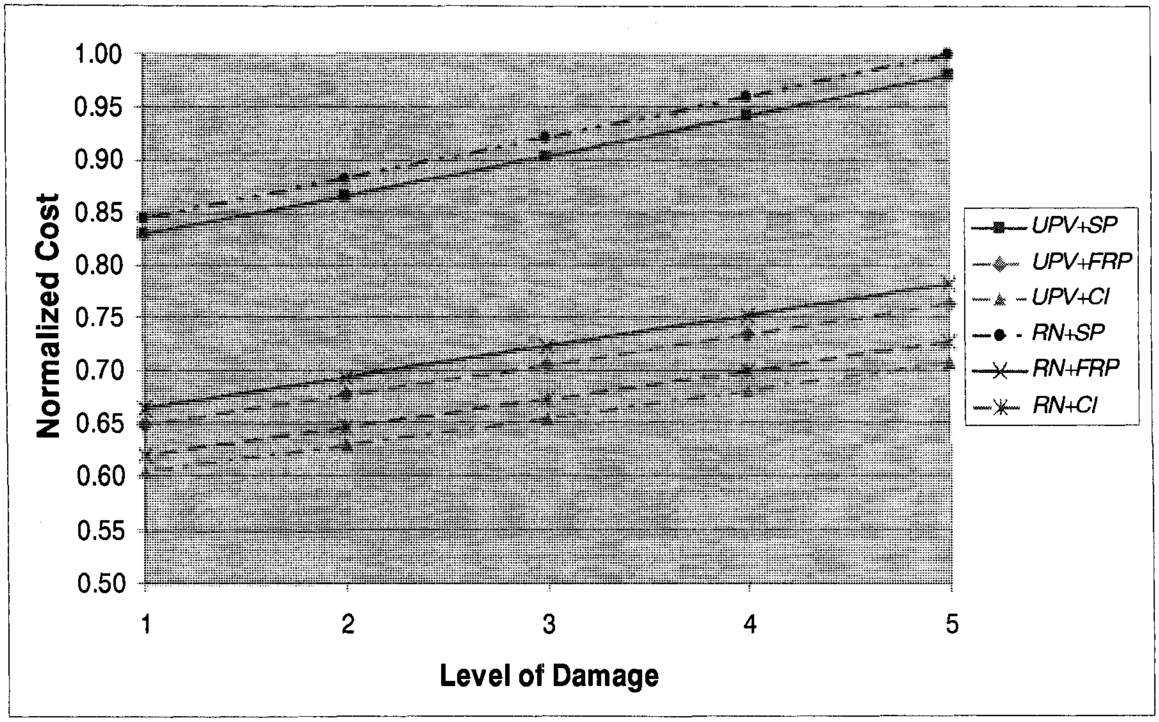


Figure 4.21: Sensitivity Information/Analysis of Structural Condition Evaluation Methods and Repair Methods for Serious Damage Case.

CHAPTER 5

CONCLUSIONS, CONTRIBUTIONS, AND RECOMMENDATIONS

The method presented in this dissertation provides a basic approach to aid in the determination of what type of condition evaluation and repair strategy to undertake, given a certain set of structural and site conditions, and performance requirements. This is the first study to attempt to provide decision support for evaluation method selection that accounts for the level of structural damage and the possible failure of detection. The procedure, as it stands herein, is not yet practical, but it is envisioned that if a fuzzy input software package could be developed or if the procedure could be used to develop rule-based charts or tables for common cases/situations, then the approach can be applied. Perhaps the most significant contribution of this method will be found in early resource allocation such as when a state needs to determine how to allocate their bridge rehabilitation budget by region without full inspection reports, or with only limited information.

It can be concluded that using the method described herein, that there is a strong dependence on the availability factors described earlier. It is also observed that the method provides consistent results and the trends are logical thus indicating that this type of approach is possible, but ultimately must be confirmed with something other than hypothetical data.

Based on the work herein, the following research is recommended:

1) Expand the study to include more detailed spatial analysis (results) of inspections.

2) Determine/identify the linkage quantitatively between the objective function and various parameters such as labor, materials, and equipment. Specifically, it is envisioned that performance functions will be developed that address axial, torsion, moment, and shear for bridge beams and influence factors such as labor and materials are integrated directly into those functions.
3) Develop a general set of guideline for bridge in various states of disrepair under the most common site condition, e.g. rural or urban. This set of guidelines will allow application without software for these cases.
4) Extension of the method, once refined, to include fuzzy sets. For example, fuzzy sets could account for input such as the availability of *FRP* contractors is "good", "average", or "poor".

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