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LAMINAR HEAT CONVECTION IN PIPES AND DUCTS

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FOREWORD

This report is No. 5 of a series written for the Colorado Agricultural Research Foundation Project No. 137 entitled Diffusion of Momentum, Vapor, and Heat under contract with the Office of Naval Research. This project is presently being carried out by the Fluid Mechanics Laboratory of the Colorado Agricultural and Mechanical College, Fort Collins, Colorado, under the supervision of Dr. M. L. Albertson, Head of Fluid Mechanics Research of the Civil Engineering Department.

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LAMINAR HEAT CONVECTION IN PIPES AND DUCTS

ABSTRACT

Laminar heat convection in pipes and ducts is treated rather exhaustively for different boundary conditions. Eigenvalues and eigenfunctions are computed for certain simple boundary conditions. Extensions to more complicated boundary conditions are explicitly given.

1. Laminar Heat Convection in Pipes

a. The Graetz Solution

With x taken along the pipe and r radially from the center of the pipe section, the temperature distribution in the pipe when the wall is kept at a temperature T_o for $x \leq 0$ and another temperature T_1 for $x > 0$ and when the flow is laminar has been solved by Graetz (1, 1885). Since in the following sections Graetz's general development will be followed, his solution will be briefly presented here.

With T denoting the temperature at any point, the parameter

$$\theta = \frac{T - T_1}{T_o - T_1}$$

is required to satisfy the boundary-layer equation of heat diffusion:

$$u_{max} \left(1 - \frac{r^2}{a^2}\right)\frac{\partial \theta}{\partial x} = \alpha \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right) \tag{2}$$

where u_{max} is the maximum velocity in the pipe, a is the radius of the pipe, and α the thermal diffusivity of the fluid. Using the new variables

$$\xi = \frac{x}{a} \qquad \eta = \frac{r}{a}$$

one can transform Eq 2 to the form

$$P(1-\eta^2)\frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial \theta}{\partial \eta} \tag{3}$$

where P is the Peclet number $u_{max} \cdot a/\alpha$. Assuming

$$\theta(\xi, \eta) = X(\xi) R(\eta) \tag{4}$$

one has, upon substitution into Eq 3,

$$\frac{PX'}{X} = \frac{R'' + \frac{1}{\eta} R'}{(1 - \eta^2)R} = -\lambda^2$$

which gives

$$X = e^{-\lambda^2 \xi / P} \tag{5}$$

and

$$R'' + \frac{1}{\eta} R' + \lambda^2 (1 - \eta^2) R = 0 \tag{6}$$

Eq 6 is to be solved with the conditions that

(i) $R'(0) = 0$

(ii) $R(1) = 0$

the latter of which determines the eigenvalues $\lambda_0, \lambda_1, \lambda_2 \cdots$, of which the first three have been found by Graetz, Drew, and Nusselt to be

$$\lambda_0 = 2.705, \qquad \lambda_1 = 6.66 \qquad \lambda_2 = 10.6$$

The corresponding eigenfunctions are given in Table 1 and represented in Fig. 1, which are taken from p. 455 of Jakob's extensive book on heat transfer (2,1949). The solution is then

$$\theta(\xi, \eta) = A_0 e^{-\lambda_0^2 \xi / P} R_0(\eta) + A_1 e^{-\lambda_1^2 \xi / P} R_1(\eta) + A_2 e^{-\lambda_2^2 \xi / P} R_2(\eta) + \cdots \tag{7}$$

where the coefficients A_0, A_1, A_2, etc. are determined in such a way that $\theta(0, \eta) = 1$ and the first three are found to be equal to 1.477, -0.810, and 0.385 respectively. Concerning the calculation of these coefficients and of the eigenvalues and eigenfunctions, one is referred to the footnotes on pages 453 and 454 of (2). This determination is possible since the eigenfunctions $R_n(\eta)$ are orthogonal for $n = 0, 1, 2, \ldots$.

The Sturm-Liouville system consisting of Eq 6 and the conditions

(i) $R'(0) = 0$ (8)

(ii) $bR(1) + cR'(1) = 0$ (9)

will now be studied. In Graetz's solution $c = 0$. There are other physically significant cases where $c = 0$, or $b = 0$, or b and c both differ from zero.

b. Generalization of Graetz's Result

--Variable Wall Temperature

If the temperature on the wall has the distribution

$$T = T_0 \qquad (\xi \le 0)$$

$$T = T_1(\xi) \qquad (\xi > 0)$$

with $T_1(0) = T_0$, the temperature distribution in the pipe can be obtained from Graetz's solution by superposition. Since

$$\frac{T - T_0}{T_1 - T_0} = 1 - \frac{T - T_1}{T_0 - T_1} = 1 - \theta(\xi, \eta) \tag{9}$$

where $\theta(\xi,\eta)$ is given by Eq 7, the solution for the present problem is

$$T - T_0 = \int_0^{\xi} [1 - \theta(\xi - \zeta, \eta)] \frac{dT_1(\zeta)}{d\zeta} d\zeta$$

$$= T_1(\xi) - T_0 - \int_0^{\xi} \theta(\xi - \zeta, \eta) \frac{dT_1(\zeta)}{d\zeta} d\zeta$$

or

$$T_1(\xi) - T = \int_0^{\xi} \theta(\xi - \zeta, \eta) \frac{dT_1(\zeta)}{d\zeta} d\zeta \tag{10}$$

Physically, this solution amounts to slicing the temperature curve $T = T_1(\xi)$ $(\xi > 0)$ into strips each of thickness $dT_1(\xi)$ starting from $\xi = 0$, and superposing the effects due to all the infinitesimal strips the effect of each of which bring given by Graetz's solution.

This case of course still corresponds to $c = 0$ in the Sturm-Liouville system mentioned in the last section.

c. Insulated Pipe

Since there is no definite T_1 when the pipe is insulated for $x > 0$, the meaning of Θ should be redefined. The temperature flux into the insulated section is

$$H = 2\pi a^2 \int_0^1 u_{max} (1-\eta^2)\, T(0,\eta)\, \eta\, d\eta \tag{11}$$

The asymptotic temperature for very large ξ will be uniform and have the magnitude H/Q where Q is the discharge through the pipe:

$$Q = 2\pi a^2 u_{max} \int_0^1 (1-\eta^2)\, \eta\, d\eta = \left(\frac{u_{max}}{2}\right) \pi a^2 \tag{12}$$

The asymptotic temperature T_a is therefore

$$T_a = 4 \int_0^1 (1-\eta^2)\, T(0.\eta)\eta d\eta \tag{13}$$

One defines

$$\Theta - \frac{T - T_a}{T_a} \tag{14}$$

so that asymptotically $\Theta \to 0$ and initially ($\xi = 0$) Θ is known.

With the Θ thus defined and remembering Eq 4, Eqs 6, 8, and 9 are to be solved with $b = 0$. The eigenvalues can be determined from $R'(1) = 0$. The first three eigenvalues have been calculated to be $\lambda_0 = 5.07$, $\lambda_1 = 9.17$, $\lambda_2 = 13.271$, and the pertaining eigenfunctions are given numerically in Table 2 and graphically in Fig. 2. The coefficients A_o, A_1, A_2 etc. in Eq 7 (which constitutes the solution) are determined in virtue of the orthogonality of the eigenfunctions from the preassigned value $\Theta(0,\eta)$. From the calculated residues for the values of $R'(1)$, it seems that the first two eigenvalues are slightly too large, and the third one slightly too small. The residues for the three eigenvalues are respectively -0.0002, $+0.006$, and $+0.0030$.

d. Pipe with Finite Wall-Thickness

When a pipe has a finite wall-thickness, it is impossible or at least impracticable to keep the inner surface of the pipe at a certain temperature T_1.

One considers the simplest case when the outside surface of the pipe is kept at a constant temperature T_1 for $x > 0$. When $x \leq 0$ the wall as well as the fluid is supposed to have the temperature T_o. The resulting temperature distribution in the fluid is sought.

First, one defines Θ by Eq 1. If the pipe is sufficiently long in comparison with its thickness, longitudinal conduction can be neglected. This is especially true if the pipe is longitudinally insulated at $\xi = 0$ where there is a sudden change of temperature. If, therefore, only the radial conduction is considered, the contribution to the inner-wall temperature being R for any eigenfunction, the heat transfer through the wall contributed by this eigenfunction is proportional to $+ kR(1)/\ln(1 + h)$ where k is the thermal conductivity of the pipe material and $h = t/a$, t being the thickness of the pipe wall. The proof is as follows:

One writes the "boundary-layer" equation of conduction

$$\frac{d^2\Theta}{d\eta^2} + \frac{1}{\eta}\frac{d\Theta}{d\eta} = 0 \tag{15}$$

where ordinary differentiation is used since longitudinal conduction is neglected. Solving Eq 15 with the boundary conditions (for any ξ)

$$\Theta(1) = R(1), \quad \Theta(1 + h) = 0$$

one obtains

$$\Theta = R\left(1 - \frac{\ln r}{\ln(1 + h)}\right)$$

from which

$$\left.\frac{d\Theta}{d\eta}\right)_{\eta=1} = \frac{R(1)}{\ln(1 + h)}$$

Hence one has the required rate of heat transfer.

On the other hand, the rate of heat transfer from the fluid to the inner surface of the wall is proportional to $-k'R'(1)$ (the factor of proportionality being the same as for $k\,R(1)/\ln(1+h)$), where k' is the thermal conductivity of the fluid. By continuity, one has

$$\frac{K}{\ln(1+h)} R(1) + R'(1) = 0 \tag{16}$$

where $K = k/k'$. This equation is a special form of Eq 9 ($b \neq 0$ and $c \neq 0$) and determines the eigenvalues.

After the eigenvalues are obtained, the solution is found as before by determining the coefficients A_o, A_1, A_2, etc. in Eq 7.

2. Laminar Heat Convection in Ducts

a. The Yih-Cermak Solution

Let the distance between the two parallel plates be 2a, the abscissa x be taken in the direction of flow, and the ordinate y be taken perpendicular to the plates, with the origin mid-way between the plates. If the approaching fluid has a temperature T_o and at $x > 0$ the inner surfaces of the duct are kept at a constant temperature $T_1 \neq T_o$, the temperature in the fluid will gradually change from T_o to T_1 and is a function of both x and y. The solution has been given by Yih and Cermak (3), a brief account of which is given below.

With θ defined by Eq 1, and

$$\xi = \frac{x}{a}, \quad \eta = \frac{y}{a}$$

The boundary-layer equation of heat diffusion can be written

$$P(1-\eta^2)\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\eta^2} \tag{17}$$

where $P = u_{max} \cdot a/\alpha$ is again the Peclet number. Assuming

$$\theta(\xi, \eta) = X(\xi)\, Y(\eta) \tag{18}$$

one has, upon substitution into Eq 17,

$$\frac{PX'}{X} = \frac{Y''}{(1-\eta^2)Y} = -\lambda^2$$

which gives

$$X = e^{-\lambda^2\xi/P}$$

and

$$Y'' + \lambda^2(1-\eta^2)Y = 0 \tag{19}$$

Eq 19 is to be solved with the conditions that

(i) $Y'(0) = 0$

(ii) $Y(1) = 0$

the latter of which determines the eigenvalues $\lambda_0, \lambda_1, \lambda_2$ - - - , of which the first three have been found to be

$$\lambda_0 = 1.6815, \qquad \lambda_1 = 5.6699, \qquad \lambda_2 = 9.6687$$

the corresponding eigenfunctions being given numerically in Table 3, and graphically in Fig. 3. The solution is then

$$\theta(\xi,\eta) = A_0 e^{-\lambda_0^2 \xi/P} Y_0(\eta) + A_1 e^{-\lambda_1^2 \xi/P} Y_1(\eta) + A_2 e^{-\lambda_2^2 \xi/P} Y_2(\eta) + \cdots \qquad (20)$$

where the coefficients A are determined by the condition $\theta(0,y) = 1$, and the first three are found to be

$$A_0 = 1.2008 \qquad A_1 = -0.2993 \qquad A_2 = 0.1596$$

The orthogonality of the eigenfunctions has made this determination possible.

The Sturm-Liouville system consisting of Eq 19 and the conditions

(i) $b_1Y(d) + b_2Y'(d) = 0$ (21)

(ii) $c_1Y(1) + c_2Y'(1) = 0$ (22)

will now be studied. In the Yih-Cermak solution $b_1 = c_2 = d = 0$. There are other physically significant cases where d may be zero or minus 1 and where the four coefficients b_1, b_2, c_1 and c_2 may not vanish or different combinations of them vanish.

b. Generalization of the Yih-Cermak Solution
--Variable Wall Temperature

The generalization follows that of 1 b. If on the inner surface of the pipe

$$T = T_0 \qquad \xi \le 0$$

$$T = T_1(\xi) \qquad \xi > 0$$

the resulting temperature distribution is given by

$$T_1 - T = \int_0^{\xi} \theta(\xi-\varsigma,\eta)\,\frac{dT_1(\varsigma)}{d\varsigma}\,d\varsigma \tag{23}$$

where $\theta(\xi,\eta)$ is given by the Yih-Cermak solution.

c. Insulated Plates

The development follows that of 1c. In this case

$$H = 2au_{max}\int_0^1 (1-\eta^2)\,T(0,\eta)\,d\eta \tag{24}$$

$$Q = 2au_{max}\int_0^1 (1-\eta^2)\,d\eta = \frac{4}{3}au_{max} \tag{25}$$

$$T_a = H/Q = \frac{3}{2}\int_0^1 (1-\eta^2)\,T(0,\eta)\,d\eta \tag{26}$$

One defines θ by Eq 14 and solve Eqs 19, 21 and 22 with $b_1 = c_1 = 0$, $d = -1$, remembering Eq 18. If the initial value of $\theta(0,\eta)$ is even or odd, Eq 21 can be respectively replaced by

$$Y'(0) = 0 \tag{27}$$

or

$$Y(0) = 0 \tag{28}$$

In the former case $Y(\eta)$ is even; in the latter, odd. If the initial value of $\theta(0,\eta)$ is neither even nor odd, both even and odd eigen-functions should be used to approximate $\theta(0,\eta)$. The first two eigenvalues for the even eigenfunctions are found to be approximately

$$\lambda_0 = 4.2872 \qquad\qquad \lambda_1 = 8.3042$$

and the corresponding eigenfunctions are given numerically in Table 4 and graphically in Fig. 4. The residues for the values of $Y'(1)$ are respectively + 0.00004 and -0.006 for the two eigenvalues given above.

d. Duct with Finite Wall-Thickness

Suppose that the thickness of the plates is t and that the outside surface of the plates is kept at T_1, the temperature of approach being T_0. Let θ be defined by Eq 1. The development is similar to that of 1 d, with the difference that in the present case the distribution of θ in the plate is linear with respect to y. Consequently, Eq 16 is replaced by

$$\frac{K}{h} Y(1) + Y'(1) = 0 \tag{27}$$

where K again is the ratio of the thermal conductivity of the plate to that of the fluid, and $h = t/a$. Eq 27 is a special case of Eq 21. Eq 20 can be replaced by

$$Y'(0) = 0$$

due to symmetry. One therefore obtains the eigenvalues from Eq 27, taking only the even part of the solution of Eq 19 for the function $Y(\eta)$. After the eigenfunctions are obtained, the solution is found as usual by the proper determination of the coefficients A in Eq 20.

e. Unsymmetric Boundary Conditions

Consider the case when the temperature of approach is T_o. The lower plate is kept at T_o throughout, and the upper plate is kept at T_o for $x \leq 0$, and T_1 for $x > 0$. With θ defined as

$$\theta(\xi, \eta) = \frac{T - T_1}{T_o - T_1} - \frac{1 - \eta}{2} \tag{29}$$

and remembering Eq 18, Eq 19 is to be solved with

$$Y(-1) = 0 \tag{30}$$
$$Y(1) = 0$$

which are special cases of Eqs 21 and 22. Either the even or the odd part of the solution of Eq 19 can be taken for $Y(\eta)$, and if so Eq 30 is contained in Eq 31, which gives the eigenvalues. The eigenfunctions are

$$Y_o(\eta) \quad Z_o(\eta), \quad Y_1(\eta), \quad Z_1(\eta)$$

where $Y(\eta)$ is even and $Z(\eta)$ is odd. The solution is

$$\theta(\xi,\eta) = \sum_{n=0}^{\infty} e^{-\lambda_n^2 \xi/P} \left(A_n Y_n(\eta) + B_n Z_n(\eta)\right) \tag{32}$$

where A_n and B_n are to be determined such that

$$\Theta(0,\eta) = 1 - \frac{1-\eta}{2} = \frac{1+\eta}{2} \tag{33}$$

The first three of the eigenvalues for the even functions have been found to be

$$\lambda_0 = 1.6815 \qquad \lambda_1 = 5.6699 \qquad \lambda_2 = 9.6687$$

and those for the odd function have been f ound to be

$$\lambda_0' = 3.6723 \qquad \lambda_1' = 7.6688 \qquad \lambda_2' = 11.5957$$

The pertaining coefficients are

$$A_0 = 0.6004 \qquad A_1 = -0.1496 \qquad A_2 = 0.0798$$

$$B_0 = 0.9102 \qquad B_1 = -0.8057 \qquad B_3 = 0.1601$$

The result just obtained can be generalized. Let the temperature distribution on the upper plate be

$$T = T_0 \qquad x \leq 0$$

$$T = f(\xi) \qquad x > 0$$

and that on the lower plate be

$$T = T_0 \qquad x \leq x_0$$

$$T = g(\xi) \qquad x > x_0$$

where x_0 is a certain value of x. Then since T_1 is not constant in this case, one uses the parameter $(T - T_0)/T_0$ and observes that

$$\frac{T - T_0}{T_0} = \left(\frac{T - T_1}{T_0 - T_1} - 1\right)\frac{T_0 - T_1}{T_0} = \left[1 - \Theta(\xi,\eta) - \frac{1-\eta}{2}\right]\frac{\Delta T}{T_0}$$

where $\Delta T = T_1 - T_0$ if a constant T_1 exists but stands for $df(\xi)$ or $dg(\xi)$ when T_1 is variable, and where $\Theta(\xi,\eta)$ is given by Eq 29. Thus considering the influence of the upper plate, and utilizing the principle of superposition,

$$\frac{T - T_0}{T_0} = \frac{1}{T_0}\int_0^{\xi}\left[\frac{1+\eta}{2} - \Theta(\xi - \zeta, \eta)\right]\frac{df(\zeta)}{d\zeta}\, d\zeta \tag{34}$$

where $\Theta(\xi,\eta)$ is given by Eq 32. In considering the influence of the lower plate, one notices that if the direction of η were turned π radians an expression similar to the one given in Eq 34 would be the solution, which means, using the existing coordinates, the η in Eq 34 should be changed to $-\eta$. Furthermore, since the change of temperature for the lower plate occurs at $\xi_0 = x_0/a$, the lower limit of the integral in Eq 34 should be changed to ξ_0. Thus, considering the influence of the lower plate,

$$\frac{T - T_0}{T_0} = \frac{1}{T_0} \int_{\xi_0}^{\xi} \left[\frac{1-\eta}{2} - \theta(\xi-\zeta, -\eta)\right] \frac{dg(\zeta)}{d\zeta} d\zeta \qquad (35)$$

If ξ_0 is negative or zero, then for $\xi > \xi_0$

$$\frac{T - T_0}{T_0} = \frac{1}{T_0}\left\{ \int_0^{\xi} \left[\frac{1+\eta}{2} - \theta(\xi-\zeta, \eta)\right] \frac{df(\zeta)}{d\zeta} d\zeta \right.$$

$$\left. + \int_{\xi_0}^{\xi} \left[\frac{1-\eta}{2} - \theta(\xi-\zeta, -\eta)\right] \frac{dg(\zeta)}{d\zeta} d\zeta \right\} \qquad (36)$$

If $\xi_0 > 0$, then $(T - T_0)/T_0$ is given by Eq 34 for $0 < \xi < \xi_0$, and by Eq 36 for $\xi > \xi_0$.

3. Conclusion

Problems in laminar heat transfer in pipes and ducts can be solved by the generalized Fourier analy is if there is symmetry. Even some unsymmetrical cases can be solved by the same general method, as presented in 2e.

Bibliography

1. Graetz, L.: Annalen d. physik (N.F.) Vol. 25, p. 337, 1885.

2. Jakob, Max: Heat Transfer, John Wiley and Sons, 1949.

3. Yih, C. S., and Cermak, J.E.: Heat Transfer in Plane Poiseuille Flow, to be published.

Table 1

η	$R_0(\eta)$	$R_1(\eta)$	$R_2(\eta)$
0	1	1	1
0.1	0.9818	0.8923	0.753
0.2	0.9290	0.6067	0.206
0.3	0.8456	0.2367	-0.290
0.4	0.7382	-0.1062	-0.407
0.5	0.6147	-0.3399	-0.204
0.6	0.4833	-0.4317	0.104
0.7	0.3506	-0.3985	0.278
0.8	0.2244	-0.3051	0.278
0.9	0.1069	-0.1637	0.144
1.0	0	0	0

Table 2

η	$R_0(\eta)$	$R_1(\eta)$	$R_2(\eta)$
0	1	1	1
0.1	0.937	0.801	0.607
0.2	0.761	0.325	-0.117
0.3	0.505	-0.154	-0.407
0.4	0.230	-0.402	-0.122
0.5	-0.032	-0.357	0.250
0.6	-0.213	-0.186	0.305
0.7	-0.382	0.135	0.109
0.8	-0.459	0.312	-0.199
0.9	-0.488	0.388	-0.328
1.0	-0.492	0.406	-0.336

Table 3

η	$Y_0(\eta)$	$Y_1(\eta)$	$Y_2(\eta)$
0	1	1	1
0.1	0.98592	0.9438	0.5685
0.2	0.94435	0.4262	-0.3513
0.3	0.87731	-0.1205	-0.9843
0.4	0.78762	-0.6345	-0.8413
0.5	0.67934	-0.9832	-0.0746
0.6	0.55665	-1.1013	0.7542
0.7	0.42514	-0.9973	1.1669
0.8	0.28489	-0.7311	1.0498
0.9	0.14294	-0.3787	0.5550
1.0	0	0	0

Table 4

η	$Y_0(\eta)$	$Y_1(\eta)$
0	1	1
0.1	0.9096	0.6751
0.2	0.6564	-0.0857
0.3	0.2889	-0.7970
0.4	-0.1270	-1.0174
0.5	-0.5245	-0.7062
0.6	-0.8518	-0.0178
0.7	-1.0812	0.6847
0.8	-1.2004	1.1659
0.9	-1.2623	1.3716
1.0	-1.2697	1.4055

Table 5

η	$Z_0(\eta)$	$Z_1(\eta)$	$Z_2(\eta)$
0	0	0	0
0.1	0.0978	0.0905	0.0791
0.2	0.1827	0.1312	0.0644
0.3	0.2443	0.1011	-0.0253
0.4	0.2764	0.0192	-0.0879
0.5	0.2776	-0.0725	-0.0617
0.6	0.2509	-0.1342	0.0239
0.7	0.2027	-0.1476	0.0919
0.8	0.1405	-0.1178	0.0892
0.9	0.0713	-0.0628	0.0640
1.0	0.0000	0.0000	0.0081

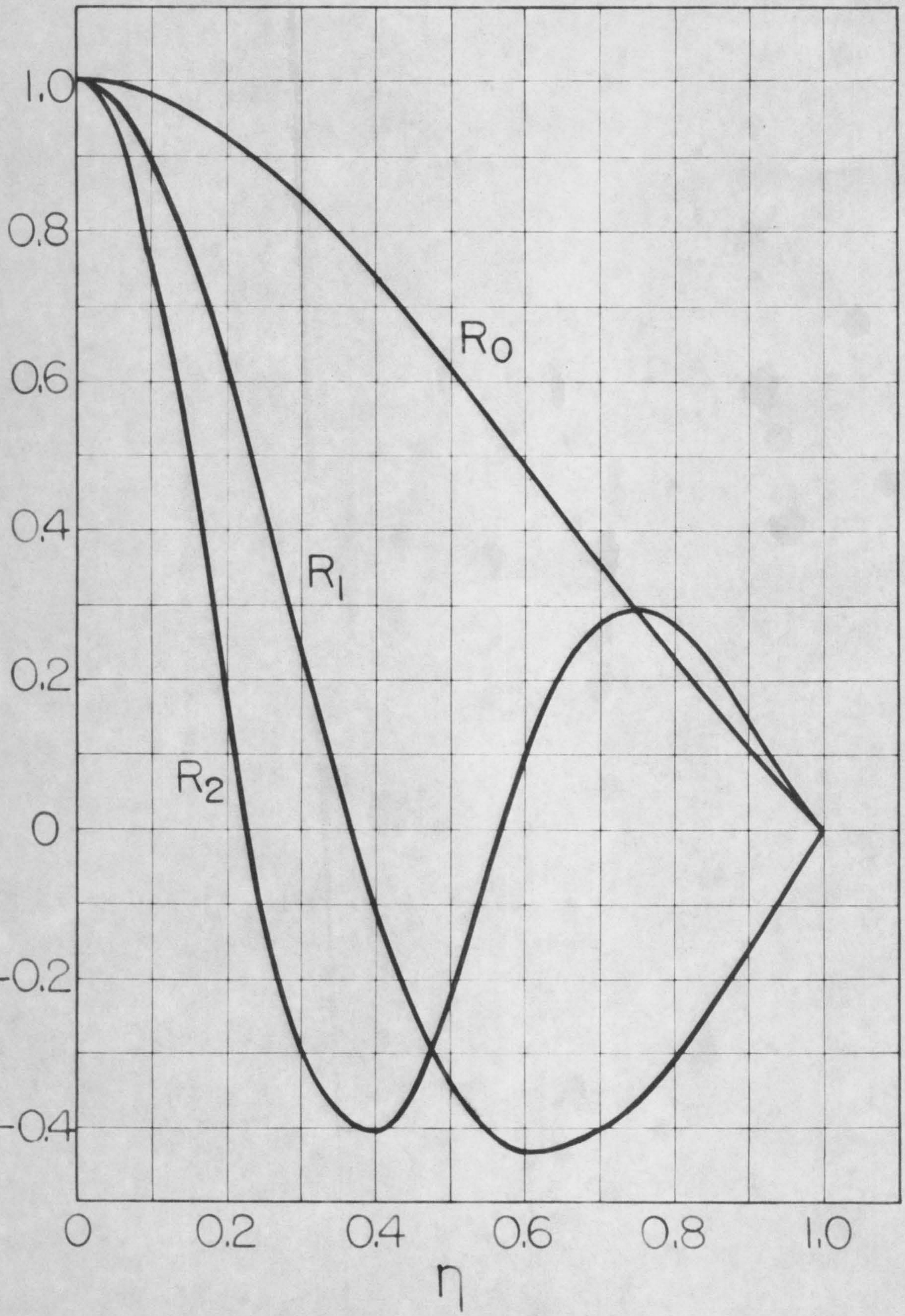

Fig. 1

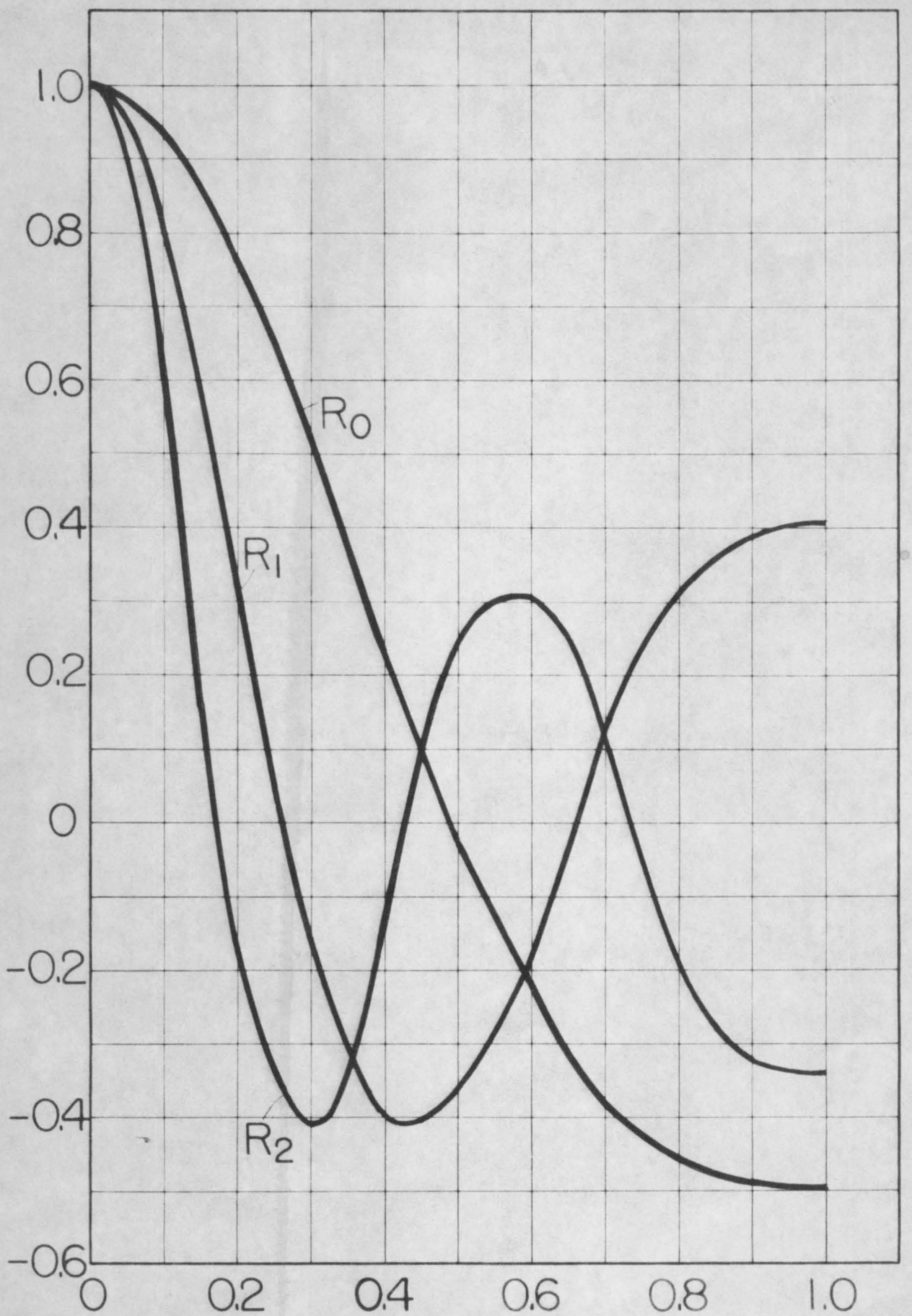

Fig. 2

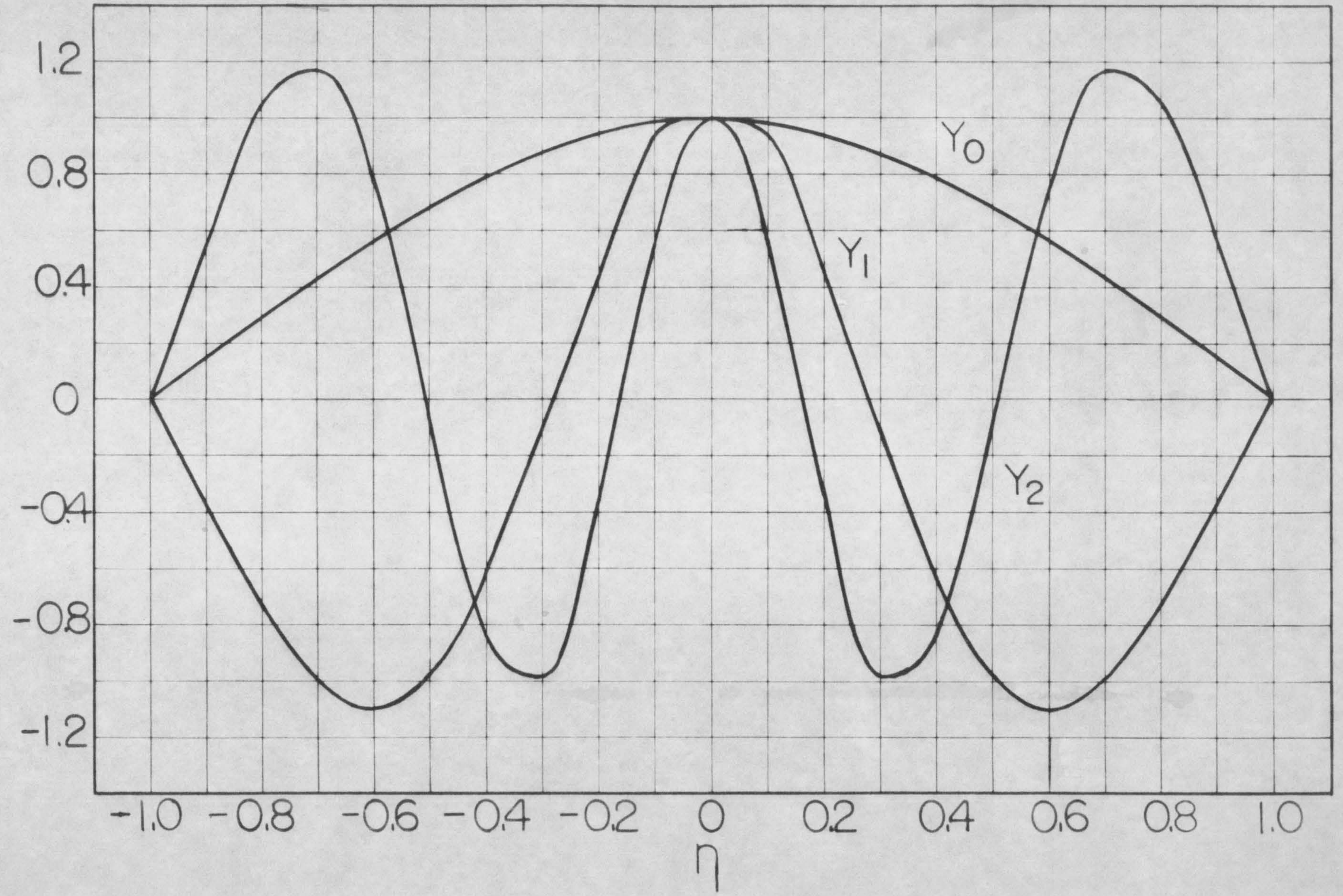

Fig. 3

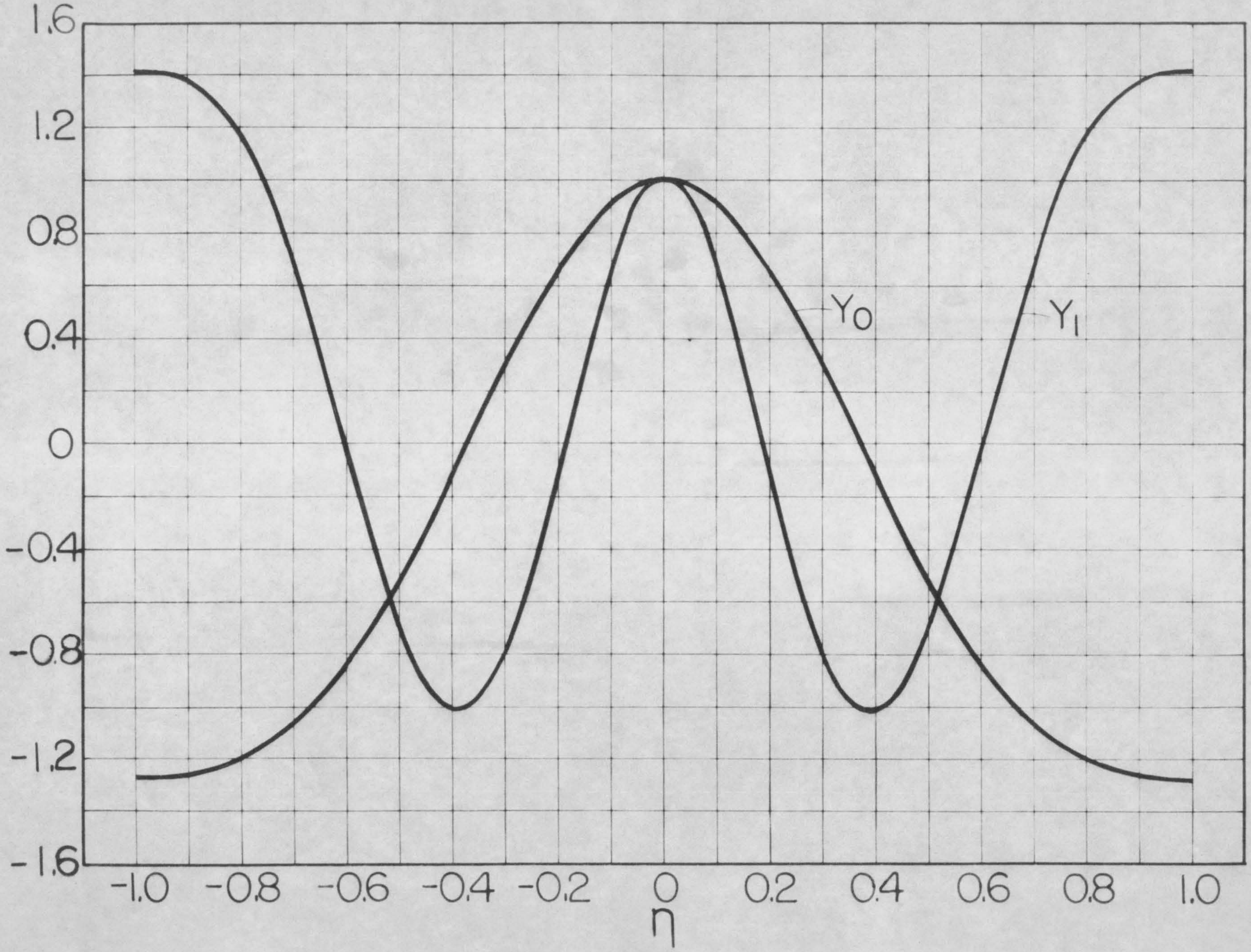

Fig. 4

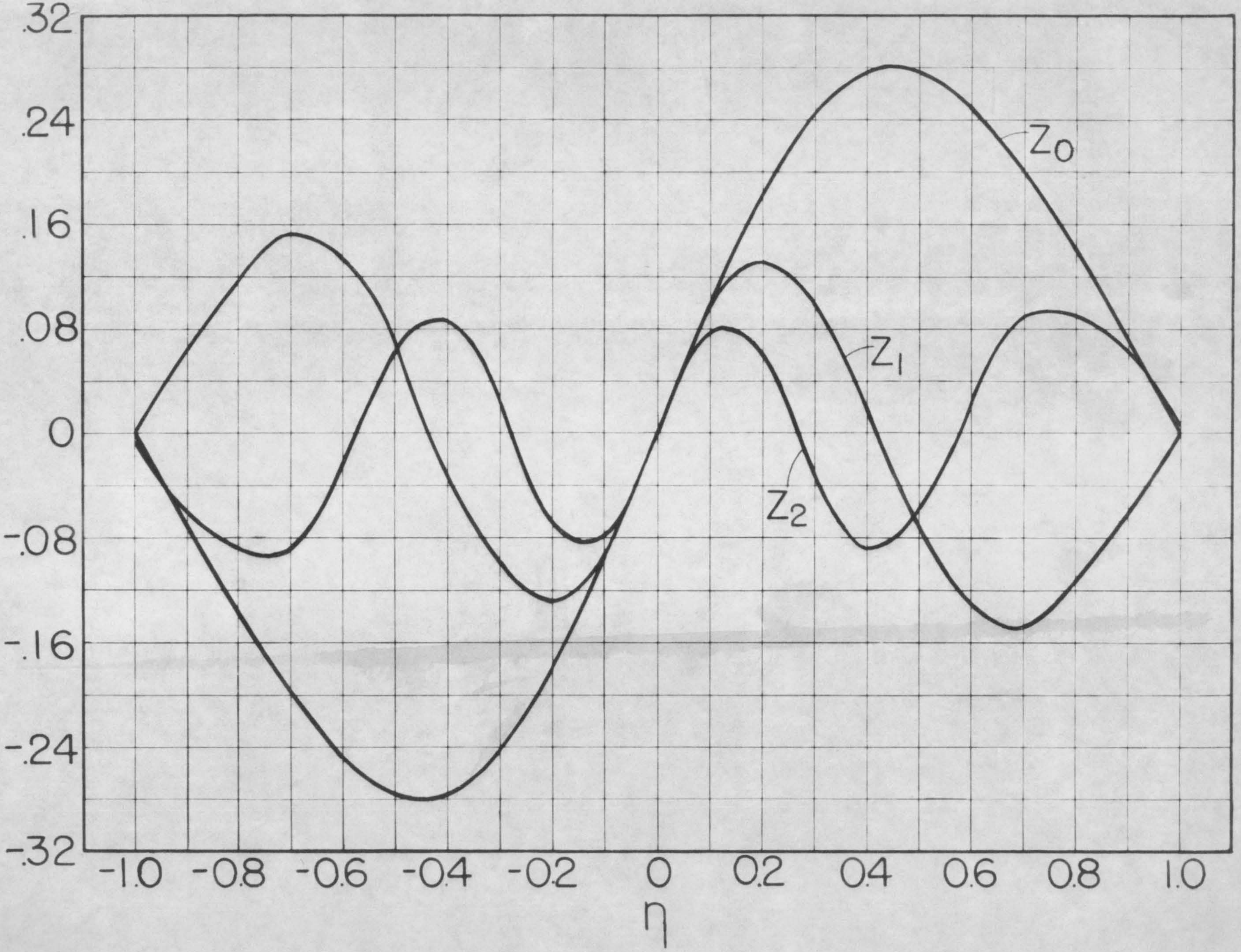

Fig. 5