THESIS

SEMI-ANALYTICAL TOOL FOR OPTIMAL MANAGEMENT OF ALLUVIAL AQUIFERS HYDRAULICALLY CONNECTED TO STREAMS

Submitted by

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In partial fulfillment of the requirements

For the Degree of Master of Science

Colorado State University

Fort Collins, Colorado

Spring 2013

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ABSTRACT

SEMI-ANALYTICAL TOOL FOR OPTIMAL MANAGEMENT OF ALLUVIAL AQUIFERS HYDRAULICALLY CONNECTED TO STREAMS

Conjunctive water resources use is becoming an important tool in water management, especially with the increase in demands in all life sectors, and the decrease in available water resources with all the evolving obstacles of climate change, growing populations in addition to the conflicts over water resources in some areas of the world.

A groundwater/surface water conjunctive management problem of a hydraulically connected aquifer/stream system is addressed in this research under the prior appropriation doctrine of water allocation practiced in the western states of the USA including Colorado. One approach for applying the concept of conjunctive groundwater/surface water management is achieved by techniques of artificial recharge of aquifers, where water is injected and stored in aquifers when surface water surplus is available for that purpose and pumped in the future when there is a need. Within the prior allocation doctrine, groundwater users in Colorado historically started extracting water from the aquifers underlying their agricultural lands after surface water rights were fully allocated. Consequently, in a system of hydraulically connected aquifers and streams as in the South Platte River Basin, ground water users are junior water right holders, who are allowed to divert surface water only when all senior water right holders have had their full allocation. From this perspective, the objective of the groundwater management problem is to minimize the impact of artificial recharge injection and extraction operations on the stream connected to the

targeted aquifer, meaning, when extracting water from the aquifer, the pumped amount should be equal to the injected volumes, else wise the aquifer will compensate for the difference by depleting the stream. An important effect characterizing artificial recharge and groundwater pumping is the change in aquifer head levels during operations, as excessive injection might cause water mounds and over pumping might result in a stressed aquifer.

In this study, groundwater pumping and artificial recharge effects on aquifers are simulated using the semi-analytical models describing the effect of an operating well in the aquifer and the interconnected stream. These models are derived from the formulated analytical solutions for aquifer drawdown and stream depletion obtained by Theis (1935) and Glover and Balmer (1945)

In the first part of this research, a number of semi-analytical models are derived and implemented in MATLAB codes to simulate the response of both the aquifer and the stream to cyclically operating wells. These models can handle the cases of laterally infinite aquifers, semi-infinite aquifers limited by a stream or an impermeable boundary, and finite aquifer comprised between an impermeable boundary and a stream or between two streams. In the second part of the research, these models are used to solve a groundwater management problem that seeks to minimize the absolute value of the volume of stream depletion/accretion over a given time period while meeting prescribed constraints on aquifer water levels, irrigation demands and injection water availability. This problem is tackled using linear programming algorithms, which is proven to be effective in providing first-hand estimations of optimal injection-extraction schemes for the management of systems characterized by large numbers of operating wells, within a reasonably small computation time.

ACKNOWLEDGEMENTS

In the name of Allah, the Most Gracious and the Most Merciful.

Foremost, all praises to Allah for the strengths and His blessing in completing this thesis.

I would like to express my deep and sincere gratitude to my advisor Dr. Domenico Baú for the continuous support of my master’s study and research, for his patience, motivation and immense knowledge. His invaluable guidance helped me in all the time of research and writing of this thesis.

Besides my advisor, I would like to thank the rest of my thesis committee Dr. Neil S. Grigg and Dr. Michael J. Ronayne.

My sincere thanks also go to all of the kind people I met in Colorado, who supported me through my studies.

Last but not least, I would like to thank my beloved family, my parents Salwa El-Banna and Salah El-Kurdi, my brother Wael and my two sisters Sarah and Raghda for supporting me throughout my life; for their understanding & endless love.

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CHAPTER ONE: INTRODUCTION

1.1Water Availability and Demand

The available water in the world is 97% of salt water in the oceans and 3% fresh water. Two-thirds of the fresh water is ice stored in the Arctic area and about 2% of the liquid fresh water is surface water, much of which is replenished by groundwater, which represents about 98% of the world’s liquid fresh water (Bouwer, 2002), however, the available groundwater resources in some areas are deep and require deep-drilled wells and pumps. Yet demands on water supplies in all life sectors are increasing significantly, along with concerns for growing population, wars and conflicts over pastures and agricultural lands, climate change, environmental protection and governing laws and regulations for water allocations and use.

The agriculture sector has a special importance in this problem, because of the competition over water particularly from the municipal and industrial sectors, which affect the efficient use of the available water resources, causing growing concerns on food security.

All of these issues raise the need for adequate, inexpensive, unsophisticated and socially accepted measures from the management perspective to sustain the growing needs, examples include: treating water and wastewater, desalination, barrier for saltwater intrusion, and groundwater/surfacewater conjunctive use.

The groundwater/surfacewater conjunctive use is defined as the efficient utilization of ground water and surface water limited resources simultaneously to meet the demands.

1.2 Artificial recharge

A popular tool of groundwater/surface water conjunctive use is groundwater artificial recharge, which is the augmentation of a groundwater reservoir with surplus surface water for later use. It is achieved by infiltration via recharge ponds (surface flooding) and subsequent movement to aquifers, by infiltration shafts in the unsaturated zone and by placing surface water into the subsurface storage by direct injection to the aquifer through wells. The injection well approach of artificial recharge is used when suitable conditions for surface flooding and infiltration shafts are not available, or when the targeted aquifers are confined or located at a large depth. Artificial recharge may also be achieved by a combined system of the mention methods. Determining the suitable recharge method requires detailed knowledge of geological and hydrological features of the area. Artificial recharge also occurs through natural and incidental activities such as irrigation surplus and leaking in canals and water pipes (O'Hare,1986).

Besides conservation of the surplus surface water for future use, artificial recharge is used for many other purposes such as: (a) water reuse through the so called “soil/aquifer treatment,” or geopurification that improve the quality of the injected waste water and remove the impurities as it infiltrates through soils to the aquifer, (b) reduce seawater intrusion by providing a hydraulic barrier in aquifers that are in coastal areas, where lowering water levels by pumping causes a reverse flow from the sea to the aquifer, and (c) replenish aquifers that are being depleted due to excessive supply or when the natural ground water resource fails to provide the required demand (Bouwer, 2002).

1.2.1 Surface storage vs. subsurface storage

Artificial recharge is preferred over surface water storage (in reservoirs) because of the disadvantages of the latter method, such as, the storage losses due to evaporation, the accumulation of sediments, development and maintenance high costs, structure failure, and risk of contamination of the stored stagnant water that causes of human diseases. It is important to note that, in addition to overcoming these disadvantages, subsurface reservoirs provide long-term storage, which will become necessary as the increasing progress in global climatic changes caused by the carbon dioxide and other greenhouse gases in the atmosphere (O'Hare, 1986).

1.2.2 Artificial recharge system requirements

On the other hand, artificial recharge might cause water quality deterioration in the aquifer, if the injected water is not of similar or acceptable properties, consequently, the applied water quality must be evaluated prior to injection. Low quality injected water might also cause clogging of the recharge structure surface, for these reasons, a degree of pretreatment is needed to meet the required standard of injected water based on the aquifer water quality and the use of the pumped water. Also when using the surface flooding and infiltration shafts approaches the unsaturated zone must be checked for pollutants.

Requirements regarding the targeted aquifer must also be considered, as some aquifers has a limited potential for successful artificial recharge. An ideally suitable aquifer will absorb, store and conduct the recharged volumes of water without significant quick release, excessive build up or chemical degradation of that water, and with adequate economical conditions to create the subsurface reservoir. That requires a pre-knowledge of the aquifer characteristics, such as the depth of the aquifer, the aquifer's transmissivity, storage capacity and porosity. Knowledge of these properties is gained by site investigation and by modeling groundwater flow and transport. That is in addition to the predetermination and infiltration rates of the permeability of the surface flooding area soils and unsaturated zone (Central Groundwater Board Ministry of Water Resources, 2000). The other features and parameters to be investigated are: geological and hydrological boundaries; inflow and outflow to the aquifer; water resources available for recharge and water balance.

1.2.3 Infiltration Surface Clogging

Plugging of the pores of the infiltration structure surface (the bottoms and sides of basins and trenches or the well-aquifer interfaces in recharge wells), and subsequent decline in the hydraulic conductivity, which results in drawdown flow reduction is the main bane in artificial recharge of groundwater. It is caused by a number of processes, such as, the build up of the precipitated suspended solids, sediment and salts in the recharge water, by the microbial growth and accumulation of biomass layers, by the entrapped gases in the soil and geochemical difference between the injected water and the water existing in the aquifer (Bouwer, 2002). Some remedial solutions are usually used to minimize the formation of the clogging layers, but no terminal solution can be introduced to permanently eliminate it. In surface flooding the adopted solutions are: (a) reduction of suspended solids by pretreating the applied water, (b) drying the infiltration surface to crack the clogging layer, and (c) physical removal of the layer. In the case of direct injection through wells, redevelopment of the well can be performed to remove the clogging layer, as well as a frequent backwash. This backwashing technique, which, of course, requires installation of a pump in the well, often prevents serious clogging.

1.3 Aquifer storage and recovery (ASR)

ASR is a relatively new and rapidly-spreading practice. It is the same approach as direct injection method of artificial recharge except that when using the ASR technology, the same well is used for both operations, that is, setting the well to perform injection to store water in the aquifer during the availability times, and setting it to pump the aquifer for recovery and supplying seasonal peak demands. ASR is the economical form of direct AR through wells, since these wells are anyhow provided with pumps for backwash of the clogging layer, and it is often cheaper than the use of treatment plants and surface reservoirs. ASR method is developed for industrial, irrigational and environmental purposes, but the main purpose it is being used for so far is to store and provide potable water supplies, where the technique guarantees the placement of a specific water in the aquifer and ideally the extraction of essentially the same water, which remains potable and the only possibly needed treatment is chlorination.

ASR methods are not only employed for direct use of treated water, but also used for storing good-quality raw water surplus supplies and pumping it to the water treatment plant when there is a need for that water. This advantage is particularly important in parts of Europe, Australia, and other countries where people prefer groundwater, yet it is being depleted during dry seasons and must be replenished when there is more surface flow.

1.4 Rain water harvesting and subsurface reservoirs

The surface/ground water conjunctive management is simulated by another technique known as the rainwater harvesting and subsurface dams. This technique is widely used in semi-arid regions, where precipitation rates vary considerably during the year, from very high rates that might result in flooding during the rain season first days to negligible rates during the dry season, in addition to the significant length of the dry season which causes the small surface reservoirs and catchment areas to dry up. Usually in these areas water flow from the catchment area through seasonal paths, where it is harvested by creating dams or barriers to arrest the natural flow of the seasonal paths and to store water in surface reservoirs for the purpose of direct use during the season, as well as storing it in subsurface natural aquifers, this is done through introducing subsurface dames or semi-permeable walls below and across the seasonal river bed to raise the aquifer water levels closer to the surface and make it easier to be extracted, it may also serve as a divergence structure to help replenishing adjacent aquifers. There are other applications of the rainwater harvesting approach practiced worldwide that do not involve subsurface storing (Water Conservation Technical Briefs, *2009*).

1.5 Water laws

One of the issues that water management and provision are subject to are the laws governing water use, supply and allocation in addition to the health, safety and environmental regulations.

The water allocation laws constrict the diversion of surface water to that practiced after obtaining the legal right. In the USA there are two main practiced water allocation laws: (a) the riparian doctrine followed in the eastern states, and (b) the appropriation doctrine practiced in the western states.

In the riparian doctrine, the user has the right of full allocation of the surface water adjacent to his owned land as long as there is no corruption on the other users rights. The appropriation doctrine has a rule statin that users are categorized as senior and junior right holders according to the time they acquired the right (first in time first in right). The prior appropriation system is of a special importance when applying groundwater/surfacewater conjunctive management; as pumping from underground aquifers is considered a junior water right that must not affect the surface water availability for senior water users (Grigg, 2005). Further discussion about application of groundwater/surface water conjunctive management under the prior appropriation doctrine is presented in Chapter 3.

1.6 Background of the Semi-analytical Models presented to simulate aquifer/stream systems

Based on the Glover analytical solution (Glover and Balmer, 1954), the Stream Depletion Factor (SDF) presented by Jenkins (1968a, 1970) is defined as the time when stream depletion is equal to 28% of the volume pumped at a given location. The SDF model is a useful, widely used tool to calculate stream depletion volume due to a well operation in an alluvial aquifer hydraulically connected to a stream. In this model, the SDF is numerically calculated even for the mathematically non-ideal conditions that Glover solution cannot solve, such as, variable transmissivities and the presence of aquifer boundaries and then it is used as an input for the Glover equation to calculate stream depletion rate and stream depletion volume. Jenkins also showed that the residual effect of pumping after the operation has ceased (aquifer recovery) is greater than the effect during well operation. In the paper by Jenkins (1968a), user friendly charts of stream depletion rate and stream depletion volume along with the computations are presented as well.

Theis (1935) and Glover and Balmer (1954) solutions were further modified by Hantush (1965). The modifications was added to calculate aquifer drawdown and stream depletion rate and stream depletion volume due to a pumping well nearby the stream for the case of partially penetrating stream with a demi-pervious bed. Hantush proposed that the resistance to flow due to semi-perviousness and partial penetration of the stream can be simulated by introducing a semi-pervious layer with no significant water storage capability between the aquifer and the stream. In Hantush (1956) paper, a comparison between an old technique proposed to solve the same problem, and his proposed solution was presented graphically, this old technique was suggesting that a solution may be reached by adding extra horizontal length to the aquifer to account for the flow resistance by the semi-pervious stream bed. The comparison results showed that disregarding the presence of the semi-pervious layer as proposed before Huntsh study, caused underestimated values of drawdown and overestimated values of stream depletion volumes, however, with the proposed approach results in fair estimates of those amounts.

Hunt (1998) carried on Huntush (1965) work by considering the case of slightly penetrating stream with small streambed dimensions in comparison of aquifers dimensions, along with the clogging of the stream bed. Hunt proposed modifying streambed leakance in Huntush (1965) study to simulate the condition, and he obtained an expression to calculated aquifer drawdown.

In this topic, Butler (2001) also suggested that the degree of stream depletion volume overestimation is also affected by the normalized distance from the pumping well to the stream as well as the stream leakance, in addition to the stream width. Wide streams have a high value of leakance, however, a very wide stream is not necessarily accurately simulated by a models based on the assumption of a fully penetrating stream. Butler (2001) discussed also the starting assumption of the Glover analytical solution (1954) is that the aquifer is limited by a stream in one direction and has an infinite extension at the other direction; he showed that aquifer width must be hundreds times the stream widths for the assumption of a laterally infinite aquifer to be true for stream-depletion calculations. This ideal assumption makes the application of the model not suitable for many of the natural systems with limited lateral extension. In his study, Butler (2001) presented an approximation of negligible stream penetration in relation to aquifer thickness, which provides reasonable results for the most of natural systems (up to 85%). Attention should be paid to the errors that appear when applying this solution to cases of significant stream penetration. This approximation is sensitive as well to the cases where stream leakance is large, and the pumping well is close to the stream

Results obtained by Theis (1935), Glover and Balmer (1954), Hantush (1965) and Hunt (1998) were further studied by Tartakovsky (2005) who obtained a solution for drawdown in shallow aquifer and stream depletion from a penetrating stream was obtained to simulate the effect of streambed leakage and aquifer leakage. The extended results show that the stream/aquifer hydraulic connection decides upon the maximum stream depletion rate, that is the maximum fraction of the pumping rate given by the stream. These results show also that stream depletion may support groundwater drawdown from a pumping well in leaky aquifers to only small extend. The obtained solutions may be used for assessment of stream-aquifer water allocations.

1.7 Objective and organization of the research

In this research, we address the management problem of the conjunctive use of a hydraulically connected aquifer/stream system, by presenting a linear programming approach to solve the problem with the assistance of models built to simulate the reaction of the aquifer/stream system to operation of wells.

In Chapter 2, detailed illustration of the models developed to study the effect of operating wells (extracting and injecting), operating both continuously and cyclically, on both the aquifer and the stream is presented. These effects are the change in aquifers hydraulic head levels, examined through Theis solution (1935) and the depletion of the connected stream, represented by Glover and Palmer's solution (1945). Aquifers with different boundary conditions are considered, such as, an aquifer with an infinite areal extraction, a semi-infinite aquifer bounded laterally by either a recharge boundary or a no-flow boundary and a laterally finite aquifer comprised between a no-flow and a recharge boundaries and comprised between two recharge boundaries.

In Chapter 3, the aspects of the linear programming technique adopted to simulate and solve the groundwater/surface water conjunctive management problem is introduced, as well as a detailed description of the requirements and limitations (constraints) on wells operation rates, aquifer head levels, demand and availability of the extracted and the injected water and the stream depletion volume. Examples of the applications of the linear programming technique are discussed in chapter three as well. These examples are: Aquifer Storage and Recovery groundwater management problem and Aquifer Pumping and Recharge Groundwater management problem, they are discussed in both semi-infinite aquifers case and finite aquifers bounded by no-flow boundary and recharge boundary case.

CHAPTER TWO: AQUIFER MODELS

2.1. Saturated Groundwater Flow

2.1.1 Darcy's Law

The equations governing groundwater flow in saturated porous media rely on Darcy's law, which is an empirical law stating that the rate of flow through a porous medium is proportional to the energy losses and inversely proportional to the distance between start and end points (Willis and Yeh 1987). This law is expressed as follows:

$$Q = q \cdot A = -K \cdot \frac{dh}{dl} \cdot A \tag{2.1}$$

where Q is the flow rate (L^3/T), q is Darcy's velocity (or specific discharge or Darcy's flux) (L/T); which represents the volume flux or volume of discharge per unit bulk area per unit time, A is the cross-sectional area (L^2), h is the piezometric head (L), K is the hydraulic conductivity (L/T) and l is the distance (L) (McWhorter and Sunada 1995).

The piezometric head h is an indicator of the energy per unit mass of water at any point in the aquifer, and is given by the following equation:

$$h = \frac{p}{\rho g} + z \tag{2.2}$$

where p is the water pressure relative to atmospheric pressure ($ML^{-1}T^{-2}$), g is the gravity acceleration (L/T^2) and ρ is the density of fluid (M/L^3)

Hydraulic conductivity is a hydro-geological parameter of the aquifer; which combines both the fluid and porous medium properties and represents the ability of the medium to conduct water (McWhorter and Sunada 1995). The hydraulic conductivity is given by (Todd and Mays 2005):

$$K = \frac{k\rho g}{\mu} \tag{2.3}$$

where μ is the dynamic viscosity (M/ L) and k is the intrinsic permeability (L^2.T). In a three dimensional system, the Darcy's law is written as:

$$\boldsymbol{q} = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = -\boldsymbol{K} \cdot \boldsymbol{\nabla} h = -\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} . \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{bmatrix} \tag{2.4}$$

where $\boldsymbol{q}$ is the specific discharge vector, $\boldsymbol{\nabla}$ is the differential operator and $\boldsymbol{\nabla} h$ is the gradient vector of the head $h = (\frac{\partial h}{\partial x}\bar{\boldsymbol{\imath}} + \frac{\partial h}{\partial y}\bar{\boldsymbol{\jmath}} + \frac{\partial h}{\partial z}\bar{\boldsymbol{k}})$ where $\bar{\boldsymbol{\imath}}, \bar{\boldsymbol{\jmath}}\ and\ \bar{\boldsymbol{k}}$ are the unit vectors in coordination with *x* ,*y* and *z* axes, such that $\bar{\boldsymbol{\imath}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \bar{\boldsymbol{\jmath}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\ and\ \bar{\boldsymbol{k}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. The matrix ***K*** is the hydraulic conductivity tensor, which becomes a diagonal matrix $\begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix}$ when the coordinate axes selected for analysis are collinear with the principal directions of hydraulic conductivity (Willis and Yeh, 1987).

According to its hydraulic conductivity an aquifer is homogenous if ***K*** is independent of position (or uniform in space). Instead, if ***K*** is dependent of position, the aquifer is said to be heterogeneous. An aquifer is isotropic if its hydraulic conductivity is independent of direction ($Kxx = Kyy = Kzz$). Vice versa, an aquifer is anisotropic if its hydraulic conductivity depends on direction ($Kxx \neq Kyy \neq Kzz$). In condition of isotropy, the hydraulic conductivity tensor $\boldsymbol{K}$ is reduced to a scalar coefficient *K*.

2.1.2 Continuity Equation

The continuity equation represents the mass balance for a fluid in a closed system, in a three-dimensional form it is written as follows:

$$\boldsymbol{\nabla}[\boldsymbol{K}\ \boldsymbol{\nabla} h] + f = S_s \cdot \frac{\partial h}{\partial t} \tag{2.5}$$

where f is the source/sink term (forcing terms for water extraction or injection) and S_s is the specific elastic storage (1/L) (the volume of water released from storage per unit volume of the aquifer per unit decline in pressure head). In the case of a diagonal hydraulic conductivity tensor, Equation (2.5) becomes:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) + f = S_s \frac{\partial h}{\partial t} \tag{2.6}$$

Equation (2.6) applies to heterogeneous and anisotropic confined aquifers. It can be modified to reflect other conditions of aquifer's hydraulic conductivities. For example, for homogenous and anisotropic aquifers, Equation (2.6) takes on the form:

$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} + f = S_s \frac{\partial h}{\partial t} \tag{2.7}$$

For homogenous and isotropic aquifers, Equation (2.6) is further simplified to the form:

$$K\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) + f = S_s \frac{\partial h}{\partial t} \tag{2.8}$$

The integration of the continuity Equation (2.5) requires assigning initial and boundary conditions.

The initial conditions necessitate knowing the hydraulic head in the aquifer before the changes made by the external influences (e.g. operating wells) applied, that is written as:

$$h(x, y, z; 0) = h_0(x, y, z), \forall\ (x, y, z) \in\ \Omega \tag{2.9}$$

where Ω is the aquifer domain.

For boundary conditions, there are three types representing them: Dirichlet boundary, Neumann boundary and Cauchy boundary. As described in Willis and Yeh (1987), boundary conditions mathematically represent a head or a flow/flux state along the aquifer boundary.

Dirichlet boundary conditions are used when the hydraulic head is known at any time over a given portion Γ_D of the domain boundary:

$$h(x, y, z; t) = h_D(x, y, z; t), \forall(x, y, z) \in \Gamma_D, \forall\ t > 0 \tag{2.10}$$

where h_D is the boundary head.

Neumann boundary conditions are prescribed when the flow across a portion Γ_N of the aquifer boundary is known at any time:

$$-\boldsymbol{K} \cdot \boldsymbol{\nabla} h \cdot \boldsymbol{n} = g_N(x, y, z; t), \forall(x, y, z) \in \Gamma_N, \forall t > 0 \tag{2.11}$$

where $g_N(x, y, z;\ t)$: is the flow normal to the boundary Γ_N and *n* is the unit vector normal to the boundary.

Cauchy boundary conditions consist of a linear conribution of Dirichlet and Numann conditions, imposed over a prescibed portion Γ_c of the aquifer domain

$$\delta_D \cdot \boldsymbol{K} \cdot \nabla h \cdot \boldsymbol{n} + \delta_N h = C(x, y, z; t), \forall (x, y, z) \epsilon \Gamma_c, \forall t > 0 \tag{2.12}$$

where δ_D and δ_N are coefficients and C s the Cauchy potential function.

Differential equations governing the flow in aquifers can be solved analytically only under highly simplified assumptions for the aquifer setting (e.g. Theis, 1935 and Glover and Balmer, 1954), which limits the application of these methods to the ideal conditions tailored to enable using them. For the realistic complex systems, an acceptable approximate solution is preferably reached numerically (e.g. USGS's MODFLOW (Harbaugh,1996)). Semi-analytical methods (e.g. SDF (Jenkins, 1968 and Miller et at, 2007)) may also be more efficient when the numerical solutions become computationally expensive.

2.2 Fundamental Analytical Solutions

2.2.1 The Theis Equation

Theis (1935) derived the solution to the Equation (2.5) for an infinite, horizontal, constant-thickness, homogeneous and isotropic confined aquifer under the effect of a radial unsteady flow due to a fully penetrating well, located at the center of the aquifer, and operating with a constant operation rate Q. The initial conditions under which Theis solution is solved require the initial head to be uniform over the aquifer domain. Boundary conditions state that the hydraulic head remains undisturbed and equal to the initial head h_0 at infinite distance from the well during pumping operation.

According to Theis assumptions, the spatial dependency on head is reduced to the horizontal coordinates x and y, and the vertical coordinate z is dropped given the condition of ideal horizontal flow.

Figure 2.1 shows the features of the perfectly confined aquifer setting addressed by Theis (1935), with a fully penetrating pumping well. The dashed lines represent the position of the potentiometric surface initially and during pumping.

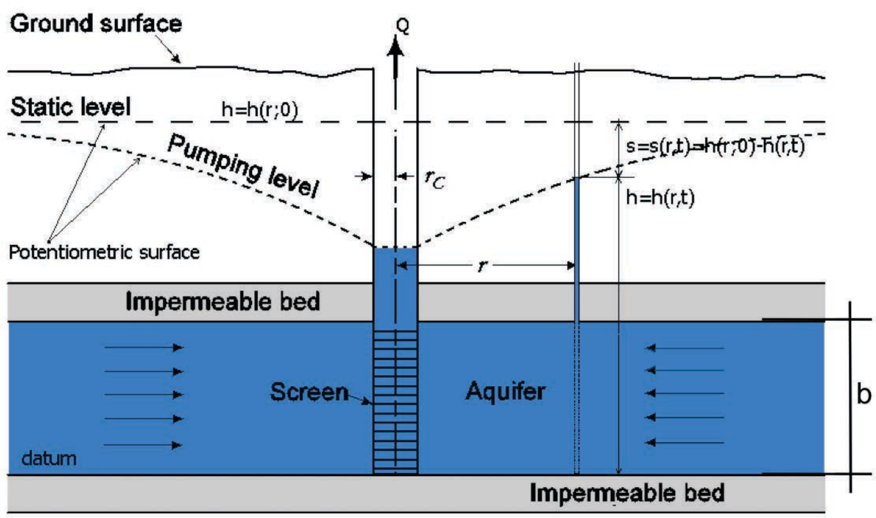


Figure 2.1 the system used to derive Theis solution (1935), a confined aquifer affected by the operation of a pumping well

When studying the effect of radial flow, the equations are written in terms of aquifer drawdown instead of hydraulic head, where the drawdown is given by:

$$s(x, y, t) = h_o - h(x, y, t) \tag{2.13}$$

Under the conditions of radial symmetry in the aquifer around the well considered by Theis (1935), the differential equation governing unsteady flow in a confined aquifer may be simplified as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) = \frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + f = \frac{S}{T}\frac{\partial s}{\partial t} \tag{2.14}$$

In Equation (2.14), S is the Storage coefficient or storativity (/) (the volume of water released from a column of a unit cross-sectional area per a unit decline in pressure head), T is the transmissivity (L^2/T) (the rate at which water is transmitted through a unit width of aquifer under a unit hydraulic gradient) and r is the distance between the operating well (origin of the Cartesian system) and the observation point *(x,y)*.

The initial conditions associated with equation (2.14) require the drawdown at time *t=0* to be zero everywhere:

$$s(r;0) = 0 \; \forall \, r \tag{2.15}$$

The prescribed boundary conditions require a zero drawdown at infinite distance from the pumping well:

$$s(r;t) \xrightarrow{r\to\infty} 0 \tag{2.16}$$

Note that condition (2.16) is a Dirichlet boundary condition. In addition, the following Neumann condition must be imposed at the pumping well location:

$$\lim_{r\to 0} r \cdot \frac{\partial s}{\partial r} = \frac{Q}{2\pi T} \tag{2.17}$$

Condition (2.17) is in practice derived by applying the Darcy's law across the lateral surface of a cylinder of infinitesimal radius centered on the well.

The Theis solution is derived by introducing the following unitless variable:

$$u(r;t) = \frac{S}{4T} \cdot \frac{r^2}{t} \tag{2.18}$$

which is known as the Boltzman variable. Using this variable the first and second derivatives of the drawdown *s* with respect to *r* in Equation (2.14), can be calculated using the chain rule:

$$\frac{\partial s}{\partial r} = \frac{ds}{du} \cdot \frac{\partial u}{\partial r} = \frac{ds}{du} \cdot \frac{2u}{r} \tag{2.19}$$

$$\frac{\partial^2 s}{\partial r^2} = \frac{\partial}{\partial r}\left(\frac{ds}{du} \cdot \frac{2u}{r}\right) = \frac{ds}{du} \cdot \frac{\partial}{\partial r}\left(\frac{2u}{r}\right) + \frac{2u}{r} \cdot \frac{\partial}{\partial r}\left(\frac{ds}{du}\right) \tag{2.20}$$

Equation (2.20) can be further developed as in the following:

$$\frac{\partial^2 s}{\partial r^2} = \frac{ds}{du} \cdot \frac{2u}{r^2} + \frac{2u}{r} \cdot \frac{d^2 s}{du^2} \cdot \frac{2u}{r} = \frac{2u}{r^2} \cdot \frac{ds}{du} + \left(\frac{2u}{r}\right)^2 \cdot \frac{d^2 s}{du^2} \tag{2.21}$$

The derivative of *s* with respect to *t* is obtained as:

$$\frac{\partial s}{\partial t} = \frac{ds}{du} \cdot \frac{\partial u}{\partial t} = -\frac{ds}{du} \cdot \frac{u}{t} \tag{2.22}$$

Substituting Equations (2.19), (2.21) and (2.22) into equation (2.14) gives:

$$\left(\frac{2u}{r}\right)^2 \cdot \frac{d^2 s}{du^2} + 2\left(\frac{2u}{r^2}\right) \cdot \frac{ds}{du} + \frac{S}{T} \cdot \frac{ds}{du} \cdot \frac{u}{t} = 0 \tag{2.23}$$

By dividing both sides of Equation (2.23) by $\left(\frac{2u}{r}\right)^2$ and rearranging its terms, the following Ordinary Differential Equation (ODE) is obtained:

$$\frac{d^2 s}{du^2} + \frac{ds}{du} \cdot \left(1 + \frac{1}{u}\right) = 0 \tag{2.24}$$

Based on Equation (2.15), the initial conditions for the ODE are:

$$t \to 0, \qquad u \to +\infty, \qquad s(u \to +\infty) = 0 \tag{2.25}$$

Based on Equations (2.16) and (2.17) the boundary conditions for the ODE (2.24) are respectively:

$$r \to \infty, \qquad u \to +\infty, \qquad s(u \to +\infty) = 0 \tag{2.26}$$

$$\lim_{r\to 0} r\cdot\frac{\partial s}{\partial r} = \lim_{r\to 0} r\cdot\frac{ds}{du}\cdot\frac{\partial u}{\partial r} = \lim_{r\to 0} r\cdot\frac{ds}{du}\cdot\frac{2u}{r} = \lim_{u\to 0} 2u\cdot\frac{ds}{du}$$
$$= \frac{Q}{2\pi T} \Rightarrow \lim_{u\to 0} u\cdot\frac{ds}{du} = \frac{Q}{4\pi T} \tag{2.27}$$

Assuming $P = \frac{ds}{du}$, the integration of the ODE (2.27) proceeds as follows:

$$\frac{dP}{du} + \left(\frac{1}{u}+1\right)\cdot P = 0 \Rightarrow \frac{dP}{P} + \left(\frac{1}{u}+1\right)\cdot du = 0$$

$$\Rightarrow \int\frac{dP}{P} = -\int\left(\frac{1}{u}+1\right)\cdot du \Rightarrow \ln P = -u - \ln u + C \tag{2.28}$$

$$\Rightarrow P = e^{(-u-\ln u)+c} = e^{c}\,\frac{e^{-u}}{e^{lnu}} = e^{c}\,\frac{e^{-u}}{u}$$

Introducing the constant $k = e^{c}$, Equation (2.28) becomes:

$$P = \frac{ds}{du} = \frac{k.e^{-u}}{u} \tag{2.29}$$

Equation (2.29) can be integrated by separation of variables:

$$ds = k\,\frac{e^{-u}}{u}\,du \Rightarrow \int_{s(u)}^{s(u\to 0)} ds = k\int_{u}^{\infty}\frac{e^{-u}}{u}\,du \tag{2.30}$$

From which

$$\Rightarrow s(\infty) - s(u) = k\int_{u}^{\infty}\frac{e^{-u}}{u}\,du \tag{2.31}$$

Based on condition (2.25), Equation (2.31) becomes:

$$s(u) = -k\int_{u}^{\infty}\frac{e^{-u}}{u}\,du \tag{2.32}$$

where the constant k can be obtained by imposing condition (2.26):

$$u\,\frac{ds}{du}\xrightarrow{u\to 0}\frac{Q}{4\pi T}\Rightarrow\frac{ds}{du}=\left[k\frac{e^{-u}}{u}\right]_u^{\infty}=0-\left(-k\frac{e^{-u}}{u}\right)=k\,\frac{e^{-u}}{u}$$

$$\Rightarrow\; u\,\frac{ds}{du}=k\,e^{-u}\xrightarrow{u\to 0}k=\frac{Q}{4\pi T} \tag{2.33}$$

From Equations (2.32) and (2.33), the drawdown function is thus (Theis, 1935):

$$s(r;t)=\frac{Q}{4\pi T}\cdot\int_u^{\infty}\frac{e^{-u}}{u}\cdot du \tag{2.34}$$

Equation (2.34) represents the drawdown distribution in time and space due to a single well operating at a constant rate Q in a homogenous, isotropic, horizontal, constant thickness confined aquifer. The exponential integral in Equation (2.34) is known as Theis well function:

$$W(u)=\int_u^{\infty}\frac{e^{-u}}{u}\;du \tag{2.35}$$

Note that in Equation (2.34), drawdown is positive if Q is positive, that is, if water is extracted from the aquifer. In this research, we adopt an apposite sign rule so that Q is positive if injected and negative if extracted. Accordingly with this assumption, the Theis Equation (2.34) is rewritten as:

$$s(r;t)=\frac{-Q}{4\pi T}.W(u) \tag{2.36}$$

Figure 2.2 shows the exponential well function development with the inverse of u.

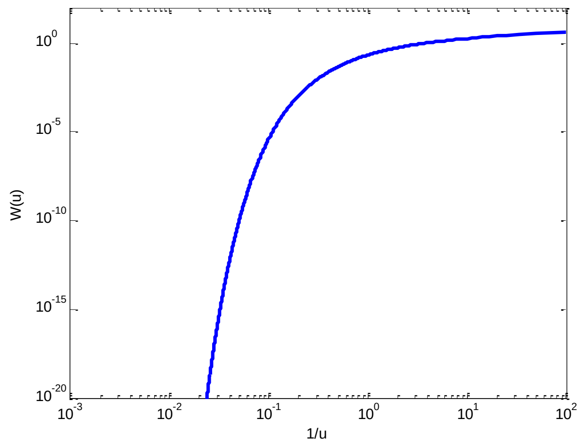


Figure 2.2 W(u) Vs 1/u

Note that, in Equation (2.18) the value of *u* decreses with the increase of *T* then thedrawdwon at distance *r* from the operating well increses and the cone of depresseion extened further in the aquifer, this is the same as the decrease in storativity (when the aquier tends to yield the water storage easly), however when storativity increases and the transmissivity decreases, value of u increses then drawdown at the same distance *r* decreses and the cone of depression extenson shrinks within the quifer area.

2.2.2 Application of Superposition of Solutions

This section presents the application of Theis solution (Equation (2.35)) in conditions where some of its assumptions, such as the constant pumping rate or the single operating well, are violated. In order to remove these assumptions the principle of superposition is introduced.

The principle of superposition of effects or solutions in physics states that the total response of a linear system governed by linear differential equations can be evaluated as the sum of individual, elementary, linear responses in space and time caused by multiple source/sink terms. Given that the solution of the ground water flow problem involves satisfying initial and boundary conditions, to be able to apply the principle of superposition these conditions have to be linear as well.

In the case of the Theis equation, the principle of superposition may be used in the calculation of the drawdown at a certain point in space and time due to the effect of: a) a well field, that is, a set of pumping wells operating simultaneously; b) a single operating well with a varying operation rates and; c) no-flow and recharge boundaries that render the aquifer semi-infinite.

a) Well field

In this case, head drawdowns or build-ups in the aquifer occur as a response to spatially distributed operating wells. In the two dimensional extent of the aquifer, the drawdowns at point *(x,y)* and time *t* due to a well field with n_{ow} number of wells, each one with an operating rate Q_m and operation starting time τ_m can be calculated as:

$$s(x, y; t) = \sum_{m=1}^{now} \left[\frac{-Q_m}{4\pi T} \cdot W\left(\frac{S}{4T} \cdot \frac{r_m^2}{t - \tau_m} \right) \right] \tag{2.37}$$

where r_m is the distance from well *m* to the observation point *(x,y)* which is calculated by:

$$r = \sqrt{(x - x_m)^2 + (y - y_m)^2} \tag{2.38}$$

Note here that Equation (2.37) is valid for $t - \tau > 0$, otherwise the well function is to be set equal to zero.

An application example of Equation (2.37) is given in Figure 2.3 which shows the contour line plot of the drawdown distribution, obtained at a time *t* = 120 days in an aquifer with the hydrogeological parameters given in Table 2.1. The wells locations, operation rates, starting time and operation period are listed in Table 2.2. The plot in Figure 2.3 is obtained with a Matlab code that implements Equation (2.37). This Matlab code, called Finite.Drawdwon.2D.m is provided in Appendix A.

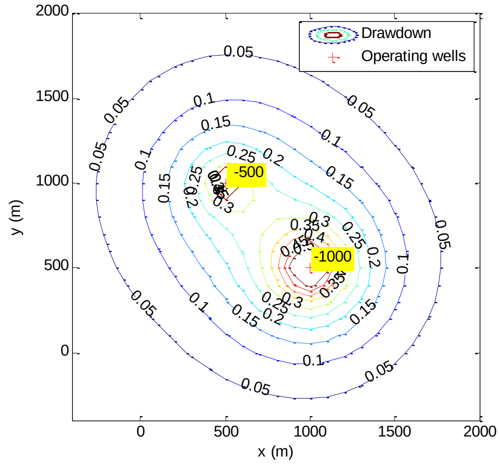


Figure 2.3 Drawdown contour due to an operating well field of two wells located at (500,1000) and (1000,500), operate with a constant rate -500 m^3/day and -1000 m^3/day, respectively, the contours are plotted at a time *t=120* days

Table 2.1: Aquifer hydrogeological parameters

T (m^2/day)	*S (/)*	*h_0 (m)*
622.08	*0.2*	*30*

Table 2.2: Operating wells data

Well	*Location (x,y)*	*Operation starting time (day)*	*Operation period (day)*	*Rate (m^3/day)*

1	*(500,1000)*	*0*	*180*	*-500*
2	*(1000,500)*	*60*	*120*	*-1000*

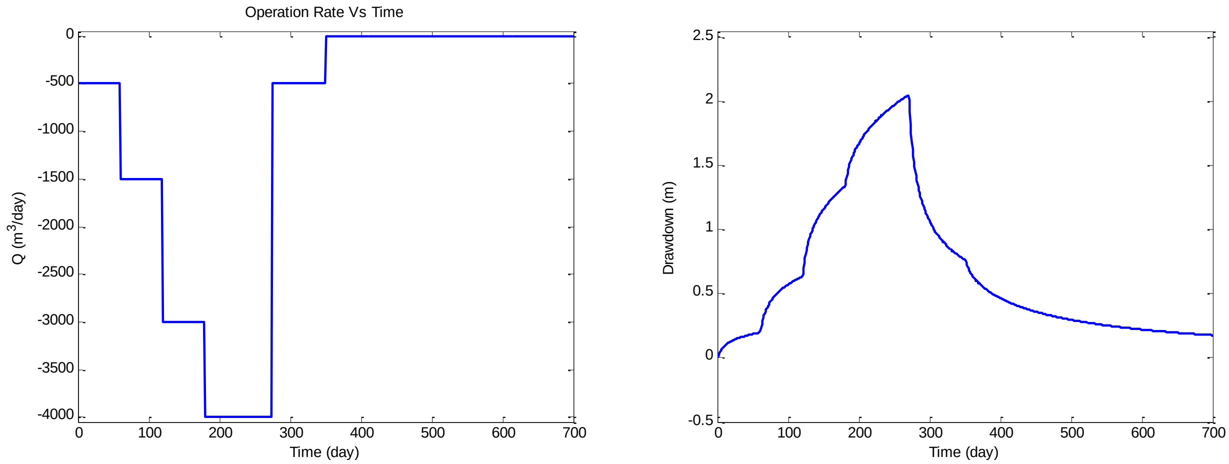
b) Time-varying operation rates

Since the Theis solution does not allow for the change in operating rates, the principle of superposition can be used to deal with cases that violate this limitation, where the operation rate is treated as a time dependent function, and the drawdown is computed using the following "convolution" integral:

$$s(r;t) = -\int_{o}^{t} \left[\frac{1}{4\pi T} \frac{\partial Q(\tau)}{\partial \tau} W \left(\frac{S}{4T} \frac{r^2}{t-\tau} \right) . d\tau \right] \tag{2.39}$$

where $\bar{\tau}$ is the infinitesimal increment of well's operation time. Again, Equation (2.39) is used if $t > \tau$, otherwise the well function W must be set equal to zero. A Matlab code infinite.Drawdwon.Time.m is developed to simulate the results of Equation (2.39). This code is provided in Appendix B.

Figure 2.4a shows an example of time varying operation (extraction) rate. Figure 2.4b shows the corresponding drawdown profile obtained at an observation point located at a distance *r* equal to *140* m from the operating well, using the Matlab code infinite.Drawdown.Time.m. Aquifer parameters used in this example are given in Table 2.1.



(a) (b)

Figure 2.4 Time varying rates of an extracting well in subpanel (a) and in subpanel (b) the resulting drawdown profile over time in an observation point located at *r = 140* m from the operating well

Equations (2.13), (2.37) and (2.39) can be combined together to provide a general equation for the hydraulic head in an aquifer subject to pumping from a well field with varying pumping rates:

$$h(x,y;t) = h_o + \sum_{m=1}^{n_{ow}} \int_o^t \left[\frac{1}{4\pi T} \frac{\partial Q_m(\tau_m)}{\partial \tau_m} W\left(\frac{S}{4T} \frac{r_m^2}{t-\tau_m} \right) . d\tau \right] \tag{2.40}$$

c) Effect of recharge and no-flow boundaries

A recharge boundary is considered as a boundary subject to a Dirichlet constant head condition, where the drawdown is maintained constant and equal to zero over time. Conversely, a no-flow boundary consists of an impervious boundary across which the groundwater cannot flow. The Theis Equation (2.36) can be extended to dealing with cases in which these boundaries are rectilinear.

To create a mathematically equivalent condition for either a recharge boundary or a no-flow boundary and restore the infinite aquifer condition, an image well may be introduced to the system, located at a point symmetrical to the real well with respect to the boundary. In the case of recharge boundary, the image well performs simultaneously the opposite type of operation of the actual well with the same rate to keep the state of zero drawdown at the boundary. In the case of a no-flow boundary, the image well performs the same type of operation to create zero constant flux condition on the impermeable boundary line (McWhorter and Sunada, 1995).

Figure 2.5 shows layouts for a semi-infinite aquifer with the actual and image wells for cases of a recharge boundary (subpanel a) and a no-flow boundary (subpanel b). Subpanels (c) and (d) illustrate the heads levels due to the operation of each well and the resulting combined head levels.

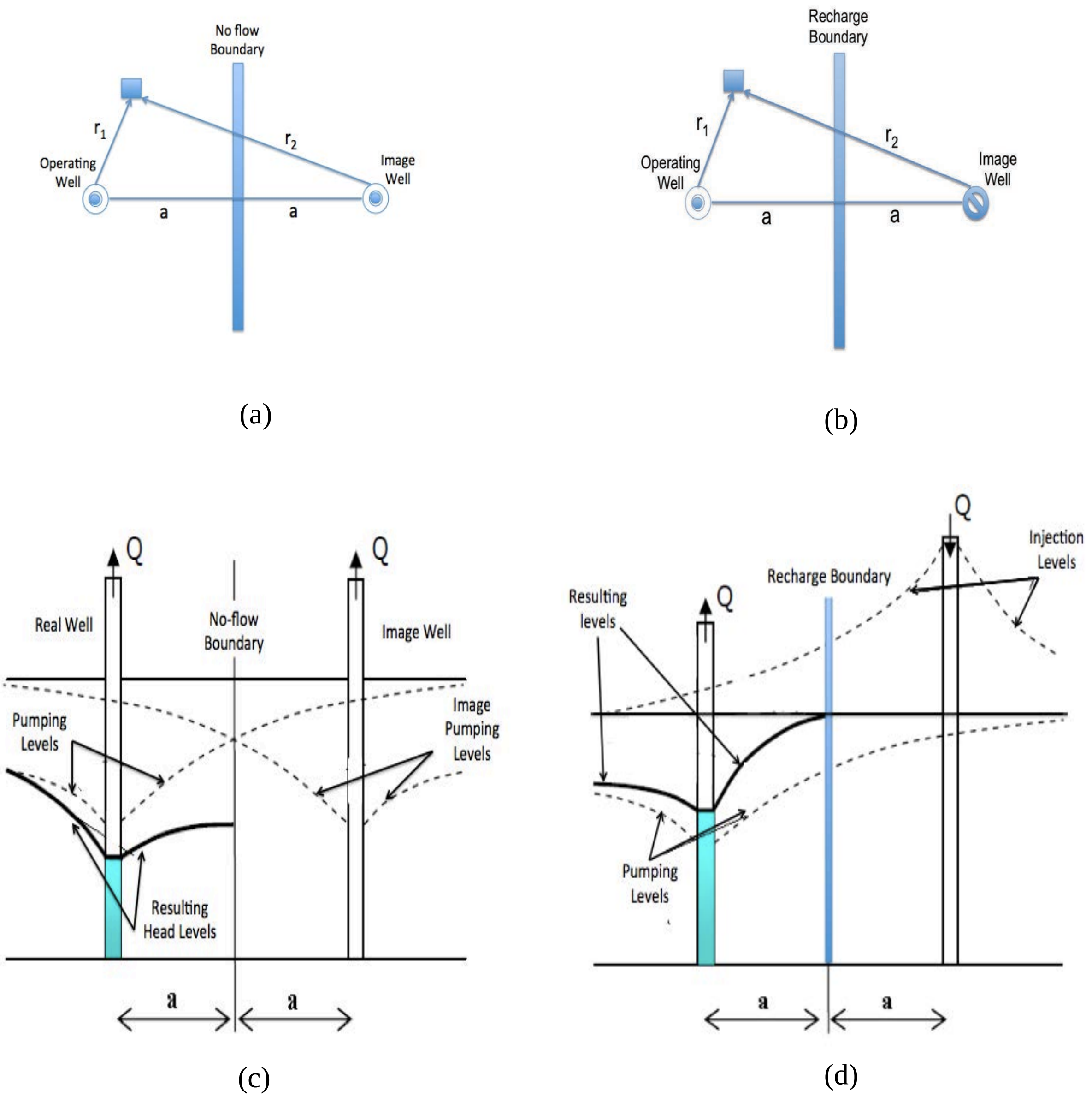


Figure 2.5 layouts of semi-infinite aquifer with a recharge boundary in subpanel (a) and no-flow boundary in subpanel (b). Subpanels (c) and (d) show the resulting head profiles due to a well operating in proximity of a no-flow and a recharge boundary, respectively

The general drawdown equation due to a single well operating in proximity of a recharge boundary is thus given by the sum of the effects of the real well and the image well:

$$s(x,y;t) = -Q \cdot \left[\frac{1}{4\pi T} \cdot \left[\mathrm{W}\left[\mathrm{u}(\mathrm{r}_1; t-\tau)\right] - \mathrm{W}\left[\mathrm{u}(\mathrm{r}_2; t-\tau)\right]\right]\right]$$
$$= -Q \cdot \mathrm{W_R}\left[\mathrm{u}(\mathrm{r}_1, \mathrm{r}_2, \tau; t)\right] \tag{2.41}$$

The well function for the operating well $W\ [u(r_1; t-\tau)]$ and the well function for the image well $W\ [u(r_2; t-\tau)]$ are both calculated using Equation (2.37), r_1 and r_2 are the distance of the monitoring point *(x,y)* from the real well and image well, respectively.

The general drawdown equation due to a single well operating in proximity a no-flow boundary is the following:

$$s(x,y;t) = -Q \cdot \left[\frac{1}{4\pi T}\left[\mathrm{W}\left[\mathrm{u}(\mathrm{r}_1; t-\tau)\right] + \mathrm{W}\left[\mathrm{u}(\mathrm{r}_2; t-\tau)\right]\right]\right]$$
$$= -Q \cdot \mathrm{W_N}\left[\mathrm{u}(\mathrm{r}_1, \mathrm{r}_2, \tau; t)\right] \tag{2.42}$$

The drawdown distribution due to a well field operating in an aquifer delimited by either a recharge or a no-flow rectilinear boundary is obtained as:

$$s(x,y;t) = \sum_{m=1}^{n_{ow}} -Q_m \cdot \mathrm{W_B}\left[u\left(r_{m,1}, r_{m,2}, \tau_m; t\right)\right] \tag{2.43}$$

where the function $\mathrm{W_B}\left[u\left(r_{m,1}, r_{m,2}, \tau_m; t\right)\right]$ is equal to $\mathrm{W_R}$ (Equation (2.41)) for a recharge boundary, or $\mathrm{W_N}$ (Equation (2.42)) for a no-flow boundary.

Figure 2.6 shows the drawdown distribution at time *t* = 120 days in a semi-infinite aquifer subject to extraction from 2 wells at *(x,y) = (500,1000)* and *(1000,500)*. In subpanel (a) a recharge boundary is located at *x = 0* (the y-axis), where in subpanel (b) a no-flow boundary is present at *x = 0*. Aquifer parameters used in these scenarios are given in Table 2.1. Detailed information about the extracting wells (schedule and pumping rates) is given in Table 2.2. These plots are obtained using a Matlab code SI.Drawdown.2D.m which calls two different subroutines; Theis.Recharge.m represents the well function in Equation (2.41) and Theis.No-flow.m represents the well function in equation (2.42). This code is provided in Appendix C.

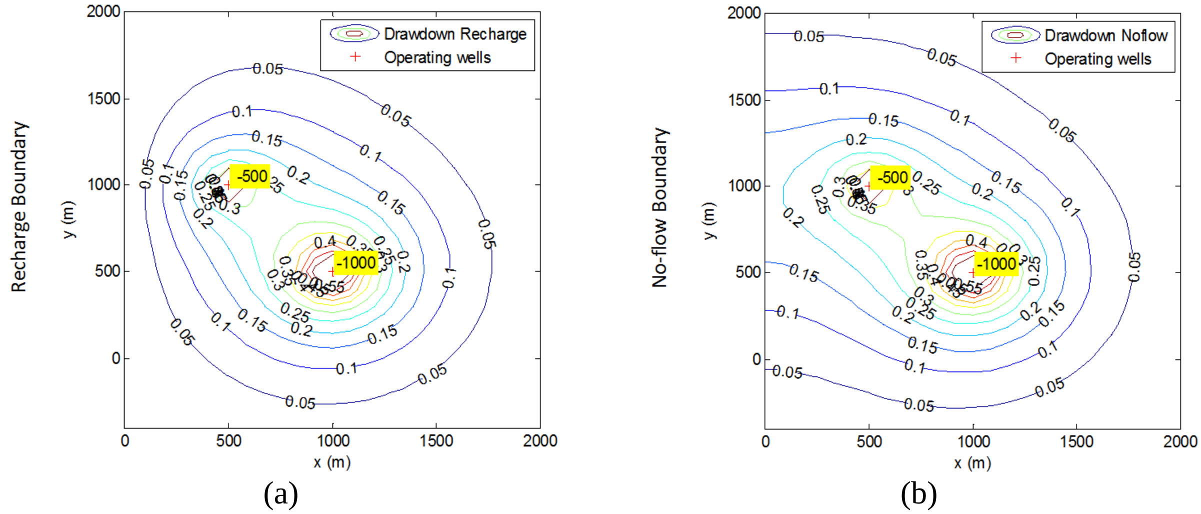


Figure 2.6 Drawdown contours at *t* = *120* days caused by two extracting wells located at *(500,1000)* and *(1000,500)*, respectively, in semi-infinite aquifer with a recharge boundary (subpanel a) and with a no-flow boundary (subpanel b), extraction rates are *-500* m/day for the first operating well and *-1000* m^3/day for the second well 2

In Figure 2.6 subpanel (a) note that the drawdown contours do not intersect with the recharge boundary; that is because, in the flow net shown in the figure, both of the drawdown contours and the recharge boundary are equi-potential lines. Instead in subpanel (b), with the no-flow boundary being a flow line, the contours intersect with it at a right angle.

2.2.3 The Theis Solution in Unconfined Aquifers

Unconfined aquifers differ from confined aquifers in that their upper boundary, known as water table, constitutes a free surface boundary at which the relative pressure is equal to zero. In an unconfined aquifer, the change in the water storage occurs as a response to drainage or recharge of the pores within the cone of depression; and, different from the confined aquifers, the saturated thickness changes with time. (McWhorter and Sunada, 1995)

In unconfined aquifers, solution of the saturated ground water flow Equation (2.5) is difficult because of the unknown location of the water table, which would be required as a known boundary condition.

In practice, the Theis solution can still be extended to model flow in unconfined aquifers under the assumption of prevalent horizontal flow (Dupuit approximation). Polubarinova-Kochina (1962) shows that this assumption is sufficiently accurate if the aquifer drawdown is small compared to the initial saturated thickness of the aquifer. With this condition aquifer's transmissivity and storativity can be considered constant, and the vertical velocities are neglected in relation to horizontal velocities within the aquifer. However, corrections should be applied to the Theis solution in vicinity of operating wells, where the vertical component of the pore velocity may be significant and the occurring drawdown is typically large.

It is worth noting that storativity values for unconfined aquifers are practically equal to the apparent specific yield, S_{ya} (the ratio of the volume of water added or removed directly from the saturated zone of the aquifer to the resulting change in the volume of aquifer below water). In this respect, the storativity value of 0.2 given in Table 2.1 is truly appropriate for unconfined aquifers.

2.2.4 The Glover Equation:

Glover and Balmer's (1954) solution was developed to compute the effects of depletion/ accretion due to an operating well on a stream hydraulically connected to the aquifer. In this situation, the stream may be seen as a recharge boundary, which provides a constant head boundary condition for the semi-infinite aquifer. The solution was derived based on approximation the proposed by Theis (1941) to evaluate stream depletion by integrating the Darcy's flux (in terms of drawdown) per unit width of the stream over the entire length of the stream. The Glover and Balmer (1954) solution is still widely used in ground-water/surface-water conjunctive management.

The assumptions underlying the Glover model are the following**:** (a) the aquifer is semi-infinite, horizontal, homogeneous and isotropic; (b) stream and aquifer are initially at equilibrium (the initial head h_0 in aquifer is constant and the same as the stage level in the river); (c) aquifer transmissivity is uniform and constant over time (saturated thickness does not change significantly); (d) stream stage remains constant over time; (e) the stream forms a straight line and fully penetrates the aquifer (flow is horizontal); and (f) the stream is perfectly connected to the aquifer (no resistance to flow is caused by fine sediments at the streambed).

Note a significant part of these assumptions is the same as in the Theis solution. The Glover model is derived for the case of a single well operating at a constant extraction rate Q and starting time $t = 0$, located at the origin of the Cartesian system in an aquifer with transmissivity T and storativity S. The stream is represented by the straight line located at $x = a$.

Figure 2.7 shows the layout of aquifer/stream system used to derive Glover's model.

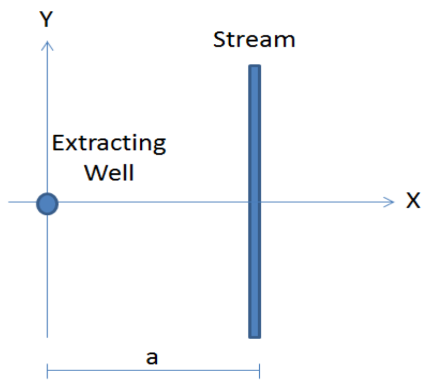


Figure 2.7 Aquifer/stream system layout used to derive the Glover equation

Let $q'(a, y; t)$ be the Darcy's discharge (in terms of drawdown) per unit width along the x direction on the generic point located on the stream line $x = a$:

$$q'_x(a, y; t) = T \cdot \frac{\partial s}{\partial x}(a, y; t) \tag{2.44}$$

Rewriting Equation (2.18) in the Cartesian system and using the chain rule, the partial derivative at the right hand side of Equation (2.44) can be expressed as:

$$\frac{\partial s}{\partial x}(a, y; t) = \frac{\partial s}{\partial u}(a, y; t) \cdot \frac{\partial u}{\partial x}(a, y; t) \tag{2.45}$$

where:

$$\frac{\partial s}{\partial u}(a,y;t) = \frac{\partial\left[\frac{Q}{4\pi T}\cdot\int_u^{+\infty}\frac{e^{-u'}}{u'}\cdot du'\right]}{\partial u} = \frac{Q}{4\pi T}\cdot\left[\frac{e^{-u'}}{u'}\right]_u^{+\infty}$$

$$= -\frac{Q}{4\pi T}\cdot\frac{e^{-u'}}{u'} = -\frac{Q}{4\pi T}\cdot\frac{e^{-\frac{S}{4T}\cdot\frac{(a^2+y^2)}{t}}}{\frac{S}{4T}\cdot\frac{(a^2+y^2)}{t}} \tag{2.46}$$

and

$$\frac{\partial u}{\partial x}(a,y;t) = \frac{\partial\left[\frac{S}{4T}\cdot\frac{(a^2+y^2)}{t}\right]_{x=x_1}}{\partial x} = \frac{S}{4T}\cdot\frac{2\cdot a}{t} \tag{2.47}$$

Substitution of Equations (2.46) and (2.47) into Equation (2.44) yields:

$$q_x'(a,y;t) = -T\cdot\frac{Q}{4\pi T}\cdot\frac{e^{-\frac{S}{4T}\cdot\frac{(a^2+y^2)}{t}}}{\frac{S}{4T}\cdot\frac{(a^2+y^2)}{t}}\cdot\frac{S}{4T}\cdot\frac{2\cdot a}{t}$$

$$= -\frac{Q}{2\pi}\cdot\frac{e^{-\frac{S}{4T}\cdot\frac{(a^2+y^2)}{t}}}{(a^2+y^2)}\cdot a \tag{2.48}$$

Note that in Equation (2.48), q_x' is negative if both Q and a are positive. Indeed, groundwater flow will have a negative horizontal component, that is, opposite to the verse of the x-axis.

The stream depletion rate can be obtained by using the method of images presented in section 2.2.2 (c). In practice, the stream provides a discharge per unit length equal to two times that given by Equation (2.48). The total stream depletion rate is thus given by:

$$Q_r = 2\cdot\int_{-\infty}^{+\infty} q_x'(a,y;t)\cdot dy = \frac{Q}{\pi}\int_{-\infty}^{+\infty}\frac{e^{-\frac{S}{4T}\cdot\frac{(a^2+y^2)}{t}}}{(a^2+y^2)}\cdot a\cdot dy \tag{2.49}$$

In Equation (2.49), the negative sign of the right-hand side of Equation (2.48) is intentionally overlooked since the stream depletion rate is to be considered positive if the well withdraws water from the aquifer ($Q_r > 0$ if $Q > 0$ and vice versa).

To calculate the integral in Equation (2.49), the fraction Pr between the stream depletion rate Q_r and the pumping rate Q is considered, and the parameter $\alpha = \frac{S}{4T \cdot t}$ is substituted in (2.49):

$$P_r = \frac{Q_r}{Q} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-\alpha \cdot (a^2 + y^2)}}{(a^2 + y^2)} \cdot a \cdot dy \tag{2.50}$$

The integral in Equation (2.50) may be calculated by observing that:

$$\begin{aligned} \frac{\partial P_r}{\partial \alpha} &= \frac{1}{\pi} \cdot \frac{\partial \left[\int_{-\infty}^{+\infty} \frac{e^{-\alpha \cdot (a^2 + y^2)}}{(a^2 + y^2)} \cdot a \cdot dy \right]}{\partial \alpha} \\ &= \frac{1}{\pi} \cdot \int_{-\infty}^{+\infty} \frac{\partial \left[\frac{e^{-\alpha \cdot (a^2 + y^2)}}{(a^2 + y^2)} \cdot a \right]}{\partial \alpha} \cdot dy \\ &= -\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-\alpha \cdot (a^2 + y^2)}}{(a^2 + y^2)} \cdot a \cdot (a^2 + y^2) \cdot dy \\ &= -\frac{a}{\pi} \int_{-\infty}^{+\infty} e^{-\alpha \cdot (a^2 + y^2)} \cdot dy \\ &= -\frac{a}{\pi} \cdot e^{-\alpha . a^2} \cdot \int_{-\infty}^{+\infty} e^{-\alpha \cdot y^2} \cdot dy \end{aligned} \tag{2.51}$$

The integral in Equation (2.51) may be solved by introducing the variable $z = \sqrt{\alpha} \cdot y$ where $dz = \sqrt{\alpha} \cdot y$ and using the Gauss integral $\int_{-\infty}^{+\infty} e^{-z^2} \, dz = \sqrt{\pi}$:

$$\int_{-\infty}^{+\infty} e^{-\alpha \cdot y^2} . dy = \frac{1}{\sqrt{\alpha}} \cdot \int_{-\infty}^{+\infty} e^{-z^2} \cdot dz = \sqrt{\frac{\pi}{\alpha}} \tag{2.52}$$

Thus, Equation (2.51) becomes:

$$\frac{\partial P_r}{\partial \alpha} = -\frac{a}{\pi} \cdot\ e^{-\alpha\,.a^2}\ \cdot \int_{-\infty}^{+\infty} e^{-\alpha\,.y^2} \cdot dy = -\frac{a}{\pi} \cdot e^{-\alpha\,.a^2}\ \cdot \sqrt{\frac{\pi}{\alpha}}$$

$$= -\frac{a}{\sqrt{\pi \cdot \alpha}} \cdot e^{-\alpha \cdot a^2} \tag{2.53}$$

Note that the integration "in *dy*" has been taken care of in Equation (2.53). Equation (2.51) is now integrated with respect to α to obtain P_r Equation (2.50). To do so, the variable $\chi = \sqrt{\alpha}$ is introduced (note that $d\chi = \frac{d\alpha}{2.\sqrt{\alpha}}$):

$$P_r = \frac{\partial P_r}{\partial \alpha} \cdot d\alpha = -\int \frac{a}{\sqrt{\pi.\alpha}} \cdot e^{-\alpha\,.a^2} \cdot d\alpha =$$

$$-\frac{a}{\sqrt{\pi}} \int\ \frac{e^{-\alpha\,.a^2}}{\sqrt{\alpha}} d\alpha = -\frac{a}{\sqrt{\pi}} \int\ \frac{e^{-(\chi.a)^2}}{\chi} \cdot 2 \cdot \chi \cdot d\chi \tag{2.54}$$

$$= -\frac{2}{\sqrt{\pi}} \int\ e^{-(\chi.a)^2}\ \cdot d(\chi \cdot a)$$

Since the error function is defined as $\mathrm{erf}(v) = \frac{2}{\sqrt{\pi}} \cdot \int_0^x e^{-v^2} \cdot dv$, Equation (2.54) may be rewritten as:

$$P_r = -\frac{2}{\sqrt{\pi}} \int\ e^{-(\chi.a)^2}\ \cdot d(\chi \cdot a) = -erf(\chi \cdot a) + C$$

$$= -erf\left(\sqrt{\alpha} \cdot a\right) + C \tag{2.55}$$

where C is a constant of integration that may be calculated from the condition: $\mathrm{P_r}(\alpha \to \infty) = 0$ (note that $\alpha \to \infty$ corresponds to $t \to 0$). Since the error function is such that $\mathrm{erf}(v \to +\infty) = \frac{2}{\sqrt{\pi}} \cdot\ \int_0^{+\infty} e^{-v'^2} \cdot dv' = \frac{2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} = 1$, the constant C is determined as follows:

$$P_r(\alpha \to +\infty) = -\lim_{\alpha \to \infty} erf\left(\sqrt{\alpha}\,.x_1\right) + C = -1 + C = 0\ \Rightarrow C = 1$$

The function P_r is thus determined to be equal to:

$$P_r = 1 - erf\left(\sqrt{\alpha} \cdot a\right) = erfc\left(\sqrt{\alpha} \cdot a\right) = erfc\left(\sqrt{\frac{S}{4 \cdot T \cdot t}} \cdot a\right)$$
$$= erfc\left(\sqrt{\frac{S}{4T} \cdot \frac{a^2}{t}}\right) \tag{2.56}$$

From Equation (2.51), the stream depletion flow rate is thus:

$$Q_r(Q, T, S, a; t) = Q \cdot erfc\left(\sqrt{\frac{S}{4T} \cdot \frac{a^2}{t}}\right) \tag{2.57}$$

Equation (2.57) constitutes the Glover model (Glover and Balmer.1954) and it can be integrated over time to obtain an equation for stream depletion volume (e.g. Miller et al, 2007):

$$V_r(Q, T, S, a; t)$$
$$= Q \cdot t$$
$$\cdot \left[\left(\frac{S}{2T} \cdot \frac{a^2}{t} + 1\right) \cdot erfc\left(\sqrt{\frac{S}{4T} \cdot \frac{a^2}{t}}\right) - \left(\sqrt{\frac{S}{4T} \cdot \frac{a^2}{t}}\right) \tag{2.58}\right.$$
$$\left. \cdot \left(\frac{2}{\sqrt{\pi}}\right) \cdot \exp\left(\frac{S}{4T} \cdot \frac{-a^2}{t}\right)\right]$$

Equations (2.58) and (2.57) can be rearranged to obtain the ratio between the stream depletion rate and the well pumping rate starting at generic time τ:

$$Q_{ratio}(T, S, a, \tau; t) = \frac{Q_r(Q, T, S, a; t)}{Q} = erfc\left(\sqrt{\frac{S}{4T} \cdot \frac{a^2}{t - \tau}}\right) \tag{2.59}$$

and the ratio between the stream depletion volume and the volume of groundwater pumped by the well:

$$V_{ratio}(T,S,a,\tau;t) = \frac{V_r(Q,T,S,a;t)}{Q\cdot(t-\tau)}$$

$$= \left(\frac{S}{2T}\cdot\frac{a^2}{t-\tau}+1\right).erfc\left(\sqrt{\frac{S}{4T}\cdot\frac{a^2}{t-\tau}}\right) \tag{2.60}$$

$$-\left(\sqrt{\frac{S}{4T}\cdot\frac{a^2}{t-\tau}}\right)\cdot\left(\frac{2}{\sqrt{\pi}}\right)\cdot\exp\left(\frac{S}{4T}\cdot\frac{-a^2}{t-\tau}\right)$$

A Matlab code (SI.Glover) is built to calculate and plot Equations (2.59) and (2.60) with time. This code is provided in Appendix E, along with instructions and examples for its use.

Figure 2.8 shows the profiles of the stream depletion rate ratio $Q_{ratio}(T,S,a;t)$ (represented by the solid line) and the stream depletion volume ratio $V_{ratio}(T,S,a;t)$ (represented by the dashed line) as a function of time, respectively, due to a well located *1000* m away from the stream, for a simulation period of ten years (*3650* days)of continuous well operation. These profiles are obtained using the SI.Glover.m code. Aquifer properties used to plot this figures are presented in Table 2.1.

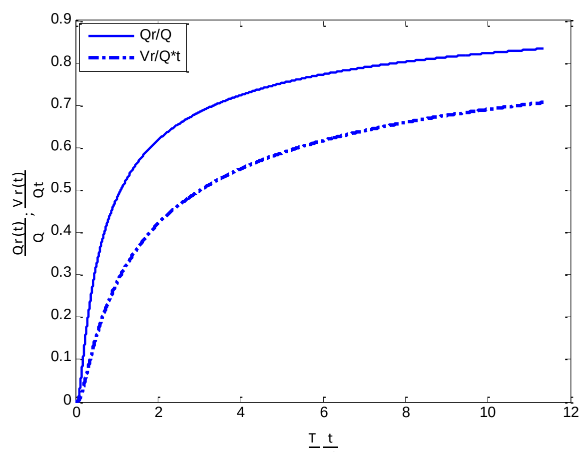


Figure 2.8 Stream depletion rate (solid line) and stream depletion volume (dashed line) due to a well operating continuously for *3650* days. The well is located at *500* m from the stream

Note that, in Figure 2.8 the stream depletion volume function defined as the ratio between stream depletion volume and the actual pumped volume, tends to reach a value of 1.0 (a steady state) with time.

2.2.5 Superposition of solutions: Stream Depletion

Similar to the Theis solution, Equation (2.57) and (2.58) (Glover and Balmer, 1954) indicate that both Q_r and V_r are linearly proportional to the well pumping rate Q. In practice, the principal of superposition of solution can be applied to the Glover model to calculate stream depletion rates and volumes from a generic well field with time varying pumping rates.

The stream depletion rate due to a well field can be calculated as:

$$Q_{r,total} = \sum_{m=1}^{n_{ow}} [Q_m \cdot Q_{ratio}(T, S, a_m, \tau_m; t)] \tag{2.61}$$

Note that in Equation (2.61) the Q_{ratio} function is given by Equation (2.59) if $t_m > 0$, or it is equal to zero otherwise.

The stream depletion rate due to a single well with time varying pumping rates is calculated as:

$$Q_r(t) = \int_o^t \frac{\partial Q(\tau)}{\partial \tau} \cdot Q_{ratio}(T, S, a, \tau; t) \cdot d\tau \tag{2.62}$$

The stream depletion rate due to a well field with time varying pumping rates is obtained combining Equation (2.61) and (2.62):

$$Q_{r,total}(t) = \sum_{m=1}^{n_{ow}} \int_o^t \frac{\partial Q_m(\tau_m)}{\partial \tau_m} . Q_{ratio}(T, S, a_m, \tau_m; t). d\tau_m \tag{2.63}$$

Equations similar to (2.61), (2.62) and (2.63) can be obtained for the stream depletion volume:

In the case of a well field, the stream depletion volume is given by:

$$V_{r,total}(t) = \sum_{m=1}^{n_{ow}} Q_m \cdot (t - \tau_m) \cdot V_{ratio}(T, S, a_m, \tau_m; t) \tag{2.64}$$

The stream depletion volume due to a single operating well with time varying pumping rate is obtained by:

$$V_r(t) = \int_0^t \frac{\partial Q(\tau)}{\partial \tau} \cdot (t - \tau) \cdot V_{ratio}(T, S, a, \tau; t) \cdot d\tau \tag{2.65}$$

The stream depletion volume in a well field with a time varying pumping rates is calculated as:

$$V_{r,total}(t) = \sum_{m=1}^{n_{ow}} \int_0^t \frac{\partial Q_m(\tau_m)}{\partial \tau_m} \cdot (t - \tau_m) \cdot V_{ratio}(T, S, a_m, \tau_m; t) \cdot d\tau_m \tag{2.66}$$

Using the stream depletion equations listed above; one can study the ratio between stream depletion volume during operation period and after it has been ceased. Let us consider, for example, the case of five operating wells in an alluvial aquifer. The aquifer is in a hydraulic contact with a stream located at $x = 0$ (the y-axis), and all wells are extracting with the same constant operation rate of *-1000* m^3/day for *180* days and are shut off afterwards. Table 2.3 shows the distances between the operating wells and the stream, listed in the second column.

A simulation is performed for a total time of five year (1825 days), where the wells are activated individually and their impact on stream (stream depletion volume) is calculated during pumping and after the well is shut off, as well as the ratio between them. The results are shown in Table 2.3. The third column represents the stream depletion volume during operation, the fourth column gives the values of stream depletion at end of simulation duration, and in the fifth column, the ratio between the two volumes is reported.

Table 2.3 operating wells distances from the stream and simulation results (stream depletion volume during operation, stream depletion volume at the end of simulation duration and ratio between the two volumes)

Well	Distance *a* (m)	Vr_1 m^3 (during operation)	Vr_2 m^3 (at end of simulation time)	Ration (Vr_1/ Vr_2)
1	200	-130968.34	-171259.28	0.764737
2	800	-45156.620	-145358.63	0.310657
3	1200	-19784.90	-128668.62	0.153766
4	2000	-2798.70	-97659.76	0.028658
5	5000	-0.02852	-23039.15	1.24E-06

It is interesting to note that, since the stream is the recharge source of the aquifer, it provides a continuous supply even after the shutting off of the extracting well. However the ratio between the stream depletion volume during and after well operation varies significantly, from 0.764737 to 1.24E-06, as the distance of the well to the stream is increased, and all the pumped volume ends being extracted completely from the stream, that is why, one can note that in Figure 2.8 the stream depletion volume ratio reaches a value of 1.0. The total time needed for this process to end, and the volume extracted directly from the stream during operation time depend largely on the distance between the operating well and the stream.

Figure 2.9 shows the V_{ratio} vs. time profiles for the five wells considered in the example, these profiles show that at larger times the ratio between the stream depletion volume and the total pumped water tends to be 1, which complies with the conclusion that the extracted amount of water ends coming completely from the stream.

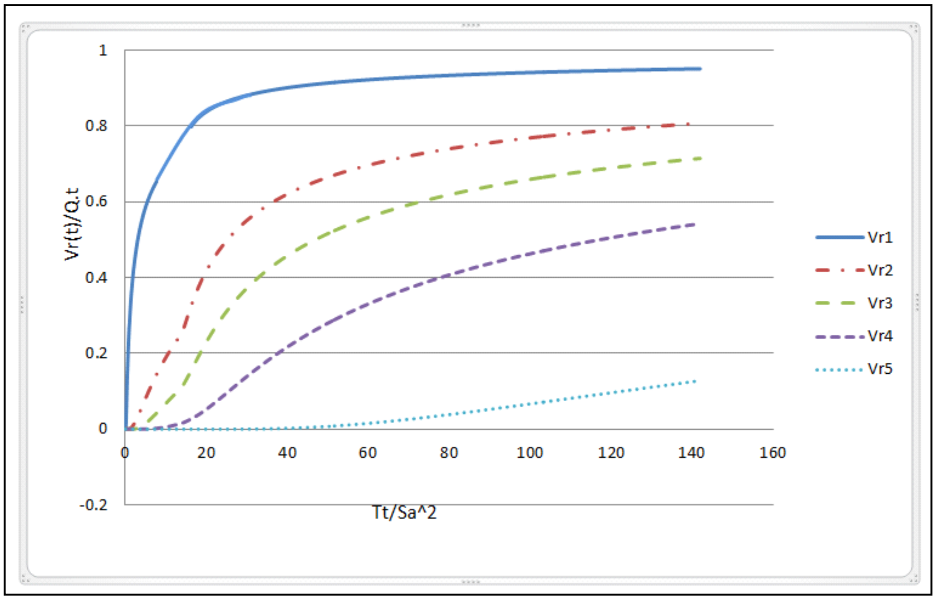


Figure 2.9 stream depletion volume ration V_{ratio} vs. unitless time $\frac{4T}{S} \cdot \frac{t}{a^2}$ for the five wells used for study, each curve (with a different shape) represents a well as shown in the legend

2.3 Pumping in a Finite/Bounded Aquifer

In this work, an aquifer is considered finite if characterized by a finite areal extension due to the presence of two parallel boundaries in a 2-dimensional domain. The aquifer has a width (*w*) equal to the distance between the two boundaries, and the operating well is located within this width. This estate yields a number of image wells across the two boundaries. As mentioned above in Section 2.2.2, superposition of solution provides a useful approach to apply the Theis and Glover equations in order to calculate the distribution of drawdown in the aquifer, and the impact on the stream flow due to well operations.

In this section, we consider two configurations of bounded aquifers: a) an aquifer comprised between a recharge boundary and a no-flow boundary, and b) an aquifers comprised between two recharge boundaries.

Figure 2.10 shows practical examples of the two configurations mentioned above. In Figure 2.10a the alluvial aquifer is comprised between a no-flow boundary represented by a physical boundary, at which the aquifer terminates, and a recharge boundary represented by a stream. In Figure 2.10b the aquifer is bounded between an irrigation ditch and a stream, in this case, both boundaries constitutes recharge, constant-head boundaries.

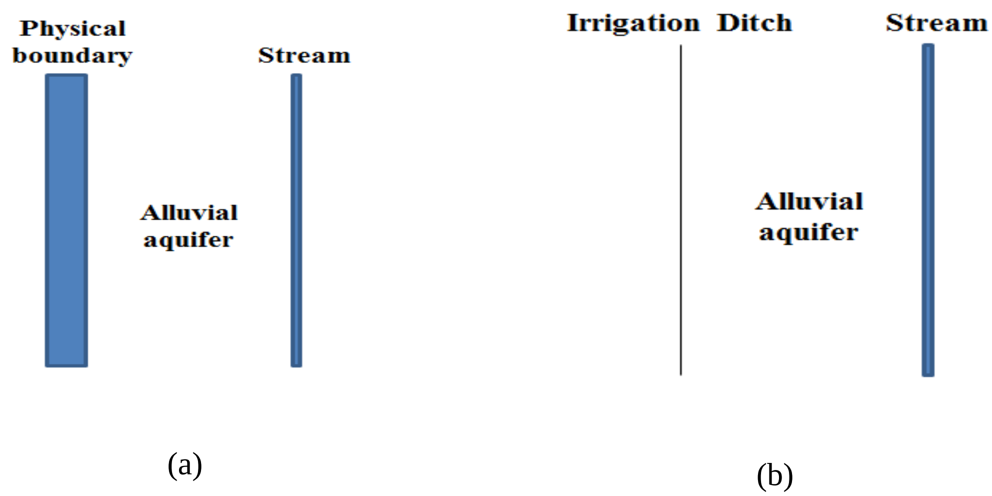


Figure 2.10 Examples of finite constant width aquifers: (a) an aquifer between a no-flow boundary (the physical boundary) and a recharge boundary (the stream), and (b) an aquifer comprised between two recharge boundaries (an irrigation ditch and a stream)

2.3.1 Superposition of Solutions: Drawdown

Let us study first the case of a finite aquifer comprised between a no-flow/physical boundary and a recharge boundary (a stream). Figure 2.11 shows the well layout scheme obtained by applying the method of images accounting for the effect of the two boundaries.

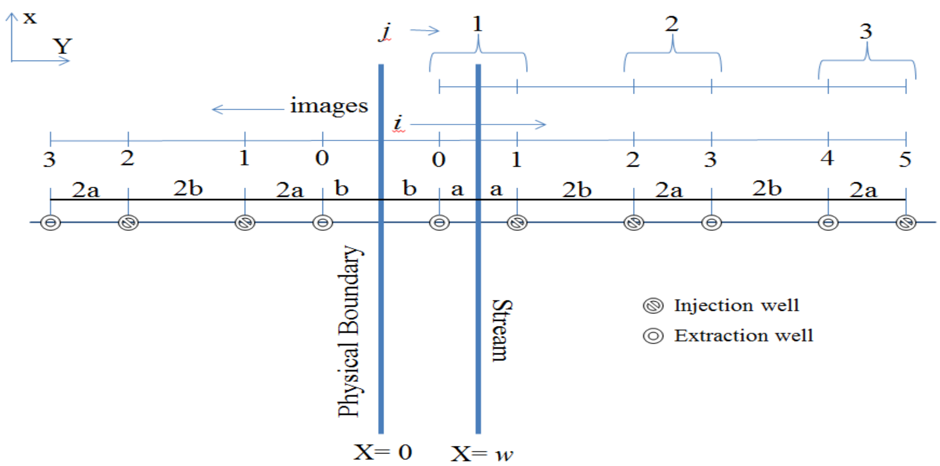


Figure 2.11 Distribution of image wells with their operation type for an operating well at *x* = *b*, in an aquifer bounded between a physical boundary and a stream

As shown in Figure 2.11, the real well i_0 produces two images across the two boundaries, which are in turn going to produce further images; this process creates an infinite series of image wells across the two boundaries of the finite aquifer. The wells are located symmetrically with respect to the no-flow boundary. On one side of this boundary, the wells are grouped into couples ordered sequentially according to the index *j* (*j*=*1, 2, 3…*). Each couple is characterized by wells with flow rates that are opposite in sign. If *j* is an odd integer the first well is extracting and the second well is injecting, whereas if *j* is an even integer the first well is injecting and the second well is extracting.

In Figure 2.11, the coordinates of wells in group *j* with the indices $i=2 \cdot j-2$ and $i = 2 \cdot j-1$ are given by:

$$x_{w,2\cdot j-2} \;=\; (2\cdot j - 1)\cdot\; w - a \tag{2.67}$$

$$x_{w,2\cdot j-1} \;=\; (2\cdot j - 1)\cdot\; w + a \tag{2.68}$$

and the coordinates of image wells with respect to the axis of symmetry are:

$$x_{w,I2\cdot j-2} \;=\; -x_{w,2\cdot j-2} \tag{2.69}$$

$$x_{w,2\cdot j-1} \;=\; -x_{w,2\cdot j-1} \tag{2.70}$$

All wells have the same y coordinate, y_w. With the wells layout given in Figure 2.11, the drawdown at a generic time t due to a single well operating continuously with a rate Q, starting at time τ, in a bounded aquifer characterized by transmissivity T and storativity S, is thus given by:

$$\begin{aligned} s(x,y;t) = -Q \Bigg\{ \sum_{j=1}^{\infty} (-1)^{j+1} \cdot \frac{1}{4\pi T} \\ \cdot \big(W\big[u\big(r_{2\cdot j-2}; t-\tau\big)\big] - W\big[u\big(r_{2\cdot j-1}; t-\tau\big)\big] \\ + W\big[u\big(r_{I2\cdot j-2}; t-\tau\big)\big] - \; W\big[u\big(r_{I2\cdot j-1}; t-\tau\big)\big]\big) \Bigg\} \\ = \; -Q \cdot W_{NR}\big[S, T, r_{2\cdot j-2}, r_{2\cdot j-1}, r_{I2\cdot j-2}, r_{I2\cdot j-1}, \tau;\; t\big] \end{aligned} \tag{2.71}$$

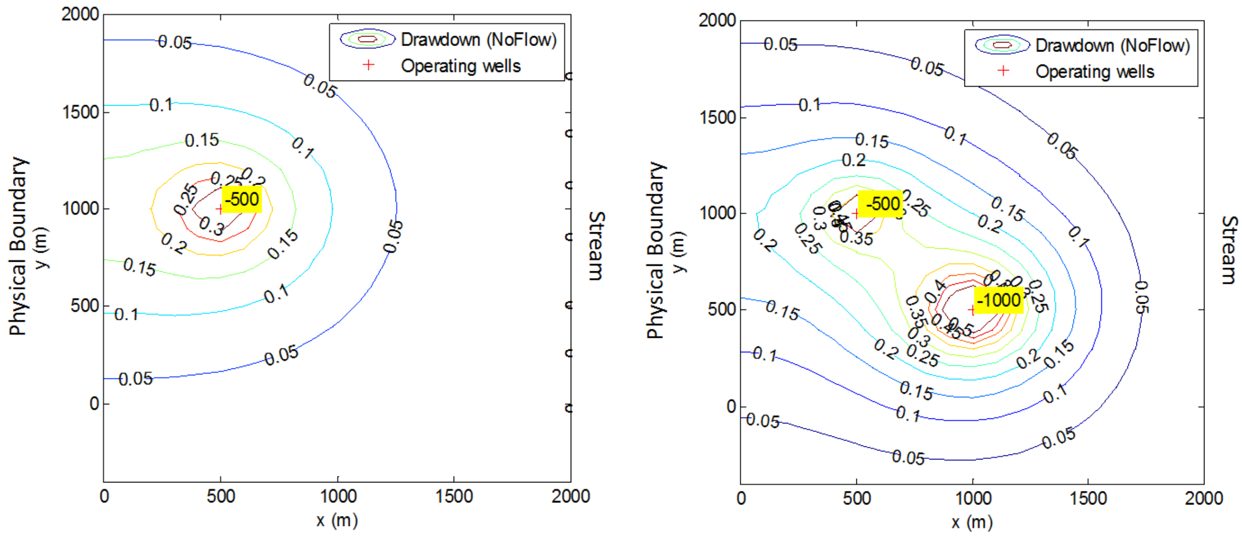
Where: $W_{NR}\big[S, T, r_{2\cdot j-2}, r_{2\cdot j-1}, r_{I2\cdot j-2,} r_{I2\cdot j-1}, \tau;\; t\big]$ represents an equivalent well function equal to the sum of the four functions of well group j and their images, forming the contribution to drawdown in a bounded aquifer between a no-flow boundary (N) and a recharge boundary (R). $r_{2\cdot j-2}$ and $r_{2\cdot j-1}$ are the distances from the operating well locations *(* $x_{w,2\cdot j-2}$ *,* y_w*)* and *(* $x_{w,2\cdot j-1}$ *,* y_w*)* to the observation point *(x,y)*. Similarly, $r_{I2\cdot j-2}$ and $r_{I2\cdot j-1}$ are the distances of image wells at *(* $x_{w,I2j\cdot-2}, y_w$*)* and *(* $x_{w,I2\cdot j-1}, y_w$*)* to observation point *(x,y)*. All distances are calculated using Equation (2.17). The well functions in Equation (2.72) are calculated using the Theis well function (Equation (2.35)) and they are valid only if $t - \tau > 0$, or equal to zero otherwise. Such condition is applied to all well functions introduced in this section.

Equation (2.71) can be generalized to calculate drawdown due to a well field with constant-rate pumping wells:

$$s(x, y; t) = \sum_{m=1}^{n_{ow}} -Q_m \cdot W_{NR}\left[S, T, r_{m,2j\cdot-2}, r_{m,2\cdot j-1}, r_{m,I2\cdot j-2}, r_{m,I2\cdot j-1}, \tau_m; t\right] \tag{2.72}$$

Equations (2.71) and (2.72) are implemented in a Matlab code Finite.Drawdown.2D.m described in Appendix F. The code is built with a subroutine Theis.NR.m that calculates the well function $W_{NR}\left[S, T, r_{m,2j\cdot-2}, r_{m,2\cdot j-1}, r_{m,I2\cdot j-2}, r_{m,I2\cdot j-1}, \tau_m; t\right]$.

Figures 2.12a and 2.12b show the drawdown distributions obtained using Finite.Drawdown.2D.m, at a snap shot time equal to *120* days due to a single continuously extracting well obtained using Equation (2.71), and due to two-well well field obtained using Equation (2.72), respectively. Wells locations and extraction rates are labeled in each plot. Other well's data are given with details in Table 2.2 and aquifer properties are indicated in Table 2.1. The no-flow boundary is located at the y-axis and the recharge boundary is at *x* = *2000* m (the aquifer width is 2000 m).



(a) (b)

Figure 2.12 Drawdown contours in a finite aquifer bounded by a physical boundary at *x* = *0* and a stream at *x* = *2000* m (aquifer width *2000* m), due to (a) a single continuously extracting well and (b) two extracting wells well field, both captures at *t* equals *120* days, wells operation details are given in Table 2.1

Now let us look at the configuration of finite aquifers bounded between two recharge boundaries. Figure 2.13 presents the well layout scheme created using the method of images to model the presence of the two boundaries.

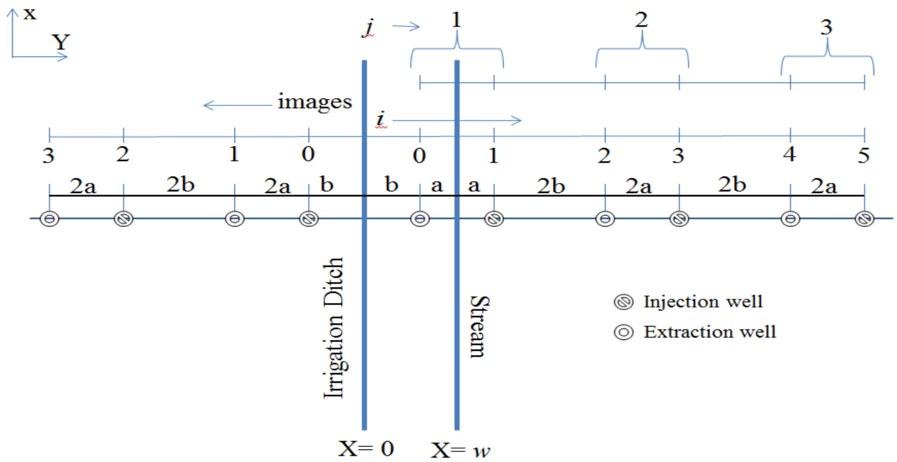


Figure 2.13 Distribution of image wells with their operation type of a an operating well at x = b in an aquifer bounded between an irrigation ditch and a stream

In this case, the no-flow boundary in Figure 2.11 is substituted by a secondary constant head boundary, for example, an irrigation ditch. This boundary represents the symmetry axis for the well layout. The image wells created across the axis of symmetry will have different operation types (in order) than their equivalents across a no-flow boundary. This condition causes the two wells in well group *j* to have the same order of operation types for all groups *j*; that is, extraction for the first well and injection for the second. Wells coordinates are obtained using Equations (2.67), (2.68), (2.69) and (2.70) and all wells distances to the observation point *(x,y)* are the same as in the former case. The drawdown general equation for the configuration in Figure 2.13 is as follows:

$$s(x,y;t) = -Q\left\{\sum_{j=1}^{\infty}\frac{1}{4\pi T}\cdot W\left[u\left(r_{2\cdot j-2};t-\tau\right)\right]\right.$$

$$-W\left[u\left(r_{2\cdot j-1};t-\tau\right)\right]-W\left[u\left(r_{I2\cdot j-2};t-\tau\right)\right] \tag{2.73}$$

$$\left.+\ W\left[u\left(r_{I2\cdot j-1,};t-\tau\right)\right]\right\}$$

$$= -Q\cdot W_{RR}\left[S,T,r_{2\cdot j-2},r_{2\cdot j-1},r_{I2\cdot j-2,}r_{I2\cdot j-1},\tau;\ t\right]$$

where $W_{RR}\left[S,T,r_{2\cdot j-2},r_{2\cdot j-1},r_{I2\cdot j-2,}r_{I2\cdot j-1},\tau;\ t\right]$ is the sum of the four functions of well group j and their images forming the contribution to drawdown in a system bounded between two recharge boundaries (RR).

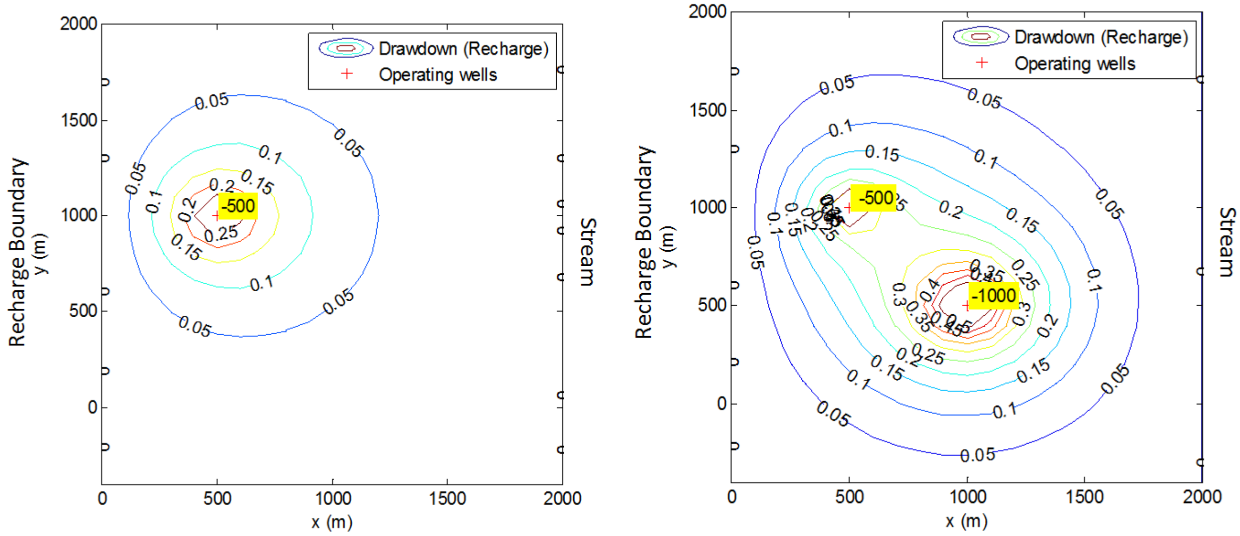
Consequently, the drawdown due to a well field of m wells is calculated by:

$$s(x,y;t)$$

$$= \sum_{m=1}^{n_{ow}} = -Q_m \tag{2.74}$$

$$\cdot W_{RR}\left[S,T,r_{m,2\cdot j-2},r_{m,2\cdot j-1},r_{m,I2\cdot j-2,}r_{m,I2\cdot j-1},\tau_m;t\right]$$

Figure 2.14a shows the drawdown distribution due to a single continuously extracting well, and Figure 2.14b shows the drawdown distributions due to two extracting wells. Both distributions are plotted using the Matlab code Finite.Drawdown.2D.m which, in this case, calls a subroutine Theis.RR.m to calculate the well function $W_{RR}\left[S,T,r_{m,2\cdot j-2},r_{m,2\cdot j-1},r_{m,I2\cdot j-2,}r_{m,I2\cdot j-1},\tau_m;t\right]$, detailed description of the code is in Appendix F. The labels in the figures indicate wells locations and extraction rates. Table 2.2, gives the detailed wells' data, whereas aquifer properties are given in Table 2.1. The irrigation ditch is located on the y-axis and the stream is at *x = 2000* m (the aquifer width is 2000 m). The snap shot time is 120 days.



(a) (b)

Figure 2.14 Drawdown contours in a finite aquifer bounded by a ditch located on the y-axis and a stream at *x* = 2000 m (aquifer width), due to (a) a single continuously extracting well and (b) a two extracting wells well field with operation details listed in Table 2.1, the figures are plotted at *t* equals 120 days

2.3.2 Superposition of solutions: Steam Depletion

To develop stream depletion general equations for a stream being one of the two bodies bounding a finite aquifer, the principle of superposition of effects is applied to all image wells introduced to the system to simulate the presence of the two boundaries. Figure 2.15 shows the layout scheme for image wells of an operating well located at $x = b$. In this case, the no-flow boundary represents symmetry axis for the well layout, and it is located on the y-axis.

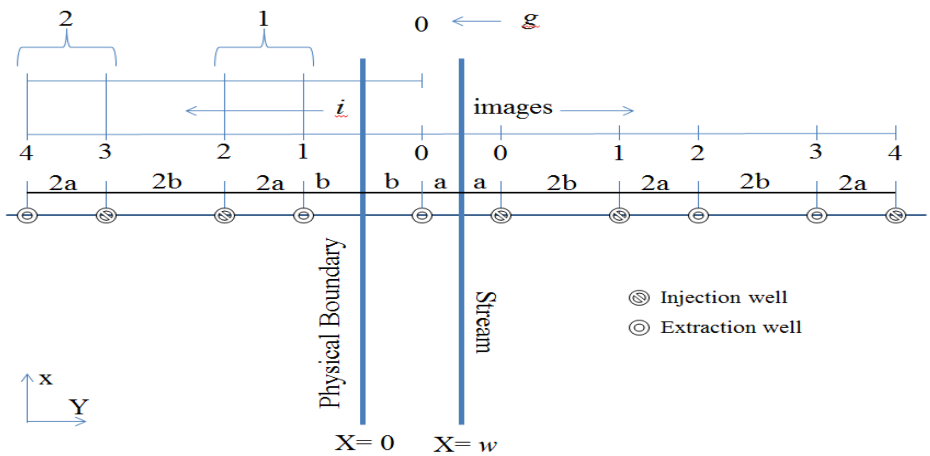


Figure 2.15 Image wells layout scheme with their operation type for a well in a finite aquifer bounded between a physical (no-flow) boundary on the y-axis and a recharge boundary located at *x*=*w* (the width of the aquifer)

As shown in Figure 2.15, wells to the left side of the symmetry axis are grouped in couples named Glover groups g $(g= 1, 2, 3, \ldots)$. Wells in group g have a different order (operation wise) from one group to the next. Wells coordinates are obtained using equations similar to Equations (2.67), (2.68), (2.69) and (2.70).

The total stream depletion rate can be obtained by applying superposition of solutions for the well system represented in Figure 2.15:

$$Q_r(Q,S,T,w,a;t) = Q \cdot \left\{ Q_{ratio}(S,T,w,a,\tau;t) + \sum_{g=1}^{\infty} [(-1)^{g+1}(Q_{ratio}[S,T,w,(2g.w - a),\tau;t] - Q_{ratio}[S,T,w,(2g.w+a),\tau;t])] \right\} = Q \cdot BNQ_{ratio}(S,T,w,a,\tau;\ t) \quad (2.75)$$

Analogously, the stream depletion volume is obtained as:

$$V_r\ (Q,S,T,w,a;t) = Q \cdot (t-\tau) \cdot \sum_{g=1}^{\infty} [(-1)^g \cdot C(S,T,w,a,g,\tau;t) + (-1)^g \cdot D(S,T,w,a,g,\tau;t)] = Q \cdot (t-\tau) \cdot BNV_{ratio}(S,T,w,a,\tau;\ t) \quad (2.76)$$

In Equation (2.75) and (2.76), the term $(-1)^{g+1}$is added to account for the change in wells extraction/injection order of group *g*. BNQ_{ratio} is the stream depletion rate ratio for a stream in a bounded aquifer with a no-flow boundary as the axis of symmetry, representing the sum of stream depletion rate function of the two well in group *g*. Likewise, $BNV_{ratio}(S,T,W,a,\tau;t)$ in Equation (2.76) is the stream depletion volume ratio, which equals the sum of stream depletion volume function of the two wells of group *g* and their images. The functions *C* and *D* in Equation (2.76) are calculated as:

$$C(S,T,w,a,g,\tau;t) = V_{ratio}[S,T,w,(2g.w+a),\tau;\ t] \quad (2.77)$$

$$D(S,T,w,a,g,\tau;t) = V_{ratio}[S,T,w,(2w+2g.w-a),\tau;t] \quad (2.78)$$

The application of Equations (2.75) and (2.76) are simulated using a Matlab code Finite.Glover.m with the manual given in Appendix H.

Figure 2.16 shows the stream depletion function profile with unit less time $\frac{4T}{S}.\frac{t}{a^2}$ for stream depletion rate ratio (represented by the dashed line) and stream depletion volume ratio (represented by the solid line), due to a well pumping continuously for simulation period of 1 year *(365 days)*. The well is located at *500* m away from the stream in an aquifer bounded between a no-flow boundary and a stream, and the aquifer width is *2000* m. These profiles are obtained using the Matlab code Finite.Glover.m which calls the function BNRQratio.m and BNRVratio.m to calculate stream depletion ratios in Equations (2.75) and (2.76). Aquifer properties used to develop this plot are listed in Table 2.1.

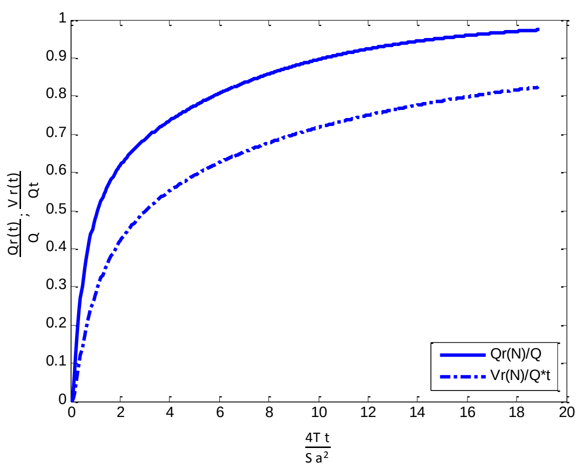


Figure 2.16 Stream depletion rate ratio (solid line) and stream depletion volume ratio (dashed line) caused by a well located at *500* m from a stream in a finite aquifer bounded between a no-flow boundary and a stream with a width *2000* m, the well operates continuously for *1* years (*365* days)

Figure 2.17 shows the stream depletion volume functions profile with unit less time $\frac{4T}{S}.\frac{t}{a^2}$ for a well operating in a semi-infinite aquifer (dashed line) and in a finite aquifer bounded between a no-flow boundary and a stream (solid line) with a width *2000* m, the well is located at *500* m from the stream in both cases and it is pumping continuously for *10* years (*3650* days). Aquifer properties used to develop this plot are listed in Table 2.1.

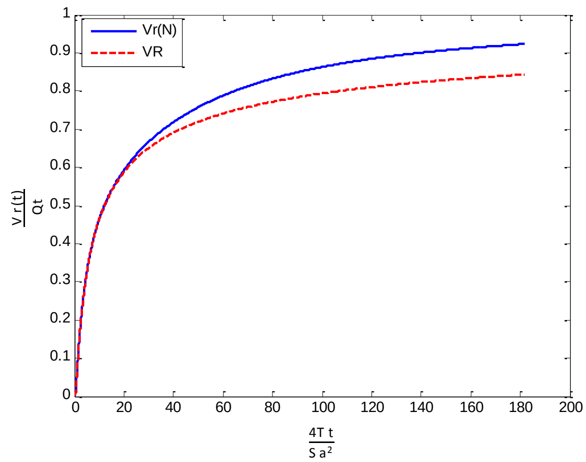


Figure 2.17 stream depletion volume due to a well operating continuously for *10* years (*3650* days) located at *500* m away from a stream in as semi-infinite aquifer (dashed line) and a stream in a finite aquifer bounded between a no-flow boundary (solid line) and a stream with a width of *2000* m

In Figure 2.17, the ratio between stream depletion volume and the actual pumped volume in a finite aquifer (solid line) reaches the steady state (when the ratio equals 1.0) faster than its equivalent in semi-infinite aquifer, due to the limitation on the lateral area in the former case, which constricts the expansion of the cone of depression within the aquifer in the direction of the no-flow boundary and force it to reach the stream in a less time. Stream depletion general equations for the case of a well field are obtained by applying the principle of superposition. The stream depletion rate is given by:

$$Q_{r,total} = \sum_{m=1}^{n_{ow}} Q_m \cdot BNQ_{ratio}\ (S, T, w, a_m, \tau_m; t) \tag{2.79}$$

The stream depletion volume is calculated as:

$$V_{r,total} = \sum_{m=1}^{n_{ow}} Q_m \cdot (t - \tau_m) \cdot BNV_{ratio}\ (S, T, w, a_m, \tau_m; t) \tag{2.80}$$

In Equations (2.79) and (2.80), the functions BNQ_{ratio} and BNV_{ratio} are equal to zero if $t \leq \tau_m$

Application of super-position of effects to stream depletion due to image wells in a bounded aquifer between two recharge boundaries (an irrigation ditch and a stream) produces the well layout shown in Figure 2.18. Glover groups similar to those created for the former configuration Figure 2.18 are going to be used here, whereby the wells order (injection/extraction) does not change from one group to the other, due to the type of the boundary representing the axis of symmetry characterized by the irrigation ditch.

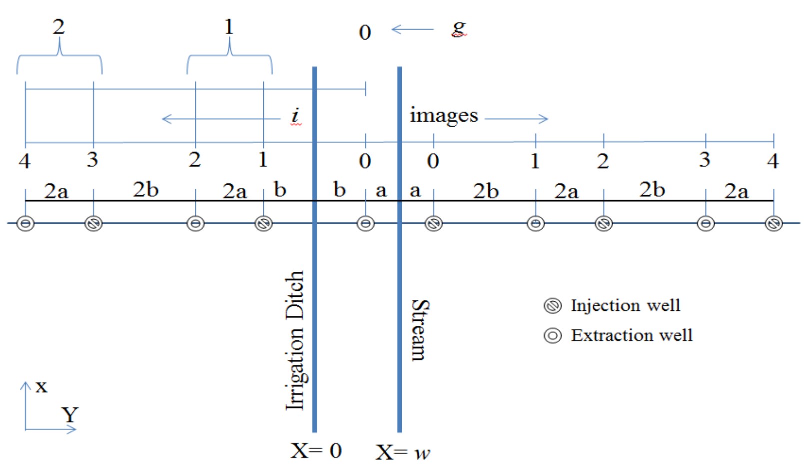


Figure 2.18 Image wells layout scheme with their operation type of a well at *x* = *b* in a bounded aquifer between an irrigation ditch and a stream

Given the well layout presented in Figure 2.18, the stream depletion rate due to a single operating well is thus given as:

$$
\begin{aligned}
Q_r(Q, S, T, w, a; t) \\
&= Q \\
&\cdot \left(Q_{ratio}(S, T, w, a, \tau; t) \right. \\
&+ \sum_{g=1}^{\infty} [-(Q_{ratio}[S, T, w, (2g.w - a), \tau;\ t] \\
&\left. + Q_{ratio}[S, T, w, (2g.w + a), \tau;\ t])] \right) \\
&= Q \cdot BRQ_{ratio}(S, T, w, a, \tau; t)
\end{aligned}
\tag{2.81}
$$

Correspondingly, the volume of stream depletion is:

$$
\begin{aligned}
V_r\ (Q, S, T, w, a; t) \\
&= Q \cdot (t - \tau) \\
&\cdot \sum_{g=0}^{\infty} [\ C(S, T, w, a, g, \tau;\ t) - D(S, T, w, a, g, \tau;\ t)] \\
&= Q \cdot (t - \tau) \cdot BRV_{ratio}(S, T, w, a, \tau;\ t)
\end{aligned}
\tag{2.82}
$$

where BRQ_{ratio} is the stream depletion ratio function, and BRV_{ratio} is the stream depletion volume function, in an aquifer bounded between two recharge boundaries. Both functions are equal to zero if $t \leq \tau$.

Functions $BRQ_{ratio}(S, T, w, a, \tau; t)$ and $BRV_{ratio}(S, T, w, a, \tau;\ t)$ are implanted in Matlab subroutines BRRQratio.m and BRRVratio.m, respectively, both subroutines are called by the main code Finite.Glover.m to calculate stream depletion. Code details are given in Appendix H. Figure 2.19 shows the stream depletion function profile with unit less time $\frac{4T}{S} \cdot \frac{t}{a^2}$ for stream depletion rate ratio (represented by the dashed line) and stream depletion volume ratio (represented by the solid line), due to a well pumping continuously for simulation period of 1 year *(365 days)*. The well is located at *500* m away from the stream in an aquifer bounded between an irrigation ditch and a stream, and the aquifer width is *2000* m. these profiles are obtained using the Matlab code Finite.Glover.m. Aquifer properties used to develop this plot are listed in Table 2.1.

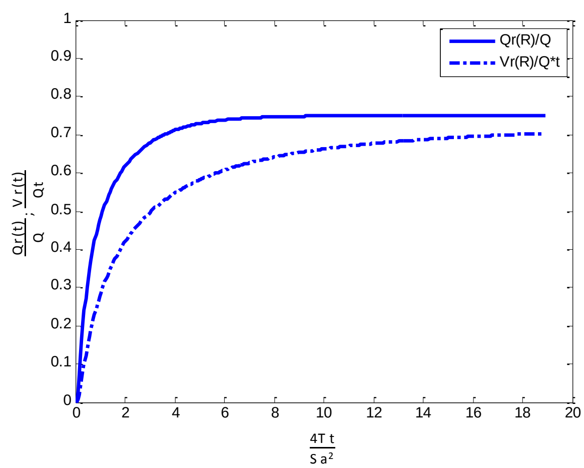


Figure 2.19 Stream depletion rate ratio (solid line) and stream depletion volume ratio (dashed line) caused by a well located at *500* m from a stream in a finite aquifer bounded between an irrigation ditch and a stream with a width *2000* m, the well operates continuously for *1* years (*365* days)

Figure 2.20 shows the stream depletion volume functions profile with unit less time $\frac{4T}{S} \cdot \frac{t}{a^2}$ for a well operating in a semi-infinite aquifer (dashed line) and in a finite aquifer bounded between a an irrigation ditch and a stream (solid line) with a width *2000* m, the well is located at *500* m from the stream in both cases and it is pumping continuously for *10* years (*3650* days). Aquifer properties used to develop this plot are listed in Table 2.1.

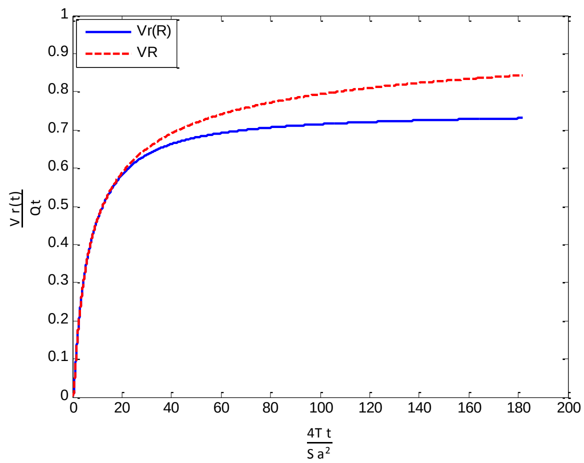


Figure 2.20 stream depletion volume due to a well operating continuously for *10* years (*3650* days) located at *500* m away from a stream in as semi-infinite aquifer (dashed line) and a stream in a finite aquifer bounded between an irrigation ditch (solid line) and a stream with a width of *2000* m

Note that, in the bounded aquifer configuration considered here, the other recharge boundary (irrigation ditch) represents a supply source for the aquifer besides the stream under study, so the actual pumped volume will eventually equal the sum of the stream depletion volume from both of the boundaries, that is why a delay can be noticed between the point where the ration between stream depletion volume and the actual pumped volume attains the steady state in the case of semi-infinite aquifer and its equivalent in the current case.

In the case of well field, the total stream depletion rate is calculated as:

$$Q_{r,total} = \sum_{m=1}^{n_{ow}} Q_m \cdot BRQ_{ratio}\,(S, T, w, a_m, \tau_m; t) \tag{2.83}$$

Likewise, stream depletion volume is obtained by:

$$V_{r,total} = \sum_{m=1}^{n_{ow}} Q_m \cdot (t - \tau_m) \cdot BRV_{ratio}\,(S, T, w, a_m, \tau_m; t) \tag{2.84}$$

Note that, Equations (2.81) and (2.83) are used to estimate the impact of well pumping on the stream, which, in Figure 2.17 is represented by the straight line $x=w$. the rates and volumes of depletion produced on the secondary recharge boundary, that is, the irrigation ditch, can be calculated using the same equations presented above, after changing the stream/well distance to b instead of a.

Figure 2.21 shows the stream depletion volume functions profile with unit less time $\frac{4T}{S} \cdot \frac{t}{a^2}$ for a well operating in a finite bounded between a no-flow boundary and a stream (dashed line) and in a finite aquifer bounded between a an irrigation ditch and a stream (solid line) with a width *2000* m, the well is located at *500* m from the stream in both cases and it is pumping continuously for *10* years (*3650* days). Aquifer properties used to develop this plot are listed in Table 2.1.

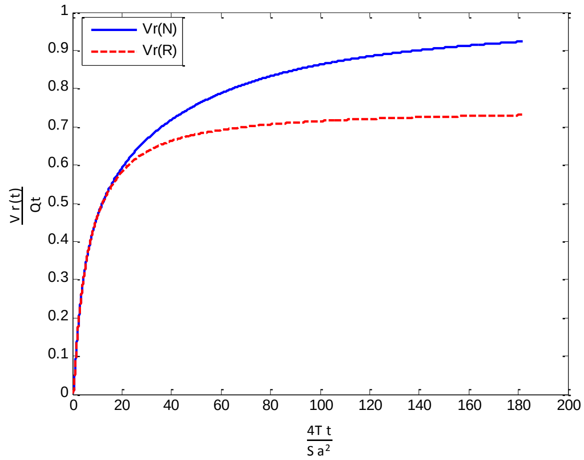


Figure 2.21 The stream depletion volume functions profile with unit less time $\frac{4T}{S}\cdot\frac{t}{a^2}$ for a well operating in a finite aquifer bounded between a no-flow boundary and a stream (solid line) and in a finite aquifer bounded between a an irrigation ditch and a stream (dashed line) with a width *2000* m, the well is located at *500* m from the stream in both cases and it is pumping continuously for *10* years (*3650* days). Aquifer properties used to develop this plot are listed in Table 2.1.

2.4 Cyclical Operation of Wells

In this section, we consider the case of a periodic operation of wells. Figure 2.22 illustrates the cyclic operation rate profile over time for a single well. The well operates at a constant rate Q over a given period Δt_{on} and it is shut off during a period Δt_{off}. The length of the full cyclic period Δt is given by the sum of Δt_{on} and Δt_{off}.

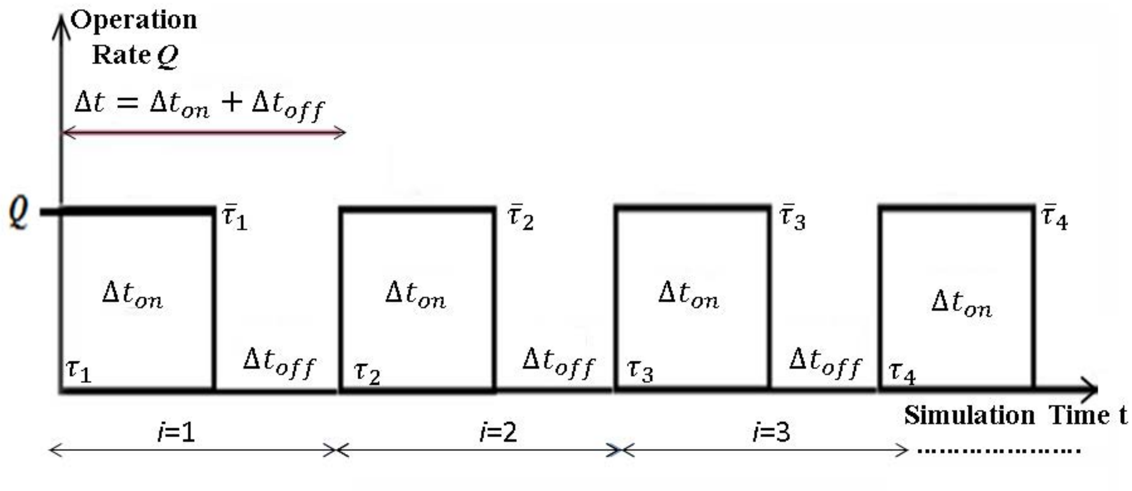


Figure 2.22 the schedule plan for a well operating cyclically until *t: [3Δt* + Δ*t*$_{on}$ ≤ *t* ≤ *4Δt]*. The well operates with a constant rate *Q*, for a period Δ*t*$_{on}$ during each operating cycle, starting at τ_1

$$\Delta t = \Delta t_{on} + \Delta t_{off} \tag{2.85}$$

As previously mentioned, continuous operation is one of the limitations of the Theis solution (Equation (2.34)) and Glover solution (Equation (2.57)). The principle of superposition may be used to overcome this obstacle; whereby there is a variation of the pumping rate, one can simulate the activation of a new imaginary well, located at the same position of the real well with a rate equal to the change of the pumping rate and opposite sign. For example, for the cyclic operation depicted in Figure 2.22, at any given time *t*, the effect occurred on the system during the generic interval of time Δt_{on} within the operation full cycle *i (i=1,2,3…)* can be simulated by two wells: the first well starts operating continuously at time $\tau_i = (i-1) \cdot \Delta t$ with a rate Q, and the second wells starts operating continuously at time $\bar{\tau}_i = (i-1) \cdot \Delta t + \Delta t_{on}$ with a rate –*Q*. At the generic time *t*, the number of full cycles (on and off) that have been completed is given by:

$$n = int\left(\frac{t}{\Delta t}\right) \tag{2.86}$$

where $int\ (x)$ is the integer part of the real number *x*.

2.4.1 Superposition of Effects: Drawdown

The drawdown general equation for the cyclic operation of a well is developed using the principle of superposition for the effects of the *n* couples of wells operating during *n* full cycles, plus an extra term accounting for the well operation during the current cycle. This extra contribution has a different expression depending if *t* falls within the period Δt_{on} or within the period Δt_{off}. In a laterally infinite aquifer the resulting drawdown is thus given by the flowing equation:

$$\begin{aligned} s(r;t) &= -Q \cdot W_C[S,T,r,\Delta t,\Delta t_{on},\tau,\bar{\tau};t\,] \\ &= -Q\sum_{i=1}^{n}\frac{1}{4\pi T}\Bigg[\big[W[u(r;t-\tau_i)] - W[u(r;t-\bar{\tau}_i)]\big] \\ &+ \begin{cases} W[u(r;t-\tau_{n+1})] & if \quad \mathrm{n}\cdot\Delta t < t \le \mathrm{n}\cdot\Delta t + \Delta t_{on} \\ W[u(r;t-\tau_{n+1})] - W[u(r;t-\bar{\tau}_{n+1})] & if \quad \mathrm{n}\cdot\Delta t + \Delta t_{on} < t \le (\mathrm{n}+1)\cdot\Delta t \end{cases}\Bigg] \end{aligned} \tag{2.87}$$

where $W_C[S,T,r,\Delta t,\Delta t_{on},\tau,\bar{\tau};\ t\,]$ is the overall well function, representing the accumulated response of the system to the operation of the wells couples during *n* full cycles plus an extra term accounting for the well operation during the most current period Δt. All the well functions in this section are valid only when $t - \tau > 0$, or otherwise they are set to quale zero.

The drawdown general equation for a system of wells operating cyclically within the same period Δt is the following:

$$s(x,y;t) = \sum_{m=1}^{n_{ow}} -Q_m \cdot\ W_C\big[S,T,r_m,\Delta t,\Delta t_{on,m},\tau_m,\bar{\tau}_m;t\,\big] \tag{2.88}$$

In Equation (2.88), r_m is the distance between the generic well *m* and the observation point *(x,y)*. Each well has a generic operation starting time. Figure 2.23 shows the drawdown profiles over time for a ten-year long simulation total duration (*t=10* years), calculated at two observation points located at a distance *r* equals *15* m (represented by the dashed line) and *r* equals *150* m (represented by the solid line). The well operates cyclically at a rate of *-500* m^3/day, for a period of *180* days in each cycle starting at time τ *=0.* The length of the operation cycle Δt is 365 days (one year). The profiles are obtained using the Matlab code Infinite.Drawdown.Time.m presented in Appendix B. Aquifer properties are given in Table 2.1.

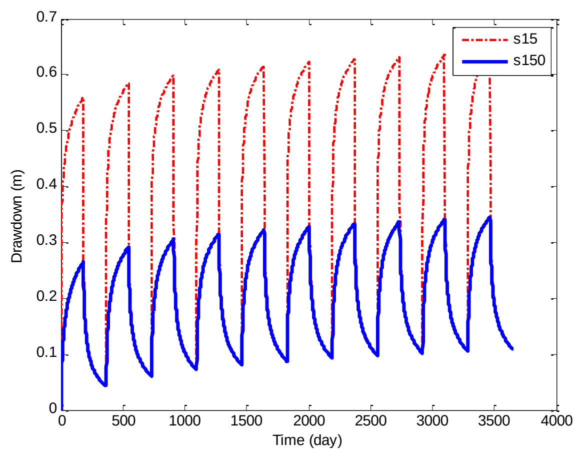


Figure 2.22 Drawdown profiles over time in two observation points distant *15* m (s*15*, represented by the dashed line) and *150* m (s*150*, represented by the solid line) from a well operating cyclically starting at τ = 0 for a total simulation period of ten years (t=*10* years), the well operates with a rate of *-500* m³/day during *180* days in each cycle, where the cycle length Δt is *365* days

Figures 2.24a and 2.24b show the drawdown spatial distribution obtained by the Matlab code Infinite.Drawdown.2D.m at a time t = *100* days, and t = *200* days, respectively, due to a well operating cyclically, for a period Δt_{on} = *180* days every year. The drawdown distribution shown in Figure 2.23c and 2.23d are due to a two-well well field for the same times as in subpanel (a) and (b) respectively, these snapshots times are taken during the operation period and after the operation has been ceased, respectively. Wells' locations and operation rates are labeled in each plot. Table 2.1 contains the aquifer properties, and Table 2.2 contains the detailed well field data, the operation rates and wells locations are labeled in the figures. Appendix A has the detailed description of the code Infinite.Drawdown.2D.m.

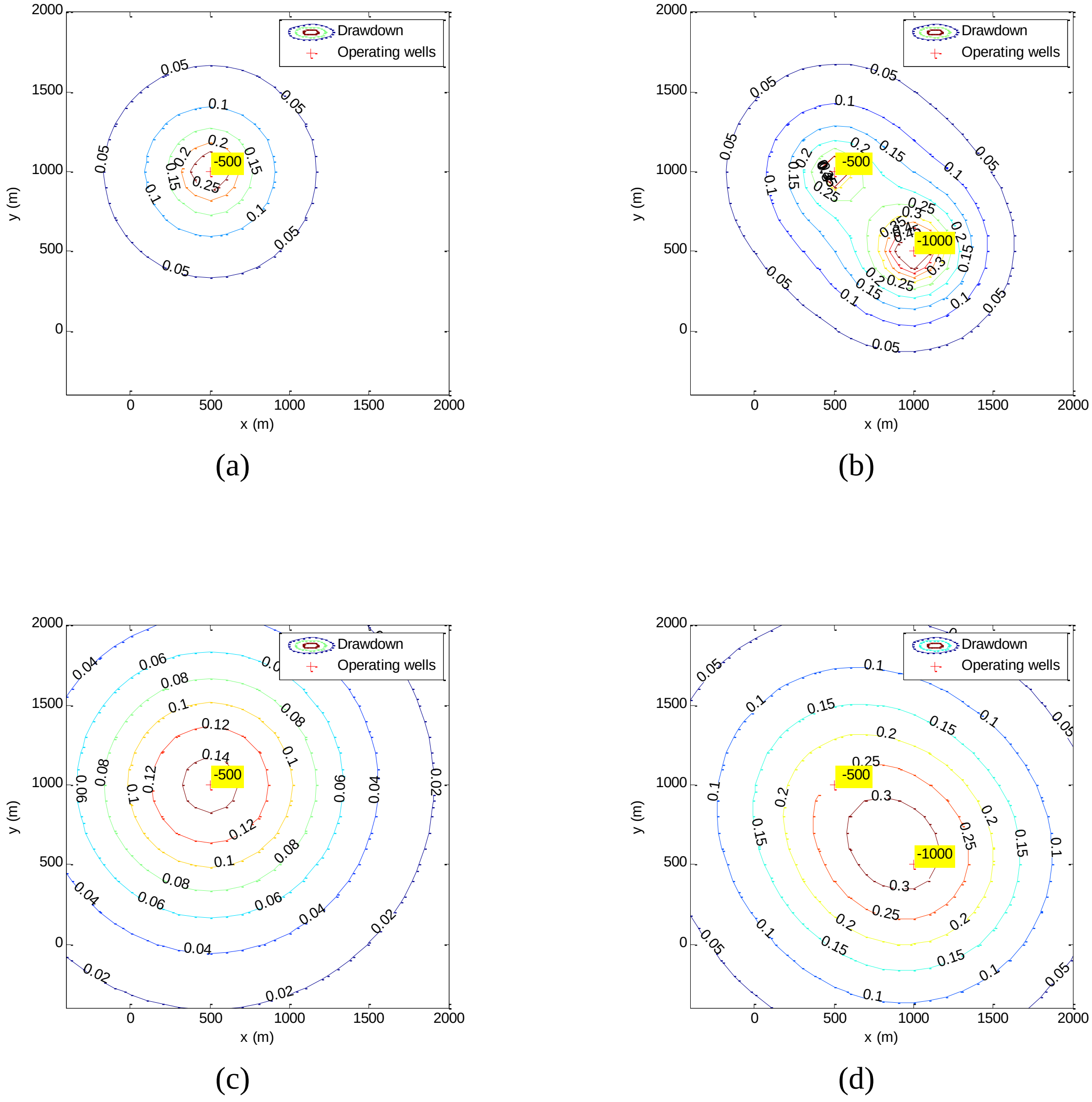


Figure 2.24 Drawdown contours caused by a single well operating cyclically at a rate of -*500* m^3/day for Δt_{on} = *180* days in subpanels (a) and (b), the images are taken at a snap shot times t =*100* days (during Δt_{on}) and t = *200* days (during Δt_{off}), respectively, for a cycle length Δt =*365* days. In (c) and (d) the drawdowns are caused by a two-well well field for the same times as in (a) and (b) respectively, wells data are given in details in Table 2.2

In the case of semi-infinite aquifers bounded by either a recharge or a no-flow boundary, an equation similar to (2.87) applies, except that the well function W_C must be substituted by the following well function:

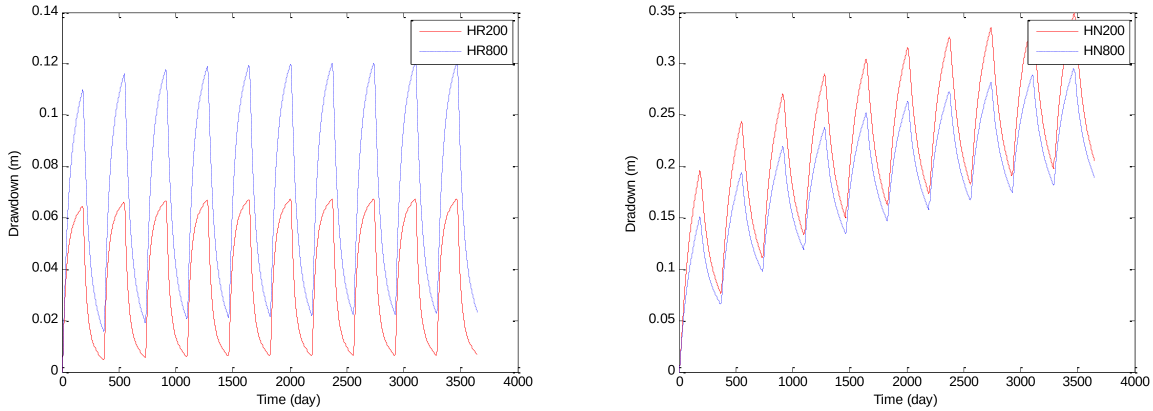
$$W_{CSI}[S,T,r,\Delta t,\Delta t_{on},\tau,\bar{\tau};t\,]$$
$$= \sum_{i=1}^{n} \frac{1}{4\pi T} \left[\left[W_{R/N}(u(r_1,r_2;t-\tau_i)) - W_{R/N}(u(r_1,r_2;t-\bar{\tau}_i))\right] \right. \qquad (2.89)$$
$$\left. + \begin{cases} W_{R/N}(u(r_1,r_2;t-\tau_{n+1})) \\ if \quad \mathrm{n}\cdot\Delta t < t \le \mathrm{n}\cdot\Delta t + \Delta t_{on} \\ W_{R/N}(u(r_1,r_2;t-\tau_{n+1})) - W_{R/N}\big(u(r_1,r_2;t-\bar{\tau}_{n+1})\big) \\ if \quad \mathrm{n}\cdot\Delta t + \Delta t_{on} < t \le (\mathrm{n}+1)\cdot\Delta t \end{cases} \right]$$

where $W_{CSI}[S,T,r,\Delta t,\Delta t_{on},\tau,\bar{\tau};\ t\,]$ is the overall well function for the group of wells simulating the cyclical operation in a semi-infinite aquifer with a recharge boundary $W_R[r_1,r_2,\tau;t]$ (Equation (2.41)) or with a no-flow boundary $W_N[r_1,r_2,\tau;t]$ (Equation (2.42)), along with their equivalent well functions ($W_R[r_1,r_2,\bar{\tau};t]$) and ($W_N[r_1,r_2,\bar{\tau};t]$) included to calculate the effect of the second well and its image well located at r_1 and r_2 from the observation point, respectively, which start operating at $\bar{\tau}_i$. These well functions are valid only if $t-\tau>0$, otherwise the function is set to be equal to zero.

Accordingly, the drawdown equation for a well field in a semi-infinite aquifer, where each well operates cyclically is obtained by superposition of solutions as follows:

$$s(x,y;t) = \sum_{m=1}^{n_{ow}} -Q_m.\,W_{CSI}\left[S,T,r_m,\Delta t,\Delta t_{on,m},\tau_m,\bar{\tau}_m;t\,\right] \qquad (2.90)$$

where r_m is the distance between the generic well *m* and the observation point *(x,y)*, τ_m is the generic operation starting time for well *m* and $\bar{\tau}_m$ is the operating starting time for the imaginary well *m.* Figure 2.25a shows the drawdown vs. time profiles over ten years plotted using the Matlab code (SI.Drawdown.Time) for two observation points located at distances *200* m (represented by the solid line) and *800* m (represented by the dashed line) from the recharge boundary, respectively, in a semi-infinite aquifer. The two observation points are at the same distance from the operating well *(r= 425* m*)* that operates cyclically for *180* days every year (Δt = *365*) starting at time τ*=0*, with a cyclic extraction rate of *-500* m^3/day. The well is located at *500* m from the recharge boundary. Similarly, Figure 2.25b shows the drawdown profiles vs. time, for the same setting as in subpanels (a) in a semi-infinite aquifer is with a no-flow boundary. Appendix D includes the detailed description of the Matlab code SI.Drawdown.Time.m.



(a) (b)

Figure 2.25 Drawdwon profiles in a semi-infinte aquifer with (a) a recharge boundary, and (b) a no-flow boundary, at two observation points located at *200* m (represented by the solid line) and *800* m (represented by the dashed line) from the boundary, both of the observation points are at *425* m from the operating well, which is located at *500* m from the boundary. The well starts operating at time *τ=0*, with a cyclic extraction rate of -*500* m^3/day and operation period of *180* days during each cycle, with a cycle length of (*365* days).

Figure 2.26 shows the drawdwon distributions obtained by applying the Matlab code SI.Drawdwon.2D.m at time t = *200* days (during operation period Δt_{off}), caused by an extracting well field of two wells, in a semi-infinite aquifer with (a) a recharge boundary at *x=0* (the y-axis), and (b) a no-flow boundary at the y-axis. The aquifer properties are given in Table 2.1, and in Table 2.2, the extraction rates and operation sechedule of the well field are listed. Figure 2.6 illustrates the drawdwon distributions for the same setting as in Figure 2.26a and 2.26b at a snap shot t=100 days (during operation period Δt_{on}), both figures are obtained using Matlab code SI.Drawdwon.2D.m presented in Appendix C.

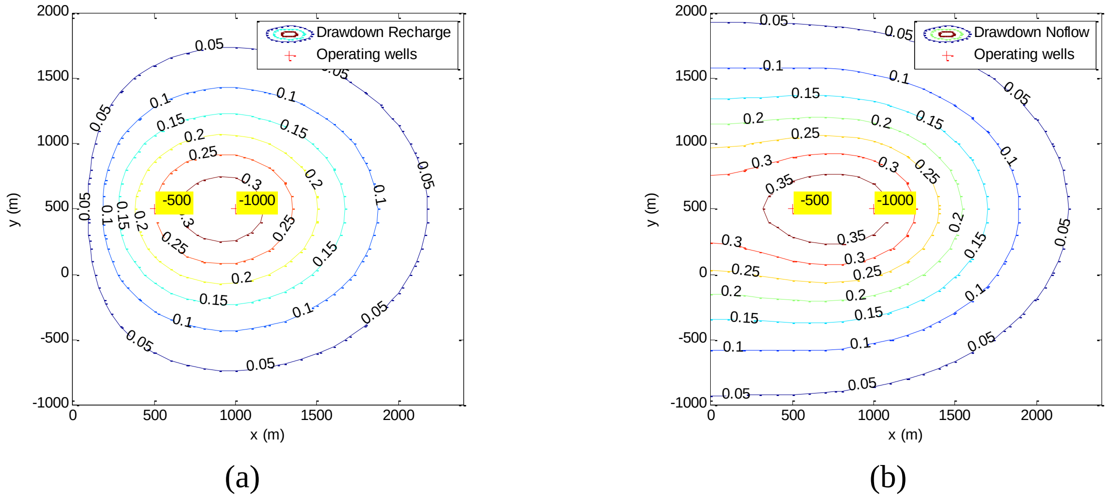


Figure 2.26 Drawdown contours occuring during the cyclic extraction Δt_{off} period (t=200 days), due to a two-well well field in (a) in a semi-infinite aquifer with a recharge boundary (b) in a semi-infinite aquifer with a no-flow boundary. Well fild data are listed in details in table 2.2.

2.4.2 Superposition of Effects: Stream depletion

Equations similar to those derived in section 2.4.1 can be obtained to assess the impact on stream depletion flow of cyclic well pumping in an aquifer hydraulically connected to a stream. Similar to Equation (2.87), the stream depletion rate Q_r and the stream depletion volume V_r are given by the two following equations:

$$Q_r(Q,S,T,a;t) = Q.Q_{c,ratio}[S,T,a,\Delta t,\Delta t_{on},\tau,\bar{\tau};t] \qquad (2.91)$$

$$V_r(Q,S,T,a;t) = Q.V_{c,ratio}[S,T,a,\Delta t,\Delta t_{on},\tau,\bar{\tau};t] \qquad (2.92)$$

where:

$$Q_{c,ratio}[S,T,a,\Delta t,\Delta t_{on},\tau,\bar{\tau};t]$$

$$= \sum_{i=1}^{n}[Q_{ratio}(S,T,a;t-\tau_i) - Q_{ratio}(Q,S,T,a;t-\bar{\tau}_i)]$$

$$+ \begin{cases} Q_{ratio}(S,T,a;t-\tau_{n+1}) \\ if \quad \mathrm{n}\cdot\Delta t < t \leq \mathrm{n}\cdot\Delta t + \Delta t_{on} \\ Q_{ratio}(S,T,a;t-\tau_{n+1}) - Q_{ratio}(Q,S,T,a;t-\bar{\tau}_{n+1}) \\ if \quad \mathrm{n}\cdot\Delta t + \Delta t_{on} < t \leq (\mathrm{n}+1)\cdot\Delta t \end{cases} \tag{2.93}$$

and

$$V_{c,ratio}[S,T,a,\Delta t,\Delta t_{on},\tau,\bar{\tau};t]$$

$$= \sum_{i=1}^{n}[(t-\tau_i)\cdot V_{ratio}(S,T,a;t-\tau_i) - (t-\bar{\tau}_i)\cdot V_{ratio}(Q,S,T,a;t-\bar{\tau}_i)]$$

$$+ \begin{cases} (t-\tau_{n+1}).V_{ratio}(S,T,a;t-\tau_{n+1}) \\ if \quad \mathrm{n}\cdot\Delta t < t \leq \mathrm{n}\cdot\Delta t + \Delta t_{on} \\ (t-\tau_{n+1})\cdot V_{ratio}(S,T,a;t-\tau_{n+1}) - (t-\bar{\tau}_{n+1})\cdot V_{ratio}(Q,S,T,a;t-\bar{\tau}_{n+1}) \\ if \quad \mathrm{n}\cdot\Delta t + \Delta t_{on} < t \leq (\mathrm{n}+1)\cdot\Delta t \end{cases} \tag{2.94}$$

Equations (2.91) and (2.92) can be generalized to calculate stream depletion due to a cyclically operating well filed as following:

$$Q_{r,total}(t) = \sum_{m=1}^{n_{ow}} Q_m.\,Q_c ratio\big[S,T,a_m,\Delta t,\Delta t_{on,m},\tau_m,\bar{\tau}_m;t\big] \tag{2.95}$$

$$V_{r,total}(t) = \sum_{m=1}^{n_{ow}} Q_m.\,V_c ratio\big[S,T,a_m,\Delta t,\Delta t_{on,m},\tau_m,\bar{\tau}_m;t\big] \tag{2.96}$$

Note that, all the stream depletion function ratios and stream depletion volume ratios mention above are equal to zero if $t \leq \tau$.

Figure 2.27 shows the stream depletion volume profile with time for ten years long simulation obtained using SI.Glover.m a Matlab code given in appendix E, due to a cyclical operation of a well located at *500* m from the stream, obtained by using the Matlab code

SI.Glover.m. The well is extracting cyclically with a rate of *-500* m^3/day for a period of *180* days during each cycle *(Δt=365)* starting at time τ= 0. Aquifer properties are listed in Table 2.1.

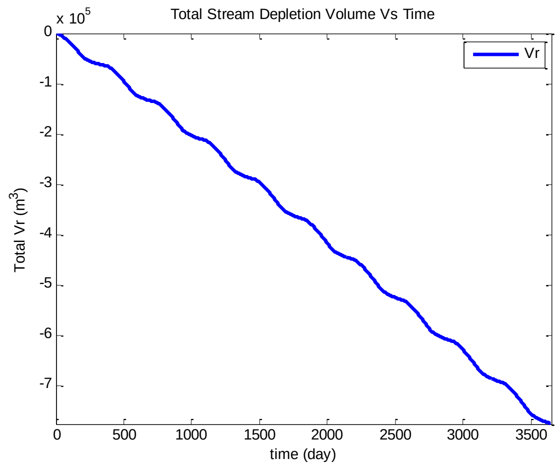


Figure 2.27 The stream depletion volume profile with time for ten years long simulation, due to a cyclical operation of a well located at *500* m from the stream. The well is extracting cyclically with a rate of *-500* m^3/day for a period of *180* days during each cycle *(Δt=365)* starting at time τ= 0

2.5 Cyclic Operation in a Finite Aquifer

This case addresses two limitations of the Theis and Glover solutions, that is, the condition of infinite areal extension of the aquifer and the condition of continuous constant-rate operation of the well. Once more, superposition of solutions is the approach used to remove these hypotheses. The response of the system is thus obtained by assuming the presence of two groups of wells: the first group represents image wells that simulate the presence of the aquifer boundaries, and the second group represents the imaginary wells that simulate the cyclic well operation.

2.5.1 Superposition of Effects: Drawdown

In a constant width finite aquifer characterized by one of the two configurations presented in Figure 2.11 and 2.13, the drawdown at a generic point *(x,y)* caused by a cyclically operating well with an operation schedule such as that graphed in Figure 2.21 is given by:

$$s(x,y;t) = -Q \cdot W_{FC}(S,T,r_{2\cdot j-2},r_{2\cdot j-1},r_{I2\cdot j-2},r_{I2\cdot j-1},\Delta t,\Delta t_{on},\tau,\bar{\tau};t)$$

$$= -Q \,.\sum_{i=1}^{n}\frac{1}{4\pi T}$$

$$\cdot\left[\left\{\sum_{j=1}^{\infty}\left[W_F(S,T,r_{2\cdot j-2},r_{2\cdot j-1},r_{I2\cdot j-2},r_{I2\cdot j-1};t-\tau_i)-W_F(S,T,r_{2j-2},r_{2j-1},r_{I2j-2},r_{I2j-1};t-\overline{\tau_i})\right]\right\}\right. \quad (2.97)$$

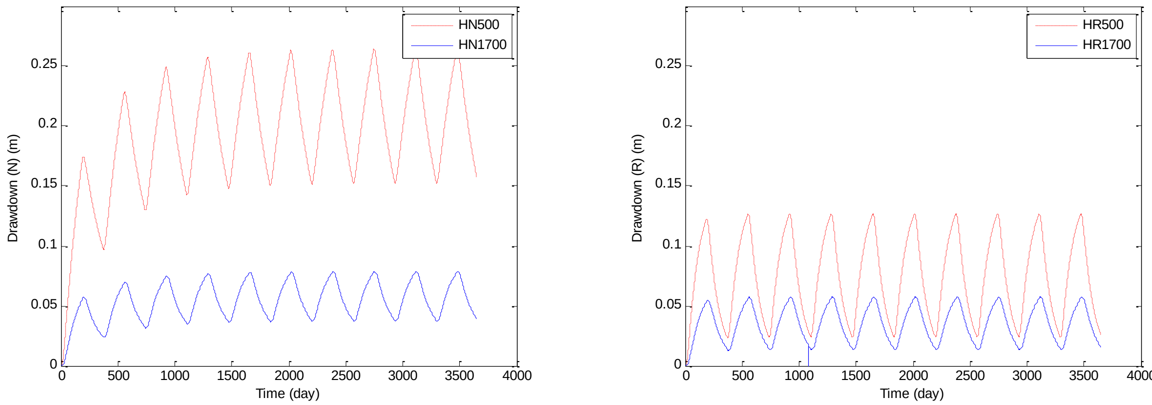
$$\left.+\begin{cases}\displaystyle\sum_{j=1}^{\infty}W_F(S,T,r_{2j-2},r_{2j-1},r_{I2j-2},r_{I2j-1};t-\tau_{n+1}) \\ If \quad \mathrm{n}\cdot\Delta t < t \le \mathrm{n}\cdot\Delta t+\Delta t_{on} \\ \displaystyle\sum_{j=1}^{\infty}W_F(S,T,r_{2j-2},r_{2j-1},r_{I2j-2},r_{I2j-1};t-\tau_{n+1})-W_F(S,T,r_{2j-2},r_{2j-1},r_{I2j-2},r_{I2j-1};t-\overline{\tau_{n+1}}) \\ If \quad \mathrm{n}\cdot\Delta t+\Delta t_{on} < t \le (\mathrm{n}+1)\cdot\Delta t\end{cases}\right]$$

where $W_{FC}(S,T,r_{2\cdot j-2},r_{2\cdot j-1},r_{I2\cdot j-2},r_{I2\cdot j-1},\Delta t,\Delta t_{on},\tau,\bar{\tau};\ t)$ is the overall well function for a cyclically operating well in a finite aquifer. In Equation (2.97) this function is calculated based on $W_F(S,T,r_{2j\cdot-2},r_{2\cdot j-1},r_{I2\cdot j-2},r_{I2\cdot j-1},\tau;\ t)$, which equals $W_{NR}[S,T,r_{2\cdot j-2},r_{2\cdot j-1},r_{I2\cdot j-2,}r_{I2\cdot j-1},\tau;\ t]$ (Equation (2.71)) for an aquifer comprised between a no-flow boundary and a recharge boundary, or $W_{RR}[S,T,r_{2\cdot j-2},r_{2\cdot j-1},r_{I2\cdot j-2,}r_{I2\cdot j-1},\tau;\ t]$(Equation (2.73)) for an aquifer comprised between two recharge boundaries.

Equation (2.97) can be generalized to calculate drawdown due to a cyclically operating well field of *m* wells in a finite aquifer, resulting in the following equation:

$$s(x, y; t) = \sum_{m=1}^{n_{ow}} -Q_m W_{FC}\left(S, T, r_{m,2j\cdot-2}, r_{m,2\cdot j-1}, r_{m,I2\cdot j-2}, r_{m,I2\cdot j-1}, \Delta t, \Delta t_{on,m}, \tau_m, \bar{\tau}_m; t\right) \quad (2.98)$$

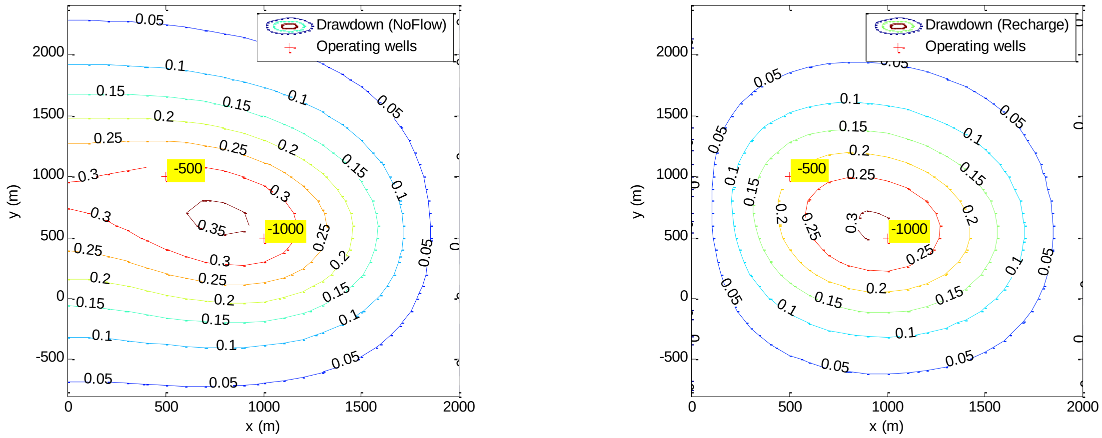
Figure 2.28a shows the drawdown profiles with time for ten years long simulation, due to a cyclically operating well located at *1000* m from the no-flow boundary in an aquifer bounded between a no-flow boundary and a stream with a width *2000* m. The well is extracting at a constant rate of -*1000* m^3/day for a period of *180* days every year starting at a time τ= 0, the cycle length Δt is *365* days. The profiles are obtained in two observation points located at *500* m (represented by the solid line) and *1700* m (represented by the dashed line) from the no-flow boundary, and both of these observation points are at *707* m and *990* m away from the operating well, respectively. The aquifer parameters are given in Table 2.1. Figure 2.28b shows the same setting as in subpanel (a) for a finite aquifer comprised between irrigation ditch and a stream with *2000* m distance between them. Both figures are obtained using Matlab code Finite.Drawdown.Time.m introduced in Appendix G.



(a) (b)

Figure 2.28 Drawdwon profiles due to a well operating cyclically in a finite aquifer with a constant width of *2000* m bounded between (a) a no-flow and a recharge boundary, and (b) an irrigation ditch and a stream. The profiles are obtained at two observation points located at *500* m (represented by the dashed line) and *1700* m (represented by the solid line) from the no-flow boundary in subpanel (a) and the irrigation ditch in subpanel (b). The operating well has a cyclic extraction rate of *-1000* m^3/day and operation period of *180* days during each cycle (Δt =*365* days) starting at a time τ= 0. The observation points are at distances from the operating well *r= 707* and *r= 990*,respectively, and the well is at *1000* m away from the symmetry axis in each of the two plots

Figure 2.29 shows the drawdown distributions at a snap shot time equal to *200* days (during the operation off time Δt_{off} for Δt=*365* days) due to a cyclically extracting well filed obtained using Equation (2.98), in a finite aquifer with a constant width of *2000* m bounded between (a) a no-flow boundary located at the y-axis and a stream, and (b) an irrigation ditch located at the y-axis and a stream. Wells locations and extraction rates are labeled in each plot. Other well's data are given with details in Table 2.2 and aquifer properties are indicated in Table 2.1. Drawdown distributions during operation time Δt_{on} are shown in Figures 2.12 and 2.14 for the same finite aquifer configurations as in Figure 2.29a and 2.29b, respectively, all figures are obtained using the Matlab code Finite.Drawdown.2D.m presented in Appendix F.



(a) (b)

Figure 2.29 Drawdown contours in a constant width finite aquifer (*w=2000* m) bounded between a no-flow boundary (located at the y-axis) and a stream, and an irrigation ditch (located at the y-axis) and a stream in subpanels (a) and (b), respevtively. Drawdwon distributions are shown during the cyclic extraction period of Δt_{off} (snap shot time is *200* days) due to an extarctig well field of two wells with the operation detailes listed in Table 2.2, operation rates and wells locations are labeled in each plot

2.5.2 Superposition of Solutions: Stream depletion

Based on superposition of effects, the depletion rate from a stream constituting a boundary for a finite-width aquifer caused by a cyclically operating well may be expressed as:

$$Qr = Q.\,Q_{FC,ratio}\,(S, T, w, a, \Delta t_{on}, \Delta t, \tau; t) \qquad (2.99)$$

where $Q_{FC,ratio}\,(S, T, w, a, \Delta t_{on}, \Delta t, \tau;\ t)$ is the accumulated sum of stream depletion rate ratio functions for a cyclically operating well. The function is calculated for a stream hydraulically connected to an aquifer bounded between the stream and another boundary of either no-flow type or recharge type. This function is calculated as:

$$\begin{aligned} &Q_{FC,ratio}\ (S,T,w,a,\Delta t_{on},\Delta t,\tau,\bar{\tau};t) \\ &= \sum_{i=1}^{n}[BQ_{ratio}(S,T,w,a;t-\tau_i) - BQ_{ratio}(S,T,w,a;t-\bar{\tau}_i)] \\ &+ \begin{cases} BQ_{ratio}(S,T,w,a;t-\tau_{n+1}) \\ if \quad \mathrm{n}\cdot\Delta t\ < t \le \mathrm{n}\cdot\Delta t + \Delta t_{on} \\ BQ_{ratio}(S,T,w,a;t-\tau_{n+1}) - BQ_{ratio}(S,T,w,a;t-\bar{\tau}_{n+1}) \\ if \quad \mathrm{n}\cdot\Delta t + \Delta t_{on}\ < t \le (\mathrm{n}+1)\cdot\Delta t \end{cases} \end{aligned} \tag{2.100}$$

The function BQ_{ratio} in Equation (2.100) is given by Equation (2.75) if the aquifer is limited, in addition to the stream, by a no-flow boundary, or by Equation (2.81) if the aquifer is limited by another constant head boundary. Similar to the stream depletion rate (Equation (2.99)), the stream depletion volume is obtained as:

$$Vr = Q.V_{FC,ratio}\ (S,T,w,a,\Delta t_{on},\Delta t,\tau,\bar{\tau};t) \tag{2.101}$$

where $V_{FC,ratio}\ (S,T,w,a,\Delta t_{on},\Delta t,\tau,\bar{\tau};t)$ is the overall stream depletion volume ratio function representing the response to a well operating cyclically in a finite aquifer, bounded between the stream and another boundary of either no-flow type (Equation(2.76)) or recharge type (Equation(2.82)). This function is:

$$\begin{aligned} &V_{FC,ratio}\ (S,T,w,a,\Delta t_{on},\Delta t,\tau,\bar{\tau};t) \\ &= \sum_{i=1}^{n}[(t-\tau_i)\cdot BV_{ratio}(S,T,w,a;t-\tau_i) - (t-\bar{\tau}_i)\cdot BV_{ratio}(S,T,w,a;t-\bar{\tau}_i)] \\ &+ \begin{cases} (t-\tau_{n+1})\cdot BV_{ratio}(S,T,w,a;t-\tau_{n+1}) \\ if \quad \mathrm{n}\cdot\Delta t\ < t \le \mathrm{n}\cdot\Delta t + \Delta t_{on} \\ (t-\tau_{n+1})\cdot BV_{ratio}(S,T,w,a;t-\tau_{n+1}) - (t-\bar{\tau}_{n+1})\cdot BV_{ratio}(S,T,w,a;t-\bar{\tau}_{n+1}) \\ if \quad \mathrm{n}\cdot\Delta t + \Delta t_{on}\ < t \le (\mathrm{n}+1)\cdot\Delta t \end{cases} \end{aligned} \tag{2.102}$$

Figure 2.30 shows the stream depletion volume obtained using Matlab code Finite.Glover.m introduced in Appendix H. for a well operating cyclically in a constant-width finite aquifer (*w=2000* m) bounded between the steam under study and a no flow boundary (represented by the dashed line), and a finite aquifer bounded between the stream and another recharge boundary (represented by the solid line). The operating well is located at *1000* m from the stream and it is extracting with a rate *-1000* m^3/day for *180* days operation period for a cycle length (Δt) equal to *365* days, over ten years simulation period, starting at time τ*=0.*

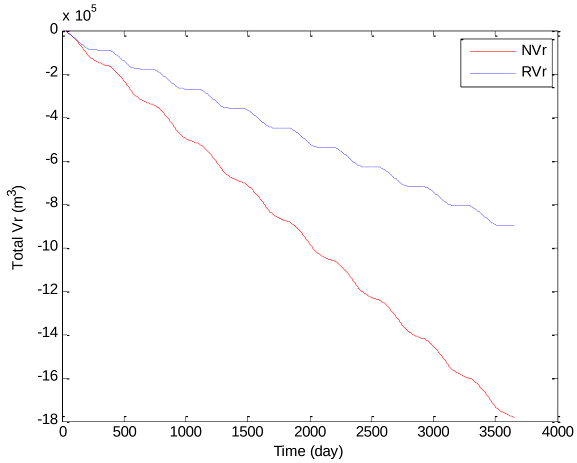


Figure 2.30 The stream depletion volume obtained for a well operating cyclically in a constant-width finite aquifer (*w=2000* m) bounded between the steam under study and a no flow boundary (represented by the dashed line), and a finite aquifer bounded between the stream and another recharge boundary (represented by the solid line). The operating well is located at *1000* m from the stream and it is extracting with a rate *-1000* m^3/day for *180* days operation period for a cycle length (Δt) equal to *365* days, over ten years simulation period, starting at time τ*=0.*

CHAPTER THREE: GROUNDWATER MANAGEMENT

3.1 Groundwater Management

The conjunctive management of water deals with the coordinated combined consumptive use of surface water and ground water resources, in order to efficiently meet the demands during times of water deficiency as well as availability. It is subject to laws regulating the water use; such as the prior appropriation system (also known as the priority doctrine) widely practiced in the western US.

The phrase “first in time, first in right” describes the doctrine of prior appropriation, according to which, water users with earlier appropriation decrees or “senior right holders” have a superior right in full water allocation before “junior right holders”, who can get water supply only if that does not impact its availability for senior users. According to prior appropriation law, well water users are junior right holders; since historically they were granted use rights much later than when surface water use rights were fully allocated. Therefore, the water conjunctive management becomes a complex task when applied to a hydraulically connected stream/aquifer system under the prior appropriation system. Consequently, groundwater can be pumped from the aquifer for junior use (e.g. irrigation) under the condition of maintaining the minimal effect on the senior rights of the surface water (Grigg, 2005)

An example of the application of such a system is the non-tributary and not non-tributary aquifers within the South-Platte river basins in northern Colorado. Pumping of these aquifers is permitted for overlaying landowners at a rate of 1% a year to avoid affecting the connected surface water up until 100 years; otherwise, groundwater pumping is allowed provided that the stream is recharged with an amount of water equivalent to that extracted (*Colorado Foundation for Water Education, 2003)*.

3.1.1. Optimization of Groundwater Use

This chapter addresses the groundwater management problem of an agricultural land irrigated by extracting water from the underlying aquifer, which is hydraulically connected to a stream. Given the fact that the stream will be compensating for the volume extracted from the aquifer, pumping groundwater is a junior act, which potentially affects the senior water right on the surface water, thus, the management goal is to minimize the impact of water pumping on stream flows while satisfying the required irrigation needs. Withdrawing water from aquifer storage, not only affects the stream but also the hydraulic head levels in the aquifer. If the aquifer is over pumped it may not recover properly and will eventually fall short of providing required quantity of water, adding another constraint on the desired objective. Satisfying these conditions while meeting water demand is possible by replacing the extracted amount of groundwater back to the aquifer to keep heads levels slightly unchanged and to replenish the stream. This process is done through: (a) aquifer storage and recovery (ASR); where each operating well is provided with a pump able to extract water during periods of water need for irrigation and inject water when surface water is available for storage, or through (b) aquifer pumping and artificial recharge (APR), where, after being pumped, the aquifer is recharged with surface water at prescribed locations.

This management problem can be formulated in mathematical language, as an optimization problem with an objective that requires minimizing the total depletion/accretion of the stream caused by both pumping (extraction) and recharge (injection) with constraints to represent requirements of the irrigation demand, the available water to inject in the aquifer, maximum and minimum allowed aquifer head levels, and maximum and minimum values of well operation rates established by well capacities.

Before proceeding to the description of the optimization problem setting, we have to distinguish the two groundwater management problem formulations to be considered: (a) aquifer storage and recovery (single well operation), and (b) aquifer pumping and recharge (dual well operation).

(a) For aquifer storage and recovery (ASR) the wells are operating in an operation mode, in which they are set to perform one operation type during a certain period of time (extraction during growing season), and then reverse it for the rest of simulation cycle (injection during off season). Given this assumption, operating wells have a cyclic operation schedule similar to that presented in Figure 3.1. Each operating well extracts with a rate Q_E during Δt_E starting at τ_E, and injects for a period of Δt_I with a rate Q_I starting at τ_I. As in Chapter 2, $\Delta t = \Delta t_E + \Delta t_I$ is the length of the single operation cycle.

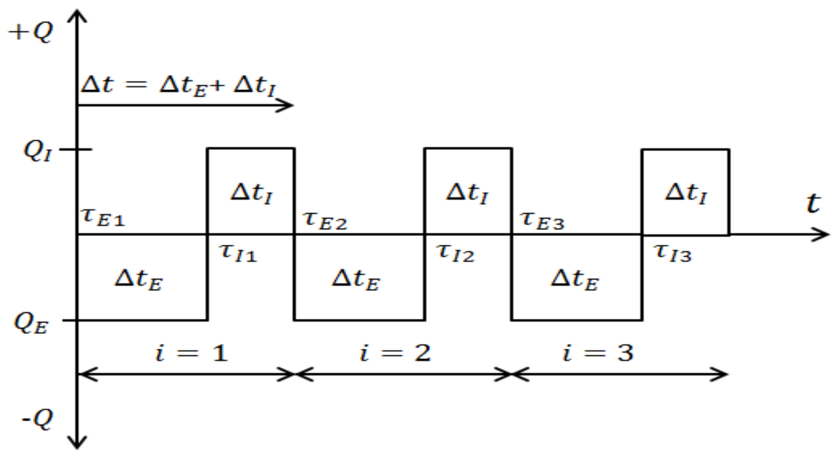


Figure (3.1) Schedule plan for aquifer storage and recovery

(b) Aquifer pumping and recharge (APR): in this formulation, there are two different operation groups. These groups consist of pumping wells and injection wells or generic recharge facilities. The two groups may be operating during generic periods, which may or may not be overlapping. That is why it is described as a dual well formulation. Figure (3.2) shows an example of the schedule plan for two cyclically operating wells. The first well extracts with a rate Q_E starting at τ_E for a period Δt_E, whereas the second well injects starting from τ_I up until Δt_I with a rate Q_I. Both wells are shut off during the part of the operation cycle Δt outside of the operation season.

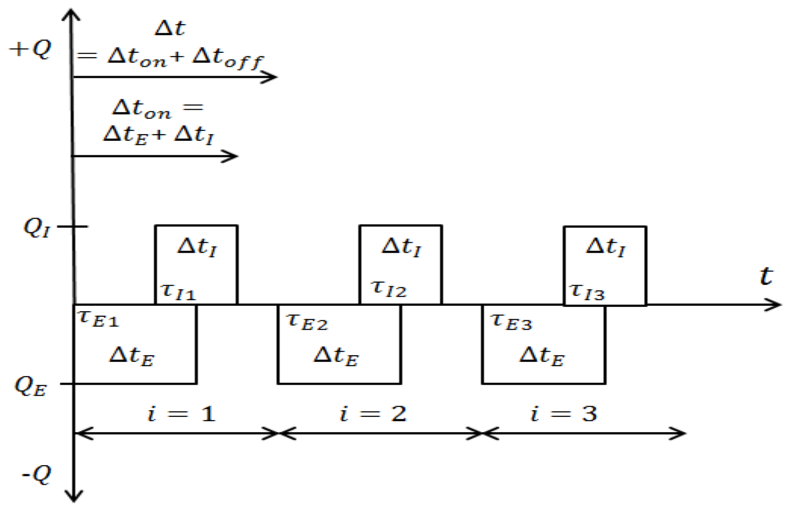


Figure (3.2) Schedule plan for aquifer pumping and recharge

3.1.2 Linear Optimization Approach

The linear semi-analytical models presented in Chapter 2 for assessing the stream depletion/accretion and the aquifer drawdown can be applied to simulate the ground water management problem presented above, and solve it using linear programming techniques. In this case the "independent" decision variables of the problem consist of the pumping rates, Q, at a number of prescribed well locations. The solution of a linear optimization problem requires expressing the objective function and the constraints

$$Optimize\ \{\boldsymbol{c}^{\mathrm{T}}\boldsymbol{Q}\}$$

$$subject\ to\ \boldsymbol{AQ} \leq \boldsymbol{b}$$

Where $\boldsymbol{Q}$ represents the decision variable (operation rate) vector, $\boldsymbol{c}$ *and* $\boldsymbol{b}$ are vectors of known coefficients and $\boldsymbol{A}$ is a matrix of known coefficients.

In this section, we present the formulation of the groundwater management problem under study into a linear optimization problem.

Objective Function: Equation (2.93) which estimates the effect on stream flow due to a cyclically operating well, in an aquifer hydraulically connected to such a stream, can be applied to estimate the stream depletion volume, $V_{r(ex)}$ or the stream accretion volume $V_{r(in)}$ over a given time horizon t:

$$V_r = \begin{cases} V_{r(ex)} & if Q = Q_E < 0 \\ V_{r(in)} & if Q = Q_I > 0 \end{cases} \tag{3.1}$$

that is:

$$V_{r,total} = \begin{cases} V_{r,depletion} = Q_E \, V_{cratio}[S, T, a, \Delta t_E, \Delta t; t] & if \ Q = Q_E < 0 \\ V_{r,accretion} = Q_I \, V_{cratio}[S, T, a, \Delta t_I, \Delta t; t] & if \ Q = Q_I > 0 \end{cases} \tag{3.2}$$

When planning the use of ground water with prior appropriation rule, the optimization objective may be to minimize the sum of the effects on the stream, so eventually there will be a minimum injection volume loss to the stream and a minimum extraction volume from it, this objective can be expressed in terms of minimizing the absolute value of the total volume of stream depletion, $V_{r,total}$, over the investigated time horizon

$$Min \left|V_{r,total}\right| = Min \left| \sum_{j=1}^{n_E} Q_j . V_c ratio\left[S, T, a_m, \Delta t_{E,j}, \Delta t; t\right] + \sum_{j=n_E+1}^{n_E+n_I} Q_j . V_c ratio\left[S, T, a_m, \Delta t_{I,j}, \Delta t; t\right] \right| \tag{3.3}$$

Where Q_j well injection or extraction rate (negative for $1 \leq j \leq n_E$, positive for $n_E + 1 \leq j \leq n_E + n_I$, $n_E$is the number of extracting wells and n_I is the number of injecting wells. Solving the optimization problem (3.4) would yield an ideal answer V_{rtotal} equal to zero, which means that well operation has no net impact on stream flow. This formulation of the optimization problem has a complication brought up by introducing an absolute value operator in the objective function, which causes a loss of linearity such that the optimization problem cannot be tackled using a linear optimization method. This problem, however, can be solved by substituting the original objective function (3.4) with another objective function $\overline{V_r}$ equal exactly to the absolute function of $V_{r,total}$:

$$Min\left|V_{r,total}\right| = \; Min\,\overline{V_r} \tag{3.4}$$

and adding the two following constraints:

$$V_{r,total} \leq \; \overline{V_r} \tag{3.5}$$

$$-V_{r,total} \leq \; \overline{V_r} \tag{3.6}$$

These constraints have the effect of forcing $\overline{V_r}$ to equal to V_r upon being minimized, so that the objective function (3.5) is equivalent to the original objective function (3.4). Note that in this formulation $\overline{V_r}$ acts as both objective function and additional decision variable. Since this problem statement does not contain the absolute value operator, it can be solved using linear programming.

The vector of decision variables for a generic system made up by n_E extraction wells and nI injection wells can be thus expressed as:

$$\boldsymbol{Q} = \; \left[Q_1, Q_2, \dots, Q_{n_E};\; Q_{n_E+1}, Q_{n_E+2}, \dots, Q_{n_E+n_I}; \overline{V_r}\right]^{\intercal} \tag{3.7}$$

where $Q_1, \dots, Q_{n_E}$ are extraction rate values (negative) and $Q_{n_E+1}, \dots, Q_{n_E+n_I}$ are injection rate values (positive). The objective function (3.5) can thus be rewritten in vector product notation as:

$$Min\left\{\mathrm{I}_{n_E+n_I+1} \cdot Q\right\} \tag{3.8}$$

where $\mathrm{I}_{n_E+n_I+1}$ is 1 by $(n_E + n_I + 1)$ row vector, whose coefficients are equal to zero except the last one, corresponding to $\overline{V_r}$, which equals 1.

Linear Inequality Constraints: the objective function is to be optimized under a number of constraints on: (a) operation rates; (b) hydraulic head values at prescribed control points; and (c) irrigation demand and recharge availability

(a) Operation rate constraints are prescribed based upon minimum (maximum extraction) and maximum (maximum injection) flow rate values. For each generic pumping well j $(j=1, 2,\ldots, n_E)$, the flow rate must be such that

$$Q_{Emin,j} \leq Q_j \leq 0 \tag{3.9}$$

where $Q_{Emin,j}$ (< 0) represents the maximum extraction rate at which the well can be operated. Similarly for each injection unit, which may be either an injection well or a recharge facility, the injection rate Q_j j $(j= n_E +1, n_E+2,\ldots, n_E+n_I)$ must be such that:

$$0 \leq Q_j \leq Q_{Imax,j} \tag{3.10}$$

where $Q_{Imax,j}$ (> 0) is the maximum injection rate at which the well or recharge unit can be operated using matrix-vector notation the constraints (3.9) and (3.10) may be rewritten as follows:

$$\boldsymbol{A_L} \cdot \boldsymbol{Q}$$

$$= \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ -1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & -1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & -1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & 0 & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{nE} \\ Q_{nE+1} \\ Q_{nE+2} \\ \vdots \\ Q_{nE+nI} \\ \overline{V}_r \end{bmatrix} \quad (3.11)$$

$$\leq \begin{bmatrix} 0 \\ -Q_{E1,min} \\ 0 \\ -Q_{E2,min} \\ \vdots \\ 0 \\ -Q_{E,nE,min} \\ -Q_{I,nE+1,max} \\ 0 \\ -Q_{I,nE+2,max} \\ 0 \\ \vdots \\ -Q_{I,nE+nI,max} \\ 0 \end{bmatrix} = \boldsymbol{b_L}$$

where the matrix $\boldsymbol{A_L}$ has a size $2 \cdot (n_E + n_I) \times (n_E + n_I + 1)$ and the vector $\boldsymbol{b_L}$ has a size $(n_E + n_I + 1) \times 1$.

(b) Hydraulic head constraints require that maximum and minimum allowable heads are checked at a number n_{mw} of prescribed control points in the aquifer where monitoring wells are located. At the generic monitoring well *m* $(m=1, 2, \ldots, n_{mw})$ the hydraulic head at a given time t_m can be calculated as:

$$h(x_m, y_m; t_m) = h_o + \sum_{j=1}^{n_{Ew}} Q_j \cdot W_{CSI}\left[S, T, r_{m,j}, \Delta t, \Delta t_{E,j}, \tau_j, \bar{\tau}_j; t_m\right] + \sum_{j=n_{Ew}+1}^{n_{Iw}} Q_j \cdot W_{CSI}\left[S, T, r_j, \Delta t, \Delta t_{I,j} \tau_j, \bar{\tau}_j; t_m\right] \tag{3.9}$$

where the coefficients W_{CSI} are calculated using Equation (2.90) and will be denoted as $\beta_{m,j}$. Constraints on the head are thus expressed as:

$$h_0 - h_{min} \leq \sum_{j=1}^{nE} \beta_{m,j} \cdot Q_j + \sum_{nE+1}^{nE+nI} \beta_{m,j} \cdot Q_j \leq h_{max} - h_0 \tag{3.10}$$

where h_{min} and h_{max} are the minimum and maximum hydraulic head allowed, respectively. Using matrix-vector notation, hydraulic head constraints at the n_{mw} monitoring wells can be indicated as

$$\boldsymbol{A_H} \cdot \boldsymbol{Q} = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1,nE} & \beta_{1,nE+1} & \beta_{1,nE+2} & \cdots & \beta_{1,nE+nI} & 0 \\ -\beta_{11} & -\beta_{12} & \cdots & -\beta_{1,nE} & -\beta_{1,nE+1} & -\beta_{1,nE+2} & \cdots & -\beta_{1,nE+nI} & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \beta_{nmw,1} & \beta_{nmw,2} & \cdots & \beta_{nmw,nE} & \beta_{nmw,nE+1} & \beta_{nmw,nE+2} & \cdots & \beta_{nmw,nE+nI} & 0 \\ -\beta_{nmw,1} & -\beta_{nmw,2} & \cdots & -\beta_{nmw,nE} & -\beta_{nmw,nE+1} & -\beta_{nmw,nE+2} & \cdots & -\beta_{nmw,nE+nI} & 0 \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{nE} \\ Q_{nE+1} \\ Q_{nE+2} \\ \vdots \\ Q_{nE+nI} \\ \bar{V}_r \end{bmatrix} \leq \begin{bmatrix} h_{max} - h_0 \\ h_0 - h_{min} \\ \vdots \\ h_{max} - h_0 \\ h_0 - h_{min} \end{bmatrix} = \boldsymbol{b_H} \tag{3.14}$$

where $\boldsymbol{A_H}$ is a $2 \cdot n_{mw} \times (n_E + n_I + 1)$ matrix and $\boldsymbol{b_H}$ is a column vector of size $2 \cdot n_{mw} \times 1$. It is worth noting that the index m identifies a control point where the head value is checked at a given time. If at the same monitoring well, heads must be checked at a different time, then an additional constraint is to be added.

(c) Irrigation Demand and recharge availability constraints require that during the pumping season the total sum of the (negative) extraction rates is less than or equal to the total (negative) demand rate Q_{demand}, thus:

$$\sum_{j=1}^{n_{Ew}} Q_j \leq Q_{demand} \leq 0 \tag{3.13}$$

And during the period in which surface water is made available for aquifer recharge, the total sum of injection well rates is less than or equal to total available recharge rate $Q_{available}$:

$$\sum_{j=n_{Ew}+1}^{n_{nE}+n_{Iw}} Q_j \leq Q_{available} \tag{3.14}$$

Following the matrix-vector notation, the irrigation demand and recharge availability constraints can be expressed as follows:

$$\boldsymbol{A_{DA}} \cdot \boldsymbol{Q} = \begin{bmatrix} 1 & 1 & \dots & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{nE} \\ Q_{nE+1} \\ Q_{nE+2} \\ \vdots \\ Q_{nE+nI} \\ \overline{V_r} \end{bmatrix} \tag{3.15}$$

$$\leq \begin{bmatrix} Q_{dem} \\ Q_{ava} \end{bmatrix} = \boldsymbol{b_{DA}}$$

where $\boldsymbol{A_{DA}}$ is a $2 \times (n_E + n_I + 1)$ matrix and $\boldsymbol{b_{DA}}$ is a column vector of size 2×1.

Two additional inequalities are necessary to prescribe the constraints (3.6) and (3.7) introduced in order to remove the absolute value from the objective function (3.4), inequalities (3.6) and (3.7) can thus be rewritten, respectively, as:

$$\sum_{j=1}^{n_E} Q_j \cdot V_c ratio\left[S, T, a_j, \Delta t_{E,j}, \Delta t, \tau_j, \bar{\tau}_j; t\right] + \sum_{j=nE+1}^{n_{E+nI}} Q_j \cdot V_c ratio\left[S, T, a_j, \Delta t_{I,j}, \Delta t, \tau_j, \bar{\tau}_j; t\right] \leq \bar{V}_r \tag{3.15}$$

and

$$-\sum_{j=1}^{n_E} Q_j \cdot V_c ratio\left[S, T, a_j, \Delta t_{E,j}, \Delta t, \tau_j, \bar{\tau}_j; t\right] - \sum_{j=nE+1}^{n_{E+nI}} Q_j \cdot V_c ratio\left[S, T, a_j, \Delta t_{I,j}, \Delta t, \tau_j, \bar{\tau}_j; t\right] \leq \bar{V}_r \tag{3.16}$$

where the coefficients $V_{c,ratio}$ are calculated using Equation (2.95) and are indicated as α_j in the following. Using matrix-vector notation, the two constraints (3.6) and (3.7) can thus be rewritten as:

$$\boldsymbol{A_{OF}} \cdot \boldsymbol{Q} = \begin{bmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_{nE} & \alpha_{nE+1} & \alpha_{nE+2} & \dots & \alpha_{nE+nI} & -1 \\ -\alpha_1 & -\alpha_2 & \dots & -\alpha_{nE} & -\alpha_{nE+1} & -\alpha_{nE+2} & \dots & -\alpha_{nE+nI} & -1 \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{nE} \\ Q_{nE+1} \\ Q_{nE+2} \\ \vdots \\ Q_{nE+nI} \\ \bar{V}_r \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \boldsymbol{\phi} \tag{3.20}$$

where the matrix $\boldsymbol{A_{OF}}$ has a size $2\times(n_E + n_I + 1)$ matrix, and the zero vector $\boldsymbol{\phi}$ has a size 2×1.

The linear optimization problem into which the groundwater management is formulated can thus be structured as:

$$min\left\{\mathbf{I}_{n_E+n_I+1}\cdot\boldsymbol{Q}\right\}$$

subject to

$$\boldsymbol{A}\cdot\boldsymbol{Q}\leq\boldsymbol{b}$$

where $\boldsymbol{A}\equiv[\boldsymbol{A_L}\ \boldsymbol{A_H}\ \boldsymbol{A_{DR}}\ \boldsymbol{A_{OF}}]^{\mathrm{T}}$, and $\boldsymbol{b}\equiv[\boldsymbol{b_L}\ \boldsymbol{b_H}\ \boldsymbol{b_{DR}}\ \boldsymbol{\phi}]^{\mathrm{T}}$ the size of matrix **A** is $2\cdot(n_E+n_I+n_{mw}+2+2)\times(n_E+n_I+1)$, and the size of b is $2\cdot(n_E+n_I+n_{mw}+2+2)\times 1$.

A similar linear optimization setting can be formulated to minimize stream depletion/accretion from a stream that represents a boundary in a finite aquifer bounded between the stream under study and another boundary of either type (a no-flow boundary or a recharge boundary) using the equations presented in Chapter 2 Section 2.5 to estimate stream depletion volume V_r and drawdown s due to a cyclically operating (extracting or injecting) well.

In the following, we are going to discuss the application of the linear optimization problem described above in semi-infinite aquifers as well as finite aquifers comprised between a no-flow boundary and a stream. The examples considered here are simulations of both of the management settings of (APR) and (ASR). Despite the fact that, the semi-analytical models presented in Chapter two and used here in the linear programming setting of the management problem, are devolved for only the case of operating wells (extraction and injection), and in the case of APR the recharge is achieved by surface infiltration using ponds, these codes can be used to provide an acceptable results in this case as well. Molden et al (1984) have proposed an approach to calculate recharge volumes due to surface infiltration. Such approximation may be used to improve the results of the developed semi-analytical models codes.

3.2 Groundwater management problem setting

Let us consider the 8-km by 6-km stretch of an alluvial aquifer limited by a stream located at the y-axis shown in Figure 3.3. Aquifer properties are listed in table 3.1

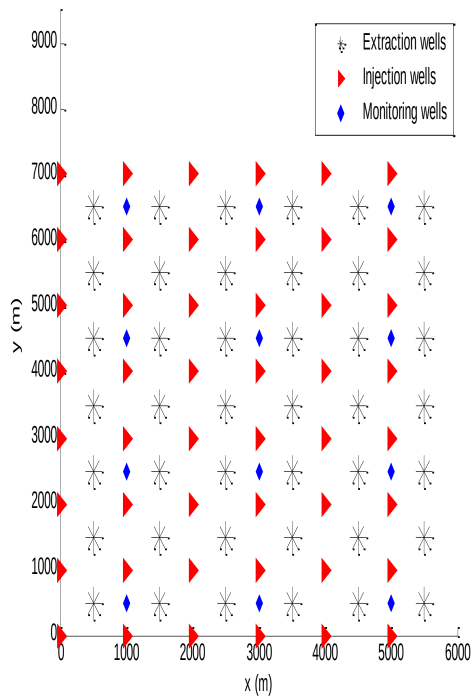


Figure 3.3 the candidate wells and recharge ponds locations in a semi-infinite aquifer limited by a stream located at the y-axis (x=0)

Table 3.1 Aquifer properties

h_0 (m)	K (m/day)	S (/)
30	*86.4*	*0.2*

The management problem requires providing water for irrigation extracted from the aquifer during the growing season, for consumptive use of 1 m per the total duration of irrigation, to irrigate a cultivated area of 0.3 of the total area of the aquifer, and recharge the aquifer with an equivalent amount of surface water to offset potential stream over pumping. This consumptive use is representative of corn crop type.

Table 3.2 shows the operation details of the extracting and injecting wells, demand and availability in cubic meters per year and in cubic meters per day for both APR and ASR problems.

Table 3.2 Extraction and injection details used in the management problem

Operation		Operation period	Demand/Availability (m^3/year)	Demand/Availability (m^3/day)
Pumping	APR	120 days (March 15–July 15)	1.2×10^7	100000
	ASR			
Recharge	APR	180 days (October 01-March 01)	1.2×10^7	≈ 70000
	ASR			

The conjunctive management aim is to determine the spatial distribution of extraction and injection wells that achieves the objective of the management problem of minimizing the absolute value of the stream depletion volume over the 10 years operation period. Constraints described in the previous section are imposed on the demand and water availability for recharge as shown in table 3.2, for the maximum operation rates of pumping wells and recharge ponds, and aquifer's head levels at monitoring locations during both operations durations over the simulation period (for three considered scenarios), the imposed constraints are as follows:

Table 3.3 Optimization problem constraints

Constraint		Minimum	Maximum
Operation rate (m^3/day)	Pumping	-5000	0
	Recharge	0	5000
Hydraulic Head (m)	Scenario 1	29	31
	Scenario 2	29.5	30.5
	Scenario 3	25	31

For the third scenario, the constraint on recharge water availability is different than the other two scenarios, as the available recharge water volume is set to be 85% of the irrigation demand. In the ASR groundwater management problem, all the candidate operation facilities are used as pumping wells during growing season, and all of them reverse the operation to recharge (direct injection wells) outside of the season.

3.2.1 Results in semi-infinite aquifers

A Matlab code SI_Opt.m is built to calculate the optimal operation rates based on the constraints and operation plan of APR and ASR. This code uses the Matlab linear optimization solver linprog. Detailed description of the code is presented in Appendix I. In Table 3.4 lists the results of the APR case for the three considered scenarios. Column one shows the cumulative recharge rate in cubic meters per day, column two shows the cumulative extraction rate in cubic meters per day and in column three listed the net cumulative stream depletion volume $\overline{V_r}$ in cubic meters per day at the end of simulation period.

Table 3.4 APR case results for a semi-infinite aquifer

Scenario	Recharge (m^3/day)	Extraction (m^3/day)	Net Cumulative Stream Depletion Volume $\overline{V_r}$ (m^3/day)
1) $29 \leq h \leq 31$	6.65×10^{4}	-1.0×10^{5}	1.34×10^{-8}
2) $29.5 \leq h \leq 30.5$	6.99×10^{4}	-9.98×10^{4}	6.694×10^{-9}
3) $25 \leq h \leq 31$	5.27×10^{4}	-1.004×10^{5}	2.547×10^{-9}

Figure 3.4 shows the optimal wells and recharge ponds distribution for the first scenario of the APR along with the operation rates.

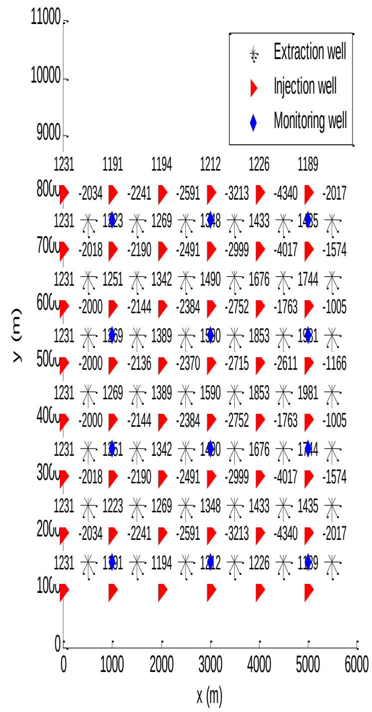


Figure 3.4 active wells and recharge ponds and their operation rates shown above each marker, this layout is for the first scenario of APR. The presented aquifer is limited by a stream located at x=0

Note that all candidate wells are active during the operation period and they operate with the rates noted above each marker in Figure 3.4. As shown in Table 3.4, this operation plan results in a net volume of stream depletion of almost a zero m^3/day at the end of simulation period, and all irrigation demand is met. Water volume lost to the stream is minimal, with recharge amounts almost as the same as the pumped demands. Note that to meet the objective of minimum stream depletion volume, extracting wells rates increase away from the stream and recharge ponds rates increase towards it.

Figure 3.5 shows the optimal distribution of operating facilities (pumping wells and recharge ponds) for the second scenario of the APR case.

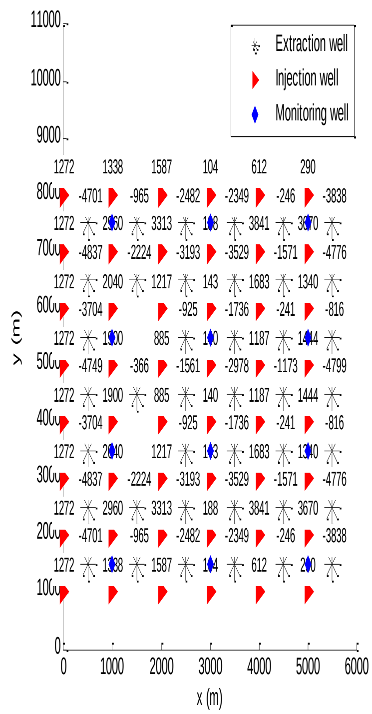
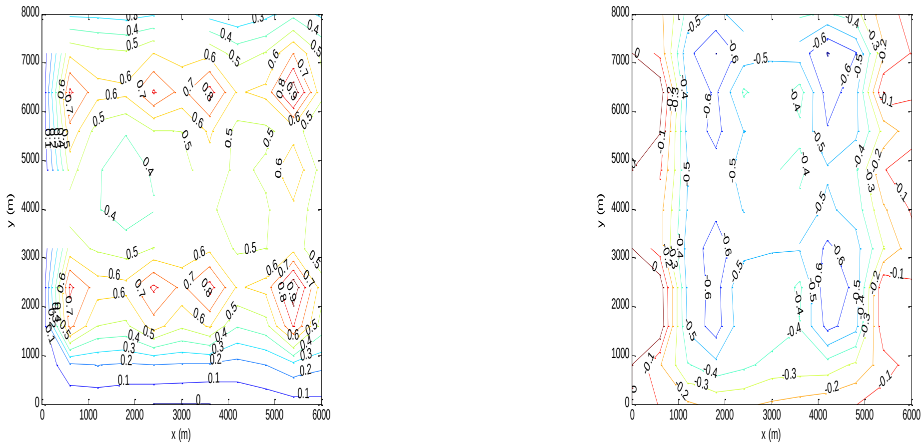


Figure 3.5 active wells and recharge ponds and their operation rates shown above each marker, this layout is for the second scenario of APR. The presented aquifer is limited by a stream located at x=0

Note the change in the optimal layout as not all the wells are activated this time. The irrigation demand is met with the available recharge volume causing almost a zero stream depletion volume even with the small allowed range of aquifer head changes. In contrary to the case in the first scenario, and in order to meet the head constraint of this scenario, high capacity wells are located closer to the stream to guarantee obtaining the demand volume. As well, high capacity recharge ponds are activated closer to the high extraction rates away from the stream to balance drawdown.

Figure 3.6 shows the drawdown distribution in APR second scenario, at two head checks times: a) 2020 days (the end of extraction period in the fifth operation cycle); and b) 2280 days (the end of injection period in the sixth operation cycle).



(a) **(b)**

Figure 3.6 resulting drawdwon contours at two head check times 2020 days and 2280 days in (a) and (b), respectively, for the APR second scenario.

Note in Figure 3.6a, the high drawdown values around locations of high capacity wells closer to the stream, in the upper and lower parts of the middle part of the aquifer, and at the far end of it from the stream. On the other hand note the build-up values closer to the stream and in the middle of the aquifer in Figure 3.6b. These build up values are responsible of replenishing the stream and smoothing head changes caused by extraction.

Figure 3.7 shows the optimal distribution of operation rates (pumping recharge) for the third scenario of APR.

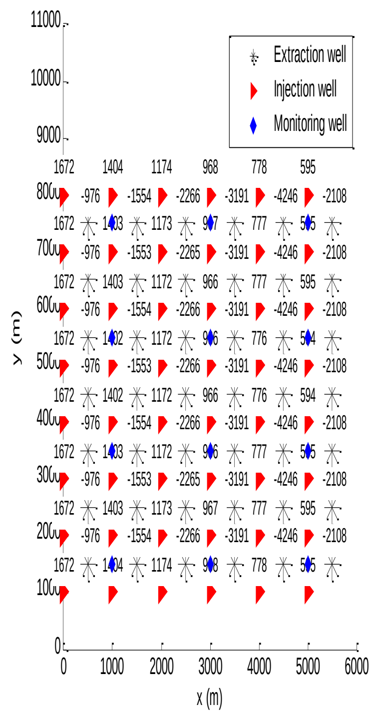


Figure 3.7 active wells and recharge ponds and their operation rates shown above each marker, this layout is for the third scenario of APR. The presented aquifer is limited by a stream located at x=0

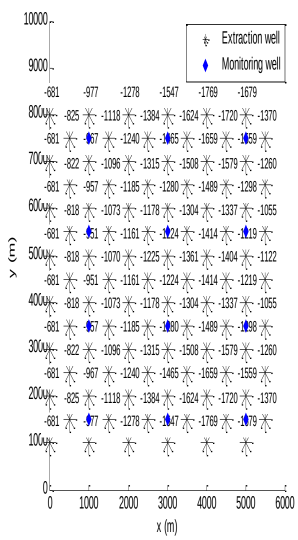
Again, all demands are met in this scenario with cumulative stream depletion volume at the end of the 10 years period equal to zero even though recharge capacity is 85% of groundwater demand and it wasn't fully consumed, because in this case, demands can be met directly from the aquifer storage during irrigation season, since a much loose range of head change is allowed here.

Table 3.5 shows the results of the three considered scenarios in ASR. The listed results are the cumulative recharge rate in cubic meters per day, the cumulative extraction rate in cubic meters per day and the net cumulative stream depletion volume at the end of the 10 years simulation period in cubic meters per day.

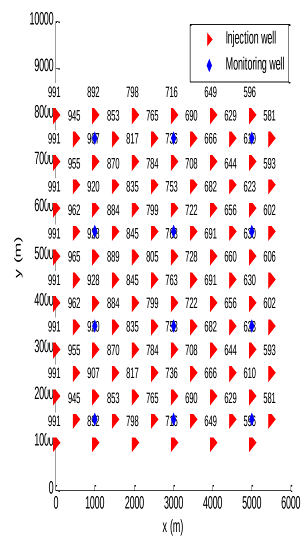
Table 3.5 ASR results in semi-infinite aquifer case

Scenario	Recharge (m^3/day)	Extraction (m^3/day)	Net Cumulative Stream Depletion Volume $\overline{V_r}$ (m^3/day)
1) $29 \leq h \leq 31$	6.9×10^{4}	-1.09×10^{5}	6.57×10^{-9}
2) $29.5 \leq h \leq 30.5$	6.99×10^{4}	-1.05×10^{5}	5.91×10^{-9}
3) $25 \leq h \leq 31$	5.9×10^{4}	-1.02×10^{5}	2.74×10^{-9}

Figure 3.8a and 3.8b illustrate the optimal results of ASR scenario 1 for pumping and recharge rates respectively.



(a)

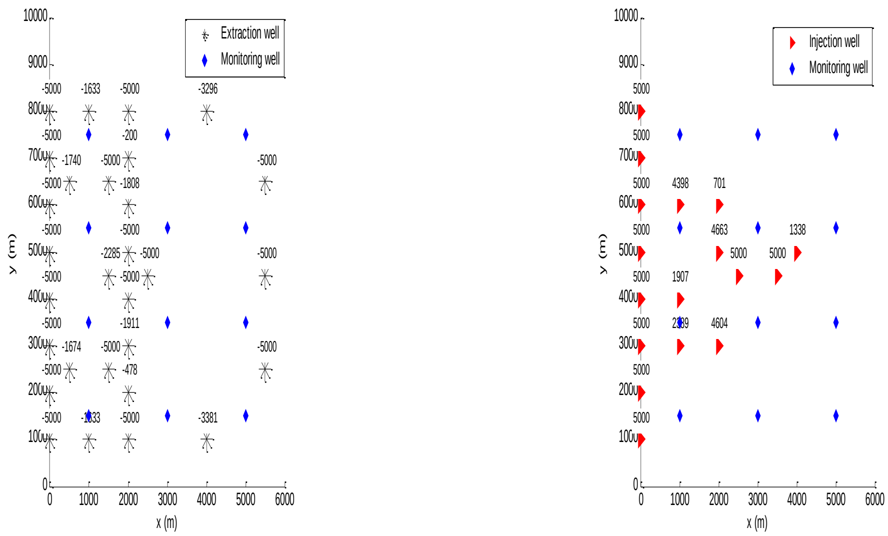


(b)

Figure 3.8 optimal rates of pumping and recharge of senario 1 in the semi-infinte alluvial aquifer under study in subpanel (a) and subpanel (b), respectively. The aquifre is limited by stream located at the y-axis

It is interesting to note that in Figure 3.8, the pumping wells optimal rates values decline towards the stream to meet the objective of minimum stream depletion, the maximum pumping rate is even less than the half of the maximum capacity of wells. On the other hand the optimal recharge rates increase closer to the stream with a maximum recharge rate less than a 1000 m^3/day. As listed in Table 3.5, the irrigation demand is met in this scenario with the same amount of available recharge water, and no stream depletion or accretion occurred at the end of simulation period.

Figure 3.9a and 3.9b illustrate the optimal results of the ASR scenario 2 for pumping and recharge rates, respectively.

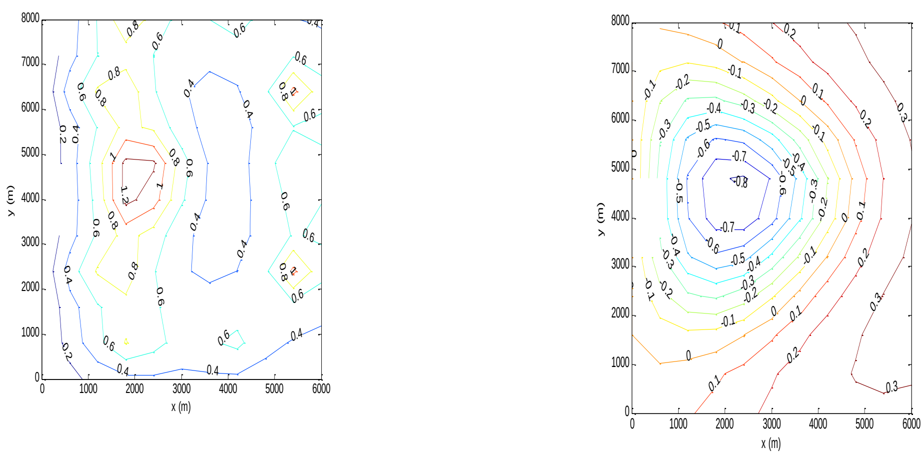


(a) **(b)**

Figure 3.9 optimal rates of pumping and recharge of senario 2 in the semi-infinte alluvial aquifer under study in subpanel (a) and subpanel (b), respectively. The aquifre is limited by stream located at the y-axis

In this scenario, about 27% of the total number of pumping wells (25 out of 90 operating wells) is activated, with 18 of them pumping with their maximum capacities. Despite of the presence of these high capacity wells closer to the stream, no stream depletion volume noticed at the end of the 10 years simulation period, because, during the operation period of the recharge ponds, 11 out of 18 operating recharge ponds operate with the maximum capacity and 9 of them are located at the stream reach. This noticed difference in wells distribution between the first and second scenarios, happened to ensure meeting the strict constraint in aquifer heads. As listed in Table 3.5 the demands are met with the available recharge water.

Figures 2.10a and 2.10b show the drawdown distribution in the second scenario of ASR, at the two head check times 2020 days and 2280, respectively.

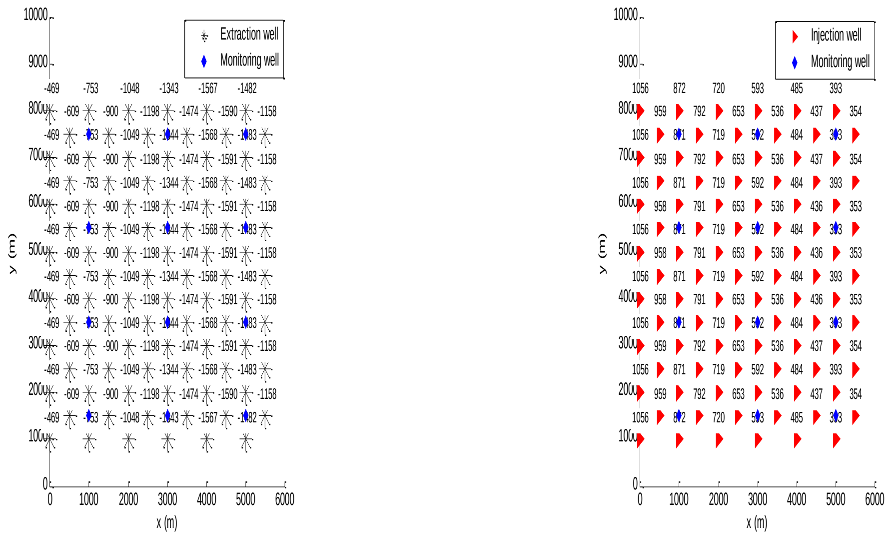


(a)

(b)

Figure 3.10 resulting drawdwon distributions at two head check times 2020 days and 2280 days in (a) and (b), respectively, for the ASR second senario

Even though the high capacity wells are located in the stream reach, the drawdown values are small closer to it in Figure 3.10a, because of the fact of the stream being a recharge boundary. A high value of drawdown occurs at the end of the aquifer where there is a couple of high capacity well and less impact of stream supply. In Figure 3.10b, high build up values are noticed about the middle of the aquifer due to the combined effect of the high capacity recharge ponds and the presence of the stream. Figure 3.11a and 3.11b illustrate the optimal results of the ASR scenario 3 for pumping and recharge rates, respectively.



(a) **(b)**

Figure 3.11 optimal rates of pumping and recharge of senario 3 in the semi-infinte alluvial aquifer under study in subpanel (a) and subpanel (b), respectively. The aquifre is limited by stream located at the y-axis

One can note the resemblance between the first and the third scenarios in terms of the operating wells distribution and the number of active wells and recharge ponds, as the stretched range of change in aquifer heads allowed in this scenario, the demands are met without depleting the stream even with less available recharge water.

3.2.2 Results in finite aquifer comprised between a no-flow boundary and a recharge boundary

The Matlab code BNF_Opt.m is developed for the case of a finite aquifer comprised between a physical boundary and a stream to calculate optimal operation rates applying the linear optimization approach presented in the previous section and using the linear optimization solver Linprog of Matlab. Details and manual of the code are given in Appendix J.

The same aquifer setting presented to study the semi-infinite aquifer case is going to be used to evaluate stream depletion and operation rates in finite aquifer case, as well as the APR and ASR operation plans and constraints. In the 8-km by 6-km stretch of the alluvial aquifer under study, the no-flow boundary is located at the y-axis (x=0) and the stream is located at x=6 km. Table 3.6 shows the results of the three scenarios of APR. These results are the cumulative recharge rate in cubic meters per day, the cumulative extraction rate in cubic meters per day and the cumulative stream depletion volume at the end of simulation period in cubic meters, respectively.

Table 3.6 finite aquifer (no-flow/recharge case) APR results

Scenario	Recharge (m^3/day)	Extraction (m^3/day)	Net Cumulative Stream Depletion Volume $\overline{V_r}$ (m^3/day)
1) $29 \leq h \leq 31$	6.999×10^{4}	-9.99×10^{4}	5.2×10^{-9}
2) $29.5 \leq h \leq 30.5$	6.817×10^{4}	-1.000×10^{5}	5.824×10^{6}
3) $25 \leq h \leq 31$	5.932×10^{4}	-1.031×10^{5}	1.863×10^{-9}

For scenario 1, all pumping wells and recharge ponds are activated resulting in a net stream depletion volume at the end of the 10 years simulation period equal to zero as listed in Table 3.6, meaning that, aquifer storage is extracted to provide irrigation demand with an equal amount to the aquifer recharge.

In scenario 2, the irrigation demands are perfectly met with the available recharged amounts and the stream is depleted with about 5.9×10^{6} m^3/day at the end of simulation period as shown in Table 3.6. With the strict limitation on aquifer head levels at monitoring wells the results show that the need to minimize aquifer's level change is as effecting as the need of minimizing stream depletion volumes.

Recalling that in scenario 3 there is no strict constraint on the head levels, where ($25 < t < 31$ m) but the recharge capacity is changed to 85% of the irrigation demand, the demands are met with less conservative extraction from the aquifer to an adequate extend to not deplete the stream even with less available recharge amounts, results are shown in Table 3.6.

Figure 3.12 shows the optimal rates results and the active wells and recharge ponds for scenario 1 of APR

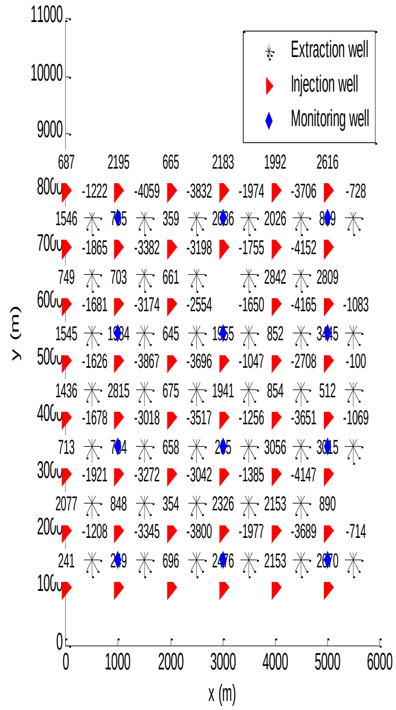


Figure 3.12 active wells and recharge ponds and their operation rates shown above each marker, this layout is for the first scenario of APR. The presented aquifer is limited by a stream located at x=0

Again, one can notice that the extraction rates are low in proximity of the stream to achieve the optimization objective. With the presence of the no-flow boundary, the optimal result tend to locate wells with low pumping rates in proximity of the no-flow boundary to meet the heads constraints.

Figure 3.13 displays the optimal wells/recharge ponds layout with rates above markers for scenario 2 of APR

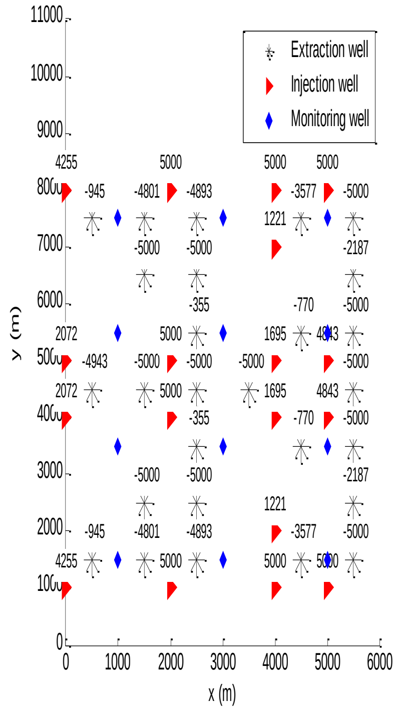
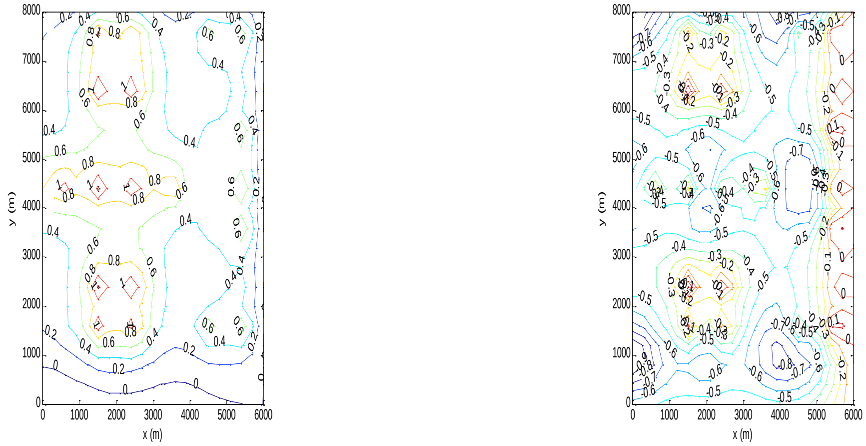


Figure 3.13 active wells and recharge ponds and their operation rates shown above each marker, this layout is for the second scenario of APR. The presented aquifer is limited by a stream located at x=0

Note that in Figure 3.13, some high capacity wells are located in proximity of the stream indicating that, these wells are responsible for stream depletion. The recharge ponds depicted at the same figure in proximity of the no-flow boundary are set to smooth the drawdown in that area since it retains the maximum values. Figure 2.14 shows the drawdown distribution in the second scenario of APR, at two head check times at 2020 days (the end of extraction period in the fifth operation cycle) and 2280 days (the end of injection period in the sixth operation cycle) in (a) and (b), respectively.



(a) **(b)**

Figure 3.14 drawdown contours in APR second scenario, at two head check times: (a) 2020 days, and (b) 2280 days.

In Figure 3.14a, note the high drawdown values in proximity of then no-flow boundary and the operating wells, while low extraction occurs closer to the stream. In Figure 3.14b, there are a couple of build-up high values closer to the stream and to the no-flow boundary, and in the middle of the aquifer, these values are minimal.

Figure 3.15 shows the active wells and recharge ponds along with the optimal rates results for scenario 3 of APR.

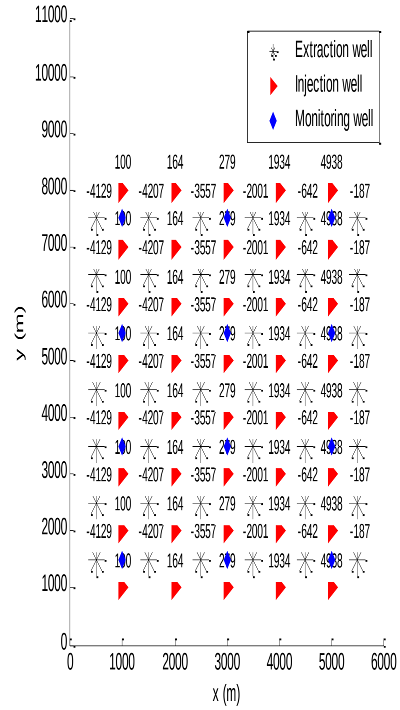


Figure 3.15 active wells and recharge ponds and their operation rates shown above each marker, this layout is for the second scenario of APR. The presented aquifer is limited by a stream located at x=0

Note, in Figure 3.15 a typical distribution of pumping wells with low rates closer to the stream that increases away from it to achieve the objective of minimum stream depletion, and recharge ponds that operates with high capacities in proximity of the stream to augment its flows. In this scenario, the demands are met with the extracted volumes from the aquifer since the head constrain is less conservative, in addition to the recharge availability which is 85% of the demands, even though, no stream depletion noticed at the end of simulation period.

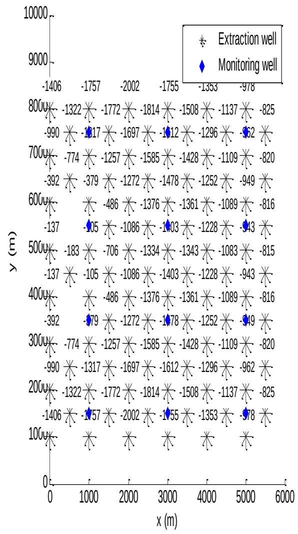
Table 3.7 shows the ASR groundwater management case results for a finite aquifer pounded between a no-flow boundary and a stream.

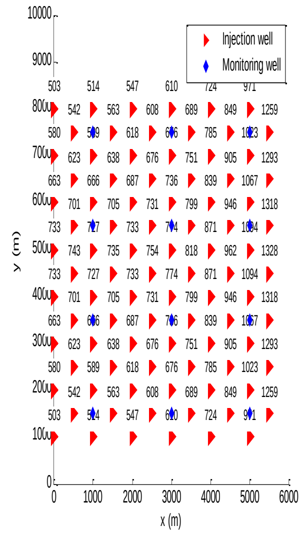
Table 3.7 ASR results

Scenario	Recharge (m^3/day)	Extraction (m^3/day)	Net Cumulative Stream Depletion Volume $\overline{V_r}$ (m^3/day)
1) $29 \le h \le 31$	6.9×10^{4}	-1.0×10^{5}	2.9×10^{-9}
2) $29.5 \le h \le 30.5$	6.3×10^{4}	-9.9×10^{4}	1.3×10^{-9}
3) $25 \le h \le 31$	5.8×10^{4}	-1.1×10^{5}	1.07×10^{-9}

Again, the ASR case is a typical illustration of achieving the demands while applying constraints. The demands in the three scenarios are met with no significant stream depletion as shown in table 3.7 even with less recharge availability and strict head levels constraints. The pumping wells is distributed in way that minimizes the stream depletion and recharge pond are distributed in way that helps replenishing the stream and balance the drawdown in the aquifer.

Figure 3.16 shows the active wells and recharge ponds along with the optimal rates results for scenario 1 of ASR

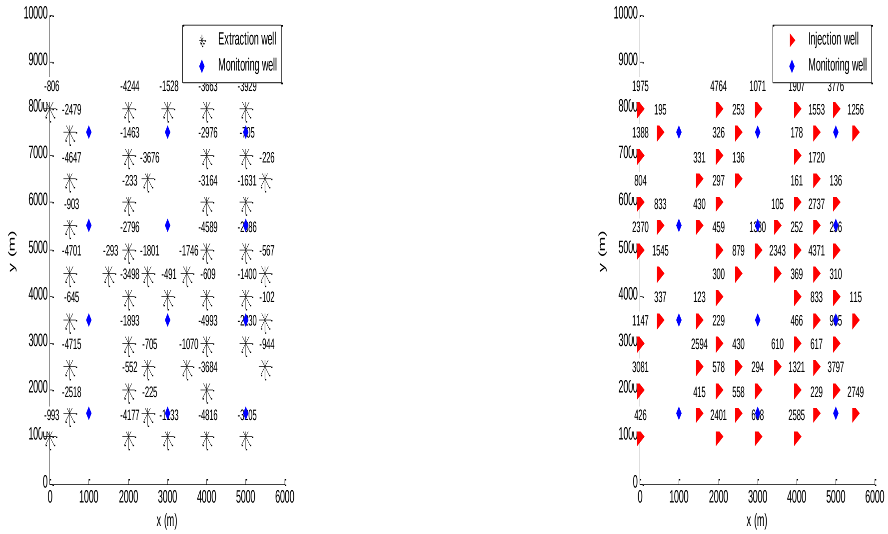




(a) **(b)**

Figure 3.16 optimal layout of pumping wells in (a) and recharge ponds in (b) for ASR scenario one

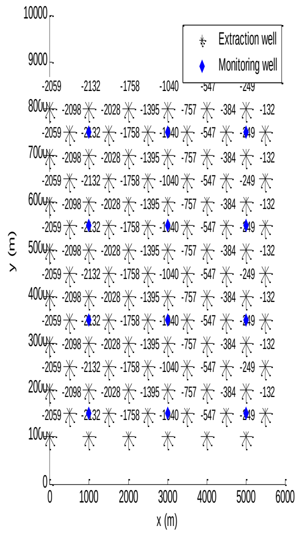
Figure 3.17 shows the active wells and recharge ponds along with the optimal rates results for scenario 2 of ASR



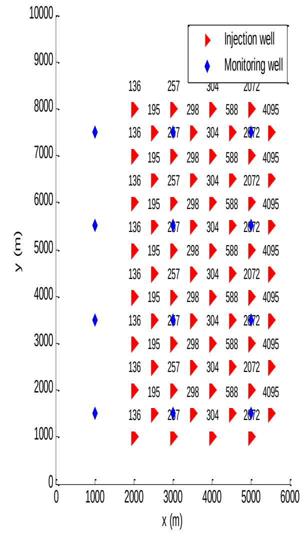
(a) **(b)**

Figure 3.17 optimal layout of pumping wells in (a) and recharge ponds in (b) for ASR scenario two.

Figure 3.18 shows the active wells and recharge ponds along with the optimal rates results for scenario 3 of ASR



(a)



(b)

Figure 3.18 optimal layout of pumping wells in (a) and recharge ponds in (b) for ASR scenario three

Figure 3.19 shows the profiles of total stream depletion rate and the total cumulative stream depletion volume with time in APR Scenarios 1, 2, and 3, in (a)\and (b) respectively, and in subpanel (c) and (d) the profiles are for ASR case

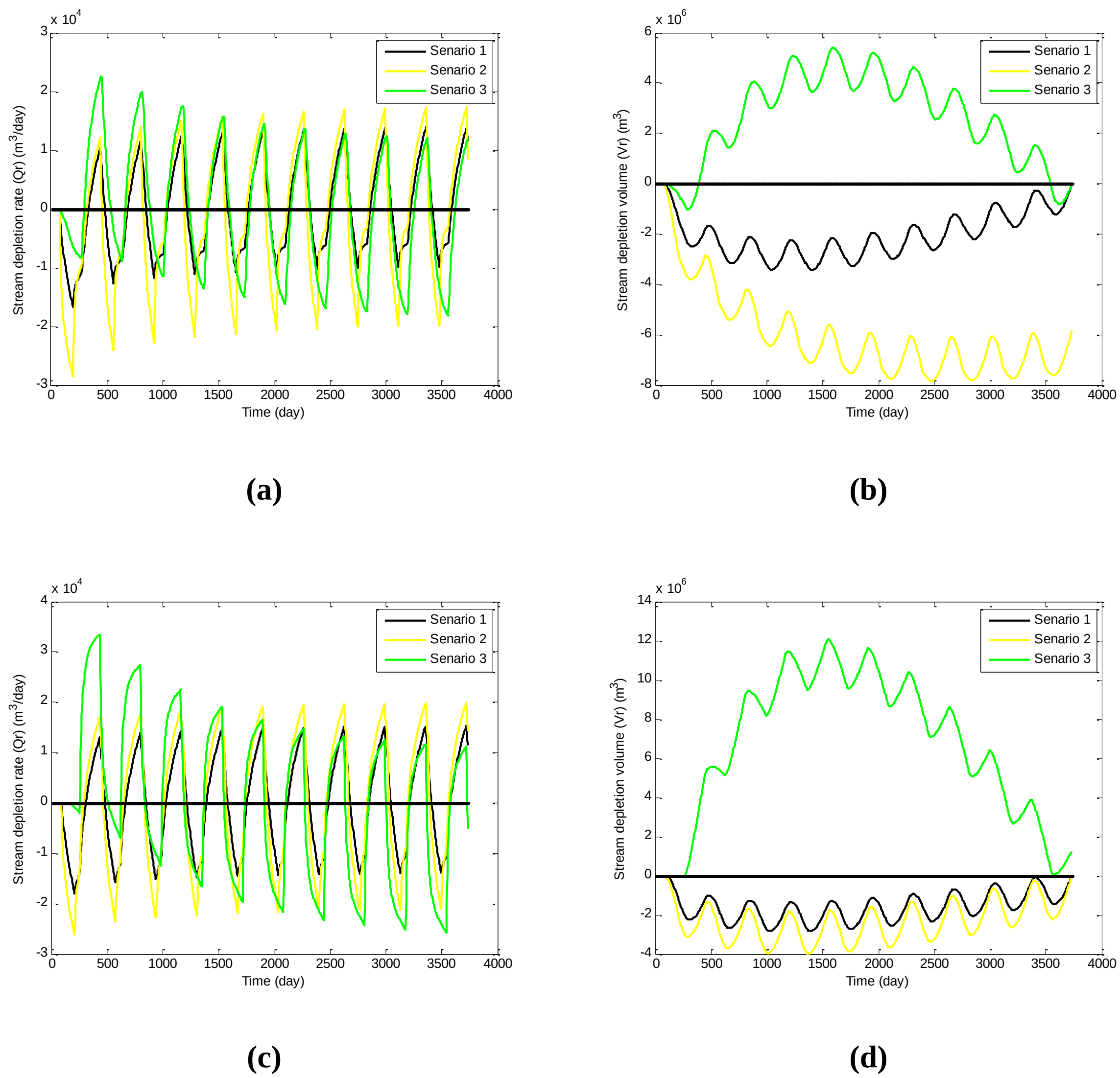


Figure 3.19 the time series of the total stream depletion rate and the total cumulative stream depletion volume in APR Scenarios 1, 2, and 3, in (a)\and (b) respectively, and in subpanel (c) and (d) the profiles are for ASR case

Note, in Figure 3.19b, even though the stream depletion volume at the end of the 10 years simulation period is zero, the stream is being depleted for almost the entire duration of scenario one. On the other hand, the stream is being replenished for the entire simulation duration in scenario 3, but in scenario 2 it is depleted during wells operation and at the end of simulation time. For the case of ASR, the profiles are almost the same for the three scenarios, but the depletion is almost zero at the end of simulation. The doubled number of operating wells/recharge ponds in this case, causes the increase in recharge volumes in all of the three scenarios.

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APPENDIX A

Matlab code: **Infinite.Drawdown.2D**

This code calculates and plots the spatial distribution of head at a specified time t_{fin} for a number of wells operating cyclically or continuously in an aquifer with an infinite areal extension.

The input file **Aquifer.Parms** includes simulation time parameters and aquifer parameters:

- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t = \Delta t_{on} + \Delta t_{off}$ (e.g., 1 year)

- the aquifer parameters
  - initial hydraulic head (aquifer's saturated thickness) h_0
  - hydraulic conductivity K;
  - storativity S.

The input file **Grid.parms** includes wells data:

  - grid lower left corner abscissa (x_{min})
  - grid lower left corner ordinate (y_{min})
  - grid upper right corner abscissa (x_{max})
  - grid upper right corner ordinate (y_{max})
  - n. of gridblocks along x (n_x)

n. of gridblocks along y (n_y)

The input file **Wells.dat** includes wells data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - the pumping rate (Q_w);
  - the time at which well operation starts (t_{st}),
  - the total operation period (Δt_{on})
  - well location x_w and y_w.

Examples:

Aquifer.Parms

100. or 200.	365.	30.	622.08	0.2

t_{fin} (day)	Δt=Δt_{on}+Δt_{off} (day)	b(m)	K (m/day)	S (/)

Grid.Parms

0.	0.	2000.	2000.	20	20
x_{min} (m)	y_{min} (m)	x_{min} (m)	y_{min} (m)	n_x	n_y

Wells.dat

Description:	2 wells operating cyclically			
2 (n_w)				
-500.	0.	180.	500.	1000.
-1000.	60.	120.	1000.	500.
Q_w (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

Infinite.Drawdown.2D

```
clc
clear all
% Reading Data
% 1- General Data
fid1 = fopen ('parameters.txt','r');
Temp = fscanf(fid1,'%f %f %f %f %f',[1,5]);
tfin = Temp(1); % Final Time of Simulation (Days)
delt = Temp(2); % Cycle Time = One Year (days)
ho   = Temp(3); % Aquifer Initial Heard (M)
K    = Temp(4); % Hydraulic Conductivity (M^2/day)
Sy   = Temp(5); % Aquifer Specific Yeild (/)
T    = K*ho;    % Transmissivity
fclose(fid1);
%
% 2- Wells Data
fid2 = fopen ('wells.txt','r');
Temp = fscanf(fid2,'%f',[1,1]);
now   = Temp(1);% Wells Number
for m = 1:now
Temp    = fscanf(fid2,'%f %f %f %f %f',[1,5]);
```

```
Qw(m)   = Temp(1); % Well Pumping Rate
tst(m)  = Temp(2); % Pumping Start Time
dton(m) = Temp(3); % Pumping Period
xw(m)   = Temp(4); % Well Location X Coordinate
yw(m)   = Temp(5); % Well Location Y Coordinate
end
fclose(fid2);
%
% 3- Reading Grid Data
fid3 = fopen ('Aqui_Param.txt','r');
Temp = fscanf(fid3,'%f %f %f %f %f %f',[1,6]);
xmin = Temp(1); % Minimum Value of X in the Grid
ymin = Temp(2); % Minimum Value of Y in the Grid
xmax = Temp(3); % Maximim Value of X in the Grid
ymax = Temp(4); % Maximum Value of Y in the Grid
nx   = Temp(5); % Number of X Divisions
ny   = Temp(6); % Number of Y Divisions
fclose(fid3);
% %
% Creating the Grid %
nxx = nx+1;
nyy = ny+1;
dx = (xmax-xmin)/nx;
dy = (ymax-ymin)/ny;
for i= 1:nyy
   for j= 1:nxx
      x(i,j)= xmin+ dx*(j-1);
   end
end
%
for j= 1:nxx
   for i= 1:nyy
      y(i,j)= ymin+dy*(i-1);
   end
end
%
% Computing Drawdown Distribution for number of operating wells
 fid3 = fopen('results.txt','w');
 s_sum = zeros(nyy,nxx);% Initial Drawdown
 for i= 1:nyy;
    for j= 1:nxx;
       for m =1:now
          t = tfin-tst(m);
          if t>0
             s_sum(i,j)=s_sum(i,j)-
Qw(m)*RC_fun(t,delt,dton(m),Sy,T,x(i,j),y(i,j),xw(m),yw(m));
```

```
            % CYC_THEIS calculates the drawdown for one operating well and its image
and imaginary wells
          end
       end
       temp = [x(j), y(i), s_sum(i,j)];
       fprintf(fid3,'%15.6E %15.6E %15.6E\n',temp);
       fwrite(fid3,temp);
    end
 end
fclose(fid3);
% %
str1 = num2str(Qw');
cell1 = cellstr(str1);
% Figure
figure;
contour(x,y,s_sum);
[C,h] = contour(x,y,s_sum);
clabel(C,h);
% title('Drawdown M, Snapshot Time 400 days');
xlabel('x (m)');
ylabel('y (m)');
axis square;
hold on
scatter(xw,yw,50,'r+')
text(xw+20,yw+50,cell1,'BackgroundColor',[1 1 0],'FontSize',10);
legend('Drawdown','Operating wells')
```

RC_fun:

```
function [RC] = RC_fun(t,delt,dton,Sy,T,xm,ym,xw,yw)
% Initial value
RC=0.;
% Time parameters
frac= t/delt;
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t;
% Distance to wells
r=sqrt((xm-xw)^2+(ym-yw)^2);
for i=1:n
   t1=t-(i-1)*delt;
   t2=t1-dton;
   u_O=(Sy/(4*T))*(r^2/t1); % Operating Well
  % Well Function
   wu_O=expint(u_O);
   uim_O=(Sy/(4*T))*(r^2/t2); % Imaginary Operating Well
   % Well Function
```

```
   wuim_O=expint(uim_O);
  % Total Well Function
   RC=RC+(wu_O-wuim_O)/(4*pi()*T);
end
%
if rest_t > 0 && rest_t <= dton
   % Stream Constant Head and infinite Aquifer Effects
   u_O=(Sy/(4*T))*(r^2/rest_t); % Operating Well
   % Well Function
   wu_O=expint(u_O);
   RC=RC+wu_O/(4*pi()*T);
end
if  rest_t > dton
   t1=rest_t;
   t2=t1-dton;
   % Stream Constant Head and infinite Aquifer Effects
   u_O=(Sy/(4*T))*(r^2/t1); % Operating Well
   % Well Function
   wu_O=expint(u_O);
   % Imaginary Compensation Wells
  uim_O=(Sy/(4*T))*(r^2/t2); % Imaginary Well
   % Well Function
   wuim_O=expint(uim_O);
   RC=RC+(wu_O-wuim_O)/(4*pi()*T);
end
```

APPENDIX B

Matlab code: **Infinite.Drawdown.Time**

This code calculates and plots the head time series for t$\boxed{\in}$$(0,t_{fin})$ for a number of wells operating cyclically or continuously in an aquifer with an infinite areal extension.

The input file **Aquifer.Parms** includes simulation time parameters and aquifer parameters:

- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)

- the aquifer parameters
  - initial hydraulic head (aquifer's saturated thickness) h_0
  - hydraulic conductivity K;
  - storativity S.

The input file **Monitoring.Wells.dat** includes monitoring well data.

- total number of monitoring wells n_{mw}

- for each monitoring well, each of the following lines provides well location coordinates x_w and y_w.

The input file **Wells.dat** includes wells data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - the pumping rate (Q_w);
  - the time at which well operation starts (t_{st}),
  - the total operation period (Δt_{on})
  - well location x_w and y_w.
  -

Examples:

Aquifer.Parms

730.	365.	30.	622.08	0.2
t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Wells.dat

Description:	2 wells operating cyclically			
2 (n_w)				
-500.	0.	180.	500.	1000.
-1000.	60.	120.	1000.	500.
Q_w (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

MonitoringWells.dat

Description:	2 Monitoring wells
2 (n_m)	
490.	990.
900.	400.
x_w (m)	y_w (m)

Infinite.Drawdown.Time

```
clc
clear all
% Reading Data Files
% 1- Parameters File
fid1 = fopen ('parameters.txt','r');
Temp = fscanf(fid1,'%f %f %f %f %f',[1,5]);
tfin = Temp(1); % Final Time of Simulation (Days)
delt = Temp(2); % Cycle Time = One Year (days)
ho   = Temp(3); % Aquifer Initial Heard (M)
K    = Temp(4); % Hydraulic Conductivity (M/day)
Sy   = Temp(5); % Aquifer Specific Yeild (/)
fclose(fid1);
%
% 2- Wells Data file
fid2 = fopen ('wells.txt','r');
Temp    = fscanf(fid2,'%f',[1,1]);
now= Temp(1); % Well Pumping Rate
```

```
 for m=1:now
    Temp    = fscanf(fid2,'%f %f %f %f %f',[1,5]);
    Qw(m)   = Temp(1); % Well Pumping Rate
    tst(m)  = Temp(2); % Pumping Start Time
    dton(m) = Temp(3); % Pumping Period
    xw(m)   = Temp(4); % Well Location X Coordinate
    yw(m)   = Temp(5); % Well Location Y Coordinate
 end
fclose(fid2);
%
% 3- Monitoring Wells data file
fid3 = fopen ('Mon_Wells.txt','r');
Temp = fscanf(fid3,'%f',[1,1]);
nmw   = Temp(1);% Monitoring Wells Number
for mm = 1:nmw
    Temp   = fscanf(fid3,'%f %f',[1,2]);
    xm(mm) = Temp(1);
    ym(mm) = Temp(2);
end
fclose(fid3);
% %
% Calculations
% 1- General
T = K*ho;
to = 0.;
dt = 1;
t = to:dt:tfin; % time matrix
nt = length(t);  % number of time steps(/)
index = (tfin/365)-1;
% 2- Computing Drawdown Distribution for number of operating wells
 fid4 = fopen('results(t).txt','w');
 s_sum = zeros(nt,nmw);% Initial Drawdown
 for i= 1:nt
     for mm = 1:nmw
         for m= 1:now
             tt = t(i)-tst(m);
             if (tt>=0.)
                 s_sum(i,mm)=s_sum(i,mm)-
Qw(m)*RC_fun(tt,delt,dton(m),Sy,T,xm(mm),ym(mm),xw(m),yw(m));
             end
             temp = [t(i), s_sum(i,mm)];
             fprintf(fid4,'%15.6E %15.6E\n',temp);
             fwrite(fid4,temp);
         end
     end
 end
```

```
 fclose(fid4);
 q= zeros(nt,1);
fid5 = fopen ('Q.txt','r');
Temp = fscanf(fid5,'%f',[1,1]);
n= Temp(1);
 for i=1:n
    Temp = fscanf(fid5,'%f',[1,1]);
    q(i,1)= -Temp(1);
 end
fclose(fid5);
%
[s_MAX, iMAX] = max(s_sum);
[s_MIN, iMIN] = min(s_sum);
[q_max, iqMAX] = max(q);
[q_min, iqMIN] = min(q);
% Fiqures
figure
plot(t,s_sum(:,1),'-.r','LineWidth',1.5)
xlabel('Time (day)');
ylabel('Drawdown (m)');
% xlim ([0 tfin]);
% ylim ([s_MIN-0.5 s_MAX+0.5]);
hold on
plot(t,s_sum(:,2),'LineWidth',2.5)
legend('s15','s150')
```

APPENDIX C

Matlab code: **SI.Drawdown.2D**

This code calculates and plots the spatial distribution of head at a specified time t_{fin} for a number of wells operating cyclically or continuously in a semi-infinite aquifer limited by either a stream boundary or a no-flow boundary. The boundary is represented by the $y=y_s$ straight line at $x=0$

The input file **Aquifer.Parms** includes simulation time parameters and aquifer parameters:

- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)

- the aquifer parameters
  - initial hydraulic head (aquifer's saturated thickness) h_0
  - hydraulic conductivity K;
  - storativity S.

The input file **Grid.parms** includes wells data:

  - grid lower left corner abscissa (x_{min})
  - grid lower left corner ordinate (y_{min})
  - grid upper right corner abscissa (x_{max})
  - grid upper right corner ordinate (y_{max})
  - n. of gridblocks along x (n_x)

n. of gridblocks along y (n_y)

The input file **Wells.dat** includes wells data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - the pumping rate (Q_w);
  - the time at which well operation starts (t_{st}),
  - the total operation period (Δt_{on})
  - well location x_w and y_w.

Examples:

Aquifer.Parms

100. or 200.	365.	30.	622.08	0.2

t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Grid.Parms

0.	0.	2000.	2000.	20	20
x_{min} (m)	y_{min} (m)	x_{min} (m)	y_{min} (m)	n_x	n_y

Wells.dat

Description:	2 wells operating cyclically			
2 (n_w)				
-500.	0.	180.	500.	1000.
-1000.	60.	120.	1000.	500.
Q_w (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

SI.Drawdown.2D

```
% Reading Data
% 1- General Data
fid1 = fopen ('Aquifer.Parms','r');
Temp = fscanf(fid1,'%f %f %f %f %f',[1,6]);
tfin = Temp(1); % Final Time of Simulation (day)
delt = Temp(2); % Cycle time step of simulation (day)
ho   = Temp(3); % Aquifer Initial Head (saturated thickness)(m)
K    = Temp(4); % Hydraulic Conductivity (m^2/day)
Sy   = Temp(5); % Storativity (/)
ys   = Temp(6); % stream location (m)
T    = K*ho;    % Transmissivity (m^2/day)
fclose(fid1);
% 2- Reading Grid Data
fid2 = fopen ('Grid.parms','r');
Temp = fscanf(fid2,'%f %f %f %f %f %f',[1,6]);
xmin = Temp(1); % Minimum Value of X in the Grid
ymin = Temp(2); % Minimum Value of Y in the Grid
xmax = Temp(3); % Maximim Value of X in the Grid
ymax = Temp(4); % Maximum Value of Y in the Grid
nx   = Temp(5); % Number of X Divisions
ny   = Temp(6); % Number of Y Divisions
```

```
fclose(fid2);
%
% 3- Wells Data
fid3 = fopen ('Wells.dat','r');
Temp = fscanf(fid3,'%f',[1,1]);
now   = Temp(1);% Wells Number
for m = 1:now
Temp    = fscanf(fid3,'%f %f %f %f %f',[1,5]);
Qw(m)   = Temp(1); % Well Pumping Rate
tst(m)  = Temp(2); % Pumping Start Time
dton(m) = Temp(3); % Pumping Period
xw(m)   = Temp(4); % Well Location X Coordinate
yw(m)   = Temp(5); % Well Location Y Coordinate
end
fclose(fid3);
%
% Creating the Grid %
nxx = nx+1;
nyy = ny+1;
dx = (xmax-xmin)/nx;
dy = (ymax-ymin)/ny;
for j= 1:nxx
    for i= 1:nyy
        x(i,j)= xmin+dx*(j-1);
        y(i,j)= ymin+dy*(i-1);
    end
end
%
% Computing Drawdown Distribution for number of operating wells
 ssum_rech   = zeros(nyy,nxx);% Initial Drawdown
 ssum_noflow = zeros(nyy,nxx);% Initial Drawdown
 for j= 1:nxx;
     for i= 1:nyy;
         for m =1:now;
             t = tfin-tst(m);
             if t>0
ssum_rech(i,j)=ssum_rech(i,j)+CYC_THEIS_RECHARGE(t,delt,Sy,T,ys,x(i,j),y(i,j),dton(
m),xw(m),yw(m),Qw(m));
ssum_noflow(i,j)=ssum_noflow(i,j)+CYC_THEIS_NOFLOW(t,delt,Sy,T,ys,x(i,j),y(i,j),dton
(m),xw(m),yw(m),Qw(m));
             end
         end
     end
 end
fid4 = fopen('results.recharge.dat','w');
fid5 = fopen('results.noflow.dat','w');
```

```
for j= 1:nxx;
   for i= 1:nyy;
      temp = [x(i,j), y(i,j), ssum_rech(i,j)];
      fprintf(fid4,'%15.6E %15.6E %15.6E\n',temp);
      %
      temp = [x(i,j), y(i,j), ssum_noflow(i,j)];
      fprintf(fid5,'%15.6E %15.6E %15.6E\n',temp);
   end
end
fclose(fid4);
fclose(fid5);
% %
str1 = num2str(Qw');
cell1 = cellstr(str1);
% Figure
figure;
contour(x,y,ssum_rech);
[C,h] = contour(x,y,ssum_rech);
clabel(C,h);
title('Drawdown (m) @ ??? days');
xlabel('x (m)');
ylabel('y (m)');
hold on
scatter(xw,yw,50,'r+')
text(xw+20,yw+50,cell1,'BackgroundColor',[1 1 0],'FontSize',10);
legend('Drawdown','Operating wells')
figure;
contour(x,y,ssum_noflow);
[C,h] = contour(x,y,ssum_noflow);
clabel(C,h);
title('Drawdown (m) @ ??? days');
xlabel('x (m)');
ylabel('y (m)');
hold on
scatter(xw,yw,50,'r+')
text(xw+20,yw+50,cell1,'BackgroundColor',[1 1 0],'FontSize',10);
legend('Drawdown','Operating wells')
```

CYC_THEIS_NOFLOW

```
function [dh] = CYC_THEIS_NOFLOW(t,delt,Sy,T,ys,xm,ym,dton,xw,yw,Qw)
dh=0.;
frac= t/delt;
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t+1;
```

```
% Distance to wells
r1=sqrt((xm-xw)^2+(ym-yw)^2);
r2=sqrt((xm-xw)^2+(2*ys-ym-yw)^2);
for i=1:n-1
   t1=t-(i-1)*delt;
   t2=t1-dton;
   % No-Flow Boundary Effect
   u1_O=Sy/(4*T)*(r1^2/t1); % Operating Well
   u1_I=Sy/(4*T)*(r2^2/t1); % Image Well
   % Well Function
   wu1_O=expint(u1_O);
   wu1_I=expint(u1_I);
   % Imaginary Compensation Wells
   u2_O=Sy/(4*T)*(r1^2/t2); % Imaginary Operating Well
   u2_I=Sy/(4*T)*(r2^2/t2); % Imaginary Image Well
   % Well Function
   wu2_O=expint(u2_O);
   wu2_I=expint(u2_I);
  % Total Well Function
   wu1 = wu1_O+wu1_I;
   wu2 = wu2_O+wu2_I;
   % Drawdown
   dh=dh+(Qw/(4*pi()*T))*(wu1-wu2);
end
%
if rest_t > 0 && rest_t <= dton
   % No-Flow Boundary Effect
   u1_O=Sy/(4*T)*(r1^2/rest_t); % Operating Well
   u1_I=Sy/(4*T)*(r2^2/rest_t); % Image Well
   % Well Function
   wu1_O=expint(u1_O);
   wu1_I=expint(u1_I);
   % Total Well Function
   wu1 = wu1_O+wu1_I;
   % Drawdown
   dh=dh+(Qw/(4*pi()*T))*wu1;
end
if rest_t>0 && rest_t > dton
   t1=rest_t;
   t2=t1-dton;
   % No-Flow Boundary Effect
   u1_O=Sy/(4*T)*(r1^2/t1); % Operating Well
   u1_I=Sy/(4*T)*(r2^2/t1); % Image Well
   % Well Function
   wu1_O=expint(u1_O);
   wu1_I=expint(u1_I);
```

```
   % Imaginary Compensation Wells
   u2_O=Sy/(4*T)*(r1^2/t2); % Imaginary Well
   u2_I=Sy/(4*T)*(r2^2/t2); % Imaginary Image Well
   % Well Function
   wu2_O=expint(u2_O);
   wu2_I=expint(u2_I);
   % Total Well Function
   wu1 = wu1_O+wu1_I;
   wu2 = wu2_O+wu2_I;
   % Drawdown
   dh=dh+(Qw/(4*pi()*T))*(wu1-wu2);
end
```

CYC_THEIS_RECHARGE

```
function [dh] = CYC_THEIS_RECHARGE(t,delt,Sy,T,ys,xm,ym,dton,xw,yw,Qw)
dh=0.;
frac= t/delt;
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t+1;
% Distance to wells
r1=sqrt((xm-xw)^2+(ym-yw)^2);
r2=sqrt((xm-xw)^2+(2*ys-ym-yw)^2);
for i=1:n-1
   t1=t-(i-1)*delt;
   t2=t1-dton;
   % Constant Head Boundary Effects
   u1_O=(Sy/(4*T))*(r1^2/t1); % Operating Well
   u1_I=(Sy/(4*T))*(r2^2/t1); % Image Well
   % Well Function
   wu1_O=expint(u1_O);
   wu1_I=expint(u1_I);
   % Imaginary Compensation Wells
   u2_O=Sy/(4*T)*(r1^2/t2); % Imaginary Operating Well
   u2_I=Sy/(4*T)*(r2^2/t2); % Imaginary Image Well
   % Well Function
   wu2_O=expint(u2_O);
   wu2_I=expint(u2_I);
  % Total Well Function
   wu1 = wu1_O-wu1_I;
   wu2 = wu2_O-wu2_I;
  % Drawdown
   dh=dh+(Qw/(4*pi()*T))*(wu1-wu2);
end
%
```

```
if rest_t > 0 && rest_t <= dton
    % Constant Head Boundary Effects
    u1_O=(Sy/(4*T))*(r1^2/rest_t); % Operating Well
    u1_I=(Sy/(4*T))*(r2^2/rest_t); % Image Well
    % Well Function
    wu1_O=expint(u1_O);
    wu1_I=expint(u1_I);
    % Total Well Function
    wu1 = wu1_O-wu1_I;
   % Drawdown
    dh=dh+(Qw/(4*pi()*T))*wu1;
end
if rest_t>0 && rest_t > dton
    t1=rest_t;
    t2=t1-dton;
    % Constant Head Boundary Effects
    u1_O=(Sy/(4*T))*(r1^2/t1); % Operating Well
    u1_I=(Sy/(4*T))*(r2^2/t1); % Image Well
    % Well Function
    wu1_O=expint(u1_O);
    wu1_I=expint(u1_I);
    % Imaginary Compensation Wells
    u2_O=(Sy/(4*T))*(r1^2/t2); % Imaginary Well
    u2_I=(Sy/(4*T))*(r2^2/t2); % Imaginary Image Well
    % Well Function
    wu2_O=expint(u2_O);
    wu2_I=expint(u2_I);
    % Total Well Function
    wu1 = wu1_O-wu1_I;
    wu2 = wu2_O-wu2_I;
   % Drawdown
    dh=dh+(Qw/(4*pi()*T))*(wu1-wu2);
end
```

APPENDIX D

Matlab code: **SI.Drawdown.Time**

This code calculates and plots the head time series for t$\boxed{\in}$$(0,t_{fin})$ for a number of wells operating cyclically or continuously in a semi-infinite aquifer limited by either a stream boundary or a no-flow boundary. The boundary is represented by the y axis (x=0).

The input file **Aquifer.Parms** includes simulation time parameters and aquifer parameters:

- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)

- the aquifer parameters
  - initial hydraulic head (aquifer's saturated thickness) h_0
  - hydraulic conductivity K;
  - storativity S.

The input file **Monitoring.Wells.dat** includes monitoring well data.

- total number of monitoring wells n_{mw}

- for each monitoring well, each of the following lines provides well location coordinates x_w and y_w.

The input file **Wells.dat** includes wells data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - the pumping rate (Q_w);
  - the time at which well operation starts (t_{st}),
  - the total operation period (Δt_{on})
  - well location x_w and y_w.
  -

Examples:

Aquifer.Parms

3650.	365.	30.	622.08	0.2

t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Wells.dat

Description:	2 wells operating cyclically			
2 (n_w)				
-500.	0.	180.	500.	1000.
-1000.	60.	120.	1000.	500.
Q_w (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

MonitoringWells.dat

Description:	2 Monitoring wells
2 (n_m)	
200.	200.
800.	800.
x_w (m)	y_w (m)

SI.Drawdown.Time

```
clc
clear all
% Reading Data Files
% 1- Parameters File
fid1 = fopen ('Aquifer.Parms','r');
Temp = fscanf(fid1,'%f %f %f %f %f',[1,5]);
tfin = Temp(1); % Final Time of Simulation (day)
delt = Temp(2); % Cycle time step of simulation (day)
ho   = Temp(3); % Aquifer Initial Head (saturated thickness)(m)
K    = Temp(4); % Hydraulic Conductivity (m/day)
Sy   = Temp(5); % Aquifer Storativity (/)
fclose(fid1);
%
% 2- Monitoring Wells data file
fid2 = fopen ('Monitoring.Wells.dat','r');
Temp = fscanf(fid2,'%f',[1,1]);
nmw   = Temp(1);% Monitoring Wells Number
```

```
for m = 1:nmw
   Temp  = fscanf(fid2,'%f %f',[1,2]);
   xm(m) = Temp(1);
   ym(m) = Temp(2);
end
fclose(fid2);
% 3- Wells Data file
fid3 = fopen ('Wells.dat','r');
Temp = fscanf(fid3,'%f',[1,1]);
nw   = Temp(1);% Wells Number
for w = 1:nw
   Temp    = fscanf(fid3,'%f %f %f %f %f',[1,5]);
   Qw(w)   = Temp(1); % Well Pumping Rate
   tst(w)  = Temp(2); % Pumping Start Time
   dton(w) = Temp(3); % Pumping Period
   xw(w)   = Temp(4); % Well Location X Coordinate
   yw(w)   = Temp(5); % Well Location Y Coordinate
end
fclose(fid3);
%
% Calculations
% 1- General
T = K*ho;
to = 0;
dt = 1.;
t = to:dt:tfin; % time matrix
nt = length(t);  % number of time steps(/)
%
% 2- Computing Drawdown Distribution for number of operating wells
% Initialize Drawdown Arrays
H_sum = zeros(nt,nmw);
HN_sum = zeros(nt,nmw);
HR_sum = zeros(nt,nmw);
 for i= 1:nt
    for j = 1:nmw
        H_sum(i,j) = ho;
        HN_sum(i,j) = ho;
        HR_sum(i,j) = ho;
        for w = 1:nw
            dt = t(i)-tst(w);
            if (dt>=0.)
                H_sum(i,j)  = H_sum(i,j)
+CYC_THEIS(dt,delt,Sy,T,xm(j),ym(j),dton(w),xw(w),yw(w),Qw(w));
                HN_sum(i,j) =
HN_sum(i,j)+CYC_THEIS_NOFLOW(dt,delt,Sy,T,xm(j),ym(j),dton(w),xw(w),yw(w),Qw(
w));
```

```
            HR_sum(i,j) =
HR_sum(i,j)+CYC_THEIS_RECHARGE(dt,delt,Sy,T,xm(j),ym(j),dton(w),xw(w),yw(w),Q
w(w));
          end
        end
    end
 end
 fid4 = fopen('Drawdown.VS.Time.dat','w');
 fid5 = fopen('Drawdown.VS.Time.NoFlow.dat','w');
 fid6 = fopen('Drawdown.VS.Time.Recharge.dat','w');
 for i= 1:nt
    for j = 1:nmw
        temp = [t(i), H_sum(i,j)];
        fprintf(fid4,'%15.6E %15.6E\n',temp);
        temp = [t(i), HN_sum(i,j)];
        fprintf(fid5,'%15.6E %15.6E\n',temp);
        temp = [t(i), HR_sum(i,j)];
        fprintf(fid6,'%15.6E %15.6E\n',temp);
    end
 end
 fclose(fid4);
 fclose(fid5);
 fclose(fid6);
 %
[H_MAX, iMAX] = max(H_sum);
[H_MIN, iMIN] = min(H_sum);
[HN_MAX, iNMAX] = max(HN_sum);
[HN_MIN, iNMIN] = min(HN_sum);
[HR_MAX, iRMAX] = max(HR_sum);
[HR_MIN, iRMIN] = min(HR_sum);
% Fiqures
figure
plot(t,H_sum(:,1),'-r',t,ho,'.-b')
title('Head at monitoring well location (???,???) Vs Time');
xlabel('Time (day)');
ylabel('Head (m)');
xlim ([0 tfin]);
ylim ([H_MIN-0.01 H_MAX+0.01]);
legend('H,')
figure
plot(t,HN_sum(:,1),'-r',t,ho,'.-b')
title('Head at monitoring well location (Noflow) (???,???) Vs Time');
xlabel('Time (day)');
ylabel('Head (m)');
xlim ([0 tfin]);
ylim ([HN_MIN-0.01 HN_MAX+0.01]);
```

```
legend('H,')
figure
plot(t,HR_sum(:,1),'-r',t,ho,'.-b')
title('Head at monitoring well location (Recharge) (???,???) Vs Time');
xlabel('Time (day)');
ylabel('Head (m)');
xlim ([0 tfin]);
ylim ([HR_MIN-0.01 HR_MAX+0.01]);
legend('H,')
```

APPENDIX E

Matlab code: SI.**Glover**

This code calculates and plots the stream depletion rate, Q_r, and stream depletion volume, V_r, vs Time, t, produced by a generic number of pumping wells operating either continuously or cyclically in a semi-infinite aquifer bounded by a recharge (stream) boundary. The stream is located on the y axis of the reference system (x=0).

The input file **Data.txt** includes simulation time parameters and aquifer parameters:

- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)

- the aquifer parameters
  - aquifer's saturated thickness b;
  - hydraulic conductivity K;
  - storativity S.

The input file **Wells.txt** includes wells data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - the pumping rate (Q_w);
  - the time at which well operation starts (t_{st}),
  - the total operation period (Δt_{on})
  - well location x_w then y_w.
  -

Examples:

Data.txt

3650.	365.	30.	622.08	0.2
t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Wells.dat

Description:	2 wells operating cyclically
2 (n_w)	

-500.	0.	180.	500.	1000.
-1000.	60.	120.	1000.	500.
Q_w (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

MonitoringWells.dat

Description:	2 Monitoring wells
2 (n_m)	
500.	500.
1700.	1700.
x_w (m)	y_w (m)

SI.Glover

```
% Reading Data Files
% 1 - General Data
fid1 = fopen ('Data.txt','r');
Temp = fscanf(fid1,'%f %f %f %f %f',[1,5]);
tfin = Temp(1);
% tfin = Final Time of Simulation (days)
delt = Temp(2);
% delt = dton+dtoff (e.g., 365 days)
h = Temp(3);
% h = Thickness of the Aquifer (m)
K = Temp(4);
% K = Hydraulic Conductivity (m/day)
Sy = Temp(5);
% Sy = Storativity (/)
fclose(fid1);
%
% 2 - Operating Wells Data
fid2 = fopen ('wells.txt','r');
Temp = fscanf( fid2,'%f',[1,1]);
now = Temp(1);
for iw = 1:now
   Temp = fscanf(fid2,'%f %f %f %f %f',[1,5]);
   Qw(iw)  = Temp(1);
   tst(iw) = Temp(2);
   dton(iw)= Temp(3);
```

```
   xw(iw)  = Temp(4);
   yw(iw)  = Temp(5);
end
fclose(fid2);
%
% Calculations
% 1 - General calculations
T = K*h;
to = 0.;
dt = 5.;
t = to:dt:tfin; % time matrix
nt = length(t); % number of time steps(/)
%
% 2 - Operation off period for operating wells
for iw = 1:now
   dtoff(iw)=delt-dton(iw);
end
%
% 3 - Calculating Qr (stream depletion rate)and Vr (stream depletion volume)
Qr_sum = zeros(nt,1);
Vr_sum = zeros(nt,1);
for i= 1:nt
   for iw= 1:now
      tt = t(i)-tst(iw);
      if tt>0.
         Qr_sum(i)=Qr_sum(i)+rate_sol(tt,dton(iw),delt,Qw(iw),T,Sy,xw(iw));
         Vr_sum(i)=Vr_sum(i)+vol_sol(tt,delt,dton(iw),Qw(iw),T,Sy,xw(iw));
      end
   end
end
%
Qr_MAX = max(Qr_sum);
if Qr_MAX>=0.
   Qr_MAX=Qr_MAX+100.;
else
   Qr_MAX=0;
end
Qr_MIN = min(Qr_sum);
if Qr_MIN<=0.
   Qr_MIN=Qr_MIN-100.;
else
   Qr_MIN=0;
end
Vr_MAX = max(Vr_sum);
Vr_MIN = min(Vr_sum);
% Fiqures
```

```
figure
plot(t,Qr_sum,'-r')
%title('Total Stream Depletion Rate Vs Time');
xlabel('Time (day)');
ylabel('Total Qr (m^3/day)');
xlim ([0 tfin+1]);
ylim ([Qr_MIN Qr_MAX]);
%legend('Qr')
%
figure
plot(t,Vr_sum)
%title('Total Stream Depletion Volume Vs Time');
xlabel('Time (day)');
ylabel('Total Vr (m^3)');
xlim ([0 tfin+1]);
ylim ([0 Vr_MAX+1000]);
%legend('Vr')
%
% Output Results
fid3 = fopen('Time.Qr.Vr.dat','w');
for i= 1:nt
   temp = [t(i),Qr_sum(i),Vr_sum(i)];
   fprintf(fid3,'%15.6E %15.6E %15.6E\n',temp);
end
```

Qratio:

```
function QR = Qratio (T,Sy,y,t)
u = (Sy*y^2)/(4*T*t);
QR = erfc(sqrt(u));
```

rate_sol:

```
function [Qr] = rate_sol(t,dton,delt,Q,T,Sy,xw)
% Calculating Qr (stream depletion rate)
Qr = 0.;
frac=t/delt;
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t+1;
for i=1:n-1
   t1=t-(i-1)*delt;
   t2=t1-dton;
   Qr =Qr+Q*(Qratio(T,Sy,xw,t1)-Qratio(T,Sy,xw,t2));
end
if rest_t >0 && rest_t <= dton
   Qr =Qr+Q*Qratio(T,Sy,xw,rest_t);
end
```

```
if rest_t > dton
   Qr =Qr+Q*(Qratio(T,Sy,xw,rest_t)-Qratio(T,Sy,xw,rest_t-dton));
end
```

Vratio:

```
function VR = Vratio(T,S,y,t)
u = (S*y^2)/(4*T*t);
p = (S*y^2)/(2*T*t);
VR = (1+p)*erfc(sqrt(u))- (sqrt(u)*2/sqrt(pi()))*(exp(-u));
```

vol_sol:

```
function [Vr] = vol_sol(t,delt,dton,Q,T,Sy,xw)
% Calculating Vr (depletion volume)
Vr = 0.;
frac=(t)/delt;
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t+1;
for i=1:n-1
   t1=t-(i-1)*delt;
   t2=t1-dton;
   Vr =Vr+Q*(t1* Vratio(T,Sy,xw,t1)-t2*Vratio(T,Sy,xw,t2));
end
if rest_t >0 && rest_t <= dton
   Vr=Vr+Q*rest_t*Vratio(T,Sy,xw,rest_t);
end
if rest_t > dton
   t1=rest_t;
   t2=t1-dton;
   Vr=Vr+Q*(t1*Vratio(T,Sy,xw,t1)-t2*Vratio(T,Sy,xw,t2));
end
```

APPENDIX F

Matlab code: **Finite.Drawdown.2D**

This code calculates and plots the spatial distribution of head at a specified time t_{fin} for a number of wells operating cyclically or continuously in a finite aquifer comprised between either a no-flow boundary and a stream or by two recharge boundaries

The input file **Aquifer.Parms** includes simulation time parameters and aquifer parameters:

- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)

- the aquifer parameters
  - initial hydraulic head (aquifer's saturated thickness) h_0
  - hydraulic conductivity K;
  - storativity S.
  - location of the no-flow or recharge boundary
  - location of the stream

The input file **Grid.parms** includes wells data:

- grid lower left corner abscissa (x_{min})
- grid lower left corner ordinate (y_{min})
- grid upper right corner abscissa (x_{max})
- grid upper right corner ordinate (y_{max})
- n. of gridblocks along x (n_x)

n. of gridblocks along y (n_y)

The input file **Wells.dat** includes wells data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - the pumping rate (Q_w);
  - the time at which well operation starts (t_{st}),
  - the total operation period (Δt_{on})
  - well location x_w and y_w.

Examples:

Aquifer.Parms

100. or 200.	365.	30.	622.08	0.2
t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Grid.Parms

0.	0.	2000.	2000.	20	20
x_{min} (m)	y_{min} (m)	x_{min} (m)	y_{min} (m)	n_x	n_y

Wells.dat

Description:	2 wells operating cyclically			
2 (n_w)				
-500.	0.	180.	500.	1000.
-1000.	60.	120.	1000.	500.
Q_w (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

Finite.Drawdown.2D

```
clc
clear all
% Reading Data File
% 1-Aquifer Parameter
fid1 = fopen ('Aqui_Param.txt','r');
Temp = fscanf(fid1,'%f %f %f %f %f',[1,5]);
ho        = Temp(1); % Aquifer Thickness (M)
K         = Temp(2); % Hydraulic Conductivity (M^2/day)
Sy        = Temp(3); % Apparent Specific Yeild (/)
x_boundary = Temp(4); % Stream Location (M)
x_stream   = Temp(5); % Boundary Location (M)
fclose(fid1);
%
% 2-Wells Data file
fid2 = fopen ('Wells.txt','r');
Temp = fscanf( fid2,'%f',[1,3]);
tfin = Temp(1); % Final Time of Simulation (Days)
delt = Temp(2); % Cycle Time = One Year (days)
now = Temp(3);   % Wells Number
for m = 1:now
```

```
Temp = fscanf( fid2,'%f %f %f %f',[1,5]);
Qw(m)   = Temp(1);  % Well Pumping Rate (M^3/Day)
tst(m)  = Temp(2);  % Well Location X Coordinate
dton(m) = Temp(3);  % Well Location Y Coordinate
xw(m)   = Temp(4);  % Well Operation Starting Time  (Days)
yw(m)   = Temp(5);  % Pumping Period (Days)
end
fclose(fid2);
%
% 3-Grid Data file
fid3 = fopen ('Grid_Data.txt','r');
Temp = fscanf(fid3,'%f %f %f %f %f %f',[1,6]);
xmin = Temp(1); % Minimum Value of X in the Grid
ymin = Temp(2); % Minimum Value of Y in the Grid
xmax = Temp(3); % Maximim Value of X in the Grid
ymax = Temp(4); % Maximum Value of Y in the Grid
nx = Temp(5);   % Number of X Divisions
ny = Temp(6);   % Number of Y Divisions
fclose(fid3);
%
% Creating the Grid
nxx = nx+1;
nyy = ny+1;
dx  = (xmax-xmin)/nx;
dy  = (ymax-ymin)/ny;
%
for i= 1:nyy
   for  j= 1:nxx
      x(i,j)= xmin+ dx*(j-1);
   end
end
%
for  j= 1:nxx
   for i= 1:nyy
     y(i,j)= ymin+dy*(i-1);
   end
end
%
% Calculations
% 1-General
w = x_stream - x_boundary; % Aquifer Width (M)
T = K*ho; % Transmissivity
% 2-Computing Drawdown Distribution for a number Operating Wells
 fid4 = fopen('results(RECH).txt','w');
 ssum_rech = zeros(nyy,nxx);% Initial Drawdown
 fid5 = fopen('results(NOFLOW).txt','w');
```

```
 ssum_noflow = zeros(nyy,nxx);% Initial Drawdown
 for i= 1:nyy;
    for j= 1:nxx;
        for m =1:now
            a = x_stream -xw(m);
            t = tfin-tst(m);
            if t>0
               ssum_noflow(i,j)=ssum_noflow(i,j)-
Qw(m)*BRC_NOFLOW(t,delt,dton(m),Sy,T,w,a,yw(m),x(i,j),y(i,j));
                ssum_rech(i,j)  =ssum_rech(i,j)-
Qw(m)*BRC_RECH(t,delt,dton(m),Sy,T,w,a,yw(m),x(i,j),y(i,j));
                % B_CYC_THEIS calculates the drawdown for a number of cyclically operating
well
                % and all of their image and imaginary wells for a Bounded Aquifer
            end
        end
        temp = [x(i,j), y(i,j), ssum_rech(i,j)];
        fprintf(fid4,'%15.6E %15.6E %15.6E\n',temp);
        temp = [x(i,j), y(i,j), ssum_noflow(i,j)];
        fprintf(fid5,'%15.6E %15.6E %15.6E\n',temp);
    end
 end
fclose(fid4);
fclose(fid5);
% %
str1 = num2str(Qw');
cell1 = cellstr(str1);
% Figure
figure;
contour(x,y,ssum_noflow);
[C,h] = contour(x,y,ssum_noflow);
clabel(C,h);
% title('Drawdown in a Bounded aquifer Between a Recharge and a No-flow Boundaries
M');
xlabel('x (m)');
ylabel('y (m)');
axis square;
hold on
scatter(xw,yw,50,'r+')
text(xw+20,yw+50,cell1,'BackgroundColor',[1 1 0],'FontSize',10);
legend('Drawdown (NoFlow)','Operating wells')
figure;
contour(x,y,ssum_rech);
[C,h] = contour(x,y,ssum_rech);
clabel(C,h);
% title('Drawdown in a Bounded Aquifer Between Two Recharge Boundaries M');
```

```
xlabel('x (m)');
ylabel('y (m)');
axis square;
hold on
scatter(xw,yw,50,'r+')
text(xw+20,yw+50,cell1,'BackgroundColor',[1 1 0],'FontSize',10);
legend('Drawdown (Recharge)  ','Operating wells')
```

BRC_NOFLOW

```
function [BRC] = BRC_NOFLOW(t,delt,dton,Sy,T,w,a,yw,x,y)
BRC=0.; % Initial Drawdown in a Bounded Aquifer
TOL=1.e-9; % Tolerance value at which the loop stops
DELTA=1.0; % Starting value of DELTA
% Loop over the number of Wells Groups
j=1; % (While) Loop Counter
while abs(DELTA)>= TOL && j<=30
   sign = (-1)^(j+1);
   coff = 1/(4*pi()*T);
   DELTA = sign*coff* Four_Wells(t,delt,dton,Sy,T,w,a,yw,x,y,j);
   BRC = BRC+DELTA;
   j=j+1;
end
%
function [W_Fun]= Four_Wells(t,delt,dton,Sy,T,w,a,yw,x,y,j)
  W_Fun= 0.;
% Wells x Coordinate
xw1 = (2*j-1)*w-a;
xw2 = (2*j-1)*w+a;
% %
% Wells Distance From Observation Well
r1= sqrt((x-xw1)^2+(y-yw)^2);
r2= sqrt((x-xw2)^2+(y-yw)^2);
% Image Wells
rI1= sqrt((x+xw1)^2+(y-yw)^2);
rI2= sqrt((x+xw2)^2+(y-yw)^2);
%
% Cycle Effect
frac =t/delt;
int_t =fix(frac);
rest_t =t-int_t*delt;
n =int_t+1; % Number of complete Operation Cycles
% Loop over the number of Operation Cycles
for i=1:n-1
   % For a Number of Complete Cycles
   t1=t-(i-1)*delt;
```

```
   t2=t1-dton;
   % Stream Constant Head and infinite Aquifer Effects
   u1=(Sy/(4*T))*(r1^2/t1);
   u2=(Sy/(4*T))*(r2^2/t1);
   % Image Wells
   uI1=(Sy/(4*T))*(rI1^2/t1);
   uI2=(Sy/(4*T))*(rI2^2/t1);
   % Well Function
   wu1=expint(u1);
   wu2=expint(u2);
   % Image Well Function
   wuI1=expint(uI1);
   wuI2=expint(uI2);
   % Operating And Image Wells
   wo = wu1-wu2+wuI1-wuI2;
   % Imaginary Compensation Wells (for the continuous pumping)
   u1_im=(Sy/(4*T))*(r1^2/t2);
   u2_im=(Sy/(4*T))*(r2^2/t2);
   % Image Wells
   uI1_im=(Sy/(4*T))*(rI1^2/t2);
   uI2_im=(Sy/(4*T))*(rI2^2/t2);
   % Well Function
   wu1_I=expint(u1_im);
   wu2_I=expint(u2_im);
   % Image Well Function
   wuI1_im=expint(uI1_im);
   wuI2_im=expint(uI2_im);
   % Imaginary And Image Wells
   wim = wu1_I-wu2_I+wuI1_im-wuI2_im;
   % Total Well Function
   W_Fun =W_Fun+(wo-wim);
end
%
if rest_t>0 && rest_t <= dton % During Operation Time
   % Stream Constant Head and Infinite Aquifer Effects
   u1=(Sy/(4*T))*(r1^2/rest_t);
   u2=(Sy/(4*T))*(r2^2/rest_t);
   % Image Wells
   uI1=(Sy/(4*T))*(rI1^2/rest_t);
   uI2=(Sy/(4*T))*(rI2^2/rest_t);
   % Well Function
   wu1=expint(u1);
   wu2=expint(u2);
   % Image Well Function
   wuI1=expint(uI1);
   wuI2=expint(uI2);
```

```
    % Operating And Image Wells
    W_Fun = W_Fun+(wu1-wu2+wuI1-wuI2);
end
if rest_t > dton  % During time when operation stops
    t1=rest_t;
    t2=t1-dton;
    % Stream Constant Head and infinite Aquifer Effects
    u1=(Sy/(4*T))*(r1^2/t1);
    u2=(Sy/(4*T))*(r2^2/t1);
    % Image Wells
    uI1=(Sy/(4*T))*(rI1^2/t1);
    uI2=(Sy/(4*T))*(rI2^2/t1);
    % Well Function
    wu1=expint(u1);
    wu2=expint(u2);
    % Image Well Function
    wuI1=expint(uI1);
    wuI2=expint(uI2);
    % Operating And Image Wells
    wo = wu1-wu2+wuI1-wuI2;
    % Imaginary Compensation Wells (for continous pumping)
    u1_im=(Sy/(4*T))*(r1^2/t2);
    u2_I=(Sy/(4*T))*(r2^2/t2);
    % Image Wells
    uI1_im=(Sy/(4*T))*(rI1^2/t2);
    uI2_im=(Sy/(4*T))*(rI2^2/t2);
    % Well Function
    wu1_I=expint(u1_im);
    wu2_I=expint(u2_I);
    % Image Well Function
    wuI1_im=expint(uI1_im);
    wuI2_im=expint(uI2_im);
    % Imaginary And Image Wells
    wim = wu1_I-wu2_I+wuI1_im-wuI2_im;
    % Total Well Function
    W_Fun =W_Fun+(wo-wim);
end
```

BRC_RECH

```
function [Bs] = BRC_RECH(t,delt,dton,Sy,T,w,a,yw,x,y)
Bs=0.; % Initial Drawdown in a Bounded Aquifer
TOL=1.e-9; % Tolerance value at which the loop stops
DELTA=1.0; % Starting value of DELTA
% Loop over the number of Wells Groups
j=1; % (While) Loop Counter
```

```
while abs(DELTA)>= TOL && j<=30
   coff = 1/(4*pi()*T);
   DELTA = coff* Four_Wells(t,delt,dton,Sy,T,w,a,yw,x,y,j);
   Bs = Bs+DELTA;
   j=j+1;
end
%
function [W_Fun]= Four_Wells(t,delt,dton,Sy,T,w,a,yw,x,y,j)
  W_Fun= 0.;
% Wells x Coordinate
xw1 = (2*j-1)*w-a;
xw2 = (2*j-1)*w+a;
% %
% Wells Distance From Observation Well
r1= sqrt((x-xw1)^2+(y-yw)^2);
r2= sqrt((x-xw2)^2+(y-yw)^2);
% Image Wells
rI1= sqrt((x+xw1)^2+(y-yw)^2);
rI2= sqrt((x+xw2)^2+(y-yw)^2);
% Cycle Effect
frac =t/delt;
int_t =fix(frac);
rest_t =t-int_t*delt;
n =int_t+1; % Number of complete Operation Cycles
% Loop over the number of Operation Cycles
for i=1:n-1
   % For a Number of Complete Cycles
   t1=t-(i-1)*delt;
   t2=t1-dton;
   % Stream Constant Head and infinite Aquifer Effects
   u1=(Sy/(4*T))*(r1^2/t1);
   u2=(Sy/(4*T))*(r2^2/t1);
   % Image Wells
   uI1=(Sy/(4*T))*(rI1^2/t1);
   uI2=(Sy/(4*T))*(rI2^2/t1);
   % Well Function
   wu1=expint(u1);
   wu2=expint(u2);
   % Image Well Function
   wuI1=expint(uI1);
   wuI2=expint(uI2);
   % Operating And Image Wells
   wo = wu1-wu2-wuI1+wuI2;
   % Imaginary Compensation Wells (for the continuous pumping)
   u1_im=(Sy/(4*T))*(r1^2/t2);
   u2_im=(Sy/(4*T))*(r2^2/t2);
```

```
   % Image Wells
   uI1_im=(Sy/(4*T))*(rI1^2/t2);
   uI2_im=(Sy/(4*T))*(rI2^2/t2);
   % Well Function
   wu1_im=expint(u1_im);
   wu2_im=expint(u2_im);
   % Image Well Function
   wuI1_im=expint(uI1_im);
   wuI2_im=expint(uI2_im);
   % Imaginary And Image Wells
   wim = wu1_im-wu2_im-wuI1_im+wuI2_im;
   % Total Well Function
   W_Fun =W_Fun+(wo-wim);
end
%
if rest_t>0 && rest_t <= dton % During Operation Time
   % Stream Constant Head and Infinite Aquifer Effects
   u1=(Sy/(4*T))*(r1^2/rest_t);
   u2=(Sy/(4*T))*(r2^2/rest_t);
   % Image Wells
   uI1=(Sy/(4*T))*(rI1^2/rest_t);
   uI2=(Sy/(4*T))*(rI2^2/rest_t);
   % Well Function
   wu1=expint(u1);
   wu2=expint(u2);
   % Image Well Function
   wuI1=expint(uI1);
   wuI2=expint(uI2);
   % Operating And Image Wells
   W_Fun = W_Fun+(wu1-wu2-wuI1+wuI2);
end
if rest_t > dton  % During time when operation stops
   t1=rest_t;
   t2=t1-dton;
   % Stream Constant Head and infinite Aquifer Effects
   u1=(Sy/(4*T))*(r1^2/t1);
   u2=(Sy/(4*T))*(r2^2/t1);
   % Image Wells
   uI1=(Sy/(4*T))*(rI1^2/t1);
   uI2=(Sy/(4*T))*(rI2^2/t1);
   % Well Function
   wu1=expint(u1);
   wu2=expint(u2);
   % Image Well Function
   wuI1=expint(uI1);
   wuI2=expint(uI2);
```

```
    % Operating And Image Wells
    wo = wu1-wu2-wuI1+wuI2;
    % Imaginary Compensation Wells (for continous pumping)
    u1_im=(Sy/(4*T))*(r1^2/t2);
    u2_im=(Sy/(4*T))*(r2^2/t2);
    % Image Wells
    uI1_im=(Sy/(4*T))*(rI1^2/t2);
    uI2_im=(Sy/(4*T))*(rI2^2/t2);
    % Well Function
    wu1_im=expint(u1_im);
    wu2_im=expint(u2_im);
    % Image Well Function
    wuI1_im=expint(uI1_im);
    wuI2_im=expint(uI2_im);
    % Imaginary And Image Wells
    wim = wu1_im-wu2_im-wuI1_im+wuI2_im;
    % Total Well Function
    W_Fun =W_Fun+(wo-wim);
end
```

APPENDIX G

Matlab code: **Finite.Drawdown.Time**

This code calculates and plots the head time series for $t \in (0, t_{fin})$ for a number of wells operating cyclically or continuously in a finite aquifer comprised between either a no-flow boundary and a stream or by two recharge boundaries

The input file **Aquifer.Parms** includes simulation time parameters and aquifer parameters:

- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)

- the aquifer parameters
  - initial hydraulic head (aquifer's saturated thickness) h_0
  - hydraulic conductivity K;
  - storativity S.
  - location of the no-flow or recharge boundary
  - location of the stream

The input file **Monitoring.Wells.dat** includes monitoring well data.

- total number of monitoring wells n_{mw}

- for each monitoring well, each of the following lines provides well location coordinates x_w and y_w.

The input file **Wells.dat** includes wells data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - the pumping rate (Q_w);
  - the time at which well operation starts (t_{st}),
  - the total operation period (Δt_{on})
  - well location x_w and y_w.
  -

Examples:

Aquifer.Parms

100. or 200	365.	30.	622.04	0.2

t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Grid.Parms

0.	0.	2000.	2000.	20	20
x_{min} (m)	y_{min} (m)	x_{min} (m)	y_{min} (m)	n_x	n_y

Wells.dat

Description:	2 wells operating cyclically			
2 (n_w)				
-500.	0.	180.	500.	1000.
-1000.	60.	120.	1000.	500.
Q_w (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

Finite.Drawdown.Time

```
clc
clear all
% Reading Data Files
% 1- Parameters File
fid1 = fopen ('Aqui_Param.txt','r');
Temp = fscanf(fid1,'%f %f %f %f %f',[1,5]);
ho        = Temp(1); % Aquifer Initial Heard (M)
K         = Temp(2); % Hydraulic Conductivity (M/day)
Sy        = Temp(3); % Apparent Specific Yeild (/)
x_boundary = Temp(4); % Stream Location (M)
x_stream   = Temp(5); % Boundary Location (M)
fclose(fid1);
%
% 2- Wells Data file
fid2 = fopen ('Wells.txt','r');
Temp = fscanf( fid2,'%f',[1,3]);
tfin = Temp(1); % Final Time of Simulation (Days)
delt = Temp(2); % Cycle Time = One Year (days)
now = Temp(3);   % Wells Number
for m = 1:now
   Temp    = fscanf(fid2,'%f %f %f %f %f',[1,5]);
   Qw(m)   = Temp(1); % Well Pumping Rate
   tst(m)  = Temp(2); % Pumping Start Time
```

```
   dton(m) = Temp(3); % Pumping Period
   xw(m)   = Temp(4); % Well Location X Coordinate
   yw(m)   = Temp(5); % Well Location Y Coordinate
end
fclose(fid2);
%
% 3- Monitoring Wells data file
fid3 = fopen ('Mon_Wells.txt','r');
Temp = fscanf(fid3,'%f',[1,1]);
nmw   = Temp(1);% Monitoring Wells Number
for mm = 1:nmw
   Temp   = fscanf(fid3,'%f %f',[1,2]);
   xm(mm) = Temp(1);
   ym(mm) = Temp(2);
end
fclose(fid3);
% %
% Calculations
% 1- General
w = x_stream - x_boundary; % Aquifer width
T = K*ho;
to = 0.01;
dt = 1;
t = to:dt:tfin; % time matrix
nt = length(t);  % number of time steps(/)
% 2- Computing Drawdown Distribution for number of operating wells
 fid4 = fopen('results(Nt).txt','w');
 fid5 = fopen('results(Rt).txt','w');
 % Initial Drawdown
 BHN_sum = zeros(nt,nmw);
 BHR_sum = zeros(nt,nmw);
 for i= 1:nt
    for j = 1:nmw
%         BHN_sum(i,j) = ho;
%         BHR_sum(i,j) = ho;
       for m = 1:now
          a = x_stream -xw(m);
          tt = t(i)-tst(m);
          if (tt>=0.)
             BHN_sum(i,j) = BHN_sum(i,j)-
Qw(m)*BRC_NOFLOW(tt,delt,dton(m),Sy,T,w,a,yw(m),xm(j),ym(j));
             BHR_sum(i,j) = BHR_sum(i,j)-
Qw(m)*BRC_RECH(tt,delt,dton(m),Sy,T,w,a,yw(m),xm(j),ym(j));
          end
       temp = [t(i), xm(j),ym(j), BHN_sum(i,j)];
      fprintf(fid4,'%15.6E %15.6E %15.6E %15.6E\n',temp);
```

```
        temp = [t(i), xm(j),ym(j), BHR_sum(i,j)];
        fprintf(fid5,'%15.6E %15.6E %15.6E %15.6E\n',temp);
        end
    end
 end
 fclose(fid4);
 fclose(fid5);
%
[HN_MAX, iNMAX] = max(BHN_sum);
[HN_MIN, iNMIN] = min(BHN_sum);
% Fiqures
figure
plot(t,BHN_sum(:,1),'-.r')
% title('Head at monitoring well location during the simulation time Vs Time');
xlabel('Time (day)');
ylabel('Drawdown (N) (m)');
% xlim ([0 tfin+10]);
ylim ([0 0.3]);
hold on
plot(t,BHN_sum(:,2),'-b')
legend('HN500','HN1700')
[HR_MAX, iRMAX] = max(BHR_sum);
[HR_MIN, iRMIN] = min(BHR_sum);
% Fiqures
figure
plot(t,BHR_sum(:,1),'-.r')
% title('Head at monitoring well location during the simulation time Vs Time');
xlabel('Time (day)');
ylabel('Drawdown (R) (m)');
% xlim ([0 tfin+10]);
ylim ([0 0.3]);
hold on
plot(t,BHR_sum(:,2),'-b')
legend('HR500','HR1700')
```

APPENDIX H

Matlab code: **Finite.Glover**

This code calculates and plots the stream depletion rate, Q_r, and stream depletion volume, V_r, vs. Time t for a number of operating wells operating either cyclically or continuously in a bounded aquifer comprised between a recharge stream and another boundary, which can be either constant-head or impermeable. The stream is parallel to on the x axis at $y=y_{stream.}$ The second boundary is located at $y=y_{boundary}< y_{stream}$.

The input file **Data.txt** includes simulation time parameters and aquifer parameters:

- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)

- the aquifer parameters
  - aquifer's saturated thickness b;
  - hydraulic conductivity K;
  - storativity S;
  - aquifer boundary coordinates $y_{boundary}$ and y_{stream}.

The input file **Wells.txt** includes wells data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - the pumping rate (Q_w);
  - the time at which well operation starts (t_{st}),
  - the total operation period (Δt_{on})
  - well location x_w and y_w.
  -

Examples:

Data.txt

3650.	365.	30.	622.04	0.2	0.	2000.
t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)	$y_{boundary}$	$y_{boundary}$

Wells.dat

Description:	2 wells operating cyclically

2 (n_w)				
-500.	0.	180.	500.	1000.
-1000.	60.	120.	1000.	500.
Q_w (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

MonitoringWells.dat

Description:	2 Monitoring wells
2 (n_m)	
490.	990.
990.	490.
x_w (m)	y_w (m)

Finite.Glover

```
% Reading  data
% 1 - General data
fid1 = fopen ('Data.txt','r');
Temp = fscanf(fid1,'%f %f %f %f %f %f %f',[1,7]);
% tfin = Final Time of Simulation (day)
tfin = Temp(1);
% delt = dton+dtoff [e.g. 365](day)
delt = Temp(2);
%  b = Thickness of the Aquifer (m)
b = Temp(3);
%  K = Hydraulic Conductivity (m/Day)
K = Temp(4);
%  Sy = storativity (/)
Sy = Temp(5);
% Boundary Location (m)
y_boundary = Temp(6);
% Stream Location (m)
y_stream = Temp(7);
fclose(fid1);
%
% 2 - Well Field data
fid2 = fopen ('Wells.dat','r');
Temp = fscanf( fid2,'%f',[1,1]);
now = Temp(1);
```

```
for ow = 1:now
   Temp = fscanf(fid2,'%f %f %f %f %f',[1,5]);
   Qw(ow)  = Temp(1);
   tst(ow) = Temp(2);
   dton(ow)= Temp(3);
   xw(ow)  = Temp(4);
   yw(ow)  = Temp(5) ;
end
fclose(fid2);
%
% General calculations
T = K*b;
to = 0;
w = y_stream-y_boundary;
index = (tfin/delt)-1;
dt = 1;
t = to:dt:tfin; % time matrix
nt = length(t);  % number of time steps(/)
%
% Off time for operating wells
for ow = 1:now
   dtoff(ow) = delt-dton(ow);
end
%
% Calculating stream depletion rate (Qr)and stream depletion volume (Vr)
Qr_Nsum = zeros(nt,1);
Vr_Nsum = zeros(nt,1);
Qr_Rsum = zeros(nt,1);
Vr_Rsum = zeros(nt,1);
%
for i= 1:nt
   for iw= 1:now
      a = y_stream -yw(iw);
      if (t(i)-tst(iw)>=0.)
         %
         Qr_Nsum(i)=Qr_Nsum(i)+ BNrate_sol_CYC(t(i)-
tst(iw),dton(iw),delt,Qw(iw),T,Sy,a,w);
         Vr_Nsum(i)=Vr_Nsum(i)+ BNvol_sol_CYC(t(i)-
tst(iw),delt,dton(iw),Qw(iw),T,Sy,a,w);
         %
         Qr_Rsum(i,1)=Qr_Rsum(i)+ BRrate_sol_CYC(t(i)-
tst(iw),dton(iw),delt,Qw(iw),T,Sy,a,w);
         Vr_Rsum(i,1)=Vr_Rsum(i)+ BRvol_sol_CYC(t(i)-
tst(iw),delt,dton(iw),Qw(iw),T,Sy,a,w);
         %
      end
```

```
   end
end
%
Qr_NMAX = max(Qr_Nsum);
Qr_NMIN = min(Qr_Nsum);
if Qr_NMAX>0.
   Qr_NMAX=Qr_NMAX+100.;
else
   Qr_NMAX=0;
end
if Qr_NMIN<0.
   Qr_NMIN=Qr_NMIN-100.;
else
   Qr_NMIN=0;
end
Vr_NMAX = max(Vr_Nsum);
Vr_NMIN = min(Vr_Nsum);
%
% Fiqures
figure
plot(t,Qr_Nsum,'-r')
title('Total Stream Depletion Rate (No-Flow) Vs Time');
xlabel('Time (day)');
ylabel('Total Q_r (m^3/days)');
xlim ([0 tfin]);
ylim ([Qr_NMIN Qr_NMAX]);
%legend('Qr')
%
figure
plot(t,Vr_Nsum)
title('Total Stream Depletion Volume (No-flow) Vs Time');
xlabel('Time (day)');
ylabel('Total V_r (m^3)');
xlim ([0 tfin]);
ylim ([Vr_NMIN Vr_NMAX]);
%legend('Vr')
%
Qr_RMAX = max(Qr_Rsum);
Qr_RMIN = min(Qr_Rsum);
if Qr_RMAX>0.
   Qr_RMAX=Qr_RMAX+100.;
else
   Qr_RMAX=0;
end
if Qr_RMIN<0.
   Qr_RMIN=Qr_RMIN-100.;
```

```
else
   Qr_RMIN=0.;
end
Vr_RMAX = max(Vr_Rsum);
Vr_RMIN = min(Vr_Rsum);
% Fiqures
figure
plot(t,Qr_Rsum,'-r')
title('Total Stream Depletion Rate (Recharge) Vs Time');
xlabel('Time (day)');
ylabel('Total Q_r (m^3/days)');
xlim ([0 tfin]);
ylim ([Qr_RMIN Qr_RMAX]);
%legend('Qr')
%
figure
plot(t,Vr_Rsum)
title('Total Stream Depletion Volume (Recharge) Vs Time');
xlabel('Time (day)');
ylabel('Total V_r (m^3)');
xlim ([0 tfin]);
ylim ([Vr_RMIN Vr_RMAX]);
%legend('Vr')
%
% Output Results
fid3 = fopen('Time.Qr.Vr.dat','w');
for i= 1:nt
   temp = [t(i),Qr_Nsum(i),Vr_Nsum(i),Qr_Rsum(i),Vr_Rsum(i)];
   fprintf(fid3,'%15.6E %15.6E %15.6E %15.6E %15.6E\n',temp);
end
```

BRQRatio

```
function BQRR=BRQRatio(T,S,a,w,t)
BQRR = erfc(sqrt(S/4/T*a^2/t))+FQRatio(T,S,a,w,t);
function FQR=FQRatio(T,S,a,w,t)
TOL=1.e-9; i=1; DELTA=1.0; FQR=0;
while abs(DELTA) >=TOL && i<=50
   DELTA=A(T,S,a,w,t,i);
   FQR=FQR+DELTA;
   i=i+1;
end
function Ai=A(T,S,a,w,t,i)
```

```
Ai=-erfc(sqrt(S/4/T*(2*w*i-a)^2/t))+erfc(sqrt(S/4/T*(2*w*i+a)^2/t));
```

BRrate_sol_CYC:

```
function [Qr] = BRrate_sol_CYC(t,dton,delt,Q,T,S,a,w)
% Calculating Qr (stream depletion rate)
Qr = 0.;
frac= t/delt;
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t+1;
for i=1:n-1
   t1=t-(i-1)*delt;
   t2=t1-dton;
   Qr =Qr+Q*(BRQRatio(T,S,a,w,t1)-BRQRatio(T,S,a,w,t2));
end
if rest_t>0 && rest_t <= dton
   Qr =Qr+Q*BRQRatio(T,S,a,w,rest_t) ;
end
if  rest_t>0 && rest_t > dton
   t1=rest_t;
   t2=t1-dton;
   Qr =Qr+Q*(BRQRatio(T,S,a,w,t1)-BRQRatio(T,S,a,w,t2));
end
```

BNQRatio

```
function BQRR=BNQRatio(T,S,a,w,t)
BQRR = erfc(sqrt(S/4/T*a^2/t))+FQRatio(T,S,a,w,t);
function FQR=FQRatio(T,S,a,w,t)
TOL=1.e-9; i=1; DELTA=1.0; FQR=0;
while abs(DELTA) >=TOL && i<=30
   DELTA=(-1)^(i+1)*A(T,S,a,w,t,i);
   FQR=FQR+DELTA;
   i=i+1;
end
function Ai=A(T,S,a,w,t,i)
Ai=erfc(sqrt(S/4/T*(2*w*i-a)^2/t))-erfc(sqrt(S/4/T*(2*w*i+a)^2/t));
```

BNrate_sol_CYC:

```
function [Qr] = BNrate_sol_CYC(t,dton,delt,Q,T,S,a,w)
% Calculating Qr (stream depletion rate)
Qr = 0.;
frac= t/delt;
```

```
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t+1;
for i=1:n-1
   t1=t-(i-1)*delt;
   t2=t1-dton;
   Qr =Qr+Q*(BNQRatio(T,S,a,w,t1)-BNQRatio(T,S,a,w,t2));
end
if rest_t>0 && rest_t <= dton
   Qr =Qr+Q*BNQRatio(T,S,a,w,rest_t) ;
end
if  rest_t>0 && rest_t > dton
   t1=rest_t;
   t2=t1-dton;
   Qr =Qr+Q*(BNQRatio(T,S,a,w,t1)-BNQRatio(T,S,a,w,t2));
end
BRVRatio
function BGVR=BRVRatio(T,S,a,w,t)
TOL=1.e-9; i=0; DELTA=1.0; BGVR=0;
while abs(DELTA) >=TOL && i<=30
   DELTA=C(T,S,a,w,t,i)-D(T,S,a,w,t,i);
   BGVR=BGVR+DELTA;
   i=i+1;
end
function Ci=C(T,S,a,w,t,i)
Ci=(1+S/2/T*(2*w*i+a)^2/t)*erfc(sqrt(S/4/T*(2*w*i+a)^2/t))-
2/sqrt(pi())*sqrt(S/4/T*(2*w*i+a)^2/t)*exp(-S/4/T*(2*w*i+a)^2/t);
function Di=D(T,S,a,w,t,i)
Di=(1+S/2/T*(2*w*(i+1)-a)^2/t)*erfc(sqrt(S/4/T*(2*w*(i+1)-a)^2/t))-
2/sqrt(pi())*sqrt(S/4/T*(2*w*(i+1)-a)^2/t)*exp(-S/4/T*(2*w*(i+1)-a)^2/t);
```

BRvol_sol_CYC

```
function [Vr] = BRvol_sol_CYC(t,delt,dton,Q,T,S,a,w)
% Calculating Vr (depletion volume)
Vr = 0.;
frac=t/delt;
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t+1;
for i=1:n-1
   t1=t-(i-1)*delt;
   t2=t1-dton;
   Vr =Vr+Q*(t1*BRVRatio(T,S,a,w,t1)-t2*BRVRatio(T,S,a,w,t2));
end
if rest_t>0 && rest_t <= dton
```

```
   Vr=Vr+Q*rest_t*BRVRatio(T,S,a,w,rest_t);
end
if rest_t>0 && rest_t > dton
   t1=rest_t;
   t2=t1-dton;
   Vr=Vr+Q*(t1*BRVRatio(T,S,a,w,t1)-t2*BRVRatio(T,S,a,w,t2));
end
```

BNVRatio

```
function BGVR=BNVRatio(T,S,a,w,t)
TOL=1.e-9; i=0; DELTA=1.0; BGVR=0;
while abs(DELTA) >=TOL && i<=30
   DELTA=(-1)^i*(C(T,S,a,w,t,i)+D(T,S,a,w,t,i));
   BGVR=BGVR+DELTA;
   i=i+1;
end
function Ci=C(T,S,a,w,t,i)
Ci=(1+S/2/T*(2*w*i+a)^2/t)*erfc(sqrt(S/4/T*(2*w*i+a)^2/t))-
2/sqrt(pi())*sqrt(S/4/T*(2*w*i+a)^2/t)*exp(-S/4/T*(2*w*i+a)^2/t);
function Di=D(T,S,a,w,t,i)
Di=(1+S/2/T*(2*w*(i+1)-a)^2/t)*erfc(sqrt(S/4/T*(2*w*(i+1)-a)^2/t))-
2/sqrt(pi())*sqrt(S/4/T*(2*w*(i+1)-a)^2/t)*exp(-S/4/T*(2*w*(i+1)-a)^2/t);
```

BNvol_sol_CYC

```
function [Vr] = BNvol_sol_CYC(t,delt,dton,Q,T,S,a,w)
% Calculating Vr (depletion volume)
Vr = 0.;
frac=t/delt;
int_t=fix(frac);
rest_t=t-int_t*delt;
n = int_t+1;
for i=1:n-1
   t1=t-(i-1)*delt;
   t2=t1-dton;
   Vr =Vr+Q*(t1*BNVRatio(T,S,a,w,t1)-t2*BNVRatio(T,S,a,w,t2));
end
if rest_t>0 && rest_t <= dton
   Vr=Vr+Q*rest_t*BNVRatio(T,S,a,w,rest_t);
end
if rest_t > dton
   t1=rest_t;
   t2=t1-dton;
   Vr=Vr+Q*(t1*BNVRatio(T,S,a,w,t1)-t2*BNVRatio(T,S,a,w,t2));
End
```

APPENDIX I

Matlab code: **SI_OPT**

This code optimizes the absolute value of the stream depletion volume over a given time window (0, t_{fin}) due to a number of operating wells at given locations in a semi-infinite aquifer limited by the stream under study. Wells can be activated either cyclically or continuously. The pumping rate is the decision variable so that the algorithm chooses whether a well is used for injection (Q>0) or extraction (Q<0) in the case of Aquifer Pumping and Recharge, or all the wells are chosen for injection during a period of time, then all of them are chosen for extraction during another period of time for the case of Aquifer Storage and Recovery (no overlap between operation periods)

Constraints are imposed such that each well is characterized by a minimum ($Q_{min} \leq 0$) and a maximum pumping rate ($Q_{max} \geq 0$).

Constraints on maximum and minimum drawdown are imposed at a number of monitoring wells n_{mw}.

All the other required parameters are read from (Param.txt) it contains: hydraulic conductivity of the aquifer, Storativity, finial time of simulation, delta time, initial heads level in the aquifer, maximum allowable heads, minimum allowable heads, available daily rate for injection, Irrigation demands daily rate, location of the boundary and location of the stream.

The input file **Aquifer.Parms.dat** includes simulation time parameters and aquifer parameters:

- the aquifer parameters
  - hydraulic conductivity K;
  - storativity S.
- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)
- hydraulic parameters and constraints
  - initial hydraulic head (saturated thickness) h_0
  - maximum hydraulic head (saturated thickness) h_{max}
  - minimum hydraulic head (saturated thickness) h_{min}
  - available injection rate (>0) Q_{rech}
  - irrigation demand rate (<0) Q_{dem}
  - stream boundary location ($y_{stream} > 0$)

The input file **Operation.Wells.data** includes pumping well data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - well location x_w and y_w.
  - the time at which well operation starts t_{st}
  - the operation period Δt_{on}
  - maximum extraction rate $Q_{w,min}$ (<0)
  - maximum injection rate $Q_{w,max}$ (>0)

The input file **Monitoring.Wells.data** includes monitoring well data:

- total number of monitoring wells n_{mw}

- for each operating well, each of the following lines provide:
  - well location x_{mw} and y_{mw}.
  - number of hydraulic head checks $n_{t,ch}$
  - times at which heads checks are performed

Examples:

Aquifer.Parms

3740.	365.	30.	86.4	0.2
t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Grid.Parms

0.	0.	6000.	8000.	60	80
x_{min} (m)	y_{min} (m)	x_{min} (m)	y_{min} (m)	n_x	n_y

Wells.dat

Description:		2 wells operating cyclically (APR)			
2 (n_w)					
-5000.	0.	75.	140.	1000.	1000.
0.	+5000	270.	180.	1000.	3000.
Q_{min} (m^3/day)	Q_{min} (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

Wells.dat

Description:		2 wells operating cyclically (ASR)			
2 (n_w)					
-5000.	0.	75.	140.	1000.	1000.
0.	+5000	255.	180.	1000.	3000.
Q_{min} (m^3/day)	Q_{min} (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

SI_OPT

```
clear
clc
% Reading Data
% 1- Simulation and Aquifer Parameters
fid1 = fopen ('Aquifer.Parms.dat','r');
Temp = fscanf(fid1,'%f %f %f %f %f %f %f %f %f %f',[1,10]);
% Hydraulic Conductivity (m/day)
K =Temp(1);
% Storativity (/)
S = Temp(2);
% Simulation Time Horizon (day)
tfin = Temp(3);
% Cycle simulation time  (e.g. 365 days)
delt = Temp(4);
% Aquifer Initial Head (saturated thickness)(m)
ho = Temp(5);
% Aquifer Maximum Head (m) - constraint
hmax = Temp(6);
% Aquifer Minimum Head (m) - constraint
hmin = Temp(7);
% Available water for Injection (>0) (m^3/day)
Q_ava = Temp(8);
% Irrigation Extraction Demand (<0) (m^3/day)
Q_dem = Temp(9);
% No-Flow Boundary Location (m)
x_stream = Temp(10);
fclose(fid1);
%
% 2- Operating Wells
% fid2 = fopen ('BAPR_Operation.Wells.dat','r');
fid2 = fopen ('BASR_Operation.Wells.dat','r');
Temp = fscanf (fid2 ,'%f',[1,2]);
```

```
new = Temp(1); % Number of Extraction Wells
niw = Temp(2); % Number of Injection Wells
now=new+niw;
now
%
xw = zeros(now);
yw = zeros(now);
tst = zeros(now);
dt = zeros(now);
Qw_min = zeros(now);
Qw_max = zeros(now)
%
for j = 1:new
   Temp = fscanf( fid2,'%f %f %f %f %f %f %f',[1,7]);
   xw(j)    = Temp(2);
   yw(j)    = Temp(3);
   tst(j)   = Temp(4); % Operation starting time (day)
   dt(j)    = Temp(5); % Operation Period (day)
   Qw_min(j) = Temp(6); % Minimum extraction rate (m^3/day)
   Qw_max(j) = Temp(7); % Maximum extraction rate (m^3/day)
   if (Qw_max(j)>0.)
     disp('Warning: extraction rate may be positive.')
   end
end
for j = new+1:now
   Temp = fscanf( fid2,'%f %f %f %f %f %f %f',[1,7]);
   xw(j)    = Temp(2);
   yw(j)    = Temp(3);
   tst(j)   = Temp(4); % Operation starting time (day)
   dt(j)    = Temp(5); % Operation Period (day)
   Qw_min(j) = Temp(6); % Minimum injection rate (m^3/day)
   Qw_max(j) = Temp(7); % Maximum injection rate (m^3/day)
   if (Qw_min(j)<0.)
      disp('Warning: injection rate may be negative.')
   end
end
fclose(fid2);
%
% 3- Monitoring Wells
fid3 = fopen ('Monitoring.Wells.dat','r');
Temp = fscanf( fid3,'%f',[1,1]);
nmw = Temp(1); % Number of Monitoring Wells
%
xm = zeros(nmw);
ym = zeros(nmw);
ntch = zeros(nmw);
```

```
tch= zeros(nmw,10);
%
for mw = 1:nmw % mw = monitoring wells
   Temp     = fscanf( fid3,'%f %f %f',[1,3]);
   xm(mw)   = Temp(2);
   ym(mw)   = Temp(3);
   Temp     = fscanf( fid3,'%f',[1,1]);
   % Number of Head Check Times
   ntch(mw)   = Temp(1);
   % Read in Head Check Times
   for ich=1:ntch(mw)
      tch(mw,ich)= fscanf( fid3,'%f',[1,1]);
   end
end
fclose(fid3);
%
% Start Calculations
%
T  = K*ho;
h1 = hmax-ho; % Maximum Allowed Head Increase (m)
h2 = ho-hmin; % Maximum Allowed Head Decrease (m)
%
% Calculate Cumulative number of head check times
cntch=0;
for i= 1:nmw
  cntch = cntch + ntch(i);
end
%
% Assembling Linear Optimization Problem Coefficients
%
% Inequatilty matrix A allocation
% The total number of rows of this matrix is given by the sum of:
% -> 2*now (at each candidate well, the pumping rate must be below the maximum and
above the minimum,
%          which requires two inequalities per well)
% -> 2*cntch (at each monitoring well, the head must be below the maximum and above
the minimum
%            at each prescribed check time, which requires two inequalities per well per check
time)
% -> 2 (at any time, the sum of extraction rates (<0) must be below Q_dem (<0),
%               and the sum of injection rates (>0) must be below Q_ava)
% -> 2 (extra inequalities for objective function modification accounting
%       for the module of the volume of stream depletion, Vr')
nrow = 2*now + 2*cntch + 2 + 2;
%sprintf('N. of Constraint Inequalities: %d',nrow)
% The total number of columns of this matrix is given by the total number of
```

```
% candidate operating wells plus one (for Vr')
now
ncol = now + 1;
sprintf('N. of Decision Variables: %d',ncol)
% Allocate matrix A
A  = zeros(nrow,ncol);
% Allocate Inequality RHS Vector: the total number of rows of this vector
% is the same as in Matrix A:nrow = 2*now + 2*cntch + 2 + 2;
b  = zeros(nrow,1);
%
% Calculate and assemble matrix A and RHS Vector b coefficients
irow=0;
for j = 1:now
   % set coefficients for two pumping rate constraints at each candidate well
   irow=irow+2;
   A(irow-1,j) = +1.;
   A(irow,j)   = -1.;
   % pumping rate constraints at each candidate well
   b(irow-1)   = +Qw_max(j);
   b(irow)     = -Qw_min(j);
end
%
for i=1:nmw
   % set coefficients for two hydraulic head constraints at each monitoring
   % well at each check time
   for ich = 1:ntch(i)
      irow=irow+2;
      for j = 1:now
         % calculate unit response coefficient for operating well j
         % obtained using BRC_CYC(a Theis derived solution for a well
         % operating cyclically in a bounded aquifer)
         if (tch(i,ich)>tst(j))
            A(irow-1,j) = RC_SI(S,T,xw(j),yw(j),delt,dt(j),tch(i,ich)-tst(j),xm(i),ym(i));
            A(irow ,j)  = -A(irow-1,j);
         end
         % hydraulic head constraints at each monitoring well at each check time
         b(irow-1) = h1;
         b(irow)   = h2;
      end
   end
end
%
% set coefficients for sum of extraction rates to be below Q_dem <0;
irow=irow+1;
for j = 1:new
   A(irow,j) = +1.;
```

```
end
b(irow) = Q_dem;
% set coefficients for sum of injection rates to be below Q_ava >0;
irow=irow+1;
for j = new+1:now
   A(irow,j) = +1.;
end
b(irow) = Q_ava;
%
% Set up inequality coefficients for objective function modification
% accounting for the module of the volume of stream depletion, Vr')
irow=irow+2;
for j = 1:now
   if ((tfin-tst(j))>0.)
      A(irow-1,j) = vol_solver(tfin-tst(j),delt,dt(j),T,S,xw(j));
      A(irow  ,j) = -A(irow-1,j);
   end
end
% set up Vr' cofficients
A(irow-1,ncol) = -1;
A(irow,ncol)   = -1;
% Constraints for objective function modification accounting for the module
% of the volume of stream depletion, Vr', are equal to zero.
b(irow-1)      = 0.;
b(irow)       = 0.;
%
% Matrix of Linear Objective Function Cofficients
f = zeros(ncol,1);
f(ncol) = 1; % for Vr' column
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Solution of the formulated Linear Optimization Problem
%
options = optimset('LargeScale','off','Simplex','on');
[Qw,fval,exitflag,output,lambda] = linprog(f,A,b,[],[]);
%
disp('Exit Condition:')
if (exitflag==1)
   exitflag
   disp('optimum is found')
else
   exitflag
   disp('optimum is NOT found: check linprog help')
end
%disp('Optimal Pumping Rate Set:')
%Qw
```

```
fprintf('Objective Function Value at Optimum: Minimum Stream Volume Depletion')
sprintf('fval= %f',fval)
%
%output
%lambda
% Calculated Aquifer Recharge and Extraction Cumulative Rates,
% Stream Recharge Volume over the simulated period and
% Reduce Optimal Solution by eliminating non active wells
naow=0;
naew=0;
naiw=0
Q_extract=0.;
Q_recharge=0.;
Vr=0.;
Qlim=100.;
for i=1:now
   Vr=Vr+A(nrow-1,i)*Qw(i);
   if (abs(Qw(i))>Qlim)
      naow=naow+1;
      if (Qw(i)<0.)
         naew=naew+1;
         Q_extract=Q_extract+Qw(i);
      end
      if (Qw(i)>0.)
         naiw=naiw+1;
         Q_recharge=Q_recharge+Qw(i);
      end
   end
end
% Output File
% fid4 = fopen('SIAPR.results_S1.dat','w');
% fid4 = fopen('SIAPR.results_S2.dat','w');
% fid4 = fopen('SIAPR.results_S3.dat','w');
fid4 = fopen('SIASR.results_S1.dat','w');
% fid4 = fopen('SIASR.results_S2.dat','w');
% fid4 = fopen('SIASR.results_S3.dat','w');

temp = [naow,naew,naiw];
fprintf(fid4,'%d %d %d \n',temp);
fid5 = fopen('optimal.scheme.dat','w');
for i=1: now
   %     temp = [Qw(i),xw(i),yw(i),tst(i),dt(i),Qw_min(i),Qw_max(i)];
   %     fprintf(fid4,'%15.6E %15.6E %15.6E %15.6E %15.6E %15.6E %15.6E\n',temp);
   if (abs(Qw(i))>Qlim)
      temp = [Qw(i),tst(i),dt(i),xw(i),yw(i)];
      fprintf(fid4,'%15.6E %15.6E %15.6E %15.6E %15.6E\n',temp);
```

```
      temp = [xw(i),yw(i),Qw(i)];
      fprintf(fid5,'%15.6E %15.6E %15.6E\n',temp);
   end
end
fprintf(fid4,'Net Cumulative Stream Recharge Volume (Vr_prime) (m^3/day)= %15.3E
\n',fval)
fprintf(fid4,'Cumulative Stream Recharge Volume (Vr) (m^3)= %15.3E \n',Vr)
fprintf(fid4,'Cumulative Extraction Rate (m^3/day) = %15.3E \n',Q_extract)
fprintf(fid4,'Cumulative Recharge Rate (m^3/day) = %15.3E \n',Q_recharge)
fclose(fid4);
fclose(fid5);
%
```

APPENDIX J

Matlab code: **BNFR_OPT**

This code optimizes the absolute value of the stream depletion volume over a given time window (0, t_{fin}) due to a number of operating wells at given locations in a finite aquifer bounded between a no-flow boundary and the stream under study. Wells can be activated either cyclically or continuously. The pumping rate is the decision variable so that the algorithm chooses whether a well is used for injection (Q>0) or extraction (Q<0) in the case of Aquifer Pumping and Recharge, or all the wells are chosen for injection during a period of time, then all of them are chosen for extraction during another period of time for the case of Aquifer Storage and Recovery (no overlap between operation periods)

Constraints are imposed such that each well is characterized by a minimum ($Q_{min} \leq 0$) and a maximum pumping rate ($Q_{max} \geq 0$).

Constraints on maximum and minimum drawdown are imposed at a number of monitoring wells n_{mw}.

All the other required parameters are read from (Param.txt) it contains: hydraulic conductivity of the aquifer, Storativity, finial time of simulation, delta time, initial heads level in the aquifer, maximum allowable heads, minimum allowable heads, available daily rate for injection, Irrigation demands daily rate, location of the boundary and location of the stream.

The input file **Aquifer.Parms.dat** includes simulation time parameters and aquifer parameters:

- the aquifer parameters
  - hydraulic conductivity K;
  - storativity S.
- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)
- hydraulic parameters and constraints
  - initial hydraulic head (saturated thickness) h_0
  - maximum hydraulic head (saturated thickness) h_{max}
  - minimum hydraulic head (saturated thickness) h_{min}
  - available injection rate (>0) Q_{rech}
  - irrigation demand rate (<0) Q_{dem}
  - no-flow Boundary Location $y_{no\text{-}flow}$
  - stream boundary location y_{stream} (> $y_{no\text{-}flow}$)

The input file **Operation.Wells.data** includes pumping well data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - well location x_w and y_w.
  - the time at which well operation starts t_{st}
  - the operation period Δt_{on}
  - maximum extraction rate $Q_{w,min}$ (<0)
  - maximum injection rate $Q_{w,max}$ (>0)

The input file **Monitoring.Wells.data** includes monitoring well data:

- total number of monitoring wells n_{mw}

- for each operating well, each of the following lines provide:
  - well location x_{mw} and y_{mw}.
  - number of hydraulic head checks $n_{t,ch}$
  - times at which heads checks are performed

Examples:

Aquifer.Parms

3740.	365.	30.	86.4	0.2
t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Grid.Parms

0.	0.	6000.	8000.	60	80
x_{min} (m)	y_{min} (m)	x_{min} (m)	y_{min} (m)	n_x	n_y

Wells.dat

Description:		2 wells operating cyclically (APR)			
2 (n_w)					
-5000.	0.	75.	140.	1000.	1000.
0.	+5000	270.	180.	1000.	3000.
Q_{min} (m^3/day)	Q_{min} (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

Wells.dat

Description:			2 wells operating cyclically (ASR)		
2 (n_w)					
-5000.	0.	75.	140.	1000.	1000.
0.	+5000	255.	180.	1000.	3000.
Q_{min} (m^3/day)	Q_{min} (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

BNFR_OPT

```
clear
clc
% Reading Data
% 1- Simulation and Aquifer Parameters
fid1 = fopen ('Aquifer.Parms.dat','r');
Temp = fscanf(fid1,'%f %f %f %f %f %f %f %f %f %f %f',[1,11]);
% Hydraulic Conductivity (m/day)
K =Temp(1);
% Storativity (/)
S = Temp(2);
% Simulation Time Horizon (day)
tfin = Temp(3);
% Cycle simulation time  (e.g. 365 days)
delt = Temp(4);
% Aquifer Initial Head (saturated thickness)(m)
ho = Temp(5);
% Aquifer Maximum Head (m) - constraint
hmax = Temp(6);
% Aquifer Minimum Head (m) - constraint
hmin = Temp(7);
% Available water for Injection (>0) (m^3/day)
Q_ava = Temp(8);
% Irrigation Extraction Demand (<0) (m^3/day)
Q_dem = Temp(9);
% No-Flow Boundary Location (m)
x_boundary = Temp(10);
% Stream Boundary Location (M)
x_stream = Temp(11);
fclose(fid1);
%
% 2- Operating Wells
% fid2 = fopen ('BAPR_Operation.Wells.dat','r');
```

```
fid2 = fopen ('BASR_Operation.Wells.dat','r');
Temp = fscanf (fid2 ,'%f',[1,2]);
new = Temp(1); % Number of Extraction Wells
niw = Temp(2); % Number of Injection Wells
now=new+niw;
now
%
xw = zeros(now);
yw = zeros(now);
tst = zeros(now);
dt = zeros(now);
Qw_min = zeros(now);
Qw_max = zeros(now);
%
for j = 1:new
   Temp = fscanf( fid2,'%f %f %f %f %f %f %f',[1,7]);
   xw(j)    = Temp(2);
   yw(j)    = Temp(3);
   tst(j)   = Temp(4); % Operation starting time (day)
   dt(j)    = Temp(5); % Operation Period (day)
   Qw_min(j) = Temp(6); % Minimum extraction rate (m^3/day)
   Qw_max(j) = Temp(7); % Maximum extraction rate (m^3/day)
   if (Qw_max(j)>0.)
     disp('Warning: extraction rate may be positive.')
   end
end
for j = new+1:now
   Temp = fscanf( fid2,'%f %f %f %f %f %f %f',[1,7]);
   xw(j)    = Temp(2);
   yw(j)    = Temp(3);
   tst(j)   = Temp(4); % Operation starting time (day)
   dt(j)    = Temp(5); % Operation Period (day)
   Qw_min(j) = Temp(6); % Minimum injection rate (m^3/day)
   Qw_max(j) = Temp(7); % Maximum injection rate (m^3/day)
   if (Qw_min(j)<0.)
      disp('Warning: injection rate may be negative.')
   end
end
fclose(fid2);
%
% 3- Monitoring Wells
fid3 = fopen ('Monitoring.Wells.dat','r');
Temp = fscanf( fid3,'%f',[1,1]);
nmw = Temp(1); % Number of Monitoring Wells
%
xm = zeros(nmw);
```

```
ym = zeros(nmw);
ntch = zeros(nmw);
tch= zeros(nmw,10);
%
for mw = 1:nmw % mw = monitoring wells
   Temp     = fscanf( fid3,'%f %f %f',[1,3]);
   xm(mw)   = Temp(2);
   ym(mw)   = Temp(3);
   Temp     = fscanf( fid3,'%f',[1,1]);
   % Number of Head Check Times
   ntch(mw)   = Temp(1);
   % Read in Head Check Times
   for ich=1:ntch(mw)
      tch(mw,ich)= fscanf( fid3,'%f',[1,1]);
   end
end
fclose(fid3);
%
% Start Calculations
%
T  = K*ho;
h1 = hmax-ho; % Maximum Allowed Head Increase (m)
h2 = ho-hmin; % Maximum Allowed Head Decrease (m)
w = x_stream-x_boundary; % Aquifer Width (m)
%
% Calculate Cumulative number of head check times
cntch=0;
for i= 1:nmw
  cntch = cntch + ntch(i);
end
%
% Assembling Linear Optimization Problem Coefficients
%
% Inequatilty matrix A allocation
% The total number of rows of this matrix is given by the sum of:
% -> 2*now (at each candidate well, the pumping rate must be below the maximum and
above the minimum,
%          which requires two inequalities per well)
% -> 2*cntch (at each monitoring well, the head must be below the maximum and above
the minimum
%            at each prescribed check time, which requires two inequalities per well per check
time)
% -> 2 (at any time, the sum of extraction rates (<0) must be below Q_dem (<0),
%              and the sum of injection rates (>0) must be below Q_ava)
% -> 2 (extra inequalities for objective function modification accounting
%       for the module of the volume of stream depletion, Vr')
```

```
nrow = 2*now + 2*cntch + 2 + 2;
%sprintf('N. of Constraint Inequalities: %d',nrow)
% The total number of columns of this matrix is given by the total number of
% candidate operating wells plus one (for Vr')
now
ncol = now + 1;
sprintf('N. of Decision Variables: %d',ncol)
% Allocate matrix A
A  = zeros(nrow,ncol);
% Allocate Inequality RHS Vector: the total number of rows of this vector
% is the same as in Matrix A:nrow = 2*now + 2*cntch + 2 + 2;
b  = zeros(nrow,1);
%
% Calculate and assemble matrix A and RHS Vector b coefficients
irow=0;
for j = 1:now
   % set coefficients for two pumping rate constraints at each candidate well
   irow=irow+2;
   A(irow-1,j) = +1.;
   A(irow,j)   = -1.;
   % pumping rate constraints at each candidate well
   b(irow-1)   = +Qw_max(j);
   b(irow)     = -Qw_min(j);
end
%
for i=1:nmw
   % set coefficients for two hydraulic head constraints at each monitoring
   % well at each check time
   for ich = 1:ntch(i)
      irow=irow+2;
      for j = 1:now
         % calculate unit response coefficient for operating well j
         % obtained using BRC_CYC(a Theis derived solution for a well
         % operating cyclically in a bounded aquifer)
         if (tch(i,ich)>tst(j))
            aw=x_stream-xw(j);
            am=x_stream-xm(i);
            A(irow-1,j) = BRC_NOFLOW(S,T,w,yw(j),aw,delt,dt(j),tch(i,ich)-tst(j),ym(i),am);
            A(irow ,j)  = -A(irow-1,j);
         end
         % hydraulic head constraints at each monitoring well at each check time
         b(irow-1) = h1;
         b(irow)   = h2;
      end
   end
end
```

```
%
% set coefficients for sum of extraction rates to be below Q_dem <0;
irow=irow+1;
for j = 1:new
   A(irow,j) = +1.;
end
b(irow) = Q_dem;
% set coefficients for sum of injection rates to be below Q_ava >0;
irow=irow+1;
for j = new+1:now
   A(irow,j) = +1.;
end
b(irow) = Q_ava;
%
% Set up inequality coefficients for objective function modification
% accounting for the module of the volume of stream depletion, Vr')
irow=irow+2;
for j = 1:now
   if ((tfin-tst(j))>0.)
      A(irow-1,j) = BNvol_sol_CYC(tfin-tst(j),delt,dt(j),T,S,x_stream -xw(j),w);
      A(irow  ,j) = -A(irow-1,j);
   end
end
% set up Vr' cofficients
A(irow-1,ncol) = -1;
A(irow,ncol)   = -1;
% Constraints for objective function modification accounting for the module
% of the volume of stream depletion, Vr', are equal to zero.
b(irow-1)      = 0.;
b(irow)       = 0.;
%
% Matrix of Linear Objective Function Cofficients
f = zeros(ncol,1);
f(ncol) = 1; % for Vr' column
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Solution of the formulated Linear Optimization Problem
%
options = optimset('LargeScale','off','Simplex','on');
[Qw,fval,exitflag,output,lambda] = linprog(f,A,b,[],[]);
%
disp('Exit Condition:')
if (exitflag==1)
   exitflag
   disp('optimum is found')
else
```

```
   exitflag
   disp('optimum is NOT found: check linprog help')
end
%disp('Optimal Pumping Rate Set:')
%Qw
fprintf('Objective Function Value at Optimum: Minimum Stream Volume Depletion')
sprintf('fval= %f',fval)
%
%output
%lambda
% Calculated Aquifer Recharge and Extraction Cumulative Rates,
% Stream Recharge Volume over the simulated period and
% Reduce Optimal Solution by eliminating non active wells
naow=0;
naew=0;
naiw=0;
Q_extract=0.;
Q_recharge=0.;
Vr=0.;
Qlim=100.;
for i=1:now
   Vr=Vr+A(nrow-1,i)*Qw(i);
   if (abs(Qw(i))>Qlim)
      naow=naow+1;
      if (Qw(i)<0.)
         naew=naew+1;
         Q_extract=Q_extract+Qw(i);
      end
      if (Qw(i)>0.)
         naiw=naiw+1;
         Q_recharge=Q_recharge+Qw(i);
      end
   end
end
% Output File
% fid4 = fopen('BAPR.results_S1.dat','w');
% fid4 = fopen('BAPR.results_S2.dat','w');
% fid4 = fopen('BAPR.results_S3.dat','w');
% fid4 = fopen('BASR.results_S1.dat','w');
% fid4 = fopen('BASR.results_S2.dat','w');
fid4 = fopen('BASR.results_S3.dat','w');
temp = [naow,naew,naiw];
fprintf(fid4,'%d %d %d \n',temp);
fid5 = fopen('optimal.scheme.dat','w');
for i=1: now
   %      temp = [Qw(i),xw(i),yw(i),tst(i),dt(i),Qw_min(i),Qw_max(i)];
```

```
  %     fprintf(fid4,'%15.6E %15.6E %15.6E %15.6E %15.6E %15.6E %15.6E\n',temp);
  if (abs(Qw(i))>Qlim)
     temp = [Qw(i),tst(i),dt(i),xw(i),yw(i)];
     fprintf(fid4,'%15.6E %15.6E %15.6E %15.6E %15.6E\n',temp);
     temp = [xw(i),yw(i),Qw(i)];
     fprintf(fid5,'%15.6E %15.6E %15.6E\n',temp);
  end
end
fprintf(fid4,'Net Cumulative Stream Recharge Volume (Vr_prime) (m^3/day)= %15.3E
\n',fval)
fprintf(fid4,'Cumulative Stream Recharge Volume (Vr) (m^3)= %15.3E \n',Vr)
fprintf(fid4,'Cumulative Extraction Rate (m^3/day) = %15.3E \n',Q_extract)
fprintf(fid4,'Cumulative Recharge Rate (m^3/day) = %15.3E \n',Q_recharge)
fclose(fid4);
fclose(fid5);
%
```

APPENDIX K

Matlab code: **BRR_OPT**

This code optimizes the absolute value of the stream depletion volume over a given time window (0, t_{fin}) due to a number of operating wells at given locations in a finite aquifer bounded between a recharge boundary and the stream under study. Wells can be activated either cyclically or continuously. The pumping rate is the decision variable so that the algorithm chooses whether a well is used for injection (Q>0) or extraction (Q<0)) in the case of Aquifer Pumping and Recharge, or all the wells are chosen for injection during a period of time, then all of them are chosen for extraction during another period of time for the case of Aquifer Storage and Recovery (no overlap between operation periods)

Constraints are imposed such that each well is characterized by a minimum (Q_{min} $\boxed{\leq}$ 0) and a maximum pumping rate (Q_{max} $\boxed{\geq}$ 0).

Constraints on maximum and minimum drawdown are imposed at a number of monitoring wells n_{mw}.

```
All the other required parameters are read from (Param.txt) it contains:
hydraulic conductivity of the aquifer, Storativity, finial time of simulation,
delta time, initial heads level in the aquifer, maximum allowable heads, minimum
allowable heads, available daily rate for injection, Irrigation demands daily
rate, location of the boundary and location of the stream.
```

The input file **Aquifer.Parms.dat** includes simulation time parameters and aquifer parameters:

- the aquifer parameters
  - hydraulic conductivity K;
  - storativity S.
- time parameters
  - final simulation time t_{fin};
  - cyclical time step of simulation $\Delta t=\Delta t_{on}+\Delta t_{off}$ (e.g., 1 year)
- hydraulic parameters and constraints
  - initial hydraulic head (saturated thickness) h_0
  - maximum hydraulic head (saturated thickness) h_{max}
  - minimum hydraulic head (saturated thickness) h_{min}
  - available injection rate (>0) Q_{rech}
  - irrigation demand rate (<0) Q_{dem}
  - recharge Boundary Location $y_{no\text{-}flow}$

- stream boundary location y_{stream} (> $y_{no\text{-}flow}$)

The input file **Operation.Wells.data** includes pumping well data:

- total number of operating wells n_w

- for each operating well, each of the following lines provide:
  - well location x_w and y_w.
  - the time at which well operation starts t_{st}
  - the operation period Δt_{on}
  - maximum extraction rate $Q_{w,min}$ (<0)
  - maximum injection rate $Q_{w,max}$ (>0)

The input file **Monitoring.Wells.data** includes monitoring well data:

- total number of monitoring wells n_{mw}

- for each operating well, each of the following lines provide:
  - well location x_{mw} and y_{mw}.
  - number of hydraulic head checks $n_{t,ch}$
  - times at which heads checks are performed

Examples:

Examples:

Aquifer.Parms

3740.	365.	30.	86.4	0.2
t_{fin} (day)	$\Delta t=\Delta t_{on}+\Delta t_{off}$ (day)	b(m)	K (m/day)	S (/)

Grid.Parms

0.	0.	6000.	8000.	60	80
x_{min} (m)	y_{min} (m)	x_{min} (m)	y_{min} (m)	n_x	n_y

Wells.dat

Description:			2 wells operating cyclically (APR)		
2 (n_w)					
-5000.	0.	75.	140.	1000.	1000.
0.	+5000	270.	180.	1000.	3000.
Q_{min} (m^3/day)	Q_{min} (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

Wells.dat

Description:			2 wells operating cyclically (ASR)		
2 (n_w)					
-5000.	0.	75.	140.	1000.	1000.
0.	+5000	255.	180.	1000.	3000.
Q_{min} (m^3/day)	Q_{min} (m^3/day)	t_{st} (day)	Δt_{on} (day)	x_w (m)	y_w (m)

BRR_OPT

```
clear
clc
% Reading Data
% 1- Simulation and Aquifer Parameters
fid1 = fopen ('Aquifer.Parms.dat','r');
Temp = fscanf(fid1,'%f %f %f %f %f %f %f %f %f %f %f',[1,11]);
% Hydraulic Conductivity (m/day)
K =Temp(1);
% Storativity (/)
S = Temp(2);
% Simulation Time Horizon (day)
tfin = Temp(3);
% Cycle simulation time  (e.g. 365 days)
delt = Temp(4);
% Aquifer Initial Head (saturated thickness)(m)
ho = Temp(5);
% Aquifer Maximum Head (m) - constraint
hmax = Temp(6);
% Aquifer Minimum Head (m) - constraint
hmin = Temp(7);
```

```
% Available water for Injection (>0) (m^3/day)
Q_ava = Temp(8);
% Irrigation Extraction Demand (<0) (m^3/day)
Q_dem = Temp(9);
% No-Flow Boundary Location (m)
x_boundary = Temp(10);
% Stream Boundary Location (M)
x_stream = Temp(11);
fclose(fid1);
%
% 2- Operating Wells
% fid2 = fopen ('BAPR_Operation.Wells.dat','r');
fid2 = fopen ('BASR_Operation.Wells.dat','r');
Temp = fscanf (fid2 ,'%f',[1,2]);
new = Temp(1); % Number of Extraction Wells
niw = Temp(2); % Number of Injection Wells
now=new+niw;
now
%
xw = zeros(now);
yw = zeros(now);
tst = zeros(now);
dt = zeros(now);
Qw_min = zeros(now);
Qw_max = zeros(now);
%
for j = 1:new
   Temp = fscanf( fid2,'%f %f %f %f %f %f %f',[1,7]);
   xw(j)     = Temp(2);
   yw(j)     = Temp(3);
   tst(j)    = Temp(4); % Operation starting time (day)
   dt(j)     = Temp(5); % Operation Period (day)
   Qw_min(j) = Temp(6); % Minimum extraction rate (m^3/day)
   Qw_max(j) = Temp(7); % Maximum extraction rate (m^3/day)
   if (Qw_max(j)>0.)
     disp('Warning: extraction rate may be positive.')
   end
end
for j = new+1:now
   Temp = fscanf( fid2,'%f %f %f %f %f %f %f',[1,7]);
   xw(j)     = Temp(2);
   yw(j)     = Temp(3);
   tst(j)    = Temp(4); % Operation starting time (day)
   dt(j)     = Temp(5); % Operation Period (day)
   Qw_min(j) = Temp(6); % Minimum injection rate (m^3/day)
   Qw_max(j) = Temp(7); % Maximum injection rate (m^3/day)
```

```
   if (Qw_min(j)<0.)
      disp('Warning: injection rate may be negative.')
   end
end
fclose(fid2);
%
% 3- Monitoring Wells
fid3 = fopen ('Monitoring.Wells.dat','r');
Temp = fscanf( fid3,'%f',[1,1]);
nmw = Temp(1); % Number of Monitoring Wells
%
xm = zeros(nmw);
ym = zeros(nmw);
ntch = zeros(nmw);
tch= zeros(nmw,10);
%
for mw = 1:nmw % mw = monitoring wells
   Temp     = fscanf( fid3,'%f %f %f',[1,3]);
   xm(mw)   = Temp(2);
   ym(mw)   = Temp(3);
   Temp     = fscanf( fid3,'%f',[1,1]);
   % Number of Head Check Times
   ntch(mw)   = Temp(1);
   % Read in Head Check Times
   for ich=1:ntch(mw)
      tch(mw,ich)= fscanf( fid3,'%f',[1,1]);
   end
end
fclose(fid3);
%
% Start Calculations
%
T  = K*ho;
h1 = hmax-ho; % Maximum Allowed Head Increase (m)
h2 = ho-hmin; % Maximum Allowed Head Decrease (m)
w = x_stream-x_boundary; % Aquifer Width (m)
%
% Calculate Cumulative number of head check times
cntch=0;
for i= 1:nmw
  cntch = cntch + ntch(i);
end
%
% Assembling Linear Optimization Problem Coefficients
%
% Inequatilty matrix A allocation
```

```
% The total number of rows of this matrix is given by the sum of:
% -> 2*now (at each candidate well, the pumping rate must be below the maximum and
above the minimum,
%         which requires two inequalities per well)
% -> 2*cntch (at each monitoring well, the head must be below the maximum and above
the minimum
%           at each prescribed check time, which requires two inequalities per well per check
time)
% -> 2 (at any time, the sum of extraction rates (<0) must be below Q_dem (<0),
%              and the sum of injection rates (>0) must be below Q_ava)
% -> 2 (extra inequalities for objective function modification accounting
%      for the module of the volume of stream depletion, Vr')
nrow = 2*now + 2*cntch + 2 + 2;
%sprintf('N. of Constraint Inequalities: %d',nrow)
% The total number of columns of this matrix is given by the total number of
% candidate operating wells plus one (for Vr')
now
ncol = now + 1;
sprintf('N. of Decision Variables: %d',ncol)
% Allocate matrix A
A  = zeros(nrow,ncol);
% Allocate Inequality RHS Vector: the total number of rows of this vector
% is the same as in Matrix A:nrow = 2*now + 2*cntch + 2 + 2;
b  = zeros(nrow,1);
%
% Calculate and assemble matrix A and RHS Vector b coefficients
irow=0;
for j = 1:now
   % set coefficients for two pumping rate constraints at each candidate well
   irow=irow+2;
   A(irow-1,j) = +1.;
   A(irow,j)   = -1.;
   % pumping rate constraints at each candidate well
   b(irow-1)   = +Qw_max(j);
   b(irow)     = -Qw_min(j);
end
%
for i=1:nmw
   % set coefficients for two hydraulic head constraints at each monitoring
   % well at each check time
   for ich = 1:ntch(i)
      irow=irow+2;
      for j = 1:now
         % calculate unit response coefficient for operating well j
         % obtained using BRC_CYC(a Theis derived solution for a well
         % operating cyclically in a bounded aquifer)
```

```
        if (tch(i,ich)>tst(j))
           aw=x_stream-xw(j);
           am=x_stream-xm(i);
           A(irow-1,j) = BRC_RR(S,T,w,yw(j),aw,delt,dt(j),tch(i,ich)-tst(j),ym(i),am);
           A(irow ,j)  = -A(irow-1,j);
        end
        % hydraulic head constraints at each monitoring well at each check time
        b(irow-1) = h1;
        b(irow)   = h2;
      end
   end
end
%
% set coefficients for sum of extraction rates to be below Q_dem <0;
irow=irow+1;
for j = 1:new
   A(irow,j) = +1.;
end
b(irow) = Q_dem;
% set coefficients for sum of injection rates to be below Q_ava >0;
irow=irow+1;
for j = new+1:now
   A(irow,j) = +1.;
end
b(irow) = Q_ava;
%
% Set up inequality coefficients for objective function modification
% accounting for the module of the volume of stream depletion, Vr')
irow=irow+2;
for j = 1:now
   if ((tfin-tst(j))>0.)
      A(irow-1,j) = BRvol_sol_CYC(tfin-tst(j),delt,dt(j),T,S,x_stream -xw(j),w);
      A(irow  ,j) = -A(irow-1,j);
   end
end
% set up Vr' cofficients
A(irow-1,ncol) = -1;
A(irow,ncol)   = -1;
% Constraints for objective function modification accounting for the module
% of the volume of stream depletion, Vr', are equal to zero.
b(irow-1)      = 0.;
b(irow)       = 0.;
%
% Matrix of Linear Objective Function Cofficients
f = zeros(ncol,1);
f(ncol) = 1; % for Vr' column
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Solution of the formulated Linear Optimization Problem
%
options = optimset('LargeScale','off','Simplex','on');
[Qw,fval,exitflag,output,lambda] = linprog(f,A,b,[],[]);
%
disp('Exit Condition:')
if (exitflag==1)
   exitflag
   disp('optimum is found')
else
   exitflag
   disp('optimum is NOT found: check linprog help')
end
%disp('Optimal Pumping Rate Set:')
%Qw
fprintf('Objective Function Value at Optimum: Minimum Stream Volume Depletion')
sprintf('fval= %f',fval)
%
%output
%lambda
% Calculated Aquifer Recharge and Extraction Cumulative Rates,
% Stream Recharge Volume over the simulated period and
% Reduce Optimal Solution by eliminating non active wells
naow=0;
naew=0;
naiw=0;
Q_extract=0.;
Q_recharge=0.;
Vr=0.;
Qlim=100.;
for i=1:now
   Vr=Vr+A(nrow-1,i)*Qw(i);
   if (abs(Qw(i))>Qlim)
      naow=naow+1;
      if (Qw(i)<0.)
         naew=naew+1;
         Q_extract=Q_extract+Qw(i);
      end
      if (Qw(i)>0.)
         naiw=naiw+1;
         Q_recharge=Q_recharge+Qw(i);
      end
   end
end
```

```
% Output File
% fid4 = fopen('BAPR.results_S1.dat','w');
% fid4 = fopen('BAPR.results_S2.dat','w');
% fid4 = fopen('BAPR.results_S3.dat','w');
% fid4 = fopen('BASR.results_S1.dat','w');
% fid4 = fopen('BASR.results_S2.dat','w');
fid4 = fopen('BASR.results_S3.dat','w');
temp = [naow,naew,naiw];
fprintf(fid4,'%d %d %d \n',temp);
% fid5 = fopen('optimal.scheme.dat','w');
for i=1: now
   %      temp = [Qw(i),xw(i),yw(i),tst(i),dt(i),Qw_min(i),Qw_max(i)];
   %      fprintf(fid4,'%15.6E %15.6E %15.6E %15.6E %15.6E %15.6E %15.6E\n',temp);
   if (abs(Qw(i))>Qlim)
      temp = [Qw(i),tst(i),dt(i),xw(i),yw(i)];
      fprintf(fid4,'%15.6E %15.6E %15.6E %15.6E %15.6E\n',temp);
%        temp = [xw(i),yw(i),Qw(i)];
%        fprintf(fid5,'%15.6E %15.6E %15.6E\n',temp);
   end
end
fprintf(fid4,'Net Cumulative Stream Recharge Volume (Vr_prime) (m^3/day)= %15.3E
\n',fval)
fprintf(fid4,'Cumulative Stream Recharge Volume (Vr) (m^3)= %15.3E \n',Vr)
fprintf(fid4,'Cumulative Extraction Rate (m^3/day) = %15.3E \n',Q_extract)
fprintf(fid4,'Cumulative Recharge Rate (m^3/day) = %15.3E \n',Q_recharge)
fclose(fid4);
% fclose(fid5);

%
```