On the Frequency Spectrum of Wind Generated Waves

by

G.M. Hidy
National Center for Atmospheric Research
Boulder, Colorado

and

E.J. Plate
Fluid Dynamics and Diffusion Laboratory
Colorado State University
Fort Collins, Colorado

An interesting phenomenon in fluid dynamics is the generation of water waves by wind. The properties of wind generated waves, and the mechanism for their growth have been investigated by workers in many fields from civil engineering to oceanography. It has been found that a wavy surface may be described statistically in part by the spectral energy function $\Phi(f)$, where f denotes the frequency. $\Phi(f)$ represents the Fourier transform of the calculated autocorrelation function for water waves. The average potential energy E contained in the train of waves is proportional to the variance σ^2 of the surface:

$$E \propto \sigma^2 = \int_0^\infty \Phi(f)\,df. \qquad (1)$$

Thus, $\Phi(f)$ denotes that part of the total potential energy which corresponds to waves of frequency f.

For a particular frequency band df, with center frequency f, Φ increases with wind speed V, and fetch F up to a certain value. As the wind speed and fetch continue to increase, the high frequency region of Φ tends to approach an equilibrium distribution. Phillips[1], on dimensional grounds, found that the equilibrium range for gravity waves has the form:

$$\Phi \propto g^2 f^{-5} \quad (2)$$

where g is the gravitational acceleration. On the other hand, Hicks[2] has suggested that the spectrum for "pure" capillary waves, which should depend only on the ratio of the surface tension T to the mass density ρ and the frequency, can be described as:

$$\Phi \, (T/\rho)^{2/3} f^{-7/3}. \quad (3)$$

Spectral measurements of waves on lakes and on the ocean have produced considerable evidence for the existence of the equilibrium range for gravity waves. However, to the authors' knowledge, no data have given direct verification for the $f^{-7/3}$ range in capillary waves. The spectra of Cox, as calculated by Hicks[3], suggest that high frequency components differ from the behavior predicted by Eq.(2), but these measurements appear to be inconclusive.

Spectra showing equilibrium behavior are frequently found for ocean waves at moderate conditions of wind speed and fetch. Phillips[1] has suggested that equilibrium requires breaking waves of white caps on the water surface. However, several observed spectra have approached the f^{-5} rule at high frequencies without the surface displaying white caps. New measurements of small amplitude waves generated by wind over water standing in a channel also indicate that white caps are not necessary for equilibrium conditions in the high frequency regime. Furthermore, an extensive range fitting the f^{-5} rule can be found for small waves generated in the channel at short fetch and for moderate wind speeds.

Wind conditions and fetch lengths required for establishing equilibrium spectra for surface waves so far have been rather poorly defined. The measurements of waves in the channel now suggest definite lower limits for wind speed and fetch to generate equilibrium conditions.

The observations were made in the wind-water tunnel at Colorado State University.[4] The experiments were undertaken for wind velocities, taken approximately 0.2 meter above the water, of 0 - 15 mps. The fetch length along the channel ranged from 0 - 12 meters. The mean water depth was varied from 0.025 - 0.105 meters. The variation in water depth was measured continuously with a capacitance gauge similar to Tucker and Charnock's.[5] The response of the capacitance gauge was recorded on an oscillograph. These records were digitized, and the frequency spectra were calculated on a high speed computer by the method of Blackman and Tukey.[6]

A typical spectrum for waves in the channel is shown in Fig. 1. A sharp increase to a peak is observed along with a somewhat more gradual decrease in the high frequency range, which tends to follow the f^{-5} rule. The existence of the secondary peaks in the spectrum, along with visual observations and regular oscillations in the autocorrelations[7] suggest that the water surfaces may tend to develop a nearly regular, periodic component on which smaller random waves are superimposed. These features also are often characteristic of spectra calculated for ocean waves.

The fluctuations in Φ at high frequencies can be attributed partly to the calculation procedure used to determine Φ, and partly to the random errors introduced during reading of the oscillograph charts. The errors resulting from the calculating scheme easily can be found by directly filtering out the Fourier components with filters applied to the original signal. That is, the spectrum can be determined directly using an electronic spectrum analyzer on the output voltage of the capacitance gauge. This is currently being undertaken. The random errors in the digital spectra can be smoothed out and an estimate of the noise level introduced by the random "signals" can be determined by increasing the band-width of filtering for spectral estimates at high frequencies. This has been carried out for the spectral data by the following procedure. The spectral estimates, Φ, were retained over intervals of 0.069 cps from 0.5 - 1.5 f_m, where f_m denotes the frequency where Φ is a maximum. From 1.5 f_m to 5.0 f_m, Φ was averaged over 0.69 cps intervals. For frequencies greater than 5.0 f_m, averaged intervals of 1.38 cps were used.

For the spectrum shown in Fig. 1A, the result of smoothing ($\bar{\Phi}$) is indicated in Fig. 1B. The overall features of the original data are retained, but they stand out more clearly as a result of the averaging technique. The smoothed spectrum approached an asymptotic value of approximately 3.6 x 10^{-9} m^2 - sec. This value was assumed to correspond to the (white) noise level of the spectrum, $\bar{\Phi}_n$. The data were corrected for this noise by subtracting appropriate values of $\bar{\Phi}_n$ from all values of $\bar{\Phi}$.

An important feature exhibited by many of the spectra observed for waves in the channel was a tendency to develop a similarity shape. The shape of the frequency spectrum can be expressed, with Eq. (1) in normalized form, as:

$$\frac{f_m \Phi}{\sigma^2} = \psi(f/f_m), \qquad (4)$$

where ψ is a dimensionless quantity representing a "universal" spectral density function.

Typical distributions of ψ, which have been smoothed and corrected for noise level, have been calculated, and these are shown in Fig. 2. These data correspond to small gravity waves generated at moderate wind speeds and short fetches. The conditions of V, F, and d for these spectra are shown in Table I along with values of σ, f_m and $\bar{\Phi}_n$. For all these cases, the spectra develop a similarity shape which tends to approach the f^{-5} dependence over approximately two decades.

The data indicate that the similarity form tends to develop rather quickly in the spectra of wind generated waves in a channel. Specifically, similarity as predicted by Eq. (4) appears for $V \gtrsim 4.5$ mps, for $F \gtrsim 5$ meters, and for water depths $>$ .05 meters.

It is interesting to note that, in the highest frequencies, there is a tendency for the normalized spectra to develop a slope less that f^{-5}. Capillary wave behavior should appear above $f \approx 13$ cps in the frequency spectra. Only two cases, 163 and 188, actually reach this range. In Fig. 2, capillary waves should appear for Case 163 at $f/f_m = 2.7 - 3$, and for Case 188, at $f/f_m = 6.8 - 7$. Hence, in Fig. 2, these spectra may indicate the beginnings of a transition to the $f^{-7/3}$ range.

Unfortunately, the existence of the $f^{-7/3}$ rule cannot be verified generally in the present results because this region is hidden at high frequencies in the noise. Even in cases 163 and 188 the data only reach $f \approx 19.5$ cps. Therefore, these data cannot be considered conclusive evidence for the existence of an extensive equilibrium range in capillary waves.

The authors are grateful to Colorado State University for the use of the wind-water tunnel facility. The measurements at CSU were made by C. Goodwin, and H. Shokouh. The spectral calculations were made on the NCAR-CDC 3600 computer; R. Biro did the programming. This study was supported by the National Science Foundation through its contract with NCAR.

Notes and References

[1] O.M. Phillips, J. Fluid Mech. 4, 426 (1958).

[2] O.M. Phillips, J. Mar. Res. 16, 226 (1958).

[3] B.L. Hicks, in Ocean Wave Spectra, National Academy of Sciences (Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963), p. 95.

[4] E.J. Plate, "A research facility with concurrent air and water flows," to be published in Houille Blanche.

[5] M.J. Tucker and H. Charnock, " A capacitance-wire recorder for small waves," in Proc. of the 5th Conference on Coastal Engineering (Council on Wave Research, Univ. of California, Berkeley, California, 1955).

[6] R.B. Blackman and J.W. Tukey, The Measurement of Power Spectra (Dover, New York, 1958).

[7] G.M. Hidy and E.J. Plate, "Wind generated waves in water standing in a channel," in preparation.

Captions for Table and Figures

Table I

Properties of waves generated by wind blowing over water standing in a channel.

Fig. 1

A typical example of a frequency spectrum for wind generated waves on water in a channel. A. Uncorrected spectrum, Case 212; B. Smoothed spectrum, Case 212.

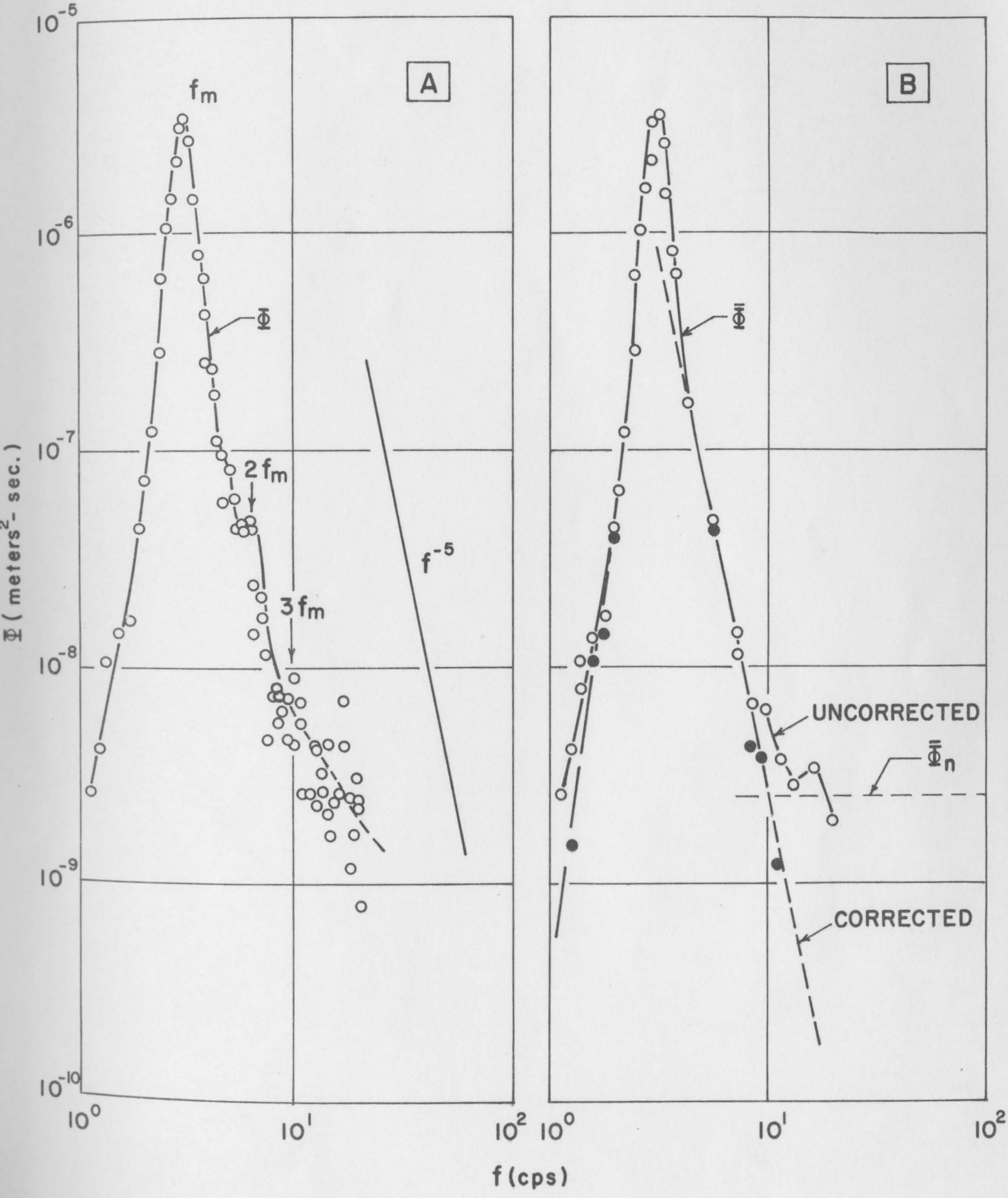
Fig. 2

The similarity spectrum for wind generated waves on water in a channel.

Table I

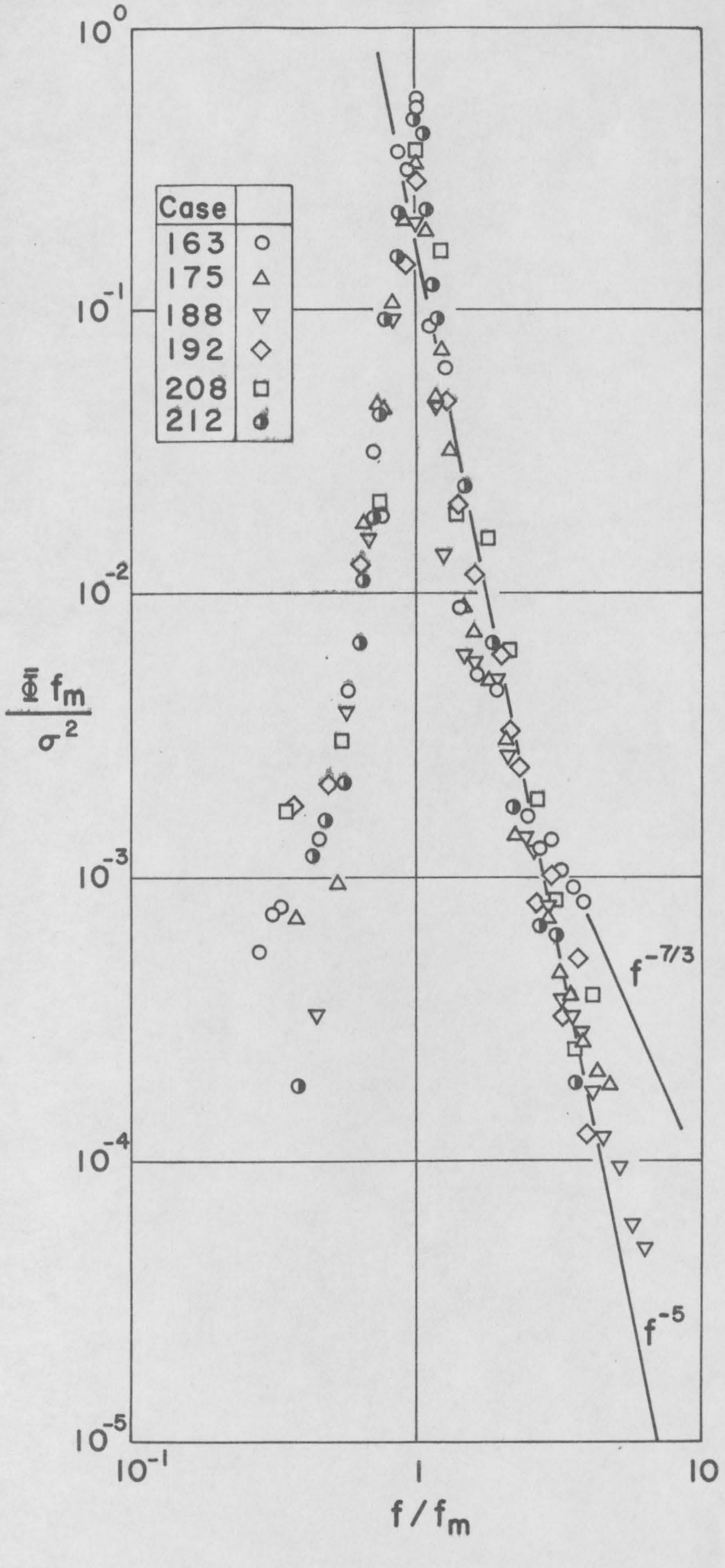
Case	d(meters)	V(mps)	F(meters)	$\sigma \times 10^2$(meters)	f_m(cps)	$\bar{\Phi}_n$(m^2-sec)
163	0.0254	6.10	5.24	0.131	4.83	---
175	0.102	10.7	10.7	0.767	2.36	3.9×10^{-9}
188	0.102	17.4	8.16	1.36	1.93	8.4×10^{-9}
192	0.0508	10.7	11.5	0.538	2.33	5.5×10^{-9}
208	0.0508	9.15	7.86	0.614	2.48	3.6×10^{-9}
212	0.102	10.7	5.74	0.457	3.17	1.6×10^{-9}

Figure 1



A
B
f_m
Φ
$2f_m$
$3f_m$
f^{-5}
$\bar{\Phi}$
UNCORRECTED
$\bar{\Phi}_n$
CORRECTED
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
Φ (meters^2- sec.)
10^0
10^1
10^2
f (cps)

Figure 2



Case
163
175
188
192
208
212
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\frac{\Phi_{\Pi} f_m}{\sigma^2}$
$f^{-7/3}$
f^{-5}
10^{-1}
1
10
f/f_m