DESIGN OF TILE DRAINAGE FOR FALLING WATER TABLES

Discussion by R. H. Brooks

November 1963

To R. H. Brooks
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DESIGN OF TILE DRAINAGE FOR FALLING WATER TABLES [a]

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R. H. Brooks[15] __ van Schilfgaarde is to be complimented for extending Glover's solution of the case of equally-spaced drainage tile on an impermeable layer to the case where the tile may be located anywhere above the impermeable layer.

It should be pointed out here that Glover considered two specific cases of flow toward equally spaced drains in the paper by Dumm[4]. In the first case, the partial differential equation based upon the Dupuit-Forcheimer assumptions (van Schilfgaarde's equ (1)) was linearized and solved to give solutions in which the thickness of the water bearing stratum is assumed to be of uniform thickness. This case was later extended by Brooks[16] by solving the non-linear differential equation (van Schilfgaarde's equ. (1)). Brooks gave solutions in which the thickness of the water bearing stratum was treated as a variable. The initial condition for these two solutions by Glover and Brooks was a horizontal water table.

The second case considered by Glover was for tile drains located on an impermeable layer. He assumed a curved water table for his initial condition as also is assumed by van Schilfgaarde. It should be pointed out van Schilfgaarde's solution is an extension of Glover's second case and not an extension of the first. The two cases differ by the initial conditions.

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a. June, 1963, by Jan van Schilfgaarde (Proc. paper 3453)

15. Research Agricultural Engineer; Northern Plains Branch, Soil and Water Conservation Research Division, Agriculture Research Service, USDA and Colorado Agricultural Experiment Station, Fort Collins, Colorado.

16. "Unsteady flow of Ground Water into drain tile", by R. H Brooks. Proceedings, ASCE, Vol. 87, No. IR 2, June 1961. pp 27-37.

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One gets the impression Glover's first case was improved by van Schilfgaarde when figure 4 and table I are used as a comparison of the two solutions. One may make any comparison he chooses. However, one would not expect Glover's first case to agree with van Schilfgaarde's solution, even if the differential equations for the two cases were the same, since the initial conditions are different. As would be expected, van Schilfgaarde's solution agrees exactly with Glover's solution of the second case for the tile on the impermeable layer.

Since van Schilfgaarde and Brooks solved the same differential equation for two different initial conditions, a comparison of these two solutions in light of two different initial conditions seems in order.

For ease of comparison, van Schilfgaarde's origin of coordinates is transposed to the same origin defined by Brooks[16]. Let $D = d + \frac{m_o}{2}$ and $m = m' + \frac{m_o}{2}$ where m' is the height to the water table midway between drains from the new origin. Then, van Schilfgaarde's equation (14) can be transformed into the dimensionless equation

$$\frac{\alpha t}{S^2} = \left[\frac{\left(1 - \frac{2m'}{m_o}\right)}{9A^2\left(\frac{D}{m_o} + \frac{m'}{m_o}\right)\left(\frac{D}{m_o} + \frac{1}{2}\right)} \right] , \qquad (17)$$

where $\alpha = \frac{KD}{f}$. Since the right side of equation (17) is only a function of $\frac{m'}{m_o}$ and $\frac{D}{m_o}$, then equation (17) may be rewritten as

$$\frac{\alpha t}{S^2} = f\left(\frac{D}{m_o}, \frac{m'}{m_o}\right) \qquad (18)$$

Brooks' equation (8)[16] cannot be solved explicitly for spacing, however, a graphical solution was presented and his solution may be written in the same form as equation (18), ie,

$$\frac{\alpha t}{L^2} = g\left(\frac{D}{H_o}, \frac{h}{H_o}\right) \tag{19}$$

where L is the spacing between drain tiles in Brooks' equation. The terms L, H_o and h in equation (19) are equal to S, m_o and m^1 respectively in equation (18)

Dividing equation (18) by (19) gives:

$$\frac{L}{S} = \left[\frac{f\left(\frac{D}{m_o}, \frac{m'}{m_o}\right)}{g\left(\frac{D}{m_o}, \frac{m'}{m_o}\right)}\right]^{1/2} . \tag{20}$$

It is convenient to compare equations (18) and (19) graphically as shown by figure 5. Both solutions have been plotted in terms of dimensionless variables. The solution of equation (20) is readily obtained by reading off values of $\frac{\alpha t}{S^2}$ and $\frac{\alpha t}{L^2}$ for a given value of $\frac{m'}{m_o}$ and $\frac{D}{m_o}$ from the respective curves where S and L denote van Schilfgaarde's and Brooks' curves respectively.

Equation (20) has been plotted in figure 6 for some values of $\frac{m'}{m_o}$ and for three values of $\frac{D}{m_o}$. Large differences in computed spacings can result if spacing calculations are made for small or large draw downs. As one would expect; by assuming an initial horizontal water table, computed spacings are generally smaller than for computed spacings assuming an initial curved water table. This is not always true for systems in which the tile is close to the impermeable layer as shown by the curve for

for $\frac{D}{m_o} = 1/2$ in figure 6.

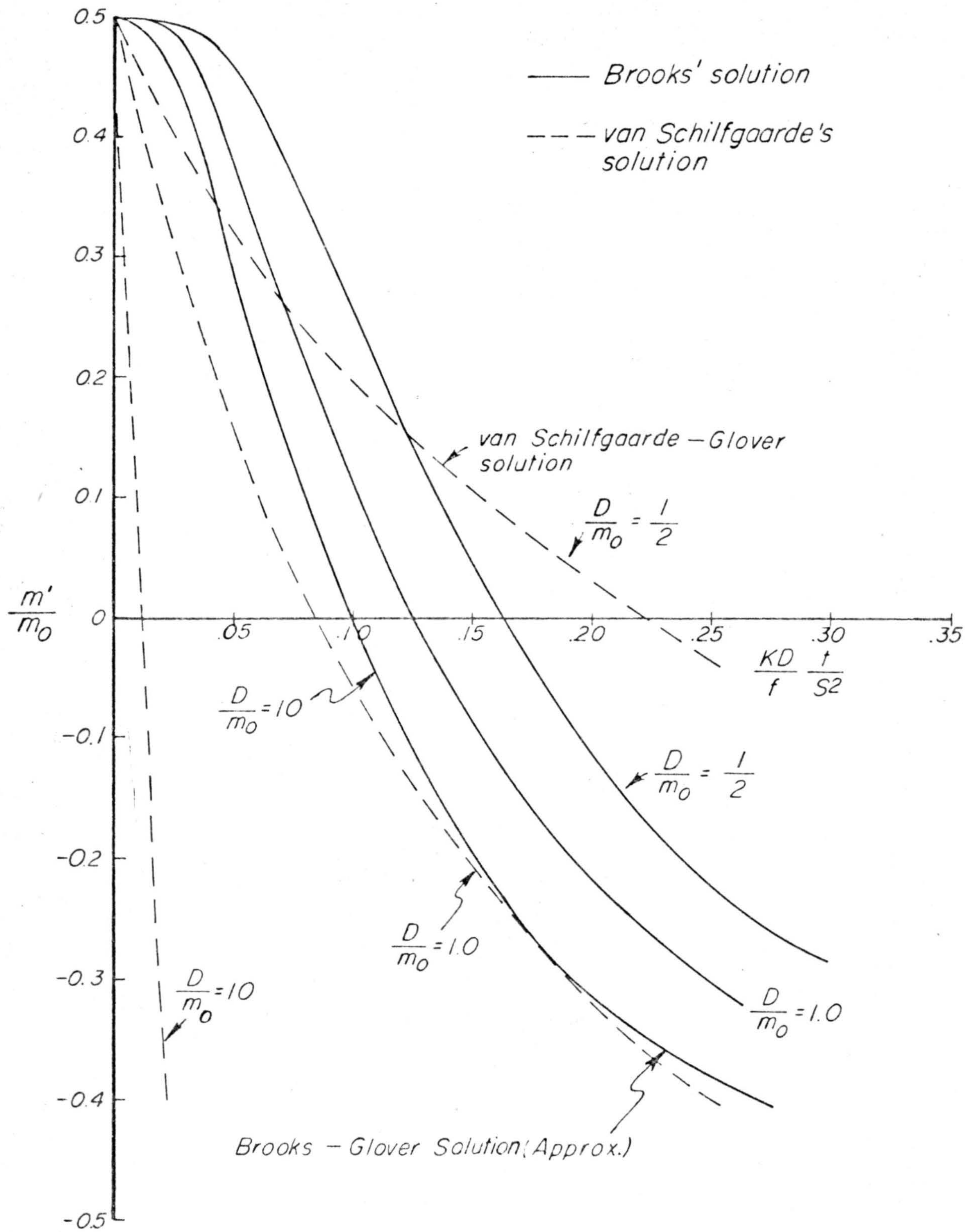

Fig. 5 A comparison of theoretical water table drawdown curves for two different initial conditions.

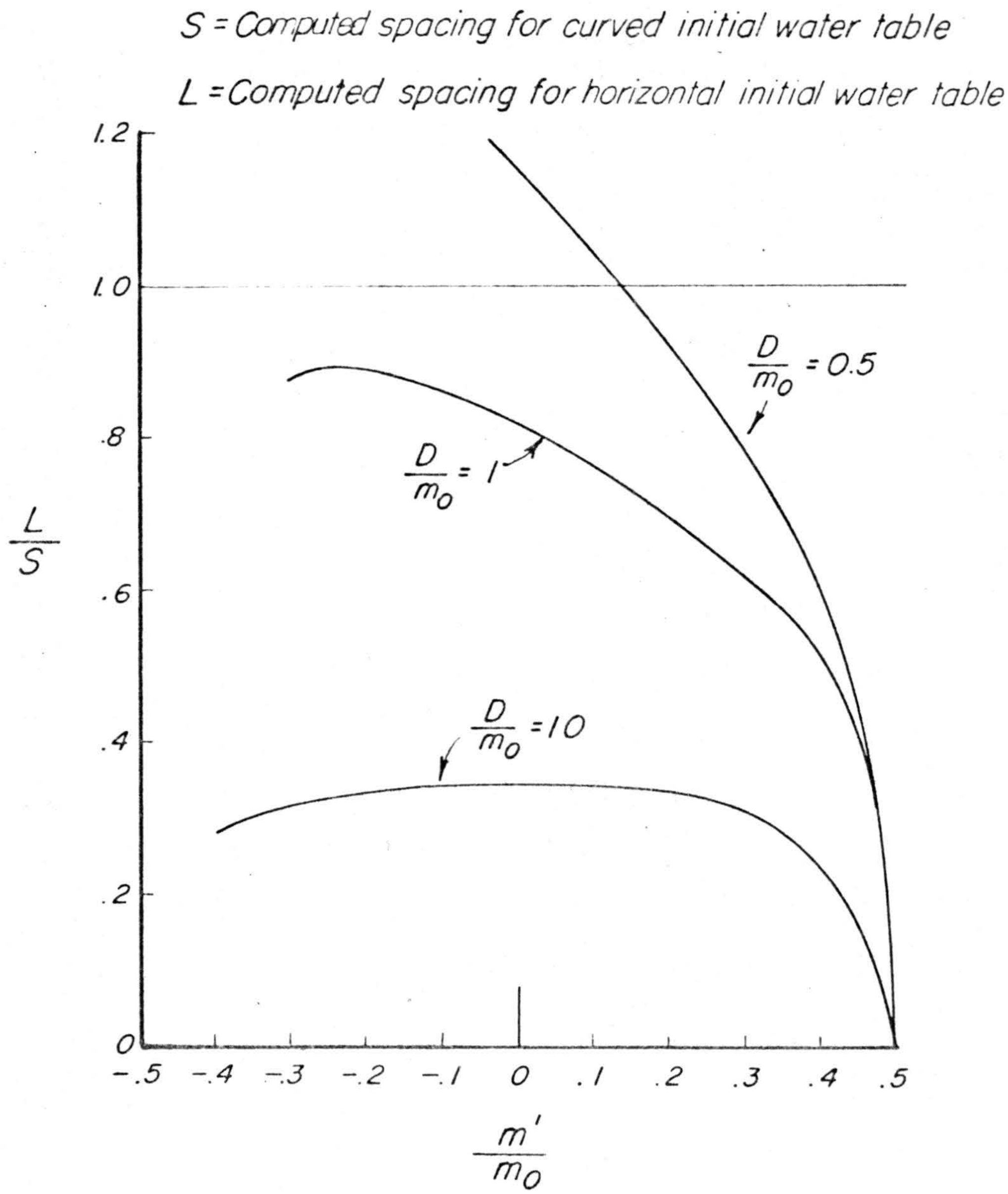

Fig. 6 The ratio of drain spacings as computed from two different initial conditions as a function of relative water table drawdowns.

Sufficient information is not available in the literature, the concerning shapes of water table recession curves to help the engineer decide which initial conditions are appropriate. This writer suspects that the two initial conditions discussed herein are conditions which closely fit many field situations. Some of the unpublished data of Klinge[18] and Isherwood and Pillsbury[19] tend to support this thesis. Additional field data concerning the affects of external water sources (irrigation water and precipitation) and soil conditions around the tile (the disturbed backfill trench above the tile line) upon the shape of the water table are needed.

The early work of Boussinesq[20] should not be over-looked in connection with this problem. Boussinesq solved equation (1) for the case of drainage tile on an impermeable layer. His solution did not exactly specify the initial conditions, in that any initial shape of the water table could be used so long as it could be expressed as a function of x.

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18. "Physical characteristics of Clermont Silt Loam Soil in Relation to tile drainage" by A. F. Klinge, M. S. Thesis, Purdue University, 144 pp. June 1955.

19. "Shallow Ground Water and Tile Drainage in the Oxnard Plain" by J. D. Isherwood and A. F. Pillsbury, Trans AGU, Vol. 39, No. 6, pp. 1101-1110, 1958

20. "Recherches theoriques sur l' ecoutement des nappes d' eau infiltrees dans le sol," by J. Boussinesq, Journal of Pure and Applied Mathematics, Series 5, Vol. 10, 1904.