Technical Note

on

ESTIMATES OF BENDING MOMENTS

AND PRESSURES DUE TO SLAMMING

by

R. E. Glover

Prepared for

David Taylor Model Basin

Department of the Navy

under

Contract Nonr 1610(02)

Through the

Colorado State University Research Foundation

Civil Engineering Section
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ABSTRACT

The bending moments which can be applied to the ship's structure and the pressures which can be applied to the ship's plates as a result of slamming are estimated from hydrodynamical considerations.

Introduction

The estimates of bending moments and pressures described in the following paragraphs have been made to obtain theoretical values which can be used for comparison with similar values obtained experimentally. Such values should prove helpful in the interpretation of experimental data and in design.

The bending moments are computed by idealizing the ship's structure as a "free-free" beam and evaluating the effect of the vibrations set up by a slam. Evaluations are made for the case where the forefoot emerges.

Pressures imposed on the ship's plating by slamming are estimated by taking the elasticity of the water and the flexibility of the plating into account.

Reference Moments

In the design of ship structures consideration is commonly given to the "sagging" and "hogging" conditions. The first condition occurs when the ship is crossing waves which have a wave length approximately equal to the length of the ship and there is a crest at the bow and at the stern and a trough amidships. The second condition is observed when there is a trough at the bow and at the stern, with a crest amidships. Under such circumstances the buoyancy is not uniformly distributed along the length of the ship and bending moments are produced in the ship's structure. If the waves have a wave length equal to the length of the ship and produce a sinusoidal buoyancy distribution along the ship's length, having a maximum amidships and zero at the bow and stern, the bending moment produced amidships will be

$$M = \frac{WL}{2\pi^2} \tag{1)*$$

where W represents the ship's displacement, L the ship's length and $\pi = 3.14159$. This result is presented here as a standard of comparison. It may be noted in passing that a concentration of buoyancy amidships should be possible in a confused sea which would produce a bending moment of the same order of magnitude.

<u>Slamming</u>

It is probable that a confused sea can impose forces of a certain type on a ship which are of greater magnitude than those imposed by any other type of sea. Such forces take the form of heavy blows which may occur fortuitously when the ship is being tossed about and is being presented with a wide variety of surface configurations. If it happens that the ship's motions and the water surface configurations conspire to bring some portion of the hull into contact with a sea surface which conforms closely to the shape of the hull, so that contact is made almost simultaneously over some area, the relative motions of ship and water must be suddenly obliterated within the area. The necessary velocity changes in the water will

* This moment is commonly computed for a wave of height equal to 1/20 of the ship's length. This computation does not bring the ship's displacement into consideration as it is desirable to do in this case.

require that the ship give up to the water a part of its store of kinetic energy, in order to set into motion an equivalent mass of water which may be some appreciable portion of the mass of the ship. The blow delivered to the ship in this way is generally audible, the motion of the ship is altered abruptly and the structure of the ship is set into violent vibration. Such an episode is termed a slam. The violence of the blow delivered to the ship depends upon the relative velocity of ship and water at the instant of impact and the area over which contact is made. The severity of the slams received by a vessel traversing confused seas are matters of chance, and although the blows may be heavy, a blow approaching the maximum that the physical conditions are capable of delivering must be a rarity. High pressures are imposed upon the ship's plating at the instant of impact. An increase of ship speed may also contribute to the violence of the slams.

The continual buoyancy shifts produced by a heavy confused sea will themselves impose heavy stresses upon the ship's structure. Additional stresses of considerable magnitude produced by slams can be added to the stresses which were already present at the instant of slamming. The stresses induced by slamming are associated with vibrations of the ship's structure and are therefore subject to rapid alternations of sign. At some phase of the vibration these stresses will therefore add to any which were originally present.

It is possible to estimate the magnitude of the stresses which bow emergence slams can produce. To this end, assimilate the ship's structure to a uniform beam having the mass of the ship and its cargo uniformly distributed along the length of the beam. To account for the equivalent mass of the water in contact with the bottom of the hull, add to the above masses a uniformly distributed mass equal to the ship's displacement. Then the mass of the equivalent beam will be twice that of the loaded ship. As a basis for estimating the magnitudes which might be involved it may be assumed that the ship can be lifted by a wave crest sufficiently to bring the forefoot out of the water for about 1/10 of the length of the ship and on re-entry a mass of water equal to 1/10 of the displacement must be set into motion. The kinetic energy needed to accomplish this must be exchanged with the part of the hull over which contact is re-established. At the instant contact is made, this part of the hull will have a velocity imparted to it while the motion of the rest of the hull remains, for the moment, unaltered. After contact is made the water whose velocity has been changed by the re-entry of the forefoot now contributes its share to the equivalent mass mentioned above.

The effects produced by dynamical situations of this kind can be calculated from the theory of beam vibrations. The procedure involves a development of the initial velocity distribution in terms of a series of the mode functions. When this has been done the bending moments at the center of the hull can be computed. In making this computation the procedure outlined in Rayleigh, paragraph 168, Vol. 1, (1) may be followed. This development treats the vibration induced in a beam by a blow which imparts an initial velocity to the part struck.

If we follow Rayleigh and let

w	represent the area of a cross section	ft^2
q	Young's modulus	lb/ft^2
k	the radius of gyration of a cross section	ft
k^2w	the moment of inertia of the cross section	$(ft)^4$
x	distance along the beam	(ft)
y	displacement normal to x	ft
ρ	volume density	$\frac{lb/sec^2}{ft^4}$
$b^2 = q/\rho$		ft^2/sec^2
$B = qk^2w$		
L	length of the beam	(l in Rayleigh)
u	a normal function	
m	an abstract number	
δ	the length of beam set into motion initially	ft
$Y = \int \dot{y}_o \rho \, w dx$	is the momentum of the blow	lb. sec.
v_o	the initial velocity of the part set in motion	(ft/sec)

Then if rotatory inertia is neglected the differential equation to be satified is:

$$\frac{\partial^2 y}{\partial t^2} + k^2 b^2 \frac{\partial^4 y}{\partial x^4} = 0 \qquad (2)$$

For a blow applied at the end $x = L$ of a free-free bar the solution is

$$y = \frac{L^2 Y}{kb\rho wL}\left[\frac{u_1(L)\,u_1(x)}{\frac{m_1^2}{L}\int u_1^2 dx}\sin\left(\frac{kb}{L^2}m_1^2 t\right) + \cdots \right.$$

$$\left. \frac{u_r(L)\,u_r(x)}{\frac{m_r^2}{L}\int u_r^2 dx}\sin\left(\frac{kb}{L^2}m_r^2 t\right) + \cdots \right] \quad (3)$$

The bending moment is:

$$q k^2 w \frac{d^2 y}{dx^2} = \frac{qkY}{b\rho L}\left[\frac{u_1(L)\,u_1''(x)}{\frac{1}{L}\int u_1^2 dx}\sin\left(\frac{kb}{L^2}m_1^2 t\right) + \cdots + \right.$$

$$\left. \frac{u_r(L)\,u_r''(x)}{\frac{1}{L}\int u_r^2 dx}\sin\left(\frac{kb}{L^2}m_r^2 t\right) + \cdots \right] \quad (4)$$

where according to Rayleigh:

$$u(x) = (\sin m - \sinh m)\left(\cos\frac{mx}{L} + \cosh\frac{mx}{L}\right)$$

$$-(\cos m - \cosh m)\left(\sin\frac{mx}{L} + \sinh\frac{mx}{L}\right) \quad (5)$$

$$u^{11}(x) = (\sin m - \sinh m)\left(-\cos\frac{mx}{L} + \cosh\frac{mx}{L}\right)$$

$$-(\cos m - \cosh m)\left(-\sin\frac{mx}{L} \quad \sinh\frac{mx}{L}\right) \quad (6)$$

The first three values of m are

$$m_1 = 4.7300$$

$$m_2 = 7.8532$$

$$m_3 = 10.9956$$

For a free-free beam

$$\frac{1}{L}\int u_r^2\,dx = \frac{u_r^2(L)}{4} \tag{7}$$

For the moment at the center of the beam equation 3 reduces to:

$$g k^2 w \frac{d^2y}{dx^2} = \frac{gkY}{b\rho L}\left[-3.18\sin\left(\frac{kb}{L^2}m_1 t\right) - 0.0156\sin\left(\frac{kb}{L^2}m_2 t\right)\right] \tag{8}$$

If all terms except the first are discarded, the maximum moment at the center of the beam is:

$$M_1 = \frac{gkY}{b\rho L}\left[\pm 3.18\right] \tag{9}$$

If we let R represent the ratio of the moment due to the slam to the hogging moment then

$$R = \frac{M_1}{M} \tag{10}$$

and let

$$c_w = \sqrt{\frac{g\lambda}{2\pi}} \tag{11}$$

represent the celerity of a wave of wave length λ . With

$$v_o = a\sqrt{\frac{2\pi g}{\lambda}} \tag{12}$$

which makes the initial velocity that of the orbital velocity of a surface particle of a wave of amplitude a, and

$$\lambda = L$$

$$k = \frac{L}{25}$$

$$b = 8000 \text{ ft/sec}$$

$$\delta = \frac{L}{40}$$

$$a = \frac{L}{40}$$

then $C_w = 46.2$ ft/sec (Corresponding to a wave length of about 416 feet)

then

$$R = \frac{b}{160C_w} \tag{13}$$

On the above basis this yields $R = 1.10$.

Even though no great precision can be expected from a computation based upon as extensive idealizations as were used here, the results obtained do give support to the idea that slams which might reasonably be expected to occur could produce bending stresses in the ship's structure of the order of magnitude of those produced by the hogging condition.

Limiting Local Pressures Applied to a Ship's Plating Due to Slamming

When an abrupt contact occurs between the water surface and the hull of a ship under conditions which require their relative velocities to be simultaneously extinguished over some area, the elasticity of the plating and the compressibility of the water may be the factors which limit the pressures generated. To form some idea of the magnitude of these pressures, we may consider the velocity change in the water to be propagated as a plane wave. In so doing we may follow the treatment in paragraph 276 of Chapter X of Lamb (2). At the instant of impact a compressional wave is initiated which moves away from the contact surface at the velocity c^*. The value placed on this velocity in the footnote of page 477 of the above reference is 4956 feet per second at 17°C in sea water. Behind this wave the new conditions are established while ahead of it the old conditions remain. The transition from the old to the new conditions occurs abruptly at the wave front. The pressure and velocity changes are related in the manner.

$$p^* = \frac{K}{c^*} u \quad , \tag{14}$$

where p^* represents the pressure change (lb/ft^2), K the bulk modulus of water (lb/ft^2), c^* the velocity of wave propagation (ft/sec), and u the velocity change associated with the pressure change p^* (ft/sec). With $K = 42700000\ lb/ft^2$ and $c^* = 4956$ ft/sec, the above equation leads to $p^* = 8620\ u$.

This means that for each foot per second of velocity extinguished a pressure of 8620 pounds per square foot will appear if the compressibility of the sea water is the only mitigating influence. This is equivalent to about 60 pounds per square inch for each foot per second of velocity extinguished.

The life of these impact pressures is brief since they can exist at maximum intensity only for what time is necessary for the compressional wave to reach a point of relief and a wave of restoration to return from the point of relief to the impact area. Relief will generally be found at the sea surface because the pressure transmitted by the incoming wave cannot be sustained there. It must be realized that the phenomena described here will quickly assume a three-dimensional character and that the one dimensional aspect of the phenomena described above will prevail only for the brief period of time required for the compression wave to traverse a distance equal to approximately the diameter of the area of impact. Because a point of relief will generally be close, the round trip travel time, as described, may be sufficient to establish the duration of the maximum pressure phase closely enough for our purposes.

This estimate of duration is useful for the purpose of deciding whether the yielding of the ship's plating can be a factor in reducing the maximum pressures developed by the blow. The plating to which the pressure is applied will have a certain geometry and will be supported in some way by a structure in the interior of the hull. The details of the geometry and the support will determine what the natural period of vibration of the plate in its gravest mode will be. Because the plating is in contact with water on the outside, this period will be longer than it would be if the plating had air on both sides instead of only one side. The effect of the water can be accounted for approximately by assuming that a thickness of 0.41d[1] of water moves with the plate, where d represents the narrow dimension between supports in a rectangular bay whose long dimension is ℓ.

The question as to whether the flexibility of the plating can mitigate the pressure due to the blow can be answered in the affirmative if the round trip travel time of the pressure wave in the sea water is short compared to 1/4 of the period of the gravest mode of vibration of the plating. If the round trip travel time is equal to or long compared to the quarter period then no appreciable mitigation can be expected.

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1 See, for example, Rayleigh's "Theory of Sound" Vol. II, page 180, paragraph 307. His investigation relates to a circular pipe of radius R for which he obtains $0.82\ R$ as the addition to the length of a pipe to account for the flow pattern beyond the opening at one end. This value is appropriate if the pressure is applied locally so that the bay receiving the pressure moves and the adjacent parts do not move.

It may be noted that a dynamic factor is present in the structural behavior of the plate since the stresses due to a load suddenly applied are double those due to the same load applied gradually.

These considerations apply when contact is made over an area in a time which is short compared to the time required for a compressional wave to traverse the area. When the contact is made gradually the incompressible fluid idealizations will normally apply. Gradual entry should be the usual experience. In the unusual case compressibility sets a limit to the attainable pressures.

Checks

The beam formulas and computations described herein have been checked by Mr. Bernard d'Utruy.

Comments

The estimates made here assume that contact between the ship and the sea surface occurs simultaneously over some area. The probability that this should happen is, of course, quite remote and these estimates represent, therefore, limiting conditions which might be realized if the physical conditions took their most unfavorable forms. The possibilities are not infinitely remote, however, because they can be closely realized if only the time of application of the forces is small compared to the period of the gravest mode of vibration of the structure to which they are applied. For a liberty ship, for example, a period of 0.56 sec. is estimated. If the slamming forces were applied within an interval of the order of 0.1 sec., therefore, they would affect such a ship about as strongly as though they were actually applied instantaneously. This statement applies to the amidship bending moment caused by the slam.

In the overwhelming majority of cases the slamming forces would not be applied simultaneously. In reference 3 to 8 inclusive, the investigators Szebehely, Todd, Lum and Bledsoe, have by well designed experiments and consumate mathematical skill, explored the consequences of progressive applications of slamming forces. It seems certain that the types of pressure patterns they find and the forces accompanying them are the types commonly experienced. Some further support for the belief that the slamming conditions usually experienced are of the progressive type is obtained from the result of measurements of bending stresses made on ships at sea. A summary of such (5) (10) results indicates that the bending stresses due to slamming may increase the stresses produced amidship by normal wave action by some 30 per cent.

In the case of local pressures due to slamming, the progressive development of contact between water and hull can hold the pressures much below the maximum which would be physically possible.

Even though the possibilities of realization of the maximum slamming forces which nature is capable of delivering to a ship may be remote it seems to the writer worthwhile to realize that such possibilities exist. Welded ships would appear to be vulnerable to damage or destruction through the sudden applications of slamming stresses (9) especially when they are operating in cold waters under storm conditions. The initiation of a brittle fracture under such conditions can result in the ship being cut in two (9).

These estimates have been based on the slamming due to emergence of the forefoot. It seems probable, however, that different kinds of slams could be produced by seas of the confused type. The discovery of such possibilities and identification of the mechanism which produces them, either by observation of the behavior of ships at sea or by model testing, would add an important item to our knowledge of the stresses which may be imposed upon a ship in a seaway.

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