ELEMENTS OF OPEN CHANNEL FLOW AND SEDIMENT TRANSPORT

Technical Training School
Quality of Water Branch

by

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CONTENTS

ELEMENTS OF SEDIMENT TRANSPORT

INTRODUCTION

The following discussion deals with open channel flow and the mechanics of transporting sediment by the flow of water in open channels. The concepts and methods discussed herein are the more important ones related to sediment transport. Much supporting and detailed explanations must be omitted because of limited time. A more complete description of sediment transport concepts and theory can be found in the literature cited and listed at the end of this paper.

The science of fluid mechanics deals with the laws that govern the movement of fluids and this assists in explaining the movement of sediment (Rouse, 1946).

PROPERTIES OF FLUIDS

The following physical properties of fluids influence fluid motion and help explain sediment transport.

Mass is the amount of substance in matter measured by its resistance to the application of force.

Density is mass per unit volume and is commonly symbolized by the Greek letter ρ (rho).

Weight is the force that gravity exerts on a mass; $W = gM$, where g is the acceleration of gravity.

Specific Weight is the weight per unit volume and is symbolized by the Greek letter γ (gamma); $\gamma = \rho g$.

Viscosity is property of fluids that resists deformation and is commonly symbolized by the Greek letter μ (mu).

Problem No. 1:

A container with dimensions of 10 x 4 x 2.5 (feet) is filled with water.

(A) What force (in pounds) does gravity apply to the mass of liquid? (Water weighs 62.5 lbs per foot^3)

(B) What is the mass of the liquid in slugs?

A - (10) (4) (2.5) (62.5) = 62.50 lbs

B - $\frac{6250}{32.2}$ = 194 slugs

Shear is a property of fluid motion that is closely related to viscosity. It is the tangential force or stress per unit area that is transmitted through a unit thickness of a fluid. Shear, τ (tau), is related to viscosity by the equation $\tau = \mu\, dv/dy$ and is explained by Fig. 1.

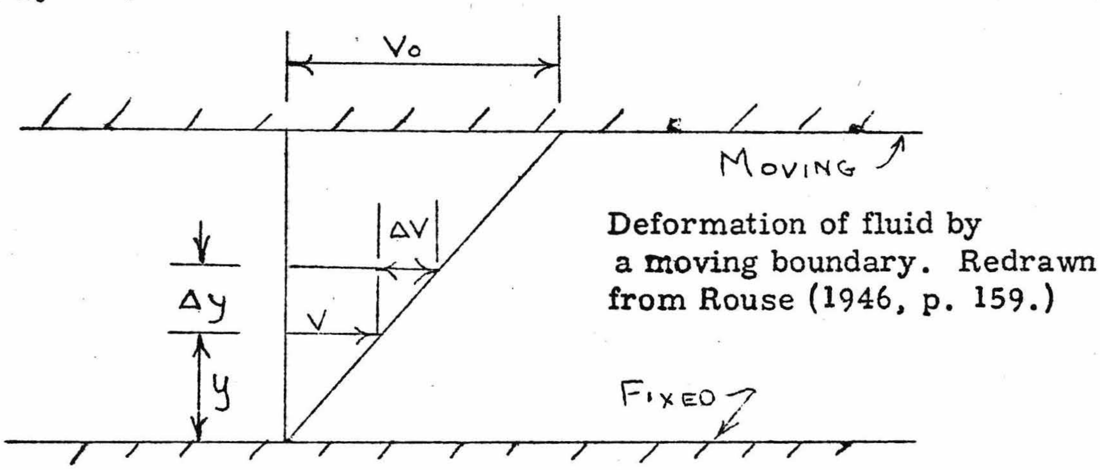


Figure 1

Problem No. 2

In Fig. 1, the upper boundary is moving uniformly at 10 feet per second. The spacing between the top and bottom plates is .01 foot. Water temperature is 60°F ($\mu = 2.34 \times 10^{-5}$ lb sec/ft²). Compute the shear (τ) in the fluid. Remember that the water filament next to the moving plate has the same velocity as the plate and the filament adjacent to the fixed side does not move. Assume that the velocity gradient thru the thickness of the fluid is uniform - a straight line.

$$\tau = \mu \frac{dv}{dy}$$

$$\frac{dv}{dy} = \frac{10}{.01} = 1000$$

$$\tau = (1000)\ (2.34 \times 10^{-5}) = 0.023 \text{ lbs/ft}^2$$

Temperature affects the density of liquids slightly and the viscosity significantly. That is, water is essentially incompressible. The viscosity of liquids decrease with increasing temperature. Water temperature in open channels can vary as much as 40° F within a 24 hr period.

Blasticity and Surface Tension have little effect on flow in open channels including sediment transport.

FLUID MOTION

The following discussion of fluid motion is limited to flow in open channels.

Types of Flow

Because flow in open channels involves a free surface or interface, it has more degrees of freedom than the flow in closed conduits flowing full. This fact results in additional types of flow which must be clearly defined and understood. These types include uniform flow and non-uniform (or varied) flow; steady flow and unsteady flow; laminar flow and turbulent flow; and subcritical flow, supercritical flow, and ultrarapid flow.

Uniform flow in open channels, like that in pipes, depends upon there being no change with distance in either the magnitude or the direction of the velocity along a streamline -- that is, both $\partial v/\partial s = 0$ and $\partial v/\partial n = 0$. Non-uniform flow in open channels occurs when either $\partial v/\partial s \neq 0$ or $\partial v/\partial n \neq 0$. The particular type of non-uniform flow which occurs when $\partial v/\partial s \neq 0$ in open channels is usually called varied flow. An example of uniform flow is flow in a straight canal or flume having a constant depth of flow, a constant slope, and a constant cross section throughout. Obviously, this condition seldom exists exactly. Examples of non-uniform flow wherein $\partial v/\partial n \neq 0$ are bends or curving sides of the channel. When $\partial v/\partial s \neq 0$ the flow is varied and occurs when there is a change in depth of flow due either to a change in slope, a barrier or drop, or a change in side or bottom so that the velocity increases or decreases in the direction of flow.

Steady flow occurs when the velocity at a point does not change with time -- that is, $\partial v/\partial t = 0$. When the flow is unsteady, $\partial v/\partial t = 0$, as in the case of pipe, unsteady flow is difficult to analyze unless the change with respect to time is sufficiently slow to permit a step type of analysis. Examples of unsteady flow are traveling surges and flood waves in an open channel.

Whether laminar flow or turbulent flow exists in an open channel depends upon the Reynolds number Re of the flow just as it does in pipes. In other words, laminar flow depends upon the viscous forces being predominant compared with the inertial forces and turbulent flow depends upon the inertial forces being great compared with the forces of viscosity. With laminar flow the mixing of the fluid is by molecular activity whereas with turbulent flow the mixing is by finite masses of molecules.

Turbulent flow may be over a smooth boundary, where Reynolds number is relatively small and the laminar sublayer covers the boundary roughness; over a rough boundary, where

Reynolds number is relatively large and the laminar sublayer is destroyed by the roughness; or over a transition boundary, where Reynolds number is intermediate and the boundary is both partly smooth and partly rough.

Unlike laminar and turbulent flow, subcritical flow and supercritical flow exists only with a free surface or interface which means these types of flow can occur in open channels and pipes flowing partly full but not in pipes flowing full. The criterion for subcritical and supercritical flow rests in the Froude number Fr. When the Froude number is one the flow is critical, when it is less than one the flow is subcritical and when it is greater than one the flow is supercritical. Ultrarapid flow involves slugs or waves superposed over the uniform flow pattern which makes the flow both non-uniform and unsteady.

Either one of the two flow types described in each of the foregoing pairs of types may exist simultaneously with either one from each of the other pairs in any combinations. That is, although the flow cannot be both uniform and non-uniform (one pair) at the same time, it can be simultaneously either uniform or non-uniform; and either steady or unsteady; and either laminar or turbulent and either subcritical or supercritical.

The various types of flow are illustrated in Fig. 2.

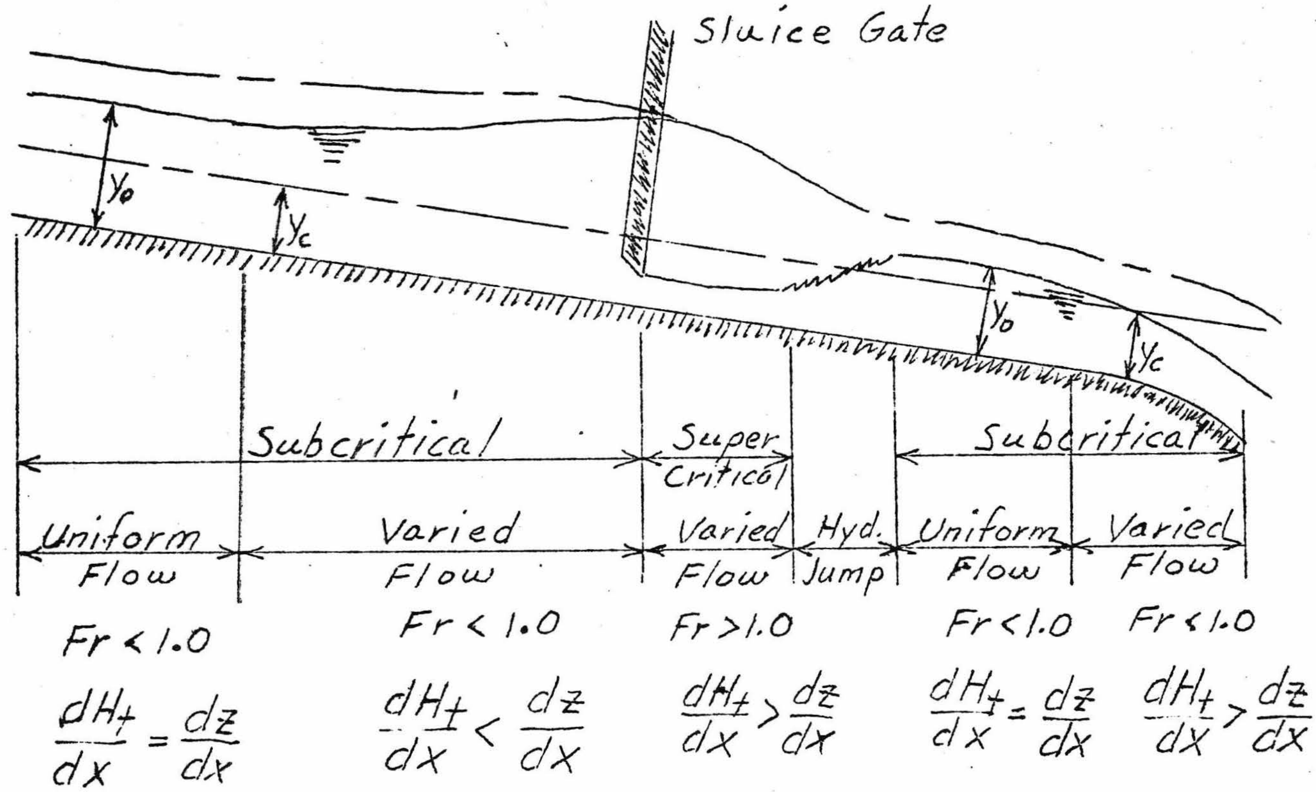


Figure 2

Various Types of Flow in Open Channels on a Mild Slope

<u>Laminar Flow</u>

As pointed out fluid motion may occur as laminar or turbulent flow. In laminar flow, each fluid element moves in a straight line with a uniform velocity. There is no diffusion between the layers or elements of flow, and accordingly no turbulence. The energy used in maintaining viscous flow is dissipated in the form of heat from the friction within the fluid.

With laminar flow, the shear stress being transmitted through each unit of depth varies uniformly from zero at the surface to a maximum at the stream bed while the velocity curve is parabolic in shape with its vertex at the surface as illustrated in Fig. 3.

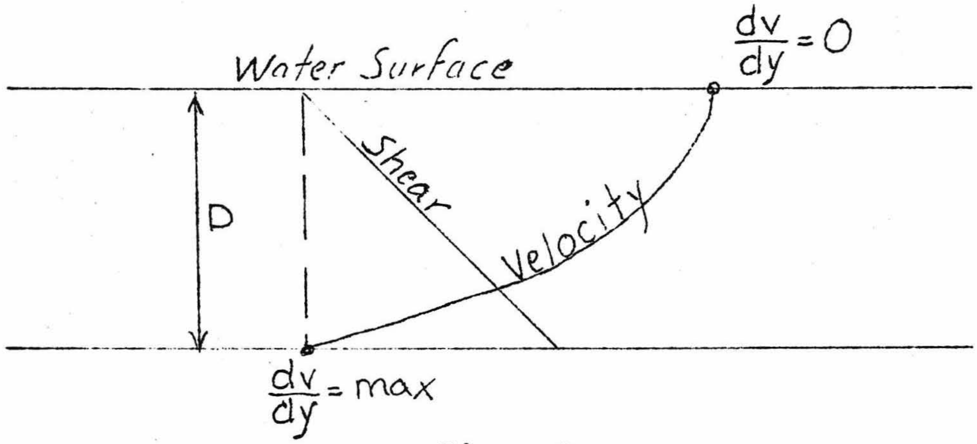


Figure 3

In stream flow, disturbances are present in such magnitude that laminar flow is rarely found. As velocity or depth increases, a given condition of laminar flow will reach a critical condition and become turbulent flow. Values of a flow parameter called Reynolds number Re can be used to predict the type of flow. This dimensionless number includes the effects of the flow characteristics, velocity, and depth, and the fluid properties density and viscosity.

$$Re = \frac{VD\rho}{\mu}$$

The ratio $\frac{\mu}{\rho}$ is a fluid property called the kinematic viscosity commonly designated ν (nu). Using this property

$$Re = \frac{VD}{\nu}$$

With the value of Reynolds number less than 2,000, laminar flow will prevail, whereas, with values in excess of 12,000 turbulent flow will prevail for smooth boundary conditions. For natural channels, the critical value will be near 2,000 due to bed roughness.

As suggested in the preceding paragraphs Re is defined as

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

That is, Re is an index of the relative importance of viscous forces in a hydraulic problem. Using Newton's second law of motion to define the inertial force and the expression $\tau = \mu \frac{dv}{dy}$ to define the viscous force

$$Re = \frac{\rho \frac{L^3 L}{T^2}}{\mu \frac{L\, L^2}{TL}}$$

substituting

$$V = \frac{L}{T} \quad \text{and } L = D$$

$$Re = \frac{\rho V^2 D^2}{\mu VD} = \frac{VD \rho}{\mu}$$

Many other dimensionless parameters which have the same form as the foregoing Reynolds number are utilized in the analysis of open channel flow problems such as:

$$\frac{wd}{\nu}$$

$$\frac{U_* d}{\nu}$$

where

w = fall velocity of sediment or bed material

d = median diameter of the sediment or bed material

U_* = shear velocity which is equal to $\sqrt{gDS}$

<u>Problem No. 3:</u>

A sheet of water 0.25′ deep is flowing over a smooth surface at 1.0 feet per second. Compute the Reynolds number (Re) of the flow. Will the flow likely be laminar or turbulent? (Kinematic viscosity ( ) = 1.21×10^{-5} feet²/second).

$$Re = (1.0)\ (0.25)\ (10^5) = \underline{\underline{20,700}} \quad \text{Turbulent}$$

Froude Number

The Froude number Fr is another dimensionless parameter frequently used to describe flow conditions. It is an index of gravity influence in flow situations where air or another fluid forms part of the boundary of the moving water — such as in an open channel. The Froude number is usually defined as

$$Fr = \left(\frac{\text{Inertia force}}{\text{Gravity force}}\right)^{1/2}$$

or

$$Fr = \frac{\text{Velocity of flow}}{\text{Velocity of a small wave in still water}}$$

Referring to the first definition

$$Fr = \left(\frac{\rho L^3 \frac{L}{T^2}}{\Delta\gamma\, L^3}\right)^{1/2}$$

Substituting

$$V = \frac{L}{T}$$

$$Fr = \left(\frac{\rho V^2 L^2}{\Delta\gamma L^3}\right)^{1/2} = \frac{V}{\sqrt{\frac{\Delta\gamma L}{\rho}}}$$

Where

L = a length dimension

$\Delta\gamma$ = difference in specific weight of the fluids — usually air and water

V = average velocity of flow

ρ = mass density of the fluid

In open channel flow $\Delta\gamma$ is essentially the same as γ for water alone since the density of air is so small. If D (depth) is used for the L dimension and γ/ρ replaced by its equivalent g, then the Froude number becomes:

$$Fr = \frac{V}{\sqrt{Dg}}$$

A Froude number of 1 indicates critical flow; less than 1 indicates the common variety of turbulent flow; and greater than 1 "rapid flow". In keeping with the second definition of Fr the $\sqrt{gD}$ term is the velocity at which a small wave travels in still water of depth D.

Problem No. 4:

Compute the Froude number of an open channel flow where the mean velocity is 5 ft/sec and the depth is 1.5 feet. Describe the flow with respect to critical flow.

$$Fr = \frac{V}{\sqrt{gD}}$$

$$Fr = \frac{5}{\sqrt{(32.2)(1.5)}} = \underline{\underline{0.73 \text{ Tranquil or subcritical}}}$$

Turbulent Flow

Turbulence, as a complicated pattern of eddies, produces small velocity fluctuations at random in all directions with an average time value of zero. Energy dissipation is high in turbulent flow due to the continuous interchange of finite masses of fluid between neighboring zones of flow. The resistance increases with approximately the square of the velocity.

Turbulent flow, as a result of this mixing and exchange of energy, has a more uniform distribution of velocity from top to bottom than laminar flow. The velocity distributions for laminar and turbulent flow are compared qualitatively in Fig. 4.

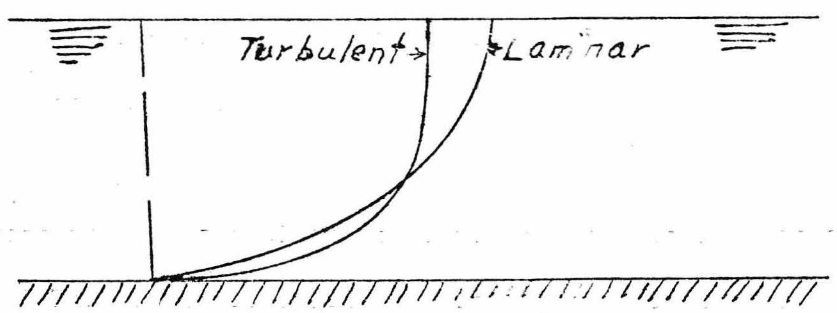


Fig. 4-Comparison of Velocity Distribution in Laminar and Turbulent Flow.

Several parameters have been developed to describe turbulent flow. Shear in turbulent flow is defined as $\tau = \eta \frac{dv}{dy}$, in which η (eta) is termed the eddy viscosity. A parameter used to describe the magnitude of turbulent velocity fluctuations is the root-mean-square ($\sqrt{V_1^2}$) of the deviations from the mean velocity. The mean size of the turbulent eddies is measured by the mixing length (l) - the distance through which fluid elements move before diffusing with the surrounding fluid. The diffusion coefficient, $\epsilon = l\sqrt{V_{,}^{3}}$, is a measure of the mixing process. The general pattern of variation of these parameters in turbulent flow is shown in Fig. 5.

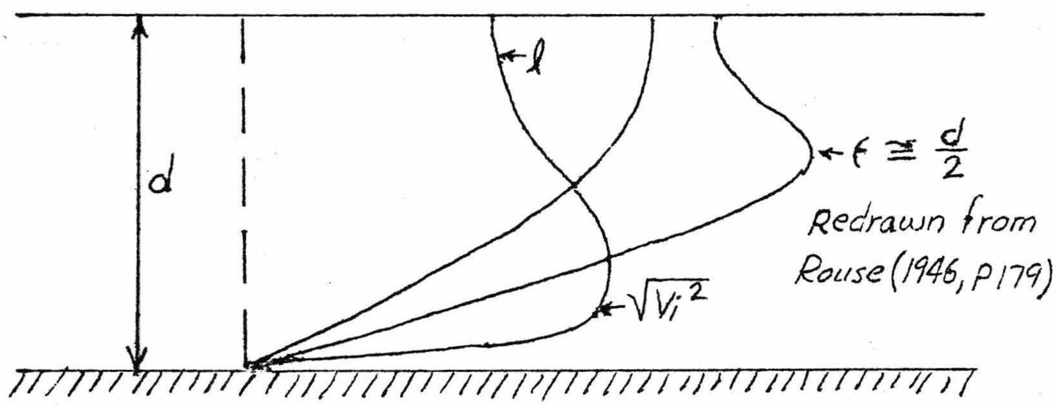


Fig. 5 - Velocity Distribution in Turbulent Flow

From a combination of experimental study and theory, the distribution of velocity in turbulent flow over smooth boundaries has been determined. A thin layer of laminar flow persists at the boundary surfaces. This layer is called the laminar sub-layer. The theoretical velocity curve is a composite of the logarithmic turbulent flow pattern and the nearly linear laminar pattern joined by a transition curve as shown in Fig. 6.

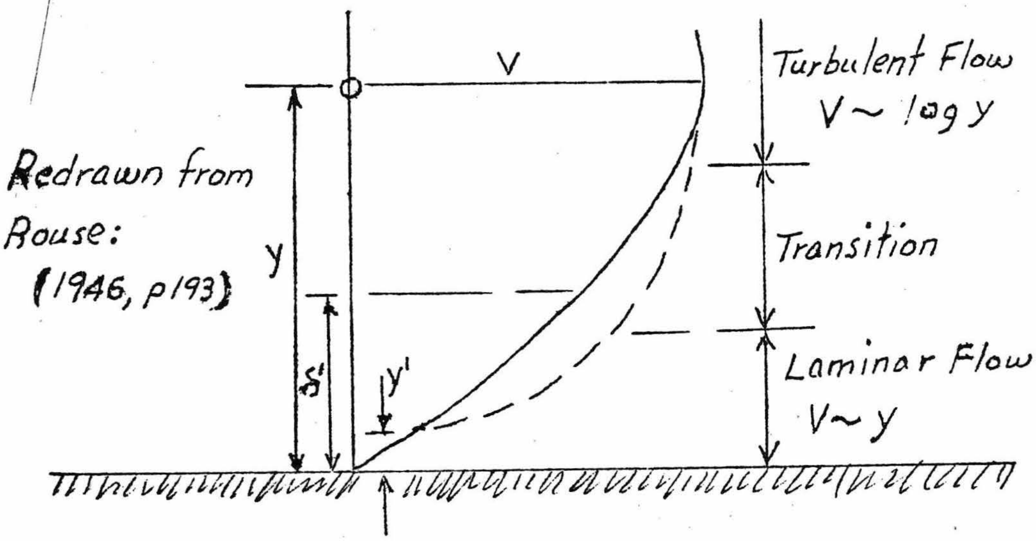


Figure 6

In the illustration δ' is the thickness of the laminar sub-layer and has been found to be defined by the equation $\delta' = \frac{11.6\,\nu}{\sqrt{\frac{\tau}{\rho}}} = \frac{11.6\,\nu}{u_*}$.

If extended downward toward the bed, the logarithmic form of the turbulent velocity distribution will yield zero velocity at a distance y' above the bed. Experiments show that $y' = \delta'/107$.

The term $\sqrt{\frac{\tau}{\rho}}$ is a common parameter called the shear velocity (u_*), and is equal to $\sqrt{RSg}$.

The Karman-Prandtl equation describes the velocity distribution of turbulent flow over a smooth bed as

$$\frac{v}{u_*} = 5.75 \log_{10} \frac{u_* y}{\nu} + 5.5 \text{ for the turbulent zone, and}$$

$$\frac{v}{u_*} = \frac{u_* y}{\nu} \text{ for the laminar zone.}$$

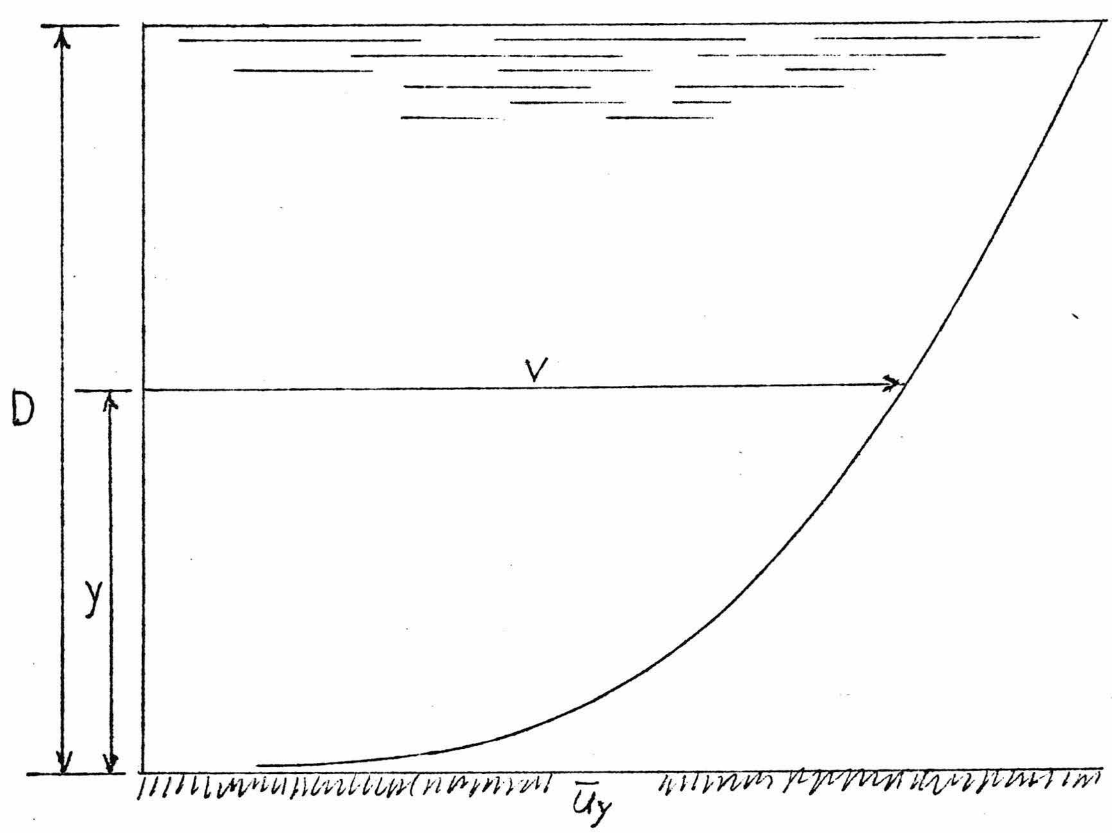
The two equations are presumed to be joined by a smooth transition curve at the distance δ' above the bed.

<u>Velocity Distribution over Rough Beds</u>

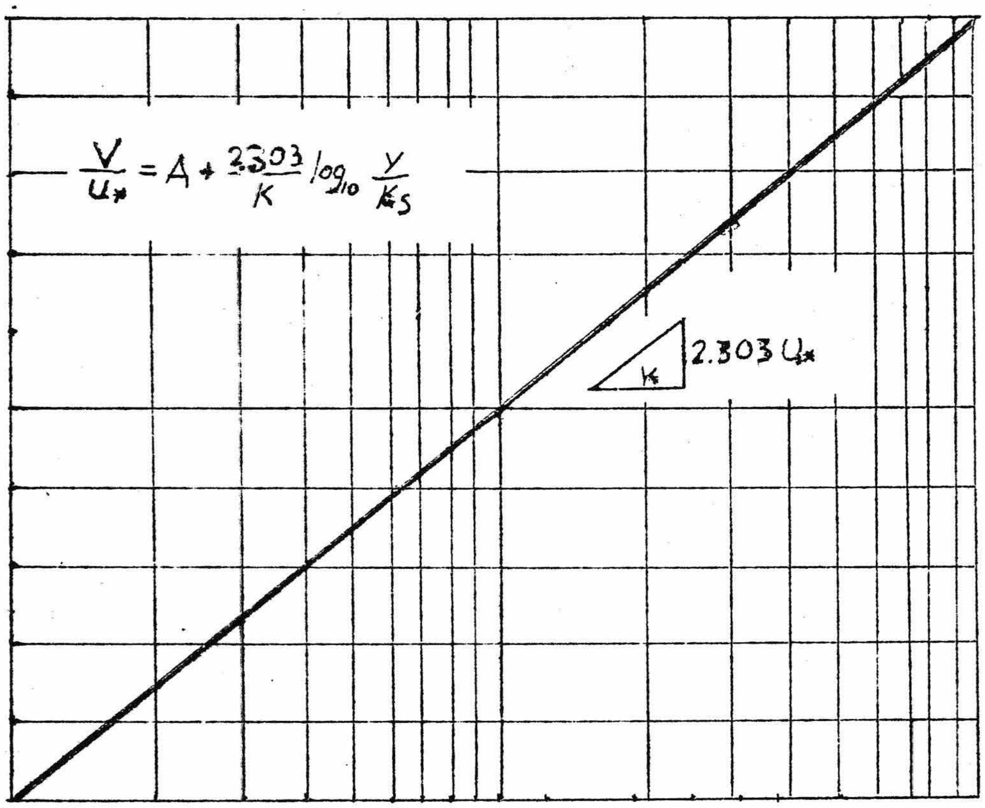
Stream channels have rough beds. The roughness is expressed in terms of K_s which is equivalent to the diameter of the sediment grains which compose the bed. The dimension of K_s is larger than that of δ', and therefore the sub-layer ceases to exist for practical purposes. Turbulent flow is assumed to occur throughout the depth. The Karman-Prandtl velocity equation for rough beds is:

$$\frac{v}{u_*} = 5.75 \log_{10} \frac{y}{K_s} + 8.5$$

The distribution of velocity in accordance with this equation is illustrated in Fig. 7.



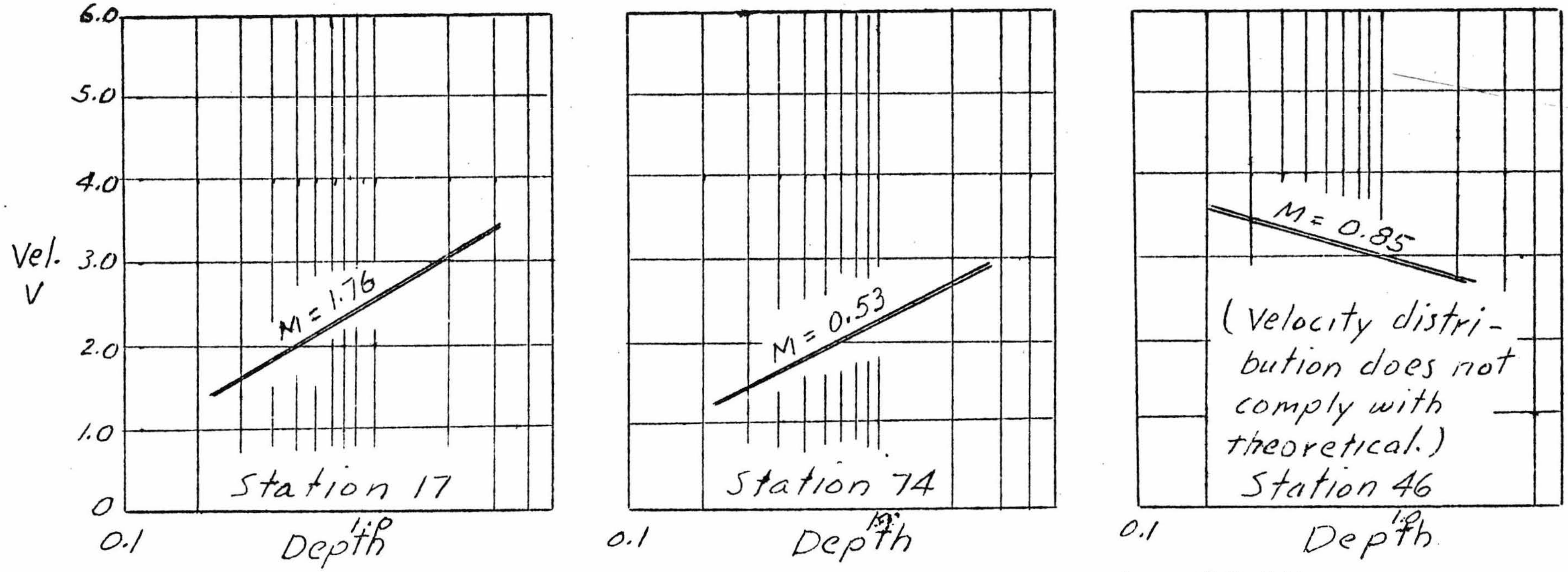
(a)



(b)

Fig. 7-Theoretical Distribution of Velocity in a Vertical Section.

To illustrate that the actual velocity distribution does not always comply with the theoretical, refer to the following actual velocity distribution curves as observed in the Middle Loup River at Denning, Nebraska, see Fig. 8. The letter M signifies the slope of the semi-log plot of V versus D. In this instance the kappa values K are reasonably close to the theoretical value 0.4.



Station	$K = \frac{2.303 U_*}{M}$
17	0.39
74	0.46
46	—

Water discharge 374 cfs
Manning n 0.030
Undulating bed with Large Dunes

Fig. 8. - Vertical distribution of velocity at Section C_2, Middle Loup River at Dunning, Nebraska, October 23, 1953.

Tractive Force

The resistance to flow in open channels involves the same principles as the resistance to flow in pipes, or closed conduits flowing full. Therefore, a free body of a segment of the full width of the channel may be selected for study as shown in Fig. 9.

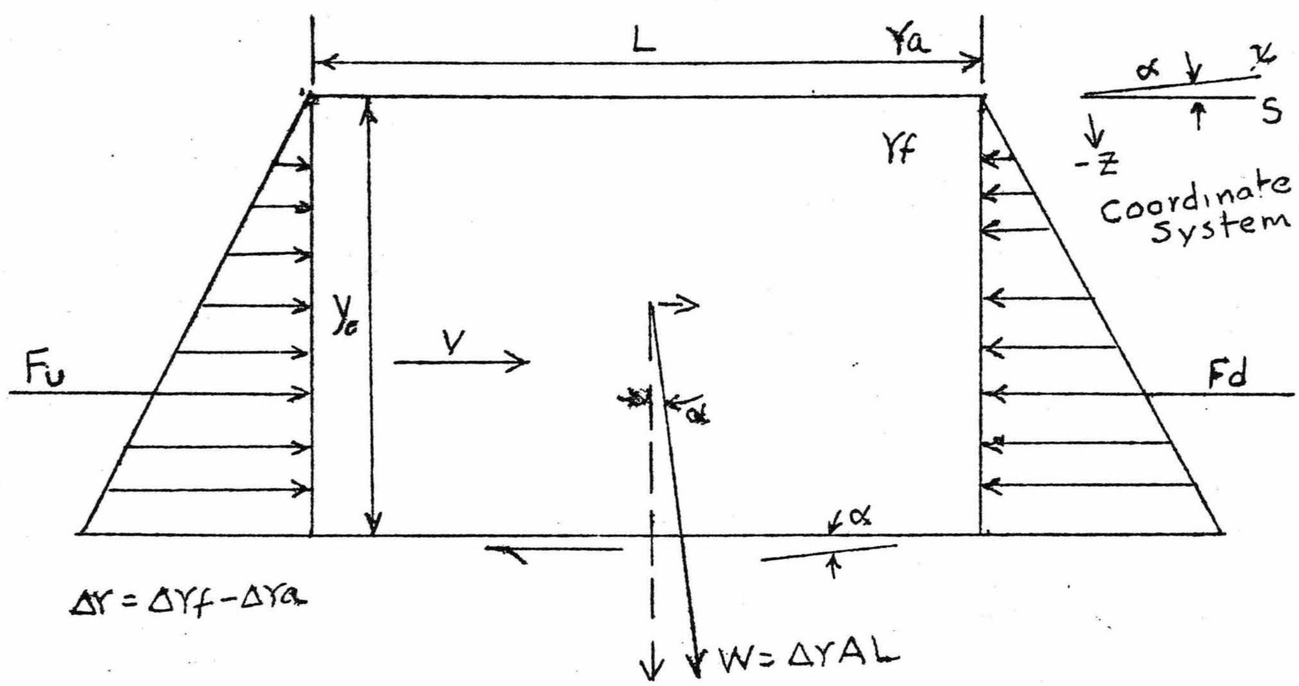


Fig. 9 - Free-body Diagram of Segment of Open Channel Flow

Equating the forces in the s-direction yields

$$F_u + W \sin\alpha = F_d + \tau_o \, wp \, L$$

where:

W is the weight of the entire segment of fluid

wp is the wetted perimeter — that is, the length of the cross sectional boundary which is in contact with the fluid flowing in the channel.

F_u and F_d are the upstream and downstream hydrostatic forces acting on the free body. Since the flow is uniform $F_u = F_d$

τ_o is the average boundary shear which is retarding the flow

A is the cross sectional area of the flow

L is the length of the free body segment

α is the angle which the channel slope makes with the horizontal, and

$\Delta\gamma$ is the difference between the specific weight of the fluid flowing and the specific weight of the ambient fluid, normally the air. f a

The product $\Delta\gamma AL$ may be substituted for the weight W and the equation rearranged to solve for the boundary shear

$$\tau_o = \Delta\gamma \frac{A}{wp} \sin\alpha = \Delta\gamma RS \tag{1}$$

where

R is called the hydraulic radius which is the area A divided by the wetted perimeter wp and

S is $\sin\alpha = dz/ds$ which for relatively flat slopes may be considered more conveniently as $\tan\alpha = dz/dx$.

Eq 1, it may be noted, evaluates the boundary shear in terms of the static characteristics of the geometry and the fluid.

It now remains to evaluate the shear τ_o in terms of the dynamic variables of resistance, viscosity and velocity.

$$\tau_o = f\rho\frac{V^2}{8} \tag{2}$$

which may be applied equally well to flow in open channels if the Darcy-Weisbach resistance coefficient f is evaluated properly for the different flow characteristics and the different shapes found in open channels.

Equating Eq 1 to Eq 2 and rearranging gives

$$V = \sqrt{\frac{8 \Delta \gamma}{f \rho}} \sqrt{RS}$$

or

$$V = C \sqrt{RS} \tag{3}$$

where C is the Chezy discharge coefficient.

<u>The Manning equation</u> -- In an effort to correlate and systematize existing data from natural and artificial channels, Manning in 1889 proposed an equation

$$V = \frac{1.5}{n} R^{2/3} S^{1/2} \tag{4}$$

or

$$Q = AV = A \frac{1.5}{n} R^{2/3} S^{1/2}$$

where n is the Manning roughness coefficient which has the dimensions of $L^{1/6}$. By comparing Eq 3 with Eq 4 it may be seen that the Chezy discharge coefficient C

$$C = 1.5 \frac{R^{1/6}}{n}$$

is related to the Manning coefficient n and the hydraulic radius R. The Manning n was developed empirically as a coefficient which remained a constant for a given boundary condition regardless of slope of channel, size of channel, or depth of flow. As a matter of fact, however, each of these factors causes n to vary to some extent. In other words, the Reynolds number, the shape of the channel, and the relative roughness have an influence on the magnitude of Mannings n. Furthermore, for a given alluvial bed of an open channel, the size, pattern, and spacing of the dunes varies with slope, discharge, and cross sectional shape of an open channel so that n also varies. Despite the shortcomings of the Manning roughness coefficient, however, it has an amazing utility and is used extensively in Europe, India, Egypt, and the United States.

Several other equations for turbulent flow in open channels with rough rigid boundaries have been developed.

(1) Keulegan (1938): $C/\sqrt{g} = 5.75 \log_{10} R/k + 6.25$

where: C = Chezy coefficient
g = gravity constant
k = roughness term equal to diameter of roughness particles on boundary

(2) Powell (1951): $C/\sqrt{g} = 7.4 \log_{10} R/\epsilon$

where: ϵ = roughness term computed from texture of boundary material and shape of channel

(3) Robinson and Albertson (1952): $C/\sqrt{g} = 5.75 (\log_{10} D/a + 0.5$

where: D = depth of flow
a = height of roughness baffles

These equations are suitable for channels where roughness is relatively constant. No suitable equations have been developed for alluvial channels where form roughness (bed configuration) is influenced by several parameters. Leopold and Maddock (1953), Brooks (1955), and others have explored the problem. The current division research project at Colorado State University is directed to this problem.

Problem No. 5:

A wide stream flowing over a gravel bed has a depth of 1 foot and a slope of .001. The bed gravel has a median diameter of 0.1 foot. Assume the depth is equal to the hydraulic radius.

(A) Compute the velocity of flow according to Keulegans equation. Recall that $\frac{C}{\sqrt{g}} = \frac{V}{\sqrt{grs}} = \frac{V}{U_*}$.

(B) Compute the value of n that would apply in Mannings equation for this stream.

$$A - \frac{C}{\sqrt{g}} = \frac{V}{\sqrt{grs}} = 5.75 \log \frac{R}{K} + 6.25$$

$$\frac{C}{\sqrt{g}} = 5.75 \log \frac{1}{0.1} + 6.25 = 12.0$$

$$C = \underline{\underline{68.0}}$$

$$B - C = \frac{1.50\, R^{1/6}}{n} \quad \text{from which} \quad n = \frac{1.50\, R^{1/6}}{C}$$

$$n = \frac{(1.50)\ (1)}{68} = \underline{\underline{0.022}}$$

Classes of Roughness

Einstein (1942) proposed that a hydraulic radius be computed for each of three systems of roughness, bank roughness, grain roughness, and form or dune roughness, and thereby enable the use of separate equations for each system. On most streams, the bank effect is small.

The mechanism of dune resistance is poorly understood - the subject for much future research. Observations show that turbulence occurs in the wake of and downstream from the dune crest and thereby, according to Einstein, does not affect the movement of bed load appreciably. It does, however, affect the flow parameters significantly and thus, indirectly, the rate of bed-material movement. Turbulence generated in the immediate vicinity of the particles in the bed has the principal effect on bed load motion - a function of the bed material particle size.

Dissipation of Energy

The potential energy available for inducing flow along a channel is "consumed" or dissipated by internal friction induced by resistance of the channel boundaries.

Backhmeteff (1946) shows that the flow energy is drawn principally from the central portion of the stream, is transmitted by turbulent processes to zone near the channel boundaries, where 80-90 percent of the total flow energy is converted into turbulent energy. The converted flow energy of laminar flow and much of the converted flow energy of turbulent flow appears as molecular turbulence which becomes molecular heat in its place of conversion. This turbulent energy does not influence sediment suspension. A smaller part of turbulent energy, perhaps as little as 5 percent, emerges as vortices in which finite masses of water move randomly across flow lines and influence sediment suspension. The vortices become diffused soon after formation and decay into molecular turbulence and thence heat. Thus, essentially all consumed energy is converted into heat.

PROPERTIES OF SEDIMENT

The way in which sediment particles react to fluid forces and movement is affected by three properties of the particles - size, density, or specific gravity, and shape. Size is the more important of the three. The method of classifying particles for size by measuring their terminal settling velocity when falling in still water also integrates into the size analyses the smaller effect of the two properties. The fall velocity or size based on fall velocity is an adequate description of the particle in sediment transport equations. The measurement of size by sieves is convenient particularly for coarse sediment but is less useful because it reflects shape only in part and density not at all.

Size Classes

Size classes are essentially arbitrary but necessary in statistical and mathematical computations of sediment. An extension of the Wentworth grade scale was proposed by the Subcommittee on Sediment Terminology of the American Geophysical Union (Lane 1947) and has been used by the Division. The separation sizes of the scale are arranged in a geometric series with a ratio of two.

Sediment Grade Scale

Class Group	Size range mm
Boulders (4 size classes)	4096 - 256
Cobbles (2 " ")	256 - 64
Gravel (5 " ")	64 - 2
Sand (5 " ")	2 - 0.062
Silt (4 " ")	0.062- 0.004
Clay (4 " ")	0.004- 0.00024

Most of our interest and effort is directed to sediment in the clay, silt, and sand sizes. The great range in fall velocity emphasizes the importance of size to the behavior of the particle in a stream. The following table lists the approximate fall velocity of the indicated spherical particles in still water at 20° C.

Diameter of particle mm	Fall Velocity cm/sec
0.001	0.0001
0.01	0.001
0.1	1.0
1.0	16
10.0	74

Fall Velocity

For small sediment particles, the rate of fall is regulated principally by the viscous forces of the fluid. The equation by Stokes applies to the fall velocity of spherical particles smaller than approximately 0.062 mm.

$$w = \frac{gd^2\,(S_s - S_f)}{18\,\nu}$$

w = fall velocity

d = particle diameter

S_s = specific gravity of particle

S_f = specific gravity of fluid

g = gravity constant

ν = kinematic viscosity of fluid

Gravity forces become increasingly more important for particles larger than 0.062 mm and Stokes law is no longer adequate. For the larger particles, the fall velocity can be derived from the general equation

$$w^2 = \frac{4gd\,(S_s - S_f)}{3C_D}$$

where C_D is the drag coefficient of the particle that must be determined experimentally.

Problem No. 6:

Compute the terminal fall velocity of a quartz sphere with a diameter of 0.01 mm (specific gravity 2.65) in still water at 70° F. (Viscosity $\nu = 1.059 \times 10^{-5}$ lb-sec/ft²).

$$w = \frac{g\,d^2\,(S_s - S_f)}{18\,\nu}$$

$$d = \frac{0.01}{(10)(2.54)(12)} = 0.0000328 \text{ ft}$$

$$V_s = \frac{(32.2)(0.0000328)^2(1.65)(10^5)}{(18)(1.059)} = \underline{\underline{0.0003 \text{ ft/sec}}}$$

THE THEORY OF SEDIMENT SUSPENSION

In order to understand suspended sediment transport, it is helpful also to understand the mechanics of turbulent flow. Turbulence makes it possible for streams to carry non-colloidal sediment in suspension and for the particles to be transported from one region to another. The purpose of this section is to present theoretical and dimensional considerations for turbulent flow, turbulent transfer of sediment, and sediment distribution in the vertical.

Introduction

For the purpose of this presentation, let turbulent flow be understood to be flow in which any velocity component at a point fluctuates with respect to time. Let u, v, and w equal the instantaneous velocity components in the directions of x, y, and z.

Let U, V, and W equal the corresponding mean velocity and u', v', and w' equal the corresponding velocity fluctuation at any instant.

For a study of steady, uniform flow in wide channels, let x be taken along the axis of the channel, y be taken vertically, and z be taken across the channel. V and W will be comparatively small and may be neglected. However, v' and w' do not vanish, but cause secondary movements which give rise to mixing of the fluid. In the derivation of the equation for flow in an infinitely wide channel carrying sediment the assumption is made that there is little variation in the conditions of flow with respect to the lateral or longitudinal directions, but that in the vertical direction, due to the presence of a gravitational field, there is a variation of sediment concentration with respect to depth.

The momentum transfer in turbulent flow is characterized by the equation

$$\frac{dv}{dy} = \frac{1}{ky}\sqrt{\frac{\tau_o}{\rho}}$$

wherein the Karman coefficient k has been found to be approximately equal to 0.4 for certain conditions. However, k is an indication of mixing and if any disturbing factor, other than the usual bottom drag, enters into the flow to increase or decrease the mixing, k might be expected to vary.

It has been found that an increase in the load of suspended sediment will decrease k. As the sediment load increases, it will take an increasing amount of energy to keep the sediment in suspension. Thus the turbulence will not extend upward into the flow as far as in clear water and the velocity distribution will have a greater variation with depth and k will be smaller than with clear water flow.

Side effects entering into the flow set up a different pattern of mixing than in the two-dimensional system. Normally, these side effects will increase the mixing through secondary circulation. Secondary circulation seems to be in the form of reverse spiral flow. The strength of the spiral flow and the number of pairs or cells in the channel will depend upon the width, the depth, the bed roughness, the side roughness, and the mean velocity of the flow. The cause of the secondary circulation is not entirely known .

Turbulent Transfer of Sediment

Consider the condition in which sediment, with a specific gravity greater than that of water, is in the flow. Then, there will be more sediment in the lower part of the channel cross section than in the upper part. Let v' flow upward through a unit area. Because of continuity, there will be downward flow equal to the upward flow. The upward fluctuation will carry more sediment upward than the downward fluctuations will carry downward, because the concentration of sediment is greater in the lower part of the channel.

Therefore, the net transfer of sediment by the flow is upward. Under conditions of equilibrium the downward transfer of sediment due to gravity is equal to the net upward transfer by the flow.

The net upward transfer of sediment varies with the gradient of the curve of mean concentration and may be expressed as

$$\epsilon_s \frac{dc}{dy}$$

where ϵ_s is the sediment transfer coefficient and c is the sediment concentration. For equilibrium, the upward transfer must be balanced by the settling of sediment for which the volume settling through a unit area is $w \times c \times 1$ where w is the fall velocity of the sediment. The equation for sediment transfer is, therefore,

$$wc + \epsilon_s \frac{dc}{dy} = 0 \tag{5}$$

Similar reasoning may be applied to the transfer of momentum, except that the slope of the velocity curve is positive, and the net transfer of momentum is downward. The effect of this transfer of momentum is a forward tangential stress, called turbulent shear, acting on the fluid directly below. The equation for shear in turbulent flow may be written

$$\tau = \epsilon_m \frac{dv}{dy}$$

where ϵ_m is the momentum transfer coefficient.

Sediment Distribution in a Vertical

Let an element of flow in equilibrium be considered. Therefore,

$$\frac{d\tau}{dy} = -\gamma S$$

where S is the slope of the uniform flow and is the specific weight of the water-sediment mixture. Upon integration

$$\tau = -\gamma S y$$

Denoting the distance in the vertical as depth D minus the vertical distance y from the bottom

$$\tau = \gamma S(D-y)$$

For the bottom shear

$$\tau_o = \gamma DS$$

and

$$\tau = \tau_o \left(\frac{D-y}{D}\right)$$

From

$$\tau = \rho \epsilon_m \frac{dv}{dy}$$

$$\epsilon_m = \frac{\frac{\tau}{\rho}}{\frac{dv}{dy}}$$

According to Karman, in turbulent flow

$$\frac{dv}{dy} = \frac{1}{ky} \sqrt{\frac{\tau_o}{\rho}}$$

Substituting gives

$$\epsilon_m = \frac{\frac{\tau_o}{\rho} \frac{D-y}{D}}{\frac{1}{ky} \sqrt{\frac{\tau_o}{\rho}}} = k \sqrt{\frac{\tau_o}{\rho}} \left(\frac{D-y}{D}\right) y$$

Assuming

$$\epsilon_s = \epsilon_m$$

Eq 5 gives

$$wc + k \sqrt{\frac{\tau_o}{\rho}} \left(\frac{D-y}{D}\right) y \frac{dc}{dy} = 0$$

Separating variables

$$\int_{ca}^{c} \frac{dc}{c} = - \frac{wD}{k\sqrt{\frac{\tau_o}{\rho}}} \int_{a}^{y} \frac{dy}{y(D-y)}$$

Which leads to

$$\frac{c}{ca} = \left(\frac{D-y}{y}\right) \left(\frac{a}{D-a}\right)^{Z} \qquad (6)$$

Where

$$Z = \frac{w}{k\sqrt{\frac{\tau_o}{\rho}}} = \frac{w}{k\sqrt{gDS}} \qquad (7)$$

D = depth of flow

C = the concentration of particles of a given size at distance y above the bed

c_a = the concentration of particles of a given size at distance a above the bed

w = settling velocity of the particles

k = is Karman's universal constant

Eq 6 enables one to compute the sediment concentration at any point in the vertical at a distance y above the bed, when the concentration ca at a distance a above the bed is known.

The exponent Z can be determined by two methods, (1) from Eq 7 designated by Z, and (2) from the slope of the log plot of concentration versus depth designated by Z_1.

The ratio of the exponent Z_1 to the exponent Z is designated β.

Several assumptions are made in the derivation of Eq 6. These assumptions and their possible effects are discussed below.

1. The flow is steady, uniform flow. This assumption would probably have its greatest effect on the determination of the slope.
2. The density ρ and the specific weight γ are independent of y. The validity of this assumption will differ with the amounts of sediment in the flow. However, for most practical purposes its variation with depth should be insignificant. This statement does not apply to the unmeasured load where ρ and γ may vary considerably with depth.
3. The velocity is proportional to the log of the depth. This assumption should not introduce any appreciable error. The effective elevation of the channel bottom, however, may need to be adjusted.
4. The sediment transfer coefficient ϵ_s is equal to the momentum transfer coefficient ϵ_m. This assumption is probably the most important one made in the derivation. There is good evidence, to show that ϵ_s does not equal ϵ_m. It is thought to vary with sediment size, roughness, and secondary circulation. The sediment transfer coefficient can be either greater than or less than the momentum transfer coefficient.

The distribution of coarse sediments in streams does not follow Eq 6. Colby and Hembree (1955) have found Z to vary with about the 0.7 power of w. Einstein and Chein (1954) have developed a "second approximation" of Z which agrees better with stream measurements.

The Von Karman constant (K) enters into equations of velocity and sediment distribution. For clear water in flumes and pipes a consistent value of 0.4 is computed. For natural sediment laden streams the value of K varies appreciably from 0.4. Benedict (1957) has shown that where prominent dunes are present on the bed, values of K have been found to vary from 0.39 to 2.59. It appears that the large scale turbulence or "Kolk action" associated with dunes may have a marked effect on vertical distribution of velocity and sediment.

The modified Einstein procedure is limited, as are the open channel flow equations for alluvial streams, by lack of understanding of factors that control roughness. It appears adequate for computation of total sediment movement under conditions existing at the time of data collection but cannot explain satisfactorily why the observed conditions occurred as they were found.

Some typical data illustrating sediment distribution is shown in Fig. 10.

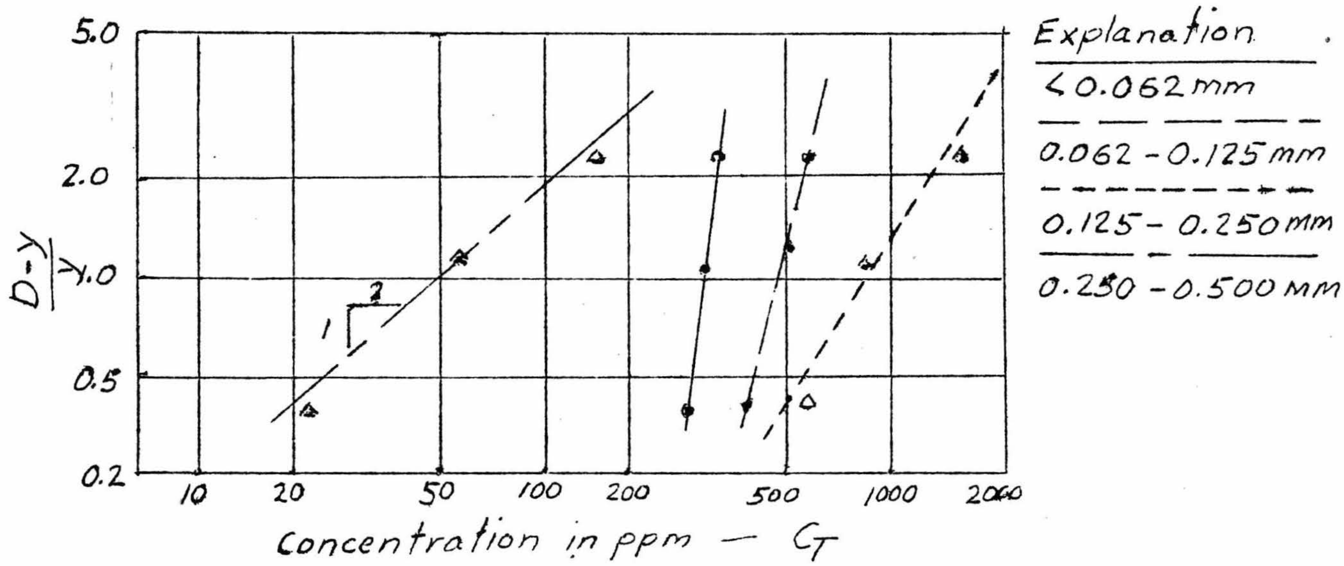


Figure 10

Several properties of sediment particles influence their behavior in moving fluids. These properties have already been discussed. It has been found that the influence of the several properties can be combined into the one parameter of fall velocity (V_s) in equations of sediment suspension.

Some typical concentration curves showing the relative influence of turbulence, particle size, and viscosity on vertical distribution of sediment are shown in Fig. 11.

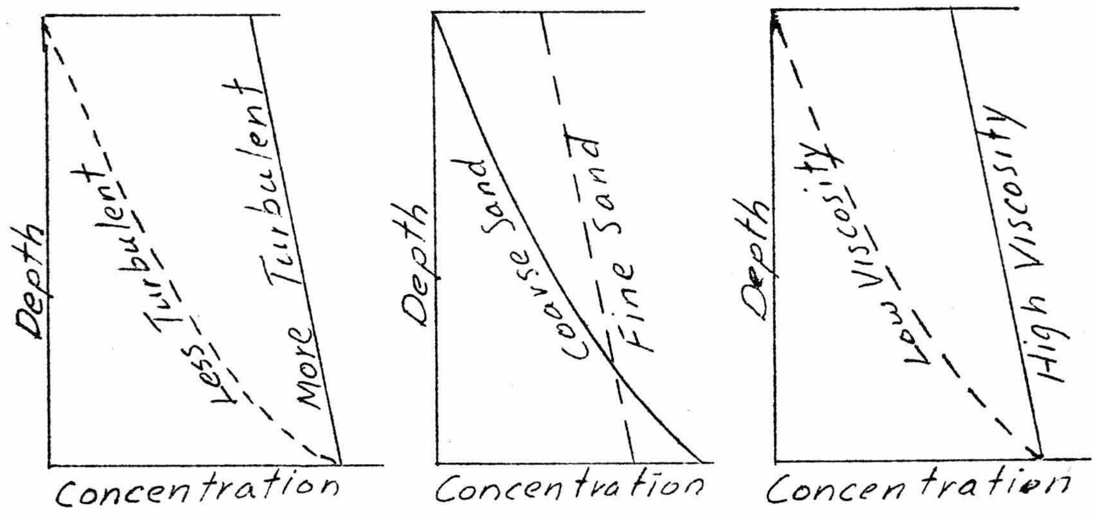


Figure 11

TRANSPORT OF SEDIMENT

Einstein (1950) has classified the total sediment load of a stream into fine material or wash load (consisting of particles finer than those in the bed material) and bed-material load (particles found abundantly in the bed). Different factors in part limit the rate of transport for each class.

Silts and Clays

The particles of wash load can be moved by small fluid forces and therefore are nearly continuously in suspension. The quantity of such material in a stream at any time depends on the rate at which these fine particles become available from the upland areas and not on the ability of the flow to transport them. Controlling factors include rainfall intensity, quantity and distribution, soil type, land use, vegetal cover, relief, channel density, and other factors.

Bed-Material Load

It is assumed that the transport rate for sediment present in unlimited amounts in a stream bed will equal the transporting capacity of the stream. It is possible then to compute the movement of this material from the characteristics of flow, channel, and bed material.

The vertical distribution equation for suspended sediment breaks down near the bed because of the interference with the mixing length of the turbulent eddies. Einstein assumes that particles near the bed roll and slide along in a layer with a thickness of two particle diameters. The concentration at the top of this layer is C_a in the vertical distribution equation and joins the suspended load concentration with that moving on the bed. The total sediment load is the sum of the wash load, the suspended bed-material load, and the bed layer movement.

Total Load Methods

The method developed by Dr. Einstein (1950) is the only one of many that clearly attempts to compute material in suspension as well as that moving on and near the bed. His original method does not compute the wash load - fine suspended material not present in the bed. The modified Einstein method (Colby and Hembree, 1955) is designed to compute all of the sediment moving through the streams section including the wash load.

A recent method by Dr. Bagnold (1956) is being studied intensively in the Division. It is similar in some respects to the Einstein approach and brings sound physical concepts and the great experience of Dr. Bagnold with wind transport of sands to the problem. Its full impact on sediment methodology must await current testing of the concept in streams.

Bed Load Equations

Numerous other equations have been developed in this country and in Europe to compute the movement of sediments in streams. It is generally not clear whether or not the equations are intended to compute suspended sediments as well as bed load. The Dubois, Straub, and Schoklitsch equations have wide use (Colby and Hembree, 1955, pp. 57-60). The Schoklitsch equation reads

$$G = \frac{86.7}{d_{50}0.5} Se^{1.5} \left(Q - 0.00532 \frac{wD_{50}}{Se^{1.33}}\right)$$

where:

- G = discharge of bed material, in pounds per second
- d_{50} = median diameter of the particles, in inches
- Se = slope of the energy gradient
- W = width of stream in feet
- Q = water discharge in cfs

The other equations are similar in form but differ in detail. These equations do not appear reliable for streams that have a significant percentage of suspended load. They are believed to be reliable for computing the transport of coarse sediments - gravel and boulders.

SEDIMENT DISTRIBUTION IN A STREAM SECTION

The vertical distribution of sediment has been described above. Figs. 12 and 13 illustrate the vertical distribution of several size classes in the Mississippi River at St. Louis and the Niobrara River near Cody, Nebraska.

The transverse distribution of sediment on most deep streams is fairly uniform with the exception of those having irregular cross sections and those located just downstream from important tributaries. Lateral mixing forces operate in the same manner as vertical ones - gravity does not oppose the process. However, varying flow patterns across the sections maintain different vertical distribution patterns. The distribution then results from an equilibrium between distinctive vertical distributions and the mixing forces of lateral turbulence. Wide streams with coarse sediments have large lateral variations while narrow streams with fine sediments have only small lateral variations. See Fig. 14.

Sediment Distribution With Time

Turbulence causes nearly instantaneous fluctuations while watershed, streamflow, or rainfall characteristics cause more gradual concentration variations. Fig. 15 illustrates typical variations

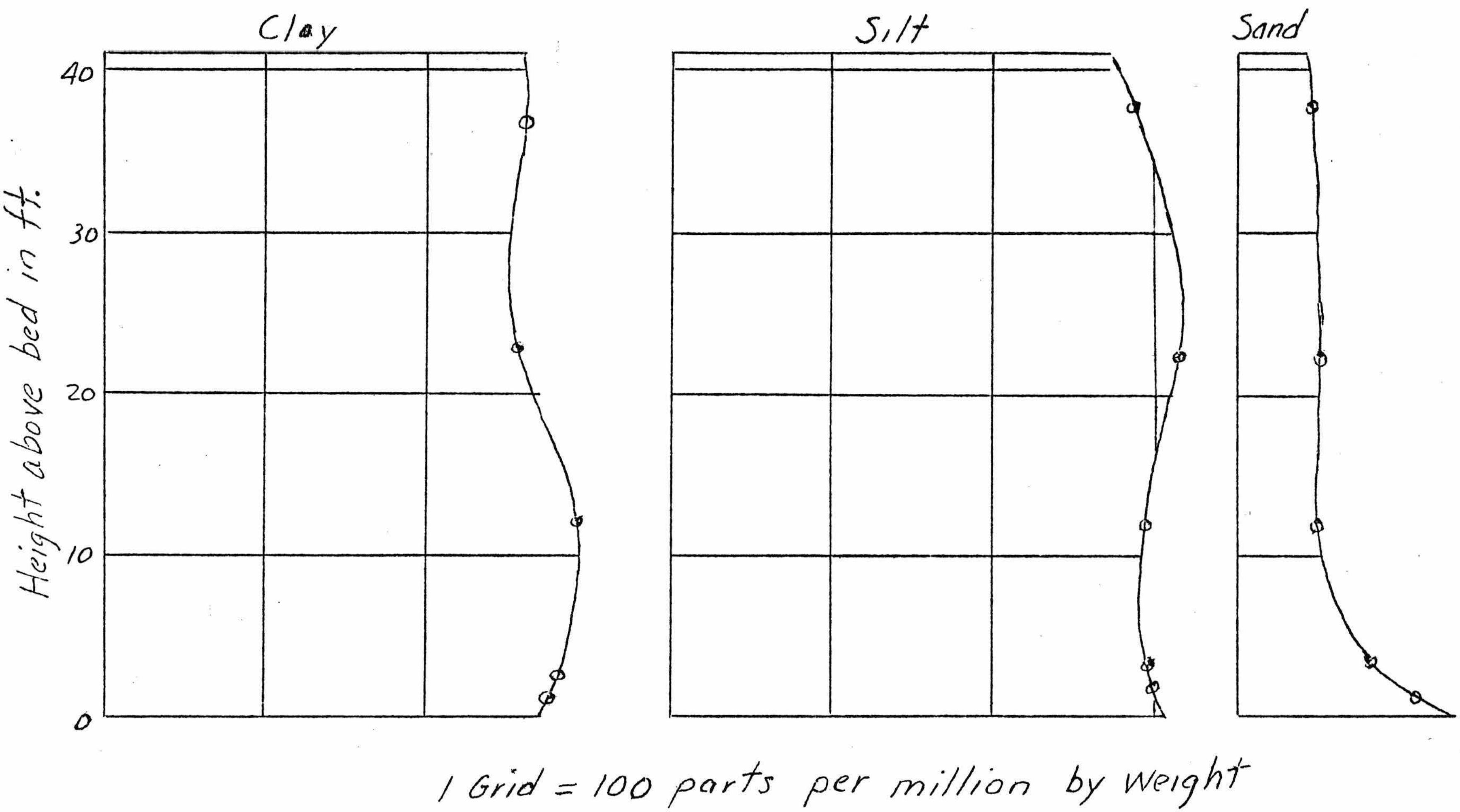


Fig. 12. Particle size of suspended sediment with depth for 3 verticals in the Mississippi River at St. Louis, Mo. on July 20, 1950. (Discharge 281,000 cfs, size analysis in distilled water)

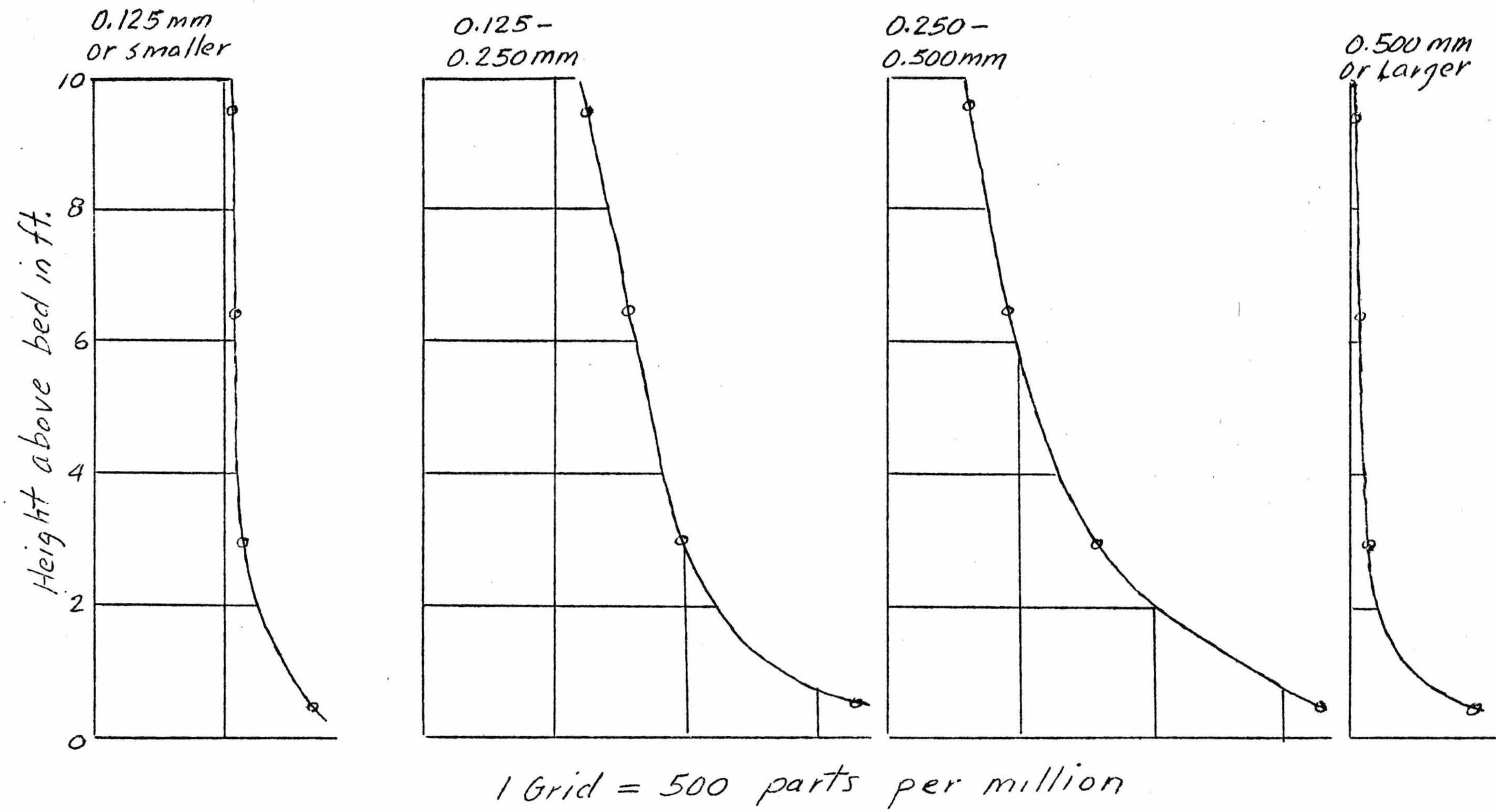
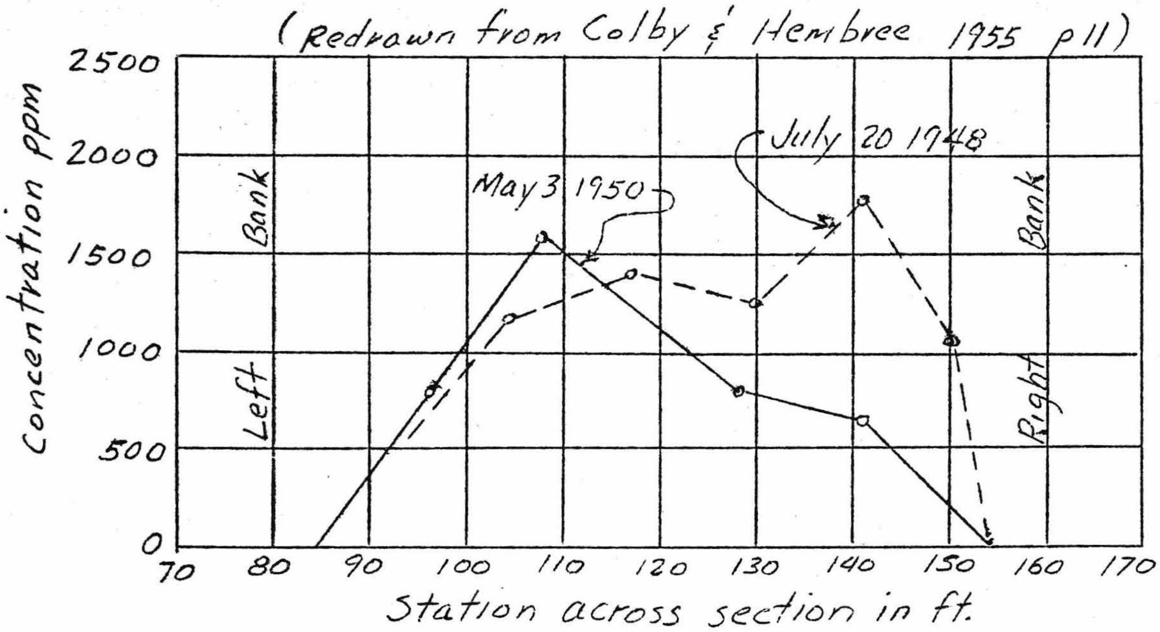
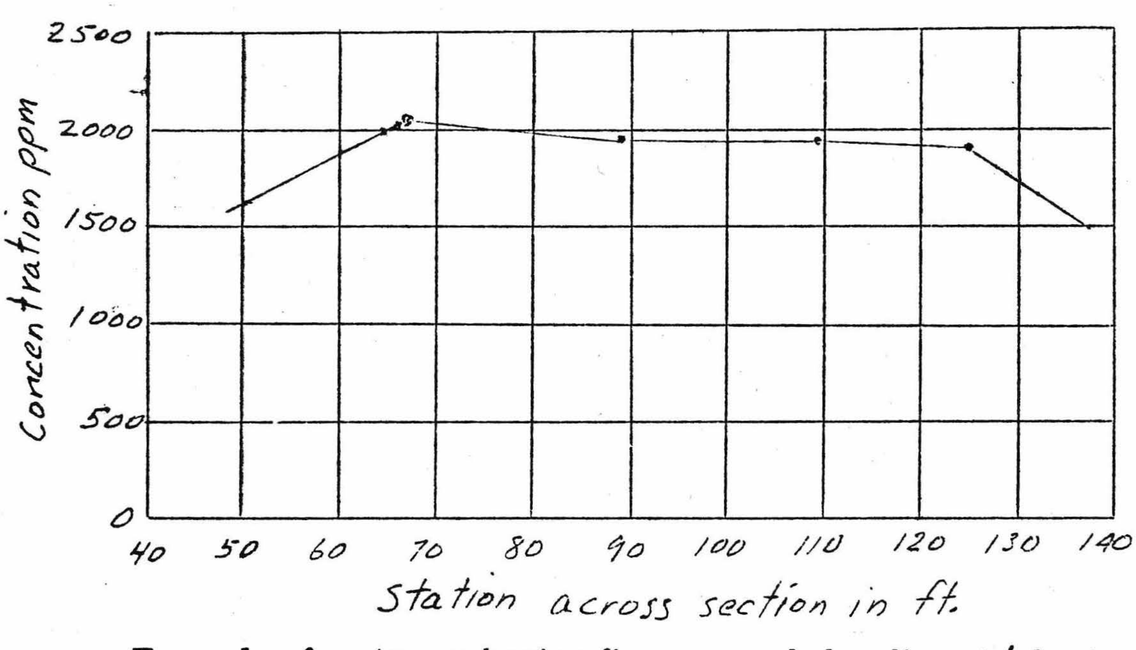


Fig. 13. -- Particle size of suspended sediment with depth at a single vertical on the Niobrara River near Cody, Nebraska on April 27, 1951, at the contracted section.



Niobrara River near Cody, Nebr. (gaging-station section) having coarse suspended sediment (40 to 60 per cent coarser than 0.125 mm.)



Example of a stream having fine suspended sediment (about 2 per cent coarser than 0.062 mm.)

Fig. 14.-- Lateral distribution of suspended sediment in typical channel cross sections having relatively coarse and fine sediments.

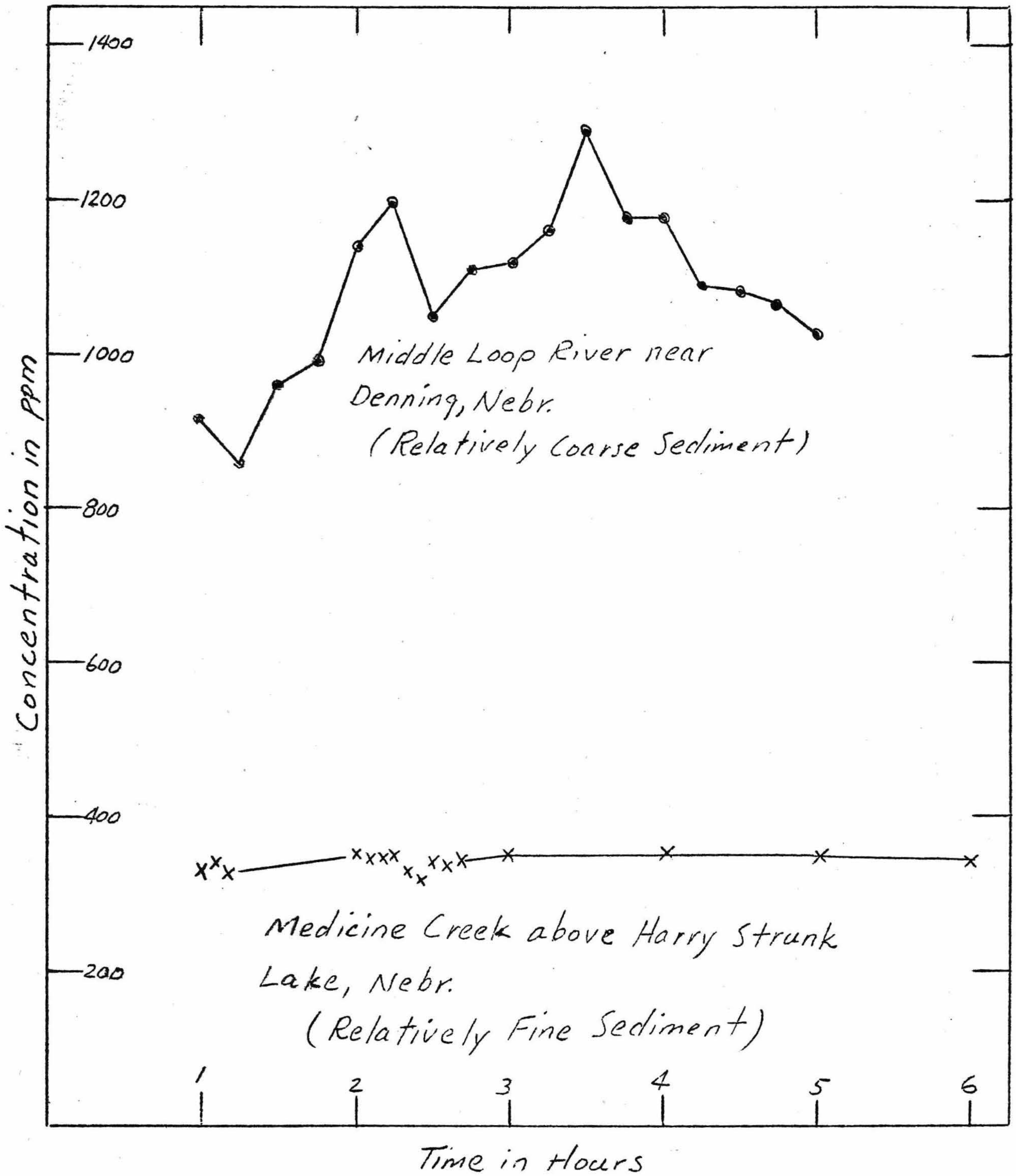


Fig. 15. -- Variations in suspended sediment concentration of samples with time for two streams having relatively fine and coarse sediment.

for two streams. Streams that transport coarse sediments have greater short-term variation than streams with fine sediments.

Fine Sediments

The concentration and discharge of fine sediments vary with time due to the influence of many factors. At a particular station, there is marked variation in the pattern for successive run-off events. At different locations the pattern ranges from the near absence of sediment to saturated mud flows. It has been noted that the transport of fine sediments is controlled by factors of supply in the watershed. Some of these factors are:

(1) Kind and condition of soil cover

(2) Condition of vegetative cover

(3) Topography of watershed

(4) Nature and density of channel system

(5) Distribution and intensity of precipitation

(6) Distance of channel travel

(7) Rate of water discharge

(8) Amount of bank erosion

Fig. 16 illustrates the effect of the distance of travel on the lag of the concentration peak with respect to the flow peak. The lag is believed to occur because suspended sediment travels at the same velocity as the mean water velocity which is known to be less than that of a flood wave. Fig. 17 illustrates the variation of concentration with discharge.

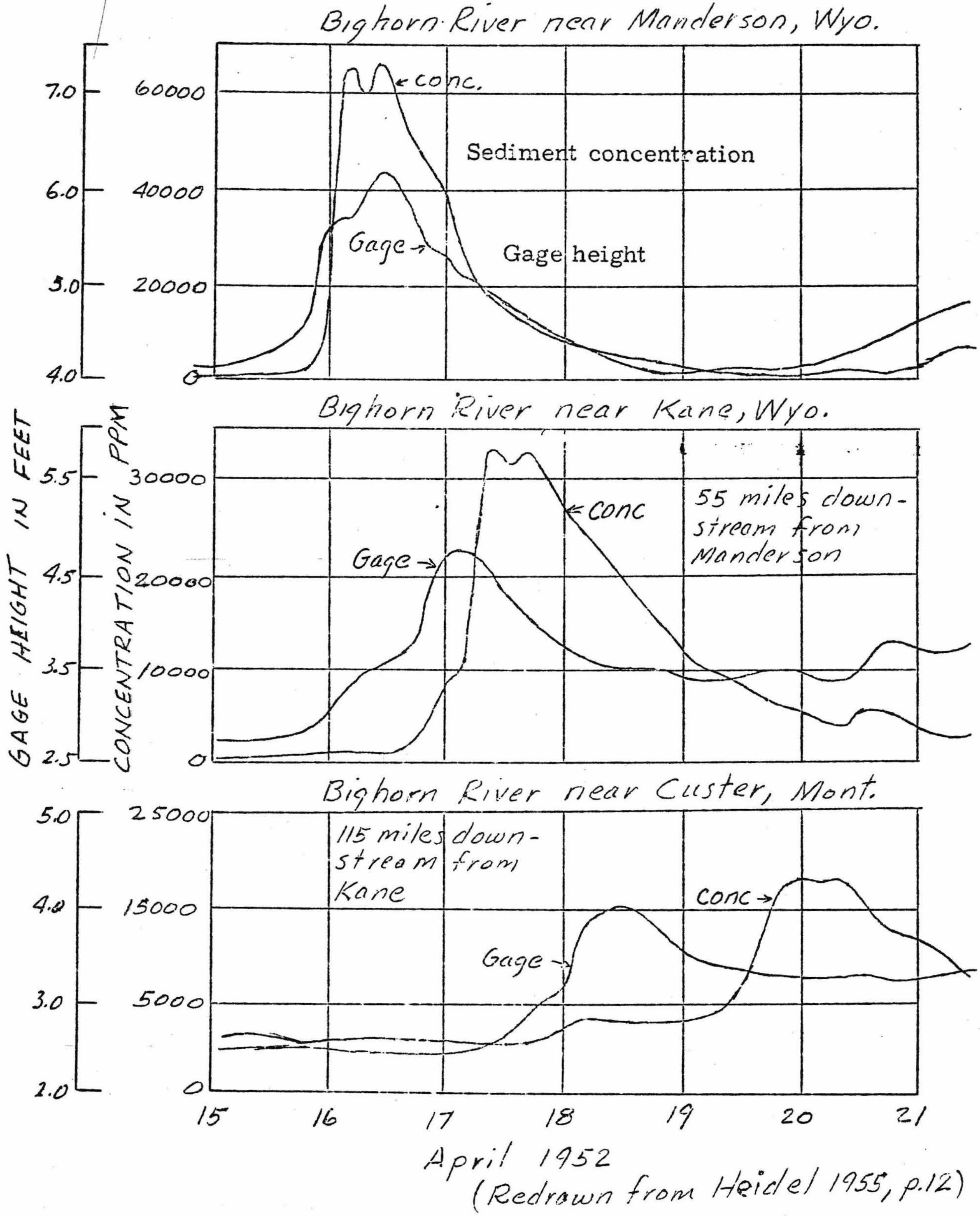


Fig. 16.-- Gage heights and sediment concentrations of the Bighorn River.

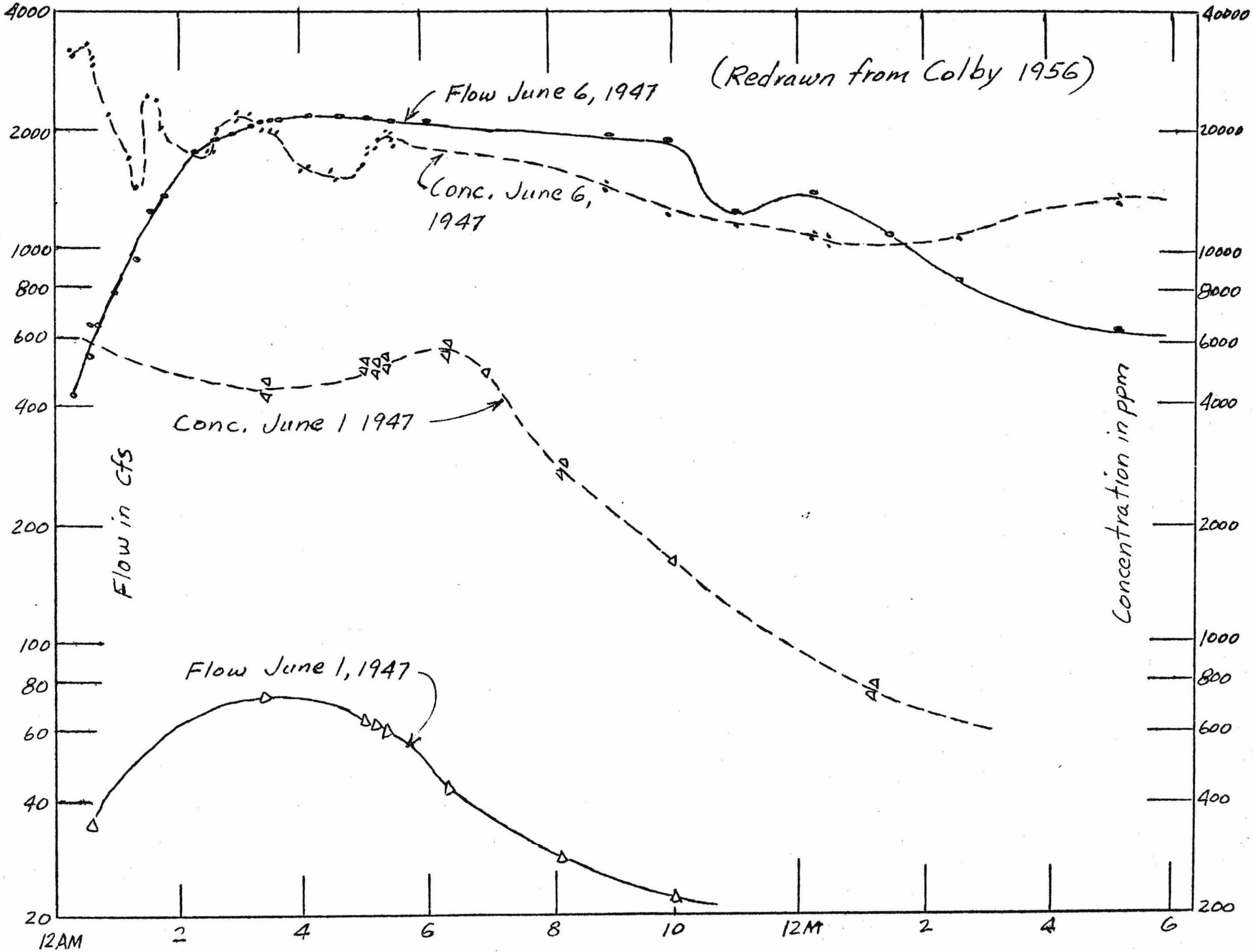


Fig. 17 -- Variations in sample concentrations. Prarie Dog Creek at Norton, Kansas.

Bed Material Discharge

The variations of discharge for the bed material load is related much closer to the quantity and turbulence of water discharge than to the factors noted above for fine sediments.

The quantity of bed material discharge, especially for fine sands, may vary considerably with seasons due to the effect of temperature on the viscosity of the water and the fall velocity of the particles. The following table from Colby (1956) shows the theoretical increase for a drop in water temperature from 80° to 40° F to be:

Particle size mm	Increase in sediment discharge, percent
0.016-0.062	57
.062-0.125	199
.125-0.250	254
.250-0.500	105
.500-1.000	17

The discharge of bed material sediments exhibits seasonal variations on typical western streams due to the changing texture of the bed material. Summer run-off feeds in tributary waters with high sediment concentrations part of which may be deposited in the bed. The spring snowmelt run-off, practically free of sediment, picks up its load from the finer fraction of the bed. Thus, the bed becomes relatively coarse toward the end of the snowmelt period.

In summary, procedures for measuring sediment discharge must average or define variations of sediment concentration that occur in the section and that occur with time. The search for effective and efficient procedures must be a continuing objective.

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Problem Set

1. An alluvial channel has a slope of energy gradient equal to 0.002, and a hydraulic radius R of 3.25 ft when carrying 525 cfs. Compute the average tractive force exerted on the wetted perimeter of the channel by the flowing water.

2. A trapezoidal channel has side slopes of 1:1 and a bottom width of 100 ft. If the depth of water in the channel is 5.0 ft, compute the hydraulic radius. Note that for wide channels R and D are nearly equal.

3. Compute and plot the theoretical velocity distribution for a channel (assume a rough boundary) if the median diameter of the bed material is 1.0 mm, the depth of flow is 5 ft and the slope of energy gradient is 0.001.

4. The concentration of suspended sediment C_a = 500 ppm at a distance a = 0.5 ft above the bed of an alluvial stream which has a depth of 3.0 ft. The slope of energy gradient is 0.001, and the fall velocity w of sediment is 0.2 ft/sec. Compute the suspended sediment distribution in the remainder of the vertical.