THESIS

EMPIRICAL MODELING OF AUTOMTIVE DAMPER CURVES AND DEVELOPMENT OF SHAPE FACTORS

Submitted By

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ABSTRACT

EMPIRICAL MODELING OF AUTOMOTIVE DAMPER CURVES AND DEVELOPMENT OF SHAPE FACTORS

Automotive dampers are a complex system developed with integration of simple mechanisms. The system comprises of a cylinder filled with hydraulic fluid, a piston dividing cylinder into two chambers known as compression and rebound chambers; a coil spring in some case and nitrogen gas. When a vehicle moves, automotive Damper system deals with damping vibration and giving occupants a comfortable ride. While the system ensures a smooth ride by absorbing all the road vibration, hydraulic fluid inside the Damper system goes through various transformations. Changes and variations happen in properties like pressure and temperature inside the Damper system with time because of displacement of piston which leads to generation of heat which leads to change in damping coefficient. While calculating energy equation, it was observed that constant damping coefficient was used and was not a function of time. In a real world scenario this assumption is not correct because with changes in above mentioned properties, damping coefficient values are affected and they change constantly with time.

Research was commenced on coming up with an empirical model that can give information regarding energy inside a Damper system, types of suspension behavior and various characteristics regarding Damper system for any range of velocities. A full-fledged Damper system empirical model will have various constants, parameters, shape factors and form factors. If these parameter values are obtained and used properly, they can give help determining the behavior of any Damper system, different settings that can be used to get required behavior by any Damper system. For a full-fledged empirical model, lots of efforts, resources and years of

research will be required. So, to start with, an empirical model was worked upon that can act as a shape factor for any Damper system for given range of velocities.

Any Damper system will give three different types of curves a progressive curve, a linear curve and a digressive curve. The three curve shows three kinds of suspension behavior which is required depending upon the application. Keeping this in mind, a model was created which was able to depict all three damper curves for certain range of values for both compression chambers and rebound chamber inside a damper. The model is a function of various trigonometric functions like sine, arctans, hyperbolic functions and property like velocity. There are constant parameters and changing their values will change shape of damper curve as per the requirement. Developed Empirical Model is able to fulfill the initial requirement of becoming a proper shape factor and can be used to predict behavior of Damper system for both compression and rebound chamber. Main reasons behind obtaining damper curves are the constant parameters associated with it. Those parameters can be related to any characteristics or properties associated with function and performance of a Damper system.

Developed Empirical model acts as a foundation for next stage of research because once the shape factor is achieved; the parameters associated with it can be given a meaning based on Damper system's properties and characteristics.

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CHAPTER 1: INTRODUCTION

"Energy can neither be created nor destroyed; it gets transformed from one form to another"

1.1: Overview

Today's world is facing its biggest and gravest challenges in the form of rising pollution and global warming. The world is facing the consequences of an uncertain climate. Countless ecological species are facing extinction and the human race is staring at an uncertain future. Need for alternative, clean energy has arisen and is now more important than ever. Scientists and engineers around the world are now on the hunt for clean alternative energy which will, in future, replace existing conventional systems, making it possible to depend less on crude oil and conventional yet inefficient systems. The automotive sector plays the role of being one of the largest contributors or pollution. "Efficiency" has become the new synonym for "automotive". Today, companies around the world are coming up with new technologies to propel vehicles like hybrid vehicles, hydrogen fuel cell technology and lithium ion batteries. Other than new technologies, major overhauling is done in traditional designing and materials used.

Today, one can see more aerodynamic and sleek cars with the use of materials like composites and carbon fiber which have mostly captured high speed and luxurious car manufacturing. New technologies like regenerative systems which can capture and re-use vehicle energy from various possible sources, are getting lots of attention and gaining importance. Systems like Kinetic Energy Recovery System (KERS) are used to capture a vehicle's kinetic energy, which is wasted as heat in braking, and reuse it to power the vehicle. Just like a KERS, a similar kind of concept can be used in automotive Damper systems. A Damper system, while doing its work, generates energy inside it and one can, with the proper technology and system, harvest that energy for reuse. However, if one is able to capture that energy, the feasibility of such a system needs to be investigated. This research started with an idea of coming up with a regenerative Damper system which would capture the kinetic energy generated because of road shocks in some form, i.e. either in the form of electrical (batteries) or mechanical (flywheel). This energy will then be re-utilized to give additional power to the vehicle or to run auxiliary systems depending upon energy available and requirements. But as the research progressed, the topic of this thesis was diverted to having an empirical model that can give information regarding the damping coefficient, energy inside the Damper system, and its behavior.

Automotive suspensions are a highly complex system with simple mechanisms. Although the workings of automotive systems are understood to some extent, it is still a mystery why the properties of these systems change. The factors affecting performance of Damper systems and how a proper performance can be programmed or set in it is not yet completely understood. This lead to the conceptualization of the idea that if a model is generated based on some empirical relations, it can predict various properties of the Damper system, like the amount of energy generated within and a damper curve. Although years of research will be required to come up with a model that can do all the aforementioned duties, the aim of this thesis is to develop a shape factor which will be able to give the basic characteristics of a Damper system.

1.2: Vehicle Damper system

Vehicles are designed to give their occupants a smooth, comfortable, vibration-free ride on the way to a destination. In order to achieve this, a Damper system is developed and incorporated which does not allow disturbance between vehicle and road to reach the occupant. Suspensions of many types are available on the market and are used by the automotive industry to meet specific needs. Most of these suspensions are semi-active comprising a coil-winded spring and damper system used mainly because of its cheap but effective operation. With development of new technologies, there are new systems available like electromagnetic suspensions, pneumatic suspensions and many supporting systems like Electronic Damper Control (EDC).

1.3: Suspension Harvesting- The Concept

The major research in the automotive industry right now is concentrating on batteries and other energy sources to propel vehicles. Though these technologies have started coming into market in vehicles like Toyota Prius, Chevy Volt, Nissan Leaf and Mahindra E2o, it will take decades to completely phase out the conventional I.C. engine-propelled vehicles. Even if their production stops, they will still be on the roads for years, so introducing technologies to increase their efficiency and mileage will always be seen as a positive step to help the environment and preserve resources. Capturing energy is not a new concept. In late 1800’s and early 1900’s people tried to capture waste energy and tried to use it to propel vehicle. Due to lack of support from industries, there was not much advancement.

But the energy crisis in the mid-1970 forced industries to look for efficient technologies which would increase the efficiency of vehicles and reduce fuel consumption while delivering the power required by the vehicle. The Damper system inside an automotive vehicle plays an important role in resisting road shocks and not allowing them to reach the vehicle’s occupants, thus making the journey smooth and protecting vehicle components from vibrations. As a Damper system does its work, forces generated inside it in compression and rebound chambers will generate kinetic energy that are dissipated in the form of heat. These energies can be high for a vehicle which is running at high speed or on rough terrains. An article by Browne and Hamburg suggests this energy can go up to 40-50 W per damper, which means a total of 200W, will be available. 200W is quite a good amount of energy and with proper technology; it can be used to power the auxiliary systems of a vehicle which is otherwise powered by an engine. There are various other articles stated in the next chapter which support this theory. If this is possible then, such a system can be a good boost for the vehicle. The idea is to use the damped energy and instead of wasting it as heat, transform it into a useful form and use it. If, there is a significant amount of energy available to capture, and if one is able to do it, then the efficiency of the vehicle will be positively affected.

1.4: Concept of Empirical Model

As mentioned in the previous section, this research started with the concept of having a Damper system able to recover energy generated inside it and then got diverted into developing an empirical model for the Damper system. The idea of developing an empirical model started while researching the calculation of energy generated inside a Damper system. When the calculation was done, the damping coefficient (Cd) was left constant in the equation$\mathrm{Es} = \int_0^{2\pi} \mathrm{Cd} * \mathrm{x}^2 * \omega^2 * \cos^2 \omega \mathrm{t} * \mathrm{dt}$. It is pretty clear that a damping constant can never remain constant during the work done by the Damper system and that it is completely dependent on both external and internal factors. External factors like road surface, tire properties and vehicle mass, and internal factors like oil viscosity, oil temperature, piston diameter, piston length, nitrogen gas pressure and more, influence the damping coefficient. So, using a constant damping coefficient is inaccurate while calculating suspension energy. Thus, an idea to develop an empirical model started emerging. When researched, there was no empirical model that could predict an accurate behavior of a Damper system and damper curve. It is known that in the world of tires Hans B.

Pacejka developed an empirical model also known as magic tire model or Pacejka Tire Model that can predict and show accurate tire curves.

The same model can also give information regarding longitudinal force, lateral force and slip angle. Although it requires years of research to come up with proper empirical relations that can act as shape factors, correction factors, and many other parameters that will accurately predict and model the behavior of damper curves, the emphasis in this thesis was given to coming up with a model that would give shapes or graphs which behave like damper curves.

Just like Hans B. Pacejka and his researching team took decades to come up with an empirical model that can predict automotive tire properties, extensive research and resources will be required to modify and completely develop the empirical model for Damper systems. As a part of this master's thesis, the main aim was to develop shape factors and a model that would give the basic three damper curves, i.e. progressive curve, digressive curve and a linear curve. This was successfully done; the model has the ability to generate damper curves. There are various parameters and constants used, and if proper values are used then the model can give any of the required curves. The model, for a given velocity, will more or less give the behavior of the curves. Chapter 3 will give a detailed idea about how and why the decision was made to work on an empirical model.

Further research should lead to the development of a model that can give the amount of energy inside a Damper system, behavior of a damper curve, and correction factor, as well as a physical model that can be used to validate the empirical model.

CHAPTER 2: LITERATURE REVIEW

2.1: Overview

The work was commenced on the literature by looking for journals and articles on suspension and work being done on the concept of energy harvesting. The initial step was to look into articles related to road energy dissipation and the factors affecting it, researching on any existing model that could be used to predict damper characteristics. The literature work was done related to road roughness, damper characteristics and at the end on development of empirical model. These articles gave information regarding various parameters affecting suspension and damper system and development of empirical model for a damper system.

2.2: Road and Energy

The energy in the suspension comes from the road. So, before looking into suspension, it is important to understand the roads on which vehicles will be driven and how various road parameters affect suspension.

2.2.1: Adaptive control of Vehicle Suspension[1]

The article was by A. Hac published in 1987 in journal Vehicle System Dynamics gave information regarding development of adaptive control system for automotive suspension system and changing suspension parameters due to external forces like tire and road deflection. The article was necessary to understand various factors affecting performance of automotive dampers and change in properties of the same due to many factors affecting it like different roads like asphalt road or dirt road or paved road and developing quarter car model.

2.2.2: Vehicle Energy Dissipation Due to Road Roughness [2]

Article "Vehicle Energy Dissipation Due to Road Roughness" by Steven A. Velinsky and Robert A. White was referred to know about development of quarter car model and equation of motion for automotive suspension system. This article looks into aspects of how road roughness and texture affects tire rolling resistance and thus the energy dissipation between the road and the tire contact surface.

2.3: Pacejka Tire Model "The Magic Model" [3]

The article published on Pacejka tire model "The magic formula Tyre model" in Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, 1992 and subsequently many related articles gave idea about the only successful empirical model used in automotive tire world which using some trigonometric functions and coefficients was able to generate all three tire curves. The Model developed by Hans B. Pacejka and Egbert Bekker was:

$$Y(x) = D * sin\big(C * arctan\big\{BX - E\{BX - arctan(BX)\}\big\}\big) + S$$

Depending upon the forces calculated, the equation is written in three different formats:

$$Fx = Dx * sin(Cx * arctan\{BxKx - Ex\{BxKx - arctan(BxKx)\}F\}) + Sx$$

$$Fy = Dy * sin(Cy * arctan\{By\alpha y - Ey\{By\alpha y - arctan(By\alpha y)\}\}) + Sy$$

$$Mz0 = -t * Fyo + Mzr$$

$$t(\alpha t) = Dt * sin(Ct * arctan\{Bt\alpha t - Et\{Bt\alpha t - arctan(Bt\alpha t)\}\}) * cos\alpha$$

Here F_x, F_y, $t(\alpha_t)$ are the three forces i.e. Longitudinal Force (N), Lateral Force(N) and Self Aligning Torque(N*m) and X is replaced by Kx which is slip ratio, αy which is slip angle and αt which is slip angle.

Values of coefficients B, C, D and E are chosen based on the forces calculated. B acts as stiffness factor, C acts as shape factor, D acts as peak value factor and E acts as curvature factor. In order to find each and every coefficients there are sub equations that needs to be solved and in order to solve that, there are various parameters that needs to found.

Fx, Dx, Cx, Ex, Kx and Sx are the coefficients that needs to be calculated in order to find the Longitudinal Force. Similarly there are other similar coefficients available for remaining equations that needs to be solved to solve the entire equation.

Taking example of one such coefficient, formula to find the value of Dx is given by

Dx= Mx*Fz

Mx= Longitudinal friction at Fz

Fz= Nominal tire vertical load

In order to find Mx and Fz, there are various other equations and parameters on needs to find. To find all the coefficients of Longitudinal Force there are there are 10 equations and 14 parameters to be solved.

Similarly, to find lateral force there are 10 equations and 18 parameters and to find self-aligning torque there are 13 equations and 25 parameters to be solved.

Pacejka tire model provided the direction to development of empirical model for automotive damper system.

CHAPTER 3: CONCEPTS AND IDEAS

3.1: Overview

The idea, as mentioned before, was to generate a regenerative damper system which would capture and store energy that would then be used as required. However in order to do that, one need to look into how a Damper system works. The basic workings of an automotive Damper system comprise the cylinders filled with hydraulic fluid and a piston separating it into two different chambers, the compression and rebound chambers. It may or may not contain spring coils and when the vehicle moves, depending on various road surfaces and abnormalities that vehicle encounters, this system will allow the suspension to compress or expand and damp a majority of the vibrations generated on the vehicle. While the Damper system is working, the displacement of the piston creates pressure between compression chambers, and the energy thus generated is converted into the form of heat due to friction between molecules. Checking all three parameters, displacement, pressure, and heat, will help give a clear idea on the factors that affect the working and properties of a Damper system.

I. Displacement:

   Displacement in suspension is the movement when shocks act upon it. The displacement causes pressure change inside the hydraulic chamber. This is an important factor due to the reason that even a small displacement leads to high impact speed. Roughly, deflection of 50mm can create an impact speed of 1 m/s in a damper[4]. The overall deflection inside the damper is very small but, due to high impact resulting from it, displacement plays an important role[4].

II. Pressure:

When the displacement occurs, the hydraulic fluid inside undergoes a change in pressure. This pressure is high in magnitude and can be used to generate energy. Even while utilizing pressure, there are two fluids which experience pressure difference. Among all the three factors, magnitude of pressure is very high. Thus, effective use of pressure may be the key to having an efficient regenerative damper system.

There are two main fluids in a damper system that undergoes constant pressure change, even when the vehicle is under constant acceleration. One is the hydraulic fluid and another one is the compressed nitrogen gas. Any twist and turn by the vehicle can cause pressure change inside. The hydraulic fluid inside the suspension is firstly divided by the piston. It is this piston which maintains pressure between the compression and rebound chambers. Other than the piston, there is nitrogen gas filling at the bottom which plays the role of absorbing heat and acts as a medium which helps the vehicle to adapt with suppleness and maintain the equilibrium inside the suspension. Nitrogen gas is almost six times more flexible than oxygen and helps in self-leveling in a Damper system.

III. Heat:

Heat is the first form of loss that comes into the picture when one is looking into various forms and means of capturing energy. Heat is generally produced when the suspension does its work of damping road vibrations and shocks. The main reason for heat is the displacement of hydraulic fluid when under action. When displacement occurs, the hydraulic fluid undergoes drastic change in pressure. Variation in pressure occurs frequently and this leads to heat. Displacement of the piston inside the Damper system causes the hydraulic fluid to compress. There is another chamber inside the system that opposes this compression and exerts force in the direction opposite, also known as rebound force. While this happens, the work generated is converted in the form of heat which finally gets dissipated.

While coming up with a concept of having a regenerative Damper system, the Damper systems that were looked at were a double tube Damper system and a single tube Damper system. In a double tube, there is a pair of concentric tubes and in the exterior of the annulus there are gases like air which helps bring the suspension to accommodation with the deflections and attain equilibrium[4].

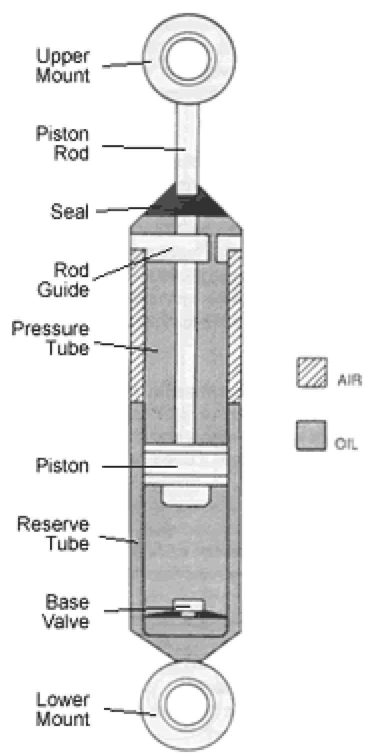

Figure 3. 1:Double tube damper system[5]

The single tube damper Damper system is used more often than the double tube due to its simplicity. This kind of Damper system has springs followed by a damper. The damper contains fluid and a piston with orifices. There are two kinds of chambers, one being the compression chamber and the other being the rebound chamber. Fluids passes through these orifices and a pressure difference is created which eventually will act to damp the vibrations.

Figure 3. 2: Single Tube Conventional damper system[6]

Having talked about the suspension types, various concepts were envisioned for both the suspension types. While thinking about designing a regenerative Damper system, the parameters taken into account were:

I. Optimum Ride Quality
II. Maximum Energy Recovery

Once the Damper system is understood, the next step is to have an evaluation done of the energy generated inside the Damper system. This will give an idea regarding whether there is adequate energy generated inside the Damper system or not, and also whether it is feasible to recapture the energy or not. A detailed explanation on energy inside Damper systems is done in the next chapter, but, to give a brief idea, the idea of generating an empirical model for a damper system emerged because of the equation obtained while calculating energy generated per cycle in a damper. In the equation, in order to calculate the energy generated per cycle inside the damper system, the damping coefficient was kept constant throughout.

This theory that energy in a damping coefficient will remain constant is not possible in an actual damper system. When work is done, due to constant compression and expansion of hydraulic oil, the damping coefficient will change with time. It was this truth that encouraged the development of a formula, a model that can predict the damping coefficient at every instant of time. And although it will take years of practice to come up with such a model, a model that can predict and give different shapes replicating characteristics of a Damper system is the primary goal of this thesis.

3.2: Idea of an Empirical Model

The general equation for calculating energy inside a damper system is given by the formula:

Energy in suspension per cycle is given by

$$\mathbf{Ec = 2\pi^2 * Cd * X^2 * f} \qquad (3.1)$$

Here, Cd is the damping coefficient, f is frequency and X is the amplitude. While doing the calculation, the damping coefficient was kept constant and energy was calculated. In actuality, the damping coefficient will never remain constant. The values will change constantly with time. It was this fact that laid that foundation of this thesis.

This thesis, which initially started with the development of a regenerative damper system that would have the ability to store energy in the form of electrical energy or mechanical energy just like Kinetic Energy Recovery System did, was totally diverted to coming up with an empirical model that has the ability to generate damper curves and give exact values regarding energy generated inside the damper system. To start with, research was done to determine whether any such model exists for a damper system which has the ability to give proper values regarding damping coefficient and performance characteristics and it was found that no such empirical model exists. Practically, a Shock Dynamometer is used to check the force generated by a Damper system, as shown in figure 3.3.

Figure 3. 3: Shock Dynamometer[7]

A motor is attached to the damper system, and there are sensors and load cells attached to the system. Analytical software like MatLAB or LabView will then convert the signals obtained from these sensors and give data regarding forces acting on these systems. Figure 4 shows a working diagram of a Shock Dynamometer.

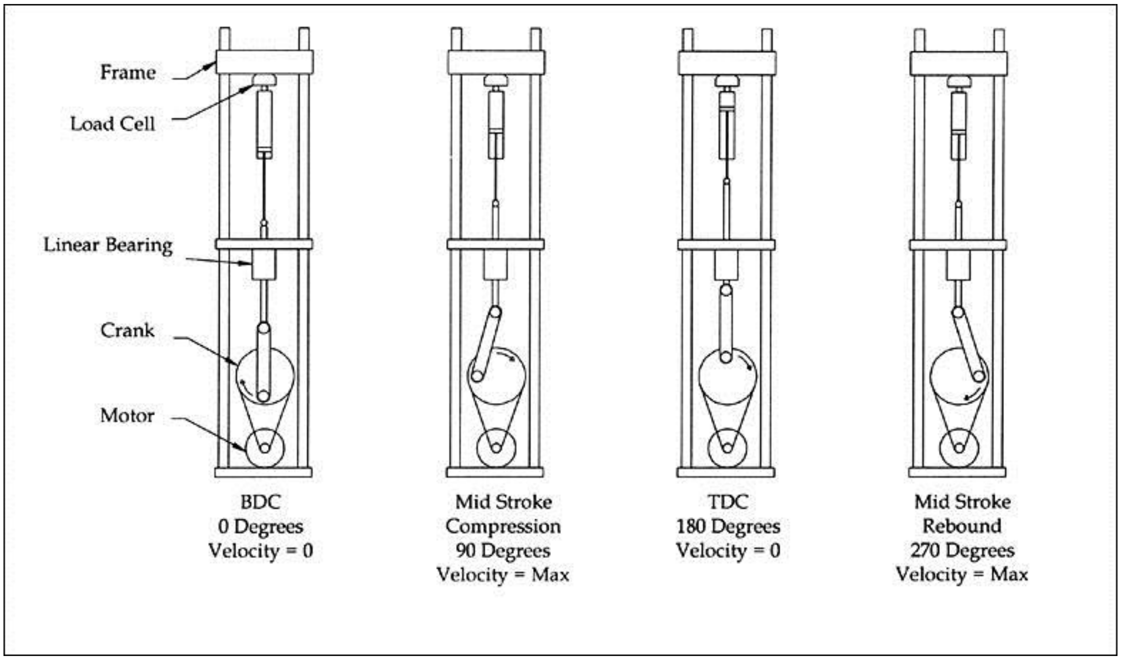

Figure 3. 4: Working of Shock Dynamometer[8]

In this manner, it was possible to get values regarding the force acting on the damper system in both compression and rebound chambers. Once the values are obtained, the damper curve will give information regarding the nature of the Damper system. The performance of the damper system will project three main characteristics. These characteristics can be shown in the form of graph and mainly 3 types of graphs are obtained. A Progressive Damper Curve, Linear Damper Curve and Digressive Damper curve. Detailed explanations regarding each damping curves are given in the coming chapter. Depending upon the usage and application, suspension properties are changed which will give either of the three above mentioned graphs.

The graph one can obtain from these Shock Dynamometers will look like the following:

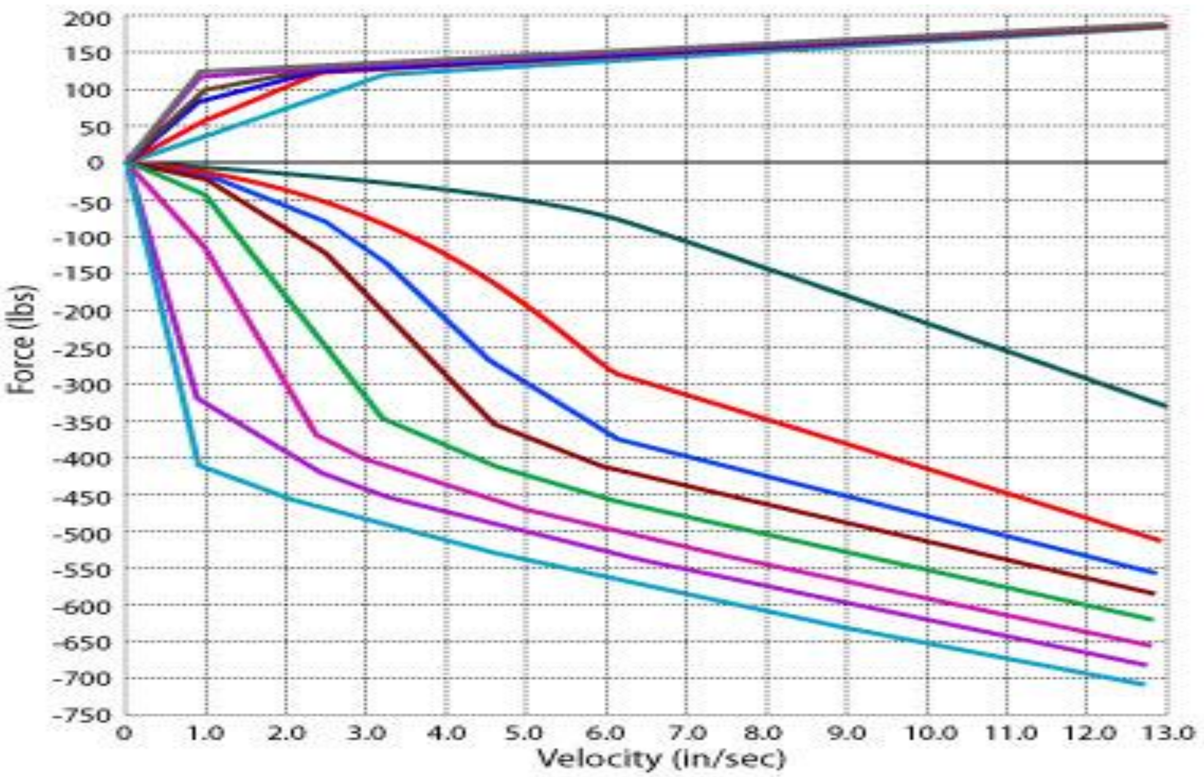

Figure 3. 5: Example of damper curves generated with a shock dynamometer[9]

The graph shows force generated inside a damper system for a range of velocities for both compression and rebound chambers. However, theoretically, when calculating, the graph obtained will be a linear curve because of constant terms in the energy equation. The idea to develop and empirical model for Damper systems came from the desire to bring these theoretical calculations closer to reality. This model will be able to predict the behavior and characteristics for any range of velocity. Further details regarding empirical modeling and detailed explanations are given in coming chapters.

However, in the world of automotive tires, there is an empirical model that accurately predicts the relationship between longitudinal force vs. slip ratio, lateral force vs. slip angle and self-aligning torque vs. slip angle. These three are the basic characteristics important for tire manufacturing. Developed by Hans B. Pacejka and his team from Delft University, Netherlands, the Pacejka Tire Model accurately predicts the relation between velocity and forces like lateral and longitudinal, and also between velocity and slip angle.

The model developed is:

$$\mathbf{Y(x) = D * sin(C * arctan\{BX - E\{BX - arctan(BX)\}\})}$$

The model was a set of trigonometric functions like sine, arctans and some constant parameters like B, C, D and E. These constants were chosen based on various tire properties like whether the tire is radial ply or biased ply, thread properties, width, and rim diameter. For each property, there were a set of values that could be used to obtain the final value. For each parameter, there were sub-formulas which were, again, a set of trigonometric functions. Using this, one was able to obtain the required tire properties.

Because of this, Pacejka Tire Model was taken as a starting point for a model created for a Damper system. The developed model is going to generate shape factors and will give a range of values which, when used, will show three characters of a Damper system. However, as mentioned earlier, it is just the first step towards developing a full-fledged model which, along with damper curves, will be able to tell the amount of energy generated at that velocity.

CHAPTER 4: ROAD AND ENERGY

4.1. Overview

Before one comes to a conclusion of how to recover energy, one needs to know how much energy is generated between road and suspension. When a vehicle is driven, any irregularity or turn which tends to cause deflection can be considered as work done by the suspension. The work is considered to be done by the suspension between the time at which the suspension moves from its initial position due to forces acting upon it to the time at which it comes back to resting position. Note that, during this one complete process, net work done by the suspension is zero as the initial and final displacement is the same. If the vehicle is moving on uneven terrain, the overall work done will be different. There will be some change in work (ΔW) acting on suspension in addition to the required work (W). Research claims that there can be as much energy as 200 watts in a car or as little as 10 watts on a straight, even road. Also, this value can change with terrains, speed, overall vehicle weight, and driving style. Depending on these factors, the damper curve is predicted. There are three types of curves.

I. Progressive Curve
II. Digressive Curve
III. Exponential Curve

Each damper curve depicts some performance characteristics of Damper system. The following gives detailed explanations of the three damper curves.

Progressive Damping Curve

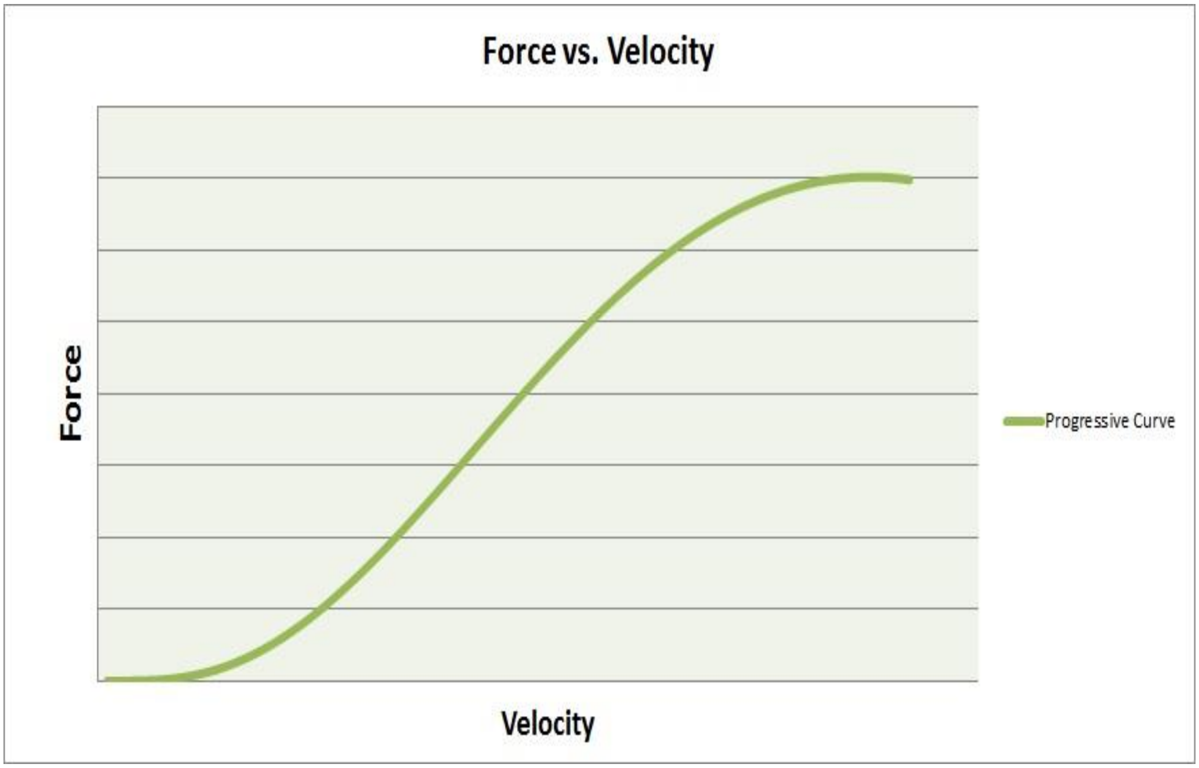

Figure 4. 1: Progressive Damping Curve

A progressive damping profile becomes progressively stiffer as suspension velocities increase. A progressive damping profile produces a compliant low speed suspension that is good for rocks, roots and ruts, at the expense of potentially vague handling when entering or exiting a turn. A progressive damping profile produces high damping forces at low suspension velocities and reduced damping forces at high speed.

Features: Low damping rates at high suspension velocities allow the suspension to float through the rough sections of the course, partially relieving the strain on the Damper system. High damping rates at low suspension velocities cause the suspension to rapidly settle as soon as the vehicle exits a rough section. More importantly, stiff low speed damping gives the driver more road feel and feedback when entering or exiting a turn and provide the stiff low speed damping rates needed to damp the chassis and the reduced high speed damping forces needed for damping the un-sprung mass of the wheels.

- Drawbacks: As velocity tends to increase, suspension will get progressively stiffer, thus giving little feedback to driver when entering or exiting turns at high speed. Handling due to this will be vague and high vibration will be felt by the driver or occupant.

Linear Damping Curve

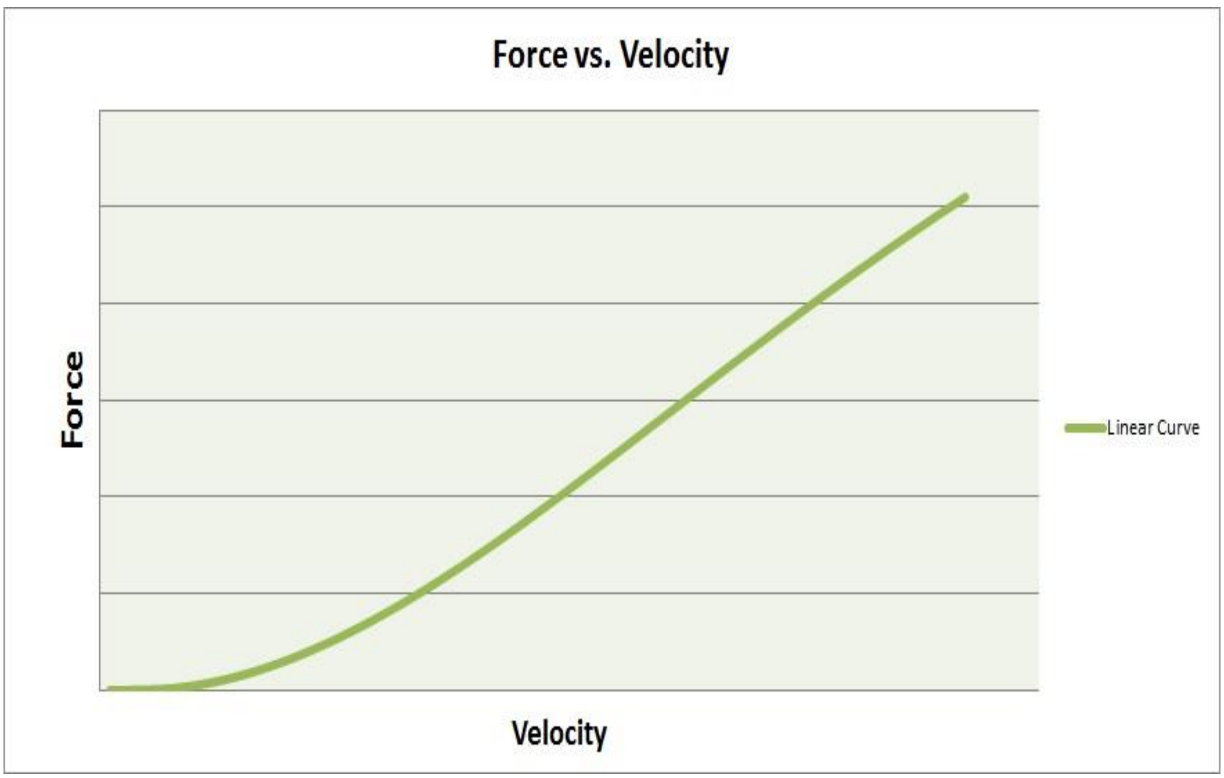

Figure 4. 2: Linear Damping Curve

Features: Maintains a constant relationship between damping force and momentum of the sprung and un-sprung suspension components across the entire range of suspension speeds. This gives the suspension a consistent feel through the entire stroke. Starting from a linear damping curve if the suspension has a tendency to tuck on corner entrances, you can run the clickers a couple of clicks in to produce a digressive damping profile and correct that behavior. Conversely if the suspension is too stiff at low speed and deflects off of rocks, you can run the clickers a couple of clicks out to produce a progressive damping profile.

Digressive Damping Curve

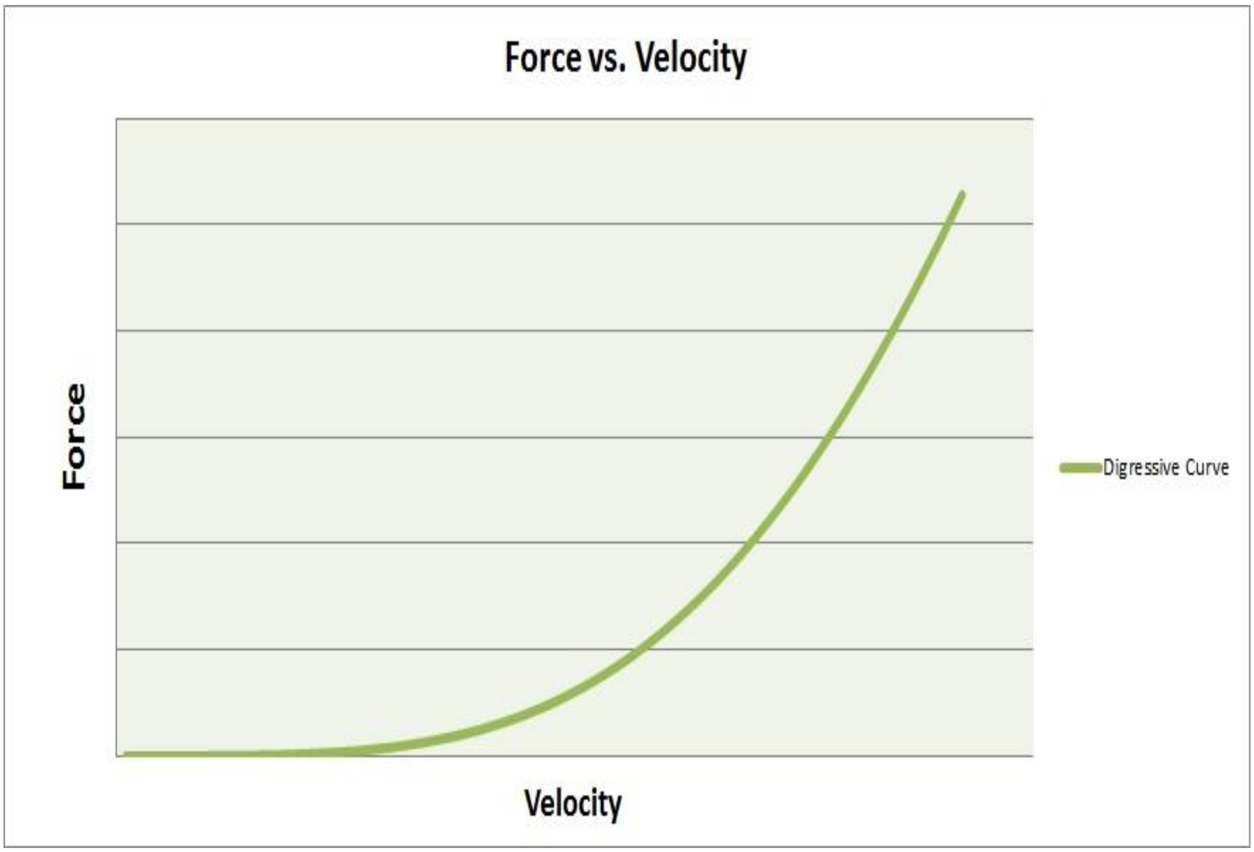

Figure 4. 3: Digressive Damping Curve

A digressive curve is generated by a damper that has the property to be stiff initially and as the suspension velocity increases, it gets progressively soft. These kinds of damper systems are used for application where vehicle requires better handling with speed.

4.2: Energy in Suspension [4]

For a normal sinusoidal profile of the graph of a simple linear damper, one can calculate the energy generated per cycle. In this, we are assuming that the damping coefficient is Cd. Let the stroke length be S and amplitude S/2.

Though we are considering the damping coefficient to be constant, one should understand that in a practical scenario, C_d is a function of V(t) where V is change in velocity with time. But, as the velocity is kept constant and in order to simplify the calculations, Cd is treated as constant. Also, we are neglecting the initial jerk acting on suspension due to road shocks as it is very low in magnitude.

Assuming Amplitude $\mathbf{X = S/2}$

Natural Frequency is given by $\boldsymbol{\omega = 2 * \pi * f}$……………………………………………….4.01

Assuming the wave to be sinusoidal and assuming amplitude to be a function of sine:

$\mathbf{X = Xo * sin\boldsymbol{\omega} t}$ ……………………………………………………………………….4.02

Differentiating amplitude w.r.t. time

Velocity $\mathbf{V = Xo * \boldsymbol{\omega} * cos\boldsymbol{\omega} t}$ ………………………………………………………….4.03

So, Force exerted in a suspension is product of damping coefficient and velocity at that time.

Therefore,

$\mathbf{Fd = Cd * Xo * \boldsymbol{\omega} * cos\boldsymbol{\omega} t}$ ……………………………………………………………….4.04

For a full cycle of work, energy produced is a suspension is:

$\mathbf{Ec} = \oint \mathbf{Fd} *$

$\mathbf{dx}$……………………………………………………………………………………4.05

=$\oint$ $\mathbf{Cd} * \boldsymbol{\omega} * \mathbf{Xo} * \mathbf{cos\ \omega t} * \mathbf{dX}$…………………………………………………………………………

4.06

Velocity $\dot{X} = \mathbf{dX/dt}$

So, Replacing $\mathbf{dX\ by\ V} * \mathbf{dt}$ in equation 4.06,

$\mathbf{Ec} = \oint \mathbf{Cd} * \mathbf{x^2} * \boldsymbol{\omega}^2 * \mathbf{cos^2\ \omega t} * \mathbf{dt}$………………………………………………………………..4.07

Further development of the above equations is discussed in next section.

4.3: Damping Coefficient and Deriving Empirical equations

Continuing with equation 4.07, two different scenarios were encountered. There can be two different cases which are addressed in the following subsections and based on that, there are empirical relations developed which can be utilized to calculate energy from any Damper system. Once, the energy is calculated, one can determine how much energy is available to utilize and how much of that it is actually possible to regenerate by eliminating all the losses and frictional forces generated inside it.

From equation 4.07, there can be two different cases that can be addressed.

I. CASE (A):

Cd is constant and the equation can be simplified.

So, equation 4.07 will be,

$$Ec = \oint Cd * x^2 * \omega^2 * cos^2\,\omega t * dt$$

$$=Cd*\omega * X^2 * \oint cos^2\,\omega t * d\omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.08$$

Using identity $\cos^2 \omega t = 1 - \cos 2\omega t$ and expanding,

$$Ec = Cd * X^2 * \omega * \pi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.09$$

$$Ec = \oint Cd * X^2 * \omega^2 * cos^2\,\omega t * dt$$

Using equation 4.1,

$$Ec = 2\pi^2 * Cd * X^2 * f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.10$$

So, based on eqn. 4.10; Suspension energy is a function of damping coefficient, amplitude and frequency.

But, as it is clearly understood that the damping coefficient cannot be constant and it is a function of velocity, this leads directly into CASE (B).

II. CASE (B)

In this case, Cd is considered a function of time and further expansion of the equation is done.

Re-writing equation 4.08 we will get,

$$Ec = Cd * \omega * X^2 * \oint cos^2\,\omega t * d\omega t$$

The energy thus generated inside suspension system can be divided into two different parts based on the damping coefficient. One part is where damping coefficient Cd is constant and another part is where damping coefficient is a function of time.

PART-1:

Considering Cd= A sin(B*dx/dt), we are assuming Cd is a velocity and sinusoidal.

The idea of using such an equation came from Pacejka's Empirical Tire Model, where the tire curves were derived based on equations with constants. Pacejka's Tire Model is used on any kind of tires.

$$\boldsymbol{Ec = \omega * X^2 * \int_0^{2\pi} Asin\left(B * \frac{dx}{dt}\right) * cos^2\, \omega t * d\omega t} \quad \text{....................................4.11}$$

Where A and B are constants for the given equation.

$$\boldsymbol{Ec = \frac{1}{4} * A * \omega * X^2 * \int_0^{2\pi} 4 * sin\left(B * \frac{dx}{dt}\right) * cos^2\, \omega t * d\omega t} \quad \text{..............................4.12}$$

$$\boldsymbol{Ec = \frac{1}{4} * A * \omega * X^2 * \int_0^{2\pi} (sin(BX\omega cos\omega t + 2\omega t) + 2\, sin(BX\omega cos\omega t) + sin(BX\omega cos\omega t - 2\omega t))\, d\omega t} \quad \text{..........4.13}$$

$$\boldsymbol{Ec = \frac{1}{4} * A * \omega * X^2 * \int_0^{2\pi} sin(BX\omega cos\omega t + 2\omega t)\, d\omega t + \int_0^{2\pi} 2\, sin(BX\omega cos\omega t) + \int_0^{2\pi} sin(BX\omega cos\omega t d\omega t - 2\omega t)\, d\omega t} \quad \text{..4.14}$$

The above equation can be solved by Bessel Functions.

Using identity $\mathbf{Jn(x) = \frac{2}{\pi} * \int_0^{\pi/2} cos(xsin\theta - n\theta)\, dx}$ and $\mathbf{Jo(x) = \frac{2}{\pi} * \int_0^{\pi/2} cos(xcos\theta)\, dx}$ we can re-write the above equation as

$$\mathbf{Ec = \frac{\pi}{4} * A * \omega^2 * X^2 * [\frac{3n^2}{2}\left\{\frac{1}{2!} - \frac{n^2}{2^2 * 1! * 3!} + \frac{n^4}{2^4 2!\,4!} - \frac{n^6}{2^6 3!\,5!} + \cdots\right\}}$$

$$\mathbf{+\left\{2n - \frac{n^3}{2 * 1!\,2!} + \frac{n^5}{2^3 * 2!\,3!} - \frac{n^7}{2^5 3!\,4!} + \cdots\right\}]}$$

Where, n = B*ω*X ……………4.15

PART -2:

$\mathbf{Cd = D\ sin(arctan\,B * dx/dt)}$…………………………………………………… 4.16

Using above equation in equation 4.08,

$\boldsymbol{Ec = \omega * X^2 * \int_0^{2\pi} D\ sin(arctan\,B * dx/dt) * cos^2\,\omega t * d\omega t}$…………….………… 4.17

$\boldsymbol{Ec = \frac{1}{4} * \omega * X^2 * D * \int_0^{2\pi}\ 4 * sin(arctan\,B * dx/dt) * cos^2\,\omega t *}$

$\boldsymbol{d\omega t}$…………………….……………..…………………………………………………..4.18

$\boldsymbol{Ec =}$

$\boldsymbol{\frac{1}{4} * \omega * X^2 * D * \int_0^{2\pi}\ sin(arctan\,B(X\omega\ cos\omega t) + 2\omega t) + sin(arctan\,B(X\omega\ cos\omega t) -}$

$\boldsymbol{2\omega t) +}$

$\boldsymbol{2 * sin(arctan\,B(X\omega\ cos\omega t))\ \ d\omega t}$………………………………………………4.19

Assuming $\mathbf{B(X\omega\ cos\omega t) = z}$ and using identity $\mathbf{arctan\,z = z - \frac{z^3}{3} + \frac{z^5}{5} - \frac{z^7}{7} +}$ ...

Es==¼ $\mathbf{\omega * X^2 * D * \int_0^{2\pi}[sin\Big(\ B(X\omega\ cos\omega t) - \frac{B^3(X\omega\ cos\omega t)^3}{3} + \frac{B^5(X\omega\ cos\omega t)^5}{5} - \frac{B^7(X\omega\ cos\omega t)^7}{7} +}$

$\mathbf{\cdots + 2\omega t\Big) + \qquad sin\Big(B(X\omega\ cos\omega t) - \frac{B^3(X\omega\ cos\omega t)^3}{3} + \frac{B^5(X\omega\ cos\omega t)^5}{5} - \frac{B^7(X\omega\ cos\omega t)^7}{7} + \cdots -}$

$$\mathbf{2\omega t}\Big) + \ \mathbf{2} * \mathbf{sin}\left(\mathbf{B}(\mathbf{X\omega\ cos\omega t}) - \frac{\mathbf{B^3(X\omega\ cos\omega t)^3}}{\mathbf{3}} + \frac{\mathbf{B^5(X\omega\ cos\omega t)^5}}{\mathbf{5}} - \frac{\mathbf{B^7(X\omega\ cos\omega t)^7}}{\mathbf{7}} + \cdots +\right)\Big]\ \mathbf{d\omega t}$$

………………… 4.20

However, none of these assumptions were giving any values, so a new approach was used to find the damping coefficient. As shown in chapter 5, various models were tried and certain ranges of values were used to check the kind of graph that was able to be achieved.

CHAPTER 5: THE ROAD TO EMPIRICAL MODEL

5.1: Overview

Deriving the equation for calculating energy in a Damper system led to the idea of coming up with an empirical model that can predict exactly the shape and characteristics of the curve. When calculating the equation 4.10 for Energy in suspension, Cd was kept constant as it was discussed in the previous chapter that the damping coefficient cannot be constant. It's a function of time, but not constant. This truth and various discussions regarding this led to the idea of developing an empirical model that will accurately predict the damping coefficient and will give damper curves. The developed empirical model will show the exact behavior of suspension characteristics under various conditions. In a Damper system, work is done based on the external factor that is affecting it. There are mainly 3 reasons or factors.

i. Vehicle Terrain
ii. Vehicle Velocity (Function of Time)
iii. Vehicle weight

Depending upon the terrain, energy generated inside the Damper system can vary. A normal automotive damper curve can behave in 3 different ways. The curve can be linear, progressive or digressive.

A typical damper curve is shown in the figure below. As the figure shows, the work done by the suspension occurs when it is under compression and rebound. The work done is mainly because of displacement, oil pressure, velocity of the vehicle, nitrogen gas pressure, piston properties, frequency generated inside the Damper system, damping coefficient and spring coil.

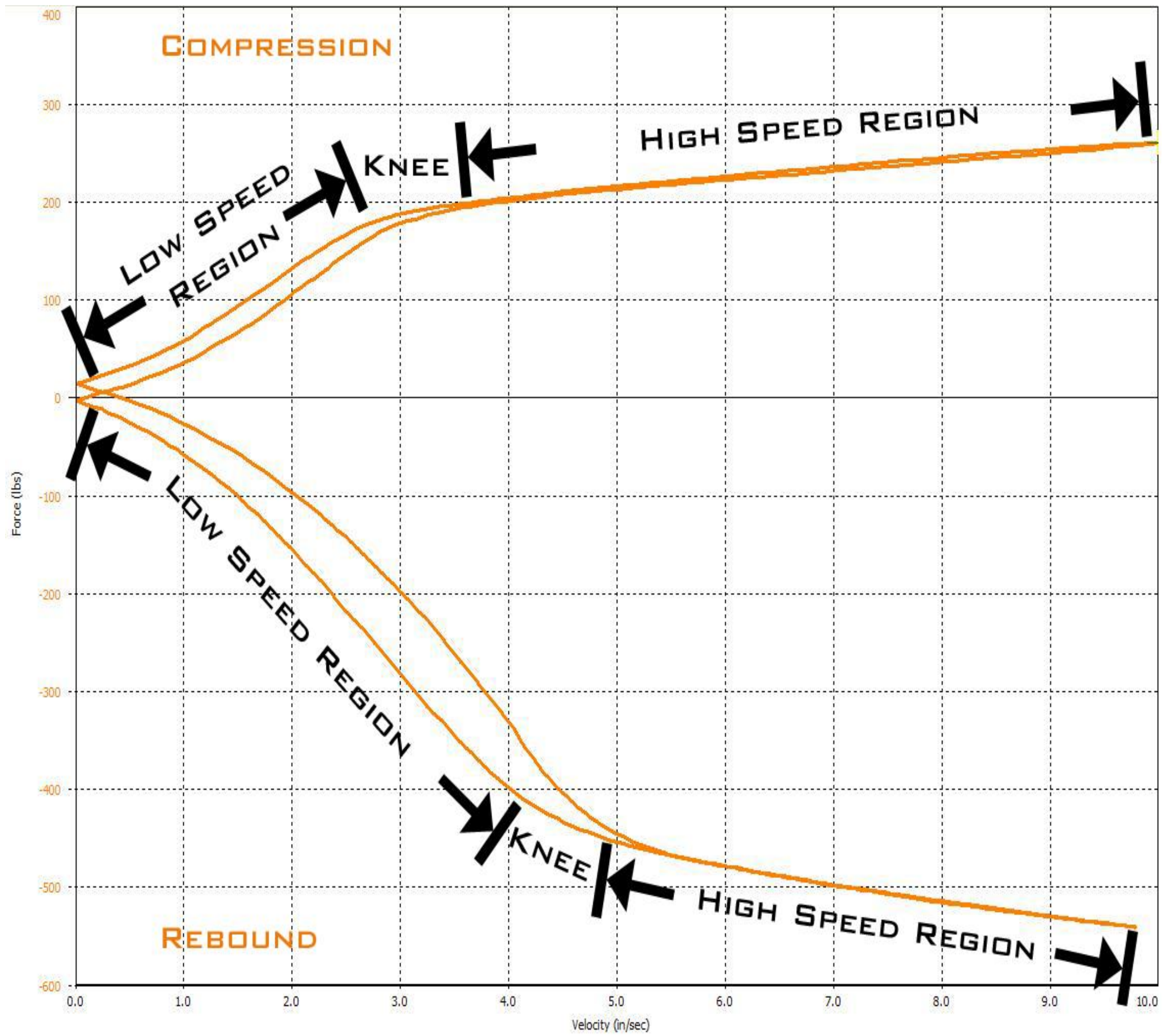

Figure 5. 1: Automotive Damper curve[10]

The objective was to develop an empirical model that can depict this property. This is just one of the three characteristics that the Damper system shows. Figure 9 shows a progressive behavior of the Damper system. The other two behaviors are the digressive curve and exponential curve. Each curve is generated because of some very specific properties or behavior that the Damper system shows. All these properties are explained in the previous chapter. The empirical model is different from the formula generally used to calculate energy inside the Damper system because the damping coefficient is now a factor of time. It will change at every instant and that will give a more accurate result and explanation regarding the behavior of the Damper system. This chapter will further discuss the development of this empirical model, including the various attempted models which lead to its finalization, the trigonometric functions used in its development, and the reason behind the model.

5.2: Empirical Model Development

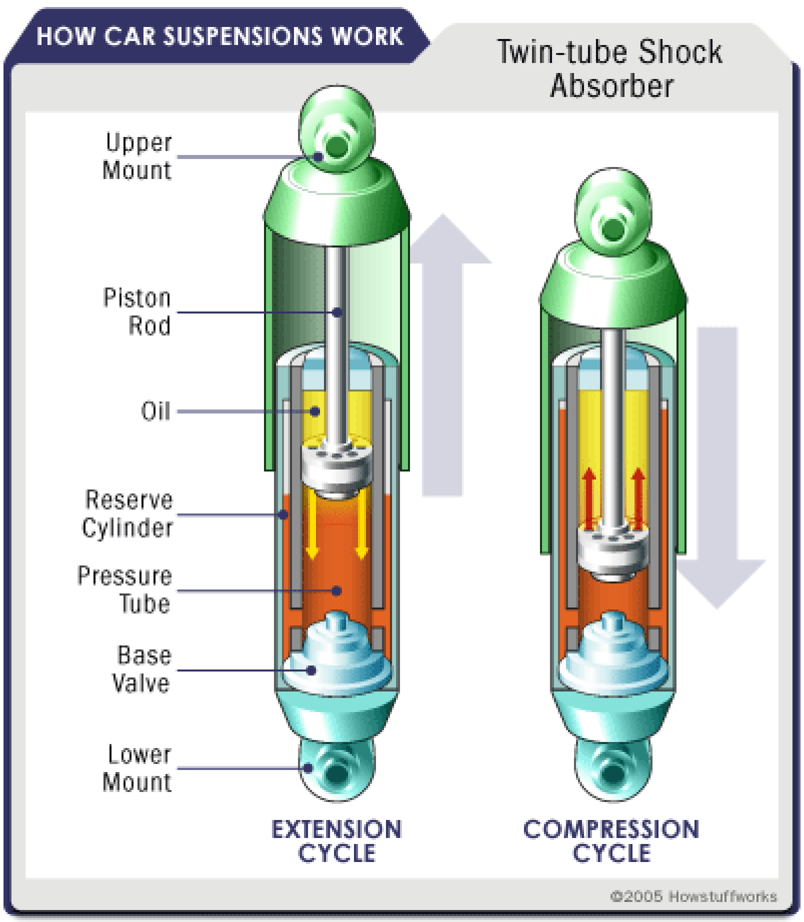

Figure 5. 2: Automotive Damper system[11]

The empirical model was for damping coefficient, Cd, which is now a function of time. Following sub-sections show different kinds of empirical model that was tried before reaching the

5.2.1: Model 1: Cd= BX*sin(C*tanhx*atan(D*(sinhx-atan*(E-BX)))

One such example of an empirical formula is shown above. The damping coefficient is shown as a function of trigonometric functions, arctan, hyperbolic functions and various constant parameters. Here, Cd is a function of X which is velocity; B, C, D, and E which are various parameters; sine function; arc tan function; and hyperbolic functions of tan and sine. For every model that has been tried and selected, these functions will be seen. Velocity is selected because that makes the entire formula a function of time. B, C, D and E will play the role of giving the required shape to the damper curve. Sine function is selected as it is an odd function and has domain between (-∞, ∞), and the range is from [-1, 1]. Arctan function is selected as it has the ability to look a lot like a damper curve. Arctan curve is shown in the following figure.

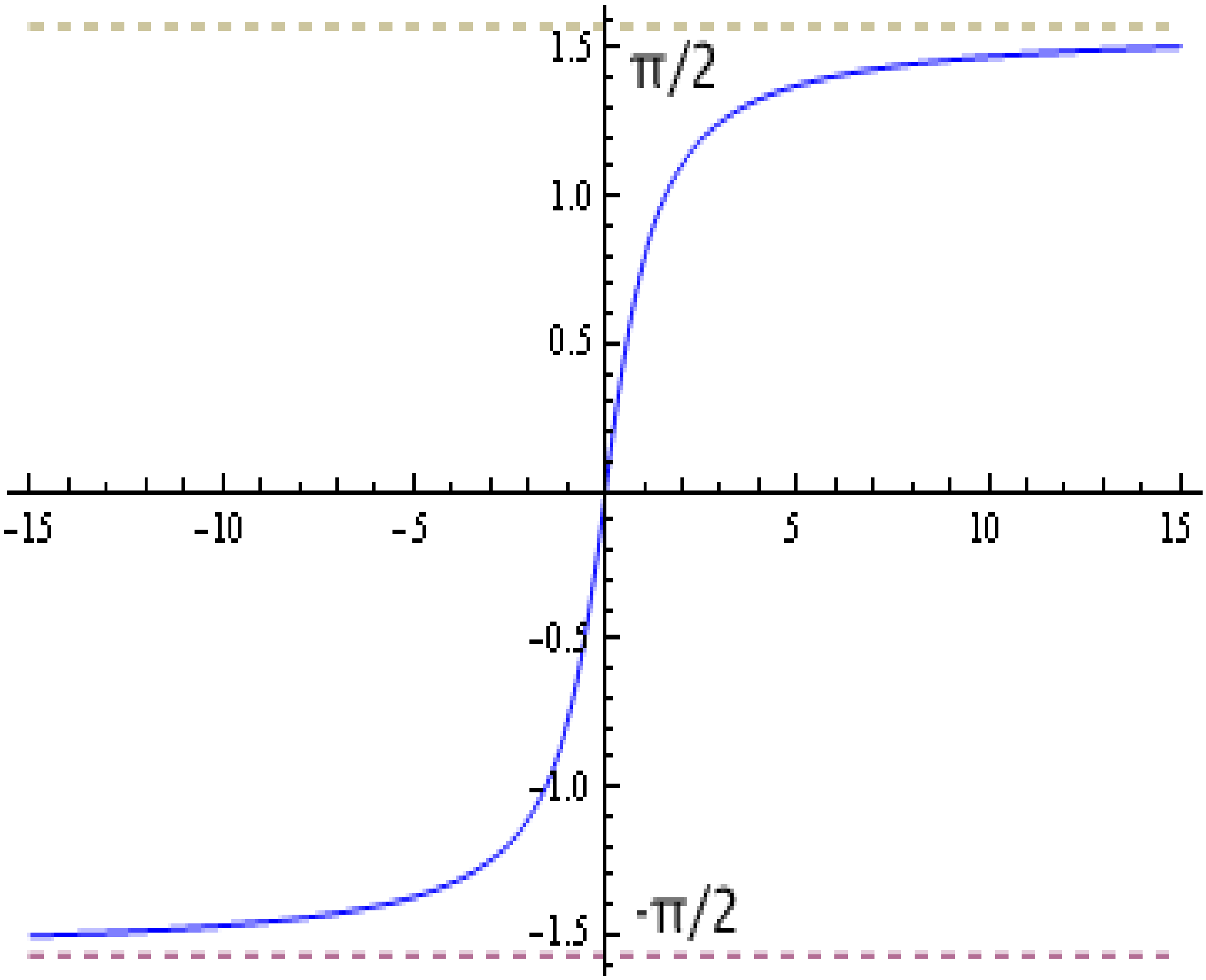

Figure 5. 3: Arctan Curve[12]

Hyperbolic Functions of Tan and Sine have the tendency to create the following shape:

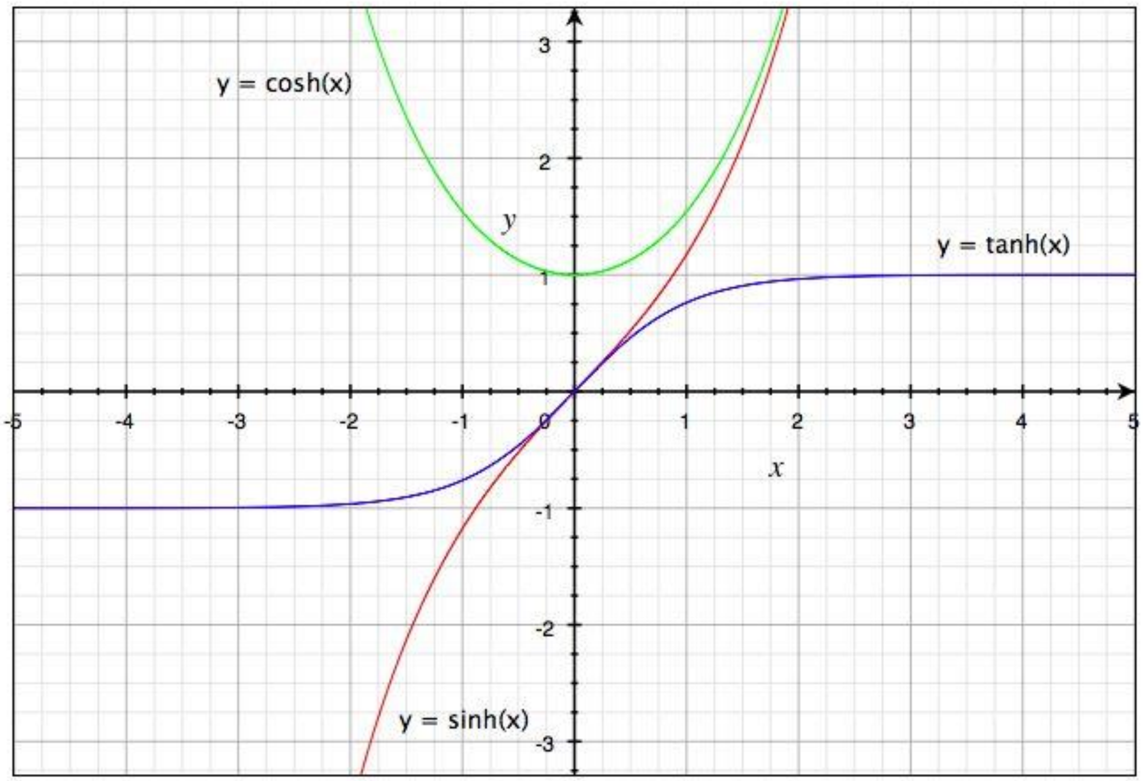

Figure 5. 4: Hyperbolic Function graph[13]

From the graph, it can be seen that both sine and tan function have characteristics resembling the damper curves. Also, sinhx has a range from (-∞, ∞) whereas tanhx has range from (-1, 1). The graph of force (N) versus velocity (in/s) needs a function which can show damper curve characteristics and also has a range that will help plot value of forces from 0 to any range.

The parameters B, C, D and E are, for now, some random values which will be used to obtain different damper curves. This is just one of many unsuccessful attempts to create an empirical formula that was tried to see the nature of the graph generated. The software used was Excel. For every empirical model, the following format of table was created.

Table 5. 1: Example of Empirical Model parameters and functions

X	BX	tanhX	sinhX	atanBX	Cd
Velocity	Constant(B) * Velocity	Hyperbolic tan of velocity	Hyperbolic sine of velocity	Arctan of BX	Damping coefficient which is equal to empirical formula based of these parameters and other constants. For example, in this case, Cd= BX*sin(C*(tanhx*atan(D*sinhx-atan*(E-BX)))

Here, values of parameters C, D and E were given externally and changing these parameters, which are real numbers, will change the shape of the curve. Also, the graph generated in such a way is the required damper curve. Now B, C, D and E can be anything. Any one of these parameters can become a shape factor, a factor affecting energy in the suspension, an offset factor, or a correction factor. Right now though, concentration is focused towards creating an empirical formula which will, irrespective of any of these factors, help create the damper curve.

Coming back to the formula, Cd= BX*sin(C*tanhx*atan(D*(sinhx-atan*(E-BX))), the following values were used to try to create the Damper system curve.

Table 5. 2: Values used in empirical formula to generate Damper curve

Values	B	C	D	E
i.	1	1	1	1
ii.	1	2	1	1

For Values of list (i), the following graph was created:

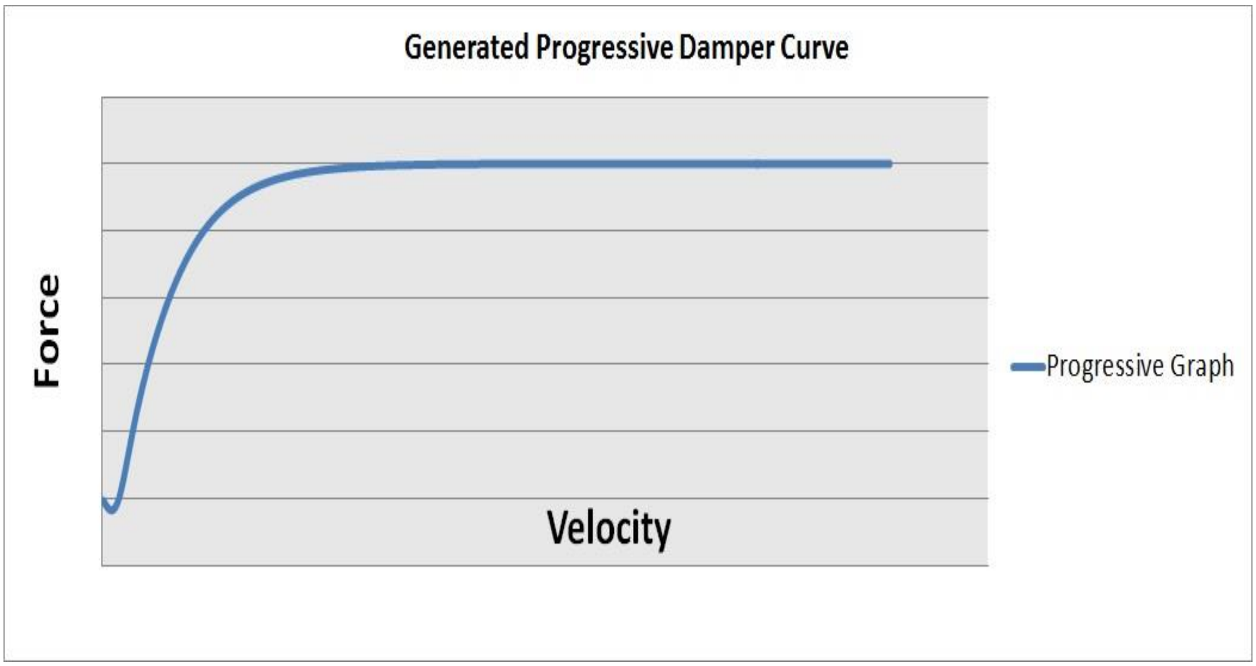

Figure 5. 5: Graph using values of B, C, D and E are 1 respectively in model 1

For, B =1, C=2, D=1, E=1, the graph obtained was:

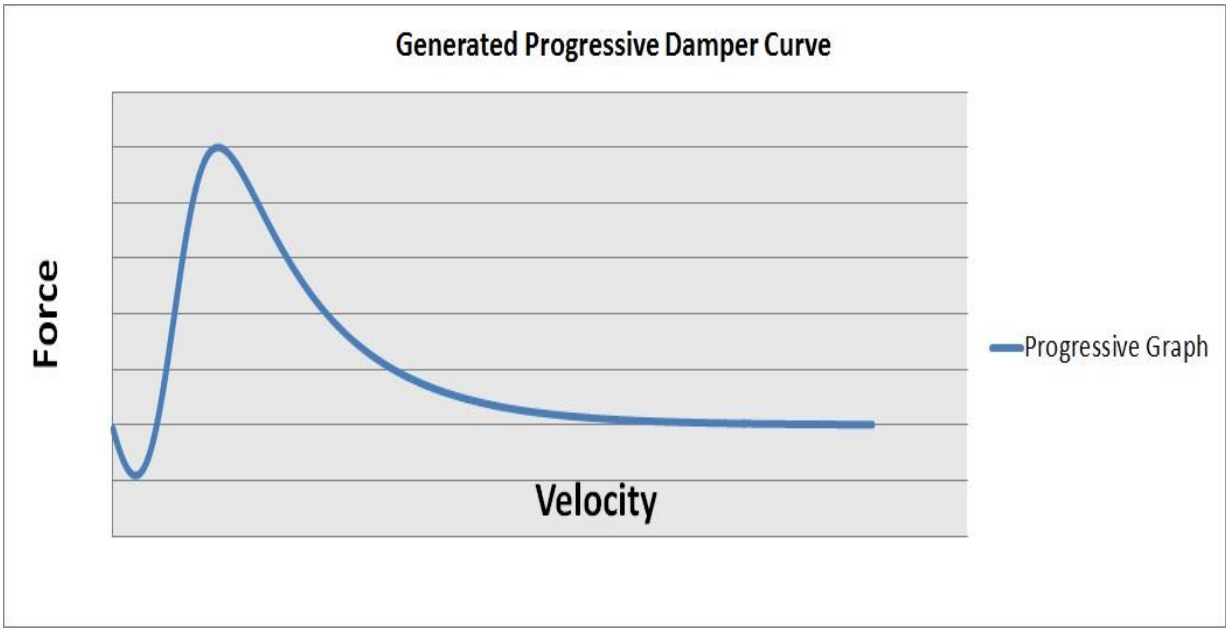

Figure 5. 6: Graph obtained for 1st Empirical Formula

If the values of B are kept around 1, the graph will be similar to what is seen in Figure 5.5. Here, abscissa is damper velocity and ordinate is longitudinal force. Note, the abscissa rises suddenly for a short period of time and then stabilizes. Similarly, if the value of B is increased from 1 to 2, and also checked for any value between 1 and 2, the nature of graph will be as shown in Figure 5.6. When the values are between 1 and 2, the abscissa reaches peak, then it starts decreasing and form a shape similar to sine wave. For values of B above 1, i.e. 2, 3, 4… the graph will be a sine wave.

This graph proves that this model is not suitable as it cannot fulfill the requirement of a proper formula which can create all three damper curves. But this formula turned out to be the first building block on the road to an empirical model which would be used to create a proper formula which will be mentioned in the coming section and will fulfill the expectations.

5.2.2: Model 2: Cd=BX*sin (tanhx*atan(C*sinhx*sinh(atan(B*atan(D*BX-atan(EX)))))

This model is a combination of velocity with tan, atan, sine and tan hyperbolic functions and parameters like B, C, D, and E, just like the first formula that was tried. As, usual the graph was created with initial values of B, C, D and E equal to unity. Then the parameter values were changed and then graph nature was analyzed. The issue with the formula was that the nature of graph was remaining either a digressive or linear curve. The progressive curve which was expected to be created was unfortunately not accomplished. Negative values and decimal values were also tried to check if at some point of time a progressive curve could be created, but in vain.

In the formula, one can see that there was utilization of arctan several times as attempts were made to create a curve that looked like a progressive curve in nature. The thought was this: the more arctans that are used, the more possibility there will be to create a curve that actually looks like a progressive damping curve, and adding some values and parameters will lead to changing that progressive curve to a digressive and linear curve.

The graph created is shown below.

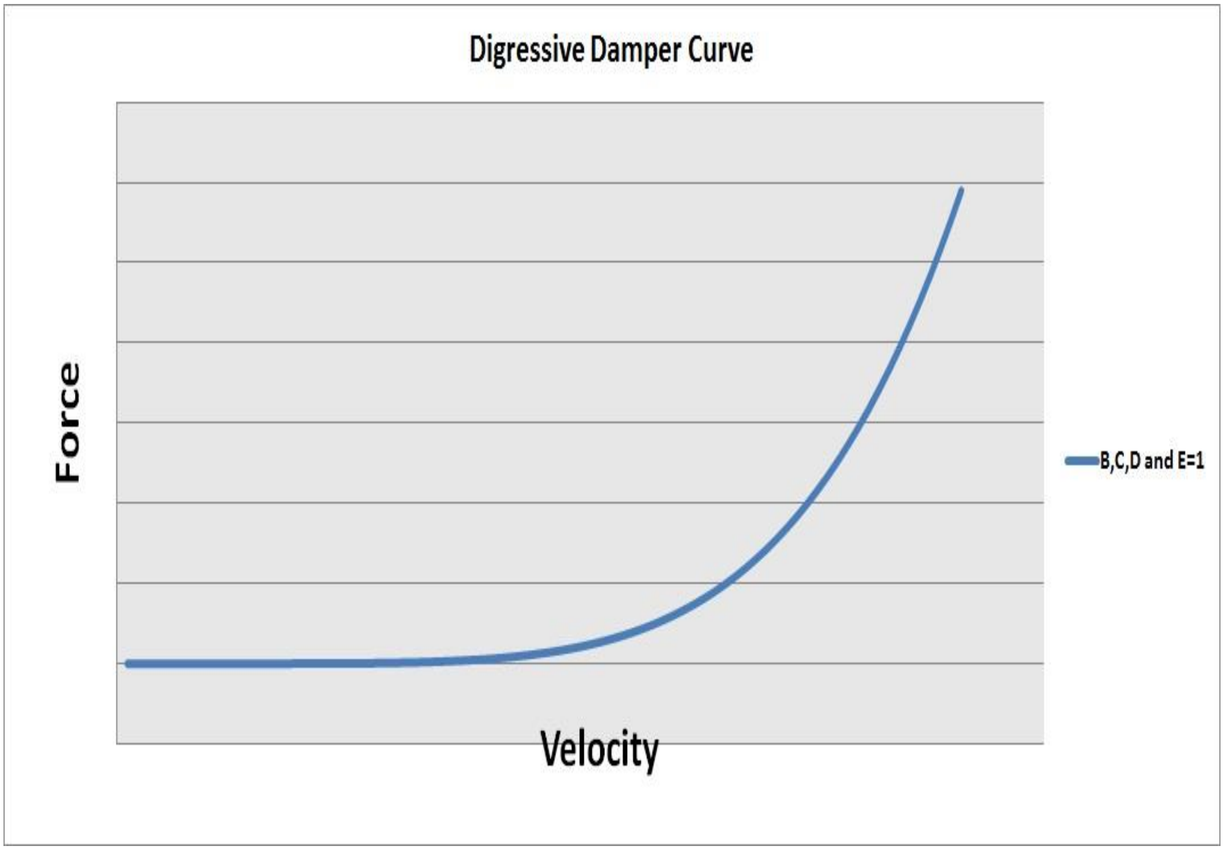

Figure 5. 7: Digressive curve created using Formula 2 with values of coefficients B, C, D and E equal to unity

The graph shows an acceptable digressive curve which is happening during the entire course of the journey of the vehicle when it is hitting maximum velocity. However, as the parameters are changed, the curve tends to go from a digressive to a linear curve. The linear curve generated by the model is as shown.

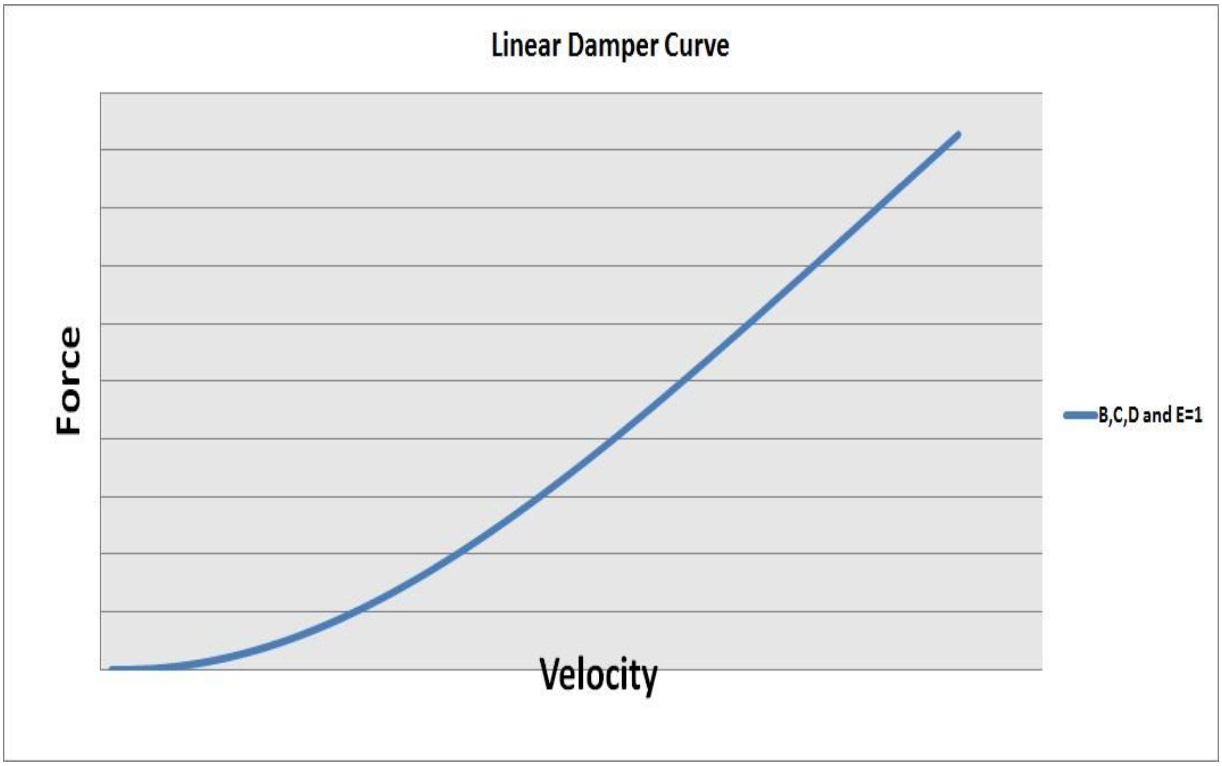

Figure 5. 8: Linear Curve generated using model

The linear damping curve is created. Therefore, this formula creates two of the three required curves, i.e. the digressive curve and the linear curve, but as further assessment of the model was made, the third curve, the progressive curve, was checked and the nature of graph didn't change. The model started with a digressive curve, then progressed to a linear damping curve and stayed there. This attempted model does not satisfy the requirement. Although this model had arctans, hyperbolic functions, sine functions, and some additional parameters and constants, and it was made a function of velocity, the combination was not a perfect fit to fulfill the purpose because of the model's inability to generate all three the of the curves.

5.2.3: Model 3: Cd= BX*sin(D*tanhx*sinh(C*sinhx*X-atan(B*Eatan(BX-E))))

This model is different from the previous two that were tried as this model does not have multiple sine hyperbolic functions. Still, just like the previous formulas, this one also falls short when it comes to creating the graphs. The issue, as before, is that the models are good enough to create two of the three curves, but due to its inability to generate third required damper curve, the model is rejected. Another behaviour that was witnessed when various permutation combinations of formulas were tried was that, as the values of coefficients were increased, they tended to generate a curve somewhat similar to Figure 5.10.

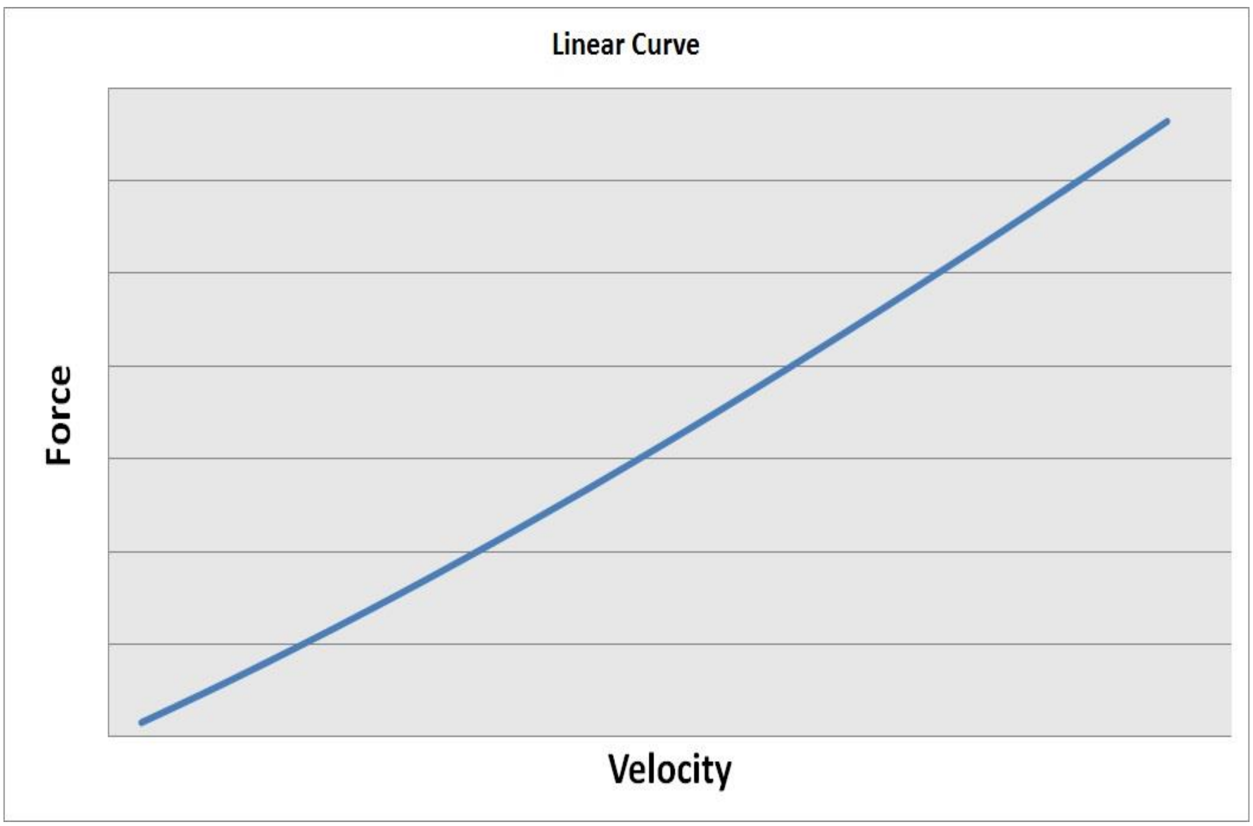

Figure 5. 9: The Linear curve created with the help of the equation developed

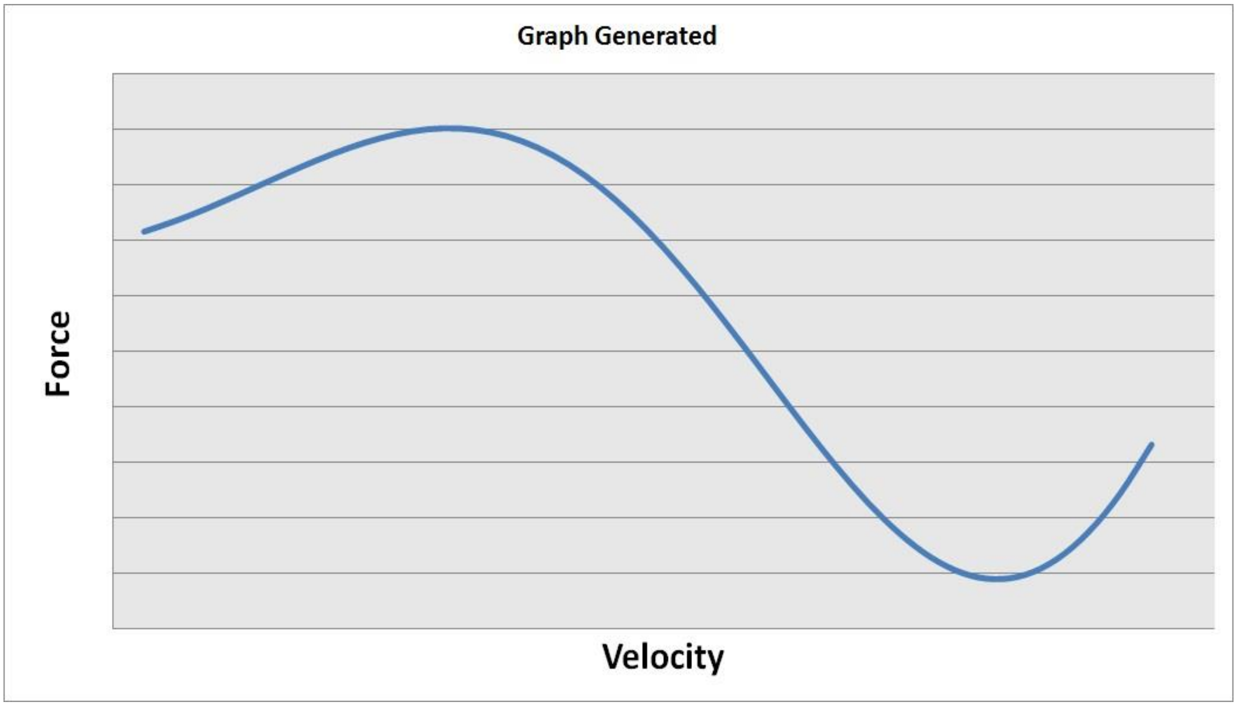

Figure 5. 10: A curve generated using the model

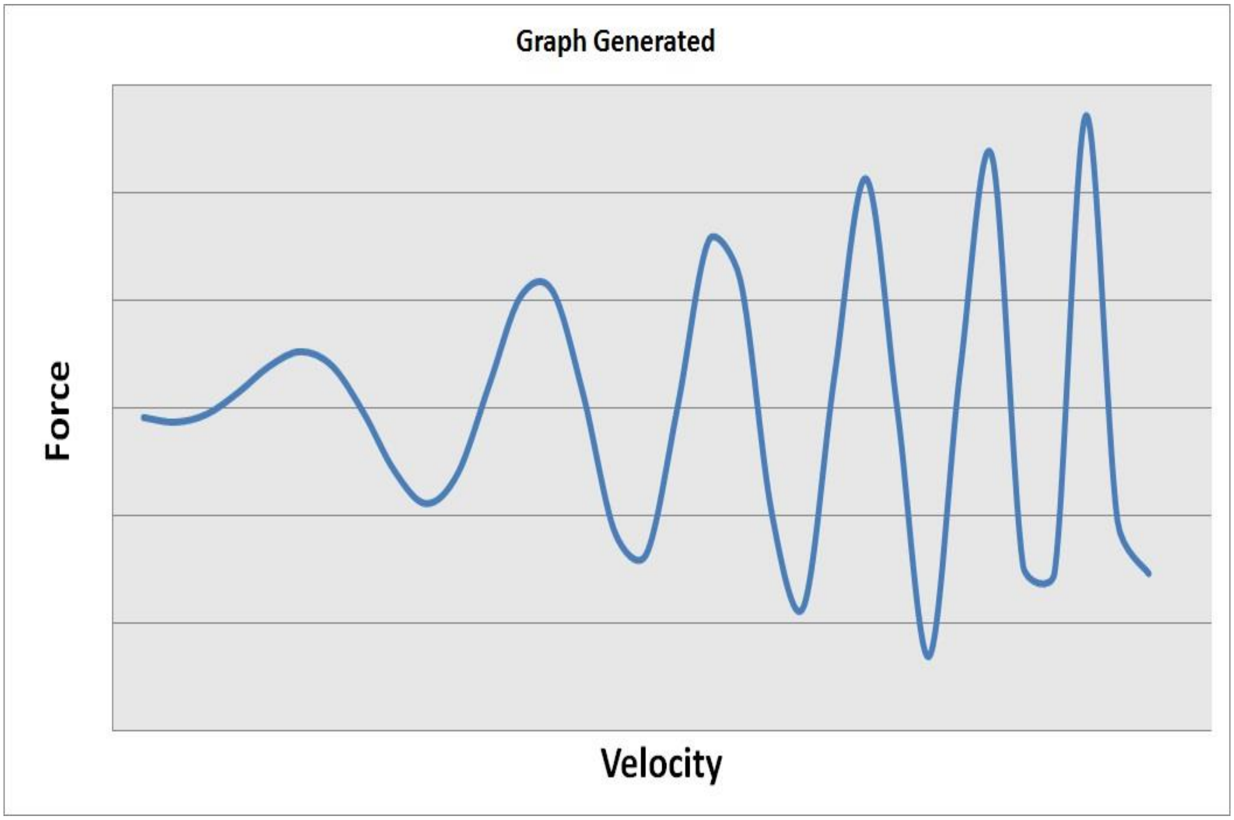

Figure 5. 11: A graph generated using the model

5.2.4: Model 4: Cd=BX*cos (Catan(D*tanhx*Atan*X-E*(BX*sinhx-atan(BX)))))

This formula, when tested, was the first formula to give a curve with properties that could be used as a reference to create a model that had the potential to generate a graph that could fulfill the goal of a damper curve. The curves achieved by this empirical model are given below. There are progressive and linear curves achieved by changing the coefficients. Then, instead of giving a digressive curve, the model generates a curve that replicates a sine curve as seen in Figure 5.13 and Figure 5.14. However, if Figure 5.13 is broken into two, one can see a curve which looks partially like a digressive curve. In the first part of Figure 5.14, where the values of the curve are between 0 to almost 42, the behavior looks somewhat like a digressive curve.

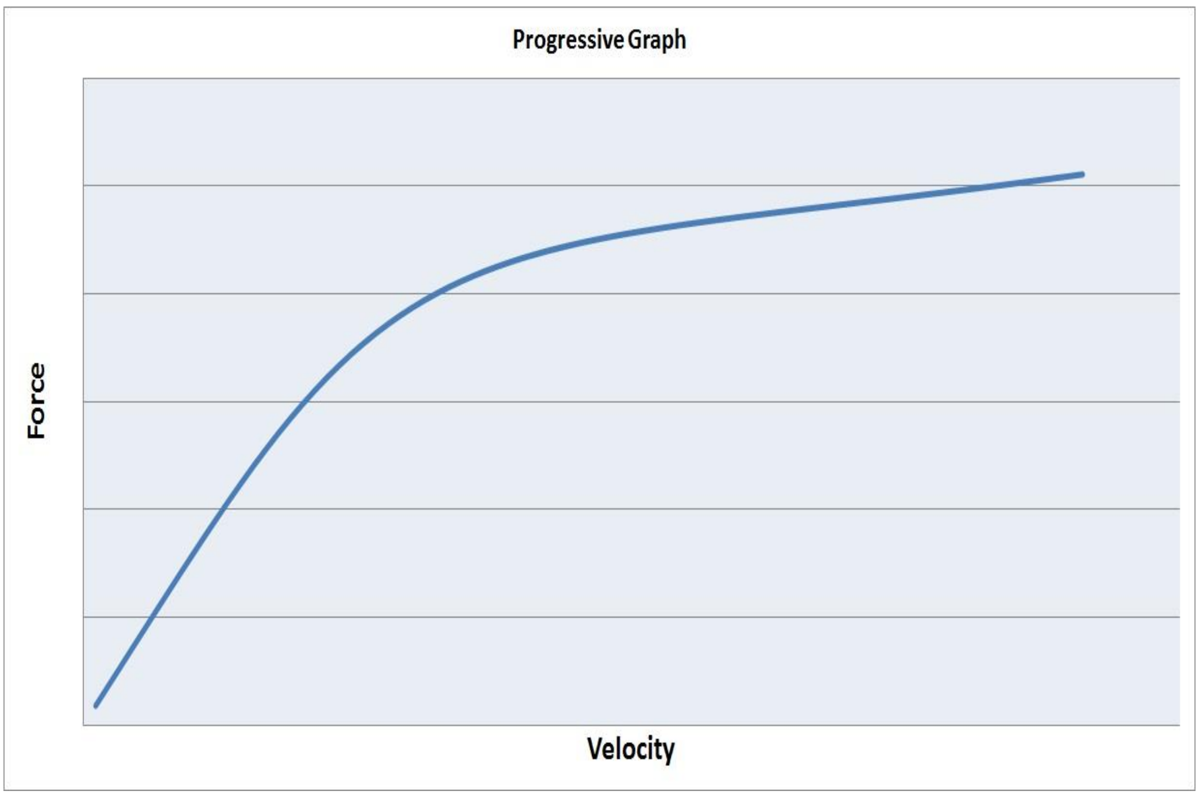

Figure 5. 12: Progressive curve generated by empirical model

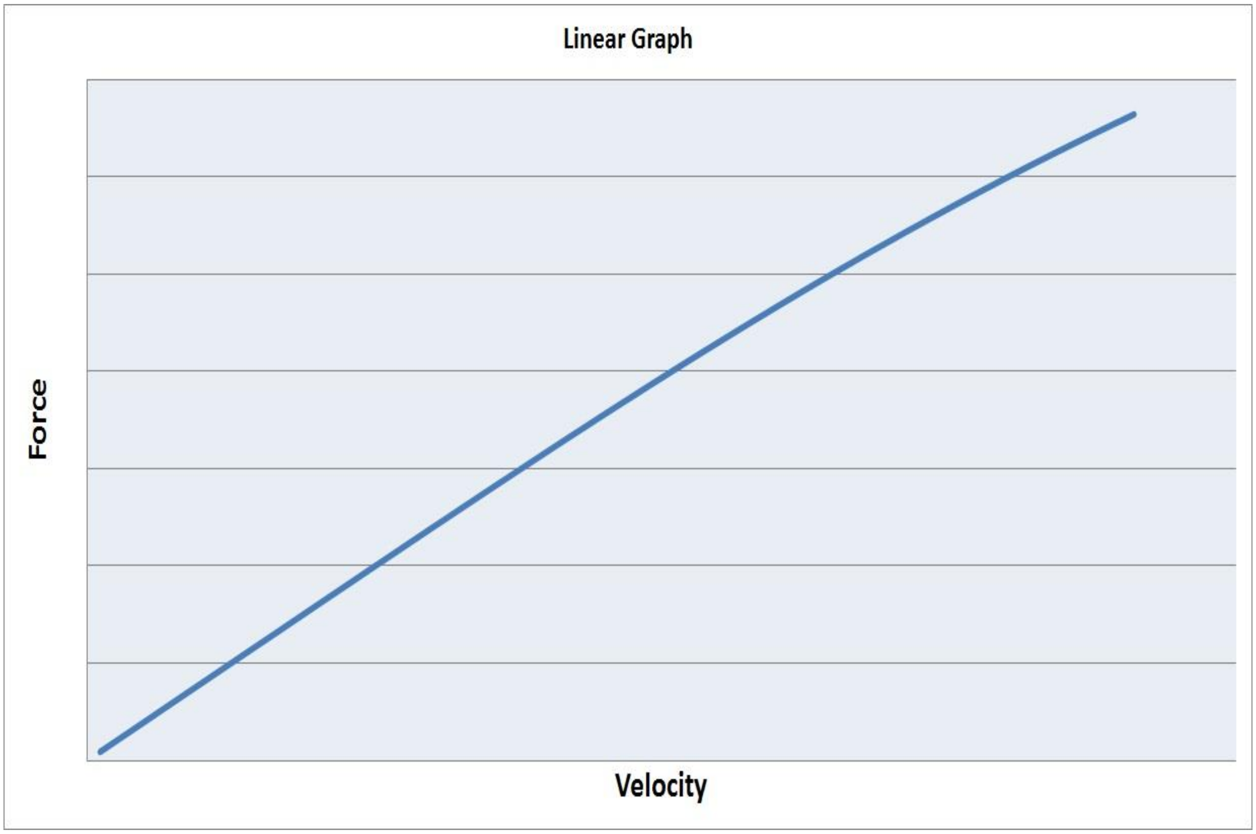

Figure 5. 13: Linear curve developed by the help of empirical model

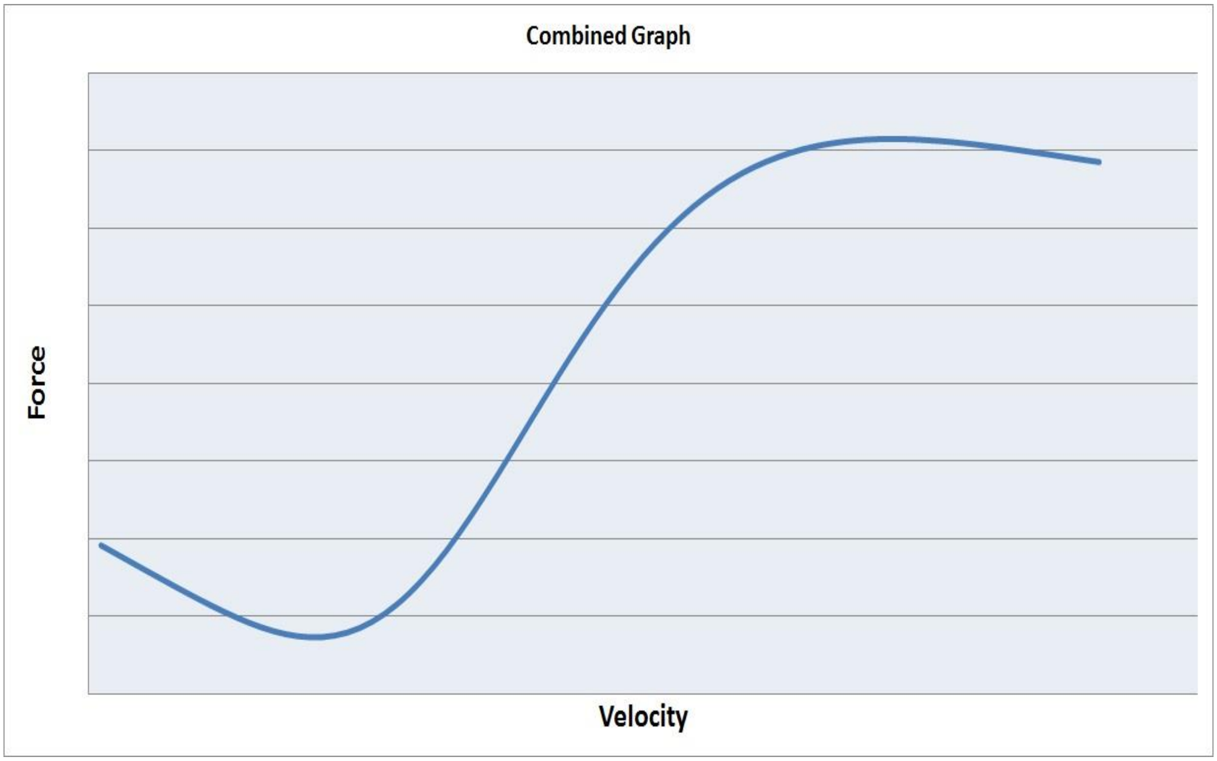

Figure 5. 14: Graph achieved with the empirical model that has properties of both progressive and digressive curve

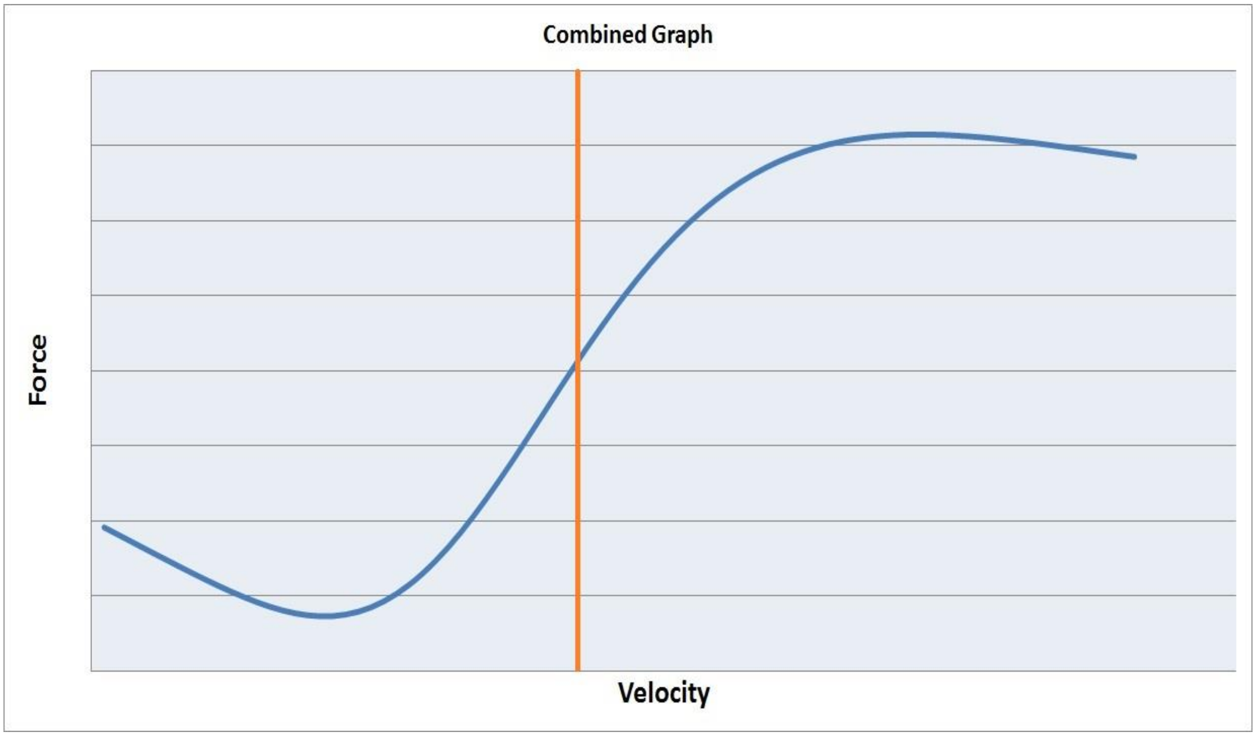

Figure 5. 15: Graph differentiating digressive and progressive curve

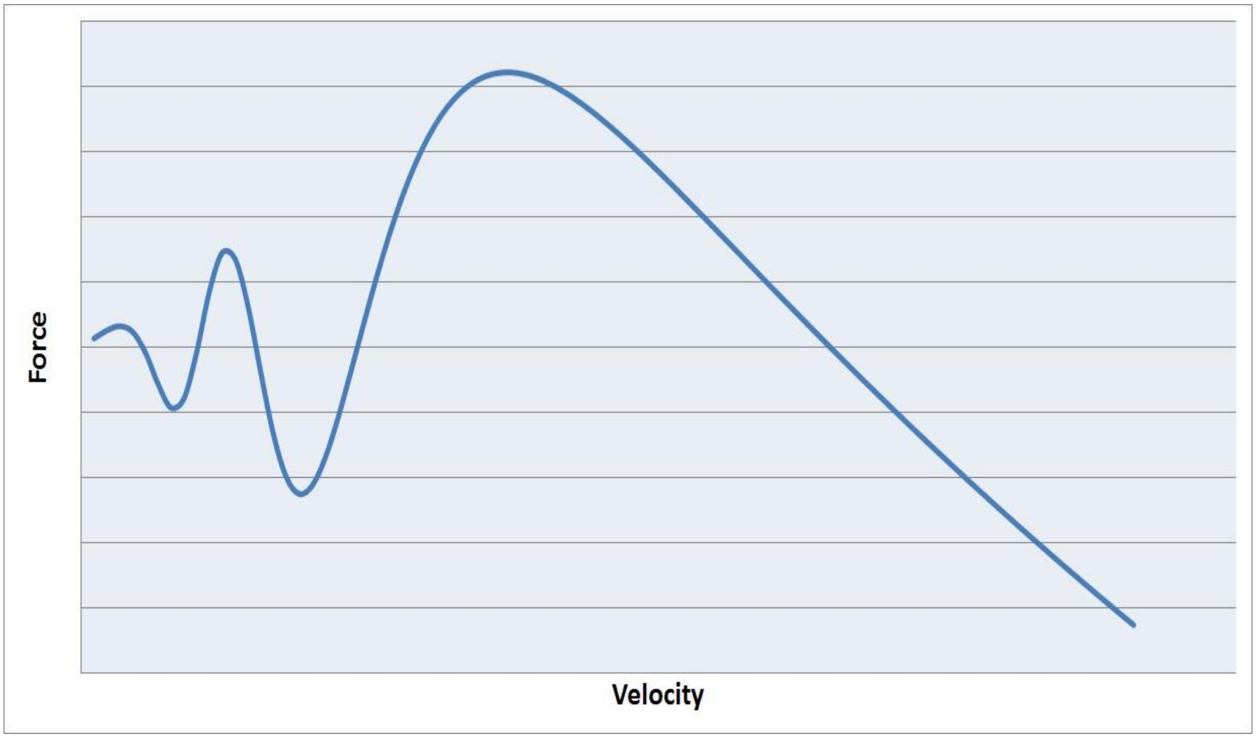

Figure 5. 16: Another graph generated with the help of the model by changing the coefficients

Figure 5.14 shows the transition of the curve to a sine wave as the coefficients are changed. Thus, although this model cannot fulfill the requirement of forming a damper curve, it can be considered a good reference.

5.2.5: Model 5: Cd= BX*sin (Catan(D*tanhx*Atan*X-E*(BX*sinhx-atan(BX)))))

The first similarity one can see between this model and the previous one is the use of sine function instead of cosine. As mentioned for the previous model, because the graphs achieved were progressive, linear and partially digressive, the use of the same equation as a reference was thought to be a good idea. Instead of using a cosine function, the function was changed to sine and the graph was created for various parameter values. The model was able to generate linear, progressive and rebound curves, but again it fell short of creating a digressive curve. When the values of B, C, D and E were 1 each, a linear curve was created. This changed to a progressive curve which was created once B, D and E values were changed to greater than 1. C was kept 1 as increasing its value tended to bring the graph towards the negative Y axis.

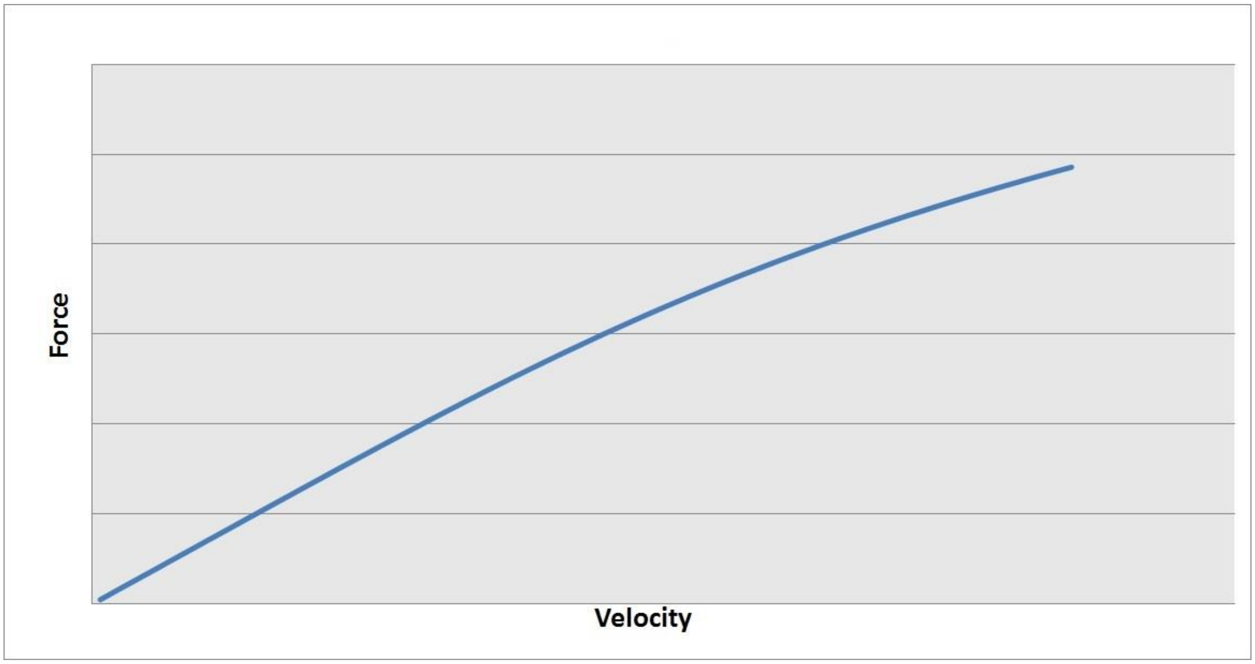

Figure 5. 17: Linear Curve

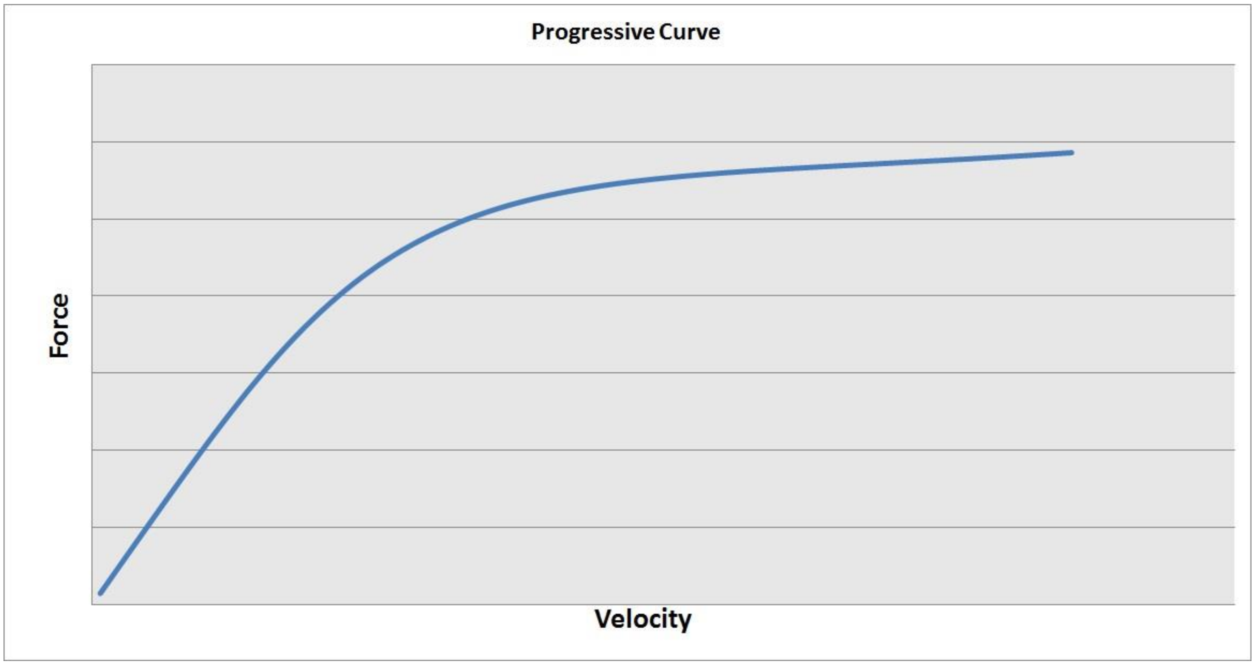

Figure 5. 18: Progressive Curve

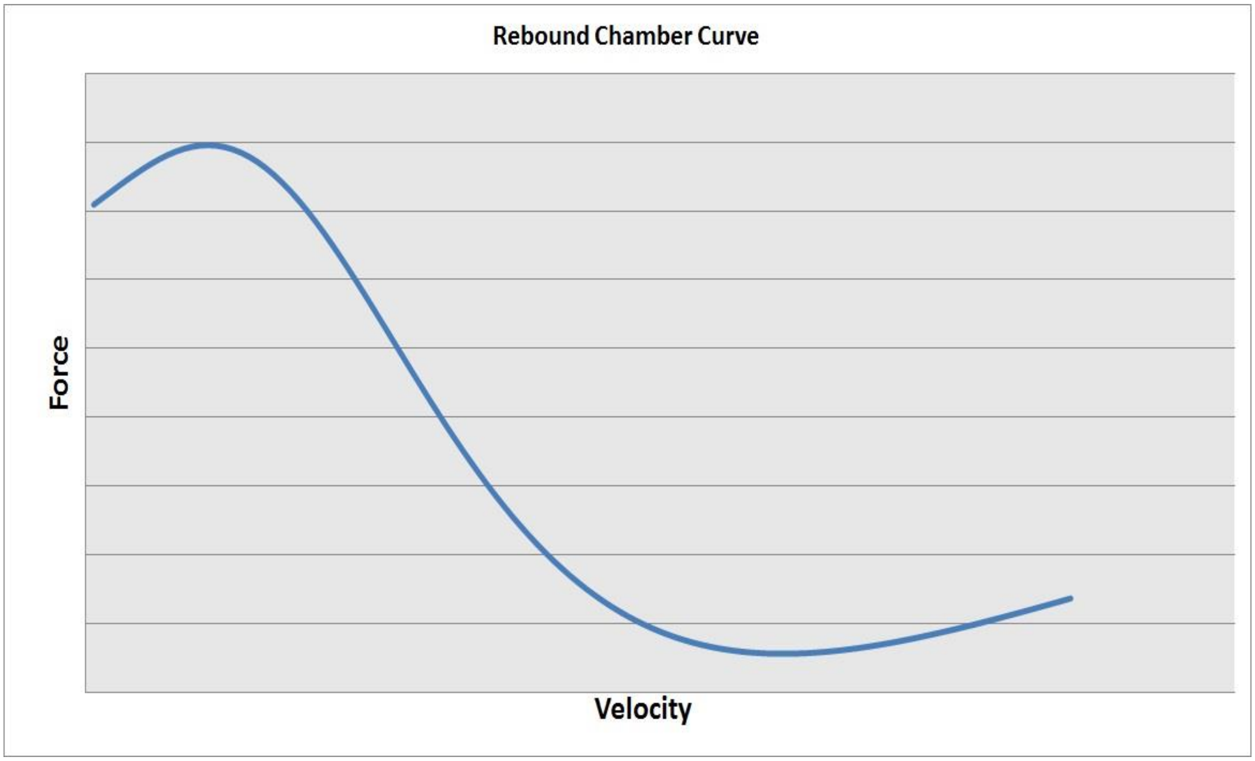

Figure 5. 19: Rebound chamber curve

One shape that was created was the rebound curve. It was achieved by keeping C between 2 and 3 and also by increasing values of B, D and E. After that, it again behaves like a sine wave. Because it was unable to create a digressive curve, the model was mainly discarded. Another reason for it being rejected is that, although the rebound curve was created, the curve was achieved because of increasing the value of various coefficients. Because rebound happens during deceleration, the aim was to have a model that could create a rebound curve just by changing the sign of one of the parameters. For example, if B, C, D or E is positive, it would give either a compressive or rebound curve and reversing the sign of either of these parameters would give the same curve on the opposite axis.

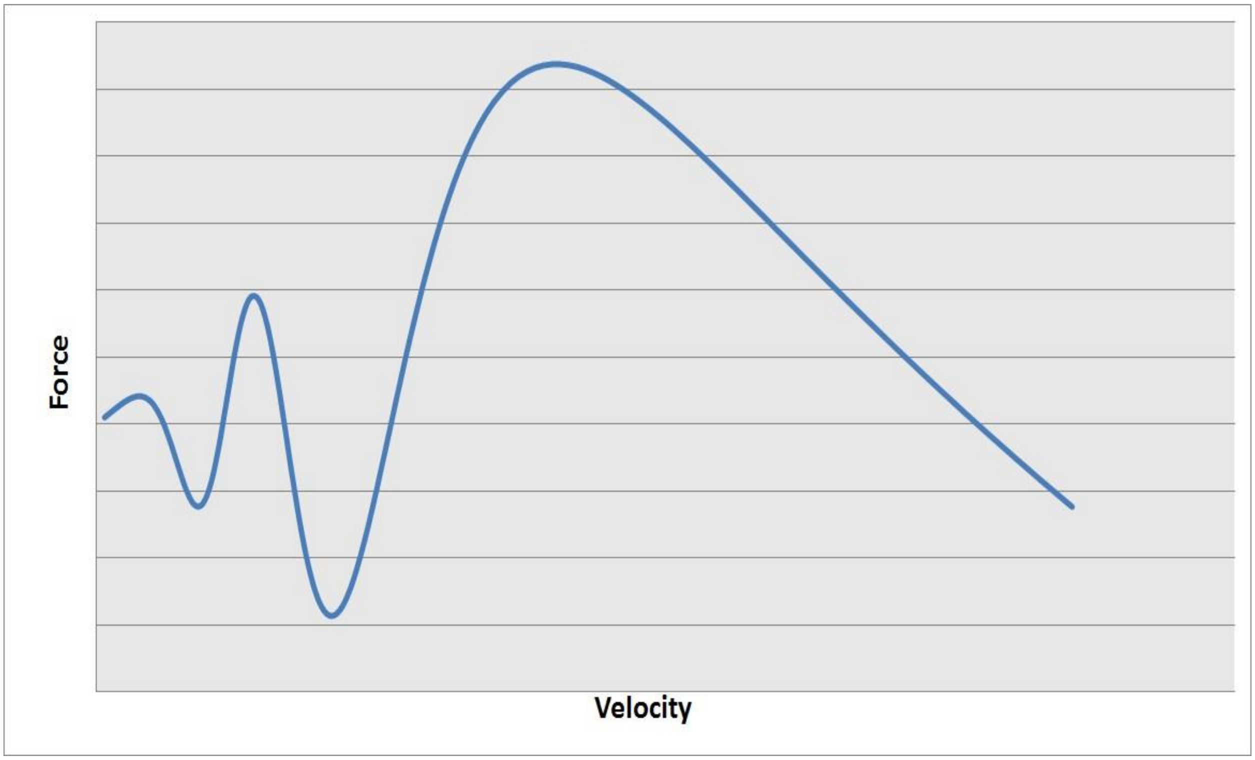

Figure 5. 20: Typical graph shape generated with change in parameters

CHAPTER 6: THE FINAL EMPIRICAL MODEL

6.1: Overview

The aim of starting this thesis was to create a model that would help predict the behavior and show energy inside the Damper system of a vehicle at any instant of time. Furthermore, this thesis concentrated on the area of creating a model that can give three basic shapes: progressive curve, linear curve and digressive curve. The model will act as a foundation to the next versions of model which, by adding meaning to B, C, D, E or any other parameters, like vehicle type, suspension type, terrain, viscosity, piston properties, coil property, installation angle and so on, will be able to predict exact values of suspension energy. In order to achieve this goal, many permutations and combinations of various trigonometric functions were tried, as explained in the previous chapter. Other than the models mentioned in the previous chapter, many others were tried and finally an empirical model was successfully created which possessed the capability to produce the damper curves. It was able to create the rebound curve by changing the sign of one of the parameters. The following sections will explain the model in detail.

6.2: The Model

The model that was able to fulfill the purpose of this thesis was:

$$\mathbf{Cd} = \mathbf{BX} * \mathbf{sin}\,(\mathbf{B} * \mathbf{tanhX} * \mathbf{atan}(\mathbf{DsinhX} * \mathbf{atan}(\mathbf{CtanhX} * \mathbf{sinhX} * \mathbf{atan}(\mathbf{EX} - \mathbf{Catan}(\mathbf{BX} - \mathbf{E})))))$$

As explained in the previous chapter, the model generated was checked for its ability to generate the three damper curves. Once the three graphs were obtained, the second round of tests was done on the range of values that needed to be used to obtain those graphs. The third step was to check the same curves for the rebound chamber of the Damper system. Once all these were achieved, then effort was taken to check whether the model was able to give the exact values of energy generated inside the Damper system at every instant of time. The tested velocity was in the range of 0 to 82 mph, because that is the maximum speed most commercial vehicles reach on highways. The velocity mentioned here is just as an example and one should take it as an index. Also, the energy inside the Damper system on a straight smooth asphalt road is as low as 40 watts. The possibility of generating maximum energy by the vehicle is possible only if it is going over off-road terrains or bumpy roads, or if the vehicle has a heavy load and is encountering vibration. Although the main purpose of the thesis was to get the shape factor that can act as a foundation for future development of models which can accurately predict the energy inside the Damper system and also can give the three curves of the Damper system.

Figure 26 is the theoretical value generated by the formula:

Power per cycle of Damper system

$\mathbf{P_m = \frac{C_d * V^2}{2}}$ Where, Cd= Damping Coefficient = 2 kNs/m

And V= Velocity of Vehicle in mph.

From this equation, the graph obtained was as shown below.

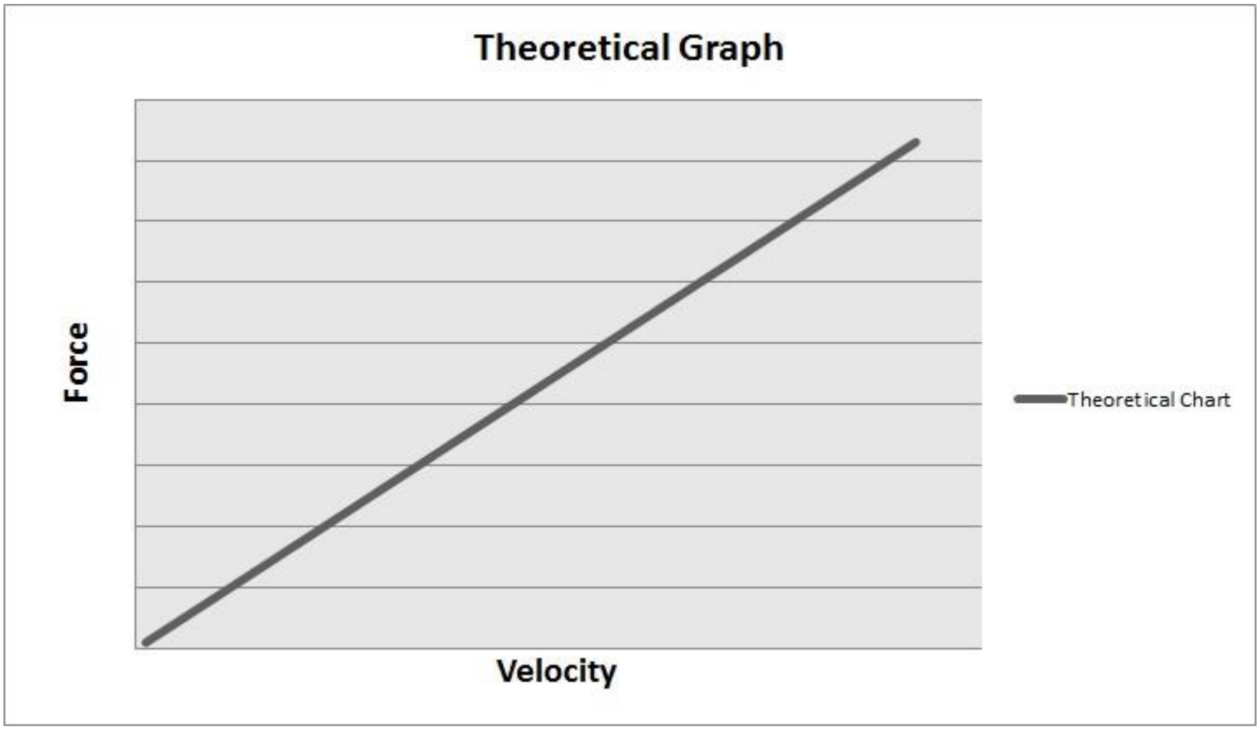

Figure 6. 1: Theoretical Graph obtained by using the equation, Power Pm= ½ * Cd* Vo2

As expected, the graph shows a linear curve because of the formula used. It had two variables, the damping coefficient and velocity. The only changing factor was the velocity. So, as per the formula, the linear curve was created which shows longitudinal force acting in the same proportion for each velocity to which it is attached. However, in a practical scenario, a linear graph is very hard to achieve as a suspension is either soft or stiff depending on its use and the terrain it travels on. For example, for a vehicle built for off-road terrains, the suspension is stiff, while for a vehicle like a rally car, which requires quicker road reaction and feedback, the suspension is kept soft.

The linear Damper system is similar to one available in a commercial vehicle for day to day use. However, if a real-world system is to be considered, when a car is unoccupied, the suspension can be considered in an ideal rest position. When passengers sit inside an automotive meant for commercial purpose which has a Damper system that can behave like a linear curve, the suspension compresses. Then when the vehicle is started, the engine and transmission force act on the vehicle. Therefore the moment vehicle starts running, the component forces along with the drag force acting on vehicle body tend to make the Damper system softer.

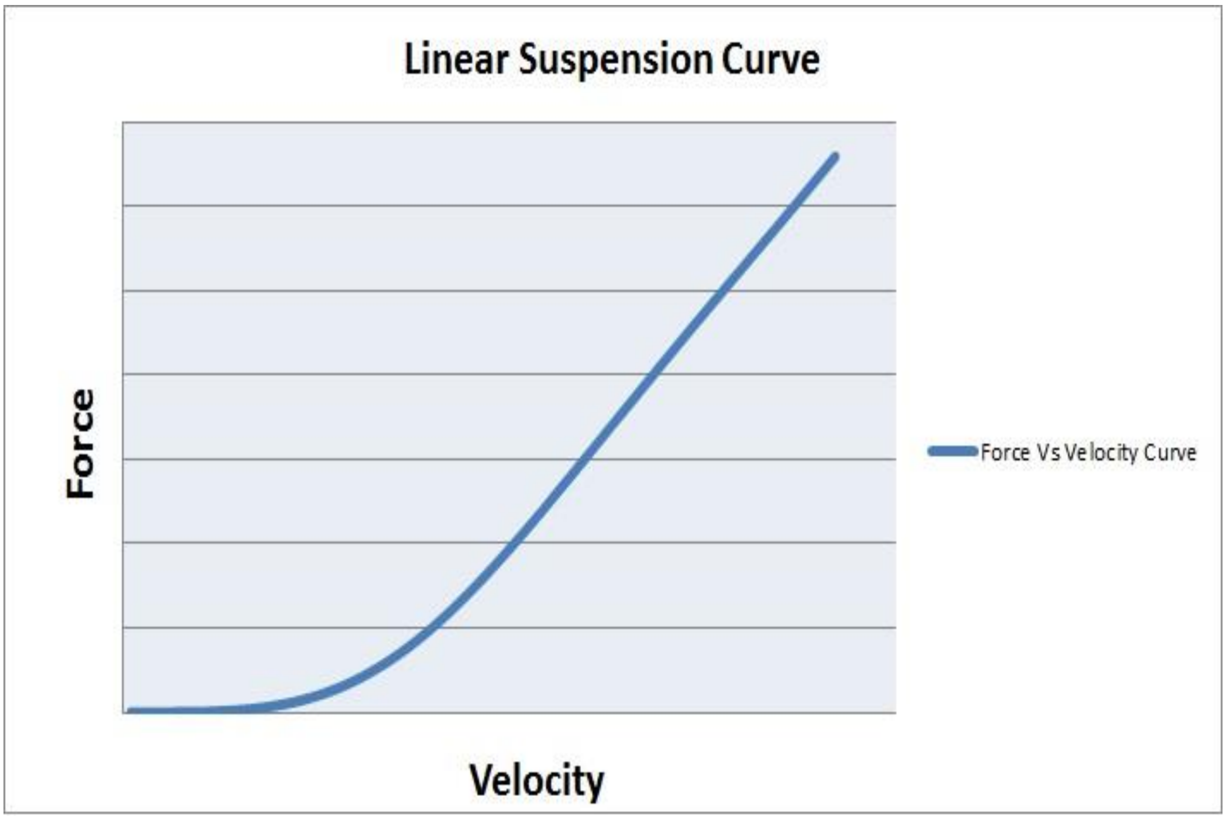

Figure 6. 2: Linear Curve achieved through the model

As the vehicle velocity increases gradually, the Damper system tends to recalibrate itself and tends to go back to the linear characteristics. The curve in such circumstances is similar to the one seen in Figure 6.2. The shape obtained is similar to the theoretical curve. However, as explained, it is not completely linear. The model initially has a very small increase in longitudinal force. Then after reaching a certain velocity, the graph advances in an upward direction and maintains linear characteristics.

Although the values obtained on the positive Y axis for the theoretical and empirical model are different, it is not important because the emphasis is on creating damper curves as compared to a model that will also give the force generated at any particular velocity.

The next section will give information regarding how the model is able to create all three curves of the Damper system.

6.3: The explanation of Empirical Model

As explained before, the model was developed for replacing a damping coefficient which is considered constant for the theoretical calculation.

The equation for energy per cycle in a Damper system is:

$$\mathbf{Ec} = \oint \mathbf{Cd} * \mathbf{X^2} * \boldsymbol{\omega}^2 * \mathbf{cos^2}\, \boldsymbol{\omega}\mathbf{t} * \mathbf{dt}$$

Normally, the values used for Damping Coefficient tend to be around 2 to 4 kNs/m

While checking the formula, the graph obtained is as shown in figure 26.

The equation $\mathbf{Cd} = \mathbf{BX} * \mathbf{sin}\,(\mathbf{B} * \mathbf{tanhx} * \mathbf{atan}(\mathbf{Dsinhx} * \mathbf{atan}(\mathbf{Ctanhx} * \mathbf{sinhx} * \mathbf{atan}(\mathbf{EX} - \mathbf{C} * \mathbf{atan}(\mathbf{BX} - \mathbf{E})))))$ is now used to replace the typical Damping Coefficient.

Once the formula was created and checked for the damping coefficient, it was used to calculate energy per cycle and power per cycle for the Damper system.

The new equation for energy per cycle is:

$$\boldsymbol{Ec = \oint X^2 * \omega^2 * cos^2\, \omega t * BX * sin\left(B * tanhx * atan\left(Dsinhx * atan\left(Ctanhx * sinhx * atan\left(EX - C * atan(BX - E)\right)\right)\right)\right) * dt} \quad \ldots 6.1$$

In equation 6.1, B, C, D and E are the parameters. Different values are used to generate curves. The curve shape transition is possible from one to another by changing the values. X is the velocity from 0 to 82 mph and f is the frequency.

So the table generated for Damping Coefficient graph is:

Table 6. 1: Generation of Damper Curve

Velocity(mph)	tanhX	sinhX	BX	atanBX	Cd
0,1,2… 82	=tanh(0,1,2… 82)	=sinh(0,1,2… 82)	B* (0,1,2… 82)	atan(B* 0,1,2… 82)	Model Created by use of many mathematical functions

The Damping Coefficient Values thus obtained for velocity between 0 to 82 mph is used to generate a graph. To start with, B, C, D, E were given values equivalent to 1. The graph was generated was as follows.

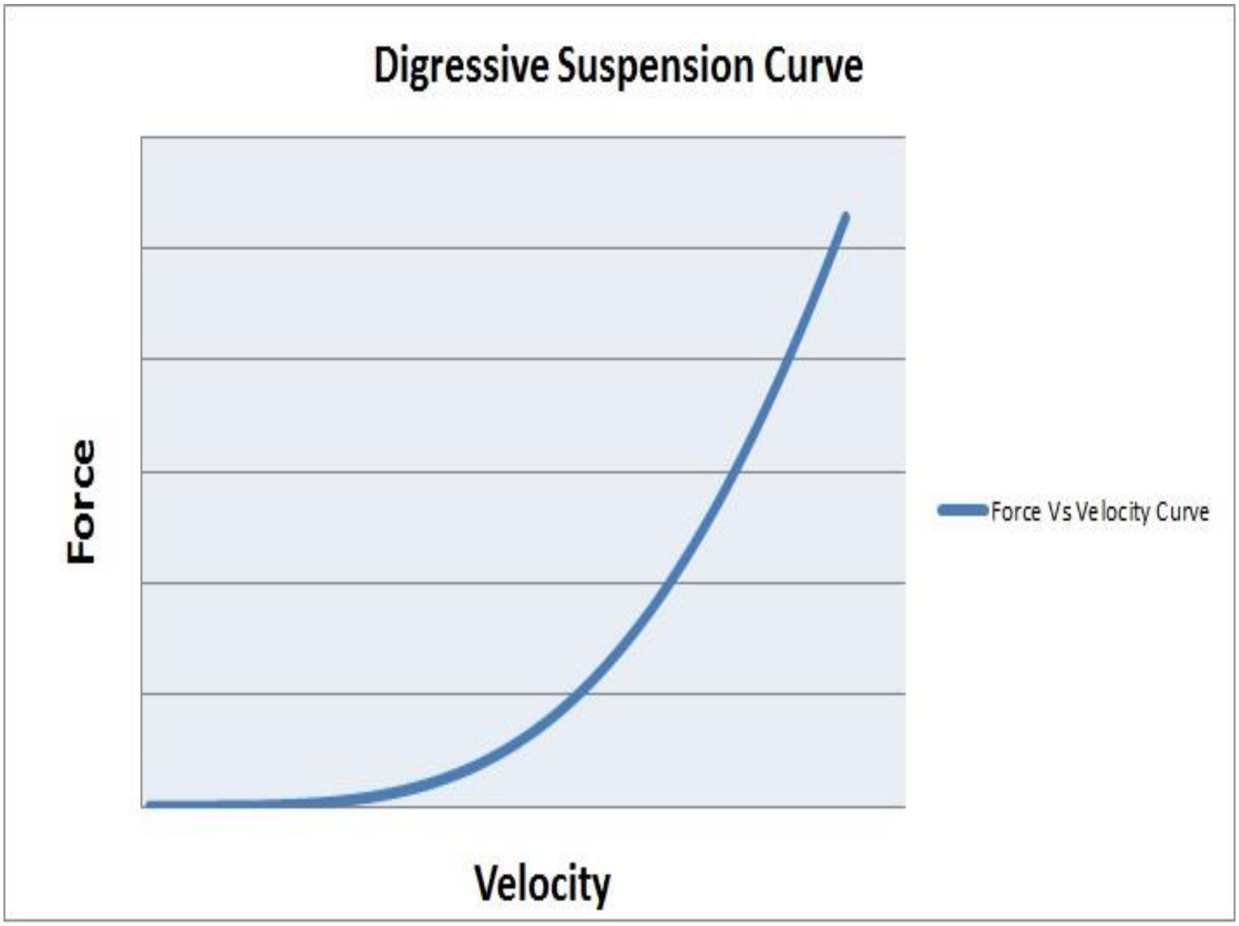

Figure 6. 3: The digressive curve generated using coefficient values equal to 1

When all the values of the parameters are 1, the graph is digressive. Then the values are changed for the same equation to check the change in the curve shape.

6.3.1: Progressive Damper curve

After doing various permutations and combinations, a range of values were obtained and used to get the combined graphs which show the transition from the initial digressive curve to the progressive curve. In figure 29, one can see the transition happening from an initial digressive curve to a progressive curve. This was done using the values that are described in table 6.2.

Table 6. 2: Parameter values used to obtain the combined graph showing transition of digressive to progressive curve

Graph #	B	C	D	E
Graph 1	2.3	1	1	2
Graph 2	2.3	2	2	7
Graph 3	2.3	10	5	7
Graph 4	2.3	8	9	7

Firstly, it was seen that, in order to have a curve with progressive features, the value of B needs to be between 1.8 and 2.5. After 2.5, the value seems to drop. This is irrespective of values of C, D and E. In order to achieve a progressive curve, the value of B should be kept in a range between 2 2.3; the value of E needs to be above 2. Thus, to start with, the values of B and E are kept at an optimal value which starts changing a digressive curve to a progressive curve. C and D are assigned values equal to 1.

The graph shown indicates that the digressive curve is changing its form. Then, the values of C, D, and E are gradually increased and curve characteristics are checked. In table 6.2, values of D are increasing and in Figure 32, one can see that the progressive curve is different in shape. Particularly in the initial section where the force is gradually increasing, all four graphs behave differently. This is because, with an increase in the value of D, more force is acting on the compression chamber. This fact can be used while using the model to get energy data. For different types of Damper system, different ranges of C and D can be used based upon which the graph can be computed. Using negative D values will give the same curve in the rebound chamber. Thus, D can be used as an accelerating and decelerating factor.

Figure 6. 4: Combined graph generated for progressive curve

6.3.2: Linear Damper curve

Just like the progressive curve, the coefficients of the empirical model were changed to test for the linear curve. By using the numbers mentioned in Table 6.3, the linear curve was achieved.

Table 6. 3: Parameter Values used to generate graph

Graph #	B	C	D	E
Graph 1	1.4	1	2	1
Graph 2	1.4	2	2	5
Graph 3	1.4	4	4	4
Graph 4	1.4	10	15	10

Based on the values above, the linear graph was obtained in various forms. As seen in the table, the value of B was kept constant as it was providing a good linear curve slope. Values of C, D and E were increased gradually, which increased the amount of force generated. This shows that the nature of the graph will be showing characteristics of a progressive curve with an increase in output values. Although similar to previous graphs, there is a stagnant point after which the increase in values is minimal. The linear graph generated based on these values is shown below.

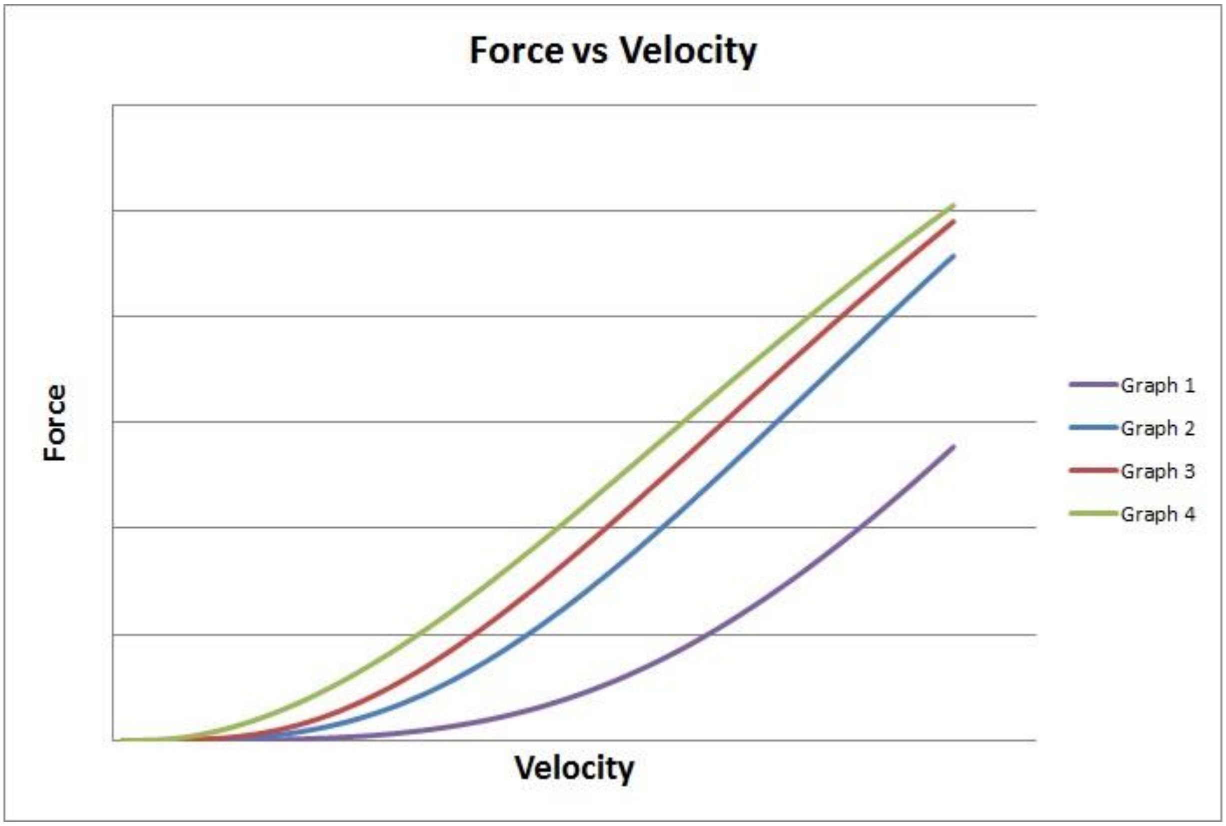

Figure 6. 5: Combined graph generated for linear curve

6.3.3: Digressive Damper curve

The digressive damper curve is the third curve after the progressive and linear curves. This particular characteristic of a Damper system is required by high-speed cars which need a rigid-type suspension. The response will be little as compared to the progressive and linear curves because of the suspension settings. Just like progressive and linear damper curves, a digressive curve was achieved using a certain range of values. The table below gives the values under which various digressive curves, such as those shown in figure 6.6, are achieved.

Table 6. 4: Values used to achieve digressive damper curve

Graph #	B	C	D	E
Graph 1	0.8	1.3	1	2
Graph 2	1	1.4	1	2
Graph 3	1	1.5	2	2
Graph 4	1	1.6	2	3

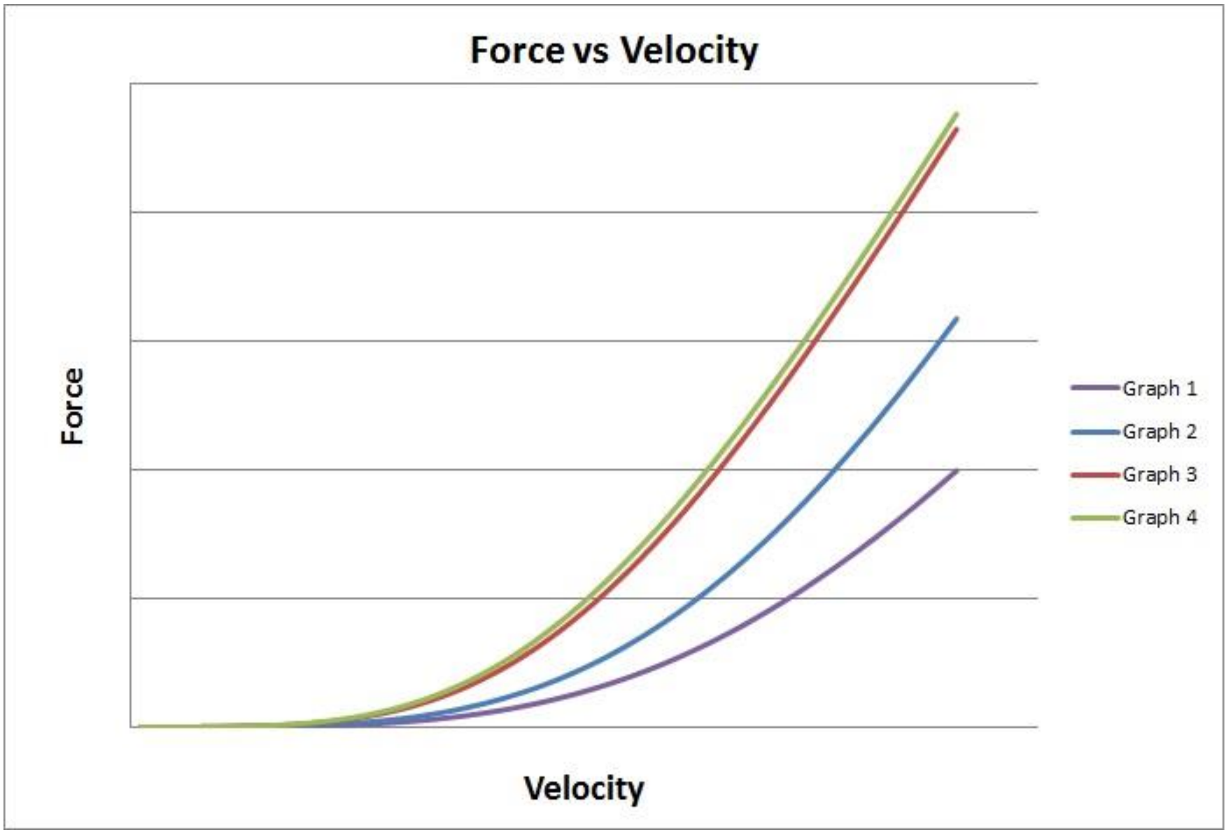

Figure 6. 6: Combined graph generated for digressive curve

This empirical model showed the three required damper curves, which was the aim of this thesis research. Just like the compression chamber, the rebound chamber is also important. As work is done in the compression chamber when the vehicle starts moving, the rebound chamber also generates force opposite to the direction of damper piston displacement. The behavior of force by rebound chamber is also similar to compression chamber and it also has the ability to generate three damper graphs just like compression chamber. In the compression chamber, the factors influencing the work are the velocity of the vehicle, the weight of the vehicle, the coil spring stiffness (if a coil spring is present), oil viscosity, piston diameter, oil temperature and piston rod length. The rebound chamber is affected by the nitrogen gas, oil viscosity and oil temperature. The graph shown is similar to the compression chamber graph but in the opposite direction. The following diagrams will show some of the comparisons between the compression and rebound chamber curves achieved using the model mentioned in this chapter.

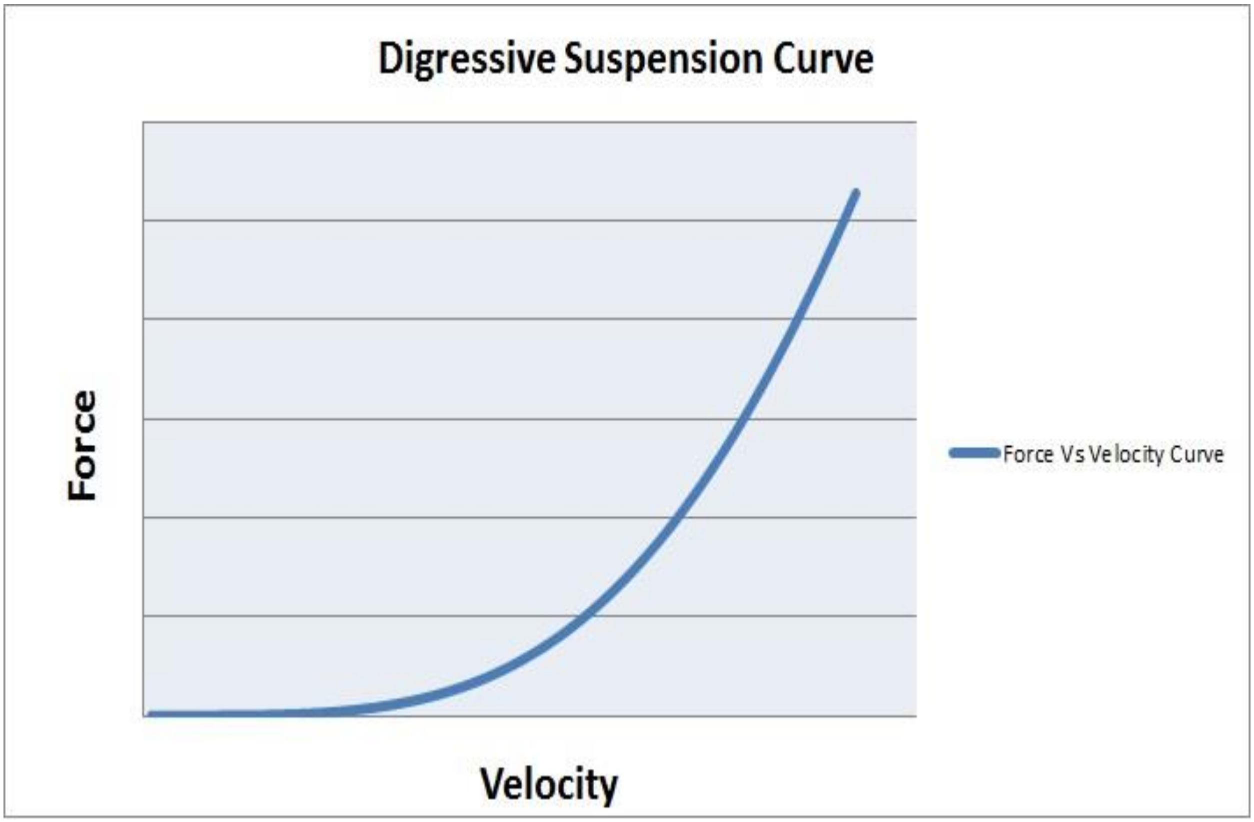

Figure 6. 7: Digressive Curve created through model

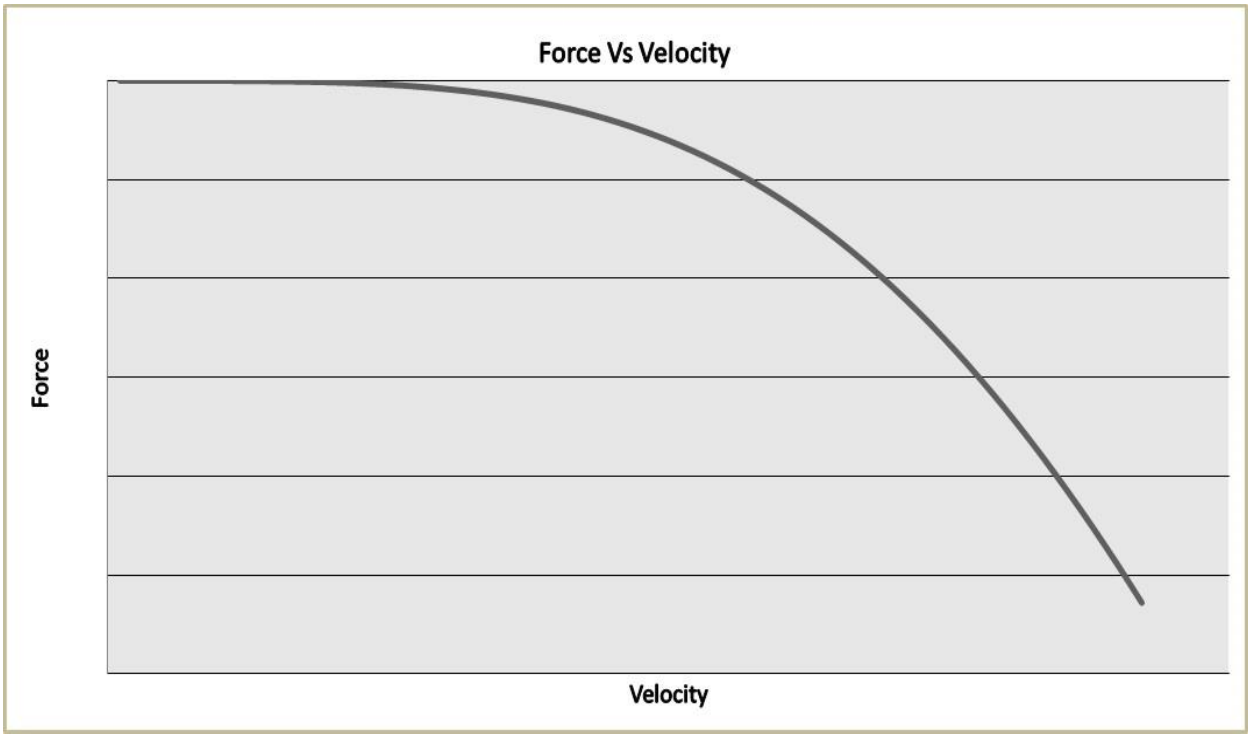

Figure 6. 8: Digressive Curve created by model in rebound section

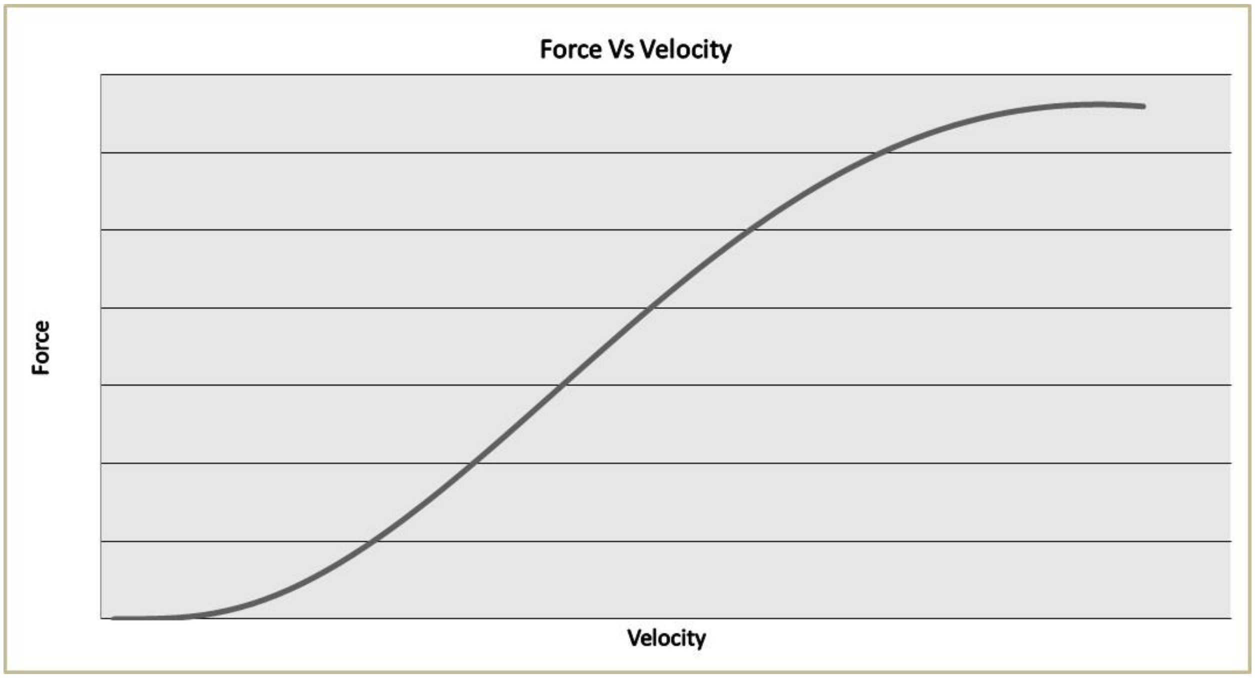

Figure 6. 9: Progressive Curve created through model

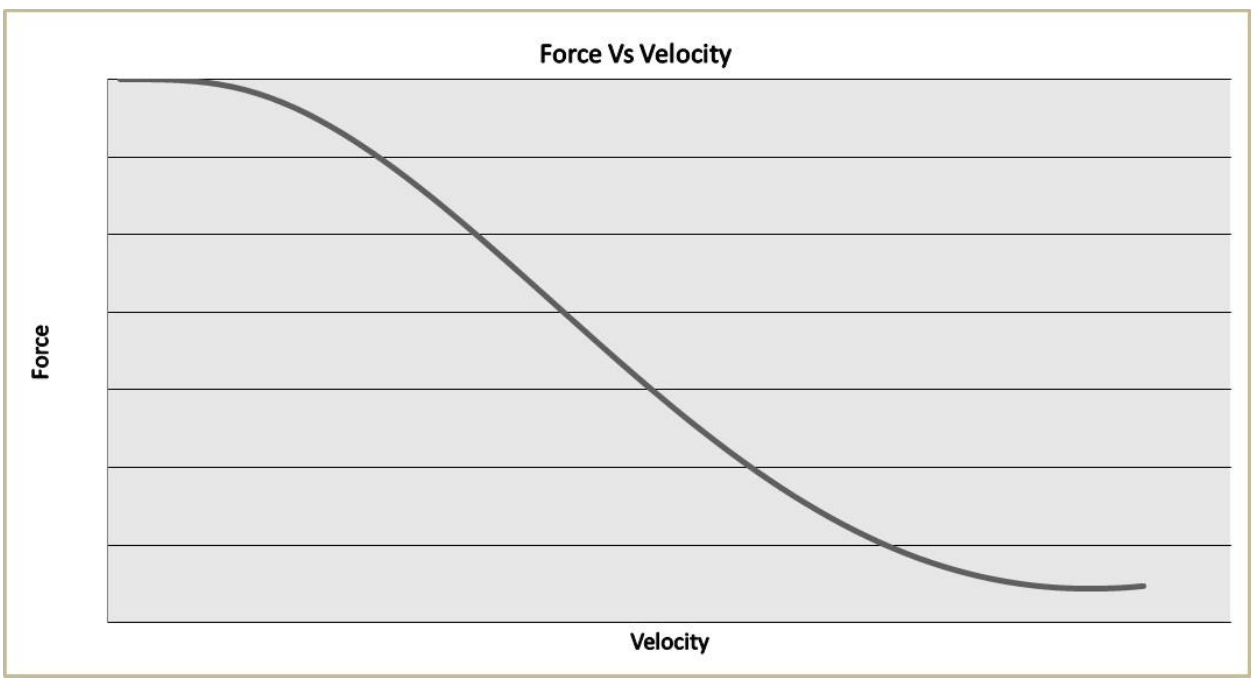

Figure 6. 10: Progressive Curve created in Rebound Section by model

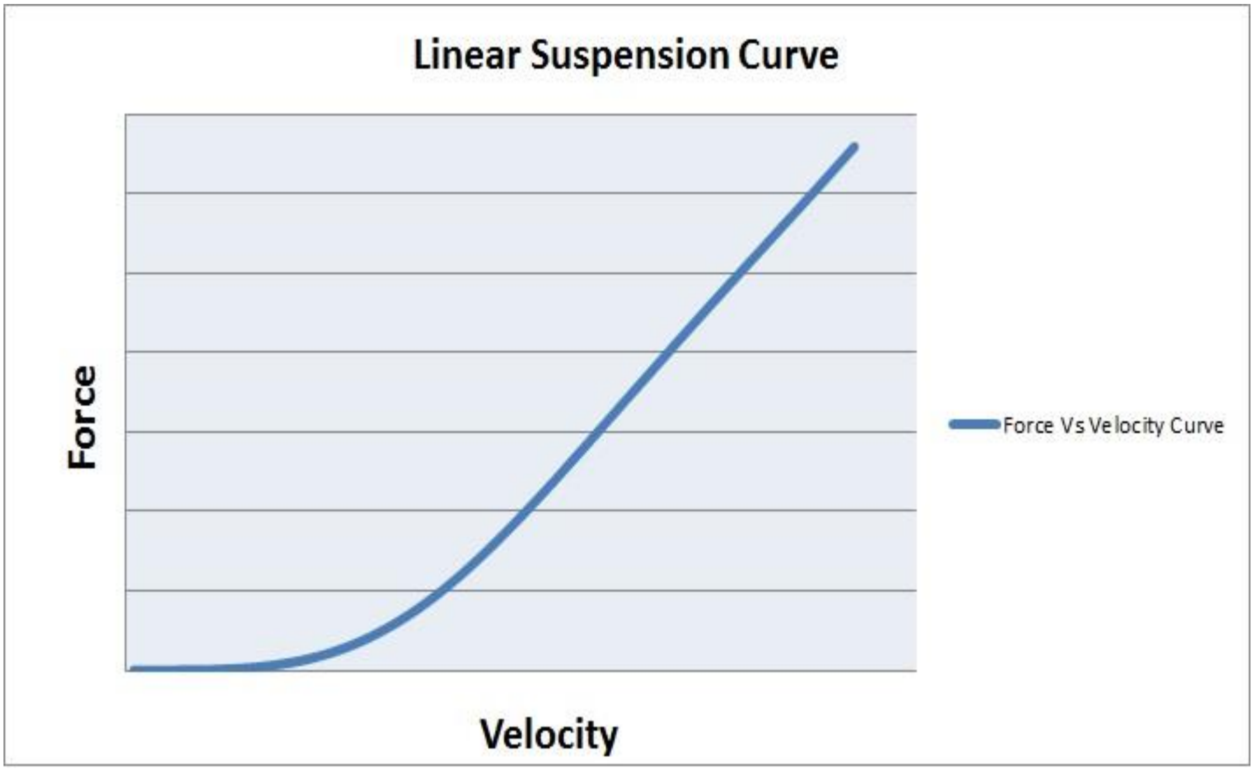

Figure 6. 11: Linear Graph in Compression Chamber

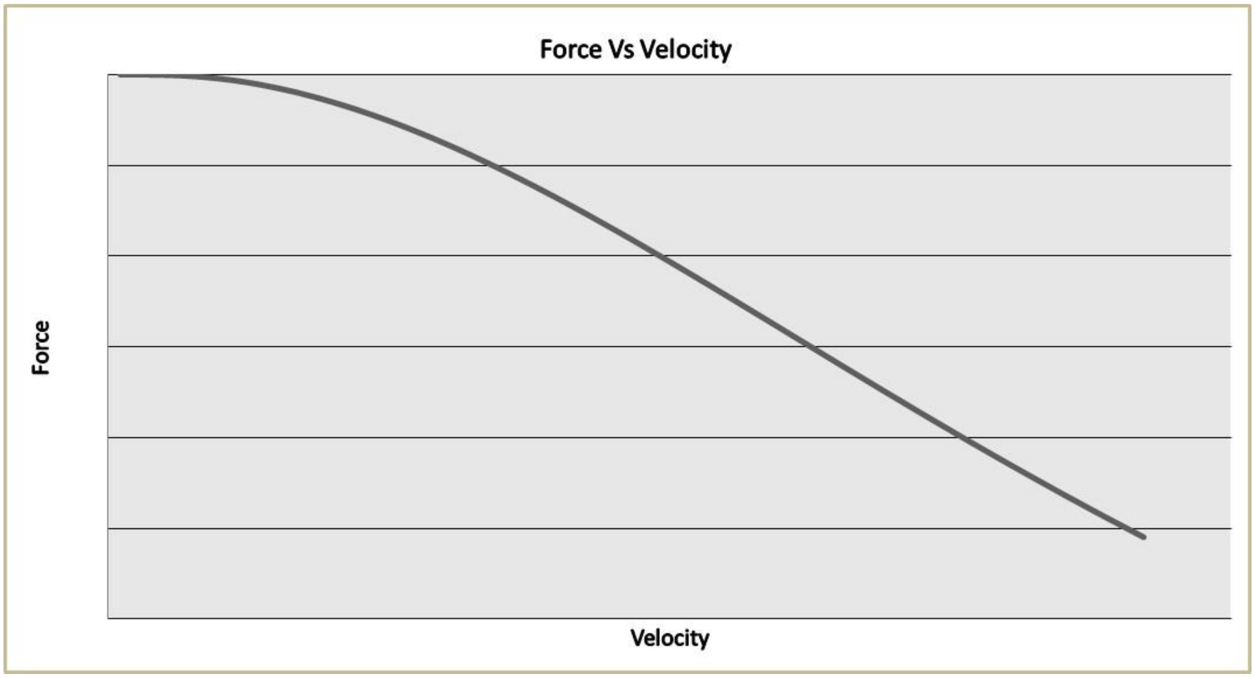

Figure 6. 12: Linear Damper curve in rebound chamber

CHAPTER 7: CONCLUSION

7.1: Overview

This thesis started with an idea of developing a Damper system that would have the capacity of using the kinetic energy produced inside by turning it into useful regenerative energy. However, as the research progressed, attention was instead given to creating a model that would be able to give a damper curve which would give information about energy generated inside a suspension at any point in time. The empirical model developed, which is now a function of time, will be replaced by the constant damping coefficient used in the energy equation and energy per cycle will be calculated. The model used no force calculations or any equation of motion or similar calculation to get to that goal. Using trigonometric functions like arctan, sine and cosine, hyperbolic functions and velocity, along with some constant parameters, creation of the damper curve was possible.

The model used was:

Damping Coefficient

$$Cd = BX * sin\,(B * tanhxatan(Dsinhxatan(Ctanhx * sinhxatan(EX - Catan(BX - E)))))$$

By using this model in the energy equation instead of a constant damping coefficient, the generation of all three damper curves was possible. B, C, D, E were the coefficients which were altered to create these damper curves. The aim of the thesis research was to generate a model that had the ability to create a damper curve, which was successfully achieved with this model. The model was able to generate all three types of curve, the progressive, linear and digressive curves for both compression and rebound chambers. Just like the Pacejka Tire Model, which has the ability to generate different tire properties that reflect tire behavior, the model developed has the ability to generate a damper curve which can show suspension characteristics, and create damper curves for both compression and rebound chambers.

7.2: Conclusion

The empirical generated is able to portray all three damper curves and was able to represent basic damper characteristics of an automotive damper system. Thus, one can conclude for the thesis that, like Pacejka's Tire Model, an empirical model is possible for automotive damper system using certain trigonometric functions, damping coefficients and using proper integers. The empirical model is able to generate graph of suspension velocity vs. force generated inside it. Changing the value of coefficient D from positive to negative inside the empirical model is able to generate rebound chamber curves. Thus, in the empirical model, coefficient D acts as the automotive damper chamber differentiator.

7.3: Future Works

The research done here is an important stepping stone to have a full-fledged mathematical model that can be used to predict automotive Damper system behavior and characteristics. The thesis shows how this model can generate damper curves. However, just generating damper curves based on various parameters does not give any idea about damper characteristics. As seen in the model, there are many constant parameters like B, C, D and E which are undefined. These parameters can be given a proper definition and they can be a function of various factors which have the ability to influence the work done by the Damper system. These parameters can be direct property values like oil viscosity or coil stiffness, or they can be another set of formulas; for instance B= $f(\alpha)$ under the influence of the abovementioned properties. Just like the damping coefficient was a function of trigonometric functions, hyperbolic functions, velocity, constant parameters and sine function, similarly these constant parameters can be a function of some other properties and functions. Thus, research is needed in the direction of modifying the model and making it expansive enough that it can give precise information regarding energy inside the Damper system and produce damper curves.

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