The Application of Shear Functions in the Study of the Meso- and Microstructure of the Atmosphere

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ABSTRACT

Initial results of an investigation of the applicability of shear functions in the analysis of meso- and microscale atmospheric motions are presented. It is shown that if the structure function of a given variable follows a power law of the form $D(r) = c_1^2 r^p$, the mean magnitude of the difference of a velocity component, V, over scale r also follows a power law which is of the form $\overline{|\Delta V|} = c_o c_1 r^{p/2}$ (the shear function). The power law is similar to the one derived empirically by Essenwanger (1963). A statistical model is presented to show that the c_o may be a function of the intermittency of the variable being analyzed. If c_o is known, the power spectrum may be derived directly from the shear function.

The application of these findings to detailed vertical wind soundings, pillow balloon soundings and to longitudinal gust data and temperature data collected by aircraft shows that although the shear function is generally a well-behaved, exponentially increasing function of differencing interval, r, the intermittent nature of the data from which the shears are derived presents difficulties in deducing the spectrum from the shear function alone. Spectra derived from shear functions with known intermittency characteristics show significant deviations from those derived from the Fourier transformation of the autocorrelation function. One of the primary causes of these differences appears to be the difficulty in correctly estimating the exponent of the shear function power law at small scales. It is concluded (subject to further study) that the shear function should not be used in the direct estimation of the spectrum. It is shown, however, that shear functions may be useful in the analysis of the mesoscale and that c_o, the ratio between the shear function and the square root of the structure function may be considered as a quantitative measure of the intermittency of microscale turbulence. Further study of shear function applications to mesoscale analysis is recommended.

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I. Introduction

One aim of the on-going research effort entitled, "The Structure of Turbulence in the Free Atmosphere"* is to investigate the applicability of the shear function in the study of clear air turbulence (CAT). Initial results of that investigation are reported in this interim report.

In a recent paper, Essenwanger and Reiter (1969) have suggested that the mean magnitude of wind shear vectors may be a useful parameter in describing the turbulent state of the atmosphere. Their proposal is based on certain relationships between shear magnitudes as a function of differencing interval (the shear function) and the structure function.

In Part II of the present paper, the shear function-structure function relationship and its implications as to the spectrum of turbulence are derived in detail. In Part III, the applications of the shear function to detailed vertical wind soundings, to horizontal trajectories of superpressure balloons and to CAT data gathered by aircraft are considered. Examples of the behavior of the shear function for the different data types are presented and attempts are made to derive the spectrum from shear function computations alone for two of the data types.

II. The Shear Function

Recognizing the inability of the ordinary rawinsonde to resolve vertical wind shears over thin layers, Essenwanger has investigated the relationship between the vertical shear of the horizontal vector

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wind and the shear interval. He found (Essenwanger, 1963, 1965) by utilizing detailed rocket soundings that the mean vector shear magnitude, $\overline{|\Delta \mathbb{V}|}$ (Units: LT^{-1}), is related to the shear interval, Δh, as

$$\overline{|\Delta V|} = a_o (\Delta h)^{a_1} \tag{1}$$

where Δh, the shear interval, is restricted to scales less than about 1 km. Further, the standard deviation of the shear magnitudes, $\sigma_{|\Delta \mathbb{V}|}$, is related linearly to the shear; i. e.,

$$\sigma_{|\Delta \mathbb{V}|} = A_o + A_1 \overline{|\Delta \mathbb{V}|} \tag{2}$$.

In equations (1) and (2), a_o, a_1, A_o and A_1 are parameters which vary with climatological conditions (Essenwanger and Billions, 1965) and have units of $L^{1-a_1}T^{-1}$, L^oT^o, LT^{-1} and L^oT^o, respectively. The determination of these parameters from a large sample of detailed soundings for a given geographical location allows one to predict shear distributions over small thickness layers from less detailed soundings which are taken on a routine basis.

Equations (1) and (2) have been verified in statistical studies of large numbers of balloon soundings by Armendariz and Rider (1966) and Belmont and Shen (1966). These investigators found a_1 (equation (1)) to be approximately 1/2 for mean vector shears and 1/3 for mean extreme vector shears in agreement with Essenwanger (1963); however, A_o (equation (2)) was close to zero for the data analyzed. As shown by Essenwanger (1965), a_1 for mean vector shears derives its value from the persistence of the mesostructure of the wind profile. In this case the shear vectors are distributed according to the bivariate normal distribution (circular). Mean extreme shears deviate from this distribution, resulting in a smaller value of a_1.

Expressions similar to (1) and (2) may be derived in a different manner. The structure function, $D_f(\tau)$, defined generally as

$$D_f(\tau) \equiv \overline{[f(t+\tau) - f(t)]^2} \tag{3}$$

has been used frequently in the study of atmospheric turbulence, particularly by Soviet investigators (e. g., see Tatarski, 1961). In (3), f(t) is a random function of time, t, and τ is a time lag. Tatarski has shown that an outstanding quality of these statistics is that for values of τ which are not too large, $D_f(\tau)$ may be treated as a stationary random process although f(t) may not be stationary. A structure function may also be defined for the analysis of the spatial structure of vector (or scalar) quantities; i. e.,

$$D_f(\vec{r}) = \overline{[f(\vec{r} + \vec{r}_1) - f(\vec{r}_1)]^2} \tag{4}$$

where $\vec{r}$ and $\vec{r}_1$ are position vectors. Although the meaning of the structure function is not always clear, especially under non-isotropic, non-homogeneous conditions, observations have shown (Tatarski, 1961; Fichtl et al., 1969) that at horizontal and vertical atmospheric scales which are not too large, a velocity structure function of the form

$$D_f(r) = c_1^2\, r^p \tag{5}$$

is often applicable. In (5), c_1 and p are constant (units: $L^{1-p/2} \cdot T^{-1}$, L^oT^o, respectively) for a particular set of data and r is the differencing interval.

If a population of wind shears, ΔV_i, of a component of the wind over some scale r is considered, then the variance of the ΔV_i is given by

$$\sigma^2_{\Delta V} = \frac{1}{N} \sum_{i=1}^{N} (\Delta V_i - \overline{\Delta V})^2 = \overline{\Delta V^2} - \overline{\Delta V}^2 \tag{6}$$

where N is the population size and the overbar represents an arithmetic average. Assuming $\overline{\Delta V} = 0$, (6) becomes

$$\sigma^2_{\Delta V} = \overline{\Delta V^2} \tag{7}$$

where $\overline{\Delta V^2}$ has the form of a structure function, i. e.,

$$\overline{\Delta V^2} = D_f(r) \tag{8}$$.

From statistical theory (e. g., Kendall and Stuart, 1958), the mean deviation for the population of ΔV_i (or, in this case, the component shear function) may be defined as

$$\overline{|\Delta V|} = \int_{-\infty}^{\infty} |\Delta V| \, f(\Delta V) \, d(\Delta V) \tag{9}$$

where $f(\Delta V)$ is the probability density function of ΔV and the vertical parallel lines indicate the absolute value. If ΔV is normally distributed, the integration of (9) gives the result

$$\overline{|\Delta V|} = \sqrt{\frac{2}{\pi}} \, \sigma_{\Delta V} \simeq 0.8 \, \sigma_{\Delta V} \tag{10}$$.

Cornu's test for the normality of a sample (Brooks and Carruthers, 1953) is based on the ratio of the mean deviation to the standard deviation. If this ratio is significantly different from $\sqrt{2/\pi}$ then it may be shown that the variable, ΔV_i is not normally distributed. The converse, however, is not necessarily true (see also Kendall and Stuart, 1958).

Since (as will be shown) the quantitative significance of the relation of the shear function to the spectrum function rests in the existence of an equation of the form of (10), it will be useful to pause briefly and examine the observed distributions of ΔV_i. The important questions are: first, are the ΔV_i distributed normally? or, put another way, does equation (10) hold? Secondly, if the ΔV_i do not have a Gaussian distribution, can an expression similar to (10), i. e., an equation of the form

$$\overline{|\Delta V|} = c_o \sigma_{\Delta V} \tag{11}$$

be derived? Dutton et al. (1969) have presented examples of the distributions of gust components in CAT conditions. A notable characteristic of these distributions was their tendency to be leptokurtic, i. e., the frequencies of gust velocities exceeded the Gaussian distribution at the origin and in the tails. Dutton et al. point out that if a gust record is such that it is composed of "bursts" of turbulence, such that each "burst" is characterized by a Gaussian distribution but with a different variance, the combined distribution will be as observed, leptokurtic. The basis of such a model is the observed patchy or intermittent nature of CAT. Townsend (1948) proposed a quantitative measure of intermittency on the basis of the kurtosis of the velocity derivatives. Batchelor (1953) has pointed out that the intermittent nature of turbulence lies in the tendency for high frequency disturbances to concentrate their energy in confined regions of space. Novikov and Stewart (1964) have proposed a physical-statistical model of turbulence based on intermittency characteristics. Evidence supporting the existence of similar distributions for velocity differences is given by Batchelor (1953). Also, the majority of distributions of gust velocity differences computed in the present study display kurtosis values larger than the Gaussian value of three. Thus, there is much laboratory and theoretical evidence which suggests that the statistical model discussed by Dutton et al. (1969) is valid not only for gust velocities, but also for velocity differences.

If it is assumed that the record of velocity differences has a "burst" character such as that discussed by Dutton et al. (1969) then it can be shown that the kurtosis, μ_4, of the distribution of the velocity differences (zero mean), defined as

$$\mu_4 = \frac{\overline{\Delta V^4}}{(\overline{\Delta V^2})^2} \tag{12}$$

is greater than three ($\mu_4 = 3$ for a Gaussian distribution). Furthermore, c_o in equation (11) is given by

$$c_o \leq \sqrt{\frac{2}{\pi}} \tag{13}$$

where the equality holds for the normal distribution. Details of the derivation of (12) and (13) may be found in the Appendix. Thus, by utilizing the knowledge that CAT tends to be intermittent, it has been possible to generalize the relation between the shear function and the standard deviation. However, in doing so, it has become apparent that c_o in equation (11) is a function of the intermittency.

One must also realize that if the model proposed above is not valid, the shear and structure functions at a given r can only be related through the expression

$$\sigma^2_{|\Delta V|} = \overline{|\Delta V|^2} - \overline{|\Delta V|}^2 \tag{14}$$

where

$$\overline{|\Delta V|^2} = \overline{\Delta V^2} = D_f(r) \tag{15}.$$

In the following development, we will assume that equation (11) is a correct general form so that by virtue of (7) and (8)

$$\overline{|\Delta V|} = c_o [D_f(r)]^{\frac{1}{2}} \tag{16}$$

where c_o is given by (13). Utilizing (5), equation (16) may be rewritten as

$$\overline{|\Delta V|} = c_o c_1 r^{p/2} \tag{17}.$$

The variance of the magnitude of the shear, $|\Delta V|_i$, at some scale, r, is defined by (14) which with (15) may be written as

$$\sigma^2_{|\Delta V|} = \overline{\Delta V^2} - \overline{|\Delta V|}^2 \tag{18}.$$

Since $\overline{\Delta V^2} = \sigma^2_{\Delta V}$ (7), we find

$$\sigma^2_{|\Delta V|} = \sigma^2_{\Delta V} - \overline{|\Delta V|}^2 \qquad (19)$$

or with (11)

$$\sigma^2_{|\Delta V|} = (c_o^{-2} - 1)\ \overline{|\Delta V|}^2 \qquad (20)$$

so that

$$\sigma_{|\Delta V|} = c_2\ \overline{|\Delta V|} \quad , \quad c_2 = (c_o^{-2} - 1)^{\frac{1}{2}} \qquad (21),$$

where c_2 is non-dimensional.

Equations (17) and (21) are of the same form as (1) and (2), respectively, with $a_o = c_o c_1$, $a_1 = p/2$, $\Delta h = r$, $A_1 = c_2$ and $A_o = 0$. Therefore, based on the observed behavior of the structure function at the small scales and allowing for the intermittency of the record, equations have been developed which are similar to those derived empirically by Essenwanger (1963) and others. Essenwanger and Reiter (1969), in recognizing this similarity, have pointed out that the relationship of the shear and structure function allows the former to be expressed directly in terms of the spectrum function. Tatarski (1961) has shown that a structure function of the form of (5) may be transformed directly to the spectrum function, i. e.,

$$E(k) = \frac{\Gamma(p+1)}{2\pi} \left(\sin\frac{\pi p}{2}\right) c_1^2\, k^{-(p+1)}, \quad 0 < p < 2 \qquad (22)$$

where E(k) is the spectral density, Γ represents the Gamma function, k is the wave number and c_1^2 and p are given by (5). In terms of the shear function [equations (1) and (17)]

$$E(k) = \frac{\Gamma(p+1)}{2\pi} \left(\sin\frac{\pi p}{2}\right) \left(\frac{a_o}{c_o}\right)^2 k^{-(2a_1-1)} ,$$

$$0 < a_1 < 1 \qquad (23).$$

It follows that a computation of the shear function of the form of (1) or (17)(with the knowledge of the intermittent nature of the shears) or the computation of the structure function allows a direct determination of the spectrum.

For the special case of local isotropy (Kolmogorov, 1941; Obukhov, 1941; Tatarski, 1961), the structure function is given by

$$D_{rr}(r) = C\epsilon^{2/3} \; r^{2/3}$$

and (24)

$$D_{tt}(r) = \frac{4}{3} C\epsilon^{2/3} \; r^{2/3}$$

where $D_{rr}(r)$ and $D_{tt}(r)$ are, respectively, the longitudinal and transverse structure functions, C is a universal constant and ϵ is the rate of dissipation of turbulent energy. The corresponding shear functions are

$$\overline{|\Delta V|}_{rr} = c_o C^{1/2} \epsilon^{1/3} r^{1/3} \qquad (25)$$

and

$$\overline{|\Delta V|}_{tt} = c_o \left(\frac{4}{3} C\right)^{1/2} \epsilon^{1/3} r^{1/3} \qquad (26).$$

The rate of energy dissipation, ϵ, can thus be determined from shear function computations under conditions of local isotropy.

Utilizing the theory developed above, Essenwanger and Reiter (1969) have suggested explanations of the shear function slopes found by Essenwanger (1963, 1965), Essenwanger and Billions (1965), Armendariz and Rider (1966) and Belmont and Shen (1966) on the basis of spectrum slopes derived by Kolmogorov (1941), Bolgiano (1959, 1962) and Phillips (1967). Because of the agreement of the spectral slopes with the shear function slopes with the assumption of the presence of a specific physical mechanism, it has been suggested that the treatment of shear functions could be considered as a simple alternative to standard methods of spectrum analysis.

III. Shear Function Applications

A. Some Basic Problems

Despite the apparent success of Essenwanger and Reiter (1969) in relating observed shear function slopes directly to spectrum slopes, the quantitative application of the shear function is restricted by a number of theoretical and practical problems. Some of the more important of these are:

(1) Shear function parameters (e. g., a_1 and a_o in equation (1)) found by Essenwanger (1963) and others were based on large, climatological samples of detailed vertical wind soundings. The study of CAT would require analyses of individual vertical and horizontal soundings. Also, the horizontal and vertical scales over which a shear function of the form of equation (1) is valid have not been established for individual soundings.

(2) The effect of intermittency on the shear-structure function relationship (11) would normally disallow the complete determination of the spectrum function from the shear function alone. Unless the intermittency is known quantitatively or may be estimated with some confidence, then, theoretically, only the slopes of the spectrum function may be implied directly from the shear function. This suggests that structure functions may be more applicable than shear functions for spectrum determinations. Also, c_o, the ratio between the shear and structure functions must be nearly constant with differencing interval. According to the statistical model of intermittency which has been adopted, this requires that the distribution of ΔV should not change appreciably from one lag to the next.

(3) One implication of the study by Essenwanger and Reiter (1969) is that the use of the shear function in the treatment of individual detailed wind profiles may be a simple and economic approach to the study of CAT. This application, however, is limited by the vertical stratification of the atmosphere, i. e., CAT is usually confined to thin layers (Reiter, 1968). One of the most accurate wind sounding systems available, the FPS-16/Jimsphere system, has a minimum vertical resolution of the order of 25-75 m depending on the data reduction method (Niemann, 1969; Fichtl et al., 1969). If one considers a turbulence layer to be typically of only a few hundred meters thickness, it becomes obvious that the confidence in shear function calculations and the associated spectral statistics in CAT layers would be small.

(4) The use of structure functions in the statistical treatment of scalar parameters, such as temperature (Tatarski, 1961; Fichtl et al., 1969) suggests that the shear function concept (within certain limitations) may apply to a wider group of variables. This possibility has not been investigated.

A number of the problems cited above may be examined through derivations of spectra from shear functions and the independent computations by other methods (e. g., Blackman and Tukey, 1958). Also, the possibility that the relationship between the shear and structure functions may be controlled by the intermittency of the samples indicates that the added computation of the structure function may be useful in the study of the intermittent characteristics of a given record. A series of experiments was conducted for these purposes. The data which were analyzed are listed in Table 1.

Data Type	Data System	Location	Date	Data Interval	Data Source
Horizontal wind velocity	FPS-16/Jimsphere	Cape Kennedy Florida	12/29/64	25 m (vertical)	NASA [1]
Vertical displacement	M-33/Pillow balloon	Fort Collins Colorado	1/ 5/68	~1000 m (horizontal)	CSU [2]
True air speed	NCAR Queen Air 80	Boulder Colorado	2/19/68	~ 10 m (horizontal)	NCAR [3]
Temperature	NCAR Queen Air 80	Boulder Colorado	2/19/68	~ 10 m (horizontal)	NCAR [3]

1. National Aeronautics and Space Administration
2. Colorado State University
3. National Center for Atmospheric Research

Table 1: Data Subjected to Shear Function Analysis

B. Examples of Shear Function Characteristics

In the determination of the shear function for individual horizontal and vertical profiles as would be done in turbulence investigations the computational procedure differs slightly from that utilized in the determination of equation (1). Essenwanger (1963) and others determined the mean magnitude of the shear vector, $\overline{|\Delta \mathbb{V}|}$, for a layer of given thickness, as

$$\overline{|\Delta \mathbb{V}|} = \frac{1}{N} \sum_{i=1}^{N} |\Delta \mathbb{V}_i| \tag{27}$$

where the $\Delta \mathbb{V}_i$ are individual vector shear magnitudes sampled from non-overlapping shear intervals and drawn from a large sample of soundings. The computations presented below are based on lagged differences (maximum lag equal to 10% of sample size) which allow overlapping intervals. Since the shear function computed for individual soundings by the latter technique is not common to meteorological literature, examples are presented below.

As noted in Section II A (Item (2)), FPS-16/Jimsphere soundings do not lend themselves to the study of CAT directly. However, they do provide excellent information with respect to the mesoscale vertical structure of the atmosphere (Scoggins, 1963; Weinstein et al., 1966; DeMandel and Scoggins, 1967; Fichtl et al., 1969; Endlich et al., 1969). Since it is the mesostructure of the atmosphere which provides the environment for the production of CAT (Reiter, 1969; Vinnichenko and Dutton, 1969), the analysis of this mesoscale plays an important role in turbulence investigations. Therefore, the first illustration of shear functions is taken from the analysis of a series of four FPS-16/Jimsphere soundings.

The data were collected at Cape Kennedy, Florida, between 1600 GMT and 2200 GMT December 29, 1964 and extended from 0.25 km to near 16 km MSL. Synoptic maps for this period indicate that a ridge of high pressure dominated the southeastern United States during the period of the soundings. Wind velocities were generally northerly (0-5 mps) near the surface and west to northwesterly (10-20 mps) near the tropopause.

Both vector and scalar shear analyses were performed on each profile in its entirety. As an illustration of the general characteristics of the calculated shear functions, the 1900 GMT analyses have been reproduced in figure 1. The main features of interest are:

(1) The mean vector shear magnitudes are approximately 1.5 times the mean scalar shear magnitudes for small shear intervals. This factor decreases slightly at larger differencing intervals to about 1.3.

(2) Both vector and scalar shear functions have slopes of the order of 0.85 at scales of less than 150 m. Slopes decrease for larger shearing intervals, approaching 0.5 at scales near 1 km.

(3) Because the slope of the shear function decreases with increasing differencing interval beyond 150 m, an equation of the form of (1) may be applied only to shears over smaller intervals.

(4) The characteristics of the scalar shears, as noted above, are in agreement with the observed behavior of structure functions computed from similar data by Fichtl et al. (1969). This assumes, of course, that the exponent p in (5) is twice the exponent of the shear function power law in (1).

In order to examine the behavior of shear functions for horizontally-gathered data, computations were made from two pillow balloon flights through lee waves (Wooldridge and Lester,

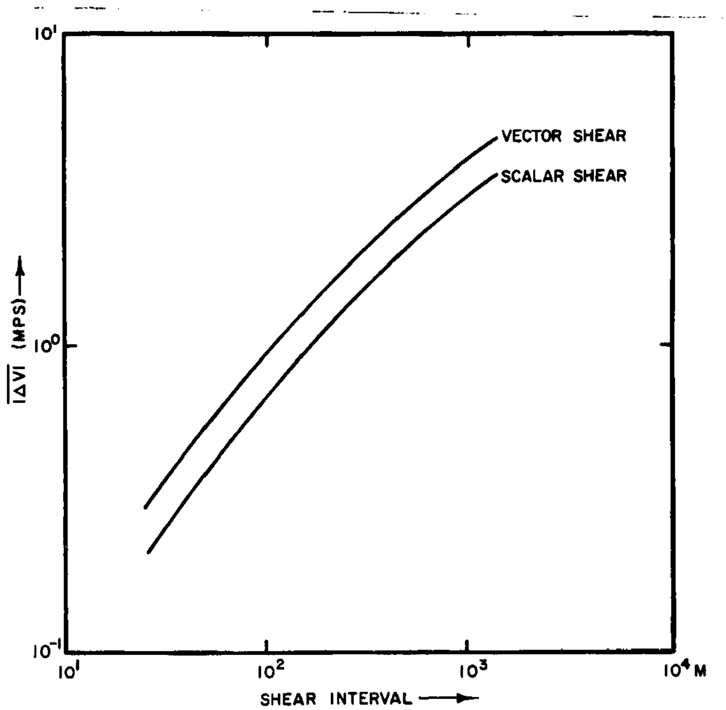

Figure 1. Vector (upper curve) and scalar (lower curve) shear analyses of FPS-16/Jimsphere data. Cape Kennedy, Florida, December 29, 1964, 1900 GMT.

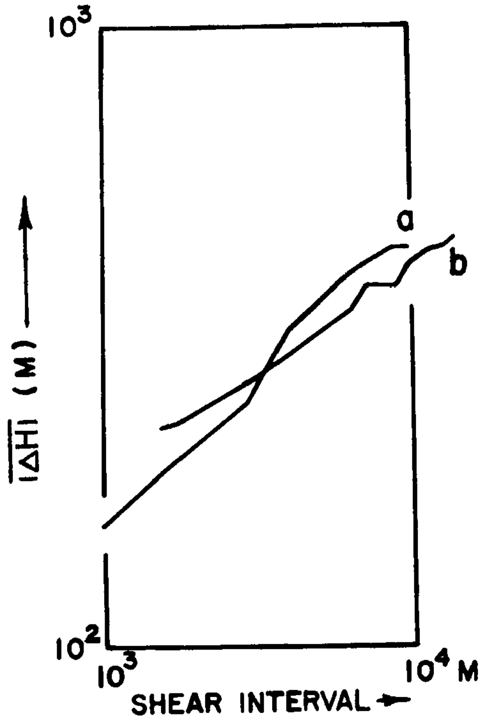

Figure 2. Shear function analysis of pillow balloon data. Fort Collins, Colorado, January 5, 1968. Curve 'a' 1622 GMT, curve 'b' 1850 GMT.

1969). Although only the mesoscale (1 km to 10 km) was resolvable in these data, it is well-known (Reiter and Foltz, 1967; Scorer, 1967, 1969) that lee waves often provide a favorable environment for CAT. The "shear" function in these cases takes on a different meaning, i.e., height differences along the balloon trajectory as a function of differencing interval. In figure 2, curve 'a' was derived from the radar tracking of a pillow balloon which was launched 2 1/2 hours before the flight upon which curve 'b' is based.

The primary characteristics of these analyses are:

(1) Shears for both cases are generally increasing functions of the shear interval although they do not display the smoothness which is characteristic of the vertical shears in figure 1. The causes for this feature are not known although one may attempt certain speculations. A trend due to balloon leakage was removed prior to analyses by fitting a quadratic function to the data by the least squares method. It is clear that the height fluctuations occurred at lower levels during the last half of the flight corresponding with curve 'a'. A variation of lee wave amplitude and/or wave length with height would then contaminate the data. Also, the time to complete the flights was of the order of two hours, a period during which significant changes in wave length may have occurred. This possibility is especially evident in the comparison of the two curves which are based on data samples separated in time by 2 1/2 hours.

(2) The mean slope of 'a' is about 0.5 while 'b' is 0.33. The rapid change of the slope of 'a' between 3 and 4 km could be due to one of the problems noted above or the presence of a well-developed lee wave mode at a slightly larger scale.

(3) The application of an expression of the form of equation (1) is only a rough approximation at scales larger than a few kilometers.

Analyses were also performed on aircraft data in order to examine the applicability of the shear function to smaller scale motions and to temperature fluctuations in the horizontal. Data were gathered by a Queen Air 80 (Anonymous, 1969) during the 1968 Colorado Lee Wave Experiment (Kuettner and Lilly, 1968; Lilly and Toutenhoofd, 1969). True air speed (TAS) and temperature data, calibrated and placed on magnetic tape at 0.125 second time intervals ($\overline{TAS}$ = 85 mps) were furnished by the National Center for Atmospheric Research (NCAR). Shear analyses were performed on a data sample collected during a downwind flight through a rotor located between the Continental Divide and Denver, Colorado on February 19, 1968. CAT was reported as severe within the rotor but was non-existent immediately upstream and 44 km downstream of the rotor position. TAS data were subjected to a high pass filter of the Martin-Graham type (Crooks et al., 1968) to remove unrepresentative TAS fluctuations with periods greater than 5-7 seconds ($\sim$ 500 m). The shear function example presented in figure 4 is an analysis of a 15-minute TAS data sample which included the rotor (see figure 3). The major characteristics of the analysis are:

(1) There is a smooth increase of the mean shear (longitudinal) to a scale slightly greater than 200 m.

(2) The slope of the shear function is approximately 1/3 at scales less than about 100 m.

(3) An equation of the form of (1) is applicable for differencing intervals less than about 100 m.

Unfiltered temperature data, also gathered by aircraft, were subjected to the same analysis and the results are shown in figure 5. The data sample was slightly larger than the TAS

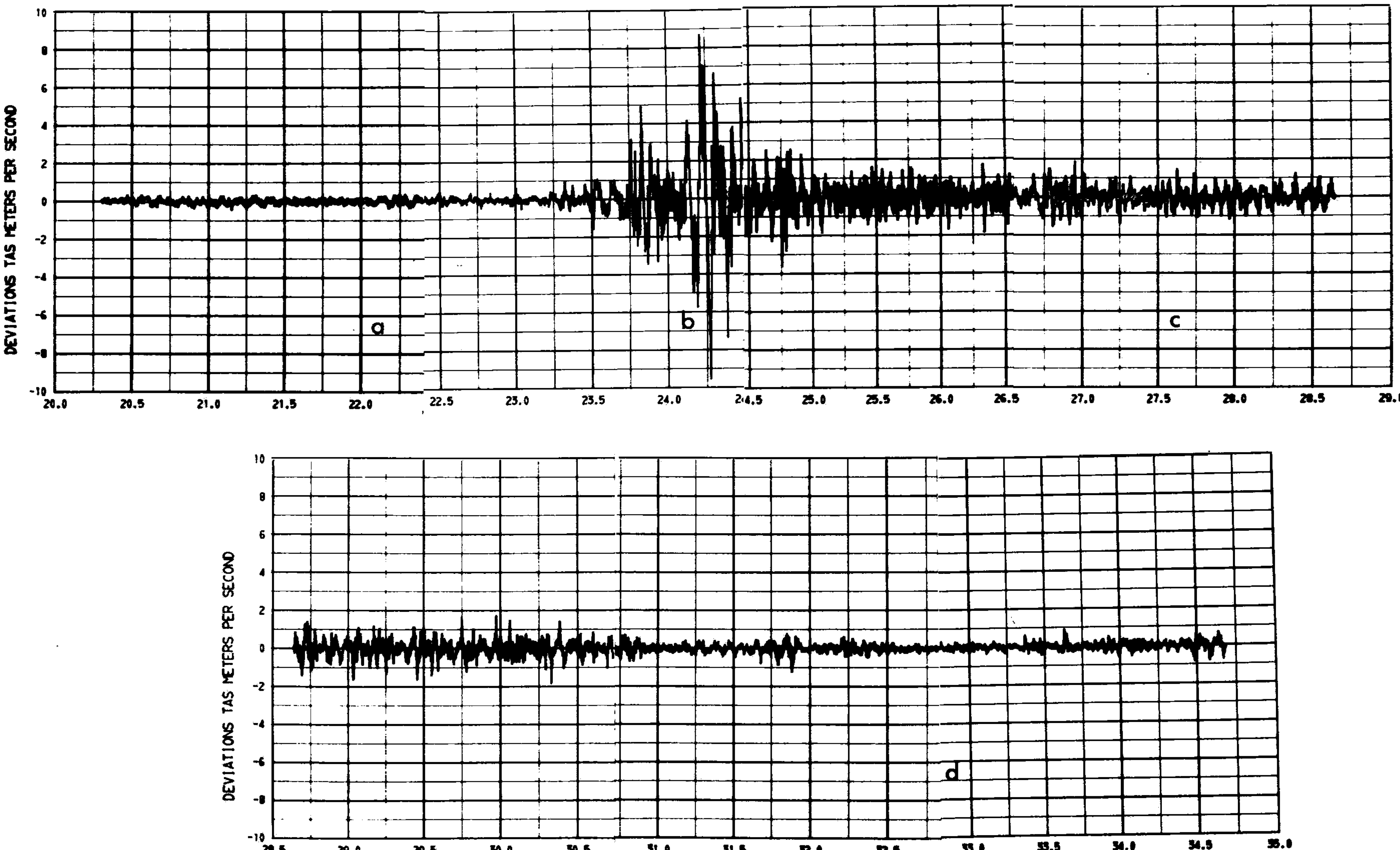

Figure 3. Filtered sample of longitudinal gust data collected during a downwind flight through a rotor. Boulder, Colorado, February 19, 1968. Abscissa is labeled in minutes after 1500 LST. The letters 'a', 'b', 'c' and 'd' are located at the center points of four 71-second samples selected for analysis.

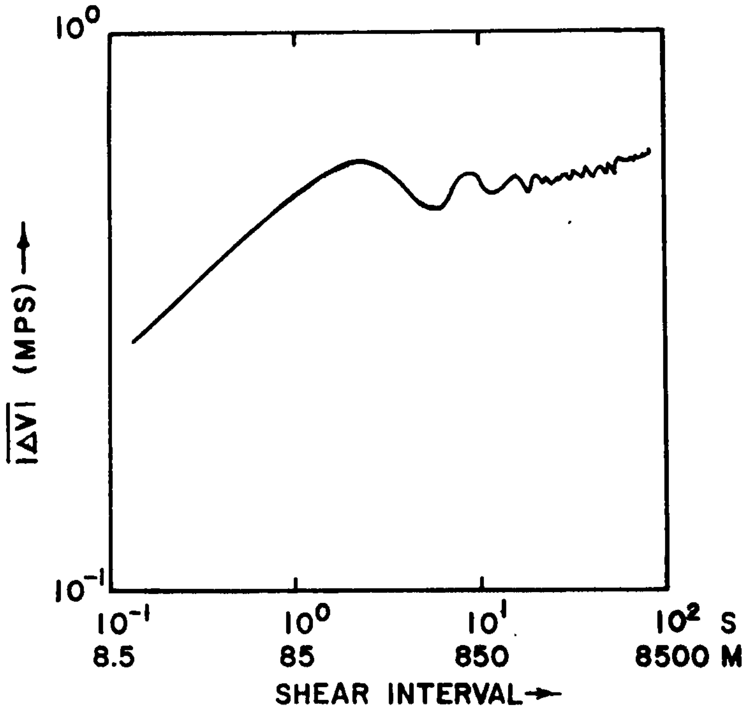

Figure 4. Shear function analysis of longitudinal gust data sampled by aircraft. Boulder, Colorado, February 19, 1968. 15-minute sample.

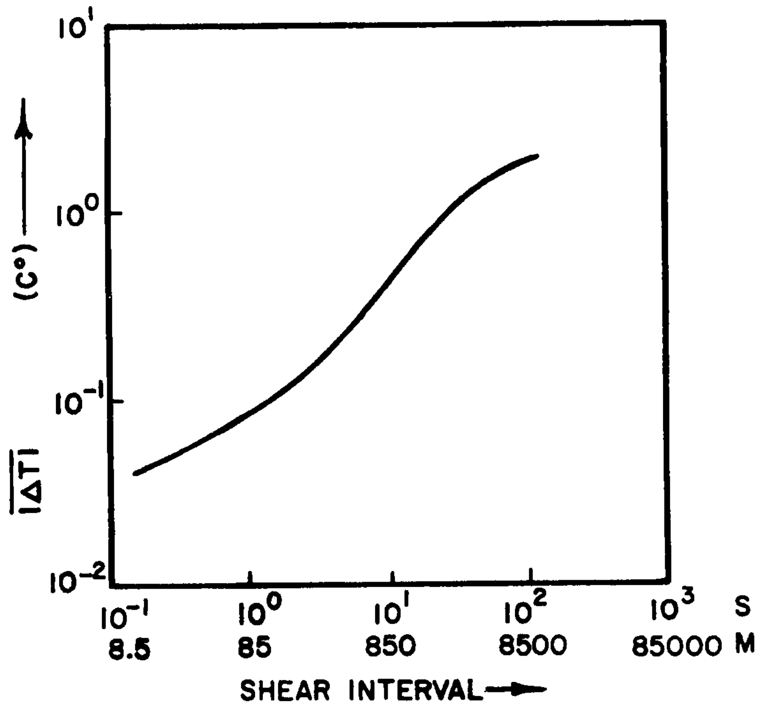

Figure 5. Shear function analysis of ambient air temperature sampled by aircraft. Boulder, Colorado, February 19, 1968. 20-minute sample.

sample, being 20 minutes in length, but it contained the same mesoscale features. The primary characteristics of figure 5 are:

(1) The smooth, monotonically increasing character of the temperature "shear". Mean temperature differences increase by more than an order of magnitude from scales near 8.5 m to 8.5 km.

(2) The slope of the shear function in the double logarithmic plot is 1/3 over the small scale range, as was the TAS shear (figure 4). At larger scales the slope increases to 2/3 and finally flattens at the large scale end to near 1/3.

(3) The behavior of the curve at scales larger than 85 m may not be representative as shown. The time scale has been converted to a distance scale by using the mean TAS. At larger scales, the mean ground speed (~ 100 mps) is more applicable because of the suspected standing-wave character of these large eddies. It is interesting to note, however, that the behavior of the shear function in figure 5 is similar to curve 'a' in figure 2 at similar differencing intervals.

The analyses presented in figures 1-5 have demonstrated that for the cases analyzed, the shear function when defined as in Section II is generally a well-behaved monotonically increasing function of the differencing interval. An exponential law, similar to that developed by Essenwanger (1963) appears to be valid at smaller scales. One possible exception to these results is found in the analysis of mesoscale pillow balloon data. The shear functions in those analyses are relatively irregular, reducing the confidence in the application of a power law. The exponent of the power law for all cases varied from 0.33 to 0.85, depending on the type of data and the conditions under which the data were collected. Since this range of exponents satisfies the condition $0 < a_1 < 1$ (equation (23)), spectra derived from the shear functions will be presented and compared with spectra determined independently from the Fourier transformation of the autocorrelation function (Blackman and Tukey, 1958). Because the quantitative determination of the spectra depends on the value of c_o in equation (11), the behavior of that "constant" for each case will also be examined.

C. Shear Functions and Spectrum Functions

In the development of the relationship between the shear function and the spectrum in Section II, it was shown that when the mean shear ($\overline{\Delta V}$) is zero, that c_o, the ratio between the shear function and the square root of the structure function or standard deviation (11), has a value $c_o \leq \sqrt{2/\pi}$. The validity of this result and its physical meaning depend on the applicability of the statistical model of intermittency which has been proposed (Appendix). A computed c_o in the range specified above does not necessarily imply that the statistical nature of the turbulence fulfills the requirements of the model. It can only be stated that if the record being analyzed possesses the hypothesized "burst" characteristics, then c_o will behave as stated and actually be a measure of the intermittency. This will be assumed as a first approximation.

Figures 6 through 9 show the computed shear function and the square root of the structure function for white noise and for some of the cases discussed in the preceding section. A common result for all of these analyses is that $\overline{\Delta V} = 0$ and therefore the square root of the structure function is equal to the standard deviation. Figure 6 was derived from the analysis of 2048 random numbers which were distributed normally. It illustrates the parallel behavior of the shear and structure functions (or the standard deviation) and their relation in the case of normality without the superimposed effect of scale dependence. c_o, as would be expected, is approximately equal to $\sqrt{2/\pi}$ in this case.

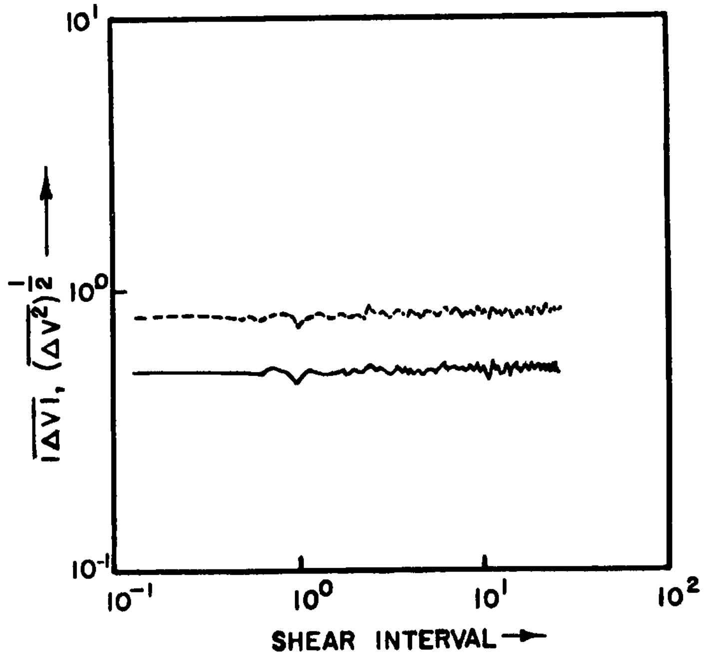

Figure 6. Shear function (solid line) and the square root of the structure function (dashed line) for Gaussian white noise.

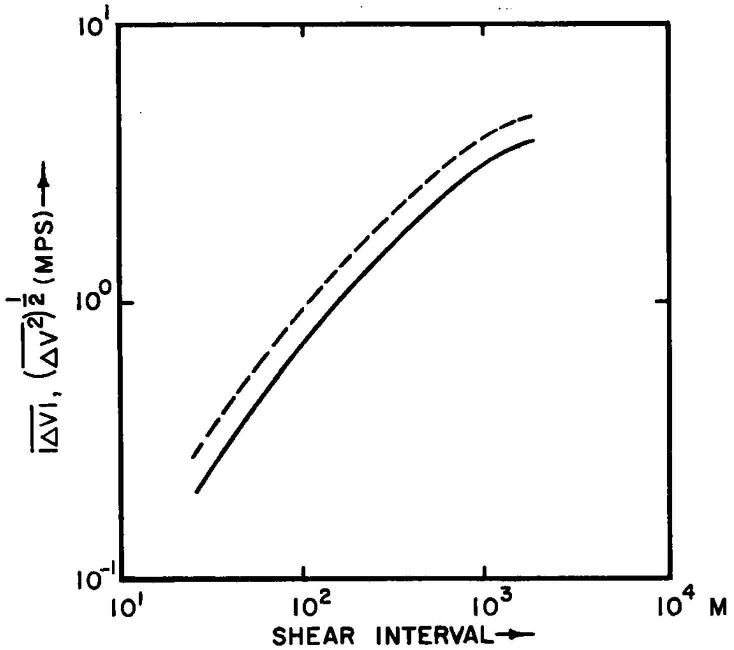

Figure 7. Shear function (solid line) and square root of the structure function (dashed line) for FPS-16/Jimsphere scalar shears. Cape Kennedy, Florida, December 29, 1964.

Figure 7 is a plot of the same variables for the FPS-16/Jimsphere scalar wind profile. Although the vertical mesostructure, rather than turbulence, is the primary physical parameter in this case, one may think of the macrostructure as being composed of "bursts" in the sense of superimposed jets (Scoggins, 1963) and other mesostructural features (Weinstein et al., 1966). On this basis the proposed statistical model may also be applied as a first approximation (see Table 2). Such an assumption, however, is evidently not applicable in the case of the superpressure balloon data (figure 8) although one might also be tempted to consider shorter lee waves or billows (of the order of 2 km) as "turbulence" on the longer waves, the distribution of deviations produced by a sinusoidal phenomenon is not necessarily Gaussian (e.g., see Bendat and Piersol, 1966). It might be argued that, for the same reason, the profiles of the Jimsphere data should not be considered, i.e., the mesostructure of the vertical sounding is simply the effect of atmospheric wave motions. However, the stratification of the atmosphere, the inclusion of the total profile (0.25 - 16 km) and the treatment of the scalar wind speeds apparently cause the mesoscale "bursts" to approach distributions which are closer to Gaussian. c_o is nearly constant with scale in figure 7, while in figure 8, c_o is an apparent function of the differencing interval, particularly for the curves labeled 'a'.

For a more detailed analysis of the TAS data derived from the flight through rotor turbulence, four smaller and more homogeneous samples were drawn from the original sample (figure 3). Shear and structure function analyses for these samples appear in figure 9. Sample 'a' was collected 10 km upstream of the rotor, 'b' within the rotor, and 'c' and 'd' were, respectively, 20 km and 45 km downstream. Each sample is 71 sec (6-7 km, 568 data points) long. A number of interesting features comes to light when the record is broken up in this manner. Mean longitudinal

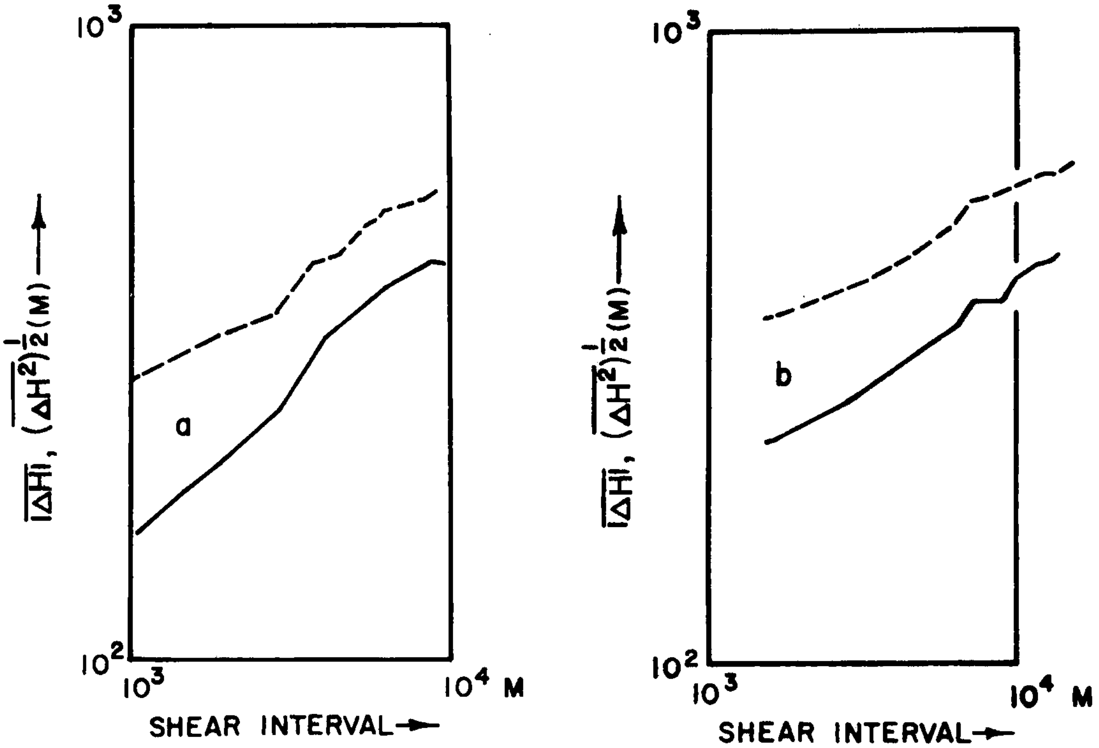

Figure 8. Shear functions (solid lines) and square roots of structure functions (dashed lines) for two superpressure, pillow balloon, flights. Fort Collins, Colorado, January 5, 1968. Curve 'a', 1622 GMT, curve 'b', 1850 GMT.

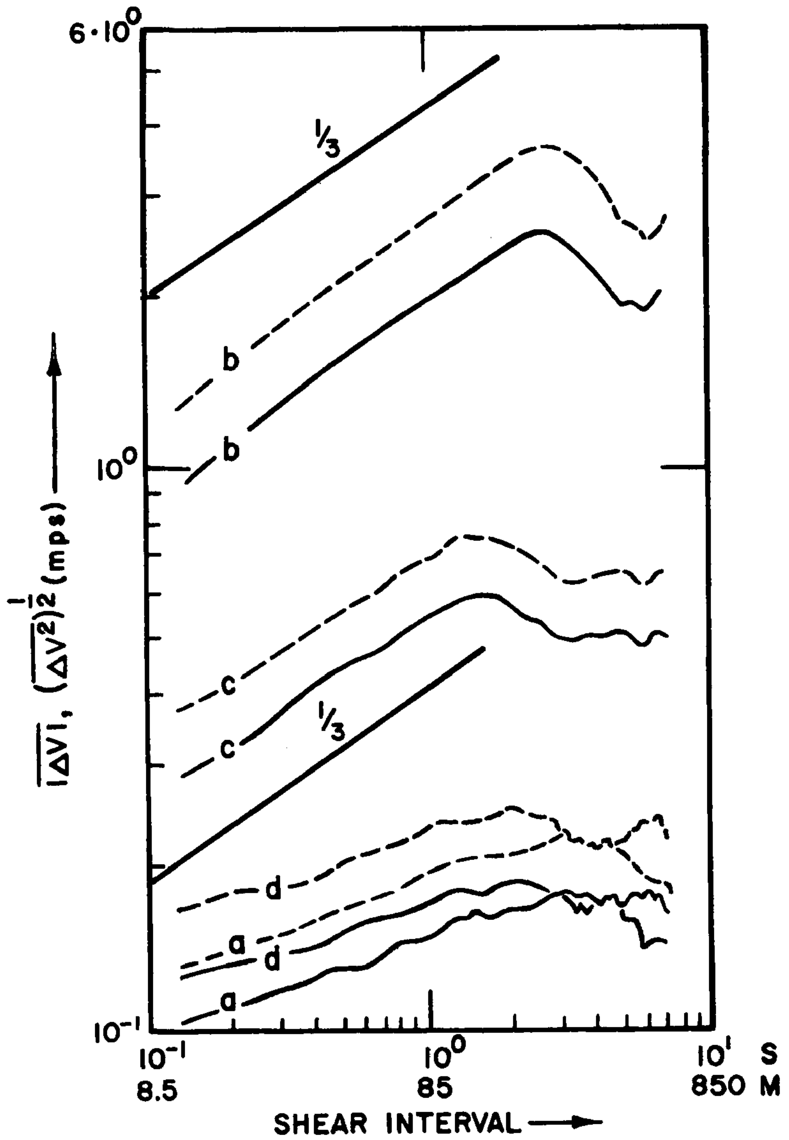

Figure 9. Shear functions (solid lines) and square roots of structure functions (dashed lines) for four 71-second filtered samples of longitudinal gust data collected by aircraft. Boulder, Colorado, February 19, 1968.

shear magnitudes are about an order of magnitude less in the reported non-turbulent regions ('a' and 'd') than in the rotor ('b') where severe turbulence was reported. Curve 'c' corresponds to light-moderate turbulence intensity. The shear function slope increases with turbulence intensity as does its smoothness and degree of parallelism with the square root of the structure function. In comparison with the shear analysis for the total sample (figure 4), slopes at small scales for the sub-samples vary from 0.40 for curve 'b' to about 0.15 for 'a' and 'd'. In fact, curves 'a' and 'd' appear to be similar to white noise (compare with figure 6).

Table 2 summarizes the findings shown graphically in figures 6 through 9. It is apparent that an assumption of normality ($c_o = \sqrt{2/\pi} \cong 0.8$) would lead to large errors in the relationship between the shear and structure function for the pillow balloon data and the large samples of TAS and temperature data gathered by aircraft. Also, the constancy of c_o with lag does not hold for the pillow balloon data and temperature data.

The tabulated c_o and first four statistical moments for the TAS data show the marked non-normality for the entire sample (in terms of c_o and μ_4) and the tendency towards the Gaussian distribution for the smaller sub-samples. It is noted that the severe CAT sample ('b') deviates from the normal distribution to a greater degree than 'c' (light-moderate CAT) and 'a' and 'd' (no CAT). Cornu's test (Brooks and Carruthers, 1953) indicates that the distributions of ΔV for sample 'b' at lags 1, 8 and 20 are significantly different from normal at the 95% level. The values of μ_4 for these cases show that the deviations from normal are leptokurtic in nature. Sample 'c', however, indicates that the distribution of ΔV at lag 20 is platykurtic ($\mu_4 < 3$). Although c_o for this lag is not significantly different from $\sqrt{2/\pi}$ at the 95% level, as was mentioned earlier this does not necessarily imply

Case	Data Interval	Sample Size	Lag	c_o^*
Gaussian white noise	1	2048	1	.80
			8	.79
			20	.80
FPS-16/Jimsphere data				
1600 GMT	25 m	630	1	.73
			8	.75
			20	.75
1731 GMT	25 m	630	1	.75
			8	.80
			20	.80
1900 GMT	25 m	630	1	.76
			8	.76
			20	.77
2200 GMT	25 m	630	1	.69
			8	.79
			20	.77
Pillow balloon data				
'a'	~800 m	97	1	.56
			8	.76
'b'	~1.5 km	97	1	.65
			8	.73
Aircraft data				
Temperature (unfiltered)	~10 m	9600	1	.55
			8	.52
			20	.56
			40	.59
			80	.60
			160	.64
			360	.67

$^* \sqrt{2/\pi} \simeq 0.80$

Table 2: Summary of shear function-structure function ratios (c_o) for all cases and statistical moments for selected cases. To obtain differencing interval, multiply lag by data interval.

Case	Data Interval	Sample Size	Lag*	c_o	μ_1**	μ_2	μ_3	μ_4
TAS (filtered)								
Entire sample	~10 m	6900	0	-	$-2.1\cdot 10^{-4}$	$5.0\cdot 10^{-1}$	$3.5\cdot 10^{-3}$	$4.0\cdot 10^{1}$
			1	0.56	$-2.9\cdot 10^{-5}$	$2.2\cdot 10^{-1}$	$6.5\cdot 10^{-1}$	$2.4\cdot 10^{1}$
			8	0.56	$-4.4\cdot 10^{-4}$	$8.8\cdot 10^{-1}$	$-6.9\cdot 10^{-2}$	$2.5\cdot 10^{1}$
Sample 'a'	~10 m	568	0	-	$-5.1\cdot 10^{-4}$	$2.0\cdot 10^{-2}$	$1.6\cdot 10^{-1}$	3.1
			1	0.79	$1.1\cdot 10^{-4}$	$1.7\cdot 10^{-2}$	$9.1\cdot 10^{-3}$	3.1
			8	0.77	$1.5\cdot 10^{-3}$	$3.6\cdot 10^{-2}$	$-5.4\cdot 10^{-3}$	3.3
			20	0.79	$-4.7\cdot 10^{-4}$	$4.7\cdot 10^{-2}$	$6.5\cdot 10^{-2}$	3.0
Sample 'b'	~10 m	568	0	-	$-6.3\cdot 10^{-3}$	4.5	$1.0\cdot 10^{-2}$	5.9
			1	0.70	$-5.9\cdot 10^{-4}$	1.6	$4.0\cdot 10^{-1}$	5.3
			8	0.73	$-2.2\cdot 10^{-3}$	7.4	$-4.4\cdot 10^{-2}$	4.2
			20	0.71	$2.5\cdot 10^{-2}$	13.9	$7.2\cdot 10^{-2}$	5.5
Sample 'c'	~10 m	568	0	-	$-3.7\cdot 10^{-3}$	$2.0\cdot 10^{-1}$	$2.9\cdot 10^{-2}$	3.0
			1	0.76	$-3.7\cdot 10^{-4}$	$1.3\cdot 10^{-1}$	$2.9\cdot 10^{-1}$	4.8
			8	0.79	$4.1\cdot 10^{-4}$	$5.5\cdot 10^{-1}$	$5.5\cdot 10^{-2}$	3.2
			20	0.82	$6.1\cdot 10^{-3}$	$4.3\cdot 10^{-1}$	$-5.8\cdot 10^{-2}$	2.5
Sample 'd'	~10 m	568	0	-	$1.2\cdot 10^{-3}$	$2.5\cdot 10^{-2}$	$1.7\cdot 10^{-1}$	3.9
			1	0.76	$2.7\cdot 10^{-4}$	$2.7\cdot 10^{-2}$	$-1.2\cdot 10^{-1}$	4.6
			8	0.77	$1.5\cdot 10^{-4}$	$5.2\cdot 10^{-2}$	$4.0\cdot 10^{-2}$	4.6
			20	0.77	$-1.5\cdot 10^{-3}$	$5.7\cdot 10^{-2}$	$1.7\cdot 10^{-1}$	3.8

** μ_1: mean, μ_2: variance, μ_3: Skewness, μ_4: Kurtosis

Table 2: Continued.

that the distribution is normal. It has been suggested that turbulence velocity distributions which have a kurtosis less than 3 may be indicative of wave motions (Finn and Sandborn, 1964). For example, a sine wave and a triangular wave yield, respectively, 1.5 and 1.8 for kurtosis values. Preliminary analyses of the mesoscale environment at the time of the lee wave flight have indicated that conditions favorable for shearing-gravity waves were present downwind of the rotor in the region of sample 'c'. This possibility is being subjected to further study.

Figures 7-9 and Table 2 have shown that for each case, with the exception of the pillow balloon data, c_o is approximately constant with differencing interval for small vertical and horizontal scales. From this it follows that equation (11) is valid for these cases and, as anticipated, c_o may assume values less than $\sqrt{2/\pi}$. These results also suggest that the subjective selection of homogeneous samples utilized here has served to bring c_o closer to $\sqrt{2/\pi}$, which is equivalent to the assumption of a Gaussian distribution. Thus, although the assumption of normality may not necessarily be correct, an assumed shear function-structure function relationship based on normality would show little error.

Computed and derived spectra for the vertical wind speed profile and for the TAS data are presented in figures 10 and 11, respectively. "Computed" spectra were estimated via the Blackman and Tukey (1958) method, while "derived" spectra were estimated from the previously-presented shear functions and equation (23). Actual values of c_o (Table 2) were utilized so that the spectra derived from the shear function agree exactly with those derived from the structure function (equation (22)). Spectra of the superpressure balloon data and the temperature data are not presented because of the variation of c_o with scale in these cases (Table 2). The effect of high pass filtering is

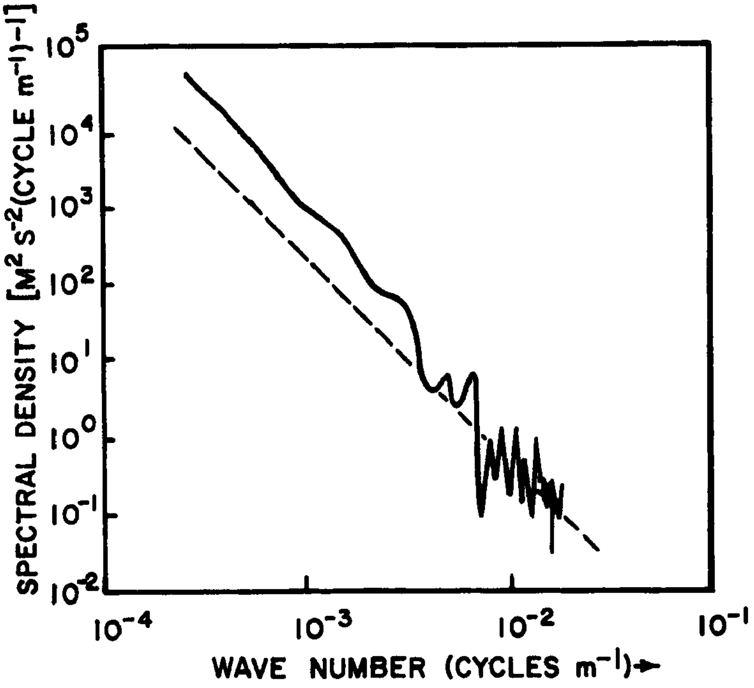

Figure 10. Derived (dashed line) and computed (solid line) spectra for FPS-16/Jimsphere data. Cape Kennedy, Florida, December 29, 1964, 1900 GMT.

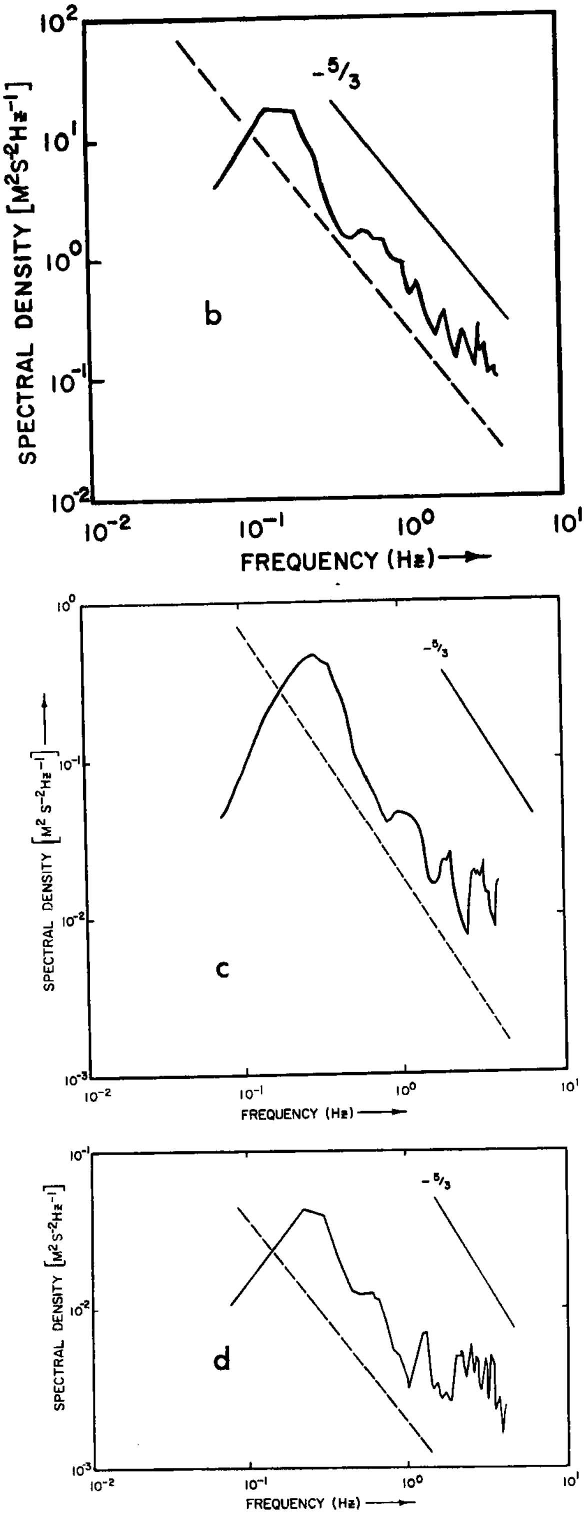

Figure 11. Derived (dashed line) and computed (solid line) spectra for three 71-second samples of longitudinal gust data. Boulder, Colorado, February 19, 1968. Sample 'b': severe CAT reported, sample 'c': light to moderate CAT reported, sample 'd': no CAT reported.

evident in figure 11 in the decrease of estimated power spectral densities at the lowest frequencies.

Qualitatively the derived spectral estimates have slopes and magnitudes which are similar to the computed spectra. This is especially true of the high wave number end of the spectrum in figure 10. However, the similarity does not extend to the detailed features. The computed spectra of the longitudinal gust component in figure 11 deviate from the derived spectra as the frequency increases and the intensity of the turbulence decreases.

A rough approximation of how well the derived spectral estimates match the computed spectral estimates is found by placing confidence limits on the latter. Blackman and Tukey (1958) state that "no estimate will be more stable than chi-square on $2n/m$ degrees of freedom", where n is one less than the sample size and m is the maximum lag. For the analyses presented in figures 10 and 11 ($2n/m \simeq 20$) the estimated power will be between 0.55 and 1.57 of the average power at the 90% confidence level. If we assume that the average power is given by the computed spectrum, then all derived spectra are significantly different than the computed spectra with the possible exception of the spectrum appearing in figure 10.

There are several possible explanations for these discrepancies. A re-examination of figures 1 and 9 indicates that the mean magnitudes of the vertical shears deviate from a straight line only slightly at large scales as compared to the longitudinal (TAS) shears which were subjected to high pass filtering. Further, it is observed that the slopes and smoothness of the TAS shear functions decrease as the intensity of the turbulence decreases. These effects decrease the confidence in the applicability of a shear function power law over a relatively wide range. A better agreement with slopes might be sought by applying equation (17) over narrower ranges, however as the exponent of the shear function ($p/2$ in (17)) becomes equal to or less than zero, the derived spectrum (equation (23)) is no longer applicable. This is especially apparent in sample 'd' of figure 11 where the computed spectrum is flat in the highest frequencies. The physical cause for this behavior in the TAS data has been traced to instrument noise. The sharp deviation of slopes and magnitudes of derived and computed spectra (figure 11) in the low frequencies is not unexpected since the shear function at scales corresponding to these frequencies display negative or zero slopes due to data filtering (figure 9).

It was stated earlier that the straight line fit to the shear function on the double logarithmic plot appeared to be a good approximation at the smallest scales, however, Stewart (1970) has pointed out recently that attempts to fit curves to measured structure functions frequently lead to erroneous results. Due to the effect of limited time constants of instruments and the viscosity of the fluid, real data suffers a loss of variance associated with the smallest scales. The result is that at these smallest scales, structure function slopes, estimated by the objective fitting of a curve through the origin ($D_f(r) = 0$, $r = 0$), will be incorrect. Since, as shown earlier, the shear function may be related directly to the structure function and curves were fitted over the smallest scales, the problem cited by Stewart may possibly apply to the present analyses.

The intermittency of the TAS data gathered during the rotor flight is illustrated by the marked decrease in spectral density from sample 'b' to sample 'd' in figure 11. Had the derived spectra for the entire sample (not shown) been determined on the basis of a Gaussian assumption ($c_o = \sqrt{2/\pi}$) these derived spectral density estimates would have been approximately 50% less than values based on c_o as a function of intermittency. This, of course, is an extreme case. When the data to be analyzed were selected subjectively on the basis of their homogeneous appearance, the differences between spectral densities computed with the actual c_o and those computed with $c_o = \sqrt{2/\pi}$ were greatly reduced. For example, the Gaussian assumption for samples 'b', 'c' and 'd' would have led to deviations of 13% or less to the low side of the derived spectra which are shown in figure 11.

The method which has been utilized here to derive spectra from the shear function (or structure function) alone is not sensitive to significant variations in the distribution of variance with frequency. This is most likely due to the suppression of small but important fluctuations of the shear function in the double logarithmic plot and to the problem pointed out by Stewart (1970).

IV. Summary and Conclusions

The shear function-spectrum function relationship proposed by Essenwanger and Reiter (1969) has been examined both theoretically and experimentally. It has been shown that equations similar to those developed empirically by Essenwanger (1963, 1965) for the shear function and its standard deviation may also be derived from the consideration of the structure function and its observed behavior over small scales. However, it was also shown that the derivation is dependent on the probability distribution of shears for the record in question. The adoption of a statistical model based on the suggestions by Dutton et al. (1969) indicates that the shear function-spectrum function relationship depends on the degree of intermittency of the record.

Shear functions were determined for individual vertical and horizontal soundings which included meso- and macroscale wind and temperature fluctuations. Analyses indicate that the mean magnitude of the shear is an exponentially increasing function of the differencing interval over the scales considered ($\sim 10^1 - 10^4$ m). A power law of the form $\overline{|\Delta V|} = a_o \Delta h^{a_1}$ is valid for shears over smaller scales ($\sim 10^1$ to 10^2 m). For these scales a_o and a_1 are constants ($a_1 < 1$) for a given sample of data, but vary from sample to sample depending on the type of data, the intensity of the fluctuations and the intermittency of the fluctuations. For larger scales, a_o and a_1 become functions of the differencing interval.

An exception to these results is found in the shear analyses of pillow balloon data which were gathered under lee wave conditions. Although shear functions for these cases are increasing functions of differencing interval, they are relatively irregular in comparison to other data types. Also, c_o ($a_o = f(c_o)$), the ratio between the shear and structure function, show significant decreases with differencing interval for both pillow balloon data and temperature data gathered by aircraft over scales of 1-10 km. It was conjectured that the non-random nature of the lee waves caused this behavior. The treatment of a highly intermittent sample of TAS gust data revealed that subjective selection of approximately homogeneous sub-pieces from the larger sample markedly reduced the intermittency. Shear distributions which are highly non-normal for the larger sample approach the Gaussian distribution for the smaller homogeneous samples. This suggests that the shear function-structure function relationship can be assumed with little error ($c_o = \sqrt{2/\pi}$) for approximately homogeneous samples. However, it should also be considered that the intermittency is an important characteristic of the turbulence and as such should <u>not</u> be eliminated by the arbitrary selection of homogeneous samples.

Spectra were derived from shear functions for the detailed wind profiles and for the longitudinal gust velocities. Slopes in the region of interest are only generally correct and detailed variations could not be predicted by the method utilized. Derived spectral densities appear to be significantly less than the computed spectral densities for the gust velocities, although they are of the same order of magnitude overall.

The results summarized above indicate that the shear function approach to the spectrum function as suggested by Essenwanger and Reiter (1969) suffers from the following shortcomings:

(1) The relationship of the shear function to the structure function and thus to the spectrum function appears to be dependent on the intermittency of the record to be analyzed. Unless the intermittency characteristics are known, the relationship cannot be stated directly.

(2) Derived spectral slopes based on shear function slopes do not show the important slope variations which are revealed by the Fourier transformation of the autocorrelation function. Also, the magnitudes of spectra derived from shear functions with known intermittency characteristics may be significantly less than the

magnitudes of the computed spectra. One important cause of this problem appears to be the unrepresentativeness of measured shear (structure) functions at small scales.

(3) The shear function does not appear to have any advantage over the structure function in terms of physical interpretation. The magnitude and slope of the shear function increase with the intensity of the turbulence, paralleling the behavior of the structure function.

(4) The possibility of utilizing the shear function approach as a quick and economic method to determine the spectrum of atmospheric turbulence from detailed balloon soundings is presently limited by the problems cited above, by the small vertical extent of turbulent layers in the free atmosphere and by the inability of the FPS-16/Jimsphere system to resolve shears over layers less than about 25 m.

It appears that a better approach to the study of intermittent, small scale atmospheric turbulence would be to compute the spectrum directly and to consider intermittency in terms of the probability density functions and the higher statistical moments.

Despite these shortcomings, the analysis of shear functions and the shear-function structure function relationship has indicated that the shear function for individual soundings of atmospheric variables may be useful in other ways. For example,

(1) The application of structure function and spectral analysis techniques to detailed vertical soundings has been carried out by Scoggins (1963), Endlich et al. (1969), Fichtl et al. (1969) and others to determine characteristics of the vertical mesostructure of the atmosphere. At these and larger scales, the FPS-16/Jimsphere has excellent resolution qualities. For these data the shear function may give added information about the mesoscale which, in turn, is the environment in which turbulence is produced. The computation of only the mean vertical shear ($\overline{\Delta V}$)

and the mean magnitude of the vertical shear ($\overline{|\Delta V|}$) of a component over a given differencing interval allows the determination of $\sigma^2_{|\Delta V|}$, $\sigma^2_{\Delta V}$ and the structure function ($\overline{\Delta V^2}$) by means of equations (11), (6) and (14) with $c_o \simeq \sqrt{2/\pi}$. If the mean shear is zero, then the desired qualities may be determined from (11) and (6) alone. The assumption that $c_o = \sqrt{2/\pi}$ for the scalar shears is valid with an error of a few per cent in the present investigation, but this result is based upon only a few cases. The procedure suggested above should be subjected to further study by utilizing soundings taken under different synoptic conditions and by considering layers of the order of 1-3 km in thickness.

(2) The application of the shear function to pillow balloon data and temperature data gathered during horizontal flights through lee waves indicates that the shear function might aid in characterizing mesoscale systems whose structure tends to be deterministic. This possibility should be investigated.

(3) Although the spectra derived for the cases of longitudinal gust velocities are not satisfactory, c_o, the ratio between the shear function and the square root of the structure function appears to be a quantitative measure of the intermittency of the data. This conclusion, of course, is dependent on the applicability of the statistical model which was adopted (see Appendix). Assuming that the model is realistic, c_o is sensitive to frequency distributions which show a large concentration about the origin while the kurtosis of the distribution is most sensitive to the flatness of the flanks of the distribution curve. Thus, the shear-structure function ratio may be a useful parameter in the investigation of atmospheric intermittency.

The results of the present study as summarized above are based upon only a few analyses and should be interpreted with caution. This investigator strongly recommends further study into the applicability of the shear function, especially with respect to the vertical mesostructure of the atmosphere and the intermittency of small scale turbulence. Currently, the shear function concept is being extended to the study of the intermittency of clear air turbulence. Details of that investigation will follow in a later report.

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VII. Appendix

At the recent Symposium on "Spectra of Meteorological Variables" (Stockholm, 1969), the following definition of intermittency was proposed:

> "A record is said to be intermittent if the sample variance is distributed in a distinctly non-uniform manner so that a relatively large fraction of the total variance comes from a relatively small fraction of the total record. An associated characteristic of many atmospheric records is that the important and intermittent events occur randomly and apparently independently."

A stationary record possessing these characteristics may be simulated in the following manner. For simplicity let x_t be a time dependent variable such that

$$x_t = y_t z_t \tag{A-1}$$

where y_t is a stationary Gaussian process with a zero mean and unit variance, approaching Gaussian white noise, i. e., $\rho(\tau) \to 0$ for all $|\tau| > \epsilon$, where $\rho(\tau)$ is the autocorrelation function, τ is the lag and ϵ is a small value; z_t is a series of step functions with jumps which occur at the onset and conclusion of a burst of activity; y_t and z_t are independent. In other words, it is assumed that the record of x_t is composed of turbulent bursts or patches such that x_t is distributed normally within each burst, but the intensity (the variance) of each burst may be different and the turbulent patches are uncorrelated.

The mean μ_1, of x_t is given by

$$\mu_1 = E(x_t) \tag{A-2}$$

where, in general, the operator (the expectation), E( ), for the variable x_t is given by

$$E(\ ) = \int_{-\infty}^{\infty} (\ )\, f(x_t)\, dx_t \tag{A-3}$$

Since

$$(c_o)_{y_t} = \frac{E(|y_t|)}{[E(y_t^2)]^{1/2}} = \frac{\int_{-\infty}^{\infty} |y_t| f(y_t)\, dy_t}{[\int_{-\infty}^{\infty} y_t^2\, f(y_t)\, dy_t]^{1/2}} = \sqrt{\frac{2}{\pi}} \quad \text{(A-15)},$$

where $f(y_t)$ is given by (A-8), and since, again by Schwartz's inequality,

$$E(z_t^2)^{1/2} \geq E(|z_t|)$$

it follows that

$$c_o \leq \sqrt{\frac{2}{\pi}} \quad \text{(A-16)}.$$

Identifying the gust velocity with the variable x_t in the previous development, it follows that a gust velocity record which is intermittent in the sense of the proposed model will have a frequency distribution which is symmetric about a zero mean but will exceed the Gaussian distribution at the origin and in the tails. The deviation of the actual distribution from the normal is a function of the degree of intermittency, i. e., the distribution of the variances of the turbulent bursts. If x_t is equated with the velocity difference, ΔV, at a given lag, τ, then in addition to the properties mentioned above, c_o, the ratio between the velocity shear function and the square root of the velocity structure function, is also dependent on the intermittency of the record of ΔV.