Development of a Numeric Model, with Explicit Solution, to Study Flood Wave Propagation

Chagas, Patrícia[1]
Department of Environmental and Hydraulics Engineering, Federal University of Ceará

Souza, Raimundo[1]
Department of Environmental and Hydraulics Engineering, Federal University of Ceará

Abstract. The knowledge of the propagation of flood waves, in natural channels, through Saint Venant's equations, has been object of studies on the part of engineers and scientists, along years. With the progress of the digital computers, the mathematical models become a great option, in the analysis of this class problem. In this context, the uses of numerical models, in the treatment of the hydrodynamic models, has allowed more select studies of the behavior of the flood routing to be accomplished. This work treats of the study of a flood wave, through a simplified methodology, for solution of the equations of the hydrodynamic that allows the use of a numerical model with explicit solution. The influence of hydraulic parameters, inherent to the natural river, it is considered. The results show that this methodology had a great acting and it constitutes in a good way for the study of this class of problem.

Keywords: Flood Control; Hydrodynamic Models; River Mechanics.

1. Introduction

The Saint-Venant equations represent a good way to describe problems concerning with flood waves propagations in open channels. This is a physical process of high complexity, caused by an intense rain or the breaking of any control structure, which represents an interesting problem to be studied.

The solution of this kind of problem passes, invariably, for the development of methods that allow solving the equations of Saint-Venant. These non linear equations contain, in its mathematical representation, all elements that, directly or indirectly, is related with the behavior of the flow in the channel. Through those equations it can be determined all the hydrodynamic of the system and it can be verified the possible risk of occurrence of inundations. It is enough that, for that, the function is modeled appropriately. Evidently, the modeling of such functions implicates in more efforts in the process of solution of the equations.

In this research a hydrodynamic model was developed to simulate the propagation of a flood wave. The model solves the equations of Saint-Venant using the difference method, with an explicit discretization, proposed by Chow (1988).

[1] Department of Environmental and Hydraulics Engineering, Federal University of Ceará Campus do Pici – Bloco 713 – P. O. Box 6018, CEP 60.451-970, Fortaleza – CE – Brasil. Tel: (55) (85) 3288-9771; e-mail: pfchagas@yahoo.com & rsouza@ufc.br

The results have shown that the computational program answered satisfactorily to the used data. To verify the capacity of the model, it was made comparisons with a classic hydrodynamic model proposed by Chow (1988).

2. Methodology

As, in this work, it intends to evaluate a series of hydraulic parameters of a natural river, there is the necessity to know the flow field along of the channel. This will be gotten through the mathematical modeling of the physical process of the dynamic wave propagation.

In the process of solution of the model some fundamental conditions will be observed. The flow will be considered one dimensional, so that, the momentum equation will be applied just in the x direction, along the longitudinal channel. The pressure has a hydrostatic distribution, and the channel will be considered with a rectangular section.

The flow field, in the river, is obtained through the numeric solution of the Saint-Venant equations. Those equations, of the continuity and of the momentum, are described to proceed, according to Keskin (1997):

- **Continuity Equation**

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

- **Momentum Equation**

$$\frac{\partial Q}{\partial t} + \frac{\partial \left(Q^2 / A\right)}{\partial x} + gA(\frac{\partial y}{\partial x} - S_0) + gAS_f = 0 \tag{2}$$

Where x is the longitudinal distance along the channel (m), t is the time (s), A is the cross section area of the flow (m^2), y is the surface level of the water in the channel (m), S_0 is the slope of bottom of the channel, S_f is the slope of energy grade line, B is the width of the channel (m), and g is the acceleration of the gravity ($m.s^{-2}$).

In order to calculate S_f, the Manning formulation will be used. Thus,

$$V = \frac{1}{n} R^{2/3} S_f^{\ 1/2} \tag{3}$$

Where V is the mean velocity (m/s), R is the hydraulic radius (m) e n is the roughness coefficient.

Operating algebraically (1), (2) and (3), Keskin (1997), one can find,

$$\frac{\partial Q}{\partial t} + \alpha \frac{\partial Q}{\partial x} + \beta = 0 \tag{4}$$

Where,

$$\alpha = 2\frac{Q}{A} + \frac{\frac{gA}{B} - \frac{Q^2}{A^2}}{\frac{Q}{A}\left(\frac{5}{3} - \frac{4R}{3B}\right)} \tag{5}$$

and

$$\beta = gA(S_f - S_0) \tag{6}$$

In this hydrodynamic model it will certain two dependent variables. The first refers to the cross section area A(x,t), along the channel, for each interval of time. The second one refers to the flow field Q(x,t) along the channel, for the same previous conditions. As the investigation demands the knowledge of two dependent variables, there is the necessity of two differential equations: the equation (1) and the equation (4) will compose the model.

- **Initial Conditions:**

$$Q(x,0)=Q_0 \tag{7}$$

$$A(x,0)=A_0 \tag{8}$$

Where Q_0 is the steady state flow of the channel, and the A_0 is the cross section area for the steady state conditions.

- **Boundary Conditions:**

$$Q(0,t) = f(t) \tag{9}$$

Where f(t) is the hydrograph.

3. Numeric Solution of the Hydrodynamic Model

With respect the numeric solution of the differential equations of the dynamic wave, the explicit discretization formulation will be used and defined through the relationship:

$$\frac{Q_{i+1}^{j+1} - Q_{i+1}^{j}}{\Delta t} + \alpha_m \left(\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x}\right) + \beta_m = 0 \tag{10}$$

Making an arrangement of the equation (10) it is possible to find:

$$Q_{i+1}^{j+1} = \frac{Q_{i+1}^{j} + \frac{\Delta t}{\Delta x}\alpha_m Q_i^{j+1} - \beta_m \Delta t}{1 + \alpha_m \frac{\Delta t}{\Delta x}} \tag{11}$$

Where α_m and β_m are defined, respectively:

$$\alpha_m = \frac{\alpha_i^{j+1} + \alpha_{i+1}^{j}}{2} \tag{12}$$

$$\beta_m = \frac{\beta_i^{j+1} + \beta_{i+1}^{j}}{2} \tag{13}$$

Finally with the Q calculate for the next time step, it is possible calculate the A(x,t), through:

$$A_{i+1}^{j+1} = A_{i+1}^{j} - \frac{Dt}{Dx}(Q_{i+1}^{j+1} - Q_i^{j+1}) + \frac{Dx}{2}(q_{i+1}^{j+1} + q_{i+1}^{j}) \tag{14}$$

4. Results

The hydrodynamic model developed in this work was used to accomplish several simulations, where the bed slope was varied, for the same initial and boundary conditions, to evaluate the propagation of a wave, along the channel.

In this numeric simulation, it was considered the channel with rectangular section, with the following characteristics: length of the channel, 50.000 meters; width of the channel, 50 meters; and roughness coefficient, 0.01. The upper boundary condition was considered a sinusoidal function, which represents the hydrograph that is coming into the channel.

In the numeric modeling, the discretização, regarding the distance x, is made in 50 spaces with length of 1000 meters each, totaling a length of 50 km. Regarding discretização of the time, it became separated in 360 intervals of 150 seconds, totaling a time of 15 hours of simulation.

The figure 1 shows the propagation of a dynamic wave, along the channel, for different intervals of time, considering the bed slope of 0.0001 m/m. Through the figure it is possible to see a strong reduction in the picks of the waves for different times. For example, in one hour the pick of the flow is close to 200.0 m^3/s, while for the fourth hours it comes to close to 85.0 m^3/s.

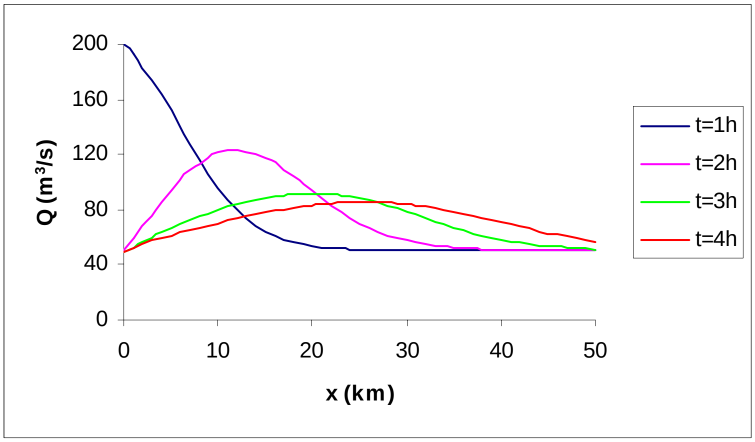

Figure 1. Propagation of the wave, for different time, with n=0.01 e S_0=0.0001.

The figure 2 shows the behavior of the flood wave for different bed slope, considering the same time of two hours. The results show that the smaller the bed slope is, there are significant reductions in the picks of the waves. For instance, for values of S_0 equal to 0.0001 the maximum value registered for the flow is of 120m^3/s, while for S_0 equal 0.00002, the found maximum value is close to 90m^3/s. This result shows that the propagation of a flood wave is strongly influenced by the bed slope of the channel, implicating, in such way, in a bigger or smaller inundation probability, in the risk areas.

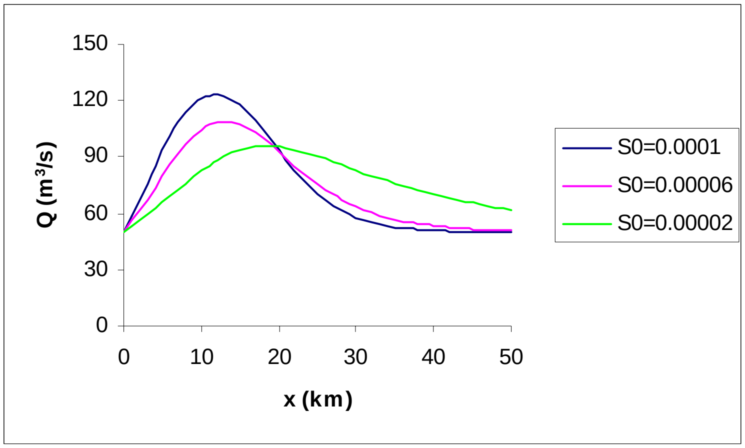

Figure 2. Behavior of the wave for different bed slope, considering n=0.01.

The figure 3 shows the propagation of the wave for different bed slope, in the section distant 12 km from the origin, in function of the time. This simulation shows the arrival of a wave in a fixed point of the channel, showing that the speed of propagation of the wave does not suffer great influence of this parameter. Through the figure, it is possible to see that the time that the pick of wave reaches the section, 12 km from the origin, is, approximately, of 1:50 hours, for all simulations.

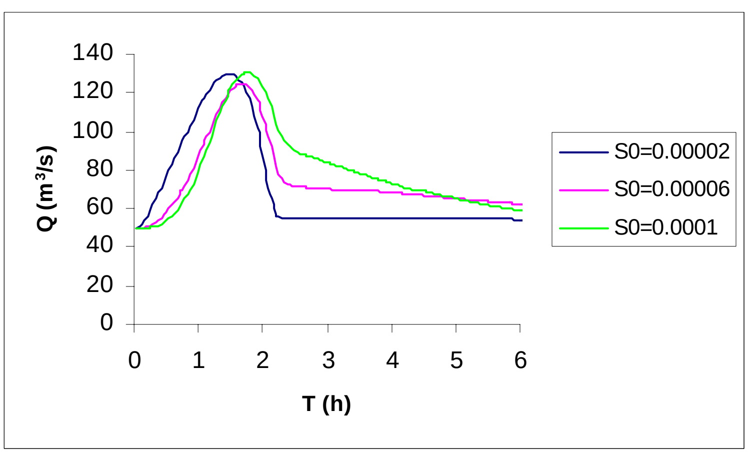

Figure 3. Wave reaching the section, 12km from the origin, with n=0.01.

5. Conclusions

The objective of this research was constituted in evaluating the propagation of a dynamic wave, through a set of simulations accomplished with the aid of a computational program, developed for this research. The program was developed with base in a hydrodynamic model, composed by Saint Venant's equations. In such way, it was possible to conclude that the model presented quite satisfactory results, answering fully to the objectives of the work.

Whit respect to the behavior of the flood wave, it was verified that, for different bed slope of the channel, the propagation of the wave suffers important influence of this parameter, allowing, so, to conclude that as larger the bed slope is, as bigger will be the pick of the flood wave. This allows concluding that the bed slope of the channel plays an important game in the control of inundations for risk areas.

Finally, the results showed that this parameter does not make a great influence on the celerity of the flood wave. It was verified that for a same distance of the origin, the picks of the wave arrive, approximately, at the same time. Evidently, these results deserve other investigations.

References

Bajracharya, K. Barry, D.A. (1999). Accuracy Criteria for Linearised Diffusion Wave Flood Routing, Journal of Hydrology, v. 195, p. 200-217, Elsevier.

Barry, D.A.; Bajracharya, K. (1995). On the Muskingum-Cunge Flood Routing Method. Environmental International, v. 21, n. 5, p. 485 - 490, Elsevier.

Chalfen, M.; Niemiec, A. (1996). Analytical and Numerical Solution of Sain-Venant Equations. Journal of Hydrology, v. 86, p. 1 – 13.

Chow, V. T. Applied Hydrology, (1988). New York: McGraw-Hill, 572p.

Keskin, M. E.; Agiralioglu, N. (1997). A Simplified Dynamic Model for Flood Routing in Rectangular Channels. Journal of Hydrology, v. 202, p. 302-314, Elsevier.

Moussa, R.,; Bocquillon, C. (1996). Criteria for the choice of Flood-Routing Methods in Natural Channels. Journal of Hydrology, 186, p. 1-30, Elsevier.

Silva, R. C. V.; Mascarenhas, F. C. B.; Miguez, M.G. (2003). Hidráulica Fluvial, v. I, Rio de Janeiro: COPPE/UFRJ, 304p.