HIGHWAYS IN THE RIVER ENVIRONMENT
HYDRAULIC AND ENVIRONMENTAL DESIGN CONSIDERATIONS

Training and Design Manual

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PARTIAL LIST OF SYMBOLS

A	Cross-sectional area of flow
a	Acceleration
a	Half-distance of wave amplitude
B	Constant
B'	Constant
C	Concentration of material in suspension in the fluid
C	Chezy discharge coefficient
$C/\sqrt{g}$	Chezy dimensionless discharge coefficient, $C/\sqrt{g} = V/\sqrt{gRS}$
C_D	Coefficient of drag
C_f	Concentration of fine sediment (washload)
C_s	Concentration of suspended sediment discharge
C_T	Concentration of bed-material discharge
C_v	Free vortex constant
C_λ	Celerity of the wave
c	Proportionality constant, concentration of sediment or subscript for critical conditions
D	Diameter of sediment particles
D_x	Diameter of sediment particles for which "x" percent of the particles are finer
D_m	Effective sediment diameter
D_{50}	Median diameter of sediment particles for which 50 percent are finer
Fr	Froude number, $V/\sqrt{gy}$
f	Darcy-Weisbach resistance coefficient
f_b'	Darcy-Weisbach bed resistance coefficient for the grain roughness
f_s	Seepage force
G	Gradation coefficient
g	Acceleration of gravity
H	Specific head, $V^2/2g + y_o$
H	Horizontal distance for one unit of vertical distance for trapezoidal channels
H_L	Head loss (total), $H_L = h_f + h_L$
H_T	Total head
h	Head
h_f	Friction head loss

h_L	Form loss
i	Subscript for inside or initial
i_B	Fraction of the bed load represented by a given grain size d
i_s	Fraction of the suspended load represented by a given grain size d
i_T	Fraction of the total bed material load represented by a given grain size d
K	Arbitrary constant
K_b	Strickler's bed roughness parameter
K_r	Strickler's particle roughness parameter
k_s	Height of the roughness element
L	Length
ℓ	Prandtl mixing length
M	Mass
m	Subscript for model
n	Manning's roughness coefficient
n	Coordinate normal to flow direction
P	Wetted perimeter
P	Sinuosity
P_i	Percentage by weight of that fraction of the bed material with geometric mean size D_i
ppm	Parts per million
Q	Discharge
Q_b	Water discharge determining bed-load discharge
Q_B	Total bed load
Q_s	Sediment discharge
Q_T	Total bed-material load
Q_{ss}	Total suspended load
q	Discharge per unit width
q_B	Bed-load discharge per unit width
q_s	Suspended load discharge per unit width
q_s	Sediment discharge per unit width
q_T	Total bed material discharge per unit width
R	Hydraulic radius, A/P
R_b	Hydraulic radius of the bed

R_b'	Hydraulic radius of the bed for the grain roughness
R_b''	Hydraulic radius of the bed for the form roughness
Re	Reynolds number, Vy/ν
r	Radius
r	Cylindrical coordinate
r	Subscript for ratio
r_o	Radius, outside of bend
r_c	Radius, center of bend
r_i	Radius, inside of bend
S	Slope
S_c	Shape factor for the cross section of a river
S_f	Slope of energy grade line
S.F.	Safety factor
S_m	Safety factor for riprap on a side slope with no flow
S_o	Slope of the bed
S_p	Shape factor of the sediment particles
S_R	Shape factor for the reach of a river
S_s	Specific gravity of solids
S_w	Slope of the water surface
s	Coordinate in the direction of flow
T	Temperature
t	Time variable
V	Mean velocity in the vertical
V	Mean velocity, Q/A
$\forall$	Volume
V_*	Shear velocity, $V_* = \sqrt{gRS}$
V_*'	Shear velocity for the grain roughness, $V_*' = \sqrt{gR_b'S}$
V_*''	Shear velocity for the form roughness, $V_*'' = \sqrt{gR_b''S}$
v	Velocity at a point, $v = \overline{v} + v'$
$\overline{v}$	Mean velocity at a point
v'	Velocity fluctuations
W	Width of stream
We	Weber number
w	Weight
w	Subscript for wave

x,y,z	Cartesian coordinate system
y	Depth
y_o	Normal depth of flow
y_o	Local depth of flow
y_c	Critical depth of flow
Z	Rouse number
z	Vertical distance
α	Kinetic energy coefficient
α	Slope angle of a channel
β	Momentum coefficient
β	Wave front angle
γ	Specific weight of water-sediment mixture
γ_s	Specific weight of sediment (approximately 165.4 pounds per cubic foot)
γ_w	Specific weight of water (approximately 62.4 pounds per cubic foot)
Δ	Small increment
δ'	Laminar sublayer thickness for V_*', $\delta' = \frac{11.6\nu}{V_*'}$
ε	Eddy viscosity
η	Stability number for riprap on a plane bed
η'	Stability number for riprap on a side slope
θ	Inclination angle
θ	Contraction angle
θ	Central angle
κ	von Karman universal velocity coefficient
λ	Wave length
μ	Dynamic viscosity
ν	Kinetic viscosity
ρ	Mass density of fluid
ρ_s	Mass density of sediment
Σ	Summation symbol
σ	Surface tension
π	Circular circumference-diameter ratio
τ	Shear stress

τ_o	Shear stress at the boundary
Φ_*	Dimensionless measure of bed load transport defined by Einstein
ϕ	Angle of repose of cohesionless materials
ψ_*	Dimensionless measure of shear on a particle defined by Einstein
ω	Fall velocity of sediment particles

ENGLISH TO SI (METRIC) CONVERSION FACTORS

To convert	To	Multiply by
inches (in.)	millimeters (mm)	25.40
inches (in.)	centimeters (cm)	2.540
inches (in.)	meters (m)	0.0254
feet (ft)	meters (m)	0.305
miles (miles)	kilometers (km)	1.61
yards (yd)	meters (m)	0.91
square feet (sq ft)	square meters (m^2)	0.093
square yards (sq yd)	square meters (m^2)	0.836
acres (acre)	square meters (m^2)	4047.
square miles (sq miles)	square kilometers (km^2)	2.59
cubic feet (cu ft)	cubic meters (m^3)	0.028
pounds (lb)	kilograms (kg)	0.453
tons (ton)	kilograms (kg)	907.2
pounds per square foot (psf)	newtons per square meter (N/m^2)	47.9
pounds per square inch (psi)	kilonewtons per square meter (kN/m^2)	6.9
gallons (gal)	liter (dm^3)	3.8
acre-feet (acre-ft)	cubic meters (m^3)	1233.
gallons per minute (gpm)	cubic meters/minute (m^3/min)	0.0038

Chapter I

INTRODUCTION

1.1.0 OBJECTIVES

The purpose of this chapter is to lay the groundwork for application of the concepts of open-channel flow, fluvial geomorphology, and river mechanics to the design, maintenance, and related environmental problems associated with highway crossings and encroachments.

Basic definitions of terms and notations adopted for use herein have been presented in the preceding section for easy use and rapid reference. Additionally, these important terms and variables are defined and explained as they are encountered.

1.2.0 CLASSIFICATION OF RIVERS, RIVER CROSSINGS AND ENCROACHMENTS

There is a wide variety of types of rivers, river crossings and encroachments. *Encroachment is any occupancy of the river and floodplain for highway use.* The objective herein is to consider the fluvial, hydraulic, geomorphic, and environmental aspects of highway encroachments, including bridge locations, bridge alignment training, longitudinal encroachments, stabilization works and road approaches. *Encroachments are usually no problem during normal flows but require special protection against floods.* Flood protection requirements vary from site to site. Some bridges must accommmodate the passage of livestock and farm equipment underneath during periods of low flow. Other bridges require low embankments for aesthetic appeal, especially in populated areas. Still other bridges require short spans with long approaches and numerous piers for economic reasons. All of these factors and many more contribute to the difficulty in generalizing the design for all highway encroachments.

A classification of encroachments based on prominent features is helpful. Classifying the regions requiring protection, the possible types of protection, the possible flow conditions, the possible channel shapes, and the various geometric conditions aids the engineer in selecting the design criteria for the conditions he has encountered.

1.2.1 Types of encroachment

In the vicinity of rivers, highways generally impose a degree of encroachment. In some instances, particularly in mountainous regions or in river gorges and canyons, river crossings can be accomplished with absolutely no encroachment on the river. The bridge and its approaches are located far above and beyond any possible flood stage. More commonly, the economics of crossings require substantial encroachment on the river and its floodplain, the cost of a single span over the entire floodplain being prohibitive. *The encroachment can be in the form of earth fill embankments over the floodplain or into the main channel itself, reducing the required bridge length; or in the form of piers and abutments in the main channel* of the river.

There are also longitudinal encroachments not connected with river crossings. Floodplains often appear to provide an attractive low cost alternative for highway location, even when the extra cost of flood protection is included. As a consequence, *highways*, including interchanges, *often encroach on a floodplain over long distances*. In some regions, river valleys provide the only feasible route for highways. This is true even in areas where a floodplain does not exist. In many locations the highway must encroach on the main channel itself and the channel is partly filled to allow room for the roadway. In some instances this encroachment becomes severe, particularly as older highways are upgraded and widened. There is often also the need to straighten a stretch of the river, eliminating meanders, to accommodate the highway.

1.2.2 Types of rivers

By way of classification, rivers can be divided into those with floodplains and those without. Floodplains are usually not the direct result of large flood flows but rather the result of lateral movement of the river from one side of the plain to the other through geologic time. Rivers which have downcut in their valleys have left former floodplains high above modern-day flood levels. These former floodplains are called terraces. By definition the floodplain is low enough to be completely inundated by floods with fairly short recurrence periods.

Whether or not floodplains exist, *rivers can also be classified as either braided, straight or meandering*. The character of each classification is shown in Fig. 1.2.1. Braided rivers may be quite stable to the extent that the associated islands support farms and even urban communities. Under other geographic conditions braided rivers are extremely unstable with the channels shifting with each sharp change in discharge. The potential width of a braided river may be much greater than casual observation indicates. Unpredicted channel shifting has been the cause of many crossing failures.

As seen in Fig. 1.2.1, even straight rivers are to some degree sinuous. The sinuosity is a measure of this meandering feature. *The sinuosity is defined as the ratio of the length of the river's thalweg to the length of the valley proper. The thalweg is the path of deepest flow.* Rivers with sinuosity less than 1.5 are usually considered straight. Meandering rivers are commonly associated with erodible floodplains, although very regular and highly developed meanders have occurred in rivers incised in solid rock valleys.

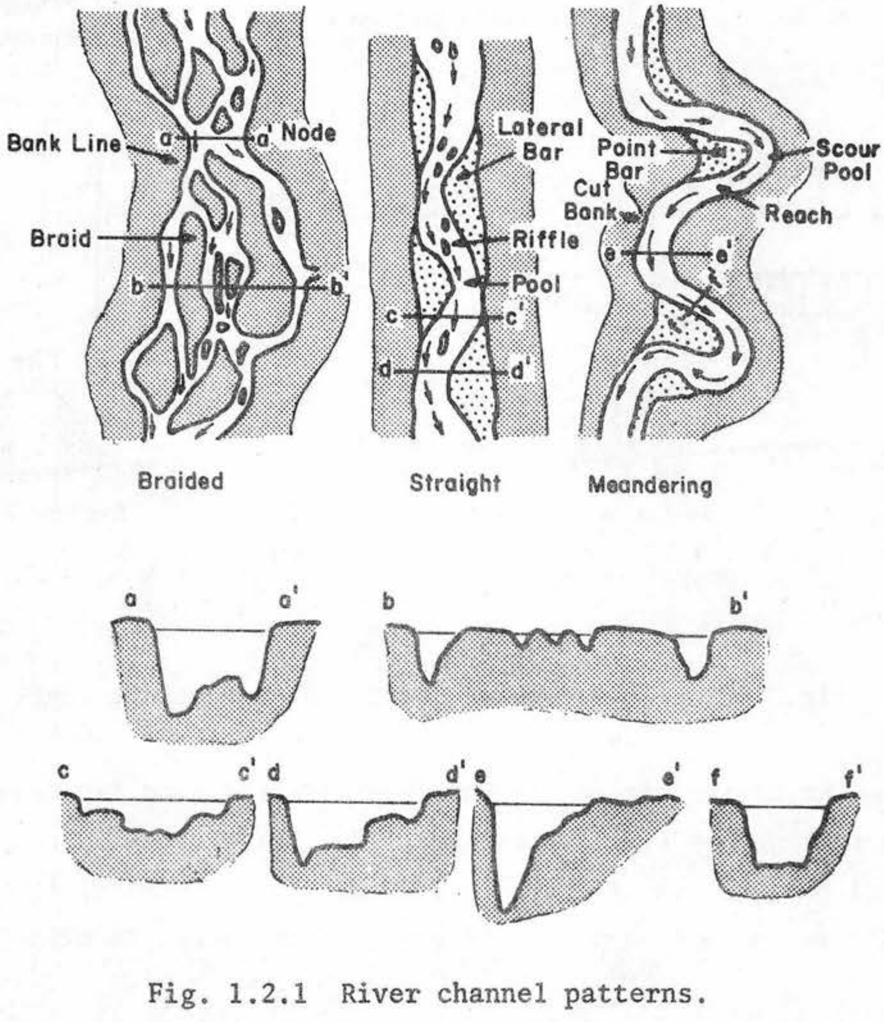

Fig. 1.2.1 River channel patterns.

1.2.3 Geometry of bridge crossings

The bridge crossing is the most common type of river encroachment. The geometric properties of bridge crossings illustrated in Fig. 1.2.2 are commonly used depending on the conditions at the site. *The approaches may be skewed or normal (perpendicular) to the direction of flow, or one approach may be longer than the other, producing an eccentric crossing.* Abutments used for the overbank-flow case may be set back from the low-flow channel banks to provide room to pass the flood flow or simply to allow passage of livestock and machinery, or the abutments may extend up to the banks or even protrude over the banks, constricting the low-flow channel. Piers, dual bridges for multi-lane freeways, channel bed conditions, and spur dikes add to the list of geometric classifications.

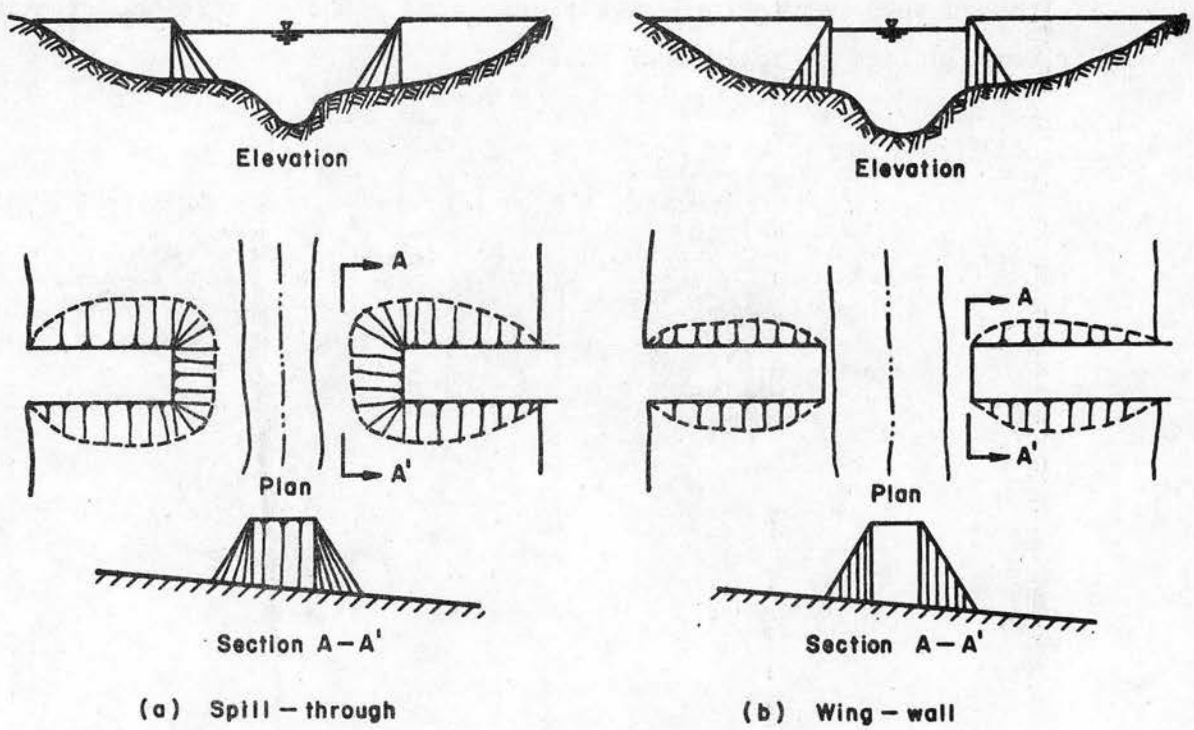

Fig. 1.2.2 Geometric properties of bridge crossings.

The design procedures have been derived from laboratory and field observations of bridge crossings. *The design procedures include allowances made for the effects of skewness, eccentricity, scour, abutment setback, channel shape, submergence of the superstructure, debris,* *spur dikes, wind waves, ice, piers, abutment types, and flow conditions.* These design procedures take advantage of the large volume of work that has been done by many people in describing the hydraulics and scour characteristics of bridge crossings.

1.3.0 DYNAMICS OF NATURAL RIVERS AND THEIR TRIBUTARIES

Frequently, environmentalists, river engineers, and others involved in transportation, navigation, and flood control consider a river to be static; that is, unchanging in shape, dimensions, and pattern. However, *an alluvial river generally is continually changing its position and shape as a consequence of hydraulic forces acting on its bed and banks. These changes may be slow or rapid and may result from natural environmental changes or from changes by man's activities.* When an engineer modifies a river channel locally, this local change frequently causes modification of channel characteristics both up and down the stream. The response of a river to man-induced changes often occurs in spite of attempts by engineers to keep the anticipated response under control.

The point that must be stressed is that a river through time is dynamic, that man-induced change frequently sets in motion a response that can be propagated for long distances, and that in spite of their complexity all rivers are governed by the same basic forces. The highway engineer must understand and work with these natural forces. It is absolutely necessary for the design engineer to have at hand competent knowledge about: (1) geological factors, including soil conditions; (2) hydrologic factors, including possible changes in flows, runoff, and the hydrologic effects of changes in land use; (3) geometric characteristics of the stream, including the probable geometric alterations that will be activated by the changes his project and future projects will impose on the channel; and (4) hydraulic characteristics such as depths, slopes, and velocity of streams and what changes may be expected in these characteristics in space and time.

1.3.1 Historical evidence of the natural instability of fluvial systems

In order to emphasize the inherent dynamic qualities of river channels, evidence is cited below to demonstrate that most alluvial rivers are not static in their natural state. Indeed, scientists concerned with the history of landforms (geomorphologists), vegetation (botanists), and the past activities of man (archaeologists), rarely consider the landscape as unchanging. Rivers, glaciers, sand dunes, and seacoasts are highly susceptible to change with time. Over a relatively short period of time, perhaps in some cases as long as man's lifetime, components of the landscape may be relatively stable. Nevertheless stability cannot be automatically assumed. *Rivers are, in fact, the most actively changing of all geomorphic forms.*

Evidence from several sources demonstrate that river channels are continually undergoing changes of position, shape, dimensions, and pattern. In Fig. 1.3.1 a section of the Mississippi River as it was in 1884 is compared with the same section as observed in 1968. In the lower 6 miles of river, the surface area has been reduced approximately 50 percent during this 84-year period. Some of this change has been natural and some has been the consequence of river development work.

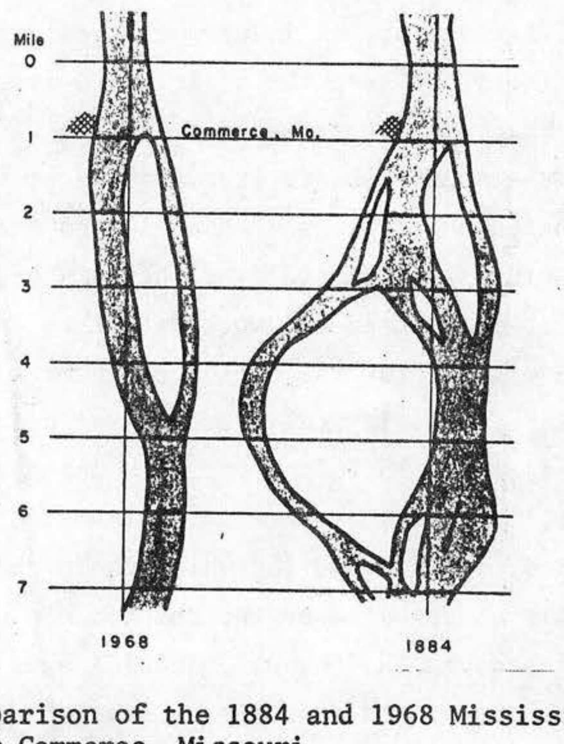

Fig. 1.3.1 Comparison of the 1884 and 1968 Mississippi River Channel near Commerce, Missouri.

In alluvial river systems, it is the rule rather than the exception that banks will erode, sediments will be deposited and floodplains, islands, and side channels will undergo modification with time.

Changes may be very slow or dramatically rapid. Risk's (1944) report on the Mississippi River and his maps showing river position through time are sufficient to convince everyone of the innate instability of the Mississippi River. The Mississippi is our largest and most impressive river and because of its dimensions it has sometimes been considered unique. This is, of course, not so. Hydraulic and geomorphic laws apply at all scales of comparable landform evolution. The Mississippi may be thought of as a prototype of many rivers or as a much larger than prototype model of many sandbed rivers.

Rivers change position and morphology (dimensions, shape, pattern) as a result of changes of hydrology. Hydrology can change as a result of climatic change over long periods of time, or as a result of natural stochastic climatic fluctuations (droughts, floods), or by man's modification of the hydrologic regime. For example, the major climatic changes of recent geological time (the last few million years of earth history) have triggered dramatic changes in runoff and sediment loads with corresponding channel alteration. Equally significant during this time were fluctuations of sea level. During the last continental glaciation, sea level was on the order of 400 feet lower than at present, and this reduction of base level caused major incisions of river valleys near the coasts.

In recent geologic time, major river changes of different types occurred. These types are deep incision and deposition as sea level fluctuated, changes of channel geometry as a result of climatic and hydrologic changes, and obliteration or displacement of existing channels by continental glaciation. Climatic change, sea level change, and glaciation are interesting from an academic point of view but are not considered as cause of modern river instability. *The movement of the earth's crust is one geologic agent causing modern river instability.* The earth's surface in many parts of the world is undergoing continuous measurable change by upwarping, subsidence or lateral displacement. As a result, the study of these ongoing changes (called neotectonics) has become a field of major interest for many geologists and geophysicists. Such gradual surface changes can affect stream channels dramatically. For example, Wallace (1967) has shown that many small streams are clearly offset laterally along the San Andreas fault in California. Progressive lateral movement of this fault on the order of an inch per year has been measured. The rates of movement of faults are highly variable, but an average rate of mountain building has been estimated by Schumm (1963) to be on the order of 25 feet per 1000 years. Seemingly insignificant in human terms, this rate is actually 0.3 inches per year or 3 inches per decade. For many river systems, a change of slope of 3 inches would be significant. (The slope of the energy gradient on the Lower Mississippi River is about 3 to 6 inches per mile.)

Of course, the geologist is not surprised to see drainage patterns that have been disrupted by uplift or some complex warping of the earth's surface. In fact, complete reversals of drainage lines have been documented. In addition, convexities in the longitudinal profile of both rivers and river terraces (these profiles are concave under normal development) have been detected and attributed to upwarping. Further, the progressive shifting of a river toward one side of its valley has resulted from lateral tilting. Major shifts in position of the Brahmaputra River toward the west are attributed by Colman (1969) to tectonic movements. Hence, neotectonics should not be ignored as a possible cause of local river instability.

Long-term climatic fluctuations have caused major changes of river morphology. Floodplains have been destroyed and reconstructed. The history of semiarid and arid valleys of the western United States is one of alternating periods of channel incision and arroyo formation followed by deposition and valley stability which have been attributed to climatic fluctuations.

It is clear that rivers can display a remarkable propensity for change of position and morphology in time periods of a century. Hence rivers from the geomorphic point of view are unquestionably dynamic, but does this apply to modern rivers? It is probable that *during a period of several years, neither neotectonics nor a progressive climate change will have a detectable influence on river character and behavior. What then causes a stable river to appear relatively unstable from the point of view of the highway engineer or the environmentalist? It is the slow but implacable shift of a river channel through erosion and deposition at bends, the shift of a channel to form chutes and islands, and the cutoff of a bend to form oxbow lakes.* Wolman and Leopold (1957) state that lateral migration rates are highly variable; that is, a river may maintain a stable position for long periods and then experience rapid movement. Much therefore depends on flood events, bank stability, permanence of vegetation on banks, and floodplain land use.

A compilation of data by Wolman and Leopold shows that rates of lateral migration for the Kosi River of India range up to approximately 2500 feet per year. Rates of lateral migration for two major rivers in the United States are as follows: Colorado River near Needles, California, 10 to 150 feet per year; Mississippi River near Rosedale, Mississippi, 158 to 630 feet per year.

Archaeologists have also provided clear evidence of channel changes that are completely natural and to be expected. For example, the number of archaeologic sites on floodplains decreases significantly with age because the earliest sites are destroyed as floodplains are modified by river migration. Lathrop (1968), working on the Rio Ucayali in the Amazon headwaters of Peru, estimates that on the average a meander loop begins to form and cuts off in 5000 years. These loops have an amplitude of 2 to 6 miles and an average rate of meander growth of approximately 40 feet per year.

A study by Schmudde (1963) shows that about one-third of the floodplain of the Missouri River over the 170-mile reach between Glasgow and St. Charles, Missouri, was reworked by the river between 1879 and 1930. On the Lower Mississippi River, bend migration was on the order of 2 feet per year, whereas in the central and upper parts of the river below Cairo it was at times 1000 feet per year (Kolb, 1963). On the other hand, a meander loop pattern of the lower Ohio River has altered very little during the past thousand years (Alexander and Nunnally, 1972).

Although the dynamic behavior of perennial streams is impressive, the modification of rivers in arid and semiarid regions and especially of ephemeral (flowing occasionally) stream channels is startling. A study of floodplain vegetation and the distribution of trees in different age groups led Everitt (1968) to the conclusion that about half of the Little Missouri River floodplain in western North Dakota was reworked in 69 years.

Historical and field studies by Smith (1940) show that floodplain destruction occurred during major floods on rivers of the Great Plains. An exceptional example of this is the Cimarron River of southwestern Kansas, which was 50 feet wide during the latter part of the 19th and first part of the 20th centuries (Schumm and Lichty, 1963). Following a series of major floods during the 1930's it widened to 1200 feet, and the channel occupied essentially the entire valley floor. During the decade of the 1940's a new floodplain was constructed, and the river width was reduced to about 500 feet in 1960. Equally dramatic changes of channel dimensions have occurred along the North and South Platte Rivers in Nebraska and Colorado as a result of man's control of flood peaks by reservoir construction. Natural changes of this magnitude due to changes in flood peaks are perhaps exceptional, but emphasize the mobility of rivers.

Another somewhat different type of channel modification which testifies to the rapidity of fluvial processes is described by Shull (1922, 1944). During a major flood in 1913, a barge became stranded in a chute of the Mississippi River near Columbus, Kentucky. The barge induced deposition in the chute and an island formed. In 1919, the island was sufficiently large to be homesteaded, and a few acres were cleared for agricultural purposes. By 1933, the side channel separating the island from the mainland had filled to the extent that the island became part of Missouri. The island formed in a location protected from the erosive effects of floods but susceptible to deposition of sediment during floods. For these reasons the channel filling was rapid and progressive. It cannot be concluded that islands will always form and side channels fill at such rapid rates, but island formation and side-channel filling appear to be the normal course of events in any river transporting moderate or high sediment loads regardless of the river size.

In summary, archaeological, botanical, geological, and geomorphic evidence supports the conclusion that most rivers are subject to constant change as a normal part of their morphologic evolution. Therefore, stable or static channels are the exception in nature.

1.3.2 Introduction to river hydraulics and river response

In the previous section it was established that rivers are dynamic and respond to changed environmental conditions. The direction and extent of the change depends on the forces acting on the system. The mechanics of flow in rivers is a complex subject that requires special study which is unfortunately not included in basic courses of fluid mechanics. The major complicating factors in river mechanics are: (a) the large number of interrelated variables that can simultaneously respond to natural or imposed changes in a river system and (b) the continual evolution of river channel patterns, channel geometry, bars and forms of bed roughness with changing water and sediment discharge. In order to understand the responses of a river to the actions of man and nature, a few simple hydraulic and geomorphic concepts are presented here.

Rivers are broadly classified as straight, meandering, braided or some combination of these classifications, but any changes that are imposed on a river may change its form. The dependence of river form on the slope which may be imposed independent of the other river characteristics is illustrated schematically in Fig. 1.3.2. By changing the slope, it is possible to change the river from a meandering one that is relatively tranquil and easy to control to a braided one that varies rapidly with time, has high velocities, is subdivided by sandbars and carries relatively large quantities of sediment. Such a change could be caused by a natural or artificial cutoff. Conversely, it is possible that a slight decrease in slope could change an unstable braided river into a meandering one.

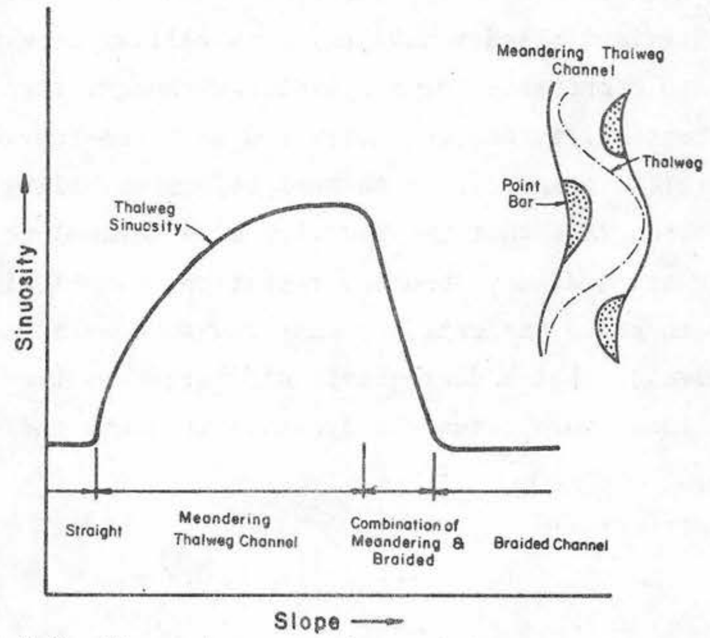

Fig. 1.3.2 Sinuosity vs. slope with constant discharge. Channel patterns are illustrated in Fig. 1.2.1.

Based on research results of Lane (1955), Leopold and Maddock (1953), Santos-Cayudo and Simons (1973) and Schumm (1971), the following general statements concerning a river's response to altered water discharge and sediment load can be made:

(1) Depth is directly proportional to discharge and inversely proportional to the bed-material discharge.

(2) Channel width is directly proportional to discharge and to sediment load.

(3) Channel shape (width-depth ratio) is directly related to sediment load.

(4) Meander wavelength is directly proportional to discharge and to sediment load.

(5) Gradient is inversely proportional to discharge and directly proportional to sediment load and grain size.

(6) Sinuosity is proportional to valley slope and inversely proportional to sediment load.

Gradient is considered in the above to be a dependent variable in that a river can reduce the gradient by becoming more sinuous. It is important to remember that the relations given above pertain to natural rivers and not necessarily to artificial channels with bank materials that are not representative of sediment load; however, the relations help to determine the response of any water conveying channel.

The significantly different channel dimensions, shapes, and patterns associated with different quantities of discharge and amounts of sediment load indicate that as these independent variables change, major adjustments of channel morphology can be anticipated. Further, if changes in sinuosity and meander wavelength as well as in width and depth are required to compensate for a hydrologic change, then a long period of channel instability can be envisioned with considerable bank erosion and lateral shifting of the channel before stability is restored. One is led to conclude that the reaction of a channel to changes in discharge and sediment load may result in channel dimension changes contrary to those indicated by many regime equations. For example, it is conceivable that a decrease in discharge together with an increase in sediment load could actuate a decrease in depth and an increase in width.

Changes in sediment and water discharge at a particular point or reach in a stream may have an effect ranging from some distance upstream to a point downstream where the hydraulic and geometric conditions can have absorbed the change. Thus, it is well to consider a channel reach as part of a complete drainage system. Artificial controls that could benefit the reach may, in fact, cause problems in the system as a whole. For example, flood control structures can cause downstream flood damage to be greater at reduced flows if the average hydrologic regime is changed so that the channel dimensions are actually reduced. Also, where major tributaries exert a significant influence on the main channel by introduction of large quantities of sediment, upstream control on the main channel may allow the tributary to intermittently dominate the system with deleterious results. If discharges in the main channel are reduced, sediments from the tributary that previously were eroded will no longer be carried away and serious aggradation with accompanying flood problems may arise.

An insight into the direction of change, the magnitude of change, and the time involved to reach a new equilibrium can be gained by studying the river in a natural condition; having knowledge of the sediment and water discharge; being able to predict the effects and magnitude of man's future activities; and applying to these a knowledge of geology, soils, hydrology, and hydraulics of alluvial rivers.

The current interest in ecology and the environment have made people aware of the many problems that mankind can cause. Previous to the present interest in environmental impact, very few people interested in rivers ever considered the long-term changes that were possible. It is imperative that anyone working with rivers, either with localized areas or entire systems, have an understanding of the many factors involved, and of the potential for change existing in the river system.

Two methods of predicting response are employed. They are the physical and the mathematical models. Engineers have long used small scale hydraulic models to assist them in anticipating the effect of altering conditions in a reach of a river. With proper awareness of the large scale effects that can exist, the results of hydraulic model testing can be extremely useful for this purpose. A more recent and perhaps more elegant method of predicting short-term and long-term changes in rivers involves the use of mathematical models. To study a transient phenomena in natural alluvial channels, the equations of motion and continuity for sediment laden water and the continuity equation for sediment can be used. These equations are powerful analytical tools for the study of unsteady flow problems. However, because of mathematical difficulties, many practical solutions can only be obtained by numerical analysis using iteration procedures and digital computers. The potential of numerical mathematical models for flood and sediment routing, degradation and aggradation studies, and long-term channel development studies is now being realized.

1.4.0 EFFECTS OF HIGHWAY CONSTRUCTION ON RIVER SYSTEMS

Highway construction can have significant general and local effects on the geomorphology and hydraulics of river systems. Hence, it is necessary to consider induced short-term and long-term responses of the river and its tributaries, the impact on environmental factors, the aesthetics of the river environment and short-term and long-term effects of erosion and sedimentation on the surrounding landscape and the river. The biological response of the river system should also be evaluated and considered.

1.4.1 Immediate responses

Let us consider a few of the numerous and immediate responses of rivers to the construction of bridges, training and channel stabilization works and approaches.

In the preceding paragraphs we indicated that local changes made in the geometry or the hydraulic properties of the river may be of such a magnitude as to have an immediate impact upon the entire river system. More specifically, *contractions due to the construction of encroachments generally cause general and local scour, and the sediments removed from this location are usually dropped in the immediate reach downstream.* In the event that the contraction is extended further downstream, the river may be capable of carrying the increased sediment load an additional distance but only until a reduction in gradient and a reduction in transport capability is encountered. The increased velocities caused by encroachments may also affect the general lateral stability of the river downstream.

In addition, the development of crossings and the contraction of river sections may have a significant effect on the water level in the vicinity and upstream of the bridge. Such changes in water level upstream of the bridge are called backwater effects. The highway engineer must be in a position to accurately assess the effects of the construction of crossings upon the water surface profile.

To offset increased velocities and to reduce bank instabilities and related problems, one ends up, in many instances, with stabilizing or channelizing the river to some degree. When it is necessary to do this, every effort should be made to do the channelization in a manner which does not degrade the river environment including its aesthetic value.

As a consequence of construction, many areas become highly susceptible to erosion. The transported sediment is carried from the construction site by surface flow into the minor rills, which combine within a short distance to form larger channels leading to the river. The water flowing from the construction site is usually a consequence of rain. The surface runoff and the accompanying erosion can significantly increase the sediment yield to the river channel unless careful control is exercised. The large sediment particles transported to the main channel may reside in the vicinity of the construction site for a long period of time or may be slowly moved away. On the other hand, the fine sediments are easily transported and generally pollute the whole cross section of the river. The fine sediments are transported downstream to the nearest reservoir or to the sea. As will be discussed later, the sudden injection of the larger sediments into the channel may cause local aggradation, thereby steepening the channel, increasing the flow velocities and possibly causing instability in the river at that site. Over a long period of time after the injection has ceased, the river would return to its former geometry.

The suspended fine sediments can have very significant effects on the biomass of the stream. Certain species of fish can only tolerate large quantities of suspended sediment for relatively short periods of time. This is particularly true of the eggs and fry. This type of biological response to development normally falls outside of the competence of the engineer. Yet his work may be responsible for the discharge of these sediments into the system and if he is unable to cope with the problem, the engineer should utilize adequate technical assistance from experts in fisheries, biology, and other related areas to overcome the consequences of sediment pollution in a river. Only with such knowledge can he develop the necessary arguments to sell his case that erosion control measures must be exercised to avoid significant deterioration of the stream environment not only in the immediate vicinity of the bridge but in many instances for great distances downstream.

Another possible immediate response of the river system to construction is the loss of the recreational use of the river. In many streams, there may be an immediate drop in the quality of the fishing due to the increase of sediment load, or other changed hydraulic characteristics within the channel. Most natural rivers consist of a series of pools and riffles. Both form an important part of the environment from the viewpoint of fisheries. The introduction of larger quantities of sediment into the channel and changes made in the geometry of the channel may result in the loss of these pools and riffles. Along the same lines, construction work within the river may cause a loss of food essential to fish life and often it is difficult to get the food chain re-established in the system.

Construction and operation of highways in water-supply watersheds present very real problems and require special precautionary designs to protect the water supplies from highway residue. These residues may be largely sedimentary and may increase the turbidity of the water. There have been instances, however, where other unwelcome materials such as asphalt distillates have been traced to highway operation.

The preceding discussion is related to only a few immediate responses to construction along a river. However, they are responses that illustrate their importance to design and the environment.

1.4.2 Delayed response of rivers to development

In addition to the example of possible immediate responses discussed above, there are important delayed responses of rivers to highway development. As part of this introductory chapter, consideration is given to some of the more obvious effects that can be induced on a river system over a long time period by highway construction.

Often it is necessary to employ training works in connection with bridges to favorably align the flow with the bridge openings. When such training works are used, they generally straighten the channel, shorten the flow line, and increase the velocity within the channel. Any such changes made in the system that cause an increase in the gradient may cause an increase in velocities. The increase in velocity increases local and general scour with subsequent deposition downstream where the channel takes on its normal characteristics. If significant lengths of the river are trained and straightened, there can be a noticeable decrease in the elevation of the water surface profile for a given discharge in the main channel. Tributaries emptying into the main channel in such reaches are significantly affected. Having a lower water level in the main channel for a given discharge means that the tributary streams entering in that vicinity are subjected to a steeper gradient and higher velocities which cause degradation in the tributary streams. In extreme cases, degradation can be induced of such magnitude as to cause failure of structures such as bridges on the tributary systems. In general, any increase in transported materials from the tributaries to the main channel causes a reduction in the quality of the environment within the river. More specifically, as degradation occurs in the tributaries, bank instabilities are induced and the sediment loads are greatly increased. Increased sediment loads usually result in a deterioration of the given environment.

1.5.0 THE EFFECTS OF RIVER DEVELOPMENT ON HIGHWAY STRUCTURES

Some of the possible immediate and delayed responses of rivers and river systems to the construction of bridges, approaches, channel stabilization and the utilization of training works have been mentioned. *It is necessary also to consider the effects of highway structures on river development works.* These works may include, for example, water diversions from the river system, water diversions to the river system, construction of reservoirs, flood control works, cutoffs, levees, navigation works, and the mining of sand and gravel. It is essential to consider the possible or probable long-term plans of all agencies and groups as they pertain to a river when designing crossings or when dealing with the river in any way. Let us consider a few typical responses of a crossing to different types of water resources development.

Cutoffs may develop naturally in the river system or cutoffs can be constructed by man. The general consequence of cutoffs is to shorten the flow path and steepen the gradient of the channel. The local steepening can significantly increase the velocities and sediment transport. Also, this action can induce significant instability such as bank erosion and degradation in the reach. The material scoured in the reach effected by the cutoff is probably carried only to the adjacent downstream reach where the gradient is flatter. In this region of slower velocities the sediment drops out rapidly. The deposition can have significant detrimental effect on the downstream reach of river, increasing the flood stage in the river itself and increasing the base level for the tributary stream, thereby causing aggradation in the tributaries.

Consider a classic example of a cutoff that was constructed on a large bend in one of the tributaries to the Mississippi. Along this bend, small towns had developed and small tributary streams entered the main channel within the gooseneck bend. It was decided to develop a cutoff across the gooseneck to shorten the flow line or the river, reduce the flood stage and generally improve poor conditions in that location. Several interesting results developed.

In the vicinity of the cutoff the bankline eroded and degradation was initiated. Within the gooseneck bend, the small tributaries continued to discharge their water and sediment. Because of the flat gradient in the bend, this channel section could not convey the sediment from the small systems through it and aggradation was initiated. Within a short period of time sufficient aggradation had occurred so as to jeopardize water intakes, sewage outfalls and so forth. As a consequence of the adverse action in the vicinity of the cutoff and within the gooseneck itself, it was finally decided that it would be more beneficial to restore the river to its natural form through the gooseneck. This action was taken and the serious problems were alleviated.

In such a program of river development, the highway engineer would be hard pressed to maintain and plan for his highway system along and over this reach of river.

Another common case occurs with the development of reservoirs for storage and flood control. These reservoirs serve as traps for the sediment normally flowing through the river system. With sediment trapped in the reservoir, essentially clear water is released at the dam site. This clear water has the capacity to transport more sediment than is immediately available. Consequently the channel begins to supply this deficit with resulting degradation of the bed. This degradation may significantly affect the safety of bridges in the immediate vicinity. Again, the degraded main channel causes steeper gradients on tributary streams in the vicinity of the main channel. The result is degradation in the tributary streams. It is entirely possible, however that the additional sediments supplied by the tributary streams would ultimately offset the degradation in the main channel. It must be recognized that downstream of storage structures the channel may either aggrade or degrade and the tributaries will be affected in either case.

There are important responses induced upstream of reservoirs as well as downstream. When the stream flowing into a reservoir encounters the ponded water, its sediment load is deposited forming a delta. This deposition in the reservoir flattens the gradient of the channel upstream. The flattening of the upstream channel induces aggradation causing the bed of the river to rise, threatening highway installations and other facilities. For example, *Elephant Butte Reservoir, built on the Rio Grande, has caused the Rio Grande to aggrade many miles upstream of the reservoir site.* At Albuquerque, New Mexico, the riverbed has aggraded until it is presently several feet above the level of the city. This degree of change in bed level can have very significant effects upon bridges, other hydraulic structures and all types of training and stabilization works. Ultimately the river may be subjected to a flow of magnitude sufficient to overflow existing banks, causing the water to seek an entirely new channel. With the abandonment of the existing channel there would be a variety of bridges and hydraulic structures that would also be abandoned at great expense to the public. Further, there are investigations underway that may lead to the construction of a storage reservoir upstream of Albuquerque. With the construction of this reservoir, clear water would be released which could initiate degradation in the channel in the immediate vicinity of the reservoir but would supply even greater sediment downstream where the channel is affected by Elephant Butte.

The clear-water diversion into South Boulder Creek in Colorado is another example of river development that affects bridge crossings and encroachments as well as the environment in general. Originally the North Fork of South Boulder Creek was a small but beautiful scenic mountain stream. The banks were nicely vegetated; there was a beautiful sequence of ripples and pools which had all the attributes of a good fishing habitat. Approximately ten years ago, water was diverted from the Western Slope of the rockies through a tunnel to the North Fork of South Boulder Creek. The normal flow in that channel was increased by a factor of 4 to 5. The extra water caused significant bank erosion and channel degradation. In fact, the additional flow gutted the river valley, changing the channel to a straight raging torrent capable of carrying large quantities of sediment. Degradation in the system had reached as much as 15 to 20 feet before measures were taken to stabilize the creek.

Stabilization was achieved by flattening the gradient by constructing numerous drop structures and by reforming the banks with riprap. The system has stabilized but it is a different system. The channel is straight, much of the vegetation has been washed away, and the natural sequence of ripples and pools has been destroyed. The valley may never again have the natural form and beauty it once possessed. It is necessary for us to bear in mind that diversions to or from the natural river system can greatly alter its geometry, beauty and utility. The river may undergo a complete change, giving rise to a multitude of problems in connection with the design and maintenance of hydraulic structures, encroachments and bridge crossings along the affected reach.

In the preceding paragraphs possible immediate and long-term responses of river systems to various types of river development have been described. Nothing has been indicated about how to determine the magnitude of these changes. This important aspect of response of rivers to development will be treated more quantitatively in other chapters.

1.6.0 ENVIRONMENTAL CONSIDERATIONS

There is a 15 mile canyon above Glenwood Springs, Colorado, in which the Colorado River flows. The natural river was narrow and flowed within the canyon walls extending essentially down to the river banks. In the early 1900's, a two-lane highway was constructed on one side of the river and a railroad was constructed on the other side in this 15 mile reach. Because of the lack of floodplain, the space for the highway and railroad was developed by cutting into the valley wall and encroaching on the river channel. This development converted the river from one with a natural sequence of pools and ripples to essentially a straight man-made channel. This development greatly reduced the value of the river from the environmental point of view. On the other hand, the development did meet the transportation needs at that time.

Presently we have a four-lane divided highway both upstream and downstream of this canyon reach. Hence, the possibility of further encroachment on the river in the canyon to provide space for a four-lane highway has been investigated. This could be done but may further deteriorate the quality of the river. Environmental considerations have become the major issue in the development of this four-lane highway through this reach.

As the preservation of environmental quality has become a matter of national interest and priority, the decisions made on the location, design and construction of an encroachment should, as far as possible, avoid or minimize the adverse effects on the quality of the environment, including effects on scenic, natural, historical, archaeological, recreational and social values and resources of the project area. To do this, the engineer must have sufficient knowledge to recognize potential problems.

A general methodology can be put forth for bringing environmental impact into the decision process. *The first step* is to locate the several alternative areas where the bridge or highway could feasibly be located. These areas would be chosen to provide a choice of environmental considerations as well as a range of purely technical requirements. An inventory and analysis of the environmental resources of each site would then be prepared. The inventory would show the uniqueness of the area for supporting specific vegetation, wildlife and aquatic life, especially rare or endangered species. Relevant specialists need to be consulted to define the interdependencies and chains that exist between biological groups and species and to evaluate the response of the species to stress conditions. Existing and future possible uses of the area must be set down, whether it is urban residential, industrial, farm, recreation or preservation for natural scenic beauty. The area may possess a community unity that would be destroyed by highway development or it may possess unique qualities of significant historical or cultural importance. All these and more are environmental considerations to inventory.

The second step is to conduct an interdisciplinary study of the impact of the alternative plans on the environmental factors identified in the first step. The alternatives would ideally include not only different sites but alternative forms of construction that would differ in their environmental impact. The impact would not be all negative. Highways and bridges are means of access, and access to areas of exceptional recreational or scenic value can be a positive impact. The impact of scarring the landscape most often is negative, but a major highway cut west of Denver exposed colorful geological strata that now is a center of attraction for tourists, students and serious geologists.

The third step is to prepare preliminary plans and cost estimates for those alternatives that provide the best compromise between function cost and environmental impact. These plans would include rehabilitation plans to ameliorate the expected harmful impacts and to achieve the most productive and environmentally harmonious future use. The final choice then can be made based on an understanding of the impact of the crossing.

The specific considerations pertinent to bridge engineering are described in more detail in the following sections.

1.6.1 Site selection

While the site selection of an encroachment is usually closely dictated by the location planning of the proposed road, the encroachment must be sited with full knowledge of its environmental impact.

To minimize the environmental impact, the site should be located:

(1) Where satisfactory geological and soil conditions exist, as determined through extensive investigation.

(2) Where a minimum of scour or fill of hydraulic sediments are expected to occur at or near the crossing.

(3) Away from reaches of highly unstable channel.

(4) Where possible adverse effects on the other existing bridges and hydraulic structures can be avoided.

(5) Where it is possible to minimize the hazards from floods, landslides, tornadoes or hurricanes, tsunamis, avalanches, earthquakes or subsidence.

(6) Where river banks are stable.

(7) Where ecological impact is acceptable.

(8) Where aesthetic considerations are favorable.

1.6.2 Recreation, fish and wildlife

Preserving and enhancing the quality of recreation, fish and wildlife areas is a major goal of the current national environmental movement. Proximity to sensitive areas requires an understanding of the effect of the encroachment and attention to possible remedial measures.

The effects of construction activities on fish and wildlife should not be overlooked. Dredging, excavating, and storage of materials may disturb aquatic and wildlife areas and should be minimized. If disruption of natural habitat is necessary, it should be restored to its natural state as soon as possible.

1.6.3 Identification of the existing ecosystem

Pre-project consideration of the total ecosystem must be an integral part of the plan. Ecological studies are necessary to document the present characteristics of the environment, to estimate the effect of the construction and operation of the proposed bridge on the environment, and to provide the basis for selecting measures which minimize any projected adverse effects.

The measures taken to assure that ecological studies are adequate include:

(1) Identifying important and supportive biological species.

(2) Formulating the ecological studies, including data collection techniques.

(3) Guiding the ecological studies.

1.6.4 Construction effects

Although construction may be of short duration as compared to the operating life of the project, some changes during construction could have long-term damaging effects.

The impact of each of the construction activities on the environment must be assessed and measures should be planned and carried out to minimize such impact. Erosion control and other pollution control measures, the impact on area water supplies, and restoration of the landscape after completion of construction must be considered. The effects of construction on navigation, the biota, water quality, aesthetics, recreation, water supply, flood damage prevention, ecosystems and, in general, the needs and welfare of the people require careful attention.

The environment of the site during construction is of importance and should be considered. Specifically, adverse environmental and aesthetic impacts induced by the construction can be minimized by:

(1) Following natural topography to reduce construction scarring. Cutting, filling and clearing should usually be held to a minimum.

(2) Closely monitoring the use of mobile heavy equipment during the various construction phases so that damage to the environment can be minimized.

(3) Timing the clearing operation to minimize damage to critical areas.

(4) Properly draining rainwater from approach or access roads to prevent erosion.

(5) Controlling the air-emission pollution from all construction vehicles.

(6) Preventing pollution from the burning of waste, littering or disposing excess excavation material into water channels.

(7) Transporting construction equipment to and from the site without causing inconvenience to traffic flow or damage to the environment.

(8) Consulting with the land management agencies and complying with their requirements.

(9) Revegetating borrow and spoil areas as soon as practicable after disturbance.

1.7.0 TECHNICAL ASPECTS

Effects of river development, flood control measures and channel structures built during the last century have proven the need for taking into account delayed and far-reaching effects of any alteration man makes in a natural alluvial river system.

Because of the complexity of the processes occurring in natural flows and the erosion and deposition of material, an analytical approach to the problem is very difficult and time consuming. Most of our river process relations have been derived empirically. Nevertheless, if a greater understanding of the principles governing the processes of river formation is to be gained, the empirically derived relations must be put in the proper context by employing the analytical approach. In that way the distinct limitations of the empirical relations can be removed.

Mankind's attempts at controlling large rivers has often led to the situation described by J. Hoover Mackin (1937) when he wrote:

> "the engineer who alters natural equilibrium relations by diversion or damming or channel improvement measures will often find that he has the bull by the tail and is unable to let go - as he continues to correct or suppress undesirable phases of the chain reaction of the stream to the initial 'stress' he will necessarily place increasing emphasis on study of the genetic aspects of the equilibrium in order that he may work *with* rivers, rather than merely *on* them."

Through such experiences, man realizes that, to prevent or reduce the detrimental effects of any modification of the natural processes and state of equilibrium on a river, he must gain an understanding of the physical laws governing them, and become knowledgeable of the far-reaching effects of any attempt to control or modify a river's course.

1.7.1 Variables affecting river behavior

Variables affecting alluvial river channels are numerous and interrelated. Their nature is such that, unlike rigid boundary hydraulic problems, it is not possible to isolate and study the role of any individual variable.

Major factors affecting alluvial stream channel forms are:

(1) Stream discharge.
(2) Sediment load.
(3) Longitudinal slope.
(4) Bank and bed resistance to flow.
(5) Vegetation.
(6) Geology including types of sediments.
(7) Works of man.

The fluvial processes involved are very complicated and the variables of importance are difficult to isolate. Many laboratory and field studies have been carried out in an attempt to relate these and other variables to the present time. The problem has been more amenable to an empirical solution than an analytical one.

In an analysis of flow in alluvial rivers, the flow field is complicated by the constantly changing discharge. Significant variables are, therefore, quite difficult to relate mathematically. It is desirable to list measurable or computable variables which effectively describe the processes occurring and then to reduce the list by making simplifying assumptions and examining relative magnitudes of variables, striving toward an acceptable balance between accuracy and limitations of obtaining data. When this is done, the basic equations of fluid motion may be simplified (on the basis of valid assumptions) to describe the physical model.

It is the role of the succeeding chapters to present these variables, define them, show how they interrelate, quantify their interrelations where feasible, and show how they can be applied to achieve the successful design of river crossings and encroachments.

1.7.2 Basic knowledge required

In order for the engineer to cope successfully with river engineering problems, it is necessary that he have an adequate background in engineering with an emphasis on hydrology, hydraulics, erosion and sedimentation, river mechanics, soil mechanics, structures, economics, the environment and related subjects. In fact, as the public has demanded more comprehensive treatment of river development problems, the highway engineer should further improve his knowledge, and the application of it, by soliciting the cooperative efforts of the hydraulic engineer, hydrologist, geologist, geomorphologist, meteorologist, mathematician, statistician, computer programmer, systems engineer, soil physicist, soil chemist, biologist, water management staff, and economist. Professional organizations involving these talents should be encouraged to work cooperatively to achieve the long range research needs and goals relative to river development and application of knowledge on a national and international basis. Through an appropriate exchange of information between scientists working in these fields, opportunities to do a better job with all aspects of river development should be greatly enhanced.

1.7.3 Data requirements

Large amounts of data pertaining to understanding the behavior of rivers have been acquired over a long period of time. Nevertheless, data collection efforts are sporadic. Agencies should take a careful look at present data requirements needed to solve practical problems along with existing data. Perhaps a careful analysis of data requirements would make it possible to more efficiently utilize funds to collect data in the future. The basic type of information that is required includes: water discharge as a function of time, sediment discharge as a function of time, the characteristics of the sediments being transported by streams, the characteristics of the channels in which the water and sediment are transported, and the characteristics of watersheds and how they deliver water and sediment to the stream systems. Environmental data is also needed so that proper assessment can be made of the impact of river development upon the environment and vice versa. The problem of data requirements at river crossings is of sufficient importance that it is treated in greater detail in subsequent chapters.

1.8.0 FUTURE TECHNICAL TRENDS

When considering the future, it is essential to recognize the present state of knowledge pertaining to river hydraulics and then identify inadequacies in existing theories and encourage further research to help correct these deficits of knowledge. In order to correct such deficits there is need to take a careful look at existing data pertaining to rivers, future data requirements, research needs, training programs and methods of developing staff that can apply this knowledge to the solution of practical problems.

1.8.1 Adequacy of current knowledge

The basic principles of fluid mechanics involving application of continuity, momentum and energy concepts are well known and can be effectively applied to a wide variety of river problems. Considerable work has been done on the hydraulics of rigid boundary open channels and excellent results can be expected. The steady-state sediment transport of nearly uniform sizes of sediment in alluvial channels is well understood. There is good understanding of stable channel theory in noncohesive materials of all sizes. The theory is adequate to enable us to design stable systems in the existing material or, if necessary, designs can be made for appropriate types of stabilization treatments for canals and rivers to have them behave in a stable manner. A good understanding of plane bed fluvial hydraulics exists. There have been extensive studies of the fall velocity of noncohesive sediments in static fluids to provide knowledge about the interaction between the particle and fluid so essential to the development of sediment transport theories.

In conclusion, *available concepts and theories which can be applied to the behavior of rivers are extensive.* However, in many instances only empirical relationships have been developed and these are pertinent to specific problems only. Consequently, a more basic theoretical understanding of flow in the river systems needs to be developed.

With respect to many aspects of river mechanics, it can be concluded that knowledge is available to cope with the majority of river problems. On the other hand, *the number of individuals who are cognizant of existing theory and can apply it successfully to the solution of river problems is limited.* Particularly, the number of individuals involved in the actual solution of applied river mechanics problems is very small. There is a specific reason for this deficit of trained personnel. Undergraduate engineering educators in the universities in the United States, and in the world for that matter, devote only a small amount of time to teaching hydrology, river mechanics, channel stabilization, fluvial geomorphology, and related problems. It is not possible to obtain adequate training in these important topics except at the graduate level, and only a limited number of universities and institutions offer the required training in these subject areas. A great need at this time (1974) is to adequately train people to cope with river problems.

1.8.2 Research needs

As knowledge of river hydraulics is reviewed, it becomes quite obvious that many things are not adequately known. *Research needs are particularly urgent and promise a rather quick return.* The classification of rivers needs additional research. Different kinds of rivers should be studied separately because the factors governing their behavior may not be the same. Stabilization of rivers and bank stability of river systems needs further consideration. Also, the study of bed forms generated by the interaction between the water and sediment in the river systems deserves further study. The types of bed forms have been identified but theories pertaining to their development are inadequate. Simple terms have been used to describe the characteristics of alluvial material of both cohesive and noncohesive types; a comprehensive look at the characteristics of materials is warranted.

Other important research problems include the fluid mechanics of the motion of particles, secondary currents, two-dimensional velocity distributions, fall velocity of particles in turbulent flow and the application of remote sensing techniques to hydrology and river mechanics. The physical modeling of rivers followed by prototype verification, mathematical modeling of river response followed by field verification, mathematical modeling of water and sediment yield from small watersheds and studies of unsteady sediment transport are areas in which significant advances can be made.

Operational research on decision making considering, cost and risk criteria to determine the hydrologic and hydraulic design of highway structures and project alternatives, is another pressing research area. Insufficient data is frequently a problem of river mechanics analysis. A comprehensive study on information theory is needed to cope with such difficulties.

Finally the results of these efforts must be presented in such a form that it can be easily taught and easily put to practical use.

1.8.3 Training

It has been pointed out that engineering training is somewhat inadequate in relation to understanding the developments of rivers. There is need to consider better ways to train engineers to disseminate existing knowledge in this important area. The training of individuals could be accomplished by conducting seminars, conferences or short courses in institutions in the spirit of continuing education. There should be an effort to improve the curriculum of university education made available to engineers particularly at the undergraduate level. At the very minimum such a curriculum should strive to introduce concepts of fluvial geomorphology, river hydraulics, erosion and sedimentation, environmental considerations and related topics.

Manuals, handbooks and reference documents should be prepared for practicing engineers in order to overcome the deficit of knowledge. Publications of material pertaining to rivers should be encouraged. This material can be and is being published to some degree in the proceedings of conferences, in journals and textbooks. Better use of informative films could be made, a technique that would be of assistance in teaching effectively, efficiently and economically. Similarly, television and video tapes can be economically prepared and utilized in instructional situations. Television cameras are available that enable the teacher to record and take field situations directly into the classroom for class consideration.

Formal training should be supported with field trips and laboratory demonstrations. Laboratory demonstrations are an inexpensive method of quickly and effectively teaching the fundamentals of river mechanics and illustrating the behavior of structures. These demonstrations should be followed by field trips to illustrate similarities and differences between phenomena in the laboratory and in the field.

Finally, larger numbers of disciplines should be involved in the training programs. Cooperative studies should involve research personnel, practicing engineers and people from the many different disciplines with an interest in rivers.

1.9.0 APPLICATION

River problems are complex and important. Furthermore, it has been illustrated that in many instances inadequate knowledge exists to appropriately cope with these problems. In the following chapters of this manual there is an effort to present the current state of knowledge on rivers and river crossings using fundamentals of fluid mechanics, geomorphology, hydraulics, and river mechanics. The final chapter is devoted to application of theories, concepts and techniques presented in this manual to the solution of practical river crossings and encroachments by highways.

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Chapter II

OPEN CHANNEL FLOW

2.1.0 INTRODUCTION

In this chapter the fundamentals of rigid boundary open channel flow are described. In open channel flow, the water surface is not confined; surface configuration, flow pattern and pressure distribution within the flow depend on gravity. In rigid boundary open channel flow, no deformations or movements of the bed and banks are considered. Mobile boundary hydraulics is discussed in Chapters III, IV and V. In this chapter, we restrict ourselves to one-dimensional analysis: that is, the direction of velocity and acceleration are large only in one direction and are so small as to be negligible in all other directions.

Open channel flow can be classified as: (1) uniform or nonuniform flow, (2) steady or unsteady flow, (3) laminar or turbulent flow, and (4) tranquil or rapid flow. *In uniform flow, the depth and discharge remain constant with respect to space.* Also, the velocity at a given depth is the same everywhere. *In steady flow, no change occurs with respect to time.* In *laminar flow*, the flow field can be characterized by layers of fluid, one layer not mixing with adjacent ones. *Turbulent flow* on the other hand is characterized by random fluid motion. *Tranquil flow* is distinguished from *rapid flow* by a dimensionless number called the Froude number, Fr. *If $Fr < 1$, the flow is tranquil; if $Fr > 1$, the flow is rapid, and if $Fr = 1$, the flow is called critical.*

Open channel flow can be nonuniform, unsteady, turbulent and rapid at the same time. Because the classifying characteristics are independent, sixteen different types of flow can occur. These terms, uniform or nonuniform, steady or unsteady, laminar or turbulent, rapid or tranquil, and the two dimensionless numbers (the Froude number and Reynolds number) are more fully explained in the following sections.

2.1.1 Definitions

Velocity: The velocity of a fluid particle is the time rate of displacement of the particle from one point to another. Velocity is a vector quantity. That is, it has magnitude and direction. In cartesian coordinates, the mathematical representation of the fluid velocity is

$$V = \frac{ds}{dt} = \frac{\partial x}{\partial t} + \frac{\partial y}{\partial t} + \frac{\partial z}{\partial t} \tag{2.1.1}$$

Streamline: An imaginary line within the flow which is everywhere tangent to the velocity vector is called a streamline.

Acceleration: Acceleration is the time rate of change of the velocity vector, either of magnitude or direction or both. Mathematically, acceleration is expressed by the total derivative of the velocity vector or

$$\frac{DV}{Dt} = \frac{dv_s}{dt} + \frac{dv_n}{dt} = \frac{\partial v_s}{\partial t} + \frac{1}{2}\frac{\partial (v_s^2)}{\partial s} + \frac{\partial v_n}{\partial t} + \frac{1}{2}\frac{\partial (v_n^2)}{\partial n} \tag{2.1.2}$$

where the subscript s is along the streamline and n refers to the direction normal to the streamline. *The tangential acceleration* component is

$$a_s = \frac{\partial v_s}{\partial t} + \frac{1}{2}\frac{\partial (v_s^2)}{\partial s} \tag{2.1.3}$$

and the *normal acceleration* component is

$$a_n = \frac{\partial v_n}{\partial t} + \frac{1}{2}\frac{\partial}{\partial n}(v_n^2) \tag{2.1.4}$$

The first terms in Eq. 2.1.3 and 2.1.4 are the change in velocity, both magnitude and direction, with time at a point. This is called the *local acceleration*. The second term in each equation is the change in velocity, both magnitude and direction, with distance. This is called *convective acceleration*.

Uniform flow: In uniform flow, there is no change in velocity along a streamline with distance; that is,

$$\frac{\partial v_s}{\partial s} = 0$$

and

$$\frac{\partial v_n}{\partial n} = 0$$

Nonuniform flow: In nonuniform flow, velocity varies with position so

$$\frac{\partial v_s}{\partial s} \neq 0$$

and

$$\frac{\partial v_n}{\partial n} \neq 0$$

Flow around a bend ($\partial v_n/\partial n \neq 0$) and flow in expansions or contractions ($\partial v_s/\partial s \neq 0$) are examples of nonuniform flow.

Steady flow: In steady flow, the velocity at a point does not change with time; that is,

$$\frac{\partial v_s}{\partial t} = 0$$

and

$$\frac{\partial v_n}{\partial t} = 0$$

Unsteady flow: In unsteady flow, the velocity at a point varies with time so

$$\frac{\partial v_s}{\partial t} \neq 0$$

and

$$\frac{\partial v_n}{\partial t} \neq 0$$

Examples of unsteady flow are channel flows with waves, flood hydrographs, and surges. Unsteady flow is difficult to analyze unless the time changes are small.

Laminar flow: In laminar flow, the mixing of the fluid and momentum transfer is by molecular activity.

Turbulent flow: In turbulent flow the mixing of the fluid and momentum transfer is by random fluctuations of finite "lumps" of fluid.

The flow is laminar or turbulent depending on the value of the Reynolds number ($Re = \frac{\rho VL}{\mu}$), which is a dimensionless ratio of the inertial forces to the viscous forces. Here ρ and μ are the density and dynamic viscosity of the fluid, V is the fluid velocity and L is a characteristic dimension, usually the depth in open channel flow. In laminar flow, viscous forces are dominant and Re is relatively small. In turbulent flow, Re is large; that is, inertial forces are very much greater than viscous forces.

In turbulent flow over a *hydraulically smooth boundary* (see Section 2.3.2), viscous forces near the boundary are the dominant resistance to flow. With a *hydraulically rough boundary*, form drag is more significant than viscous drag and is the dominant reason for resistance to flow. Between these two types of roughnesses there is an intermediate condition where viscosity and form drag affect the flow.

Turbulent flows are predominant in nature. Laminar flow occurs very infrequently in open channel flow.

Tranquil flow: In open channel flow, the flow pattern, surface configuration, depth, and changes in these quantities in response to changes in channel geometry depend on the Froude number ($Fr = V/\sqrt{gL}$), which is the ratio of inertia forces to gravitational forces. The Froude number is also the ratio of the flow velocity to the velocity of a small gravity wave in the flow. When $Fr < 1$, the flow is tranquil, and surface waves (with velocity $\sqrt{gL}$) propagate upstream as well as downstream. Control of tranquil flow depth is always downstream.

Rapid flow: When $Fr > 1$, the flow is rapid and surface disturbances can propagate only in the downstream direction. Control of rapid flow depth is always at the upstream end of the rapid flow region. When $Fr = 1.0$, the flow is critical and surface disturbances remain stationary in the flow.

2.2.0 BASIC PRINCIPLES

2.2.1 Introduction

The basic equations of flow in open channels are derived from the three conservation laws. These are: (1) *the conservation of mass*, (2) *the conservation of linear momentum, and* (3) *the conservation of energy*. The conservation of mass is another way of stating that (except for mass-energy interchange) matter can neither be created nor destroyed. The principle of conservation of linear momentum is based on Newton's second law of motion which states that a mass (of fluid) accelerates in the direction of and in proportion to the applied forces on the mass.

In the analysis of flow problems, much simplification can result if there is no acceleration of the flow or if *the acceleration is primarily in one direction*, the accelerations in other directions being negligible. However, a very inaccurate analysis may occur if one assumes accelerations are small or zero when in fact they are not. The developments in this manual assume *one-dimensional flow* and the derivations of the equations utilize a control volume. *A control volume is an isolated volume in the body of the fluid, through which mass, momentum, and energy can be convected.* The control volume may be assumed fixed in space or moving with the fluid.

2.2.2 Conservation of mass

Consider a short reach of river shown in Fig. 2.2.1 as a control volume. The boundaries of the control volume are the upstream cross section, designated section 1, the downstream cross section, designated section 2, the free surface of the water between sections 1 and 2, and the interface between the water and the banks and bed.

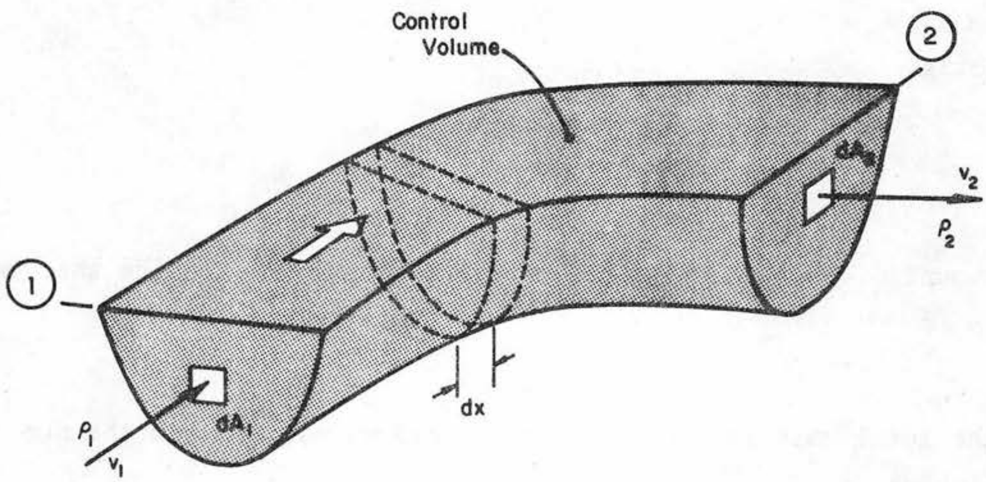


Fig. 2.2.1 A river reach as a control volume.

The statement of the conservation of mass for this control volume

Mass flux out of the control volume	-	Mass flux into the control volume	+	Time rate of change in mass in the control volume	= 0

Mass can enter or leave the control volume through any or all of the control volume surfaces. Rainfall would contribute mass through the surface of the control volume and seepage passes through the interface between the water and the banks and bed. In the absence of rainfall, evaporation, seepage and other lateral mass fluxes, mass enters the control volume at section 1 and leaves at section 2.

At section 2, the mass flux out of the control volume through the differential area dA_2 is $\rho_2 v_2\ dA_2$. The values of ρ_2 and v_2 can vary from position to position across the width and throughout the depth of flow at section 2. The total mass flux out of the control volume at section 2 is the sum of all $\rho_2 v_2\ dA_2$ through the differential areas that make up the cross-section area A_2, and may be written as

$$\sum_{A_2} \rho_2 v_2 dA$$

Therefore

$$\begin{array}{l}\text{Mass flux}\\ \text{out of the}\\ \text{control volume}\end{array} = \sum_{A_2} \rho_2 v_2\ dA_2$$

Similarly

$$\begin{array}{l}\text{Mass flux}\\ \text{into the}\\ \text{control volume}\end{array} = \sum_{A_1} \rho_1 v_1\ dA_1$$

The amount of mass inside a differential volume $d\forall$ inside the control volume is

$$\rho\ d\forall$$

and the total mass inside the control volume $\forall$ is then the sum of the mass inside or

$$\begin{array}{l}\text{Mass inside}\\ \text{the}\\ \text{control volume}\end{array} = \sum_{\forall} \rho\ d\forall$$

The statement of conservation of mass for the control volume calls for the time rate of change in mass. In mathematical notation, the rate of change is

$$\frac{\partial}{\partial t} \{\sum_{V} \rho \; dV\}$$

so that

$$\begin{array}{l}\text{Time rate of change} \\ \text{in mass in the} \\ \text{control volume}\end{array} = \frac{\partial}{\partial t} \{\sum_{V} \rho \; dV\}$$

For the reach of river, the statement of the conservation of mass becomes

$$\sum_{A_2} \rho_2 v_2 \; dA_2 - \sum_{A_1} \rho_1 v_1 \; dA_1 + \frac{\partial}{\partial t} \{\sum_{V} \rho \; dV\} = 0 \qquad 2.2.1$$

It is often convenient to work with average conditions at a cross section so we define an average velocity V such that

$$V = \frac{1}{A} \sum_{A} v \; dA \qquad 2.2.2$$

or in integral form,

$$V = \frac{1}{A} \int_{A} v \; dA \qquad 2.2.3$$

The velocity V is the *average velocity* at the cross section.

If the density of the fluid does not change from position to position in a cross section or in the reach, $\rho_1 = \rho_2 = \rho$. When the flow is steady

$$\frac{\partial}{\partial t} \{\sum_{V} \rho \; dV\} = 0$$

and Eq. 2.2.1 reduces to the statement that inflow equals outflow or

$$\rho V_2 A_2 - \rho V_1 A_1 = 0$$

That is,

$$V_1A_1 = V_2A_2 = Q \qquad 2.2.4$$

where Q is the volume flow rate or the discharge.

Eq. 2.2.4 is the familiar form of the conservation of mass equation for river flows. It is applicable when the fluid density is constant, the flow is steady and there is no lateral inflows or seepage.

2.2.3 Conservation of linear momentum

The curved reach of river shown in Fig. 2.2.1 is rather complex to analyze in terms of Newton's Second Law because of the curvature in the flow. Therefore, as a starting point, the differential length of reach dx is isolated as a control volume.

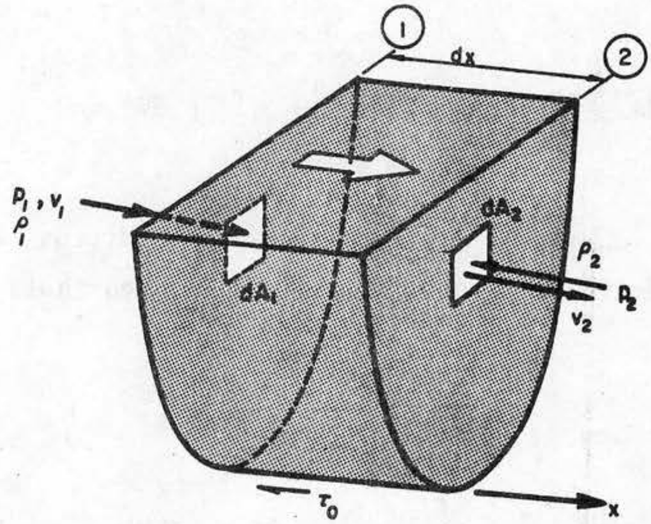


Fig. 2.2.2 The control volume for conservation of linear momentum.

For this control volume shown in Fig. 2.2.2, the statement of conservation of linear momentum is

Flux of momentum out of the control volume	-	Flux of momentum into the control volume

+	Time rate of change of momentum in the control volume	=	Sum of the forces acting on the fluid in the control volume

The terms in the statement are vectors so we must be concerned with direction as well as magnitude.

Consider the conservation of momentum in the direction of flow (the x-direction in Fig. 2.2.2). At the outflow section (section 2), the flux of momentum out of the control volume through the differential area dA_2 is

$$\rho_2 v_2 \, dA_2 \, v_2$$

Here $\rho_2 \, v_2 \, dA_2$ is the mass flux (mass per unit of time) and $\rho_2 v_2 \, dA_2 \, v_2$ is the momentum flux through the area dA_2. The total momentum flux out through section 2 is

$$\sum_{A_2} \rho_2 v_2 \, dA_2 \, v_2$$

so

$$\begin{array}{l} \text{Flux of momentum} \\ \text{out of the control} \\ \text{volume} \end{array} = \sum_{A_2} \rho_2 v_2 \, dA_2 \, v_2$$

Similarly, at the inflow section (section 1)

$$\begin{array}{l} \text{Flux of momentum} \\ \text{into the} \\ \text{control volume} \end{array} = \sum_{A_1} \rho_1 v_1 \, dA_1 \, v_1$$

The amount of momentum in the control volume is

$$\sum_{V} \rho v \, dV$$

so

$$\begin{array}{l} \text{Time rate of} \\ \text{change of momentum} \\ \text{in the control volume} \end{array} = \frac{\partial}{\partial t} \, \{\sum_{V} \rho v \, dV\}$$

At the upstream section, the force acting on the differential area dA_1 of the control volume is

$$p_1 dA_1$$

where p_1 is the pressure from the upstream fluid on the differential area.

The total force in the x-direction at section 1 is

$$\sum_{A_1} p_1 \, dA_1$$

Similarly, at section 2, the total force is

$$-\sum_{A_2} p_2 \, dA_2$$

There is a fluid shear stress acting along the interface between the water and the bed and banks. The shear on the control volume is in a direction opposite to the direction of flow and results in a force

$$-\tau_o P \, dx$$

where τ_o is the average shear stress on the interface area, P is the average wetted perimeter and dx is the length of the control volume. The term $P\,dx$ is the interface area.

If the x-direction is normal to the direction of gravity, the statement of conservation of momentum in the x-direction for the control volume is

$$\sum_{A_2} \rho_2 v_2 v_2 \, dA_2 - \sum_{A_1} \rho_1 v_1 v_1 \, dA_1 + \frac{\partial}{\partial t} \{\sum_{\forall} \rho v \, d\forall\}$$

$$= \sum_{A_1} p_1 dA_1 - \sum_{A_2} p_2 dA_2 - \tau_o P \, dx \qquad 2.2.5$$

In the limit, the summations can be replaced with integrals so that Eq. 2.2.5 becomes

$$\int_{A_2} \rho_2 \, v_2^2 \, dA_2 - \int_{A_2} \rho_1 v_1^2 \, dA_1 + \frac{\partial}{\partial t} \{\int_{\forall} \rho v \, d\forall\}$$

$$= \int_{A_1} p_1 \, dA_1 - \int_{A_2} p_2 \, dA_2 - \tau_o P \, dx \qquad 2.2.6$$

which is the integral form of the momentum equation.

Again, as with the conservation of mass equation, it is convenient to use average velocities instead of point velocities. We define a momentum coefficient β so that when average velocities are used instead of point velocities, the correct momentum flux is considered.

The momentum coefficient is

$$\beta = \frac{1}{\rho V^2 A} \int_A \rho v^2 \, dA \qquad 2.2.7$$

which reduces to

$$\beta = \frac{1}{V^2 A} \int_A v^2 \, dA \qquad 2.2.8$$

if there is no variation in fluid density at a cross section. By assuming $\rho_1 = \rho_2 = \rho$, Eq. 2.2.6 is reduced to

$$\rho\beta_2 V_2^2 A_2 - \rho\beta_1 V_1^2 A_1 + \rho \frac{\partial}{\partial t} \{\int_{V\!\!\!\!-} v \, dV\!\!\!\!-\}$$

$$= \int_{A_1} p_1 \, dA_1 - \int_{A_2} p_2 \, dA_2 - \tau_o P \, dx \qquad 2.2.9$$

If the flow is steady

$$\frac{\partial}{\partial t} \{ \int_{V\!\!\!\!-} v \, dV\!\!\!\!-\} = 0 \qquad 2.2.10$$

The pressure force and shear force terms on the right-hand side of Eq. 2.2.10 are usually abbreviated as $\sum F_x$ or

$$\sum F_x = \int_{A_1} p_1 \, dA_1 - \int_{A_2} p_2 \, dA_2 - \tau_o P \, dx \qquad 2.2.11$$

Then, for steady flow of constant density fluid, the conservation of momentum equation becomes

$$\rho\beta_2 V_2^2 A_2 - \rho\beta_1 V_1^2 A_1 = \sum F_x \qquad 2.2.12$$

For steady flow with constant density, the conservation of mass equation (Eq. 2.2.4) was

$$V_1 A_1 = V_2 A_2 = Q$$

With this expression, *the steady flow conservation of linear momentum equation takes on the familiar form*

$$\rho Q \, (\beta_2 V_2 - \beta_1 V_1) = \sum F_x \qquad 2.2.13$$

2.2.4 Conservation of energy

The First Law of Thermodynamics can be written

$$\dot{Q} - \dot{W} = \frac{dE}{dt} \qquad 2.2.14$$

where $\dot{Q}$ = the rate at which heat is added to a fluid system

$\dot{W}$ = the rate at which a fluid system does work on its surroundings

E = the energy of the system

Then dE/dt is the rate of change of energy in the system.

The statement of conservation of energy for a control volume is then

$$\begin{pmatrix}\text{Flux of energy} \\ \text{out of the} \\ \text{control volume}\end{pmatrix} - \begin{pmatrix}\text{Flux of energy} \\ \text{into the} \\ \text{control volume}\end{pmatrix} + \begin{pmatrix}\text{Time rate of} \\ \text{change of energy} \\ \text{in the control volume}\end{pmatrix} = \dot{Q} - \dot{W}$$

The choice of a control volume is arbitrary. Because of the complexities resulting from having to integrate over the cross-sectional area, the control volume which includes the entire cross section of the river is inconvenient. Therefore, the control volume is reduced to the size of a streamtube connecting dA_1 and dA_2 as shown in Fig. 2.2.3. The streamtube is bounded by streamlines through which no mass or momentum enters.

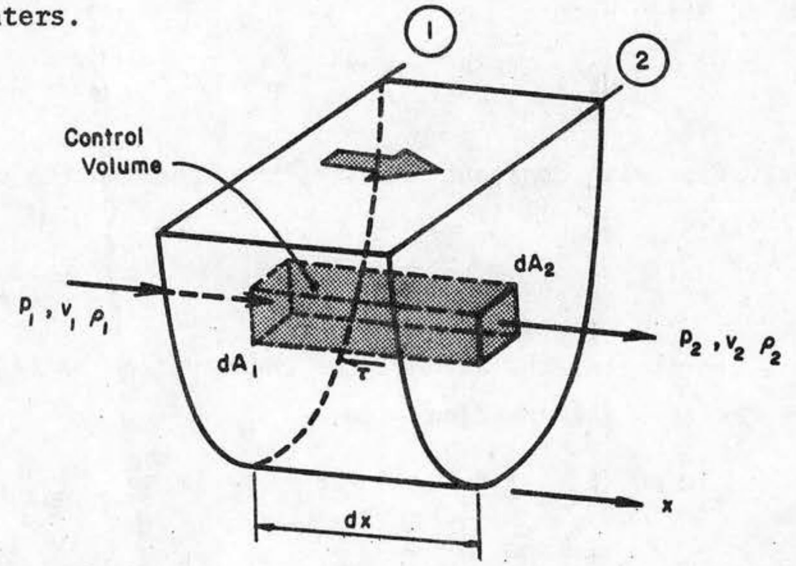


Fig. 2.2.3 The streamtube as a control volume.

For steady constant density flow in the streamtube

$$\text{Flux of energy out of the control volume} = \rho_2 e_2 \, dA_2 \, v_2$$

and

$$\text{Flux of energy into the control volume} = \rho_1 e_1 \, dA_1 \, v_1$$

Here e is the energy per unit of mass. Accordingly, the total energy in a control volume of size V is

$$E = \int_V \rho e \, dV \qquad 2.2.15$$

Because the flow is steady

$$\text{Time rate of change of energy in the control volume} = 0$$

Unless one is concerned with thermal pollution, evaporation losses, or problems concerning the formation of ice in rivers, the rate at which heat is added to the control volume can be neglected; that is,

$$\dot{Q} \simeq 0 \qquad 2.2.16$$

The work done by the fluid in the control volume on its surroundings can be in the form of pressure work W_p, shear work W_τ, or shaft work (mechanical work) W_s. For the streamtube shown in Fig. 2.2.3, no shaft work is involved.

The rate at which the fluid pressure does work on the control volume boundary dA_1 in Fig. 2.2.3 is

$$-p_1 \, dA_1 \, v_1$$

and on boundary dA_2, the rate of doing pressure work is

$$p_2 \, dA_2 \, v_2$$

At the other boundaries of the streamtube, there is no pressure work because there is no fluid motion normal to the boundary. Hence, for the streamtube

$$\dot{W}_p = p_2\, dA_2\, v_2 - p_1\, dA_1\, v_1 \qquad 2.2.17$$

Along the interior boundaries of the streamtube there is a shear stress resulting from the condition that the fluid velocity inside the streamtube may not be the same as the velocity of the fluid surrounding the streamtube. The rate at which the fluid in the streamtube does shear work on the control volume is

$$\dot{W}_\tau = \tau P\, dx\, v \qquad 2.2.18$$

where τ is the average shear stress on the streamtube boundary, P is the average perimeter of the streamtube, dx is the length of the streamtube and V is the fluid velocity at the streamtube boundary. The product $P\, dx$ is the surface of the streamtube subjected to shear stresses.

Then for steady flow in the streamtube, the statement of the conservation of energy in the streamtube shown in Fig. 2.2.3 is

$$\rho_2 e_2 v_2\, dA_2 - \rho_1 e_1 v_1\, dA_1 = p_1 v_1 dA_1 - p_2 v_2\, dA_2 - \tau P v\, dx \qquad 2.2.19$$

If the density is constant in the streamtube, the conservation of mass for the streamtube is (according to Section 2.2.2)

$$v_2\, dA_2 = v_1\, dA_1 = dQ$$

and

$$\rho_2 = \rho_1 = \rho\ . \qquad 2.2.20$$

Now Eq. 2.2.19 reduces to

$$(\rho e_1 + p_1)dQ - (\rho e_2 + p_2)dQ = \tau P v\, dx \qquad 2.2.21$$

The energy per unit mass e is the sum of the internal, kinetic and potential energies or

$$e = u + \frac{v^2}{2} + gz \qquad 2.2.22$$

where u = the internal energy associated with the fluid temperature

v = the velocity of the mass of fluid

g = the acceleration due to gravity

z = the elevation above some arbitrary reference level.

This expression for e is substituted in Eq. 2.2.21 to yield

$$u_1 + \frac{v_1^2}{2} + gz_1 + \frac{p_1}{\rho} = u_2 + \frac{v_2^2}{2} + gz_2 + \frac{p_2}{\rho} + \frac{\tau P v dx}{\rho dQ} \qquad 2.2.23$$

By dividing through by g and calling the term

$$\frac{u_2 - u_1}{g} + \frac{\tau P v dx}{\rho g dQ}$$

the head loss h_ℓ, the energy equation for the streamtube becomes

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_\ell \qquad 2.2.24$$

If there is no shear stress on the streamtube boundary and if there is no change in internal energy ($u_1 = u_2$), *the energy equation reduces to*

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 \qquad 2.2.25$$

which is *the Bernoulli Equation.*

Generally, there is not sufficient information available to do a differential streamtube analysis of a reach of river so appropriate changes must be made in the energy equation. A reach of river such as that shown in Fig. 2.2.1 can be pictured as a bundle of streamtubes. We know the statement of the conservation of energy for a streamtube. It is Eq. 2.2.24 which can be rewritten

$$\left(\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1\right) v_1\, dA_1$$

$$= \left(\frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2\right) v_2\, dA_2 + h_\ell\, vdA \qquad 2.2.26$$

because $\qquad v_1\, dA_1 = v_2\, dA_2 = vdA$

for the streamtube. By summing the energies in all the streamtubes making up the reach of river, we can write

$$\frac{\alpha_1 V_1^2}{2g} + \frac{\bar{p}_1}{\gamma} + \bar{z}_1 = \frac{\alpha_2 V_2^2}{2g} + \frac{\bar{p}_2}{\gamma} + \bar{z}_2 + H_L \qquad 2.2.27$$

Eq. 2.2.27 is the common form of the energy equation used in open channel flow. It is derived from Eq. 2.2.26 by integrating Eq. 2.2.26 over the cross-section area; that is

$$\int_A \left(\frac{v^2}{2g} + \frac{p}{\gamma} + z\right) vdA = \left\{ \frac{\alpha V^2}{2g} + \frac{\bar{p}}{\gamma} + \bar{z}\right\} Q \qquad 2.2.28$$

where α is the *kinetic energy correction factor* defined by the expression

$$\alpha = \frac{1}{V^3 A} \int_A v^3 \, dA \qquad 2.2.29$$

to allow the use of average velocity V rather than point velocity v. The average pressure over the cross section is $\bar{p}$, defined as

$$\bar{p} = \frac{1}{VA} \int_A p \, vdA \qquad 2.2.30$$

The term $\bar{z}$ is the average elevation of the cross section defined by the expression

$$\bar{z} = \frac{1}{VA} \int_A z \, vdA \qquad 2.2.31$$

and Q is the volume flow rate or the discharge. By definition

$$Q = \int_A vdA \qquad 2.2.32$$

Also

$$H_L = \frac{1}{VA} \int_A h_\ell \, vdA \qquad 2.2.33$$

In summary, *the expression for conservation of energy for steady flow in a reach of river is* written

$$\frac{\alpha_1 V_1^2}{2g} + \frac{\bar{p}_1}{\gamma} + \bar{z}_1 = \frac{\alpha_2 V_2^2}{2g} + \frac{\bar{p}_2}{\gamma} + \bar{z}_2 + H_L \qquad 2.2.34$$

The tendency in river work is to neglect the energy correction factor even though its value may be as large as 1.5. Usually it is assumed that the pressure is hydrostatic and the average elevation head $\bar{z}$ is at the centroid of the cross-sectional area. However, it should be kept in mind that Eqs. 2.2.29, 2.2.30 and 2.2.31 are the correct definitions of the terms in the energy equation.

2.2.5 Hydrostatics

When the only forces acting on the fluid are pressure and fluid weight, the differential equation of motion in an arbitrary direction x is

$$-\frac{\partial}{\partial x}\left(\frac{p}{\gamma} + z\right) = \frac{a_x}{g} \qquad 2.2.35$$

In steady uniform flow (and for zero flow), the acceleration is zero and we obtain the *equation of hydrostatics*

$$\frac{p}{\gamma} + z = \text{Constant} \qquad 2.2.36$$

However, when there is acceleration, the piezometric head varies in the flow. That is, the piezometric head is not constant in the flow. This is illustrated in Fig. 2.2.4. In Fig. 2.2.4a the pressure at the bed is equal to γy_o whereas in the curvilinear flow (Fig. 2.2.4b) the pressure is larger than γy_o because of the acceleration resulting from a change in direction.

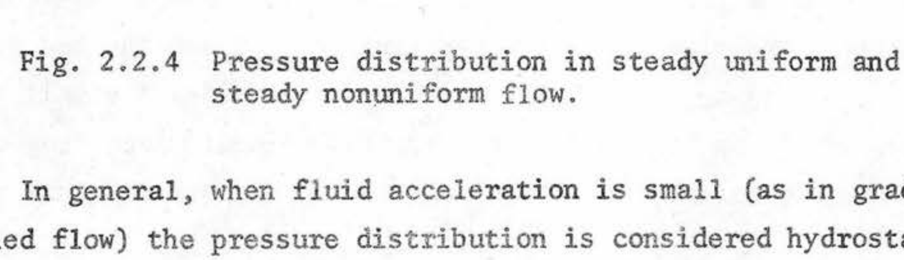


Fig. 2.2.4 Pressure distribution in steady uniform and in steady nonuniform flow.

In general, when fluid acceleration is small (as in gradually varied flow) the pressure distribution is considered hydrostatic. However, for rapidly varying flow where the streamlines are converging, expanding or have substantial curvature (curvlinear flow), fluid accelerations are not small and the pressure distribution is not hydrostatic.

In Eq. 2.2.36, the constant is equal to zero for gage pressure at the free surface of a liquid, and for flow with hydrostatic pressure throughout (steady, uniform flow or gradually varied flow) it follows that the pressure head p/γ is equal to the vertical distance below the free surface. In sloping channels with steady uniform flow, the pressure head p/γ at a depth y below the surface is equal to

$$\frac{p}{\gamma} = y \cos\theta \tag{2.2.37}$$

Note that y is the depth (perpendicular to the water surface) to the point, as shown in Fig. 2.2.5. For most channels, θ is small and $\cos\theta \simeq 1$.

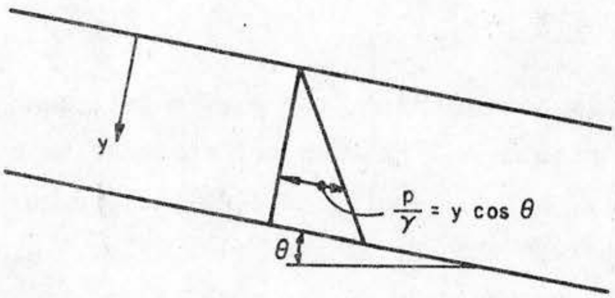


Fig. 2.2.5 Pressure distribution in steady uniform flow on large slopes.

2.3.0 STEADY UNIFORM FLOW

2.3.1 Introduction

In steady, uniform open channel flow there are no accelerations, streamlines are straight and parallel, and the pressure distribution is hydrostatic. The slope of the water surface S_w and the bed surface S_o and the energy gradient S_f are equal. Consider the unit width of channel shown in Fig. 2.3.1 as a control volume. According to Eq. 2.2.34, the conservation of energy for this control volume is

$$\frac{\alpha_1 V_1^2}{2g} + \frac{\bar{p}_1}{\gamma} + \bar{z}_1 = \frac{\alpha_2 V_2^2}{2g} + \frac{\bar{p}_2}{\gamma} + \bar{z}_2 + H_L$$

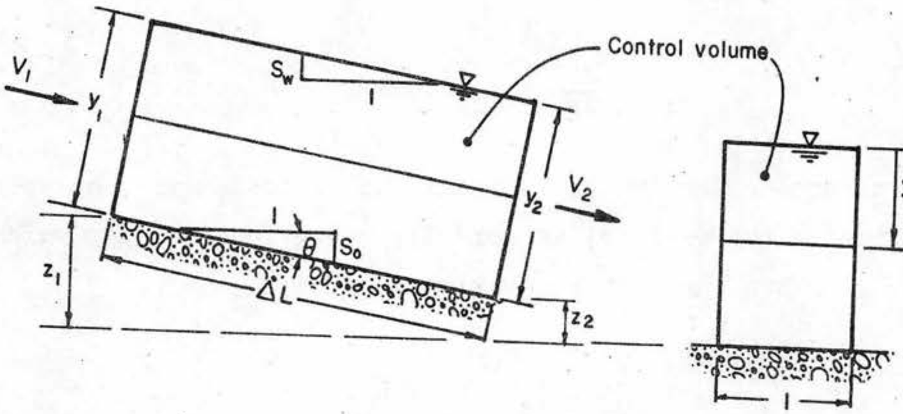


Fig. 2.3.1 Steady uniform flow in a unit width channel.

The pressure at any point y below the surface is y cosθ. Then according to Eq. 2.2.30

$$\overline{p}_1 = \frac{1}{V_1 y_1} \int_o^{y_1} \gamma y \cos\theta \; v_1 \, dy$$

Assuming only small variations in the point velocity v with y,

$$\overline{p}_1 \simeq \frac{\gamma y_1 \cos\theta}{2}$$

Similarly

$$\overline{p}_2 \simeq \frac{\alpha y_2 \cos\theta}{2}$$

Also according to Eq. 2.2.31

$$\overline{z}_1 \simeq z_1 + \frac{y_1 \cos\theta}{2}$$

and

$$\overline{z}_2 \simeq z_2 + \frac{y_2 \cos\theta}{2}$$

With the above expressions for $\overline{p}_1$, $\overline{p}_2$, $\overline{z}_1$, and $\overline{z}_2$ the energy equation for this control volume reduces to

$$\frac{\alpha_1 V_1^2}{2g} + \frac{y_1 \cos\theta}{2} + z_1 + \frac{y_1 \cos\theta}{2} = \frac{\alpha_2 V_2^2}{2g} + \frac{y_2 \cos\theta}{2} + z_2 + \frac{y_2 \cos\theta}{2} + H_L$$

or

$$\frac{\alpha_1 V_1^2}{2g} + y_1 \cos\theta + z_1 = \frac{\alpha_2 V_2^2}{2g} + y_2 \cos\theta + z_2 + H_L$$

For most natural channels θ is small and $y \cos\theta \simeq y$. The velocity distribution in the vertical is normally a log function for which $\alpha_1 \simeq \alpha_2 \simeq 1$. Then the energy equation becomes

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + H_L \qquad 2.3.1$$

and the slopes of the bed, water surface and energy gradeline are respectively

$$S_o = \sin\theta = \frac{z_1 - z_2}{\Delta L} \qquad 2.3.2$$

$$S_w = \frac{(z_1 + y_1) - (z_2 + y_2)}{\Delta L} \qquad 2.3.3$$

and

$$S_f = \frac{H_L}{\Delta L} = \frac{(\frac{V_1^2}{2g} + y_1 + z_1) - (\frac{V_2^2}{2g} + y_2 + z_2)}{\Delta L} \qquad 2.3.4$$

Steady uniform flow is an idealized concept for open channel flow and is difficult to obtain even in laboratory flumes. For many applications, the flow is steady and the changes in width, depth or direction (resulting in nonuniform flow) are so small that the flow can be considered uniform. In other cases, the changes occur over such a long distance the flow is a gradually varied flow.

Variables of interest for steady uniform flow are: (1) the mean velocity V, (2) the discharge Q, (3) the velocity distribution $v(y)$ in the vertical, (4) the head loss H_L through the reach, and (5) the shear stress, both local τ and at the bed τ_o. The variables are interrelated and which variable we determine and how we determine it depends on the data available. For example, if the discharge and cross-sectional area are known, the mean velocity is easily determined for some suitable equation such as Manning's or Chezy's equation.

2.3.2 Shear-stress and velocity distribution

Shear stress τ is the internal fluid stress which resists deformation. The shear stress exists only when fluids are in motion. It is a tangential stress in contrast to pressure, which is a normal stress.

The local shear stress at the interface between the boundary and the fluid can be determined quite easily if the *boundary is hydraulically smooth;* that is, if the roughness at the boundary is submerged in a viscous sublayer as shown in Fig. 2.3.2. Here, the thickness of the

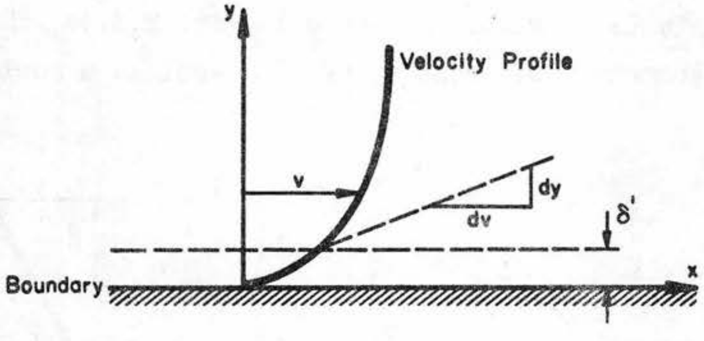


Fig. 2.3.2 Hydraulically smooth boundary.

laminar sublayer is δ'. In laminar flow, the shear stress at the boundary is

$$\tau_o = \mu\left(\frac{dv}{dy}\right)_{y=0} \qquad 2.3.5$$

The velocity gradient is evaluated at the boundary. The dynamic viscosity μ is the proportionality constant relating boundary shear and velocity gradient in the viscous sublayer.

When the *boundary is hydraulically rough,* there is no viscous or laminar sublayer. The paths of fluid particles in the vicinity of the boundary are shown in Fig. 2.3.3.

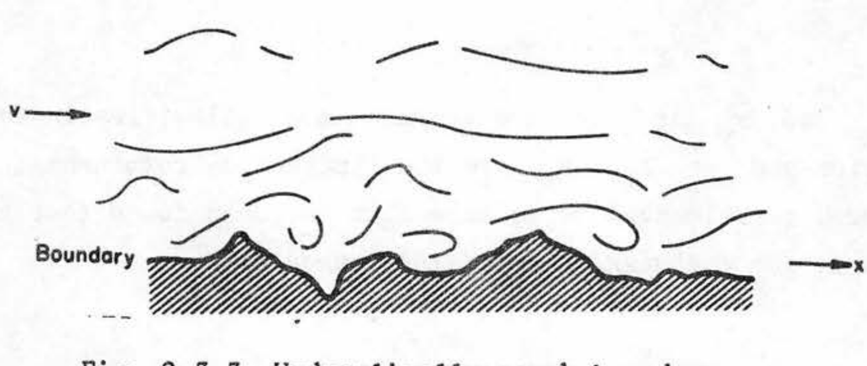


Fig. 2.3.3 Hydraulically rough boundary.

The velocity at a point near the boundary fluctuates randomly about a mean value. *The random fluctuation in velocity is called turbulence.* For the hydraulically rough boundary,

$$\tau_o \neq \mu \frac{dv}{dy}$$

so another method of expressing τ_o is required. So far, the only satisfactory way of determining the boundary shear stress at a rough boundary has been experimentally.

If we follow a unit of mass of fluid in the flow near a rough boundary, the path is erratic. As shown in Fig. 2.3.4a, the particle has a vertical component of velocity v_y as well as a horizontal component v_x.

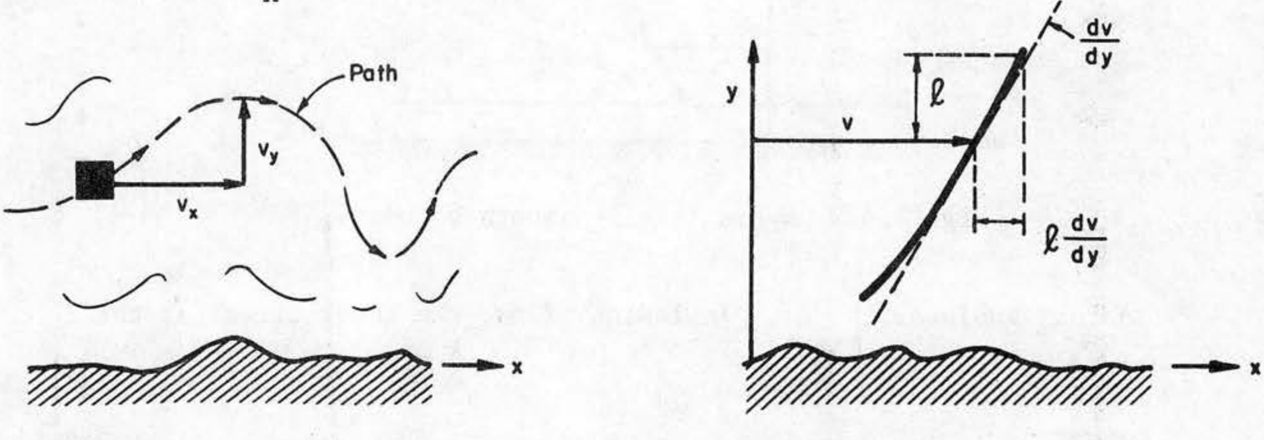


Fig. 2.3.4 Velocities in turbulent flow.

The two components of velocity in Fig. 2.3.4a can be written as

$$v_x = \overline{v}_x + v_x' \qquad 2.3.6$$

and

$$v_y = \overline{v}_y + v_y' \qquad 2.3.7$$

where $\overline{v}_x$ and $\overline{v}_y$ are the time-averaged mean velocities in the x and y direction and v_x' and v_y' are the fluctuating components.

Through experimental correlations it has been found that the *boundary shear stress for turbulent flow at the boundary is*

$$\tau_o = -\rho \overline{v_x' v_y'} \qquad 2.3.8$$

The term $\overline{v_x' v_y'}$ is the time-average of the product of v_x' and v_y' at a point in the flow. It is called the Reynolds shear stress.

Prandtl (1925) suggested that v_x' and v_y' are related and furthermore that v_x' is related to the velocity gradient dv/dy shown in Fig. 2.3.4. He proposed to characterize the turbulence with a dimension called the "mixing length", ℓ. Accordingly,

$$v_x' \sim \ell \left(\frac{dv}{dy}\right) \qquad 2.3.9$$

$$v_y' \sim \ell \left(\frac{dv}{dy}\right) \qquad 2.3.10$$

and

$$\tau \sim \rho\, \ell^2 \left(\frac{dv}{dy}\right)^2 \qquad 2.3.11$$

If it is assumed that the mixing length can be represented by the product of a constant κ and y (i.e; $\ell = \kappa y$), then for steady uniform turbulent flow,

$$\tau = \rho \kappa^2 y^2 \left(\frac{dv}{dy}\right)^2 \qquad 2.3.12$$

Using different reasoning Von Kármán (1930) derived the same equation. Eq. 2.3.12 can be rearranged to the form

$$\frac{dv}{dy} = \frac{\sqrt{\tau_o/\rho}}{\kappa y} \qquad 2.3.13$$

where κ is the Von Kármán universal velocity coefficient. For rigid boundaries κ has the average value of 0.4. The term τ_o is the shear stress at the bed. The term $(\tau_o/\rho)^{1/2}$ has the dimensions of velocity and is called the shear velocity. Integration of 2.3.13 yields

$$\frac{v}{\sqrt{\tau_o/\rho}} = \frac{1}{\kappa} \ln \frac{y}{y'} = \frac{2.31}{\kappa} \log \frac{y}{y'} \qquad 2.3.14$$

Here ln is the logarithm to the base e and log is the logarithm to the base 10. The term y' results from evaluation of the constant of integration assuming $v = 0$ at some distance y' above the bed.

The term y' depends on the flow and has been experimentally determined. The many experiments have resulted in characterizing turbulent flow into three general types:

(1) *Hydraulically smooth boundary turbulent flow* where the velocity distribution, mean velocity and resistance to flow are independent of the boundary roughness of the bed but depend on fluid viscosity. Then where δ' is equal to $11.6\nu/\sqrt{\tau_o/\rho}$, $y' = \delta'/107$.

(2) *Hydraulically rough boundary turbulent flow* where velocity distribution, mean velocity and resistance to flow are independent of viscosity and depend entirely on the boundary roughness. For this case, $y' = k_s/30.2$ where k_s is the height of the roughness element.

(3) *Transition* where the velocity distribution, mean velocity and resistance to flow depends on both fluid viscosity and boundary roughness. Then

$$\frac{\delta'}{107} < y' < \frac{k_s}{30.2}$$

There are many forms of Eq. 2.3.14 depending on the experimental and the method of expressing y'. In this manual, the Einstein method of expressing y' is employed. *The Einstein form of the Karman-Prandtl velocity distribution, mean velocity and resistance to flow equations are:*

$$\frac{v}{V_*} = 5.75 \log \left(30.2 \frac{xy}{k_s}\right) = 2.5 \ln \left(30.2 \frac{xy}{k_s}\right) \qquad 2.3.15$$

$$\frac{V}{V_*} = \frac{C}{\sqrt{g}} = 5.75 \log \left(12.27 \frac{xy_o}{k_s}\right) = 2.5 \ln \left(12.27 \frac{xy_o}{k_s}\right) \qquad 2.3.16$$

where

x = a coefficient given in Fig. 2.3.5

k_s = the height of the roughness elements. For sand channels, k_s is the D_{65} of the bed material

v = the local mean velocity at depth y

y_o = the depth of flow

V = the depth-averaged velocity

V_* = the shear velocity $\sqrt{\tau_o/\rho}$ and for steady uniform flow is $\sqrt{gRS_f}$

τ_o = the shear stress at the boundary and for steady uniform flow $= \gamma RS_f$

R = the hydraulic radius equal to the area divided by the wetted perimeter.

S_f = the slope of the energy gradeline

δ' = the thickness of the viscous sublayer

$$= \frac{11.6\nu}{V_*} \qquad 2.3.17$$

$C/\sqrt{g}$ = the Chezy discharge coefficient in the equation

$$V = C\sqrt{RS_f} \quad \text{or} \quad C = \left(\frac{8g}{f}\right)^{1/2} \qquad 2.3.18$$

f = the Darcy-Weisbach resistance coefficient and is given by the expression

$$f = \frac{8\tau_o}{\rho V^2} \qquad 2.3.19$$

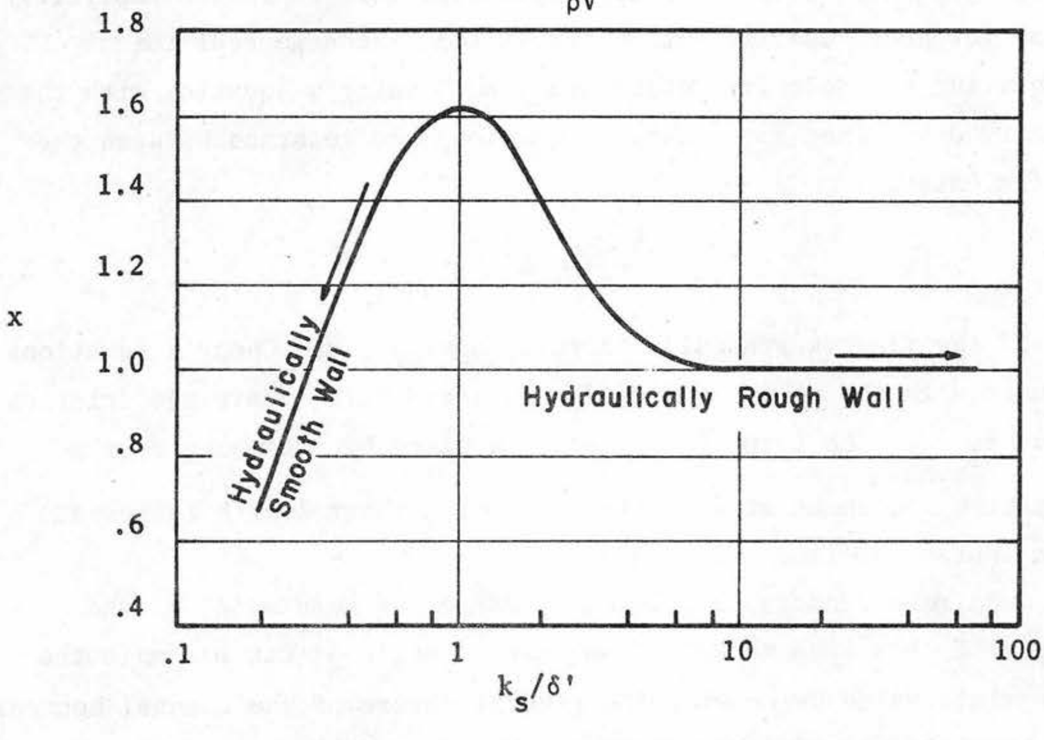


Fig. 2.3.5 Einstein's multiplication factor x in the logarithmic velocity equations (Einstein, 1950).

2.3.3 Empirical velocity equations

Because of the difficulties involved in determining the shear stress and hence the velocity distribution in turbulent flows, *the empirical approach to determine mean velocities in rivers has been prevalent.* Two such empirical equations are in common use. They are *Manning's equation*

$$V = \frac{1.486}{n} R^{2/3} S_f^{1/2} \qquad 2.3.20$$

and *Chezy's equation*

$$V = CR^{1/2} S_f^{1/2} \qquad 2.3.21$$

where

- V = the average velocity in the cross section
- n = Manning's roughness coefficient
- R = the hydraulic radius equal to the cross-sectional area A divided by the wetted perimeter P
- S_f = the energy slope of the channel
- C = Chezy's discharge coefficient known as Chezy's C.

In these equations, the boundary shear stress is expressed implicitly in the roughness coefficient n or in the discharge coefficient C. By equating the velocity determined from Manning's equation with the velocity determined from Chezy's equation, the relation between the coefficients is

$$C = \frac{1.486}{n} R^{1/6} \qquad 2.3.22$$

If the flow is gradually varied, Manning's and Chezy's equations are used with the energy slope S_f replaced with an average friction slope $S_{f_{ave}}$. The term $S_{f_{ave}}$ is determined by averaging over a short time increment at a station or over a short length increment at an instant of time, or both.

Over many decades, a catalog of values of Manning's n and Chezy's C has been assembled so that an engineer can estimate the appropriate value by knowing the general nature of the channel boundaries. An abbreviated list of Manning's roughness coefficients is given in Table 2.3.1. Additional values are given by Barnes (1967) and V. T. Chow (1959). Manning's n for sandbed channels is discussed in detail in Chapter III.

Table 2.3.1 Manning's roughness coefficients for various boundaries.

Rigid Boundary Channels	Manning's n
Very smooth concrete and planed timber	0.011
Smooth concrete	0.012
Ordinary concrete lining	0.013
Wood	0.014
Vitrified clay	0.015
Shot concrete, untrowelled, and earth channels in best condition	0.017
Straight unlined earth canals in good condition	0.020
Rivers and earth canals in fair condition-some growth	0.025
Winding natural streams and canals in poor condition-considerable moss growth	0.035
Mountain streams with rocky beds	0.040-0.050
Alluvial Sandbed Channels (no vegetation)[1/]	
Tranquil flow, Fr < 1	
plane bed	0.014-0.020
ripples	0.018-0.030
dunes	0.020-0.040
washed out dunes or transition	0.014-0.025
plane bed	0.010-0.013
Rapid flow, Fr $\simeq$ 1	
standing waves	0.010-0.015
antidunes	0.012-0.020

[1/] Data is limited to sand channels with D_{50} < 1.0 mm.

2.3.4 Average boundary shear stress

The shear stress at the boundary τ_o for steady uniform flow is determined by applying the conservation of mass and momentum principles to the control volume shown in Fig. 2.3.6. Recall that the statement of the conservation of mass is

$$\begin{array}{c} \text{Mass flux} \\ \text{out of the} \\ \text{control volume} \end{array} - \begin{array}{c} \text{Mass flux} \\ \text{into the} \\ \text{control volume} \end{array} + \begin{array}{c} \text{Time rate of change} \\ \text{in mass in the} \\ \text{control volume} \end{array} = 0$$

and the statement of the conservation of linear momentum is

Momentum flux out of the control volume	-	Momentum flux into the control volume

+	Time rate of change of momentum in the control volume	=	Sum of the forces acting on the fluid in the control volume

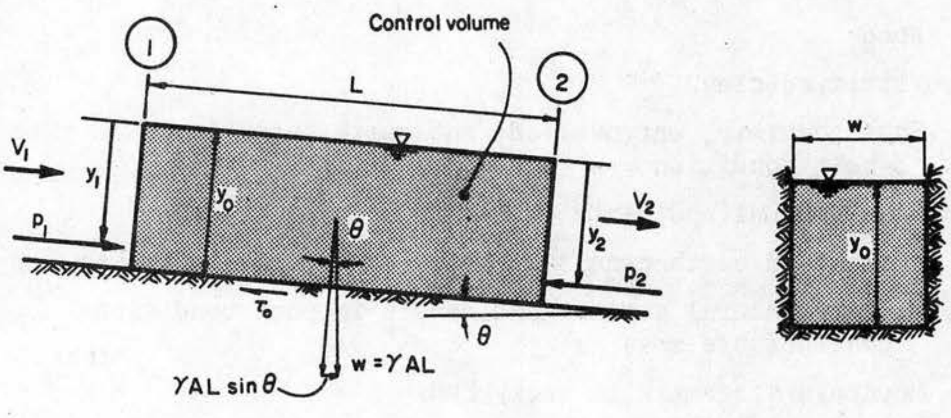


Fig. 2.3.6 Control volume for steady uniform flow.

As the flow is steady

Time rate of change in mass and momentum in the control volume $= 0.$

Mass flux into the control volume $= \rho y_o W\, V_1$

and

Mass flux out of the control volume $= \rho y_o W\, V_2$

The conservation of mass is then

$$\rho y_o W\, V_2 - \rho y_o W\, V_1 = 0$$

or

$$V_1 = V_2 \tag{2.3.23}$$

The conservation of momentum in the downstream direction is composed of the terms

Momentum flux out of the control volume $= \rho V_2\, y_o W\, V_2$

where ρV_2 is the momentum of a unit volume at the outflow section, and $y_o W$ is the outflow cross-section area.

Similarly

Momentum flux into the control volume $= \rho V_1\, y_o W\, V_1$

The pressure force acting on the control boundary at the upstream section is

$$F_1 = \int_o^{y_o} p_1 W\, dy$$

As the flowlines are parallel

$$p_1 = \gamma y$$

so

$$F_1 = \int_o^{y_o} \gamma W y\, dy = \frac{\gamma y_o^2\, W}{2} \tag{2.3.24}$$

Similarly at the downstream section the force acting on the boundary is

$$F_2 = \frac{-\gamma y_o^2\, W}{2} \tag{2.3.25}$$

The body force is the weight of the fluid in the control volume

$$\gamma AL$$

and the downstream component of this body force is

$$\gamma AL \sin\theta$$

where θ is the slope angle of the channel bed. The average boundary shear stress is τ_o acting on the wetted perimeter P. The shear force F_s in the x-direction is

$$F_s = \tau_o PL \qquad 2.3.26$$

With the above expressions for the components, the statement of conservation of linear momentum becomes

$$\rho V_2 y_o W V_2 - \rho V_1 y_o W V_1 = \gamma AL \sin\theta + \frac{\gamma y_o^2 W}{2} - \frac{\gamma y_o^2 W}{2} - \tau_o PL \qquad 2.3.27$$

which reduces to

$$\tau_o = \gamma \frac{A}{P} \sin\theta \qquad 2.3.28$$

The term A/P is called the hydraulic radius R. If the channel slope angle is small

$$\sin\theta \simeq S_o \qquad 2.3.29$$

and the *average shear stress on the boundary is*

$$\tau_o = \gamma R S_o \qquad 2.3.30$$

If the flow is gradually varied nonuniform flow, the average boundary shear stress is

$$\tau_o = \gamma R S_f \qquad 2.3.31$$

where S_f is the slope of the energy gradeline.

2.4.0 UNSTEADY FLOW

Unsteady flows of interest to the designer of waterway crossings and encroachments are: (1) waves resulting from disturbances of the water surface by wind and boats, (2) waves resulting from the surface instability that exists for flows with Froude numbers close to 1.0, (3) waves resulting from flow disturbance resulting from change in direction of flow with Froude numbers greater than 2.0, (4) surges or bores resulting from sudden increase or decrease in the flow by opening or closing of gates or the movement of tides on coastal streams, (5) standing waves and antidunes that occur in alluvial channel flow, and (6) flood waves resulting from the progressive movement downstream of stream runoff or gradual release from reservoirs.

Waves are an important consideration in bridge hydraulics when designing slope protection of embankments and dykes, and channel improvements. In the following paragraphs, only the basic one-dimensional analysis of waves and surges is presented. Other aspects of waves are presented in other sections.

2.4.1 Gravity waves

The general equation for the celerity C of a small amplitude gravity wave (velocity of the wave relative to the velocity of flow) is

$$C = \left\{\frac{g\lambda}{2\pi} \tanh \frac{2\pi y_o}{\lambda}\right\}^{1/2} \qquad 2.4.1$$

where the terms are defined in Fig. 2.4.1.

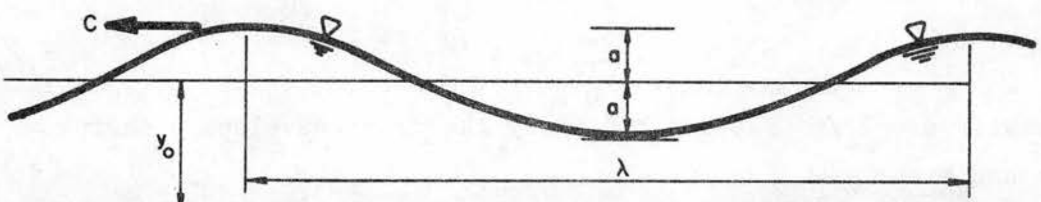


Fig. 2.4.1 Definition sketch for small amplitude waves.

For deep water waves (short waves)

$$\frac{y_o}{\lambda} > \frac{1}{2}$$

and

$$C = \{\frac{g\lambda}{2\pi}\}^{1/2} \qquad 2.4.2$$

For shallow water waves (long waves)

$$\frac{y_o}{\lambda} < \frac{1}{20}$$

and

$$C = \{gy_o\}^{1/2} \qquad 2.4.3$$

If T is the time (period) of travel of one water crest to another at a given point, then

$$C = \frac{\lambda}{T} \qquad 2.4.4$$

In Eq. 2.4.2, the celerity is independent of depth and depends on gravity and wave length. This is the celerity of ocean waves. In Eq. 2.4.3, the celerity is a function of gravity and depth and is used for small amplitude waves in open channels. These equations apply only to small amplitude waves; that is, $\frac{a}{\lambda} << 1$.

The celerity of finite amplitude shallow water waves has been determined both analytically using Bernoulli's equation and experimentally and is given by the expression

$$C = \{\frac{(y_o + 2a)^2}{(y_o + a)y_o} \; gy_o\}^{1/2} \qquad 2.4.5$$

When $2a$ is small in comparison to y_o

$$C = \{(1 + \frac{2a}{y_o})\; gy_o\}^{1/2} \qquad 2.4.6$$

Generally as $2a/y_o$ approaches unity the crest develops a sharp peak and breaks.

In the above equations, C is measured relative to the fluid. If the wave is moving opposite to the flow then, when $C > V$, the waves move upstream; when $C = V$, the wave is stationary; and when $C < V$, the wave moves downstream. When V equals C for small amplitude flow,

$$V = \sqrt{gy_o}$$

The *definition of the Froude number is*

$$Fr = \frac{V}{\sqrt{gy_o}} \qquad 2.4.8$$

Thus, *the Froude number is the ratio of the velocity of flow to the celerity of a small-amplitude wave.* When $Fr < 1$ (tranquil flow), a small amplitude wave moves upstream. When $Fr > 1$ (rapid flow), a small amplitude wave moves downstream and when $Fr = 1$ (critical flow), a small amplitude wave is stationary. The fact that waves or surges cannot move upstream when the Froude number is equal to or greater than 1.0 is important to remember when determining the control points for gradually varied flows and for determining when the stage-discharge relation at a cross section can be affected by downstream conditions.

2.4.2 Surges

A surge is a rapid increase in the depth of flow. A surge may result from sudden release of water from a dam, or from an incoming tide. If the ratio of wave height $2a$ to the depth y_o is less than unity, the surge has an undulating wave form. If $2a/y_o$ is greater than one, the first wave breaks and produces a discontinuous surface. The breaking wave dissipates energy and the previous equations for wave celerity do not hold. However, by applying the momentum and continuity equations for a control volume encompassing the surge (Fig. 2.4.2),

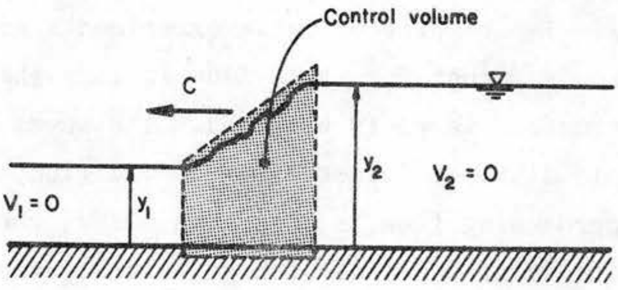


Fig. 2.4.2 Sketch of a surge and its control volume.

the equation

$$C = \{gy_1 \, [\frac{1}{2} \frac{y_2}{y_1} (\frac{y_2}{y_1} + 1)]\}^{1/2} \qquad 2.4.9$$

for the velocity of the surge can be derived.

Equation 2.4.9 gives the velocity of a surge as it moves upstream as the result of a sudden total or partial closure of a gate, or of an incoming tide, or of a surge that moves downstream as the result of a sudden opening of a gate. The lifting of a gate in a channel not only causes a position surge to move downstream, it also causes a negative surge to move upstream. Equation 2.4.9 is approximately correct for the celerity of the negative surge if the height of the surge is small compared to the depth. As it moves upstream a negative surge quickly flattens out.

2.4.3 Hydraulic jump

When the oncoming velocity of flow is rapid or supercritical the surge is a moving hydraulic jump. When V_1 equals the celerity of the surge the jump is stationary and Eq. 2.4.9 is the equation for a hydraulic jump. Equation 2.4.9 can be rearranged to the form

$$\frac{V_1}{\sqrt{gy_1}} = Fr_1 = \{\frac{1}{2}\frac{y_2}{y_1}\ (\frac{y_2}{y_1} + 1)\}^{1/2} \qquad 2.4.10$$

or

$$\frac{y_2}{y_1} = \frac{1}{2}\{(1 + 8Fr_1^2)^{1/2} - 1\} \qquad 2.4.11$$

Equation 2.4.11 has been experimentally verified along with the dependence of the jump length and energy dissipation on the Froude number of the approaching flow. The results of these experiments are given in Fig. 2.4.3.

When the Froude number for rapid flow is less than two, an undulating jump with large surface waves is produced. The waves are propagated for a considerable distance downstream. In addition, when the Froude number of the approaching flow is less than three, the energy dissipation of the jump is not large and jets of high velocity flow can exist for some distance downstream of the jump. These waves and jets can cause erosion a considerable distance downstream of the jump.

2.4.4 Roll waves

Shallow flow on steep slopes may surge or pulsate. The waves formed are called roll waves. They are observed in shallow flow over spillways, in steep alluvial channels and in steep lined channels.

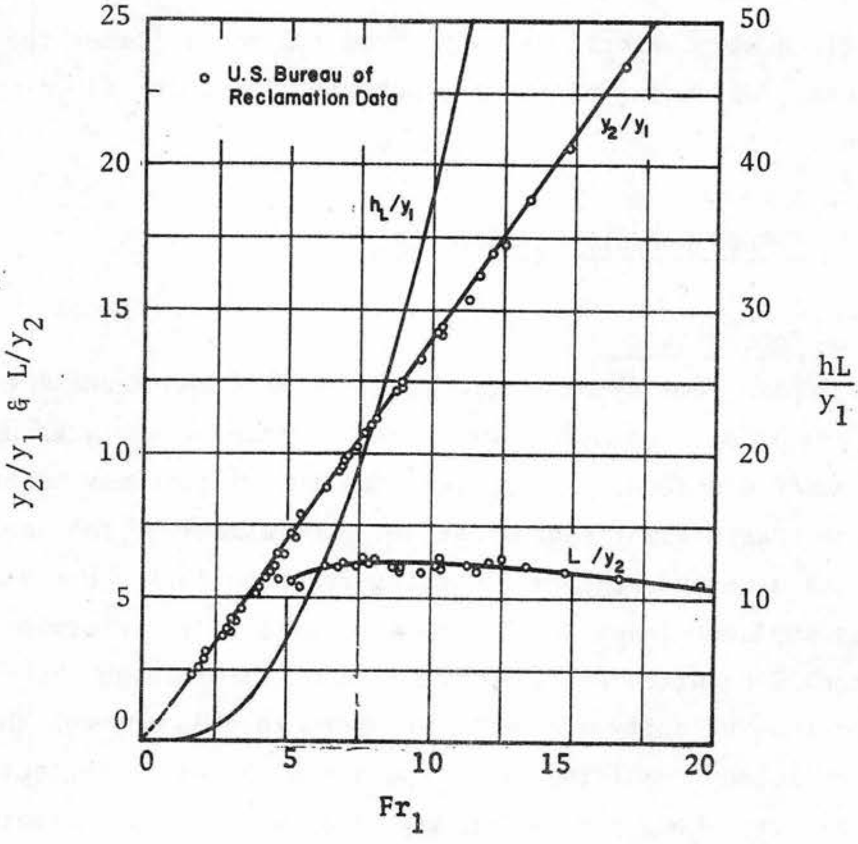


Fig. 2.4.3 Hydraulic jump characteristics as a function of the Froude number.

There is no simple criterion for determining when roll waves form, their velocity or their size. Their formation, size and velocity depend on the roughness and slope of the channel, the initial depth of flow, the length of channel and the nature of any disturbance that triggers them. Roll waves form when the Froude number is greater than two or the slope is approximately four times greater than the critical slope. They can cause the resistance to flow to increase.

2.4.5 Flood waves

Methods of determining the velocity of flood waves and of routing floods through a channel or river reach is beyond the scope of this manual. Various methods and computer programs are available. In general, the methods are based on solutions of the basic differential equations for unsteady flow or on hydrologic methods that make no direct use of the equations of motion but use the continuity equation. In general, the front of a flood wave travels downstream with a greater speed than the mean water velocity at any cross section in the wave.

The flood wave velocities range from 1.2 to 1.7 times the mean water velocity, depending on the characteristics of the flood waves and the channel.

2.5.0 STEADY RAPIDLY VARYING FLOW

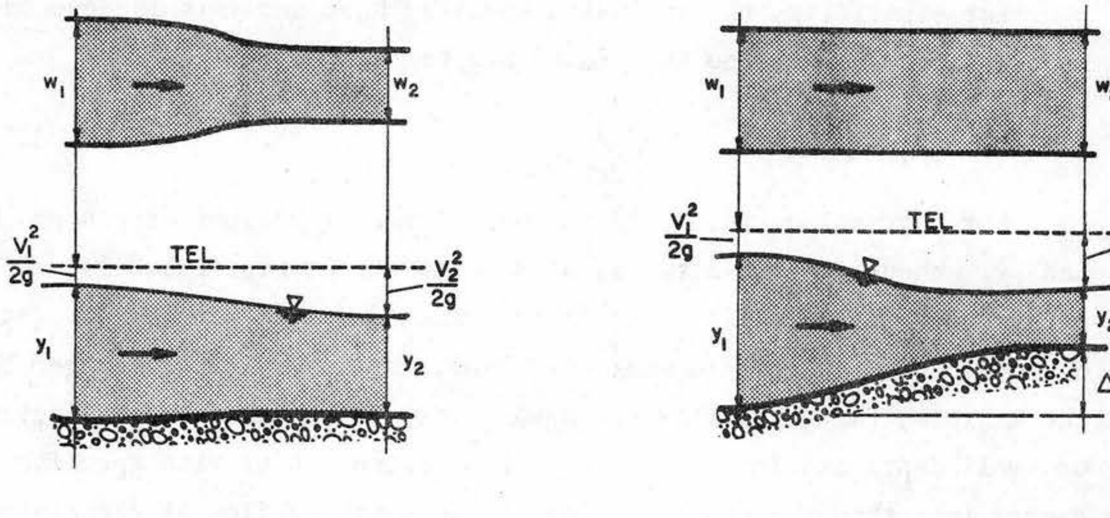
2.5.1 Introduction

Steady flow through relatively short transitions where the flow is uniform before and after the transition can be analyzed using the Bernoulli equation. Energy loss due to friction may be neglected, at least as a first approximation. Refinement of the analysis can be made as a second step by including friction loss. For example, the water surface elevation through a transition is determined using the Bernoulli equation and then modified by determining the friction loss effects on velocity and depth in short reaches through the transition. Energy losses resulting from separation cannot be neglected, and transitions where separation may occur need special treatment which may include model studies. Contracting flows (converging streamlines) are less susceptible to separation than expanding flow. Also, any time a transition changes velocity and depth such that the Froude number approaches unity, problems such as waves, blockage or choking of the flow may occur. If the approaching flow is rapid (supercritical), a hydraulic jump may result. Transitions for rapid flow are discussed in Section 2.8.0.

Transitions are used to contract or expand a channel width (Fig. 2.5.1a); *to increase or decrease in bottom elevation* (Fig. 2.5.1b); *or to change both the width and bottom elevation.* The first step in the analysis is to use the Bernoulli equation (neglecting any head loss resulting from friction or separation) to determine the depth and velocity changes of the flow through the transition. Further refinement depends on importance of freeboard, whether flow is rapid, and whether flow approaches critical.

The Bernoulli equation for flow in Fig. 2.5.1b is

$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + \Delta z \qquad 2.5.1$$



(a) Width contraction (b) Bed rise

Fig. 2.5.1 Transitions in open channel flow.

or

$$H_1 = H_2 + \Delta z \tag{2.5.2}$$

where
$$H = \frac{V^2}{2g} + y \tag{2.5.3}$$

The term H is called the specific head and is the height of the total head above the channel bed.

In the analysis of flow through transitions, the Bernoulli equation gives a numerical solution to the problem but very little descriptive information of the depth variation. Only after the analysis is completed will it be known if the depth will increase or decrease as the fluid passes through the transition or even if the flow is physically possible. On the other hand, by investigating the various interrelations between the variables (H, V and y) in the specific head equation, the variation of depth through a transition can be predicted.

There are two conditions for analyzing the flow through transitions. In the first condition, the width is constant and the elevation of the stream bed changes; that is, $q = Q/W$ is constant and H and y vary (Fig. 2.5.1b). In the second, the width changes and the elevation of the stream bed (neglecting the slope) is constant; that is, H is constant.

2.5.2 Specific head diagram

For simplicity, the following specific head analysis is done on a unit width of channel so that Eq. 2.5.3 becomes

$$H = \frac{q^2}{2gy^2} + y \qquad 2.5.4$$

For a given q, Eq. 2.5.4 can be solved for various values of H and y. When y is plotted as a function of H, Fig. 2.5.2 is obtained. *There are two possible depths called alternate depths for any H larger than a specific minimum.* Thus, for specific head larger than the minimum, the given flow may have a large depth and small velocity or small depth and large velocity. Flow cannot occur with specific energy less than the minimum. The single depth of flow at the minimum specific head is called the critical depth y_c and the corresponding velocity, the critical velocity $v_c = q/y_c$. To determine y_c the derivative of H with respect to y is set equal to 0.

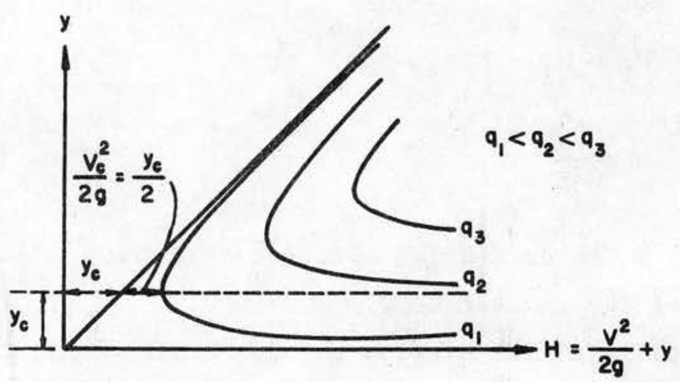


Fig. 2.5.2 Specific head diagram.

$$\frac{dH}{dy} = - \frac{q^2}{gy^3} + 1 = 0 \qquad 2.5.5$$

and

$$q = (gy_c^3)^{1/2} \qquad 2.5.6$$

or

$$y_c = (\frac{q^2}{g})^{1/3} = 2\ \frac{V_c^2}{2g} \qquad 2.5.7$$

Note that $$V_c^2 = y_c g \qquad 2.5.8$$

or $$\frac{V_c}{\sqrt{gy_c}} = 1 \qquad 2.5.9$$

But $$\frac{V}{\sqrt{gy}} = Fr \qquad 2.5.10$$

Also $$H_{min} = \frac{V_c^2}{2g} + y_c = \frac{3}{2} y_c \qquad 2.5.11$$

Thus flow at minimum specific energy has a Froude number equal to one. Flows with velocities larger than critical ($Fr > 1$) are called rapid or supercritical and flows with velocities smaller than critical ($Fr < 1$) are called tranquil or subcritical. These flow conditions are illustrated in Fig. 2.5.3, where a rise in the bed causes a decrease in depth when

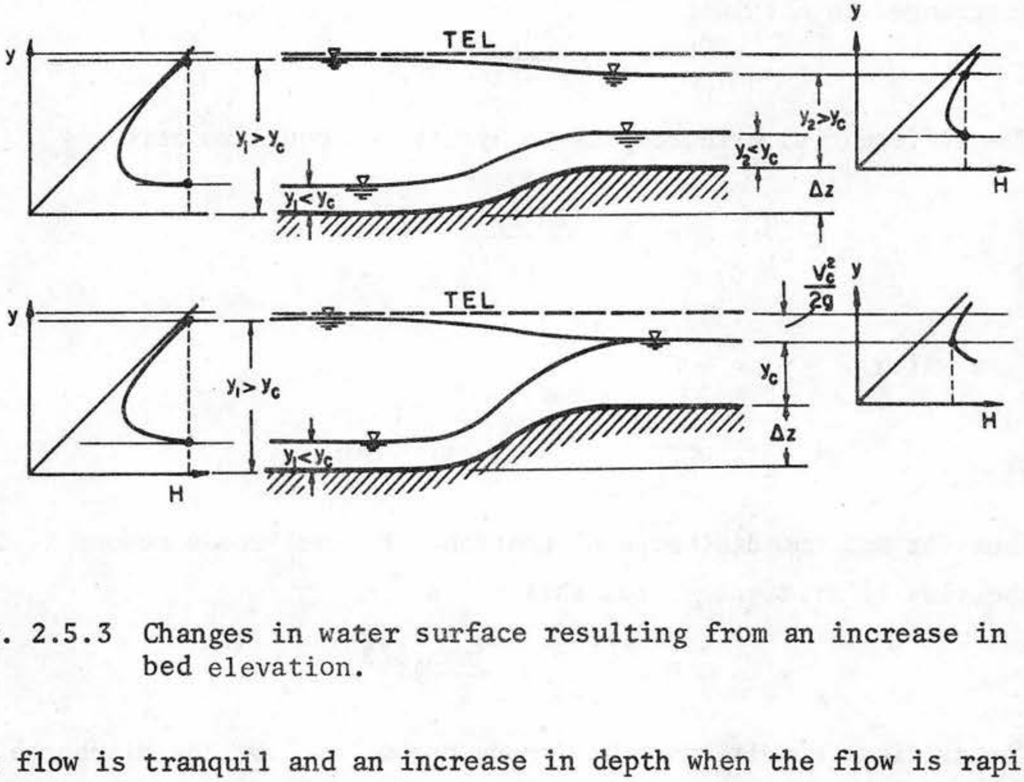


Fig. 2.5.3 Changes in water surface resulting from an increase in bed elevation.

the flow is tranquil and an increase in depth when the flow is rapid. Furthermore there is a maximum rise in the bed for a given H_1 where the given rate of flow is physically possible. If the rise in the bed is increased beyond Δz_{max} for H_{min} then the approaching flow depth y_1 would have to increase (increasing H) or the flow would have to be decreased. *Thus, for a given flow in a channel, a rise in the bed level can occur up to a* Δz_{max} *without causing backwater.*

2.5.3 Discharge diagram

For a constant H, Eq. 2.5.4 can be solved for y as a function of q. By plotting y as a function of q, Fig. 2.5.4 is obtained and for

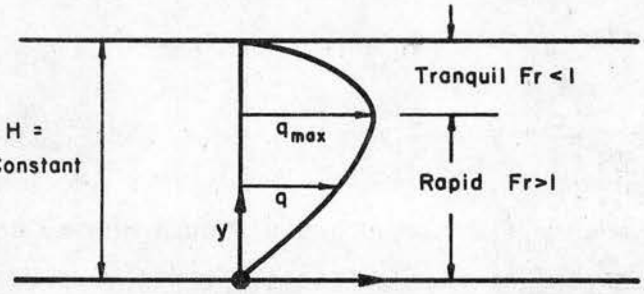


Fig. 2.5.4 Specific discharge diagram.

any discharge smaller than a specific maximum, two depths of flow are possible. To determine the value of y for q_{max} Eq. 2.5.4 is rearranged to obtain

$$q = y\sqrt{2g(H-y)} \tag{2.5.12}$$

The differential with respect to y is set equal to zero.

$$\frac{dq}{dy} = 0 = \sqrt{\frac{g}{2}}\ \frac{(2H-3y)}{(H-y)^{1/2}} \tag{2.5.13}$$

from which $$y_c = \frac{2}{3} H = 2\frac{V_c^2}{2g} \tag{2.5.14}$$

or $$V_c = \sqrt{gy_c} \tag{2.5.15}$$

Thus for maximum discharge at constant H, the Froude number is 1.0 and the flow is critical. From this

$$y_c = \frac{2}{3} H = 2\ \frac{V_c^2}{2g} = (\frac{q_{max}^2}{g})^{1/3} \tag{2.5.16}$$

For critical conditions, the Froude number is 1.0, the discharge is a maximum for a given specific head and the specific head is a minimum for a given discharge.

Flow conditions for constant specific head for a width contraction are illustrated in Fig. 2.5.5. The contraction causes a decrease in flow depth when the flow is tranquil and an increase when the flow is rapid. The maximum contraction possible for these flow conditions is to the critical depth. Then the Froude number is one, the discharge per foot of width q is a maximum, and y_c is $\frac{2}{3}H$. A further decrease in width causes backwater. That is, an increase in y_1 upstream to get a larger specific energy and increase y_c in order to get the flow through the decreased width.

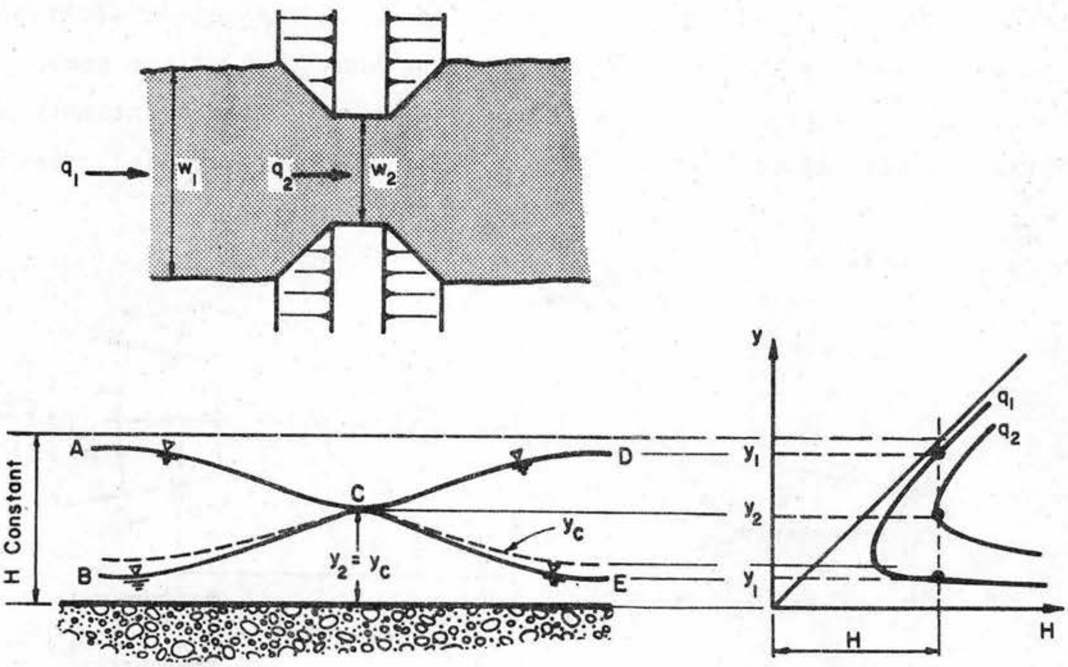


Fig. 2.5.5 Change in water surface elevation resulting from a change in width.

The flow in Fig. 2.5.5 can go from point A to C and then back to D or down to E depending on the boundary conditions. An increase in slope of the bed downstream from C and no separation would allow the flow to follow the line A to C to E. Similarly the flow can go from B to C and back to E or up to D depending on boundary conditions. Fig. 2.5.5 is drawn with the side boundary forming a smooth streamline. If the contraction were a bridge abutment, the upstream flow would follow a natural streamline to a verna contracta but then downstream the flow would probably separate. Tranquil approach flow could follow line A-C but the downstream flow probably would not follow either line C-D or C-E but would have an undulating hydraulic jump. There would be interaction of the flow in the separation zone and considerable energy would be lost. If the slope downstream of the abutments was the same as upstream, then the flow could not be sustained with this amount of energy loss. Backwater would occur, increasing the depth in the constriction and upstream, until the flow could go through the constriction and establish uniform flow downstream.

2.6.0 STEADY FLOW AROUND BENDS

Because of the change in flow direction with results in centrifugal forces, *there is a super elevation of the water surface in river bends.* The water surface is higher at the concave bank than at the convex bank. The resulting transverse slope can be evaluated quantitatively. Using cylindrical coordinates (Fig. 2.6.1), the differential pressure

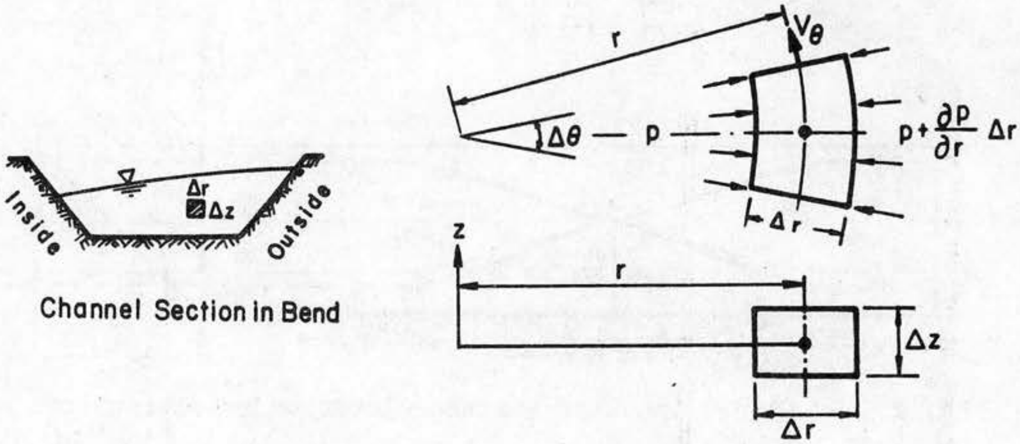


Fig. 2.6.1 Definition sketch of dynamics of flow around a bend.

in the radial direction arises from the radial acceleration or

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{V_\theta^2}{r} . \qquad 2.6.1$$

The total superelevation between the outer and inner bend is

$$\Delta z = \frac{1}{\rho g}\int_{p_i}^{p_o} dp = \frac{1}{g}\int_{r_i}^{r_o}\frac{V_\theta^2}{r}\, dr . \qquad 2.6.2$$

Two assumptions are made:

(1) The radial and vertical velocities are small compared to the tangential velocities such that $V_\theta \approx V$.

(2) The pressure distribution in the bend is hydrostatic, i.e., $p = \gamma y$.

Then $$\Delta z = \frac{1}{g} \int_{r_i}^{r_o} \frac{V^2}{r} \, dr \, . \qquad 2.6.3$$

To solve Eq. 2.6.3, the transverse velocity distribution along the radius of the bend must be known or assumed. The results obtained assuming various velocity distribution are as follows:

Woodward (1920) assumed V equal to the average velocity Q/A and r equal to the radius to the center of the stream r_c and obtained

$$\Delta z = \frac{1}{g} \int_{r_i}^{r_o} \frac{V^2}{r_c} \, dr \qquad 2.6.4$$

or

$$\Delta z = z_o - z_i = \frac{V^2}{g r_c} (r_o - r_i), \qquad 2.6.5$$

in which z_i and r_i are the water surface elevation and the radius at the inside of the bend, and z_o and r_o are the water surface elevation and the radius at the outside of the bend.

By assuming the velocity distribution to approximate that of a free vortex, Shukry (1950) obtained

$$\Delta z = \frac{1}{g} \int_{r_i}^{r_o} \frac{C^2}{r^3} \, dr = \frac{C^2}{2g} \left\{\frac{1}{r_i^2} - \frac{1}{r_o^2}\right\} \qquad 2.6.6$$

in which $C = rV$, the free vortex constant. By assuming the depth of flow upstream of the bend equal to the average depth in the bend, Ippen and Drinker (1962) reduced Eq. 2.6.6 to

$$\Delta z = \frac{V^2}{2g} \; \frac{2W}{r_c} \; \left\{\frac{1}{1-(\frac{W}{2r_c})^2}\right\} \qquad 2.6.7$$

For situations where high velocities occur near the outer bank of the channel, a forced vortex may approximate the flow pattern. With this assumption and assuming a constant average specific head, Ippen and Drinker (1962) obtained,

$$\Delta z = \frac{V^2}{2g}\frac{2W}{r_c}\left\{\frac{1}{1+\frac{W^2}{12\,r_c^2}}\right\} \qquad 2.6.8$$

By assuming that the maximum velocities are close to the centerline of the channel in the bend and that the flow pattern inward and outward from the centerline can be represented as forced and free vortices, respectively, then:

$$\Delta z = \frac{1}{g}\int_{r_i}^{r_c}\frac{C_i^2\,r^2}{r}\,dr + \frac{1}{g}\int_{r_c}^{r_o}\frac{C_o^2}{r^3}\,dr, \qquad 2.6.9$$

and when $r = r_c$, $V = V_{max}$

Therefore, $C_i = \frac{V_{max}}{r_c}$ and $C_o = V_{max}\,r_c$

and Eq. 2.6.9 becomes:

$$\Delta z = \frac{V_{max}^2}{2g}\left\{2 - \left(\frac{r_i}{r_c}\right)^2 - \left(\frac{r_c}{r_o}\right)^2\right\} \qquad 2.6.10$$

The differences in superelevation that are obtained by using the different equations are small, and in alluvial channels the resulting erosion of the concave bank and deposition on the convex bank leads to further error in computing superelevation. Therefore, it is recommended that Eq. 2.6.5 be used to compute superelevation. For lined canals with sharp radii of curvature, superelevation should be computed using Eqs. 2.6.7 and 2.6.10.

2.7.0 GRADUALLY VARIED FLOW

2.7.1 Introduction

Thus far, two types of steady flow have been considered. They are uniform flow and rapidly varying nonuniform flow. In uniform flow, acceleration forces are zero and energy is converted to heat as a result of viscous forces within the flow; there are no changes in cross section or flow direction and the depth (called normal depth) is constant. In rapidly varying flow, changes in cross section, direction, or depth take place in relatively short distances; acceleration forces are not zero; viscous forces can be neglected (at least in the first approximation).

Different conditions prevail for each of these two types of steady flow. In steady uniform flow, the slope of the bed, the slope of the water surface and the slope of the energy gradeline are all parallel and are equal to the head loss divided by the length of channel in which the loss occurred. In rapidly varying flow through short streamlined transitions, resistance is neglected and changes in depth due to acceleration are dominant. In this section, a third type of steady flow is considered. In this type of flow, *changes in depth and velocity take place slowly over large distances*, resistance to flow dominates and acceleration forces are neglected. This type of flow is called *gradually varied flow,* and the study involves 1) the determination of the general characteristics of the water surface and 2) the elevation of the water surface or depth of flow.

In gradually varied flow, the actual flow depth y is either larger than or smaller than the normal depth y_o and either larger than or smaller than the critical depth y_c. The water surface profiles which are often called backwater curves, depend on the magnitude of the actual depth of flow y in relation to the normal depth y_o and the critical depth y_c. *Normal depth* y_0 *is the depth of flow that would exist for steady-uniform flow* as determined using the Manning or Chezy velocity equations, and the critical depth is the depth of flow when the Froude number equals 1.0. Reasons for the depth being different than the normal depth are changes in slope of the bed, changes in cross section, obstruction to flow, imbalances between gravitational forces accelerating the flow and shear forces retarding the flow.

In working with gradually varied flow, the first step is to determine what type of backwater curve would exist. The second step is to perform the numerical computations.

2.7.2 Classification of flow profiles

The classification of flow profiles is obtained by analyzing the change of the various terms in the total head equation in the x-direction. The total head is

$$H_T = \frac{V^2}{2g} + y + z \qquad 2.7.1$$

or

$$H_T = \frac{Q^2}{2gA^2} + y + z \qquad 2.7.2$$

Then assuming a wide channel for simplicity

$$\frac{dH_T}{dx} = - \frac{q^2}{gy^3}\frac{dy}{dx} + \frac{dy}{dx} + \frac{dz}{dx} \qquad 2.7.3$$

The term dH_T/dx is the slope of the energy gradeline S_f, it is assumed. For short distances and small changes in y the energy gradient can be evaluated using the Manning or Chezy velocity equations.

When Chezy's equation (Eq. 2.3.21) is used the expression for dH_T/dx is

$$-\frac{dH_T}{dx} = S_f = \frac{q^2}{C^2y^3} \qquad 2.7.4$$

The term dy/dx is the slope of the water surface S_w and dz/dx is the bed slope S_o. For steady flow, the bed slope is (from Eq. 2.3.21)

$$S_o = \frac{q^2}{C_o^2 y_o^3} \qquad 2.7.5$$

where the subscript "o" indicates the steady uniform flow values.

When Eqs. 2.7.4 and 2.7.5 are substituted into Eq. 2.7.3, *the familiar form of the gradually varied flow equation*

$$\frac{dy}{dx} = S_o \left\{\frac{1 - \left(\frac{C_o}{C}\right)^2 \left(\frac{y_o}{y}\right)^3}{1 - \left(\frac{y_c}{y}\right)^3}\right\} \qquad 2.7.6$$

is obtained.

If Manning's equation is used to evaluate S_f and S_o, Eq. 2.7.6 becomes

$$\frac{dy}{dx} = S_o \left\{ \frac{1 - (\frac{n}{n_o})^2 (\frac{y_o}{y})^{10/3}}{1 - (\frac{y_c}{y})^3} \right\} \qquad 2.7.7$$

The slope of the water surface $\frac{dy}{dx}$ depends on the slope of the bed S_o, the ratio of the normal depth y_o to the actual depth y and the ratio of the critical depth y_c to the actual depth y. The difference between flow resistance for steady uniform flow n_o to flow resistance for steady nonuniform flow n is small and the ratio is taken as 1.0. With $n = n_o$, there are twelve types of water surface profiles. These are illustrated in Fig. 2.7.1 and summarized in Table 2.7.1.

Table 2.7.1 Characteristics of water surface profiles.

Class	Bed Slope	Depth	Type	Classification
Mild	$S_o>0$	$y>y_o>y_c$	1	M_1
Mild	$S_o>0$	$y_o>y>y_c$	2	M_2
Mild	$S_o>0$	$y_o>y_c>y$	3	M_3
Critical	$S_o>0$	$y>y_o=y_c$	1	C_1
Critical	$S_o>0$	$y<y_o=y_c$	3	C_3
Steep	$S_o>0$	$y>y_c>y_o$	1	S_1
Steep	$S_o>0$	$y_c>y>y_o$	2	S_2
Steep	$S_o>0$	$y_c>y_o>y$	3	S_3
Horizontal	$S_o=0$	$y>y_o$	2	H_2
Horizontal	$S_o=0$	$y_c>y$	3	H_3
Adverse	$S_o<0$	$y>y_c$	2	A_2
Adverse	$S_o<0$	$y_c>y$	3	A_3

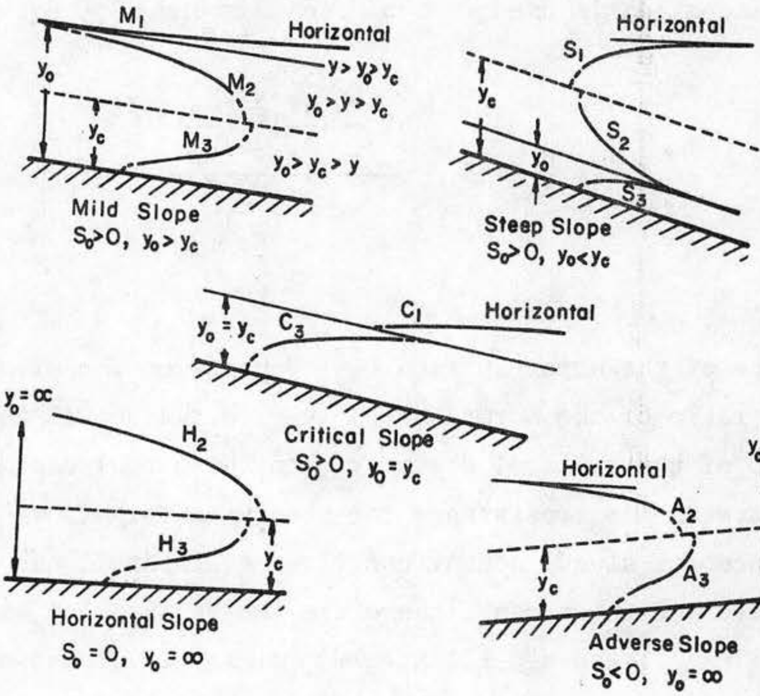


Fig. 2.7.1 Classification of water surface profiles.

Note:

(1) With a *type 1 curve* (M_1, S_1, C_1), the actual depth of flow y is greater than both the normal depth y_o and the critical depth y_c. Because flow is tranquil, control of the flow is downstream.

(2) With a *type 2 curve* (M_2, S_2, A_2, H_2), the actual depth y is between the normal depth y_o and the critical depth y_c. The flow is tranquil for M_2, A_2 and H_2 and thus the control is downstream. Flow is rapid for S_2 and the control is upstream.

(3) With a *type 3 curve* (M_3, S_3, C_3, A_3, H_3), the actual depth y is smaller than both the normal depth y_o and the critical depth y_c. Because the flow is rapid control is upstream.

(4) For a *mild slope*, S_o is smaller than S_c and $y_o > y_c$.

(5) For a *steep slope*, S_o is larger than S_c and $y_o < y_c$.

(6) For a *critical slope*, S_o equals S_c and $y_o = y_c$.

(7) For an *adverse slope*, S_o is negative.

(8) For a *horizontal slope*, S_o equals zero.

(9) The case where $y \to y_c$ is of special interest because the denominator in Eq. 2.7.6 approaches zero.

When $y \to y_c$, the assumption that acceleration forces can be neglected no longer holds. Equations 2.7.6 or 2.7.7 indicate that $\frac{dy}{dx}$ is perpendicular when $y \to y_c$. For cross sections close to the cross section where the flow is critical (a distance from 50 to 10 ft), curvilinear flow analysis and experimentation must be used to determine the actual values of y. When analyzing long distances (100 to 1000 ft or longer) one can assume qualitatively that y reaches y_c. In general, when the flow is rapid ($Fr \geq 1$), the flow cannot become tranquil without a hydraulic jump occurring. In contrast, tranquil flow can become rapid (cross the critical depth line). This is illustrated in Fig. 2.7.2.

When there is a change in cross section or slope at an obstruction to the flow, the qualitative analysis of the flow profile depends on locating the control points, determining the type of curve upstream and downstream of the control points, and then sketching the backwater curves. It must be remembered that *when flow is rapid ($Fr \geq 1$), the control of the depth is upstream and the backwater proceeds in the downstream direction. When flow is tranquil ($Fr < 1$), the depth control is downstream and in the computations must proceed upstream.* The backwater curves that result from a change in slope of the bed are illustrated in Fig. 2.7.2.

2.7.3 Computation of water surface profiles

There are many computer programs available for the computation of the elevation or depth of flow for water surface profiles. Herein, *the standard step method* is described. However, as with most computer programs, a qualitative analysis of the general characteristics of the backwater curves as described in the preceding section must be made. This is necessary in order to know whether the analysis proceeds upstream or downstream. Most available computer programs cannot solve the water surface profile equations when the flow changes from rapid to tranquil or vice versa.

The standard step method is derived from the energy equation

$$\frac{V_1^2}{2g} + y_1 + \Delta z = \frac{V_2^2}{2g} + y_2 + H_L \qquad 2.7.8$$

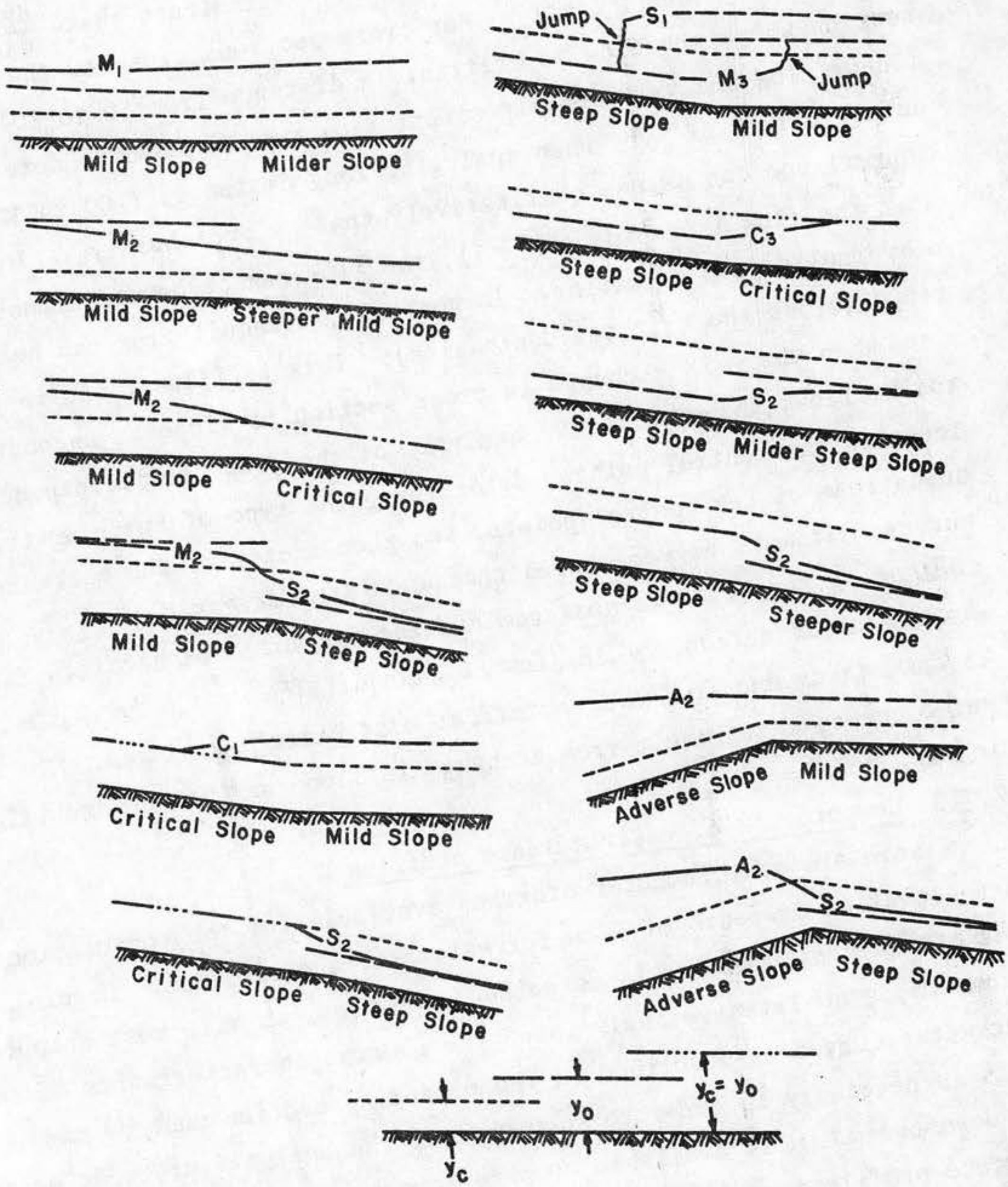


Fig. 2.7.2 Examples of water surface profiles.

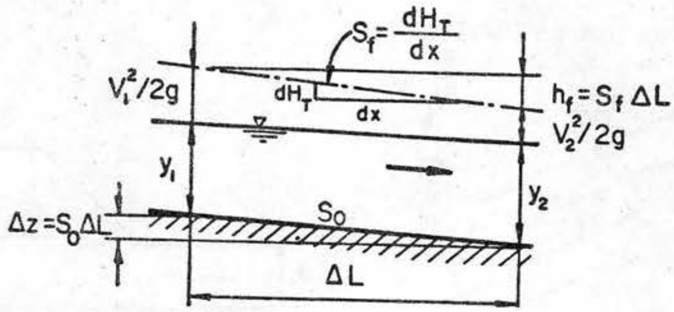


Fig. 2.7.3 Definition sketch for step method computation backwater curves.

From Fig. 2.7.3

$$\frac{V_1^2}{2g} + y_1 + S_o \, \Delta L = \frac{V_2^2}{2g} + y_2 + S_f \, \Delta L \qquad 2.7.9$$

$$H_1 + S_o \Delta L = H_2 + S_f \, \Delta L \qquad 2.7.10$$

and

$$\Delta L \doteq \frac{H_2 - H_1}{S_o - S_f} \qquad 2.7.11$$

The procedure is to start from some known y, assume another y either upstream or downstream depending on whether the flow is tranquil or rapid, and compute the distance ΔL to the assumed depth using Eq. 2.7.11.

2.8.0 RAPID FLOW IN BENDS AND TRANSITIONS

2.8.1 Bends

Rapid flow or supercritical flow in a curved *prismatic channel produces cross wave disturbance patterns which persist for long distances in a downstream direction.* These disturbance patterns are the result of nonequilibrium conditions which persist because the disturbances cannot propagate upstream or even propagate directly across the stream. Therefore, the turning effect of the walls is not felt on all filaments of the flow at the same time and the equilibrium of the flow is destroyed.

The waves produced form a series of troughs and crests in the water surface along the channel walls.

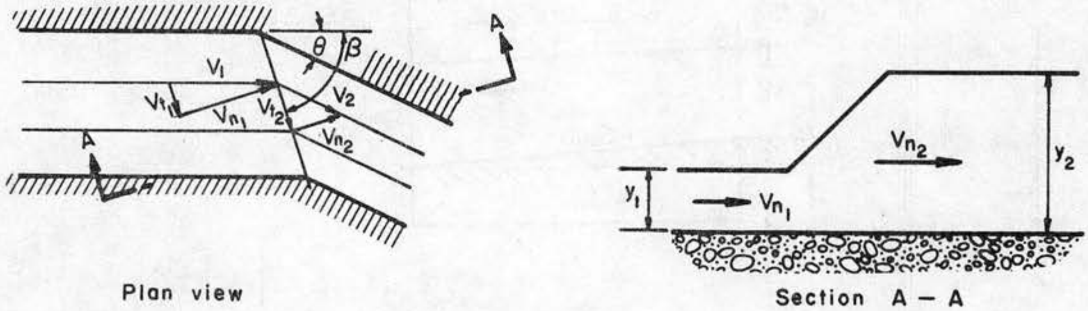


Fig. 2.8.1 Definition sketch for rapid flow in a bend.

Fig. 2.8.1 is a definition sketch to aid in the analysis of cross wave patterns in a bend with rapid flow. The water surface elevation in a bend can be computed if the following major assumptions are made: (1) the flow is two-dimensional; (2) the velocity is constant throughout the cross section; (3) the channel is horizontal; (4) there are no boundary shear stresses; and (5) the channel walls are vertical. The outer wall which turns the flow inward produces an oblique hydraulic jump and a corresponding positive disturbance line or positive wave front propagates across the channel. The inner or convex wall causes an oblique expansion or negative wave to propagate across the channel with a corresponding negative disturbance line or wave front. From analysis of Fig. 2.8.1 and the hydraulic jump equation the following formulas can be derived.

The initial velocity perpendicular to the wave front is given by

$$V_{n1} = \left\{\frac{gy_2}{2}\left(1 + \frac{y_2}{y_1}\right)\right\}^{1/2} \qquad 2.8.1$$

The wave front angle is given by:

$$\sin\beta \ \frac{V_{n1}}{V_1} = \frac{\sqrt{gy_1}}{V_1}\left\{\frac{y_2}{2y_1}\left(1 + \frac{y_2}{y_1}\right)\right\}^{1/2}$$

or

$$\sin\beta \simeq \frac{1}{Fr_1} \qquad 2.8.2$$

The relationship of the deflection angle θ and the Froude number is given by:

$$\theta = \sqrt{3}\tan^{-1}\{(\frac{3}{Fr^2-1})^{1/2}\} - \tan^{-1}\{(\frac{1}{Fr_1^2-1})^{1/2}\} + \text{const.} \qquad 2.8.3$$

where the constant may be determined from the condition that for $\theta = 0$, the depth y is the initial depth y_1.

For practical applications, Eq. 2.8.3 is very involved and inconvenient to use even with graphical charts. Knapp (1951) developed a much simpler equation which gives adequate results. The depth at the first maximum may be computed from

$$y = \frac{V^2}{g}\sin^2(\beta + \frac{\theta}{2}) \qquad 2.8.4$$

Equation 2.8.4 results from experimental observations of a constant velocity occurring at a cross section. The locations of the first maximum may be found from:

$$\theta = \tan^{-1}\{\frac{2W}{(2r_c + W)\tan\beta}\} \qquad 2.8.5$$

where r_c is the radius of curvature and W is the channel width as shown in a plan view of the cross wave pattern given in Fig. 2.8.2. The disturbance wave pattern oscillates about a plane located at the normal depth. The distance along the wall to the first maximum subtends a central angle, θ, and this distance represents half a wave length.

The amplitude of the disturbance pattern in the downstream tangent is dependent on whether the new disturbance pattern created in the change of flow from curved to straight reinforces or damps out the disturbance pattern already in existence. When the curve has central angles of θ, 3θ, 5θ, etc., where θ is given by Eq. 2.8.5., the two disturbance patterns reinforce each other and the resulting disturbance pattern in the tangent section oscillates about the normal depth with an amplitude approximately V^2W/r_cg. By adopting central angles of 2θ, 4θ, 6θ, etc., the disturbance pattern generated by the change from a straight to curved channel will cancel out the disturbance created by the initial curve in the channel.

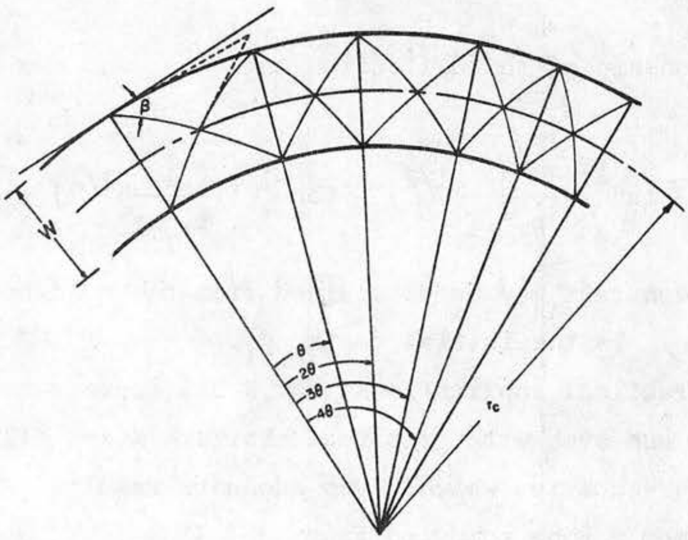


Fig. 2.8.2 Plan view of cross wave pattern for rapid flow in a bend.

Two methods have been used in the design of curves for rapid flow in channels. One method is to *bank the floor of the channel* and the other is to *provide curved vanes* in the flow. Banking of the floor produces lateral forces which act simultaneously on all filaments and causes the flow to turn without destroying the flow equilibrium. Curved vanes break up the flow into a series of small channels and since the superelevation is directly proportional to the channel width, each small channel has a smaller superelevation.

2.8.2 Transitions

Contractions and expansion in rapid flow produce cross wave patterns similar to those observed in curved channels. The cross waves are symmetrical with respect to the centerline of the channel. Ippen and Dawson (1951) have shown that in order to minimize the disturbance downstream of a contraction, the length of the contraction should be:

$$L = \frac{W_1 - W_2}{2 \tan\theta} \qquad 2.8.6$$

where W is the channel width and the subscripts 1 and 2 refer to sections upstream and downstream from the contraction. The contraction angle is θ and should not exceed 12°. The relationship between the channel widths and depths, y, can be determined from the continuity of the flow $W_1 y_1 V_1 = W_2 y_2 V_2 = Q$ or

$$\frac{W_1}{W_2} = \left(\frac{y_2}{y_1}\right)^{3/2} \left(\frac{Fr_2}{Fr_1}\right) \qquad 2.8.7$$

For an expansion, Rouse et al. (1951) have found experimentally that the most satisfactory boundary form is given by

$$\frac{z}{W_1} = \frac{1}{2} \left(\frac{x}{W_1 Fr_1}\right)^{3/2} + \frac{1}{2} \qquad 2.8.8$$

where x is the longitudinal distance measured from the start of the expansion or outlet section and z is the lateral coordinate measured from the channel centerline. A boundary developed from this equation diverges indefinitely. Therefore, for practical purposes, the divergent walls are followed by a transition to parallel lines.

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2.A1.0 BRIDGE CONSTRICTIONS WITH NO BACKWATER (Neglecting energy losses)

A stream is rectangular in shape and 100 ft wide. The design discharge is 5000 cfs and the uniform depth for this discharge is 10 ft. Neglecting energy losses *what is the maximum amount of constriction that can be imposed without causing backwater at the design discharge?*

The upstream flow rate per unit width is

$$q = \frac{Q}{W} = \frac{5000}{100} = 50 \text{ cfs/ft}$$

the average velocity is

$$V = \frac{Q}{A} = \frac{5000}{1000} = 5.00 \text{ fps}$$

and the specific head is (from Eq. 2.5.3)

$$H = \frac{V^2}{2g} + y = \frac{5^2}{64.4} + 10 = 10.39 \text{ ft}$$

According to Section 2.5.3, the maximum unit discharge that can occur with this specific head is (from Eq. 2.5.16)

$$q_{max} = \{g\ (\tfrac{2}{3}\ H)^3\}^{1/2}$$

$$= \{32.2(\tfrac{2}{3} \times 10.39)^3\}^{1/2}$$

$$= 103.4 \text{ cfs/ft}$$

Therefore, the width of channel which will accommodate this unit discharge is

$$W = \frac{Q}{q_{max}} = \frac{5000}{103.4} = 48.3 \text{ ft.}$$

and the amount of the constriction is 100 - 48.3 = 51.7 ft.

Note, as discussed on page II-41, this contraction could cause an undulating hydraulic jump downstream. When energy losses are considered there will be some backwater at this constriction. This backwater is evaluated by conventional methods such as those given in "Hydraulics of Bridge Waterways" (Federal Highway Administration, 1970).

2.A2.0 BACKWATER FROM A DOWNSTREAM DIVERSION DAM

A small diversion dam is to be placed across a stream downstream of a highway bridge. The purpose of the dam is to head up water for diversion into a canal. At the bridge, the design flood discharge was 5000 cfs. The river is 100 ft wide and has a uniform flow depth of 10 ft for the design discharge. *What is the maximum height of the dam that will not cause backwater at the bridge?*

The unit discharge in the river at design flood discharge is

$$q = \frac{Q}{W} = \frac{5000}{100} = 50 \text{ cfs/ft}$$

the velocity is

$$V = \frac{q}{y_o} = \frac{50}{10} = 5.00 \text{ fps}$$

and the specific head is (from Eq. 2.5.3)

$$H = \frac{V^2}{2g} + y_o = \frac{5^2}{2g} + 10 = 10.39 \text{ ft}$$

As a first approximation assume no energy loss in the reach. Then at the dam, the elevation of the total energy line is 10.39 ft above the bed (see Fig. 2.A2.1). At the dam,

$$H_{min} + \Delta Z_{max} = 10.39 \text{ ft}$$

that is, the dam can be built to a height of ΔZ_{max} which decreases the specific head at the dam to H_{min}. From Eq. 2.5.11

$$H_{min} = \frac{3}{2} y_c$$

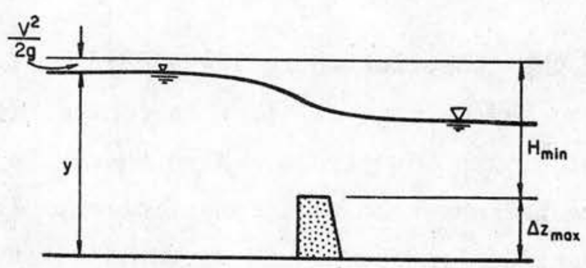


Fig. 2.A2.1 Backwater curve upstream of the dam.

and from Eq. 2.5.7

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{50^2}{32.2}\right)^{1/3} = 4.27 \text{ ft}$$

so

$$H_{min} = \frac{3}{2}\,(4.27) = 6.40 \text{ ft}$$

Thus

$$\Delta Z_m = 10.39 - 6.40 = 4.0 \text{ ft}$$

If the dam is built to a crest elevation 4.0 ft above the bed, critical flow will occur at the dam for a flow of 5000 cfs and the dam will cause no backwater.

How much backwater will the dam cause for a flow of 1000 cfs if the normal depth for this discharge is 5 ft and the dam height is 4.0 ft?

Upstream of the dam,

$$q = \frac{Q}{W} = \frac{1000}{100} = 10 \text{ cfs/ft}$$

and

$$V_o = \frac{q}{y_o} = \frac{10}{5} = 2 \text{ fps}$$

At the dam the flow is critical so from Eq. 2.5.7

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{10^2}{32.2}\right)^{1/3} = 1.46 \text{ ft}$$

and from Eq. 2.5.11

$$H_{min} = \frac{3}{2}\, y_c$$

$$= \frac{3}{2}\,(1.46) = 2.19 \text{ ft}$$

The specific head upstream of the dam is then (assuming no energy loss)

$$H = H_{min} + \Delta Z$$

or

$$H = 2.19 + 4.00 = 6.19 \text{ ft}$$

Also, the specific head upstream of the dam is (from Eq. 2.5.4)

$$H = \frac{q^2}{2gy^2} + y$$

Therefore

$$\frac{q^2}{2gy^2} + y = 6.19$$

or

$$y^3 - 6.19\, y^2 + \frac{10^2}{64.4} = 0$$

The solution is

$$y = 6.14 \text{ ft}$$

As the normal depth is only 5 ft, the backwater is

$$\Delta y = 6.14 - 5.00 = 1.14 \text{ ft}$$

That is, the depth upstream of the dam is increased 1.14 ft by the 4.0-ft high dam when the flow is 1000 cfs.

2.A3.0 STANDARD STEP METHOD FOR BACKWATER COMPUTATIONS

Consider an abrupt change of slope in a lined rectangular channel 100 ft in width. The discharge is 5000 cfs. Upstream of the slope change, the flow is at the normal depth of 10 ft. The normal depth in the downstream reach is 3 ft. Manning's n for both reaches is 0.012.

In both the upstream and downstream reaches, the flow per unit width is

$$q = \frac{5000}{100} = 50 \text{ cfs/ft}$$

and the critical depth is (from Eq. 2.5.7)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{50^2}{32.2}\right)^{1/3} = 4.27 \text{ ft.}$$

Upstream where the flow is at normal depth, $y = y_o > y_c$ so the flow is tranquil here.

The bed slope is obtained from Manning's equation for normal flow (from Eq. 2.3.20)

$$S_o = \frac{n^2 V_o^2}{2.21 R_o^{4/3}}$$

Here

$$V_o = \frac{q}{y_o} = \frac{50}{10} = 5.00 \text{ fps}$$

$$A_o = y_o W = 10\ (100) = 1000 \text{ sq ft}$$

$$P_o = 2y_o + W = 2\ (10) + 100 = 120 \text{ ft}$$

$$R_o = \frac{A_o}{P_o} = \frac{1000}{120} = 8.33 \text{ ft}$$

$$S_o = \frac{(0.012 \times 5)^2}{2.21(8.33)^{4/3}} = 0.000095$$

Downstream where the flow has attained its normal depth, $y = y_o < y_c$ so in the downstream reach flow is supercritical.

The bed slope in the downstream reach is obtained as follows:

$$V_o = \frac{q}{y_o} = \frac{50}{3} = 16.67 \text{ fps}$$

$$A_o = y_o W = 3 \times 100 = 300 \text{ sq ft}$$

$$P_o = 2y_o + W = 2\ (3) + 100 = 106\ \text{ft}$$

$$R_o = \frac{A_o}{P_o} = \frac{300}{106} = 2.83\ \text{ft}$$

$$S_o = \frac{n^2\ V_o^2}{2.21\ R_o^{4/3}} = \frac{(0.012\ \text{x}\ 16.67)^2}{2.21\ (2.83)^{4/3}}$$

$$= 0.004523$$

At the change in slope the flow must pass through the critical depth. Then, in the reach immediately upstream $y_o > y > y_c$ so the backwater curve in this reach is an M_2 type (Table 2.7.1).

Downstream, $y_o < y < y_c$ so the backwater curve in this reach is a S_2 type (Table 2.7.1).

The two backwater curves are sketched in Fig. 2.A3.1.

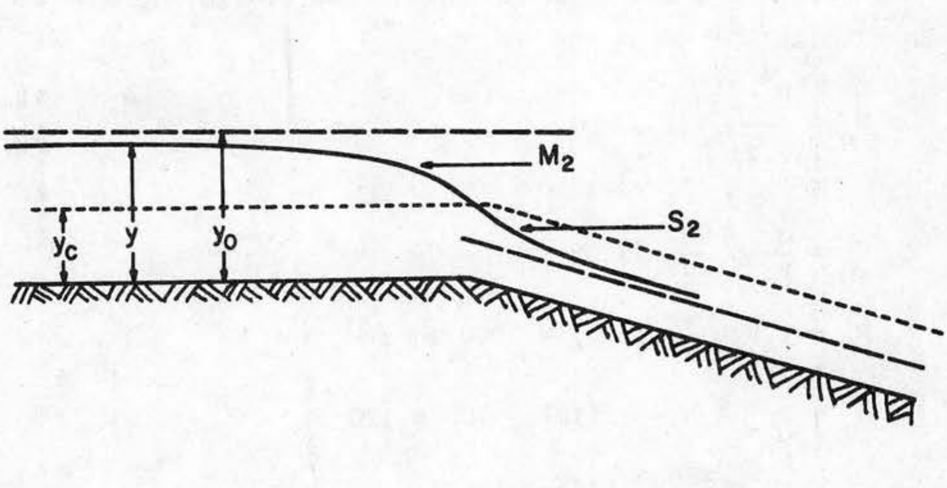


Fig. 2.A3.1 Sketch of backwater curves.

For the upstream reach, flow is subcritical so the standard step method computations start at the change in slope and proceed upstream.

At the change in slope

$$y = y_c = 4.27 \text{ ft}$$

$$A = yW = 4.27\ (100) = 427 \text{ sq ft}$$

$$V = \frac{q}{y} = \frac{50}{4.27} = 11.71 \text{ fps}$$

$$\frac{V^2}{2g} = \frac{11.71^2}{64.4} = 2.13 \text{ ft}$$

$$H = \frac{V^2}{2g} + y = 4.27 + 2.13 = 6.40 \text{ ft}$$

$$P = 2y + W = 2\ (4.26) + 100 = 108.52 \text{ ft}$$

$$R = \frac{A}{P} = \frac{427}{108.54} = 3.93 \text{ ft}$$

$$S_f = \frac{n^2\ V^2}{2.21\ R^{4/3}} = \frac{(0.012 \times 11.71)^2}{2.21(3.93)^{4/3}} = 0.001438$$

Let's compute the distance upstream to where the flow is 4.50 ft deep. The flow conditions at this section are computed with the same equations employed at the change in slope section; i.e.,

$$y = 4.50 \text{ ft}$$

$$A = 4.50\ (100) = 450 \text{ sq ft}$$

$$V = \frac{50}{4.50} = 11.11 \text{ fps}$$

$$\frac{V^2}{2g} = \frac{11.11^2}{64.4} = 1.92 \text{ ft}$$

$$H = 4.50 + 1.92 = 6.42 \text{ ft}$$

$$P = 2\ (4.50) + 100 = 109 \text{ ft}$$

$$R = \frac{450}{109} = 4.13 \text{ ft}$$

$$S_f = \frac{(0.012 \times 11.11)^2}{2.21\ (4.13)^{4/3}} = 0.001214$$

Now between these two sections where y = 4.27 ft and y = 4.50 ft the average friction slope is

$$S_{f_{ave}} = \frac{0.001438 + 0.001214}{2} = 0.001326$$

The distance between the two sections is (from Eq. 2.7.11)

$$\Delta L = \frac{H_2 - H_1}{S_o - S_{f_{ave}}}$$

$$= \frac{6.40 - 6.42}{.000095 - 0.001326} = 16.2 \text{ ft}$$

That is, the section where the depth is 4.50 ft is 16.2 ft upstream of the section where the slope changes.

In a similar manner, the distance between the sections where the depths are 4.50 ft and 5.00 ft (arbitrary choice) is computed. The results are listed in Table 2.A3.1. It is found that the flow is normal a distance approximately 44,000 ft above the change in slope.

The backwater calculations for the downstream reach are also presented in Table 2.A3.1. Here the computations start at the change in section and proceed downstream because the flow is supercritical. The computations show that the normal depth is reached approximately 1600 ft below the change in slope.

Table 2.A3.1 Computation of the backwater curve.

y ft	A sq ft	V fps	$V^2/2g$ ft	H ft	P ft	R ft	n	S_f	$S_{f_{ave}}$	$H_2 - H_1$ ft	ΔL ft	L ft
At the change in slope												
4.27	427	11.71	2.13	6.40	108.5	3.93	0.012	.001438	--	--	--	--
S_2 backwater curve												
4.20	420	11.90	2.20	6.40	108.4	3.87	0.012	.001519	.001478	-0.00	0.0	0.0
4.00	400	12.50	2.43	6.43	108	3.70		.001779	.001649	-0.03	10.9	10.9
3.50	350	14.28	3.17	6.67	107	3.27		.002738	.002258	-0.24	105.96	116.86
3.00	300	16.67	4.31	7.31	106	2.83		.004523	.003630	-0.64	716.7	833.56
M_2 backwater curve												
4.50	450	11.11	1.92	6.42	109	4.13	0.012	.001214	.001326	-0.02	16.2	16.2
5.00	500	10.00	1.55	6.55	110	4.54		.000867	.001040	-0.13	137.6	153.8
6.00	600	8.33	1.08	7.08	112	5.36		.000482	.000674	-0.53	914.6	1068.4
8.00	800	6.25	0.61	8.61	116	6.90		.000194	.000338	-1.53	6296.3	7364.7
10.00	1000	5.00	0.39	10.39	120	8.33		.000095	.000144	-1.78	36326.5	43691.2

$A = Wy$ $\quad V = \frac{Q}{A}$ $\quad P = 2y + W$ $\quad R = \frac{A}{P}$ $\quad S_f = \frac{n^2 V^2}{2.21 R^{4/3}}$ $\quad \Delta L = \frac{H_2 - H_1}{S_o - S_{f_{ave}}}$ $\quad L = \sum \Delta L$

2.A4.0 ENERGY AND MOMENTUM COEFFICIENTS FOR RIVERS

In open channel flow problems it is common to assume that the energy coefficient α and the momentum coefficient β are unity. *What are values of* α *and* β *for river channels?*

From Eqs. 2.2.29 and 2.2.28

$$\alpha = \frac{1}{V^3 A} \int_A v^3 \, dA \qquad 2.A4.1$$

and

$$\beta = \frac{1}{V^2 A} \int_A v^2 \, dA \qquad 2.A4.2$$

In many wide channels, the distribution of velocity in the vertical is given by Eq. 2.3.15 which is

$$\frac{v}{V_*} = 2.5 \ln(30.2 \frac{y}{k_s}) \qquad 2.A4.3$$

for fully turbulent flow (x = 1.0). The average velocity in the vertical is

$$V = \frac{1}{y_o} \int_0^{y_o} v \, dy \qquad 2.A4.4$$

and by employing Eq. 2.A4.3

$$V \simeq \frac{2.5V_*}{y_o - \delta} \int_\delta^{y_o} \ln(\frac{y}{\delta}) \, dy \qquad 2.A4.5$$

Here, the upper limit of integration is y_o, the depth of flow and the lower limit is

$$\delta = \frac{k_s}{30.2} \qquad 2.A4.6$$

the value of y for which Eq. 2.A4.3 gives a zero velocity.
The integration of Eq. 2.A4.5 yields

$$\frac{V}{V_*} = 2.5\{\frac{y_o}{y_o - \delta} (\ln (\frac{y_o}{\delta}) - 1)\} \qquad 2.A4.7$$

For a vertical section of unit width, the momentum coefficient is

$$\beta' = \frac{1}{V^2 (y_o - \delta)} \int_{\delta}^{y_o} v^2 \, dy \qquad 2.A4.8$$

If we substitute Eqs. 2.A4.3 and 2.A4.7 into Eq. 2.A4.8 and integrate the result is the expression

$$\beta' = \frac{1}{(\ln 11.11 \frac{y_o}{k_s})^2} \frac{y_o}{y_o - \delta} \{(\ln \frac{y_o}{\delta})^2 - 2 \ln \frac{y_o}{\delta} + 2 - \frac{2\delta}{y_o}\} \qquad 2.A4.9$$

Similarly, the energy coefficient for a vertical section unit width is

$$\alpha' = \frac{1}{V^3 (y_o - \delta)} \int_{\delta}^{y_o} v^3 \, dy \qquad 2.A4.10$$

or

$$\alpha' = \frac{1}{(\ln 11.11 \frac{y_o}{k_s})^2} \frac{y_o}{y_o - \delta} \{(\ln \frac{y_o}{\delta})^2 - 3(\ln \frac{y_o}{\delta})^2$$

$$+ 6 \ln \frac{y_o}{\delta} - 6 + \frac{6\delta}{y_o}\} \qquad 2.A4.11$$

These equations (Eqs. 2.A4.9 and 2.A4.10) are rather complex, so a graph of α' and β' vs y_o/k_s has been prepared. The relations are shown in Fig. 2.A4.1.

For the entire river cross section (shown in Fig. 2.A4.2) Eq. 2.2.29 can be written

$$\alpha = \frac{1}{(\frac{Q}{A})^3 A} \int_0^W \int_0^{y_o} v^3 \, dy \, dz \qquad 2.A4.12$$

where W is the top width of the section, z is the lateral location of any vertical section, y_o is the depth of flow at location z, and v is the local velocity at the position y,z. The total discharge is Q and the total cross-sectional area is A.

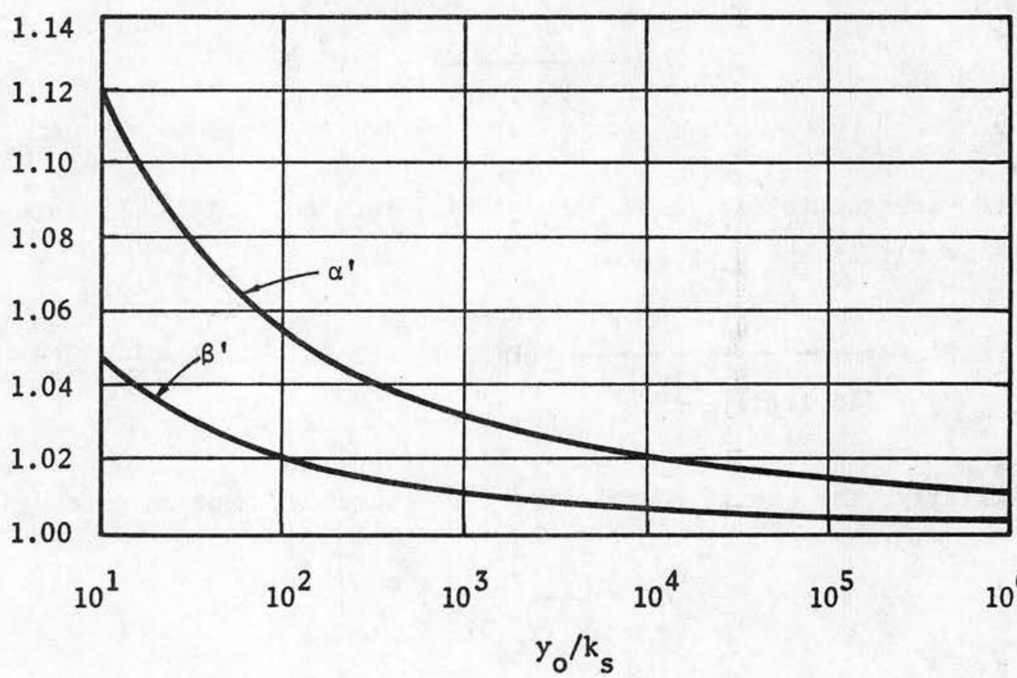


Fig. 2.A4.1 Energy and momentum coefficients for a unit width of river.

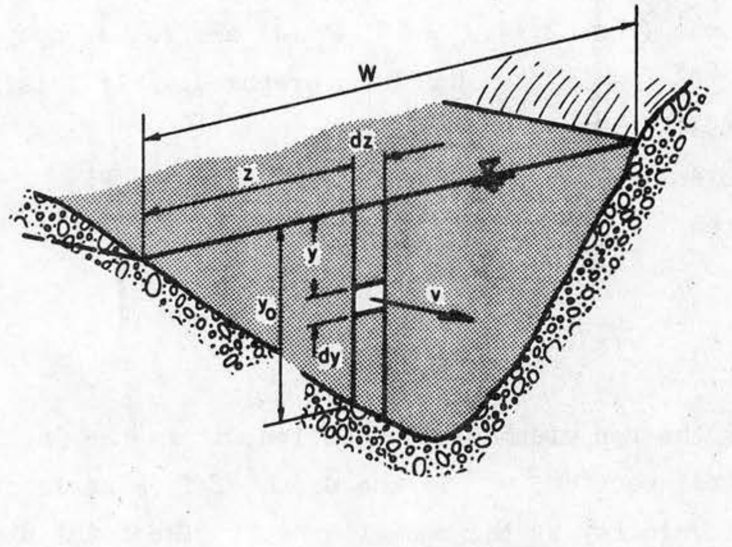


Fig. 2.A4.2 The river cross section.

In Eq. 2.A4.12 the portion

$$\int_0^{y_o} v^3 \, dy$$

is recognized as the integral portion of Eq. 2.A4.10. That is,

$$\int_0^{y_o} v^3 \, dy \simeq \int_\delta^{y_o} v^3 \, dy$$

and

$$\int_\delta^{y_o} v^3 \, dy = \alpha' V^3 (y_o - \delta) \qquad 2.A4.13$$

Here α' is the energy coefficient for the vertical section dz wide and y_o deep, V is the depth-averaged velocity in this vertical section and $\delta = k_s/30.2$ (from Eq. 2.A4.6).

Now, Eq. 2.A4.12 can be written

$$\alpha = \frac{A^2}{Q^3} \int_0^W \alpha' V^3 (y_o - \delta) dz \qquad 2.A4.14$$

Except for cases of low flow in gravelbed rivers, the term δ is very small compared to y_o so

$$\alpha = \frac{A^2}{Q^3} \int_0^W \alpha' V^3 y_o dz \qquad 2.A4.15$$

The discharge at a river cross section is determined in the field by measuring the local depth and two local velocities at each of approximately 20 vertical sections. In accordance with this general stream gaging procedure, Eq. 2.A4.15 should be written

$$\alpha = \frac{A^2}{Q^3} \sum_i \alpha_i' V_i^3 y_{oi} \Delta zi$$

or

$$\alpha = \frac{A^2}{Q^3} \sum_i \alpha_i' V_i^2 \Delta Q_i \qquad 2.A4.16$$

Here, the subscript i refers to the i-th vertical section, and ΔQ_i is the river discharge associated with the i-th vertical or

$$\Delta Q_i = V_i y_{oi} \Delta z_i$$

In a similar manner, the expression for β is

$$\beta = \frac{A}{Q^2} \sum_i \beta_i' V_i \Delta Q_i \qquad 2.A4.17$$

Now, *with Eqs. 2.A4.16 and 2.A4.17, and Fig. 2.A4.1 we are in a position to compute* α *and* β *for any river cross section given the discharge measurement notes.* An example is given below.

The information in Table 2.A4.1 is taken from the discharge measurement notes for Measurement No. 16 on the Rio Tigre at Las Piedritas in Venezuela.

The discharge measurement was made on August 18, 1969 during the peak flood event for the year. From Table 2.A4.1, the following values are obtained:

$Q = 5370$ cfs

$A = 1485$ sq ft

$W = 163$ ft

$\Sigma V_i \Delta Q_i = 21{,}070 \text{ ft}^4/\text{sec}^2$

$\Sigma V_i^2 \Delta Q_i = 85{,}500 \text{ ft}^5/\text{sec}^3$

The bed material at this gaging station[1] has a D_{50} of 0.33 mm and a D_{67} of 0.45 mm and a gradation coefficient G of 3.27. If the value of D_{67} is used for k_s, then for $y_o = 12.8$ ft (the maximum depth)

$$\frac{y_o}{k_s} = \frac{12.8}{0.45} (304.8) = 8700$$

and for $y_o = 1.1$ ft (the smallest non-zero depth)

$$\frac{y_o}{k_s} = \frac{1.1}{0.45} (304.8) = 750$$

[1]Simons, D. B., Richardson, E. V., Stevens, M. A., Duke, J. H., and Duke, V. C., Geometric and hydraulic properties of the rivers, Hydrology Report, Vol. III, Venezuelan International Meteorological and Hydrological Experiment, Civil Engineering Department, Colorado State University, October, 1971.

Table 2.A4.1 Discharge measurement notes[1].

y_{oi} ft	Δz_i ft	V_i fps	ΔA_i sq ft	ΔQ_i cfs	$V_i \Delta Q_i$ ft^4/sec^2	$V_i^2 \Delta Q_i$ ft^5/sec^3
0.0	4.0	0.00	0.0	0.00	0.00	0.00
1.1	8.0	0.98	8.8	8.62	8.45	8.28
2.6	8.0	0.54	20.8	11.23	6.06	3.27
4.5	8.0	0.64	36.0	23.04	14.75	9.44
8.5	8.0	2.40	68.0	163.20	391.68	940.03
11.0	8.0	3.17	88.0	278.96	884.30	2803.24
11.6	8.0	4.02	92.8	373.06	1499.70	6028.80
12.0	8.0	4.06	96.0	389.76	1582.43	6424.65
12.8	8.0	3.78	102.4	387.07	1463.12	5530.61
12.6	8.0	3.74	100.8	376.99	1409.94	5273.19
12.4	8.0	3.78	99.2	374.98	1417.42	5357.86
11.6	8.0	4.71	92.8	437.09	2058.69	9696.45
11.4	8.0	4.30	91.2	392.16	1686.29	7251.04
10.8	8.0	4.90	86.4	423.36	2074.46	10164.87
10.6	8.0	4.63	84.8	392.62	1817.83	8416.56
10.9	8.0	4.32	87.2	376.70	1627.34	7030.13
11.4	8.0	3.89	91.2	354.77	1380.06	5368.42
11.8	8.0	3.10	94.4	292.64	907.18	2812.27
9.8	8.0	3.02	78.4	236.77	715.05	2159.44
6.4	7.0	1.69	44.8	75.71	127.95	216.24
3.8	5.5	0.00	20.9	0.00	0.00	0.00
0.0	2.5	0.00	0.0	0.00	0.00	0.00
Total	163.0		1484.9	5368.74	21072.71	85494.77

[1]Simons, D. B., Richardson, E. V., Stevens, M. A., Duke, J. H., and Duke, V. C., Stream flow, groundwater and ground response data, Hydrology Report, Vol. II, Venezuelan International Meteorological and Hydrological Experiment, Civil Engineering Dept., Colorado State University, August 1971.

If we use a mean y_o/k_s of approximately 5000, then from Fig. 2.A4.1, the average values for the energy and momentum coefficients are

$$\alpha' = 1.024$$

and

$$\beta' = 1.008$$

As it has been assumed that α' and β' are constant across the river (for convenience), Eqs. 2.A4.16 and 2.A4.17 become

$$\alpha = \alpha' \frac{A^2}{Q^3} \sum_i V_i^{\,2} \Delta Q_i \qquad 2.A4.18$$

and

$$\beta = \beta' \frac{A}{Q^2} \sum_i V_i \Delta Q_i \qquad 2.A4.19$$

With the values computed in Table 2.A4.1

$$\alpha = 1.024 \frac{(1485)^2}{(5370)^3} (85500) = 1.247$$

$$\beta = 1.008 \frac{(1485)}{(5370)^2} (21070) = 1.094$$

These values for α (1.247) and β(1.094) differ from unity by appreciable amounts. The difference may be important in many river channel calculations. *If no data are available, the assumptions that $\alpha = 1.25$ and $\beta = 1.1$ should be used for river channels.*

2.A5.0 AVERAGE PRESSURE AND ELEVATION AT A RIVER CROSS SECTION

In the absence of heat transfer, and shaft and shear work, the energy convected through a cross section of river with area A is

$$\int_A \gamma\left(\frac{v^2}{2g} + z\right)v\ dA$$

and the pressure work done on this cross section is

$$\int_A pv\ dA$$

The sum of the pressure work and the convected energy is

$$\int_A \gamma\left(\frac{v^2}{2g} + \frac{p}{\gamma} + z\right)v\ dA$$

Instead of using the definition Eq. 2.2.28, we could write

$$\int_A \gamma\left(\frac{v^2}{2g} + \frac{p}{\gamma} + z\right)v\ dA = \left\{\frac{\alpha V^2}{2g} + \Omega(z_o + y_o)\right\}\gamma Q \qquad 2.A5.1$$

where α is the kinetic energy correction factor defined by Eq. 2.2.29 and Ω is a correction factor to be applied to the piezometric head. The terms z_o and y_o are the elevation of the bed above datum and the depth of flow respectively. According to Eqs. 2.2.29 and 2.A5.1, the expression for Ω must be

$$\Omega = \frac{1}{(z_o + y_o)VA} \int_A \left(\frac{p}{\gamma} + z\right)v\ dA \qquad 2.A5.2$$

In straight reaches of river, the piezometric head does not vary appreciably from point to point in a cross section. Then the piezometric head at any point on the cross section can be used as the reference piezometric head. For example, at the water surface

$$\frac{p}{\gamma} + z = 0 + z_o + y_o \qquad 2.A5.3$$

Using this equation in Eq. 2.A5.2, we obtain

$$\Omega = \frac{1}{(z_o + y_o)VA} \int_A (z_o + y_o) v \, dA = \frac{1}{VA} \int_A v \, dA$$

or

$$\Omega = 1 \qquad 2.A5.4$$

If the water surface at the cross section is not horizontal (as in a bend), then the piezometric head should be referenced to the point on the water surface which is at the average elevation of the water surface. Then $\Omega = 1$ for this case also.

In general, the assumption that

$$\Omega = 1$$

is satisfactory for rivers. Therefore, Eq. 2.2.34 can be written

$$\frac{\alpha_1 V_1^2}{2g} + y_1 + z_1 = \frac{\alpha_2 V_2^2}{2g} + y_2 + z_2 + H_L \qquad 2.A5.5$$

for a reach of river. Here $y_1 + z_1$ is the water surface level at Section 1 and $y_2 + z_2$ is the water surface elevation at Section 2.

Chapter III

FUNDAMENTALS OF ALLUVIAL CHANNEL FLOW

3.1.0 INTRODUCTION

Most rivers that a highway will cross or encroach upon are alluvial. That is, the rivers are formed in cohesive or non-cohesive materials that have been and can be transported by the stream. The non-cohesive material generally consists of silt (0.004 mm - 0.062 mm), sand (0.062 mm - 2.0 mm), gravel (2.0 mm - 64 mm), or cobbles (64 mm - 256 mm), or any combination of these sizes. Silt generally is not present in appreciable quantities with non-cohesive stream boundaries. Cohesive boundary material consists of clays (sizes less than .004 mm) forming a binder with silts and sand. Under most conditions clays are more resistant to erosion than non-cohesive material.

In alluvial rivers, the channel bed can scour to undermine bridge piers and abutments; or the sediment in transport can deposit in the cross section, decreasing the flow capacity of the bridge opening, approach channel or the encroached channel. Bed configuration and resistance to flow in alluvial rivers are a function of the flow and can change to increase or decrease the water surface level. The river channel can shift its location so that the bridge is unfavorably located with respect of the direction of flow. The moveable boundary of the alluvial river thus adds another dimension to the design and environmental problems associated with bridge crossings. Therefore, the design of highway crossings and encroachments in the river environment requires knowledge of the mechanics of alluvial channel flow.

This chapter presents the fundamentals of alluvial channel flow. It covers flow in sandbed channels, prediction of bed forms, Manning's n for sandbed and other natural streams, how bed-form changes affect highways in the river environment, properties of alluvial material, methods of measuring properties of alluvial materials, beginning of motion, sediment transport, flow in coarse-material streams and modeling alluvial channel flow. These fundamentals of alluvial channel flow are used in later chapters to develop design considerations for highway crossings and encroachments in river environments.

3.2.0 FLOW IN SANDBED CHANNELS

3.2.1 Introduction

Most streams flow on sandbeds for the greater part of their length and nearly all large rivers have sandbeds. Thus there are potentially many more opportunities for highway crossings or encroachments on sandbed streams than in cohesive or gravel streams. In sandbed rivers, the sand material is easily eroded and is continually being moved and shaped by the flow. The mobility of the sandbed creates problems for the safety of any structure placed in or over the stream, for the protection of private property along these streams and in the preservation and enhancement of the stream environment.

The interaction between the flow of the water-sediment mixture and the sandbed creates different bed configurations which change the resistance to flow and rate of sediment transport. The gross measures of channel flow, such as the flow depth, river stage, bed elevation and flow velocity change, with different bed configurations. In the extreme case, the change in bed configuration can cause a three-fold change in resistance to flow and a 10-to-15 fold change in concentration of bed-material transport. For a given discharge and channel width, a three-fold increase in Manning's n results in a doubling of the flow depth.

The interaction between the flow and bed material and the interdependency among the variables makes the analysis of flow in alluvial sandbed streams extremely complex. However, with an understanding of the different types of bed forms that may occur and a knowledge of the resistance to flow and sediment transport associated with each bed form, the engineer can analyze alluvial channel flow.

3.2.2 Bed configuration

The bed configurations (roughness elements) that may form in an alluvial channel *are plane bed without sediment movement, ripples, ripples on dunes, dunes, plane bed with sediment movement, antidunes, and chutes and pools*. These bed configurations are listed in their order of occurrence with increasing values of stream power $(V\gamma y_o S)$ for bed materials having D_{50} less than 0.6 mm. For bed materials coarser than 0.6 mm, dunes form instead of ripples after beginning of motion at small values of stream power. The typical forms of each bed configuration are shown in Fig. 3.2.1 and the relation of bed form to water surface is shown in Fig. 3.2.2.

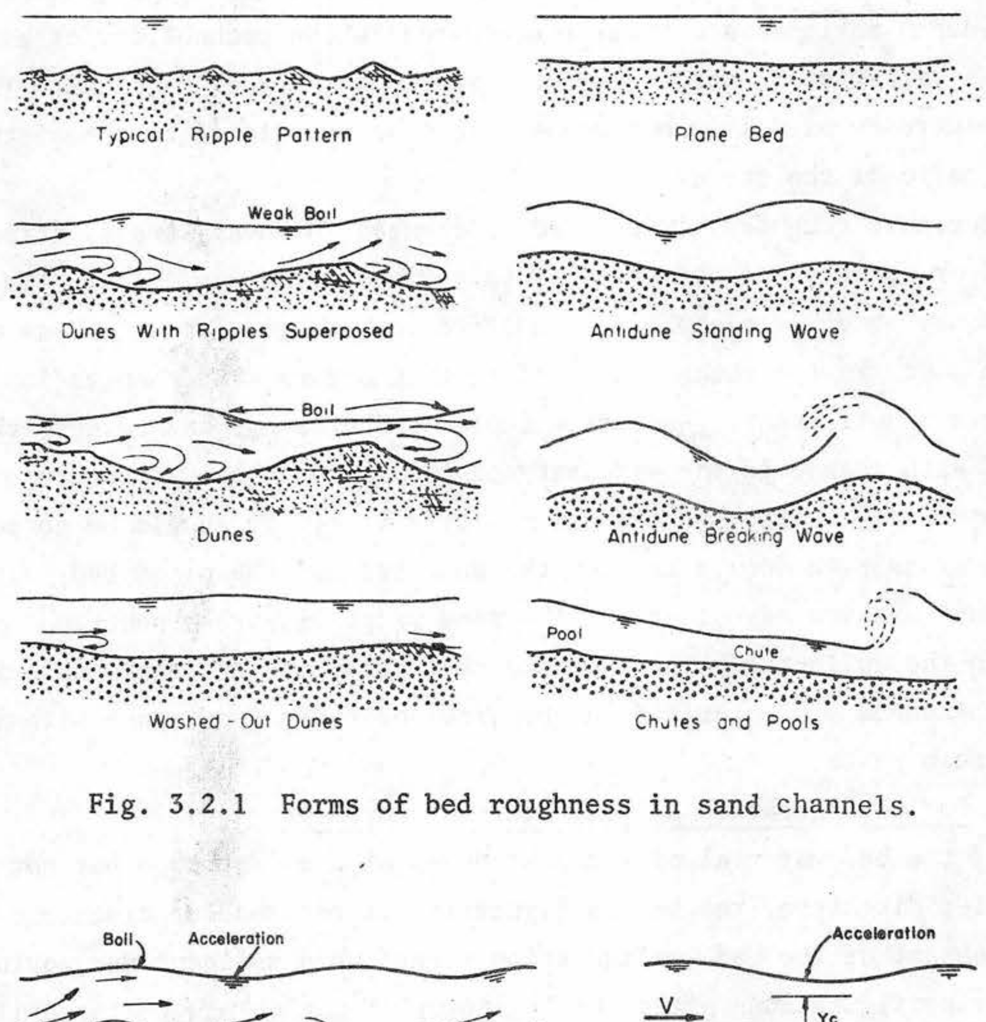


Fig. 3.2.1 Forms of bed roughness in sand channels.

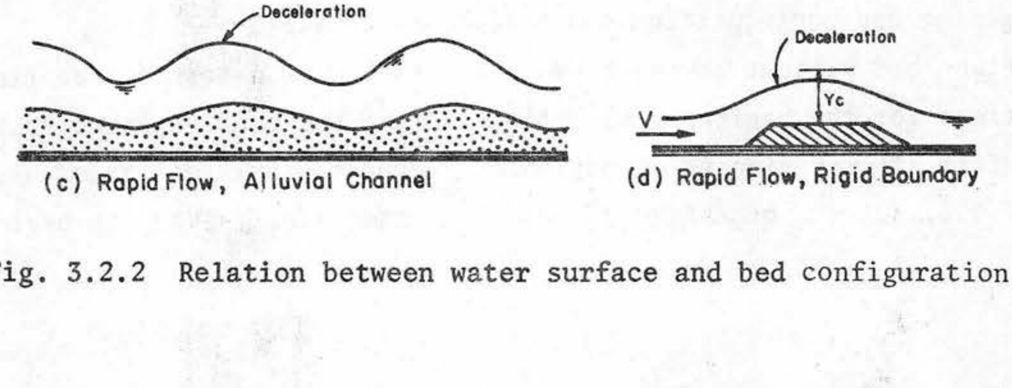


Fig. 3.2.2 Relation between water surface and bed configuration.

The different forms of bed-roughness are not mutually exclusive in time and space in a stream. *Bed-roughness elements may form side-by-side in a cross section or reach of a natural stream, giving a multiple roughness; or they may form in time sequence, producing variable roughness.*

Multiple roughness is related to variations in shear stress ($\gamma y_o S$) and stream power ($V\gamma y_o S$) in a channel cross section. The greater the width-depth ratio of a stream, the greater is the probability of a spatial variation in shear stress, stream power or bed material. Thus, the occurrence of multiple roughness is closely related to the width-depth ratio of the stream.

Variable roughness is related to changes in shear stress, stream power, or reaction of bed material to a given stream power over time. A commonly observed example of the effect of changing shear stress or stream power is the change in bed form that occurs with changes in depth during a runoff event. Another example is the change in bed form that occurs with change in the viscosity of the fluid as the temperature or concentration of fine sediment varies over time. It should be noted that a transition occurs between the dune bed and the plane bed; either bed configuration may occur for the same value of stream power.

In the following paragraphs bed configurations and their associated flow phenomena are described in the order of their occurrence with increasing stream power.

3.2.3 Bed configuration without sediment movement

If the bed material of a stream moves at one discharge but not at a smaller discharge, the bed configuration at the smaller discharge will be a remnant of the bed configuration formed when sediment was moving. The bed configurations after the beginning of motion may be those illustrated in Fig. 3.2.1, depending on the flow and bed material. *Prior to the beginning of motion, the problem of resistance to flow is one of rigid-boundary hydraulics. After beginning of motion, the problem relates to defining bed configuration and resistance to flow.*

Plane bed without movement has been studied to determine the flow conditions for the beginning of motion and the bed configuration that would form after beginning of motion. *In general, Shields' relation, Fig. 3.2.3, for the beginning of motion is adequate.* After the beginning of motion, for flat slopes and low velocity, the plane bed will change to ripples for sand material smaller than 0.6 mm, and to dunes for coarser material. Resistance to flow is small for a plane bed without sediment movement and is due solely to the sand grain roughness. Values of Manning's n range from 0.012 to 0.014 depending on the size of the bed material.

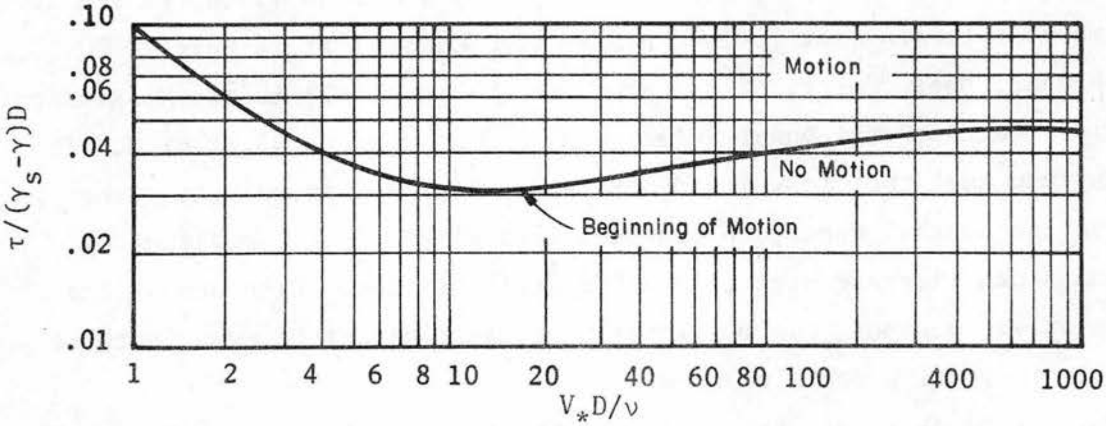


Fig. 3.2.3 Shields' relation for beginning of motion [Adapted from Gessler (1971)].

3.2.4 Ripples

Ripples are small triangle-shaped elements having gentle upstream slopes and steep downstream slopes. Length ranges from 0.4 ft to 2 ft and height from 0.02 ft to 0.2 ft (See Fig. 3.2.1). Resistance to flow is relatively large (with Manning's n ranging from 0.018 to 0.030). There is a relative roughness effect associated with a ripple bed and the resistance to flow decreases as depth increases. The ripple shape is independent of sand size and at large values of Manning's n the magnitude of grain roughness is small relative to the form roughness. The length of the separation zone downstream of the ripple crest is about ten times the height of the ripple. *Ripples cause very little, if any, disturbance on the water surface, and the flow contains very little suspended bed material.* The bed-material discharge concentration is small, ranging from 10 to 200 ppm.

3.2.5 Dunes

When the shear stress or the stream power is increased for a bed having ripples (or, a plane bed without movement if the bed material is coarser than 0.6 mm), waves called dunes form on the bed. At smaller shear-stress values, the dunes have ripples superposed on their backs. These ripples disappear at larger shear values, particularly if the bed material is coarse sand with $D_{50} > 0.4$ mm.

Dunes are large triangle-shaped elements similar to ripples (Fig. 3.2.1). Their lengths range from two feet to many hundreds of feet, depending on the scale of the flow system. Dunes that formed in the eight-foot wide flume used by Simons and Richardson (1963) ranged from two feet to ten feet in length and from 0.2 to 1 ft in height; whereas, those described by Carey and Keller (1957) in the Mississippi River were several hundred feet long and as much as 40 ft high. The maximum amplitude to which dunes can develop is approximately the average depth. Hence, in contrast with ripples, the amplitude of dunes can increase with increasing depth of flow. With dunes, the relative roughness can remain essentially constant or even increase with increasing depth of flow.

Field observations indicate that dunes can form in any channel, irrespective of the size of bed material, if the stream power is sufficiently large to cause general transport of the bed material without exceeding a Froude number of unity.

Resistance to flow caused by dunes is large. Manning's n ranges from 0.020 to 0.040. The form roughness for flow with dunes is equal to or larger than the sand grain roughness.

Dunes cause large separation zones in the flow. These zones, in turn, cause large boils to form on the surface of the stream. Measurements of flow velocities within the separation zone show that velocities in the upstream direction exist that are 1/2 to 1/3 the average stream velocity. Boundary shear stress in the dune trough is sometimes sufficient to form ripples oriented in a direction opposite to that of the primary flow in the channel. With dunes, as with any tranquil flow over an obstruction, the water surface is out of phase with the bed surface (see Fig. 3.2.2).

3.2.6 Plane bed with movement

As the stream power of the flow increases further, the dunes elongate and reduce in amplitude. This bed configuration is called the transition or washed out dunes. The next bed configuration with *increasing stream power is plane bed with movement.* Dunes of fine sand (low fall velocity) are washed out at lower values of stream power than are dunes of coarser sand. With coarse sands larger slopes are required to effect the change from transition to the plane bed and the result is larger velocities and larger Froude numbers. In flume studies with fine sand, the plane-bed condition commonly exists after the transition and persists over a wide range of Froude numbers ($0.3 \leq F_r \leq 0.8$). If the sand is coarse and the depth is shallow, however, transition may not terminate until the Froude number is so large that the subsequent bed form may be antidunes rather than plane bed. In natural streams, because of their greater depths, the change from transition to plane bed may occur at a much lower Froude number than in flumes.

3.2.7 Antidunes

Antidunes form as a series or train of inphase (coupled) symmetrical sand and water waves (Fig. 3.2.1). The height and length of these waves depend on the scale of the flow system and the characteristics of the fluid and the bed material. In the flume where the flow depth was about 0.5 ft deep, the height of the sand waves ranged from 0.03 ft to 0.5 ft. The height of the water waves was 1.5 to 2 times the height of the sand waves and the length of the waves, from crest to crest, ranged from five to ten feet. In natural streams, such as the Rio Grande or the Colorado River, much larger antidunes form. In these streams, surface waves 2 to 5 ft high and 10 to 40 ft long have been observed.

Antidunes form as trains of waves that gradually build up from a plane bed and a plane water surface. The waves may grow in height until they become unstable and break like the sea surf or they may gradually subside. The former have been called breaking antidunes, or antidunes; and the latter, standing waves. As the antidunes form and increase in height, they may move upstream, downstream, or remain stationary. Their upstream movement led Gilbert (1914) to name them antidunes.

Resistance to flow due to antidunes depends on how often the antidunes form, the area of the stream they occupy, and the violence and frequency of their breaking. *If the antidunes do not break, resistance to flow is about the same as that for flow over a plane bed. If many antidunes break, resistance to flow is larger because the breaking waves* *dissipate a considerable amount of energy.* With breaking waves, Manning's n may range from 0.012 to 0.020.

3.2.8 Chutes and pools

At very steep slopes, alluvial-channel flow changes to chutes and pools (Fig. 3.2.1). In the 8-foot-wide flume at Colorado State University, this type of flow and bed configuration was studied using fine sands. The flow consisted of a long chute (10 to 30 ft) in which the flow was rapid and accelerating followed by a hydraulic jump and a long pool. The chutes and pools moved upstream at velocities of about one to two feet per minute. The elevation of the sandbed varied within wide limits. Resistance to flow was large with Manning's n of 0.018 to 0.035.

The relation between stream power, velocity and bed configuration is shown in Fig. 3.2.4. This relation pertains to a fine sand and was determined in the 8-ft flume at Colorado State University.

3.2.9 Bars

In natural or field size channels, some other bed configurations are also found. These bed configurations are generally called bars and are related to the plan form geometry and the width of the channel. *Bars are bed forms having lengths of the same order as the channel width or greater and heights comparable to the mean depth of the generating flow.* Several different types of bars are observed. They are classified as:

(1) *Point Bars which occur adjacent to the convex banks of channel bends.* Their shape may vary with changing flow conditions, but they do not move relative to the bends.

(2) *Alternating Bars which occur in somewhat straighter reaches of channels and tend to be distributed periodically along the reach, with consecutive bars on opposite sides of the channel.* Their lateral extent is significantly less than the channel width. Alternating bars move slowly downstream.

(3) *Transverse Bars which also occur in straight channels.* They occupy nearly the full channel width. They occur both as isolated and as periodic forms along a channel, and move slowly downstream.

(4) *Tributary Bars which occur immediately downstream from points of lateral inflow into a channel.*

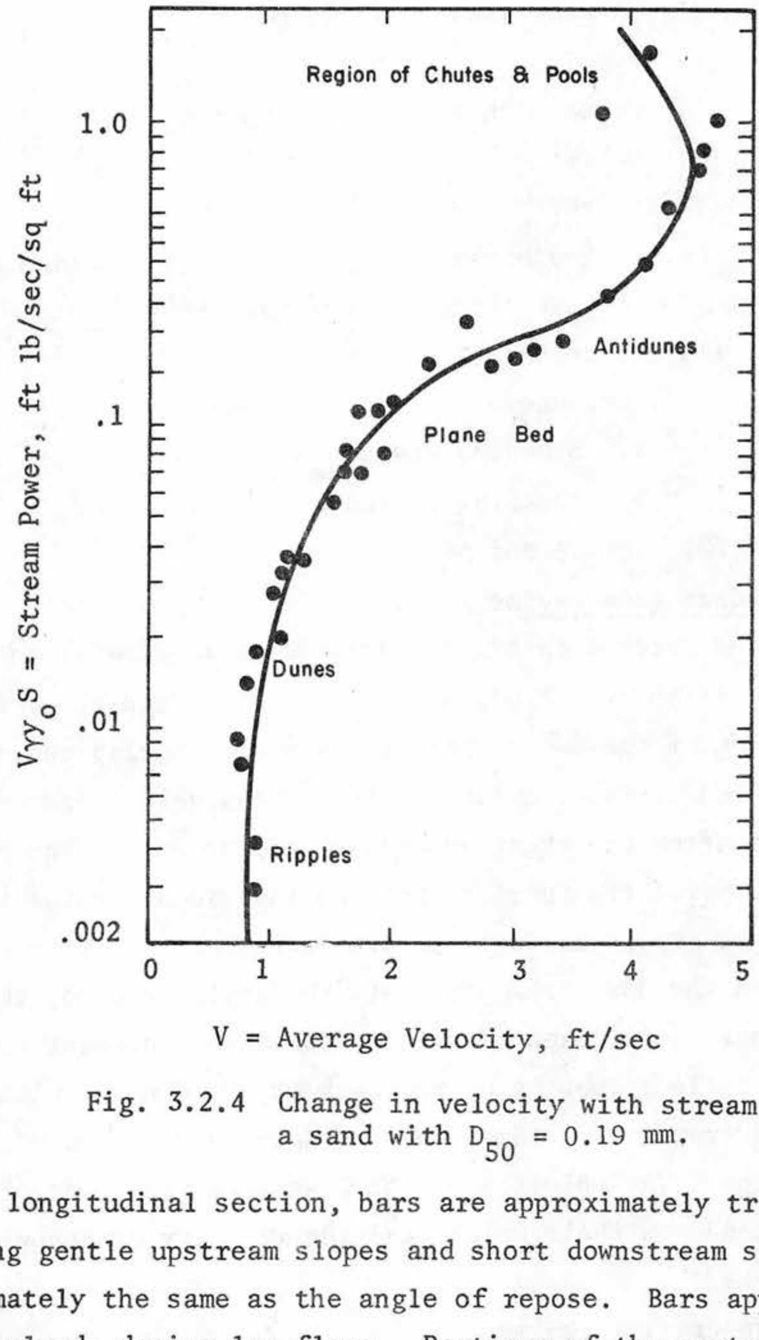


Fig. 3.2.4 Change in velocity with stream power for a sand with D_{50} = 0.19 mm.

In longitudinal section, bars are approximately triangular, with very long gentle upstream slopes and short downstream slopes that are approximately the same as the angle of repose. Bars appear as small barren islands during low flows. Portions of the upstream slopes of bars are often covered with ripples or dunes.

3.2.10 Regimes of flow in alluvial channels

The flow in alluvial channels is divided into two flow regimes with a transition zone between (Simons and Richardson, 1963). These two flow regimes are characterized by similarities in the shape of the bed configuration, mode of sediment transport, process of energy dissipation, and phase relation between the bed and water surfaces. The two regimes and their associated bed configurations are:

Lower flow regime (small stream power).

(1) Ripples.

(2) Dunes with ripples superposed.

(3) Dunes.

Transition zone.

The bed roughness ranges from dunes to plane bed or antidunes.

Upper flow regime (large stream power).

(1) Plane bed.

(2) Antidunes.

a. Standing waves.

b. Breaking antidunes.

(3) Chutes and pools.

3.2.11 Lower flow regime

In the lower flow regime, resistance to flow is large and sediment transport is small. The bed form is either ripples or dunes or some combination of the two. The water-surface undulations are out of phase with the bed surface, and there is a relatively large separation zone downstream from the crest of each ripple or dune. The most common mode of bed-material transport is for the individual grains to move up the back of the ripple or dune and avalanche down its face. After coming to rest on the downstream face of the ripple or dune, the particles remain there until exposed by the downstream movement of the dunes; then the cycle of moving up the back of the dune, avalanching, and storage is repeated. Thus, most movement of the bed-material particles is in steps. The velocity of the downstream movement of the ripples or dunes depends on their height and the velocity of the grains moving up their backs.

3.2.12 Upper flow regime

In the upper flow regime, resistance to flow is small and sediment transport is large. The usual bed forms are plane bed or antidunes. The water surface is inphase with the bed surface except when an antidune breaks, and normally the fluid does not separate from the boundary. A small separation zone may exist downstream from the crest of an antidune prior to breaking. Resistance to flow is the result of grain roughness with the grains moving, of wave formation and subsidence, and of energy dissipation when the antidunes break. The mode of sediment transport is for the individual grains to roll almost continuously downstream in sheets one or two grain diameters thick; however, when antidunes break, much bed material is briefly suspended, then movement stops temporarily and there is some storage of the particles in the bed.

3.2.13 Transition

The bed configuration in the transition zone is erratic. It may range from that typical of the lower flow regime to that typical of the upper flow regime, depending mainly on antecedent conditions. If the bed configuration is dunes, the depth or slope can be increased to values more consistent with those of the upper flow regime without changing the bed form; or, conversely, if the bed is plane, depth and slope can be decreased to values more consistent with those of the lower flow regime without changing the bed form. Often in the transition from the lower to the upper flow regime, the dunes decrease in amplitude and increase in length before the bed becomes plane (washed-out dunes). *Resistance to flow and sediment transport also have the same variability as the bed configuration in the transition.* This phenomenon can be explained by the changes in resistance to flow and, consequently, the changes in depth and slope as the bed form changes. Resistance to flow is small for flow over a plane bed; so the shear stress decreases and the bed form changes to dunes. The dunes cause an increase in resistance to flow which increases the shear stress on the bed and the dunes wash out forming a plane bed, and the cycle continues. It was the transition zone, which covers a wide range of shear values, that Brooks (1958) was investigating when he concluded that a single-valued function does not exist between velocity or sediment transport and the shear stress on the bed.

3.3.0 VARIABLES AFFECTING ALLUVIAL CHANNEL FLOW

Resistance to flow in alluvial channels is complicated by the large number of variables and by the interdependency of these variables.

It is difficult, especially in field studies, to tell which variables are governing the flow and which variables are the result of this flow.

The slope of the energy grade line of an alluvial stream illustrates the changing role of a variable. If a stream is in equilibrium with its environment, slope is an independent variable. In such a stream, the average slope over a period of years has adjusted so that the flow is capable of transporting only the amount of sediment supplied at the upper end of the stream and by the tributaries. If for some reason a larger or smaller quantity of sediment is supplied to the stream than the stream is capable of transporting, the slope would change and would be dependent on the amount of sediment supplied.

In the following sections the variables affecting resistance to flow are discussed. The effects produced by different variables change under different conditions. These changing effects are discussed along with approximations to simplify the analysis of alluvial channel flow.

The variables that describe alluvial channel flow are:

V = velocity
y_o = depth
S_f = slope of the energy grade line
ρ = density of water-sediment mixture
μ = apparent dynamic viscosity of the water-sediment mixture
g = gravitational constant
D = representative fall diameter of the bed material
G = measure of the size distribution of the bed material
ρ_s = density of sediment
S_p = shape factor of the particles
S_R = shape factor of the reach of the stream
S_c = shape factor of the cross section of the stream
f_s = seepage force in the bed of the stream
C_T = the bed-material concentration
C_f = the fine-material concentration
ω = the terminal fall velocity of the particles.

In general, the river problems are confined to flow of water over beds consisting of quartz particles with constant ρ_s. The value of g is also constant in the present context. The effect of other variables on the flow in alluvial channels is qualitatively discussed in the following sections. Most of this presentation is based on laboratory studies and has been supplemented by field experience when available.

3.3.1 Depth

With a constant slope and bed material, an increase in depth can change a plane bed (without movement) to ripples, and ripple-bed configuration to dunes, and a dune bed to a plane bed or antidunes. Also, a decrease in depth may cause a plane bed or antidunes to change to a dune-bed configuration. A typical break in a depth-discharge relation caused by a change in bed form from dunes to plane bed or from plane bed to dunes is shown in Fig. 3.3.1.

Often there is a gradual change in bed form and a gradual reduction in resistance to flow and this type of change prevents the break in the stage-discharge relation. Nevertheless, *it is possible to experience a large increase in discharge with little or no change in stage.* For this and related reasons the development of dependable stage-discharge relations in alluvial channels is very difficult.

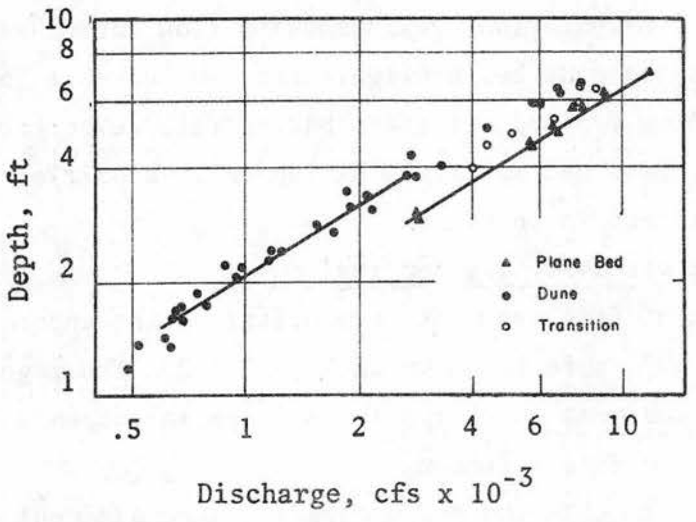


Fig. 3.3.1 Relation of depth to discharge for Elkhorn River near Waterloo, Nebraska [after Beckman and Furness (1962)].

Resistance to flow varies with depth even when the bed configurations do not change. When the bed configuration is plane bed, either with or without sediment movement or ripples, there is a decrease in resistance to flow with an increase in depth. That is, a relative roughness effect.

When the bed configuration is dunes, field and laboratory studies indicate that resistance to flow may increase or decrease with an increase in depth, depending on the size of bed material and magnitude of the depth. Additional studies are needed to define the variation of resistance to flow for flow over dune beds.

When the bed configuration is antidunes, resistance to flow increases with an increase in depth to some maximum value, then decreases as depth is increased further. This increase or decrease in flow resistance is directly related to changes in length, amplitude, and activity of the antidunes as depth is increased.

3.3.2 Slope

The slope is an important factor in determining the bed configuration which will exist for a given discharge. The slope provides the downstream component of the fluid weight, which in turn determines the fluid velocity and stream power. The relation between stream power, velocity and bed configuration has been illustrated in Fig. 3.2.4.

Even when bed configurations do not change, resistance to flow is affected by a change in slope. For example, with shallow depths and the ripple-bed configuration, resistance to flow increases with an increase in slope. With the dune-bed configuration, an increase in slope increases resistance to flow for bed materials having fall velocities greater than 0.20 fps. For those bed materials having fall velocities less than 0.20 fps, the effect is uncertain.

3.3.3 Apparent viscosity and density

The effect of fine sediment (bentonite) on the apparent kinematic viscosity of the mixture is shown in Fig. 3.3.2. The magnitude of the effect of fine sediment on viscosity is large and depends on the chemical make up of the fine sediment.

In addition to changing the viscosity, fine sediment suspended in water increases the mass density of the mixture (ρ) and, consequently,

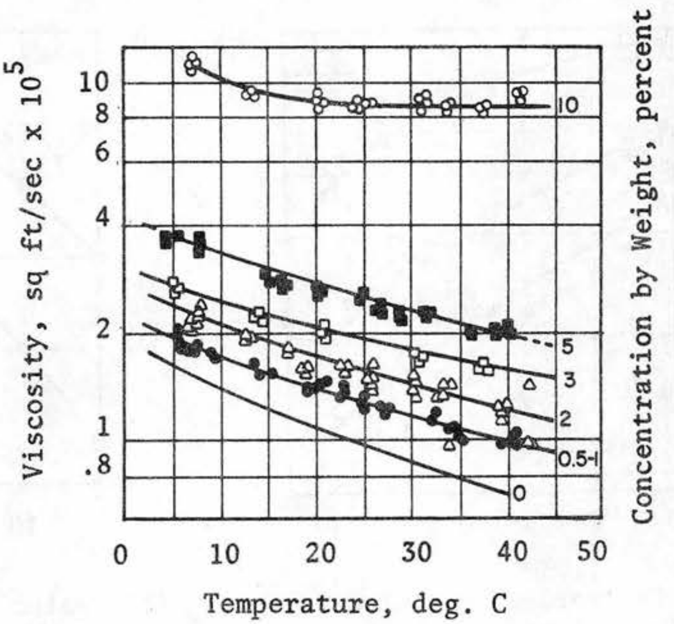


Fig. 3.3.2 Apparent kinematic viscosity of water-bentonite dispersions.

the specific weight (γ). The specific weight of a sediment-water mixture is computed from the relation,

$$\gamma = \frac{\gamma_w \gamma_s}{\gamma_s - C_s(\gamma_s - \gamma_w)} \qquad 3.3.2$$

where γ_w = specific weight of the water (about 62.4 lb per cu ft)

γ_s = specific weight of the sediment (about 165.4 lb per cu ft) and

C_s = concentration by weight (in fraction form) of the suspended sediment.

A sediment-water mixture, where C_s = 10 percent, has a specific weight (γ) of about 66.5 lb per cu ft, and any change in γ affects the boundary shear stress and the stream power.

Changes in the fall velocity of a particle caused by changes in the viscosity and the fluid density resulting from the presence of suspended bentonite clay in the water are shown in Fig. 3.3.3a. For comparative purposes, the effect of temperature on the fall velocity of two sands in clear water is shown in Fig. 3.3.3b.

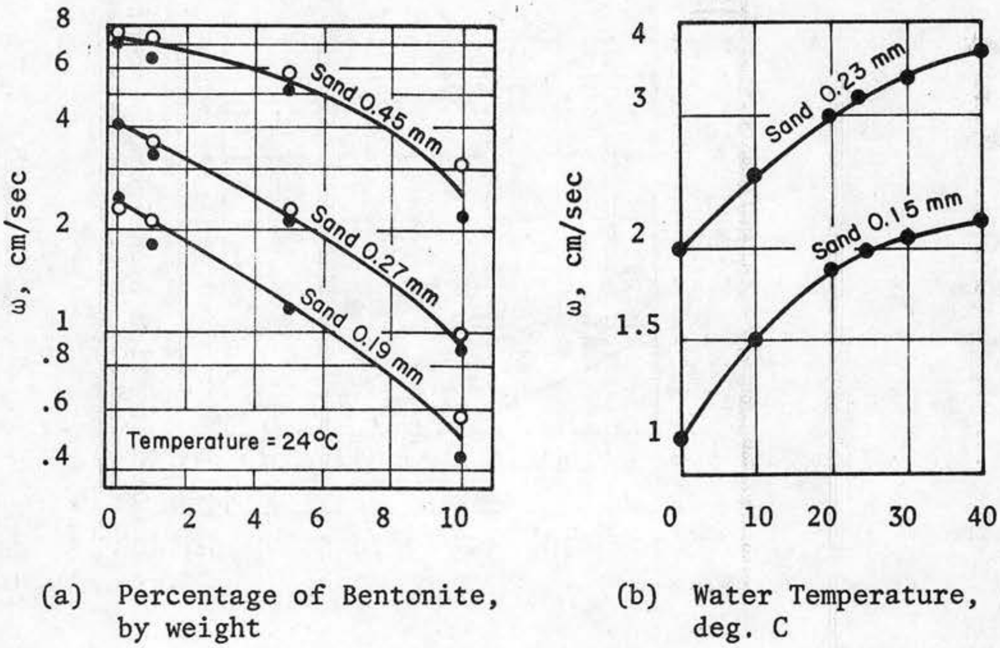


Fig. 3.3.3 Variation of fall velocity of several sand mixtures with percent bentonite and with temperature.

3.3.4 Size of bed material

The effects of the physical size of the bed material on resistance to flow are (1) *its influence on the fall velocity*, which is a measure of the interaction of the fluid and the particle in the formation of the bed configurations, (2) *its effect on grain roughness*, and (3) *its effect on the turbulent structure and the velocity field of the flow.*

The physical size of the bed material, as measured by the fall diameter or by sieve diameter, is a primary factor in determining fall velocity. *Use of the fall diameter instead of the sieve diameter is advantageous because the shape factor and density of the particle can be eliminated as variables.* That is, if only the fall diameter is known, the fall velocity of the particle in any fluid at any temperature can be computed; whereas, to do the same computation when the sieve diameter is known, knowledge of the shape factor and density of the particle are also required.

The physical size of the bed material determines the friction factor mainly for the plane-bed condition and for antidunes when they are not actively breaking. The breaking of the waves, which increases with a decrease in the fall velocity of the bed material, causes additional dissipation of energy.

The physical size of the bed material for a dune-bed configuration also has an effect on resistance to flow. The flow of fluid over the back of dunes is affected by grain roughness, although the dissipation of energy by the form roughness is the major factor. The form of the dunes is also related to the fall velocity of the bed material.

3.3.5 Size gradation

The gradation of sizes of the bed material affects bed form and resistance to flow. Flume experiments indicate that uniform sands (sands of practically the same size) have larger resistance to flow (except plane bed) than graded sands for the various bed forms. Also the transition from upper flow regime to lower flow regime occurs over a narrower range of shear values for the uniform sand. For plane bed with motion, resistance to flow is about the same for either uniform or graded sand.

3.3.6 Fall velocity

Fall velocity is the primary variable that determines the interaction between the bed material and the fluid. For a given depth and slope, the fall velocity determines the bed form that will occur, the actual dimensions of the bed form and, except for the contribution of the grain roughness, the resistance to flow.

Observations of natural streams have shown that the bed configuration and resistance to flow change with changes in fall velocity when the discharge and bed material are constant. For example, the Loup River near Dunning, Nebraska has bed roughness in the form of dunes in the summer when the water is warm and less viscous but has a nearly plane bed during the cold winter months. Similarly, two sets of data collected by Harms and Fahnestock (1965) on a stable branch of the Rio Grande at similar discharges show that when the water was cold, the bed of the stream was plane, the resistance to flow was small, the depth was relatively shallow, and the velocity was large; but when the water was warm, the bed roughness was dunes, the resistance to flow was large, the depth was large, and the velocity was low.

3.3.7 Shape factor for the reach and cross section

The shape of the reach and the shape of the cross section affect the energy losses resulting from the nonuniformity of the flow in a natural stream caused by the bends and the nonuniformity of the banks. Study of these losses in natural channels has long been neglected. Also, flow phenomena, bed configuration, and resistance to flow vary with the width of the stream. In narrow channels dunes and antidunes are more two-dimensional and resistance to flow is larger than for a wide channel. Also, in wide channels more than one bed form can occur in the cross section.

3.3.8 Seepage force

A seepage force occurs whenever there is inflow or outflow through the bed and banks of a channel in permeable alluvium. *The seepage flow affects the alluvial channel phenomena by altering the velocity field in the vicinity of the bed particles and by changing the effective weight of the bed particles.* Seepage may have a significant effect on bed configuration and resistance to flow. If there is inflow, the seepage force acts to reduce the effective weight of the sand and consequently, the stability of the bed material. If there is outflow, the seepage force acts in the direction of gravity and increases the effective weight of the sand and the stability of the bed material. As a direct result of changing the effective weight, the seepage forces can influence the form of bed roughness and the resistance to flow for a given channel flow. For example, under shallow flow a bed material with median diameter of 0.5 mm will be molded into the following forms as shear stress is increased: Ripples, dunes, transition, standing sand and water waves, and antidunes. If this same material was subjected to a seepage force that reduced its effective weight to a value consistent with that of medium sand (median diameter, $D = 0.3$ mm), the forms of bed roughness would be ripples, dunes, transition, plane bed, and antidunes for the same range of flow conditions.

A common field condition is outflow from the channel during the rising stage; this process increases the stability of the bed and bank material but stores water in the banks. During the falling stage, the situation is reversed; inflow to the channel reduces the effective weight and stability of the bed and bank material and influences the form of bed roughness and the resistance to flow.

3.3.9 Concentration of bed-material discharge

The concentration of bed-material discharge (C_T) affects the fluid properties by increasing the apparent viscosity and the density of the water-sediment mixture. However, the effect of the sediment on viscosity μ and density ρ in any resistance to flow relation is accounted for by using their values for the water-sediment mixture instead of their values for pure water. The presence of sediment in the flow causes a small change in the turbulence characteristics, velocity distribution and resistance to flow.

3.3.10 Fine-sediment concentration

Fine sediment or wash load is that part of the total sediment discharge that is not found in appreciable quantities on the bed. If much sediment is in suspension, its effect on the viscosity of the water-sediment mixture should be taken into account. The effect of fine sediment on resistance to flow is a result of its effect on the apparent viscosity and the density of the water-sediment mixture. Generally the fine sediment is uniformly distributed in the stream cross section. The method of defining and treating the fine-material load computations is subsequently discussed in this chapter.

3.4.0 PREDICTION OF BED FORM

In Fig. 3.4.1, the relation between stream power, median fall diameter of bed material, and form roughness is shown. *This relation gives an indication of the form of bed roughness one can anticipate if the depth, slope, velocity, and fall diameter of bed material are known.* Flume data were utilized to establish the boundaries separating plane bed and ripples, ripples and dunes for all sizes of bed material, and dunes and transition for the 0.93 mm bed material. The lines dividing dunes and transition and dividing transition and upper regime are based on flume data and the following field data: (1) Elkhorn River, near Waterloo, Nebraska (Beckman and Furness, 1962), (2) Rio Grande, 20 miles above El Paso, Texas, (3) Middle Loup River at Dunning, Nebraska (Hubbell and Matejka, 1959), (4) Rio Grande at Cohiti, near Bernalillo,

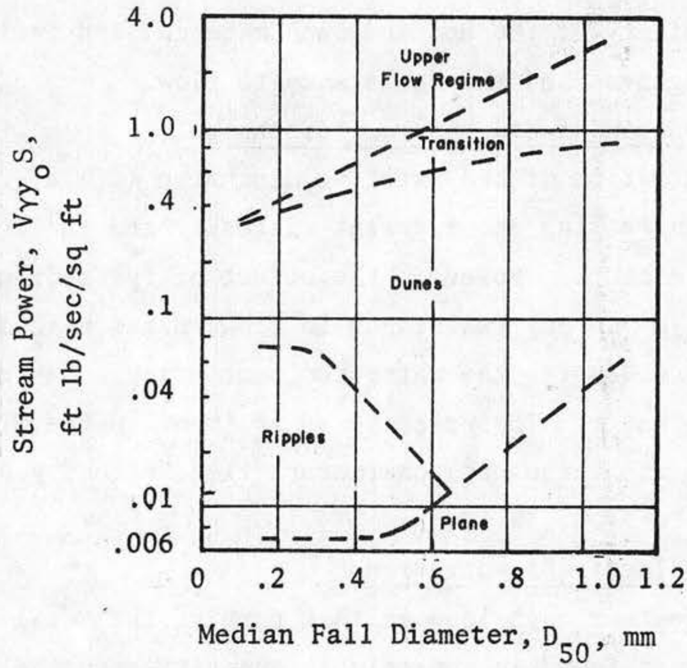


Fig. 3.4.1 Relation between stream power, median fall diameter, and bed configuration.

and at Angostura heading, N. Mexico (Culbertson and Dawdy, 1964), and (5) Punjab canal data upper regime flows that have been observed in large irrigation canals that have fine sandbeds.

3.5.0 MANNING'S n VALUES FOR NATURAL SANDBED STREAMS

Observations by the authors on natural sandbed streams with bed material having a median diameter ranging from 0.1 mm to 0.4 mm indicate that the bed planes out and resistance to flow decreases whenever high flow occurs. *Manning's n changes from values as large as 0.040 at low flow to as small as 0.012 at high flow.* An example is given in Fig. 3.5.1. These observations are substantiated by Dawdy (1961), Colby (1960), Corps of Engineers (1968) and Beckman and Furness (1962).

The range in Manning's n for the various bed configurations is as follows:

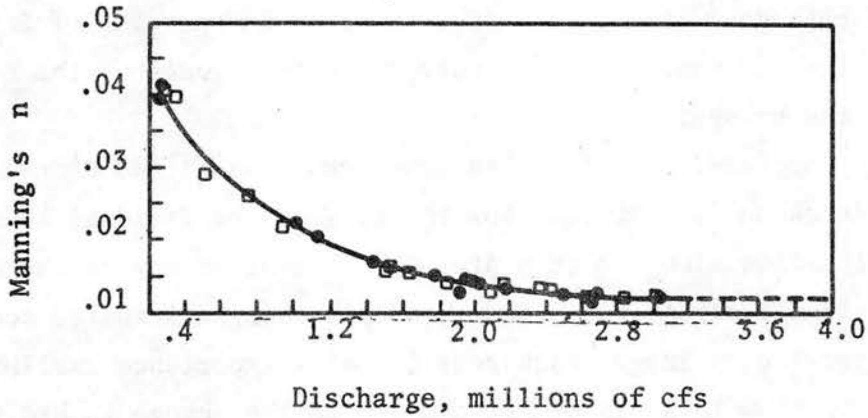


Fig. 3.5.1 Change in Manning's n with discharge for Padma River in Bangladesh.

Lower flow regime		Upper flow regime	
Ripples	$(0.018 \le n \le 0.028)$	*Plane bed*	$(0.010 \le n \le 0.013)$
Dunes	$(0.020 \le n \le 0.040)$	*Antidunes*	
		Standing waves	$(0.010 \le n \le 0.015)$
		Breaking waves	$(0.012 \le n \le 0.020)$
		Chute and pools	$(0.018 \le n \le 0.035)$

3.6.0 HOW BED-FORM CHANGES AFFECT HIGHWAYS IN THE RIVER ENVIRONMENT

At high flows, most sandbed channel streams shift from a dune bed to a transition or a plane. The resistance to flow is then decreased two to threefold. The corresponding increase in velocity can increase scour around bridge piers, abutments, spur dikes or banks and also the required size of riprap. On the other hand, the decrease in stage resulting from the planing out of the bed will decrease the required elevation of the bridge crossing, the height of embankments across the floodplain, the height of any dikes, and the height of any channel control works that may be needed.

Another effect of bed forms on highway crossings is that with dunes on the bed *there is a fluctuating pattern of scour on the bed and around the piers, abutments or spur dikes.* The average height of dunes is approximately 1/2 to 1/3 the average depth of flow and the maximum height of a dune may approach the average depth of flow. If the depth of flow is 10 feet, the maximum dune height may be of the order of 10 feet, and half of this would be below the mean elevation of the bed. With the passage of this dune through a bridge section, an increase of 5 feet in the local scour would be anticipated when the trough of the dune arrives at the bridge.

A *very important effect of bed forms and bars is the change of flow direction in channels*. At low flow the bars can be residual and cause high velocity flow along or at a pier or abutment or any of the other structures in the stream bed, causing deeper than anticipated scour. As stated previously large discharges normally experience smaller resistance to flow in a sandbed stream due to the change in bed form. However, if the bridge crossing or encroachment causes appreciable backwater, the dune bed may not plane out at large discharges and a higher resistance to flow results. This increase in resistance to flow can decrease the velocity of flow and also decrease the transport capacity of the channel so that aggradation occurs upstream of the crossing. The aggradation and the roughness increases the river stage and thus the height of any control structure or the levees. Thus, the bridge crossing can adversely affect the floodplain, due to the change in bed form that would occur.

With highways in the sandbed river environment, care must be used in analyzing the crossing in order to foresee possible changes that may occur in the bed form and what this change may do to the resistance coefficient, to the stability of the reach and its structures, and to the river environment.

3.7.0 PROPERTIES OF ALLUVIAL MATERIAL

A knowledge of the properties of the bed-material particles is essential, as they indicate the behavior of the particles in their interaction with the flow. Several of the important bed-material properties are discussed in the following sections.

3.7.1 Size

Of the various sediment properties, physical size has by far the greatest significance to the hydraulic engineer. The particle size is the most readily measured property, and other properties such as shape, fall velocity and specific gravity tend to vary with size in a roughly predictable manner. *In general, size represents a sufficiently complete description of the sediment particle for many practical purposes.*

Particle size may be defined by its volume, diameter, weight, fall velocity, or sieve mesh size. Except volume, these definitions also depend on the shape and density of the particle. The following definitions are commonly used to describe the particle size:

(1) *Nominal diameter*: The diameter of a sphere having the same volume as the particle.

(2) *Sieve diameter*: The diameter of a sphere equal to the length of the side of a square sieve opening through which measured quantities (by weight) of the sample will pass. As an approximation, the sieve diameter is equal to the nominal diameter.

(3) *Sedimentation diameter*: The diameter of a sphere with the same fall velocity and specific gravity as the particle in the same fluid under the same conditions.

(4) *Standard fall diameter*: The diameter of a sphere that has a specific gravity of 2.65 and also has the same terminal settling velocity as the particle when each is allowed to settle alone in quiescent, distilled water of infinite extent and at a temperature of 24°C.

(5) *Standard fall velocity*: The terminal settling velocity of a particle falling alone in quiescent, distilled water of infinite extent at a temperature of 24°C.

In general, *sediments have been classified into boulders, cobbles, gravels, sands, silts, and clays* on the basis of their nominal or sieve diameters. The size range in each general class is given in Table 3.7.1.

The boulder class is generally of little interest in sediment problems. The cobble and gravel class plays a considerable role in problems of local scour and resistance to flow and to a lesser extent in bed load transport. The sand class is one of the most important in alluvial channel flow. The silt and clay class is of considerable importance in the evaluation of stream loads, bank stability and problems of seepage and consolidation.

3.7.2 Shape

Generally speaking, shape refers to the overall geometrical form of a particle. *Sphericity is defined as the ratio of the surface area of a sphere of the same volume as the particle to the actual surface area of*

Table 3.7.1 Sediment grade scale.

Size				Approximate Sieve Mesh Openings per Inch		
Millimeters		Microns	Inches	Tyler	U.S. Standard	Class
4000-2000	. . . .	. . . . .	160-80	. . .	. . . . . .	Very large boulders
2000-1000	. . . .	. . . . .	80-40	. . .	. . . . . .	Large boulders
1000-500	. . . .	. . . . .	40-20	. . .	. . . . . .	Medium boulders
500-250	. . . .	. . . . .	20-10	. . .	. . . . . .	Small boulders
250-130	. . . .	. . . . .	10-5	. . .	. . . . . .	Large cobbles
130-64	. . . .	. . . . .	5-2.5	. . .	. . . . . .	Small cobbles
64-32	. . . .	. . . . .	2.5-1.3	. . .	. . . . . .	Very coarse gravel
32-16	. . . .	. . . . .	1.3-0.6	. . .	. . . . . .	Coarse gravel
16-8	. . . .	. . . . .	0.6-0.3	2 1/2	. . . . .	Medium gravel
8-4	. . . .	. . . . .	0.3-0.16	5	5	Fine gravel
4-2	. . . .	. . . . .	0.16-0.08	9	10	Very fine gravel
2-1	2.00-1.00	2000-1000	. . . . .	16	18	Very coarse sand
1-1 1/2	1.00-0.50	1000-500	. . . . .	32	35	Coarse sand
1/2-1/4	0.50-0.25	500-250	. . . . .	60	60	Medium sand
1/4-1/8	0.25-0.125	250-125	. . . . .	115	120	Fine sand
1/8-1/16	0.125-0.062	125-62	. . . . .	250	230	Very fine sand
1/16-1/32	0.062-0.031	62-31	. . . . .			Coarse silt
1/32-1/64	0.031-0.016	31-16	. . . . .			Medium silt
1/64-1/128	0.016-0.008	16-8	. . . . .			Fine silt
1/128-1/256	0.008-0.004	8-4	. . . . .			Very fine silt
1/256-1/512	0.004-0.0020	4-2	. . . . .			Coarse clay
1/521-1/1024	0.0020-0.0010	2-1	. . . . .			Medium clay
1/1024-1/2048	0.0010-0.0005	1-0.5	. . . . .			Fine clay
1/2018-1/4096	0.0005-0.00024	0.5-0.24	. . . . .			Very fine clay

(Rouse, 1950, p. 776)

the particle. Roundness is defined as the ratio of the average radius of curvature of the corners and edges of a particle to the radius of a circle inscribed in the maximum projected area of the particle. However, because of simplicity and effectiveness of correlation with the behavior of particles in flow, the most commonly used parameter to describe particle shape is the *Corey shape factor* S_p defined as

$$S_p = \frac{c}{\sqrt{ab}} \tag{3.7.1}$$

where a, b, and c are the dimensions of the three mutually perpendicular axes through a particle: a, the longest; b, the intermediate; and c, the shortest axis.

3.7.3 Fall velocity

The prime indicator of the interaction of sediment with the flow in suspension is the fall velocity of the sediment particles. *The fall velocity of a particle is defined as the velocity of that particle falling alone in quiescent, distilled water of infinite extent.* In most cases, the particle is not falling alone, and the water is not distilled or quiescent. Measurement techniques are available for determining the fall velocity of groups of particles in a finite field in fluid other than distilled water. However, the effect of turbulence on fall velocity is not known.

A particle falling at terminal velocity in a fluid is under the action of a driving force due to its buoyant weight and a resisting force due to the fluid drag. Fluid drag is the result of either the tangential shear stress on the surface of the particle, or a pressure difference on the particle or a combination of the two forces. The fluid drag on the falling particle is given by the drag equation

$$F_D = C_D \, A \, \rho \, \frac{\omega^2}{2} \tag{3.7.2}$$

The buoyant weight of the particle is

$$W_s = (\rho_s - \rho) g \forall . \tag{3.7.3}$$

Where,

C_D = coefficient of drag

ω = terminal fall velocity of the particle

A = projected area of the particle normal to the direction of flow

ρ = fluid density

ρ_s = particle density

g = acceleration due to gravity

$\forall$ = volume of the particle

The area and volume can be written in terms of the characteristic diameter of the particle D or

$$A = K_1 D^2 \tag{3.7.4}$$

and

$$\forall = K_2 D^3 \tag{3.7.5}$$

If the particle is falling at its terminal velocity, $F_D = W_s$ or

$$(\rho_s - \rho) g \forall = C_D A \rho \frac{\omega^2}{2} \tag{3.7.6}$$

By substituting Eqs. 3.7.4 and 3.7.5 into Eq. 3.7.6, the expression

$$\omega^2 = \frac{2D}{C_D} \frac{K_2}{K_1} \left(\frac{\rho_s}{\rho} - 1\right) g \tag{3.7.7}$$

is obtained.

Four dimensionless variables

$$\frac{\omega^2}{gD}, \; C_D, \; \frac{\rho_s}{\rho}, \; \frac{K_2}{K_1}$$

describing the fall velocity phenomenon result from Eq. 3.7.7. The coefficient of drag is dependent on the Reynolds number

$$R_e = \rho \frac{\omega D}{\mu}$$

the shape and the surface texture of the particle.

The ratio K_2/K_1 is usually replaced by the Corey shape factor

$$S_p = \frac{c}{\sqrt{ab}}$$

Here a, b, and c are the dimensions of the major, intermediate and minor axis of the particle, respectively.

The relation between the fall velocity of particles and the other variables are given in Figs. 3.7.1 and 3.7.2

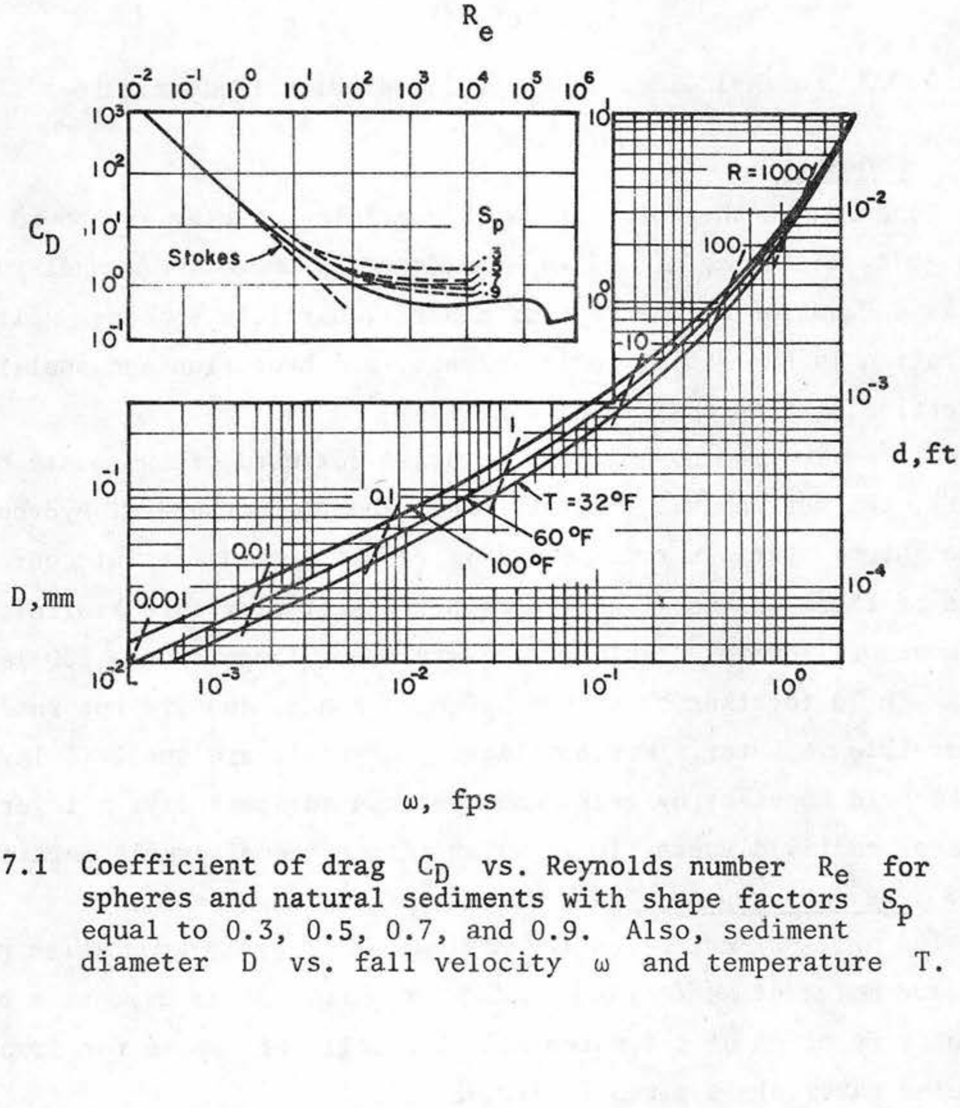


Fig. 3.7.1 Coefficient of drag C_D vs. Reynolds number R_e for spheres and natural sediments with shape factors S_p equal to 0.3, 0.5, 0.7, and 0.9. Also, sediment diameter D vs. fall velocity ω and temperature T.

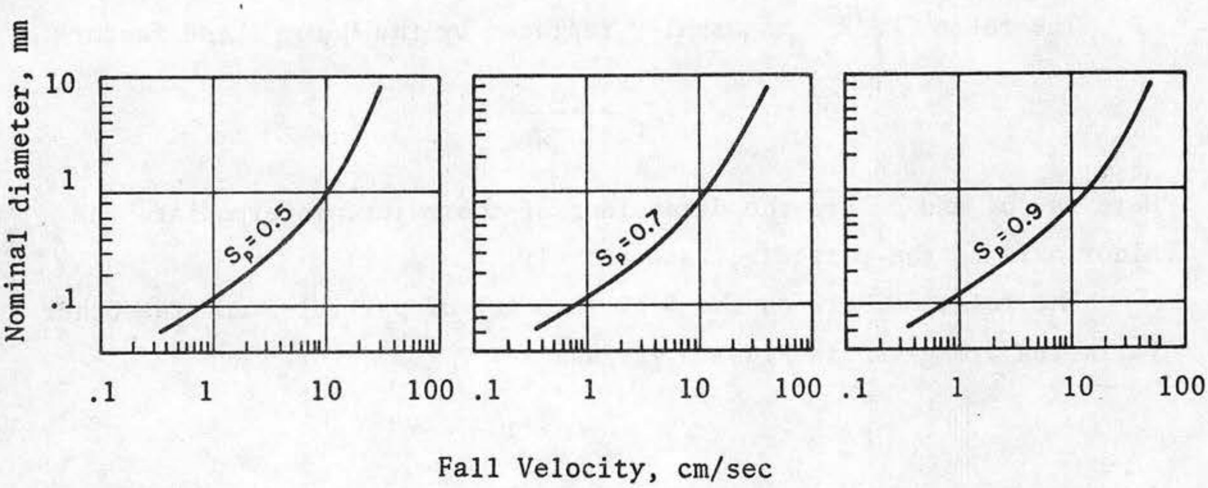


Fig. 3.7.2 Nominal diameter vs. fall velocity (Temperature = 24°C).

3.7.4 Cohesion

Cohesion is the force by which particles of clay are bound together. *This force is the result of ionic attraction among individual particles*, and is a function of the type of mineral, particle spacing, salt concentration in the fluid, ionic valence, and hydration and swelling properties of the constituent minerals.

Clays are alumino-silicate crystals composed of two basic building sheets, the tetrahedral silicate sheet and the octahedral hydrous aluminum oxide sheet. Various types of clays result from different configurations of these sheets. The two main types of clays are kaolinite and montmorillonite. Kaolinite crystals are large (70 to 100 layers thick), held together by strong hydrogen bonds, and are not readily dispersible in water. Montmorillonite crystals are small (3 layers thick) held together by weak bonds between adjacent oxygen layers and are readily dispersible in water into extremely small particles.

3.7.5 Angle of repose

The angle of repose is the maximum slope angle upon which noncohesive material will reside without moving. It is a measure of the granular friction of the material. The angle of repose for dumped granular material is given in Fig. 3.7.3.

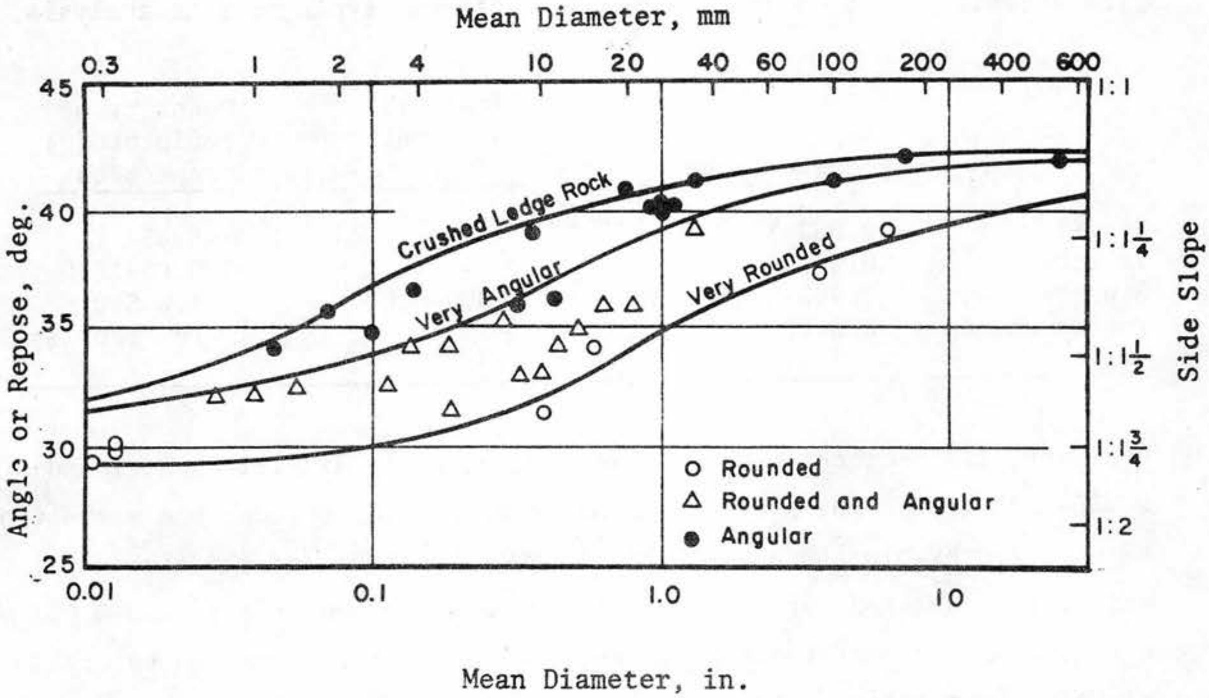


Fig. 3.7.3 Angle of repose.

3.8.0 METHODS OF MEASURING PROPERTIES OF ALLUVIAL MATERIALS

Following is a summary of selected procedures for measuring size distribution, specific weight and porosity, and cohesion of alluvial materials.

3.8.1 Size distribution

Four methods of obtaining size distribution are described herein: *sieve analysis, visual accumulation tube analysis, pebble count method, and pipette analysis.* In general, the first three methods are used for sands, gravels and cobbles. The pipette analysis is used for silts and clays. However, the methods for the size distribution analysis of coarse sediments are appropriate for only a particular range of particle sizes (see Table 3.8.1). All together the four methods provide a means of obtaining particle size distributions for most bed material samples.

3.8.2 Separation of sand from fines

If the sediment sample to be analyzed (bed material or suspended sediment) has considerable fine material ($D < .062$ mm) it must be separated prior to analysis. To separate the coarser from the finer

Table 3.8.1 Guide to size range for different types of size analysis.

	Size range		Analysis Concentration (mg/1)	Quantity of sediment (g) or pebbles
Sieves	0.062-32	mm	----	<0.05
VA tube	0.062-2	mm	----	0.05-15.0
Pipette	0.002-0.062	mm	2,000-5,000	1.0-5.0
Pebble count	0.5-40	in.	----	100 pebbles

sediment, the sediment should be wet-seived using distilled water and a 250-mesh (0.062 mm) sieve. The material passing through the sieve can be analyzed by pipette analysis if further breakdown of the fine sediment is desired, or dried and included as percent finer than 0.062 mm with the analysis of the coarser material. If it is going to be dry-sieved, the material retained on the sieve is ovendried for one hour after all visible water has been evaporated. If the material is to be analyzed by wet-sieving or with the accumulation tube it is not dried.

3.8.3 Sieves

Size distribution in the sand and gravel range is generally determined by passing the sample through a series of sieves of mesh size ranging from 4 mm to 0.062 mm. A minimum of about 1.0 gram of sand is required for an accurate sieve analysis. More is required if the sample contains particles of 1.0 mm or larger. Standard methods employed in soil mechanics are suitable for determining the sieve sizes of sand and gravel sediment samples.

3.8.4 Visual accumulation tube

The visual accumulation tube is used for determining the size distribution of the sand fraction of sediment samples ($0.062 \leq D \leq 2.0$ mm). It is a fast, economical, and accurate means of determining the fall velocity or fall diameter of the sediment. The equipment for the visual accumulation tube analysis consists of (1) a glass funnel about 25 cm long, (2) a rubber tube connecting the funnel and the main sedimentation tube, with a special clamping mechanism serving as a "quick acting" valve, (3) glass sedimentation tubes having different sized collectors, (4) a tapping mechanism that strikes against the glass tube and helps keep the accumulation of sediment uniformly packed, (5) a special recorder consisting of a cylinder carrying a chart that rotates at a constant rate and a carriage that can be moved vertically by hand on which is mounted a recording pen and an optical instrument for tracking the accumulation and (6) the recorder chart which is a printed form incorporating the fall-diameter calibration.

In the visual accumulation tube method, the particles start falling from a common source and become stratified according to settling velocities. At a given instant, the particles coming to rest at the bottom of the tube are of one "sedimentation size" and are finer than particles that have previously settled out and are coarser than those remaining in suspension.

It has been shown that particles of a sample in the visual tube settle with greater velocities than the same particles falling individually because of the effect of mutual interaction of the particles. The visual accumulation tube apparatus is calibrated to account for the effects of this mutual interaction and the final results are given in terms of the standard fall diameter of the particles.

The visual accumulation tube method may not be suitable for some streams that transport large quantities of organic materials such as root fibers, leaf fragments, and algae. Also, extra care is needed when a stream transports large quantities of heavy or light minerals such as taconite or coal. The method is explained in detail by Guy (1969).

3.8.5 Pebble count method

The pebble count method is used to obtain the size distribution of coarse bed materials (gravel and pebbles) which are too large to be sieved. These sizes are measured in situ by laying out a square grid or taking a line and either analyzing all the particles in the grid or on the line in selected class intervals or analyzing random selection of particles in the various classes. Very often the coarser material is underlain by sands. Then the underlying sands are analyzed by sieving. Depending on the type of hydraulic problem, the two classes of bed material are either combined into a single distribution or used separately.

A square-surface sample is obtained by picking up and counting all the surface pebbles in a predetermined size class within a small enclosed area of the bed. The area is taken to be representative of the whole channel bed.

The *pebble count* method entails measurement of randomly selected particles in the field, often under difficult conditions. Therefore, use of the Zeiss Particle-Size Analyzer should be considered (Ritter and Helley, 1968). For this method, a photograph of the stream bed is made, preferably at low flow, with a 35 mm camera supported by a tripod about 2 m above the stream bed, the height depending on the size of the bed materials. A reference scale, such as a steel tape or a surveyor's rod must appear in the photograph. The photographs are printed on the thinnest paper available. An iris diaphragm, illuminated from one side, is imaged by a lens onto the plane of a Plexiglas plate. By adjusting the iris diaphragm the diameter of the sharply defined circular light spot appearing on the photograph can be changed and its area made equal to that of the individual particles. As the different diameters are registered, a puncher marks the counted particle on the photograph. An efficient operator can count up to 1,000 particles in a half hour.

In the line sampling method, a line is laid out or placed either across or along the stream. Particles are picked at random intervals along the line and measured. The measured particles are classified as to size or weight and a percent finer curve or table is prepared. Usually 100 particles is sufficient to give an accurate classification of the size distribution of coarse materials.

3.8.6 Pipette analysis

The pipette method of determining gradation of sizes finer than 0.062 mm is one of the most widely accepted techniques utilizing the Oden theory and the dispersed system of sedimentation. The upper size limit of sediment particles which settle in water according to Stokes law and the lower size limit which can be determined readily by sieves is about 1/16 mm or 0.062 mm. This size is the division between sand and silt (Table 3.7.1) and is an important division in many phases of sediment phenomena.

The fundamental principle of the pipette method is to determine the concentration of a suspension in samples withdrawn from a predetermined depth as a function of settling time. Particles having a settling velocity greater than that of the size at which separation is desired will settle below the point of withdrawal after elapse of a certain time. The time and depth of withdrawal are predetermined on the basis of Stokes law.

Satisfactory use of the pipette method requires careful and precise operation to obtain maximum accuracy in each step of the procedure. Also, for routine analysis, special apparatus can be set up for the analysis of a large number of samples. A complete description of a laboratory set-up and procedure for this method is given by Guy (1969).

3.8.7 Specific weight

Specific weight is weight per unit volume. In the English system of dimensions, specific weight is usually expressed in units of pounds per cubic foot and in the metric system, in grams per cubic centimeter. In connection with granular materials such as soils, sediment deposits, or water sediment mixtures, the specific weight is the weight of solids per unit volume of the material including its voids. The measurement of the specific weight of sediment deposits is determined simply by measuring the dry weight of a known volume of the undisturbed material.

3.8.8 Porosity

The porosity of granular materials is the ratio of the volume of void space to the total volume of an undisturbed sample. To determine porosity, the volume of the sample must be obtained in an undisturbed condition. Next, the volume of solids is determined either by liquid displacement or indirectly from the weight of the sample and the specific gravity of material. The void volume is then obtained by subtracting the volume of solids from the total volume. The porosity is the ratio of volume of voids to total volume.

3.8.9 Cohesion

Several laboratory and field measurement techniques are available for determining the magnitude of cohesion, or shear strength, of clays. Among these, the vane shear test, which is performed in the field is one of the simplest. The vane is forced into the ground and then the torque required to rotate the vane is measured. The shear strength is determined from the torque required to shear the soil along the vertical and horizontal edges of the vane.

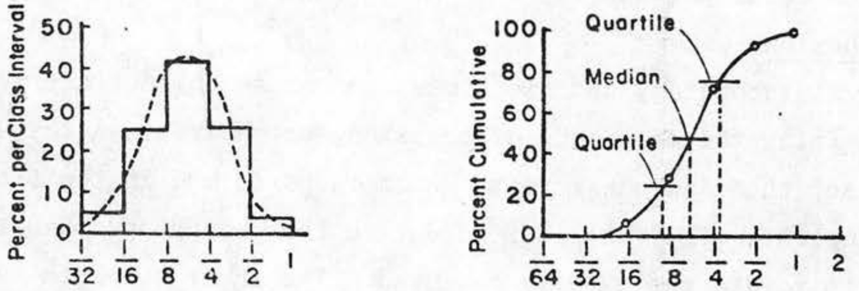
3.8.10 Methods of summarizing distributions

Sediments consist of many particles differing in size and fall velocity, in shape, and in specific gravity. In general the size distribution of the sediment is determined by performing one or more of the size analysis techniques (described earlier) on a representative sample. The results of these analyses yield either a cumulative frequency (as in the visual accumulation tube analysis) or a size-class frequency (as in the Pebble count and sieving methods). The size distribution is then reported in terms of one or more statistical parameters.

3.8.11 Frequency curves

A histogram is a graphical representation of the number, weight, or volume percentage of items in given class intervals. An example of a histogram is shown in Fig. 3.8.1a. The abscissa scale represents the class intervals, usually in geometric progression, and the ordinate scale represents either actual concentration or percent (by number, volume, or weight) of the total sample contained in each class interval. If the class intervals are small, the shape of the histogram will approach a continuous curve. The successive sizes employed in the size analysis of sediment are usually in ratios of 2 or $\sqrt{2}$.

When the ordinates of successive classes are added and plotted against the upper limit of the size class, the cumulative distribution diagram is obtained (see Fig. 3.8.1b). In this diagram, the abscissa scale (usually logarithmic) represents the intervals of the size scale and the ordinate scale is the cumulative percent of the sample up to (or percent finer than) the size in question.



(a) Histogram

(b) Cumulative Distribution Diagram

Fig. 3.8.1 Frequency curves.

3.8.12 Quartile and moment measures

In a size frequency distribution curve, it is possible to choose certain particle sizes as representing significant values, such as particles just larger than one-fourth of the distribution (the first quartile), and particles just larger than three-fourths of the distribution (the third quartile). Measures of spread are based on differences or ratios between the two quartiles. Quartile measures are confined to the central half of the frequency distribution and the values obtained are not influenced by larger or smaller sizes. Quartile measures are very readily computed, and most of the data may be obtained directly from the cumulative curve by graphic means.

In contrast to quartile measures, moment measures are influenced by each individual size class in the distribution. *The first moment of a frequency curve* is its center of gravity and *is called the arithmetic mean and is the average size of the sediment. The second moment* is a measure of the average spread of the curve and *is expressed as the standard deviation of the distribution.*

Commonly the size distribution of natural sediments plots as a straight line on log probability paper. If this is true, then a natural sediment is completely described by the median diameter (the size of sediment of which 50% is finer) and the slope of the cumulative frequency line on log probability paper. The slope of this line is proportional to the spread of the size distribution in a sediment sample. It is computed with the expression

$$G = \frac{1}{2}\left[\frac{D_{50}}{D_{16}} + \frac{D_{84}}{D_{50}}\right] \qquad 3.8.1$$

Where G = gradation coefficient

D_x = the sediment diameter particle of which x percent of sample is finer.

3.9.0 BEGINNING OF MOTION

3.9.1 Introduction

Beginning and ceasing of sediment motion is of great importance in three areas of application: (1) design of stable channels, (2) bed load transport equations (3) design of riprap.

Beginning of motion can be related to either the shear stress on the grains or the fluid velocity in the vicinity of the grains. When the grains are at the beginning of motion, these values are called the critical stress and critical velocity. The choice of shear stress or velocity depends on which one is easier to determine in the field, the precision with which the critical value is known for the particle size, and the type of problem. In sediment transport, most equations use critical shear. In stable channel design either critical shear or critical velocity is used; whereas, in the design of riprap critical velocity is commonly used.

It is not sufficient to determine the average value of the critical shear or critical velocity because both quantities are fluctuating. For the same mean values they may have larger values that act for a sufficiently long time to cause a particle to move. In addition to the forces on the particle resulting from the flowing water, waves and seepage into or out of the bed or banks affect the beginning of motion conditions.

3.9.2 Theory of beginning of motion

The forces acting on an individual grain on the bed of an alluvial channel are:

(1) The body force F_g due to the gravitational field.

(2) The external forces F_n acting at the points of contact between the grain and its neighboring grains.

(3) The fluid force F_f acting on the surface of the grain. The fluid force varies with the velocity field and with the properties of the fluid.

The relative magnitude of these forces determines whether the grain moves or not.

For the individual grain, the body force is

$$F_g = g\rho_s K_2 D^3 \tag{3.9.1}$$

where ρ_s is the density of the grain, K_2 is a coefficient and D is the grain diameter. The term K_2D^3 is the volume of the grain.

For convenience, the fluid forces acting on the grain are divided into three components. The components are:

(1) The form drag component F_d given by the expression

$$F_d = C_D K_1 D^2 \rho \frac{v^2}{2} \tag{3.9.2}$$

(2) The viscous drag component F_v given by the expression

$$F_v = C_s K_1 D^2 \tau \tag{3.9.3}$$

(3) The hydrostatic pressure component F_h is given by the expression

$$F_h = g\rho K_2 D^3 \tag{3.9.4}$$

C_D = coefficient of drag

K_1 = a coefficient associated with the area of the grain subjected to drag and shear.

D = the diameter of the grain

v = the velocity in the vicinity of the grain

C_s = coefficient of shear

τ = the average viscous shear stress

The term $K_1 D^2$ represents the cross-sectional area of the grain.

The external forces F_n depend on the values of the fluid and body forces. Under conditions of no flow, the fluid force is

$$F_f = F_h$$

There is no form or viscous drag. Then the external force is

$$F_n = F_g - F_h$$

or

$$F_n = (\rho_s - \rho) g K_2 D^3 \tag{3.9.5}$$

That is, the external force is equal to the submerged weight of the grain.

The form drag can be rewritten in terms of the shear velocity. For turbulent flow, the local velocity v is directly proportional to the shear velocity V_* according to Eq. 2.3.15. Then, Eq. 3.9.2 reduces to

$$F_d \sim \rho D^2 V_*^{\ 2} \tag{3.9.6}$$

The viscous drag is also related to the shear velocity but it is the shear velocity for laminar flow. For laminar flow

$$\tau = \mu \frac{dv}{dy} \tag{3.9.7}$$

Again, by replacing v with V_* and y with D, we can write

$$\tau \sim \frac{\mu V_*}{D} \tag{3.9.8}$$

With this expression for viscous shear, the viscous drag becomes.

$$F_v \sim \mu D V_* \tag{3.9.9}$$

Now, consider the ratio of the form drag force F_d to the viscous shear force F_v. According to Eqs. 3.9.6 and 3.9.9

$$\frac{F_d}{F_v} \sim \frac{\rho D^2 V_*^{\ 2}}{\mu D V_*}$$

or

$$\frac{F_d}{F_v} \sim \frac{D V_*}{\nu} \tag{3.9.10}$$

When the flow over the grain is turbulent, the form drag is predominant and the term DV_*/ν is large. When the flow over the grain is laminar the viscous shear force is predominant and the term DV_*/ν is small. Thus, the Reynolds' number for particle DV_*/ν is an indicator of the characteristic of the flow in the vicinity of the grain.

As both the form drag and viscous shear are proportional to the shear velocity, the ratio of the forces tending to move the grain to the forces resisting movement is

$$\frac{\rho D^2 V_*^{\ 2}}{(\rho_s - \rho) g D^3} = \frac{\tau}{(\gamma_s - \gamma) D} \tag{3.9.11}$$

(Recall that $V_*^2 = \tau/\rho$). *The relation between* $\tau/(\gamma_s-\gamma)D$ and DV_*/ν *for the condition of incipient motion has been determined experimentally by Shields and others. The relation is given in Fig. 3.2.3.* At conditions of incipient motion, the shear stress τ is designated the critical shear stress τ_c.

Figure 3.9.1 shows relationships between critical tractive force (critical shear stress) and mean diameter as determined and/or recommended by different investigators for different soil types. The difference between investigators could possibly be due to the effects of cohesion, when present, causing the particles to aggregate and therefore not act necessarily as individual particles. Chapter VI of this manual will provide more rational and better balanced design methods for cohesionless soils.

Figure 3.9.2 shows relationships between maximum allowable velocity (velocity against stone) and maximum size of riprap (stone size or equivalent diameter) as determined and/or recommended by different investigators for different applications. The difference could possibly be due to the lack of reflecting all the significant parameters. They obviously are attempting to describe all these parameters by a single index, velocity. However, Chapter VI of this manual will provide a better analysis and design for riprap.

3.9.3 Relation between shear stress and velocity

Measuring the average bottom shear stress directly in the field is tenuous. However, the average bottom shear stress can be computed for steady uniform flow from the expression

$$\tau_o = \gamma RS \qquad 3.9.12$$

The average shear stress on the bed can also be estimated by employing the velocity profile equations in Chapter II. If the local velocity v_1 at depth y_1 is known then, from Eq. 2.3.15

$$\tau_o = \frac{\rho v_1^2}{[5.75 \log(30.2 \frac{y_1}{k_s})]^2} \qquad 3.9.13$$

This equation and the ones given below are valid for fully turbulent uniform flow in wide channels with a plane bed. Alternatively, if two point velocities in a vertical profile are known

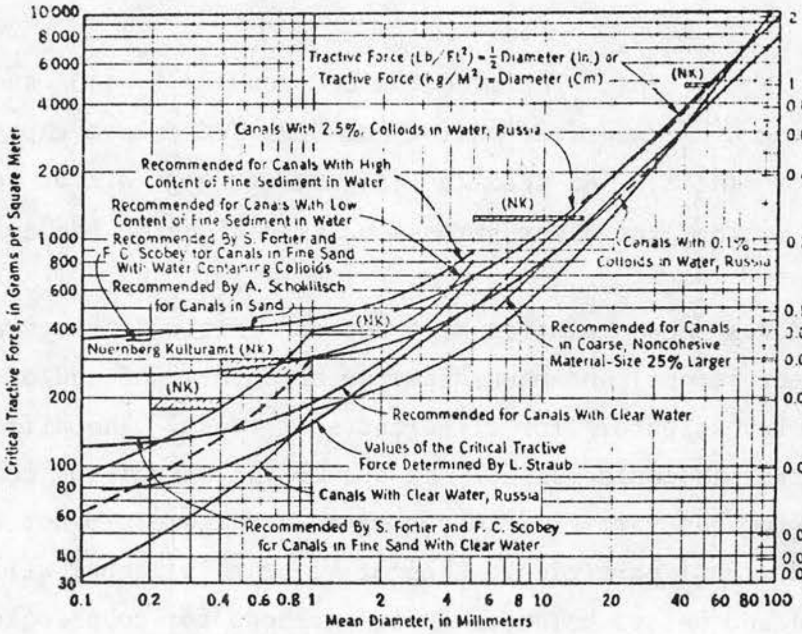


Fig. 3.9.1 Recommended limiting shear stress for canals.

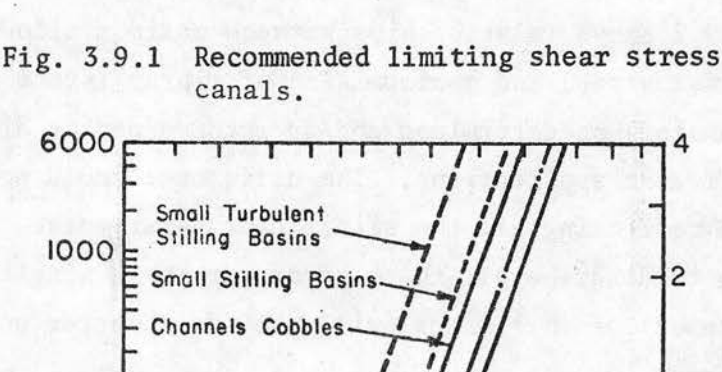


Fig. 3.9.2 Critical velocity as a function of stone size.

$$\tau_o = \frac{\rho(v_1 - v_2)^2}{[5.75 \log(\frac{y_1}{y_2})]^2} \qquad 3.9.14$$

If the depth of flow y_o, the grain size k_s and the average velocity in the vertical V are known, then according to Eq. 2.3.16

$$\tau_o = \frac{\rho V^2}{[5.75 \log(12.27 \frac{y_o}{k_s})]^2} \qquad 3.9.15$$

The preceding equations deal with average values of the shear stress or velocity.

The instantaneous value of the shear stress or local velocity v may be as much as two or three times greater than the average value. The fact that *the instantaneous shear stress at the bed* would be varying greatly *is accounted for in Shields' diagram* (Fig. 3.2.3) *if the channel is prismatic and all the turbulence is generated at the channel boundary.* (Shields' curve was determined from experimental tests.) However, if the turbulence is being generated in some other manner (by a hydraulic jump for example), then the best estimate of the average boundary shear stress is (from Eq. 2.3.8)

$$\tau_o = - \overline{\rho v_x' v_y'} \qquad 3.9.16$$

As the Reynolds' stress $\rho v_x' v_y'$ is extremely difficult to obtain at the present time, there are few good estimates of the shear stress for flow conditions such as in hydraulic jumps or in regions of flow separation. In special cases such as flow around prismatic bends, the boundary shear stress has been obtained from model tests in laboratories.

In addition to the velocity or shear stress forces, the wave forces and seepage forces must be considered in determining a critical shear stress, critical velocity or size of stone to resist motion.

3.10.0 SEDIMENT TRANSPORT

In this section the basic terms and methods of computing sediment load in alluvial channels are described.

3.10.1 Terminology

Bed layer: The flow layer, several grain diameters thick (usually taken as two grain diameters thick), immediately above the bed.

Bed load: Sediment that moves by rolling or sliding along the bed and is essentially in contact with the stream bed in the bed layer.

Bed-load discharge: The quantity of bed load passing a cross section of a stream in a unit of time.

Bed material: The sediment mixture of which the stream bed is composed.

Bed-material discharge: That part of the total sediment discharge which is composed of grain sizes found in the bed. The bed-material discharge is assumed equal to the transport capability of the flow.

Contact load: Sediment particles that roll or slide along in almost continuous contact with the stream bed.

Density of water-sediment mixture: The mass per unit volume including both water and sediment.

Discharge-weighed concentration: The dry weight of sediment in a unit volume of stream discharge, or the ratio of the discharge of dry weight of sediment to the discharge by weight of water sediment mixture normally reported in parts per million (ppm) or parts per liter (ppl).

Load (or sediment load): The sediment that is being moved by a stream.

Sediment (or fluvial sediment): Fragmentary material that originates from weathering of rocks and is transported by, suspended in, or deposited from water.

Sediment concentration (by weight or by volume): The quantity of sediment relative to the quantity of transporting fluid, or fluid-sediment mixture. The concentration may be by weight or by volume. When expressed in ppm, the concentration is always in ratio by weight.

Sediment discharge (or sediment load): The quantity of sediment that is carried past any cross section of a stream in a unit of time.

Sediment yield: The dry weight of sediment per unit volume of water-sediment mixture in place, or the ratio of the dry weight of sediment to the total weight of water-sediment mixture in a sample or a unit volume of the mixture.

Spatial concentration: The dry weight of sediment per unit volume of water-sediment mixture in place, or the ratio of the dry weight of sediment to the total weight of water-sediment mixture in a sample or a unit volume of the mixture.

Suspended load (or suspended sediment): Sediment that is supported by the upward components of turbulence in a stream and that stays in suspension for an appreciable length of time.

Suspended-sediment discharge (or suspended load): The quantity of suspended sediment passing through a stream cross section outside the bed layer in a unit of time.

Total sediment discharge: The total sediment discharge of a stream. It is the sum of the suspended-sediment discharge and the bed-load discharge, or the sum of the bed-material discharge and the wash-load discharge or the sum of the measured suspended sediment discharge and the unmeasured sediment discharge.

Unmeasured sediment discharge: Sediment discharge close to the bed that is not sampled by a suspended-load sampler.

Washload: That part of the total sediment discharge which is composed of particle sizes finer than those found in appreciable quantities in the bed material is determined by available bank and upslope supply rate.

3.10.2 General considerations

The amount of material transported or deposited in the stream under a given set of conditions is the result of the interaction of two groups of variables. In the first group are those *variables which influence the quantity and quality of the sediment brought down to that section of the stream.* In the second group are *variables which influence the capacity of the stream to transport that sediment.* A list of these variables is given below.

Group 1 - Sediment brought down to the stream depends on the geology and topography of watershed; magnitude, intensity, duration, distribution, and season of rainfall; soil condition; vegetal cover; cultivation and grazing; surface erosion and bank cutting.

Group 2 - Capacity of stream to transport sediment depends on hydraulic properties of the stream channel. These are fluid properties, slope, roughness, hydraulic radius, discharge, velocity, velocity distribution, turbulence, tractive force, viscosity and density of the fluid sediment mixture, and size and gradation of the sediment.

These variables are not all independent and, in some cases, their effect is not definitely known. The variables which control the amount of sediment brought down to the stream are subject to so much variation, not only between streams but at a given point of a single stream, that the analysis of any particular case in a quantitative way is extremely difficult. It is practicable to measure the sediment discharge over a long period of time and record the results, and from these records to determine a soil loss from the area.

The variables which deal with the capacity of the stream to transport solids are subject to mathematical analysis. These variables are closely related to the hydraulic variables controlling the capacity of the streams to carry water.

3.10.3 Source of sediment transport

Einstein (1964) stated that:

> "Every sediment particle which passes a particular cross section of the stream must satisfy the following two conditions: (1) It must have been eroded somewhere in the watershed above the cross section; (2) it must be transported by the flow from the place of erosion to the cross section.
>
> Each of these two conditions may limit the sediment rate at the cross section, depending on the relative magnitude of two controls: the availability of the material in the watershed and the transporting ability of the stream. In most streams the finer part of the load, i.e., the part which the flow can easily carry in large quantities, is limited by its availability in the watershed. This part of the load is designated as *wash load*. The coarser part of the load, i.e., the part which is more difficult to move by flowing water, is limited in its rate by the transporting ability of the flow between the source and the section. This part of the load is designated as *bed-material load*."

Thus, for engineering purposes there are *two sources of the sediment* transported by a stream: (1) the bed material that makes up *the stream bed*, and (2) the fine material that comes from *the banks and the watershed* (wash load). Geologically both materials come from the watershed. But for the engineer, the distinction is important because the bed material is transported at the capacity of the stream and is functionally related to measurable hydraulic variables. The wash load is not transported at the capacity of the stream. Instead the wash load depends on the availability and is not functionally related to measurable hydraulic variables.

There is no sharp demarcation between wash load discharge and bed-material discharge. As a rule of thumb, many engineers assume that the bed-material load is composed of sizes equal to or greater than 0.062 mm which is also the division point between sand and silt. The sediment discharge consisting of grain sizes smaller than 0.062 mm is considered the wash load. A more reasonable criterion, although not necessarily theoretically correct, is to choose a sediment size finer than the smallest 10 percent of the bed material as the dividing size between wash load and bed-material load. It is important to note that in a fast flowing mountain stream with a bed of cobbles that the wash load may consist of coarse sand sizes.

3.10.4 Mode of sediment transport

Sediment particles are transported by rolling or sliding on the bed (bed load or contact load) *or by suspension by the turbulence of the stream.* Even as there is no sharp demarcation between bed-material discharge and wash load there is no sharp line between contact load and suspended sediment load. A particle may move part of the time in contact with the bed and at other times be suspended by the flow. The distinction is important because the two modes of transport follow different laws. The equations for estimating the total bed-material discharge of a stream are based on these laws.

3.10.5 Total sediment discharge

The total sediment discharge of a stream is the sum of the bed material discharge and the fine material (wash load) discharge, or the sum of the contact load and suspended sediment load. In the former sum the total sediment discharge is based on source of the sediments and the latter sum is based on the mode of sediment transport. Whereas suspended sediment load consists of both bed material and fine material (wash load), only the bed-material discharge can be estimated by the various equations that have been developed. The fine material discharge (wash load) depends on its availability not on the transporting capacity of the flow and must be measured.

The sediment discharge that is measured by suspended-sediment samplers consists of both the wash load (fine material load) and suspended-sediment load. The contact load is not measured, and because samplers cannot travel the total distance in the vertical to the bed, part of the suspended sediment in a vertical is not measured. Generally, the amount of bed material moving in contact with the bed of a large deep, sandbed river is from 5 to 10 percent of the bed material moving in suspension. And in general the measured suspended-sediment discharge is from 90 to 95 percent of the total sediment discharge. However, in shallow sandbed streams with little or no wash load the measured suspended-sediment load may be as small as 50 percent of the total load.

The magnitude of the suspended or bed-load sediment discharge can be very large. Suspended-sediment concentrations as large as 600,000 ppm or 60 percent by weight have been observed. Concentrations of this magnitude are largely fine-material load. By changing fluid properties (viscosity and density) the fine material in the flow increases the capacity of the flow to transport bed material. The concentration and sediment loads of some streams in the United States and the world are given in Table 3.10.1.

The sediment discharge of a stream at a cross section or through a reach of a stream can be determined by measuring the suspended-sediment portion of the load using samplers and estimating the unmeasured discharge or by using one of the many methods that have been developed for computing the bed-material discharge and estimating the wash load. In many problems, only the bed-material load, both in suspension and in contact with the bed, is important. In these cases the wash load can be eliminated from the measured suspended-sediment load if the size distribution of the material is known.

There have been many equations developed for the estimation of bed-material transport. The variation in the magnitude of the bed-material discharge that these equations predict is tremendous. For the same discharge, the predicted discharge can have a 100 fold difference between the smallest and the largest value. This should not be unexpected given the number of variables, the interrelationship between them, the difficulty of measuring many of the variables and the statistical nature of

Table 3.10.1 Sediment transport in large rivers of the world [adapted from Shen (1971)].

River	Country	Drainage Basin sq. mi.	Average annual suspended load: Million Tons	Average annual suspended load: Tons per sq. mi.	Average discharge at mouth cfs
Yellow	China	260,000	2,080	7,540	53,000
Ganges	India	369,000	1,600	4,000	415,000
Brahmaputra	Bangladesh	257,000	800	3,700	430,000
Yangtze	China	750,000	550	1,400	770,000
Indus	Pakistan	374,000	480	1,300	196,000
Ching (Yellow Trib.)	China	22,000	450	20,500	2,000
Amazon	Brazil	2,230,000	400	170	6,400,000
Mississippi	U.S.A.	1,244,000	344	280	630,000
Irrawaddy	Burma	166,000	330	2,340	479,000
Missouri	U.S.A.	529,000	240	450	69,000
Lo (Yellow Trib.)	China	10,000	210	20,200	---
Kosi (Ganges Trib.)	India	24,000	190	7,980	64,000
Mekong	Thailand	307,000	187	1,240	390,000
Colorado	U.S.A.	246,000	149	1,080	5,500
Red	North Viet Nam	46,000	143	3,090	138,000
Nile	Egypt	1,150,000	122	100	100,000

bed-material transport. Nevertheless, with proper use, knowledge of the river and knowledge of the limitations of each method, useful bed-material discharge information can be obtained.

In the next sections *three methods of estimating bed-material discharge are described. These are Meyer-Peter Muller's, Einstein's and Colby's.* Then the basic suspended-sediment equation is developed. The Meyer-Peter Muller equation is applicable to streams with little or no suspended-sediment discharge and is thus used extensively for gravel and cobble bed streams. The other two methods, based to some degree on Einstein's work are used for sandbed streams. Their use depends on the amount of information available. Methods for measuring suspended-sediment discharge are not described. For information on these methods, the reader is referred to the publications by Guy (1969).

3.10.6 Suspended bed-material discharge

The suspended bed-material discharge in lbs per second per unit width of channel q_s, for steady, uniform two-dimensional flow is

$$q_s = \gamma \int_a^{y_o} v\ c\ dy \qquad 3.10.1$$

where v and c vary with y and are the time-averaged flow velocity and concentrations, respectively. The integration is taken over the depth between a level "a" above the bed and the surface of the flow "y_o". The level "a" is assumed to be 2 grain diameters above the bed layer. Sediment movement below this level is considered bed load rather than suspended load.

The discharge of suspended sediment for the entire stream cross section, Q_s, is obtained by integrating Eq. 3.10.1 over the cross section to give

$$Q_s = \gamma Q \overline{C} \qquad 3.10.2$$

where $\overline{C}$ is the average suspended sediment concentration. Both Eqs. 3.10.1 and 3.10.2 are exact.

The vertical distribution of both the velocity and the concentration vary with the mean velocity of the flow, bed roughness and size of bed material. The distributions are illustrated in Fig. 3.10.1. Also v and c are interrelated. That is, the velocity and turbulence at a point is affected by the sediment at the point and the sediment concentration at the point is affected by the point velocity. Normally this interrelation is neglected or a coefficient applied to compensate for it.

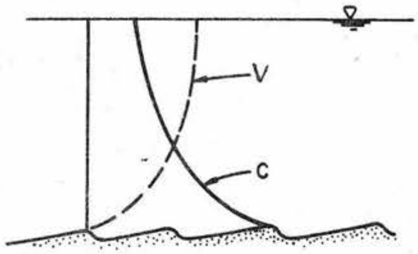


Fig. 3.10.1 Schematic sediment and velocity profiles.

To integrate Eq. 3.10.1, v and c must be expressed as functions of y. *The one-dimensional gradient type diffusion equation is employed to obtain the vertical distribution for* c *and the logarithm velocity distribution can be assumed for* v.

The one-dimensional diffusion equation is obtained from the equilibrium condition that the quantity of sediment settling across a unit area due to the force of gravity is equal to the quantity of sediment transported upwards resulting from the vertical component of turbulence and the concentration gradient. The resulting equation for a given particle size is

$$\omega\, c = -\varepsilon_s \frac{dc}{dy} \qquad 3.10.3$$

where

ω = the fall velocity of the sediment particle at a point

c = the concentration of particles

ε_s = an exchange coefficient which characterizes the magnitude of the exchange of particles across any arbitrary boundary by the turbulence. It is called the mass transfer coefficient

$\frac{dc}{dy}$ = the concentration gradient

ωc = the average rate of settling of the sediment particles

$\varepsilon_s \frac{dc}{dy}$ = the average rate of upwards sediment flow by diffusion

Integrating Eq. 3.10.3 yields

$$c = c_a \exp\{-\omega \int_a^{y_o} \frac{dy}{\varepsilon_s}\} \qquad 3.10.4$$

where c_a is a concentration of sediment with settling velocity ω at the level $y = a$ in the flow.

In order to determine the value of c at a given y, the value of c_a and the variation of ε_s with y must be known. To obtain an expression for ε_s the assumption is made that

$$\varepsilon_s = \beta\ \varepsilon_m \qquad 3.10.5$$

where ε_m is the kinematic eddy viscosity or the momentum exchange coefficient defined by

$$\tau = \rho \varepsilon_m \frac{dv}{dy} \qquad 3.10.6$$

where τ and dv/dy are the shear stress and velocity gradient, respectively, at point y.

For two-dimensional steady uniform flow

$$\tau = \gamma\ S\ (y_o - y) = \tau_o(1 - \frac{y}{y_o}) \qquad 3.10.7$$

and from Eq. 2.3.13

$$\frac{dv}{dy} = \frac{\sqrt{\tau_o/\rho}}{\kappa y} = \frac{V_*}{\kappa y} \qquad 3.10.8$$

Thus

$$\varepsilon_s = \beta\varepsilon_m = \beta\ \kappa\ V_*\ y\ (1 - \frac{y}{y_o}) \qquad 3.10.9$$

where

β = a coefficient relating ε_s to ε_m

κ = the von Karman's velocity coefficient taken as equal to 0.4

V_* = the shear velocity equal to $\sqrt{gRS}$ in steady uniform flow.

Equation 3.10.9 indicates that ε_m and ε_s are zero at the bed and at the water surface, and have a maximum value at mid-depth.
The substitution of Eq. 3.10.9 into Eq. 3.10.3 gives

$$\frac{dc}{c} = \frac{-\omega}{\beta V_*} \cdot \frac{dy}{y(1 - \frac{y}{y_o})} \qquad 3.10.10$$

and after integration

$$\frac{c}{c_a} = [\frac{y_o - y}{y} \frac{a}{y_o - a}]^Z \qquad 3.10.11$$

where

c = the concentration at a distance y from the bed

c_a = the concentration at a point a above the bed

Z = $\frac{\omega}{\beta \kappa V_*}$ the Rouse number, named after the engineer who developed the equation in 1937.

Figure 3.10.2 shows a family of curves obtained by plotting Eq. 3.10.11 for different values of the Rouse number Z. It is seen that for small values of Z, the sediment distribution is nearly uniform.

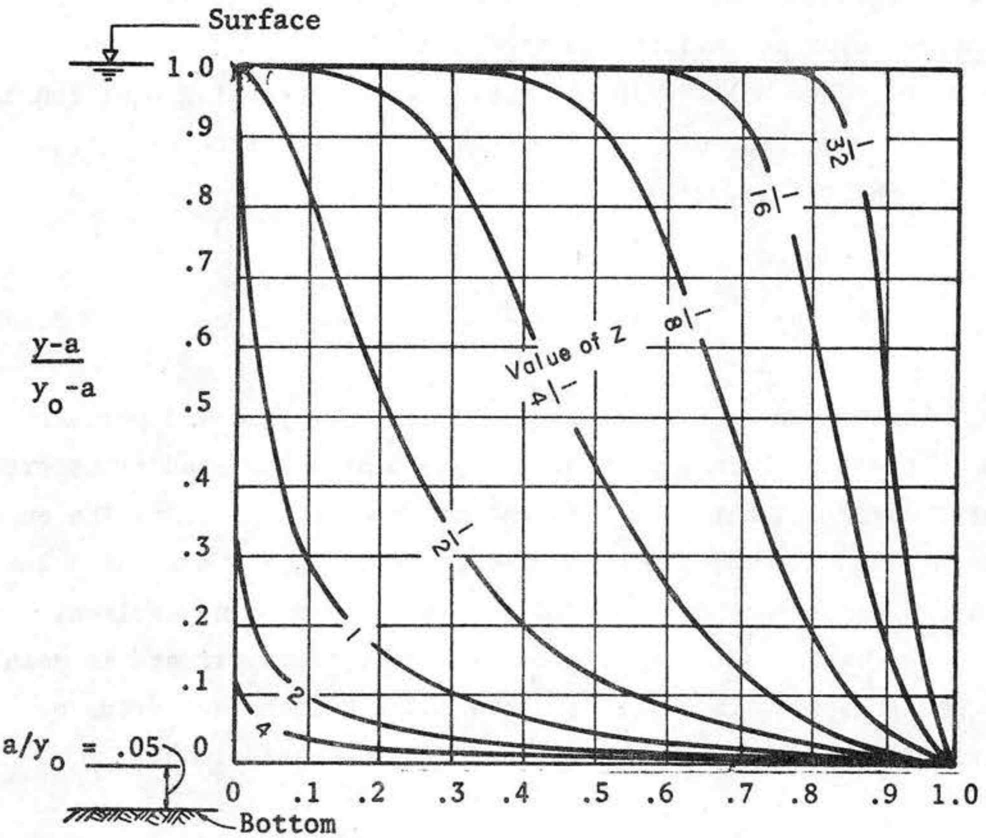


Fig. 3.10.2 Graph of suspended sediment distribution.

For large Z values, little sediment is found at the water surface. The value of Z is small for large shear velocities V_* or small fall velocities ω and large for small V_* and large ω. Thus, for small particles or for extremely turbulent flows, the concentration profiles are uniform.

The values of β and κ have been investigated. For fine particles $\beta \sim 1$. Also, it is well known that in sediment-laden water $\kappa \neq 0.4$ but decreases with increasing sediment concentration.

Using the logarithmic velocity distribution for steady uniform flow and Eq. 3.10.11 the equation for suspended sediment transport becomes

$$q_s = \gamma V_* c_a \int_a^{y_o} \left[\frac{a}{y_o - a} \cdot \frac{y_o - y}{y}\right]^Z \left[2.5 \ln\left(30.2 \frac{xy}{k_s}\right)\right] dy \qquad 3.10.12$$

This equation has been integrated by many investigators and the assumptions and integration made by Einstein are presented later.

3.10.7 Meyer-Peter and Muller Equation

Meyer-Peter and Muller (1948) developed the following equation based on experiments with sand particles of uniform sizes, sand particles of mixed sizes, natural gravel, lignite, and baryta:

$$\left(\frac{Q_b}{Q}\right)\left(\frac{K_b}{K_r}\right)^{3/2} \gamma y_o S = B' (\gamma_s - \gamma) D_m + B\left(\frac{\gamma}{g}\right)^{1/3} \left(\frac{\gamma_s - \gamma}{\gamma_s}\right)^{2/3} q_B^{2/3} \qquad 3.10.13$$

where q_B is the bed-load rate in weight per unit time and per unit width, Q_b is water discharge quantity determining bed load transport, Q is total water discharge, y_o is the depth of flow, S is the energy slope and B' and B are dimensionless constants. B' has the value 0.047 for sediment transport and 0.034 for the case of no sediment transport. B has a value of 0.25 for sediment transport and is meaningless for no transport since q_B is zero and the last term drops out. The quantities K_b and K_r are defined by the expressions

$$V = K_b R_b^{2/3} S^{1/2} \qquad 3.10.14$$

and

$$V = K_r R_b^{2/3} S'^{1/2} \qquad 3.10.15$$

where S' is the part of the total slope, S, required to overcome the grain resistance and $S-S'$ is that part of the total slope required to overcome form resistance. Therefore

$$\frac{K_b}{K_r} = \sqrt{\frac{f_b'}{8}} \; \frac{V}{\sqrt{g\, R_b S}} \qquad 3.10.16$$

where f_b' is the Darcy-Weisbach bed friction factor for the grain roughness. If the boundary is hydraulically rough ($V_* D_{90}/\nu \geq 100$), K_r is given by

$$K_r = \frac{26}{D_{90}^{1/6}} \qquad 3.10.17$$

in which D_{90} is in meters and K_r is in $m^{1/3}$sec. The quantity D_m is the effective diameter of the sediment given by

$$D_m = \frac{\sum_i P_i D_i}{100} \qquad 3.10.18$$

where P_i is the percentage by weight of that fraction of the bed material with geometric mean size, D_i.

Equation 3.10.13 is dimensionally homogeneous so that any consistent set of units may be used.

Equation 3.10.13 has been converted to units generally used in the United States in the field of sedimentation for water and quartz particles by the U.S. Bureau of Reclamation (1960). This equation is

$$q_B = 1.606 \left[3.306 \left(\frac{Q_b}{Q}\right) \left(\frac{D_{90}^{1/6}}{n_b}\right)^{3/2} y_o S - 0.627\, D_m\right]^{3/2} \qquad 3.10.19$$

where q_B is in tons per day per foot width, Q_b is the water discharge quantity determining the bed-load transport in cfs, Q is the total water discharge quantity in cfs, D_{90} and D_m are in millimeters. The quantity n_b is given by

$$n_b = n \left[1 + \frac{2y_o}{W}\left(1 - \left(\frac{n_w}{n}\right)^{3/2}\right)\right]^{2/3} \qquad 3.10.20$$

for rectangular channels and

$$n_b = n \left\{1 + \frac{2y_o(1 + H^2)^{1/2}}{W}\left[1 - \left(\frac{n_w}{n}\right)^{3/2}\right]\right\}^{2/3} \qquad 3.10.21$$

for trapezoidal channels, where, n, n_b, and n_w are roughness coefficients of the total stream, of the bed, and of the banks, respectively; and H is the horizontal side slope related to one unit vertically. The quantity Q_b/Q is given by

$$\frac{Q_b}{Q} = \frac{1}{1 + \frac{2y_o}{W}\left(\frac{n_w}{n_b}\right)^{3/2}} \qquad 3.10.22$$

for rectangular channels and

$$\frac{Q_b}{Q} = \frac{1}{1 + \frac{2y_o(1 + H^2)^{1/2}}{W}\left(\frac{n_w}{n_b}\right)^{3/2}} \qquad 3.10.23$$

for trapezoidal channels.

The Meyer-Peter and Muller formula (Eq. 3.10.13) is often written in the form

$$q_b = K\,(\tau - \tau_c)^{3/2} \qquad 3.10.24$$

where

$$K = \left(\frac{1}{B\left(\frac{\gamma}{g}\right)^{1/3}\left(\frac{\gamma_s - \gamma}{\gamma_s}\right)^{2/3}}\right)^{3/2}$$

$$\tau = \left(\frac{Q_b}{Q}\right)\left(\frac{K_b}{K_r}\right)^{3/2} \gamma y_o S$$

$$\tau_c = B' \, (\gamma_s - \gamma) D_m$$

3.10.8 Einstein's bed load function

Einstein's (1950) method for determining total bed-material discharge is to sum up the contact load and the suspended load. As mentioned earlier, there is no sharp demarcation between the contact bed material load and the suspended bed material load. However, the division is warranted by the fact that there is a difference in behavior of the two different loads which requires two physical models.

Einstein's bed-material discharge function gives the rate at which flow of any magnitude in a given channel transports the *individual sediment sizes which make up the bed material.* This makes his equations extremely valuable in many studies because one can determine the change in bed material with time that occurs because each size moves at its own rate. For each size D of the bed material, the contact load is given as

$$i_B \, q_B \qquad 3.10.25$$

and the suspended sediment load is given by

$$i_s \, q_s \qquad 3.10.26$$

and the total bed material discharge is

$$i_T q_T = i_s q_s + i_B q_B \qquad 3.10.27$$

$$Q_T = \sum i_T q_T \qquad 3.10.28$$

where i_T, i_s and i_B are the fraction of the total, suspended and contact loads q_T, q_s and q_B for a given grain size D. The term Q_T is the total bed-material transport. The suspended sediment total is related to the contact load because there is a continuous exchange of particles between the two modes of transport.

With suspended-sediment load related to the contact load, Eq. 3.10.27 becomes

$$i_T q_T = i_B q_B \ (1 + P_E \ I_1 + I_2) \qquad 3.10.29$$

where

$$i_B q_B = \frac{\phi_* \ i_b \ \gamma_s}{\left(\frac{\rho}{\rho_s - \rho} \frac{1}{gD^3}\right)^{1/2}} \qquad 3.10.30$$

and

γ_s = the unit weight of sediment
ρ = the density of the water
ρ_s = the density of the sediment
g = gravitational acceleration
ϕ_* = $f(\psi_*)$ given in Fig. 3.10.3

$$\psi_* = \xi Y \left(\frac{B}{B_x}\right)^2 \psi \qquad 3.10.31$$

$$\psi = \frac{\rho_s - \rho}{\rho} \ \frac{D}{R_b' S} \qquad 3.10.32$$

ξ = a correction factor given as a function of d/X in Fig. 3.10.4.

$$\begin{aligned} X &= 0.77\Delta \qquad \text{if } \frac{\Delta}{\delta'} > 1.8 \\ X &= 1.39\delta' \qquad \text{if } \frac{\Delta}{\delta'} < 1.8 \end{aligned} \qquad 3.10.33$$

Δ = the apparent roughness of the bed, k_s/x

x = a correction factor in the logarithmic velocity distribution equation and is given as a function of k_s/δ' in Fig. 2.3.5

$$\delta' = \frac{11.6\nu}{V_*'} \qquad 3.10.34$$

$\frac{v}{V_*'}$ = Einstein's velocity distribution equation

$$= 5.75 \log \left(30.2 \frac{y}{\Delta}\right) \qquad 3.10.35$$

V_*' = the shear velocity due to grain roughness
$= \sqrt{gR_b'S}$

R_b' = the hydraulic radius of the bed due to grain roughness,

$= R_b - R_b''$

S = the slope of the energy grade line normally taken as the slope of the water surface.

Y = another correction term given as a function of D_{65}/δ' in Fig. 3.10.5

B = log 10.6

B_x = log (10.6 X/Δ)

The preceding equations are used to compute the fraction i_B of the load. The other terms in Eq. 3.10.29 are

$$P_E = 2.3 \log 30.2 \frac{y_o}{\Delta} \qquad 3.10.36$$

I_1 and I_2 are integrals of Einstein's form of the suspended sediment Eq. 3.10.11

$$I_1 = 0.216 \frac{E^{Z-1}}{(1-E)^Z} \int_E^1 \left(\frac{1-y}{y}\right)^Z dy \qquad 3.10.37$$

$$I_2 = 0.216 \frac{E^{Z-1}}{(1-E)^Z} \int_E^1 \left(\frac{1-y}{y}\right)^Z \ln y \, dy \qquad 3.10.38$$

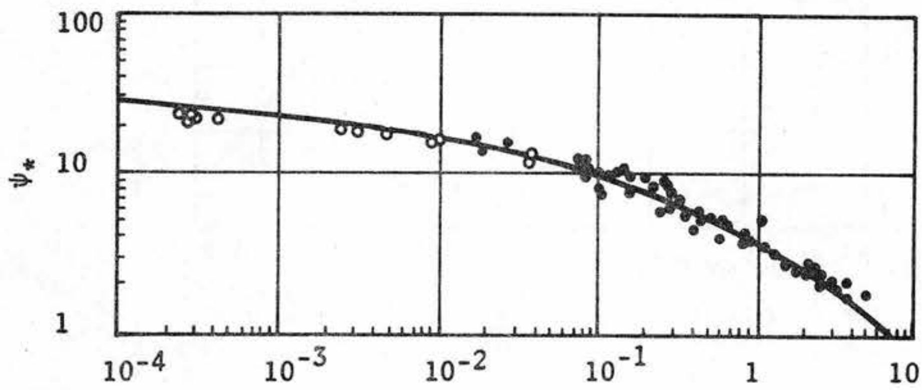


Fig. 3.10.3 Einstein's $\phi_* - \psi_*$ bed load function, (Einstein, 1950).

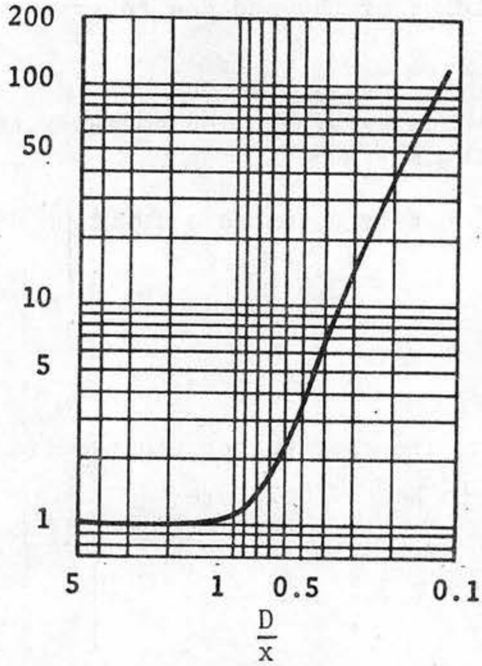


Fig. 3.10.4 Hiding factor, (Einstein, 1950).

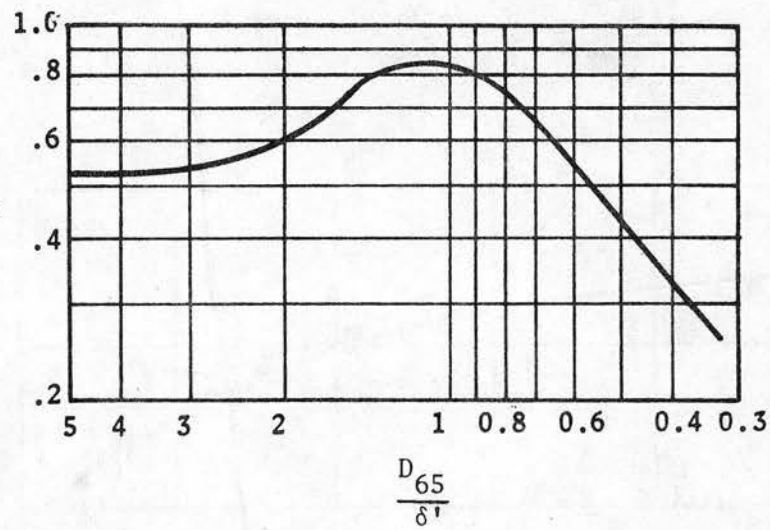


Fig. 3.10.5 Pressure correction, (Einstein, 1950).

$$Z = \frac{\omega}{.4V'_*} \qquad 3.10.39$$

ω = the fall velocity of the particle of size D

E = the ratio of bed layer thickness to flow depth, a/y_o

y_o = depth of flow

a = the thickness of the bed layer, 2d

The two integrals I_1 and I_2 are given in Figs. 3.10.6 and 3.10.7 as a function of Z and E.

In the preceding calculations for the total load, the shear velocity is based on the hydraulic radius for the bed, R'_b. Its computation is explained in the following paragraph.

Total resistance to flow is composed of two parts, surface drag and form drag. The transmission of shear to the boundary is accompanied by a transformation of flow energy into energy turbulence. The part of energy corresponding to grain roughness is transformed into turbulence which stays at least for a short time in the immediate vicinity of the grains and has a great effect on the bed load motion; whereas, the other part of the energy which corresponds to the form resistance is transformed into turbulence at the interface between wake and free stream flow, or at a considerable distance away from the grains. This energy does not contribute to the bed load motion of the particles and may be largely neglected in the sediment transportation.

Einstein's equation for mean flow velocity V in terms of V'_* is

$$\frac{V}{V'_*} = 5.75 \log (12.27 \frac{R'_b}{\Delta})$$

or 3.10.40

$$\frac{V}{V'_*} = 5.75 \log (12.27 \frac{R'_b}{k_s} x)$$

Furthermore Einstein suggested that

$$\frac{V}{V''_*} = \theta [\psi'] \qquad 3.10.41$$

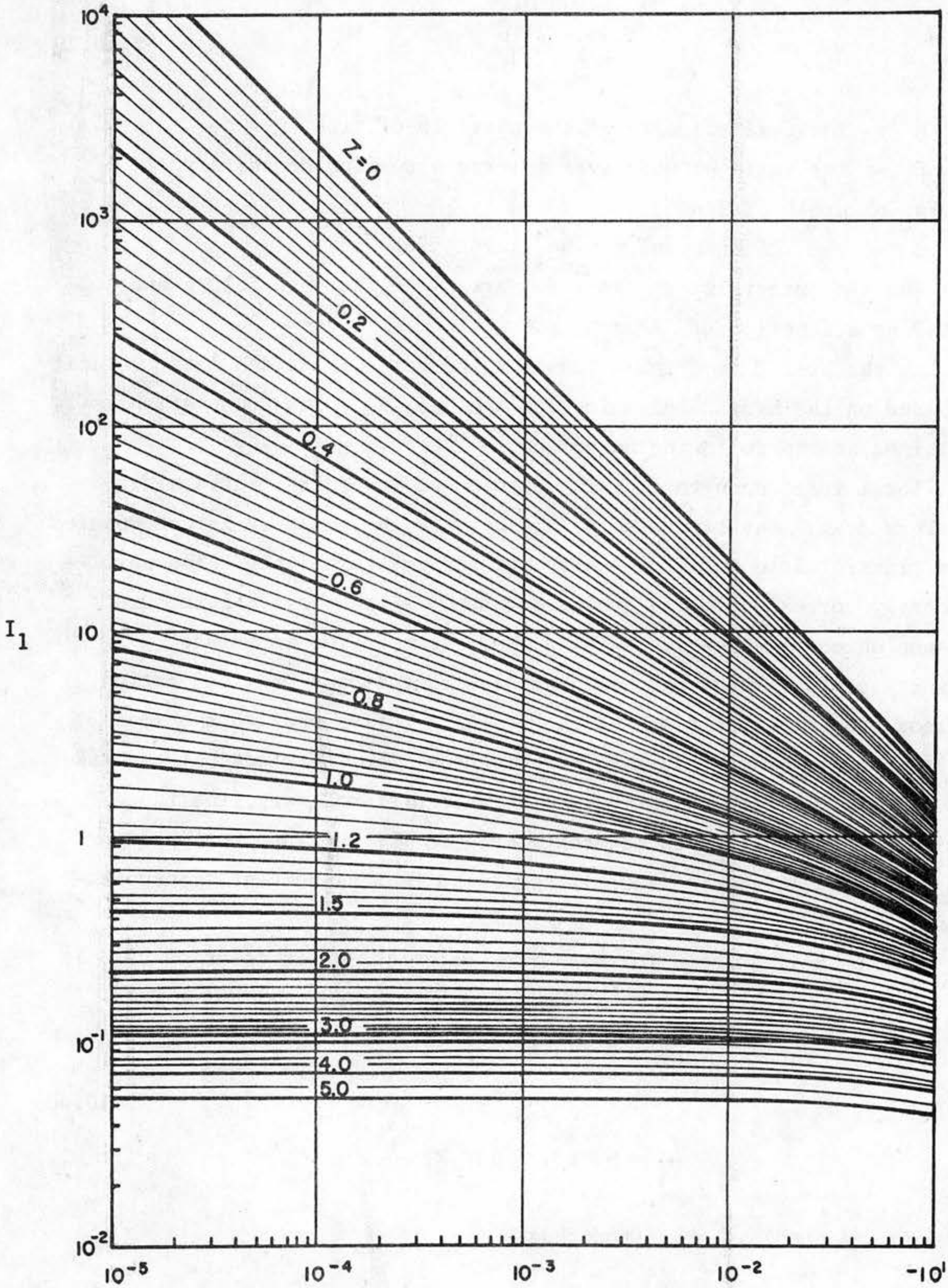


Fig. 3.10.6 Integral I_1 in terms of E and Z, (Einstein, 1950).

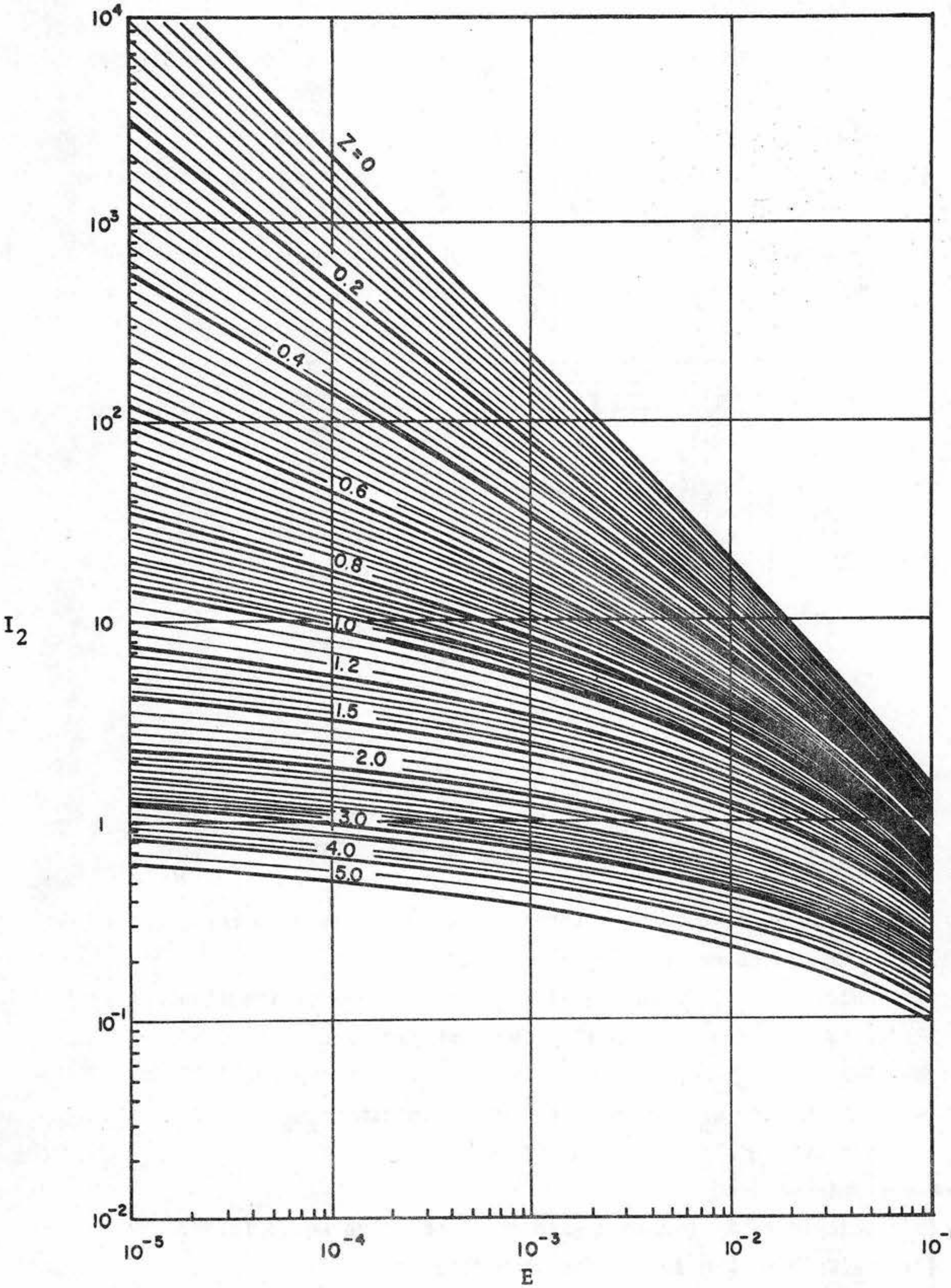


Fig. 3.10.7 Integral I_2 in terms of E and Z, (Einstein, 1950).

where

$$\psi' = \frac{\rho_s - \rho}{\rho} \frac{D_{35}}{R_b' S} \qquad 3.10.42$$

The relation for Eq. 3.10.42 is given in Fig. 3.10.8.

The procedure to follow in computing R_b' depends on the information available. If mean velocity V, slope S, cross sectional dimensions R_b and bed-material size are known, then R_b' is computed by trial and error using Eq. 3.10.40 and Fig. 3.10.8.

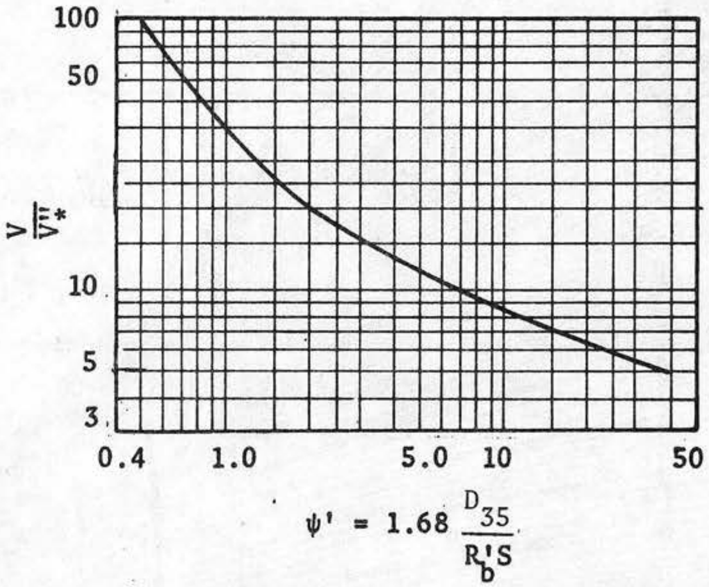


Fig. 3.10.8 V/V''_* vs. ψ' (Einstein, 1950).

The procedure for computing total bed-material load in terms of different size fractions of the bed material is:

(1) Calculate ψ_* using Eq. 3.10.31 for each size fraction.

(2) Find ϕ_* from Fig. 3.10.3 for each fraction.

(3) Calculate $i_B \, q_B$ for each size fraction using Eq. 3.10.30.

(4) Sum up the q_B across the flow to obtain $i_B Q_b$.

(5) Sum up the size fractions to obtain Q_b.

Suspended sediment load:

(6) Calculate Z for each size fraction using Eq. 3.10.39.

(7) Calculate $E = 2D/y_o$ for each fraction.

(8) Determine I_1 and I_2 for each fraction from Fig. 3.10.6 and 3.10.7.

(9) Calculate P_E using Eq. 3.10.36.

(10) Compute the suspended load from $i_B q_B (P_E I_1 + I_2)$.

(11) Sum up all the q_B and all the i_B to obtain the total suspended load Q_{ss}.

Total bed material discharge:

(12) Add the results of Step 5 and 11.

3.10.9 Application of transport functions to field channels

In applying the Einstein procedure (1950) to a particular water course, three steps should be carried out. The first step involves the choice of a river reach and the collection of the field data. The second step is to determine hydraulic parameters. The third step is the calculation of the total bed-material discharge.

3.10.10 Description of the test reach

A test reach, representative of the Big Sand Creek near Greenwood, Mississippi, was used by Einstein (1950) as an illustrative example for applying his bed-load function. His example is reproduced here. For simplicity, the effects due to bank friction are neglected. The reader can refer to the original example for the construction of the representative cross section. The characteristics of this cross section are as follows.

The channel slope was determined as $S = 0.00105$. The relation between cross-sectional area, hydraulic radius, and wetted perimeter of the representative cross section and stage are given in Fig. 3.10.9. In the case of this wide and shallow channel, the wetted perimeter is

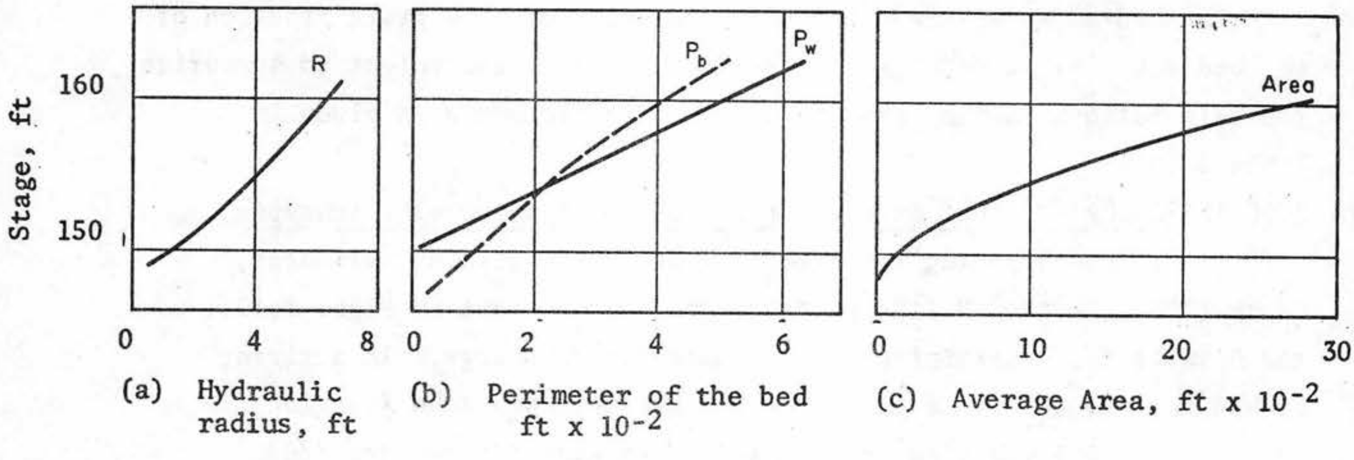


Fig. 3.10.9 Description of the average cross section (Einstein, 1950).

assumed to equal the surface width. The averaged values of the four bed-material samples are given in Table 3.10.2. Ninety-six percent of the bed material is between 0.589 and 0.147 mm, which is divided into four size fractions. The sediment transport calculations are made for the individual size fraction which has the representative grain size equal to the geometric mean grain diameter of each fraction. The water viscosity is $\nu = 1.0 \times 10^{-5}$ sq ft/sec, and the specific gravity of the sediment is 2.65.

The calculation of important hydraulic parameters is performed in Table 3.10.3. The table heading, its meaning, and calculation are explained with footnotes.

Table 3.10.2 Bed-material information for sample problem.

Grain size distribution, mm	Average grain size mm	ft	%	Settling velocity cm/sec	fps
D > 0.589	---	---	2.4	---	---
0.589 > D > 0.417	0.495	0.00162	17.8	6.25	0.205
0.417 > D > 0.295	0.351	0.00115	40.2	4.51	0.148
0.295 > D > 0.208	0.248	0.00081	32.0	3.23	0.106
0.208 > D > 0.147	0.175	0.00058	5.8	2.04	0.067
0.147 > D	---	---	1.8	---	---

D_{35} = 0.29 mm = 0.00094 ft

D_{65} = 0.35 mm = 0.00115 ft

3.10.11 Bed-material discharge calculations

The bed-material transport is calculated for each grain fraction of the bed material at each given flow depth. It is convenient to summarize the calculations in the form of tables. The procedure is given in Table 3.10.4.

3.10.12 Colby's method of estimating total bed material discharge

After investigating the effect of all the pertinent variables, Colby (1964) developed four graphical relations shown in Figs. 3.10.10 and 3.10.11 for determining the bed-material discharge. In arriving at his curves, *Colby was guided by the Einstein bed-load function* (Einstein, 1950) *and a large amount of data from streams and flumes.* However, it should be understood that all curves for 100 ft depth, most curves of 10 ft depth and part of the curves of 1.0 ft and 0.1 ft are not based on data but are extrapolated from limited data and theory.

Table 3.10.3 Hydraulic calculations for sample problem in applying the Einstein procedure (after Einstein, 1950).

R_b'	V_*'	δ'	κ_s/δ'	x	Δ	V	ψ'	$\frac{V}{V_*''}$	V_*''	R_b''	R_b	y_o	z	A	P_b	Q	X	Y	B_x	$(B/B_x)^2$	P_E
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
0.5	0.129	0.00095	1.21	1.59	0.00072	2.92	2.98	16.8	0.17	0.86	1.36	1.36	150.2	140	103	409	0.00132	0.84	1.29	0.63	10.97
1.0	0.184	0.00067	1.72	1.46	0.00079	4.44	1.49	27.0	0.16	0.76	1.76	1.76	150.9	240	136	1,065	0.00093	0.68	1.19	0.85	11.10
2.0	0.259	0.00047	2.44	1.27	0.00090	6.63	0.75	51.0	0.13	0.50	2.50	2.50	152.1	425	170	2,820	0.00069	0.56	0.91	1.27	11.30
3.0	0.318	0.00039	2.95	1.18	0.00097	8.40	0.50	87.0	0.10	0.30	3.30	3.30	153.3	640	194	5,380	0.00076	0.55	0.91	1.27	11.50
4.0	0.368	0.00033	3.50	1.14	0.00102	9.92	0.37	150.0	0.07	0.14	4.14	4.14	154.9	970	234	9,620	0.00079	0.54	0.91	1.27	11.70
5.0	0.412	0.00030	3.84	1.10	0.00104	11.30	0.30	240.0	0.05	0.07	5.07	5.07	156.9	1,465	289	16,550	0.00080	0.54	0.91	1.27	11.90
6.0	0.450	0.00027	4.26	1.08	0.00107	12.58	0.25	370.0	0.03	0.03	6.03	6.03	159.5	2,400	398	30,220	0.00082	0.54	0.91	1.27	12.04

[1] See the following for explanation of symbols, column by column:

(1) R_b' = bed hydraulic radius due to grain roughness, ft.

(2) V_*' = shear velocity due to grain roughness, fps
$= \sqrt{gR_b'S}$

(3) δ' = thickness of the laminar sublayer, ft
$= 11.62\nu/U_*'$

(4) κ_s = roughness diameter, ft
$= D_{65}$

(5) x = correction factor in the logarithmic velocity distribution, given in Fig. 2.3.5

(6) Δ = apparent roughness diameter, ft
$= \kappa_s/x$

(7) V = average flow velocity, fps
$= 5.75V_*' \log(12.27\, R_b'/\Delta)$

(8) ψ' = intensity of shear on representative particles
$= \frac{\rho_s-\rho}{\rho}\ \frac{D_{65}}{R_b'S}$

(9) V/V_*'' = velocity ratio, given in Fig. 3.10.8

(10) V_*'' = shear velocity due to form roughness, fps

(11) R_b'' = bed hydraulic radius due to form roughness, ft
$= V_*''^2/gS$

(12) R_b = bed hydraulic radius, ft
= R, the total hydraulic radius if there is no additional friction
$= R_b' + R_b''$

(13) y_o = average flow depth, ft
= R for wide, shallow streams

(14) z = stage, ft, given in Fig. 3.10.9a

(15) A = cross-sectional area, ft^2, given in Fig. 3.10.9c

(16) P_b = bed wetted perimeter, ft, given in Fig. 3.10.9b

(17) Q = flow discharge
$= AV$

(18) X = characteristic distance, ft
$= 0.77\Delta$ for $\Delta/\delta' > 1.80$
$= 1.39\delta'$ for $\Delta/\delta' < 1.80$

(19) Y = pressure correction term, given in Fig. 3.10.5

(20) B_x = coefficient
$= \log(10.6X/\Delta)$

(21) B = coefficient
$= \log 10.6$

(22) P_E = Einstein's transport parameter
$= 2.303 \log \frac{30.2\, y_o}{\Delta}$

Table 3.10.4 Bed-material load calculations for sample problem in applying the Einstein procedure (Einstein, 1950).

D (1)	i_B (2)	R_b' (3)	ψ (4)	d/X (5)	ξ (6)	ψ_* (7)	ϕ_* (8)	i_Bq_B (9)	i_BQ_B (10)	Σi_BQ_B (11)	10^3E (12)	Z (13)	I_1 (14)	$-I_2$ (15)	$P_EI_1+I_2+1$ (16)	i_Tq_T (17)	i_TQ_T (18)	Σi_TQ_T (19)
.00162	.178	0.5	5.08	1.23	1.08	2.90	1.90	0.0267	119	400	2.38	3.78	0.078	0.44	1.42	0.0380	168	670
		1.0	2.54	1.74	1.00	1.73	4.00	0.0561	330	1,335	1.84	2.65	0.131	0.74	1.71	0.0958	561	3,938
		2.0	1.27	2.35	1.00	0.90	8.20	0.115	845	3,771	1.30	1.88	0.240	1.27	2.44	0.281	2,050	30,500
		3.0	0.85	2.16	1.00	0.60	12.8	0.180	1,510	6,496	0.98	1.53	0.385	2.01	3.44	0.617	5,170	113,000
		4.0	0.63	2.05	1.00	0.43	18.0	0.253	2,560	10,745	0.78	1.33	0.560	2.80	4.75	1.20	12,100	324,000
		5.0	0.51	2.03	1.00	0.35	22.5	0.316	3,950	16,333	0.63	1.18	0.810	3.85	6.78	2.13	26,500	800,000
		6.0	0.42	1.98	1.00	0.29	27.0	0.380	6,530	27,142	0.54	1.08	1.09	4.90	9.20	3.48	59,800	1,940,000
.00115	.402	0.5	3.38	0.82	1.36	2.44	2.45	0.0471	210		1.69	2.88	0.117	0.68	1.60	0.0754	335	
		1.0	1.69	1.16	1.10	1.27	5.50	0.106	623		1.31	2.02	0.210	1.19	2.14	0.227	1,330	
		2.0	0.85	1.57	1.01	0.61	12.6	0.242	1,780		0.92	1.44	0.450	2.33	3.76	0.910	6,660	
		3.0	0.56	1.44	1.04	0.41	19.0	0.364	3,050		0.70	1.17	0.830	3.85	6.73	2.44	20,400	
		4.0	0.42	1.37	1.05	0.30	26.0	0.500	5,050		0.56	1.01	1.37	5.70	11.3	5.65	57,100	
		5.0	0.34	1.35	1.05	0.25	31.5	0.604	7,540		0.45	0.90	2.12	8.10	17.2	10.4	129,000	
		6.0	0.28	1.32	1.05	0.20	39.0	0.749	12,900		0.38	0.83	2.95	10.5	26.0	19.6	335,000	
.00081	.320	0.5	2.54	0.61	2.25	3.03	1.75	0.0155	69.0		1.19	1.94	0.230	1.29	2.23	0.0345	153	
		1.0	1.27	0.87	1.26	1.09	6.80	0.0600	353		0.92	1.36	0.520	2.60	4.16	0.250	1,460	
		2.0	0.63	1.17	1.10	0.49	15.8	0.139	1,020		0.65	0.97	1.53	6.10	12.2	1.70	12,500	
		3.0	0.42	1.08	1.12	0.33	23.5	0.207	1,730		0.49	0.79	3.35	11.0	28.7	5.95	49,700	
		4.0	0.32	1.04	1.15	0.25	31.5	0.279	2,820		0.39	0.68	6.20	17.5	56.0	15.0	157,000	
		5.0	0.25	1.01	1.17	0.20	39.5	0.349	4,360		0.32	0.61	9.80	25.5	92.0	32.0	397,000	
		6.0	0.21	0.99	1.19	0.17	46.0	0.406	6,980		0.27	0.55	15.0	36.0	146	59.5	1,020,000	
.00057	.058	0.5	1.80	0.43	5.40	5.15	0.58	0.00056	2.49		0.85	1.21	0.720	3.35	5.55	0.00312	14	
		1.0	0.90	0.61	2.28	1.39	5.10	0.00500	29.4		0.65	0.86	2.44	8.10	20.0	0.100	587	
		2.0	0.45	0.83	1.37	0.44	17.5	0.0171	126		0.46	0.61	8.40	21.5	74.4	1.26	9,350	
		3.0	0.30	0.76	1.52	0.32	25.0	0.0246	206		0.35	0.49	19.3	41.0	183	4.50	37,600	
		4.0	0.22	0.72	1.60	0.25	31.5	0.0310	313		0.28	0.43	32.0	63.0	312	9.68	97,800	
		5.0	0.18	0.71	1.65	0.20	39.5	0.0387	483		0.23	0.38	51.0	91.0	516	20.0	248,000	
		6.0	0.15	0.70	1.70	0.18	43.5	0.0426	732		0.19	0.35	70.0	122	722	30.8	526,000	

[1] See the following for explanation of symbols, column by column:

(1) D = representative grain size, ft, given in Table 3.10.2

(2) i_B = fraction of bed material, given in Table 3.10.2

(3) R_b' = bed hydraulic radius due to grain roughness, ft, given in Table 3.10.3

(4) ψ = intensity of shear on a particle

$$= \frac{\rho_s - \rho}{\rho} \frac{d}{R_b' S}$$

(5) D/X = dimensionless ratio, X given in Table 3.10.3

(6) ξ = hiding factor, given in Fig. 3.10.4

(7) ψ_* = intensity of shear on individual grain size $= \xi Y (B/B_x)^2 \psi$, (values of Y and $(B/B_x)^2$ are given in Table 3.10.3)

(8) ϕ_* = intensity of sediment transport for individual grain size, given in Fig. 3.10.3

(9) i_Bq_B = bed load discharge per unit width for a size fraction, lb/sec-ft

$$= i_B \phi_* \rho_s (gD)^{3/2} \sqrt{(\rho_s/\rho)-1}$$

(10) i_BQ_B = bed load discharge for a size fraction for entire cross section, tons/day $= 43.2\, W i_B q_B$, $W = P_b$ given in Table 3.10.3

(11) Σi_BQ_B = bed load discharge for a size fraction for entire cross section, tons/day

(12) E = ratio of bed layer thickness to water depth $= 2D/y_o$, for values of y_o See Table 3.10.3

(13) Z = exponent for concentration distribution $= \omega/(0.4U_*')$, for values of ω and U_*' see Tables 3.10.2 and 3.10.3, respectively

(14) I_1 = integral, given in Fig. 3.10.6

(15) I_2 = integral, given in Fig. 3.10.7

(16) $P_EI_1 + I_2 + 1$ = factor between bed load and total load

(17) i_Tq_T = bed material load per unit width of stream for a size fraction, lb/sec-ft $= i_Bq_B (P_E I_1 + I_2 + 1)$

(18) i_TQ_T = bed material load for a size fraction for entire cross section, tons/day

(19) Σi_TQ_T = total bed material load for all size fractions, tons/day

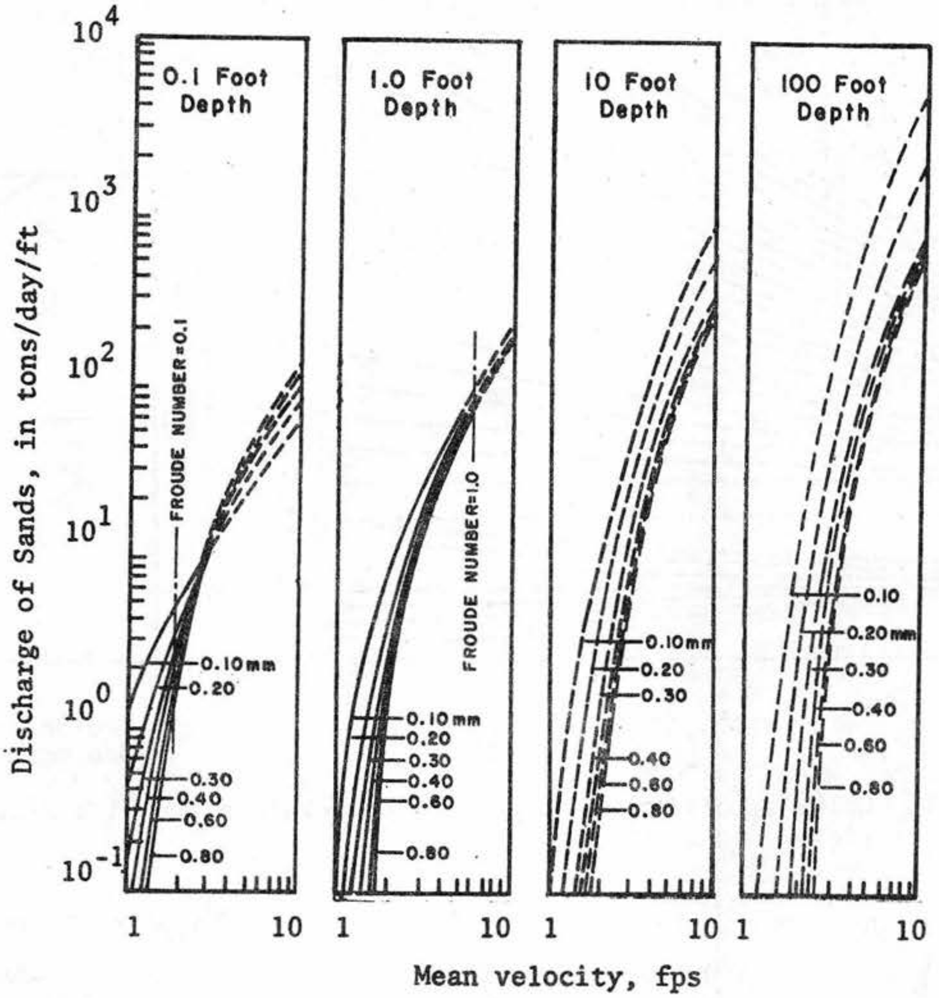


Fig. 3.10.10 Relation of discharge of sands to mean velocity for six median sizes of bed sands, four depths of flow, and a water temperature of 60°F (Colby, 1964).

In applying Figs. 3.10.10 and 3.10.11 to compute the total bed-material discharge, the following procedure is proposed: (1) the required data are the mean velocity V, the depth y_o, the median size of bed material D_{50}, the water temperature T and the fine sediment concentration C_f; (2) the uncorrected sediment discharge q_n for the given V, y_o and D_{50} can be found from Fig. 3.10.10 by first reading q knowing V and D_{50} for two depths that bracket the desired depth and then interpolating on a logarithmic graph of depth versus q_n, to get the bed-material discharge per unit width; (3) the two correction factors k_1 and k_2 shown in Fig. 3.10.11 account for the effect of water temperature and fine suspended sediment on the bed-material discharge. If the bed-material size falls outside the 0.20 mm

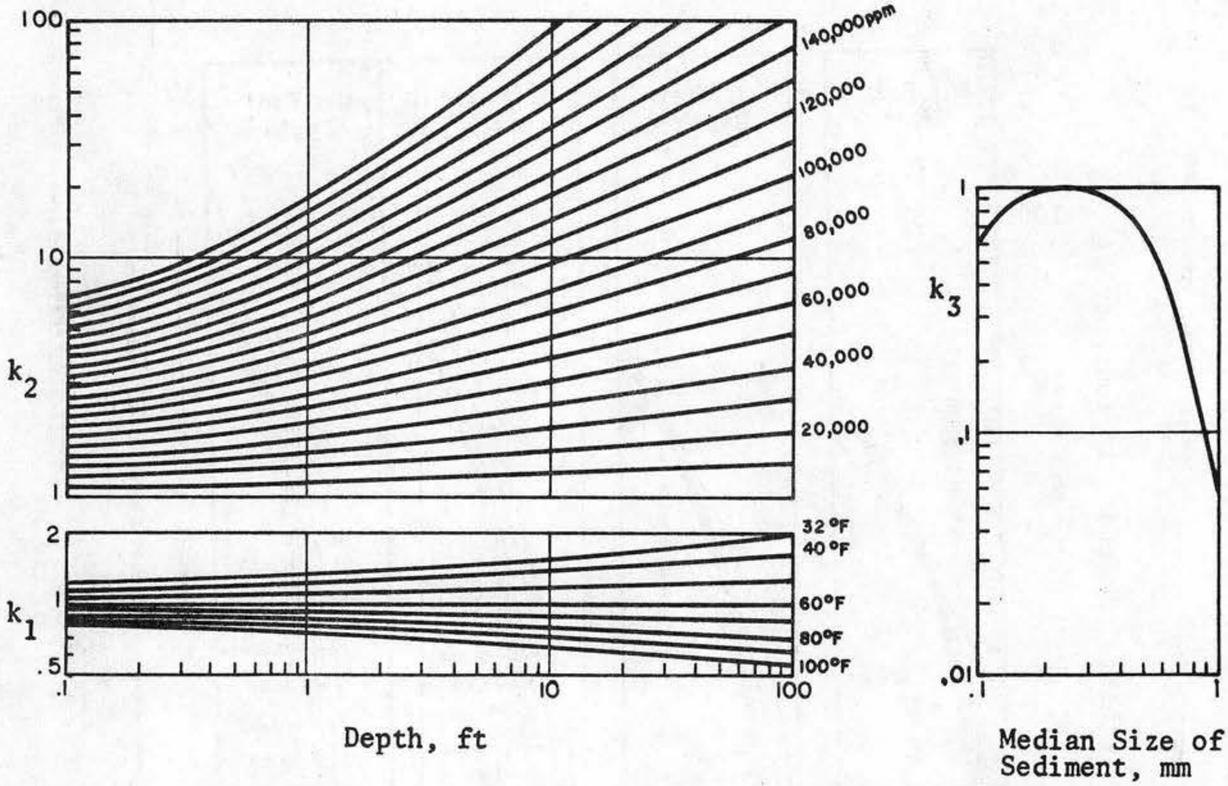


Fig. 3.10.11 Colby's correction curves for temperature and fine sediment (Colby, 1964).

to 0.30 mm range, the factor k_3 from Fig. 3.10.11 is applied to correct for the effect of sediment size; (4) the sediment discharge, q_T corrected for the effect of water temperature, presence of fine suspended sediment and sediment size is given by the equation

$$q_T = [1 + (k_1 k_2 - 1)k_3]q_n \qquad 3.10.43$$

As Fig. 3.10.11 shows, $k_1 = 1$ when the temperature is 60° F, $k_2 = 1$ when the concentration of fine sediment is negligible and $k_3 = 1$ when d_{50} lies between 0.2 mm and 0.3 mm. The total sand discharge is

$$Q_T = Wq_T \qquad 3.10.44$$

where W is the width of the stream.

In spite of many inaccuracies in the available data and uncertainties in the graphs, Colby (1964) found that "...about 75 percent of the sand discharges that were used to define the relationships were less than twice or more than half of the discharges that were computed from the graphs of average relationship. The agreement of computed and observed discharges of sands for sediment stations whose records were not used to define the graphs seemed to be about as good as that for stations whose records were used."

3.10.13 Calculation of bed-material discharge by the Colby method

In order to compare the results calculated by Colby's method, the sample problem used to illustrate the Einstein method is used. The required data are taken from Table 3.10.3. In addition, the water temperature and the fine sediment concentration are assumed to equal 70°F and 10,000 ppm, respectively. For convenience, the calculations are summarized in the form of tables. Table 3.10.5 contains the calculations over all size fractions using the median diameter of bed material; whereas, Table 3.10.6 contains the calculations for individual fractions using the bed-material size distribution.

Table 3.10.5 Bed material load calculations for sample problem by applying the Colby method (overall computations).

y_o (1)	W (2)	V (3)	q_n (4)	k_1 (5)	k_2 (6)	k_3 (7)	q_T (8)	Q_T (9)
1.36	103	2.92	14.5	0.92	1.20	.99	15.6	1,610
1.76	136	4.44	50.0	0.91	1.21	.99	55.0	7,480
2.50	170	6.63	135.	0.91	1.22	.99	150.	25,500
3.30	194	8.40	220.	0.90	1.23	.99	243.	47,100
4.14	234	9.92	325.	0.90	1.25	.99	365.	85,410

(1) y_o = mean depth ft taken from Table 3.10.3.
(2) W = surface width ft taken from Table 3.10.3.
(3) V = average velocity fps taken from Table 3.10.3.
(4) q_n = uncorrected sediment discharge tons/day/ft width taken from Fig. 3.10.10 for the given V, y_o and D_{50} by logarithmic interpolation.
(5) k_1 = correction factor for temperature, given in Fig. 3.10.11.
(6) k_2 = correction factor for fine sediment concentration, given in Fig. 3.10.11.
(7) k_3 = correction factor for sediment size, given in Fig. 3.10.11.
(8) q_T = true bed-material discharge per unit width of stream tons/day/ft width given by Eq. 3.10.43.
(9) Q_T = bed-material discharge for all size fractions for entire cross section tons/day given by Eq. 3.10.44.

Table 3.10.6 Bed-material discharge calculations for sample problem by applying the Colby method (individual size fraction computation).

D (1)	i_B (2)	y_o (3)	W (4)	V (5)	q_n (6)	k_1 (7)	k_2 (8)	k_3 (9)	q_T (10)	$i_B q_T$ (11)	$i_B Q_T$ (12)
0.495	.178	1.36	103	2.92	12	0.92	1.20	.62	13	2.3	237
		1.76	136	4.44	40	0.91	1.21	.62	43	7.7	1,050
		2.50	170	6.63	112	0.91	1.22	.62	119	21	3,570
		3.30	194	8.40	193	0.90	1.23	.62	205	37	7,180
		4.14	234	9.92	265	0.90	1.25	.62	288	51	11,900
0.351	.402	1.36	103	2.92	15	0.92	1.20	.92	16	6.4	659
		1.76	136	4.44	45	0.91	1.21	.92	49	20	2,720
		2.50	170	6.63	120	0.91	1.22	.92	132	53	9,010
		3.30	194	8.40	210	0.90	1.23	.92	230	93	18,000
		4.14	234	9.92	290	0.90	1.25	.92	323	130	30,420
0.248	.320	1.36	103	2.92	18	0.92	1.20	1.00	20	6.4	659
		1.76	136	4.94	53	0.91	1.21	1.00	58	19	2,580
		2.50	170	6.63	140	0.91	1.22	1.00	155	50	8,500
		3.30	194	8.40	240	0.90	1.23	1.00	266	85	16,500
		4.14	234	9.92	345	0.90	1.25	1.00	388	124	29,000
0.175	.058	1.36	103	2.92	23	0.90	1.25	1.00	25	1.5	155
		1.76	136	4.44	64	0.91	1.21	.97	70	4.1	558
		2.50	170	6.63	163	0.91	1.22	.97	180	10	1,700
		3.30	194	8.40	305	0.90	1.23	.97	337	20	3,880
		4.14	234	9.92	420	0.90	1.25	.97	471	27	6,320

(1) D = representative grain size, mm, given in Table 3.10.2.
(2) i_b = fraction of bed material, taken from Table 3.10.2.
(3) y_o = average flow depth, ft, taken from Table 3.10.3.
(4) W = top width, ft, taken from Table 3.10.3.
(5) V = average velocity, fps, taken from Table 3.10.3.
(6) q_n = uncorrect sediment discharge per unit width assuming the bed is composed entirely of one sand of size (d), tons/day/ft width, taken from Fig. 3.10.10 by interpolation on logarithmic paper for the given V, D, and d.
(7) k_1 = correction factor for temperature, given in Fig. 3.10.11.
(8) k_2 = correction factor for fine sediment, given in Fig. 3.10.11.
(9) k_3 = correction factor for sediment size, given in Fig. 3.10.11.
(10) q_T = corrected bed-material discharge per unit width by assuming the bed is composed entirely of one sand of size D, tons/day/ft width, given by Eq. 3.10.43.
(11) $i_B q_T$ = bed-material discharge per unit width for a size fraction, tons/day/ft width.
(12) $i_B Q_T$ = $W\, i_B q_T$, the bed-material discharge for a size fraction for entire cross section, tons/day.

The results of bed-material discharge calculations for the sample problem by using Einstein's (1950), and Colby's (1964) methods are shown in Fig. 3.10.12. The curves indicate that the sediment discharge increases rapidly with increasing water discharge. However, considerable deviations have resulted from different methods. As Einstein's method is basically derived from bed load measurements, it is questionable to apply this method to calculate bed-material load having most sediment in suspension such as the case in this sample problem.

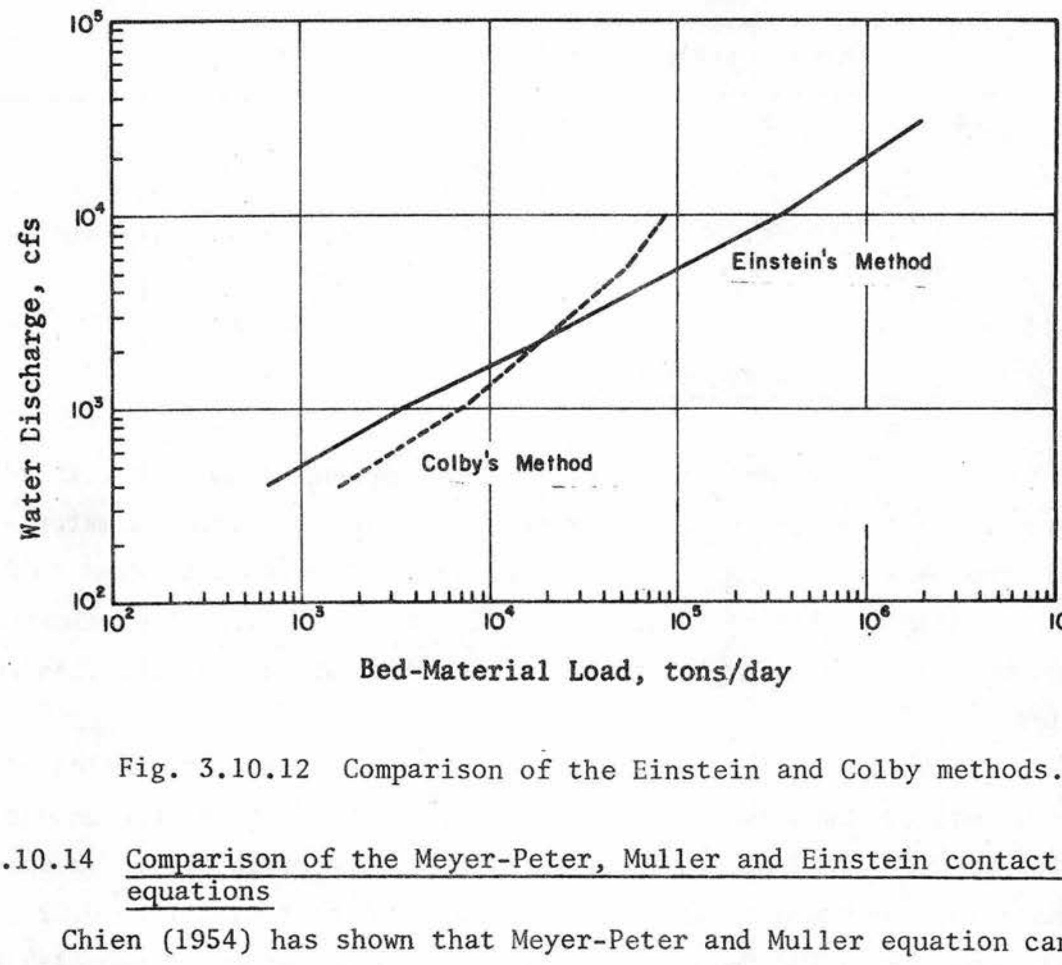


Fig. 3.10.12 Comparison of the Einstein and Colby methods.

3.10.14 <u>Comparison of the Meyer-Peter, Muller and Einstein contact load equations</u>

Chien (1954) has shown that Meyer-Peter and Muller equation can be modified into the form

$$\phi_* = \left(\frac{4}{\psi_*} - 0.188\right)^{3/2} \qquad 3.10.45$$

Fig. 3.10.13 shows the comparison of Eq. 3.10.45 with Einstein's ψ_* vs. ϕ_* relation for uniform bed material and for sediment mixtures using D_{35} in the Einstein relation and D_{50} in the Meyer-Peter and Muller relation. They show good agreement.

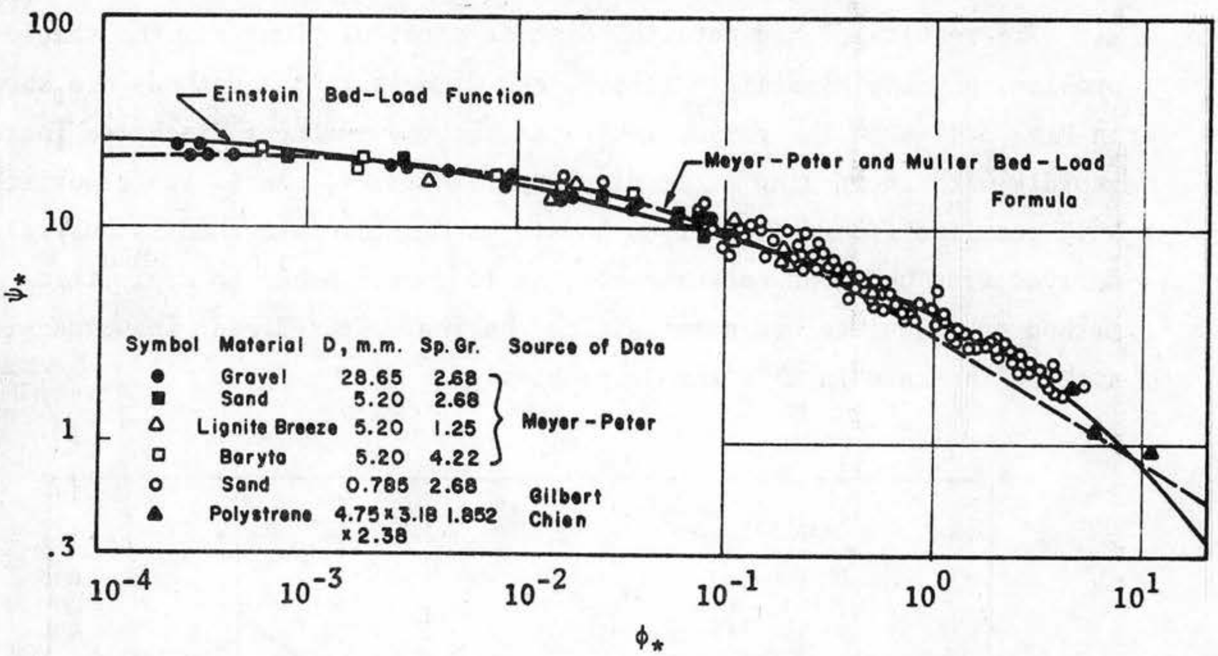


Fig. 3.10.13 Comparison of the Meyer-Peter, Muller and Einstein methods for computing contact load (Chien, 1954).

3.11.0 COARSE-MATERIAL STREAMS

The preceding discussion of alluvial channel flow is mainly related to sandbed channels; that is, channels with noncohesive bed materials of size less than 2 mm. *The other class of channels pertinent to highway engineering is that of coarse-material channels.* This classification includes all channels with noncohesive bed materials coarser than 2mm size.

The *behavior* of coarse material channels *is somewhat different* from sandbed channels. The main distinction between the two channels lies in the spread of their bed-material size distribution. In sandbed channels, for example, the bed may consist of particles from 0.02 to 2.00 mm; i.e. a 100-fold size range. In coarse-material channels, even if the maximum size is limited to cobbles (250 mm), the size range of particles may be 0.10 to 250 mm, which is a 2,500-fold size range. The armoring of a channel is, therefore, more pronounced in coarse material channels. In general, the coarse-material channels are less active and slower in bank shifting than sandbed channels.

The phenomenon of armoring in mobile bed channels occurs by the rearrangement of bed material during movement. The bed is covered by a one particle thick layer of the coarser material underlain by the finer sizes. (An example of armoring is given in Chapter VIII.) The absence of finer sizes from the surface layer is caused by the winnowing away of these sizes by the flow. As the spread of particle sizes available in the bed of coarse-material channels is large, these channels can armor their beds and behave as rigid boundary channels for all except the highest flows. The bed and bank forming activity in these channels is therefore limited to much smaller intervals of the annual hydrographs than the sandbed channels.

The general lack of mobility in coarse-material channels also means the bed forms do not change as much or as rapidly as in sandbed channels. The roughness coefficients in coarse-material channels are therefore more consistent during the annual hydrographs than in sand-bed channels. Most of the resistance to flow in coarse-material channels comes from the grain roughness and from bars. The river bed forms (dunes) are less important in the hydraulic behavior of coarse-bed channels.

Another special condition in coarse-material channels relates to the sampling and size analysis of bed material. As the particle sizes in these channels are large, fairly large volumes of material are needed to determine the particle size distribution. Also, the sudden variation in size from the surface layer to underlying material means that sampling has to be specifically oriented to either the bed layer or the material in a finite depth of the bed. These aspects of coarse-material channels are discussed in the following paragraphs.

3.11.1 Bed-material sampling in coarse-material channels

The purposes of bed-material sampling in coarse-material channels are:

(1) To determine the conditions of *incipient movement.*

(2) To assess the bed roughness related to the *resistance to flow.*

(3) To determine the *bed-material load* for a given flow.

(4) To determine the long and short time *response* of the channel to specific activities.

For objectives (1) to (3), the properties of the surface layer are needed. If it is anticipated that the bed layer will be disrupted at any given stage, it is necessary to take a scoop sample of both the surface and subsurface material.

The surface sampling can be easily done on the channel bed by counting particles on a grid, as already explained in this chapter. However, special effort should be made to obtain an objective sample. There is a tendency to select too many large particles. The scoop sample with bed-material sizes larger than an inch or so is difficult to obtain and such samples may have to be collected from bars and other exposed areas on channel perimeter.

In the size distribution analysis of coarse-bed materials, it is necessary to obtain particle counts by number, rather than by sieving or visual accumulation tube analysis for a part of the sample. Care must be taken in the interpretation of frequency distribution of part of a sample obtained by sieving. Only if the size distribution in a sample follows log-normal probability distribution can we transfer number counts to distributions by size, volume, weight or surface areas directly. For other distributions, special numerical techniques have to be used to transform the number distributions to weight or size distributions.

If the objectives of bed-material sampling include bed roughness and channel response, then the particles coarser than D_{84} or D_{90} need to be analyzed with more care. These sizes also require large samples for their determination.

3.11.2 Hydraulics of coarse-material channels

In sandbed channels, the form roughness can be much greater than the grain roughness when the bed forms are ripples and dunes. In coarse-material channels, the ripples never form and dunes are rare. The main type of bed form roughness in such channels is the pool and riffle configuration. With this configuration, the grain roughness is the main component of the channel roughness.

A coarse-material channel may have bed material that is only partly submerged during most of the flows. It is difficult to determine the channel roughness for such beds. For other cases, analysis of data from many rivers, canals and flumes (Anderson et al., 1968) shows that the channel roughness can be predicted by the equation

$$n = 0.04\ D_{50}^{\ 1/6} \qquad 3.10.46$$

where D_{50} is measured in ft and n is Manning's n.

3.11.3 Sediment transport in coarse-material channels

In considering the sediment transport in coarse-material channels, the distinction between wash load and bed material load is reemphasized. The reason is that in such channels, particles as big as coarse sand may behave as wash load. These particles may not be available in sufficient quantities in the bed surface and yet may constitute a large part of the total sediment load. It is repeated here, that the wash load in an alluvial channel cannot be related to the local flow or the channel bed and therefore is not predictable from the known channel flow properties.

The bed material load in coarse bed channels is mostly transported as bed load and not as suspended load. *For the bed-load transport, Einstein's bed-load function (without the suspended-load component) and the Meyer-Peter Muller transport function have been found to be fairly useful.*

3.11.4 Long and short term response of coarse-material channels

The time response of coarse-material channels also is different from sandbed channels in the time scale of response. This time response is dominated by two factors: (1) the difference in particle size between the surface (armor) layer of the bed and the bed material below it, and (2) the wash load may extend to coarse sand sizes. These factors are discussed below.

The formation of an armor layer on the bed may immobilize the bed for a large part of the hydrograph. However, if the conditions for incipient movement of this layer are exceeded, the underlying finer bed material will be readily picked up by the flow. The channel then establishes an armor of larger size particles for which a substantial depth of the bed may be degraded. Thus, extreme flow events in coarse-material channels are capable of inducing rapid and large bed-level changes.

The coarse sand and larger particles may behave as wash load in coarse material channels; that is, although the flow may be transporting a large quantity of these particles, the boundary shear may be large, so that these particles are not found in appreciable quantities in the armor layer. *If the boundary shear is reduced by afflux at a highway crossing, the flow may not sustain this material as wash load and rapid aggradation may occur.* In general, afflux at highway crossings induces rapid and more pronounced channel response in coarse material channels.

3.12.0 OPEN CHANNEL MODELING

In the present context, models are replication of phenomena associated with the behavior of highway crossings. The models may be physical models; that is, small-scale physical replications. They may also be mathematical when they are mathematical abstractions of the phenomena. Models are used to test the performance of a design or to study the details of a phenomenon. The performance tests of proposed structures can be made at moderate costs and small risks on small-scale (physical) models. Similarly, the interaction of a structure and the river environment can be studied in detail.

The natural phenomena are governed by appropriate sets of governing equations. If these equations can be integrated, the prediction of a given phenomenon in time and space domains can be made mathematically. In many cases related to river engineering, all the governing equations are not known. Also, the known equations cannot be directly treated mathematically for the geometries involved. In such cases, models are used to physically integrate the governing equations.

Similitude between a prototype and a model implies two conditions:

(1) To each point, time and process in the prototype, a uniquely coordinated point, time and process exists in the model.

(2) The ratios of corresponding physical magnitudes between prototype and model are constant for each type of physical quantity.

3.12.1 Rigid boundary models

To satisfy the preceding conditions in clear water, geometric, kinematic and dynamic similarities must exist between the prototype and the model. *Geometric similarity refers to the similarity of form between the prototype and its model. Kinematic similarity refers to similarity* *of motion, while dynamic similarity is a scaling of masses and forces.* For kinematic similarity, patterns or paths of motion between the model and the prototype should be geometrically similar. If similarity of flow is maintained between the model and the prototype, mathematical equations of motion will be identical for the two. Considering the equations of motion, *the dimensionless ratios of* $\frac{V}{\sqrt{gy}}$ *(Froude number)* and $\frac{Vy}{\nu}$ *(Reynolds number) are both significant parameters in models of rigid boundary clear water open channel flow.*

It is seldom possible to achieve kinematic, dynamic and geometric similarity all at the same time in a model. For instance, in open channel flow, gravitational forces predominate, and hence, the effects of the Froude number are more important than those of the Reynolds number. Therefore, the Froude criterion is used to determine the geometric scales, but only with the knowledge that some scale effects, that is, departure from strict similarity, exists in the model.

Ratios (or scales) of velocity, time, force, and other characteristics of flow for two systems are determined by equating the appropriate dimensionless number which applies to a dominant force. If the two systems are denoted by the subscript m for model and p for prototype, then the ratio of corresponding quantities in the two systems can be defined. The subscript r is used to designate the ratio of the model quantity to the prototype quantity. For example, the length ratio is given by

$$L_r = \frac{x_m}{x_p} = \frac{y_m}{y_p} = \frac{z_m}{z_p} \qquad 3.12.1$$

for the coordinate directions x, y, and z. Equation 3.12.1 assumes a condition of exact geometric similarity in all coordinate directions.

Frequently, open channel models are distorted. *A model is said to be distorted if there are variables that have the same dimension but are modeled by different scale ratios.* Thus, geometrically distorted models can have different scales in horizontal (x,y) and vertical (z) directions and two equations are necessary to define the length ratios in this case.

$$L_r = \frac{x_m}{x_p} = \frac{y_m}{y_p} \qquad 3.12.2$$

and

$$z_r = \frac{z_m}{z_p} \qquad 3.12.3$$

If perfect similitude is to be obtained the relationships that must exist between the properties of the fluids used in the model and in the prototype are given in Table 3.12.1 for the Froude, Reynolds and Weber criteria.

In free surface flow, the length ratio is often selected arbitrarily, but with certain limitations kept in mind. The Froude number is used as a scaling criteria because gravity has a predominant effect. However, if a small length ratio is used (that is, the water depths are very shallow) then surface tension forces, effects of which are included in the Weber number ($\frac{V}{\sqrt{\sigma/\rho L}}$) may become important and complicate the interpretations of results of the model. The length scale is made as large as possible so that the Reynolds number is sufficiently large and friction becomes a function of the boundary roughness and essentially independent of the Reynolds number. A large length scale also insures that the flow is turbulent in the model as it is in the prototype.

The boundary roughness is characterized by Manning's roughness coefficient, n, in free surface flow. Analysis of Manning's equation and substitution of the appropriate length ratios, based upon the Froude criterion, results in an expression for the ratio of the roughness which is given by

$$n_r = L_r^{1/6} \qquad 3.12.4$$

It is not always possible to achieve boundary roughness in a model and prototype that correspond to that required by Eq. 3.12.4 and additional measures, such as adjustment of the slope, may be necessary to offset disproportionately high resistance in the model.

Table 3.12.1 Scale ratios for similitude.

Characteristic	Dimension	Re	Fr	We
		Scale Ratios		
Length	L	L	L	L
Area	L^2	L^2	L^2	L^2
Volume	L^3	L^3	L^3	L^3
Time	T	$\rho L^2/\mu$	$(L\rho/\gamma)^{1/2}$	$(L^3\rho/\sigma)^{1/2}$
Velocity	L/T	$\mu/L\rho$	$(L\gamma/\rho)^{1/2}$	$(\sigma/L\rho)^{1/2}$
Acceleration	L/T^2	μ^2/ρ^2L^3	γ/ρ	$\sigma/L^2\rho$
Discharge	L^3/T	$L\mu/\rho$	$L^{5/2}\ \frac{\gamma}{\rho}1/2$	$L^{3/2}(\sigma/\rho)^{1/2}$
Mass	M	$L^3\rho$	$L^3\rho$	$L^3\rho$
Force	ML/T^2	μ^2/ρ	$L^3\gamma$	$L\sigma$
Density	M/L^3	ρ	ρ	ρ
Specific weight	M/L^2T^2	$\mu^2/L^3\rho$	γ	σ/L^2
Pressure	M/LT^2	$\mu^2/L^2\rho$	$L\gamma$	σ/L
Impulse and momentum	ML/T	$L^2\mu$	$L^{7/2}(\rho\gamma)^{1/2}$	$L^{5/2}(\rho\sigma)^{1/2}$
Energy and work	ML^2/T^3	$L\mu^2/\rho$	$L^4\gamma$	$L^2\sigma$
Power	ML^2/T^3	$\mu^3/L\rho^2$	$\frac{L^{7/2}\gamma^{3/2}}{\rho^{1/2}}$	$\sigma^{3/2}(L/\rho)^{1/2}$

3.12.2 Mobile bed models

In modeling highway crossings and encroachments in the river environment, three-dimensional mobile bed models are often used. These models have the bed and sides molded of materials that can be moved by the model flows. Similitude in mobile bed models implies that the model reproduces the fluvial processes such as bed scour, bed deposition, lateral channel migration, and varying boundary roughness. It has not been considered possible to faithfully simulate all of these processes simultaneously on scale models. Distortions of various parameters are often made in such models.

Two approaches are available to design mobile bed models. One is the analyzical derivation of distortions explained by Einstein and Chien (1956) and the other is based on hydraulic geometry relationships given by Lacey, Blench, and others (See Mahmood and Shen, 1971). In both of these approaches, a first approximation of the model scales and distortions can be obtained by numerical computations. The model is built to these scales and then verified for past information obtained from the prototype. In general, the model scales need adjusting during the verification stage.

The model verification consists of the reproduction of observed prototype behavior under given conditions on the model. This is specifically directed to one or more alluvial processes of interest. For example, a model may be verified for bed-level changes over a certain reach of the river. The predictive use of the model should be restricted to the aspects for which the model has been verified. This use is based on the premise that if the model has successfully reproduced the phenomenon of interest over a given hydrograph as observed on the prototype, it will also reproduce the future response of the river over a similar range of conditions.

The mobile bed models are more difficult to design and their theory is vastly complicated as compared to clear water rigid bed models. However, many successful examples of their use are available the world over. In general, all important river training and control works are invariably studied on physical models. The interpretation of results from a mobile bed model requires a basic understanding of the fluvial processes and some experience with such models. Even in the many cases where it is only possible to obtain qualitative information from a mobile bed models, this information is of great help in comparing the performance of different designs.

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3.A1.0 SEDIMENT TRANSPORT IN GRAVEL RIVERS

In Section 3.10.8, the Meyer-Peter and Muller equations for bed-material sediment transport is presented. *The equations are recommended for computations of transport rates in coarse material channels.* A sample computation is presented below.

3.A1.1 Sediment transport rate

A gravel river is 200 ft wide and has a slope of 0.0005. The discharge is 7000 cfs and the depth of flow is 9.80 ft. The bed material has the grain size distribution shown in Table 3.A1.1.

Table 3.A1.1 Bed material size distribution.

Size range mm	Percent of total weight in size range
0.002 -0.0625	0.9
0.0625-0.125	4.4
0.125 -0.250	14.2
0.250 -0.500	74.9
0.500 -1.00	5.0
1.00 -2.00	0.5
2.00 -4.00	0.3

We need to extract more information about the bed material so Table 3.A1.2 is prepared. The geometric mean size is defined to be the square root of the product of the two extreme values; e.g. the geometric mean size for the first size range = $\sqrt{(0.002)(0.0625)} = 0.011$ mm.

The effective diameter of the bed-material sediments is given by Eq. 3.10.18 or

$$D_m = \frac{\sum_i P_i D_i}{100} = \frac{34.52}{100} = 0.345 \text{ mm} = .00113 \text{ ft}$$

The size distribution is plotted on log-probability paper in Fig. 3.A1.1 and the following values are obtained:

$D_{90} = 0.46$ mm

$D_{84} = 0.42$ mm

$D_{50} = 0.31$ mm

$D_{16} = 0.24$ mm

Table 3.A1.2 Bed-material size fractions.

Size range mm	Geometric mean size, D_i mm	Percent of bed material in this size, P_i	Percent finer	P_iD_i
.002 - .0625	0.011	0.9	0.8	0.01
.0625 - .125	0.088	4.4	5.1	0.39
.125 - .250	0.177	14.2	19.3	2.51
.250 - .500	0.354	74.9	94.2	26.51
.500 -1.00	0.707	5.0	99.2	3.54
1.00 -2.00	1.41	0.5	99.7	0.71
2.00 -4.00	2.83	0.3	100.0	0.85
	Total	100.2		34.52

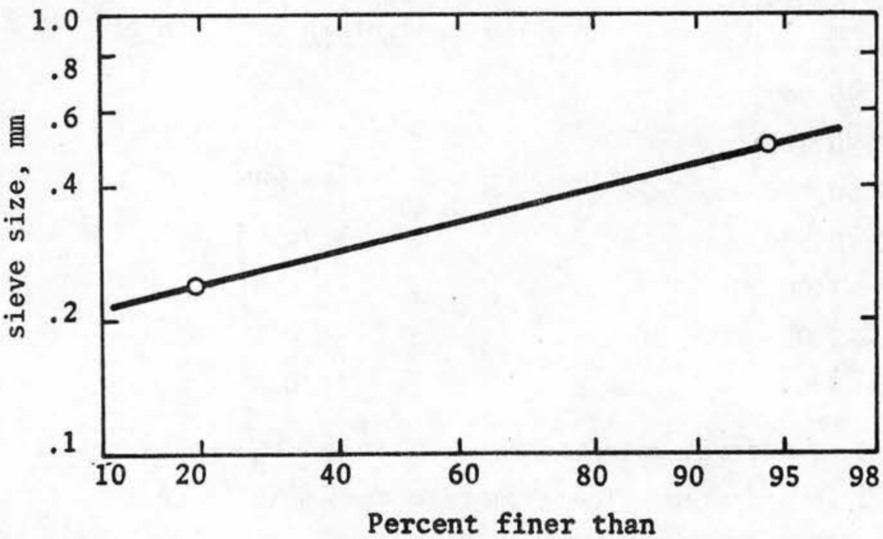


Fig. 3.A1.1 Bed-material size distribution.

and
$$G = \frac{1}{2}\left(\frac{D_{84}}{D_{50}} + \frac{D_{50}}{D_{16}}\right)$$

$$= \frac{1}{2}\left(\frac{0.42}{0.31} + \frac{0.31}{0.24}\right) = 1.32 \qquad 3.A1.1$$

This gradation coefficient is used in later calculations.

The Meyer-Peter and Muller equation for bed-material transport is Eq. 3.10.13

$$\left(\frac{Q_b}{Q}\right)\left(\frac{K_b}{K_r}\right)^{3/2} \gamma y_o S = B'(\gamma_s-\gamma)D_m + B\left(\frac{\gamma}{g}\right)^{1/3}\left(\frac{\gamma_s-\gamma}{\gamma_s}\right)^{2/3} q_B^{2/3}$$

where

γ_s, γ = specific weight of sediment and water, respectively,

Q_b = portion of total discharge, Q related to the channel bed

K_b = roughness coefficient related to bed

K_r = roughness coefficient related to the bed-material grain roughness

y_o = depth of flow

S = energy gradient

D_m = mean diameter of the bed material

g = gravitational acceleration

q_B = bedload in units of weight/unit width/unit time

The coefficients B' and B have values of 0.047 and 0.025, respectively, for the case of sediment transport (from Section 3.10.7).

The term K_r is computed by employing Eq. 3.10.17 or

$$K_r = \frac{26}{(D_{90})^{1/6}}$$

$$= \frac{26}{(.00046)^{1/6}} = 93.6$$

The Manning's roughness coefficient for the reach is (from Eq. 2.3.20)

$$n = \frac{1.486}{V} R^{2/3} S^{1/2}$$

Here, the cross-sectional area is

$$A = y_o W$$

$$= 9.8 \times 200 = 1960 \text{ sq ft}$$

the wetted perimeter is

$$P = 2y_o + W$$

$$= 2 \times 9.8 + 200 = 219.6 \text{ ft}$$

so the hydraulic radius is

$$R = \frac{A}{P}$$

$$= \frac{1960}{219.6} = 8.92 \text{ ft}$$

The average velocity is

$$V = \frac{Q}{A}$$

$$= \frac{7000}{1960} = 3.57 \text{ fps}$$

By putting these values and the value of the bed slope into Manning's equation, we obtain

$$n = \frac{1.486}{3.57} (8.92)^{2/3} (.0005)^{1/2} = 0.040$$

Assuming a rectangular channel, the Manning's roughness associated with the bed is (from Eq. 3.10.20)

$$n_b = n[1 + \frac{2y_o}{W} (1 - (\frac{n_w}{n})^{3/2})]^{2/3}$$

where n_w is the bank roughness. As n_w is not given, assume a value of 0.060; that is, the roughness of the banks is rather large. Then

$$n_b = 0.040[1 + \frac{2.(9.8)}{200} (1 - \frac{0.060}{0.040})^{3/2}]^{2/3} = 0.0378$$

With reference to Eq. 3.10.14 we can set

$$K_b = \frac{1}{n_b}$$

$$= \frac{1}{.0378} = 26.45$$

From Eq. 3.10.22

$$\frac{Q_b}{Q} = \frac{1}{1 + \frac{2y_o}{W} (\frac{n_w}{n_b})^{3/2}}$$

$$= \frac{1}{1 + \frac{2(9.8)}{200} (\frac{.060}{.0378})^{3/2}} = 0.836$$

Now, we have values for all the variables in the Meyer-Peter and Muller equation so by inserting these values

$$0.836 (\frac{26.45}{93.58})^{3/2} (62.4)(9.8)(.0005) = 0.047 (165-62.4)(.00113) + 0.25 (\frac{62.4}{32.2})^{1/3} (\frac{165-62.4}{165})^{2/3} q_B^{2/3}$$

or $\quad 0.0384 = 0.00545 + 0.227 q_B^{2/3}$

so $\quad q_B = 0.055$ lb/sec/ft

The total sediment discharge for the channel is

$$Q_B = q_B W$$

$= .055 \times 200 = 11$ lbs/sec

$= 475$ tons/day

3.A1.2 Armor coating

If the movement of bed materials out of the river reach is not accompanied by an equal influx of bed material at the upstream end, the bed degrades and develops an armor coat. The size of the armor material is determined from the Meyer-Peter and Muller equation (from 3.10.13)

$$\left(\frac{Q_b}{Q}\right) \left(\frac{K_b}{K_r}\right)^{3/2} \gamma y_o S = 0.034 \ (\gamma_s - \gamma) D_m \qquad 3.A1.2$$

for zero sediment discharge. That is, the armor-coat materials are of size D_m computed by Eq. 3.A1.2 and do not move.

For the flow conditions described in the previous section

$$\frac{Q_b}{Q} = 0.836$$

$$\frac{K_b}{K_r} = \frac{26.45}{93.58} = 0.283$$

so from Eq. 3.A1.2

$$D_m = \frac{0.836 \ (0.283)^{3/2} (62.4) \ (9.8) \ (0.0005)}{0.034 \ (165 - 62.4)}$$

$$= 0.0110 \text{ ft}$$

$$= 3.35 \text{ mm}$$

This size is larger than the larger bed-material sizes so the bed must degrade substantially to become armored.

3.A1.3 Rapid computations

If the channel is wide and the bed materials have a log-normal size distribution, the bed-material transport can be estimated very quickly.

With the assumption of a wide channel, the ratio Q_b/Q becomes unity.

If the bed materials have a log-normal distribution

$$\frac{D_m}{D_{50}} = \exp \left\{\frac{1}{2} \ (\ln G)^2\right\} \qquad 3.A1.3$$

where

$$G = \frac{1}{2} \left(\frac{D_{84}}{D_{50}} + \frac{D_{50}}{D_{16}}\right)$$

For the gravel bed river being considered in Section 3.A1.1

$$G = 1.32$$

so

$$\frac{D_m}{D_{50}} = \exp\{\frac{1}{2}(\ln 1.32)^2\} = 1.04$$

and

$$D_m = 1.04 \times 0.31 = 0.322 \text{ mm}$$

The USBR short form of the Meyer-Peter and Muller equation (Eq. 3.10.19) is

$$q_B = 1.606\,[3.306(\frac{Q_b}{Q})(\frac{D_{90}^{1/6}}{n_b})^{3/2}\; y_o S - 0.627\, D_m]^{3/2} \qquad 3.A1.4$$

Recall that

$$D_{90} = 0.46 \text{ mm}$$

$$n_b = 0.0378$$

$$y_o = 9.80 \text{ ft}$$

$$S = 0.0005$$

and

$$\frac{Q_b}{Q} = 1.0$$

Then

$$q_B = 1.606\,[3.306\,(1)\,(\frac{0.46^{1/6}}{.0378})^{3/2}\,(9.8)(.0005) - .627\,(.322)]^{3/2}$$

$$= 3.3 \text{ tons/day/ft}$$

The total bed-material discharge is

$$Q_B = q_B\, W$$

$$= 3.3 \times 200$$

$$= 660 \text{ tons/day}$$

This compares with 475 tons/day computed using the complete form of the transport equations.

Chapter IV

FLUVIAL GEOMORPHOLOGY

4.1.0 INTRODUCTION

Rivers and river systems have served man in many ways. Rivers are fundamental to agriculture particularly in the arid and semiarid parts of the world. To some degree the flooding by rivers and the deposition of sediment therefrom on the river valleys have been a means of revitalizing the river valleys to keep them productive. Rivers have provided a means of traveling inland and developing trade. This has played a significant role in the development of all countries wherever rivers of significant size exist.

Rivers have different alignments and geometry. There are meandering rivers, braided rivers, and rivers that are essentially straight. In general, braided rivers are relatively steep and meandering rivers have more gentle slopes. Meandering channels have characteristics that enable us to utilize them without experiencing extensive improvement and maintenance cost.

Meandering rivers are not subject to rapid movement, are reasonably predictable in behavior and are utilizable to man's benefit. Nevertheless, they are unstable, banks are eroded, productive land, bridges, bridge approaches, control works, buildings, and urban properties are often destroyed by floods. Bank protection works are often necessary to stabilize certain reaches of the river and to improve them for other aspects of flood control.

4.2.0 FLUVIAL CYCLES AND PROCESS

4.2.1 Youthful, mature and old streams

Various methods are used to classify rivers according to their age. One of the methods used by geomorphologists, and widely accepted by the engineering profession, classifies streams as youthful, mature, and old. *Youthful* implies the initial state of streams. As channels are first developed in the earth's surface by the flowing water, they *are generally V-shaped, very irregular and consist of fractured erosive and nonerosive* *materials*. Examples of youthful streams are mountain streams and their tributaries developed by overland flow.

There is no clean line between youthful and mature rivers. *In the case of mature channels, the river valleys have widened, the river slopes are flat, and bank cutting has largely replaced downward cutting*. The streambed has achieved *a graded condition*; that is, *the slope and the energy of the stream are just sufficient to transport the material delivered to it*. With mature channels, narrow floodplains and meanders have formed. The valley bottoms are sufficiently wide to accommodate agricultural and urban developments, and where development has occurred, usually channel stabilization works and other improvements have been made to prevent lateral migration of the river.

River channels classified as old are extensions in age of the mature channel. As erosion continues, the river valleys develop characteristics of greater width and low relief; the stream gradient has flattened further, and meanders and meander belts that have developed are not as wide as the river valley. Natural levees have formed along the stream banks. Landward of the natural levees, there are swamps. The tributaries to the main channel parallel the main channel sometimes for long distances before there is a breach in the natural levee that permits a confluence. In conjunction with an old river and its river valley, wide areas are available for cultivation, improvements of all types are built, and flood levees are generally required to protect those occupying the valley. Because of the more sophisticated development of the river valley, channel stabilization and contraction work such as revetments and dikes are generally constructed.

It should be emphasized that the preceding concept of the fluvial cycle is not accepted by all geologists. For example, some consider a channel to be mature only after the trunk stream as well as the side streams have achieved a graded condition. Some define old age as a condition when the entire river system is graded. Graded streams are referred to as those that have achieved slopes such that their energy is just sufficient to transport the sediment through the system that is delivered by the tributaries. This concept can only be applied as an average condition extending over a period of years. No stream is continuously graded. A *poised stream refers to one that neither aggrades* *or degrades its channel over time. Both graded and poised streams are delicately balanced.* Any change imposed on the river system will alter the balance and lead to actions by the stream to reestablish balance. For example, a graded or poised stream may be subjected locally to the development of a cutoff. The development of the cutoff increases the channel slope, increases velocity, and increases transport at least locally. Changes in these variables cause changes in the channel and deposition downstream. The locally steepened slope gradually extends itself upstream attempting to reestablish equilibrium.

4.2.2 Floodplain and delta formations

Over time, the highlands of an area are worn down. The streams erode their banks. The material that is eroded is utilized further downstream to build banks and to further the meandering process. Streams move laterally pushing the highlands back. Low flat valley land and floodplains are formed. As the streams transport sediment to areas of flatter slopes and in particular to bodies of water where the velocity and turbulence are too small to sustain the transport of the material, the material is deposited forming deltas. As deltas build outward the up-river portion of the channel is elevated through deposition and becomes part of the floodplain. Also, the stream channel is lengthened and the slope is further reduced. The upstream river bed is filled in and average flood elevations are increased. As it works across the river valley *this type of development causes the total floodplain to raise in elevation.* Hence, even old streams are far from static. Old rivers meander, are affected by changes in sea level, are influenced by movements of the earth's crust, are changed by delta formations or glaciation, and are subject to modifications due to climatological changes and as a consequence of man's development of them.

4.2.3 Alluvial fans

Another feature of rivers is *alluvial fans*. They *occur wherever there is a change from a steep to a flat gradient.* As the bed material and water reaches the flatter section of the stream, the coarser bed material can no longer be transported because of the sudden reduction in both slope and velocity. Consequently, a cone or fan builds out as the material is dropped. The steep side of the fan faces the floodplain. There is considerable similarity between a delta and an alluvial fan.

Both result from reductions in slope and velocity. Both have steep slopes at their outer edges. Both tend to reduce upstream slopes. Alluvial fans, like deltas, are characterized by unstable channel geometries and rapid lateral movement. An action very similar to the delta develops where a steep tributary enters a main channel. The steep channel tends to drop part of its sediment load in the main channel building out into the main stream. In some instances, the main stream can be forced to make drastic changes at the time of major floods by the stream's tributaries.

4.3.0 STREAM FORM

A study of the plan and profile of a stream is very useful in understanding stream morphology. Plan-view appearances of streams are many and varied and are the result of many interacting variables. Small changes in a variable can change the plan view and profile of a river, adversely affecting a highway crossing or encroachment. Conversely the highway crossing or encroachment can inadvertently change the plan view or profile adversely affecting the river environment. In this section the stream form is classified and the channel processes are discussed.

4.3.1 The braided stream

A *braided stream is one that consists of multiple and interlacing channels* (see Fig. 1.2.1). *One cause of braiding is the large quantity of bed load that the stream is unable to transport.* The magnitude of the bed load is more important than its size. Many geologists claim that braiding is independent of the size of the bed material at least in the sand range. If the channel is overloaded with sediment, deposition occurs, the bed aggrades, and the slope of the channel increases in an effort to obtain a graded state. As the channel steepens, the velocity increases, and multiple channels develop. These interlaced multiple channels cause the overall channel system to widen. Multiple channels are generally formed as bars of sediment are deposited within the main channel.

Another cause of braiding is easily eroded banks. If the banks are easily eroded, the stream widens at high flow and at low flow bars form which become stabilized, forming islands. In general, a braided channel has a large slope, a large bed-material load in comparison with its suspended load, and relatively small amounts of silts and clay in the bed and banks. Fig. 4.3.1 may assist to define the various conditions for multiple channel streams.

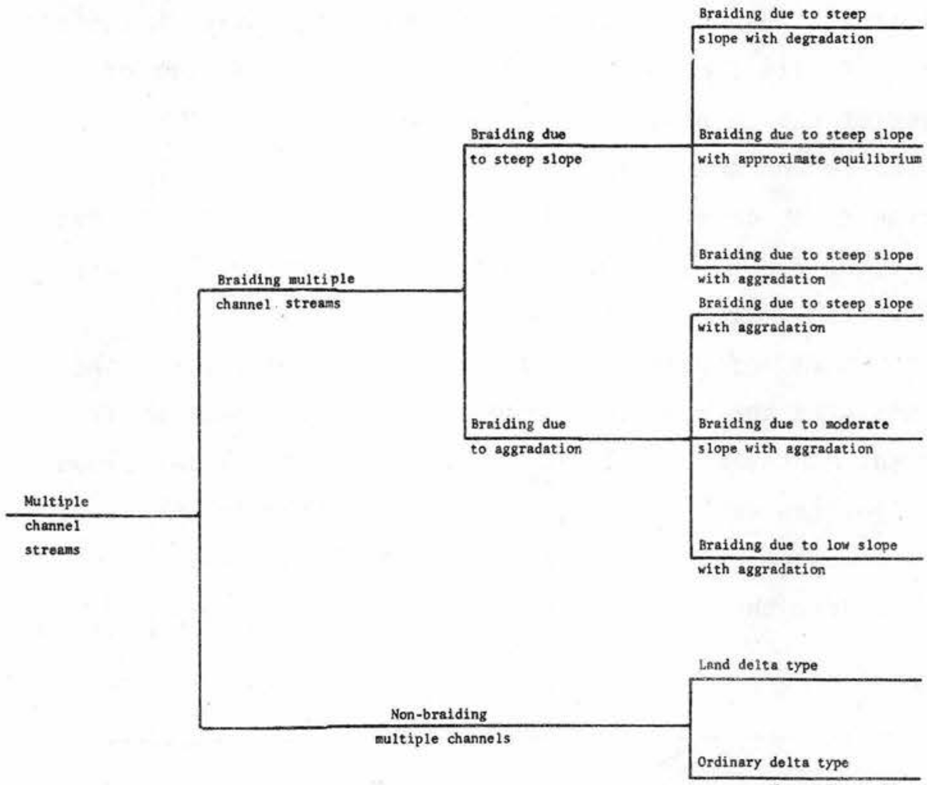


Fig. 4.3.1 Types of multi-channel streams.

The braided stream is difficult to work with in that it is unstable, changes its alignment rapidly, carries large quantities of sediment, is very wide and shallow even at flood flow and is, in general, unpredictable.

4.3.2 <u>The meandering channel</u>

The meandering channel is the one that consists of alternating bends of an S-shape. However, this is a static definition; in reality the meandering river is subjected to both lateral and longitudinal movement caused by the formation and destruction of bends. Even straight channels have a meandering current. In fact, in most straight channels there is a tendency for the current to meander therein and to develop alternate bars that may ultimately lead to the development of a meandering channel, given sufficient time. The meandering channel was defined by E. W. Lane (1957) as one whose channel alignment consists principally of pronounced bends, the shapes of which have not been determined predominately by the varying nature of the terrain through which the channel passes. For comparison, Gerald H. Matthes (1941) stated, "the term meander is here applied to any letter-S channel pattern, fashioned in alluvial materials, which is free to shift its location and adjust its shape as part of a migratory movement of the channel as a whole down the valley." Fig. 4.3.2 illustrates the meandering river form.

The meandering river consists of pools and crossings. The thalweg, or main current of the channel, flows from the pool through the crossing to the next pool forming the typical S-curve. In the pools, the channel cross section is somewhat triangular. Point bars form on the inside of the bends. In the crossings, the channel cross section is more rectangular and depths are smaller. At low flows the local slope is steeper and velocities are larger in the crossing than in the pool. At low stages the thalweg is located very close to the outside of the bend. At higher stages, the thalweg tends to straighten. More

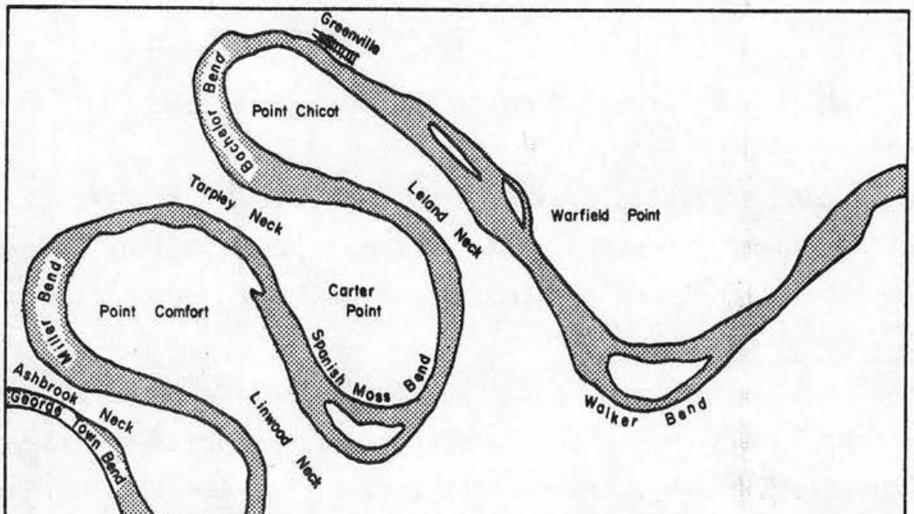


Fig. 4.3.2 Meanders in Mississippi River near Greensville, Mississippi.

specifically, the thalweg moves away from the outside of the bend encroaching on the point bar to some degree. In the extreme case, the shifting of the current causes chute channels to develop across the point bar at high stages. Fig. 4.3.3 shows the plan view and cross section of a typical meandering stream. In this figure, one can observe the position of the thalweg, the location of the point bars, alternate bars and the location of the pools and crossings. Note that in the crossing the channel is shallow compared to pools and the banks may be more subject to erosion.

4.3.3 <u>The meandering process</u>

Alluvial channels of all types deviate from a straight alignment. The thalweg oscillates transversely and initiates the formation of bends. In general, the river engineer concerned with channel stabilization should not attempt to develop straight channels. *In a straight channel the alternate bars and the thalweg (the deeps and shallows) are continually changing;* thus the current is not uniformly distributed

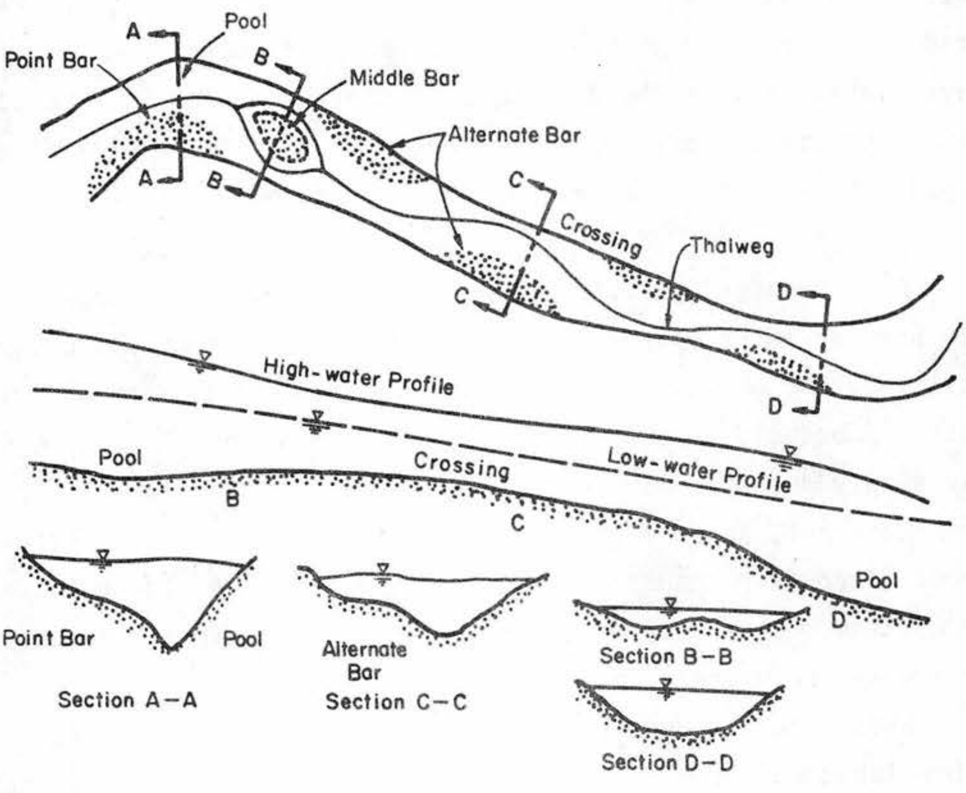


Fig. 4.3.3 Plan view and cross section of a meandering stream.

through the cross section but is deflected toward one bank and then the other. Sloughing of the banks, nonuniform deposition of bed load by debris such as trees, and the Coriolis force have been cited as causes for meandering of streams. *When the current is directed toward a bank, the bank is eroded in the area of impingement and the current is deflected and impinges upon the opposite bank further downstream.* The angle of deflection of the thalweg is affected by the curvature formed in the eroding bank and the lateral depth of erosion.

In general, *bends are formed by the process of erosion and deposition.* Erosion without deposition to assist in bend formation would result only in escalloped banks. Under these conditions the channel would simply widen until it was so large that the erosion would terminate. The material eroded from the bank is normally deposited over a period of time on the point bars that are formed downstream. The point bars constrict the bend and enable erosion in the bend to continue, accounting for the lateral and longitudinal migration of the meandering stream. Erosion is greatest across the channel from the point bar. As the point bars build out from the downstream sides of the points, the bends gradually migrate down the valley. The point bars formed in the bendways clearly define the direction of flow. The bar is generally streamlined and its largest portion is oriented downstream. If there is very rapid caving in the bendways upstream, the sediment load may be sufficiently large to cause middle bars to form in the crossing.

As a meandering river system moves laterally and longitudinally, the meander loops move at an unequal rate because of the unequal erodibility of the banks. This causes a tip or bulb to form and ultimately this tip or bulb is cut off. After the cutoff has formed, a new bend may slowly develop. Its geometry depends upon the local slope, the bank material, and the geometry of the adjacent bends. *Over time the local steep slope caused by the cutoff is distributed both upstream and downstream.* Years may be required before a configuration characteristic of average conditions in the river is attained.

When a cutoff occurs, an oxbow lake is formed (see Fig. 4.3.4). Oxbow lakes may persist for long periods of time before filling. Usually the upstream end of the lake fills quickly to bank height. Overflow during floods carries fine materials into the oxbow lake area. The lower end of the oxbow remains open and the drainage and overland flow entering the system can flow out from the lower end. The oxbow

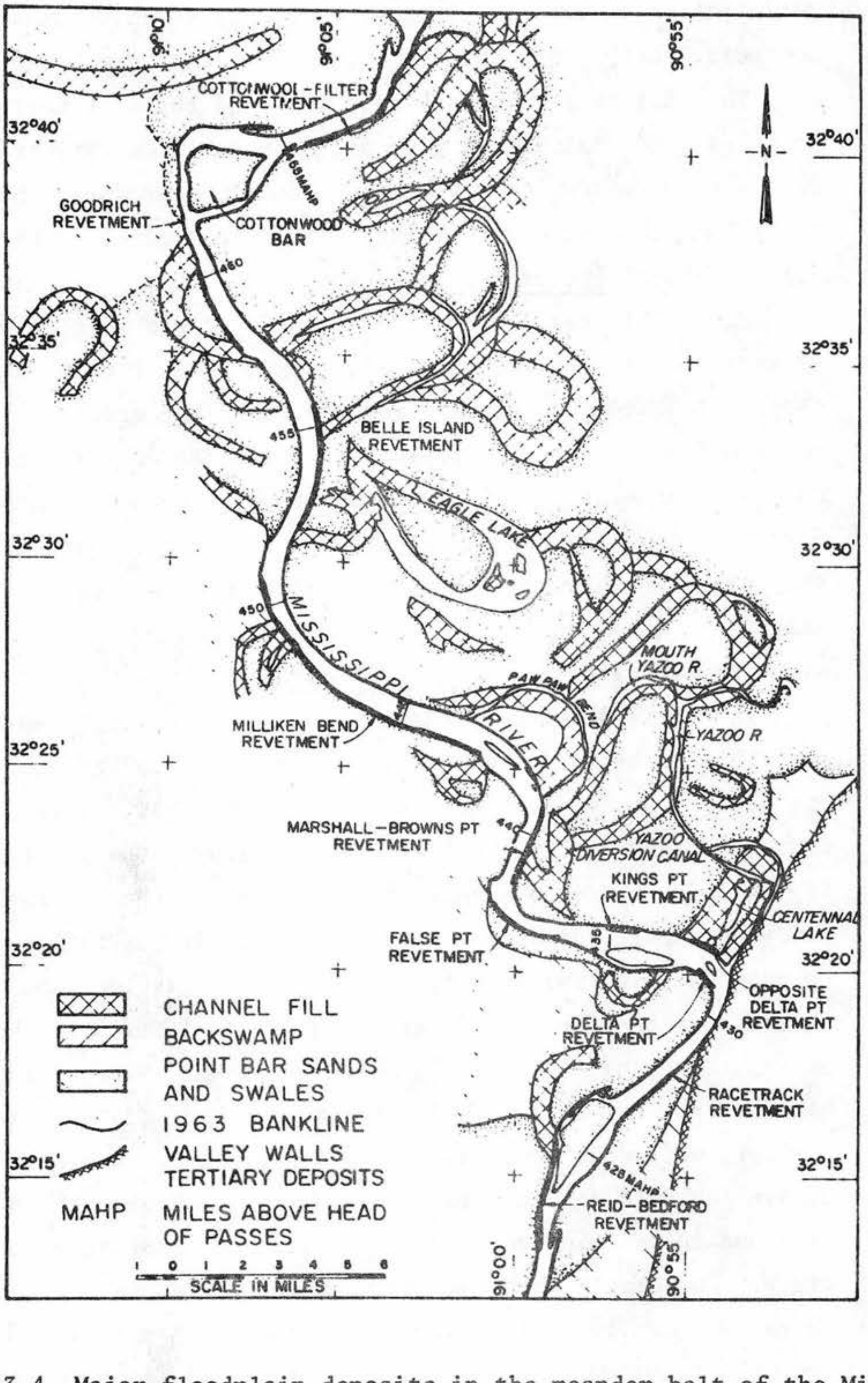


Fig. 4.3.4 Major floodplain deposits in the meander belt of the Mississippi River, after Waterways Experiment Station Potamology Investigation Report No. 12-15 (1965).

gradually fills with fine silts and clays. Fine material that ultimately fills the bendway is plastic and cohesive. As the river channel meanders it encounters old bendways filled with cohesive materials (referred to as clay plugs). These plugs are sufficiently resistant to erosion to serve as essentially semipermanent geologic controls. Clay plugs can drastically affect river geometry.

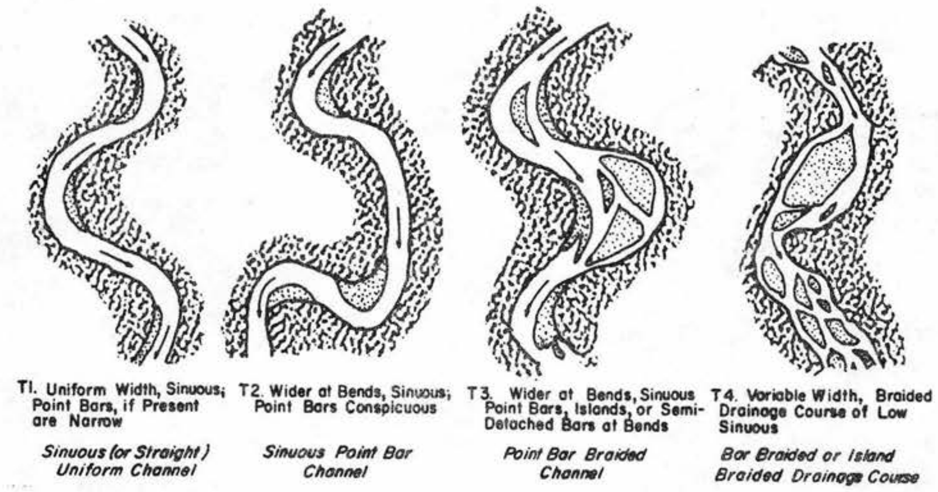
The variability of bank materials and the fact that the river encounters such features as clay plugs causes wide variety of river forms even with a meandering river. The meander belt formed by a meandering river is often fifteen to twenty times the channel width.

4.3.4 Natural levees and back swamps

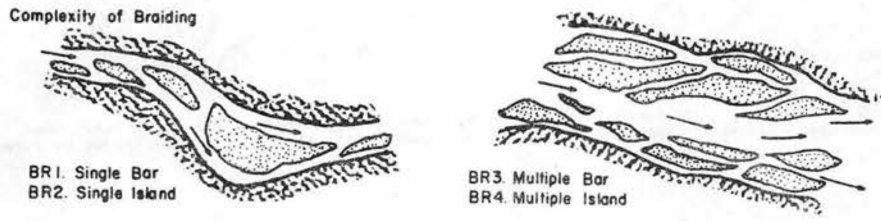
Natural levees are a characteristic of old river systems. *The natural levees near the river are rather steep because coarse material drops out quickly.* Farther from the river the gradients are flatter and the finer materials drop out. *Beyond the levees are the swamp areas.* On the lower Mississippi River, natural levees on the order of ten feet in height are common. The rate of growth of natural levees is smaller after they reach a height equal to the average annual flood stage.

4.3.5 Subclassification of river channels

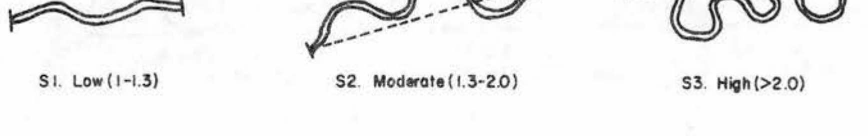
There are subclassifications within the major types of meandering, straight and braided channels that are of use to the geomorphologist and engineer. Low, moderate and high sinuosity are illustrated in Fig. 4.3.5c. Classification based on *oxbow lakes* is illustrated in Fig. 4.3.5d. In Fig. 4.3.5e, types of *meander scroll formations* are illustrated. By studying scroll formations in terms of age of vegetation it is possible to quantify rate and direction of channel migration. The *bank height* classification of rivers is given in Fig. 4.3.5f. Bank height is often an important index to age and activity of the river. Classification based on natural levees is illustrated in Fig. 4.3.5g. As pointed out earlier, well developed levees are associated with older rivers. Typical modern *floodplains* are illustrated in Fig. 4.3.5h. The floodplain that is broad in relation to the channel width is indicative of an older river. Conversely when the river valley is narrow and confined by terraces or valley walls, the river flowing therein is usually mature. Typical *vegetative patterns* that are observed along meandering channels



(a) Variability of unvegetated channel width: channel pattern at normal discharge



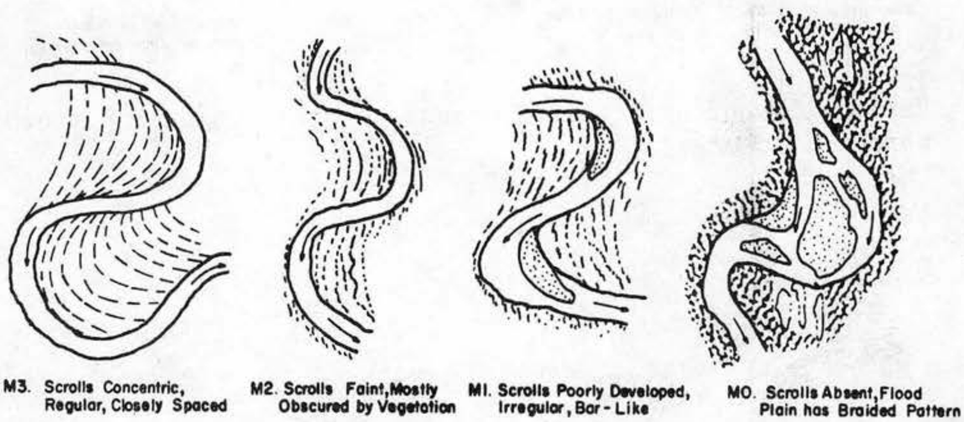
(b) Braiding patterns



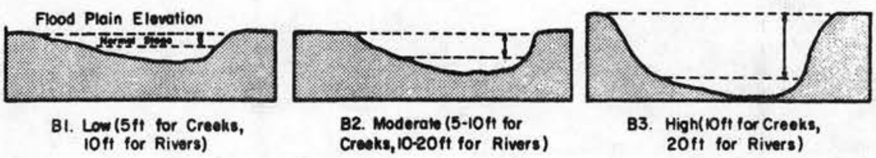
(c) Types of sinuosities

Fig. 4.3.5 Classification of river channels (after Culbertson et al., 1967).

(d) Oxbow lakes on floodplain

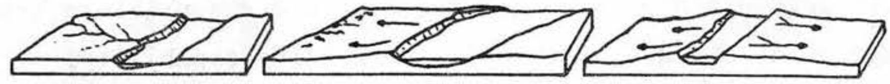


(e) Types of meander scroll formations

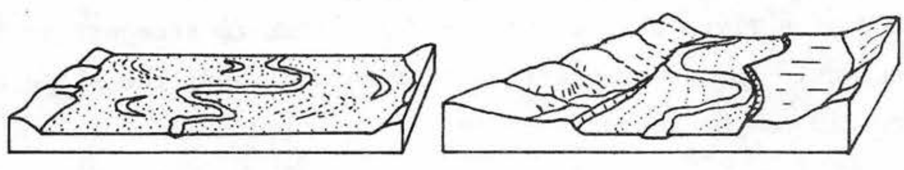


(f) Types of bank heights

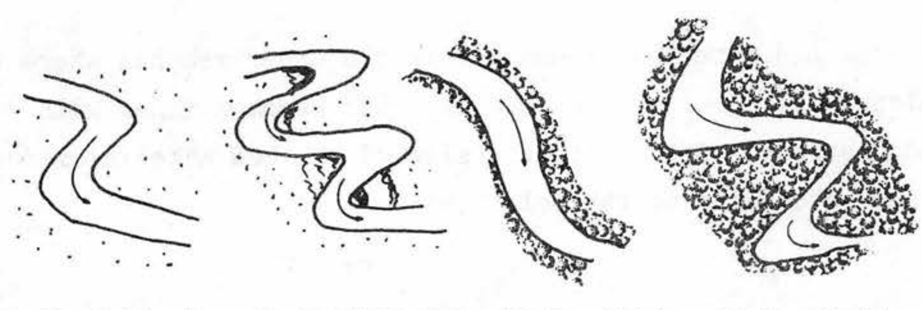
Fig. 4.3.5 Classification of river channels (after Culbertson et al., 1967) (Continued).



(g) Types of natural levee formations



(h) Types of modern floodplains



(i) Types of vegetal patterns

Fig. 4.3.5 Classification of river channels (after Culbertson et al., 1967) (Continued).

are shown in Fig. 4.3.5i. In general, the growth of vegetation is indicative of the presence of silts and clays in the river banks and the floodplain. This is particularly true if the floodplain is well drained. With good drainage the silt and clay are essential to the growth of vegetation because of their water holding capability.

A comparison of the hydraulic and morphological properties of the channels as classified in Fig. 4.3.5 is provided in Table 4.3.1. A detailed knowledge of the hydraulic characteristics of different types of streams is of great value when dealing with the location of bridges, training works, flood control works and other river structures.

4.3.6 <u>The river profile and its bed materials</u>

The slope of a river channel or a river system is steepest in the headwater regions. The river profile is concave upward; the slope of the river profile can be represented by the equation

$$S_x = S_o e^{-\alpha x} . \qquad 4.3.1$$

where S_x = the slope at any station a distance x downstream of the reference station,

S_o = the slope at the reference station, and

α = a coefficient.

Similarly, the bed material is coarser in the upper reaches where the channel slopes are steep and the bed material becomes finer with distance downstream. Generally, the size of the bed material reduces with distance according to the relationship

$$D_{50_x} = D_{50_o} e^{-\beta x} \qquad 4.3.2$$

where D_{50_x} = the size of bed material at distance x downstream of the reference station.

D_{50_o} = the size of bed material at the reference station and

β = a coefficient.

Table 4.3.1 The comparison of hydraulic and morphological properties of each type of classification.

	Uniform-width sinuous channel T1	Sinuous point bar channel T2	Point bar braided channel T3	Bar braided or island braided drainage course T4
Shape of hydrograph	The slopes of rising and falling limbs are steeper than for the sinuous point bar channel and flatter than for point bar braided channels. Groundwater fed channels have flat rise and fall-curves.	The rate of change of slopes of rising and falling limbs of the hydrograph are less than for the uniform-width sinuous channel and point bar braided channel.	The rise and fall of the hydrograph is very steep due to low sinuosity, steep slope and narrow modern floodplain of the channel.	The peak duration of the hydrograph is long. If braiding is associated with the steep slope of the channel, the rate of rise and fall of hydrograph may be steep.
Modern flood plain	The channel can be formed on a narrow or on a broad floodplain.	A broad modern floodplain is associated with this type of channel.	Generally, the modern floodplain is narrow.	The modern floodplain may be narrow if the channel slope is steep and may be broad if the slope is flat.
Sinuosity	Sinuosity low ($P<1.5$), moderate ($1.5<P\leq 2.0$) or high ($P\geq 2.0$)	moderate ($1.5<P\leq 2.0$) or high ($P\geq 2.0$)	low ($P<1.5$) or moderate ($1\leq 5<P\leq 2.0$)	low ($P<1.5$)
Pattern of Vegetation	A narrow belt of dense vegetation is found along both the banks of channel. The vegetal growth mostly on the inside of channel bends is associated with high sinuosity.	Negligible to very dense vegetation may be formed on the floodplain.	When channels have steep slopes the vegetal growth is usually negligible along both banks of the channel.	The pattern of vegetation found is generally either dense all along the area of flow or negligible.
Bank heights	Banks are cohesive and resistant to erosion. Bank heights are low to high.	Banks are relatively less cohesive than for the uniform-width sinuous channel. The banks are moderate to high.	The banks are less cohesive and may be low to moderately high.	The banks are generally low and cohesionless.
Natural levee formation	Moderate or high bank heights are generally associated with natural levees.	Natural levees are generally found on concave banks.	Natural levees are not formed by the channel.	Natural levees are not formed.
Oxbow lake formations	Oxbow lakes are generally not formed unless the sinuosity is large.	Generally, oxbow lakes are formed by this type of channel.	The oxbow lakes are not formed by the channel.	Oxbow lakes are not expected in this type of channel.
Meander scroll formation	The concentric and regular scrolls are associated with high sinuosity. The low sinuosity channel is accompanied by poorly developed scrolls.	Regular, concentric and closely spaced meander scrolls are associated with this type of channel.	Meander scrolls are either absent or poorly developed.	Meander scrolls are mostly absent or poorly developed.
Braiding	braiding absent	braiding absent	single bar braiding or single island braiding	single or multiple bar or island braiding
Mode of sediment transport	Sediment is transported mainly as suspended load consisting of wash load and bed-material load.	Similar to type T1. Sediment is mainly transported through suspension.	Sediment is mainly transported as bed load.	If slopes are steep, mode of sediment transport is similar to type T3. If slopes are flatter, sediment is transported by suspension.

4.4.0 QUALITATIVE RESPONSE OF RIVER SYSTEMS

Many rivers have achieved a state of practical equilibrium throughout long reaches. For practical engineering purposes, these reaches can be considered stable and are known as "graded" streams by geologists and as "poised" streams by engineers. However, this does not preclude significant changes over a short period of time or over a period of years. Conversely, many streams contain long reaches that are actively aggrading or degrading. *These aggrading and degrading channels may pose a definite hazard to any highway crossing or encroachment.*

Regardless of the degree of the channel stability, man's local activities may produce major changes in river characteristics locally and throughout the entire reach. All too frequently the net result of a river improvement is a greater departure from equilibrium than that which originally prevailed. *Good engineering design must invariably seek to enhance the natural tendency of the stream toward poised conditions.* To do so, an understanding of the direction and magnitude of change in channel characteristics caused by the actions of man and nature is required. This understanding can be obtained by: (1) studying the river in a natural condition, (2) having knowledge of the sediment and water discharge, (3) being able to predict the effects and magnitude of man's future activities, and (4) applying to these a knowledge of geology, soils, hydrology, and hydraulics of alluvial rivers.

To predict the response to channel development is a very complex task. There are large numbers of variables involved in the analysis that are interrelated and can respond to changes in a river system in the continual evolution of river form. The channel geometry, bars, and forms of bed roughness all change with changing water and sediment discharges. Because such a prediction is necessary, useful methods have been developed to predict the response of channel systems to changes both qualitatively and quantitatively.

4.4.1 Prediction of general river response to change

Quantitative prediction of response can be made if all of the required data are known with sufficient accuracy. Usually, however, *the data are not sufficient for quantitative estimates, and only qualitative estimates are possible.* Examples of studies that have been undertaken by various investigators for qualitative estimates follow. Lane (1955) studied the changes in river morphology caused by modifications of water and sediment discharges. Similar but more comprehensive treatments of channel response to changing conditions in rivers have been presented by Leopold and Maddock (1953), Schumm (1971), and Santos-Cayado (1972). All research results support the following general statements:

(1) Depth of flow is directly proportional to water discharge and inversely proportional to sediment discharge.

(2) Width of channel is directly proportional to water discharge and to sediment discharge.

(3) Shape of channel expressed as width-depth ratio is directly related to sediment discharge.

(4) Meander wavelength is directly proportional to water discharge and to sediment discharge.

(5) Slope of stream channel is inversely proportional to water discharge and directly proportional to sediment discharge and grain size.

(6) Sinuosity of stream channel is proportional to valley slope and inversely proportional to sediment discharge.

It is important to remember that these statements pertain to natural rivers and not necessarily to artificial channels with bank materials that are not representative of sediment load. In any event, the relations will help to determine the response of any water conveying channel to change.

Sediment bed material transport (Q_s) can be directly related to stream power ($\tau_o V$) and inversely related to the fall diameter of bed material (D_{50}).

$$Q_s \sim \frac{\tau_o V\ W}{D_{50}/C_f} \quad , \tag{4.4.1}$$

Here τ_o is the bed shear, V is the cross-sectional average velocity, W is the width of the stream and C_f is the fine material load concentration. Equation 4.4.1 can be written as

$$Q_s \sim \frac{\gamma y_o SWV}{D_{50}/C_f} = \frac{\gamma QS}{D_{50}/C_f} \ . \tag{4.4.2}$$

If specific weight, γ is considered constant and the concentration of wash load C_f can be incorporated in the fall diameter, D_{50}, the relation can be expressed as

$$QS \sim Q_s D_{50} \tag{4.4.3}$$

which is the *relation originally proposed by Lane (1955)*, except Lane used the median diameter of the bed material as defined by sieving instead of the fall diameter. The fall diameter includes the effect of temperature on the transportability of the bed material and is preferable to the use of physical diameter.

Equation 4.4.3 is very useful to qualitatively predict channel response to climatological changes, river development or both. Two simple example problems are analyzed using Eq. 4.4.3.

Consider a tributary entering the main river at point C that is relatively small but carries a large sediment load (see Fig. 4.4.1). This increases the sediment discharge in the main stream from Q_s to Q_s^+. It is seen from Eq. 4.4.3 that, for a significant increase in sediment discharge (Q_s^+) the channel gradient (S) below C must increase if Q remains constant. The line CA (indicating the original channel gradient) therefore changes with time to position C'A. Upstream of the confluence the slope will adjust over a long period of time to the original channel slope. The river bed will aggrade from C to C'.

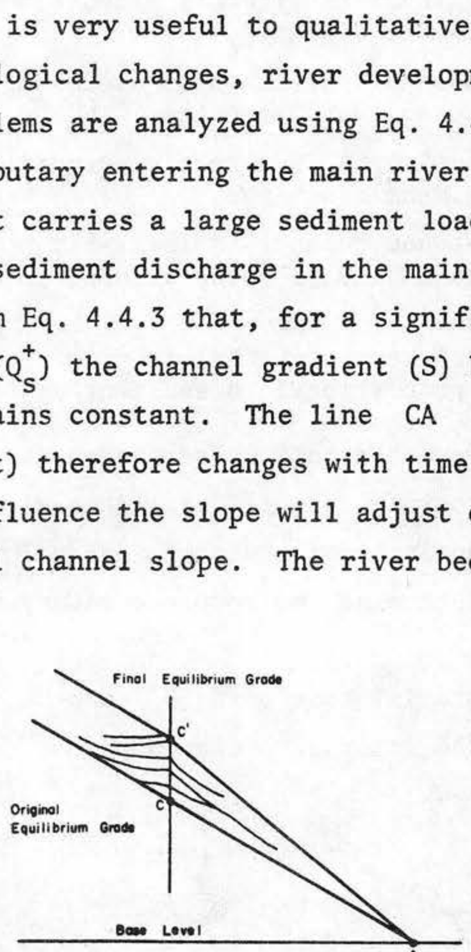


Fig. 4.4.1 Changes in channel slope in response to an increase in sediment load at point C.

Construction of a dam on a river usually causes a decrease in sediment discharge downstream. Referring to Fig. 4.4.2, and using Eq. 4.4.3 and the earlier discussion, it can be concluded that for a decrease in bed material discharge from Q_s to Q_s^-, the slope S decreases downstream of the dam. In Fig. 4.4.2, the line CA, representing the original channel gradient, changes to C'A, indicating a decrease in bed elevation and slope in the downstream channel with time. Note, however, if the dam fills with sediment so that the incoming sediment discharge passes through, that, except for local scour at the dam, the grade line C'A would return to the line CA. Also upstream of the dam the grade would return to the original equilibrium grade but would be offset vertically by the height of the dam. Thus *small dams* (storage capacity small in relation to annual discharge) *may cause degradation and then aggradation over a relatively short period of time.*

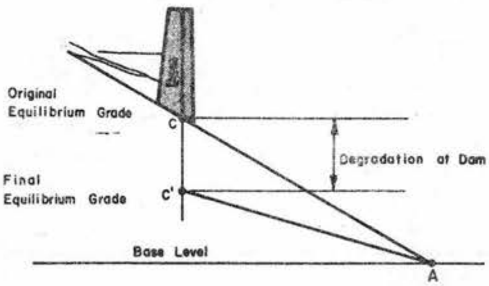


Fig. 4.4.2 Changes in channel slope in response to a dam at point C.

The engineer is also interested in quantities in addition to directions of variations. The geomorphic relation $QS \sim Q_s D_{50}$ is only an initial step in analyzing long-term channel response problems. However, this initial step is useful because it warns of possible future difficulties in designing channel improvement and flood protection works. The prediction of the magnitude of possible errors in flood protection design, because of changes in stage with time, requires the quantification of changes in stage. To quantify these changes it is necessary to be able to quantify future changes in the variables that affect the stage. In this respect, knowledge of the future flow conditions is necessary.

4.4.2 River conditions for meandering and braiding

In the preceding examples it was shown that changes in water, sediment discharge or both can cause significant changes in channel slope. The changes in sediment discharge can be in quantity Q_s or caliber D_{50} or both. Often such changes can alter the plan view in addition to the profile of a river.

Fig. 4.4.3 illustrates the dependence of river form on channel slope and discharge.

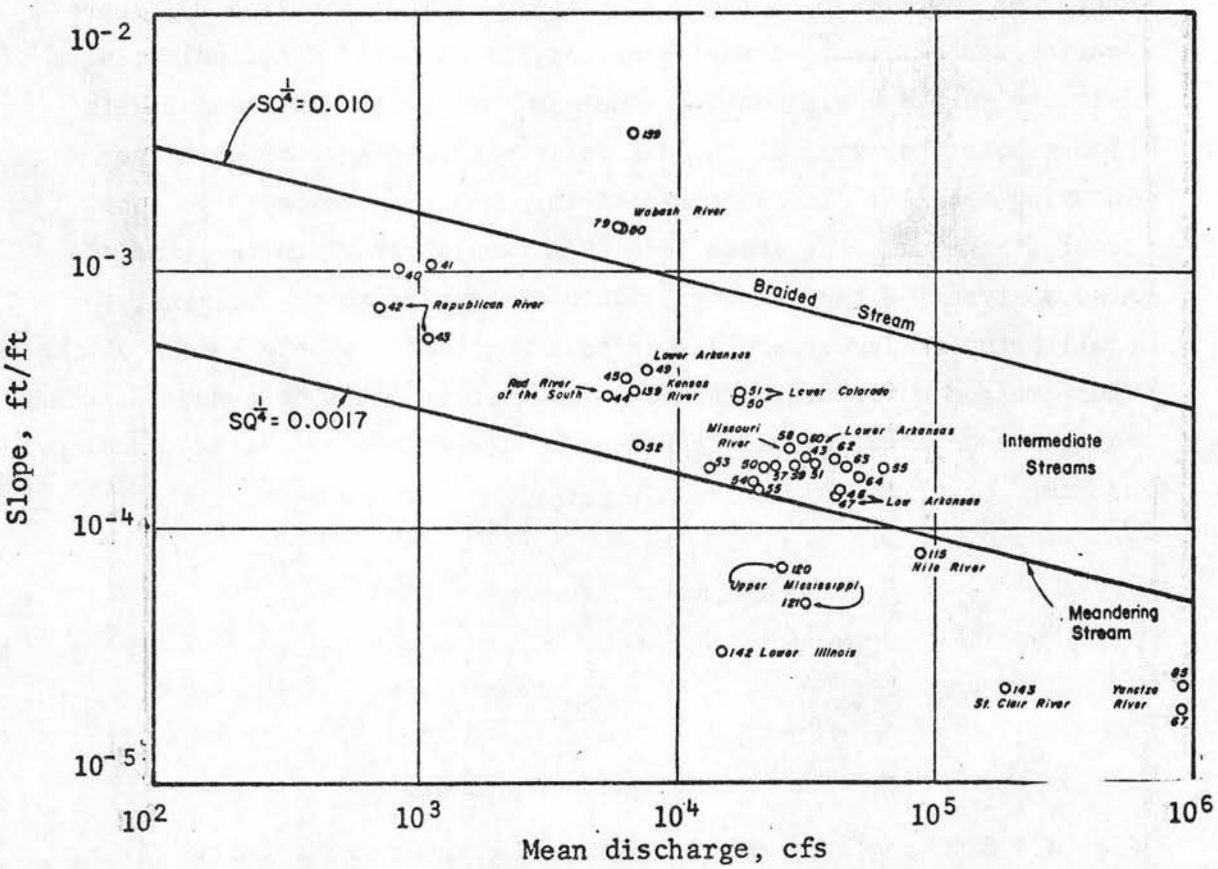


Fig. 4.4.3 Slope-discharge relation for braiding or meandering in sandbed streams (after Lane, 1957).

It shows that when

$$S\ Q^{1/4} \leq .0017 \qquad 4.4.4$$

a sandbed channel meanders. Similarly, when

$$S\ Q^{1/4} \geq .010 \qquad 4.4.5$$

the river is braided. In these equations, S is the channel slope in feet per foot and Q is the mean discharge in cfs. Between these values of $S\ Q^{1/4}$ is the transitional range and many of the U.S. rivers, classified as intermediate sandbed streams, plot in this zone between the limiting curves defining meandering and braided rivers. If a river is meandering but its discharge and slope borders on the transitional zone a relatively small increase in channel slope may cause it to change, with time, to a transitional or braided river. The reader can deduce the consequence of other changes in variables on river form by employing Eq. 4.4.2 and Table 4.4.2.

4.4.3 Hydraulic geometry of alluvial channels

Hydraulic geometry is a general term applied to alluvial channels to denote relationships between discharge Q and the channel morphology, hydraulics and sediment transport. In self-formed alluvial channels, the morphologic, hydraulic and sedimentation characteristics of the channel are determined by a large variety of factors. The mechanics of such factors is not fully understood. However, alluvial streams do exhibit some quantitative hydraulic geometry relations. *In general, these relations apply to channels within a physiographic region and can be easily derived from data available on gaged rivers. It is understood that hydraulic geometry relations express the integral effect on all the hydrologic, meteorologic, and geologic variables in a drainage basin.*

The hydraulic geometry relations of alluvial streams are useful in river engineering. The forerunner of these relations are the regime theory equations of stable alluvial canals. A generalized version of hydraulic geometry relations was developed by Leopold and Maddock (1953) for different regions in the United States and for different types of rivers. In general the hydraulic geometry relations are stated as:

$$W = a\,Q^b$$
$$y_o = c\,Q^f$$
$$V = k\,Q^m$$
$$Q_T = p\,Q^j$$
$$S = t\,Q^z$$
$$n = r\,Q^y$$

where W is the channel width, y_o is the channel depth, V is the average velocity of flow, Q_T is the total bed material load, S is the energy gradient, n is the Manning's roughness coefficient, and Q is the discharge as defined in the following paragraphs. The coefficients a, c, k, p, t, r and exponents b, f, m, j, z, y in these equations are determined from analysis of available data on one or more streams. From the definition equation $Q = Wy_oV$, it is seen that

$$a \cdot c \cdot k = 1$$

and

$$b + f + m = 1$$

Leopold and Maddock (1953) have shown that in a drainage basin, two types of hydraulic geometry relations can be defined: (1) relating W, y_o, V and Q_s to the variation of discharge at a station, and (2) relating these variables to the discharges of a given frequency of occurrence at various stations in a drainage basin. Because Q_T is not available, they used Q_s the suspended load transport rate. The former are called *at-station* relationships and the latter *downstream* relationships. The distinction between *at-station* and *downstream* hydraulic geometry relations is illustrated in Figs. 4.4.4 and 4.4.5.

The mean values of exponents b, f, m, j, z, and y as reported by Leopold et al. (1964) are given below. These values are based on an extensive analysis of stream data in the United States.

	Average At-A-Station Relations						Average Downstream Relations (bank-full or mean annual flow)					
	b	f	m	j	z	y	b	f	m	j	z	y
Average values midwestern United States	.26	.40	.34	2.5			.5	.4	.1	.8	-.49	
Brandywine Creek, Pennsylvania	.04	.41	.55	2.2	.05	-.2	.42	.45	.05		-1.07	-.28
Ephermeral Streams in semiarid United States	.29	.36	.34				.5	.3	.2	1.3	- .95	-.3
Appalachian Streams							.55	.36	.09			
Average of 158 gauging stations in United States	.12	.45	.43									
Ten gauging stations on Rhine River	.13	.41	.43									

Symbols:

Q	discharge	$W = a\,Q^b$
W	channel width	
y_o	mean depth	$y_o = c\,Q^f$
V	mean velocity	$V = k\,Q^m$ $\quad n = r\,Q^y$
Q_s	suspended load transport rate	$Q_s = p\,Q^j$
S	water-surface slope	$S = t\,Q^z$
n	roughness parameter of Manning type	

More recently hydraulic geometry relations were theoretically developed at CSU. These relations are almost identical to those proposed by Leopold and Maddock. The *at-station* relations derived at CSU are:

$$W \sim Q^{0.26} \tag{4.4.6}$$

$$y_o \sim Q^{0.46} \tag{4.4.7}$$

$$S \sim Q^{0.00} \tag{4.4.8}$$

$$V \sim Q^{0.30} \tag{4.4.9}$$

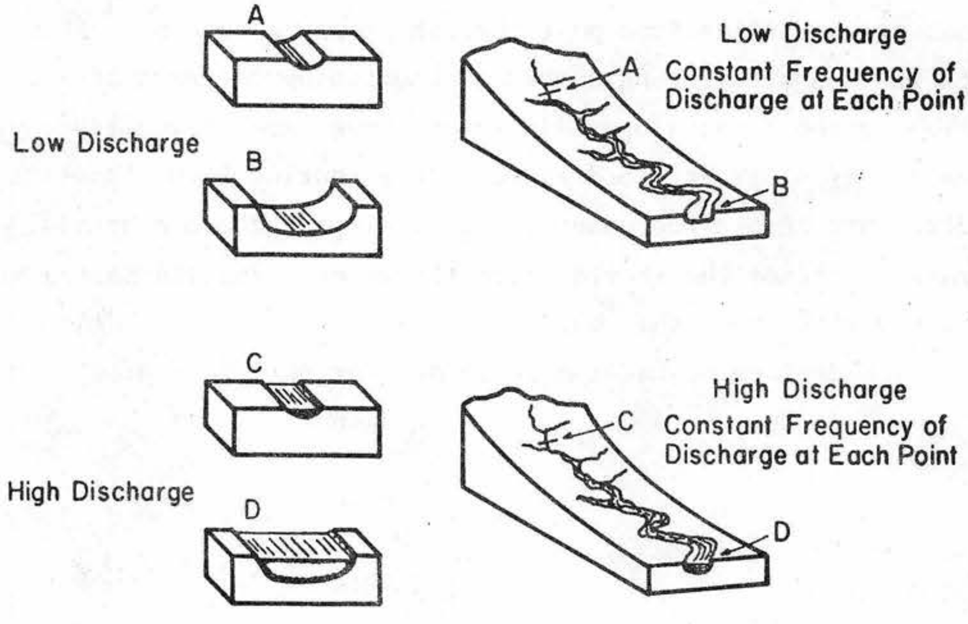


Fig. 4.4.4 Variation of discharge at a given river cross section and at points downstream (after Leopold and Maddock, 1953). *At-station* relations pertain to individual sites such as A or B. *Downstream* relations pertain to a channel (segment A-B) or drainage network for discharge of a given frequency of occurrence.

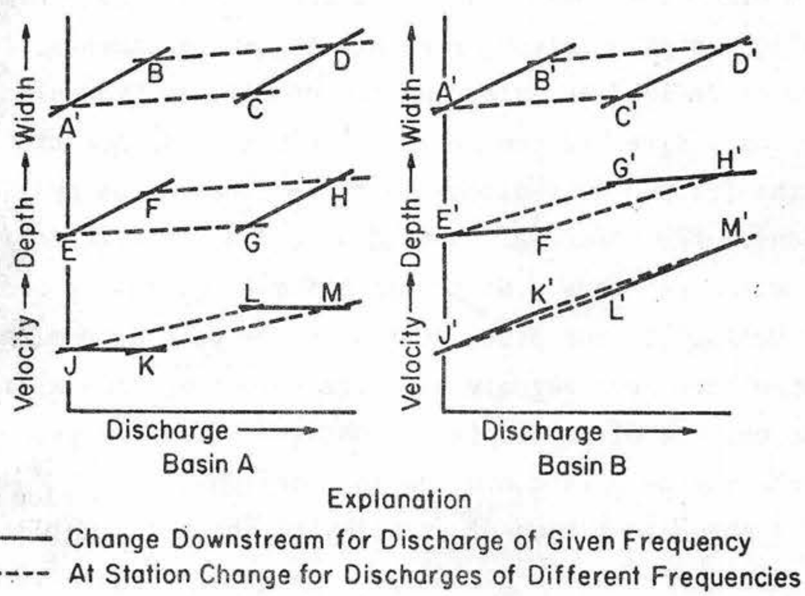


Fig. 4.4.5 Schematic variation of width, depth, and velocity with *at-station* and *downstream* discharge variation (after Leopold and Maddock, 1953).

Equation 4.4.8 implies that slope is constant at a cross section. This is not quite true. At low flow the effective channel slope is that of the thalweg that flows from pool through crossing to pool. At higher stages the thalweg straightens somewhat, shortening the path of travel and increasing the local slope. In the extreme case, river slope approaches the valley slope at flood stage. It is during high floods that the flow often cuts across the point bars, developing chute channels. This path of travel verifies the shorter path the water takes and that a steeper channel prevails under this condition.

The derived *downstream* relations for *bank-full discharge* are:

$$y_b = Q_b^{0.46} \tag{4.4.10}$$

$$W_b = Q_b^{0.46} \tag{4.4.11}$$

$$S = Q_b^{-0.46} \tag{4.4.12}$$

$$V_b = Q_b^{0.08} \tag{4.4.13}$$

Here the subscript b indicates the bank-full condition.

4.4.4 Dominant discharge in alluvial rivers

The hydraulic geometry relations discussed in 4.4.3 indicate how the channel morphology and other characteristics vary with discharge at a station or in the downstream direction in a drainage network. In the hydraulic design of river crossings and encroachments, the relations need to be defined to determine the *downstream* hydraulic geometry of the channel at a site between two gaged sites. The question then arises about the frequency of discharge to be used in the hydraulic geometry relations. *The relations expressed in 4.4.3 relate to the bank-full stage, which for many U.S. rivers has a frequency of occurrence of one in 1.5 years.* In the past, various terms such as *dominant* or *formative* discharge have been vaguely used for selecting some arbitrary discharge for the purpose of developing *downstream* hydraulic geometry relations. This arbitrariness is confusing to a designer.

The characteristics of an alluvial channel, including its hydraulic geometry, vary with the discharge. In natural rivers, the characteristics such as the bed material load, energy gradient and meander geometry can be related to channel discharge as simple power functions. The formative discharge corresponding to the average value of such characteristics can then be defined in terms of the power function and the frequency distribution of the flow as follows:

$$F(Q) = a\ Q^n$$

$$\overline{F(Q)} = \frac{1}{T} \int_0^T F(Q) \cdot dt$$

$$Q_f = [\frac{1}{a}\ \overline{F(Q)}]^{1/n}$$

where $F(Q)$ is the power function relating the phenomenon of interest, for example the bed material load, to discharge, Q; T is the time period over which the occurrence of the phenomenon is averaged and Q_f is the formative discharge for the particular phenomenon. As the functions $F(Q)$ are different for different characteristics, the value of Q_f obtained from the preceding equations will also be different. Also, when $n > 1$, Q_f will be greater than the mean discharge $\overline{Q}$. For a given site, Q_f can be expressed in terms of the frequency of occurrence or a return period.

Analyses of bed material load estimations of Q_f on sand bed rivers may show that up to 90 percent of the total transport is caused by flows that are equalled or exceeded about ten percent of the time only. *Thus, the average bed material load in a river may be described in terms of a formative discharge much larger than the mean annual flow. Also, the average channel width, depth and meander geometry may be defined in terms of different formative discharges rather than an arbitrarily chosen dominant discharge.*

The concept of frequency of occurrence of flows is important in the hydraulic design of highway crossings and encroachments. This concept can also be combined with the economic analysis to determine the design flow conditions. If this approach is used, the terminology of *dominant* and *formative* discharges loses its relevance to design considerations, except for a gross representation of the channel behavior. Both the *at-station* and *downstream* hydraulic geometry relations are especially useful when the hydraulic design is based on the frequency of occurrence of flows. After the design flow has been determined for a given highway structure, the channel hydraulics and morphology can be determined from the hydraulic geometry relations.

The hydraulic geometry relations are applicable to continuous channel behavior. In some cases, this behavior may become *discontinuous*, as the channel pattern changes from meandering to braided by the formation of cut-offs. *Caution should be used when such possibilities exist for design flows, and the channel behavior should be specially analyzed.*

4.4.5 Prediction of channel response to change

In section 4.4.1, it was illustrated that Eq. 4.4.3 could be used to predict changes in channel profiles caused by changes in water and sediment discharge. It is now possible to talk qualitatively about changes in channel profile, changes in river form and changes in river cross section both at a section and along the river channel using the other relations presented above.

This can be best illustrated by application. Referring to Table 4.4.1, consider the effect of an increase in discharge indicated by a plus sign on line (a) opposite discharge. The increase in discharge may affect the river form, energy slope, stability of the channel, cross-sectional area and river stage. Eqs. 4.4.4 and 4.4.5 (or Fig. 4.4.3) show that an increase in discharge could change the channel form in the direction of a braided form. Whether or not the channel form changed would depend on the river form prior to the increase in discharge. With the increase in discharge the stability of the channel would be reduced according to Eq. 4.4.9, which indicates an increase in velocity. On the other hand, this prediction could be affected by changes in form of bed roughness that dictate resistance to flow. This effect is discussed later.

From Chapter III recall that the wash load increases the apparent viscosity of the water and sediment mixture. This makes the bed material behave as if it were smaller. In fact, the fall diameter of the bed material is made smaller by significant concentrations of wash load. With more wash load, the bed material is more susceptible to transport and any river carrying significant wash load will change from lower to upper regime at a smaller Froude number than otherwise. Also, the viscosity is affected by changes in temperature.

Table 4.4.1 Qualitative response of alluvial channels.

Variable		Change in Magnitude of Variable	Effect on: Regime of Flow	River Form	Resistance to Flow	Energy Slope	Stability of Channel	Area	Stage
Discharge	(a)	+	+	M→B	±	-	-	+	+
	(b)	-	-	B→M	∓	+	+	-	-
Bed-Material Size	(a)	+	-	M→B	+	+	±	+	+
	(b)	-	+	B→M	-	-	±	-	-
Bed-Material Load	(a)	+	+	B→M	-	-	+	-	-
	(b)	-	-	M→B	+	+	-	+	+
Wash Load	(a)	+	+		-	-	±	-	-
	(b)	-	-		+	+	±	+	+
Viscosity	(a)	+	+		-	-	±	-	-
	(b)	-	-		+	+	±	+	+
Seepage force	(a)	Outflow	-	B→M	+	-	+	+	+
	(b)	Inflow	+	M→B	-	+	-	-	-
Vegetation	(a)	+	-	B→M	+	-	+	+	+
	(b)	-	+	M→B	-	+	-	-	-
Wind	(a)	Downstream	+	M→B	-	+	-	-	-
	(b)	Upstream	-	B→M	+	-	-	+	+

Seepage forces resulting from seepage outflow help stabilize the channel bed and banks. With seepage inflow, the reverse is true. Vegetation adds to bank stability and increases resistance to flow, reducing the velocity. Wind can retard flow, increasing roughness and depth, when blowing upstream. The reverse is true with the wind blowing downstream. Wind generated waves and their adverse influence on channel stability are the most significant effects of wind.

In many instances it is important to assess the effects of changes in water and sediment discharge on specific variables such as depth of flow, channel width, characteristics of bed materials, velocity and so forth. For this type of analysis we can use Eq. 4.4.3, and the *at-station* hydraulic geometry relation. Eq. 4.4.3 can be written in terms of width, depth, velocity concentration of bed material discharge C_s and water discharge Q or

$$QS \approx Q_s D_{50} = QC_s D_{50} \qquad 4.4.14$$

and

$$C_s D_{50} \sim S$$

These equations are helpful for detailed analysis.

4.4.6 <u>Relative influence of variables on bed material and water discharge</u>

The study of the relative influence of viscosity, slope, bed material size and depth on bed material and water discharge is examined in detail using Einstein's bed-load function (1950) and Colby's (1964) relationships. *Einstein's bed-load function* was chosen because it *is the most detailed and comprehensive treatment from the point of fluid mechanics. Colby's relations were chosen because of the large amount and range of data used in their development.*

The data required to compute the total bed material discharge using Einstein's relations are:

S = energy slope
D_{65} = size of bed material for which 65 percent is finer
D_{35} = size of bed material for which 35 percent is finer
ν = kinematic viscosity
n_w = Manning's wall friction coefficient
A = cross-sectional area

P_b = wetted perimeter of the bed.

P_w = wetted perimeter of the banks.

D_i = size of bed-material fraction i.

i_b = percentage of bed material in fraction i.

γ_s = specific weight.

V = average velocity.

To study the relative influence of variables on bed material and water discharges, the data taken by the U.S. Geological Survey from October 1, 1940 to October 1, 1970 on the Rio Grande near Bernalillo are used. The width of the channel reach was 270 ft. In the analysis the energy slope was varied from 0.7S to 1.5S, in which S is the average bed slope assumed to be equal to the average energy slope. Further, the kinematic viscosity was varied to correspond with variations in temperature from 39.2° to 100°F inclusive. The variation of D_{65}, D_{35}, D_i and i_B was accomplished by using the average bed-material distribution given by Nordin (1964) and shifting the curve representing the average bed-material distribution along a line parallel to the abscissa drawn through D_{50}. The average water temperature was assumed to be equal to 70°F and the average energy gradient of the channel was assumed to be equal to 0.00095 ft/ft = 5.0 ft/mi. The water and sediment discharges were computed independently for each variation of the variables and for three subreaches of the Rio Grande of different width near Bernalillo. The applicability of the results depend on the reliability of the modified Einstein bed-load function and Colby's relationships used in the analysis rather than on the choice of data.

The computed water and sediment discharges are plotted in Figs. 4.4.6, 4.4.7 and 4.4.8 and show the variation of sediment discharge due to changes in bed material size, slope and temperature for any given water discharge. Figure 4.4.6 shows that *when the bed material becomes finer, the sediment discharge increases considerably*. The second most important variable affecting sediment discharge is the slope variation (see Fig. 4.4.7). Temperature is third in importance (Fig. 4.4.8). The effects of variables on sediment discharge were studied over approximately the same range of variation for each variable.

Fig. 4.4.9 shows the variation of the sediment discharge due to changes in the depth of flow for any given discharge, computed using

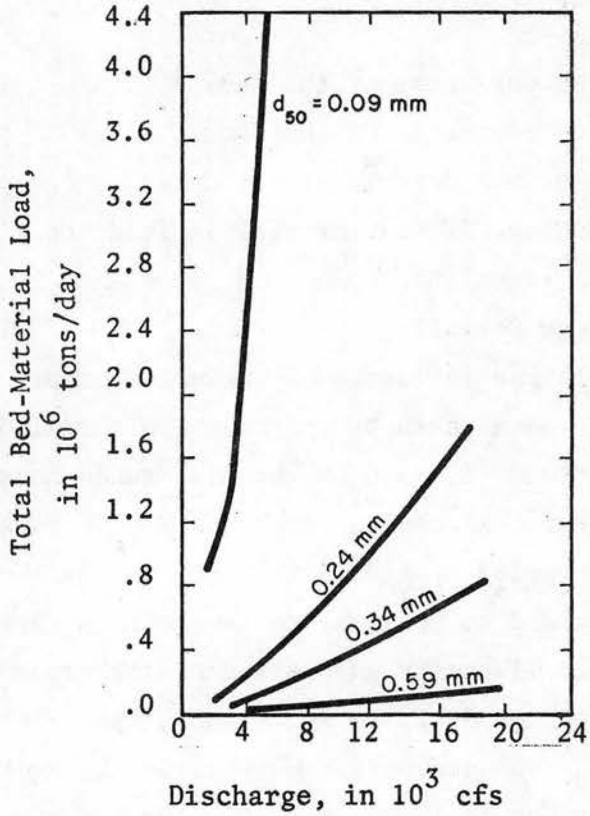


Fig. 4.4.6 Bed-material size effects on bed-material transport.

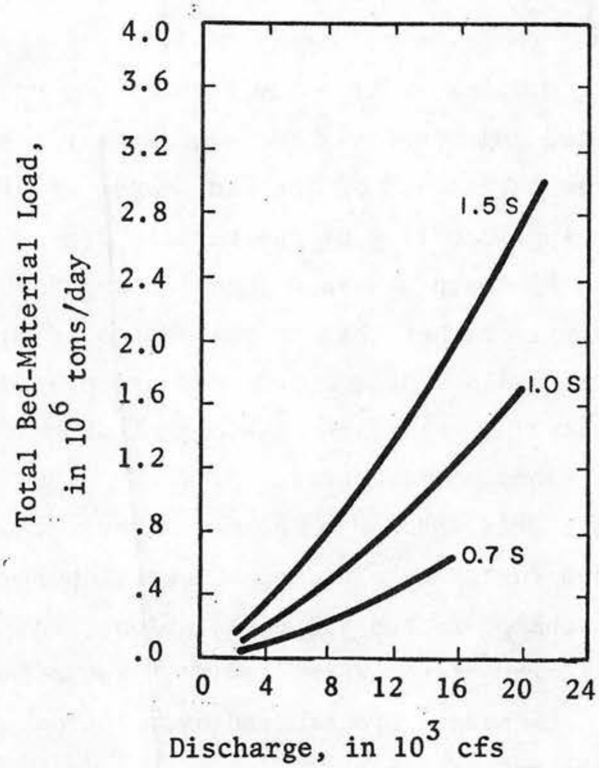


Fig. 4.4.7 Effect of slope on bed-material transport.

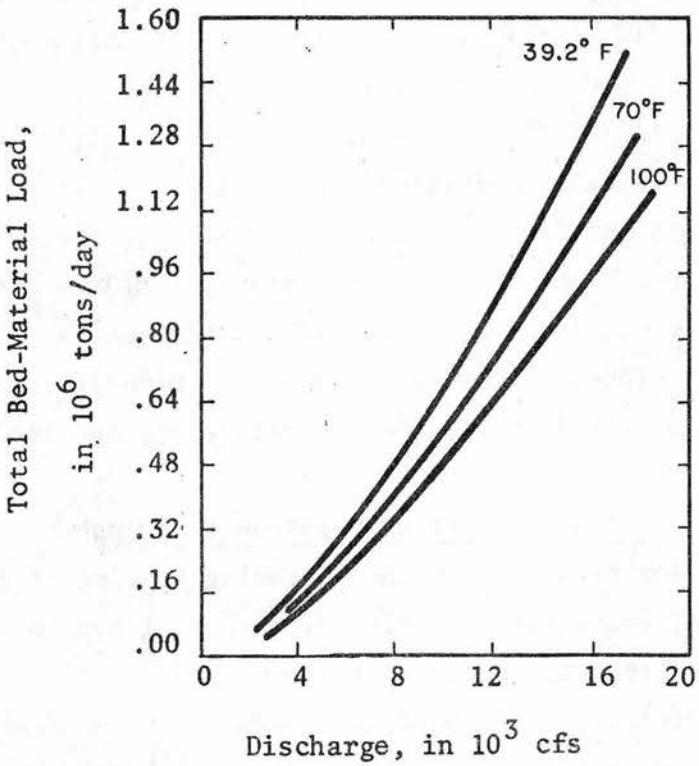


Fig. 4.4.8 Effect of kinematic viscosity (temperature) on bed-material transport.

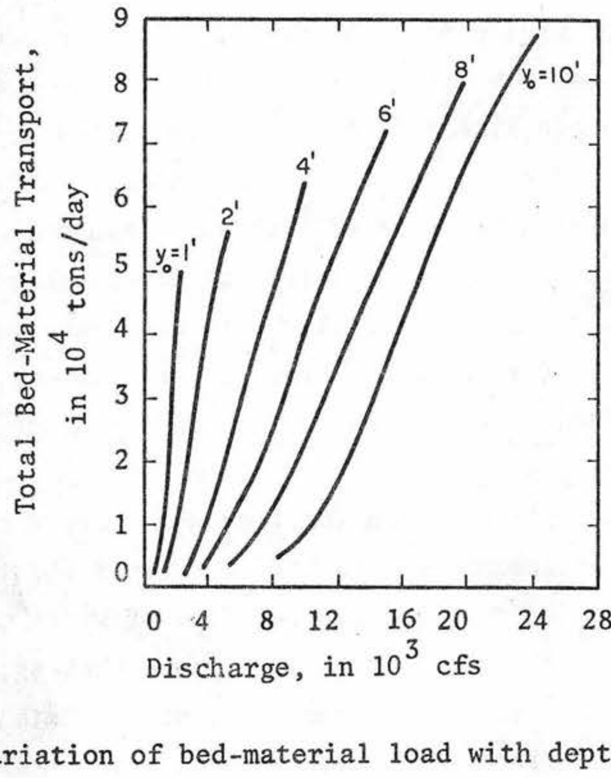


Fig. 4.4.9 Variation of bed-material load with depth of flow.

Colby's (1964) relations. The values of depth of flow varied from 1.0 to 10.0 ft, the median diameter of the bed material was maintained constant equal to 0.030 mm, the water temperature was assumed constant and the concentration of fine sediment was assumed less than 10,000 ppm. The channel width was also maintained constant at 270 ft. In Fig. 4.4.9, the curves for constant depth of flow show a steep slope. This indicates that the capacity of the stream to transport sands increases very fast for a small increase of discharge at constant depth. Similar figures can be developed for other sizes of bed material, and the relations can be modified to include the effect of wash load and viscosity effects.

4.4.7 Prediction of long term river response to change

The information presented in the preceding portion of this chapter can be used to determine the direction of change of hydraulic variables when the water and sediment discharges are varied. It is important to notice that the *Einstein's, Colby's, and Manning's equations apply to a cross section or reach and differ from some of the available geomorphic equations* that have been derived by considering a reach or total length of river. Einstein's, Colby's and Manning's equations deal with depth of flow, width of flow and energy slope whereas most geomorphic equations deal with channel depth, channel width and channel slope.

The interdependency of top width, depth of flow, energy slope, bed-material size and kinematic viscosity on the water and sediment discharge allows the establishment of the relative influence of those variables on stage-discharge relationships. Information concerning the interdependency of top width, depth of flow, energy slope, bed material size and kinematic viscosity with water and sediment discharges can be used to establish the direction of variation of hydraulic variables as a consequence of changes imposed on the water and bed-material discharge.

Neither Einstein's bed-load function nor Colby's relationships directly take into account the width of the cross section, except when transforming the sediment discharge per foot of width to the total river width. The influence of the width, nevertheless, indirectly enters any method of estimating transport, since width affects the depth of flow for a given water discharge and energy slope. With the total information provided to date, the response of a river system to changes in variables are given in Table 4.4.2. A plus (+) sign signifies an increase in the value of the variable and a minus (-) sign signifies a decrease in the value of the variable. The letter B indicates an increase in the product $SQ^{1/4}$ and a shift toward a braided condition and the letter M indicates a reduction in $SQ^{1/4}$ and a shift toward the meandering condition. No attempt is made here to determine whether or not the channel braids or meanders.

Table 4.4.2 Change of variables induced by changes in sediment discharge, size of bed material and wash load.

Equation	Tendency to Braid or Meander
$Q_s^+ D_{50}/C_f \sim S^+ V^+ y_o^- W^+$	B
$Q_s^- D_{50}/C_f \sim S^- V^- y_o^+ W^-$	M
$Q_s D_{50}^+/C_f \sim S^+ V^+ y_o^- W^+$	B
$Q_s D_{50}^-/C_f \sim S^- V^\pm y_o^\pm W^\pm$	M
$Q_s D_{50}/C_f^+ \sim S^- V^\pm y_o^\pm W^\pm$	M
$Q_s D_{50}/C_f^- \sim S^+ V^+ y_o^- W^+$	B
$Q_s^+ D_{50}^+/C_f \sim S^+ V^+ y_o^- W^+$	B
$Q_s^- D_{50}^-/C_f \sim S^- V^\pm y_o^\pm W^-$	M
$Q_s^+ D_{50}^+/C_f^- \sim S^+ V^+ y_o^- W^+$	B
$Q_s^- D_{50}^-/C_f^+ \sim S^- V^\pm y_o^\pm W^\pm$	M
$Q_s^+ D_{50}^+/C_f^+ \sim S^+ V^+ y_o^- W^+$	M

Note: An increase in the value of the variable is denoted by a + ; and a decrease is denoted by a - . As an example, in the first line, if the value of Q_s increases, the slope, velocity and width will increase and the depth of flow will decrease.

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Chapter V

RIVER MECHANICS

5.1.0 INTRODUCTION

The rivers are a dynamic geomorphic feature. Generally they are continuously changing their position, shape, and other morphological characteristics with the variation of discharges and with the passage of time. In the context of highway engineering, *it is therefore important not only to study the existing river but also its possible variations during the life of the highway project.*

Rivers are nature's way of conveying water on the surface of the earth. *The characteristics of a river are determined by water discharge, quantity and character of sediment discharge, composition of the bed and bank material of the channel, variations of these parameters in time, and man's activities.* In general, man's activities may relate to the variation of the imposed discharge and the sediment load in the channel or to the variation of the channel geometry by encroachment. These activities may pertain to the location of the highway in the river environment or farther upstream or downstream in the channel. To predict the behavior of a river in its natural state or as affected by man's activities, it is necessary to delineate the characteristics of the river as well as the mechanics of their formation.

The more apparent characteristic of a river is its *plan form geometry*. The rivers are classified as meandering, straight, and braided. The characteristics of *sectional geometry* of the river relate to the channel width, the width-depth ratio, and the form of the cross section. Characteristics of the river also relate to the *bed material* size. From the point of view of the hydraulics of a river channel, the characteristics pertaining to the bed forms in the channel are important. The river characteristics are *interrelated* so that a change in one may result in a change in others.

In this chapter, it is shown that the characteristics of individual river systems are well behaved and that a small amount of information about a particular river may be sufficient to deduce the other important characteristics.

At the present time, developments in river mechanics are such that quantitative information from one river can be transferred to another river. However, this transfer must be based on an understanding of river mechanics. The objective of this chapter is to introduce the highway engineer to the basic characteristics of rivers in a quantitative manner so that he is able to estimate and transfer information about rivers.

5.2.0 RIVER FORM

Rivers may be classified as meandering, straight, and braided. There are many transitional forms between these types. The processes involved in these forms have been presented in Chapter IV. A brief description is given below for continuity.

5.2.1 Meanders

A meandering river has more or less regular inflections that are sinuous in plan. *It consists of a series of bends connected by crossings.* In the bends, deep pools are carved adjacent to the concave bank by the relatively high velocities. Because velocities are lower on the inside of the bend, sediments are deposited in this region forming the *point bar*. The centrifugal force in the bend causes a transverse water surface slope, and in many cases, helicoidal flow occurs in the bend. Point bar building is enhanced when large transverse velocities occur. In so doing, they sweep the heavier concentrations of bed load toward the convex bank where they are deposited to form the point bar. Some transverse currents have a magnitude of about 15 percent of the average channel velocity. The bends are connected by crossings (short straight reaches) which are quite shallow compared to the pools in the bendways. At low flow, large sandbars form in the crossings if the channel is not well confined. The scour in the bend causes the bend to migrate downstream and sometimes laterally. Lateral movements as large as 2500 feet per year have been observed in alluvial rivers. Much of the sediment eroded from the outside bank is deposited in the crossing and on the point bar in the next bend downstream. Meandering rivers have relatively flat slopes.

The geometry of meandering rivers is quantitatively measured in terms of: (1) meander wavelength λ, (2) meander width W_m, (3) mean radius of curvature r_c, (4) meander amplitude A and (5) bend deflection angle ϕ. A sketch defining these quantities is shown in Fig. 5.2.1.

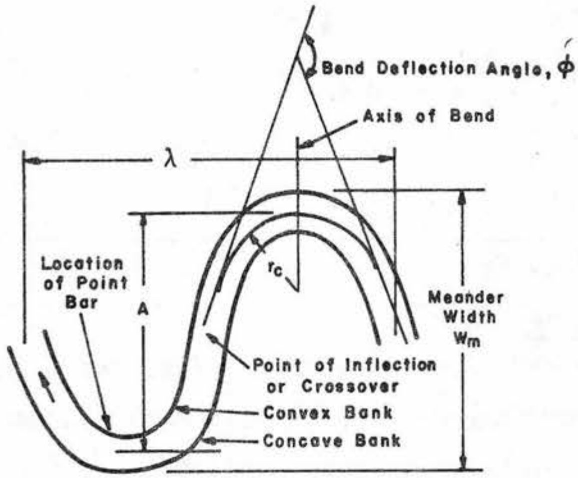

Fig. 5.2.1 Definition sketch for meanders.

The actual meanders in natural rivers are generally not as regular as indicated in Fig. 5.2.1. The precise measurement of meander dimensions is therefore difficult in natural channels and tends to be subjective. The analysis of the median meander dimension in nature shows that the meander length and meander width are both related to the width of the channels. The empirical realtionships for the meander length λ and the bank-full channel width as well as the meander amplitude A and the channel width are shown in Fig. 5.2.2 and Table 5.2.1.

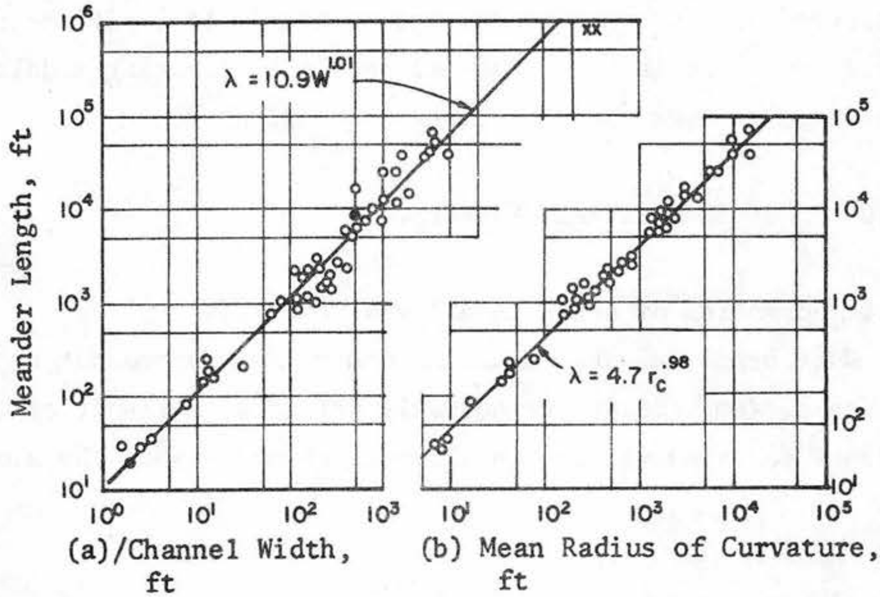

Fig. 5.2.2 Empirical relations for meander characteristics (Leopold et al., 1964).

Table 5.2.1 Empirical relations for meanders in alluvial valleys.

Meander Length to Channel Width	Amplitude to Channel Width	Meander Length to Radius of Curvature	Source
$\lambda = 6.6W^{0.99}$	$A = 18.6W^{0.99}$	-	Inglis (1949)
-	$A = 10.9W^{1.04}$	-	Inglis (1949)
$\lambda = 10.9W^{1.01}$	$A = 2.7W^{1.10}$	$\lambda = 4.7r_c^{0.98}$	Leopold and Wolman (1960

5.2.2 Braiding

The braided river channel is wide, the banks are poorly defined and unstable, and there are two or more main channels that cross one another giving the riverbed a braided appearance at low flow. Between sub-channels there are sandbars and islands. The sub-channels and sandbars change position rapidly with time and stage and in an unpredictable manner. At flood stage, the flow straightens, most of the sandbars are inundated or destroyed and the river has a canal-like appearance except that it is much wider and has a higher flow velocity. Such rivers have relatively steep slopes and carry large concentrations of sediment.

5.2.3 Straight

The straight channel has small sinuosity at bankful stage. At low stage the channel develops alternate sandbars and the thalweg meanders around the sandbars in a sinuous fashion. Straight channels are often considered as a transitional stage to meandering. If the channel is unconfined, more than one channel develops, creating middle bars as well as point bars, and the river is braided.

5.3.0 BENDS IN ALLUVIAL CHANNELS

5.3.1 Formation of bends in alluvial channels

Most bends in sandbed rivers are part of a meander or deformed meander system. Bends are normally formed as a result of the natural tendency for sinuous flow in alluvial channels when the slope of the river is less than $S = 0.0017\ \bar{Q}^{-1/4}$. (See Section 4.4.2.) This tendency for bends to develop in sand channels with flat slopes has been demonstrated by Friedkin (1945), Lane (1955) and many others.

The actual shape of the bends varies from beautifully symmetrical patterns to the deformed bends encountered most frequently in nature, particularly on large river systems.

The shape of a typical reveted bend in the Lower Mississippi River is illustrated in Fig. 5.3.1. The statistical nature of Lower Mississippi River bends is illustrated in Figs. 5.3.2 and 5.3.3. The figures show the percent occurrence of bend radii r_c and the percent occurrence of bend deflection angles ϕ in radians respectively. The most common radius is about 5,500 feet. This radius was observed on 11 percent of 179 river bends. Similarly, the most common deflection angle is about 1.15 radians. This deflection angle occurred for about 15 percent of these same bends. These distribution curves illustrate the variability of the characteristics of bends in the Mississippi river system that has been subjected to varying degrees of development.

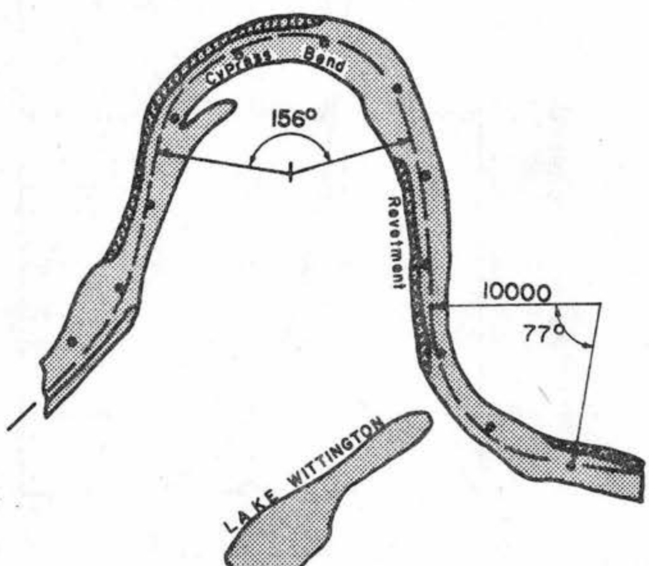

Fig. 5.3.1 Cypress Bend, Mississippi River, 1962 (after Assifi, 1966).

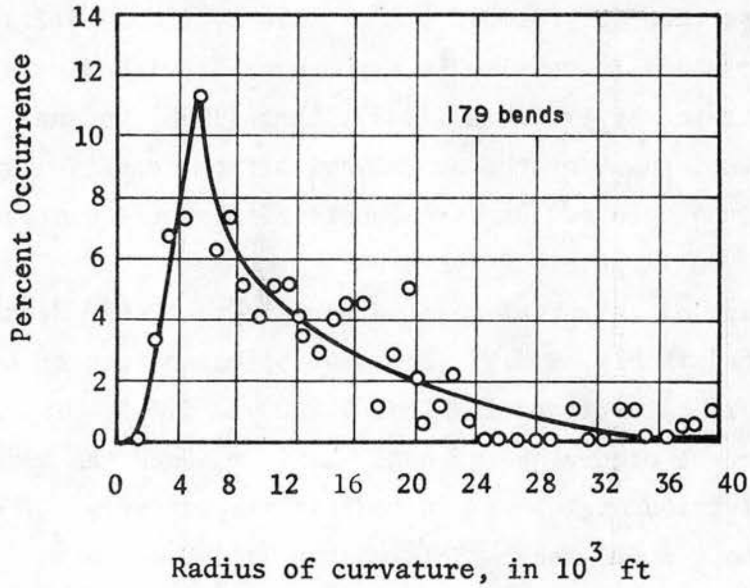

Fig. 5.3.2 Occurrence of bends in the Lower Mississippi River from the Ohio River to the Gulf (after Assifi, 1966).

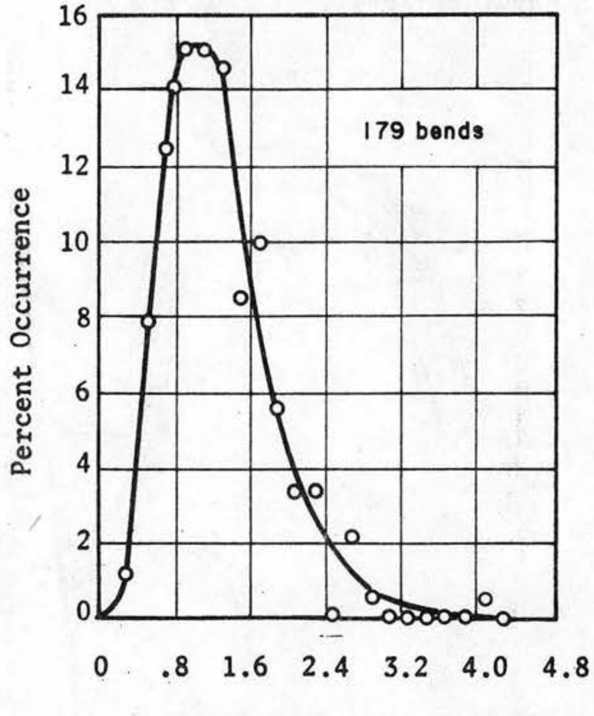

Fig. 5.3.3 Occurrence of bend deflection angles in the Lower Mississippi River from the Ohio River to the Gulf (after Assifi, 1966).

5.3.2 Types of bends

Two principal types of bends are deepened or entrenched bends and meandering surface bends. The first type includes those in which the river bends follow the curves of the valley so that each river bend includes a promontory of the parent plateau. The second type includes bends which are formed only by the river on a flat, alluvium covered valley floor, and where the slopes of the valley are not involved in the formation of such bends. This division of bends is correct and sufficiently definite with respect to external forms of the relief and the process of formation and development of bends. It is, however, incomplete from the standpoint of the work of the river and of the physical nature of this phenomenon. Both of the morphological types of bends can be put into one category--the category of freely meandering channel, i.e., meandering determined only by the interaction of the stream and the bed material. Such meandering, not disturbed by the influence of external factors, proceeds at an approximately equal rate along the length of the river.

Under natural conditions, there is often encountered a third type of bend. This bend occurs when the stream impinging on a practically noneroding parent bank forms a forced curve which is gradually transformed into a river bend of a more constricted shape.

In all cases the effect of the character (density) of the bank material is important and, to a certain degree, determines the radius of curvature of the channel. In a free bend the radius of curvature increases with the density of the material. The radius of curvature is smallest in a forced bend.

Both from the standpoint of the action of the stream and the interaction between the stream and the channel, as well as from the standpoint of the general laws of their formation, one can distinguish the following three types of bends of a natural river channel:

(1) *Free bends* - Both banks are composed of alluvial floodplain material which is usually quite mobile. The free bend corresponds to the common concept of a surface bend.

(2) *Limited bends* - The banks of the stream are composed of consolidated parent material which limits the lateral erosion by the stream. Limited bends are entrenched bends.

(3) *Forced bends* - The stream impinges onto an almost straight parent bank at a large angle (60° to 90°).

A typical feature of bends is a close relationship between the type of stream bend and the radius of curvature. *The forced bend has the smallest radius of curvature.* Next in size are the radii of free bends. *The limited bends have the greatest radii.* The average values of the ratios of the radii of curvature to the width of the stream at bankfull stage for the three types of bends are:

Free bends 4.5 to 5.0

Limited bends 7.0 to 8.0

Forced bends 2.5 to 3.0

A second characteristic feature of bends is the distribution of depths along the length of the bend. *In free bends and limited bends, the depth gradually increases and the maximum depth is found some distance below the apex of the bend. In the forced bend, the depth sharply increases at the beginning of the bend and then gradually diminishes.* The greatest depth is located in the middle third of the bend, where there appears to be a concentrated deep scour.

5.3.3 Transverse velocity distribution in bends

The theory of superelevation in open channel bends was presented in Chapter II. *This superelevation produces a transverse velocity distribution in channel bends.* The transverse velocities result from an imbalance of radial pressures on a particle of fluid traveling around the bend. In Fig. 5.3.5, a cross section through a typical bend is shown.

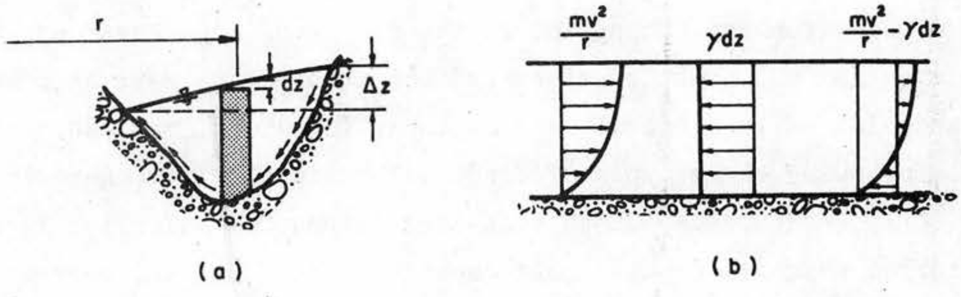

Fig. 5.3.4 Schematic representation of transverse currents in a channel bed.

The radial forces acting on the shaded control volume are the centrifugal force mv^2/r and the differential hydrostatic force γdz caused by the superelevation of the water surface dz. As shown in Fig. 5.3.4, the centrifugal force is greater near the surface where the fluid velocity v is greater and less at the bed where v is small. The differential hydrostatic force is uniform throughout the depth of the control volume. As shown in Fig. 5.3.4, the sum of the centrifugal and excess hydrostatic forces varies with depth and can cause a lateral velocity component. The magnitude of the transverse velocity is dependent on the radius of curvature and on the proximity of the banks. In the immediate vicinity of the banks, there can be no lateral velocity if the river is narrow and deep, and this bank constraint to the transverse velocity field is felt throughout the cross section.

The velocity distributions in natural stream bends are very complex. *The usual way to describe the velocity distribution in alluvial channels is by actual measurements.* In this way, accurate knowledge of the various velocity components in the cross section can be obtained.

In prismatic channels with rigid beds, it is possible to compute the velocity field in the bends. At any vertical in the bend, the variation of longitudinal velocity with respect to depth can be described by the von Karman velocity relation (Eq. 2.3.15).

$$\frac{v}{V_*} = \frac{2.303}{\kappa} \log\{30.2 \frac{xy}{k_s}\} \qquad 5.3.1$$

where

v = the velocity at depth y

V_* = the shear velocity

k_s = the diameter of the sediment grains that compose the bed

κ = the universal velocity coefficient

Extending this concept, if one can describe the longitudinal velocity distribution at several verticals in a cross section, the variation of the longitudinal velocity over the width of the stream is known.

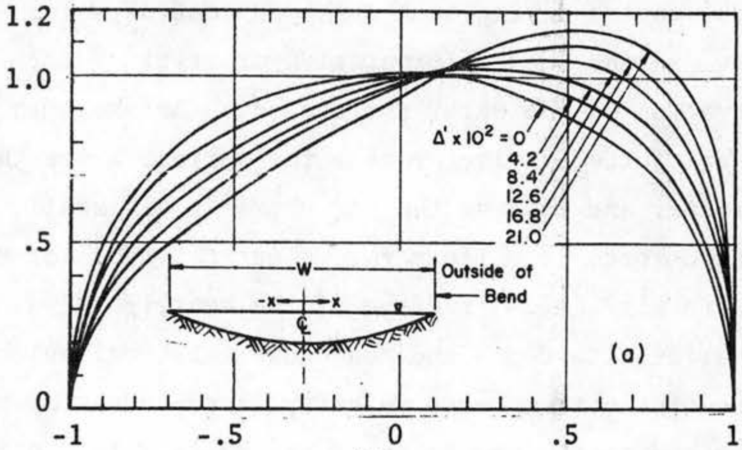

Fig. 5.3.5 Lateral distribution of velocity.

For a gentle bend of a parabolic cross section, Fig. 5.3.5 was developed. Fig. 5.3.5 shows the curves for velocities across the width of a prismatic channel for consecutive sections along a bend. In Fig. 5.3.5, V is the depth-averaged velocity in any vertical, and V_{max} is the maximum velocity in the straight channel. Define

$$\Delta' = 0.42\Delta \frac{y_{max}}{W} \frac{\sqrt{g}}{C} , \qquad 5.3.2$$

where Δ is the angle of the bend in degrees. The distribution of velocity in the straight reach is assumed to follow the form

$$\frac{V}{V_{max}} = \left(\frac{y}{y_{max}}\right)^{0.4} \qquad 5.3.3$$

The V values for sections within a bend are referenced to V_{max} in the straight reach. The depth across the width of the channel is assumed to vary as,

$$\frac{y}{y_{max}} = \left(1 - \frac{2x}{W}\right)^2 \qquad 5.3.4$$

Longitudinal velocities in natural river bends are similar to those shown in Fig. 5.3.5 but because the cross sections in river bends are not prismatic, the information in Fig. 5.3.5 cannot be readily used in rivers.

Several studies have been made of the transverse velocity field in the cross section of an open channel. The equation for transverse velocity developed by Rozovskii (1957) is

$$v_r = \frac{1}{\kappa^2} V \frac{y}{r} \left[F_1 (\eta) \frac{\sqrt{g}}{\kappa C} F_2 (\eta)\right] \qquad 5.3.5$$

in which

v_r = the radial velocity corresponding to a flow depth y

V = the average longitudinal velocity

C = the Chezy coefficient

η = the relative depth, y/y_o

κ = the von Karman coefficient

The functions $F_1(\eta)$ and $F_2(\eta)$ can be determined from Fig. 5.3.6

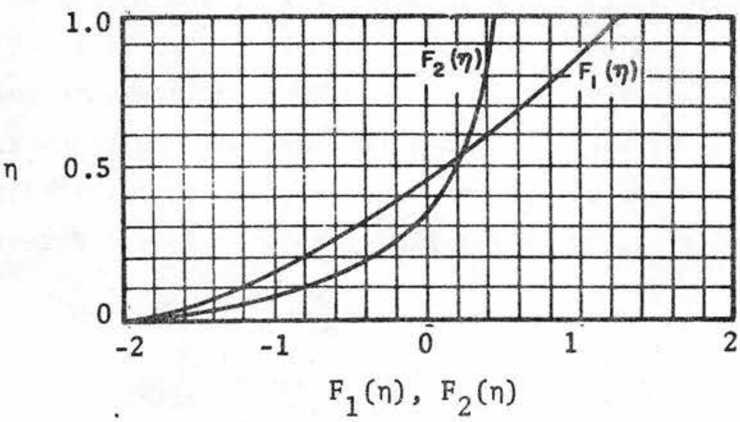

Fig. 5.3.6 Graph of functions $F_1(\eta)$ and $F_2(\eta)$.

A comparison of the predicted (Eq. 5.3.5) and observed transverse velocity distributions for a river bend is given in Fig. 5.3.7. For such sections fairly good results can be obtained. For the more irregular sections, the results are less impressive.

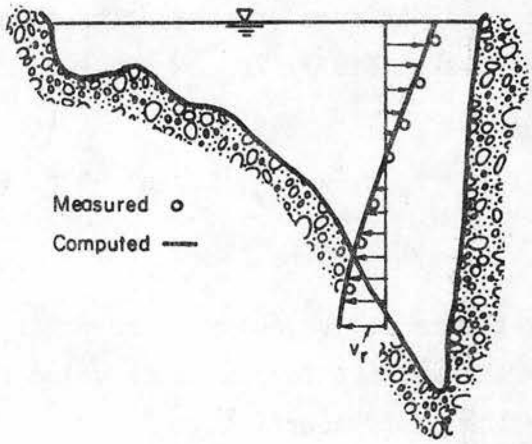

Fig. 5.3.7 Comparison of predicted and measured velocity distributions in a bend.

Another form of secondary circulation occurs in open channels. In channels with large width to depth ratios and approximately uniform depth, transverse flow cells occur, usually in pairs. One rotates clockwise and the other counterclockwise. Between alternate pairs of cells, the transverse surface flows come together and dive downward. This flow phenomena may accumulate debris, ice or other material floating on the surface into distinct parallel lines oriented in the direction of longitudinal flow.

5.4.0 ROUGHNESS CHARACTERISTICS OF ALLUVIAL RIVERS

The roughness of alluvial channels is variable and complex. Roughness is a function of such variables as channel geometry, channel irregularities, type of bed and bank material, response of bed material to flow at the bed-water interface resulting in dunes and bars, the rate of bed-material discharge in the channel, the characteristics of channel alignment and slope, the temperature of the water-sediment mixture flowing in the channel, the characteristics of wash load, the intensity of turbulence, and other factors.

5.4.1 Main channel

The resistance to flow in the main channel resulting from grain roughness and form roughness is discussed in detail in Chapter III.

5.4.2 Floodplain

The roughness characteristics on the floodplain are complicated by the presence of vegetation, natural and artificial irregularities, buildings, undefined direction of flow, varying slopes and other complexities. Resistance factors reflecting these effects must be selected largely on the basis of past experience with similar conditions. *In general, resistance to flow is large on the floodplains.* In some instances, conditions are further complicated by deposition of sediment and development of dunes and bars which affect resistance to flow and direction of flow.

5.4.3 Ice conditions

The presence of ice affects channel roughness and resistance to flow in various unique ways. When an ice cover occurs, the open channel is more nearly comparable to a closed conduit. There is an added shear stress developed between the flowing water and the ice cover. This surface shear is much larger than the normal shear stresses developed at the air-water interface. A study of ice cover by the U.S. Geological Survey has revealed that the ice-water interface is not always smooth. In many instances, the underside of the ice is deformed so that it resembles ripples or dunes observed on the bed of sandbed channels. This may cause overall resistance to flow in the channel to be further increased.

With total or partial ice cover, the drag of the ice retards flow, decreasing the average velocity and increasing the depth. Another serious effect is its influence on bank stability, in and near water structures such as docks, loading ramps, and ships. For example, the ice layer may freeze to bank stabilization materials, and when the ice breaks up, large quantities of rock and other material embedded in the ice may be floated downstream and subsequently thawed loose and dumped randomly leaving banks raw and unprotected.

5.5.0 LONGITUDINAL VELOCITIES OF ALLUVIAL RIVERS

The usual reference to velocity in natural streams is not to a velocity at a point but rather to a mean velocity for the channel.

5.5.1 Maximum velocities

A tabulation was made by the USGS of the largest measured values of velocity at a single point (not the average for the whole cross section). The maximum point velocity usually is on the order of 25 to 50 percent greater than the average velocity for the cross section. Out of 2950 measurements included in the sample, the median value was 4.11 fps, the mean 4.84 fps, and less than one percent of the total exceeded 13 fps. *One of the highest velocities ever measured by current meter by the USGS was 22 fps in a rock gorge of the Potomac River at Chain Bridge near Washington, D.C., during the flood of March, 1936.* Velocities up to 30 fps have occasionally been observed, but none have been recorded greater than this value.

5.5.2 Mean velocities

The mean velocity of river corresponds to the mean or average discharge of a stream. During the flood stages, the mean velocities in the river vary from about 6 to 10 fps. The mean velocity attained in large rivers is generally slightly larger than that in small ones. There are, of course, many local situations where, owing to the constrictions or rapids, velocities attain greater values. The figures cited above include a large majority of river channels in reaches that have no unusual features. Fig. 5.5.1 shows the variations of mean velocity along Yellowstone-Missouri-Mississippi River system. As discharge increases in the downstream river system, the velocity remains essentially constant.

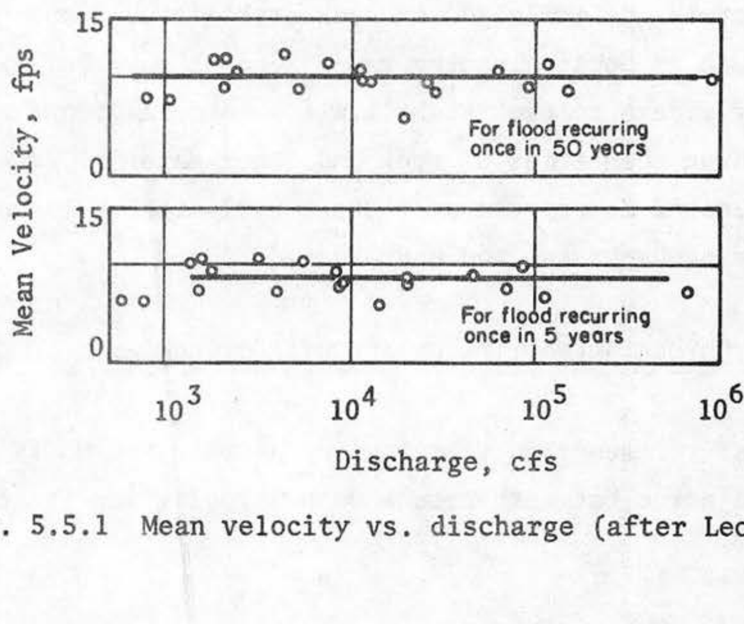

Fig. 5.5.1 Mean velocity vs. discharge (after Leopold et al., 1964).

5.5.3 Minimum velocities

The minimum velocities in alluvial rivers correspond to the average velocity of the river during the low flow. The minimum velocities range from zero to approximately 3 fps.

5.5.4 Velocity fluctuations

The instantaneous turbulent velocity in alluvial rivers can exceed the time average velocity by as much as 70 percent or more. When the turbulent fluctuating component of velocity is expressed as a ratio root-mean-square of the fluctuating component (or standard deviation) to mean average velocity, the ratio can attain a value of 30 to 40 percent. For the Mississippi River, Kalinske (1942) found that the maximum fluctuating component of velocity is about three times the standard deviation. A typical plot of fluctuating velocity against time for the Mississippi River is shown in Fig. 5.5.2. These data were reported by Tiffany (1950).

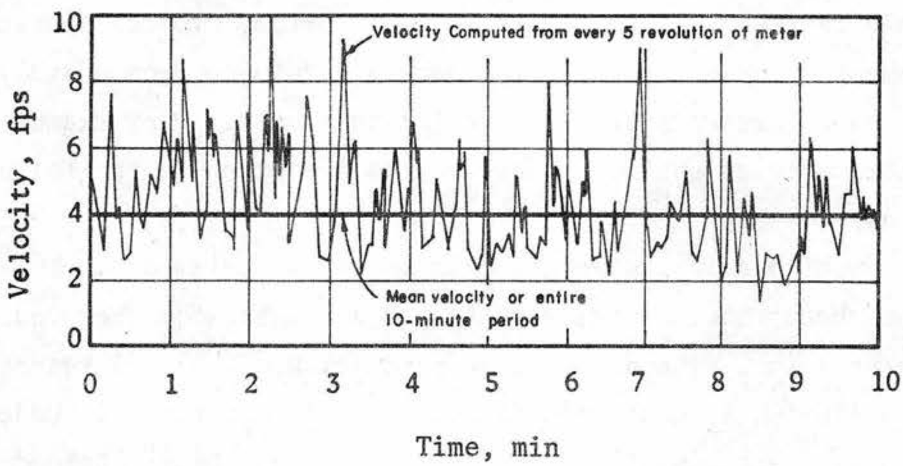

Fig. 5.5.2 Velocity fluctuations in Mississippi River at Vicksburg, Mississippi (Tiffany, 1950).

5.6.0 METHODS OF PREDICTING, CONTROLLING, AND ANALYZING THE CHARACTERISTICS OF RIVERS

5.6.1 Introduction

To predict the characteristics of a river, one has to separate the related variables into dependent and independent categories. In the case of rivers, the separation of variables becomes difficult because the relative independence of variables depends on the time scale from which one is viewing the phenomena. In geological time scales, the independent variables of a river are the geological and meteorological factors developing the drainage basin. In such time scales, all the river characteristics, as well as the discharge and sediment load, become dependent variables. Such a viewpoint is not helpful from the engineering aspects of river control. *In engineering time scales, the independent variables for a given river channel are the water discharge, the sediment load, the sediment size and particle shape and the antecedent channel conditions.*

If the channel is considered to be in equilibrium, considerable simplification of the number of variables and the relationships between such variables and the channel characteristics becomes possible. Channel equilibrium is sometimes a tenuous concept. However, *one can define the equilibrium condition of a river channel as indicating the existence of certain stationary values of average properties of the channel.* Most of the computational relationships presented in this chapter pertain to channels that are defined to be in equilibrium. In the hydraulic design of highways, these relationships can be developed from a study of the local characteristics of the river channel used. Many examples of equilibrium relations for river systems are given in the following sections.

A problem arises when the existing equilibrium of a river channel is disturbed. Qualitative results are available for the prediction of such a behavior. The qualitative prediction of channel response when its equilibrium is disturbed is discussed in Chapter IV. Under the state-of-the-art, quantitative results have not been obtained for rivers "out-of-equilibrium". However, the development of detailed mathematical models of the physical processes in rivers offers new hope for better river engineering answers to changing river forms.

5.6.2 <u>The geometry of pools and bendways</u>

In general, *meandering rivers assume a natural alignment consisting of bends and crossings. The depth of the channel increases along the concave bank of the bend, and decreases at the crossing.* The profile of the thalweg consists of successive deeps or pools in the bends, and shallows or shoals in the crossings.

The geometry of the pool is a function of water and sediment discharge, the hydraulic parameters of the stream, the geometry of the bend itself, the nature of the bank material and other less significant variables.

The characteristics of bends can be approximated by the use of the following approach of Rzhanitsyn (1960).

Let

v = average stream velocity

y_o = depth of flow

W = width of the stream

L = length of the bend

r_c = radius of curvature of the bend

ϕ = bend deflection angle, in radians

The equation for superelevation Δz in a bend (given by Eq. 2.6.7) can be simplified for the case where W/r_c is very small to

$$\Delta z = \frac{V^2 W}{g r_c} \tag{5.6.1}$$

The longitudinal drop Δz_o can be derived from the Chezy formula (Eq. 2.3.21).

$$\Delta z_o = \frac{V^2 L}{C^2 y_o} \tag{5.6.2}$$

The ratio K_o of the longitudinal drop to the superelevation is

$$K_o = \frac{g r_c L}{C^2 W y_o} \tag{5.6.3}$$

and is a function of the roughness of the channel and the geometry of the bend.

From the geometry of the bend

$$L = \phi r_c \tag{5.6.4}$$

By substituting this expression for L in Eq. 5.6.3 and then solving for r_c, we obtain

$$r_c = C\sqrt{\frac{K_o A}{g\phi}} = \frac{CA}{\sqrt{g}}\sqrt{\frac{K_o}{A\phi}} \qquad 5.6.5$$

As

$$\frac{CA}{\sqrt{g}} = \frac{Q}{V_*} \qquad 5.6.6$$

it follows that

$$r_c = \frac{Q}{V_*}\sqrt{\frac{K_o}{A\phi}} \qquad 5.6.7$$

The depth of flow in a pool is determined by a complex interaction of a number of factors, among them the stability of the river channel, the sediment concentration and the size of the river (stream order).

In Fig. 5.6.1, the relation between the relative maximum depth in a pool y_{max}/W and the radius of curvature of the bend r_c/W is presented. The information was obtained from various Russian rivers. This figure shows that y_{max}/W decreases with increasing r_c/W for relatively stable rivers, while the converse is true for less stable rivers. For values of r_c/W in excess of 12, the relative maximum depth approaches a constant value for a given stream.

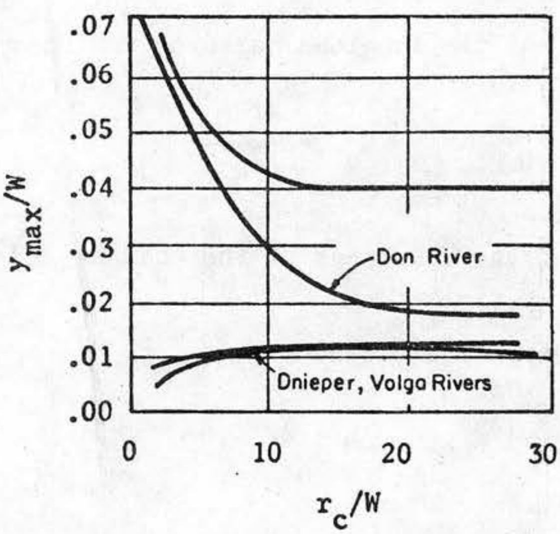

Fig. 5.6.1 The maximum depth in the pool as a function of the bend radius (after Rzhanitsyn, 1960).

For streams of the same order, the mean annual sediment concentration is an important factor in determining the maximum depth. Fig. 5.6.2 shows the variation of y_{max}/W with sediment concentration for XII to XIV order streams and r_c/W as a third variable.

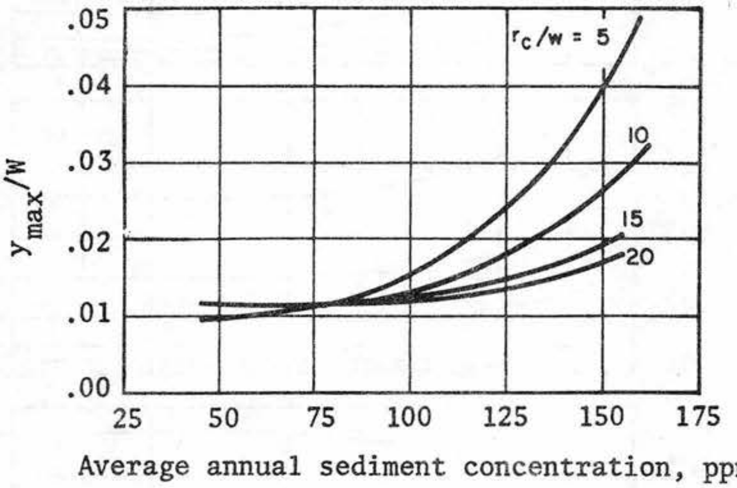

Fig. 5.6.2 The maximum depth in the pool as a function of sediment transport (after Rzhanitsyn, 1960).

In Fig. 5.6.3, the general pattern of change of y_{max}/W with r_c/W is shown for rivers of different size and degree of stability. The stability of a river can be represented by the index of stability χ defined as

$$\chi = \frac{d_{50}W}{y_o^2 S} \qquad 5.6.8$$

When χ is large the river is wide and shallow and relatively unstable. When χ is small the river is narrow and deep and relatively stable.

The curves in Fig. 5.6.3 have been obtained from data taken from rivers with low mean annual sediment concentrations, and are applicable for concentrations up to 90 ppm. For higher concentrations, use can be made of Fig. 5.6.2 extended to cover a wider range of stream sizes (From X to XIV). Figures 5.6.1, 5.6.2, and 5.6.3 were developed to determine the maximum depth in the pool. Care must be used in interpreting variations in the other parameters from these figures.

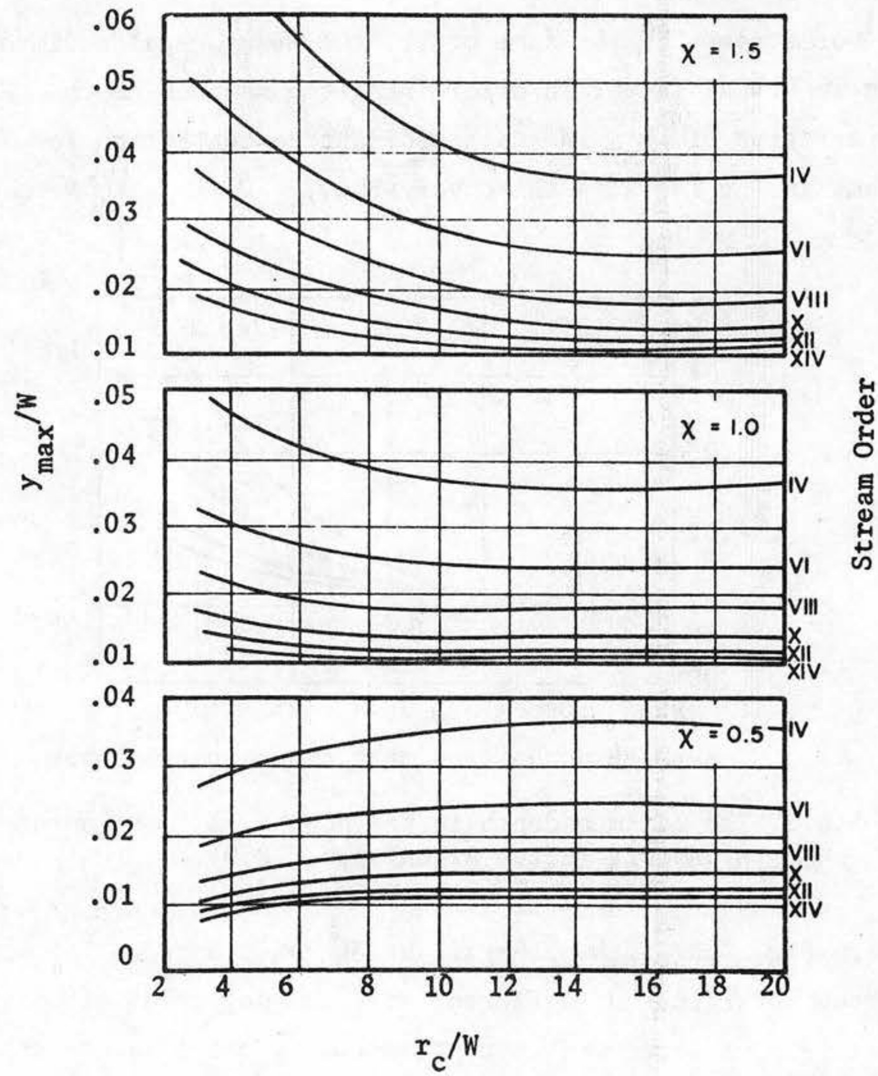

Fig. 5.6.3 Variation in pool depth with bend radius, river stability and stream order (after Rzhanitsyn, 1960).

As mentioned earlier, in a bend there is a transverse water surface slope and transverse currents. These phenomena are greatest near the end of a bend. Upon emerging into the straight reach below the curve, the induced transverse currents slowly die out. The transverse slope is reduced and at some distance from the end of the curve the stream begins to move normally without the cross currents induced by the bend. This section is the end of the pool.

The length of pool depression is closely related to the instability of the river channel and the sediment transport: the greater instability and sediment discharge, the shorter the length of the pool depression.

The relative length of a pool depression can be plotted as a function of the relative depth and the relative radius, as shown in Fig. 5.6.4. The values in this figure correspond to a stability factor $\chi = 1.5$. For other indices of stability, the length given in Fig. 5.6.4 must be multiplied by k_χ the correction coefficient for stability given in Fig. 5.6.5. Also, a correction for mean annual sediment concentration k_c given in Fig. 5.6.6 is required.

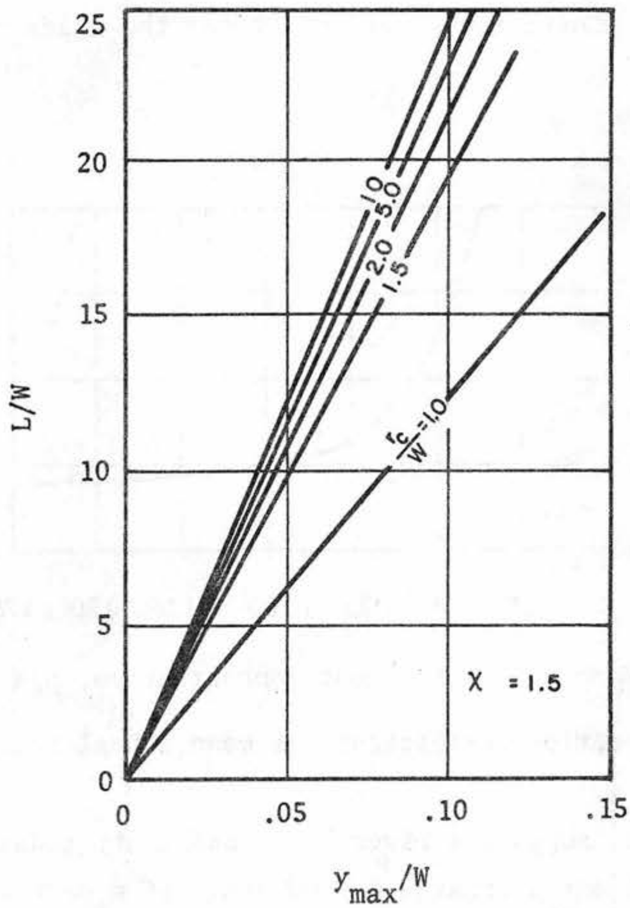

Fig. 5.6.4 Length of pools in meandering rivers (after Rzhanitsyn, 1960).

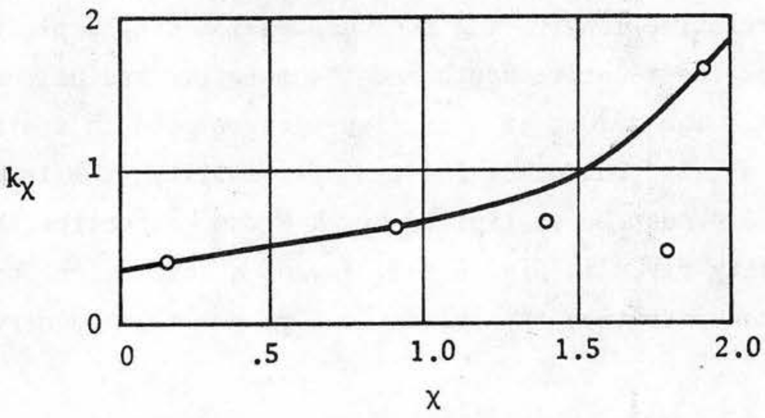

Fig. 5.6.5 Correction coefficient for the index of stability.

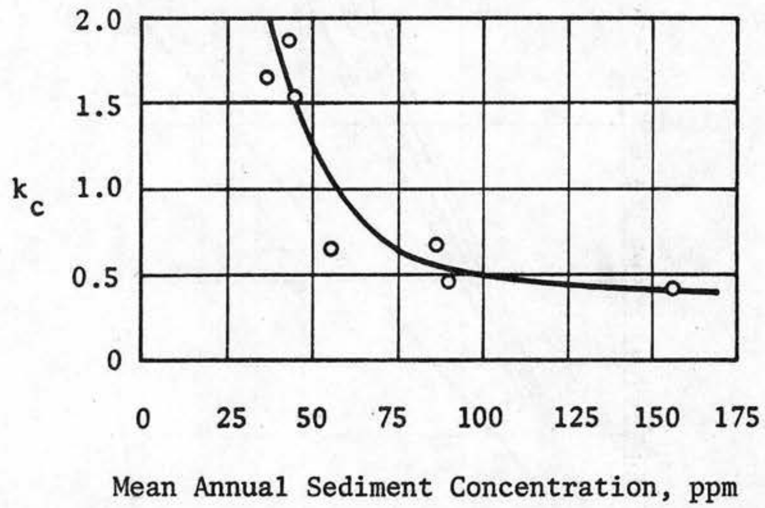

Fig. 5.6.6 Correction coefficient for mean annual sediment concentration.

For example, suppose a river has a stability index $\chi = 0.5$ and mean annual sediment discharge of 100 ppm. If a particular bend has $y_{max}/W = 0.03$ and $r_c/W = 4$, then according to Fig. 5.6.4, L/W = 7. The correction factor for stability is given in Fig. 5.6.5 as $k_\chi = 0.5$ and the correction factor for sediment concentration is given in Fig. 5.6.6 as $k_c = 0.5$. Therefore correct L/W for the bend is

$$\frac{L}{W} = 7k_{\chi}k_{c} \tag{5.6.9}$$

or

$$\frac{L}{W} = 7(.5)(.5) = 1.8$$

The *shape of the longitudinal profile* of a river channel bed in a bend is a function of the hydraulic characteristics of the stream and the bed material, the size of the stream and the mean annual sediment concentration. In general, the longitudinal profile of the pool depression bed becomes steeper in the first one-third of the pool in rivers carrying large sediment concentrations. Also, the profile then rises more rapidly to the next crossing. Fig. 5.6.7 shows how the profiles of rivers of different sizes compare, all other pertinent variables being held constant. The variables z, z_{max}, ℓ and ℓ_{max} are defined in Fig. 5.6.8. It is concluded from the analysis of this figure that stream size is not an important factor in shaping the profile of the bend.

It is useful to compare the shapes of longitudinal profiles for rivers that differ in size but are similar with respect to their bed material and mean annual concentration. The longitudinal profiles of several such rivers are shown in Fig. 5.6.7.

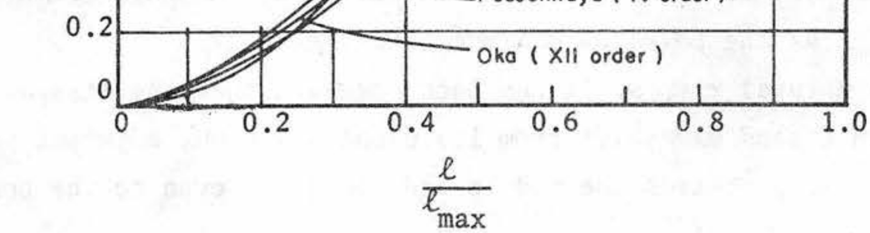

Fig. 5.6.7 Longitudinal profiles of pool depressions for rivers with similar stabilities and similar average annual sediment concentrations (after Rzhanitsyn, 1960).

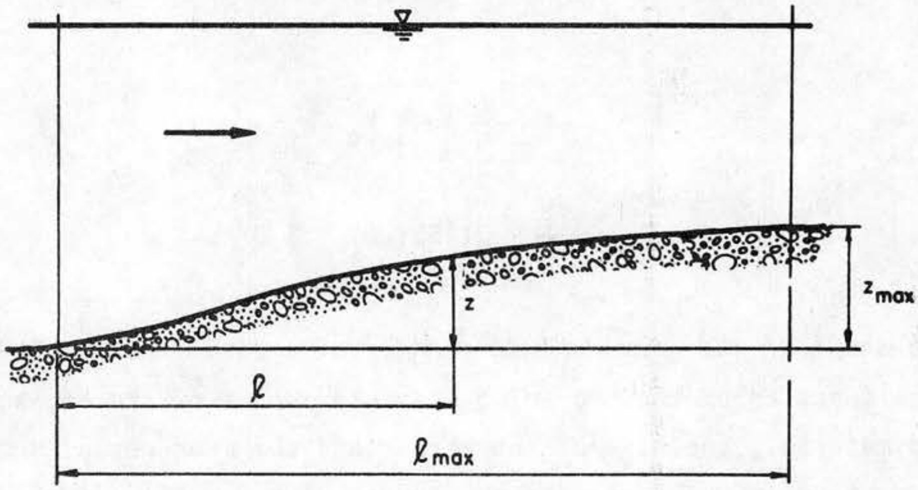

Fig. 5.6.8 Definitions of variables describing the longitudinal profile in a pool.

The analysis of these data shows that in relative terms, the shape of the longitudinal profiles of pool depressions do not depend on the size of the stream. Their shape is, therefore, similar for rivers of the same type.

5.6.3 Variation of depth of flow

To describe a river channel, it is necessary to know the maximum depth of the pool, the minimum depth in the crossing, and the position of the line of greatest depth in the channel. *This line is called the thalweg.* The greatest depth in the stream is located below the section of greatest curvature; the smallest depth is approximately the same distance below the point of inflection in the crossing. More precisely, *the deepest part of the pool and the shallowest part of the crossing are downstream of the point of greatest and least curvature by approximately a fourth of the length of the pool plus the crossing. This relation holds in the majority of cases where stream bends are of the free and limited types. For the forced bend, the greatest depth of the pool lies at the point of maximum constriction.*

In natural rivers, it has been observed that the line of greatest depth in a bend may shift from its usual position, adjacent to the concave bank, towards the middle and sometimes even to the convex bank. Therefore, *a designer should place all footings in a river channel at an elevation anticipating that the thalweg has freedom to select any position in the cross section.*

The position of the line of greatest depth is affected most by the variability in the flow and by bank conditions. A change in the flow changes the channel forming processes of the stream. With each rise in stage, changes occur in the characteristics of flow in the channel. For example, the flow lines are straightened. The appearance of flow on the floodplain often leads to basic changes in the channel forming activity. The duration of the various phases of flooding and, therefore, the duration of stages and of phases of the channel-forming process are of great importance. The discharge, channel geometry, and channel conditions all combine to determine general trends in the further development of the channel.

5.6.4 Straight reaches of a river channel

The cross section of a channel in a straight reach of the river may assume a parabolic shape in many rivers. This shape is relatively stable and is due primarily to the interaction of the longitudinal flow with the bed material. Under natural conditions, a number of factors affect the shape of the straight river channel. With changes in discharge, the velocity structure and the depth of the stream change. This leads to the intensification of the process of development of channel shape. For a given bed material, the velocity structure of the stream and its depth are associated with a definite shape of channel.

With continually changing hydraulic conditions, which are typical for natural streams, channel shapes develop. To these shapes, we attempt to relate some constant, equivalent, channel-forming discharge. During periods *when the flow is less than dominant, deposition processes predominate in the channel. Conversely, when the discharge is higher than dominant, erosional processes usually predominate.*

In streams of different size, the parabolic shape of the channel is common, but its geometric ratios are changed. For example, larger rivers have larger width to depth ratios.

The geologic structure and composition of the material of the riverbed are important in the development of channel shape. These affect the relative channel dimensions. In general, a channel in fine material has smaller width to depth ratio than one formed of coarser particles.

The sediment discharge affects the channel-forming action of the stream. With riverbeds composed of material with a small fall diameter the sediment load is greater, the channel process more intense, and the width to depth ratio is greater.

The process of river channel formation in different materials is determined principally by the effect of two related factors. These are size of particles and their cohesion, and the sediment load. The change in width to depth ratio of the river channel depends on whether the influence of one or the other of these factors predominate. The width to depth ratio is also influenced by size of the stream (stream order). This effect is shown by Fig. 5.6.9. The value of the relative depth y_o/W decreases and approaches some relatively small stable value for streams of high order. This regularly is observed in both crossings and pools. Therefore, with a decrease in the size of the river, the natural stream develops an increasingly deeper channel.

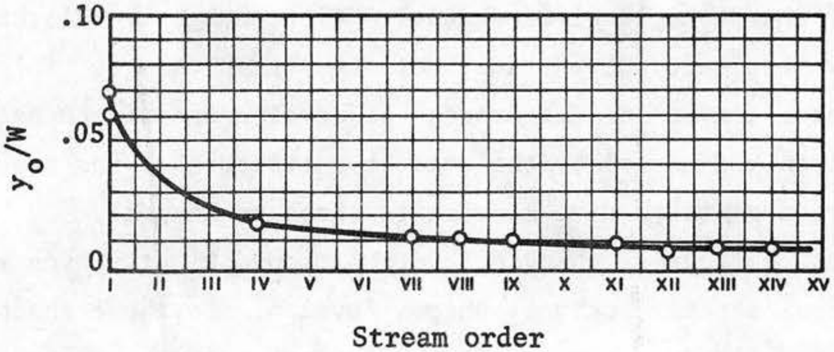

Fig. 5.6.9 Relation of relative depth of natural streams to their stream order (after Rzhanitsyn, 1960).

5.7.0 QUANTITATIVE PREDICTION OF CHANNEL RESPONSE TO CHANGE

The necessity for quantitative prediction of river channel response is increasing. The accuracy of such a prediction depends on the quality of the data. There are generally two ways of predicting response. One is the mathematical model and the other is the physical model. *Mathematical models* utilize a number of mathematical equations governing the motion of flow in the channel. In general, the equation of motion and continuity for sediment laden water, the continuity equation for sediment, and the sediment-transport equation are used to study a transient phenomena in alluvial channels. These equations are recognized as powerful tools for the study of unsteady flow problems. By solving these equations using numerical methods and digital computers, the response of the river system to various natural and man-made activities on channel improvement can be simulated by simply applying the mathematical model using different boundary conditions. Regardless of the potential of the mathematical models, up to date they have been restricted to the study of the response problems using one-dimensional approximations.

For complex channel characteristics, it is very difficult to accurately formulate mathematically what happens in a river. Studies of channel response to development are then usually made by using a *physical model*. Some aspects of physical modeling have been presented in Chapter III. To achieve similar behavior in the model and the prototype, the "relative importance of the governing parameters" must be the same in the model and prototype model. This often leads to distortion and scale effects and results in experimental uncertainties. Furthermore, the construction, operation, and modification of physical models are expensive, time consuming and laborious, especially when long-term response is investigated.

Adoption of a particular method for estimating river response depends on quality and availability of data as well as the engineer's experience. More detail on the physical and mathematical modeling can be found from works by Gessler (1971) and Chen (1973).

5.7.1 River response to confinement by dikes

The response of rivers to varying degrees of confinement is an interesting and challenging problem. The basic types of rivers, their bed configurations, the response of bed configuration to changing stage, the types and location of sandbars, the characteristics of the bed and bank materials, channel geometry, sediment transport, and historic characteristics of rivers with their variations and trends should be studied and understood. With this fundamental information, the potential changes which may result from various degrees of river development can be investigated. This requires consideration of the effect of development on channel geometry, flow characteristics, sediment discharge, aggradation, degradation, river stage, channel form, channel stability, bed and bank materials, possible changes in river alignment, and other important factors. A backwater analysis has been used to estimate the water surface profiles for rivers being confined by embankments. The increase in stage as determined from the analysis is usually based on: (1) a given design flood discharge; (2) given embankment setback distances; (3) Manning's n values based on field measurements, and (4) verification of computational procedures and assumptions by using historical flood levels and verifying them by computations. Using these considerations a realistic appraisal can be made of river response, embankment setback distance, embankment alignment, height of embankments, and other factors.

5.7.2 Observed channel changes

The plan and profile of sandbed rivers are subject to changes of varying magnitude during an annual cycle. These changes are even more noticeable when the profile and banklines are compared over several years.

The bank stability is greatly affected by the type of river, the characteristics of the sediment load, and the riverbed response to changing hydraulic conditions. The meandering river changes its position relatively slowly and it maintains its sinuous pattern. Hence *the future behavior and geometry of a meandering river are easier to predict.*

The plan and profile of braided rivers may change continuously. Erosion and deposition along the bankline and within the river channel *are much less predictable.* Both river banks may be attacked simultaneously and the pattern of alternate bars, braided channels, and middle bars and islands, experiences large and random changes with time and river stage. With the braided river and its unpredictable geometry, the embankments located adjacent to it should be set back further from the river to provide safety from changes in alignment.

Unstable and stable reaches of rivers tend to remain so except for catastrophic events such as major earthquakes. These, accompanied by subsidence or upheaval and very large floods, may cause major changes in both stable and unstable reaches. Such events may even completely change the course of a river.

Because of limited documentation of bank line migration and large potential changes in river alignment during flood events, river banklines should be defined and compared annually by aerial photography. With this information, the river changes could be studied in greater detail and used with hydrologic, hydraulic and soils data to provide knowledge of interrelations between the projected aim and the river. This kind of study would also provide a history of river changes which would help to predict river response to the development of water resources.

5.8.0 SUMMARY

At the present time, the quantitative analysis of river response to natural or man-made activities is difficult to do. No clear straightforward methods of analysis have been developed. Often, the value of an analysis is almost totally dependent on the experience of the engineer. Hopefully, in the future, the experience of the engineers can be enhanced with rapid methods of analysis that will produce accurate quantitative predictions.

In this chapter, we have tried to illustrate that the geometry of natural river systems can be described, at least in the form of graphical plots of the relations between a few key variables. Through samples of river data taken mostly from Russian literature, the basic relations have been depicted. With these types of relations and those given in Chapter IV, the response to changing activities in the river system can be estimated.

The basic relations for one river system may differ from those of another. Therefore, at least a sample of data is needed from a river system in order to assess how that river system is different or similar to other river systems for which good data is available. With this knowledge, one is in a position to assess the trends of future responses even if he cannot compute the precise amount of change.

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Chapter VI

RIVER STABILIZATION, BANK PROTECTION AND SCOUR

6.1.0 INTRODUCTION

From a study of the chapters on river mechanics and geomorphology, it should be clear that both short-term and long-term changes can be expected on river systems as a result of natural and man-made influences. Recommended structures and design methods for river control are presented in this chapter. The integrated and interactive effects of these structures with the river are discussed in Chapter VIII.

Numerous types of river control and bank stabilization devices have evolved through past experience. Concrete, brick, willow and asphalt mattresses, sacked concrete and sand, riprap, grouted slope protection, sheet piles, timber piles, steel jack and brush jetties, angled and sloped rock-filled, earth-filled, and timber dikes, automobile bodies, and concrete tetrahedrons have all been used in the practice of training rivers and stabilizing river banks. *An extensive treatise on the subject of bank and shore protection was prepared by the California Division of Highways (1970).* A large number of publications on river training and stabilization have been prepared by the Corps of Engineers and the U.S. Bureau of Reclamation. Many more publications on the subject exist in the open literature. It is not intended that an exhaustive coverage of the various types of river control structures and methods of design be made in this manual; rather, the purpose of this manual is to recommend methods and devices which provide useful alternatives to the highway engineer for the majority of circumstances which are likely to be encountered in highway practice.

Generally, changes to river alignment, river cross section, training, and bank stabilization of rivers associated with highway projects are confined to short reaches of the river. While the methods for river training and bank stabilization discussed herein are applicable to short and long reaches of the river, they are not panacea to all problems associated with highway encroachments on rivers. Handbook analyses and designs usually lead to poor solutions of specific problems. Also, solution to a particular problem may generate problems elsewhere in the river system.

6.2.0 CHANNEL IMPROVEMENT

In some circumstances it could be advantageous to change the river channel alignment because of highway encroachments. *When a river crossing site is so constrained by non-hydraulic factors that consideration to alternative sites is not possible, the engineer must attempt to improve the local situation to meet specific needs.* Also, the engineer may be forced to make channel improvements in order to *maintain and protect existing highway structures* in or adjacent to the river.

Suppose a meandering river is to be crossed with a highway, as shown in Fig. 6.2.1a. Assume that the alignment is fixed by constraints in the acquisition of the right-of-way.

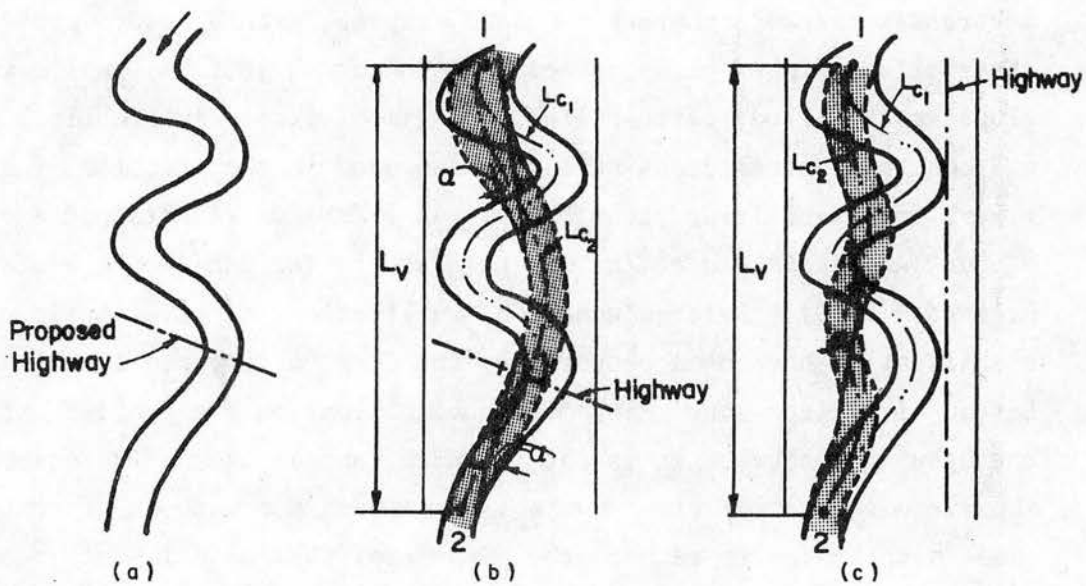


Fig. 6.2.1 Encroachment on a meandering river.

To create better flow alignment with the bridge, consideration is given to channel improvement as shown in Fig. 6.2.1b. Similarly, consideration for improvement to the channel would also be advisable for a hypothetical lateral encroachment of a highway as depicted in Fig. 6.2.1c. In either case, the designer's questions are how to realign the channel, and what criteria to use to establish the cross-sectional dimensions.

In realigning a river channel, the channel may be made straight without curves, or may include one or more curves. If curves are included, the radii of curvature, the number of bends, the limits of rechannelization (hence the length or slope of the channel) and the cross-sectional area are decisions which have to be made by the designer. Different rivers have different characteristics and historical background with regard to channel migration, discharge, stage, geometry and sediment transport, and as indicated in the previous chapters, it is important for the designer to understand and appreciate river hydraulics and geomorphology when making decisions concerning channel improvement. It is difficult to state generalized criteria for channel improvement applicable to any river. Knowledge about river systems has not yet advanced to such a state as to make this possible. Nevertheless, it is important to provide some principles and guidelines for the design engineer.

As the general rule, the radii of bends should be made about equal to the mean radii of bends, r_c, in extended reaches of the river. The angle α shown in Fig. 6.2.1 between a line drawn tangent to the inside of two successive bends and the bank line in the crossing should be approximately 20 degrees. This enables a sufficient crossing length for the thalweg to swing from one side of the channel to the other. Generally, it is necessary to stabilize the outside banks of the curves in order to hold the new alignment and depending upon crossing length, some amount of maintenance may be necessary to remove sandbars after large floods so that the channel does not develop new meander patterns in the crossings during normal flows.

The sinuosity and channel bed slope are related in the following way. The bed elevations at the ends of the reach being rechannelized, (designated 1 and 2, in Fig. 6.2.1) are established by existing conditions. Hence, the total drop in bed elevation for the new channels (subscript 2) and the old channels (subscript 1) are the same.

$$\Delta z_1 = \Delta z_2 = \Delta z \qquad 6.2.1$$

The length of channel measured along the thalweg is labeled L_c. Thus, the mean slope of the channel bed before rechannelization is

$$S_1 = \frac{\Delta z}{L_{c_1}} \tag{6.2.2}$$

and after rechannelization is

$$S_2 = \frac{\Delta z}{L_{c_2}} \tag{6.2.3}$$

Sinuosity is defined by the ratio of the length of channel to length of the valley, or

$$P = \frac{L_c}{L_v} \geq 1 \tag{6.2.4}$$

Clearly,

$$P_1 = \frac{L_{c_1}}{L_{v_1}} \tag{6.2.5}$$

$$P_2 = \frac{L_{c_2}}{L_{v_2}} \tag{6.2.6}$$

but

$$L_{v_1} = L_{v_2} = L_v \tag{6.2.7}$$

and

$$\frac{\Delta z_1}{L_{v_1}} = \frac{\Delta z_2}{L_{v_2}} = \frac{\Delta z}{L_v} \tag{6.2.8}$$

Thus,

$$P_1 S_1 = \frac{L_{c_1}}{L_v} \cdot \frac{\Delta z}{L_{c_1}} = \frac{L_{c_2}}{L_v} \cdot \frac{\Delta z}{L_{c_2}} = P_2 S_2 \tag{6.2.9}$$

The new channel slope and channel sinuosity are inversely related. If $P_2 < P_1$ then $S_2 > S_1$. The new channel alignment, hence P_2, can be chosen by the designer with due consideration given to the radii of curvature, deflection angles and tangent lengths between reversing curves. As indicated before, consideration should also be given to prevailing average conditions in the extended reach. The new slope S_2 can be calculated from Eq. 6.2.9, and the relationship (from Eq. 4.4.4)

$$S_2 Q^{1/4} \leq 0.0017 \qquad 6.2.10$$

should be satisfied. If S_1 is of such magnitude that Eq. 6.2.10 cannot be satisfied with still larger S_2, the possibility of the river changing to a braided channel because of steeper slope should be carefully evaluated. With steeper slope, there could be an increase in sediment transport which could cause degradation and the effect would be extended both upstream and downstream of the rechannelized reach. The meander patterns could change. Considerable bank protection might be necessary to contain lateral migration which is characteristic of a braided channel, and if the slope is too steep, head cuts could develop which migrate upstream with attendant effects on the plan geometry of the channel. Even when changes in slope are not very large, a short-term adjustment of the average river slope occurs, consistent with the sediment transport rate, flow velocities and roughnesses, beyond the upstream and downstream limits of channel improvement. For small changes in slope, the proportionality (Eq. 4.4.3), $QS \sim Q_s D_{50}$ tends toward equilibrium by slight increases in bed material size D_{50} and adjustment in the sediment transport rate Q_s.

A small increase in the new channel width could be considered which tends to maintain the same stream power in the old and new channels. That is,

$$(\tau_o V)_1 = (\tau_o V)_2 . \qquad 6.2.11$$

With substitution of $\tau_o = \gamma RS$, $V = Q/A$ and $R = A/P \simeq A/W$, Eq. 6.2.11 leads to

$$W_2 = \frac{S_2}{S_1} W_1 \qquad 6.2.12$$

The increase in width should be limited to about 10 to 15 percent. Wider channels would be ineffective. Deposition would occur along one bank and the effort of extra excavation would be wasted. Furthermore, bar formation would be encouraged, with resultant tendencies for changes in the meander pattern leading to greater maintenance costs of bank stabilization and removal of the bars to hold the desired river alignment.

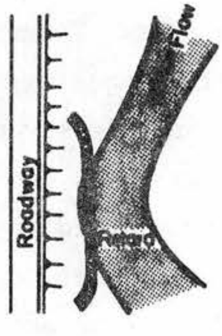
The depth of flow in the channel is dependent on discharge, effective channel width, sediment transport rate (because it affects bed form and channel roughness) and channel slope. Methods for determining flow depth are given in Chapter III.

The foregoing discussion pertains to alluvial channels with fine-sized bed materials (sands and silts). For streams with gravel and cobble beds, the concern is to provide adequate channel cross-sectional dimensions to convey flood flows. If the realigned channels are made too steep, there is an increased stream power with a consequent increase in transport rate of the bed material. The deposition of material in the downstream reaches tends to form gravel bars and encourages changes in the plan form of the channel. Short-term changes in channel slope can be expected until equilibrium is reestablished over extended reaches both upstream and downstream of the rechannelized reach. Bank stabilization may be necessary to prevent lateral migration and periodic removal of gravel bars may also be necessary.

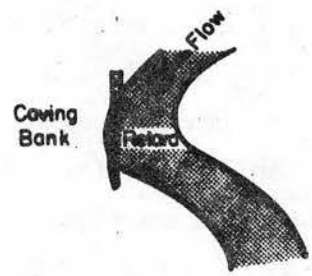
6.3.0 RIVER TRAINING AND STABILIZATION

Various devices and structures have been developed to control river flow along a preselected path and to stabilize the banks. Most have been developed through trial and error applications, aided in some instances by hydraulic model studies. *Rock riprap is probably the most widely used material* to stabilize river banks and protect the side slopes of embankments. Because of its wide use and importance in highway practice a separate section (Section 6.4.0) is devoted to rock riprap. Other materials useful in highway practice are discussed in this section.

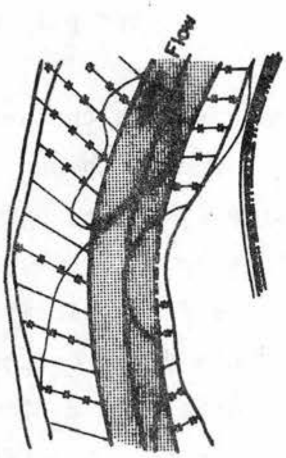
Dikes, retards, and jetties are devices used to guide the river flow and to protect the banks. Their use is illustrated in Fig. 6.3.1. In all instances, the intent of all the devices shown in Fig. 6.3.1 is to cause resistance or obstruction to flow along the channel bank, thereby creating lower velocities to stop bank erosion, contain the thalweg of the stream to the center portions of the channel, and if possible cause deposition of sediment along the bank where erosion previously occurred.



(a) Retards to protect highway embankment



(b) Retard to prevent further bank caving



(c) Jetties to train the flow and protect the bank

(d) Dikes to train the flow and protect the bank

Fig. 6.3.1 Retards, jetties and dikes to protect embankments and train channel flow.

6.3.1 <u>Dikes</u>

In general, dikes extend outward from the bank into the channel at right angles or angled thereto, depending upon the circumstances and particular success achieved in past experiences. Along straight reaches, dikes should be perpendicular to the bank. Along sharp curves the dikes should be angled slightly downstream so as to deflect the flow toward the center of the channel. Some of the dikes are terminated with extensions parallel to the flow, forming L or T shapes, and are correspondingly referred as L or T heads.

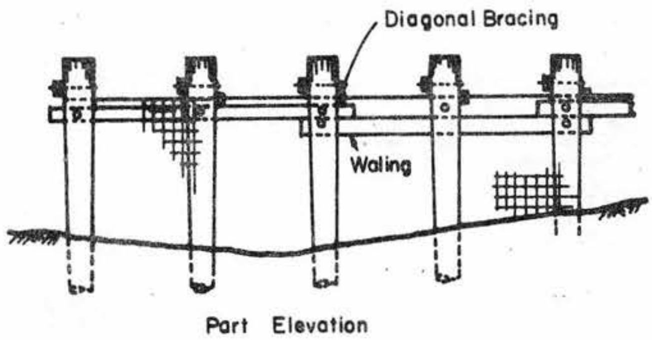
There are two principle types of dikes, permeable and impermeable. Permeable dikes are those which permit flow through the dike but at reduced velocities, thereby preventing further erosion of the banks and causing deposition of suspended sediment from the flow.

Timber pile dikes - Timber pile dikes (also retards) may consist of closely-spaced single, double, or multiple rows. There are a number of variations to this scheme. For example, wire fence may be used in conjunction with pile dikes to collect debris and thereby cause effective reduction of velocity. Double rows of timber piles can be placed together to form timber cribs, and rocks may be used to fill the space between the piles. Timber pile dikes are vulnerable to failure through scour. The piles can be driven to a large depth to achieve safety from scour or the base of the piles can be protected from scour with dumped rock in sufficient quantities to form a combination permeable and impermeable dike. The various forms of timber pile dikes are illustrated in Fig. 6.3.2.

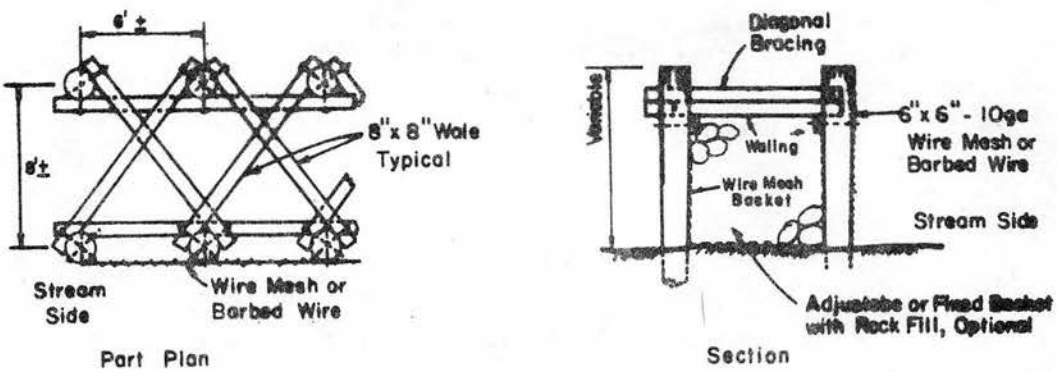
The arrangement of timber piles depends upon the velocity of flow, quantity of suspended sediment transport, and depth and width of river. If the velocity of flow is large, timber pile dikes are not likely to be very effective. Stabilization of the bank by other methods should be considered. On the other hand, in moderate flow velocities with high concentrations of suspended sediments, these dikes can be quite effective. Deposition of suspended sediments in the pile dike field is a necessary consequence of reduced velocities. If there is not sufficient concentration of suspended sediment in the flow, or the velocities in the dike fields are too large for deposition, the permeable timber pile dikes will be only partially effective in training the river and protecting the bends.

The length of each dike depends on channel width, position relative to other dikes, flow depth and available pile lengths. Generally, pile dikes are not used in large rivers where depths are great, although timber pile dikes have been used in the Columbia River. On the other hand, banks of wide shallow rivers can be protected with dikes. The spacing between dikes varies from 3 to 20 times the length of the upstream dike, with closer spacing favored for best results.

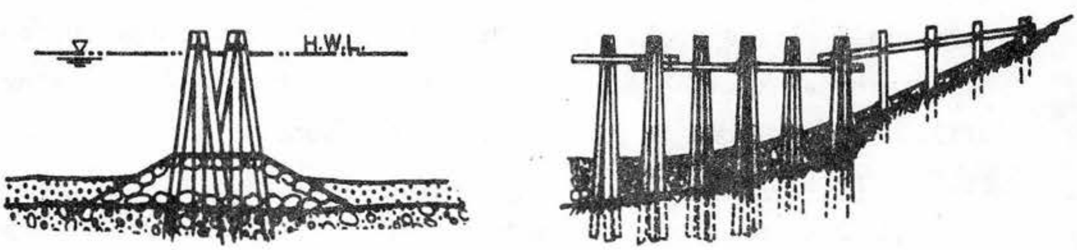
Stone-fill dikes - Stone-fill dikes are classed as impermeable dikes and do not depend on deposition of sediment between dikes nearly as much as permeable dikes. The principal function is to deflect the flow away from the bank and the dikes must be long enough to accomplish this purpose. The dikes may be angled downstream, angled upstream, or constructed normal to the bank. Variations such as a sloping dike, with



(a) Single row timber pile with wire fence



(b) Double row timber piles with rocks and wire fence



(c) Pile clusters

Fig. 6.3.2 Timber pile dikes (retards would be similar).

declining top elevation away from the bank, L or T head dikes, and curved dikes have been used. Stone-fill dikes are illustrated in Fig. 6.3.3.

The spacing between dikes may vary from three or four dike lengths to 10 or 12 dike lengths depending upon velocity and depth. Short dikes with long spacing are generally not useful for bank protection unless jacks or riprap are used to protect the bank between them.

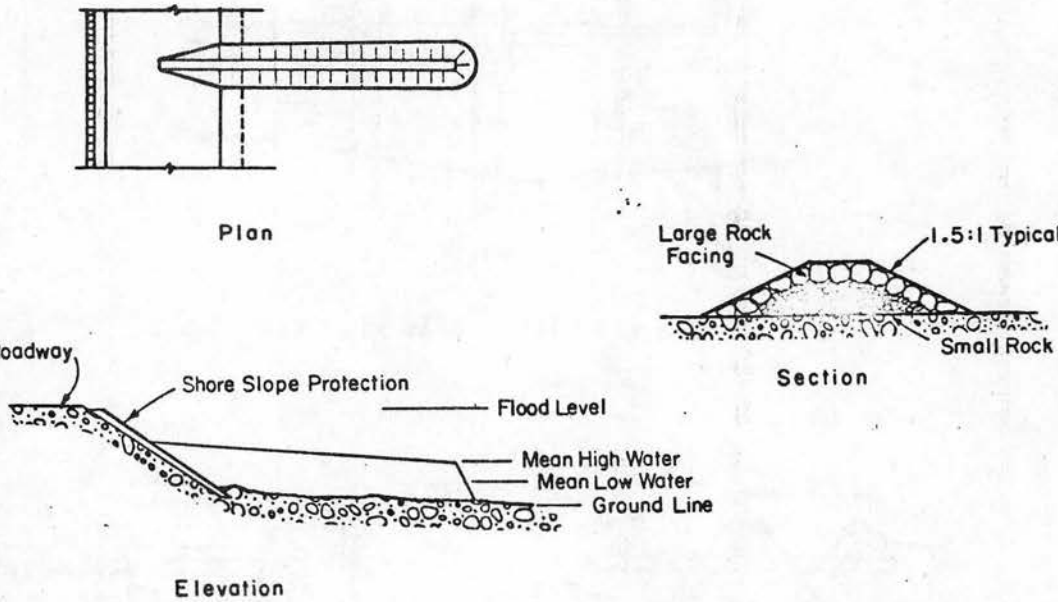


Fig. 6.3.3 Typical stone fill dike.

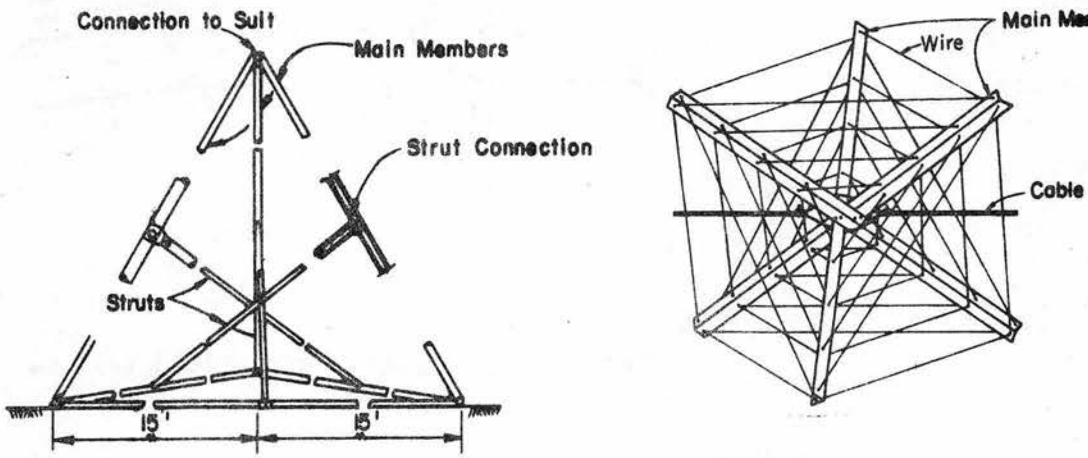
The ends of the dikes are subjected to local scour and appropriate allowance should be made for loss of dike material into the scour hole. The size of rock to be used for the dike depends on availability of material. Large rocks are generally used to cover the surface, while the internal section may be constituted with smaller rocks or earth-fill. Side slopes of 1.5:1 and 2:1 are common.

6.3.2 Retards

Retards are permeable devices placed parallel to embankments and river banks to decrease the stream velocities and prevent erosion.

Timber pile retards - The design of timber pile retards is essentially the same as timber pile dikes discussed in the previous section, and shown in Fig. 6.3.1. They may be used in combination with bank protection works such as riprap. The retard then serves to reduce the velocities sufficiently so that the riprap behind the retard is stable.

Steel jacks - These devices are basic triangular frames tied together to form a stable unit. The resulting framework is called a tetrahedron. The tetrahedrons are placed parallel to the embankment and cabled together with the ends of the cables anchored to the bank. Wire fencing may be placed along the row of tetrahedrons. In order to function well, there must be considerable debris in the stream to collect on the fence and the suspended sediment concentration must be large so that there will be deposition behind the retard. Various forms of steel jacks may be assembled. Two types are shown in Fig. 6.3.4. Jacks must be tied together with cable and must have tiebacks to deadmen set in the bank. Tiebacks should be spaced every 100 feet and space between jacks should not be greater than their width.



(a) Typical Tetrahedron (b) Kellner Jack

Fig. 6.3.4 Steel jacks.

6.3.3 Jetties

The purpose of a jetty field is to add roughness to a channel or overbank area to train the main stream along a selected path. The added roughness along the bank reduces the velocity and protects the bank from erosion. Jetty fields are usually made up of steel jacks tied together with cables. Both lateral and longitudinal rows of jacks are used to make up the jetty field as shown in Fig. 6.3.5.

The lateral rows are usually angled about 45 to 70 degrees downstream from the bank. The spacing varies, depending upon the debris and sediment content in the stream, and may be 50 to 200 feet apart. Jetty fields are effective only if there is a significant amount of debris carried by the stream and the suspended sediment concentration is high.

When jetty fields are used to stabilize meandering rivers, it may be necessary to use jetty fields on both sides of the river channel because in flood stage the river may otherwise develop a chute channel across the point bar. A typical layout is shown in Fig. 6.3.5.

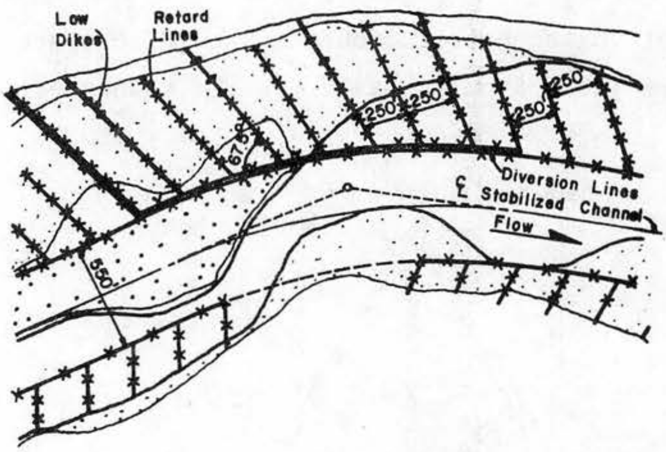


Fig. 6.3.5 Typical jetty-field layout.

6.3.4 Spur dikes

To river engineers, spur dikes are dikes placed laterally from the river bank. These were discussed previously in this chapter and were simply called dikes. Spur dikes, to highway engineers, have acquired a special meaning because of their localized interest with rivers. *In highway engineering, spur dikes are guide banks placed at or near the ends of approach embankments to guide the stream through the bridge opening.* Constructed properly, flow disturbances, such as eddies and cross-flow, will be eliminated to make a more efficient waterway under the bridge. They are also used to protect the highway embankment and reduce or eliminate local scour at the embankment and adjacent piers. The effectiveness of spur dikes is a function of river geometry, quantity of flow on the floodplain, and size of bridge opening. A typical spur dike at the end of an embankment is shown in Fig. 6.3.6.

The recommended shape of a spur dike is a quarter ellipse with a major to minor axis ratio of 2.5. The major axis should be approximately parallel to the main flow direction. For bridge crossings normal to the river, the major axis would be normal to the highway embankment. However, for skewed crossings, the spur dike should be placed at an angle with respect to the embankment with the view of streamlining the flow through the bridge opening. An illustration of spur dikes for a skewed crossing is shown in Fig. 6.3.7.

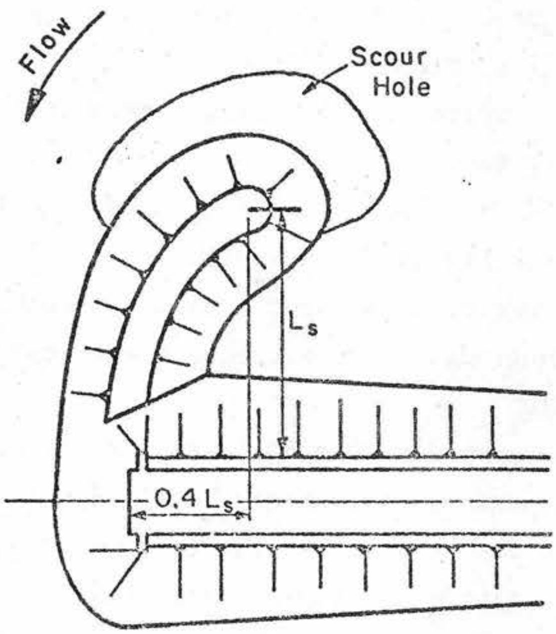


Fig. 6.3.6 Typical spur dike.

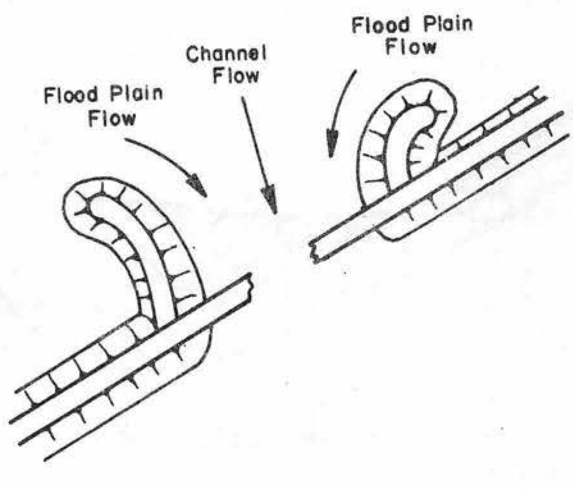


Fig. 6.3.7 Spur dikes at skewed highway crossing.

The length of the spur dike, L_s, required depends upon quantity of flow on the floodplain, width of bridge opening and skewness of the highway crossing. Shorter spur dikes may be used where floodplain flow is small or scour potential at piers and embankment ends are small.

6.3.5 Bank protection

The term "bank protection" implies that the bankline has or is about to fail. In order to design bank protection properly, we must know how the bank fails. There are four principal ways in which a bank fails. They are:

(1) The erosion of soil particles on the bank either by the river currents or by waves.

(2) Sloughing banks caused by excessive internal hydrostatic pressure in the bankline materials.

(3) Slip-circle failures caused by the undermining of the toe.

(4) Liquefaction and subsequent movement of the soil mass (called a flow slide).

The most common method of bank protection is with rock riprap. The sides of the bank or embankment are lined with large rocks to prevent erosion along the bank and at the toe. Section 6.4.0 in this chapter is devoted to riprap and further discussion is deferred. Other methods and devices are discussed in this section.

Rock-fill trenches - Rock-fill trenches are structures used to protect banks from caving caused by erosion at the toe. A trench is excavated along the toe of the bank and filled with rocks as shown in Fig. 6.3.8.

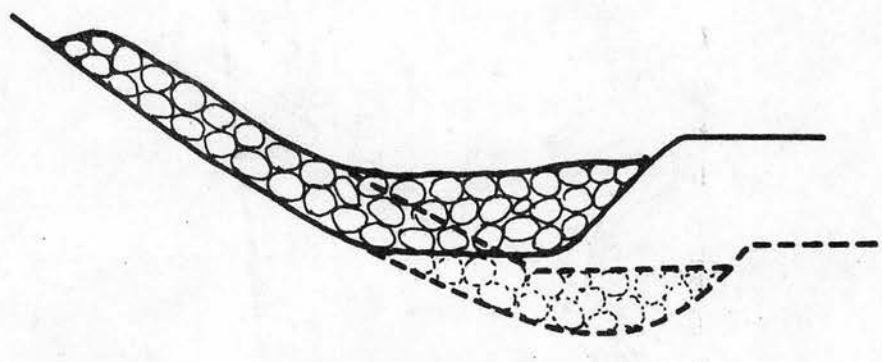


Fig. 6.3.8 Rock-fill trench.

As the stream bed adjacent to the toe is eroded, the toe trench is undermined and the rock fill slides downward to pave the bank. The size of trench to hold the rock fill depends on expected depths of scour. It is advantageous to grade the banks before paving the slope with riprap and placing rock in the toe trench. The slope should be at such an angle that the saturated bank is stable while the river stage is falling.

The rock-fill trench need not be at the toe of the bank. An alternative method is to excavate a trench above the water line along the top of the river bank and fill with rocks. Then as the bank erodes toward the trench, the rocks in the trench slide down and pave the bank. This method is applicable in areas of rapidly eroding banks of medium to large size rivers.

A variation of this method of toe protection is to pile the rocks in a "windrow" along the bank line instead of excavating a trench. Then as the bank is scoured, the rocks in the windrow drop down to pave the bank.

Rock-and-wire riprap - When adequate riprap sizes are not available, rocks of cobble sizes may be placed in wire mesh mats made of galvanized fencing and placed along the bank forming a mattress. The individual wire units are called *baskets* if the thickness is greater than 12 inches. The term *mattress* implies a thickness no greater than 12 inches. Toe protection is offered by extending the mattresses into the channel bed as shown in Fig. 6.3.9. As the bed along the toe is scoured, the mattress drops into the scour hole. Special wire baskets of manageable sizes are manufactured and sold throughout the United States. It should be noted that when rock-and-wire mattresses are used in streams transporting cobble and rocks, the wires of the basket can be cut rather rapidly, which will destroy the intended protection along the base of the bank. Rusting of the wire mesh may also be a problem.

Mats can be made up in large sizes in the field. The mats are flexible and can conform to scour holes which threaten the stability of the banks. The mats should be linked together to prevent separation as subsidence takes place.

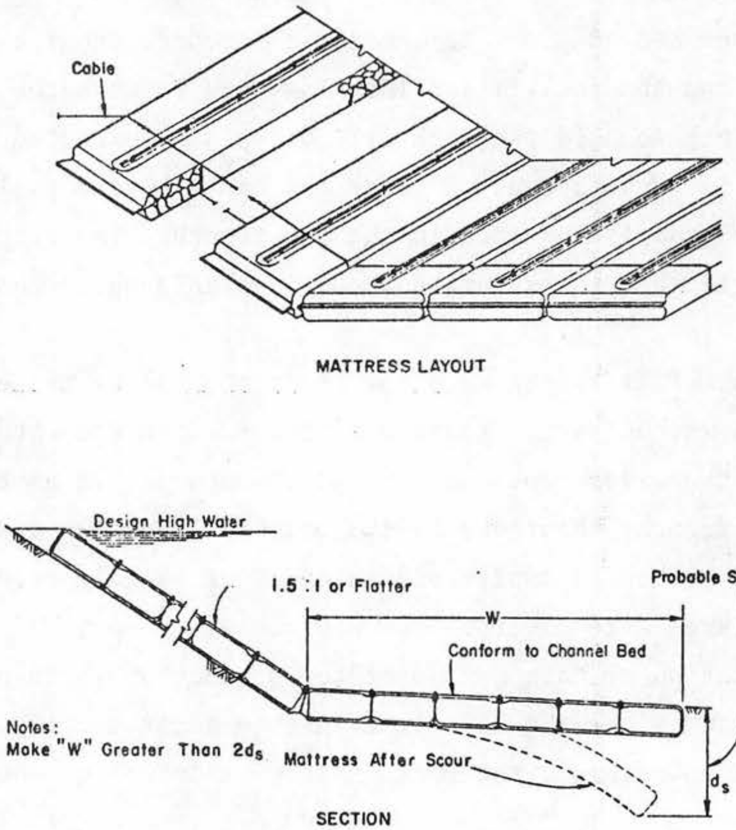


Fig. 6.3.9 Rock and wire mattress.

Articulated concrete mattress - Small precast concrete blocks held together by steel rods or cables can be used to form a flexible mat as shown in Fig. 6.3.10.

The sizes of blocks may vary to suit the contour of the bank. It is particularly difficult to make a continuous mattress of uniform sized blocks to fit sharp curves. The open spacing between blocks permits removal of bank material unless a filter blanket of gravel or plastic filter cloth is placed underneath. For embankments that are subjected only to occasional flood flows, the spaces between blocks may be filled with earth and vegetation can be established.

The use of articulated concrete mattresses has been limited primarily to the Mississippi River. This is due to the large cost of the plant required for the placement of the mattress beneath the water surface. Thus, it is economically feasible to use articulated concrete mattresses only on rivers which require extensive bank

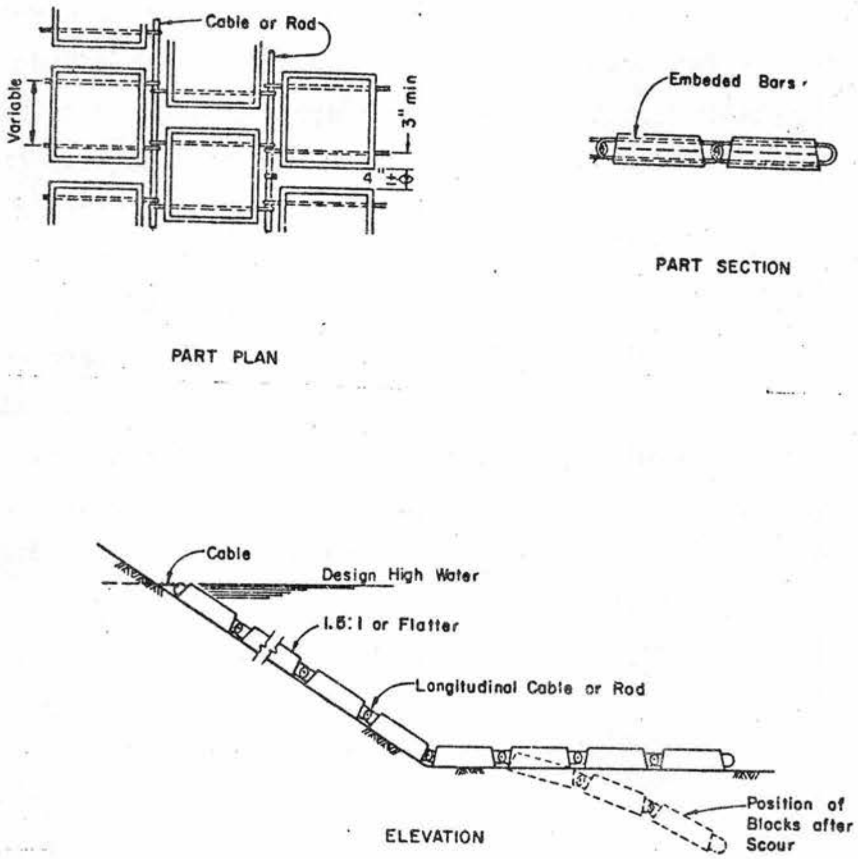


Fig. 6.3.10 Articulated concrete mattress.

protection. The expense of the installation plant is not required, however, for placement of articulated concrete mattresses above the water surface. Thus, paving the upper bank with articulated concrete mattresses has been used occasionally in the United States and Europe.

Other types of mattresses - Woven willow, brush, woven lumber, asphalt, and soil cement mattresses are other types that can be utilized. In modern highway construction practice however, occasion to use these types arises only rarely. Concrete paving slabs are occasionally used. While paving slabs may be satisfactory along high water lines, they are not satisfactory for use below normal water levels because with even minor scour of the bank or toe, the rigid paving cannot conform to the scour hole and soon becomes undermined and the paving may break up. If the paving is extended well below the stream bed, or if the entire cross section is lined with concrete paving, (especially for small channels) this form of bank paving is satisfactory.

Timber or concrete cribs - Timber and concrete cribs are sometimes used for bulkheads and retaining walls to hold highway embankments, particularly where lateral encroachment into the river must be limited. Cribs are made up by interlocking pieces together in the manner shown in Fig. 6.3.11. The crib may be slanted or vertical depending on height and the crib is filled with rock or earth. Reinforced concrete retaining walls are alternatives to timber cribs which can be considered. However, concrete retaining walls are expensive and are generally only used in special confined locations where space precludes other methods of bank protection. In constructing concrete retaining walls drainage holes (weep holes) must be provided. The foundation of these walls should be placed below expected scour depths.

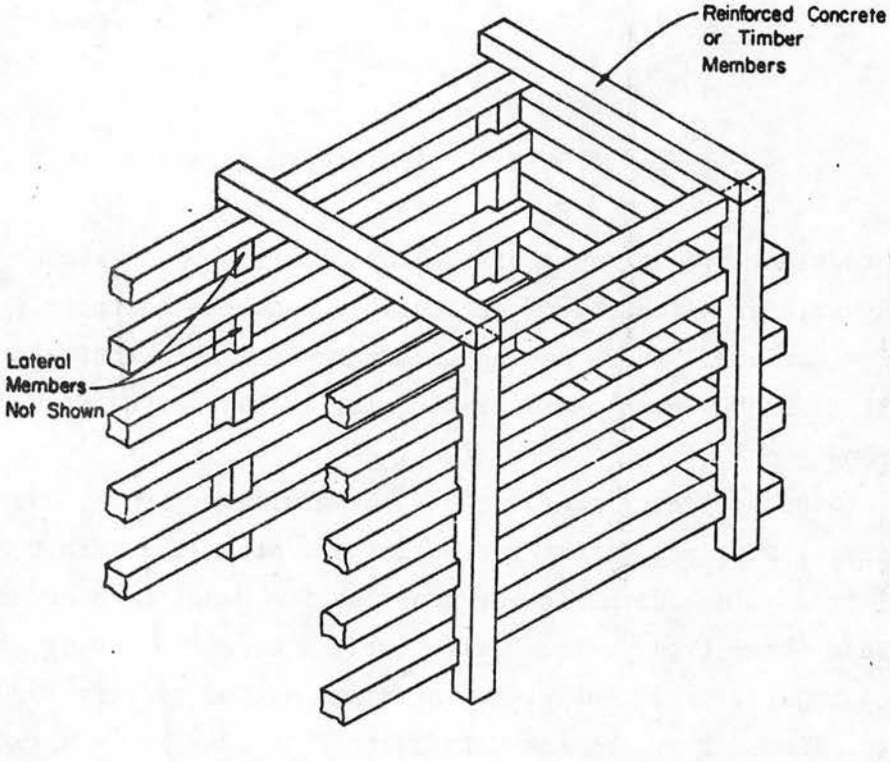


Fig. 6.3.11 Concrete or timber cribs.

6.4.0 RIPRAP SIZE AND STABILITY ANALYSIS

When available in sufficient size, rock riprap is usually the most economical material for bank protection. Rock riprap has many other advantages over other types of protection. Rock riprap protection is flexible and local damage is easily repaired. Construction must be accomplished in a prescribed manner but is not complicated. Although the riprap must be placed to the proper level in the bed, there are no foundation problems. The appearance of rock riprap is natural and after a period of time vegetation will grow between the rocks. Wave runup on rock slopes is usually less than on other types. Finally, when the usefulness of the protection is finished, the rock is salvable.

The important factors to be considered in designing rock riprap protection are:

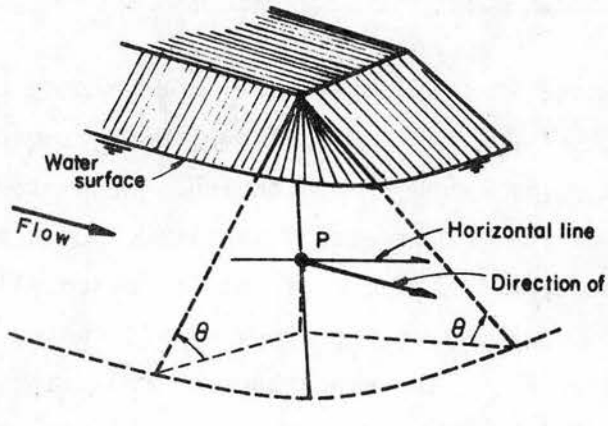
(1) The *durability* of the rock.
(2) The *density* of the rock.
(3) The *velocity* (both magnitude and direction) of the flow in the vicinity of the rock.
(4) The *slope* of the bed or bankline being protected.
(5) The *angle of repose* for the rock.
(6) The *size* of the rock.
(7) The *shape and angularity* of the rock.

The theoretical development of the relations between these important factors is presented in the Chapter VI appendix. A summary of the equations for riprap design is given below.

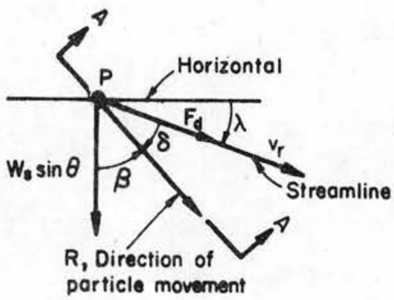
6.4.1 Oblique flow on a side slope

Consider flow along the nose of an embankment as shown in the diagrams of Fig. 6.4.1.

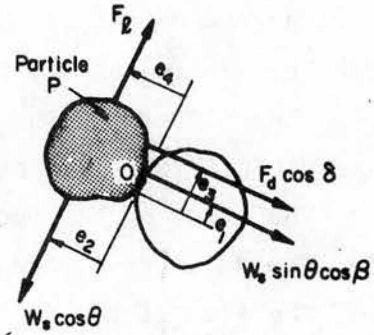
The forces on the rock particle are lift force F_ℓ, drag force F_d and weight of the particle W_s. Rock particles on side slopes tend to roll rather than slide, so it is appropriate to consider stability of rock particles in terms of moments about a contact point 0 about which rotation must take place. The components of forces relative to the plane of motion are shown in Fig. 6.4.1b.



(a) General view



(b) View normal to the side slope



(c) Section A-A

Fig. 6.4.1 Diagrams for the riprap stability analysis.

At incipient motion, there is a balance of moments such that

$$e_2 W_s \cos\theta = e_1 W_s \sin\theta \cos\beta + e_3 F_d \cos\delta + e_4 F_\ell \qquad 6.4.1$$

where e is the moment arm of each force.

The factor of safety, S.F., of particles against rotation is then determined by the ratio of the moments.

$$S.F. = \frac{e_2 W_s \cos\theta}{e_1 W_s \sin\theta \cos\beta + e_3 F_d \cos\delta + e_4 F_\ell} \qquad 6.4.2$$

By following the development given in the Chapter VI appendix, the following equations relating the safety factor for rock riprap on side slope where the flow has a non-horizontal velocity vector are obtained.

$$S.F. = \frac{\cos\theta \tan\phi}{\eta' \tan\phi + \sin\theta \cos\beta} \qquad 6.4.3$$

where

$$\eta' = \eta \left\{\frac{1 + \sin(\lambda+\beta)}{2}\right\} \qquad 6.4.4$$

$$\eta = \frac{21 \tau_o}{(S_s - 1)\gamma D} \qquad 6.4.5$$

$$\beta = \tan^{-1} \left\{\frac{\cos\lambda}{\frac{2 \sin\theta}{\eta \tan\phi} + \sin\lambda}\right\} \qquad 6.4.6$$

The angle λ shown in Fig. 6.4.1 is the angle between the horizontal and the velocity vector (drag force) measured in the plane of the side slope.

The angles θ and β are defined in Fig. 6.4.1, ϕ is the angle of repose (given in Fig. 3.7.3 for dumped riprap), τ_o is the bed shear stress, D is the representative rock size, S_s is the specific weight of the rock, and η' and η are stability numbers.

In full-scale experiments with rock riprap below culvert outlets, Stevens (1969) developed the expression

$$D = \left\{\frac{\sum_{i=1}^{10} D_i^3}{10}\right\}^{1/3} \qquad 6.4.7$$

where $D_i(i=1) = \frac{D_0 + D_{10}}{2}$

$$D_i(i=2) = \frac{D_{10} + D_{20}}{2}$$

$$\vdots$$

$$D_i(i=10) = \frac{D_{90} + D_{100}}{2}$$

for the *representative grain size of riprap*. The terms D_0, D_{10}, ... D_{100} are the sieve diameters of the rock for which zero percent, 10 percent, ..., 100 percent of the material by weight is finer. The value of D is greater than D_{50} except for uniform materials. If all the rock are one size, $D = D_{50}$. The representative grain size for riprap is discussed further in the Chapter VI appendix.

6.4.2 Horizontal flow on a side slope

In many circumstances the flow angularity with the horizontal is small, (i.e. $\lambda \approx 0$), and Eq. 6.4.6 reduces to

$$\tan\beta = \frac{\eta \tan\phi}{2 \sin\theta} \qquad 6.4.8$$

and Eq. 6.4.3 solved for η becomes

$$\eta = \left(\frac{S_m^2 - (S.F.)^2}{(S.F.)S_m^2}\right) \cos\theta \qquad 6.4.9$$

where S_m is the safety factor of rock particles from rolling down the slope with no flow. Accordingly

$$S_m = \frac{\tan\phi}{\tan\theta} \qquad 6.4.10$$

Alternatively, the safety factor may be expressed as

$$S.F. = \frac{S_m}{2} \left\{(S_m^2 \eta^2 \sec^2\theta + 4)^{1/2} - S_m \eta \sec\theta\right\} \qquad 6.4.11$$

6.4.3 Flow on a sloping bed

When considering flow along a plane bed sloping at an angle α with respect to the horizontal (see Fig. 6.4.2), the equations describing the stability of the riprap on the bed reduce to

$$S.F. = \frac{\cos\alpha \tan\phi}{\eta \tan\phi + \sin\alpha} \qquad 6.4.12$$

or $$\eta = \cos\alpha \left[\frac{1}{S.F.} - \frac{\tan\alpha}{\tan\phi}\right] \qquad 6.4.13$$

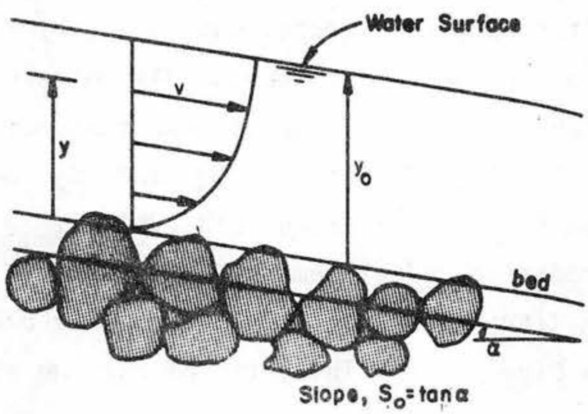


Fig. 6.4.2 Definition sketch for riprap on a channel bed.

6.4.4 Flow on a horizontal bed

The most simple case is flow on a plane horizontal bed. In that case

$$S.F. = \frac{1}{\eta} \qquad 6.4.14$$

If the particles on a plane horizontal bed are in incipient motion, S.F. = 1 so $\eta = 1$ and from Eq. 6.4.5

$$\frac{\tau_0}{(S_s - 1)\gamma D} = 0.047 \qquad 6.4.15$$

which is the Shields' criteria for incipient motion in fully developed rough turbulent flow. The use of foregoing analysis for determining riprap size is illustrated by numerical examples in the appendix at the end of this chapter.

6.4.5 University of Minnesota method for riprap size determination

A method to determine size of riprap to line the entire channel of small to intermediate sizes (6 to 1000 cfs) has been proposed by Anderson, et al. (1970). *The method applies to channels that are trapezoidal or* *triangular in shape and which are essentially straight in alignment.* The proposed equation relating size of riprap to discharge and channel geometry is

$$Q = \frac{1}{118} \frac{D_{50}^{5/2}}{S_o^{13/6}} \frac{P}{R} \qquad 6.4.16$$

in which P is the wetted perimeter and R is hydraulic radius. Equation 6.4.16 is based on maximum shear stress related to rock diameter and Manning's equation of flow. It can be seen that for fixed channel size, P/R, the riprap size is a function of Q and S_o so that a family of design curves can be made for fixed P/R values.

6.4.6 Riprap gradation and placement

Riprap gradation should follow a smooth size distribution curve such as that shown in Fig. 6.4.3. The ratio of maximum size to median size,

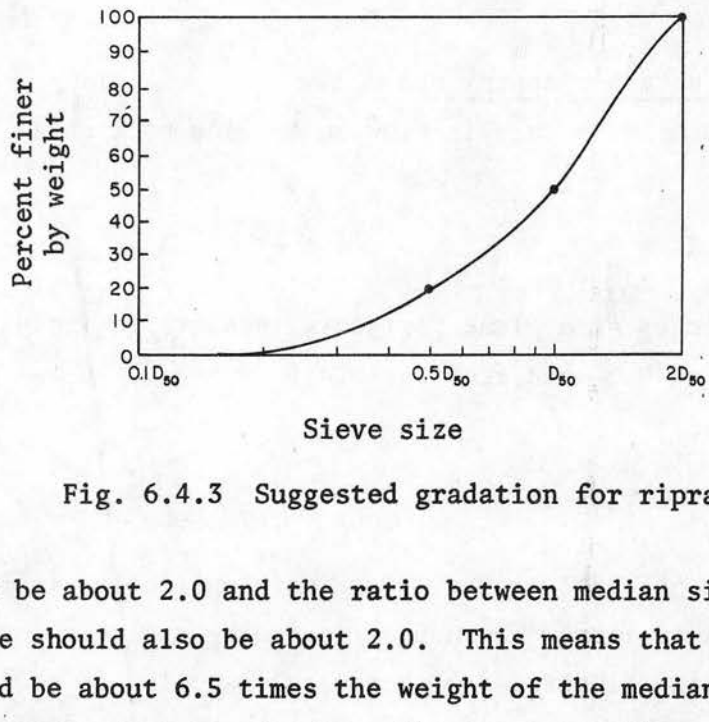


Fig. 6.4.3 Suggested gradation for riprap.

D_{50}, should be about 2.0 and the ratio between median size and the 20 percent size should also be about 2.0. This means that the largest stones would be about 6.5 times the weight of the median size and small sizes would range down to gravels. The representative rock size D for the gradation shown in Fig. 6.4.3 is 1.25 D_{50} (calculated using Eq. 6.4.7) which is approximately equal to the D_{67}.

With a distributed size range, the interstices formed by the larger stones are filled with the smaller sizes in an interlocking fashion, preventing formation of open pockets. Riprap consisting of angular stones is more suitable than that consisting of rounded stones. Control of the gradation of the riprap is almost always made by visual inspection.

If it is necessary, poor gradations of rock can be employed as riprap provided the proper filter is placed between the riprap and the bank of bed material. The representative grain size of the riprap is determined by Eq. 6.4.7 and the filter is designed in accordance with the criteria given in the next section.

Riprap should be hard, dense and durable to withstand long exposure to weathering. Visual inspection is most often adequate to judge quality, but laboratory tests may be made to aid the judgment of the field inspector.

Riprap placement is usually accomplished by dumping directly from trucks. If riprap is placed during construction of the embankment, rocks can be dumped directly from trucks from the top of the embankment. Rock should never be placed by dropping down the slope in a chute or pushed downhill with a bulldozer. These methods result in segregation of sizes. With dumped riprap there is a minimum of expensive hand work. Poorly graded riprap with slab-like rocks requires more work to form a compact protective blanket without large holes or pockets. Draglines with orange peel buckets, backhoes and other power equipment can also be used advantageously to place the riprap.

Hand placed rock riprap is another method of riprap placement. Stones are laid out in more or less definite patterns, usually resulting in a relatively smooth top surface. This form of placement is used rarely in modern practice because it is usually more expensive than placement with power machinery.

The thickness of riprap should be sufficient to accommodate the largest stones in the riprap. With a well-graded riprap with no voids, this thickness should be adequate. If strong wave action is of concern, the thickness should be increased by 50 percent.

6.4.7 Filters for riprap

Filters underneath the riprap are recommended to protect the fine embankment or riverbank material from washing out through the riprap. Two types of filters are commonly used; gravel filters and plastic filter cloths.

Gravel filters - A layer or blanket of well-graded gravel should be placed over the embankment or riverbank prior to riprap placement. Sizes of gravel in the filter blanket should be from 3/16 in. to an upper limit depending on the gradation of the riprap with maximum sizes of about 3 to 3-1/2 in. Thickness of the filter may vary depending upon the riprap thickness but should not be less than 6 to 9 inches. Filters that are one-half the thickness of the riprap are quite satisfactory. Suggested specifications for gradation are as follows:

$$(1) \quad \frac{D_{50}\ \text{(Filter)}}{D_{50}\ \text{(Base)}} < 40$$

$$(2) \quad 5 < \frac{D_{15}\ \text{(Filter)}}{D_{15}\ \text{(Base)}} < 40$$

$$(3) \quad \frac{D_{15}\ \text{(Filter)}}{D_{85}\ \text{(Base)}} < 5$$

Plastic filter cloths - Plastic filter cloths are being used beneath riprap and other revetment materials such as articulated concrete blocks with considerable success. The cloths are generally in 100 ft long rolls, 12 to 18 ft wide. Overlap of 8 to 12 inches is provided with pins at 2 to 3 ft intervals along the seam to prevent separation in case of settlement of the base material. Some amount of care must be exercised in placing riprap over the plastic cloth filters to prevent damage. Experiments and results with various cloth filters were reported by Calhoun, Compton and Strohm (1971) in which specific manufacturers and brand names are listed. Stones weighing as much as 3000 lbs have been placed on plastic filter cloths with no apparent damage.

Filters can be placed subaqueously by using steel rods as weights fastened along the edges. Additional intermediate weights would assist in sinking the cloth in place. Durability of filter cloths has not yet been established because they have been in use only since about 1967. However, inspections at various installations indicate little or no deterioration had occurred in the few (1 to 4) years that have elapsed for test installations.

6.4.8 Guide to channel improvement, river training and bank stabilization

The type of channel improvement and devices used for training and bank stabilization depend upon the size of river with regard to width, depth and discharge; type of rivers, that is, meandering, braided or straight; sediment transport in terms of concentration and size distribution; length of river to be protected; availability of materials; environmental considerations; aesthetics; legal aspects; river use with regard to navigation, recreation, agriculture, municipal and industrial purposes; and perhaps other factors. Decisions concerning highway locations near rivers or across rivers, and designs for specific devices to integrate the highways with the river systems are therefore very complex.

Table 6.4.1 is offered as a guide to assist the highway engineer with regard to decisions for channel improvement and selection of type of bank protection and river training works. The rivers are first categorized as to size and type. The descriptors large, medium and small are relative terms but should give no interpretive problem. Straight rivers are those which have sinuosity less than 1.5, but in terms of highway concerns, long reaches between meander bends which are essentially straight may be included in the straight river classification. Because they are part of meander systems, stabilization and improvements may be required. This is the interpretation to be used in the table. The X in the box indicates that consideration could be given to use of the particular device. The absence of a check mark in the box indicates that the devices are not often used, but consideration could be given to them in special circumstances. Additional remarks are noted on the table.

6.5.0 SCOUR AND DEGRADATION

Changes in bed level which affect highway construction and structures may be described by three types of interrelated phenomena. They are:

(1) *Local scour*, caused by local disturbances in the flow such as vortices and eddies. Examples are scour at the base of piers, dikes and other obstructions in a stream.

(2) *General scour* due to contractions causing increasing velocities across the entire contracted width. Examples are scour at contracted stream channels caused by spur dikes, embankments, and accumulation of debris at bridge openings.

Table 6.4.1 Guide for selection of methods and devices for river channel improvement and bank protection works.

Size of River	Type of River	Channel Improvement	Dikes			Retards		Jetties		Bank Protection					
												Mattresses			
			Timber	Stone-fill	Earth	Timber	Steel Jacks	Timber	Steel Jacks	Riprap	Rock Trench	Rock and Wire	Concrete	Other	Cribs
Large	Meandering		X	X	*	*		X		X	X	*	X	X	
	Braided	X	X	X	*	*		X		X	X	*	X	X	
	Straight		X	X		*		X		X	X	*		X	
Medium	Meandering	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	Braided	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	Straight	X	X			X		X		X	X	†		X	X
Small	Meandering	X				X	X	X	X	X	X	†		X	X
	Braided	X				X	X	X	X	X	X	†		X	X
	Straight	X								X	X	†		X	X

* Floodplain embankment protection

† Where large rocks for riprap are not available

(3) *Degradation or aggradation* of a stream channel over long lengths and time due to changes in controls, such as dams, changes in sediment content and changes in river geomorphology, such as changing from a meandering river to a braided stream.

These effects are in general additive, so that local scour can occur while scour due to contraction is occurring and degradation or aggradation of the stream is taking place. The respective time scales for scour are progressively larger.

6.5.1 Degradation and aggradation

Degradation and aggradation of the river bed has been discussed previously. A long reach of river channel may be subjected to a general lowering or raising of the bed level over a long period of time. Prediction of ultimate degradation or aggradation of a stream was discussed in Chapters IV and V. Degradation or aggradation at a bridge opening must be anticipated. Otherwise, pier and abutment foundation depths may be inadequate when degradation occurs, or the road and bridge deck levels may be too low in a few years with an aggrading stream.

6.5.2 General scour

Scour at contractions occur because the flow area becomes smaller than the normal stream and the average velocity and bed shear stress increase; hence, there is an increase in stream power at the contraction and more bed material is transported through the contracted section than is transported into the section. As the bed level is lowered, the velocity decreases, shear stress decreases and equilibrium is restored when the transport rate of sediment through the contracted section is equal to the incoming rate.

Flows confined to the channel - Consider a situation where a normal river channel is narrowed by a contraction. Scour due to the contraction may be determined in the following way (Nordin, 1971). The approach flow depth, y_1, and average approach flow velocity, V_1, results in the sediment transport rate q_{s1}. The total transport rate to the contraction is $W_1 q_{s1}$ in which W_1 is the width of the approach. If the water flow rate, $Q_1 = W_1 q_1$, in the upstream channel is equal to the flow rate at the contracted section, then by continuity

$$q_2 = \frac{W_1}{W_2} q_1 \qquad 6.5.1$$

Here $q_1 = y_1V_1$ and $q_2 = y_2V_2$ and the subscript 2 refers to conditions in the contracted section. The sediment transport rate at the contracted section after equilibrium is established must be

$$q_{s2} = \frac{W_1}{W_2} q_{s1} \qquad 6.5.2$$

The relationships of y and V at sections 1 and 2 are shown in Fig. 6.5.1 for constant q_1 and q_2.

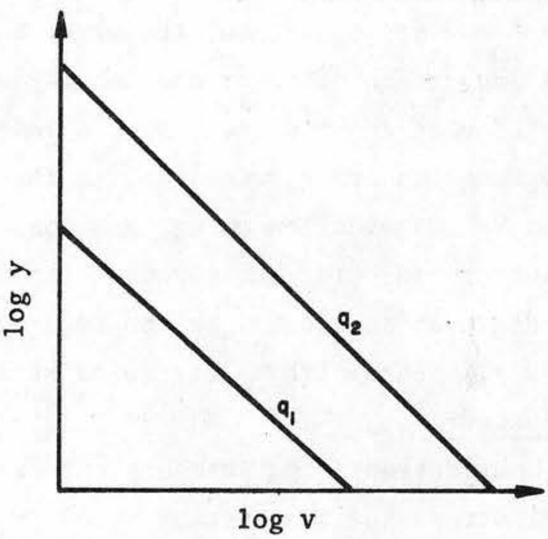


Fig. 6.5.1 Unit discharge as a function of depth and velocity.

From one of the sediment transport equations given in Chapter III it is possible to construct curves for transport rates of sediment of given median size as functions of flow depth and velocities. An illustration of such dependence is shown in Fig. 6.5.2 using the method of Colby.

Now overlap Fig. 6.5.1 with Fig. 6.5.2. The result is shown in Fig. 6.5.3.

The depth of scour due to the contraction is then

$$y_s = y_2 - y_1 \qquad 6.5.3$$

Overbank flow with flow in the channel - Laursen (1960) developed an equation for scour at a contraction, where in addition to channel flow there is overbank flow concentrating into the contracted channel

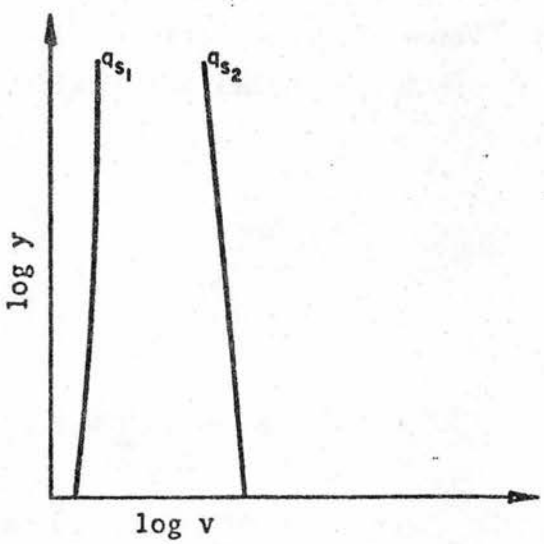


Fig. 6.5.2 Sediment transport rate as a function of depth and velocity.

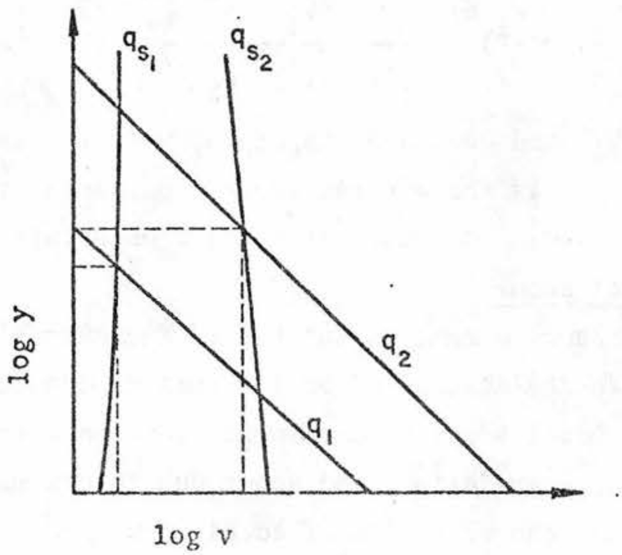


Fig. 6.5.3 Determination of scour depth.

(designated by subscript 2). The equation to predict depth of flow at section 2 is

$$\frac{y_2}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{6/7} \left(\frac{W_1}{W_2}\right)^{\frac{6(2+f)}{7(3+f)}} \left(\frac{n_2}{n_1}\right)^{\frac{6f}{7(3+f)}} \qquad 6.5.4$$

in which Q_c is the approach channel flow rate and Q_t is the contracted channel flow rate which is greater than the approach channel flow rate by the amount of flow on the floodplain. The variable n is the Manning roughness coefficient, W is the channel width and the exponent f is given below.

V_{*c}/ω	f
< 0.5	0.25
1	1
> 2	2.25

Here V_{*c} is the shear velocity in the approach channel and ω is the fall velocity of the bed material.

Overbank flow only - For scour at bridges on a floodplain where there is no sediment transport from upstream, Laursen (1963) proposed that

$$\frac{y_2}{y_1} = \left(\frac{W_1}{W_2}\right)^{6/7} \left(\frac{V_1^2}{120\, y_1^{1/3}\, D_{50}^{2/3}}\right)^{3/7} \qquad 6.5.5$$

Here y_1, V_1 and W_1 are the depth, velocity and width of the approach flow, and y_2 is the general scour flow depth at the bridge. The term D_{50} is the median diameter of the bed materials at the bridge.

6.5.3 Local scour

Local scour occurs in the bed of the channel around piers and embankments due to the actions of vortex systems induced by the obstructions to the flow. Local scour occurs in conjunction with or in the absence of degradation, aggradation, and scour due to contractions. There is need to understand the mechanism of local scour and calculation of potential scour depths, after which means may be considered in the design to eliminate or reduce its magnitude by suitable protective methods.

Mechanism of local scour - The basic mechanism causing local scour is the vortex of fluid resulting from the pileup of water on the upstream edge and subsequent acceleration of flow around the nose of the pier or embankment. The action of the vortex is to erode bed materials away from the base region. If the transport rate of sediment away from the local region is greater than the transport rate into the region, a scour hole develops. As the depth is increased, the strength of the vortex is reduced, the transport rate is reduced and equilibrium is reestablished and scouring ceases.

The flow field and vortex systems around a circular pier and approach embankment are illustrated in Figs. 6.5.4 and 6.5.5. Although the vortex system is known to be the cause of local scour, it has not been possible as yet to calculate the strength of the vortex and relate the velocity field with subsequent scour. Until further research and study makes this possible, average velocity and local depth of flow are used in the equations to predict local scour depths.

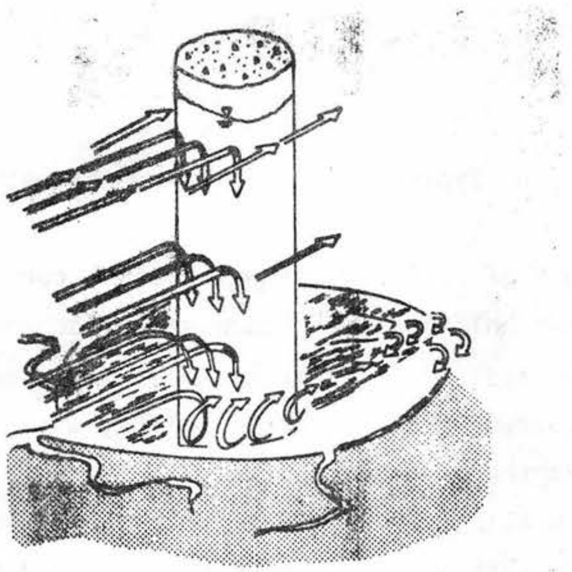


Fig. 6.5.4 Schematic representation of scour at a cylindrical pier.

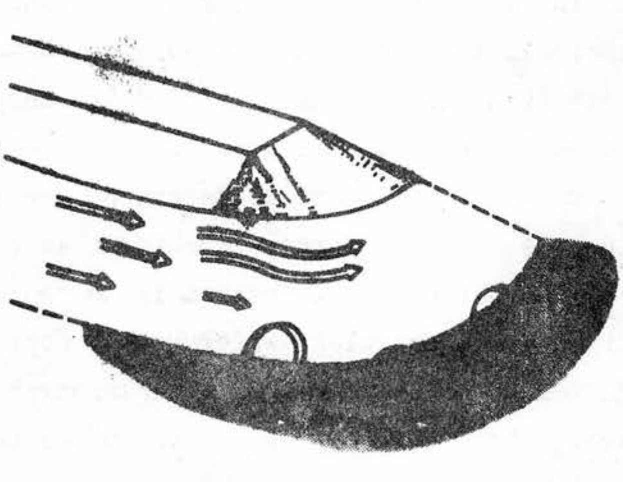


Fig. 6.5.5 Schematic representation of scour at an embankment.

Local scour around embankments - A typical scour hole at an embankment and adjacent pier is illustrated in Fig. 6.5.6.

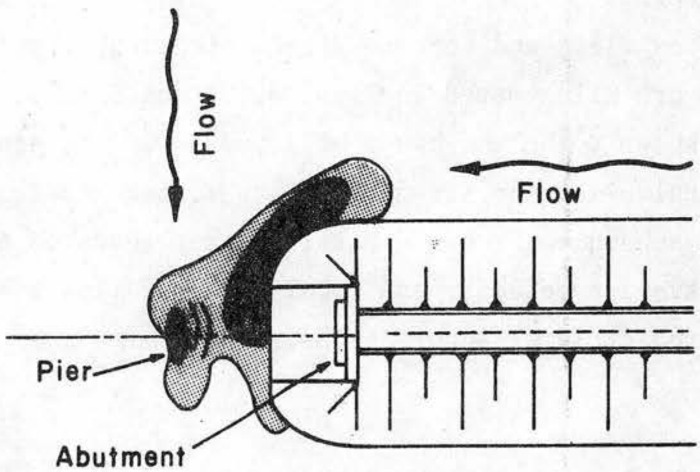


Fig. 6.5.6 Typical scour at an embankment and adjacent pier.

The depth of scour varies with time because the sediment transported into the scour hole from upstream varies, depending upon the presence or absence of dunes. The time required for dune motion is much larger than the time required for local scour. Thus, even with steady state conditions, the depth of scour is likely to fluctuate with time when there are dunes traveling on the channel bed. The depth of the scour hole is more variable with larger dunes. When the crest of the dune reaches the local scour area, the transport rate into the hole increases, the scour hole fills and the scour depth temporarily decreases. When a trough approaches, there is less sediment supply and the scour depth increases to try to reestablish equilibrium in sediment transport rates. A mean scour depth between these oscillations is referred to as equilibrium scour depth. It is not uncommon (as determined in laboratory tests) to find maximum depths to be 30 percent greater than equilibrium scour depths. The depth that would be reached if no sediment was transported into the scour hole is the "clear-water" scour depth.

Detailed studies of scour around embankments have been made mostly in laboratories. There are very few case studies for scour at field installations. According to the studies of Liu et al. (1961) *the equilibrium scour depth for local scour in sand at a spill slope when the flow is subcritical is determined by the expression*

$$\frac{y_s}{y_1} = 1.1 \left(\frac{a}{y_1}\right)^{0.40} F_{r1}^{\ 0.33} \qquad 6.5.6$$

where y_s is the equilibrium depth of scour measured from the mean bed level to the bottom of the scour hole, a is the embankment length (measured normal to the wall of a flume), y_1 is upstream depth and F_{r1} is the upstream Froude number determined as

$$F_{r1} = \frac{V_1}{\sqrt{gy_1}} \qquad 6.5.7$$

If the embankment terminates at a vertical wall and has a vertical wall on the upstream side then the scour hole depth in sand nearly doubles (Liu, 1961 and Gill, 1972). That is,

$$\frac{y_s}{y_1} = 2.15 \left(\frac{a}{y_1}\right)^{0.40} F_{r1}^{\ 0.33} . \qquad 6.5.8$$

The lateral extent of the scour hole is nearly always determinable from the depth of scour and the natural angle of repose of the bed material.

Field data for scour at embankments for various size rivers are scarce, but data collected at rock dikes on the Mississippi indicate that

$$\frac{y_s}{y_1} = 4\ F_{r1}^{\ 0.33} \qquad 6.5.9$$

determines the equilibrium scour depth for large a/y_1. The data are scattered, primarily because equilibrium depths were not measured. Dunes as large as 20 to 30 feet high move down the Mississippi and associated time for dune movement is very large in comparison to time required to form local scour holes. Nevertheless, it is believed that these data represent the limit in scale for scour depths as compared to laboratory data and enables useful extrapolation of laboratory studies to field installations.

Accordingly, it is recommended that Eq. 6.5.6 be applied for embankments with $0 < a/y_1 < 25$ and Eq. 6.5.9 be used for $a/y_1 > 25$. The equations are shown in graphical form in Fig. 6.5.7. In applying Eq. 6.5.6, the embankment length a is measured from the high water line at the valley bank perpendicularly to the end of the embankment at the bridge. If $a/y_1 > 25$, then scour depth is independent of a/y_1 and depends only on the approach Froude number and depth of flow. For $a/y_1 < 25$, as for example at dikes and short highway embankments, Eq. 6.5.6 would apply.

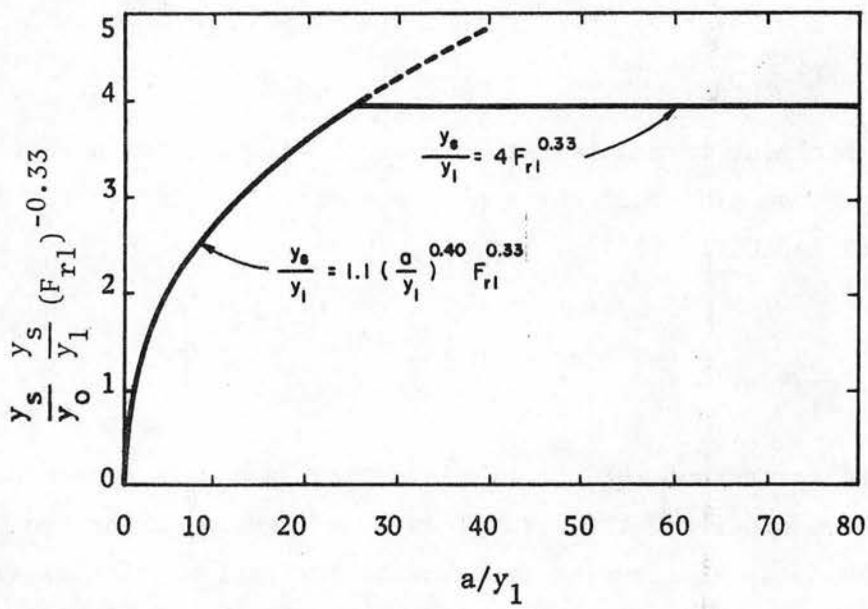


Fig. 6.5.7 Recommended prediction equation for embankment scour.

It should be recalled that maximum depth of scour is about 30 percent greater than equilibrium scour depth. The lateral extent of scour can be determined from the angle of repose of the material and scour depth.

If the embankment is angled downstream, the depth of scour is reduced because of streamlining effect. Embankments that are angled upstream have deeper scour holes. The calculated scour depth should be adjusted in accordance with the curve of Fig. 6.5.8 which is patterned after Ahmad (1953).

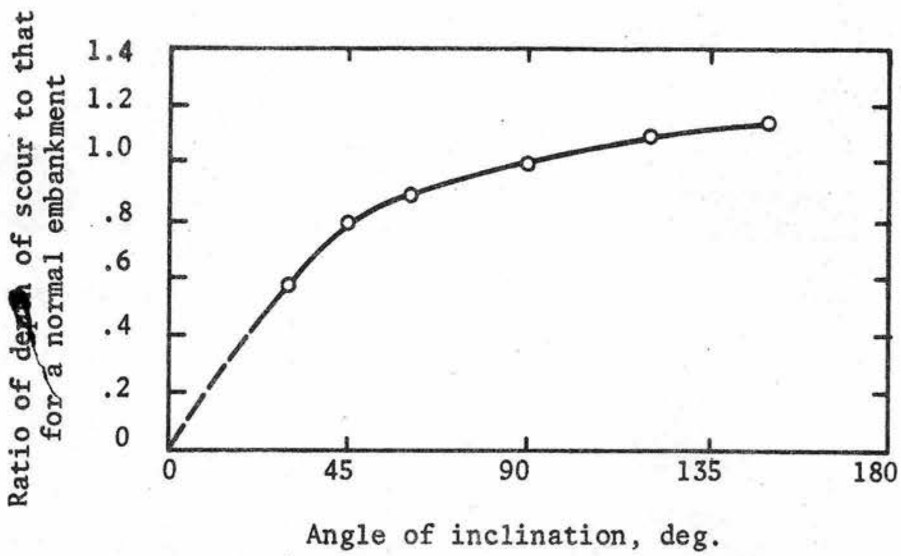


Fig. 6.5.8 Scour reduction due to embankment inclination.

Local scour around piers - Local scour at bridge piers is a result of vortex systems developed at the pier. The so-called *"horseshoe" vortex system is dominant at piers*, causing deepest scour at the nose of the pier. The axis of this vortex, or the vortex line, is horizontal, and wraps around the base of the pier in the shape of a horseshoe. The high velocities scour the bed. The wake-vortex system has vertical axes and develops because of blockage of the flow by the pier. The wake vortices are commonly seen as "eddies". This vortex system suspends the scoured material and carries it downstream with the flow. Downstream of embankments, large wake vortices or eddies are set up which scour the downstream sides of the embankments, the river bank and the stream bed. Wake vortices downstream of piers may create sufficient velocities to cause bed scour if the piers are wide. For most piers, however, very little additional scouring is caused by wake vortices.

The shape of the pier is very significant with respect to scour depth because it reflects the strength of the horseshoe vortex at the base of the pier. A blunt-nose pier causes the greatest scour depth. Streamlining the front end of the pier reduces the strength of the horseshoe vortex reducing the scour. Streamlining the downstream end of piers reduces the strength of wake vortices. Common shapes of piers are shown in Fig. 6.5.9. The width of piers are labeled "a" and length designated ℓ.

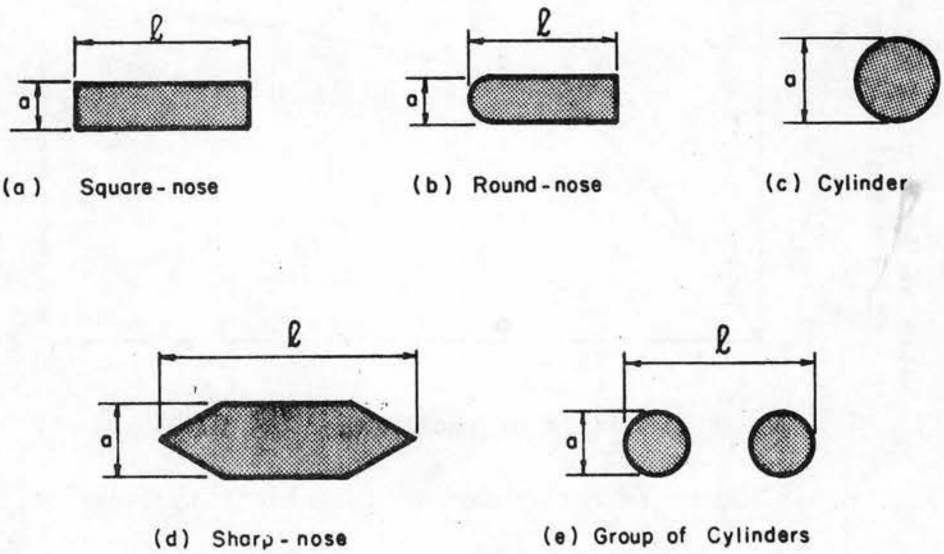


Fig. 6.5.9 Common pier shapes.

The equilibrium of scour at the nose of square-nosed piers can be estimated with the equation

$$\frac{y_s}{y_1} = 2.2 \left(\frac{a}{y_1}\right)^{0.65} F_{r1}^{\ 0.43} \qquad 6.5.10$$

where y_1 is upstream depth of flow and F_{r1} is the Froude number. The scour depth for circular cylinders is

$$\frac{y_s}{y_1} = 2.0 \left(\frac{a}{y_1}\right)^{0.65} F_{r1}^{\ 0.43} \qquad 6.5.11$$

The form of Eq. 6.5.10 for square-nosed piers and Eq. 6.5.8 for vertical-wall embankments are similar, although the variable "a" takes on different meanings. This similarity exists because the scour mechanisms for the two cases are similar.

Cylindrical piers have been widely investigated in the laboratory. The exponents in Eq. 6.5.11 were determined from laboratory data shown in Fig. 6.5.10. In this figure, the abscissa is labeled $(a/y_1)^3 F_{r1}^2$ to spread the data. Note that

$$\left\{\left(\frac{a}{y_1}\right)^3 F_{r1}^2\right\}^{0.215} = \left(\frac{a}{y_1}\right)^{0.65} F_{r1}^{\;0.43} \qquad 6.5.12$$

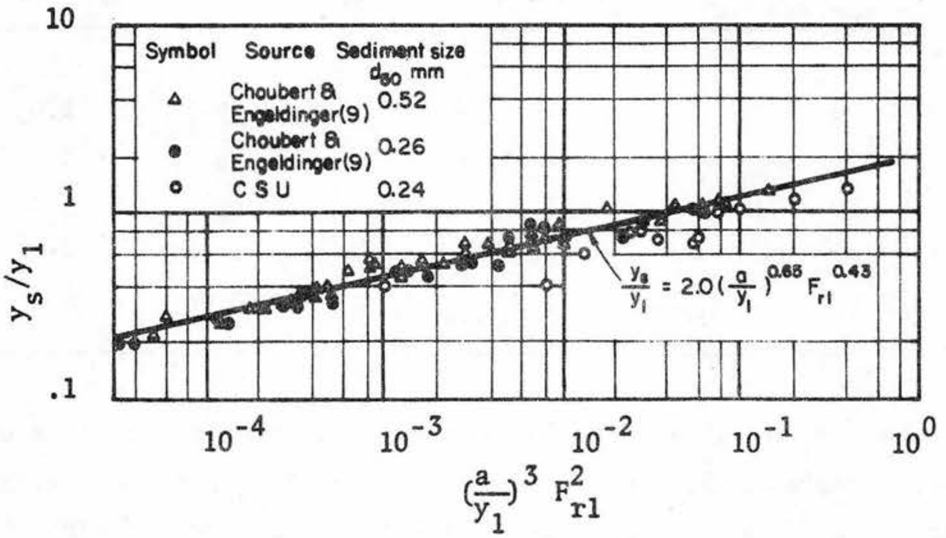


Fig. 6.5.10 Results of laboratory experiments for scour at circular piers.

The scour depth decreases as a consequence of streamlining. The reduction in scour depth can be estimated from the value given in Table 6.5.1.

Table 6.5.1 Reduction in scour depths for equal projected widths of pier.

Type of Pier	y_s/y_s(square nose)
Square nose	1.0
Cylinder	0.9
Round nose	0.9
Sharp nose	0.8
Group of cylinders	0.9

Pier alignment other than parallel with flow direction will create deeper scour holes because, in effect, the dimension "a" increases. In Table 6.5.2 multiplying factors to Eq. 6.5.10 are given. Scour at single cylindrical piers is invariant to flow direction, but groups of cylindrical piers would be affected by flow angularity.

Table 6.5.2 Multiplying factors for scour depths with skewed flow direction.

Angle of skew in degrees	$\ell/a = 4$	$\ell/a = 8$	$\ell/a = 12$
0	1.0	1.0	1.0
15	1.5	2.0	2.5
30	2.0	2.5	3.5
45	2.3	3.3	4.3
90	2.5	3.9	5.0

The concept of equilibrium scour depth also applies to scour around piers. *It should be recalled that maximum scour depth at piers could be as large as 30 percent greater than equilibrium scour depth.*

The base level to which scour depths can be referenced is not a trivial question. In some rivers, dune heights may be as large as 20 to 30 feet, and y_1 would normally be measured from some level closer to the tops of the dunes. Scour depths on the other hand should be referenced nearer the trough of the dunes. The bed level for scour due to contractions is first calculated. This establishes y_1 and F_{r1}. The local scour depth is then referenced to that bed level.

6.5.4 Protection of structures from local scour

Three basic methods may be used to protect structures from damage due to local scour. The first is to prevent damaging vortices from developing and the second is to provide protection at some level at or below the stream bed to arrest development of the scour hole. The third is to place the foundations of structures at such depth that the deepest scour hole will not threaten the stability of the structure. The last method is often very expensive, and risk is involved because of the uncertainty associated with estimating the additive effects of scour due to contraction and channel degradation.

Vortex reduction - As previously mentioned in Section 6.5.3, streamlining the piers can reduce scour depth by 10 to 20 percent. Another method of reducing the vortex strength at the pier is to construct barriers upstream of bridge piers-for instance, with a cluster of piles. While the piles are subjected to scour, failure of these piles is not damaging to the bridge. Debris can collect on the upstream piles keeping the nose of the bridge pier relatively free of debris. The pileup of water at the upstream piles reduces the dynamic pileup of water at the pier and reduces the vortex strength at the pier. Spur dikes can be placed at the ends of approach embankments to reduce local scour at the bridge. Spur dikes were discussed in Section 6.3.4.

Bed protection - Riprap piled up around the base of the pier is a common method of local scour protection. The principle is the same as toe trenches for bank protection. The region of the bed beyond the riprap pile scours and as the scour hole is formed, the riprap slides down into the scour hole, eventually armoring the side of the scour hole adjacent to the pier. An estimate of the depth of scour is needed to determine the quantity of riprap required for effective protection. Because of armoring, the effective depth of scour is less than that calculated by Eqs. 6.5.10 and 6.5.6. There are few studies to establish dependable guidelines, but 50 to 60 percent reduction in y_s may be used for an estimate of final scour depth. By frequent inspection it can be determined whether the size and quantity of riprap used initially is adequate. If additional amounts of riprap are necessary, placement from the water surface may be possible in times of low flow with consideration given to the falling path of rocks in a flowing stream.

A *structural concrete shelf* placed at about 0.5 y_s, where y_s is calculated from Eq. 6.5.10, extending laterally from the pier and completely surrounding the pier *may be effective in limiting the scour depth.* The lateral extent of the shelf should be about 0.3 $y_s \cos\phi$, where ϕ is the angle of repose of the bed material. While this method may be effective for $y_s < 20$ feet, it may become impractical for larger y_s.

Protective mattresses such as rock and wire have been suggested in the past, and have been used in a few circumstances. While they may have merit where adequate size riprap may be scarce, anchoring and stabilization of the mattresses to conform with scour holes is usually difficult. Use of mattresses in conjunction with riprap may be quite effective, if the mattress performs essentially as a flexible filter blanket which deforms as scour holes develop.

6.6.0 SUMMARY

Ordinarily, highway agencies cannot justify extensive control installations. As a consequence most highway installations will suffer damages at some time. Usually, it is less expensive to repair damages at some future time than to invest large amounts of money in the initial structures.

The information in this chapter is guidance on the principles of training or controlling relatively short reaches of river channels. Over long periods of time, normal river behavior can result in the river outflanking or destroying training structures in short reaches. Often, in the initial design, some thought as to how the river will attack a structure can result in efficient maintenance plans.

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APPENDIX

6.A1.0 SAFETY FACTORS FOR RIPRAP - THEORETICAL DEVELOPMENT

In the absence of waves and seepage, the stability of rock riprap particles on a side slope is a function of: (1) the magnitude and direction of the stream velocity in the vicinity of the particles; (2) the angle of the side slope; (3) the characteristics of the rock including the geometry, angularity and density. The functional relations between the variables is developed below. This development closely follows that given by Stevens and Simons (1971).

6.A1.1 Oblique flow on a side slope

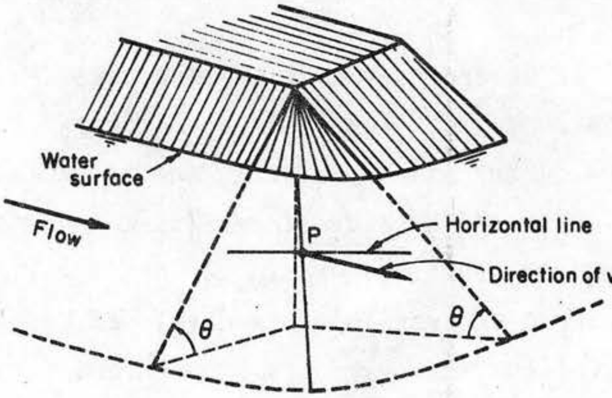
Consider flow along an embankment as shown in Fig. 6.A1.1. The fluid forces on a rock particle identified as P in Fig. 6.A1.1a result primarily from fluid pressures around the surface of the particles. The lift force F_ℓ is defined herein as the fluid force normal to the plane of the embankment. The lift force is zero when the fluid velocity is zero. The drag force F_d is defined as the fluid force acting on the particle in the direction of the velocity field in the vicinity of the particle. The drag force is normal to the lift force and is zero when the fluid velocity is zero. The remaining force is the submerged weight of the rock particle W_s.

Rock particles on side slopes tend to roll rather than slide, so it is appropriate to consider the stability of rock particles in terms of moments about the point of rotation. In Fig. 6.A1.1b the direction of movement is defined by the vector R. The point of contact about which rotation in the R direction occurs is identified as point "0" in Fig. 6.A1.1c.

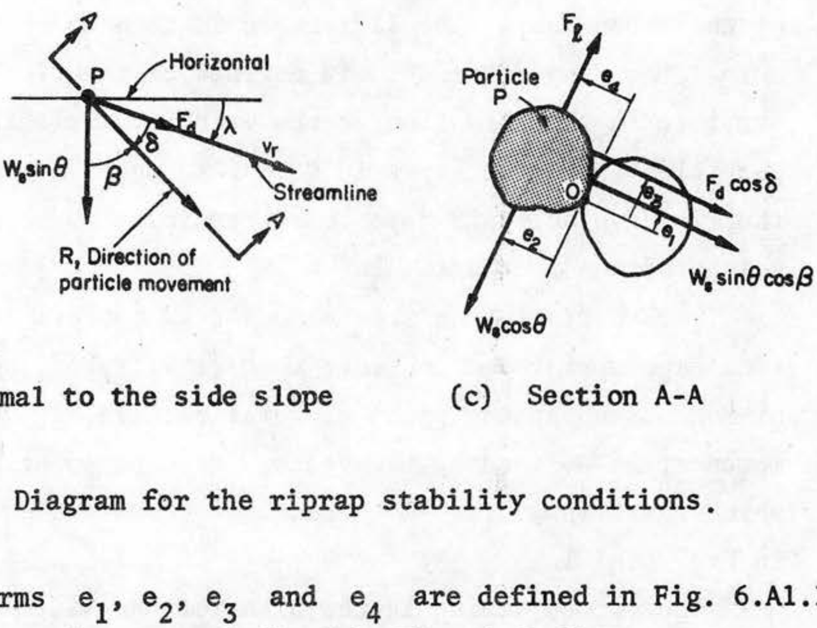
The forces acting in the plane of the side slope are F_d and $W_s \sin\theta$ as shown in Fig. 6.A1.1b. The angle θ is the side slope angle. The lift force acts normal to the side slope and the component of submerged weight $W_s \cos\theta$ acts normal to the side slope as shown in Fig. 6.A1.1c.

At incipient motion, there is a balance of moments about the point of rotation such that

$$e_2 W_s \cos\theta = e_1 W_s \sin\theta \cos\beta + e_3 F_d \cos\delta + e_4 F_\ell \qquad 6.A1.1$$



(a) General view



(b) View normal to the side slope (c) Section A-A

Fig. 6.A1.1 Diagram for the riprap stability conditions.

The moment arms e_1, e_2, e_3 and e_4 are defined in Fig. 6.A1.1c and the angles δ and β are defined in Fig. 6.A1.1b.

The factor of safety S.F. of the particle P against rotation is defined as the ratio of the moments resisting particle rotation out of the bank to the submerged weight and fluid force moments tending to rotate the particle out of its resting position. Accordingly,

$$\text{S.F.} = \frac{e_2 W_s \cos\theta}{e_1 W_s \sin\theta \cos\beta + e_3 F_d \cos\delta + e_4 F_\ell} \qquad 6.\text{A1}.2$$

If there is no flow and the side slope angle is increased to the angle of repose ϕ for the rock particles, the safety factor becomes unity. The no flow condition causes F_d and F_ℓ to become zero. Then,

$$\begin{aligned} \text{S.F.} &= 1.0 \\ \theta &= \phi \\ \beta &= 0 \text{ deg} \\ \lambda &= 0 \text{ deg} \\ \delta &= 90-\lambda-\beta \text{ (See Fig. 6.A1.1b)} \\ &= 90 \text{ deg} \end{aligned}$$

With these values, Eq. 6.A1.2 reduces to

$$\frac{e_2 W_s}{e_1 W_s} \frac{\cos\theta}{\sin\theta} = 1 \qquad 6.\text{A1}.3$$

or

$$\tan\phi = \frac{e_2}{e_1} \qquad 6.\text{A1}.4$$

That is, the ratio of the moment arms e_2/e_1 is characterized by the natural angle of repose ϕ. Further, it is assumed that the ratio e_2/e_1 is invariant to the direction of particle motion indicated by the angle β.

Dividing both numerator and denominator by $e_2 W_s$, Eq. 6.A1.2 is transformed to

$$\text{S.F.} = \frac{\cos\theta \tan\phi}{\eta' \tan\phi + \sin\theta \cos\beta} \qquad 6.\text{A1}.5$$

in which

$$\eta' = \frac{e_3 F_d}{e_2 W_s} \cos\delta + \frac{e_4 F_\ell}{e_2 W_s} \qquad 6.\text{A1}.6$$

The variable η' is called the stability number for the particles on the embankment side slope and is related to the Shields' parameter

$$\frac{\tau_o}{(S_s - 1)\gamma D}$$

Here τ_o is the average tractive force on the side slope in the vicinity of the particle P, γ is unit weight of water and D is the diameter of the rock particles.

The angle λ shown in Fig. 6.A1.1b is the angle between the horizontal and the velocity vector (or drag force) measured in the plane of the side slope. Then

$$\delta = 90-\lambda-\beta \qquad 6.A1.7$$

so

$$\cos\delta = \cos(90-\lambda-\beta)$$

$$= \sin(\lambda+\beta) \qquad 6.A1.8$$

Also $$\sin\delta = \sin(90-\lambda-\beta)$$

$$= \cos\lambda\ \cos\beta - \sin\lambda\ \sin\beta \qquad 6.A1.9$$

It is assumed that the moments of the drag force F_d and the component of submerged weight $W_s\sin\theta$ normal to the path R are balanced so that the direction of particle motion will be along R. Thus

$$e_3F_d\ \sin\delta = e_1W_s\ \sin\theta\ \sin\beta \qquad 6.A1.10$$

It follows then from Eqs. 6.A1.9 and 6.A1.10 that

$$\sin\beta = \frac{e_3F_d\ \sin\delta}{e_1W_s\ \sin\theta}$$

$$= \frac{e_3F_d(\cos\lambda\ \cos\beta - \sin\lambda\ \sin\beta)}{e_1W_s\ \sin\theta} \qquad 6.A1.11$$

or

$$\tan\beta = \frac{\cos\lambda}{\frac{e_1W_s}{e_3F_d}\sin\theta + \sin\lambda} \qquad 6.A1.12$$

The stability number η for particles on a plane bed ($\theta = 0$) with $\delta = 0$ would be

$$\eta = \frac{e_3 F_d}{e_2 W_s} + \frac{e_4 F_\ell}{e_2 W_s} \qquad 6.A1.13$$

according to Eq. 6.A1.6. Also, Eq. 6.A1.5 becomes

$$S.F. = \frac{1}{\eta} \qquad 6.A1.14$$

for flow over a plane flat bed.

For incipient motion conditions for flow over a plane flat bed, S.F. = 1.0 by definition so from Eq. 6.A1.14, $\eta = 1.0$. When the flow along the bed is fully turbulent, the Shields' parameter for incipient motion has the value 0.047 according to Gessler (1971). That is, with $\eta = 1.0$,

$$\frac{\tau_o}{(S_s - 1)\gamma D} = 0.047 \qquad 6.A1.15$$

For flow conditions other than incipient η is the ratio

$$\frac{1}{0.047} \; \frac{\tau_o}{(S_s - 1)\lambda D}$$

or

$$\eta = \frac{21\ \tau_o}{(S_s - 1)\gamma D} \qquad 6.A1.16$$

For convenience, let

$$M = \frac{e_4 F_\ell}{e_2 W_s} \qquad 6.A1.17$$

and

$$N = \frac{e_3 F_d}{e_2 W_s} \qquad 6.A1.18$$

In terms of these new variables, Eq. 6.A1.6 becomes

$$\eta' = M + N \cos\delta \qquad 6.A1.19$$

and Eq. 6.A1.13 becomes

$$\eta = M + N \qquad 6.A1.20$$

Thus η' and η are related by the expression

$$\frac{\eta'}{\eta} = \frac{\frac{M}{N} + \cos\delta}{\frac{M}{N} + 1} \qquad 6.A1.21$$

Eq. 6.A1.21 is represented graphically in Fig. 6.A1.2.

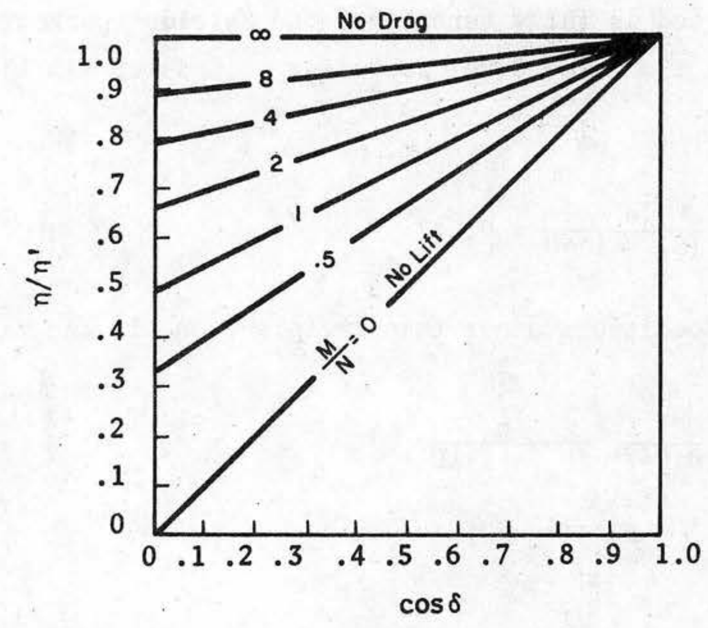


Fig. 6.A1.2 Ratio of stability factors.

The problem is to select the proper value of the ratio M/N so that the stability factor on a side slope η' can be related to the stability factor on a plane horizontal bed η, which in turn is related to the Shields' parameter. The assumption that the drag force F_d is zero means M/N is infinite, β is zero and $\eta' = \eta$. The assumption of zero lift force F_ℓ means M/N is zero and $\eta'/\eta = \cos\delta$.

In considering incipient motion of riprap particles, the ratios F_ℓ/F_d and e_4/e_3 vary depending on the turbulent conditions of the flow and the interlocking arrangement of the rock particles. In referring to Fig. 6.A1.1c assume that

$$\frac{e_4}{e_3} \simeq 2 \qquad 6.A1.22$$

then a choice for F_ℓ/F_d is

$$\frac{F_\ell}{F_d} \simeq \frac{1}{2} \qquad 6.A1.23$$

so that

$$\frac{M}{N} = \frac{e_4}{e_3}\,\frac{F_\ell}{F_d} \simeq 1. \qquad 6.A1.24$$

With $M/N = 1$, Eq. 6.A1.21 becomes

$$\frac{\eta'}{\eta} = \frac{1 + \cos\delta}{2} \qquad 6.A1.25$$

or by using Eq. 6.A1.8

$$\frac{\eta'}{\eta} = \frac{1 + \sin(\lambda+\beta)}{2} \qquad 6.A1.26$$

In Eq. 6.A1.12, the term e_1W_s/e_3F_d can be written

$$\frac{e_1W_s}{e_3F_d} = \frac{e_2W_s}{e_3F_d}\,\frac{e_1}{e_2}$$

$$= \frac{1}{N}\,\frac{1}{\tan\phi} \qquad 6.A1.27$$

according to Eqs. 6.A1.18 and 6.A1.4. For $M/N = 1$, Eq. 6.A1.20 becomes

$$N = \frac{\eta}{2} \qquad 6.A1.28$$

If we substitute Eqs. 6.A1.27 and 6.A1.28 into Eq. 6.A1.12, the expression for β becomes

$$\tan\beta = \frac{\cos\lambda}{\frac{2\sin\theta}{\eta\tan\phi} + \sin\lambda} \qquad 6.A1.29$$

In summary, the safety factor for rock riprap on side slopes where the flow has a non-horizontal velocity vector is related to properties of the rock, side slope and flow by the following equations:

$$S.F. = \frac{\cos\theta \tan\phi}{\eta'\tan\phi + \sin\theta \cos\beta} \qquad 6.A1.30$$

in which

$$\beta = \tan^{-1}\left\{\frac{\cos\lambda}{\frac{2\sin\theta}{\eta\tan\phi} + \sin\lambda}\right\} \qquad 6.A1.31$$

$$\eta = \frac{21\ \tau_o}{(S_s - 1)\gamma D} \qquad 6.A1.32$$

and

$$\eta' = \eta\ \{\frac{1 + \sin(\lambda+\beta)}{2}\} \qquad 6.A1.33$$

Given a rock size D, of specific weight S_s and angle of repose ϕ and given a velocity field at an angle λ to the horizontal producing a tractive force τ_o on the side slope of angle θ, the set of 4 equations (Eqs. 6.A1.30, 6.A1.31, 6.A1.32, and 6.A1.33) can be solved to obtain the safety factor S.F. *If S.F. is greater than unity, the riprap is safe from failure; if S.F. is unity, the rock is at the condition of incipient motion; if S.F. is less than unity, the riprap will fail.*

6.A1.2 Horizontal flow on a side slope

In many circumstances, the flow angularity with the horizontal is small; i.e., $\lambda \simeq 0$. Then Eqs. 6.A1.31 and 6.A1.32 reduce to

$$\beta = \tan^{-1} \{\frac{\eta \tan\phi}{2 \sin\theta}\} \qquad 6.A1.34$$

and

$$\eta' = \eta \{\frac{1 + \sin\beta}{2}\} \qquad 6.A1.35$$

When Eqs. 6.A1.34 and 6.A1.35 are substituted into Eq. 6.A1.30, the expression for the safety factor for horizontal flow on a side slope is

$$S.F. = \frac{S_m}{2} \{\sqrt{\xi^2 + 4} - \xi\} \qquad 6.A1.36$$

in which

$$\xi = S_m \eta \sec\theta \qquad 6.A1.37$$

and

$$S_m = \frac{\tan\phi}{\tan\theta} \qquad 6.A1.38$$

If we solve Eqs. 6.A1.36 and 6.A1.37 for η, then

$$\eta = \{\frac{S_m^2 - S.F.^2}{S.F.\ S_m^2}\} \cos\theta \qquad 6.A1.39$$

The term S_m is the safety factor for riprap on a side slope with no flow. Unless the flow is up the slope, the safety factor for the riprap cannot be greater than S_m.

6.A1.3 Flow on a plane sloping bed

Flow over a plane bed at a slope of α degrees in the downstream direction is equivalent to oblique flow on a side slope with $\theta = \alpha$ and $\lambda = 90°$.

Then, according to Eq. 6.A1.31, $\beta = 0°$ and from Eq. 6.A1.33.

$$\eta' = \eta \{\frac{1 + \sin(90° + 0°)}{2}\} = \eta \qquad 6.A1.40$$

It follows from Eq. 6.A1.30 that

$$S.F. = \frac{\cos\alpha \tan\phi}{\eta\tan\phi + \sin\alpha} \qquad 6.A1.41$$

for flow on a plane bed sloping α degrees to the horizontal. Alternatively solving for η in Eq. 6.A1.41

$$\eta = \cos\alpha \left\{\frac{1}{S.F.} - \frac{\tan\alpha}{\tan\phi}\right\} \qquad 6.A1.42$$

6.A1.4 Flow on a horizontal bed

For fully developed rough turbulent flow over a plane horizontal bed (α = 0) of rock riprap, Eq. 6.A1.41 reduces to

$$S.F. = \frac{1}{\eta} \qquad 6.A1.43$$

If the riprap particles are at the condition of incipient motion, S.F. = 1 so $\eta = 1$ and from Eq. 6.A1.32

$$\frac{\tau_o}{(S_s - 1)\gamma D} = 0.047 \qquad 6.A1.44$$

which is Shields' criteria for the initiation of motion.

6.A2.0 THE REPRESENTATIVE GRAIN SIZE FOR RIPRAP

The concept of a representative grain size for riprap is simple. A uniformly graded riprap with a median size D_{50} scours to a greater depth than a well-graded mixture with the same median size. The uniformly distributed riprap scours to a depth at which the velocity is less than that required for the transportation of D_{50} size rock. The well-graded riprap, on the other hand, develops an armor plate. That is, some of the finer materials, including sizes up to D_{50} and larger, are transported by the high velocities, leaving a layer of large rock sizes which cannot be transported under the given flow conditions. Thus, the size of rock representative of the stability of the riprap is determined by the larger sizes of rock. The representative grain size D for riprap is larger than the median rock size D_{50}.

In studies of scour below culvert outlets, Stevens (1968) obtained the following expression for the representative grain size of well-graded materials:

$$D = \left\{\frac{\sum_{i=1}^{10} D_i^3}{10}\right\}^{1/3} \qquad 6.A2.1$$

where

$$D_i(i=1) = \frac{D_0 + D_{10}}{2}$$

$$D_i(i=2) = \frac{D_{10} + D_{20}}{2}$$

$$\vdots$$

$$D_i(i=10) = \frac{D_{90} + D_{100}}{2}$$

The terms D_0, D_{10}, ..., D_{100} are the sieve diameters of the riprap for which 0 percent, 10 percent, ..., 100 percent of the material is finer by weight.

Eq. 6.A2.1 is equivalent to taking the arithmetic average of the sum of the weights of the individual rock sizes. In Stevens' studies (1968), the ratio of the representative size to the median size D/D_{50} was varied between 1.005 and 2.25.

6.A2.1 <u>The recommended representative grain size for riprap</u>

In Fig. 6.4.3, the recommended gradation for riprap is illustrated in terms of D_{50}. The computations of the representative grain size D for the recommended gradation is given in Table 6.A2.1.

Table 6.A2.1 Data for suggested gradation.

Percent finer	Sieve Dia.	i	D_i
0	0.25 D_{50}	--	--
10	0.35 D_{50}	1	0.28 D_{50}
20	0.5 D_{50}	2	0.43 D_{50}
30	0.65 D_{50}	3	0.57 D_{50}
40	0.8 D_{50}	4	0.72 D_{50}
50	1.0 D_{50}	5	0.90 D_{50}
60	1.2 D_{50}	6	1.10 D_{50}
70	1.6 D_{50}	8	1.50 D_{50}
90	1.8 D_{50}	9	1.70 D_{50}
100	2.0 D_{50}	10	1.90 D_{50}

The rock sizes in the last column in Table 6.A2.1 are used in Eq. 6.A2.1 to find the representative grain size D. The computed D is

$$D = \left\{\frac{\sum_{i=1}^{10} D_i^3}{10}\right\}^{1/3}$$

$$= 1.25\ D_{50}$$

This effective grain size of the mixture corresponds to the size D_{65} of the riprap.

6.A2.2 Comparison of representative grain sizes

In scour investigations, Stevens (1968) compared the scour produced in two widely different gradations of riprap having the same median diameter, D_{50} = 1.2 in. The riprap gradations are shown in Fig. 6.A2.1 as curves 1-2 and 3-4.

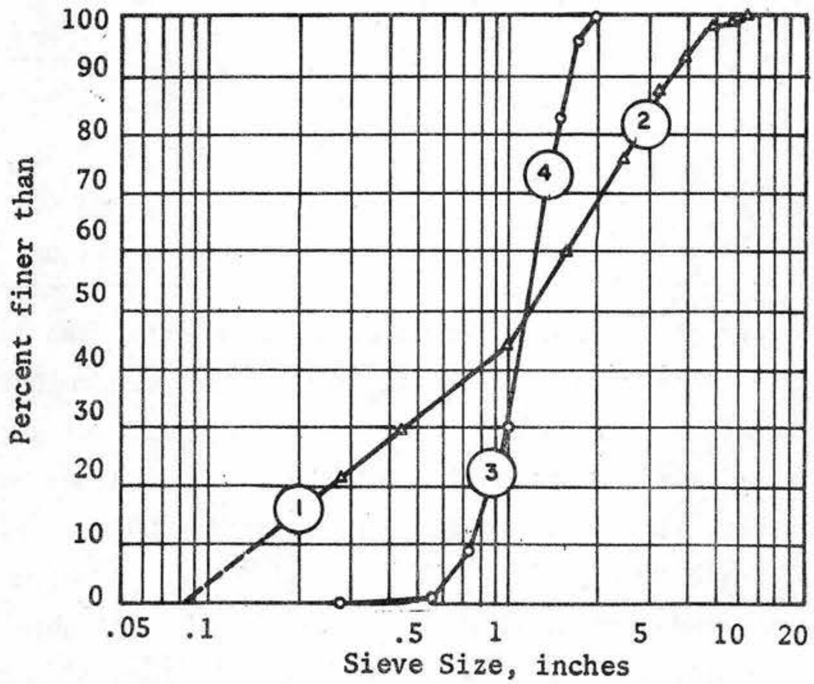


Fig. 6.A2.1 Gradation curves.

The sieve sizes of four segments of the two gradation curves., (segments 1, 2, 3 and 4) are listed in Table 6.A2.2.

Table 6.A2.2 Data for selected gradations.

Percent Finer	Curve 1 Dia. in.	Curve 2 Dia. in.	Curve 3 Dia. in.	Curve 4 Dia. in.
0	0.08	--	0.28	--
10	0.15	--	0.75	--
20	0.26	--	0.88	--
30	0.46	--	1.00	--
40	0.80	--	1.10	--
50	1.20	1.20	1.20	1.20
60	--	1.60	--	1.28
70	--	2.15	--	1.35
80	--	2.80	--	1.45
90	--	3.70	--	1.60
100	--	6.60	--	2.00

From these data, the representative grain sizes for the four gradations are computed using Eq. 6.A2.1. The representative grain size D and the ratio of representative diameter to median diameter D/D_{50} are given in Table 6.A2.3 for the four gradations (refer to Fig. 6.A2.1).

Table 6.A2.3 Representative diameter for various gradations.

Curve	D in.	D/D_{50}
1-2	2.70	2.25
3-2	2.71	2.26
1-4	1.21	1.01
3-4	1.27	1.06

Inspection of the values in Table 6.A2.3 shows that the larger sizes in the gradation have a dominant effect in the determination of the representative grain size. Thus, the large sizes of a gradation are the important sizes for stability. The fines should not be neglected, however, since lack of fines necessitates using a filter under the riprap.

If a riprap gradation has a wide range of sizes, the thickness of the riprap layer must be large enough to permit the loss of some fines (armorplating) without uncovering the protected materials (filter or bank material). The recommended thickness for the recommended gradation (Fig. 6.4.3) is $2D_{50}$ which is also D_{100}. For gradations with larger gradation coefficients, the thickness must be at least the D_{100} size. For large gradation coefficients ($G > 3.0$), the thickness can be increased to $1.5D_{100}$ to provide enough material for armorplating.

6.A2.3 <u>Effective size for log-normal gradations</u>

In section 3.A1.3, an expression was given for the mean size of the bed material based on the assumption that the bed material has a log-normal distribution. The cumulative distribution function of the log-normal distribution can be expressed as

$$F_X(x) = \int_o^x \frac{1}{\sigma_y x (2\pi)^{1/2}} \exp\{-1/2(\frac{\ln x - \mu_y}{\sigma_y})^2\} dx \qquad 6.A2.2$$

where exp means raising the base of the natural log e to the power $(-1/2((\ln x-\mu_y)/\sigma_y)^2)$.

where x = particle diameter

Y = a normally (Gaussian) distributed variable

= $\ln x$

μ_y = mean of Y

σ_y = standard deviation of Y

The median value of x can be written

$$x_{med} = D_{50} = \exp(\mu_y)$$

Thus

$$\mu_y = \ln D_{50} \qquad 6.A2.3$$

Similarly

$$\sigma_y = \ln G \qquad 6.A2.4$$

where G is the gradation coefficient. It follows that the expression for the mean size of bed material is

$$D_m = D_{50} \exp\{\frac{1}{2} (\ln G)^2\} \qquad 6.A2.5$$

Mahmood[1] has found that the distribution of other properties of the bed material such as the projected area of the particle, the volume of the particle, or the surface area of the particle can also be represented by a log-normal distribution. The volume of a particle properties can be written

$$\forall = bX^3 \qquad 6.A2.6$$

where V = the volume of the particle

b = constant

x = particle diameter

[1]Mahmood, Khalid, 1973, Lognormal size distribution of particulate matter: Jour. of Sedimentary Petrology, vol. 43, no. 4, December, pp. 1161-1166.

Assuming a constant specific weight of the sediment, the weight of the particles can be represented by Eq. 6.A2.6.

The weight of a bed-material particle is important to the stability of the particle. Thus, it is more meaningful to compute the representative particle size based on the weight of the particle than on its diameter. From the expressions given by Mahmood, the representative size of the bed material based on the weight of the particles is given by

$$D = D_{50} \exp\{\frac{3}{2} (\ln G)^2\} \qquad 6.A2.7$$

The ratio of the mean weight size D to the mean diameter size D_m is given by

$$\frac{D}{D_m} = \frac{\exp\{\frac{3}{2} (\ln G)^2\}}{\exp\{\frac{1}{2} (\ln G)^2\}} \qquad 6.A2.8$$

which is always greater than one for nonuniform grain size distributions. For uniform distributions, there is only one particle size and the means are identical.

6.A3.0 RELATION BETWEEN VELOCITY AND SHEAR

In order to design riprap, it is necessary to be able to relate the tractive force acting on the riprapped bed or bank to the fluid velocity in the vicinity of the riprap. For fully turbulent flow, the relation for the local velocity v at a distance y above the bed is given by Eq. 2.3.14 (with $y' = \frac{D}{30.2}$ for the hydraulically rough boundary) or

$$v = 2.5\ V_* \ln\left(30.2\ \frac{y}{D_c}\right) \qquad 6.A3.1$$

in which V_* is the shear velocity which is

$$V_* = \left(\frac{\tau_o}{\rho}\right)^{1/2} \qquad 6.A3.2$$

by definition. This velocity distribution equation was employed by Einstein (1950) in his bed-load function research.

If we select the velocity at a distance $y = D$ above the bed as the reference velocity v_r, then

$$v_r = 2.5\ V_* \ln 30.2$$

or

$$v_r = 8.5\ V_* \qquad 6.A3.3$$

From Eqs. 6.A3.2 and 6.A3.3, the relation between v_r and τ_o is

$$\rho v_r^2 = 72\ \tau_o \qquad 6.A3.4$$

This relation is strictly valid only for uniform flow in wide prismatic channels in which the flow is fully turbulent. For the purposes of riprap design, Eq. 6.A3.4 can be employed when the flow is accelerating- for example, on the nose of a spur dike. The equation should not be used in areas where the flow is decelerating or below energy dissipating structures. In these areas, the shear stress is larger than would be calculated by Eq. 6.A3.4.

By substituting Eq. 6.A3.4 into Eq. 6.4.5, the expression for the stability factor η becomes

$$\eta = \frac{0.30\, v_r^2}{(S_s - 1) g D} \qquad 6.A3.5$$

The average velocity in the vertical V is obtained from Eq. 2.3.16 with $x = 1$ for the large riprap sizes. The expression can be written

$$V = 2.5\, V_* \ln \left(12.3 \frac{y_o}{D}\right) \qquad 6.A3.6$$

in which y_o is the depth of flow. The ratio of the reference velocity v_r to the depth-averaged velocity is

$$\frac{v_r}{V} = \frac{2.5\, V_* \ln (30.2)}{2.5\, V_* \ln \left(12.3 \frac{y_o}{D}\right)}$$

or

$$\frac{v_r}{V} = \frac{3.4}{\ln\left(12.3 \frac{y_o}{D}\right)} \qquad 6.A3.7$$

Eq. 6.A3.7 is obtained from Eqs. 6.A3.3 and 6.A3.6. Now the expression for the stability factor η can be written in terms of the depth-averaged velocity. From Eqs. 6.A3.5 and 6.A3.7

$$\eta = \frac{\beta V^2}{(S_s - 1) g D} \qquad 6.A3.8$$

in which

$$\beta = 0.30 \left\{ \frac{3.4}{\ln\left(12.3 \frac{y_o}{D}\right)} \right\}^2 \qquad 6.A3.9$$

As shown in Fig. 6.A3.1 the value of β varies from 0.30 for relatively shallow flows to a value of 0.04 for $y_o/D = 1000$.

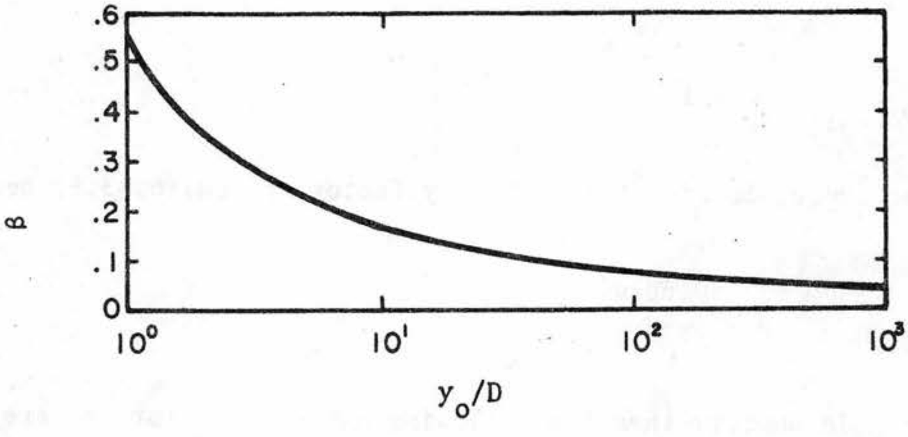


Fig. 6.A3.1 Relation between β and y_o/D.

Hydraulic Engineering Circular No. 11[1] uses the expression

$$\frac{v_s}{\overline{V}} = \frac{1}{0.958\ \log(\frac{y_o}{D}) + 1} \qquad 6.A3.10$$

to determine the velocity "against the stone". Here v_s is the velocity against the stone, $\overline{V}$ is the mean velocity in the channel and log is the logarithm to the base 10.

In wide channels, the depth-averaged velocity and the mean velocity in the channel are nearly equal; i.e., $V \simeq \overline{V}$. Then the velocity against the stone is related to the reference velocity by the expression

$$\frac{v_r}{v_s} = \frac{v_r}{V}\frac{V}{v_s}$$

or

$$\frac{v_r}{v_s} = \frac{3.4\{.958\ \log(\frac{y_o}{D}) + 1\}}{\ln(12.3\ \frac{y_o}{D})} \qquad 6.A3.11$$

[1] Searcy, J. K., 1967, Use of riprap for bank protection: Hydr. Eng. Cir. No. 11, Hydraulics Branch, Bridge Division, Office of Engineering and Operations, Federal Highway Administration, Washington, D.C.

according to Eqs. 6.A3.7 and 6.A3.10. For values of y_o/D between $1x10^0$ and $1x10^6$, the value of the v_r/v_s is nearly 1.4. By letting

$$\frac{v_r}{v_s} = 1.4 \qquad 6.A3.12$$

the expression for the stability factor η (Eq. 6.A3.5) becomes

$$\eta = \frac{0.60\ v_s^2}{(S_s - 1)gD} \qquad 6.A3.13$$

In summary then the following expressions for η are equivalent:

$$\eta = \frac{21\ \tau_o}{(S_s - 1)\gamma D} \qquad 6.4.5$$

$$\eta = \frac{0.30\ v_r^2}{(S_s - 1)gD} \qquad 6.A3.5$$

$$\eta = \frac{0.60\ v_s^2}{(S_s - 1)gD} \qquad 6.A3.13$$

and $$\eta = \frac{\beta V^2}{(S_s - 1)gD} \qquad 6.A3.8$$

in which

$$\beta = 0.30\ \left\{\frac{3.4}{\ln(12.3\ \frac{y_o}{D})}\right\}^2 \qquad 6.A3.9$$

6.A4.0 RIPRAP DESIGN ON AN EMBANKMENT

When the drawdown through a bridge opening is large there is an appreciable downslope component to the velocity vector on the nose of an embankment end or spur dike. This downslope component of velocity is illustrated in the model embankment shown in Fig. 6.A4.1. The model study was reported by Simons and Lewis[1].

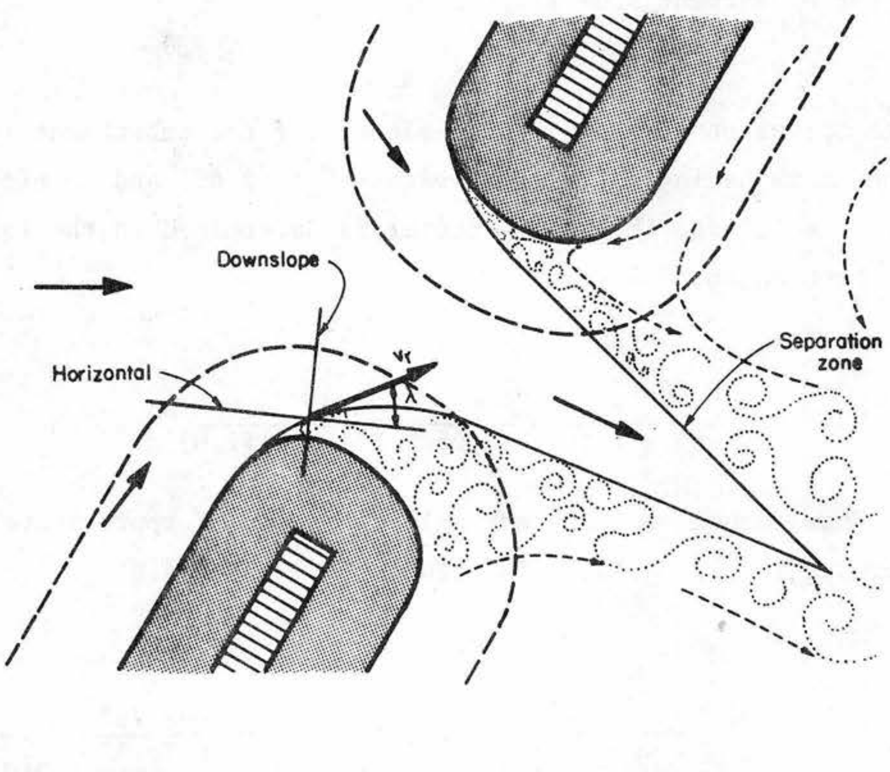


Fig. 6.A4.1 Flow around an embankment end.

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[1] Simons, D. B., and Lewis, G. L., 1971, Flood protection at bridge crossings: CER71-72DBS-GLL10, Colorado State University, Fort Collins, Colorado.

For the case shown in Fig. 6.A4.1, the angle between the horizontal and the velocity vector at the point "P" is

$$\lambda = 20°$$

The method of computing the streamlines and velocities around an embankment end is given by Lewis[1]. If the drawdown through the bridge is large, the reference velocity v_r in the vicinity of the riprap can be as large as 6 fps.

Suppose that the reference velocity is

$$v_r = 6 \text{ fps}$$

and the embankment side slope angle is

$$\theta = 18.4°$$

which corresponds to a 3:1 side slope. If the embankment is covered with dumped rock having a specific weight $S_s = 2.65$ and an effective rock size $D = 1.0$ ft, the safety factor is determined in the following manner.

From Eq. 6.A3.5

$$\eta = \frac{0.30\ v_r^2}{(S_s - 1)gD} = \frac{(0.30)(6)^2}{(2.65 - 1)(32.2)(1.0)} = 0.203$$

This dumped rock has an angle of repose of approximately 35° according to Fig. 3.7.3. Therefore, from Eq. 6.4.6

$$\beta = \tan^{-1}\left\{\frac{\cos\lambda}{\frac{2\sin\theta}{\eta\tan\phi} + \sin\lambda}\right\} = \tan^{-1}\left\{\frac{\cos 20°}{\frac{2\sin 18.4°}{0.203\tan 35°} + \sin 20°}\right\} = 11°$$

[1] Lewis, G. L., 1972, Riprap protection of bridge footings: Ph.D. Dissertation, Dept. of Civil Eng., Colorado State University, Fort Collins, Colorado.

and from Eq. 6.4.4

$$\eta' = \eta\{\frac{1 + \sin(\lambda+\beta)}{2}\} = 0.203\{\frac{1 + \sin(20° + 11°)}{2}\} = 0.154$$

The safety factor for the rock is given by Eq. 6.4.3 or

$$S.F. = \frac{\cos\theta \tan\phi}{\eta'\tan\phi + \sin\theta \cos\beta} = \frac{\cos 18.4° \tan 35°}{0.154 \tan 35° + \sin 18.4° \cos 11°} = 1.59$$

Thus, with a safety factor of 1.59, this rock is more than adequate to withstand the flow velocity.

By repeating the above calculations over the range of interest for D (with $\phi = 35°$), the curve given in Fig. 6.A4.2 is obtained. This

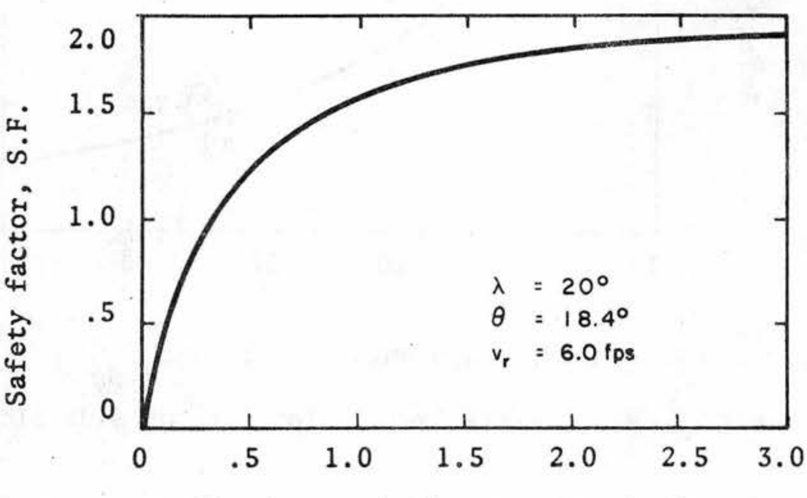

Fig. 6.A4.2 Safety factors for various rock sizes on a side slope.

curve shows that the incipient motion rock size is approximately 0.35 ft and that the maximum safety factor is less than 2.0 on the 3:1 side slope.

The safety factor of a particular side slope riprap design can be increased by decreasing the side slope angle θ. If the side slope angle is decreased to zero degrees, then Eq. 6.4.14 is applicable and

$$S.F. = \frac{1}{\eta} = \frac{1}{0.203} = 4.93$$

The curve in Fig. 6.A4.3 relates the safety factor and side slope angle of the embankment (for $\lambda = 20°$, $D = 1.0$ ft and $v_r = 6.0$ fps). The curve is obtained by employing Eqs. 6.4.6, 6.4.4 and 6.4.3 for various values of θ.

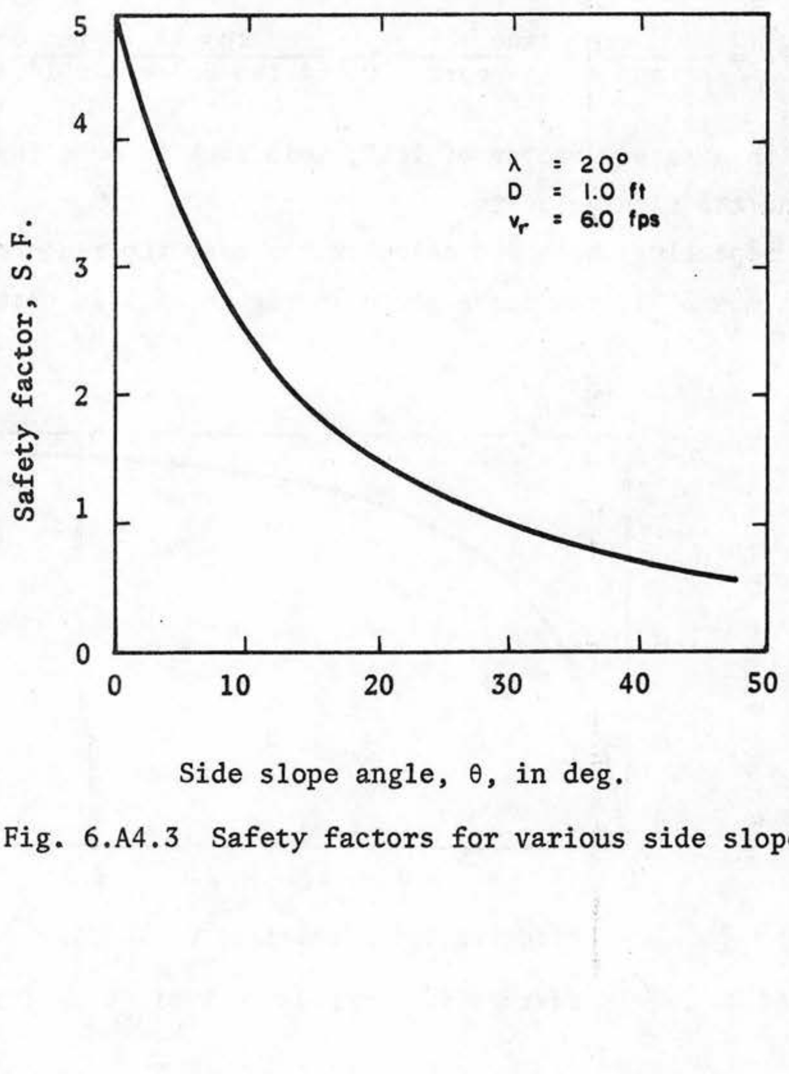

Fig. 6.A4.3 Safety factors for various side slopes.

6.A5.0 DESIGN AID FOR SIDE SLOPE RIPRAP

When the velocity along a side slope has a downslope component, the sizing of the riprap is accomplished by employing the complex equations given in Section 6.4.1. An example of sizing riprap using these equations is given in Appendix Section 6.A4.0. When the velocity along a side slope has no downslope component (i.e., the velocity vector is along the horizontal), some simple design aids can be developed.

For horizontal flow along a side slope, the equations relating the safety factor, the stability number, the side slope angle, and the angle of repose for the rock are (from Section 6.4.2)

$$\eta = \left\{\frac{S_m^2 - (S.F.)^2}{(S.F.)\ S_m^2}\right\} \cos\theta \qquad 6.A5.1$$

and

$$S_m = \frac{\tan\phi}{\tan\theta} \qquad 6.A5.2$$

The interrelation of the variables in these two equations is represented in Fig. 6.A5.1. Here, the specific weight of the rock is taken as 2.65 and a safety factor of 1.5 is employed.

The recommended safety factor for the design of riprap is

$$S.F. = 1.5$$

This recommendation is the result of studies of the riprap embankment model data obtained by Lewis. These studies were reported by Simons and Lewis[1].

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[1] Simons, D. B. and Lewis, G. L., 1971, Flood protection at bridge crossings: CER71-72DBS-GLL10, Colorado State University, Fort Collins, Colorado.

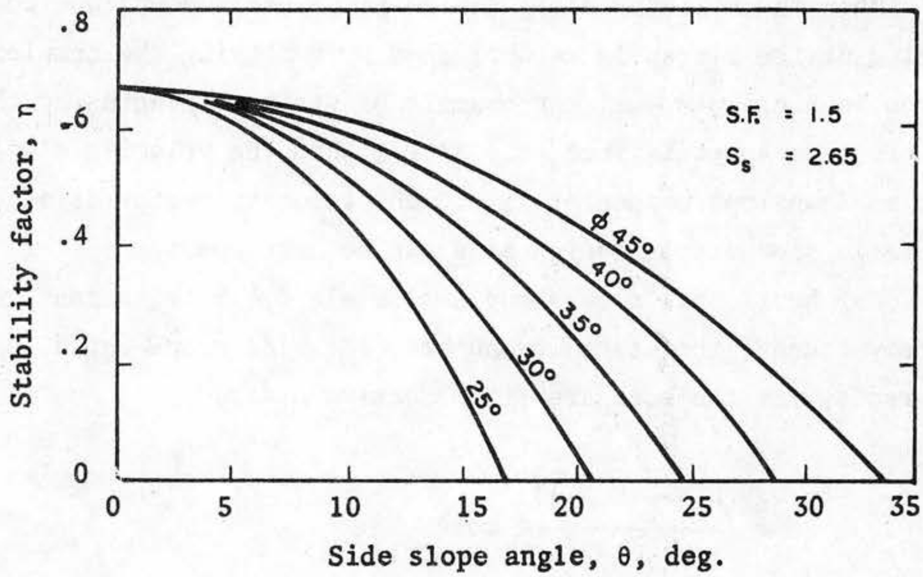

Fig. 6.A5.1 Stability factors for a 1.5 safety factor for horizontal flow along a side slope.

The curves in Fig. 6.A5.1 are computed in the following manner.

(1) Select an angle of repose ϕ. For example

$\phi = 45°$

(2) Select a side slope angle. For example

$\theta = 25°$

(3) Compute S_m from Eq. 6.A5.2

$$S_m = \frac{\tan 45°}{\tan 25°} = 2.4$$

(4) Compute η from Eq. 6.A5.1 with S.F. = 1.5

$$\eta = \{\frac{(2.14)^2 - (1.5)^2}{(1.5)(2.14)^2}\} \cos 25° = 0.29$$

(5) Repeat the above steps for the full range of interest for ϕ and θ.

In a design problem, the side slope angle is known and the angle of repose can be estimated (See Fig. 3.7.3). With these values the stability factor is obtained from Fig. 6.A5.1. For example, if

$$\phi = 30°$$

and

$$\theta = 10°$$

then

$$\eta = 0.52$$

If the shear stress on the side slope is known, then (from Eq. 6.4.5)

$$D = \frac{21\ \tau_o}{(S_s - 1)\gamma\eta} \qquad 6.A5.3$$

$$= \frac{21\ \tau_o}{(2.65 - 1)(62.4)\eta}$$

$$= 0.20\ \frac{\tau_o}{\eta}$$

In our example if

$$\tau_o = 2 \text{ psf}$$

then

$$D = \frac{(0.20)(2)}{(0.52)} = 0.77 \text{ ft.}$$

If the reference velocity on the side slope is known, then (from Eq. 6.A3.5)

$$D = \frac{0.30\ v_r^2}{(S_s - 1)g\eta}$$

$$= \frac{(0.30\ v_r^2}{(2.65 - 1)(32.2)\eta}$$

$$= .0056\ \frac{v_r^2}{\eta}$$

In our example, if

$$v_r = 6 \text{ fps}$$

$$D = \frac{(.0056)(6)^2}{0.52} = 0.40 \text{ ft.}$$

If the average velocity V and the depth y_o are known, more design aids should be prepared from Eqs. 6.A3.8 and 6.A3.9.

6.A6.0 FILTER DESIGN

The requirements for a gravel filter are given in Section 6.4.7. The gradation of a filter should be such that

$$\frac{D_{50}(\text{Filter})}{D_{50}(\text{Base})} < 40 \tag{6.A6.1}$$

$$5 < \frac{D_{15}(\text{Filter})}{D_{15}(\text{Base})} < 40 \tag{6.A6.2}$$

and

$$\frac{D_{15}(\text{Filter})}{D_{85}(\text{Base})} < 5 \tag{6.A6.3}$$

6.A6.1 Filter design example 1

Consider a riprap blanket resting on a base material. The properties of the riprap and base material are given in Table 6.A6.1. *Design a filter to be placed between the riprap and the base material.*

Table 6.A6.1 Sizes of materials.

Base Material Sand	Riprap Gravel
D_{85} = 1.50 mm	D_{85} = 24 mm
D_{50} = 0.75 mm	D_{50} = 12 mm
D_{15} = 0.38 mm	D_{15} = 6 mm

In accordance with the recommended sizes for filters, we note that

$$\frac{D_{50}(\text{Riprap})}{D_{50}(\text{Base})} = \frac{12}{0.75} = 16$$

which satisfies expression 6.A6.1. Also

$$\frac{D_{15}(\text{Riprap})}{D_{15}(\text{Base})} = \frac{6}{0.38} = 16$$

which satisfies the requirement 6.A6.2. Moreover

$$\frac{D_{15}(\text{Riprap})}{D_{85}(\text{Base})} = \frac{6}{1.5} = 4$$

which satisfies the requirement 6.A6.3. The riprap itself satisfies the requirements for the filter so no filter is needed.

6.A6.2 Filter design example 2

The following filter design is taken from Anderson et al.[1] The properties of the base material and the riprap are given in Table 6.A6.2.

Table 6.A6.2 Sizes of materials.

Base Material Sand	Riprap Rock
D_{85} = 1.5 mm	D_{85} = 400 mm
D_{50} = 0.5 mm	D_{50} = 200 mm
D_{15} = 0.17 mm	D_{15} = 100 mm

The riprap does not contain sufficient fines to act as the filter because

$$\frac{D_{15}(\text{Riprap})}{D_{85}(\text{Base})} = \frac{100}{1.5} = 67$$

which is much greater than 5, the recommended upper limit (expression 6.A6.3). Also

$$\frac{D_{15}(\text{Riprap})}{D_{15}(\text{Base})} = \frac{100}{0.17} = 600$$

which is much greater than 40, the recommended upper limit (expression 6.A6.2).

[1] Anderson, A.G., Paintal, A.S., and Davenport, J.T., 1968, Tentative design procedure for riprap lined channels: Project Report No. 96, St. Anthony Falls Hydraulic Laboratory, University of Minnesota, Minneapolis, Minnesota.

The properties of the filter to be placed adjacent to the base are as follows:

(1) $$\frac{D_{50}(\text{Filter})}{D_{50}(\text{Base})} < 40$$

so $$D_{50}(\text{Filter}) < (40)(0.5) = 20 \text{ mm}$$

(2) $$\frac{D_{15}(\text{Filter})}{D_{15}(\text{Base})} < 40$$

so $$D_{15}(\text{Filter}) < (40)(0.17) = 6.8 \text{ mm}$$

(3) $$\frac{D_{15}(\text{Filter})}{D_{85}(\text{Base})} < 5$$

so $$D_{15}(\text{Filter}) < (5)(1.5) = 7.5 \text{ mm}$$

(4) $$\frac{D_{15}(\text{Filter})}{D_{15}(\text{Base})} > 5$$

so $$D_{15}(\text{Filter}) > (5)(.17) = 0.85 \text{ mm}$$

Thus, with respect to the base

$$0.85 \text{ mm} < D_{15}(\text{Filter}) < 6.8 \text{ mm}$$

and $$D_{50}(\text{Filter}) < 20 \text{ mm}$$

The properties of the filter to be placed adjacent to the riprap are as follows:

(1) $$\frac{D_{50}(\text{Riprap})}{D_{50}(\text{Filter})} < 40$$

so $$D_{50}(\text{Filter}) > \frac{200}{40} = 5 \text{ mm}$$

(2) $$\frac{D_{15}(\text{Riprap})}{D_{15}(\text{Filter})} > 5$$

so $$D_{15}(\text{Filter}) < \frac{100}{5} = 20 \text{ mm}$$

(3) $$\frac{D_{15}(\text{Riprap})}{D_{15}(\text{Filter})} < 40$$

so $$D_{15}(\text{Filter}) > \frac{100}{40} = 2.5 \text{ mm}$$

(4) $$\frac{D_{15}(\text{Riprap})}{D_{85}(\text{Filter})} < 5$$

so $$D_{85}(\text{Filter}) > \frac{100}{5} = 20 \text{ mm}$$

Therefore, with respect to the riprap, the filter must satisfy these requirements

$$2.5 \text{ mm} < D_{15}(\text{Filter}) < 20 \text{ mm}$$

$$D_{50}(\text{Filter}) > 5 \text{ mm}$$

$$D_{85}(\text{Filter}) > 20 \text{ mm}$$

These riprap filter requirements along with those for the base material are shown in Fig. 6.A6.1. Any filter having sizes represented by the double cross-hatched area is satisfactory. For example, a good filter could have these sizes:

$$D_{85} = 40 \text{ mm}$$

$$D_{50} = 10 \text{ mm}$$

$$D_{15} = 4 \text{ mm}$$

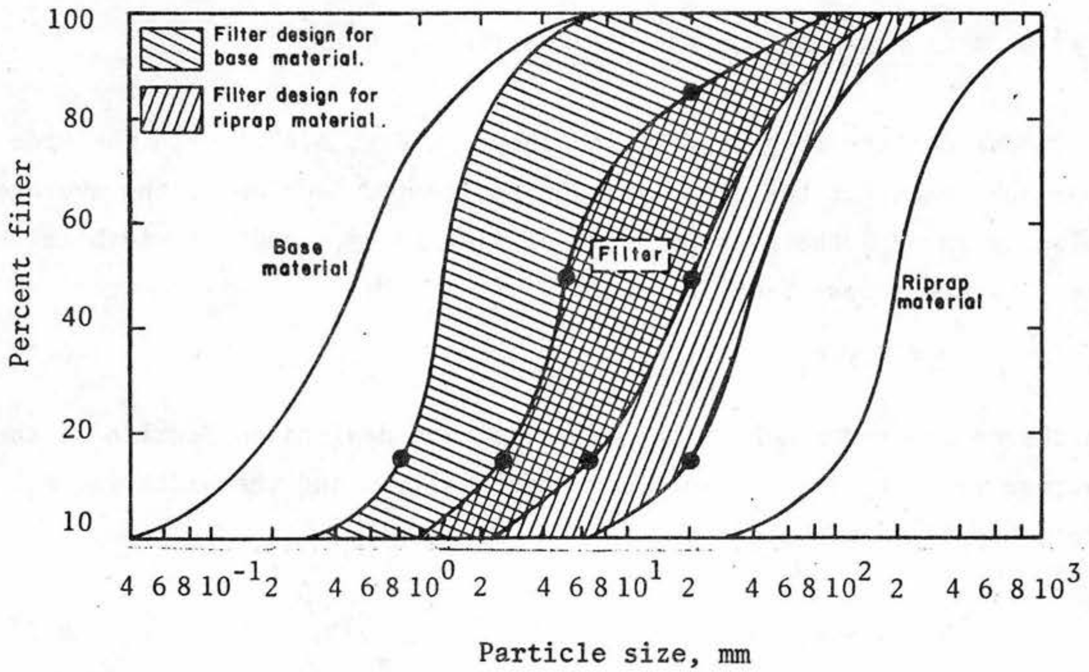

Fig. 6.A2.1 Gradations of filter blanket for example 2 (after Anderson et al., 1968).

6.A7.0 STRAUB'S EQUATION FOR CLEAR-WATER SCOUR

Consider the long contraction shown in Fig. 6.A7.1. In the wide approach reach, at the cross section designated Section 1, the average velocity is V_1, the average depth of flow is y_1 and the width is W_1. The flowrate across Section 1 is

$$Q = V_1 y_1 W_1 \qquad 6.A7.1$$

In the contracted reach at the cross section designated Section 2, the average velocity is V_2, the flow depth is y_2 and the width is W_2. The flowrate across Section 2 is

$$Q = V_2 y_2 W_2 \qquad 6.A7.2$$

For a given flowrate Q and a given contraction ratio W_2/W_1 we would like to know the depth ratio y_2/y_1 for the clear-water scour case.

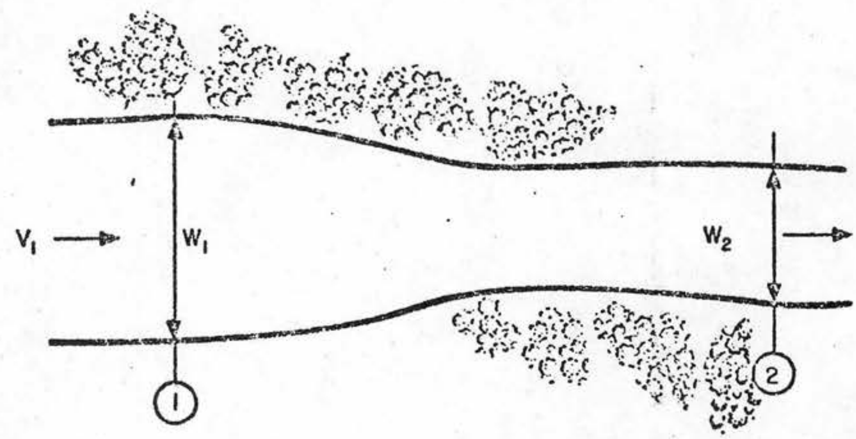

Fig. 6.A7.1 Plan view of the long contraction.

In the clear-water scour case, there is no transport in the wide upstream section. The shear stress here is less than the critical shear stress (the shear stress causing initial movement of the bed particles). That is,

$$\tau_1 = \gamma y_1 S_{f_1} < \tau_c \qquad 6.A7.3$$

Here S_{f_1} is the slope of the energy grade line at Section 1.

Assume for the time being that scour occurs in the long contraction. Scour will continue until the bed shear stress in the long contraction has been reduced to the critical shear stress. Then, at Section 2

$$\tau_2 = \tau_c = \gamma y_2 S_{f_2} \qquad 6.A7.4$$

When this condition is reached there is no longer sediment transport at Section 2. As well, there is no sediment transport at Section 1 ($\tau_1 < \tau_c$). Hence the term "clear-water scour" is employed.

By employing Eqs. 6.A7.3 and 6.A7.4, the depth ratio is

$$\frac{y_2}{y_1} = \frac{S_{f_1}}{S_{f_2}} \frac{\tau_c}{\tau_1} \qquad 6.A7.5$$

Manning's equation (Eq. 2.3.20) can be employed to determine the friction slope ratio. Accordingly

$$\frac{S_{f_1}}{S_{f_2}} = \left(\frac{n_1}{n_2}\right)^2 \left(\frac{V_1}{V_2}\right)^2 \left(\frac{y_2}{y_1}\right)^{4/3} \qquad 6.A7.6$$

so

$$\frac{y_2}{y_1} = \left(\frac{n_1}{n_2}\right)^2 \left(\frac{V_1}{V_2}\right)^2 \left(\frac{y_2}{y_1}\right)^{4/3} \left(\frac{\tau_c}{\tau_1}\right) \qquad 6.A7.7$$

The velocity ratio V_1/V_2 is obtained by equating Eqs. 6.A7.1 and 6.A7.2 (constant discharge) or

$$\frac{V_1}{V_2} = \frac{y_2}{y_1} \frac{W_2}{W_1} \qquad 6.A7.8$$

By putting this ratio into Eq. 6.A7.7, the expression

$$\frac{y_2}{y_1} = \left(\frac{n_2}{n_1}\right)^{6/7} \left(\frac{W_1}{W_2}\right)^{6/7} \left(\frac{\tau_1}{\tau_c}\right)^{3/7} \qquad 6.A7.9$$

is obtained for clear-water scour. If it is assumed that

$$n_1 = n_2$$

then Eq. 6.A7.9 reduces to

$$\frac{y_2}{y_1} = \left(\frac{W_1}{W_2}\right)^{6/7} \left(\frac{\tau}{\tau_c}\right)^{3/7} \qquad 6.A7.10$$

which is the form of the clear-water scour equation first developed by Straub[1].

[1] Straub, L.G., 1940, Approaches to the study of mechanics of bed movement: Iowa State Univ. Studies in Engin. Bul. 20.

6.A8.0 DETERMINATION OF FLOW PARAMETERS ON AN EMBANKMENT

To apply the equations presented in Section 6.4 for the design of riprap bank protection, the flow parameters τ_o , v_r, v_s or V , and λ must be evaluated. Lewis[1], developed an analytical model for determination of the complete velocity and depth distribution in the vicinity of embankments but the application of his method is complex. Lewis pointed out, based on the model and analytical studies, that the initial losses of the riprap protection occurs at one or both of two zones on the embankment. One zone is near the flow separation point located approximately at midway around the upstream spill-slope. The other zone is along the embankment toe through the constriction. Riprap losses initially occurred on the upstream spill-slope for small constrictions ($\Delta h/L$ is small) and along the embankment toe for the severe constrictions ($\Delta h/L$ is large) as shown in Fig. 6.A8.1. Here Δh is the drop in water surface elevation through the bridge opening and L is the horizontal distance shown in Fig. 6.A8.2 with ω = 45 deg.

For design purposes, the estimation of the velocities and depths in the entire bridge crossing is not necessary. It is adequate to determine the riprap required for protecting the most hazardous zones subject to failure. Then this riprap is used to protect the embankment. The flow parameters for determining the required riprap at midway around the upstream spill-slope and along the embankment toe through the constriction are determined below. The sizes of riprap required to produce a safety factor of 1.5 at those two positions are computed and the larger size rock is chosen for the embankment protection.

6.A8.1 Determination of flow parameters at the embankment toe

The flow at the toe of the embankment is approximately horizontal and parallel to the side slope (see Fig. 6.A8.2). For design purposes

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[1] Lewis, G. L., 1972, Riprap protection of bridge footings: Ph.D. Dissertation, Dept. of Civil Eng., Colorado State University, Fort Collins, Colorado.

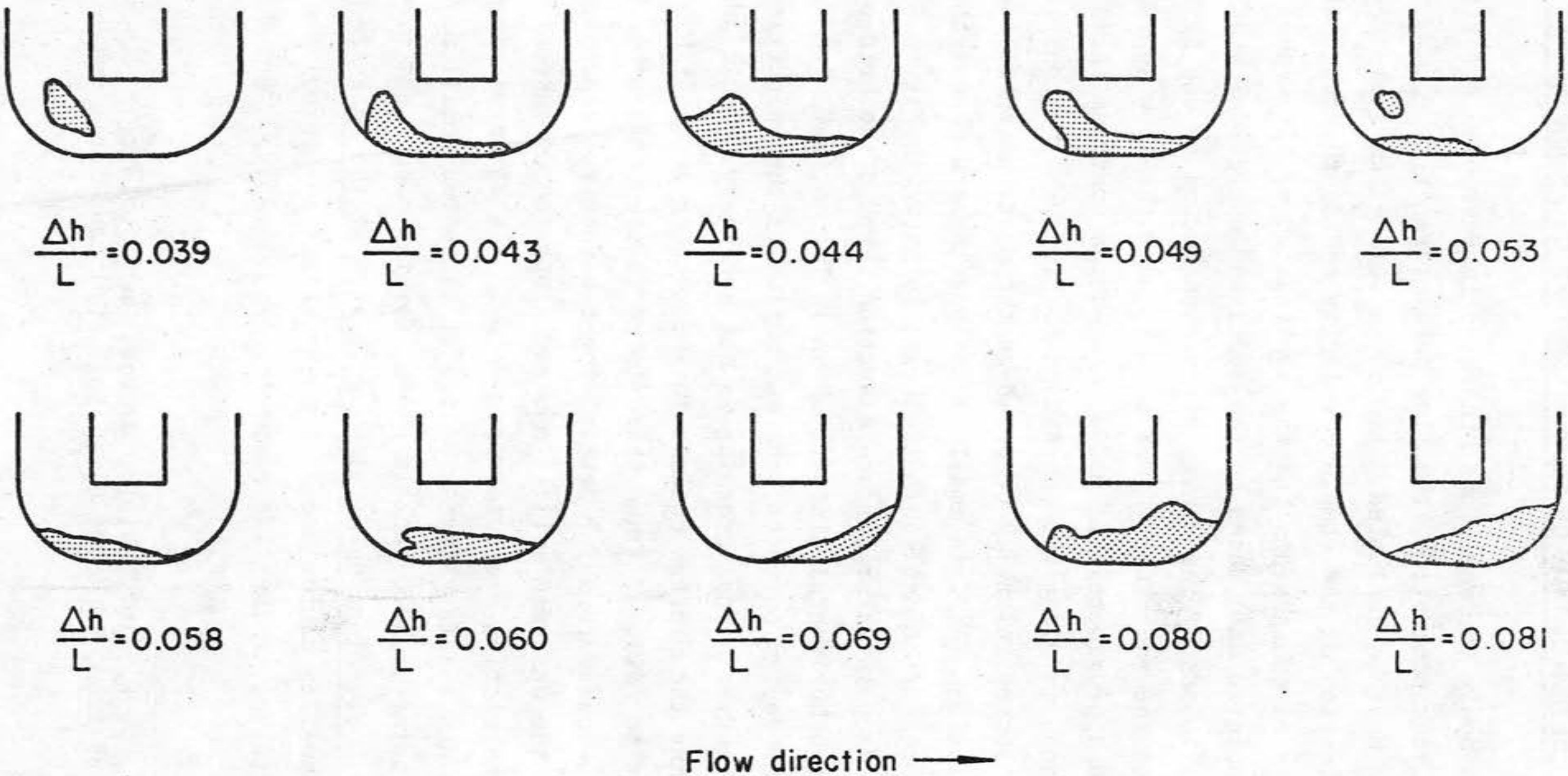

Fig. 6.A8.1 Zones of failure on riprapped embankments.

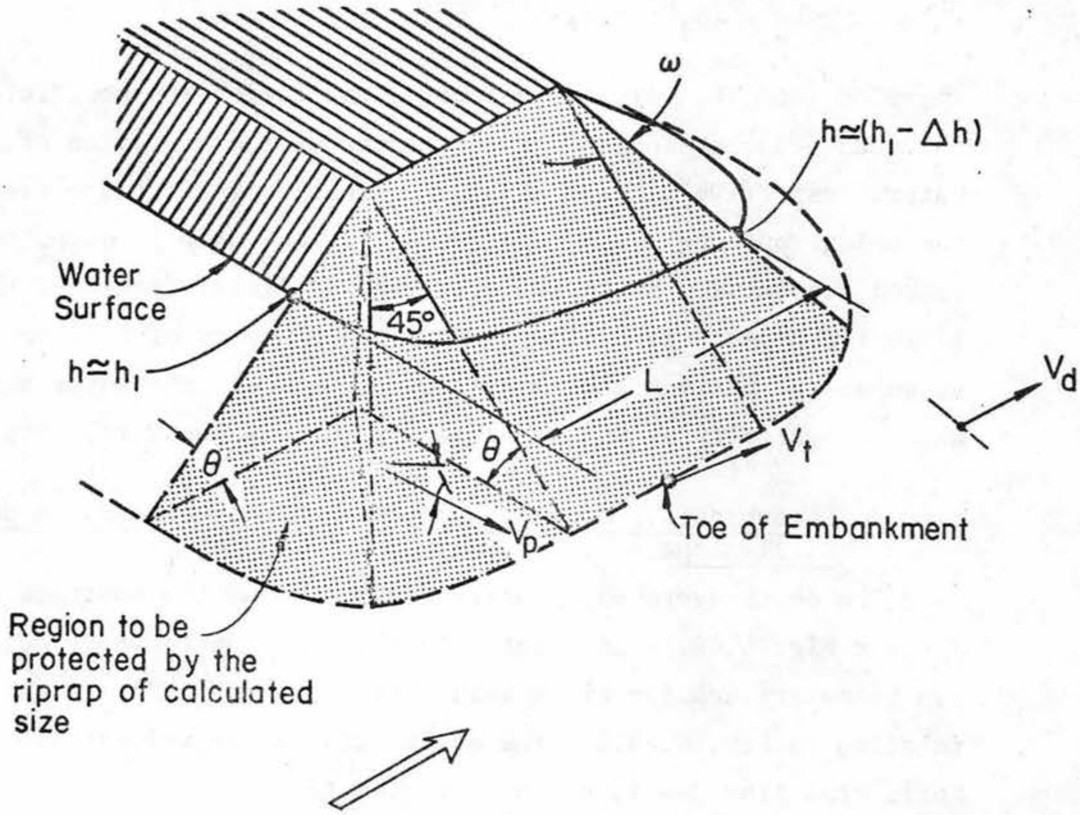

Fig. 6.A8.2 Flow around a spill-through embankment nose.

it is assumed that $\lambda = 0$ deg. The depth-averaged flow velocity at the two V_t is related to the depth-averaged velocity at the vena contracta V_d by the curves in Fig. 6.A8.3. The quantity V_d is found by using the expression

$$V_d - (\alpha_1 V_1^2 + 2g\Delta h)^{1/2} \qquad 6.A8.1$$

where α and V_1 are the velocity head correction coefficient and the mean velocity in the channel at the upstream section of maximum backwater, respectively, and Δh is the total water surface drop through the bridge opening. The total water surface drop is computed by the method recommended by the Federal Highway Administration[1]. The quantity L in Fig. 6.A8.2 is estimated from the geometry of embankment, the stage at the maximum backwater section h_1 and the water surface drop Δh. The velocity at the toe velocity is then found from Fig. 6.A8.3.

6.A8.2 Determination of flow parameters midway around the upstream spill-slope

The depth-averaged velocity midway around the upstream spill-slope V_p (see Fig. 6.A8.2) is related to the vena contracta velocity V_d and can be determined for given values of V_p and $\Delta h/L$ by using the relation in Fig. 6.A8.3. The angle between the velocity vector and the horizontal line λ is given in Fig. 6.A8.4.

6.A8.3 Application and limitations

After the values of V_t , V_p and λ are estimated from Figs. 6.A8.3 and 6.A8.4, the riprap sizes for a given safety factor are determined at the toe and midway around the upstream spill-slope by employing Eqs. 6.A3.8, 6.A3.9, 6.4.3, and 6.4.6. The quantity

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[1]Federal Highway Administration, 1970, Hydraulics of bridge waterways: Hydr. Branch, Bridge Div., Off. of Eng. and Oper., Hydr. Design Ser., n 1, 2nd ed., U.S. Gov. Printing Office, Washington, D.C.

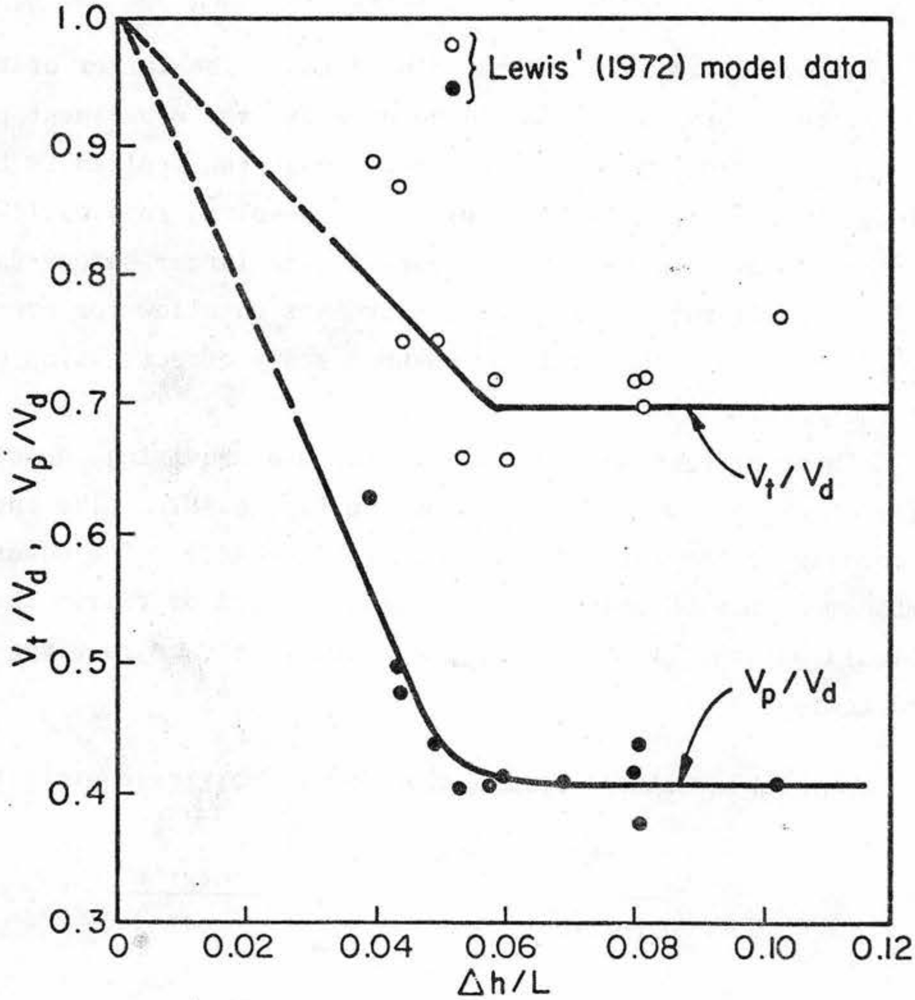

Fig. 6.A8.3 Relation between relative velocities and the drop ratio.

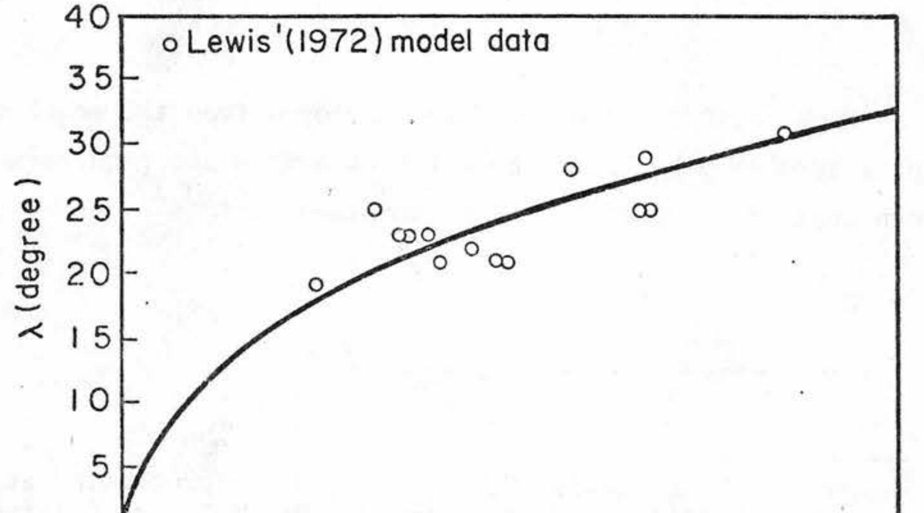

Fig. 6.A8.4 Relation between λ at midway around the upstream spill-slope and the drop ratio.

y_o in Eq. 6.A3.9 is the local flow depth. The larger of the two calculated riprap sizes should be used for the embankment protection.

The design procedure given above has been applied to Lewis' model data. It was found that the procedure resulted in a built-in safety factor of approximately 1.2. However, the larger safety factor of S.F. = 1.5 is recommended for embankments to allow for error in the hydraulic predictions and for unknown scale effects which may exist in the models.

The region around the embankment nose requiring the calculated size of riprap protection is shown in Fig. 6.A8.2. The angle ω varies according to the relation given in Table 6.A8.1. The outer region of the embankment may be protected by a smaller size of riprap than the calculated one. A method for computing this size is given by Simons and Lewis[1].

Table 6.A8.1 Termination angle for riprap protection.

$\frac{\Delta h}{L}$	ω degree
0 to .05	45
.06	55
.07	65
.08	75
.09	85
.10	90

Figs. 6.A8.3 and 6.A8.4 are developed from the model data on spill-through embankments normal to the flow and require modifications when applied to other types of embankments.

[1] Simons, D. B. and Lewis, G. L., 1971, Flood protection at bridge crossings: CER71-72DBS-GLL10, Colorado State University, Fort Collins, Colorado.

Chapter VII

NEEDS AND SOURCES FOR DATA

7.1.0 DATA NEEDS

The objective and purpose of this chapter is to identify data needed for calculations and analyses which will lead to recommendations for highway crossings and encroachments of rivers. The types and amounts of data needed for planning and designing river crossings and lateral encroachments can vary from project to project depending upon the class of the proposed highway, the type of river and geographic area.

The data, preliminary calculations, alternative route selections and analyses of these routes should be documented in a report. Such a report serves to guide the detailed designs, and provides reference background for environmental impact analysis and other needs such as application for permits and historical documentation for any litigation which may arise.

7.2.0 BASIC DATA NEEDS

7.2.1 Area maps

An area map is needed to identify the location of the entire highway project and all streams and river crossings and encroachments involved. The purpose of the map is to orient the highway project geographically with other area features. The map may be very small scale showing towns, cities, mountain ranges, railroads and other highways and roads. The area map should be large enough to identify river systems and tributaries.

7.2.2 Vicinity maps

Vicinity maps for each river crossing or lateral encroachment are needed to layout the proposed highway alignment and alternate routes. There should be sufficient length of river reach included on the vicinity map to enable identification of stream type and to locate river meanders, sand bars and braided channels. Other highways and railroads should be identified. The maps should show coarse contours and relief. Intakes for municipal and industrial water, diversions for irrigation and power, and navigation channels should be clearly identified. Recreational areas such as camping and picnic grounds, bathing beaches, and recreational boat docks should be identified. Cultivated areas and urban and industrial areas in the vicinity of towns and cities should be noted on the map. The direction of river flow should of course be clearly indicated.

7.2.3 Site maps

Site maps are needed to determine details for hydraulic and structural designs. *The site map should show detailed contours of one or two foot intervals, vegetation distribution and type, and other structures.* The site map is used to locate highway approach embankments, piers and alignment of piers, channel changes and protection works. High water lines should be indicated on the site maps for the purpose of estimating flood flows and distributions across the river cross section.

7.2.4 Aerial and other photographs

It is highly desirable in preparing vicinity and site maps that aerial photographs be obtained. Modern multi-image cameras use different ranges of the light spectrum to assist in identifying various features such as sewer outfalls, groundwater inflows, types of vegetation, sizes and heights of sandbars, river thalwegs, river controls and geologic formations, existing bank protection works, old meander channels, and other features. Detailed contours can be developed from aerial photographs for vicinity and site maps where such information is not readily available. Land photographs (as opposed to aerial photos) of existing structures are always helpful in documentation and evaluation of potential effects of highway construction. Photographs of water intake works likely to be affected by the highway project should be obtained, and specific pertaining data should be noted and briefly discussed. High water marks are useful to record photographically along with dates of occurrence. *Photographs aid the designer, who may not have the opportunity to visit the site, to visualize crossings and encroachments, and they aid documentation.*

Conditions of the river channel in the river reach of concern are easy to record photographically, and such pictures can be very helpful in analysis of the river reach. Vegetation on floodplains and seasonal variations of vegetation should be recorded photographically. Notable geologic formations should be photographed as well and supplemented with adequate notes.

7.2.5 Hydrologic data

The purpose of hydrologic data is to determine the stream discharge, flood magnitudes, and duration and frequencies of floods preparatory to analysis of river behavior and design of the river encroachments and crossings. Hydrologic data and hydraulic analyses should be documented in report form for the purpose of preparing construction plans and evaluating performance after construction. The documentation would be helpful in evaluating any damage from floods and failures in the event they occur and providing background for any litigation which may arise as a consequence.

The basic data needed are stream discharge data at the nearest gaging station, historical floods, and highwater marks. It is also desirable to prepare a drainage map for the region upstream of the proposed highway project, with delineation of size, shape, slope, land use, and water resource facilities such as storage reservoirs for irrigation and power and flood control projects. It is desirable whenever possible to obtain flood histories of the river from residents and accounts by the news media, particularly for events prior to stream gaging records. The accounts of high water and period of years estimates of flood discharge can be made which are valuable in flood-frequency analysis.

Sometimes a highway crossing and/or encroachment may have a significant effect on flood hydrographs. Sufficient hydrologic analysis should be made to determine the significance. This would involve hydrograph development and valley routings within the zone of influence of such highway structures.

7.2.6 Geologic map

A geologic vicinity map, on which geophysical features are indicated is of basic need. *The basic rock formations, outcroppings, and glacial* *and river deposits which form control points on rivers are valuable in analysis of rivers.* Soil type has important effects on sediment transport material, infiltration rates, and groundwater flows. Channel geometry and roughness are important factors in river mechanics.

Soil survey maps with engineering interpretations are available for a significant proportion of the United States. They may be helpful in selecting layouts and assessing the suitability of fill materials.

7.2.7 Field inspection

A field inspection of potential highway encroachment sites of rivers should be made prior to or during the analysis. This has been implied in the foregoing paragraphs but is emphasized again because of the underlying importance of making first hand appraisals of specific sites before conclusions and recommendations are advanced for possible highway routes. Of course, they are important in making detailed designs as well but it is not always feasible to provide opportunities for personal inspection by the entire design staff.

7.2.8 Environmental data

In making environmental impact analysis of highway projects on streams and rivers, it is necessary to obtain water quality and biological data for the streams. Such data are available for many rivers but are not readily available for many others. Municipal water and sewage treatment facilities and industrial plants utilizing river water should have recent records regarding river water quality which will be helpful in making comprehensive environmental analyses. Water quality data for certain rivers can be obtained from the U.S. Geological Survey. Wildlife information such as migration patterns of deer and elk should be determined and local game refuges should be located. Information regarding fish and fish habitat in the river should be obtainable from the state fish and game agencies. Species of trees and other vegetation should be determined, and some information regarding sensitivity of the flora to auto emissions should be obtained. Data should also be obtained in order to enable assessment of stream turbidity during and after highway construction. Information on soil type to be used in construction of embankment would be helpful in this regard.

7.3.0 AUXILIARY DATA

The basic data needed for hydrologic, hydraulic and environmental analyses have been indicated. In addition, depending upon the nature of the highway project, it may be desirable to obtain additional data.

7.3.1 Climatologic data

Stream gaging stations have been established on many streams throughout the United States. However, there are some streams where either a gaging station does not exist near the project site or a gaging station does not exist at all. In such cases *it is necessary to estimate flood flows. These estimates may be based on regionalized estimating procedures or other prediction models using meteorological and watershed data inputs.* These meteorological data are available from the National Weather Service (NWS) Data Center of the National Oceanic and Atmospheric Administration (NOAA), and estimates of average conditions can be made from rainfall data published by the NWS. Temperature records are helpful in making snowmelt estimates, and wind data are helpful in making wave height estimates on rivers, lakes and reservoirs as well as for coastal areas.

7.3.2 Hydraulic data

Whenever possible, *sediment load data should be provided as auxiliary data for river analyses.* Bed-material load, suspended load and wash load data may be obtained for some rivers in the water supply papers published by the U.S. Geological Survey, state engineers' reports, flood control and other water resources investigation reports. Information may also be obtained by direct sampling of the river.

Riverbed cross sections and profiles may be obtained with an ultrasonic depth sounder and would be helpful in sediment transport and backwater studies. It would also be helpful to know water temperatures. Direct measurement of flood flows should be made when records may be deficient. Depth and velocity measurements need to be made at a sufficient number of subsections in a cross section to determine total flow rate. Discharge measurements made at various stages at a gaging site can provide data for developing a stage-discharge rating curve.

Observations of *high water marks* along the river reach should be made. Each high water mark and relevant profile should be established.

These are helpful in calculating historical flood discharges. Also, stages achieved by ice jams at specific locations should be noted.

Channel changes which have occurred after floods are of particular interest in evaluating future effects on channel planform. Whenever possible, historic aerial photographs or equivalent maps which show river channels should be obtained.

Records of the performance of existing bridges and other drainage structures should be obtained. Data on scour at piers of existing bridges (or at bridges which have failed) in the vicinity should be obtained. For bridges which have failed, as much information as possible should be obtained relative to direction of flow (angle of attack) at the piers or embankment ends. Flood duration, debris in the river, distribution of flows, and magnitudes of scour are useful information. Historical records of damage to adjacent property and results of legal actions brought about because of damage are useful information also.

7.4.0 FLOOD-FREQUENCY ANALYSIS

A *flood-frequency curve is prepared from recorded stream flow data* and augmented by estimated discharges (using Manning's equation or equivalent) from high water marks. Several methods ranging from sophisticated stochastic analysis to simple methods have been developed. The greatest difficulty in constructing a flood-frequency curve is lack of sufficient data. Approximate methods for extrapolating the range of flood-frequency curves are available but are not discussed in detail here. (See Guidelines for Hydrology (1973) for references.)

A simple graphical method based on extreme value theory is reasonably satisfactory. The method consists of ordering the annual peak flood discharges of record from the largest to smallest, irrespective of chronological order. The annual (flood) discharge is plotted against its recurrence interval on special probability (Gumbel, or others) paper. The recurrence interval, RI is calculated from

$$RI = \frac{n+1}{m} \qquad 7.4.1$$

in which n is the number of years of record, and m is the order (largest flood is ranked 1) of the flood magnitude. Thus the highest flood discharge would have a recurrence interval of n+1 years and lowest would have a recurrence interval of (1+1/n) years. The U.S. Water Resources Council (1967) has adopted the log-Pearson III distribution for use as a base method for determining flood flow frequencies. Details of the method and plotting paper may be obtained from the U.S. Geological Survey or the Federal Highway Administration (Washington or regional offices).

When adjusting discharge records from the gaging station to the project site, the flood peaks are prorated on the basis of drainage area ratios. Depending on drainage basin characteristics, the exponent of the ratio varies from 0.5 to 0.8. Slope-area calculations for peak discharges are also used. In using this method, the conveyance of the channel is calculated using the Manning equation in which the roughness coefficient, n, needs to be estimated. An excellent reference relating n to channel conditions is presented pictorially in USGS Water Supply Paper 1849 which is published in book form. By referring to a catalog of (color) photographs, similar channel situations to the specific site can be identified and a relatively inexperienced engineer may make a reliable estimate for n.

7.5.0 CHECKLIST OF DATA NEEDS

As an aid to check data needs preparatory to analysis of rivers and highway encroachment of rivers, the relevant types of data have been listed in Table 7.5.1. There may be more data items included in this table than are needed for a given project site, and some judgment is required in delineation. For data which are not available, the checklist should be helpful for planning a field investigation or other data acquisition program.

7.6.0 DATA SOURCES

The best data sources are national data centers where the principle function is to disseminate data. But it probably will be necessary to collect data from a variety of other sources such as a field investigation, interviews with local residents, and a search through library materials. The following list of sources is provided to serve as a guide to the data collection task:

Topographic Maps:

(1) Quadrangle maps--U.S. Department of the Interior, Geological Survey, Topographic Division; and U.S. Department of the Army, Army Map Service.
(2) River plans and profiles--U.S. Department of the Interior, Geological Survey, Conservation Division.
(3) National parks and monuments--U.S. Department of the Interior, National Park Service.
(4) Federal reclamation project maps--U.S. Department of the Interior, Bureau of Reclamation.
(5) Local areas--commercial aerial mapping firms.
(6) American Society of Photogrammetry.

Planimetric Maps:

(1) Plats of public land surveys--U.S. Department of the Interior, Bureau of Land Management.
(2) National forest maps--U.S. Department of Agriculture, Forest Service.
(3) County maps--State Highway Agency.
(4) City plats--city or county recorder.
(5) Federal reclamation project maps--U.S. Department of the Interior, Bureau of Reclamation.
(6) American Society of Photogrammetry.
(7) ASCE Journal--Surveying and Mapping Division.

Aerial Photographs:

(1) The following agencies have aerial photographs of portions of the United States: U.S. Department of the Interior, Geological Survey, Topographic Division; U.S. Department of Agriculture, Commodity Stabilization Service, Soil Conservation Service and Forest Service; U.S. Air Force; various State agencies; commercial aerial survey; National Oceanic and Atmospheric Administration; and mapping firms.
(2) American Society of Photogrammetry.
(3) Photogrammetric Engineering.
(4) Earth Resources Observation System (EROS) Photographs from Gemini, Apollo, Earth Resources Technology Satellite (ERTS) and Skylab.

Transportation Maps:

(1) State Highway Agency.

Triangulation and Benchmarks:

(1) State Engineer.
(2) State Highway Agency.

Geologic Maps:

(1) U.S. Department of the Interior, Geologic Survey, Geologic Division; and State geological surveys or departments. (Note--some regular quadrangle maps show geological data also.)

Soils Data:

(1) County soil survey reports--U.S. Department of Agriculture, Soil Conservation Service.
(2) Land use capability surveys--U.S. Department of Agriculture, Soil Conservation Service.
(3) Land classification reports--U.S. Department of the Interior, Bureau of Reclamation.
(4) Hydraulic laboratory reports--U.S. Department of the Interior, Bureau of Reclamation.

Climatological Data:

(1) National Weather Service Data Center.
(2) Hydrologic bulletin--U.S. Department of Commerce, National Oceanic and Atmospheric Administration.
(3) Technical papers--U.S. Department of Commerce, National Oceanic and Atmospheric Administration.
(4) Hydrometeorological reports--U.S. Department of Commerce, National Oceanic and Atmospheric Administration, and U.S. Department of the Army, Corps of Engineers.
(5) Cooperative study reports--U.S. Department of Commerce, National Oceanic and Atmospheric Administration and U.S. Department of the Interior, Bureau of Reclamation.

Stream Flow Data:

(1) Water supply papers--U.S. Department of the Interior, Geological Survey, Water Resources Division.
(2) Reports of State engineers.
(3) Annual reports--International Boundary and Water Commission, United States and Mexico.
(4) Annual reports--various interstate compact commissions.
(5) Hydraulic laboratory reports--U.S. Department of the Interior, Bureau of Reclamation.
(6) Owners of Reclamation.
(7) Corp of Engineers, U.S. Army, Flood control studies.

Sedimentation Data:

(1) Water supply papers--U.S. Department of the Interior, Geological Survey, Quality of Water Branch.
(2) Reports--U.S. Department of the Interior, Bureau of Reclamation; and U.S. Department of Agriculture, Soil Conservation Service.
(3) Geological Survey Circulars--U.S. Department of the Interior, Geological Survey.

Quality of Water Reports:

(1) Water supply papers--U.S. Department of the Interior, Geological Survey, Quality of Water Branch.
(2) Reports--U.S. Department of Health, Education, and Welfare, Public Health Service.

(3) Reports--State public health departments.
(4) Water Resources Publications--U.S. Department of the Interior, Bureau of Reclamation.
(5) Environmental Protection Agency, regional offices.
(6) State water quality agency.

Irrigation and Drainage Data:

(1) Agricultural census reports--U.S. Department of Commerce, Bureau of the Census.
(2) Agricultural statistics--U.S. Department of Agriculture, Agricultural Marketing Service.
(3) Federal reclamation projects--U.S. Department of the Interior, Bureau of Reclamation.
(4) Reports and Progress Reports--U.S. Department of the Interior, Bureau of Reclamation.

Power Data:

(1) Directory of Electric Utilities--McGraw Hill Publishing Co.
(2) Directory of Electric and Gas Utilities in the United States--Federal Power Commission.
(3) Reports--various power companies, public utilities, State power commissions, etc.

Basin and Project Reports and Special Reports:

(1) U.S. Department of the Army, Corps of Engineers.
(2) U.S. Department of the Interior, Bureau of Land Management, Bureau of Mines, Bureau of Reclamation, Fish and Wildlife Service, and National Park Service.
(3) U.S. Department of Agriculture, Soil Conservation Service.
(4) U.S. Department of Health, Education, and Welfare, Public Health Service.
(5) State departments of water resources, departments of public works, power authorities, and planning commissions.

Environmental Data:

(1) Sanitation and public health--U.S. Department of Health, Education, and Welfare, Public Health Service; State departments of public health.
(2) Fish and wildlife--U.S. Department of the Interior, Fish and Wildlife Service; State game and fish departments.
(3) Municipal and industrial water supplies--city water departments; State universities; Bureau of Business Research; State water conservation boards or State public works departments, state health agencies, Environmental Protection Agency, Public Health Service.
(4) Watershed management--U.S. Department of Agriculture, Soil Conservation Service, Forest Service; U.S. Department of the Interior, Bureau of Land Management, Bureau of Indian Affairs.

Table 7.5.1 Checklist of data needs.

Description of data or needed information	Basic Data	Auxiliary Data	Check Whether Available	Needed
Maps and charts:				
Geographic	*			
Topographic	*			
Geologic	*			
Navigation charts		*		
Potamology surveys		*		
County and city plats		*		
Aerial and other photos:				
Large scale photos for working plans	*			
Small scale stereo pairs of river and surrounding terrain	*			
Color infrared photos for flow patterns, scour zones, and vegetation		*		
Ground photos	*			
Underwater photos		*		
Information on existing structures, bridges, dams, diversions or outfalls:				
Plans and details	*			
Construction details		*		
Alterations and repairs		*		
Foundations	*			
Piers and abutments	*			
Scour		*		
Dikes	*			
Field investigations:				
Investigating bridge structure & repairs to bridge & approach	*			
Damage due to ice or debris	*			

Table 7.5.1 Checklist of data needs (continued).

Description of data or needed information	Basic Data	Auxiliary Data	Check Whether Available	Needed
Hydraulic, Hydrology and Soils:				
Discharge records	*			
Stage-discharge records	*			
Flood frequency curves for stations near site	*			
Flow duration curves (hydrographs)		*		
Newspaper, radio, television, accounts of large floods		*		
Channel geometry:				
Main channel	*			
Side channel	*			
Navigation channel	*			
Floodplain	*			
Slopes	*			
Backwater calculation	*			
Bars	*			
Sinuosity	*			
Type (braided, meandering, straight)	*			
Controls (falls, rapids, restriction, rock outcropping dams, diversions)	*			
Sediment discharge:	*			
Size distribution	*			
Bed and bank material sizes	*			
Roughness coefficient n	*			
Ice:				
Recorded thickness		*		
Dates of freeze up and break up		*		
Flow patterns and jams		*		

Table 7.5.1 Checklist of data needs (continued).

Description of data or needed information	Basic Data	Auxiliary Data	Check Whether Available	Needed
Damage		*		
Regulating structures:				
Dams, diversions	*			
Intake, outfalls	*			
Scour survey around existing piers, abutments, spur dikes	*			
Inspect and photograph stabilization works, riprap sizes, filter blankets	*			
Check wells for groundwater levels in areas		*		
Install gaging stations		*		
Soils Information:	*			
Excavation data	*			
Borrow pits	*			
Gravel pits	*			
Cuts	*			
Tunnels	*			
Core boring logs	*			
Well drilling logs		*		
Soil tests	*			
Permeability		*		
Rocks - riprap	*			
Planned and anticipated water resources projects	*			
Lakes, tributaries, reservoirs or side channel impoundments	*			
Field surveys:	*			
Onsite inspections and photographs	*			

Table 7.5.1 Checklist of data needs (continued).

Description of data or needed information	Basic Data	Auxiliary Data	Check Whether Available	Needed
Sample sediments	*			
Measure water and sediment discharge	*			
Observe channel changes or realignment since last maps or photos	*			
Identify high water lines or debris deposits due to recent floods	*			
Check magnitude of velocities and direction of flow in vicinity of proposed structure	*			
Outcroppings	*			
Subsurface exploration	*			
Climatological data:				
Natural weather service records for precipitation	*			
Wind	*			
Temperatures	*			
Land Use:				
Zoning maps	*			
Recent aerial photographs	*			
Planning committee records	*			
Urban areas	*			
Industrial areas	*			
Recreational areas	*			
Primitive areas	*			
Forests	*			
Vegetation	*			

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[illegible] Bureau of [illegible] [illegible]

Smith, [illegible] [illegible]

Chapter VIII

HYDRAULIC AND ENVIRONMENTAL CONSIDERATIONS OF HIGHWAY RIVER CROSSINGS AND ENCROACHMENTS

8.1.0 INTRODUCTION

The objective of this chapter is to *present applications of the basic principles of hydraulics, hydrology, fluvial geomorphology and river mechanics to the hydraulic and environmental design of river crossings and highway encroachments.* In general, the design of complex problems in river engineering can be facilitated by a qualitative estimate followed by a quantitative analysis. For this reason, this chapter includes thirteen hypothetical cases of river environments and their response to river crossings and encroachments based on the geomorphological principles given in Chapter IV. These cases indicate the trend of change in river morphology for given initial conditions. The case studies are followed by twelve case histories of actual river crossings in the United States. These histories document river response to highway crossings and encroachments and serve the basic purpose of illustrating qualitative river response.

Finally, this chapter presents the main design considerations related to river crossings and highway encroachment. The application of basic principles developed in Chapters I through VI is illustrated by specific numerical examples related to the subject matter of this manual. It is believed that *the systematic approach of qualitative assessment of channel response followed by a quantitative estimate, will enable a meaningful analysis of complex river response problems.*

8.2.0 CASE HISTORIES

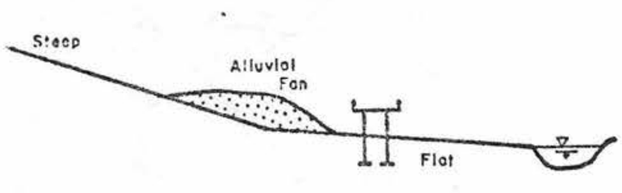
8.2.1 Introduction

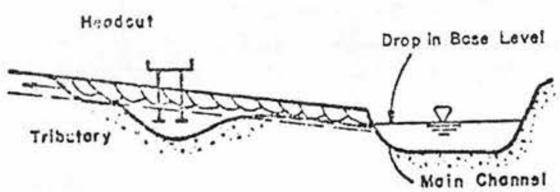
To initiate analysis of case histories, first consider some of the problems normally encountered in river crossings and encroachments. Several hypothetical cases are tabulated in Table 8.2.1. Each individual case is identified in the first column to show the physical situation that exists prior to the construction of the highway crossing. In the following three columns some of the major local effects, both upstream and downstream, resulting from construction of a particular crossing are given. It is necessary to emphasize that only the gross local upstream and downstream effects are identified in this table. In an actual design situation, it is worthwhile *first* of all to *consider the gross effects* as listed in Table 8.2.1. *The relation* $QS \sim Q_s D_{50}$ *is valuable in determining qualitative river response. Having identified the qualitative response that can be anticipated, water and sediment routing techniques coupled with river mechanics relations* given in Chapters IV and V *can be used to predict the possibility of change in river form and to estimate the magnitudes of local, upstream and downstream river response.* This approach should be kept in view as one considers each of the fourteen cases outlined in Table 8.2.1.

The initial river conditions in Table 8.2.1 are sometimes given in terms of storage dams, water diversions, etc. These examples are used as illustrations relating to common experience. In general, the effect of a storage reservoir is to cause a sudden increase of base level for the upstream section of the river. The result is aggradation of the channel upstream, degradation downstream and a modification of the flow hydrograph. Similar changes in the channel result if the base level is raised by some other mechanism, say a tectonic uplift. The effect of diversions from rivers is to decrease the river discharge downstream of the diversion with or without an overall reduction of the sediment concentration. Similarly, *changes in water and sediment input to a river stretch often occur due to river development projects upstream from the proposed crossings* or due to natural causes.

Case (1) involves the construction of a bridge across a tributary stream downstream of where the steeper tributary stream has reached the floodplain of the parent stream. *The change in gradient of the tributary stream in most cases causes significant deposition.* In the case illustrated for Case (1), an alluvial fan develops which in time can divert the river around the bridge, or even if the water continues to flow under the bridge, the waterway is significantly reduced thus endangering the usefulness and stability of the structure. In general, streams on alluvial fans shift laterally so that the future direction of the approach flow to the bridge is uncertain.

Table 8.2.1 River response to highway encroachments and to river development.

<table>
<tr><th>Bridge Location</th><th>Local Effects</th><th>Upstream Effects</th><th>Downstream Effects</th></tr>
<tr><td>


(1) *Crossing downstream of an alluvial fan.*
</td><td>1 - Fan reduces waterway
2 - Direction of flow at bridge site is uncertain
3 - Channel location is uncertain</td><td>1 - Erosion of banks
2 - Unstable channel
3 - Large transport rate</td><td>1 - Aggradation
2 - Flooding
3 - Development of tributary bar in the main channel</td></tr>
<tr><td>


(2) *Lowering of base level for the channel.*
</td><td>1 - Headcutting
2 - General scour
3 - Local scour
4 - Bank instability
5 - High velocities</td><td>1 - Increased velocity
2 - Increased bed material transport
3 - Unstable channel
4 - Possible change of form of river</td><td>1 - Increased transport to main channel
2 - Aggradation
3 - Increased flood stage</td></tr>
</table>

Table 8.2.1 River response to highway encroachments and to river development (continued).

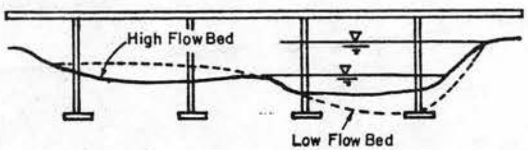
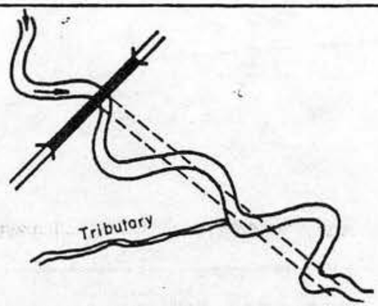
Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 (3) *Channel characterized by prolonged low flows*	1 - At low flow a low water channel develops in river bed 2 - Increased danger to piers due to channelization and local scour 3 - Bank caving	---	---
 (4) *Cutoffs downstream of crossing*	1 - Steeper slope 2 - Higher velocity 3 - Increased transport 4 - Degradation and possible head-cutting 5 - Banks unstable 6 - River may braid 7 - Danger to bridge foundation from degradation and local scour	See local effects	1 - Deposition downstream of straightened channel 2 - Increase in flood stage 3 - Loss of channel capacity 4 - Degradation in tributary

Table 8.2.1 River response to highway encroachments and to river development (continued).

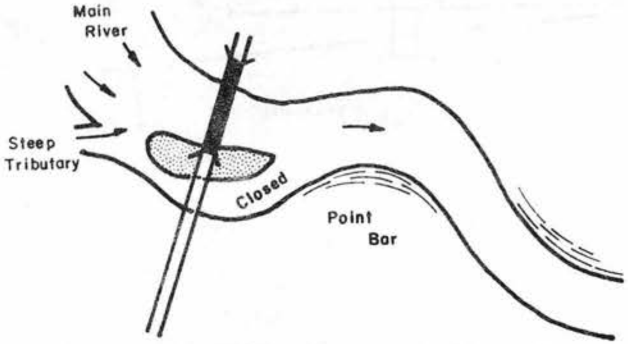
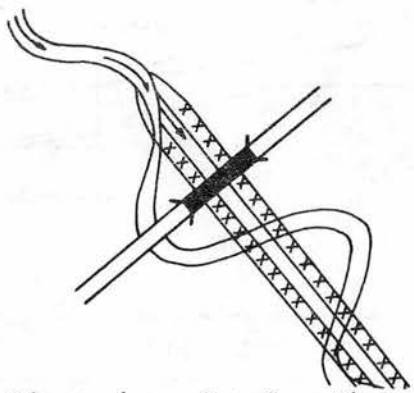
Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 (5) *Excess of sediment at bridge site due to upstream tributary*	1 - Contraction of the river 2 - Increased velocity 3 - General and local scour 4 - Bank instability	1 - Aggradation 2 - Backwater at flood stage 3 - Changed response of the tributary	1 - Deposition of excess sediment eroded at and downstream of the bridge 2 - More severe attack at first bend downstream 3 - Possible development of a chute channel across the second point bar downsteam of the bridge
 (6) *River channel relocation at crossing site*	1 - None if straight section is designed to transport the sediment load of the river and if it is designed to be stable when subjected to anticipated flow. Otherwise same as in case (4).	1 - Similar to local effects	1 - Similar to local effects

Table 8.2.1 River response to highway encroachments and to river development (continued).

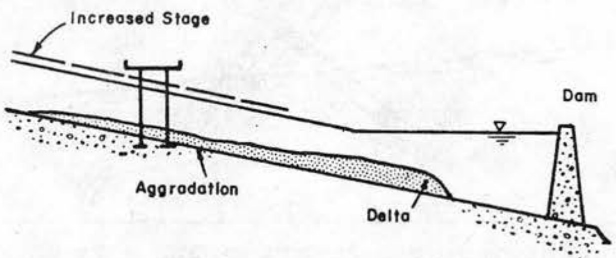
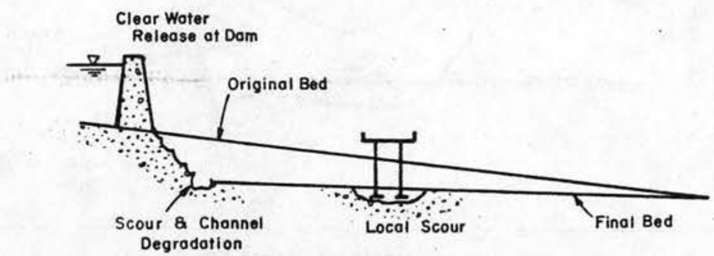
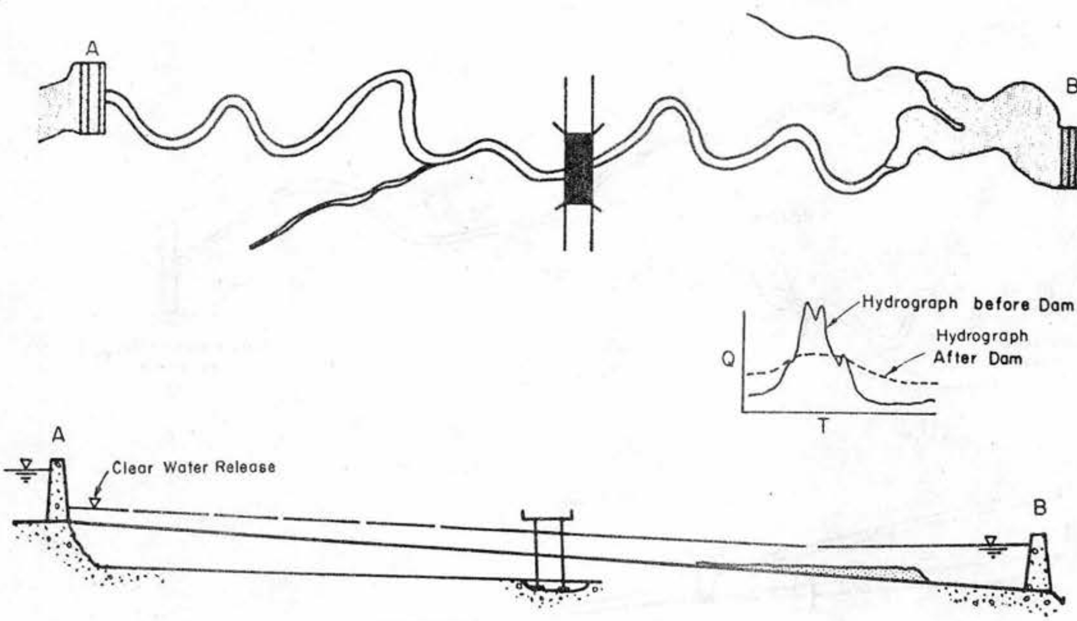
Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 (7) *Raising of river base level*	1 - Aggradation of bed 2 - Loss of waterway 3 - Change in river geometry 4 - Increased flood stage	1 - See local effects 2 - Change in base level for tributaries 3 - Deposition in tributaries near confluences 4 - Aggradation causing a perched river channel to develop or changing the alignment of the main channel	1 - See upstream effects
 (8) *Reduction of sediment load upstream*	1 - Channel degradation 2 - Possible change in river form 3 - Local scour 4 - Possible bank instability 5 - Possible destruction of structure due to dam failure	1 - Degradation 2 - Reduced flood stage 3 - Reduced base level for tributaries, increased velocity and reduced channel stability causing increased sediment transport to main channel	1 - Degradation 2 - Increased velocity and transport in tributaries

Table 8.2.1 River response to highway encroachments and to river development (continued).

Bridge location

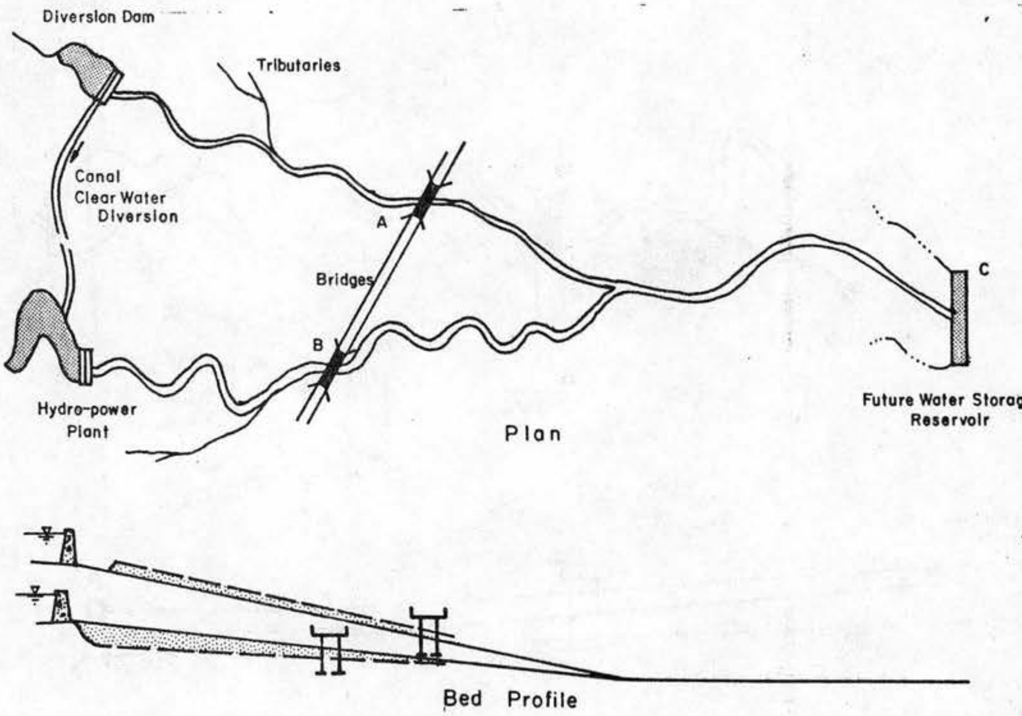


(9) *Combined increase of base level and reduction of sediment load upstream*

Local Effects	Upstream Effects	Downstream Effects
1 - Dam A causes degradation 2 - Dam B causes aggradation 3 - Final condition at bridge site is the combined effect of (1) and (2). Situation is complex and combined interaction of dams, main channel and tributaries must be analyzed using water and sediment routing techniques and geomorphic factors	1 - Channel could aggrade or degrade with effects similar to cases (7) and (8)	1 - See upstream effects

Table 8.2.1 River response to highway encroachments and to river development (continued).

Bridge location



(10) *Change in water discharge, no change in sediment load*

Local Effects	Upstream Effects	Downstream Effects
1 - Bridge A may be subjected to aggradation due to excess sediment left in the channel by diversion of clear water. 2 - Bridge B may be subjected to degradation due to increased discharge in the channel. 3 - If a storage reservoir was constructed at C it would induce aggradation in both main tributaries.	1 - Upstream of Bridge A - aggradation and possible change of river form 2 - Upstream of Bridge B - degradation and change of river form 3 - Channel instabilities 4 - Significant effects on flood stage	1 - See upstream effects 2 - Construction of reservoir C could induce aggradation in the main channel and in the tributaries Effects same as in case 7

Table 8.2.1 River response to highway encroachments and to river development (continued).

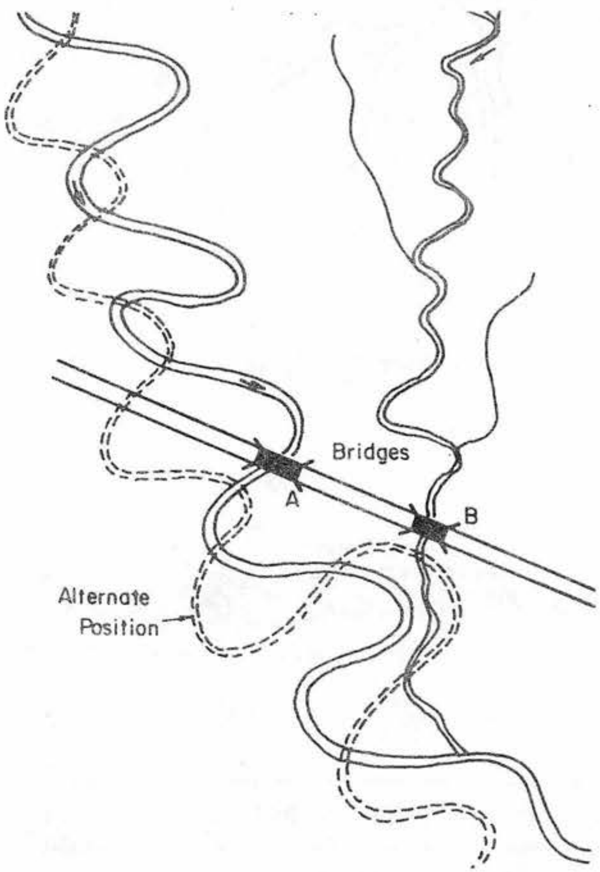
Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 (11) *Naturally shifting river channel*	1 - Rivers are dynamic (ever changing) and the rate of change with time should be evaluated as part of the geomorphic and hydraulic analysis 2 - Alignment of main channel continually changes affecting alignment of flow with respect to Bridge A. 3 - If the main channel shifts to the alternate position, the confluence shifts and the tributary gradient is significantly increased causing degradation in the tributary. Local effects on Bridge B same as 1,2,3 and 4 in Case (8). 4 - Excess sediment from the tributary, assuming (3) causes aggradation in the main channel and possible significant changes in channel alignment	1 - The river could abandon its present channel. Changing position of the main channel may require realignment of training works.	1 - See upstream effects 2 - Shifts in the position of the main channel relative to the position of the confluence with the tributary alternatively flattens or steepens the gradient of the tributary causing corresponding aggradation and degradation. 3 - Shifts in the position of the main channel causes aggradation, degradation and instabilities depending upon direction and magnitude of channel change

Table 8.2.1 River response to highway encroachments and to river development (continued).

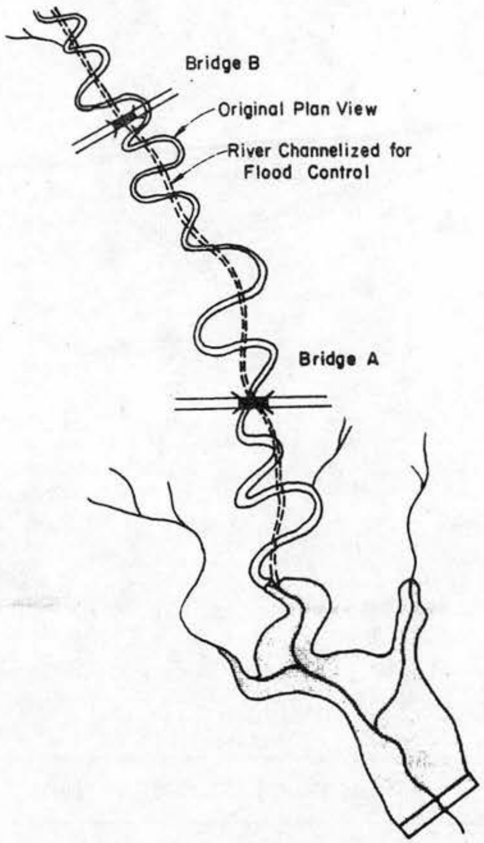
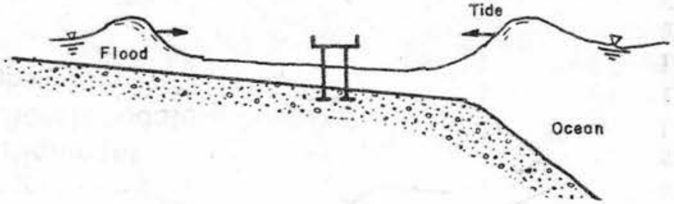
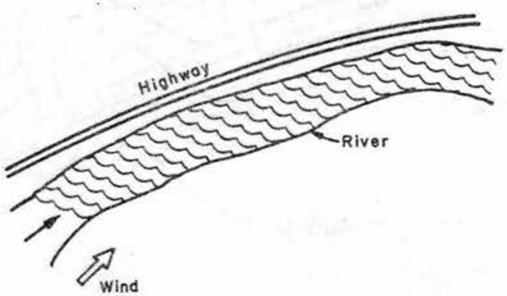
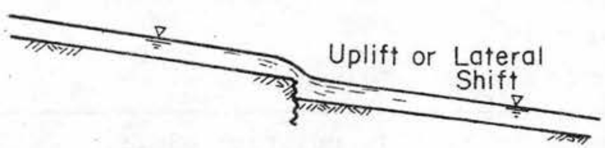
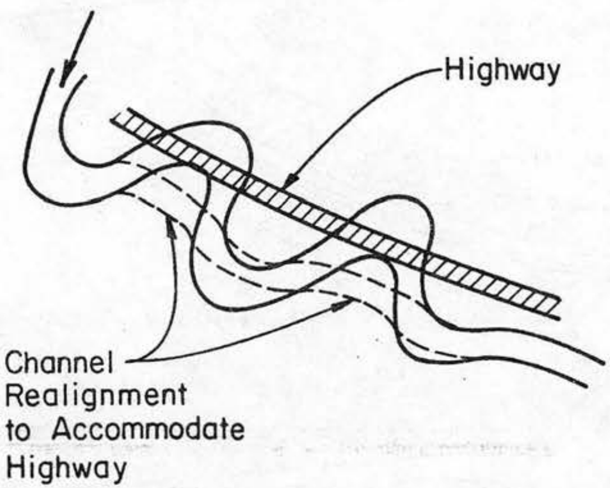
Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 (12) *Man-induced reduction of channel length*	1 - Bridge A is first subjected to degradation and then aggradation. Action can be very severe 2 - Bridge B is primarily subjected to degradation. The magnitude can be large 3 - The whole system is subjected to passage of sediment waves 4 - River form could change to braided 5 - Flood levels are reduced at B and increased at A 6 - Local and general scour is significantly affected	1 - A change of river form from meandering to braiding is possible 2 - Rate of sediment transport is increased 3 - Head cutting is induced in the whole system upstream of B 4 - Flood stage is reduced 5 - Velocity increases 6 - Tributaries respond to main channel changes	1 - For Bridge B see upstream effects 2 - For Bridge A the channel first degrades and then significantly aggrades 3 - Large quantities of bed material and wash load are carried to the reservoir 4 - Delta forms in the reservoir 5 - Wash load may affect water quality in the entire reservoir 6 - Tributaries respond to main channel changes

Table 8.2.1 River response to highway encroachments and to river development (continued).

Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 a. - Tidal Flows, Seiches, Bores, etc.	1 - Scour or aggradation 2 - Bank erosion 3 - Channel change 4 - Bed form change	1 - See local effects 2 - Channel erosion 3 - Changes in channel slope	1 - See local effects 2 - Beach erosion
 b. - Wind (Hurricanes, Tornadoes)	1 - Bank erosion 2 - Inundated highway 3 - Increase in velocity 4 - Wave action	1 - See local effects	1 - See local effects
 c. - Earthquakes (see Seismic Probability Map of U.S.)	1 - Channel changes 2 - Scour or deposition 3 - Decrease in bank stability 4 - Landslides 5 - Rockslides 6 - Mudflows	1 - See local effects 2 - Slide lakes	1 - See local effects 2 - Slide lakes

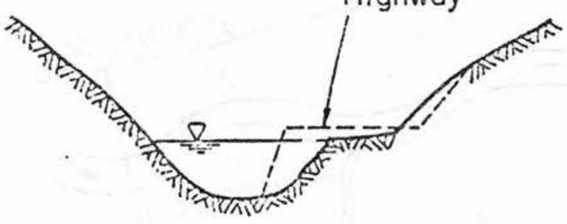
(13) *Tectonics and other natural causes*

Table 8.2.1 River response to highway encroachments and to river development (continued).

Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 a. - Meandering Channel	1 - Increased energy gradient and potential bank and bed scour 2 - Highway fill is subject to scour as channel tends to shift to old alignment. 3 - Reach is subject to bed degradation as headcut develops at the downstream end and travels upstream. 4 - Lateral drainage into the river is interrupted and may cause flooding and erosion.	1 - Energy gradient also increased in the reach upstream and may cause change of river form from meandering to braided 2 - Rate of sediment transport is increased. As the headcut travels upstream severe bank and bed erosion is possible. 3 - If tributaries in the zone of influence exist they will respond to lowering of base level.	1 - Channel will aggrade as the sediment load coming from bed and bank erosion is received. 2 - Channel may deteriorate from meandering to braided.

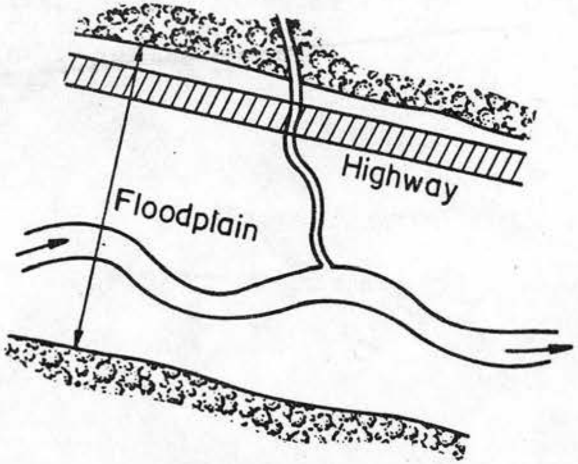
(14) *Longitudinal Encroachment*

Table 8.2.1 River response to highway encroachments and to river development (continued).

Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 b. - Incised Channel	1 - Reduced waterway causes a local obstruction to flow and higher velocities. 2 - Significant erosion problem on the highway fill and induced bed degradation 3 - Lateral drainage into the river is interrupted and may cause flooding and erosion.	1 - Backwater generated by the obstruction increases flood stage. 2 - Deposition induced by the backwater	1 - Large sediment load may cause aggradation. 2 - Local scour at end of contracted section

(14) *Longitudinal Encroachment* (continued)

Table 8.2.1 River response to highway encroachments and to river development (continued).

Bridge Location	Local Effects	Upstream Effects	Downstream Effects
 c. - Floodplain Encroachment	1 - Erosion of highway fill and submergence possible during floods 2 - Pattern of overbank spill are affected by the encroachment and in highly shifting channels may change river course downstream. 3 - Lateral drainage into the river is interrupted and may cause flooding and erosion.	1 - If significant encroachment on the floodplain waterway, backwater may be induced.	1 - If the river channel is highly shifting, the channel alignment may change. 2 - If significant erosion experienced upstream, aggradation will occur.

(14) *Longitudinal Encroachment* (continued)

Case (2) illustrates a situation where a bridge is constructed across a tributary stream. *The average water surface elevation in the main channel acts as the base level for the tributary.* It is assumed that at some point in time after the construction of the bridge, the base level has been lowered. Under the new imposed condition, the local gradient of the tributary stream is significantly increased. This increased energy gradient induces head cutting and causes a significant increase in water velocities in the tributary stream. The result is bank instability, possible major changes in the geomorphic characteristics of the tributary stream and increased local scour.

Case (3) illustrates a situation where a bridge supported by piers and footings is constructed across a channel that is subjected to long periods of low stage. *When a river is subject to long periods of low flows, there is a tendency for the low flow to develop a new low water channel in the bed of the main channel.* If the low water channel aligns itself with a given set of piers, it is possible that the depth of local scour resulting from this flow condition may be greater than the depth of local scour at high stage. There have been *several examples documented where bridges have been entirely safe in terms of local scour at high stage, but have failed or have partially failed as a consequence of the development of greater local scour during low flow periods.*

Case (4) illustrates a situation where artificial cutoffs have straightened the channel downstream of a particular crossing. It is obvious that *straightening the channel* downstream of the crossing *significantly increases the channel slope. In general, this causes higher velocities, increased bed material transport, degradation and possible head cutting in the vicinity of the structure.* This can result in unstable river banks and a braided streamform. The straightening of the main channel brings about a drop in base level and any tributary streams flowing into the affected reach of the main channel are subjected to conditions discussed in Case (2).

Case (5) illustrates the situation where a bridge is constructed across a river immediately downstream of the confluence with a steep tributary. The tributary introduces relatively large quantities of bed materials into the main channel. As a result, an island has formed in the main channel and divided flow exists. In order to reduce the cost of the bridge structure, the bridge is built across one subchannel to the island or bar formed by deposition. Such a procedure forces all of the water and sediment to pass through a reduced width. This *contraction of the river in general increases the local velocity, increases general and local scour, and may significantly increase bank instability.* In addition, the contraction can change the alignment of the flow in the vicinity of the bridge and thus would affect the downstream main channel for a considerable distance. A chute channel can develop across the second point bar downstream and its effect may extend several meander loops downstream. Upstream of the bridge, there is aggradation and its amount depends on the magnitude of water and sediment being introduced from the tributary. Also, there is significant increase in the backwater upstream of the bridge at high flows which in turn affects other tributaries farther upstream of the crossing.

Case (6) illustrates a situation where the main channel is realigned in the vicinity of the bridge crossing. A cutoff is made to straighten the main channel through the selected bridge site. As discussed in Case (4), increased local gradient, local velocities, local bed material transport, and possible changes in the characteristics of the channel are expected due to the new imposed conditions. As a result the channel may braid. On the other hand, if the straightened section is designed to transport the same sediment loads that the river is capable of carrying upstream and downstream of the straightened reach, the bank stability is ensured. Such a channel should not undergo significant change over either short or long periods of time.

It is possible to build modified reaches of main channels that do not introduce major adverse responses due to local steepening of the main channel. In order to design a straightened channel so that it behaves essentially as the natural channel in terms of velocities and magnitude of bed material transport, it is necessary, in general, to build a wider, shallower section.

Case (7) illustrates a bridge constructed across a main channel. Subsequently, the base level for the channel is raised by the construction of a dam. Whenever the base level of a channel is raised, a pool is

created extending a considerable distance upstream depending on the amount of raise. As the water and sediment being transported by the river encounters this pool, most of the sediments drop out, form
amount of raise. As the water and sediment being transported by the river encounters this pool, most of the sediments drop out, forming a delta-like structure at the mouth. The deposition of sediment at the
If the bridge lies within the effects imposed by the new base level, the following effect at the crossing will be expected: a loss of waterway at the bridge site, significant changes in river geometry, and increased flood stages. In the extreme it is possible that the river may become sufficiently perched that at some high flow it could abandon the old channel and adopt a new one.

Case (8) considers the situation where the sediment load is reduced in the channel after a bridge has been constructed. This may happen due to the construction of a storage dam upstream of the crossing. As stated in the preceding case, the raising of base level of a river, as in the development of storage by constructing a dam on a river, provides a desilting basin for the water flowing in the system. In most instances all of the sediment coming into a reservoir drops out within the reservoir. Water released from the reservoir is mostly clear. The existing river channel is the result of its interaction with normal water-sediment flows over a long period of time. *With the sediment-free flows, the channel is too steep and sediments are entrained from the bed and the banks bringing about significant degradation.* If the bridge is sufficiently close to the reservoir to be affected by the degradation in the channel, the depth due to general and local scour at the bridge may be significantly increased. Also, the channel banks may become unstable due to degradation and there is a possibility that the river, as its profile flattens, may change its form. In the extreme case, it is possible that the degradation may cause failure of the dam and the release of a flood wave.

Case (9) illustrates a more complicated set of circumstances. In this case the river crossing is affected by Dam A constructed upstream as well as Dam B constructed downstream. As documented in the preceding case, Dam A causes significant degradation in the main channel. Dam B causes aggradation in the main channel. The final condition at the bridge site is estimated by summing the affects of both dams on the main channel and the tributary flows. Normally, this analysis requires water and sediment routing techniques studying both long- and short-term effects of the construction of these dams and it is necessary to consider the extreme possiblities to develop a safe design.

In Case (10) Bridges A and B cross two major tributaries a considerable distance upstream of their confluence. Upstream of Bridge A, a diversion structure is built to divert essentially clear water by canal to the adjacent tributary on which Bridge B has been constructed. Upstream of Bridge B the clear water diverted from the other channel enters the storage reservoir and the water from the tributary plus the transfer water is released through a hydro-power plant. Ultimately, it is anticipated that a larger storage reservoir may be constructed downstream of the confluence on the main stem at C. These changes in normal river flows give rise to several complex responses at bridge sites A and B, in the tributary systems as well as on the main stem. Bridge site A may aggrade due to the excess of sediment left in that tributary when clear water is diverted. However, initially there may be lowering of the channel bed in the vicinity of the diversion structure because of the deposition upstream of the diversion dam and the release of essentially clear water for a relatively short period of time until the sediment storage capacity of the reservoir is satisfied. Bridge site B is subjected to degradation due to the increased discharge and an essentially clear water release. However, the degradation of the channel could induce degradation in the tributaries causing them to provide additional sediment to the main channel, see Case (8). This response would to some degree counteract the degrading situation in this reach of river. Such changes in river systems are not uncommon and introduce complex responses throughout the system. Any complete analysis must consider the individual effects and sum them over time to determine a safe design for the crossings.

Case (11) shows a highway that crosses the main channel at Bridge A and its tributary at Bridge B. The confluence of the main channel and its tributary is downstream of both bridges. It is assumed that the alignment of the main channel is continually changing. The rate of change in the river system will have been evaluated as part of the geomorphic and hydraulic analysis of the site. If the main channel shifts to the alternate position shown and moves the confluence closer to Bridge B, *the gradient in the tributary is significantly increased causing degradation therein as well as channel instabilities and possible changes in river form. Excess sediment from the tributary causes aggradation in the main channel and possibly significant changes in channel alignment.* Considering the possible changes in the position of the main channel, training works may be required at and upstream of Bridge A to assure a satisfactory approach of the flow to the bridge crossing. Otherwise, the river could abandon its present channel as shown in Table 8.2.1. A shift in the position of the main channel relative to the position of the confluence with the tributary also alternately flattens or steepens the gradient of the tributary causing corresponding aggradation or degradation in the tributary. This type of problem is rather difficult because of the ever changing characteristics of such river systems. Rivers of this type are usually stable for several years at a time or at least between major flows. Consequently, if crossing locations are properly selected and appropriate stabilization techniques and measures are taken, it may be possible to maintain the usefulness of the crossings for the life of the structures. However, the disadvantages associated with such locations will often require expensive solutions and these locations should be avoided if possible.

Case (12) illustrates a meandering channel with several tributaries and a major storage reservoir constructed on the main channel. Two crossings are shown on the main channel upstream of the reservoir. It is assumed that complete channelizing of the meandering river has been authorized. This, in effect, shortens the path of travel of the water by an appreciable distance. If we consider local effects at the bridges, bridge site A is first subjected to possibly severe degradation and then aggradation. Bridge site B is primarily subjected to degradation. The magnitude of this degradation can be large. With the degree of straightening indicated in the sketch, severe head cutting may be initiated up the main channel as well as the tributaries. The whole system may be subjected to passage of sediment waves and the river form can dramatically change over time. The flood level in the system and the local and general scour in the vicinity of the bridges is greatly affected by the channelization.

As a result of channelization, the river reach at bridge site B braids. Also, in this reach the rate of sediment transport is increased, head cutting is induced and flood stages are reduced. The tributaries in the upper reach are subjected to severe degradation. For the bridge at position A, the channel would probably degrade and then significantly aggrade. *Significant reactions are possible when channelization is undertaken in a river system.* A detailed analysis of all of the responses is necessary before it is possible to safely design crossings such as those at location A and B.

Case (13) is a series of situations, unrelated in some instances and combined in others, which can affect bridge crossings. *Tidal flows, seiches and bores can have significant effects on scour and depth in the channel system.* The tidal flows, seiches and bores, as well as wind waves, can rapidly and violently destroy existing bank lines. When considering earthquakes, it is of interest to look at a seismic probability map of the United States. Large portions of the United States are subjected to at least infrequent earthquakes. *Associated with earthquake activity are severe landslides, mud flows, uplifts in the terrain, and liquefaction of otherwise semi-stable materials, all of which can have a profound effect upon channels and structures located within the earthquake area.* Historically, several rivers have completely changed their course as a consequence of earthquakes. For example, the Brahmaputra River in Bangladesh and India shifted its course laterally a distance of some 200 miles as a result of earthquakes that occurred approximately 200 years ago. Although it may not be possible to design for this type of natural disaster, knowledge of the probability of its occurrence is important so that certain aspects of the induced effects from earthquakes can be taken into consideration when designing the crossings and affiliated structures.

Case (14) illustrates three examples of longitudinal encroachment. In example (a), a few bends of a meandering stream have been realigned to accommodate a highway. There are two problems involved in channel realignment. One, the length of realigned channel is generally smaller than the original channel and consequently results in a steeper energy gradient in the reach. Two, the new channel bank material in the re-aligned reaches may have a smaller resistance to erosion. As a result of these two problems, the channel may suffer instability by the formation of a headcut from the downstream end and increased bank erosion. The realigned channel may also exhibit a tendency to regain the lost sinuosity and may approach and scour the highway embankment. To counter these local effects one could design the realignment to maintain the original channel characteristics (length, sinuosity). Another way would be to control slope by a series of low check dams. In any case, bank protection by riprap, jacks or spurs will be needed. The upstream and downstream effects of the channel realignment will be the same as discussed for channel length reduction in Case 12. For example, as the degradation travels through the realigned reach, sediment load generation in the river by bed and bank erosion will cause aggradation downstream.

Example (b) illustrates encroachment on the waterway of an incised stream flowing through a narrow gorge. This is just one of many possible reasons that the highway may need to encroach on the main channel. Locally, the effect is to reduce the waterway and to increase the velocities and bank and bed erosion potential. The erosion protection of the highway slope exposed to the flow and possibly on the opposite bank are important problems. The backwater induced by this obstruction may cause upstream aggradation and higher flood levels. On the downstream side, channel aggradation may be experienced if bed erosion locally occurs in the encroached reach.

Example (c) is a case of floodplain encroachment. It is assumed that during bankful and lower stages the highway does not interact with the flow. However, during high stages the flow area is occupied by the encroachment. Locally, the highway is to be protected against inundation and erosion during flood. The effect on the river channel depends on the extent of encroachment on the waterway. If the highway occupies a significant portion of the floodplain, it may increase river stages for a given flood. If the river channel is a shifting one, the highway encroachment may alter the direction and pattern of spill onto the floodplain and back into the channel. Very often this type of encroachment has little or no effect on flood stages or on the stream upstream or downstream.

In all cases of longitudinal encroachment, the lateral drainage into the river will be intercepted. A main consideration in the design of encroachments will be to provide for this drainage.

8.2.2 Actual case histories of river encroachments

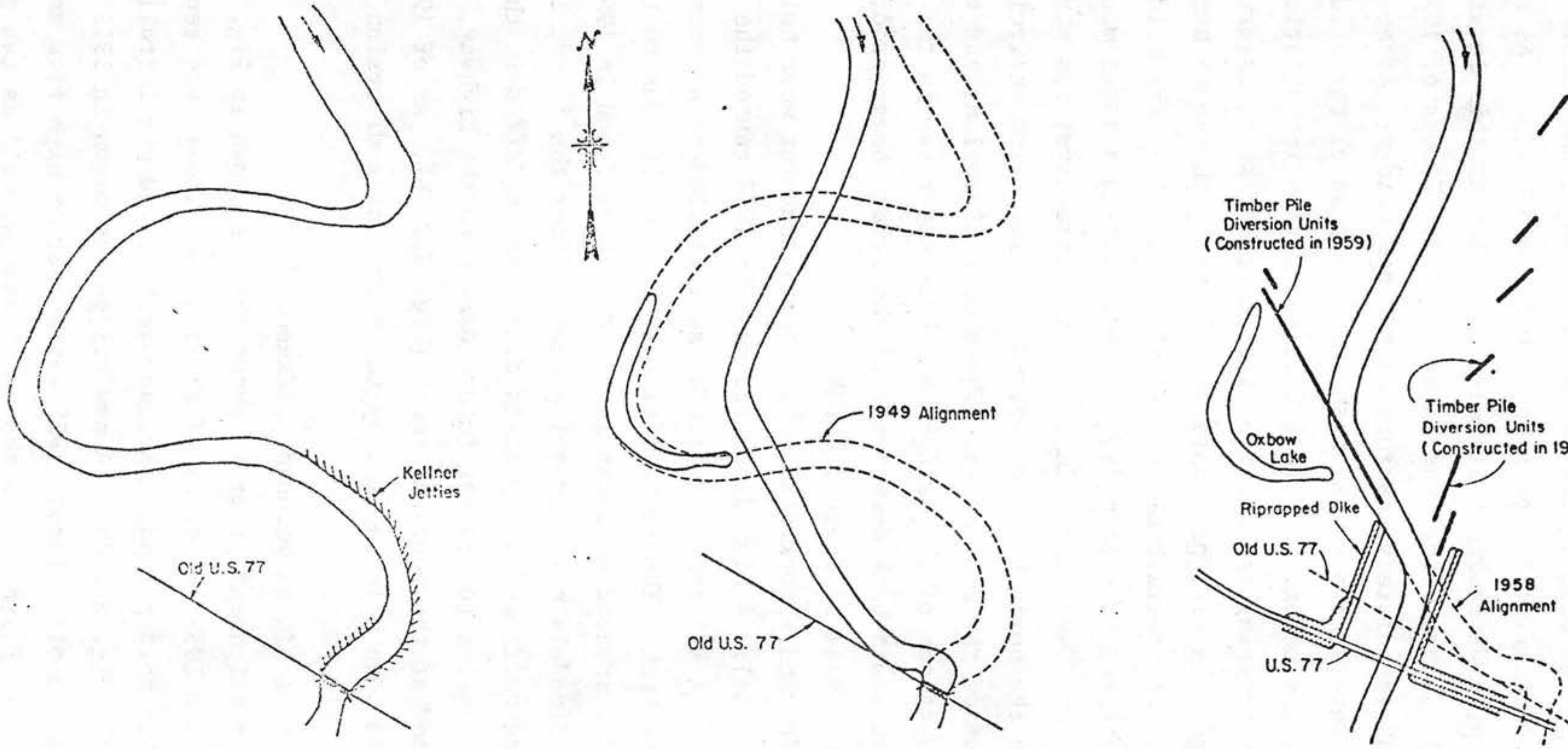
In the preceding paragraphs several possible cases were discussed that the engineer dealing with river encroachments might encounter. As a follow up, some actual river encroachments are presented. Each case considers the interactions between the river and the encroachment over a period of time. In general, these particular cases are not as complex as some of the foregoing hypothetical cases. For example, there is no consideration of water resources development throughout the basins, including construction of reservoirs, transmountain diversions, and so forth.

Washita River, west of Wynnewood, Oklahoma

During a large flood in 1949, the concave bank immediately upstream of the old bridge began eroding at an excessive rate. After the subsidence of the high flows, Kellner jetties were installed as shown in Fig. 8.2.1a to prevent further erosion of the banks. The jetty field was successful. Then, in a 50 to 60 year return period flood in 1958, *the channel developed a new alignment by cutting across several meander loops* and washed out a section of the old U.S. 77, (see Fig. 8.2.1b). In 1959, a new bridge was built over the more stable river alignment (Fig. 8.2.1c) and *riprap dikes were constructed to protect the structure.* Six timber pile diversions were constructed on the east edge of the floodplain and have been dormant since. Two timber pile diversion structures were constructed on the southwest bank of the river upstream of the bridge to prevent a meander loop from developing and encroaching on the approach to the bridge. These structures proved to be ineffective, as the river flows under the timber pile structures at all stages. Nevertheless, in 1968, the river was still being held in a proper channel location relative to the bridge.

Cimarron River, south of Perkins, Oklahoma

At some point in time, AT&SF Railroad constructed Kellner jetties on the south bank of the river to prevent it from encroaching on the



(a) Channel in 1949

(b) Channel in 1958

(c) Channel in 1968

Fig. 8.2.1 Washita River, west of Wynnewood, Oklahoma.

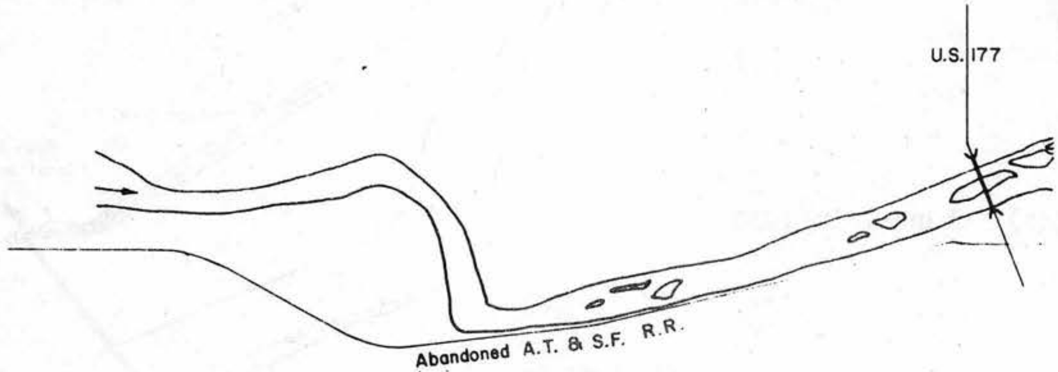
road bed. Figure 8.2.2a shows the alignment of the river in 1938. Note the location of the bend in the river. In 1949, flood waters eroded the south bank immediately upstream from the old bridge. As a result, in 1950 five pile diversion units were installed at this location (Fig. 8.2.2b). A new bridge was built in 1953. Floods of 1957 eroded the south bank immediately upstream of the new bridge. After subsidence of the flood waters, riprap was installed upstream of the bridge to prevent further erosion. Figure 8.2.2c shows this installation as well as subsequent training and stabilization that became necessary. Floods of 1959 damaged the five pile diversion units on the south bank and eroded some of the north bank immediately upstream of the bridge. *High flows were recorded from 1959 to 1962 causing a rapid movement of the meander loop downstream.* In 1963, five pile diversion structures were built on the north bank to prevent the river from encircling the north abutment of the new bridge. These pile diversions and the dike immediately upstream of the bridge have held the river in the same location. The south bank downstream of the bridge began eroding in 1971.

Cimarron River, east of Okeene, Oklahoma

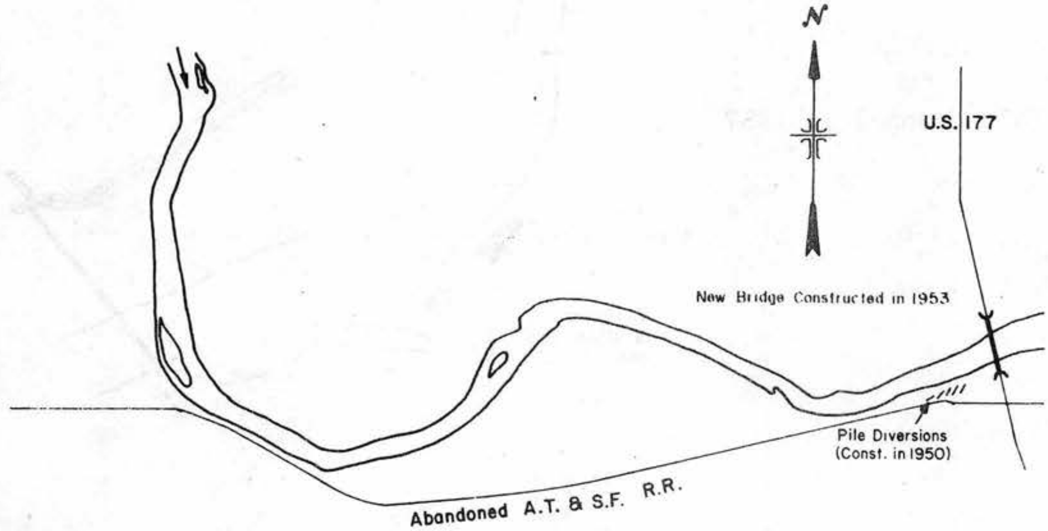
The bridge and a rock dike on its north abutment were built in 1934 (Fig. 8.2.3a). A high discharge year in 1938 caused the south bank to erode. A Kellner jetty field was installed to prevent further erosion of the bank. The jetty field was ineffective due to the lack of debris and suspended sediment load and a large flood in 1957 spread out over the floodplain in several places. After the flood, *seven timber pile diversion units and a riprapped dike were installed* in the old jetty field location to prevent future damage to the highway. Riprap was also placed at the south abutment (Fig. 8.2.3b). As of 1968, the south bank had been held in line by the timber pile diversion structures and the dike (Fig. 8.2.3c).

Cimarron River, south of Waynoka, Oklahoma

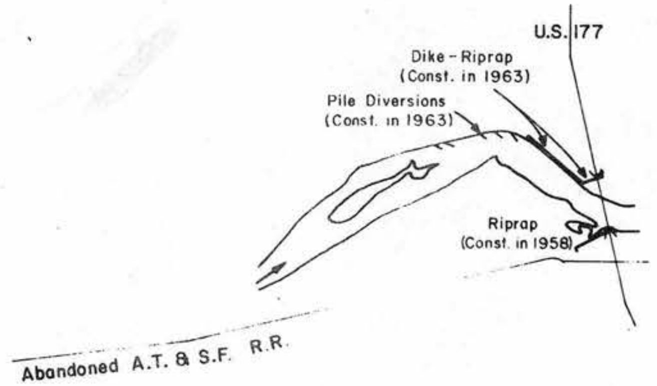
The river alignment prior to about 1942 is shown in Fig. 8.2.4a. Between 1942 and 1952, a series of above normal flows were reported. At some time in this period, a Kellner jetty field was installed on the north bank (Fig. 8.2.4b). A new bridge was begun in 1955 just upstream from the old bridge. 1955 was a year of high flow and several timber piles were installed on the north bank as well as two piles on



(a) Channel in 1938



(b) Channel in 1956



(c) Channel in 1969

Fig. 8.2.2 Cimarron River, south of Perkins, Oklahoma.

(a) Channel in 1938

(b) Channel in 1957

(c) Channel in 1968

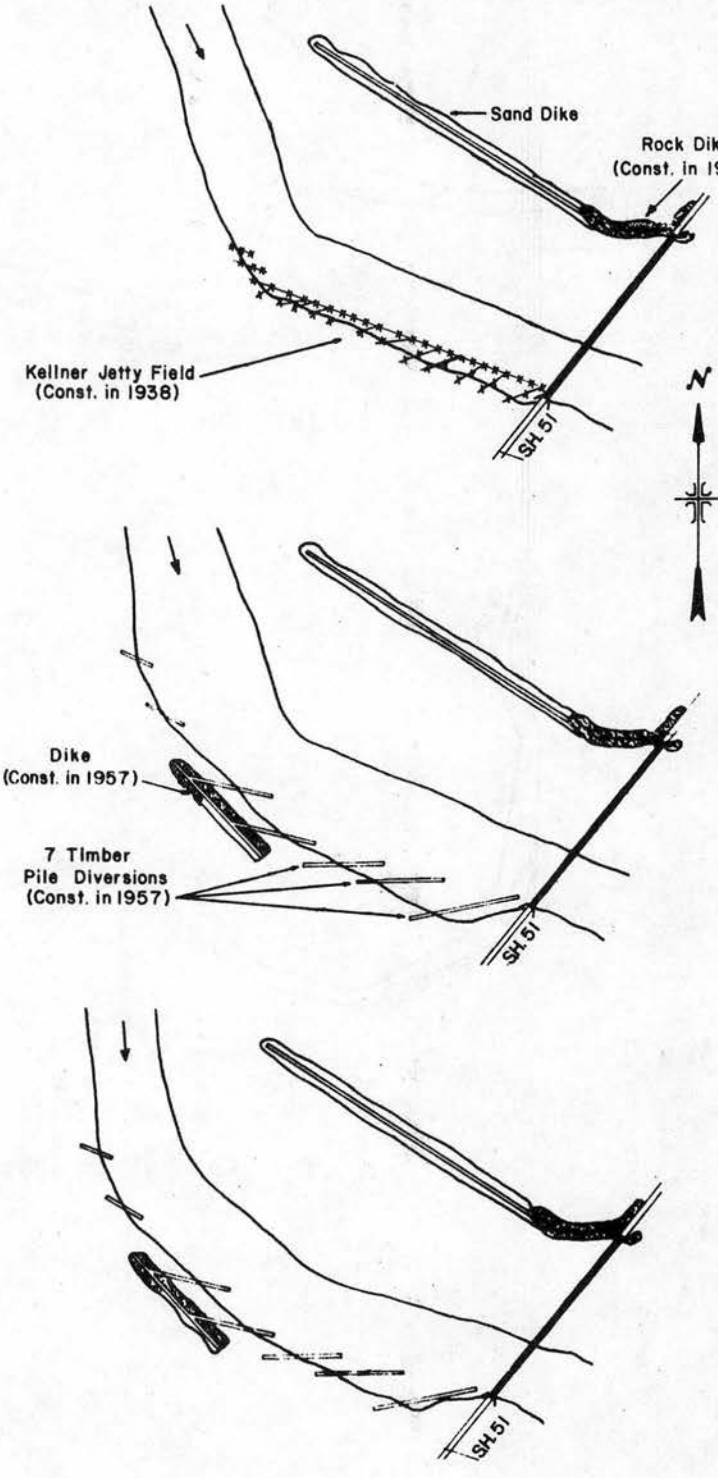


Fig. 8.2.3 Cimarron River, east of Okeene, Oklahoma.

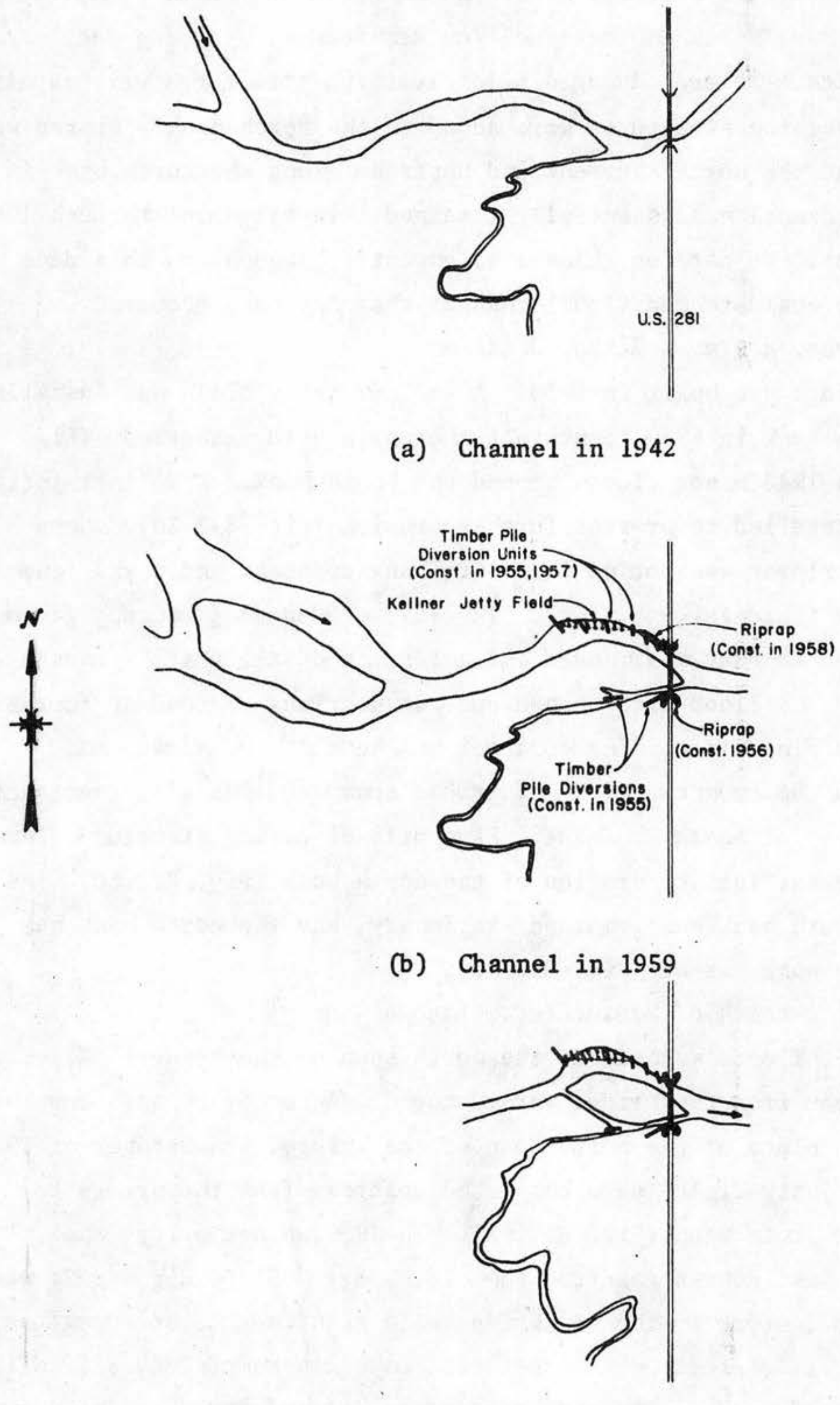
the south bank (Fig. 8.2.4b). *The highest flood on record occurred in 1957 and overturned two piers of the new bridge, dropping three spans into the water.* Several damaged pile diversion structures were repaired and pile diversion structures were added to the north bank. Riprap was placed around the north abutment and upstream along the north bank in 1958. The channel has essentially retained this alignment through 1968 (Fig. 8.2.4c). No data on channel alignment subsequent to this date were available to evaluate additional changes that may have occurred.

Arkansas River, north of Bixby, Oklahoma

The bridge was built in 1938. A Kellner jetty field was installed on the north bank in 1939 to protect the north bridge abutment (Fig. 8.2.5a). In 1948 minor floods eroded the south bank. A Kellner jetty field was installed to prevent further erosion (Fig. 8.2.5b). Some time after, riprap was put on the south bank upstream and downstream of the jetty field (Fig. 8.2.5c). *In 1959, a 50-year frequency flood eroded the north bank and washed out a section of the north approach to the bridge.* The flood also washed out two sections of roadway further north on the floodplain. *The approach was rebuilt and riprap was installed on the embankment. A riprapped spur dike was also constructed just south of the north abutment.* Five pile diversion structures were built to prevent further erosion of the north bank (Fig. 8.2.5c). As of 1968, the south bank has remained stationary, and the north bank has filled in to some extent (Fig. 8.2.5d).

Washita River, north of Maysville, Oklahoma

In 1949, floods washed out the north span of the bridge. Also, the banks upstream from the bridge were damaged. A temporary structure was installed in place of the north span of the bridge. In October of 1949, two Kellner jetty fields were completed upstream from the bridge to provide bank protection (Fig. 8.2.6a). In 1950, a new bridge was constructed just downstream from the old bridge. State Highway 74 was realigned to conform to the new bridge. *In eight months of operation, the Kellner jetty field on the northeast bank had completely silted in. This was largely due to the clay content in the suspended sediment and the large amount of drift in the stream* (Fig. 8.2.6b). The floods of 1957 did very little damage to this bridge site or the banks. Floods

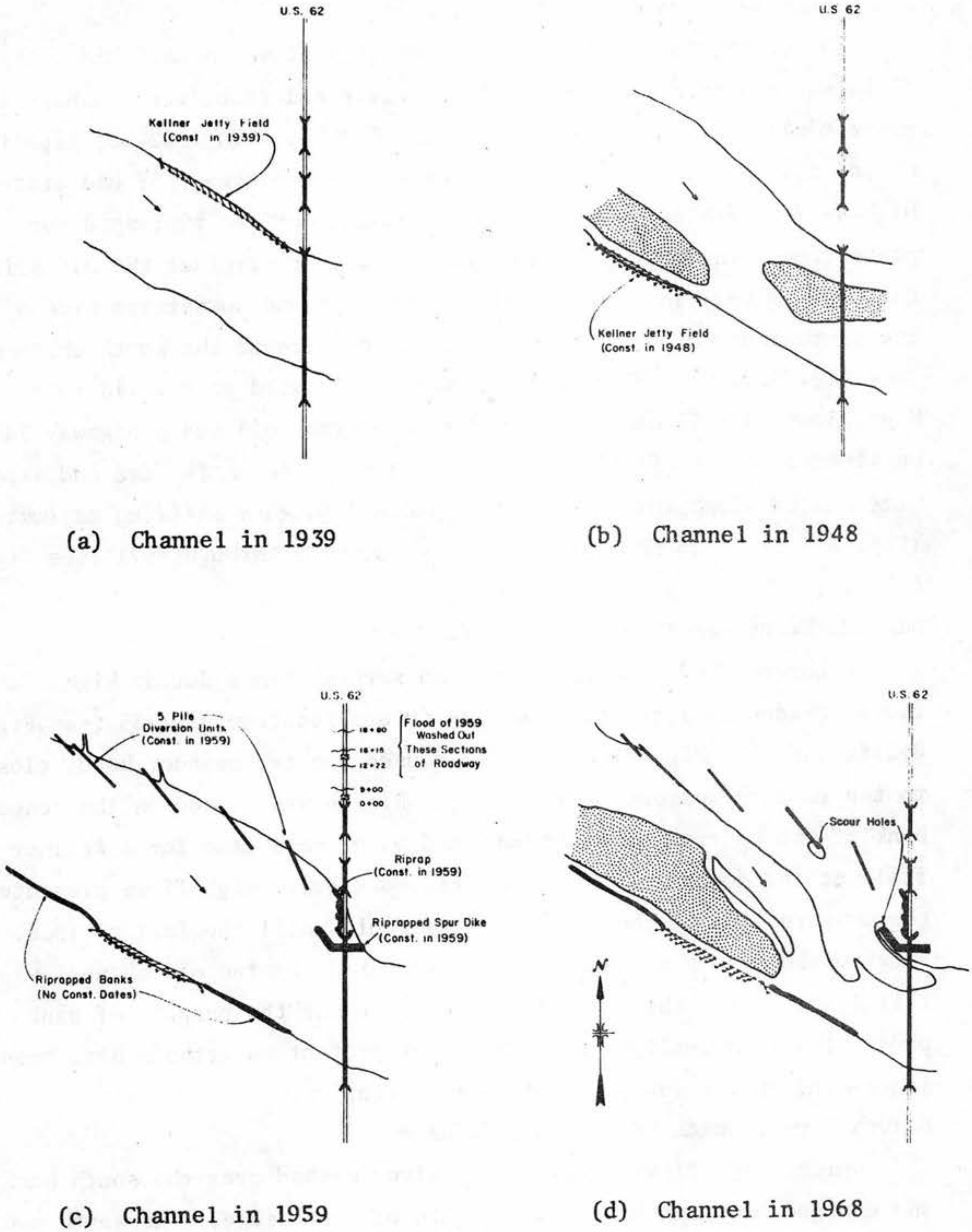


(a) Channel in 1942

(b) Channel in 1959

(c) Channel in 1968

Fig. 8.2.4 Cimarron River, south of Waynoka, Oklahoma.



(a) Channel in 1939

(b) Channel in 1948

(c) Channel in 1959

(d) Channel in 1968

Fig. 8.2.5 Arkansas River, north of Bixby, Oklahoma.

in 1968 and 1969 have caused bank erosion on the north bank upstream of the jetty field which could eventually cut in behind the jetty field (Fig. 8.2.6c).

Cimarron River, south of Crescent, Oklahoma

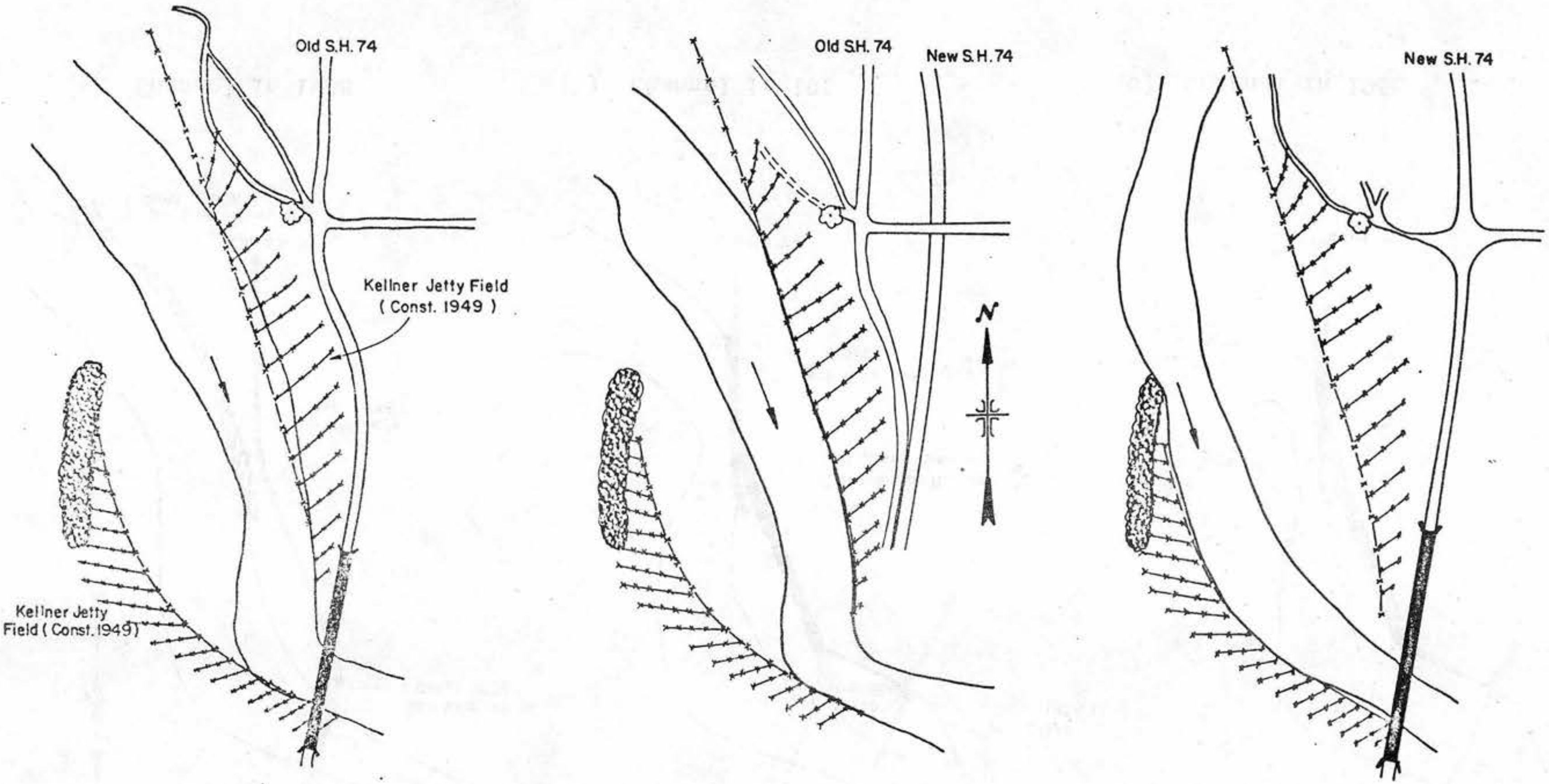
The water year 1937-38 was one of high flow. A Rayfield jetty field was constructed to control the river and stabilize the bank to protect old State Highway 74 (see Fig. 8.2.7a). In 1942-43, high flows caused the east bank to erode closer to the embankment of old State Highway 74. Sixteen hundred feet of embankment was riprapped for protection. In 1956, a new bridge was built to replace the old bridge. Earth dikes were constructed on the upstream and downstream side of the south abutment. Riprap was also placed around the south abutment (see Fig. 8.2.7b). The north abutment is located on a solid rock bluff. High flows in 1957 caused the river to overtop old State Highway 74 in an attempt to cut off the bend to the north. *An earth dike and riprap were placed along the east bank to prevent further shifting eastward* (Fig. 8.2.7b). No further damage has occurred through 1971 (see Fig. 8.2.7c).

Washita River, south of Davis, Oklahoma

Prior to 1967, the river shifted several times during high flows, gradually encroaching on the future location of I-35 (see Figs. 8.2.8a and b). High flows in 1967 caused the two meander bends closest to the road to approach even closer. Riprap was placed on the concave bank of the upstream meander bend and plans were made for a Kellner jetty field at the downstream bend (see Fig. 8.2.8c). High flows prevented the construction of the Kellner jetty field until the fall of 1968. After placement, a new channel was excavated and the old channel was filled in. Thus, the jetty field was more for the purpose of bank protection than realignment. Both bank protection methods have been successful in the few years of their operation.

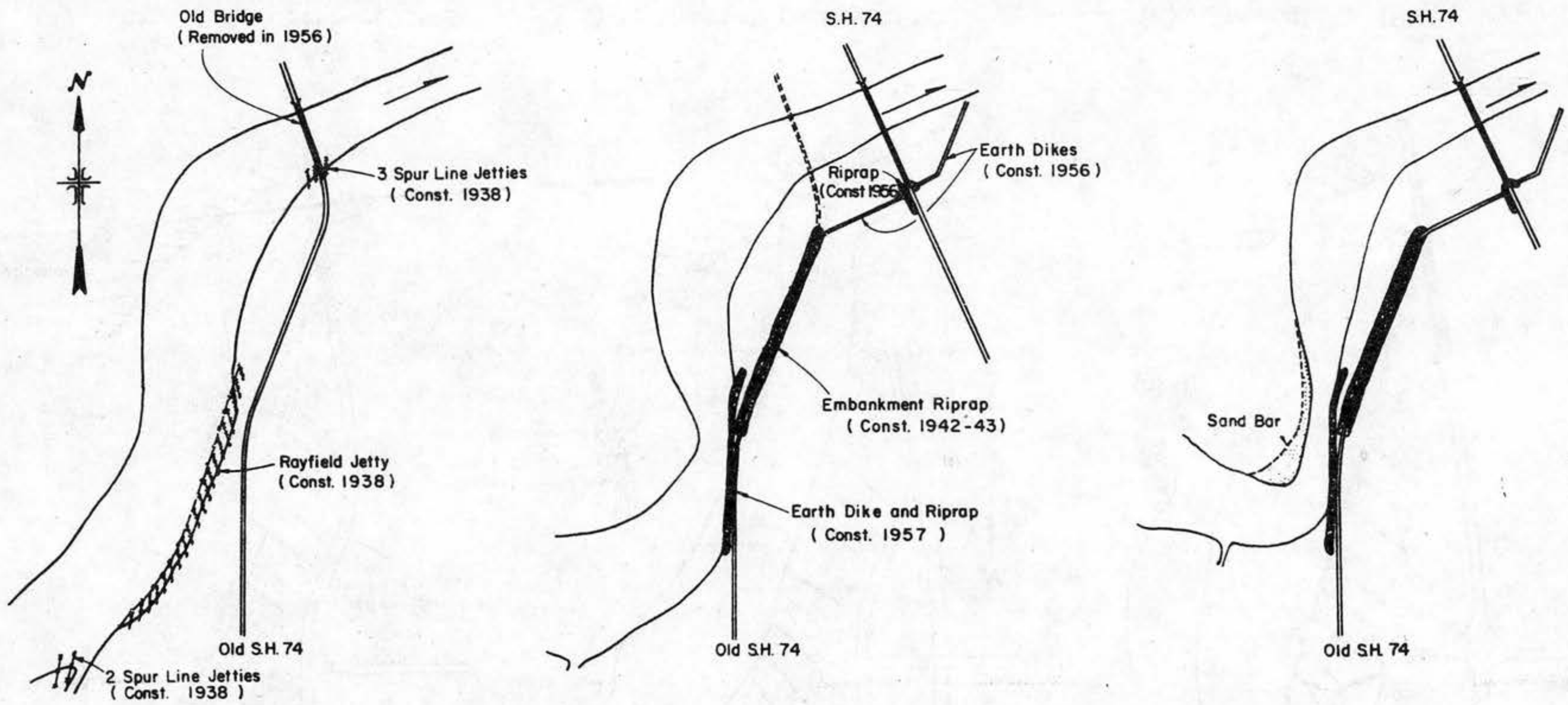
Beaver River, north of Laverne, Oklahoma

During high flows of 1938, the river washed over the south bank and damaged the approach roadway south of the bridge. The south end of the bridge was also damaged. Jetty fields were constructed in several locations upstream of the bridge in an attempt to reduce bank erosion. *Two jetty lines were constructed in a side channel downstream*



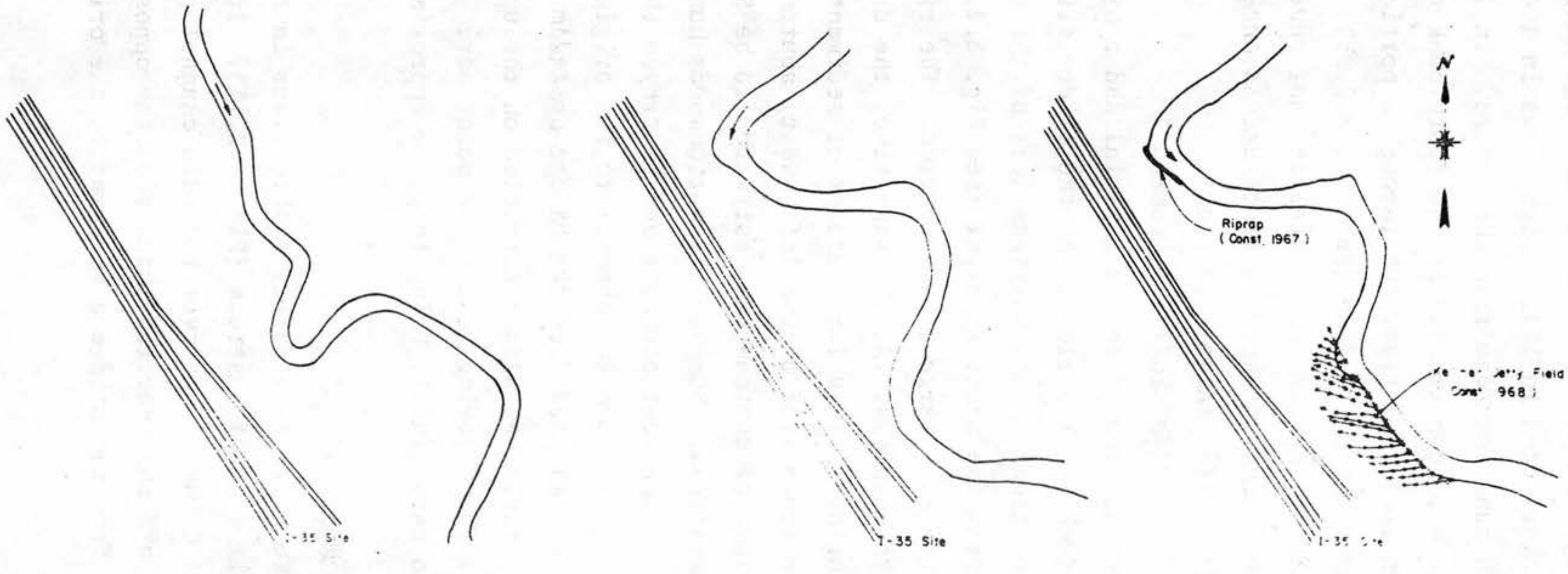
(a) Channel in October, 1949 (b) Channel in May 1950 (c) Channel in 1969

Fig. 8.2.6 Washita River, north of Maysville, Oklahoma.



(a) Channel in 1938 (b) Channel in 1957 (c) Channel in 1968

Fig. 8.2.7 Cimarron River, south of Crescent, Oklahoma.



(a) Approximate channel location in 1940

(b) Approximate channel location in 1966

(c) Approximate channel location in 1968

Fig. 8.2.8 Washita River, south of Davis, Oklahoma.

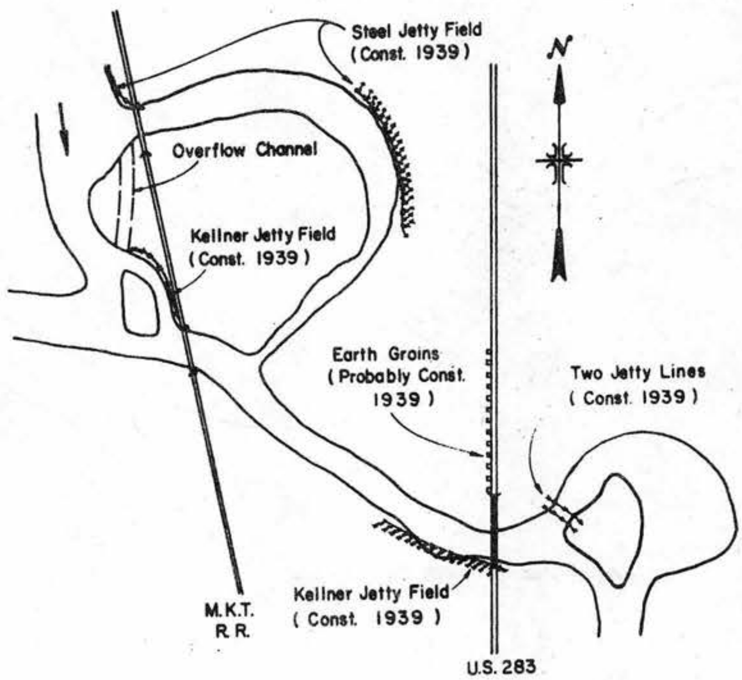
of the bridge to discourage flow in that channel to prevent eddy currents from eroding the north embankment (see Fig. 8.2.9a). A new longer bridge was constructed in 1941. High flows in 1946 caused severe erosion on the south bank upstream from the bridge. In 1949, an earth dike and jetty field were constructed on the south bank to prevent further erosion. In 1969, the river cut through a portion of the 1949 jetty field and eroded the earth dike (see Fig. 8.2.9b). *Car bodies were used as bank protection. However car bodies are not environmentally acceptable and are difficult to hold in place unless anchored with cable or weighted down with concrete or rocks.*

Powder River, 40 miles east of Buffalo, Wyoming

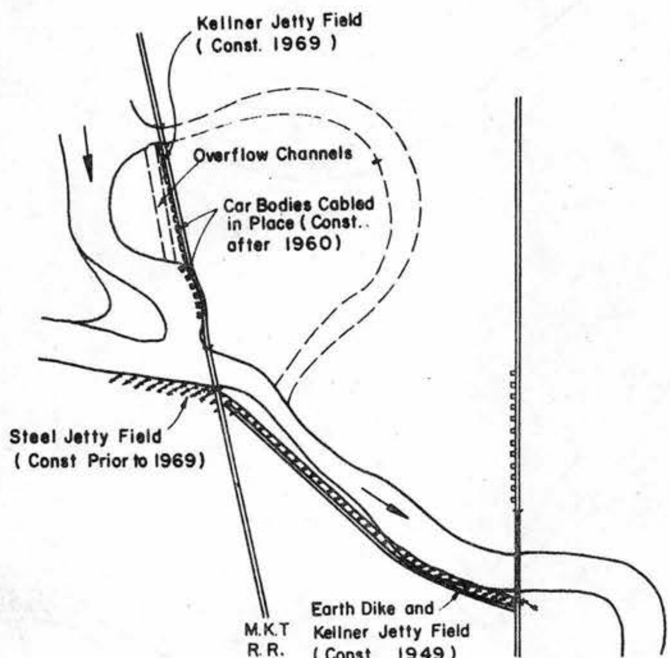
The Powder River has very fine bed material and a high sinuosity. The river contains dunes at low flows. At the bridge site, there was a grove of cottonwood trees on the upstream left of the bridge and a dry draw coming in from the upstream right (see Fig. 8.2.10a). Upon completion of the bridge, a large flood occurred. The river attempted to straighten out its meanders. At the same time, the draw on the upstream right was bringing in a large amount of sediment, forcing the stream toward the upstream side of the left (west) abutment. The flood flow uprooted the grove of cottonwoods (estimated to be 50 years old) and carried them downstream. Some of the cottonwoods hung up on the riprapped spur dike at the west abutment and destroyed the dike completely (see Fig. 8.2.10b). To restore the channel to its original alignment, *a training dike was constructed from the bridge upstream to a nearby bluff.* A jack jetty field was also constructed on the upstream meander to prevent the river from flowing across the point bar. This has been fairly successful to date (1974), being in place approximately six years (see Fig. 8.2.10c).

North Platte River, Wyoming

The North Platte River is a fairly stable river in this reach as a result of reservoir control upstream (Fig. 8.2.11). It was decided at this crossing to build the bridge over the main channel and part of the island and to block off the overflow channel on the opposite side of the island. Downstream from the bridge a spur was in the original channel

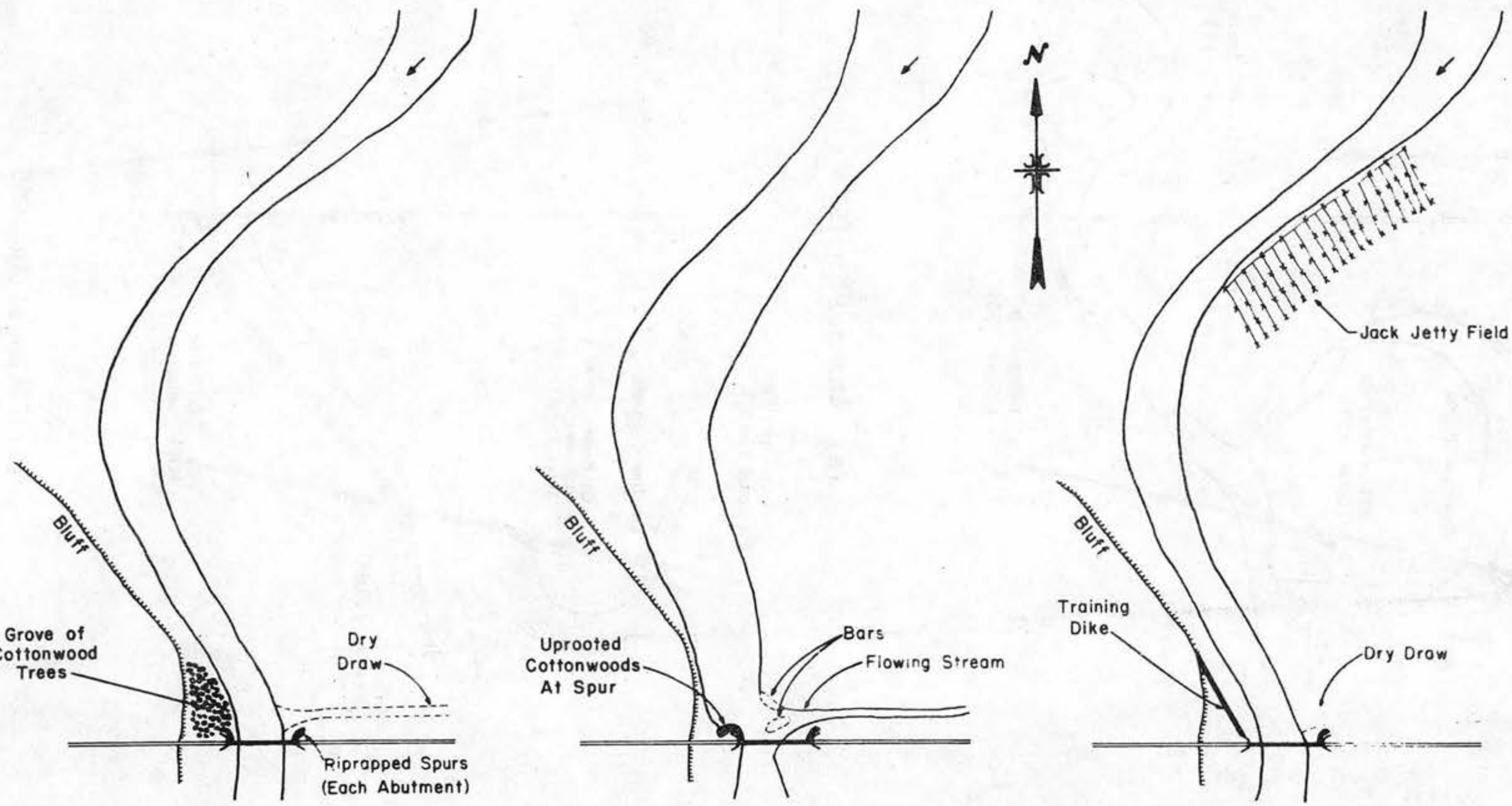


(a) Channel in 1939



(b) Channel in 1960

Fig. 8.2.9 Beaver River, north of Laverne, Oklahoma.



(a) Channel after completion of bridge.

(b) Channel during flood

(c) Channel after restoration

Fig. 8.2.10 Powder River, 40 miles east of Buffalo, Wyoming.

to protect the bank (Fig. 8.2.11a). *Two situations can occur due to this choice for the bridge crossing.* One situation is that the spur is washed out and the concave bank erodes due to the high velocities resulting from the decreased area of flow under the bridge (see Fig. 8.2.11b). In fact, with extreme flows the river could erode a chute across the point bar on the first bend downstream. The other situation which may occur is that the high velocity flow carries increased sediment load and deposits this material in the eddies downstream of the spur (see Fig. 8.2.11c). This would not cause any problems.

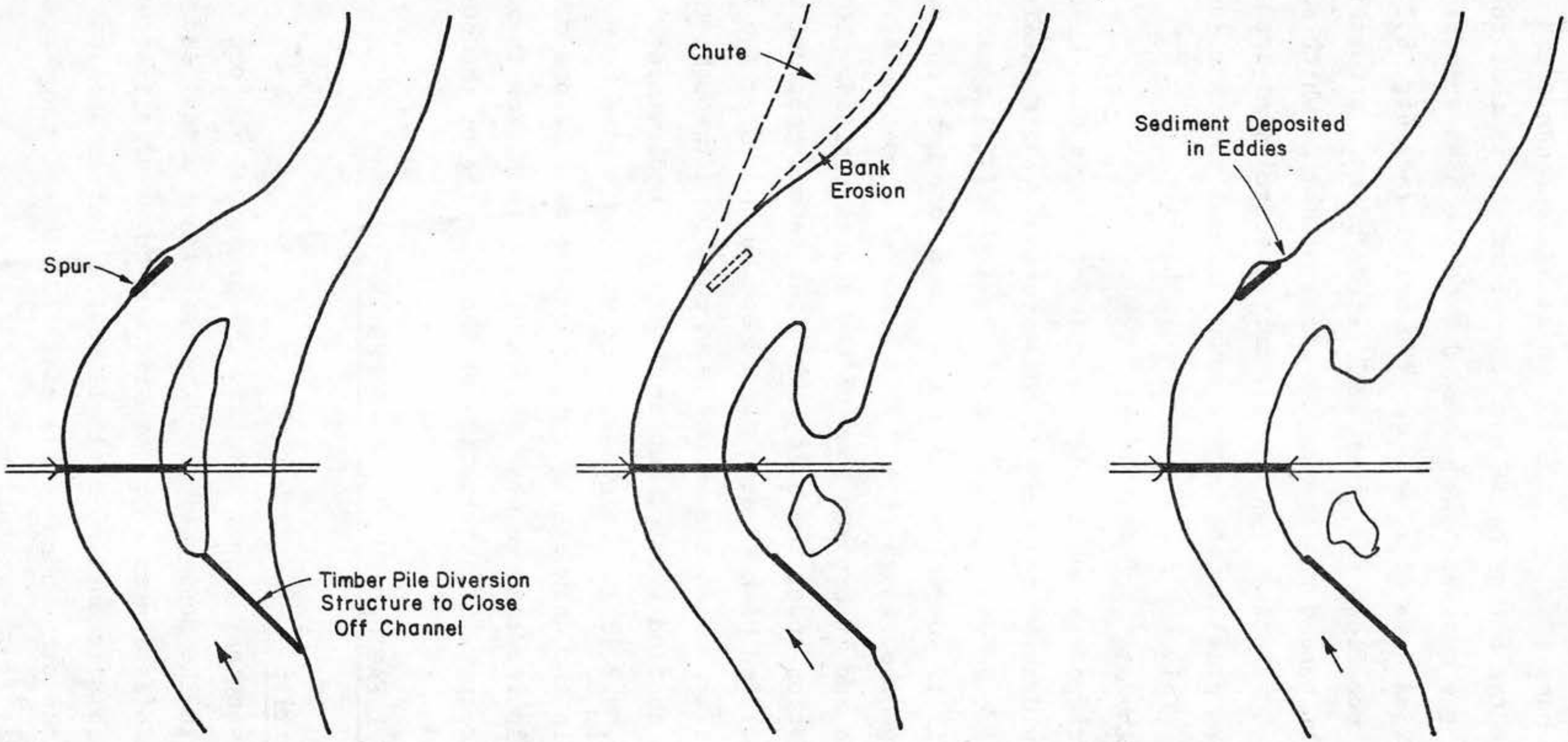
Coal Creek, tributary of Powder River, Wyoming

A small bridge was constructed over intermittent Coal Creek. Coal Creek was a dry draw at the time of construction. *There existed some head cuts downstream from the bridge at the time of its construction* (see Fig. 8.2.12a). During a subsequent flood, one head cut moved upstream through the bridge site. This head cut almost undercut the midstream piles and it exposed some of the abutment piles. To prevent further degradation under the bridge when the second head cut moves through, rock filled baskets were placed on the bed under the bridge (see Fig. 8.2.12a). When the second head cut moved through, the rock filled baskets settled slowly, but prevented the undermining of the piles (see Fig. 8.2.12b). Other alternatives would have been to excavate the head cuts in the channel through the bridge site before constructing the bridge, allowing them to move naturally upstream from there, or to set the piles deeper in anticipation of the lowering of the bed elevation.

8.3.0 PRINCIPAL FACTORS TO BE CONSIDERED IN DESIGN

8.3.1 Introduction

The following paragraphs identify the principal factors that should be considered in the design of crossings and longitudinal encroachments. Because of the differences in river size and forms at different locations, it is not possible to outline a single system that is applicable to all problems. On the other hand, *it is essential that the application of the fundamentals to these problems be understood.*

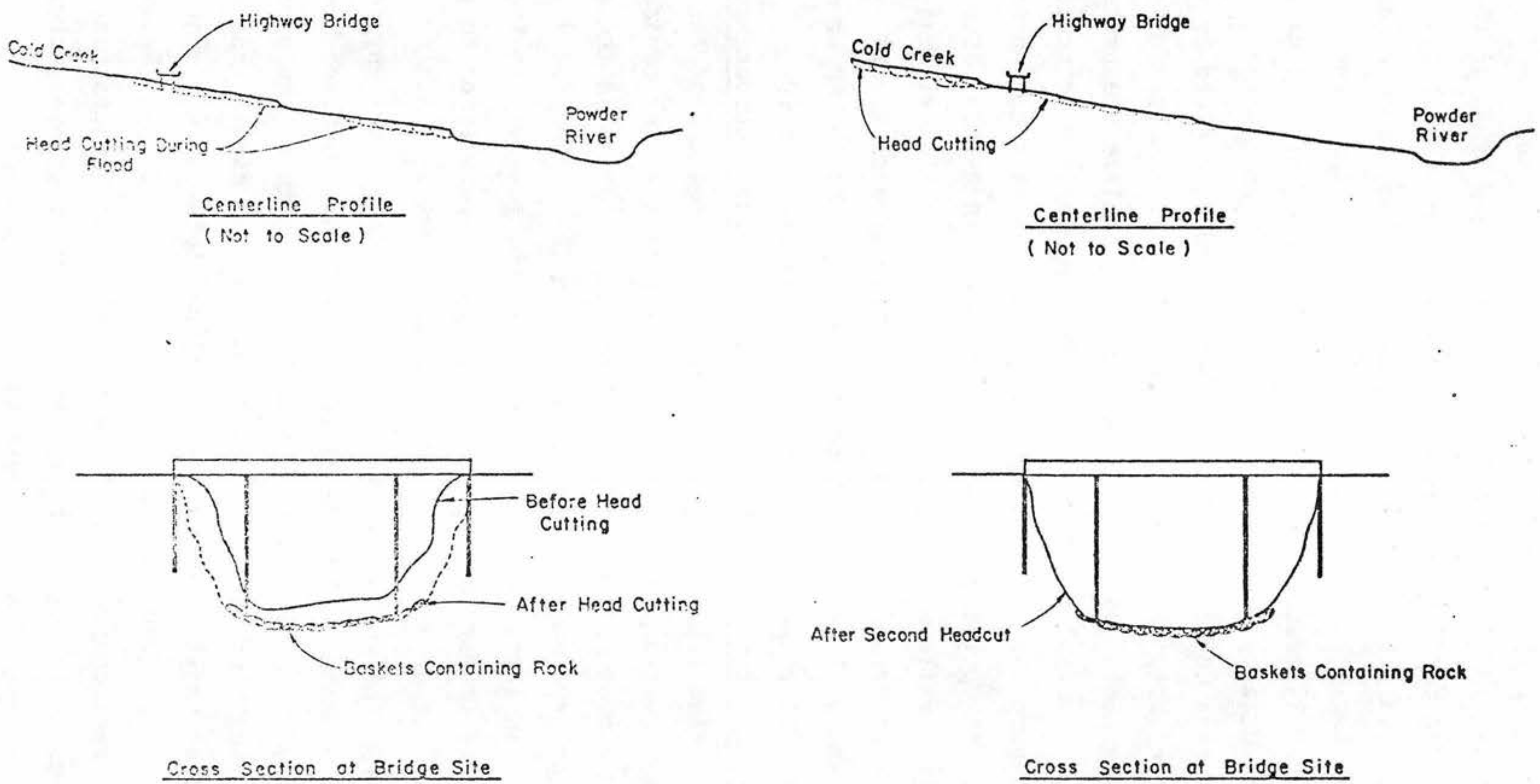


(a) Channel immediately after closure

(b) Channel after first flood (situation 1)

(c) Channel after first flood (situation 2)

Fig. 8.2.11 North Platte River.



(a) First head cut

(b) Second head cut

Fig. 8.2.12 Coal Creek, tributary of Powder River, Wyoming.

8.3.2 Types of rivers

In selecting the site for a crossing or an encroachment on a river it is necessary to give detailed consideration and study to the type of river or rivers involved. A sandbed river may be meandering, it may be essentially straight, or it may be braided. In addition, a meandering river may be small, medium, or large. The same channel can be classified as youthful, mature, or old. *Each of these different river subdivisions requires different design procedures.* For example, in designing training works for large sandbed channels, braided or meandering, it is unlikely that Kellner jetties alone will be useful to stabilize the bank alignment. It may be necessary to stabilize the banks with rock riprap and to control the overbank flows using jetties to achieve a set of specific purposes. Gravel and cobble bed channels are normally considerably steeper than sandbed channels and in general have narrower river valleys. In the extreme are torrential rivers, the beds of which are comprised of large rocks. These type of rivers usually exist in a youthful or canyon type environment near the upper end of large river systems where the slopes are relatively steep.

8.3.3 Location of the crossing or the longitudinal encroachment

In selecting the site of a crossing or a longitudinal encroachment several considerations are necessary. First of all, *the crossing or encroachmennt must mesh with the transportation system in the area.* Secondly, *the environmental considerations cited in Chapter 1 should be considered.* In fact, unless appropriate weight is given to the environmental impacts it may not be possible to obtain permission to proceed with the project at all. *Economic considerations are equally important.* Depending upon the characteristics of the rivers and the environmental considerations, the cost of a particular crossing or encroachment can be significantly affected by its location. The length of the approaches versus the length of the bridge, the cost of real estate that must be acquired to accomplish the crossing, the maintenance cost required to keep the crossing functional over its estimated life and the method of the construction are some of the more specific aspects that should be considered in locating the crossing. The cost of protective measures should also be considered in locating an encroachment.

8.3.4 River characteristics

The subclassifications of river form can be utilized to identify the range of conditions within which the particular river operates. It is necessary to determine if a river is relatively stable in form or is likely to be unstable. In Fig. 1.3.2 of Chapter I, it was pointed out that rivers can be essentially poised so that a small change in discharge characteristics can change a river from meandering to braided or vice versa. *It is important to know the sensitivity of any river system to change.* Criteria given in Chapter IV, for example Fig. 4.4.3, or Chapter III, Fig. 3.4.1, can be used to predict this sensitivity. A meandering stream whose slope and discharge plot close to the braided river line in Fig. 4.4.3 may change to a braided stream with a small increase in discharge or slope.

In addition to river form, it is important to determine other characteristics of the channel: that is, the channel may have a sand bed and sand banks; it may have a sand bed and gravel banks; it may have a sand bed and cohesive banks; it may be formed entirely in cohesive materials; it may be formed in gravel; it may be formed in cobbles or it may be formed in other combinations of these materials. Each of these river systems behave differently depending upon the characteristics of the floodplain material, the bank material, and the bed material of the river both over short time and long time. Hence, a rather detailed survey of the characteristics of the bed and bank material coupled with river form plus other pertinent information is essential to design.

8.3.5 River geometry

For planning a river crossing or an encroachment it is important to know the river geometry and its variation with discharge and time. It is essential to know the slope of the channel and preferably the energy gradient through the reach. In Chapter V, relations were presented that illustrate how width and depth vary with stage at-a-section as well as along the length of a channel. For most rivers, if the appropriate hydraulic and hydrologic data are available, it is possible to develop simple relations showing how width and depth vary with discharge.

8.3.6 Hydrologic data

It is necessary to gather all of the hydrologic data pertinent to the behavior of the river and to the design of the river crossing or encroachment. As pointed out in Chapter VII, records of the flood flows are essential. From such information, flow duration curves can be developed, seasonal variations in the river system can be considered and design discharge values can be established depending upon the discharge frequency criteria used in the design. Highway projects constructed with Federal-aid funds and projects under the direct supervision of the Federal Highway Administration should be designed for a "basic flood". The basic flood is a 100 year flood and the design standards for the basic flood are specified by the Federal Highway Administration (1974) and the Water Resources Council (1972).

Also, it is important to consider the low flows that the river channel will be subjected to and the possible changes in flow conditions that may be imposed on the river system as a consequence of water resources development in the area. As pointed out in Table 8.2.1, Case (3), sometimes low flows may lead to a more severe local scour situation at bridge piers and footings. Finally, in terms of hydrologic data it is usually necessary to synthesize some of the required data. Conventional techniques may be used to fill in missing records or it may be essential to synthesize records where few hydrological data exist. In synthesizing data it is very important to compare the particular watershed with other watersheds having similar characteristics. With this information, reasonably good estimates of what can be anticipated at the site can be established.

8.3.7 Hydraulic data

At the site of a crossing or a longitudinal encroachment it is essential to know the discharge and its variation over time. Coupled with this, it is necessary to know the velocity distribution in the river cross section and its variation in the river system. This involves determining the type of velocity distribution across the channel as well as in the vertical. Knowledge of the distribution of velocities should be coupled with a study of changes in position of the thalweg to estimate the severity of attack that may occur along the river banks and in the vicinity of the crossing. Furthermore, it is essential to develop stage-discharge relations since these relations fix key elevations of the structure in design and serve as bench-mark data when considering the channel training works that may alter the stage of the river. *Large changes in velocity can occur in a river system with changing discharge and stage.* In a sandbed river, as flow conditions bring about a switch from lower regime to upper regime, the average velocity in the cross section may actually double. From another viewpoint, changes induced in the river system, such as those due to artificial cutoffs or channelization, may sufficiently steepen the gradient so the river operates in upper regime over its whole range of discharge. These possibilities must be considered in the detailed design.

8.3.8 Characteristics of the watershed feeding the river system

The water flowing in the river system and the sediment transported therein are usually intimately related to the watershed feeding the river system. Consequently, *one needs to study the watershed considering its geology, geometry and land use.* In the case of development, land uses including recreation, industrial development, agriculture, grazing, etc., may all be important factors. Similarly, we need to consider the vegetative cover on the watershed and the response of vegetative cover to its utilization by man and to climatic changes. Significant changes in vegetative cover affect the amount of sediment delivered from the watershed to the river system. It is possible to study the sources of sediment in a watershed. One of the most common techniques is to employ aerial photography and remote sensing techniques coupled with ground investigations. The utilization of remote sensing techniques enables the skilled observer to determine which areas of the watershed are stable and which are unstable. Viewing the total watershed from this viewpoint and using water and sediment routing techniques, it is possible to evaluate the sediment yield as a function of time. In addition, such information can be used to determine the feasibility of watershed engineering to help control the water and sediment yield from the watershed to the river system.

8.3.9 Flow alignment

In order to appropriately and safely design a crossing or longitudinal encroachment, it is necessary to consider flow alignment in detail. *The direction of flow must be considered as a function of time.* The position of the thalweg will vary with low, intermediate and high stages. The changing characteristics of the river with stage, such as the change in velocity distribution, the position of the thalweg and the river form can have a significant effect on the intensity of attack on the approaches, the abutments, the piers and embankments. This detailed study of the behavior of the river over time and with varying discharge is necessary for proper design of training works. Only with this type of information can one adequately consider the intensity of attack, the duration of attack and the necessity for training works to make the river system operate within a range of conditions acceptable at the crossing or encroachment. Certainly changes over time at a particular crossing affect the channel geometry, the geometry of the crossing itself, general scour and local scour. If we know the characteristics of the flow and how they vary with time, then one can utilize the information in Chapter VI to design against excessive general and local scour in order to make the highway functional with minimum maintenance over the life of the project.

8.3.10 Flow on the floodplain

Up to this point we have principally concerned ourselves with flow in the main channel. However, *design floods usually flow in both the main channel and on the floodplain.* Only by studying the characteristics and geometry of the river and the floodplain can we determine the type of flows that are apt to occur on the floodplain. This particular topic should be studied in adequate detail so that the magnitude and intensity of the flows on the floodplain can be approximated. The characteristics of flow on the floodplain are especially relevant to the design study of longitudinal encroachments (refer to Case 14 in Table 8.2.1) As an example, consider a sinuous channel. At flood stage there is a tendency for the water to flow in the main channel in such a way as to develop chute channels across the point bars. Often, the water spills over the outsides of the bends onto the floodplain. As has been illustrated in the preceding chapters, flow conditions on the floodplain and in the main channel can be greatly different at flood stage than at low flow and these factors must be taken into consideration. A case in point is a new bridge being constructed across the Mississippi River. In this instance, the flow on the floodplain was sufficiently intense and the alignment of the approaches to the bridge in relation to the flow on the floodplain was such that a large channel was scoured along the upstream side of the approach embankment. Ultimately, a large segment of the approach embankment was lost into this channel and washed downstream as a huge sand wave on the floodplain. In the extreme case, it is entirely possible for cutoffs to form naturally in river systems and only by considering the intensity of flow in the channel and on the floodplain can we determine the probability of such an occurrence.

8.4.0 SITE SELECTION

Most of the factors cited in the preceding sections have a bearing on the final site selection. In summary, *such factors as the form of the river, the alignment of the river, variations of the river form over time, the type of bed and bank material, and the hydrologic and hydraulic characteristics of the river are all important inputs to the site selection.* In addition, it is necessary to consider the requirements of the area to be served and the economic and environmental factors that relate to the crossing. Having made a detailed study of possible alternate sites, and having determined the best site considering these important factors, one can then proceed with the determination of the geometry and length of the approaches to the crossing, the type and location of the abutments, the number and location of the piers, the depth to the footing supporting the piers to insure against danger from local scour, the location of the longitudinal encroachment in the floodplain, the amount of allowable longitudinal encroachment into the main channel, and the required river training works to insure that river flows approach the crossing or the encroachment in a complementary way.

8.5.0 CHANNEL STABILITY INVESTIGATIONS

In conjunction with the background information discussed in the preceding paragraphs *it is essential to determine the necessity for bank stabilization.* The location, design, and various types of river training works must be considered. The selection of training works is significantly affected by the characteristics of the river and the river system itself. The magnitude of local scour at the training structure must be considered. The possible necessity of holding the river in a selected alignment must also be adequately explored. With regard to these particular issues, one can apply the principles of Chapter VI to develop suitable designs for stabilizing the approaches, the spur dikes at the end of the approaches, the banks of the main channel and the design of training works that assist in controlling the alignment of the river relative to the crossing or longitudinal encroachment.

8.6.0 SHORT-TERM RESPONSE

Having completed the tentative design of the crossing or the encroachment based on river form, channel geometry, hydrologic and hydraulic data etc., it is essential to take a look at the short-term response of the river system to the construction. Similarly, the river developments upstream and downstream of the site and at the site itself should also be considered. The techniques that may be utilized to investigate the short-term response at the site or in the vicinity of the crossing or encroachment involves the utilization of qualitative geomorphic relationships followed by the application of more sophisticated analyses using the principles presented in the chapters on open channel flow, sediment transport and river mechanics. In fact, it is possible to establish a mathematical model designed to route both water and sediment through the system. If this model is appropriately designed and utilized, it is possible to evaluate the response of the river system to both the construction of the crossing or encroachment and to other river development projects in the immediate area. For example, it may be important to establish the pattern of clear water releases from a dam upstream of a crossing. Knowing the type of flow the channel would be subjected to and that the water being released is clear, one can make an estimate of the extent of degradation in the channel, the amount of sediment derived from the bed and bank, the instability of the banks and even the types of lateral shifting that may be induced in the river system as it affects the crossing or encroachment.

8.7.0 LONG-TERM RESPONSE

The long-term river response at a crossing or a longitudinal encroachment and in the river system itself should be considered based on all river development projects including the highway. This type of treatment is, in general, beyond the scope of this particular manual. Nevertheless, sufficient advances have been made pertaining to the mathematical modeling of river systems, considering both their short- and long-term response, that this approach is worth considering on important projects.

8.8.0 DESIGN EXAMPLES

8.8.1 Introduction

In this concluding section of the chapter, examples are given showing the application of the principles, methods and conceptions of previous chapters. The examples contain situations where design is determined by well established numerical procedures and also situations where design depends heavily on the judgment of the engineer. It would be wrong to treat these examples as approved design procedures. They are not intended to be examples of how a particular design problem is handled but rather as examples of how the concepts previously outlined in this manual find their application in design. For these reasons, *this section should be read and studied as an illustrative unit and not as a collection of individual design problems from which one can choose the correct prescription for the problem at hand. River problems are much too complex for a cookbook approach and it is hoped that the examples make this evident.*

The examples relate to the design of a crossing on the "Mainstream River" shown in the aerial photograph of Fig. 8.8.1. The flow is from right to left. An existing highway crossing can be seen in the photograph but the highway and its alignment are to be upgraded and one alternative for the crossing is drawn on the photograph about 2200 ft upstream. The old crossing is to be preserved for local travel. From first appearances, this proposed crossing seems to be located in a very unstable section of the river and more attractive locations are possible. But one must assume that there are factors other than those associated with the bridge location that make this alternative worthy of scrutiny. Indeed, such considerations actually dictate the location of many crossings.

There is a USGS gaging station several miles upstream of the crossing site with no intervening tributaries and only one minor diversion for irrigation. *According to 43 years of record, the mean*

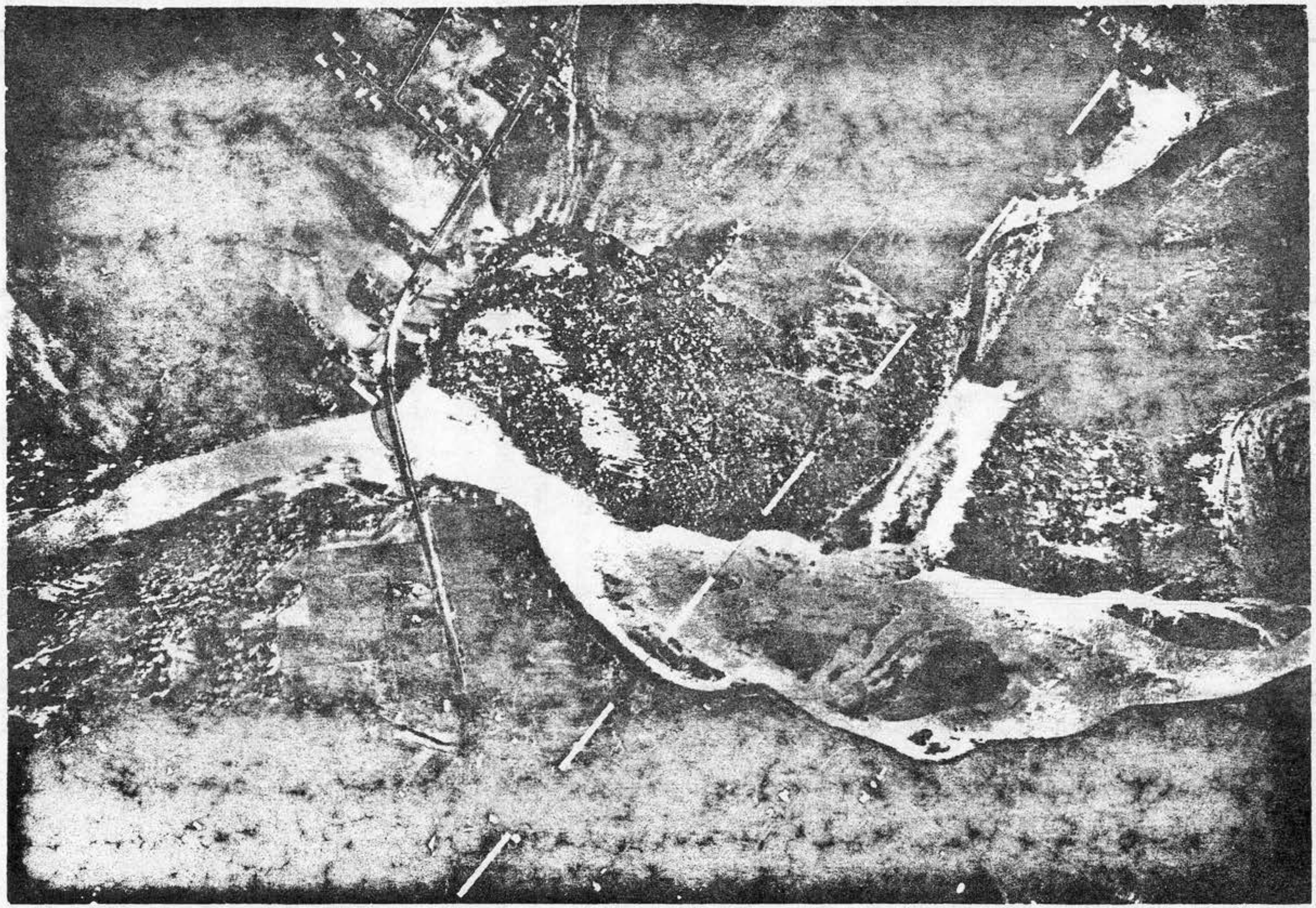


Fig. 8.8.1 Mainstream River showing existing crossing, newly proposed crossing, recently cutoff meander, and broad flat floodplain. Flow is from right to left and the scale is approximately 1 inch = 1000 ft.

river flow is 2900 cfs. Fig. 8.8.2 is a hydrograph for the water year that included the flood of record. The summer flows are typically low, less than about 500 cfs, while the winter flows are much higher, punctuated by flood peaks lasting several days. The record flood overflowed the banks and inundated a considerable expanse of the extensive and flat floodplain. *The peak instantaneous flow for this flood was 97,000 cfs, while the average flow for that day,* shown on Fig. 8.8.2, *is only 77,000 cfs.* Flood peaks are relatively short lived on this river. The daily flows of Fig. 8.8.2 are replotted in Fig. 8.8.3 in the form of the flow duration curve for one year. During that year, the flow exceeded the mean flow of 2900 cfs approximately 30 percent of the time. A flow of 1250 cfs was exceeded 50 percent of the time.

The flood of record, estimated at 105,000 cfs at the proposed *crossing, reached a stage of El. 272 in the vicinity of the crossing.* The water levels on the floodplain went as high as El. 275, however, indicating that the channel flow and the floodplain flow are poorly connected in this area. *Field estimates were 64,000 cfs channel flow and 41,000 cfs overbank flow. The bankfull discharge for this river is about 42,000 cfs.*

The floodplain has an overall slope of 0.00191 (10.09 ft/mile) in the direction of flow and the river is relatively unrestricted in lateral migration except at the localized revetment protections. Fig. 8.8.4 shows a profile of the river with a steeper section just upstream of the proposed crossing. *The river has an average slope of 0.00138 (7.29 ft/mile).* In this reach, the river is classified as very mature. One question regarding this river: is what discharge dominates in defining the character of the river's morphology? As seen from the hydrograph of Fig. 8.8.2, the mean flow of 2900 cfs is almost never realized. The flows are much higher in the winter and very much lower in the summer. One cannot imagine the low summer flows as contributing much to the morphology. The record flood, on the other hand, created an anomaly that only time will restore to equilibrium. At this stage of the development of knowledge *an experienced river engineer might well consider the mean flow for the five winter months, approximately 7000 cfs, as representing the dominant discharge for this river.*

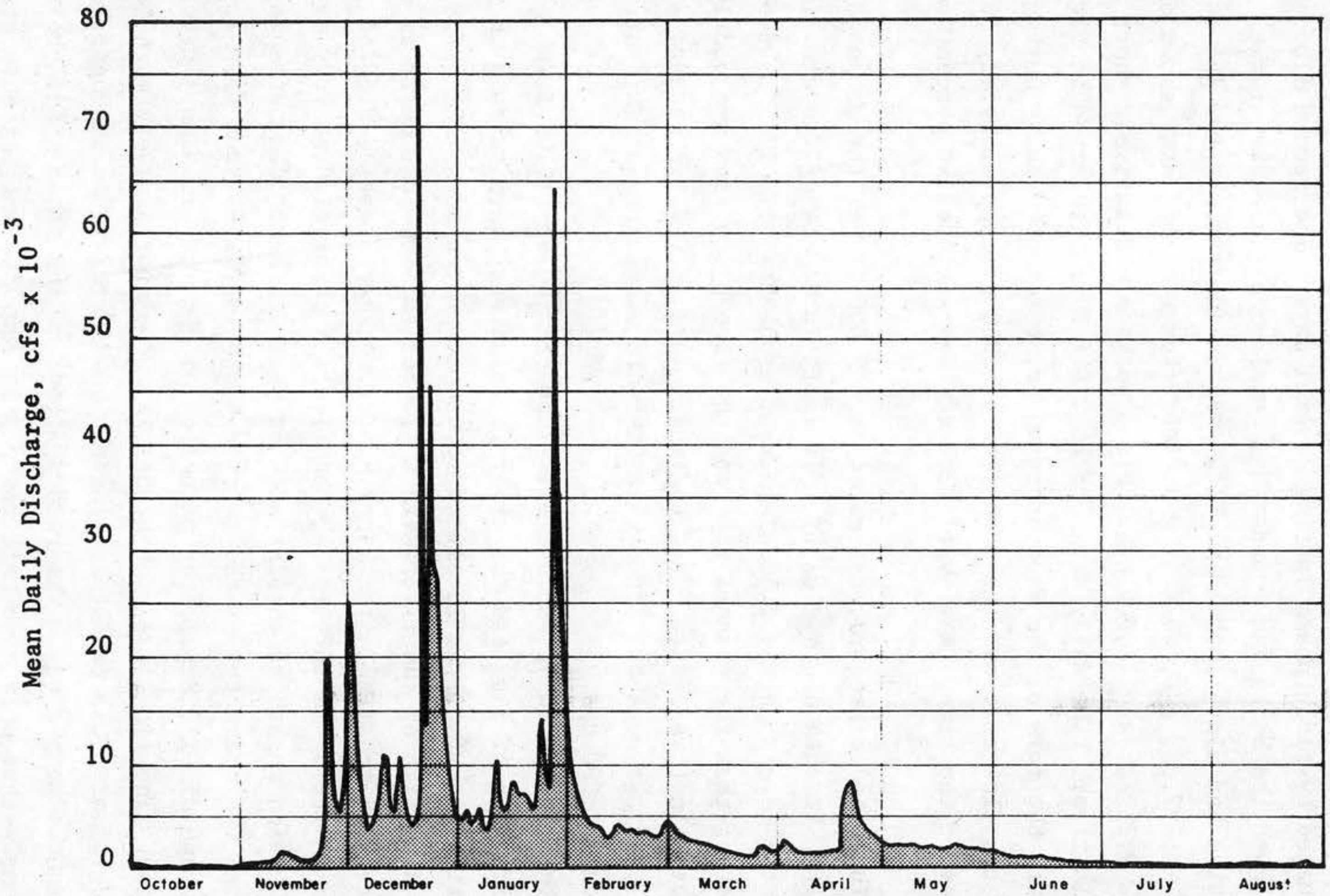


Fig. 8.8.2 Hydrograph from gaging station on Mainstream River 12 miles upstream of proposed crossing.

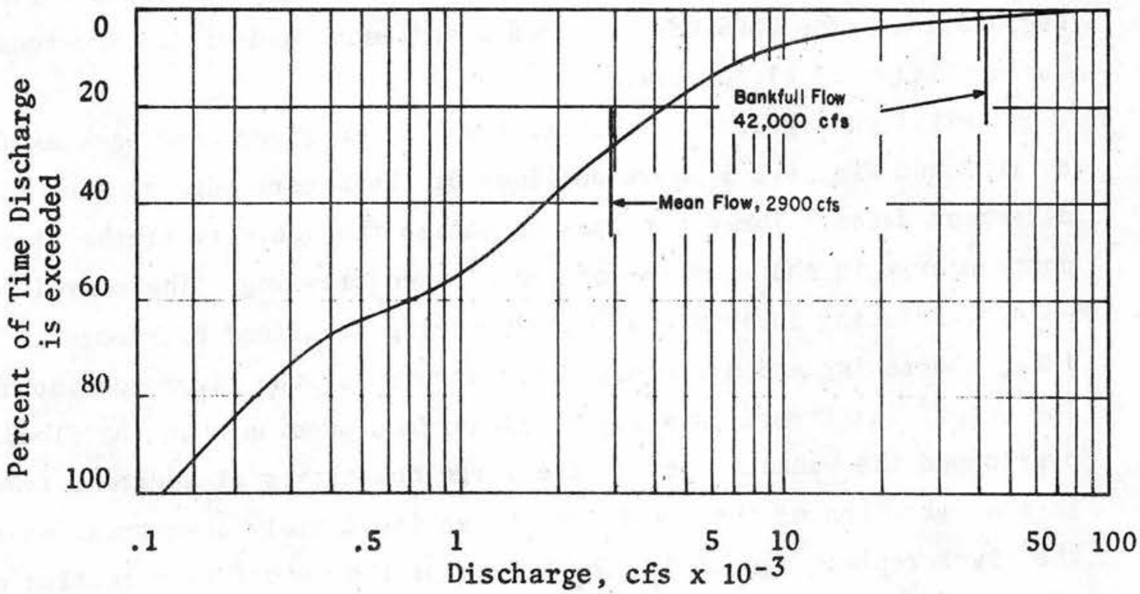


Fig. 8.8.3 Flow duration curve for Mainstream River.

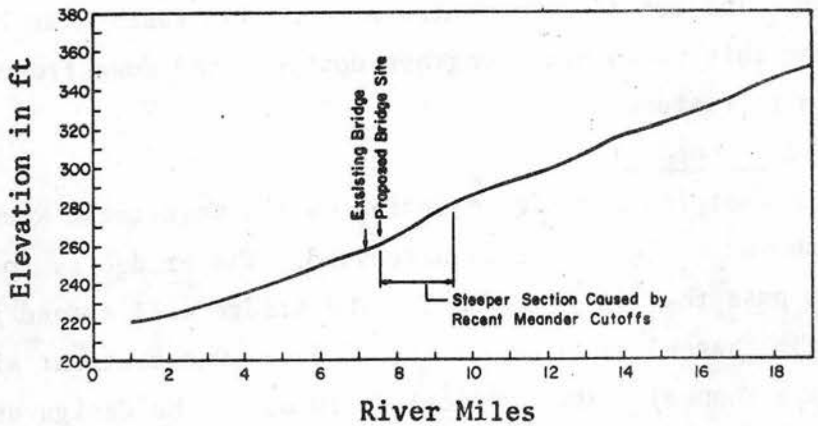


Fig. 8.8.4 Profile for the Mainstream River.

Applying the river slope of 0.00138 ft/ft and the mean discharge of 7000 cfs to Fig. 4.4.3 of Chapter IV, the characteristics of this border on the braided zone. Meanders are, however, very much in evidence in the outlines of Fig. 8.8.5. The braiding, particularly at the crossing site, appears to be a temporary condition caused by the steepening of the local slope due to cutoffs. *The sinuosity of the river, 1.38, is low,* more like that of a straight river. The bed material has a median size of 1.0 mm, typical of coarse sandbed rivers.

The D_{90} is about 15 mm and considerable armoring takes place during degradation. The USGS has recorded a sediment load of 223,000 tons/day during a flow of 41,300 cfs.

Aerial photographs of this stretch of the river date back as far as 1959 and Fig. 8.8.5 shows outlines of the waters edge at four different dates. These outlines dramatize the activity of the river particularly in the vicinity of the planned crossing. The convoluted meander existing in 1959 was cut off during the flood of record in 1964, shortening and steepening this stretch of the river considerably. The cutoff was formed as a result of surface erosion when the flood overtopped the banks. Most of the large quantities of sediment removed in the formation of the cutoff deposited immediately downstream where the river rapidly aggraded. The result of the cutoff is a section of the river steeper than the rest. The river is slowly degrading its upper end and aggrading its lower end to restore its normal slope. As this process continues, the meander character of the river is once more taking over. The cutoff has created a localized reach that is highly unstable and this reach will lengthen upstream and downstream before equilibrium is restored.

8.8.2 Design example 1

In this example, a bridge crossing on the Mainstream River at the alignment shown in Fig. 8.8.1 is discussed. The bridge is to be designed to pass the 100-year flood. The bridge will extend across 500 ft of the channel as shown in Fig. 8.8.6. One abutment will extend into the main channel. The critical features of the design as far as river mechanics are concerned are as follows.

Design flows - Design flow for bridge safety is stated to be the 100-year flood and the 100-year flood is 110,000 cfs. From estimates reported earlier on the flood of record, this can be divided into 66,000 cfs channel flow and 44,000 cfs overbank flow.

Design stage - A stage of 272 ft was recorded in the field for the 105,000 cfs flood and this is essentially the design flood. This actual measured value of stage is much superior to any that could be calculated by backwater methods.

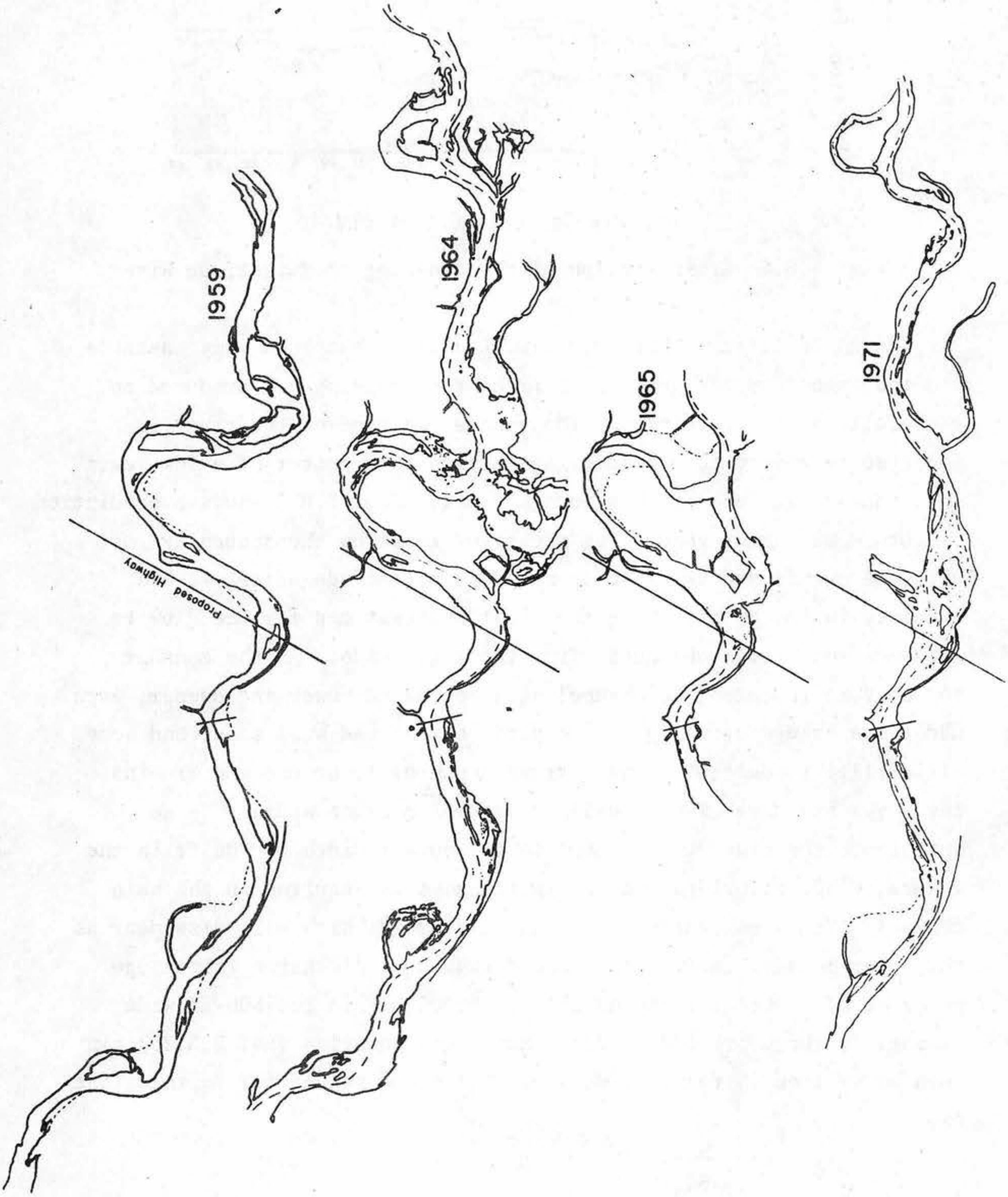


Fig. 8.8.5 Recent alignment changes of Mainstream River.

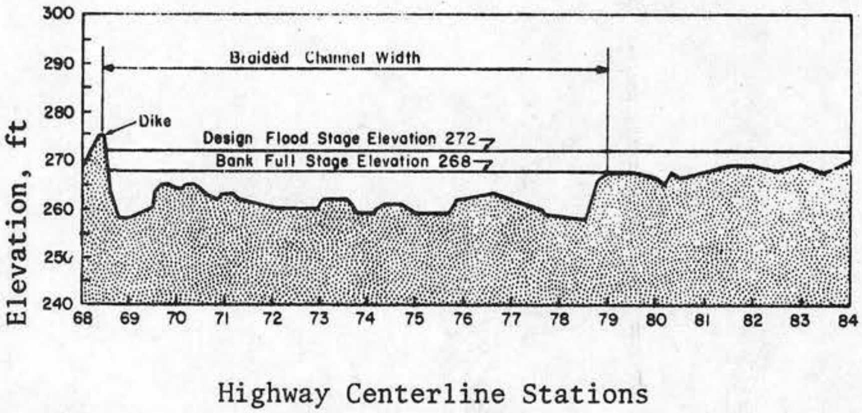


Fig. 8.8.6 Cross section of the crossing on Mainstream River.

River stability - Historically, the river has been very unstable and the recent cutoff just upstream of the crossing has produced an especially active situation. The braided section of the river is expected to revert to a meandering section in a matter of a few years and, indeed, is showing this tendency now. Fig. 8.8.7 shows a prediction of future alignment changes as the river restores the sinuosity lost with the cutoff and as existing meanders migrate downstream. The tendency is for an attack on the right abutment and for the flow to approach the bridge obliquely from the right side. As the meander tendency is restored the channel will become narrower and deeper, even under the bridge crossing. Some parts of the bed will scour and some will fill. To determine the maximum depth of scour one can examine the river and determine visually a natural minimum width. As we anticipate the river will reform to its normal width of 500 ft in the future, a 500-ft bridge opening is all that is required in the main channel. The wide braided reach is an anomoly which will disappear as the river meander redevelops. The design unit discharge (discharge per foot of width) for the estimated 66,000 cfs in the 500-ft wide channel is therefore 132 cfs/ft. Manning's equation (Eq. 2.3.20) can then be written in terms of unit discharge and solved for depth. That is,

$$y_o \approx \left\{\frac{qn}{1.486 S_f^{1/2}}\right\}^{3/5}$$

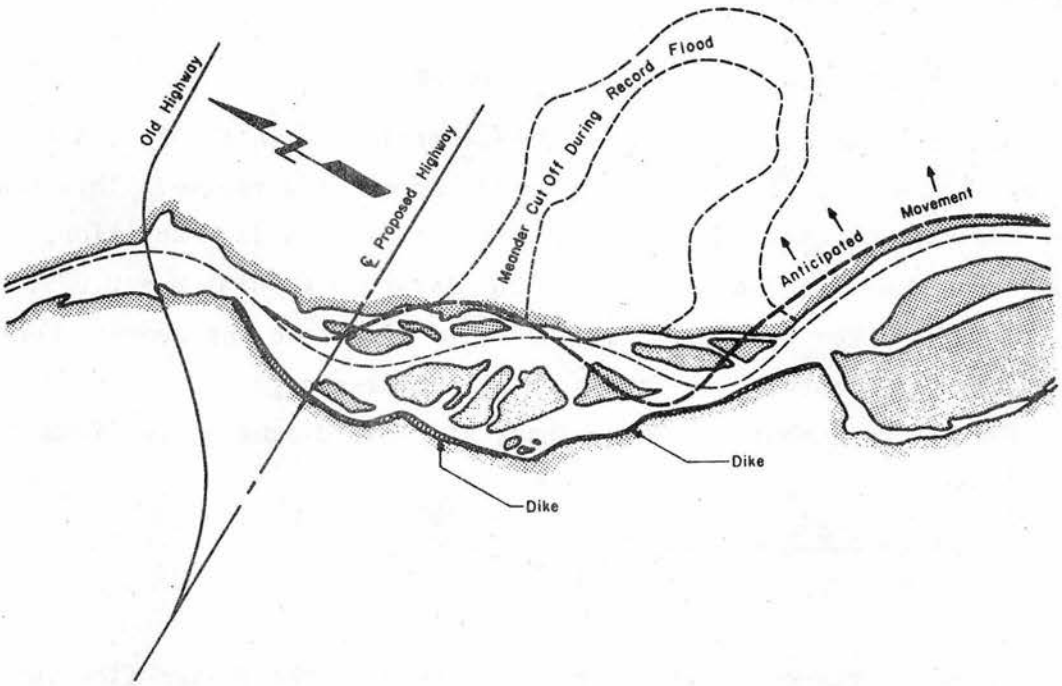


Fig. 8.8.7 Existing and anticipated channel alignments for Mainstream River.

In Table 8.8.1 and in later calculations it is assumed that the depth is approximately equal to the hydraulic radius which is an acceptable assumption for wide rivers. The friction slope S_f can be assumed equal to the river slope 0.00138. As a first approximation, assume Manning's n of 0.025 (Table 2.3.1, alluvial channel bed in transition). Also, from Eq. 3.10.46 and the D_{90} of the bed material equal to 15 mm Manning's n is 0.024 and Fig. 8.8.1 indicates large bars in the channel. Then

$$y_o = \left\{\frac{(132)(0.025)}{(1.486)(0.00138)^{1/2}}\right\}^{3/5} = 11.6 \text{ ft}$$

and

$$V = \frac{q}{y_o} = \frac{132}{11.6} = 11.4 \text{ fps}$$

Then, the average shear stress on the bed is (from Eq. 2.2.31)

$$\tau_o = \gamma y_o S_f,$$

$$= (62.4)(11.6)(0.00138) = 1.0 \text{ psf}$$

and the stream power is

$$\tau_o V = (1.0)(11.4) = 11.4 \text{ ft lb/sec/sq ft}$$

For 1 mm sand and a stream power of 11.4 ft lb/sec/sq ft, Fig. 3.4.1 indicates that the flow should be in the upper flow regime. This bed form does not check our assumption that the flow is in transition. However, because of the gravel bars and large D_{90} use $n = 0.025$.

The stage for the design discharge is 272 ft so the average bed level is approximately 260 ft for the flood discharge.

The Froude number for the channel at flood discharge is (from Eq. 2.4.8)

$$Fr = \frac{V}{\sqrt{gy_o}} = \frac{11.4}{\sqrt{(32.2)(11.6)}} = 0.59$$

The width-to-depth ratio for the channel at the design flow is

$$\frac{W}{y_o} = \frac{500}{11.6} \simeq 43$$

A summary of the calculations are given in Table 8.8.1.

The depth computed above is an average depth. Deeper sections exist near the outside of meander bends. These pools can and will exist under the crossing. The anticipated depth of flow in the meander bend must be considered in design. The geometry of pools in bendways has been discussed in Section 5.6.2, Chapter V. For a given river system, the maximum depth in a meander bend y_{max} is primarily dependent on the river width W and usually to a lesser extent on the radius of curvature r_c. (See Fig. 5.6.1). Looking back on Fig. 5.3.2 it is noted that the radii of curvature vary in a river. A study of the topograph maps will reveal the information required to determine the frequency of occurrence of bends.

For the Mainstream River, the most common value of r_c is 2500 ft. Therefore

$$\frac{r_c}{W} = \frac{2500}{500} = 5$$

In the crossings, the average depth is 11.6 ft at design discharge so the depth-to-width ratio is

Table 8.8.1 Data and computations for design example 1.

Data:

Valley slope, S_v = 0.0019 (10.0 ft/mile)
River slope, S_o = .00138
Average discharge = 2,900 cfs
Average five-month winter discharge = 7,000 cfs
Bankfull discharge = 42,000 cfs

Record discharge:	Channel =	64,000 cfs
	Overbank =	41,000 cfs
		105,000 cfs
Design discharge:	Channel =	66,000 cfs
(100-year)	Overbank =	44,000 cfs
		110,000 cfs

Design flood stage = 272 ft
Minimum channel width, W = 500 ft
Mean bed-material size, D_{50} = 1 mm

Conditions at design discharge:

Unit discharge in channel, q = 132 cfs/ft
Manning's n = 0.025
Average flow depth, y_o = 11.6 ft
Average velocity, V = 11.4 fps
Channel Froude number, Fr = 0.59
Average bed shear, τ_o = 1.0 psf
Average stream power, $\tau_o V$ = 11.4 ft lb/sec/sq ft
Width-to-depth ratio, W/y_o = 43
Flood stage elevation = 272 ft

Conditions in meander pool:

Maximum depth in pool, y_{max} = 29 ft
Bed elevation in deep part of pool = 243 ft
Average velocity in the pool = 14.2 fps
Froude number in the deep pool, Fr = 0.46

Embankment riprap for present conditions:

Angle of repose, $\phi = 37°$
Side slope angle, $\theta = 18.4°$
Shear stress on side slope, τ_o = 1.0 psf
Specific weight of riprap, S_s = 2.50
Safety factor, S.F. = 1.5
Stability number, η = 0.356
Effective grain size, D = 7.5 in
Recommended median size, D_{50} = 6 in
Recommended maximum size, D_{100} = 12 in
Minimum thickness or riprap = 12 in

Pier scour for present conditions:

Width of round nose pier, a = 5 ft
Length of pier, ℓ = 20 ft
Skew angle = 30 deg
Approach Froude number, Fr_1 = 0.59

Table 8.8.1 Data and computations for design example 1 (continued).

Approach flow depth, y_1 = 11.6 ft

maximum depth of scour, y_{smax} = 28 ft

Pier scour for future conditions:

Skew angle = 30 deg
Approach flow depth, y_1 = 29 ft
Approach Froude number, Fr_1 = 0.46
Maximum depth of scour, y_s = 34 ft

$$\frac{y_o}{W} = \frac{11.6}{500} = 0.023$$

It is possible that Fig. 5.6.3 can be used to determine the maximum depth in the pool of a meander bend. The stream order of Mainstream River can be found from topography maps, the r_c/W ratio is known and the stability factor is (from Eq. 5.6.8)

$$\chi = \frac{D_{50}W}{y_o^2 S}$$

$$= \frac{(1)(500)}{(304.8)(11.6)^2(0.00138)} = 8.8$$

The information in Fig. 5.6.3 is for rivers with $\chi \leq 1.5$ so Fig. 5.6.3 is not applicable in this design problem.

To determine the maximum depth in the meander bend, the limiting value of y_{max}/W from Fig. 5.6.1 for r_c/W equal to 5 could be used. The value of the ratio is approximately 0.06. Therefore the maximum depth would be 30 ft. The limiting value is for relative stable streams so one could assume that y_{max} would be less than 30 for unstable rivers. A depth of 30 is probably conservative (deeper) for scour determinations but is not conservative for design of riprap (gives too low a velocity).

At this point, a few field measurements would be appropriate. The bed elevation and bank elevation at the deepest part of the bends with the largest, the smallest and the median radii of curvature should be measured along with the river width at these locations. These data establish the relation between y_{max}, W and r_c for the Mainstream River equivalent to that shown in Fig. 5.6.1.

Suppose that the field measurements indicate a y_{max} of 29 ft. The elevation of the bed in the deep part of the pool is then

$$272-29 = 243 \text{ ft}$$

The velocity in the bend will be greater than in the crossing. An estimate of this velocity can be made from the information given in Fig. 5.3.5. For a long bend in a parabolic channel, the maximum velocity in the bend is only about 10 percent greater than the maximum velocity in the straight approach section. For most design problems, it is recommended that a 25 percent increase in the average velocity be used for the maximum velocity in the bend. Then, in the bendway

$$V = (1.25)(11.4) = 14.2 \text{ fps}$$

and the Froude number in the deep part of the pool is

$$Fr = \frac{14.2}{\sqrt{(32.2)(29)}} = 0.46$$

A decision must be made regarding the substantial overbank flow associated with the design flood. If the highway grade is placed above the levels of the overbank flow, the entire flow would be forced under the crossing. The increase in the flow in the main channel would be 67 percent at the design discharge. Such an increase will surely magnify the flow problems downstream. The 44,000 cfs of overbank flow would have to flow laterally across the floodplain to return to the river upstream of the crossing. This is certain to increase the depth of inundation and worsen the flooding. If the highway were placed low enough not to obstruct the overbank flow, the roadbed would be flooded whenever the flow exceeded the 42,000 cfs bankfull flow or about every two years. One solution would be to provide openings under the highway (relief bridge) to handle this flow. These would have to be extensive but it is assumed in this example that they have been provided. Example 2 examines the case where the total flow goes through the bridge.

Abutment protection - The projected river alignment changes indicate the need for protection on the right embankment of the crossing as shown in Fig. 8.8.8. The left abutment already has a protective dike that has withstood flows in excess of 100,000 cfs.

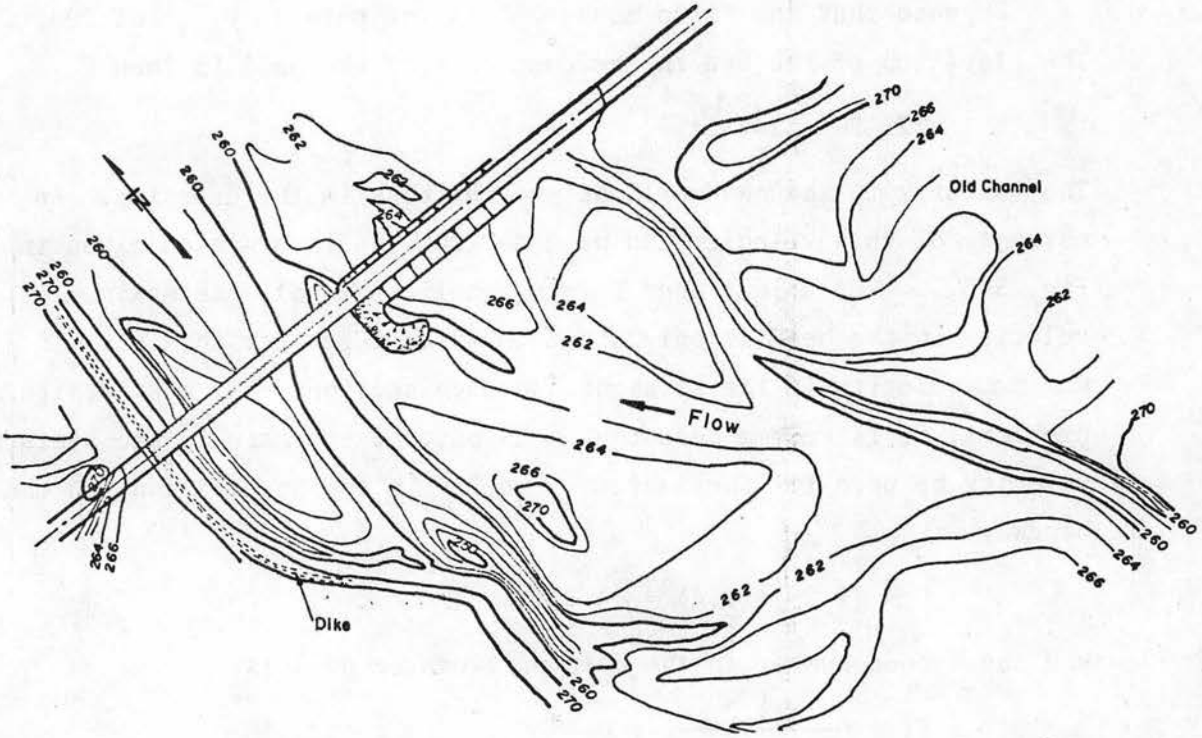


Fig. 8.8.8 Right embankment and spur dike.

A spur dike is ideally suited for this embankment end. While this reach of river is in the braided form, the spur will help direct the flow from the right side of the channel through the bridge opening. After the river has narrowed, the spur will protect the embankment by holding the meander loops away from the embankment. In addition the spur will move the local scour away from the embankment end under the bridge to the nose of the spur.

The recommended spur dike configuration is shown in Fig. 6.3.6. Here, a length of 200 ft is chosen because the spur dike will have to direct a substantial amount of channel flow while the approach river channel is braided. The spur bends outwards on a 1/4 ellipse and is terminated at (0.4)(200) = 80 ft back from the embankment end. The plan view of the spur is shown in Fig. 8.8.9.

The top of the spur is set at El. 275, three feet above design stage. This grade elevation allows one foot for aggradation and two feet for wave wash. The toe of the riprap protection is set at Elev. 260, the anticipated average bed level for a design flood. This elevation is above the low summer flow stage so that construction would not require cofferdamming. Fig. 8.8.8 shows areas of local scour to Elev. 250 in the natural river and computations mentioned above indicate scour in meander pools to reach Elev. 243. To protect against localized scour to these elevations, a rock-filled trench of riprap has been provided (see Fig. 8.8.9) that will stop any local pockets of scour.

The size of riprap required for the spur can be determined by the method in section 6.4.0 of Chapter VI. The computations are summarized in Table 8.8.1. As the embankments for this bridge do not constrict the river flow after the river narrows (relief bridges are provided on the floodplain), the flow through the bridge will not accelerate. Therefore, this is a case of horizontal flow on a side slope.

The side slope angle along the bankline is set at 3:1 to prevent slip circle failures in the soil behind the riprap on the spur. Therefore

$$\theta = 18.4°$$

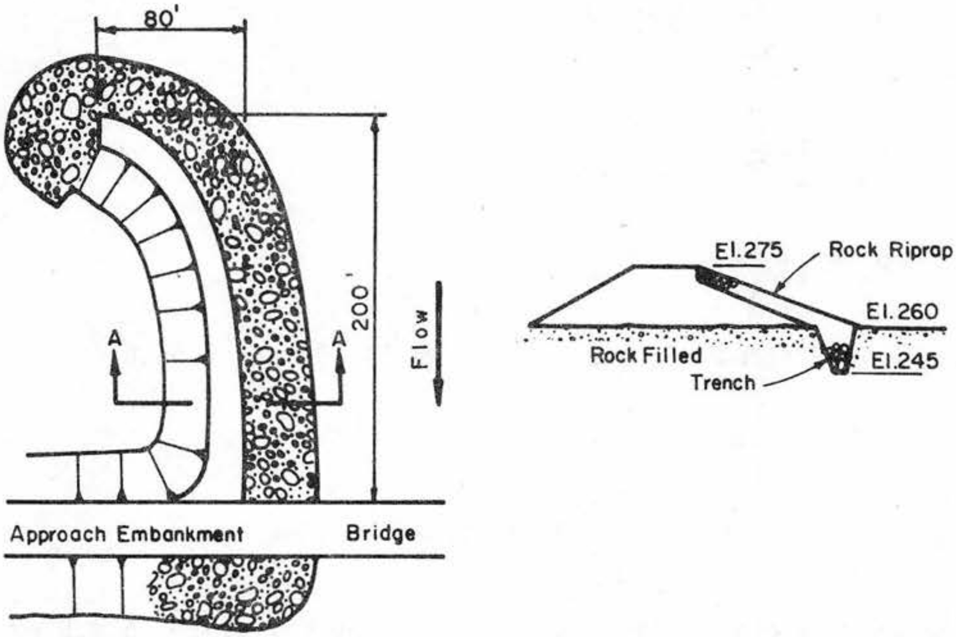


Fig. 8.8.9 Spur dike with a rock trench.

The anticipated size of riprap is in the range between 1.0 and 10 in. Then, from Fig. 3.7.3 the angle of repose for the dumped riprap will be approximately 37° or

$$\phi = 37°$$

For flow alignment as shown in Fig. 8.8.8, the average shear stress on the bed as expressed by Eq. 3.9.12 is

$$\tau_o = \gamma\, y_o S_f$$

$$= (62.5)(11.6)(0.00138) = 1.0 \text{ psf}$$

where y_o is equal to R for wide floodplain crossing. The bed shear stress is equal to the side slope shear stress at the toe of the side slope. Therefore, a shear stress of 1.0 psf is used to design the side slope riprap.

In this region, the rock available for riprap has a specific weight

$$S_s = 2.50$$

The stability number for horizontal flow along a side slope is given by Eq. 6.4.9, or

$$\eta = \{\frac{S_m^2 - (S.F.)^2}{(S.F.)S_m^2}\} \cos\theta$$

Here $$S_m = \frac{\tan\phi}{\tan\theta}$$

$$S_m = \frac{\tan 37°}{\tan 18.4°} = 2.27$$

The recommended safety factor for design is (Secton 6.A5.0)

$$S.F. = 1.5$$

so

$$\eta = \{\frac{(2.27)^2 - (1.5)^2}{(1.5)(2.27)^2}\} \cos 18.4° = 0.356$$

The rock size is related to stability number by Eq. 6.4.5 or

$$D = \frac{21\tau_o}{(S_s-1)\gamma\eta}$$

$$= \frac{(21)(1.0)}{(2.50-1)(62.4)(0.356)} = 0.63 \text{ ft}$$

or

$$D \simeq 7\ 1/2 \text{ in}$$

The effective rock size of the required riprap is then 7 1/2 in. The recommended gradation is given in Fig. 6.4.3. This gradation is such that

$$D = 1.25\ D_{50}$$

or

$$D_{50} = \frac{7.5}{1.25} = 6 \text{ in.}$$

and

$$D_{100} = 2D_{50}$$

or

$$D_{100} = (2)(6) = 12 \text{ in.}$$

The recommended minimum thickness of the riprap blanket is $2D_{50}$ or D_{100} which ever is greater. Here, use a minimum thickness of 12 in.

When a meander bend reaches the spur, the bed will scour and the maximum flow depth in the bend at the design discharge becomes 29 ft. The average velocity in this deep pool is approximately 14.5 fps. *How will the riprap be affected by these flow conditions?* The attack on the bank protection on the right abutment will become severe when the spur is at the outside downstream end of the meander bend. Assuming that y_o/D is 23, Eq. 6.A3.7 indicates the shear stress on the side slope will be approximately twice as much as in the crossing. That is

$$\tau_o \simeq (2)(1.0) = 2.0 \text{ psf.}$$

The stability number for the 6 in. diameter riprap (D = 7.5 in.) on a 3:1 side slope with a shear stress of 2.0 psf is given by Eq. 6.4.5 or

$$\eta = \frac{(21)(2)}{(2.50-1)(62.4)(0.63)} = 0.712$$

and the safety factor becomes (from Eq. 6.4.11)

$$S.F. = \frac{S_m}{2} \{(S_m^2 \eta^2 \sec^2\theta + 4)^{1/2} - S_m \eta \sec\theta\}$$

or

$$S.F. = \frac{S_m}{2} \{(\xi^2 + 4)^{1/2} - \xi\}$$

where $\xi = S_m \eta \sec \theta$

$$= (2.27)(0.712)(1.054) = 1.704$$

Then $S.F. = \frac{2.27}{2} \{[(1.704)^2 + 4]^{1/2} - 1.704\}$

$$= 1.048$$

As the safety factor is only slightly greater than unity, the riprap could move when flow in the meander bend attacks the spur protection.

The size of riprap required to give a 1.5 safety factor for the riprap under a shear stress of 2.0 psf is found by employing Eq. 6.4.9 and 6.4.5 in order. Accordingly, the stability factor is still

$$\eta = 0.356$$

but

$$D = \frac{(21)(2.0)}{(2.50-1)(62.4)(0.356)} = 1.26 \text{ ft}$$

or

$$D = 15 \text{ in.}$$

which corresponds to

$$D_{50} = 12 \text{ in.}$$

and

$$D_{100} = 24 \text{ in.}$$

Future river alignment conditions dictate that the riprap on the abutment have a D of 15 in. The recommended gradation requires that the D_{50} be approximately 12 in. and D_{100} be 24 in. Other riprap gradations can be used provided the D is equal to or greater than 15 in.

A filter is most likely required between the riprap and the spur embankment materials. A cloth filter is recommended. It would be prudent to go out in the field during a low-flow period and check the size and condition of the riprap on the left bank. This riprap has been subjected to a large flood. The inplace riprap should be at least 4 in. (D_{50}) in diameter.

Scour at the piers - The scour to be expected around the piers can be computed with the equations in Section 6.5.3, Chapter VI. For the present-day conditions at the bridge crossing the approach flow depth is

$$y_1 = y_o = 11.6 \text{ ft}$$

and the corresponding Froude number is (from Table 8.8.1)

$$Fr_1 = 0.59$$

The pier width a is five ft and the length ℓ is 20 ft. If the pier skew angle is zero degrees, then the equilibrium depth of scour is given by Eq. 6.5.11 (round nose pier) or

$$y_s = 2.0 y_1 \left(\frac{a}{y_1}\right)^{0.65} Fr_1^{\,0.43}$$

$$= (2.0)(11.6)\left(\frac{5.0}{11.6}\right)^{0.65}(0.59)^{0.43} = 10.7 \text{ ft}$$

According to the plan view on Mainstream River shown in Fig. 8.8.7, a pier skew angle (to the approach flow) of 30 deg can be anticipated. From Table 6.5.2, the scour depth would increase. For

$$\frac{\ell}{a} = \frac{20}{5} = 4$$

the multiplying factor is 2.0 and the increased depth of scour is

$$y_s = (10.7)(2.0) = 21.4 \text{ ft}$$

The maximum depth of scour can be 30 percent greater (Section 6.5.3) so the maximum depth of scour is

$$y_s = (21.4)(1.3) \simeq 28 \text{ ft}$$

The greatest contributor to this depth is the skewed approach flow. This is one of the main penalties to pay for the lack of river control.

The maximum depth of scour corresponds to a scour hole bed elevation of

$$272-11.6-28 \simeq 232 \text{ ft}$$

This pier scour would be obtained if the bed material were sand only. However, the scour hole armorplates if there is any gravel or cobbles in the underlying bed deposits. The depth of scour would be decreased if there is underlying hardpan. A look at the laboratory test results on the core hole samples obtained in the bridge pier foundation explorations is very important.

In Mainstream River, the bed is alluvium at least down to an elevation of 220 ft but there is a considerable amount of gravel and a few cobbles in the underlying bed material. The scour hole will armor plate so that the scour hole bed elevation will be above El. 232 ft.

Future conditions may result in a lower bed elevation around the piers. The bed elevation in the deep part of the pool has been estimated as El. 243 ft. (Table 8.8.1). Probably, the bed material in such pools is greater than 1 mm sand. The bed of the pool can also armorplate during a flood.

In the deep part of the pool the approach flow has a depth (from Table 8.8.1)

$$y_1 = 29 \text{ ft}$$

and the approach Froude number of

$$Fr_1 = 0.46$$

Assuming a sandbed, the equilibrium scour at the round nose pier (a = 5 ft, ℓ = 20 ft) is given by Eq. 6.5.11

$$y_s = (2.0)(29)\left(\frac{5}{29}\right)^{0.65}(0.46)^{0.43}$$
$$= 13.2 \text{ ft}$$

Again, the skew angle could be as great as 30° so the multiplying factor is 2.0 (Table 6.5.2) and the depth of scour is

$$y_s = (2.0)(13.2) = 26.4 \text{ ft}$$

and the maximum depth of scour is

$$y_s = (26.4)(1.3) \simeq 34 \text{ ft}$$

This depth of scour corresponds to a scour hole bed elevation of

$$272-29-34 = 209 \text{ ft}$$

Based on future meandering considerations, it appears prudent to specify that the pier foundations be set at an elevation of 200 ft.

8.8.3 Design example 2

In example 1, a far reaching assumption was made that the overbank flood flow passed under the highway through relief bridges so that, of the 110,000 cfs design flood, only 66,000 cfs would pass under the bridge. In the flood of record, the channel carried 64,000 cfs without serious damage to the protected embankments or to the existing bridge, so it is not expected that the new crossing of example 1 will affect the river behavior. In design example 2, the assumption is made that the new highway completely obstructs the overbank flow so that all the design flow passes under the bridge.

There are several consequences to forcing all the flow under the main bridge. First, the highway obstruction increases the depth of the overbank flow immediately upstream and so worsens the local flooding problem. The concentrated overbank flow returning to the river just upstream of the highway could cause land erosion. The increased flow in the channel increases the channel scour and bank attack until the excess flow can return to the floodplain downstream. Also, the increased flow increases the stage under the bridge which decreases the clearance. Fortunately, the design flood is not long lasting as shown in the design hydrograph of Fig. 8.8.2. This hydrograph was adapted from the flood of record measured at the USGS gage simply by multiplying those flows by a constant to get a peak of 110,000 cfs. The actual hydrograph at the crossing would be appreciably different in shape, however, because the USGS gage is in a narrow valley where there is no overbank flow. The peak at the crossing would be flattened as the first part of the flood goes into storage on the floodplain and later drains off to increase the flows towards the last of the flood.

The problems of the increased flooding and the returning flow along the highway are difficult to treat analytically but qualitative descriptions can be readily given. Fig. 8.8.10 shows a schematic drawing

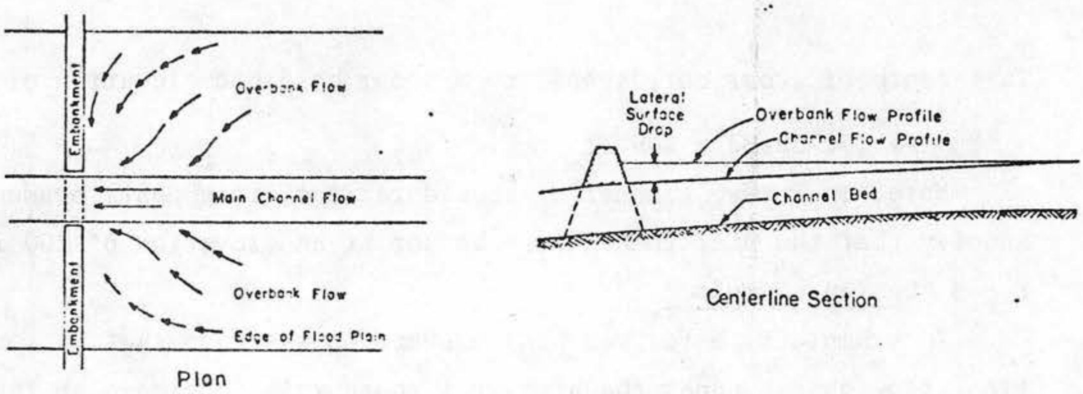


Fig. 8.8.10 Schematic of overbank and main channel flow.

of the overbank flow being impounded by a highway embankment and returning to the river. At the embankment, there is no overbank flow in the downstream direction and the slope of the longitudinal profile is necessarily level there. The profile gradually picks up slope until it parallels the main channel slope. The difference in the two profiles provides the gradient that forces the overbank flow towards the river. The intensity and velocity of flow towards the river is related to this gradient so that this intensity is highest at the embankment and tapers off slowly in the upstream direction. While these lateral flows may become quite strong, the situation is clearly not one where all the overbank flow moves like a river along the embankment. The added depth of overbank flooding depends on how much gradient is required to direct the flow back to the river.

The profile of the river channel flow is shown essentially constant in slope. This is not actually so because the overbank flow is entering the main channel with low momentum in the direction of flow and must pick up momentum at the expense of increasing the main channel slope. This increases the upstream main channel stage and consequently the amount of flooding.

The stage under the crossing is also increased over example 1 as a result of backwater due to increased channel flow. In this example, the banks are either protected or higher than average between the proposed crossing and the existing bridge so that the flow is expected to remain channelized for some 2000 ft before it can again spread onto the floodplain. As a first approximation, it is assumed that no appreciable bed scour has occurred in the several hours that it took for the flood to reach the design flow. Also, the return of the flood water to the floodplain takes some distance of the river to accomplish but it is assumed here that it all happens at one point.

What is the depth and velocity of flow in the main channel when the discharge is 110,000 cfs?

The unit discharge is

$$q = \frac{Q}{W} = \frac{110{,}000}{500} = 220 \text{ cfs/ft}$$

Assume the friction slope is

$$S_f = 0.00138$$

With the higher discharge, the bed should be plane (Manning's n 0.012 to 0.015). As a first approximation assume

$$n = 0.015$$

(See Table 2.3.1, Alluvial sandbed channels, plane bed.) Then from Eq. 2.3.20

$$y_o = \left\{\frac{(220)(0.015)}{(1.486)(0.00138)^{1/2}}\right\}^{3/5} = 11.6 \text{ ft.}$$

and the average velocity is

$$V = \frac{q}{y_o} = \frac{220}{11.6} = 19.0 \text{ fps}$$

The average shear stress on the bed is (from Eq. 2.3.31)

$$\tau_o = \gamma y_o S_f$$
$$= (62.4)(11.6)(0.00138) = 1.0 \text{ psf}$$

and the stream power is

$$\tau_o V = (1.0)(19.0) = 19.0 \text{ ft lb/sec/sq ft.}$$

For 1 mm sand and this stream power, Fig. 3.4.1 indicates that the flow is in the upper flow regime; that is, the bed form is antidunes.

Check the Froude number of the flow in the main channel.

$$Fr = \frac{19.0}{\{(32.2)(11.6)\}^{1/2}} = 0.98$$

A Froude number this large is not unacceptable for this channel. Therefore, use $n = 0.015$. The first approximation is that *the flow is critical in the main channel*. With this very high velocity flow in the main channel, the channel bed scours. The general scour decreases the main channel Froude number.

In the 2000-ft reach below the bridge, the scour is of the general type (see Section 6.5.2) and Laursen's equation (Eq. 6.5.4) is applicable. The equation is

$$\frac{y_2}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{6/7} \left(\frac{W_1}{W_2}\right)^{\frac{6(2+f)}{7(3+f)}} \left(\frac{n_2}{n_1}\right)^{\frac{6f}{7(3+f)}}$$

In this case, Q_t is the total design flood discharge or

$$Q_t = 110{,}000 \text{ cfs}$$

Q_c is the flow in the main channel upstream of the bridge or

$$Q_c = 66{,}000 \text{ cfs}$$

W_1 and W_2 are the approach channel and contracted channel reaches. Here

$$W_1 = W_2 = 500 \text{ ft}$$

The term f is dependent on the approach channel shear velocity and the fall velocity of the bed material. The approach channel shear velocity is

$$V_{*c} = \sqrt{\frac{\tau_1}{\rho}}$$

Here τ_1 is the average bed shear in the upstream channel and is computed from Eq. 2.2.31 or

$$\tau_1 = \gamma y_1 S_f$$

In the upstream approach channel the average depth is (from design example 1)

$$y_1 = 11.6 \text{ ft}$$

and

$$S_f \simeq 0.00138$$

Then $$\tau_1 = (62.4)(11.6)(0.00138) = 1.0 \text{ psf}$$

and

$$V_{*c} = \left(\frac{1.0}{1.94}\right)^{1/2} = 0.72 \text{ fps}$$

The median diameter of the bed material (sieve analysis) is

$$D_{50} = 1.0 \text{ mm}$$

Assume a shape factor of 0.7 (normal for sands) so that the fall velocity is (from Fig. 3.7.2)

$$\omega \simeq 12 \text{ cm/sec} = 0.39 \text{ fps}$$

so that

$$\frac{V_{*c}}{\omega} = \frac{0.72}{0.39} = 1.85$$

With this value of v_{*c}/ω, the value of f can be found in the table accompanying Eq. 6.5.4. Use

$$f \simeq 2.0$$

Upstream of the bridge, Manning's n for the main channel is 0.025 (from design example 1) so

$$n_1 = 0.025$$

and downstream

$$n_2 = 0.015$$

Now, the flow depth downstream is

$$y_2 = (11.6)\left(\frac{110,000}{66,000}\right)^{6/7}\left(\frac{0.015}{0.025}\right)^{\frac{(6)(2)}{7(3+2)}} = 15.0 \text{ ft}$$

The uniform flow depth in the downstream section has been computed as 11.6 ft. Therefore, the general scour should be 3.4 ft for the design flood discharge. With this flow depth the downstream velocity is

$$V = \frac{q}{y_2} = \frac{220}{15} = 14.7 \text{ fps}$$

and the new Froude number is

$$Fr = \frac{14.7}{\sqrt{(32.2)(15)}} = 0.67$$

In reality, flow conditions in the downstream reach have a Froude number somewhere between the Froude number 0.67 for the general scour assumption and the Froude number 0.98 for the no scour assumption. The Froude number of the flow at the peak of the design discharge hydrograph depends on the hydrograph rise time. If this time is long, the bed has time to scour out and general scour occurs. If the rise time is very short, the scour at the peak of the flood is much less than the general scour figure.

The flow and bed conditions during the passage of the hydrograph can be computed by applying the gradually varied nonuniform flow equations for water routing and a set of transport equations for routing sediment. Numerical programs are available to solve this water and sediment routing problem but their presentation and use is outside the scope of this manual.

According to Fig. 8.8.2, the rise time can be very short. Then the flow in the downstream channel has a Froude number close to unity. With such flow conditions, large waves form in the channel (see Section 2.4.0) and these waves could be very destructive to the channel and the downstream bridge. Also, the local depth of scour around the embankment ends and piers would be very large. The designer probably would not confine all the flow under the bridge, but would provide relief bridges on the floodplain to take a portion of the flow.

Neither design example 1 or 2 are complete. At this stage the designer would decide on whether to have relief bridges, their number location, and amount of flow they would take. These decisions would be based on economic, political, social and environmental factors that would exist for a particular site. The designer would need to continue the analysis outlined in the two examples and, in addition, would need to make a backwater analysis on the most feasible designs before going to final design.

8.8.4 Design example 3

Design example 3 is concerned with degradation of the channel. The same crossing and the same 500-ft bridge as in design example 1 are used but now a storage dam has been constructed 7 miles upstream for power generation, summer irrigation and flood control. Normal daily power releases are 10,000 cfs from the power plant for the six high demand hours and nominal releases for the remainder of the day to maintain fish stock. The irrigation diversion is far downstream of the crossing. Flood routing through the reservoir will reduce peak flow to the extent that the 100-year design flood is now only 40,000 cfs. The natural flow of sediment in the river has also been checked at the dam. The downstream control is 9 miles downstream where Mainstream River joins a much larger river.

The effect of the dam is that the time distribution of the flow is changed although the total volume is not. The flood peaks are reduced and the sediment transport is cut off. The average flow has been increased from 7000 cfs to about 10,000 cfs, ignoring the periods when the flows are very low. According to Fig. 4.4.3, increasing the mean discharge shifts the river towards the braided stream classification. Such a shift is generally a destabilizing trend. The channel will probably widen and this effect may be estimated by Eq. 4.4.6 which is

$$W \sim Q^{0.26}$$

The new width is

$$W_n = (500)\left(\frac{10,000}{7,000}\right)^{0.26}$$

$$\simeq 550 \text{ ft}$$

The design flood is very nearly the bankfull discharge so that the design stage is approximately the bankfull stage or El. 268. The depth of flow at a flow of 40,000 cfs is computed from Manning's equation (Eq. 2.3.20) or

$$y_o = \left\{\frac{Vn}{1.486S_f^{1/2}}\right\}^{3/2}$$

but since

$$V = \frac{q}{y_o}$$

$$y_o = \left\{\frac{qn}{1.486S_f^{1/2}}\right\}^{3/5}$$

The unit discharge is

$$q = \frac{Q}{W} = \frac{40,000}{550} = 72.7 \text{ cfs/ft}$$

The large D_{90}, the gravel bars and the smaller discharges (40,000 cfs) vs 66,000 cfs) will increase the value of Manning's n to a value larger than that used in example 1. Therefore, from our experience in working design examples 1 and 2, estimate Manning's n as 0.028. The friction slope is assumed equal to the bed slope 0.00138. Then

$$y_o = \left\{\frac{(72.7)(0.028)}{(1.486)(0.00138)^{1/2}}\right\}^{3/5} = 8.7 \text{ ft}$$

The average velocity is

$$V = \frac{q}{y_o} = \frac{72.7}{8.7} = 8.4 \text{ fps}$$

the average bed stress is (from Eq. 2.3.31)

$$\tau_o = \gamma\, y_o S_f$$

$$= (62.4)(8.7)(.00138) = 0.75 \text{ psf}$$

The Froude number for the channel flow is

$$Fr_1 = \frac{8.4}{\sqrt{(32.2)(8.7)}} = 0.50$$

and the average bed level is

$$268-9 = 259 \text{ ft}$$

The bed level of El. 259 can be expected to degrade as a result of a cutoff of sediment by the dam. This degradation can be very extensive on a steep sloping river such as this one. Degradation starts at the dam and progresses downstream with time and stops only when it reaches a rock or gravel ledge or where the river enters a lake or confluences with a larger river as in this case. The river scours its bed to establish an ultimate gradient such that the shear is below the critical for transport of sediment. This is not necessarily the critical shear for D_{50} because the large sizes in the bed material tend to remain to armor the bed. The D_{90} size is sometimes considered as appropriate for armoring and a grain size analysis shows this to be about 15 mm for the Mainstream River.

The critical tractive force for the D_{90} material is given by Shields' diagram (Fig. 3.2.3). Assume the flow is fully turbulent at the bed. Then

$$\frac{V_* D}{\nu} \geq 400$$

$$\frac{\tau_c}{(\gamma_s - \gamma) D_{90}} = \frac{\tau_c}{(S_s - 1)\gamma D_{90}} = 0.047$$

and

$$\tau_c = (0.047)(2.65-1)(62.4)\left(\frac{15}{304.8}\right)$$

$$= 0.24 \text{ psf.}$$

It will be the normal daily power release discharge that will degrade the channel. This flow is 10,000 cfs so

$$q = \frac{Q}{W} = \frac{10,000}{550} = 18.2 \text{ cfs/ft}$$

The Manning's n for the degraded bed will reflect the losses due to the remnant bed forms that were formed when the sediment was moving and the losses due to a higher grain roughness because the bed material becomes coarser. If there were grain roughness only, Manning's n is given by Eq. 3.10.46 assuming the D_{90} is now the D_{50} size

$$n = 0.04\ D_{50}^{1/6}$$

$$= (0.04)\left(\frac{15}{304.8}\right)^{1/6} = 0.024$$

Because there will be some form roughness from the gravel bars use

$$n = 0.028$$

Manning's equation (Eq. 2.3.20)

$$q = \frac{1.486}{n}\ y^{5/3} S_f^{1/2}$$

Here we know q, and n. Because

$$\tau_c = \gamma y S_f = 0.24$$

Then

$$S_f = \frac{0.24}{62.4y} = \frac{0.00385}{y}$$

Put this expression for S_f in Manning's equation so that

$$18.2 = \frac{1.486}{0.028}\ (0.062) y^{7/6}$$

or $\quad y = 4.3$ ft

It follows that

$$S_f = \frac{0.00385}{4.3} = 0.00090$$

$$\simeq 4.8 \text{ ft/mile}$$

The slope before degradation is 7.3 ft/mile.

The existing profile and the ultimate profile are shown in Fig. 8.8.11 along with an intermediate profile during the degradation process. These profiles are ultimately controlled at the larger river which is controlling the water surface level of the river at the point of confluence. If degradation proceeded to the limit shown, the scour at the crossing would be

$$y_s = (9)(7.3\text{-}4.8) \simeq 22 \text{ ft}$$

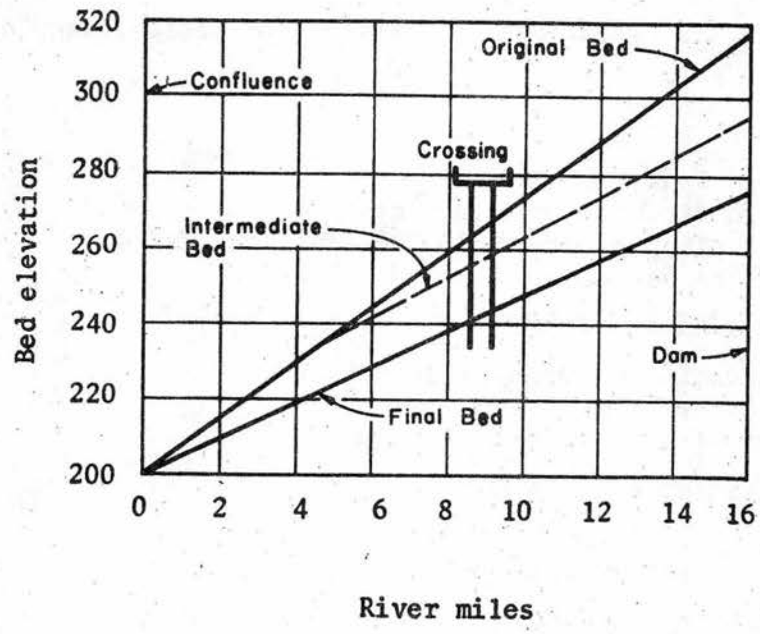


Fig. 8.8.11 Degradation due to dam upstream of the crossing.

This represents the removal of a substantial amount of material and, in the meantime, an equal amount is being captured by the upstream reservoir. The reservoir could well fill before the ultimate degradation is reached, in which case the flow of sediment would be restored.

A computation of the rate of sediment discharge will provide an estimate of the rate of degradation, especially if this computation is made in the undisturbed river just downstream of the degrading zone. A somewhat crude computation based on Fig. 3.10.10 will serve this purpose. A discharge of 10,000 cfs at a slope of .00138 in a 550 ft channel will have a depth of approximately 4.3 ft and a velocity of 4.2 fps. Using Fig. 3.10.10 for 1 mm sand gives a transport rate of approximately 35 tons per day per foot of width, but since the release is for 6 hours only, the actual transport in Mainstream river is (6/24)(130) = 9 tons per day per ft of width. If the bed material has a dry weight of 100 lbs/cu ft this represents 165 cu ft per day per foot of width. Now if the degradation zone has just reached the crossing seven miles from the dam, the average rate of degradation over this stretch of 37,000 ft is then 0.004 ft per day or about 1.6 ft per year. When degradation has progressed to the confluence 9 miles downstream, the rate will be about one-half this or 0.002 ft per day Thus, degradation proceeds at an ever decreasing rate.

Table 8.8.2 Data and computations for design example 3.

Data:

Bridge span = 500 ft
Design discharge = 40,000 cfs
Power plant release = 10,000 cfs for 6 hrs/day
Original bed slope, S = 7.3 ft/mile

Final degraded river form:

Degradation discharge = 10,000 cfs
Bed material size = 15 mm
Critical bed shear τ_c = 0.24 psf
New width, w = 550 ft
New slope, S_f = 4.8 ft/mile
Degradation at crossing = 22 ft

8.8.5 Design example 4

In this example, the problem of aggradation will be considered. A dam is located eight miles downstream of the proposed crossing. The present width of the river at the proposed crossing is 1000 feet as shown in Fig. 8.8.1. The reservoir level is at Elev. 250 during the winter, with summer drawdowns as low as Elev. 240. The design flood level of the reservoir is Elev. 260. The effects of the reservoir are to increase the design flood stage and to cause general aggradation of the bed at the proposed crossing. The aggradation is considered because the design flood stage is superimposed on the aggraded profile.

Aggradation occurs as the river flow slows down upon entering the reservoir and deposits its sediment load. The method of deposition is usually complex in rivers with fine sediment load, but in coarse sandbed rivers such as the Mainstream River the sediment is largely deposited at the entrance of the reservoir in a delta formation. The top slope of the deposit formation is generally concave as shown in Fig. 8.8.12. At the upstream end of the deposition, the river retains its normal bed slope and sediment transport rate. The sediment-carrying capacity of the river decreases downstream until it is almost zero at the end of the delta. Point a in Fig. 8.8.12 must remain at the same elevation as the delta moves into the reservoir. The aggraded reach lengthens until the reservoir is filled with sediment and sediment starts to pass

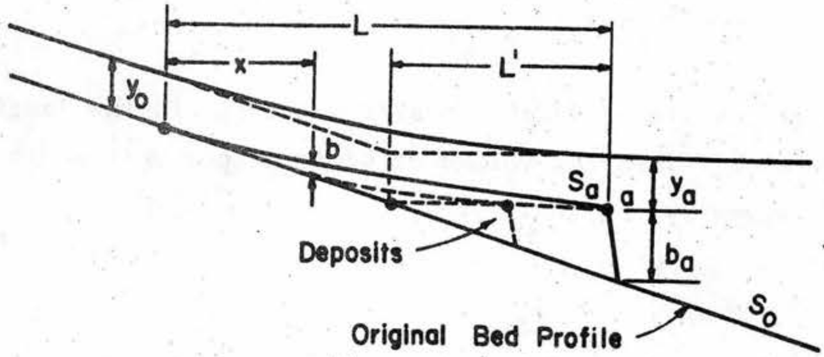


Fig. 8.8.12 Definition diagram for aggradation from deposition in a reservoir.

out with the release water. As with degradation, the increase in the bed elevation proceeds at a decreasing rate as the length of the aggraded reach grows.

A simplified evaluation of the amount of aggradation at any point can be made by assuming (1) that the slope at the end of the delta is the maximum slope for zero sediment transport, (2) that the slope increases linearly until it reaches the natural slope of the river, and (3) that the width of the delta formation is constant. The last assumption is a gross oversimplification but it results in a conservative design.

If the slope S at a distance x from the upstream end of the aggrading reach varies linearly from S_o to S_a then

$$S = S_o - \frac{x}{L}(S_o - S_a) \qquad 8.8.1$$

where the terms are defined in Fig. 8.8.12. Then the deposit thickness b changes according to

$$db = (S_o - S)\,dx = \{S_o - [S_o - \frac{x}{L}(S_o - S_a)]\}dx$$

$$= \frac{x}{L}(S_o - S_a)\,dx \qquad 8.8.2$$

Integrating over length x yields

$$\int_o^b db = \int_o^x \frac{x}{L}(S_o - S_a)\,dx$$

or $$b = \frac{x^2}{2L}(S_o - S_a) \qquad 8.8.3$$

where the slopes are considered positive and L is the length of the aggrading reach. Then the volume of sediment per unit width in the deposit is given by

$$\int_o^L b\,dx = \int_o^L \frac{x^2}{2L}(S_o - S_a)\,dx$$

$$= \frac{L^2}{6}(S_o - S_a) \qquad 8.8.4$$

If sediment per unit width is supplied by the river at a rate q_s, and deposits with a specific weight γ_s, including voids, then the time to develop the volume of aggradation in Eq. 8.8.4 is

$$t = \frac{L^2}{6}(S_o - S_a)\frac{\gamma_s}{q_s} \qquad 8.8.5$$

The downstream face of the delta moves downstream as the depth of deposit increases to keep point a of Fig. 8.8.12 at a constant level. Thus

$$\frac{b_a}{L'} = S_o \qquad 8.8.6$$

where b_a can be found from Eq. 8.8.3 with $x = L$,

$$b_a = \frac{L}{2}(S_o - S_a) \qquad 8.8.7$$

Therefore during the time in which length L develops, the distance that the face of the delta moves downstream L' is given by

$$L' = \frac{b_a}{S_o} = \frac{L}{2}\,\frac{S_o - S_a}{S_o} \qquad 8.8.8$$

In this example, the mean five-month winter flow of 7000 cfs is considered to be the dominant discharge in the aggradation process. The river channel is 500 ft wide except at the crossing, the natural slope is 1.38 ft/1000 ft and the median material size is 1 mm. The data for design example 4 is given in Table 8.8.3. *The channel width in the flow and sediment transport problems is 500 ft as opposed to the 1000 ft at the crossing.* This is because the 1000 ft wide reach is a short reach which does not affect the large scale properties of the river.

The unit discharge in the channel is

$$q = \frac{Q}{W} = \frac{7000}{500} = 14 \text{ cfs/ft}$$

Assuming that Manning's n = .025, the depth upstream of the aggrading reach can be calculated from Manning's equation (Eq. 2.3.20)

$$y_o = \left\{\frac{qn}{1.486 S_f^{1/2}}\right\}^{3/5}$$

$$= \left\{\frac{(14)(.025)}{(1.486)(0.00138)^{1/2}}\right\}^{3/5}$$

$$= 3.0 \text{ ft}$$

Thus, the velocity becomes

$$V = \frac{q}{y_o} = \frac{14}{3.0} = 4.7 \text{ fps}$$

The shear stress on the boundary is given by Eq. 2.3.30

$$\tau_o = \gamma R S_o$$

$$\simeq \gamma\, y_o\, S_o = (62.4)(3.0)(.00138) = .26 \text{ psf}$$

The stream power becomes

$$\tau_o V = (.26)(4.7) = 1.21 \text{ ft lb/sec/sq ft}$$

With a median sieve diameter of 1.0 mm and the computed stream power Fig. 3.4.1 can be used to determine the probable bed form. The bed would be in transition and from Table 2.3.1 (Alluvial sandbed channels), the assumed Manning's n of .025 is acceptable.

The slope for zero transport must be computed assuming the shear stress remains constant throughout the aggrading reach. The shear velocity can be written

$$V_* = (\frac{\tau_o}{\rho})^{1/2} = (\frac{.26}{1.94})^{1/2} = .37 \text{ fps}$$

by definition. Assuming $\nu = 1.3 \times 10^{-5}$ ft^2/sec, the grain size Reynolds number can be computed by

$$\frac{V_* D}{\nu} = \frac{(.37)(1)(.00328)}{(1.3)(10^{-5})} = 93$$

Referring to Shields' diagram (Fig. 3.2.3), for a grain size Reynolds number of 93, the dimensionless shear stress for incipient motion is .04. That is,

$$.04 = \frac{\tau_c}{(\gamma_s - \gamma)D}$$

or $\tau_c = .04\ (\gamma_s - \gamma)D$

$$= .04\ (165.4 - 62.4)(.00328) = .014 \text{ psf}$$

This value of shear stress should now be used to get the shear velocity, Reynolds number for the grain and Shields' parameter as follows

$$V_* = (\frac{\tau_o}{\rho})^{1/2} = (\frac{.014}{1.94})^{1/2} = 0.85 \text{ fps}$$

$$\frac{V_* D}{\nu} = \frac{(.085)(.00328)}{(1.3)(10^{-5})} = 21$$

and $.031 = \dfrac{\tau_c}{(\gamma_s - \gamma)D}$

Thus

$$\tau_c = .031\ (165.4 - 62.4)(.00328) = .010 \text{ psf}$$

Using Eq. 2.2.30 for the critical shear stress on the bed and Manning's equation (Eq. 2.3.20) for the critical transport conditions prevailing, the depth and slope at the end of the delta can be obtained as follows:

$$\tau_c = \gamma y_o S_c$$

and
$$q = \frac{1.486}{n} y_o^{5/3} S_c^{1/2}$$

or
$$q = \frac{1.486}{n} y_o^{5/3} \left(\frac{\tau_c}{\gamma y_o}\right)^{1/2}$$

$$= \frac{1.486}{n} y_o^{7/6} \left(\frac{\tau_c}{\gamma}\right)^{1/2}$$

In solving the above equation for the depth at the end of the delta, it is assumed that the delta is wider than the width of the river. Inspection of maps and aerial photos shows that the average width of the delta is approximately 1000 ft. Thus the unit discharge becomes

$$q = \frac{Q}{W} = \frac{7000}{1000} = 7 \text{ cfs/ft}$$

It will also be assumed that Manning's n is about 0.015 since the bed forms at the end of the delta should be small or nonexistent. Thus, the depth at the end of the delta becomes

$$y_o = \left\{\frac{n\, q\, \gamma^{1/2}}{1.486\, \tau_c^{1/2}}\right\}^{6/7}$$

$$= \left\{\frac{(.015)(7)(62.4)^{1/2}}{(1.486)(.010)^{1/2}}\right\}^{6/7} = 4.37 \text{ ft}$$

The velocity becomes

$$V = \frac{q}{y_o} = \frac{7}{4.37} = 1.6 \text{ fps}$$

Checking the assumption of n, the stream power is

$$\tau V = (.010)(1.6) = .016 \text{ ft lb/sec/sq ft}$$

According to Fig. 3.4.1, with a median fall diameter of approximately 1.0 mm, the bed form is flat with no transport. Table 2.3.1 (Alluvial sandbed channels) shows that .015 for Manning's n is a good selection.

The slope for zero sediment transport at the downstream end of the delta is determined from Eq. 2.2.30

$$\tau_c = \gamma y_o Sc$$

or $$S_c = \frac{\tau_c}{\gamma y_o} = \frac{.010}{(62.4)(4.37)} = .000037$$

Thus, the slope at the end of the delta is .037 ft/1000 ft (.19 ft/mile).

Table 8.8.3 Data for design example 4.

Hydraulics for natural river:

Slope, S_o = .00138(7.29 ft/mile)
Width, W = 500 ft
Width (at crossing), W_c = 1000 ft
Mean river flow, Q = 7000 cfs
Unit discharge, q = 14 cfs/ft
Manning's n, n = .025
Depth of flow, y_o = 3.0 ft
Velocity, V = 4.7 fps
Boundary shear stress, τ_o = .26 psf

Hydraulics for no transport:

Critical shear, τ_c = .010 psf
Unit discharge, q = 7 cfs/ft
Manning's n = 0.015
Depth of flow, y_o = 4.37 ft
Velocity, V = 1.6 fps
Slope, $S_c = S_a$ = .000037 (.19 ft/mile)

Sediment transport:

Rate of transport, q_T = 50 tons/day/ft
= 3,750,000 tons/yr.
Transport deposited on delta, q_s = 3.750 tons/yr/ft
Specific weight of deposit, γ_s = 100 pcf

Profile dimensions when the delta reaches the dam:

Distance from crossing to the dam = 42,000 ft
Bed elevation at start of delta = El. 246 ft
Distance from crossing to start of delta = 10,000 ft
Movement required of delta = 32,000 ft
Equivalent length of aggraded reach, L = 66,000 ft
Time required to deposit material, t = 13 yrs
Distance from beginning of deposit to the crossing = 24,000 ft
Depth of deposit at the crossing, b = 5.8 ft

Tabulated results of bed profile:

Distance x mi	Distance x ft	Original Bed El., ft	Deposition ft	Aggraded Bed El., ft	Aggraded Bed Slope
0	0	292	0	292	.00138
2.85	15,000	272	2	274	.00108
5.70	30,000	251	9	260	.00077
8.50	45,000	230	20.5	250.5	.00047
11.40	60,000	209	36.5	245.5	.00016
12.50	66,000	201	45	246	.00004

It is assumed that the delta initiates at that point at which the bed is 4.37 ft below the normal operating level of Elev. 250. The basis of this assumption is that the reservoir fills rapidly to the operating level and that 4.37 ft is the depth required for deposition. The depth for deposition is less than this due to the slope being the average bed slope at the initiation of the delta. Thus, use 4.0 ft, resulting in a bed Elev. 246 at the initiation of the delta. With a bed elevation of 260 ft at the crossing, the point at which the delta begins is

$$\Delta x = \frac{\Delta y}{S_o} = \frac{260 - 246}{.00138} \approx 10,000 \text{ ft}$$

downstream of the crossing. *The dam is 8 miles or 42,000 ft downstream of the crossing.* Thus the delta must move a distance

$$L' = 42,000 - 10,000 = 32,000 \text{ ft}$$

before filling the reservoir. The length of the aggrading reach when the reservoir is filled is determined from Eq. 8.8.8

$$L = 2L' \frac{S_o}{(S_o - S_a)} = (2)(32,000) \frac{(.00138)}{(.00138 - .000037)} = 66,000 \text{ ft}$$

The time required to fill the reservoir can be obtained from Eq. 8.8.5 if the sediment discharge is known.

The sediment transport can be determined using Colby's method (see Section 3.10.13). The Mainstream River carries very little fine sediment and thus $k_2 = 1.0$. Assuming a temperature of 60°F, $k_1 = 1.0$. Thus Eq. 3.10.43 reduces to

$$q_T = [1 + (k_1 k_2 - 1)k_3]\, q_n$$

$$= q_n$$

Thus, even though k_3 is different from 1.0, it does not enter into the calculations. The rate of transport can be obtained from Fig. 3.10.10. For the 1.0 foot depth and $V = 4.7$ fps, $q_n = 42$ tons/day/ft of width. For a 10.0 foot depth, $q_n = 60$ tons/day/ft of width. Using logarithmic interpolation, $q_n = 50$ tons/day/ft of width for a depth of $y_o = 3$ ft. This is equivalent to 7500 tons/ft of width for the 5 months of high flow per year or 3,750,000 tons/year. This amount of material is deposited on the delta, which was assumed to average 1000 ft wide. Thus, the unit sediment discharge depositing on the delta is 3,750 tons/year/ft of width (7,500,000 lbs/year/ft).

If it is assumed that the material deposits with a specific weight of $\gamma_s = 100$ pcf (a good assumption for coarse sediment) Eq. 8.8.5 gives

$$t = \frac{L^2}{6}(S_o - S_a)\frac{\gamma_s}{q_s}$$

$$= \left(\frac{(66{,}000)^2}{6}\right)(.00138 - .000037)\left(\frac{100}{7{,}500{,}000}\right)$$

$$= 13 \text{ years}$$

as the time required to fill the reservoir.

The aggrading bed will affect the design discharge stage at the proposed bridge site. The depth of aggradation at the proposed crossing can be obtained from Eq. 8.8.3. In this equation, x is the distance from the upstream end of the aggrading section (66,000 ft from the dam) to the crossing (42,000 ft upstream from the dam). Thus,

$$x = 66{,}000 - 42{,}000 = 24{,}000 \text{ ft}$$

and

$$b = \frac{x^2}{2L}(S_o - S_a)$$

$$= \frac{(24{,}000)^2}{(2)(66{,}000)}(.00138 - .000037) = 5.8 \text{ ft}$$

The depth of aggradation at the proposed crossing is 5.8 feet, raising the bed elevation to approximately 266 feet. The effect of this on the flood stage must now be considered.

The flood stage can be computed by a backwater calculation for the 110,000 cfs design flood over the aggraded profile. Several problems arise because of the fact that aggradation has filled in most of the channel and much of the design flood will pass over the floodplain. In this example we will consider the floodplain to be 4000 ft in width, virtually flat, normal to the river and at a level 15 ft above the original bed. A reasonable assumption for this example is that the design flood is restricted to a 1000 ft wide channel at the crossing but begins immediately to spread out into the floodplain downstream of the crossing. Also it can be assumed that the spreading is complete and the flow is uniformly distributed across the floodplain at the point c where delta formation rises above the floodplain. This occurs when the depth of aggradation equals 15 feet, the assumed level of the floodplain above the original bed level. The location of point c can be obtained from Eq. 8.8.3

$$b = \frac{x^2}{2L}(S_o - S_a)$$

or

$$x = \left\{\frac{2\,bL}{S_o - S_a}\right\}^{1/2}$$

$$= \left\{\frac{(2)(15)(66{,}000)}{(.00138 - .000037)}\right\}^{1/2} = 38{,}500 \text{ ft}$$

Thus, point c is 66,000 - 38,500 = 27,500 ft upstream from the dam. In this section *where the flow is spreading, the total design flood should be divided into channelized flow and floodplain flow and separate backwater curves computed for each.* However, provision must be made to see that there is flow from the channel to the floodplain in just the correct amounts to produce backwater profiles that are identical except for the small lateral gradients needed to accomplish this lateral flow. Thus, the problem becomes complex.

A simplification would be to arbitrarily establish the rate of lateral movement and then compute the backwater curve for the resulting channel flow. Different patterns of lateral flow could be assumed to see how sensitive the stage at the crossing is to this variable, a reasonable approach for computer work. *Herein, the assumption is that the rate of lateral flow is constant along the reach from the crossing to point* c. Then the unit discharge would vary linearly from 110 cfs per foot width at the crossing to 27.5 cfs per foot width at point c and remain at that value to the dam.

The backwater problem to be solved is one in which both unit discharge and bed slope are changing with distance x, complications not existing in the theoretical development of the backwater equation given in Section 2.7.3. Variables are functions of both x and y and the direct method of specifying Δy and computing Δx is not available. It is assumed that *the discharge over a given step can be considered uniform and equal to the average discharge in the reach,* Eq. 2.7.7 can be applied in a trial and error solution. This solution is shown in Table 8.8.5. It is further assumed in this table that the energy gradient in a given step is the same as occurring for the average flow conditions in the reach taken as steady uniform flow. So, in Eq. 2.7.7 $n = n_o$. Table 8.8.5 shows a step method in which Δx are specified rather than Δy. The dam face is at $x = 66,000$ ft, the point c is at $x = 38,500$ ft and the crossing is at $x = 24,000$ ft. The reservoir pool is at El. 260 and the aggraded bed is at El. 246 at the dam so that the depth y at the dam is 14 ft.

The computational procedure in Table 8.8.4 is as follows:

(1) Station 1 is assumed to be the location of the dam at which $x = 66{,}000$ ft and $y = 14.00$ ft are given.

(2) Choose an increment of length, Δx (the smaller the increment, the more accurate is the solution) and determine the upstream position x_{us} by

$$x_{us} = x_{ds} + \Delta x$$
$$= 66{,}000 - 5000 = 61{,}000 \text{ ft}$$

where x_{us} = the position of the upstream end of the reach
x_{ds} = the position of the downstream end of the reach
Δx = length of the reach chosen.

(3) Compute the average x over the reach:

$$x_{ave} = \frac{1}{2}(x_{us} + x_{ds}) = x_{ds} + \frac{1}{2}\Delta x$$
$$= \frac{1}{2}(61{,}000 + 66{,}000)$$
$$= 63{,}500 \text{ ft}$$

(4) Compute the average slope over the reach using Eq. 8.8.1:

$$S_{ave} = S_o - \frac{x_{ave}}{L}(S_o - S_a)$$
$$= .00138 - \frac{63{,}500}{66{,}000}(.00138 - .000037)$$
$$= .000091$$

where S_o = natural bed slope,
S_a = bed slope at the end of the delta
L = length of the aggradation.

(5) Compute the average discharge over the reach q_{ave}: The average discharge is 27.5 cfs/ft until the point is reached at which the flow is spreading onto the floodplain (x = 38,500 ft) after which q_{ave} can be computed using

$$q_{ave} = 27.5 \text{ cfs/ft} + \left(\frac{38{,}500 - x_{ave}}{38{,}500 - 24{,}000}\right)(110.0 - 27.5)$$
$$= 27.5 \text{ cfs/ft} + \left(\frac{38{,}500 - x_{ave}}{14{,}500}\right)(82.5 \text{ cfs/ft})$$

in which q = 27.5 cfs/ft denotes the unit discharge downstream of complete spreading,

Table 8.8.4 Backwater computations for example 4.

Sta.	x ft	y ft	Δx ft	x_{ave} ft	S_{ave} ft/1000 ft	q_{ave} cfs/ft	$y_{o_{ave}}$ ft	$y_{c_{ave}}$ ft
1	66,000	14.00						
			-5,000	63,500	.0878	27.5	10.36	2.84
2	61,000	13.73						
			-5,000	58,500	.1896	27.5	8.22	2.84
3	56,000	12.98						
			-3,000	54,500	.2710	27.5	7.39	2.84
4	53,000	12.31						
			-3,000	51,500	.3321	27.5	6.95	2.84
5	50,000	11.49						
			-3,000	48,500	.3931	27.5	6.61	2.84
6	47,000	10.54						
			-3,000	45,500	.4541	27.5	6.33	2.84
7	44,000	9.51						
			-2,000	43,000	.5050	27.5	6.13	2.84
8	42,000	8.48						
			-2,000	41,000	.5457	27.5	5.99	2.84
9	40,000	7.56						
			-1,500	39,250	.5813	27.5	5.88	2.84
10	38,500	7.12						
			-1,500	37,750	.6118	31.77	6.31	3.12
11	37,000	6.87						
			-2,000	36,000	.6475	41.72	7.31	3.74
12	35,000	7.10						
			-2,000	34,000	.6882	53.10	8.29	4.38
13	33,000	7.81						
			-3,000	31,500	.7390	67.33	9.36	5.13
14	30,000	9.01						
			-3,000	28,500	.8000	84.40	10.46	5.95
15	27,000	10.10						
			-3,000	25,500	.8611	101.47	11.43	6.72
16	24,000	11.10						
			-3,000	22,500	.9221	110.00	11.76	7.09
17	21,000	11.57						
			-3,000	19,500	.9832	110.00	11.53	7.09
18	18,000	11.55						
			-3,000	16,500	1.044	110.00	11.33	7.09
19	15,000	11.39						
			-3,000	13,500	1.105	110.00	11.13	7.09
20	12,000	11.20						
			-3,000	10,500	1.166	110.00	10.96	7.09
21	9,000	11.01						
			-3,000	7,500	1.227	110.00	10.79	7.09
22	6,000	10.82						
			-3,000	4,500	1.244	110.00	10.75	7.09
23	3,000	10.76						
			-3,000	1,500	1.345	110.00	10.49	7.09
24	0	10.51						

q = 110.0 cfs/ft denotes the unit discharge upstream of the initiation of spreading which occurs at the bridge site

x = 38,500 ft indicates the location of the completion of the spreading of the flow

x = 24,000 ft marks the location of the crossing which is where the spreading of the flow starts.

(6) Compute the normal depth for the reach $y_{o_{ave}}$ using Manning's equation (Eq. 2.3.20):

$$y_{o_{ave}} = \{\frac{n}{1.486} \frac{q_{ave}}{S_{ave}^{1/2}}\}^{3/5}$$

$$= \{\frac{.025}{1.486} \frac{2.75}{(.000091)^{1/2}}\}^{3/5}$$

$$= 10.27 \text{ ft}$$

(7) Compute the critical depth for the reach $y_{c_{ave}}$:

$$y_{c_{ave}} = \sqrt[3]{q_{ave}^2/g}$$

$$= \sqrt[3]{(27.5)^2/32.2}$$

$$= 2.86 \text{ ft}$$

(8) Assume a depth of flow at the upstream end of the reach y^*_{us} and compute the resulting average depth over the reach y^*_{ave}:

$$y^*_{ave} = \frac{1}{2}(y^*_{us} + y_{ds})$$

$$= \frac{1}{2}(14.00 + 14.00)$$

$$= 14.00 \text{ ft}$$

where y^*_{ave} = the average depth in the reach based on an assumed upstream depth

y^*_{us} = the assumed upstream depth

y_{ds} = the known downstream depth

(9) Compute the following quantities:

$$\{1 - (\frac{y_{o_{ave}}}{y^*_{ave}})^{10/3}\} = \{1 - (\frac{10.27}{14.00})^{10/3}\} = 0.644$$

and

$$\{1 - (\frac{y_{c_{ave}}}{y^*_{ave}})^3\} = \{1 - (\frac{2.86}{14.00})^3\} = 0.992$$

(10) Compute the change in depth over the reach Δy using Eq. 2.7.7:

$$\Delta y = S_{ave}\Delta x \left\{\frac{1 - (\frac{y_{o_{ave}}}{y^*_{ave}})^{10/3}}{1 - (\frac{y_{c_{ave}}}{y^*_{ave}})^3}\right\}$$

$$= (.000091)(-5000) \{\frac{.644}{.992}\}$$

$$= -0.30 \text{ ft}$$

(11) Compute the upstream depth of flow y_{us}

$$y_{us} = y_{ds} + \Delta y$$
$$= 14.00 + (-.30)$$
$$= 13.70 \text{ ft}$$

and check against the assumed value y^*_{us}:

$$y_{us} = 13.70 \neq y^*_{us} = 14.00$$

If the assumed depth and computed depth are approximately equal ($y^*_{us} \simeq y_{us}$), the computations for that reach are complete and y_{us} becomes the known downstream depth for the next reach.

Succeeding steps are performed in the same manner. In each case, the appropriate unit discharge and bed slope is determined first. Then the upstream depth is computed.

As shown in Fig. 8.8.13, *the backwater curve is complex*. From the dam (x = 66,000 ft) to x = 37,000 ft, the water surface profile is type M1 (see Table 2.7.1 and Fig. 2.7.2). Then from x = 37,000 ft to x = 20,000 ft, the water surface profile is type M2. The proposed crossing is in this reach. Upstream of station x = 20,000 ft, the backwater curve is again type M1. Normal depth is reached at x = 0 ft.

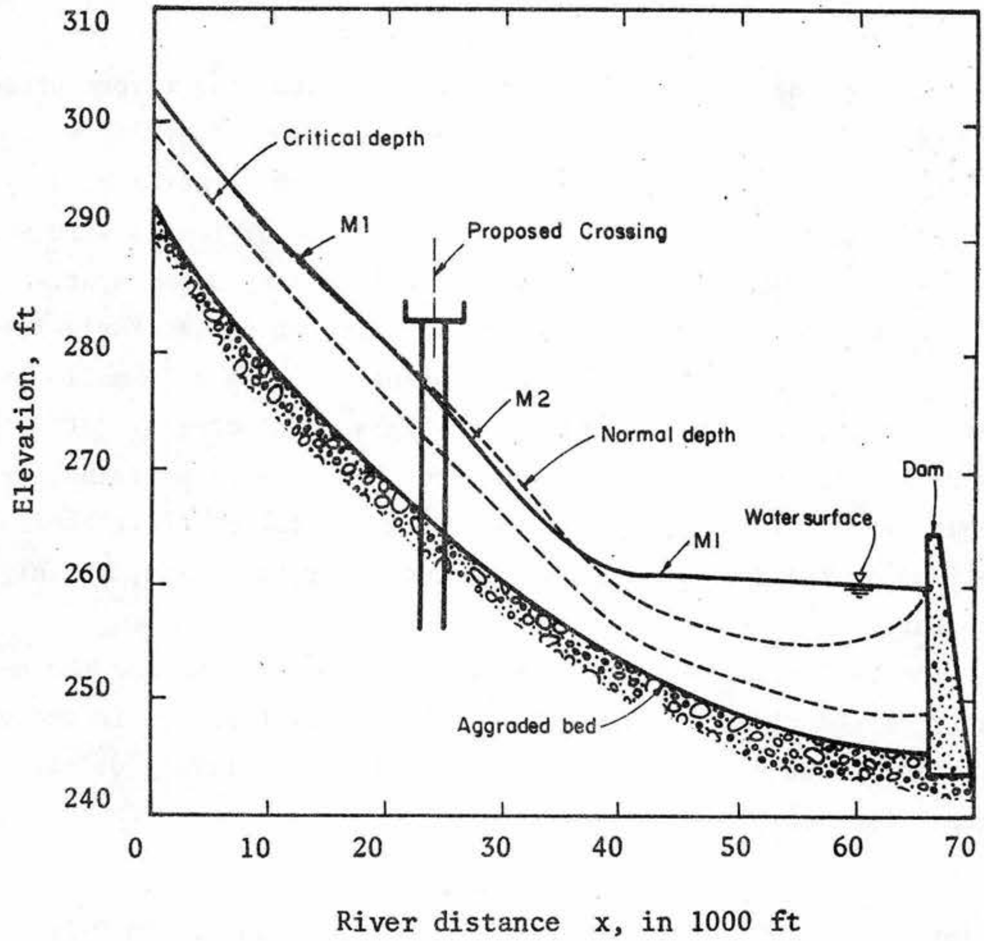


Fig. 8.8.13 Backwater with aggradation.

The depth of flow at the crossing becomes 11 ft. Considering the original bed at the crossing was at El. 260, the aggradation at the crossing was computed to be 5.8 ft and the depth of flow is 11 ft, *the stage for the design flood becomes El. 277.* The resulting velocity at the crossing is 10 fps.

Considering the increased stage due to the backwater superimposed on the aggraded bed resulting from the downstream dam, *the proposed bridge site is a poor one.* The river can become perched and very unstable in its location. If it is possible to route the highway 4 or more miles upstream, the problem of the increased stage can be alleviated.

ACKNOWLEDGMENTS

Many of the case histories of highways crossing rivers presented in this chapter have been taken from a report prepared by Joe W. Keeley titled "Bank Protection and River Control in Oklahoma", Federal Highway Administration, Oklahoma Division, 1971. The other case histories were supplied to us by Mainard Wacker of the Wyoming State Highway Department.

The type of information in Keeley's report and Wacker's files is of great benefit to highway and river engineers. The information shows the many interactions between rivers and highway structures. It is proof that rivers are dynamic, moving back and forth across floodplains, lying dormant through years of low flows and then breaking out of their banks to recarve the form of the immediate landscape including, at times, the highway structures.

The dynamic features of rivers and river systems and the natural beauty of the river scenery make the design of highways in the river environment one of the most challenging and stimulating of all engineering designs.

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