A B S T R A C T

ENERGY LOSSES
THROUGH
CONICAL DIFFUSERS

Submitted by
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ABSTRACT

Introduction

In many types of hydraulic systems it is often necessary to increase the pipe size in the direction of flow. Such a condition requires the design of a transition section connecting the pipes of different diameters. Because the fluid flow is very complex for an expanding transition, the problem of obtaining the most satisfactory design -- a design for which a minimum of energy loss will occur -- has been solved for specific cases in the laboratory. Such a procedure has resulted in a considerable quantity of uncorrelated data.

The problem

The problem for which an answer is sought in this thesis may be framed as follows: Can the energy losses of fluid flow through diffusers of a particular shape be expressed in a functional relationship between a set of variables which will cover a practical range of flow conditions and dimensions?

Problem analysis.--An examination of the above problem presents the following questions:

1. What particular form of diffuser would be most practical and revealing for an analysis?

2. What variables must be included in the analysis?

3. What range of the variables should be investigated?

4. How best can the energy losses be expressed?

5. Can the energy loss be formulated as a simple function of the important variables?

6. Can existing data be correlated with the results obtained from this study?

7. Will the energy losses obtained in this study be extendable to cases in which the dimensionless parameters have the same values but the absolute magnitude of the individual variables are different?

The diffuser form chosen for study in this thesis is a truncated cone.

Theoretical analysis

To determine what parameters are most important in diffused flow an analysis of the problem was made in accordance with the principles of dimensional homogeneity. From this analysis four dimensionless parameters were found to be of paramount importance. The general relationship may be expressed as follows:

$$f_3(\alpha,\ d_2/d_1,\ R_1,\ \theta) = 0 \qquad (3)$$

where α is the ratio of total energy lost in flow through the conical diffuser to the kinetic energy of the approaching flow, d_2/d_1 is the ratio of the pipe diameter downstream from the diffuser to the pipe diameter upstream from the diffuser, R_1 is the Reynolds number for the pipe before the diffuser, and θ is the total cone angle of the expansion. By means of the momentum relationship an expression representing an upper limit for α is obtained. The applicability of the above equation to all values of θ is discussed. The lower limit for α is obtained by assuming that no energy loss occurs during the expansion. For purposes of comparison an energy loss equation is arrived at for a pipe equal in length to the diffuser and having a diameter equal to the mean diameter of the diffuser.

Materials and methods

Because of available equipment, restrictions on cost, and limited time, the dimensionless parameters were varied as follows:

d_2/d_1	θ deg.	R_1
6.427	7.5	5,000
3.297	15	
1.706	30	to
1.949	60	
	90	
	180	150,000

The fluid used in this study was water which was pumped by a ten-stage, deep-well, turbine pump into a pressure tank fitted with a bell-mouth transition leading to a length of brass pipe before the diffuser. At the downstream end of the entrance length of pipe was fastened a conical transition formed of plastic (Lucite) which in turn opened into a downstream section of pipe. The piezometric head loss was obtained by measurements of piezometric head at piezometer openings spaced at regular intervals along both pipe sections. Each piezometer opening was fitted with a length of copper tubing leading to a small pressure manifold. Connected to the manifold was a mercurial manometer, for high heads, and a small hook-gage, for low heads. The quantity of discharge was obtained by directing the flow into a tank during a measured interval of time and then weighing it on a platform scale. Temperature was measured with a mercurial thermometer. From these measurements α was computed as was the Reynolds number for each run of a series of tests with a particular combination of cone angle θ and diameter ratio d_2/d_1. The extent of the experimental work consisted of a series of tests at different values of Reynolds number for each possible combination of cone angle θ and diameter ratio d_2/d_1 given in the foregoing paragraph.

The data were then plotted as follows:

1. α for each value of θ versus R_1 with d_2/d_1 constant.

2. α for each value of θ versus d_2/d_1 with R_1 constant.

3. α for several values of d_2/d_1 versus θ with R_1 constant.

Conclusions

The following general conclusions are arrived at from the foregoing study:

1. No practical equation can be determined which will express the function defined by Equation (3) throughout the investigated range of the variables.

2. The value of α may be taken as an average of the values in the interval of R_1 from 10,000 to 170,000 without introducing appreciable error.

3. For divergence angles of 60° to 180° having a diameter ratio greater than about 6.5, the loss coefficient α is nearly constant at a value slightly less than the kinetic energy coefficient K_1.

4. As θ is reduced from 60° to 7.5°, α approaches a limiting value much less than the kinetic energy coefficient. As θ is reduced this limiting value of α is approached at progressively smaller values of d_2/d_1 ; that is, a reduction of d_2/d_1 from approximately 6.5 at 60° to about 3 at 7.5°.

5. For divergences having cone angles of 7.5° or less, the value of α varies with Reynolds number R_1

in a manner similar to α computed for the frictional resistance of pipes.

6. At high Reynolds numbers (in the vicinity of 150,000), α for a constant d_2/d_1 varies only slightly as θ is increased from 60° to 180°.

7. The works of previous investigators agree in most cases with the data of this study and where large deviations occur logical explanations can be found.

8. In a divergence of any angle θ the energy loss coefficient α is not dependent upon the absolute magnitude of d_1 for any particular value of R_1, but upon the dimensionless ratio d_2/d_1.

9. In order to determine more extensively the variation of α with the Reynolds number R_1, it is recommended that future research be carried on in the range of R_1 below 10,000 and in the range above 170,000. In the first case the construction of a recirculating circuit using oil as the fluid would be desirable. In the second case high-head equipment (above 150 feet of water) would be required or provisions made to use air instead of water as the fluid. Of particular value would be the measurement of the velocity distribution and pressure intensity distribution at various sections along an expansion of angle θ for different values of R_1 in order that detailed analyses could be made of the flow patterns.

T H E S I S

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COLORADO AGRICULTURAL AND MECHANICAL COLLEGE

DECEMBER 1948

WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR SUPERVISION BY JACK EDWARD CERMAK

ENTITLED ENERGY LOSSES THROUGH CONICAL DIFFUSERS

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE.

CREDITS 10

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INTRODUCTION

Whenever an expanding section is made part of an hydraulic system a source of energy loss is introduced. Such an expanding section may be necessary in the design of a siphon transition for a canal, for conducting air from an air-conditioning unit, as a draft-tube on a turbine, and frequently when pipe size changes are required in a piping system. Because important losses of energy do occur, it is essential that the designer be able to predict what magnitude of loss will result from a given expansion when known flow conditions exist. This has led many investigators to attack the problem of determining the amount of energy loss attendant upon diffused flow, and also under what conditions a maximum conversion of kinetic energy to potential energy is possible.

Unfortunately, no mathematical analysis of the flow through diffusers has been accomplished which closely approximates the actual flow conditions. Because no general solution of the problem has been made, or seems likely to be made, investigators have turned to the laboratory for a solution. Here results have been obtained for specific cases and over a limited range of the variables involved.

The problem of generalizing the knowledge of diffused flow and correlating the results of previous work is an important one to the hydraulic designer. Because of the many variables involved a systematic attack of the problem must be planned. The study in this thesis is confined to diffusers of only one form -- a truncated cone. To complete the generalization on diffused flow parallel studies must be made in the future on diffusers of different shapes.

<u>The problem</u>

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An analysis of the problem presents the following questions:

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7. Will the energy losses obtained in this study be extendable to cases in which the dimensionless parameters have the same values but the absolute magnitude of the individual variables are different?

As previously stated, a conical diffuser section was chosen for study. This choice was made because of the wide practical applications of such a diffuser and its simplicity of form resulting in ease of fabrication.

Chapter II

THEORETICAL ANALYSIS

In this thesis no attempt is made to analyze the fluid motion by either the Navier-Stokes equations or the Reynold's equations. To determine the limiting values of energy loss, however, certain elementary analyses are necessary. These consist of a general analysis by the theory of dimensional homogeneity and application of the energy equation to obtain an expression for the total energy loss. For the special case of sudden expansion the momentum relationship and the energy equation are used to obtain an energy loss equation. The applicability of this equation for predicting energy loss through conical diffusers is discussed from the standpoint of momentum flux changes. Momentum flux is defined as the flow of momentum per unit of time across a given section. For comparison purposes an energy loss equation is arrived at for a pipe equal in length to the diffuser and having a diameter equal to the mean diameter of the diffuser. Lastly, equations are derived for the maximum and minimum limits of pressure recovery.

The variables of greatest prominence involved in this problem are the pressure loss p_L in force per unit of area; diameter d_1 of the entrance section; diameter d_2 of the exit section; total cone angle θ; mean velocity V_1 in entrance section; velocity distribution Z_1 in entrance section; density ρ of the fluid; the coefficient of dynamic viscosity μ; and the roughness e of the cone walls. The relationship may be expressed as follows:

$$f_1(p_L, d_1, d_2, \theta, V_1, \rho, \mu, Z_1, e) = 0 \qquad (1)$$

When d_1, V_1, and ρ are chosen as repeating variables the following function of dimensionless parameters is obtained:

$$f_2(p_L/\rho V_1^2, d_2/d_1, \theta, V_1 d_1 \rho/\mu, e/d_1) = 0 \qquad (2)$$

In this study the velocity distribution Z_1 in the entrance section varies with the Reynolds number R_1 only, because the entrance pipes are smooth and grids, vanes, or variable length of entrance pipe are not used. Therefore, Z_1 will be dropped from further consideration in the dimensional analysis. An attempt was made to keep the cones smooth so that the relative roughness e/d_1 is of little importance. The parameter $p_L/\rho V_1^2$ may be transformed to a more familiar form through multiplication by $2\gamma/\gamma$ --γ being the specific weight of water. Hereinafter the new parameter $\dfrac{p_L/\gamma}{V_1^2/2g}$ or

$\frac{h_L}{V_1^2/2g}$ will be designated by the Greek letter α. The resulting equation of dimensionless parameters may now be written as follows:

$$f_3(\alpha, \; d_2/d_1, \; R_1, \; \theta) = 0 \tag{3}$$

To obtain an expression for the energy loss h_L in terms of measurable quantities, the energy equation may be applied to the diffuser entrance section 1 and to the exit section 2. This yields the following expression:

$$h_L = h_1 - h_2 + K_1 V_1^2/2g - K_2 V_2^2/2g \tag{4}$$

where K_1 and K_2 are coefficients to correct for non-uniform velocity distributions. Because it is impractical to measure h_1 at the entrance section, this quantity may be obtained by measuring the piezometric head h_u in a piezometer slightly upstream from the entrance section and subtracting the energy loss h_{Lu} because of pipe resistance between the two points. The magnitude of h_2 may be obtained by measuring the piezometric head h_d at a piezometer over 40 pipe diameters below the diffuser exit and then adding the energy loss h_{Ld} because of pipe resistance between the upper and lower point. In order to correct the kinetic energy term a velocity distribution corresponding to

$$v = v_{max}(1 - a/r)^{1/7}$$

may be used in which v is the velocity at any point, a is the radial distance to any point measured from the pipe center, and r is the pipe radius. Integration of the kinetic energy over the entire cross-section using this velocity distribution gives coefficients, K_1 and K_2, of 1.058. The resulting equation for the energy loss is then

$$h_L = h_u - h_d - (h_{Lu} + h_{Ld}) + \frac{1.058}{2g}(V_1^2 - V_2^2) \tag{5}$$

or

$$\alpha = (h_u - h_d - h_{Lu} - h_{Ld})\frac{2g}{V_1^2} + 1.058\left[1 - (d_1/d_2)^4\right] \tag{6}$$

In the case of sudden expansions a theoretical loss may be derived by using the momentum relationship with the energy equation. First consider the change of momentum flux in passing from the expansion entrance to a point in the large pipe where the conversion process has been completed. According to Kalinske (10:370) this point is about 35 pipe diameters below the diffuser exit. Mathematically the momentum relationship may be written as

$$K_m V_2 A_2 \rho (V_2 - V_1)$$

where the vector notation has been omitted, but it must be remembered that V_1 and V_2 are in the same direction, and K_m is a coefficient to correct for a non-uniform velocity distribution. The above change in momentum flux may be equated to the resultant force in the direction of V_1 and V_2. By assuming that the pressure intensity at the expansion entrance p_1 is uniform over the area of the large pipe the resultant force becomes $p_1A_2 - p_mA_2$ where p_m is the pressure intensity (boundary resistance being neglected) in the large pipe at a section downstream from the expansion at which the conversion process has been completed. The resulting equation is then

$$p_1 A_2 - p_m A_2 = K_m V_2 A_2 \rho (V_2 - V_1) \tag{7}$$

or

$$\frac{h_1 - h_2}{V_1^2/2g} = 2 K_m \left[(d_1/d_2)^4 - (d_1/d_2)^2\right] \tag{8}$$

If in Equation (4) the right hand part of Equation (7) is substituted in place of the piezometric head difference $h_1 - h_2$ after A_2 has been eliminated and both sides divided by γ, the following expression for the head loss (Borda loss) is obtained:

$$h_L = (V_1 - V_2)^2/2g \tag{9}$$

or

$$\alpha = 1 - 2(d_1/d_2)^2 + (d_1/d_2)^4 \tag{10}$$

Here K_1, K_2, and K_m have been assumed to be unity. When the coefficients are evaluated for a velocity distribution corresponding to the 1/7th-power law K_1 and K_2 become 1.058 and K_m becomes 1.022. Equation (10) may then be written as follows:

$$\alpha = 1.058 - 2.044(d_1/d_2)^2 + 0.986(d_1/d_2)^4 \tag{11}$$

Examination of Equations (7) and (8) reveals the fact that the change in momentum flux between sections 1 and 2 for any particular value of d_2/d_1 is a fixed quantity independent of the form of transition and distribution of pressure intensity. The physical significance of this is that for any transition having the same value of d_2/d_1 the resultant force acting on the mass of fluid due to a change in momentum flux between sections 1 and 2 must be the same. The change in total head as given in Equation (11) represents the maximum possible loss due to form resistance -- this type of resistance resulting from separation and the consequent high internal shear in the fluid. A means is now at hand to predict the variation of experimental results from Equations (8) and (11) when the pressure intensity distribution assumed for their formulation is no longer valid.

It is of interest to investigate how $h_1 - h_2$ and α will vary from Equations (8) and (11) respectively for the sudden expansion when (1) the average pressure intensity for section 1 across the area $A_2 - A_1$ is greater than p_1, and (2) the average pressure intensity over the same area is less than p_1. In the first case $|p_1 - p_2|$ must be greater than the corresponding value arrived at by the assumption in Equations (8) and (11). The result is then a smaller value of α for a given value of d_2/d_1. In the second case the opposite must hold true.

As d_2/d_1 approaches 1 the agreement with Equations (8) and (11) should become better, for then the difference between A_2 and A_1 approaches zero and any variation of the pressure intensity at section 1 from the assumed will have small effect upon the value of $p_1 - p_2$ and α. As d_2/d_1 approaches infinity the difference $p_1 - p_2$ becomes so small that very little deviation of experimental results from the equations derived may be expected. In the limit as d_2/d_1 approaches infinity for the sudden expansion it is logical to expect exact agreement with the equations since the entire velocity head will be dissipated and p_1 will equal p_2.

Application of Equations (8) and (11) to expansions having angles less than 180° breaks down into three main cases as follows:

1. The angle reduced from 180° is such that the flow still separates to the extent that a backflow occurs along the boundary of the transition and the boundary shear is negative.

2. The angle is further reduced so that no backflow occurs near the boundary and the velocity gradient approaches zero at the wall. In this case the shear is zero.

3. The angle is finally reduced until the velocity gradient is similar to that for a circular pipe resulting in positive shear at the boundary.

In the first case, as in all cases, a particular value of d_2/d_1 fixes the momentum flux before and after the expansion. However, the force now causing a change of momentum flux is the difference between p_2A_2 and p_1A_1 combined with a force described by $\int_{A_2-A_1} p \cos\frac{\Theta}{2}\, dA$ instead of $p_1A_2 - p_2A_2$ as for the sudden expansion. The magnitude of $p_1 - p_2$ will consequently be a function of the latter force which in turn is a function of the pressure intensity variation along the cone. For small values of d_2/d_1 Equation (11) should be in close agreement with experimental values because $A_2 - A_1$ will be small and the force defined by the integral will have small influence. As d_2/d_1 increases the force defined by the integral will become more important provided any appreciable increase in the value of p occurs. However, for this case the pressure conversion will be very small and consequently Equation (11) should apply reasonably well. Again as d_2/d_1 approaches infinity, p_1 and p_2 will be nearly equal and Equation (11) should hold with good approximation.

For the cone angles falling in class two, it is to be expected that marked variation between Equation

(11) and experimental results will exist. Here the transition will be accomplished with very little loss due either to form resistance or to boundary resistance. Realizing that little loss exists, the force defined by the integral equation will become large for increasing values of d_2/d_1; hence, to produce a given change in momentum flux for a given value of d_2/d_1 the value of $|p_1 - p_2|$ will be much larger than that given by Equation (7). It follows, therefore, that the experimental values of α will be smaller than those given by Equation (11). Apparently, for any cone the maximum value of α will be approached at the value of d_2/d_1 for which the remaining kinetic energy is very small. This value of d_2/d_1 will be given in a following paragraph.

In the third case the momentum relationships will fail to give approximate results because of the large energy loss due to boundary resistance. It is logical to expect that the variation of α with Reynolds number will possibly be predictable by use of the pipe resistance equations. For comparison an equation for α will be developed on the basis of the resistance loss through a length of pipe equal to the length of the transition and having a diameter equal to the mean of the transition diameter. The usual pipe equation gives the head loss as follows:

$$h_L = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

where f is the resistance coefficient, L the length of pipe, D the pipe diameter, and V the mean pipe velocity. This may be written as follows in terms of α

$$\alpha = f\,\frac{L}{D}\left(\frac{V}{V_1}\right)^2$$

In the present case let f be expressed by the Blasius function $0.316/R^{\frac{1}{4}}$ and the other quantities in terms of the transition dimensions. The expression for α will then become

$$\alpha = \frac{0.316}{R_1^{1/4}}\,\frac{1}{4^{3/4}\tan\theta/2}\,\frac{\left[1+2(d_1/d_2)^2+(d_1/d_2)^4\right]\left(\frac{d_2-d_1}{d_2+d_1}\right)}{\left[1+(d_1/d_2)^2+d_2/d_1+d_1/d_2\right]^{1/4}} \qquad (12)$$

To maintain flow in this case as d_2/d_1 approaches infinity and θ approaches zero, $p_1 - p_2$ will of necessity approach positive infinity. It is not expected that this equation will give the correct magnitude for α but merely will be a guide as to the way α varies with respect to R_1.

A minimum value of α will of course occur if it were assumed that no loss occured during the transition. The pressure head gain will than equal the change in velocity head and α may be expressed as follows:

$$\alpha = 0 \qquad (13)$$

In order to investigate the trend of piezometric head losses Equation (4) may be separated into two parts as follows:

$$\alpha = \frac{h_1 - h_2}{V_1^2/2g} + 1.058\left[1 - (d_1/d_2)^4\right] \tag{14}$$

or

$$\alpha = \beta + \lambda \tag{15}$$

For the experimental data β will then represent the ratio of the difference in piezometric heads between sections 1 and 2 to the velocity head in the upstream section. This will be negative since h_2 will be larger than h_1. For any particular value of d_2/d_1 λ will be constant regardless of the cone angle θ, if the kinetic energy coefficients K_1 and K_2 are assumed to be constant. A plot of λ versus d_2/d_1 is given in Fig. 8. An examination of this plot shows that for values of d_2/d_1 equal to and larger than 2.75 the change in velocity head is practically negligible. Therefore, for cone angles of the second case the value of α will be almost a constant in the range of d_2/d_1 above 2.75.

As previously done for α, β may be expressed as a mathematical function in two limiting cases. In the case of minimum pressure recovery as given by momentum change considerations, β may be expressed as follows from Equation (8):

$$\beta = 2.044\left[(d_1/d_2)^4 - (d_1/d_2)^2\right] \tag{16}$$

Likewise a maximum limit of β may be obtained from Equation (4) when h_L is assumed to be zero. In this case β is:

$$\beta = 1.058\left[(d_1/d_2)^4 - 1\right] \qquad (17)$$

Because the expressions for β are derived from corresponding equations for α, the same reasoning applies to the expected agreement between the experimental data and Equations (16) and (17).

Chapter III

REVIEW OF LITERATURE

In diffused flow the matter of practical interest is to determine how much of the oncoming kinetic energy is transformed into potential energy for a given diffuser and known flow conditions. In this thesis the main objective is to determine the magnitude of this transformation by measuring the energy loss accompanying the process. This has also been the objective of divers experimental work during the past forty years. Recently, however, emphasis has been placed on a more fundamental problem -- what internal action results in the dissipation of energy during diffusion and how does this action vary with the form of diffuser? Since there have been many investigations carried out over a long period of time during which the knowledge of fluid mechanics of real fluids was advancing from infancy, careful examination of the methods and results of each investigator must be made.

Andres (1), Germany, in 1909 performed the first important experiments on diffused flow. In these tests the piping upstream from the cone consisted of a

2.95 inch pipe with a rounded transition into a short section of 1.58 inch pipe before the cone. Among the great variety of shapes tested -- curved wall transitions, multi-angle conical transitions, and rectangular shapes -- were four conical diffusers. In addition Andres varied the velocity profile at the cone entrance by allowing the water to flow through a series of 20 sieves in order to obtain an essentially rectangular velocity pattern and also a spiral flow was induced by placing metal vanes in the approach pipe. Although a confusingly large number of variables were introduced Andres stated the following conclusions:

1. In column 1 (referring to a table of computed values of efficiency, η) we see the least favorable η values. They refer to water, which was combed through 20 sieves before entering the nozzle. The sieves were placed behind each other at distances of 5 mm. Such a flow whose streamlines are at least very nearly parallel, has an especially strong inclination to depart from the walls, ... (1:88)

2. Referring to the type of flow in the conclusion above a maximum efficiency was found to occur for a cone angle of 4.25°.

3. A greater efficiency occurs when rotational motion is imparted to the oncoming flow.

Gibson (5,6,7), University College, Dundee, Scotland, carried out a series of experiments between 1909 and 1912. He considered conical diffusers with cone angles of 3^o, 4^o, 5^o, $7\frac{1}{2}^o$, 10^o, $12\frac{1}{2}^o$, 15^o, $17\frac{1}{2}^o$, 20^o, 30^o, 40^o, 50^o, 60^o, 90^o, and 180^o. The ratios of inlet to exit diameter d_2/d_1 were 1.5 (2" to 3"), 2(1.5" to 3"), and 3 (0.5" to 1.5" and 1" to 3"). The range of Reynolds number for the tests was 50,000 to 250,000. The energy loss was expressed as a percentage of the theoretical loss due to sudden expansion $(V_1 - V_2)^2/2g$--

$$\% H_L = \frac{(h_1 - h_2) + (V_1^2 - V_2^2)/2g}{(V_1 - V_2)^2/2g} \times 100 \tag{18}$$

where h_1 is the piezometric head at the entrance to the diffuser and h_2 is the piezometric head at the exit of the diffuser obtained by measuring the piezometric head at a point 5-12 inches downstream from the diffuser exit and extrapolating back to the exit along the normal pressure gradient. Since Professor Gibson found no uniform variation of the percentage loss with Reynolds number, his published values represent an average for the range covered. Furthermore, no attempt was made to separate the loss due to wall resistance from that due to form resistance. Two formulae were deduced from the data (7:91-2); (1) loss of head h_L in feet of water for sudden expansions ($\theta = 180^\circ$) --

$$h_L = \left[\frac{102.5 + 0.25(d_2/d_1)^2 - 2.0d_1}{100}\right](V_1 - V_2)^2/2g \tag{19}$$

where $1.5 \leq d_2/d_1 \leq 3.5$ and $0.5'' \leq d_1 \leq 6''$, and (2) loss of head h_L for conical diffusers --

$$h_L = 0.011\ \theta^{1.22}(V_1 - V_2)^2/2g \tag{20}$$

where $7.5^\circ \leq \theta \leq 35^\circ$.

The losses given by Professor Gibson have been found larger than those obtained by later tests. This discrepancy may be accounted for since the piezometric head h_2 was extrapolated from a point only 5-13 inches downstream from the diffuser exit which in the light of more complete pressure measurements on the downstream pipe is not a sufficient distance for complete conversion of energy to occur (2:1025, 14:3). This of course resulted in h_2 being too small and the corresponding percentage loss of energy too large.

Gibson also concluded that small pipes have a greater percentage loss than large pipes with the same diameter ratio d_2/d_1. Such a conclusion is not justified because the values of h_2 were not taken in a part of the downstream pipe where the turbulent energy created by expansion had been dissipated. Therefore, a change in pipe size also changed the magnitude of error in h_2.

Archer (2), University of California, Berkley, in 1913 performed a series of experiments on sudden expansions. In his tests the Reynolds number R_1 was varied between 10,000 and 250,000. The values of d_1, d_2, and d_2/d_1 were as follows:

d_1 (in.)	d_2 (in.)	d_2/d_1
2.486	2.994	1.204
1.730	2.495	1.442
1.730	2.994	1.731
0.982	2.495	2.541
0.982	2.994	3.049

The loss of head h_L was expressed as a function of the entrance velocity V_1 and the diameter ratio d_1/d_2 --

$$h_L = 1.098 \frac{V_1^{1.919}}{2g} \left[1 - (d_1/d_2)^2\right]^{1.919} \tag{21}$$

Riffart (15), Germany, in 1922 carried out experiments using air as the fluid with the goal of determining how different initial velocity distributions affect the energy losses in diffusers. The main study was on induced spiral flow for which he found a decrease in energy loss compared with non-spiral flow.

Vedernikov (16), Central Aero-Hydrodynamic Institute, Moscow, U.S.S.R., conducted a series of tests on rectangular diffusers in 1927. Here only the most efficient angle of divergence was determined for the particular flow velocity and shape desired.

Peters (14), Germany, performed a series of tests in 1934 to determine the effect of changing the velocity profile in the diffuser entrance. He accomplished this by varying the length of pipe between the rounded transition and the diffuser entrance. By this means the velocity profile was varied from practically rectangular in form to the approximate 1/7th power distribution. Furthermore a spiral flow was introduced in the second part of the experiment. All tests were carried out with a constant quantity of discharge and a constant value of diameter ratio d_2/d_1 of 1.529 (2.756" to 4.214"). Cone angles of 5.2°, 7.8°, 8.8°, 11.4°, 14.7°, 28.4°, 55.8°, 66.9°, 91.6°, and 180° were used. The Reynolds number was always near 200,000 for the upstream pipe. From these tests the diffuser efficiency was determined. The special efficiency term η_{OII} was defined as the ratio of the rise in piezometric head between the entrance and exit of the diffuser to the change in kinetic energy between the same two points corrected for boundary resistance along the cone:

$$\eta_{OII} = \frac{h_2 - h_1}{V_1^2/2g\,[K_1 - K_2(A_1/A_2)^2]} + \frac{f}{4 \tan \theta/2} \qquad (22)$$

where h_2 is extrapolated back to the exit from the point of maximum pressure, K_1 is the kinetic energy coefficient at the entrance to correct for the difference between the mean velocity squared and the mean of the point velocities squared and was determined as a function of the entrance pipe length by velocity traverse, K_2 is the kinetic energy coefficient at the exit and was taken as constant at 1.03, A_1 and A_2 are the pipe areas upstream and downstream respectively. The last term corrects for wall resistance and is obtained by integrating the expression $\int_0^L (f/r)(v^2/2g)\,dL$ where f is the loss coefficient which was assumed to be constant at 0.008, v the velocity at any point, r the radius at any section of the cone, and L the cone length. Peters concluded that with the exit length of pipe a change in velocity profile had small effect upon the efficiency; however, induced spiral flow produced a marked increase in efficiency. He also concluded that the energy loss computed as a ratio of $(V_1 - V_2)^2/2g$ was more useful than the efficiency.

Several other efficiencies were defined in the paper, but the one given seems to be of greatest significance. This efficiency is not completely sound since the factor correcting for boundary resistance has been added in an attempt to separate the boundary loss from the form loss. Because f is a function of boundary shear it is impossible to conceive that this quantity is a constant. Especially is this true for cone angles greater than 7.5° where considerable separation takes place.

In 1934, Patterson (13), reviewed the available literature on diffused flow. No new data were produced, but the important findings to date were assembled along with a bibliography as an aid to designers seeking information on the subject.

Gourzhienko (8), Central Aero-Hydrodynamical Institute, Moscow, U.S.S.R., in 1939 derived formulas for computing velocity and pressure distributions in the turbulent flow along, and perpendicular to, the axis of a diffuser of small cone angle. His analysis began with the Reynolds equations of motion. The main assumptions made were; (1) the motion takes place along straight lines intersecting at the vertex of the diffuser cone, (2) the normal components of the turbulent stress tensor are isotropic and their gradients along the diffuser are negligible in comparison with the gradient of the static pressure, and (3) that the curve of dependence of the nondimensional mixing length on the distance from the wall is absolute.

Experimental tests were conducted on diffusers 6 meters in length having an entrance diameter of 240 millimeters and cone angles of 1° and 2°. Reynolds number R_1 was varied from 77,800 to 214,000. The important conclusions arrived at are as follows:

1. The assumption of radial flow is satisfied with a large degree of approximation.

2. The formula obtained for the velocity distribution agrees with the measured distribution.

3. The computed increase in static pressure along the diffuser deviates little from the test results.

At the University of Iowa, Kalinske (10), conducted a series of experiments in 1946. In this study the objective was to determine the action by which energy is dissipated in an expansion process. Measurements were made of the velocity components in the direction of mean flow and also at right angles to the mean flow. Measurements also were made by taking motion pictures of brightly illuminated particles having the same specific weight as the water. These measurements were made at various sections of the transparent pipe and the expansion. Also pressure measurements were taken as had been done in previous experiments. Cone angles of 7.5°, 30°, and 180° were used with a constant diameter ratio d_2/d_1 of 1.73 (2.75" to 4.75") and a cone angle of 15° with a diameter ratio of 1.67 (3" to 5"). The range of Reynolds number R_1 was from 30,000 to 83,000. The most important conclusion resulting from this study is that the maximum total turbulent energy is a small part of the energy lost in the transformation process. This implies that the major energy loss occurs in the zone of intense shear between the high-velocity jet and the surrounding, relatively quiet fluid. Also it was found that large scale turbulence such as occurs in the 30° expansion is much more effective in reducing the high-velocity jet than is small scale turbulence such as occurs in the 180° expansion.

Chapter IV

MATERIALS AND METHODS

The laboratory work required for this study is concerned with determining the total energy loss which occurs with each combination of diameter ratio d_2/d_1, cone with central angle θ, and Reynolds number R_1. Three measurements must be made in order to accomplish this; (1) piezometric head gradient of the upstream and downstream pipes, (2) quantity of discharge, and (3) temperature of the water. A description of the equipment and instruments used is given followed by an account of the methods used and problems encountered in collecting data.

Because of available equipment, restrictions on cost, and limited time the dimensionless parameters were varied as follows:

d_2/d_1	d_1 (in.)	d_2 (in.)
6.427	0.626	4.023
3.297	0.626	2.064
1.706	0.626	1.064
1.949	2.064	4.023

θ (deg.)	R_1
7.5	5,000
15	
30	to
60	
90	150,000
180	

The conical transitions (Fig. 1) were constructed of transparent, sheet acrylic resin (Lucite). The developed cone was cut from a sheet of plastic, heated, and deformed until the two edges were touching throughout the entire length. The edges were then cemented together. Finished forming was accomplished by forcing the cemented section, while hot, over a wooden, conical form cut to the desired angle. Final diametral dimensions at the ends were obtained by sanding off the excess material. Each cone was then fitted with two flanges which matched those on the upstream and downstream pipes. Cones were fabricated in this manner for transitions from diameters of 1" to 2" and 2" to 4". Transitions from a diameter of 5/8" to 1" were machined from polished plastic rod. When a transition from diameters of 5/8" to 2" and 5/8" to 4" was desired two or more of the above cones having the same central angle θ were flanged together. Care at all times was exercised in fitting the cones to the pipes and one cone to another so as to avoid discontinuity in the inner surface. Furthermore, to insure a continuous surface and to prevent leaking, gaskets were replaced by a thin film of belt dressing on the flange faces. Care was also taken to remove any excess cementing material from the inner surface of the longitudinal joint of the cone.

The sudden expansion, $\theta = 180^{\circ}$, was provided for by machining several steel plates with a central hole the same diameter as the upstream pipe and a set of holes, countersunk on the downstream side, matching the flange holes of the upstream pipe flanges. Also sets of holes, countersunk on the upstream face, were made to match the flange holes of any downstream pipe flange which was larger than the upstream pipe. With these plates any two pipes of different diameter could be fastened together to form a sudden expansion.

Seamless, brass pipes were used for the diameters of 5/8", 1", and 2". This pipe was obtained in 12' lengths. Two lengths of pipe were used on the diffuser entrance section. These two pipes were fastened together by brass flanges which were soldered to the pipe. Matching dowel pins and holes were placed in the flanges so that the joint could be accurately matched. The 5/8" pipe section was 24' and the 2" upstream section was $22\frac{1}{2}$' in length. Standard steel pipe was used for the 4" diameter and the section used was composed of two eight foot lengths joined together by a threaded coupling.

The pump and accompanying piping system are shown in Fig. 3. A ten-stage, four inch, deep-well turbine pump was used. In order to obtain a non-fluctuating flow at small discharges, flow into the pressure tank was throttled at valve 1 and the by-pass valve 2 was opened. This allowed a large discharge through the pump, but only a small discharge through the experimental circuit. To quiet the incoming water, the vertical entrance pipe in the pressure tank was closed at the top and perforated on the side opposite the entrance leading to the conical transition. A bell-mouth entrance was constructed for the transition from pressure tank to entrance section of the diffuser. This transition was cast of aluminum and the inner surface machined to the desired contour. Although the tank was constructed with six outlets, only the third outlet from the bottom was used in this study. The entrance section and exit section of pipe were supported at the same elevation as the pressure tank outlet by means of a series of stands. To the downstream end of the exit section of pipe was fastened an open box with the top edge about four inches higher than the top of the exit pipe. The box was provided with an overflow chute from which the discharged water could be collected in the weighing tank. The purpose of this box was to keep the exit section of pipe full at all discharges.

The discharge was obtained by weighing the water with a platform scale measuring to the nearest ¼ pound. Time was measured with a stop watch to the nearest 0.01 second.

To determine the piezometric gradients along the pipe, piezometer holes were drilled into the pipe sections at regular intervals throughout the section length. The holes were 1/16" in diameter. Great care was taken to make the axis of the hole coincide with a pipe radius and also to eliminate all burrs from the inner surface. All piezometer holes were connected to a pressure manifold by means of 3/16" copper tubing. The tubing was fastened to the pipe by first soldering a threaded adapter plate approximately 1" square centered on the piezometer hole. Into the threaded adapter was then screwed a copper tubing fitting. Piezometric heads were measured to 0.01' of mercury with the manometer and to 0.001' of water with the hook gage.

For each Reynolds number used, the piezometric head at each piezometer was obtained. For all heads above 2' of water the mercurial manometer was used and for heads below this the hook gage was used. All readings were referred to the center-line of the pipe and index corrections for the manometer and hook gage were obtained to an accuracy of 0.001' with an engineers level. After reading all piezometers, a check was then made on the first piezometer. In the first series of tests in which the value of d_1 was 2" and d_2 was 4" an open water manometer reading to 0.01' of water was used for the lower heads. However, because of the small loss of head in the 4" pipe at low discharges this arrangement was not sensitive enough and the hook gage was installed. The disadvantage of the hook gage was that from 5-10 minutes were required for the 1" diameter column of water to reach equilibrium. All readings were then converted to feet of water and diffuser entrance section values were plotted versus the distance of the piezometer from the first upstream piezometer and exit section values were plotted versus the distance from the diffuser exit -- a sample plot is shown in Fig. 2. The value of h_{Lu} in Equation (6) was obtained by measuring the distance from where h_u was measured -- the piezometer nearest the entrance section -- to the diffuser entrance and multiplying by the piezometric head gradient. h_{Ld} was obtained by measuring the distance from where h_d had been measured -- the piezometer at the diffuser exit section end -- and multiplying by the piezometric head gradient in the downstream part of the exit section. The gradient used in determining h_{Ld} was always far enough below the diffuser to be the gradient resulting from pipe resistance only. To check the gradients a resistance coefficient for the pipe was computed for each run and compared with the established value for smooth pipes. These values are plotted in Fig. 13.

For each combination of cone and diameter ratio the tests were begun at the highest Reynolds number possible by opening the pressure tank entrance valve and closing the by-pass valve. Lower Reynolds numbers were then obtained by opening the by-pass valve in steps until completely open. To obtain even lower values the entrance valve to the pressure tank was closed in steps while the by-pass valve remained open. This procedure was adopted after the first series of tests in which d_1 was 2" and d_2 was 4" yielded large periodic pressure fluctuations during low discharge without use of a by-pass valve. Also in this initial series of tests pressure variation during a particular trial resulted from using a perforated disc to keep the exit section pipe full at all times. Small fibers and other debris clogged some of the disc openings which resulted in a reduced discharge and increased pressure intensity. This difficulty was overcome by flushing out the pump sump and using the open box, previously described, at the discharge end of the exit pipe.

The quantity of flow was obtained for each trial by averaging the result of three independent measurements. A variation between the three measurements of 0.1% to 1.0% was obtained at low discharges and high discharges respectively.

Temperature was measured in the open box at the end of the discharge pipe by a mercurial thermometer reading to the nearest 0.5° F. A reading was taken at the beginning and end of each individual run and the mean used in computing Reynolds number. The maximum variation in temperature during any one trial was 1° F.

Chapter V

ANALYSIS OF DATA

Introduction

As an aid to the analysis and solution of the problem set forth in Chapter I, the variables of Equation (3) were plotted in three different arrangements as follows:

1. α for each value of θ versus R_1 with d_2/d_1 constant (Figs. 4, 5, and 6).

2. α for each value of θ versus d_2/d_1 with R_1 constant (Fig. 7).

3. α for several values of d_2/d_1 versus θ with R_1 constant (Fig. 11).

Agreement of the data with predictions made from the theoretical analysis of Chapter II is discussed and approximate relationships between the variables pointed out. This is followed by a comparison of previous data with the data of this study. Finally, the effect of varying the absolute magnitude of d_1 and d_2 but holding d_2/d_1 constant is discussed.

Relationship between Dimensionless Parameters

In Figs. 4, 5, and 6 the variation of α with

R_1 may be examined for each combination of θ and d_2/d_1 investigated. The general trend to be noted is a decrease in α with increasing values of R_1. For θ equal to 7.5° there is agreement with Equation (12) in the rate of change of α with respect to R_1 as was predicted for small angles in case 3 of Chapter II. Apparently the major loss in this case is due to boundary resistance. In the range of θ between 7.5° and 180°, however, the variation of α with R_1 is more complex. It may be that for these larger angles (cases 1 and 2 of Chapter II) the point of separation varies with R_1 and causes α to become unpredictable. In the special case of a sudden expansion, α varies with R_1 to a smaller extent -- a situation which seems reasonable because the separation point is fixed. Equation (3) does not seem subject to analytical expression in the range of R_1 in Figs. 4, 5, and 6 because of the apparent transition in flow conditions.

To investigate more closely the variation of α with d_2/d_1 for each cone angle, the curves of Fig. 7 were prepared from Figs. 4, 5, and 6 for two values of R_1 -- 20,000 and 150,000. Comparison of the data with Equations (11) and (13) reveals, as predicted, that for the most part these equations are the upper and lower limit, respectively, of the energy loss coefficient α.

The large values of α for the 60° and 90° transitions at the small value of R_1 is probably the result of boundary resistance which is ignored in Equation (11). Of particular importance is the fact that all the curves tend to approach an horizontal asymptote. As predicted in Chapter II the limit of α for the sudden expansion and the angles in case 1, which apparently includes those angles between 60° and 180°, is very near the kinetic energy coefficient K_1. For cone angles between 7.5° and 60° the maximum value of α is much less than the kinetic energy coefficient. In this range of angles it may be seen that α is practically constant for d_2/d_1 greater than 3.

The variation of β, plotted in Figs. 9 and 10, substantiates the magnitude of pressure conversion assumed in the discussion in Chapter II of momentum flux change for the various angles of expansion. In general the same observations apply to these curves as for those of Fig. 7.

To picture the limiting values of α with respect to the parameter d_2/d_1, Fig. 11 was prepared from Fig. 7 for a Reynolds number of 150,000. Here, of course, the lower limit occurs when d_2/d_1 is equal to 1. The upper limit is approached as d_2/d_1 becomes infinite. The curve was obtained by applying the conclusions of Chapter II in regard to values of α as d_2/d_1 approaches infinity. An important feature of this family of curves is that, for a particular value of d_2/d_1, α remains practically constant throughout the range of θ between 90° and 180°. Curves of α for θ less than 7.5° are broken lines indicating estimates only. The minimum value of was estimated to occur for θ approximately equal to 6° -- Gibson (5) -- and for still smaller angles the general trend of α was obtained from the discussion in Chapter II of angles falling in case 3.

Correlation of Previous Data

Because α does not vary greatly with R_1 an average value was taken between R_1 equal to 10,000 and 170,000 for each combination of θ and d_2/d_1. These mean values were then plotted in Fig. 12. This averaging procedure, which was also used by Gibson (5, 6, 7), eliminated R_1 as a variable and made possible the comparison of the data in this study with those of other investigators. It may be seen that in all cases the average α from these data is less in magnitude than the corresponding values from Equation (11).

Data from the research of previous investigators were converted into equivalent values of α and also plotted in Fig. 12. The data of Gibson (5, 6, 7) were converted to α by the following transformation:

$$\alpha = \frac{\%H_L}{100}\left[1-2(d_1/d_2)^2+(d_1/d_2)^4\right]+(K-1)\left[1-(d_1/d_2)^4\right]$$

where $\%H_L$ is the numerical value of the percentage loss obtained by Gibson -- Equation (18). These values are given in Table 9. In general the data of Gibson resulted in larger values of α than obtained in this study. The explanation for these larger values is given in Chapter III. Equation (20) was used for θ equal to 7.5°, 15°, and 30°. In general the curves fall below those representing the data in this study. This disagreement is probably due in large measure to Gibson's failure to take into consideration the kinetic energy coefficient K.

The data from experiments performed by Archer (2) on sudden expansions, computed in the manner described in Chapter IV, are also plotted. The estimated mean value of α is tabulated in Table 6. An examination of these values plotted in Fig. 12 shows them to be in close agreement with the average values of the present study.

By the use of Equation (22), α for Peters' data may be found as follows:

$$\alpha = \left(1.058 + \frac{f}{4\tan\theta/2} - \eta_{OII}\right)(0.572)$$

in which η_{OII} was taken from (14:Fig. 25). In Fig. 12 it may be seen that the values of α are lower than those for this thesis. Although this is probably in large measure due to Peters' method of extrapolating h_2

(see Chapter III), his larger value of R_1 (200,000) would also tend to cause a lower value of α.

Data from experiments by Kalinske (10) were also used to compute α. These values are given in Table 8. A plot of the values in Fig. 12 shows excellent agreement with the results of this study for θ equal to 30° and 180°; however, for θ equal to 7.5° and 15° Kalinske's values are higher.

In summary it can be said that the discrepancies between results may be attributed to four main causes; (1) errors resulting from averaging the values of α over different ranges of Reynolds number, (2) inconsistencies due to measuring the piezometric head h_2 from different reference points in the diffuser exit section, (3) different assumptions as to kinetic energy coefficients, and (4) the variation in diffuser entrance conditions.

Effect of Changing the Magnitude of d_1

Gibson (7:91) concluded that a greater percentage loss because of expansion occurs for small pipes than for large pipes having the same diameter ratio d_2/d_1. In the following paragraphs the validity of this conclusion is tested from the standpoint of dimensional analysis and by comparison of experimental data.

In the dimensional analysis of Chapter II, Equation (3) was found to express the flow phenomena through conical expansions by four dimensionless parameters. The variable in question d_1 appears not alone, but in the ratio of d_2/d_1 and in the formulation of a Reynolds number R_1. By virtue of this alone it is logical to conclude that the only effect of varying the magnitude of d_1, while d_2/d_1 remains constant, is to change the magnitude of R_1. This points to the possibility that the conclusion of Gibson resulted from observations on tests using various pipe sizes (while holding d_2/d_1 constant) for which the Reynolds number did not remain constant.

In Fig. 12 a comparison of the data from this study for d_2/d_1 equal to 1.70 and 1.95 can be made because these values do not differ greatly. In the first case d_1 was equal to 0.626 inch and in the second case d_1 was equal to 2.064 inches. Both sets of points lie very near the smooth curve representing the experimental data throughout the range of Θ, although the values of d_1 vary by more than a factor of three. The deviation of the second set may be attributed to experimental error and the averaging process. Fig. 12 gives more variety of points for examination along the 180° curve. At diameter ratios from 1.50 to 1.75 is a group of points from the data of Kalinske, Archer, and this study. An examination of the d_1 values entered with the plot indicates that the energy loss coefficient does not vary in any regular manner with this quantity and may be regarded as constant with respect to d_1 except for small experimental errors, variations due to different Reynolds numbers, and possibly dissimilar entrance conditions.

In light of the foregoing discussion, it may be said that the statement of Gibson (7:91) is not valid -- instead the energy loss coefficient α is independent of d_1 and dependent upon the diameter ratio d_2/d_1 and R_1 for any particular cone angle θ.

Chapter VI

CONCLUSIONS

In this thesis many data were collected on the magnitude of energy losses through conical diffusers. Some of the data were collected for use in correlation with results of previous works and still other data were collected in an effort to extend the range of experimental knowledge with respect to the diameter ratio d_2/d_1 and the Reynolds number R_1. By virtue of comparison and extension of energy loss data, the following general conclusions are inferable:

1. No practical equation can be determined which will express the function defined by Equation (3) throughout the investigated range of the variables.

2. The value of α may be taken as an average of the values in the interval of R_1 from 10,000 to 170,000 without introducing appreciable error.

3. For divergence angles of 60° to 180° having a diameter ratio greater than about 6.5, the loss coefficient α is nearly constant at a value slightly less than the kinetic energy coefficient K_1.

4. As θ is reduced from 60° to 7.5°, α

approaches a limiting value much less than the kinetic energy coefficient. As θ is reduced this limiting value of α is approached at progressively smaller values of d_2/d_1; that is, a reduction of d_2/d_1 from approximately 6.5 at 60° to about 3 at 7.5°.

5. For divergences having cone angles of 7.5° or less, the value of α varies with Reynolds number R_1 in a manner similar to α computed for the frictional resistance of pipes.

6. At high Reynolds numbers (in the vicinity of 150,000), α for a constant d_2/d_1 varies only slightly as θ is increased from 60° to 180°.

7. The works of previous investigators agree in most cases with the data of this study and where large deviations occur logical explanations can be found.

8. In a divergence of any angle θ the energy loss coefficient α is not dependent upon the absolute magnitude of d_1 for any particular value of R_1, but upon the dimensionless ratio d_2/d_1.

9. In order to determine more extensively the variation of α with the Reynolds number R_1, it is recommended that future research be carried on in the range of R_1 below 10,000 and in the range above 170,000. In the first case the construction of a recirculating circuit using oil as the fluid would be

desirable. In the second case high-head equipment (above 150 feet of water) would be required or provisions made to use air instead of water as the fluid. Of particular value would be the measurement of the velocity distribution and pressure intensity distribution at various sections along an expansion of angle θ for different values of R_1 in order that detailed analyses could be made of the flow patterns.

A P P E N D I X

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Appendix A.--Figures 1-13

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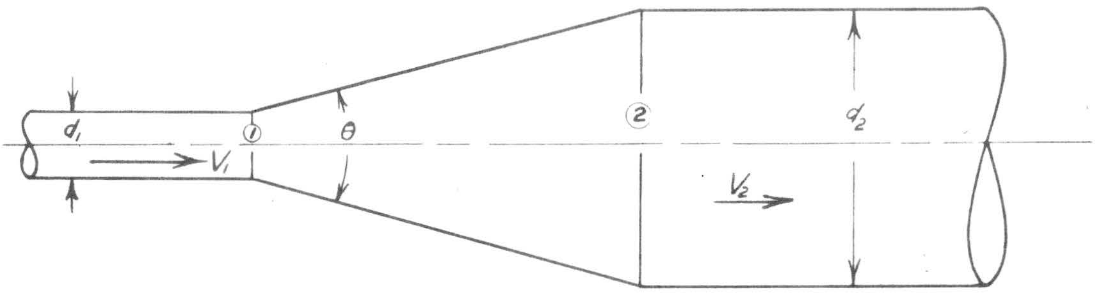

FIG. 1.-- DEFINITION SKETCH OF CONICAL TRANSITION

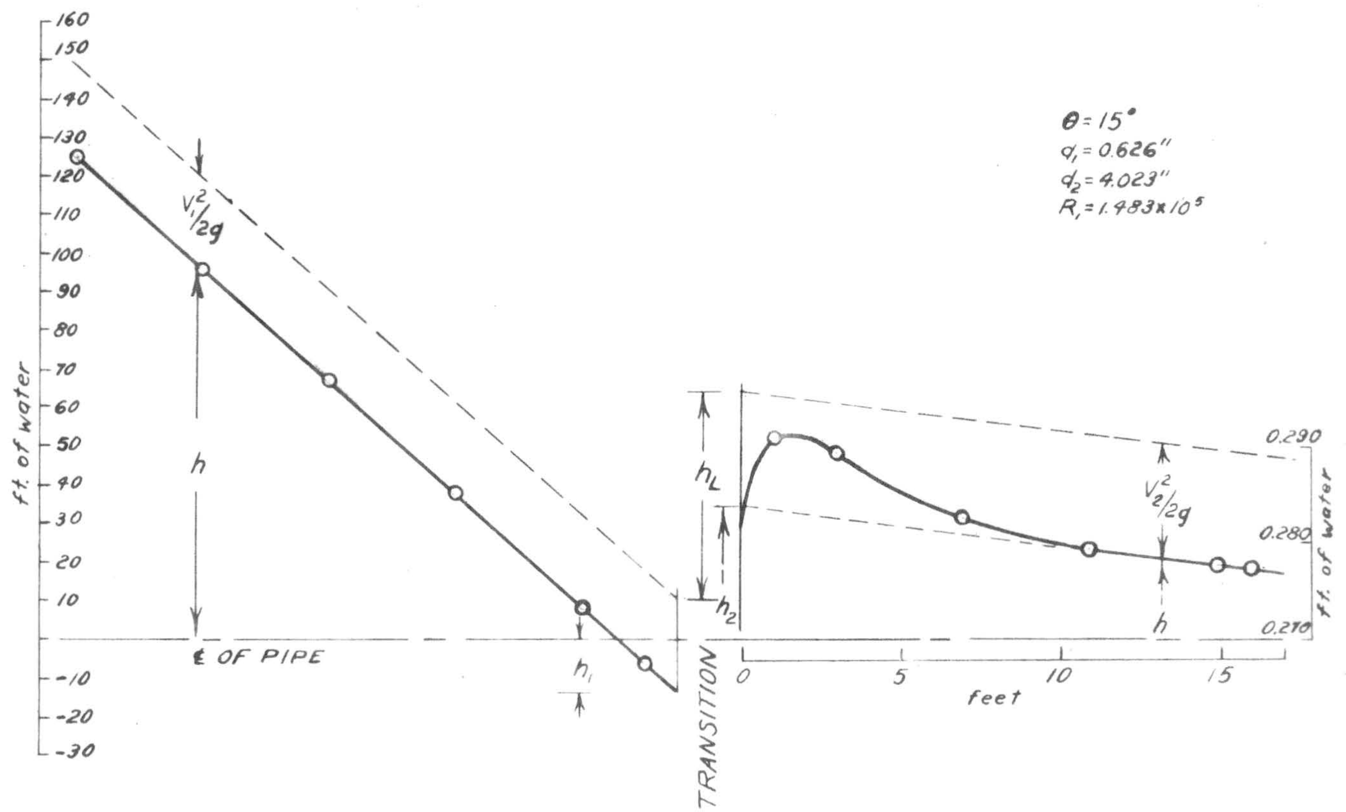

FIG. 2.--TYPICAL PIEZOMETRIC HEAD GRADIENT

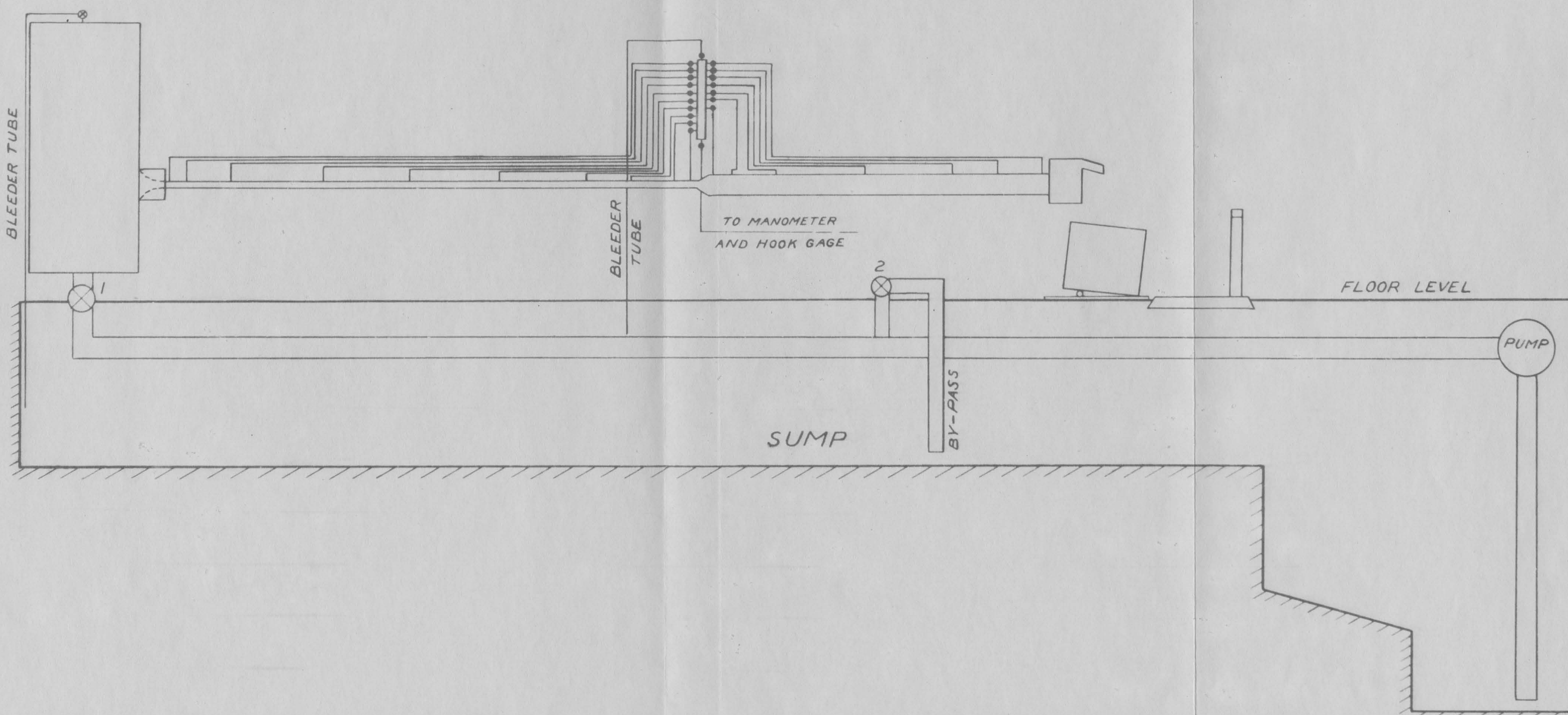

FIG. 3.--EXPERIMENTAL SETUP

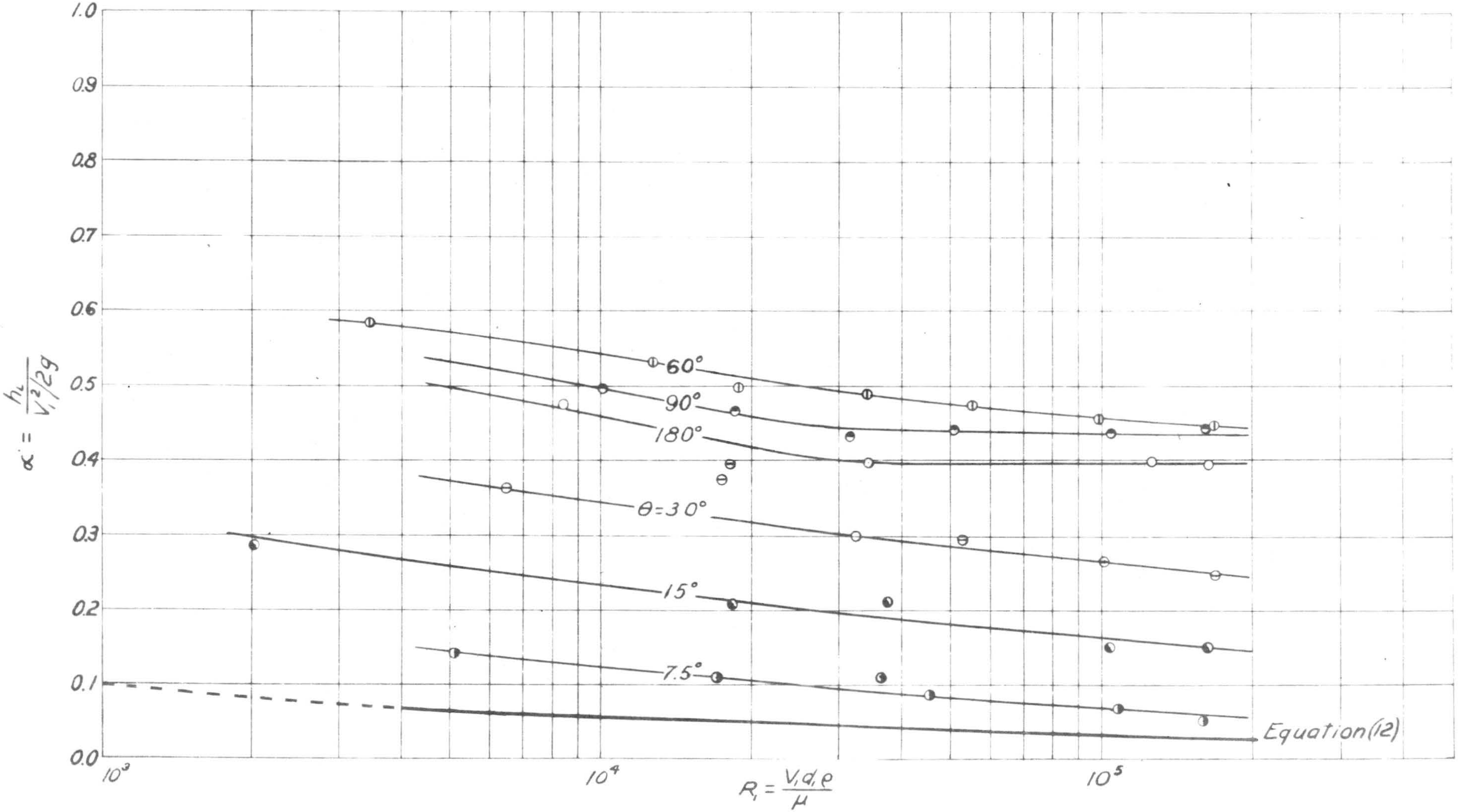

Fig. 4. Variation of α with R_1 for d_2/d_1 equal to 1.70

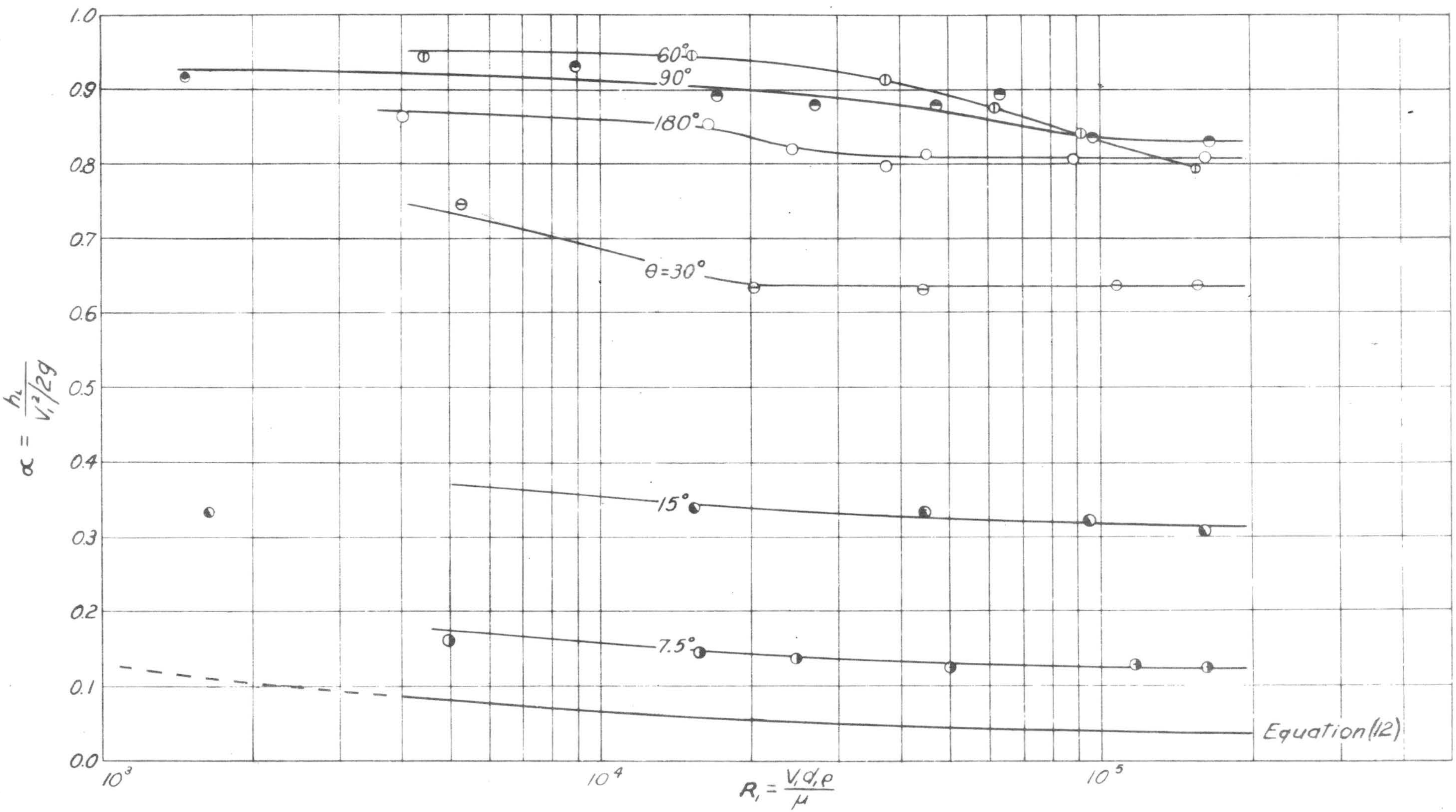

Fig. 5. Variation of α with R_1 for d_2/d_1 equal to 3.30

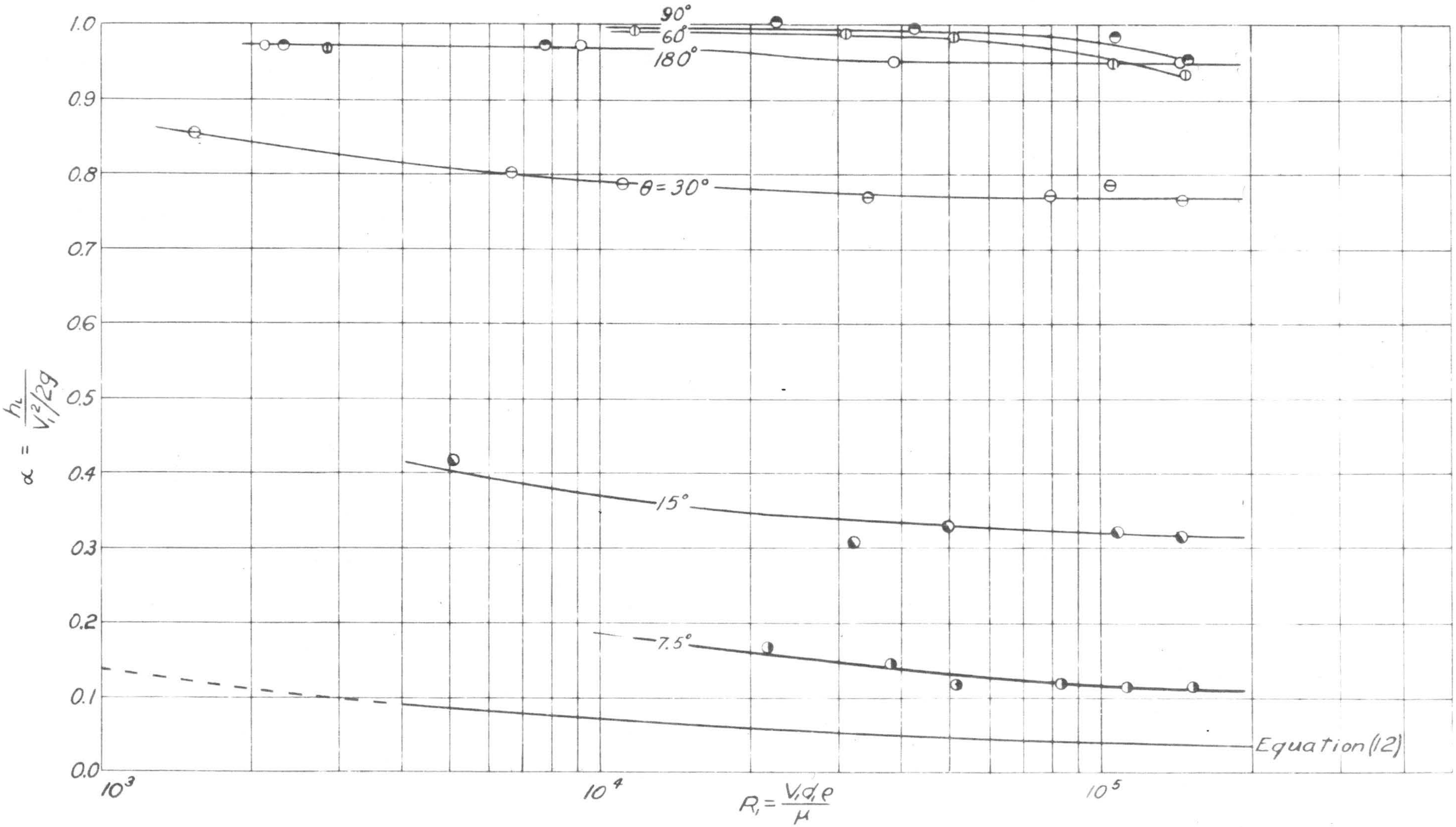

Fig. 6. Variation of α with R_1 for d_2/d_1 equal to 6.43

55

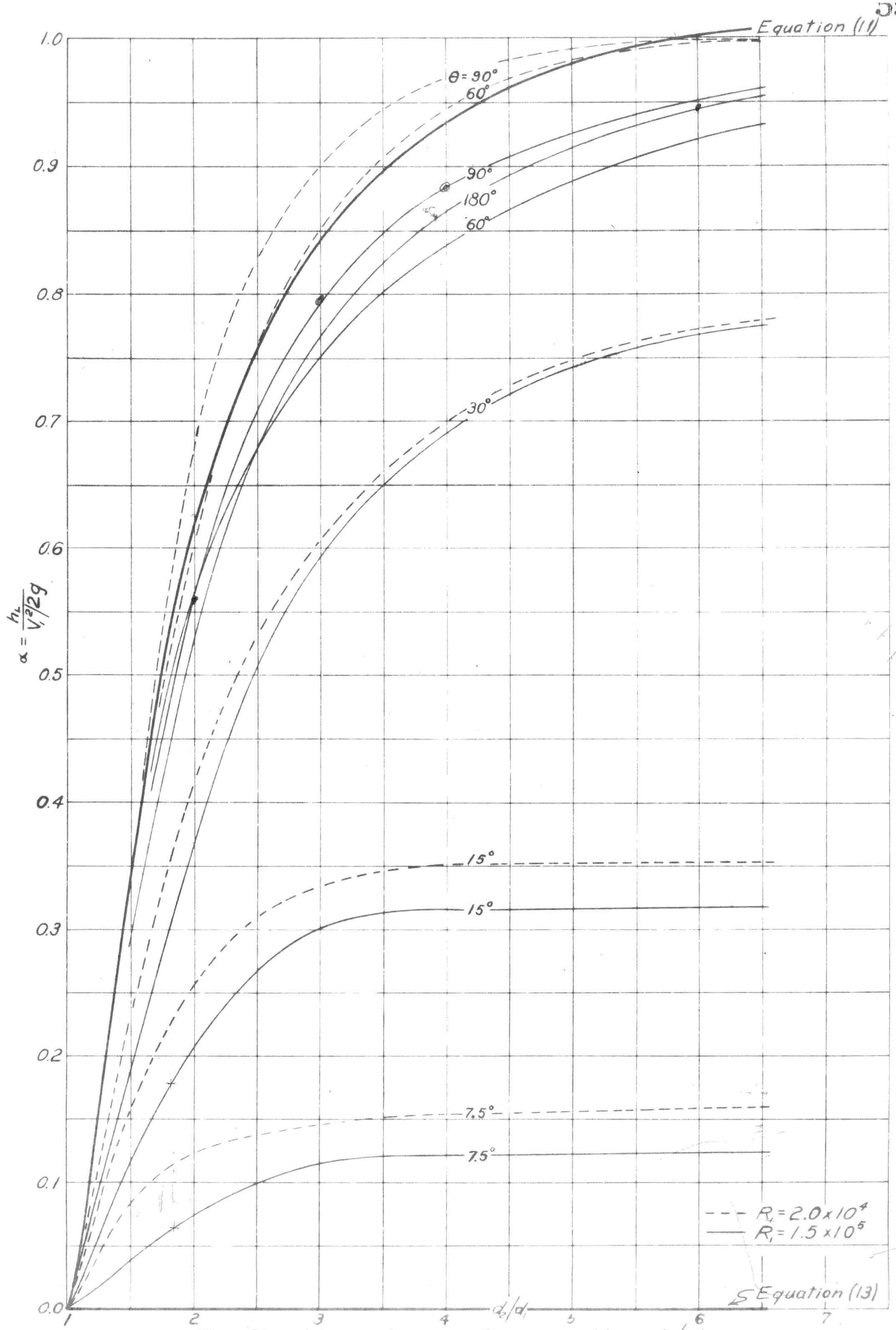

Fig. 7 Variation of α with d_2/d_1

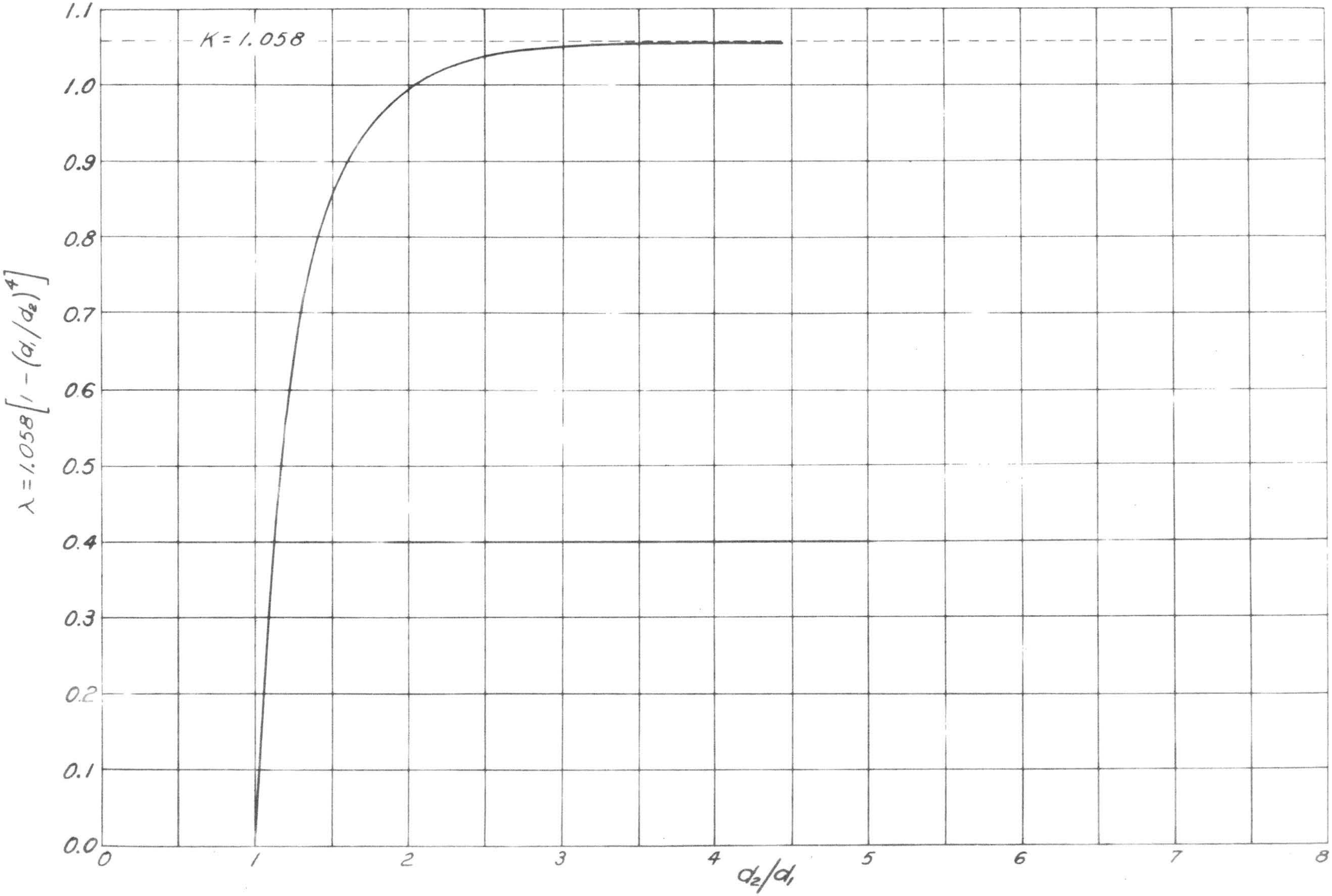

Fig. 8. Variation of λ with d_2/d_1

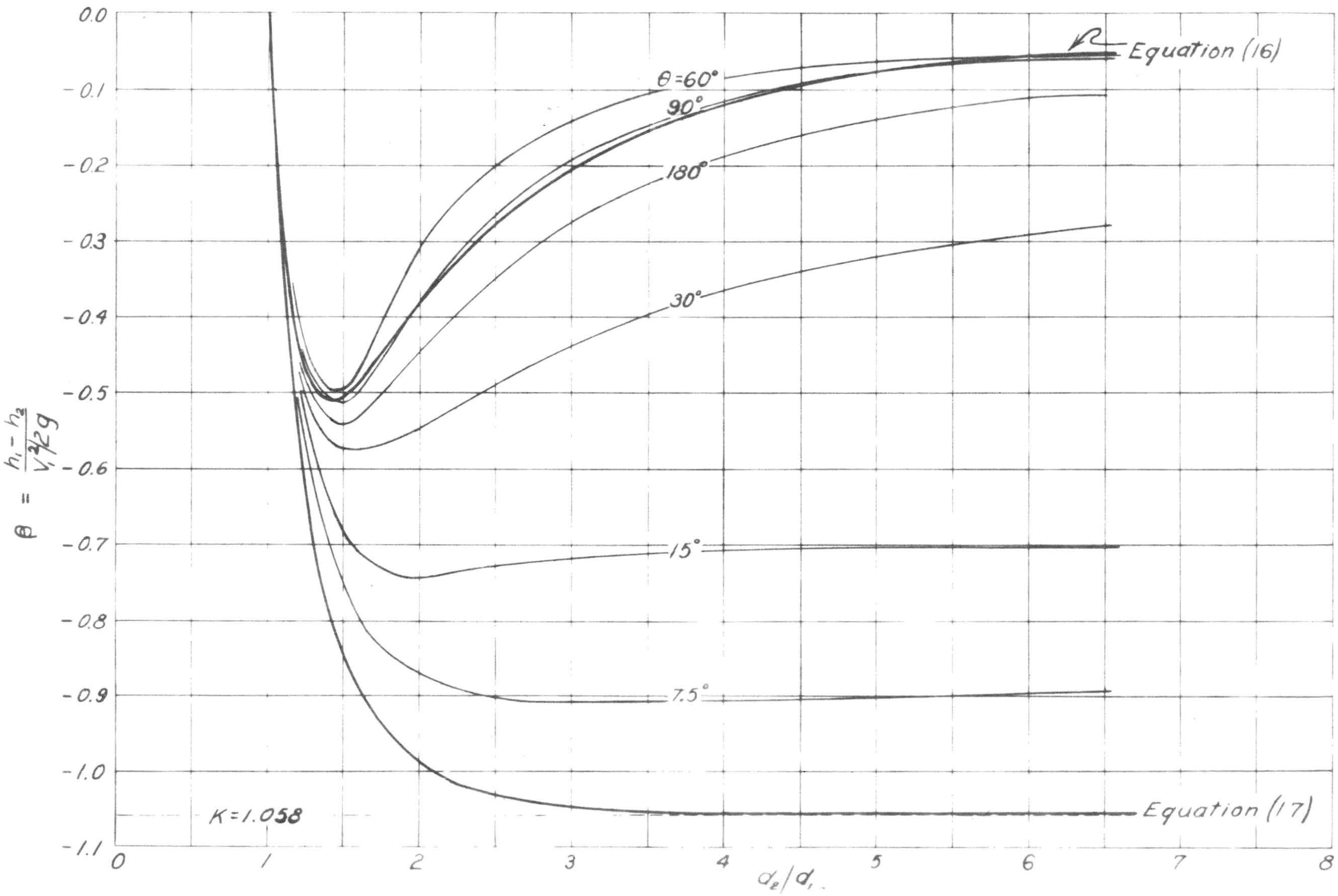

Fig. 9. Variation of ϱ with d_2/d_1 for R_1 equal to 2.0×10^4

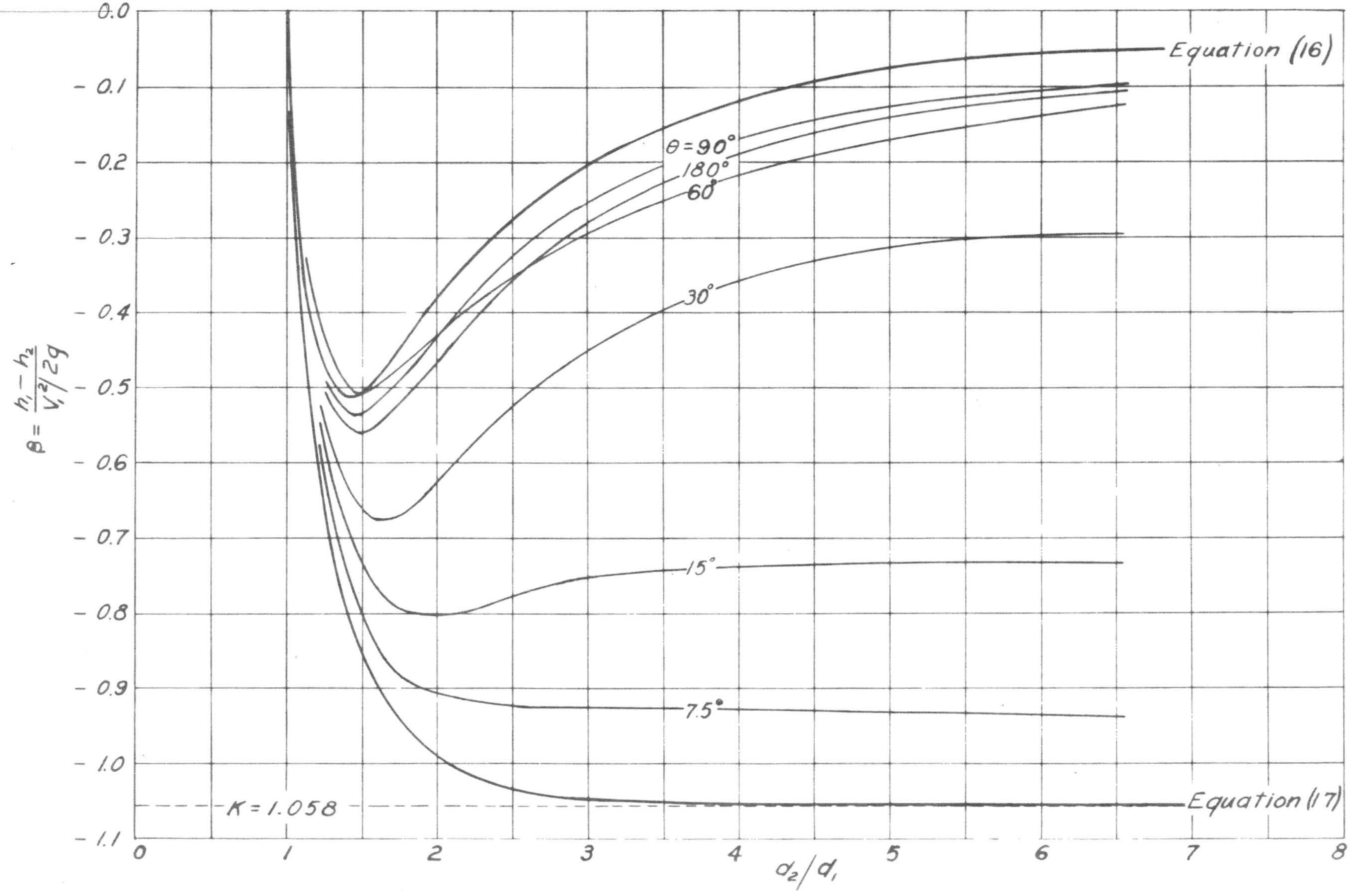

Fig. 10. Variation of β with d_2/d_1 for R_1 equal to 1.5×10^5

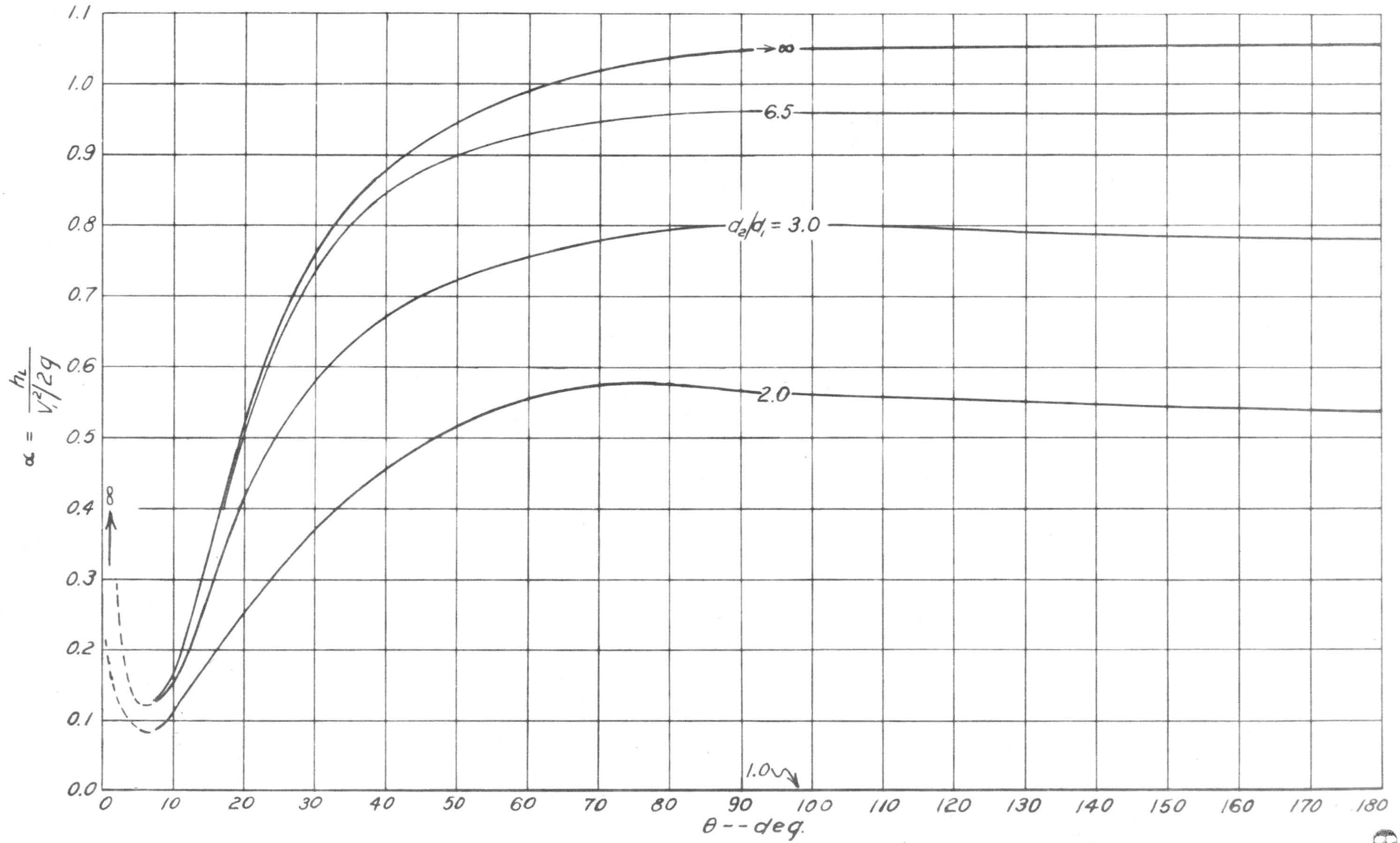

Fig. 11. Variation of α with θ for R_1 equal to 1.5×10^5

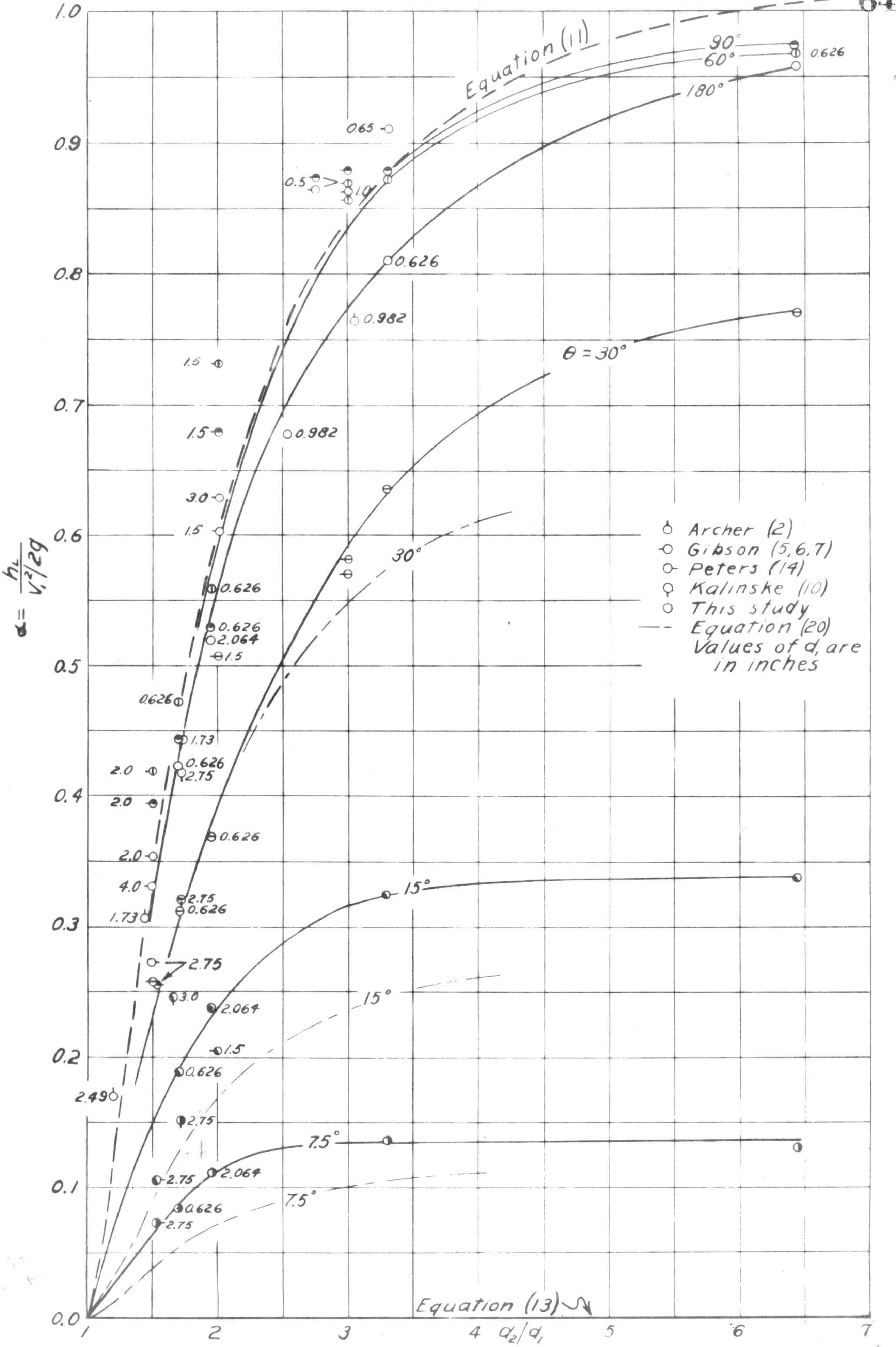

Fig. 12. Variation of the average value of α with d_2/d_1

5" by 6"

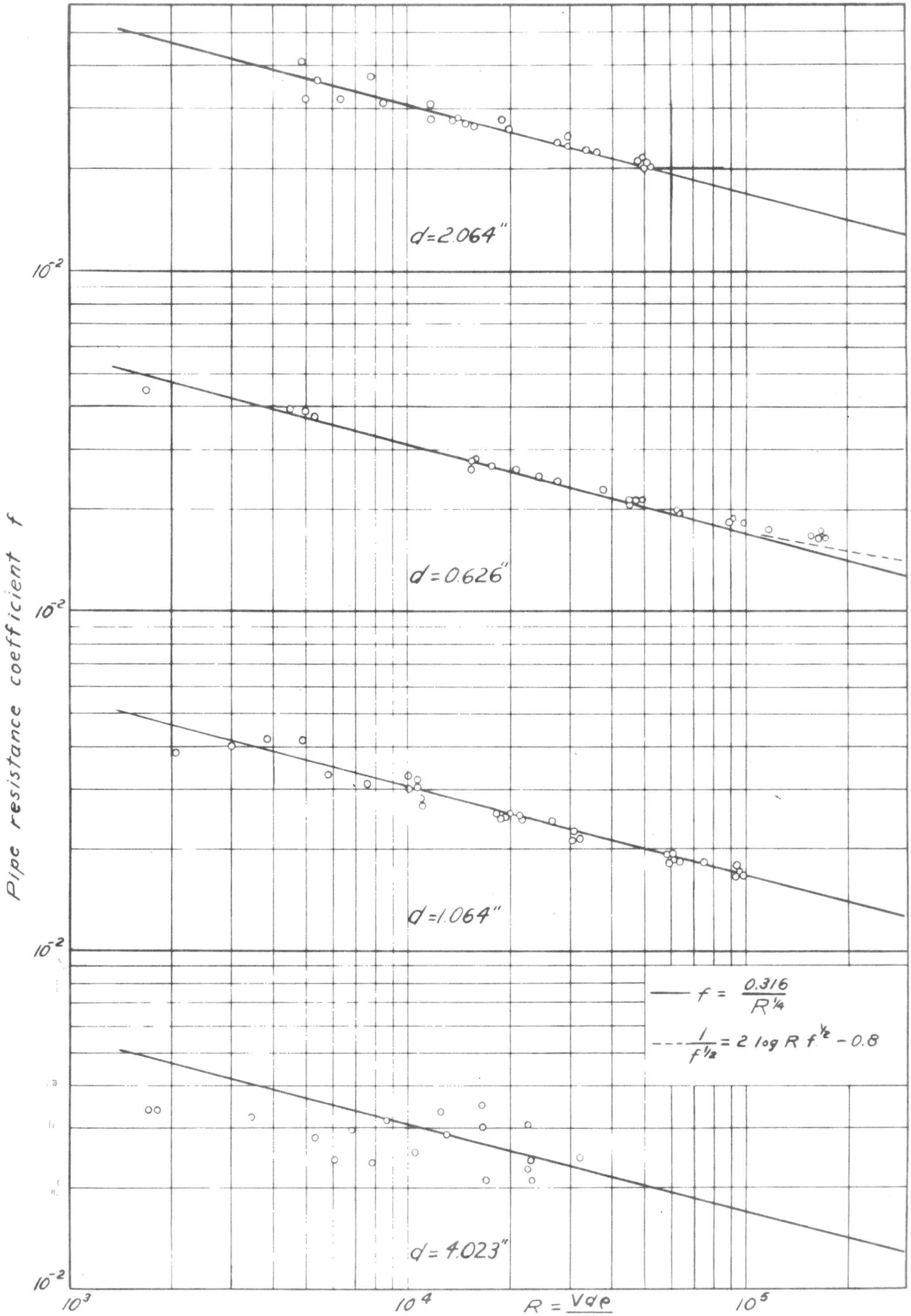

Fig. 13. Computed values of f

Appendix B.--Tables 1-5

Appendix B

LIST OF TABLES

Table 1.--EXPERIMENTAL DATA FOR d_2/d_1 EQUAL TO 1.70

θ deg.	Trial	Q ft.^{3}/sec.	Temp. deg. F.	α
7.5	1	0.07633	62.8	0.050
	2	0.05171	62.8	0.067
	3	0.02182	62.9	0.084
	4	0.01732	63.4	0.111
	5	0.00784	65.3	0.109
	6	0.00235	65.3	0.138
15	1	0.07554	64.3	0.150
	2	0.04896	64.1	0.150
	3	0.01747	64.5	0.210
	4	0.00852	64.7	0.204
	5	0.00090	66.5	0.286
30	1	0.07581	66.9	0.248
	2	0.04570	67.0	0.273
	3	0.02378	67.0	0.295
	4	0.01443	67.2	0.279
	5	0.00839	65.0	0.395
	6	0.00790	67.2	0.376
	7	0.00295	66.2	0.365
60	1	0.07521	67.5	0.445
	2	0.04472	67.5	0.463
	3	0.02442	67.6	0.471
	4	0.01506	68.1	0.484
	5	0.00855	66.5	0.498
	6	0.00581	66.7	0.527
	7	0.00157	66.5	0.582
90	1	0.07483	64.7	0.445
	2	0.04809	64.8	0.433
	3	0.02463	65.0	0.440
	4	0.01443	65.6	0.434
	5	0.00846	66.5	0.465
	6	0.00441	67.5	0.496
180	1	0.07522	65.7	0.396
	2	0.05836	65.9	0.399
	3	0.01563	66.4	0.398
	4	0.00371	68.0	0.478

Table 2.--EXPERIMENTAL DATA FOR d_2/d_1 EQUAL TO 3.30

θ deg.	Trial	Q $ft.^3/sec.$	Temp. deg. F.	α
7.5	1	0.07847	61.6	0.124
	2	0.05650	62.0	0.127
	3	0.02401	62.1	0.123
	4	0.01187	62.4	0.140
	5	0.00748	63.5	0.145
	6	0.00237	63.1	0.162
15	1	0.07694	62.8	0.307
	2	0.04542	63.0	0.321
	3	0.02121	63.1	0.336
	4	0.00732	64.0	0.340
	5	0.00077	64.7	0.333
30	1	0.07615	61.5	0.639
	2	0.05285	61.5	0.638
	3	0.02154	61.7	0.633
	4	0.00976	62.6	0.635
	5	0.00247	63.8	0.744
60	1	0.07451	68.9	0.795
	2	0.04083	67.7	0.840
	3	0.02758	68.1	0.877
	4	0.01643	68.9	0.911
	5	0.00860	69.5	0.944
	6	0.00215	62.0	0.930
90	1	0.07473	66.5	0.830
	2	0.04390	66.8	0.835
	3	0.02821	67.6	0.895
	4	0.02093	68.0	0.878
	5	0.01219	66.6	0.879
	6	0.00771	67.0	0.892
	7	0.00396	67.7	0.932
	8	0.00071	61.9	0.918
180	1	0.07533	64.4	0.807
	2	0.04178	64.5	0.802
	3	0.02168	62.8	0.811
	4	0.01769	63.6	0.796
	5	0.01151	62.8	0.820
	6	0.00777	63.5	0.853
	7	0.00189	63.5	0.861

Table 3.--EXPERIMENTAL DATA FOR d_2/d_1 EQUAL TO 6.43

θ deg.	Trial	Q $ft.^3/sec.$	Temp. deg. F.	α
7.5	1	0.07814	58.0	0.115
	2	0.05621	59.0	0.114
	3	0.04249	58.6	0.119
	4	0.02591	59.4	0.118
	5	0.01904	59.5	0.145
	6	0.01086	59.5	0.167
15	1	0.07746	56.6	0.317
	2	0.05572	57.8	0.321
	3	0.02536	58.3	0.330
	4	0.01796	57.4	0.308
	5	0.00257	60.6	0.418
30	1	0.07510	57.5	0.769
	2	0.05438	57.2	0.787
	3	0.04067	58.1	0.774
	4	0.01766	58.7	0.771
	5	0.00564	58.1	0.787
	6	0.00336	58.3	0.805
	7	0.00078	59.1	0.857
60	1	0.07456	59.2	0.937
	2	0.05366	59.4	0.946
	3	0.02545	59.8	0.984
	4	0.01503	61.4	0.984
	5	0.00569	61.3	0.993
	6	0.00137	61.2	0.969
90	1	0.07434	59.8	0.951
	2	0.05326	60.1	0.982
	3	0.02124	59.1	0.993
	4	0.01129	60.0	1.005
	5	0.00372	62.5	0.971
	6	0.00114	60.8	0.968
180	1	0.07447	58.4	0.951
	2	0.05372	59.5	0.945
	3	0.01924	60.0	0.951
	4	0.00451	60.7	0.973
	5	0.00102	62.0	0.972

Table 4.--EXPERIMENTAL DATA FOR d_2/d_1 EQUAL TO 1.95

θ deg.	Trial	Q ft.^{3}/sec.	Temp. deg. F.	α
7.5	1	0.2963	52.0	0.086
	2	0.2428	52.5	0.132
	3	0.1814	53.1	0.092
	4	0.1047	53.8	0.133
15	1	0.2827	52.0	0.201
	2	0.2263	52.5	0.261
	3	0.1486	52.7	0.267
	4	0.0440	53.1	0.223
30	1	0.2910	51.0	0.373
	2	0.2540	51.5	0.363
	3	0.2134	51.9	0.369
	4	0.1751	52.2	0.355
	5	0.1114	52.6	0.381
60	1	0.2870	52.6	0.561
	2	0.2253	52.9	0.541
	3	0.1420	52.3	0.541
	4	0.1282	54.4	0.591
90	1	0.2913	52.0	0.540
	2	0.1865	52.1	0.513
	3	0.0962	52.5	0.532
	4	0.0413	52.7	0.525
180	1	0.2933	52.8	0.538
	2	0.2509	53.1	0.486
	3	0.1773	53.7	0.534

Table 5.--AVERAGE VALUES OF α

d_1 in.	d_2 in.	d_2/d_1	θ deg.	α
0.626	1.064	1.70	7.5	0.084
			15	0.179
			30	0.311
			60	0.472
			90	0.443
			180	0.422
0.626	2.064	3.30	7.5	0.136
			15	0.326
			30	0.636
			60	0.873
			90	0.877
			180	0.812
0.626	4.023	6.43	7.5	0.130
			15	0.339
			30	0.771
			60	0.969
			90	0.973
			180	0.958
2.064	4.023	1.95	7.5	0.111
			15	0.238
			30	0.368
			60	0.559
			90	0.528
			180	0.519

Appendix C.--Tables 6-9

Appendix C

LIST OF TABLES

Table 6.--DATA FROM ARCHER (2)

d_1 in.	d_2 in.	d_2/d_1	θ deg.	α
2.486	2.994	1.204	180	0.162
1.730	2.495	1.442	180	0.306
1.730	2.994	1.731	180	0.443
0.982	2.495	2.541	180	0.677
0.982	2.994	3.049	180	0.764

Table 7.--DATA FROM PETERS (14)

d_1 in.	d_2 in.	d_2/d_1	θ deg.	α
2.756	4.214	1.529	7.8	0.074
			14.7	0.105
			91.6	0.255
			180	0.256

Table 8.--DATA FROM KALINSKE (10)

d_1 in.	d_2 in.	d_2/d_1	θ deg.	α
2.75	4.75	1.73	7.5	0.150
			30	0.320
			180	0.417
3.00	5.00	1.67	15	0.246

Table 9.---DATA FROM GIBSON (5,6,7)

d_1 in.	d_2 in.	d_2/d_1	θ deg.	α
1.5	3.0	2.0	15	0.204
			30	0.507
			60	0.732
			90	0.679
			180	0.627
2.0	3.0	1.5	30	0.257
			60	0.418
			90	0.393
			180	0.354
0.5	1.5	3.0	30	0.582
			60	0.869
			90	0.868
			180	0.869
1.0	3.0	3.0	30	0.571
			60	0.857
			90	0.879
			180	0.864
0.65	2.15	3.31	180	0.912
3.0	6.0	2.0	180	0.603
4.0	6.0	1.5	180	0.331

Appendix D.--NOTATION

NOTATION

a -- radial distance from center of circular section to any point in section

A -- area of circular section

d -- diameter of circular section

e -- wall roughness of transition

f -- pipe resistance coefficient

h -- piezometric head

h_L -- total head loss

K -- kinetic energy coefficient

K_m -- momentum flux coefficient

p -- pressure intensity

r -- radius of circular section

R -- Reynolds number

v -- velocity at a point

V -- mean velocity of section

Z -- velocity destribution parameter

α -- total head loss coefficient $\frac{h_L}{V_1^2/2g}$

β -- coefficient for piezometric head change between entrance and exit section of diffuser

$$\frac{h_1 - h_2}{V_1^2/2g}$$

γ -- specific weight of fluid

θ -- total cone angle

λ -- coefficient for velocity head change between entrance and exit section of diffuser

$$\frac{V_1^2 - V_2^2}{V_1^2/2g}$$

μ -- coefficient of dynamic viscosity

ρ -- density of fluid

B I B L I O G R A P H Y

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