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FLUID MECHANICS OF POROUS MEDIA

I. HYDRAULIC CHARACTERISTICS OF POROUS MEDIA

By

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A theory showing how the variables, capillary pressure and permeability, are related to saturation is needed to describe water movement in soils above the water table.

Burdine (1953) and Corey (1954) have presented expressions relating permeability to saturation and capillary pressure for petroleum reservoir rocks. The expressions proposed by Burdine (1953) and Corey (1954) have been verified for many consolidated rocks. Insufficient data are available to verify them for unconsolidated sands and soils. Wyllie and Gardner (1959) indicated that a modification of Corey's equations would be necessary for unconsolidated porous media. The modified equations, perhaps, should involve parameters that are characteristic of particular porous media. In the following theoretical presentation, a set of general equations are presented which contain parameters characteristic of porous media.

In order to predict (from these equations) how any (consolidated or unconsolidated) porous medium will behave with respect to the functional relationship between permeability and saturation (or fluid pressure), certain medium properties must be known.

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Perhaps the most significant characteristic of a porous medium is its pore-size distribution. Another medium characteristic which the authors feel is significant is the minimum capillary pressure at which there exists a continuous gas phase.

The objective of the research is to investigate the validity of the equations for all types of consolidated and unconsolidated, homogeneous, isotropic porous media.

THEORY

Burdine (1953) found that by making use of the capillary pressure-desaturation curves, permeability could be expressed as a function of saturation and capillary pressure by the following equation:

$$K_{rw} = S_e^{2} \frac{\int_0^{S_e} \frac{d\,S_e}{P_c^2}}{\int_{S_r}^{1.0} \frac{d\,S_e}{P_c^2}} \tag{1}$$

where K_{rw} is relative permeability; i.e., the ratio of effective permeability to the permeability at a saturation of unity.

An assumption suggested by Corey (1959) for solving Burdine's equation is

$$S_e = \left(\frac{P_d}{P_c} \right)^{\gamma}, \; P_c \geq P_d \quad . \tag{2}$$

The term P_d in equation (2) is tentatively defined as a scaling parameter, the physical significance of which is described later in the paper. Effective saturation was defined by Corey (1954) as

$$S_e = \frac{S_w - S_r}{1 - S_r} \quad .$$

The residual saturation, S_r , is the saturation at which the gas permeability does not change greatly for further decreases in saturation.

Substituting equation (2) into equation (1) and integrating results in

$$K_{rw} = (S_e)^{\frac{3\gamma + 2}{\gamma}} \tag{3}$$

or

$$K_{rw} = (S_e)^{\omega} \quad , \tag{4}$$

where $\omega = \frac{3\gamma + 2}{\gamma}$.

If equation (2) is substituted into equation (4), then,

$$K_{rw} = \left(\frac{P_d}{P_c} \right)^{\gamma\omega} \tag{5}$$

or

$$K_{rw} = \left(\frac{P_d}{P_c} \right)^{\eta} \quad , \tag{6}$$

where $\eta = \gamma\omega$. Equations (2), (4) and (6) provide relationships among the variables capillary pressure, liquid permeability and saturation.

A procedure similar to the above can be carried out for the non-wetting phase. Burdine's equation for relative permeability of the non-wetting phase is

$$K_{rnw} = (1-S_e)^2 \frac{\int_{S_e}^{1.0} \frac{d\,S_e}{P_c^{\,2}}}{\int_0^1 \frac{d\,S_e}{P_c^{\,2}}} \quad . \tag{7}$$

Substituting equation (2) into equation (7) and integrating, yields

$$K_{rnw} = (1-S_e)^2 (1-S_e^{\alpha}) \tag{8}$$

where $\alpha = \dfrac{2+\gamma}{\gamma}$.

It follows that upon substitution of equation (2) into equation (8) an expression relating permeability and capillary pressure is

$$K_{rnw} = \left[1 - \left(\frac{P_d}{P_c} \right)^{\gamma} \right]^2 \left[1 - \left(\frac{P_d}{P_c} \right)^{2+\gamma} \right] . \tag{9}$$

All of the exponents in the above equations are interrelated. It is interesting to note that an analysis of the exponent γ in the above equations provides a means for determining the range of values for the slopes of the various equations. In equation (3) and (4), for example, as γ becomes large ω approaches the limit three. In a similar manner, the lower limit for the exponent η in equation (6) is two and in equation (8) the exponent α has unity as a lower limit. As γ approaches zero; ω , η , and α increase without bound.

Considerable physical significance can be attached to these exponents. If equation (6) is plotted on log-log paper, for example, the result is a family of straight lines having a range of slopes from two to infinity. If the slope approaches infinity, the distribution of pores will be uniform in size while for a slope that approaches two, the distribution of pore sizes will be extremely non-uniform. The exponent η therefore can be used as a parameter which describes the character of the pore-size distribution.

The values of the exponents ω , η , and α for Corey's approximation of wetting and non-wetting relative permeability as functions of saturation and capillary pressure for many consolidated porous media are 4, 8 and 2 respectively. As will be shown later Corey's approximation might very well be average expressions for many porous media.

If the parameters P_d , η , and S_r can be determined for a porous medium by laboratory techniques, then all of the functional relationships between the variables can be determined. This assumes, of course, that the equations adequately describe these relationships.

Methods will now be discussed which are presently being used to test the validity of the equations and obtain the parameters mentioned for unconsolidated porous media having a wide range of media properties.

EXPERIMENTAL PROCEDURES

In order to adequately test the equations presented above, it would be highly advantageous to be able to check each equation independently and make all measurements on the same sample. This is not possible, at least for the present, for unconsolidated materials.

In each procedure given below, every effort is made to obtain the same packing arrangement on each sample. The variation in packing resulting from the techniques employed in this study are assumed to have produced little effect upon the pore-size distribution. All measurements were made for the drainage cycle.

Liquid Permeability - Capillary Pressure - Saturation Percentage

The method described by Scott and Corey (1961) was used to obtain liquid permeability as a function of capillary pressure. The method consists of packing soil or sand into a long column consisting of a large number of short sections containing tensiometer rings. Steady downward flow is established under a hydraulic gradient of unity. By measuring outflow or inflow after the run becomes steady, a permeability value is calculated for the region in the column where the capillary pressure is constant. After each succeeding run the capillary pressure is increased and the discharge through the column becomes smaller. When sufficient data are obtained over the desired range of capillary pressures, permeability is plotted as a function of capillary pressure on log-log paper.

Permeability and capillary pressure may also be obtained as a function of saturation from this same experiment if the degree of saturation can be determined in the region of constant capillary pressure without disturbing the column. The absorption of a gamma ray through a column of porous material of constant density is a convenient means for accomplishing this purpose. Equipment similar to that used by Ferguson (1959) has been set up for purposes of measuring saturation percentage in the long column method of Scott and Corey (1961). There has been some difficulty in obtaining reliable data from the gamma ray absorption equipment. Until these difficulties are overcome, another procedure will be used to obtain capillary pressure as a function of saturation on a different sample.

Gas Permeability - Capillary Pressure - Saturation Percentage

A method has been devised for measuring gas permeability as a function of both capillary pressure and saturation on small samples of porous media.

A small pressure cell was constructed from a lucite tube 2.5 cm inside diameter and 3 cm long. Provic was cemented over interconnected grooves machined around the entire surface of the inside wall of the plastic tube. A calibrated horizontal capillary tube was connected to the annular space between the provic barrier and the inside wall of the plastic tube. The end of the horizontal capillary tube was connected to a system for applying a constant suction. The calibrated horizontal capillary tube was used to measure the volume of water extracted from the sample at a given capillary pressure. Plastic caps were placed over the ends of the cell which were made air-tight by the use of "O" rings. Two holes were drilled in each cap for connection of a differential manometer and for the inflow and outflow of air. A schematic sketch of the equipment is shown in figure (1).

Soil was packed into the cell and the end caps were clamped into position. The sample was submerged in the wetting fluid and vacuum

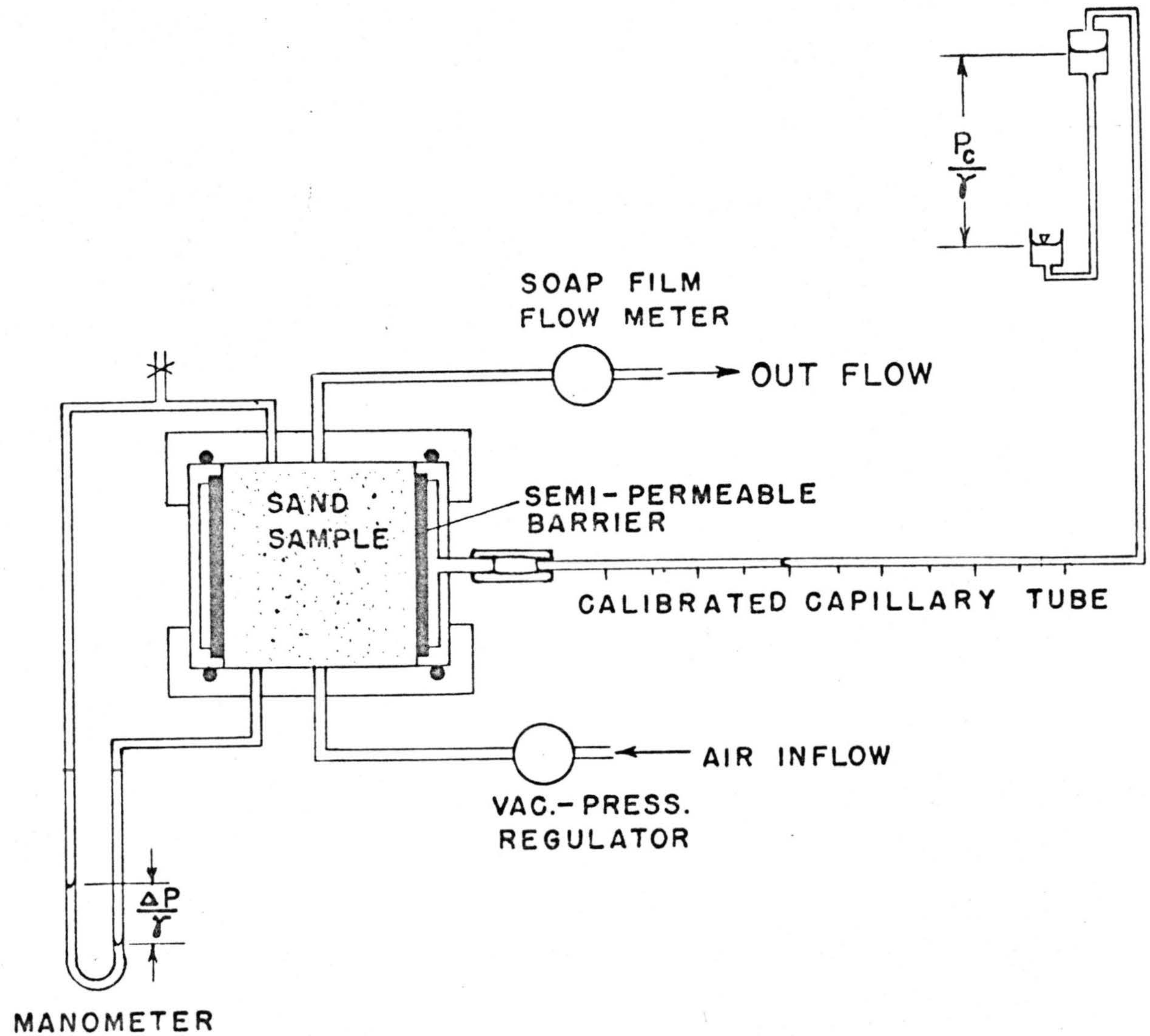

Figure 1 - Schematic diagram of equipment for measuring non-wetting permeability as a function of capillary pressure and saturation.

saturated. After the cell was completely saturated, the horizontal capillary tube was connected to the barrier and a small suction was applied to remove excess wetting fluid. An air supply was connected to the bottom of the cell and pressure was applied such that a manometer reading, $\frac{\Delta P}{\gamma}$, was equal to the length of the sample in the cell. The top of the cell was connected to a soap-film flow meter for measuring the quantity of gas flowing through the sample during the permeability measurement. When the suction on the sample was increased such that the saturation was reduced to allow air to flow through the sample, a constant pressure gradient was maintained across the sample by a vacuum pressure regulator.

The suction to the horizontal capillary tube was increased in small increments and when the outflow through the barrier into the horizontal capillary ceased for small increases in suction, the reading was recorded as a zero. Further outflow into the horizontal capillary when the suction was increased indicated a reduction in the saturation of the sample. The maximum capillary pressure for which the zero was obtained on the horizontal capillary was defined as the entry pressure P_e. This is the pressure at which the non-wetting fluid just begins to finger into the sample as the first reduction in saturation occurs. The saturation was increased in increments allowing equilibrium to take place between each increase. The outflow in the horizontal capillary and the air permeability was read at each equilibrium suction. Air did not begin to flow until the suction was such that the non-wetting phase was continuous throughout the sample. By maintaining at all times a difference in pressure head $\frac{\Delta P}{\gamma}$ across the sample equal to the length of the sample (with the top of the sample open to the atmosphere), a uniform capillary pressure was maintained in the porous medium.

After the final desired capillary pressure was reached and the air permeability was near its maximum value, the sample was removed from the cell, weighed and dried in an oven to determine the total pore volume.

From these measurements, a plot was made of gas permeability as a function of both capillary pressure and saturation and also capillary pressure as a function of saturation.

RESULTS AND DISCUSSION

Some preliminary measurements have been obtained for several sands. A liquid permeability-capillary pressure curve is shown in figure (2) for two sands. The curves show liquid permeability plotted against matrix suction, where matrix suction is defined as P_c/γ . One sand was obtained from the U. S. Salinity Laboratory, U.S.S.L. Accession No. 3445. This sand is a fine-textured silty sand of low permeability. The other sand shown in figure (2) was called white sand by Scott and Corey (1961) and is approximately ten times as permeable as the former. This sand is very clean and its particles are angular and rounded in shape as opposed to the flat platy particles of the former sand.

One usually has no difficulty in drawing a best-fit curve through the data as most of the data fall on a straight line. The intersection of the straight line with the horizontal line representing the saturated permeability K is used to determine the parameter, P_d , called the displacement pressure. Equation (6) represents the straight line portion of these curves with η and P_d/γ for the two sands shown on figure (2).

The gas permeability-saturation data for these two sands are shown in figure (3). This figure shows non-wetting permeability plotted against saturation percentage. The points represent the experimental data while the curves represent the theory as given by equation (8) where α and S_r are shown on the figure.

The gas permeability-capillary pressure data are shown in figure (4). Non-wetting permeability is plotted as a function of matrix suction P_c/γ . The data are shown as points while the theory is shown as a curve. Attention is drawn to the fact that, the gas permeability

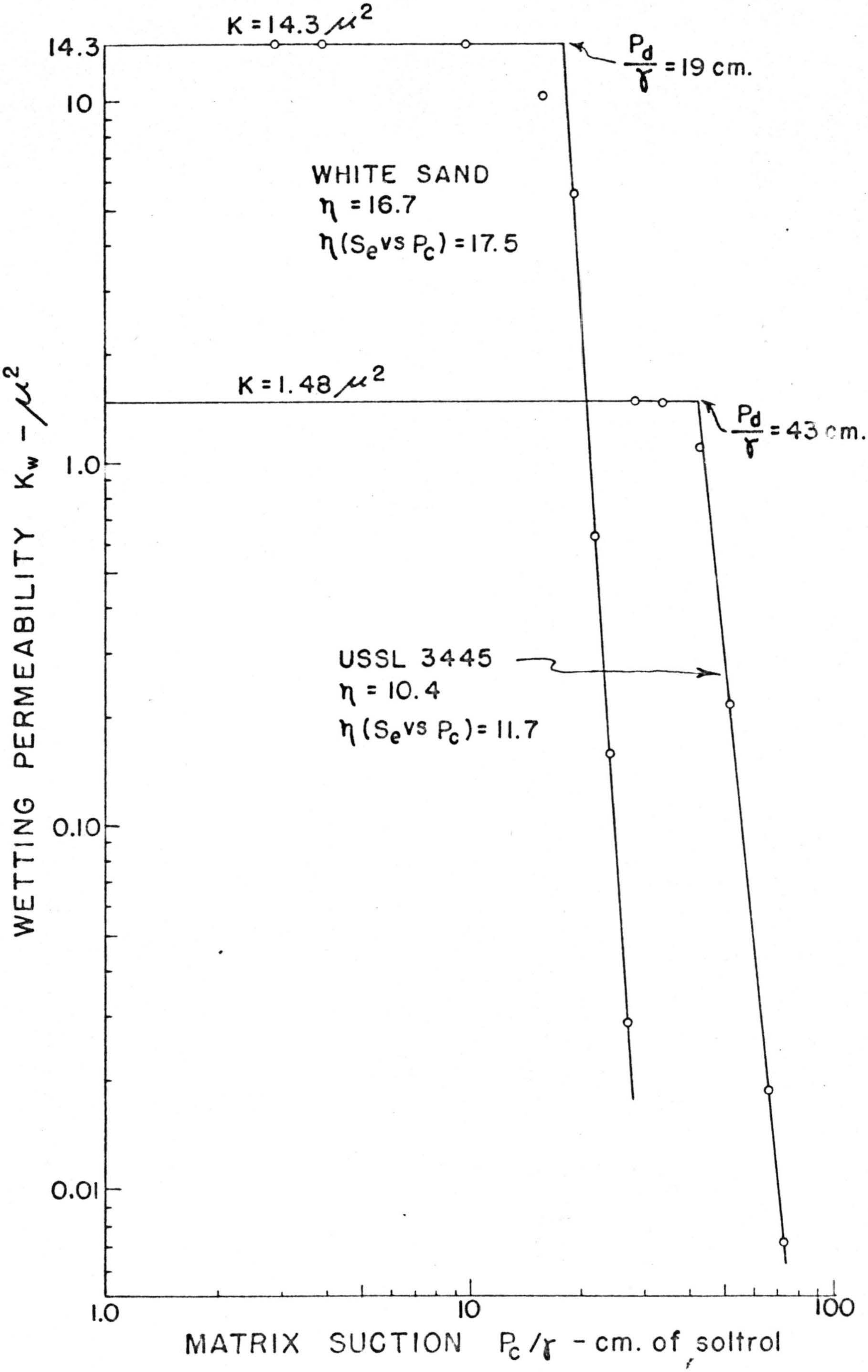

Figure 2 - Wetting permeability K_W as a function of matrix suction P_c/γ for two sands.

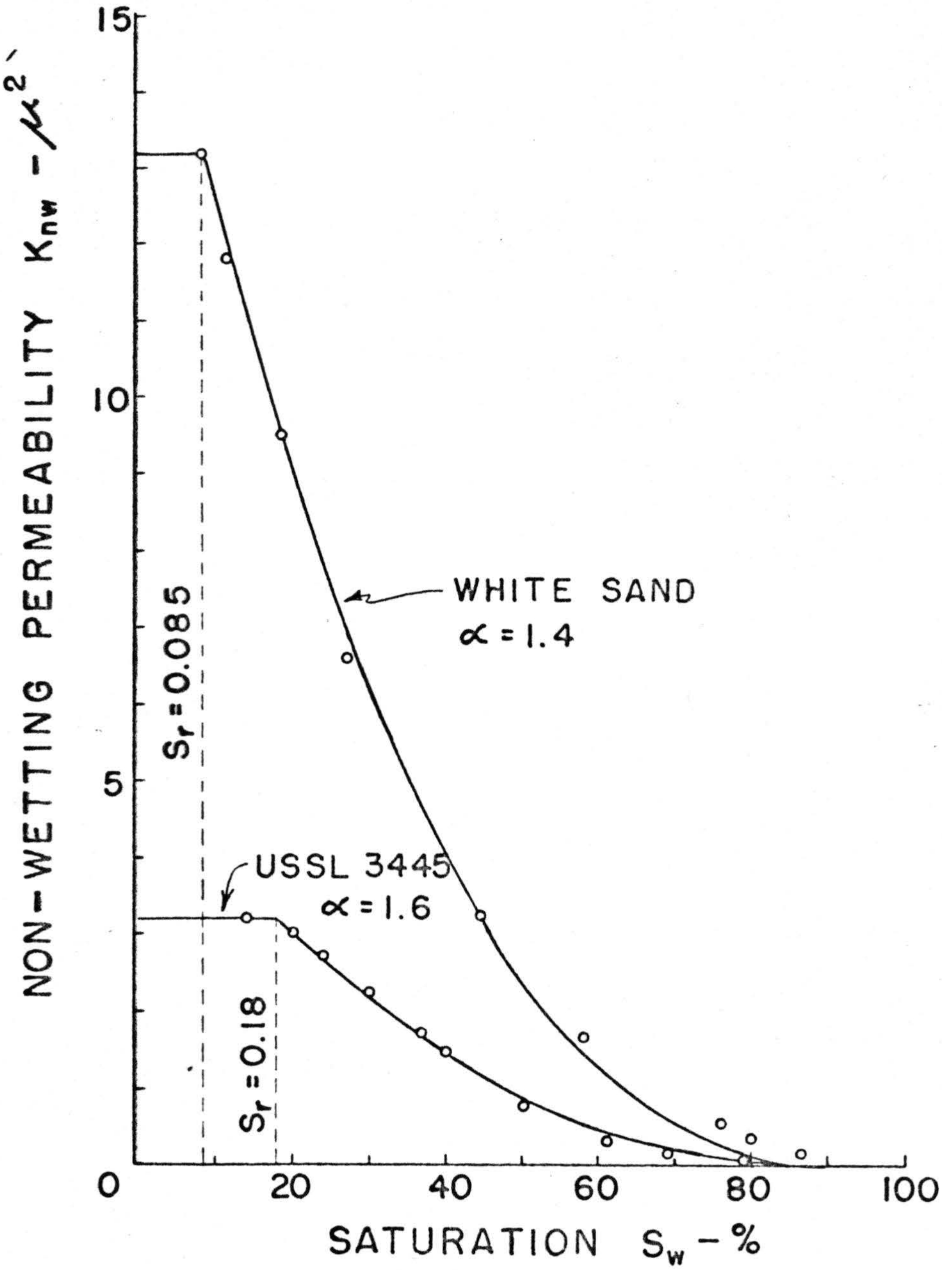

Figure 3 - Non-wetting permeability K_{nw} as a function of saturation S_w for two sands.

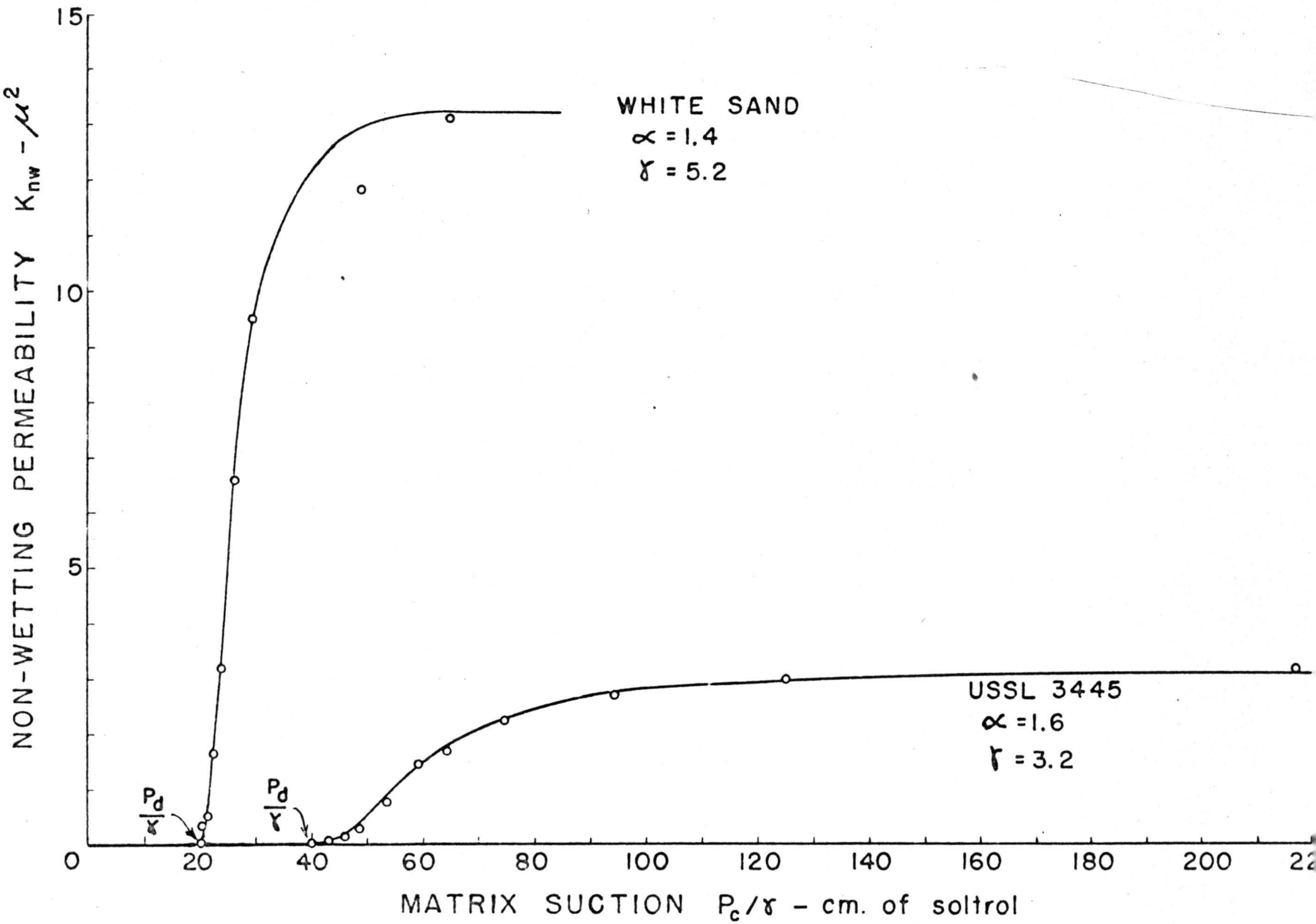

Figure 4 - Non-wetting permeability K_{nw} as a function of matrix suction P_c/γ for two sands.

remains zero until the displacement pressure P_d is reached.

Having determined P_d and the residual saturation S_r for these two sands from the air permeability data, a plot was made of effective saturation S_e as a function of matrix suction P_c/γ. The data for these two sands have been plotted on log-log paper with effective saturation as the ordinate and matrix suction as the abscissa in figure (5). By drawing a best-fit straight line through the data, the slope of the curve γ was measured and compared with the slope η of the liquid permeability-capillary pressure curve of figure (2) since

$$\eta = 2 + 3\gamma .$$

It should be noted that the parameter η as determined from the liquid saturation-capillary pressure curve was used to calculate the theoretical curves for the gas permeability data.

A comparison of the η and P_d/γ values obtained from the experimental curves is given in table I.

TABLE I

Values of η and P_d/γ as determined by two different methods for two sands.

Sample	η Values		P_d/γ values - cm of Soltrol		
	K_w vs P_c	S_e vs P_c	K_w vs P_c	S_e vs P_c	K_{nw} vs P_c
USSL 3445	10.4	11.7	43	40	40
White Sand	16.7	17.5	19	20	20

Since these values were determined on two different samples and by two different methods, the agreement is good. It appears from this preliminary data, that the P_d obtained from the extrapolated straight line portion of

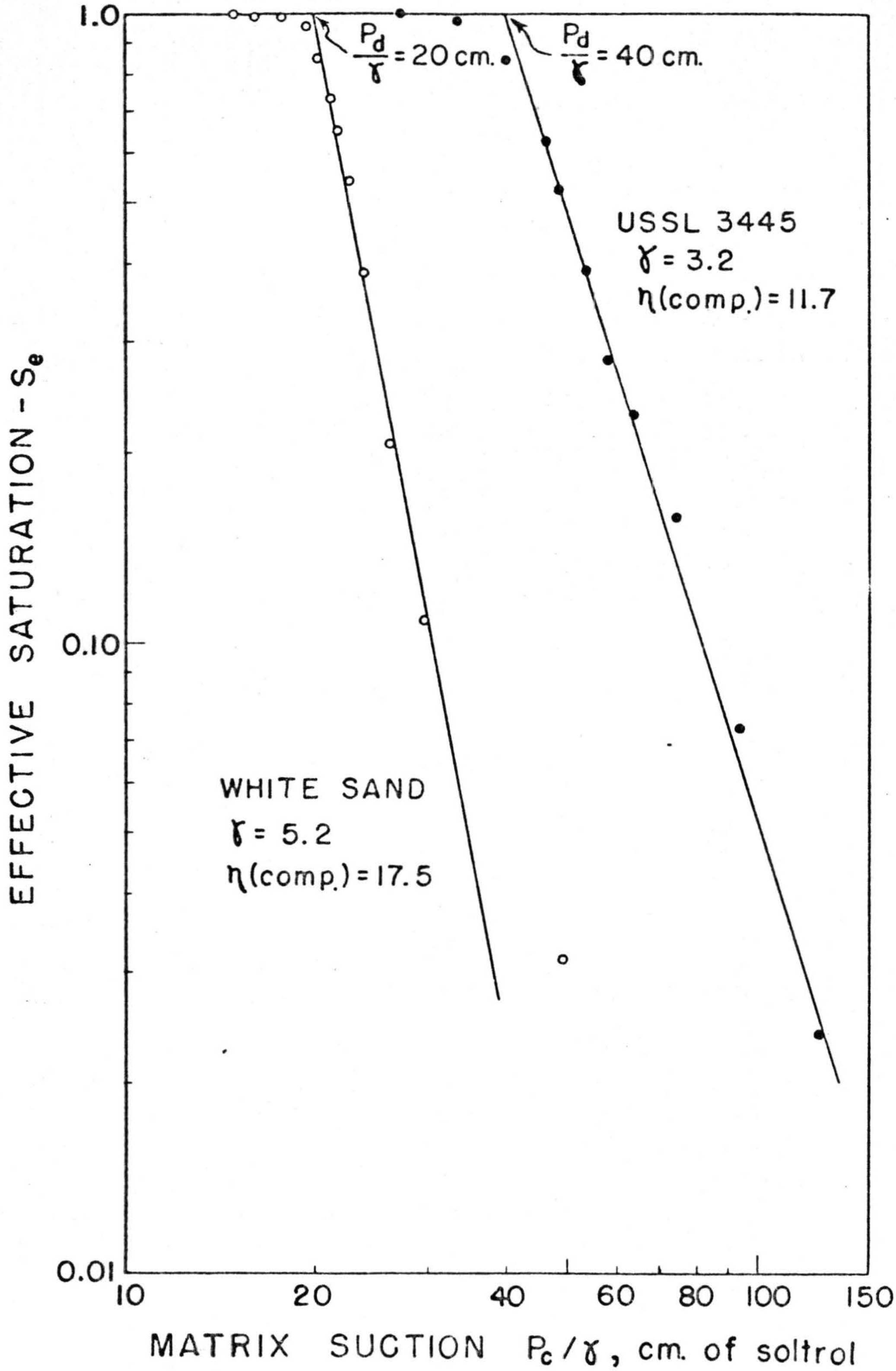

Figure 5 - Effective saturation S_e as a function of matrix suction P_c/γ for two sands.

the liquid permeability-capillary pressure curve has physical significance other than that inherent in its definition. A comparison of the P_d/γ values in table I taken from the gas permeability-capillary pressure curve, figure (4), with the P_d/γ values obtained from the liquid permeability-capillary pressure curves, figure (2), indicate that P_d is the capillary pressure when the non-wetting phase is first continuous throughout the medium on the drainage cycle.

It appears that the hydraulic properties of the porous medium can be described by three parameters: P_d which is a measure of the largest continuous pores in the medium, η which is a measure of the uniformity of the pores in the medium and K the saturated permeability.

CONCLUSIONS

The data available to date are preliminary and by no means represent a sufficient sample of unconsolidated porous media. These preliminary data do, however, agree very well with the theoretical equations. Physical significance can be attached to the parameters, P_d , η and K and they seem to characterize porous media with respect to hydraulic behavior. It appears that the method described for measuring gas permeability as a function of capillary pressure and saturation can be used for predicting liquid permeability as a function of capillary pressure or of saturation.

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