ESSAYS ON RIVER MECHANICS

Colorado State University

Presented by the Graduate Students
in CE 717 River Mechanics (Spring 1987)

Instructor: P.Y. Julien

May 1987

CER 86 - 87 PYJ-HY1

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FOREWORD

I am very pleased to honor the work of my graduate students in the class CE717 - River Mechanics with this report of their technical papers. Each student worked on a particular aspect of river engineering in order to meet the following objectives:

1) familiarize with the recent literature and new methodologies not available in textbooks,
2) compare various methods (new versus old) and discuss the advancement of engineering technology on a given topic,
3) develop skills to point out the key elements of recent technological developments,
4) share interesting results with the other students through an oral presentation and a written paper.

The requirements for this project were:

1) select a topic relevant to river mechanics and sediment transport,
2) conduct a mini literature review including papers published in the past five years,
3) compare new methodologies with those detailed in textbooks on either a theoretical basis or through comparison with an appropriate data set,
4) write a 40 page report and discuss the major findings in a 30-45 minute oral presentation,
5) summarize the analysis and the results in a 15 page paper following the ASCE editorial standards (these papers are enclosed herein)

Not only did the students show great enthusiasm in this class but the reader will certainly agree with me that the objectives were met with great success. I am personally very impressed with the overall quality of the reports presented and can only encourage them to pursue advanced studies in this field.

Pierre
P.Y. Julien

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AN INVESTIGATION OF CRITERIA USED TO PREDICT THE BRAIDING/MEANDERING THRESHOLD

By Mark E. Smith

Abstract: Nine methods - seven empirical and two theoretical - for predicting whether alluvial rivers should braid or meander are evaluated using a data set composed of 101 stream channels; comparison among the methods is achieved through use of a single threshold to separate braiding from meandering. The results indicate the importance of considering bed material size in any analysis, as well as the need to choose a method which was developed for river conditions similar to those being studied. Lane's method (1957) for use with sand-bed streams, Ferguson's method (1984) for use with gravel-bed streams, and Fredsoe's method (1978) for use with all streams yield the best results.

INTRODUCTION

The morphology of alluvial stream channels has been defined by geomorphologists and engineers in many ways. Culbertson (8), Rundquist (18), and Brice (4) have developed extensive classification schemes to describe river morphology. The transitions between many of these forms, however, are not clear and so analysis of river morphology is often hindered by the continuum which is a part of our natural environment.

One of the more commonly used classifications was presented by Leopold and Wolman (16) in 1957. They proposed that alluvial channels may be either straight, meandering, or braided. Of interest to many researchers, then, is the particular combination of hydraulic and geomorphic conditions which might cause these forms to occur.

A straight stream may be defined as one that does not follow a sinuous course; Leopold and Wolman (16) noted that straight streams in nature are rare. Because of this very qualitative definition, the classification of "straight" is often subject to controversy.

A meandering stream consists of alternating bends, which produce an S-shaped appearance in planform. The degree of curvature of the bend may vary from near straight to highly curved, depending upon conditions of flow, bed-bank material, gradient, geology, and other factors. The meandering channel is typically narrow and deep, having a relatively flat slope (21).

The braided stream is often wide and shallow, with poorly defined, unstable banks. Its channel is generally steeper than that of a meandering one, and is characterized by multiple channel divisions around alluvial bars or islands. Leopold and Wolman (1957) concluded that two primary causes of braiding are: 1) overloading of sediment in the stream, and 2) steep slopes (21).

Experiments by Friedkin (11), Lane (14), Leopold and Wolman (16), and others suggest that hydraulic thresholds exist between meandering and braided streams. Lane, Ackers and Charlton (1), and others also indicate a threshold between meandering and straight streams. Figure 1, from Schumm and Khan (20), shows a graph of discharge vs. channel slope with several thresholds which have been proposed.

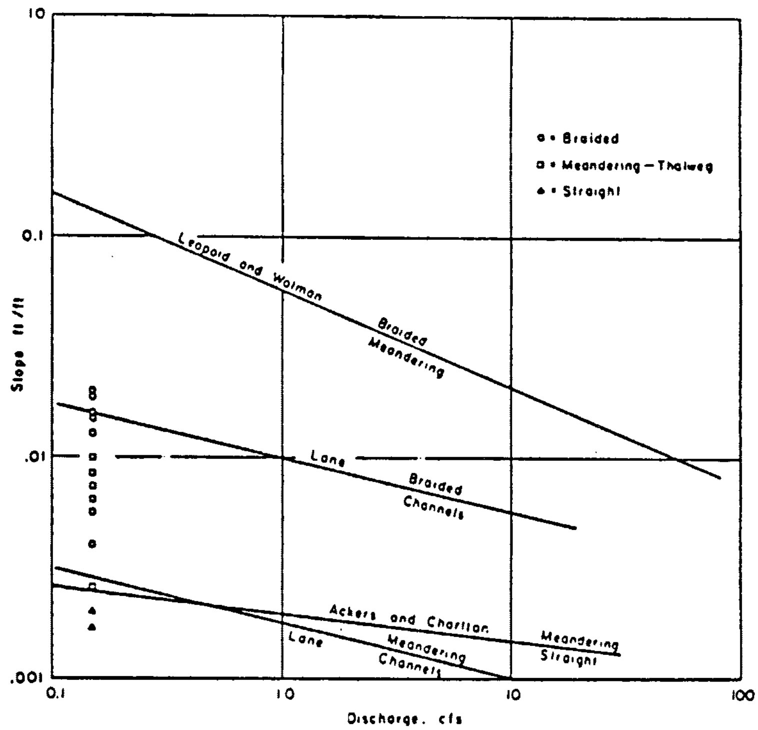

Fig. 1. Relation between slope and discharge and threshold slopes at each discharge, as defined by Lane (1957), Leopold and Wolman (1957), and Ackers and Charlton (1971). Symbols show position of experimental channels. (From Schumm and Khan, 1973.)

OBJECTIVE - This analysis will extend a study conducted by Stubblefield (22) in 1986 which compared three methods for predicting whether a natural alluvial channel should braid or meander. Straight channels are not considered because of their relative scarcity in nature. Nine methods are subsequently discussed and applied, where appropriate, to predict channel pattern. Some of the methods are very similar, some require modification of the authors' original work. The means for comparison among the methods will be the use of a single threshold (for each method) to separate braided streams from meandering ones.

DATA AND PARAMETERS

The data set (Appendix 3) consists of 93 natural streams and rivers, and 8 laboratory flume tests. 56 of the rivers were selected by Stubblefield (22) from an extensive catalogue of river data published by Church and Rood (7) in 1983. In their catalogue the authors included only data which was obtained by acceptable measurement techniques. Stubblefield's data includes North American and Canadian rivers of both sand and gravel sizes.

Of the remaining natural rivers, 35 are taken from Chitale (6), 1970, who published a set of river data including mainly India rivers and streams. Chitale's data include gravel-bed, sand-bed, and a few silt-bed channels. Two New Mexico rivers are also included, taken from Rundquist (19).

The flume data were reported by Schumm and Khan (20); bed material was composed of 0.7mm sand.

Necessary data for the present analysis include channel pattern, slope, discharge, width, depth, and D_{50}. Use of these parameters and other necessary (computed) values are discussed below:

CHANNEL PATTERN - the data herein are described as either braided or meandering. Included are 26 braided streams and 75 meandering ones.

BRAIDING - the classic definition of braiding includes rivers which are divided into separate channels by bars at low flows, but during high flows these bars are submerged (7); this definition was espoused by Lane and by Ferguson. A broader definition includes any multichannel stream having two or more separate channels. A stream may be "anastomosed", that is, it may have permanent islands which separate the channels; this definition was accepted by Leopold and Wolman. The data in this report are classified according to the broad interpretation of braiding (however, Church and Rood note 6 of the rivers as anastomosed; Chitale makes no distinction among his data).

SINUOSITY - sinuosity has been defined by Leopold-Wolman (16) as the ratio of thalweg length to valley length. Meandering is often defined by sinuosity greater than 1.5. In their analyses, some of the authors (Osterkamp, Bray) shifted their curves to account for varying sinuosity; increased sinuosity pushes a river downward on the Q-S plot. The data examined herein contains little information regarding sinuosity and the parameter is not evaluated.

SLOPE - channel slope (for uniform flow) is equal to the water surface slope and to the slope of the energy grade line.

DISCHARGE - investigators often disagree as to the definition of the dominant (channel-forming) discharge. Lane used mean annual discharge, Leopold-Wolman used bankfull discharge (at which the flow just reaches the top of the exposed bank), and Bray used the 2-year flood. The present data set includes both mean annual discharges and bankfull discharges; all methods are applied indiscriminately to the data.

WIDTH - the width of a wide channel is approximately equal to its top width.

DEPTH - the depth of a wide channel is approximately equal to its hydraulic radius, R. Depth is used as such in all calculations.

SEDIMENT SIZE, D_{50} - median grain diameter of bed material (mm). All streams and rivers considered are alluvial in nature; included are 59* sand-bed channels - $D_{50} < 2.0$mm, 41 gravel-bed channels - 2.0mm $< D_{50} <$ 64mm, and one fine silt-bed channel - D_{50} = 0.01mm. (23)

FROUDE NUMBER - dimensionless ratio of inertial force to gravitational force, defined here as:

$$F = V/\sqrt{gD} \quad \text{. . . . .} \quad (1)$$

where

F = Froude number

V = velocity (V = Q/Area)

g = acceleration due to gravity

D = depth of channel

* Note that 4 coarse silt-bed channels were included, upon inspection of the graphs, among the sand-bed streams.

BED SHEAR STRESS - the mean bed shear stress for a wide channel may be written:

$$\tau = \gamma DS \quad . . . . (2)$$

SHIELD'S COEFFICIENT - defines the beginning of motion of bed material in terms of shear stress and sediment size D_{50}:

$$\theta = \tau/(s-1)\, D_{50} . . . . (3)$$

METHODS

Several researchers have attempted to define the threshold between braiding and meandering, some using empirical equations and others a theoretical approach. The empirical relations, which are based upon log-log plots of discharge vs. channel slope, may be broadly classified into two groups. One of these, of the form $S \sim Q^{-\frac{1}{4}}$, was first reported by Lane (14) in 1957 for use with sand-bed streams. Other investigators, including Osterkamp (17) and Begin (2), have also used this relation. The second group of empirical equations, of the form $S \sim Q^{-\frac{1}{2}}$, was suggested by Leopold and Wolman (16) in 1957 based upon (mainly) gravel-bed river data. Other researchers have since used this form to classify gravel-bed rivers, proposing different constant of proportionality.

Two theoretical approaches are also examined. One, developed by Parker (18), is based upon a two-dimensional river model; the other was developed by Fredsoe (10) who used hydrodynamic stability analysis to predict the braiding/meandering threshold.

EMPIRICAL EQUATIONS OF THE FORM $S \sim Q^{-\frac{1}{4}}$

METHOD BY LANE - In 1957 Lane (14) studied the characteristics of braiding and meandering streams. He identified 8 variables which have a deterministic effect on natural channels and chose four of these - slope, discharge, bed-bank material, and geomorphic form - for his analysis. Recognizing the difficulty of relating all four variables with a single solution, Lane eliminated sediment size and geomorphic form by selecting only sand-bed streams having one pattern (braided or meandering). Hence Lane did not derive a single threshold, but a curve to define braiding and another to define meandering.

For meandering channel patterns found in the lower Mississippi River Lane concluded, based on regression analysis, that a relation between slope and discharge (plotted on a log-log scale) was defined by:

$$S = 0.0017\, Q^{-0.25} . . . . (4)$$

where
S = slope in ft./1000 ft.
Q = mean annual discharge in c.f.s.

Streams plotting below this curve should be meandering.

For braided streams, Lane used a similar analysis to determine that the relation between slope and discharge was:

$$S = 0.01\, Q^{-0.25} . . . . (5)$$

Streams plotting above this curve should be braided.

Between these two curves is a zone of transition from braiding to meandering. Lane recognized that scatter in the data was inevitable and attributed this to variable sediment size, and to variable sinuosity among meandering streams. In his analysis, Lane adopted the classic definition of braiding.

Equations 1 and 2 are shown in Fig. 2; these were generalized by Lane in the form:

$$S = KQ^{-0.25} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where K is a constant.

METHOD BY OSTERKAMP - Osterkamp (17) performed an analysis similar to that of Lane on sand-bed Kansas streams in 1978. Using simple structural analysis (rather than linear regression) he concluded that Lane's equation (6) could be used to separate braided streams from meandering ones. Osterkamp also recognized the importance of sediment size and sinuosity; he proposed various values of K to define the meandering curve depending on the degree of sinuosity of the channel.

Because of the similarity between Osterkamp's and Lane's methods, and since the data examined in this paper does not include quantitative descriptions of sinuosity, Osterkamp's equations are not evaluated.

METHOD BY BEGIN - In 1980, Begin (2) defined Lane's constant K in terms of relative shear stress. For wide channels, the mean bed shear stress is:

$$\tau = \gamma DS \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where

τ = mean shear stress

γ = specific weight of water

D = mean water depth

S = slope of energy grade line

Leopold and Maddock (15) related stream depth to mean annual discharge, Q, with the following equation:

$$D = cQ^{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where c and f are constants

Equations 4 and 5 may be combined and written in the form:

$$S = (\tau_o/\gamma c)Q^{-f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Begin then obtained an average "standard" value of the term $(\tau_o/\gamma c)$ by linearly regressing his data (Q vs. S plotted on a log-log scale); he obtained an equation similar to that of Lane above and proposed that, on average:

$$(\tau_o/\gamma c)_{ave} = K \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

where K is Lane's constant.

Assuming that γ and C remain constant and combining the above equations Begin wrote:

$$(\tau_o/\tau_{ave}) = \frac{Q^{m}S}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

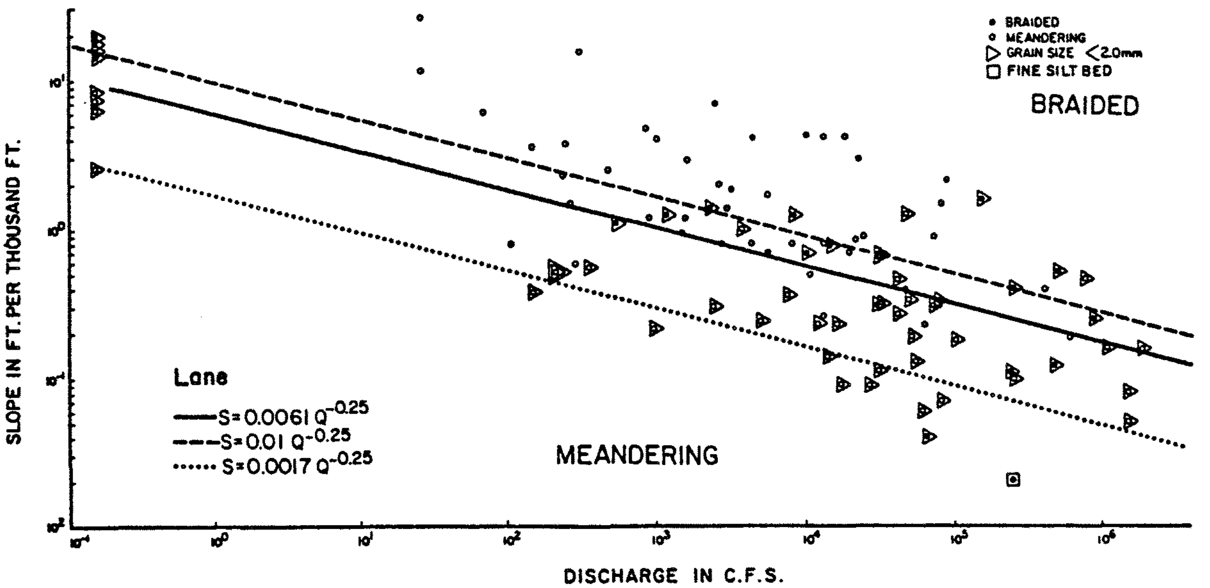

Figure 2

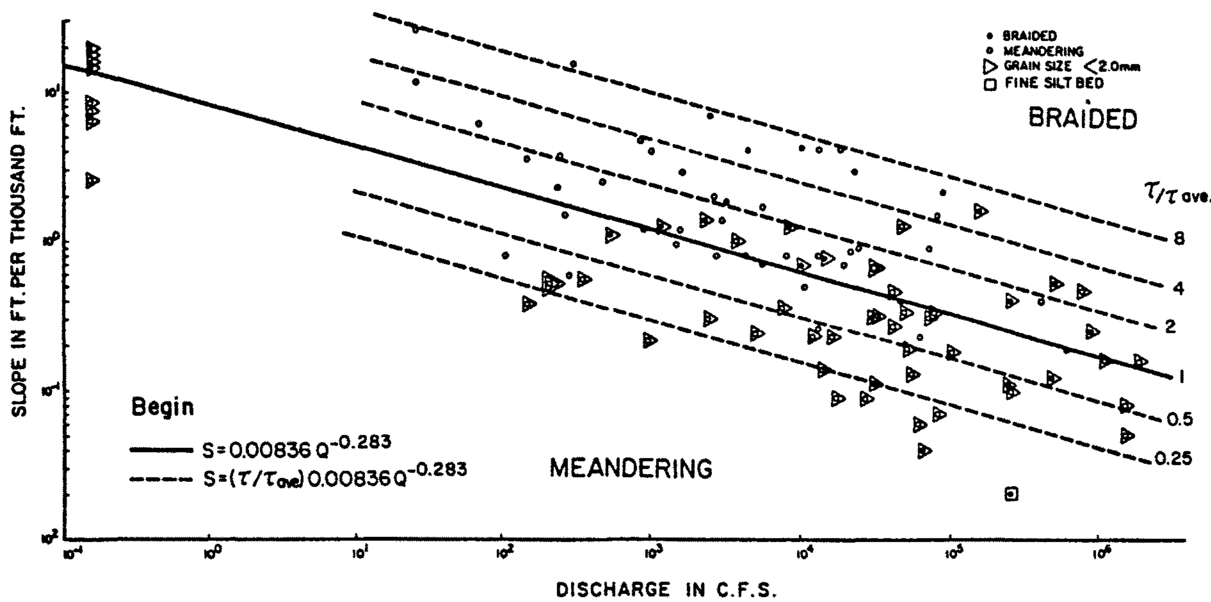

Figure 3

where m is the coefficient computed by the regression (m = 0.25 for Lane's equations).

Begin produced a family of curves from equation (10) for various values of τ/τ_{ave} (relative shear stress), and assigned a value of $\tau/\tau_{ave} = 1$ to the data regression curve. He concluded that the constant K was related to bed shear stress and that braided streams generally exhibit greater values of relative shear than do meandering ones. Comparable curves (for some values of τ/τ_{ave}) developed for the present data set are shown in Fig. 3.

EMPIRICAL EQUATIONS OF THE FORM $S \sim Q^{-\frac{1}{4}}$

METHOD BY LEOPOLD AND WOLMAN - In 1957 these researchers (16) identified at least 8 hydraulic parameters which affect natural rivers and streams. Using a set of data composed mainly of gravel-bed channels, bankfull discharge was plotted against channel slope on a log-log scale. The data were characterized morphologically as straight, meandering or braided. A threshold was fit by eye to separate meandering and braiding, and was defined by the equation:

$$S = 0.06\ Q^{-0.44} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where
S = slope in ft./1000 ft.
Q = discharge in c.f.s.

Eq. 10 is plotted in Fig. 4 (Braiding above, meandering below, this curve).

Leopold and Wolman classified streams as meandering if the channel sinuosity was greater than 1.5. Braided streams were defined as any multichannel stream, i.e. any stream consisting of two or more channels. These researchers recognized that sediment size was an important factor, but did not try to account for its effect. They also noted the importance of width-to-depth ratio, but that stream width is mainly dependent upon discharge.

METHOD BY HENDERSON - Henderson (13) attempted to quantify the effect of sediment size on channel pattern, and in 1961 redefined the equation of Leopold-Wolman by incorporating the sediment size D_{50} as an additional parameter. He related the size of the bed material (in Leopold-Wolman's data) to the distance of each data point above or below the Leopold-Wolman curve (Eq. 10) and proposed the following:

$$S = 0.63\ D_{50}^{1.14} Q^{-0.44} \quad \ldots\ldots\ldots\ldots \quad (12)$$

where
S = channel slope in ft./1000 ft.
D_{50} = median sediment diameter in ft.
Q = bankfull discharge in c.f.s.

Henderson's equation is based on Leopold-Wolman's data and thus pertains mainly to gravel-bed rivers; curves for different sediment sizes (D_{50}) are shown in Fig. 5.

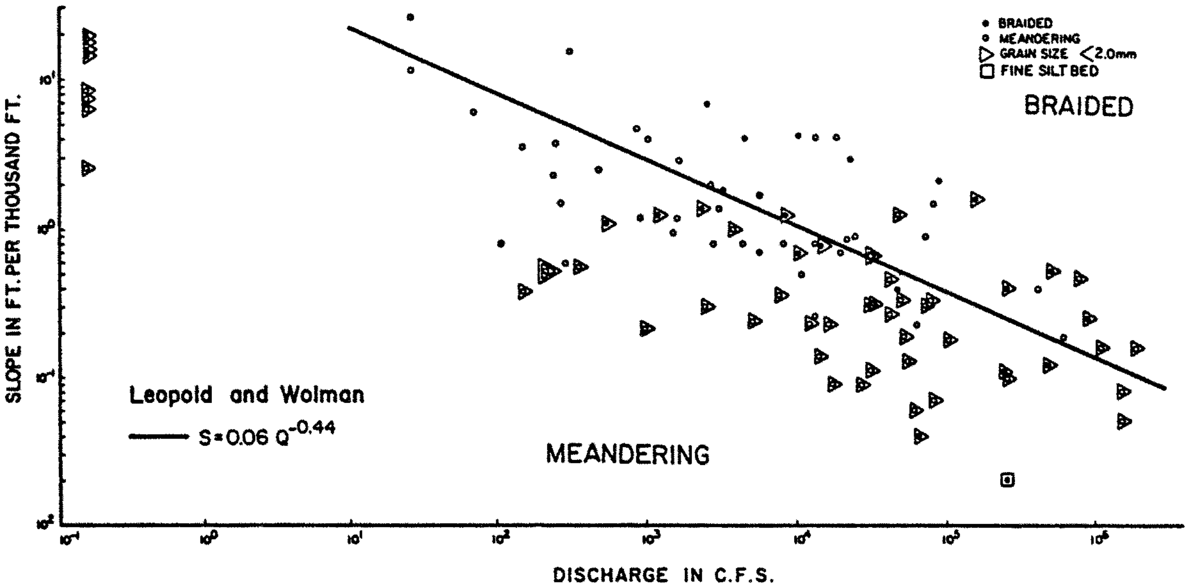

Figure 4

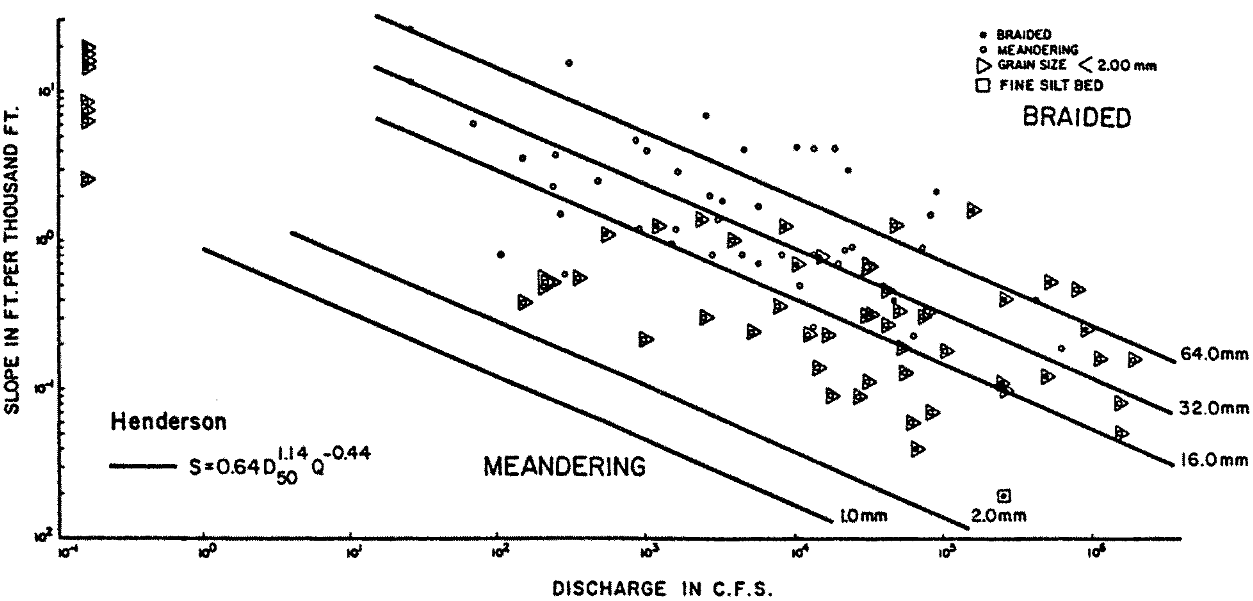

Figure 5

METHOD BY BRAY - Bray (3) based his analysis of channel pattern on the relation $S \sim Q^{-0.44}$. Restricting his classification to gravel-bed rivers, he proposed a threshold equation for braiding-meandering in 1982:

$$S = 0.16\ Q_2^{-0.44} \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

where

S = slope in ft./1000 ft.

Q_2 = discharge of the 2-year flood, in c.f.s.

Bray used the 2-year flood discharge because it produced the highest coefficient of determination (r^2) and the lease standard error (S.E.) during regression analysis. Bray's definition of braiding included any mulitchannel stream. He, too, suggested that the constant 0.16 would change for different degrees of sinuosity of meandering channels. Equation 13 is plotted in Fig. 6.

METHOD BY FERGUSON - In 1984 Ferguson (9) reevaluated the methods of Leopold-Wolman, Henderson, and Bray. With a data set composed of mostly braided and near-braided rivers, Ferguson developed a "best-fit discriminant function" which included a grain-size parameter.

Ferguson emphasized the importance of sediment size with regard to channel morphology. Not only does grain size affect the braiding/meandering threshold for sand-bed channels or for gravel-bed channels, but the transition from meandering to braiding in gravel-bed rivers should require much greater power and shear stress than this transitions in sand-bed rivers. Ferguson suggested that Leopold-Wolman's threshold was too high for sand-bed rivers but too low for gravel-bed rivers. His equation, which included a D_{50} parameter and which applies only to gravel-bed rivers is:

$$S = 0.042\ Q^{-0.49} D_{50}^{\ 0.09} \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

where

S = slope in m/m

Q = bankfull discharge in m^3/s

D_{50} = median sediment diameter in mm

Ferguson's equation applies only to stream channels with D50   2.0mm. Noting that grain size was not statistically significant in this equation, Ferguson proposed an average threshold:

$$S = 0.056\ Q^{-0.5} \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

This equation is plotted in Fig. 7. *

Unlike Leopold-Wolman, Henderson, or Bray, Ferguson limited his analysis to classically braided or near-braided channels.

Ferguson also examined Parker's (1976) mathematical analysis, which relates slope and Froude number to depth and width. He proposed a more useful relation for design purposes (since depth, velocity and width are not generally known beforehand) in the form:

$$S = 0.00491\ Q^{-0.21}\ D_{50}^{\ 0.52} \quad \ldots\ldots\ldots\ldots \quad (16)$$

This equation, referred to by Ferguson as "deductive threshold", is plotted in Fig. 7 along with Eq. 15. *

* Equations converted to English units in figure.

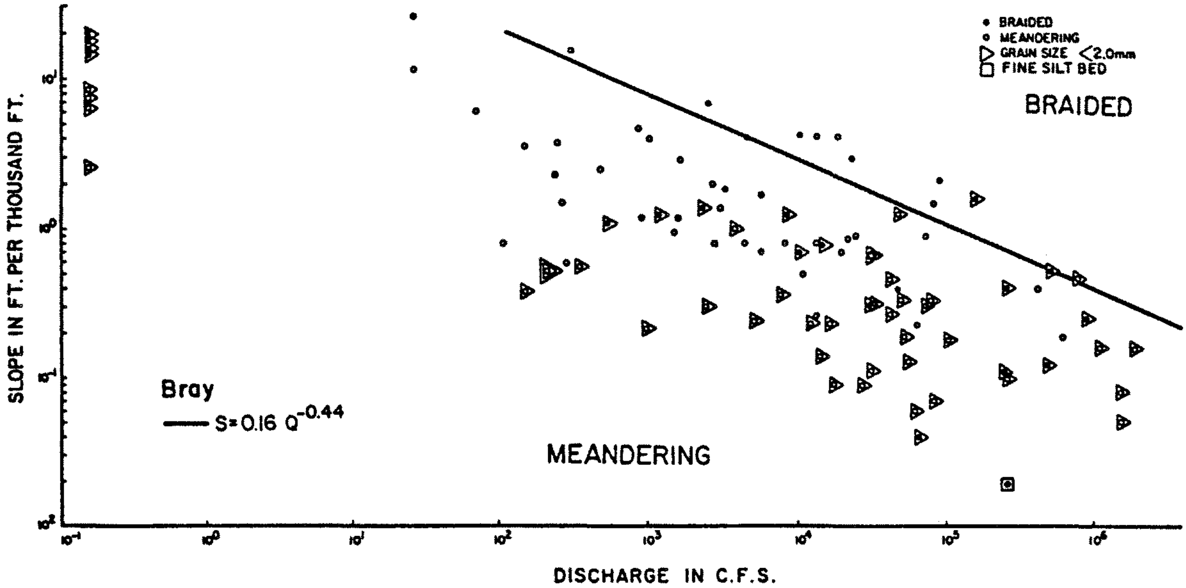

Figure 6

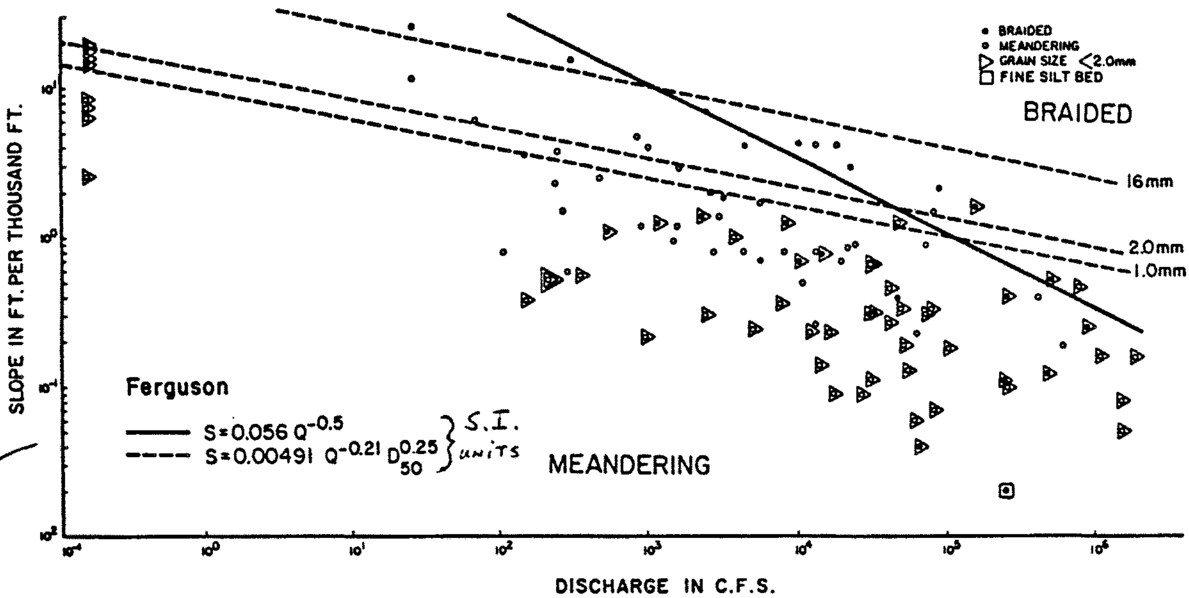

Figure 7

Ferguson's equations in English units:

——— $S = 0.333\,Q^{-0.5}$

- - - - $S = 0.0077\,Q^{-0.21}\,D_{50}^{0.25}$

THEORETICAL INVESTIGATIONS

METHOD BY PARKER - In 1976 Parker (18) used a theoretical two-dimensional river model developed by others to define a threshold between meandering and braiding. Ignoring the effects of secondary (transverse) flow in his two-dimensional approach, Parker developed equations based upon examination of submerged bedforms in a shallow, rectangular channel of constant slope with constant discharge.

Parker concluded that a transition between meandering and braiding occurs when:

$$S/F \sim D/B \qquad (17)$$

where
S = channel slope
F = Froude number $(V/\sqrt{gD})$
D = depth channel
B = width of channel

Equation 16 is plotted in Fig. 8.

When $S/F \ll D/B$ meandering should occur, and when $S/F \gg D/B$ braiding should occur.

These relations support observations by Friedkin (11) and others that meandering streams are generally characterized by gentle slopes and narrow channels, while braided streams generally exhibit steeper slopes and wider channels.

METHOD BY FREDSOE - Fredsoe (10) developed a hydrodynamic stability analysis to predict whether a channel would braid, meander, or remain straight. In 1978 the author presented a two-dimensional mathematical model of an alluvial channel with nonerodible banks. Given an originally straight channel, the model was used to examine the instability of the channel and its tendency to change form. Fredsoe produced threshold curves defining straight, meandering, and braided for flow over a dune-covered bed, for flow over a plane bed (Froude > 0.8), and for flow neglecting suspended sediments.

The stability analysis yielded plots of $\pi\cdot$depth/width vs. Shield's coefficient θ. Shield's coefficient may be defined as follows:

$$\theta = \tau/(s-1) \quad D_{50} \qquad (3)$$

where
θ = dimensionless Shield's coefficient
τ = mean shear stress at the bed
S = specific gravity of bed material
γ = specific weight of water
D_{50} = mean sediment diameter

In constructing his threshold curves, Fredsoe assumed that S = 2.65 (average specific gravity of sands), that depth/sediment size = 1000, and that the drag coefficient $C_D = 7$. He noted, however, that the analysis is very insensitive to changes in any of these values.

Use of Shield's coefficient in the analysis provides for consideration of bed material size, bed shear stress, and channel slope ($\tau = \gamma DS$). The channel width is often considered to be related to discharge (15).

Threshold curves for flow over a dune-covered bed and for flow over a

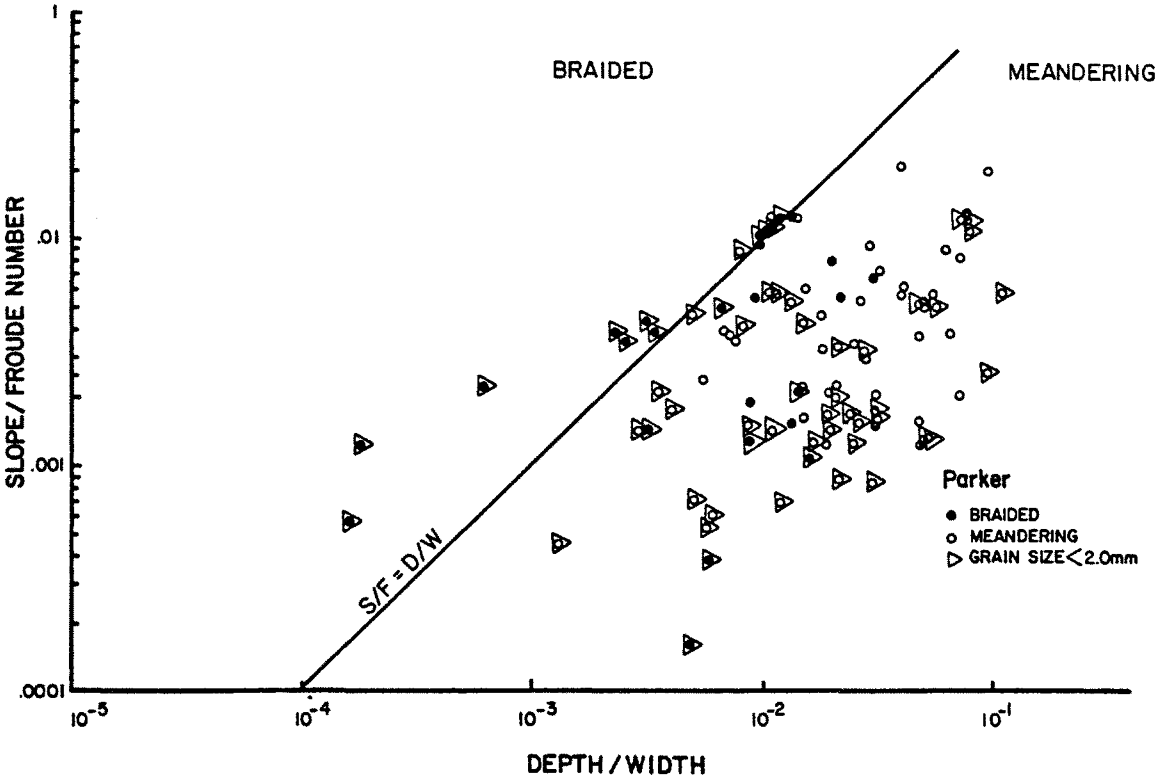

Figure 8

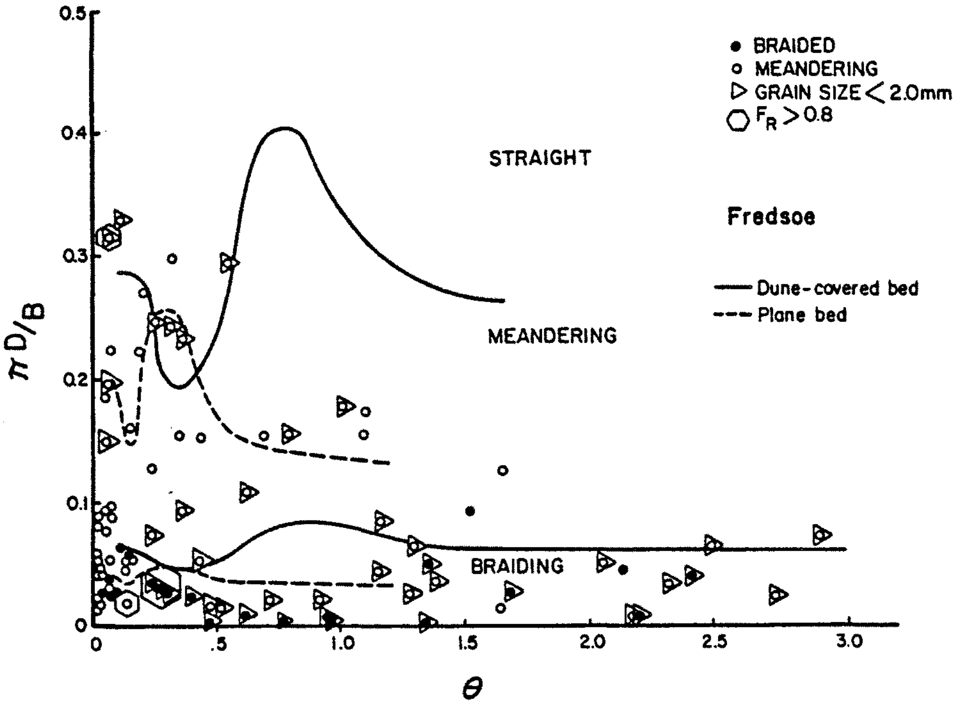

Figure 9

plane bed are shown in Fig. 9.

METHOD OF COMPARISON

The most effective comparison among the methods requires a single threshold curve to separate meandering streams from braided ones. This threshold exists for all methods evaluated except that by Lane (14). Lane proposed one curve to define meandering and another to define braiding - a single threshold must be drawn between the two. Stubblefield (22) experimented with a number of curves, including one halfway between those of Lane. Such a "best fit" is obviously dependent upon the given data, and for the present study the following equation appears reasonable:

$$S = 0.0061\, Q^{-0.25} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

where Lane's K = 0.0061.

Equation 18 is shown in Fig. 2.

Ferguson's method was reported in S.I. units, as were Stubblefield's data in 1986. All data herein are reported in English units, and Ferguson's curves for gravel-bed rivers (Eq. 15,16) has been transformed into English units.

Among all of the empirical Q vs. S thresholds, braided streams should plot above the curve (higher channel slope), while meandering streams should plot below the curve. Parker's criteria (Eq. 17) indicates that braided streams should lie to the left of the curve, while meandering ones should plot on the right.

Fredsoe's stability curves are shown in Fig. 9, indicating that braiding should occur below the lower solid line, and that meandering should occur above it. Note that 6 of the rivers show Froude > 0.8 (plane-bed curves are shown with dotted lines), but the classification of these points is unchanged from that predicted by the dune-bed curves.

The following comparison have been made:

1) All 101 streams have been classified collectively by the methods of Lane, Leopold-Wolman, Parker and Fredsoe. The results of this comparison are shown in Table 1. Methods by Bray and Ferguson have not been included here since they were expressly developed for gravel-bed rivers (visual evaluation indicates poor results among sand-bed rivers).

As previously noted, Osterkamp's curves are not evaluated because of the lack of sinuosity information. Begin's method merely assigns physical meaning to Lane's constant K, and so no numerical tabulation of his results is presented. The linear regression was performed to test his conclusions qualitatively, and the family of curves corresponding to variation of (τ/τ_{ave}) is shown in Fig. 3 ($\tau/\tau_{ave} = 1$ corresponds to the original regression equation). Henderson's method is not evaluated for the collective data set, nor for the sand-bed population, since the 1.0mm curve (Fig. 5) misclassifies almost all of the meandering streams of that sediment size range.

2) 59 of the streams may be considered sand-bed channels, such that $D_{50} < 2.0$mm (excluding the single fine silt-bed river). The results of the methods by Lane and Leopold-Wolman for sand-bed channels are shown in Table 2.

Neither Parker's nor Fredsoe's method was tested with the restricted data since no size limitations were noted by the authors.

3) Results of the methods of Leopold-Wolman, Henderson, Bray, and Ferguson* for the 41 gravel-bed rivers is shown in Table 3. Lane's method was not tabulated in light of visual inspection of the data.

RESULTS

Regression analysis of the entire set of data indicates a best-fit curve of the form:

$$S = KQ^{-f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

where
$K = 0.00836$
$f = 0.2833$

The coefficient 'f' has been found by Graf (12) to lie within the range $0.22 < f < 0.45$; the results here corroborate Lane's use of $f = 0.25$, based on his own regression analysis. These results also lend some credibility to Begin's argument that 'K' may be related to bed shear stress. Qualitative analysis of Fig. 3 indicates that braided streams tend to have higher values of relative shear (i.e. plot higher on the Q vs. S graph) than meandering ones, but that no marked threshold exists. A weakness in Begin's approach lies with his use of average shear; the predictive measure itself will vary from one data set to another, a quality which prevents unbiased analysis.

Based upon the figures in Table 1, several observations are notable. Scatter of data on the Q vs. S diagrams precludes use of any single straight-line threshold to successfully classify every river; however, the enlarged (over that of Stubblefield) data set provides better results among empirical methods than those reported by Stubblefield (22). Lane's method shows an overall accuracy of 61% using Eq. 19; accuracies computed separately for braided and meandering streams are both above 50%. Leopold-Wolman's method shows similar results - an overall accuracy of 65%. However, accuracy for braiding is only 42% using Leopold-Wolman; the fact that there are fewer braided streams (26) than meandering ones (75) is misleading with regard to the overall percentages.

Parker's method shows an overall accuracy of 83%; it correctly predicts 97% of the meandering streams, but only 42% of the braided ones. Again the overall figure is somewhat misleading. Fredsoe's method yields an overall accuracy of 60%. Among the braided streams it predicts 92% correctly, but only 49% of the meandering ones. Note that 6 of the misclassified meandering streams lie above the straight/meandering threshold. In view of the arbitrary distinction between meandering and straight channels, and since these 6 streams are far removed from the braiding threshold, it may be argued that these streams were correctly classifies - as non-braided. This would improve Fredsoe's overall accuracy to 66%.

Both Parker and Fredsoe used the depth-width ratio as part of their prediction criteria. A river's new width and depth <u>after</u> some

* tabulation of Ferguson's best-fit discriminant function (Eq. 15) only; deductive threshold (Eq. 16) was not pursued numerically, based upon visual inspection.

TABLE 1.- PREDICTION ACCURACY - ALL DATA

OBSERVED		LANE (Eq. 19)		LEOPOLD-WOLMAN		PARKER		FREDSOE	
Pattern	# Observed	# Correctly Predicted	Accuracy	# Correctly Predicted	Accuracy	# Correctly Predicted	Accuracy	# Correctly Predicted	Accuracy
BRAIDED	26	21	81%	11	42%	11	42%	24	92%
MEANDERING	75	41	55%	55	73%	73	97%	*37	49%
TOTALS	101	62	61%	66	65%	84	83%	61	60%

* Neglecting the "straight" classification of 6 meandering streams, meandering accuracy is improved to 57%; overall accuracy is improved to 66%.

TABLE 2.- PREDICTION ACCURACY - SAND-BED CHANNELS ($D_{50} < 2.0$mm) **

OBSERVED		LANE (Eq. 19)		LEOPOLD-WOLMAN	
Pattern	# Observed	# Correctly Oredicted	Accuracy	# Correctly Predicted	Accuracy
BRAIDED	17	13 *	76%	5	28%
MEANDERING	42	35	83%	35	87%
TOTALS	59	48	81%	40	68%

* Restriction to classic braiding improves Lane's braiding accuracy to 93%; overall accuracy is improved to 86%.

** Excluding Tonle Sap R. (fine silt-size sediment)

TABLE 3.- PREDICTION ACCURACY - GRAVEL - BED CHANNELS ($D_{50} > 2.0$mm)

OBSERVED		LEOPOLD-WOLMAN		HENDERSON		BRAY		FERGUSON	
Pattern	# Observed	# Correctly Predicted	Accuracy	# Correctly Predicted	Accuracy	# Correctly Predicted	Accuracy	# Correctly Predicted	Accuracy
BRAIDED	8	6	75%	8	100%	5	62.5%	* 4	50%
MEANDERING	33	19	58%	9	27%	29	88%	30	91%
TOTALS	41	25	61%	17	41%	34	83%	34	83%

* Restriction to classic braiding and near-braiding improves Ferguson's braiding accuracy to 80%, and overall accuracy to 89%.

(man-made) change are generally not known before that change occurs; hence the predictive capability of these methods is limited by the possibility that the criteria itself may not be defined. It should further be noted that Parker's method includes no sediment size parameter.

When the data are sorted according to grain size, Lane's results are markedly improved (Table 2). Among 59 sand-bed streams, Lane's method shows an overall accuracy of 81%; the meandering streams are predicted more effectively than the braided ones. Leopold-Wolman's method yields an overall accuracy of 68%, but correctly predicts only 28% of the braided streams (noting that Leopold-Wolman developed their threshold based upon gravel-bed rivers).

Exclusion of the fine silt-bed river appears justified based upon its location on the Q vs. S plot. It lies below all of the other sand-bed streams and probably requires some other method of determination.

Among the 41 gravel-bed streams (Table 3), the Leopold-Wolman curve correctly predicts 61%, showing better success with braided streams (75%) than with meandering ones (58%). Henderson's method successfully classifies 100% of the braided streams, but only 27% of the meandering ones; overall accuracy is 41%. Bray's curve properly classifies 62.5% of the braided streams and 88% of the meandering ones, for an overall accuracy of 83%. Finally, Ferguson's best-fit discriminant function correctly predicts 50% of the braided streams and 91% of the meandering ones; overall accuracy is 83%.

Among the braided channels taken from Stubblefield's data set, there are 6 rivers classified as anastomosed; this form of braiding was excluded in the analyses of both Lane and Ferguson. 3 of these 6 have $D_{50} < 2.0$mm, and when they are excluded from the sand-bed tabulation, Lane's braiding accuracy increases to 93%, with an overall accuracy of 86%. Likewise, exclusion of the 3 anastomosed gravel-bed streams improves Ferguson's statistics to 80% correct among braided streams and to 89% overall. Anastomosed channels seem, generally, to plot lower on the Q vs. S diagram than do those defined (by Church and Rood) as classically braided.

CONCLUSIONS

There are many interrelated variables which affect river morphology; discharge and slope are obviously important, but this analysis show that they are often inadequate as predictive measures.

The need to include bed material size in any threshold analysis is apparent. All Q-S methods are greatly improved by considering only sediment sizes for which the methods were developed, even when this consideration is qualitative (i.e. separation of sand-bed from gravel-bed channels). Further, knowledge of relative sinuosity and of braiding type will likely improve predictive results. This analysis demonstrates the need to utilize all available river data and to choose a method that was developed for similar conditions. Certainly the results obtained herein fall short of the accuracy desired for design purposes, but the effect of careful variable interpretation is clear.

For sand-bed rivers, Lane's method should be the preferred choice; the constant 'K' will likely require adjustment for a particular situation, based upon degree of sinuosity (Osterkamp) or upon braiding type. Fredsoe's method may also be considered, provided that river width and depth can be anticipated and that the depth-width ratio lies within the range of the model's assumptions ($B > 50D$ will always plot as braided on

Fredsoe's diagram).

Ferguson's method (Eq. 14,15) appears to be the most reliable for gravel-bed rivers; his use of a grain-size parameter allows for adjustments with different situations. This method should also be combined with sinuosity and braid-type data. Again, Fredsoe's method may be evaluated simultaneously if the aforementioned provisions for its use are met.

Carson (5) suggested in 1984 that use of mean or bankfull discharge may be inadequate for such investigations, and that perhaps valley slope should be included because of its effect on channel slope. Like Ferguson, he recognized the role of bed material; active gravel rivers must plot higher on the Q-S diagram than sand streams - regardless of pattern - because of greater power requirements for bed movement. The present study supports this claim - examination of the Q vs. S data indicates that, indeed, the gravel rivers tend to plot higher than the sand rivers. Further agreement is provided by the one silt-bed braided river, which plots much lower than all of the sand channels.

More accurate predictive measures of river morphology are needed. Improvements can come only through more detailed analyses of the many variables which determine channel form.

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APPENDIX II. - NOTATION

The following symbols are used:

- B = width of channel
- Br = braided channel
- c = constant
- D = depth of channel
- D_{50} = grain diameter of bed material
- f = constant
- F = Froude Number
- g = acceleration of gravity
- k = Lane's constant
- m = constant
- M = meandering
- Q = discharge
- s = specific gravity of sediment
- S = longitudinal channel slope
- V = average velocity
- γ = specific weight of water
- π = constant (3.1416)
- τ = average bed shear stress
- θ = Shield's coefficient

Appendix III. - Data

River Description	Pattern: B= Braided, M= Meandering	Discharge cfs	Slope Sx1000	Width ft	Depth ft	Velocity fps	Grain Size d50 mm	Froude Number
Towanda Cr. Nr Monroeton, PA	M	12995.80	4.20	154.20	7.91	10.66	45.00	0.668
Green R. at Munfordville, KY	M	20517.82	0.85	324.80	23.00	2.76	21.00	0.101
Elk R. Nr Prospect, TN	M	23590.20	0.80	226.38	19.52	5.38	14.00	0.215
Blacks Fork Nr Little America, WY	M	7981.11	0.80	190.29	5.84	7.19	18.00	0.524
White R. Nr Soldier Summit, UT	M	430.84	2.50	45.93	2.30	4.27	2.70	0.496
Sweetwater R. Nr Alcova, WY	M	2800.45	0.80	95.14	4.69	6.27	1.00	0.510
Cocolamus Ck. Nr Millerstown, PA	M	1599.75	2.90	95.14	5.61	3.02	35.00	0.225
Rio Grande at Cochiti, NM	* Br	8087.06	1.27	295.28	4.10	6.69	0.40	0.583
Valley Cr., Idaho, Nr mouth	M	999.41	4.00	78.74	2.49	4.92	30.00	0.549
N. Platte R. Nr Douglas, WY	M	1493.81	0.95	311.68	1.71	2.99	48.80	0.403
N. Platte R., N. Platte, NE	Br	2298.92	1.40	541.34	1.71	2.40	0.20	0.323
N. Platte R. Nr Sutherland, NE	Br	536.78	1.10	341.21	0.92	1.71	0.20	0.314
N. Platte R. Nr Lisco, NE	Br	1137.13	1.25	400.26	1.35	2.10	0.50	0.319
Peace R. at Ft. Vermillion, Alberta Can.	* Br	75220.24	0.04	2021.00	15.78	2.40	0.30	0.106
Peace R. at Peace Pt., Alberta Can.	M	80446.81	0.07	2109.58	14.17	2.69	0.20	0.126
Beaver R. at Gold Lk. Reserve, Alberta Can.	M	953.50	0.21	150.92	3.67	1.77	0.60	0.163
Red Deer R. Nr Empress & Nr Blindloss, Alberta Can.	M	2436.71	0.30	561.02	2.79	1.57	0.30	0.166
S. Saskatchewan R. at Hwy 41, Alberta Can.	M	7522.02	0.36	659.45	5.77	1.97	0.30	0.144
E. Prairie R. Nr Enilda, Alberta Can.	M	211.89	0.50	72.18	1.38	2.30	0.30	0.345
Medicine R. Nr Eckville, Alberta Can.	M	211.89	0.51	55.77	1.97	1.97	0.30	0.247
Atabasca R. at Embarras Airport, Alberta Can.	M	27086.35	0.09	1259.84	8.69	2.49	0.20	0.149
Red Deer R. Nr Sundre, Alberta Can.	M	847.55	4.90	167.32	1.77	2.89	28.00	0.382
Sheep R. at Otocks & Nr Aldersyde, Alberta Can.	M	247.20	3.80	91.86	1.28	1.97	31.00	0.307
Bow R. at Calgary, Alberta Can.	* Br	3284.26	1.80	331.36	3.08	3.22	40.00	0.323
Bow R. below Bassano Dam , Alberta Can.	M	4379.02	0.81	469.16	3.67	2.49	32.00	0.229
Clearwater R. Nr. Rocky Mtn. House, Alberta Can.	M	918.18	1.20	177.17	2.69	1.87	27.00	0.201
Prairie Ck. Nr Rocky Mtn. House, Alberta Can.	M	141.26	3.60	72.18	1.28	1.77	43.00	0.276
Willow Ck. Nr Claresholm, Alberta Can.	M	105.94	0.80	62.34	1.67	1.08	23.00	0.148
Milk R. at Milk River, Alberta Can.	M	252.52	0.59	88.55	1.67	1.97	19.00	0.268
Oldman R. Nr Monarch, Alberta Can.	M	1589.16	1.20	288.71	2.10	2.59	30.00	0.315
Gakona R. at Gakona, AK	* Br	4484.96	4.10	200.13	3.84	5.81	21.00	0.522
Gulkana R. at Gulkana, AK	M	18010.48	4.20	498.69	3.31	10.99	21.00	1.064
Tazlina R. Nr Glenallen, AK	Br	22248.24	3.10	354.33	7.55	8.53	40.00	0.547
Squirrel Ck. Nr Tonsina, AK	M	300.17	15.60	39.37	1.61	5.18	18.00	0.721
McLaren Ck. Nr Paxson, AK	Br	5685.66	0.70	295.28	3.90	5.09	8.50	0.454
Yukon R. at Rampart, AK	M	407990.34	0.40	1948.82	29.00	7.32	13.40	0.239
Salcha R. Nr Salchaset, AK	M	12995.80	0.60	324.80	9.15	4.49	18.80	0.262
Chena R. Nr Two Rivers, AK	M	5756.29	1.70	223.10	4.04	6.00	19.00	0.527
Little Chena R. at Fairbanks, AK	M	2577.97	2.00	121.39	11.52	1.94	13.50	0.101
Wheaton R. Nr Carcross, Yukon Terr. Can.	M	264.86	1.50	82.02	2.40	1.41	27.00	0.161

Appendix III (Cont)

River Description	Pattern: B= Braided, M= Meandering	Discharge cfs	Slope Sx1000	Width ft	Depth ft	Velocity fps	Grain Size d50 mm	Froude Number
Yukon R. above Frank Ck., Yukon Terr. Can.	M	10453.14	0.50	288.71	8.99	4.07	14.00	0.239
Yukon R. above White R., Yukon Terr. Can.	* Br	44920.26	0.40	1151.57	10.17	3.84	13.00	0.212
Stewart R. at Mayo, Yukon Terr. Can.	M	12995.60	0.26	662.73	9.48	2.07	22.00	0.118
Lubbock R. Nr Atlin, B.C., Yukon Terr. Can.	M	151.85	0.38	32.81	3.08	1.51	0.40	0.151
McClintock R. Nr Whitehorse, Yukon Terr. Can.	M	353.15	0.53	72.18	4.10	1.21	0.40	0.106
Wye R. Bredwardine Bridge, So. Britain	M	19423.07	0.70	193.57	13.75	7.22	28.00	0.343
Dean R. at Adlington Hall	M	24.72	11.50	26.25	0.66	1.54	23.80	0.336
N. Saskatchewan R. at Drayton Valley, Alberta Can.	M	80164.29	1.50	800.52	14.30	6.89	30.00	0.321
Niger R. at Jebba, Nigeria	M	52972.00	0.13	1689.63	9.84	3.28	0.50	0.184
Red Deer R. Nr Duchess Bridge, Alberta Can.	M	40011.52	0.27	741.47	12.34	4.36	0.30	0.219
Red Deer R. Nr Jenner Ferry, Alberta Can.	* Br	30017.47	0.30	610.24	9.74	5.05	0.40	0.285
Red Deer R. Nr Buffalo Bridge, Alberta Can.	M	30017.47	0.31	613.52	7.22	6.76	0.30	0.443
Chilliwack R., Ryder Ck. to Vedder Crossing, B.C. Can.	Br	2436.71	6.90	216.54	2.10	5.41	32.00	0.659
Little Grizzly Ck. above Hebron, CO	M	236.61	2.30	32.81	1.57	4.43	23.00	0.622
N. Platte R. Nr North Gate, CO	M	24.72	26.00	6.56	0.66	4.92	49.00	1.071
Little Muddy Ck. Nr Parshall, CO	M	67.10	6.10	16.40	1.02	3.87	24.00	0.677
Vatrak R., Kaira, Gujrath State India	M	75000	0.33	675.00	21.00	5.29	0.15	0.204
Yamuna R., at Dehli India	M	70000	0.31	1200.00	13.00	4.49	0.32	0.219
Indus R., at Hajipur Pakistan	M	250000	0.10	3225.00	16.00	4.84	0.18	0.214
Ramganga R., in U.P. State India	M	233500	0.11	1130.00	24.00	8.61	0.19	0.310
Sutlej R., in Samasatta Pakistan	M	100000	0.18	3875.00	11.00	2.35	0.17	0.125
Khipra R., at Ujjain in M.P. State India	M	61000	0.23	900.00	16.44	4.12	5.00	0.179
Tapi R., at Nanavarachha, Gujrath State India	M	750000	0.47	1986.00	39.00	9.68	0.25	0.273
Ganga R., at Mokameh, Bihar State India	M	1848000	0.16	13200.00	17.00	8.24	0.43	0.352
Purna R., Surat, Gujrath State India	M	70000	0.90	525.00	26.00	5.13	4.00	0.177
Vatrak R., at Mehmadabad, Gujrath State India	M	48000	0.34	585.00	17.80	4.61	0.15	0.193
Mohor R., at Kapadganj, Gujrath State India	M	40000	0.46	500.00	13.00	6.15	0.04	0.301
Yashwantpur R., at Rly. Bridge, A.P. State India	Br	10000	4.30	400.00	5.50	4.55	2.05	0.342
Orsong R., Nr Bhadeli, Gujrath State India	Br	155400	1.60	1900.00	12.50	6.54	0.32	0.326
Banas R., at Abu Road, Gujrath State India	Br	85000	2.17	610.00	16.10	7.70	4.77	0.319
Yamuna R., at Hamirpur, U.P. State India	M	1080000	0.16	3200.00	36.00	9.38	0.18	0.275
Bhagirathi R., in W. Bengal State India	M	59000	0.06	715.00	19.50	4.23	0.18	0.169
Tapi R., at Ukai, Gujrath State India	M	600000	0.19	900.00	50.00	13.33	1.60	0.332
Jhelum R., at Shrinagar, J. and K. State India	M	14000	0.14	335.00	17.00	2.46	0.04	0.105
Jhelum Out Fall Channel, J. and K. State India	M	17000	0.09	500.00	15.00	2.27	0.67	0.103
Salandi R., at Bidhyadharpur, Orissa State India	M	30000	0.67	500.00	10.00	6.00	0.96	0.334
Burhi Gandak R., at Muzaffarpur, Bihar State India	M	50000	0.19	700.00	15.00	4.76	0.10	0.217
Ranoli R., Rajasthan State India	Br	14000	0.78	940.00	2.92	5.10	0.11	0.526
Mutha R., at Poona, Maharashtra State India	M	30000	0.67	500.00	20.00	3.00	1.50	0.118
Saraswati R., at Sidhpur, Gujarat State India	M	45000	1.25	1050.00	8.50	5.04	0.30	0.305
Bennihalla R., railway bridge at Gadag-Hubli line, India	M	16000	0.23	450.00	6.00	5.93	0.04	0.426

Appendix III (Cont)

River Description	Pattern: B= Braided, M= Meandering	Discharge cfs	Slope Sx1000	Width ft	Depth ft	Velocity fps	Grain Size d50 mm	Froude Number
Redihalla R., railway bridge at Gadag-Hubli line, India	M	12000	0.23	460.00	5.00	5.00	0.04	0.394
Savannah R., Georgia U.S.A.	M	30000	0.11	350.00	17.00	5.04	0.80	0.216
Beaver R., Alberta Can.	M	5000	0.24	180.00	9.00	3.09	0.50	0.181
Mississippi R., Vicksburg to Angola, U.S.A.	M	1500000	0.05	4532.00	66.00	5.01	0.50	0.109
Mississippi R., Cairo to Memphis, U.S.A.	M	1500000	0.08	7034.00	55.00	3.88	0.30	0.092
Sone R., at Derhi, Bihar State India	Br	500000	0.52	14700.00	8.80	3.87	0.63	0.230
Gogra R., at Inchcape Rly. Bridge, Bihar State India	Br	475000	0.12	5800.00	13.44	6.09	0.30	0.293
Kosi R., at Bhimnagar, Bihar State India	Br	250000	0.40	20100.00	3.56	3.49	0.56	0.326
Bhramaputra R., at Dibrugarh, Assam State India	Br	875000	0.25	31000.00	5.00	5.65	0.30	0.445
Tonle Sap R., at Kompongchhnang Cambodia	** Br	250000	0.02	4600.00	20.00	2.72	0.01	0.107
Rio Grande R., Belen, N.M.	Br	10000	0.69	420.00	3.70	6.00	0.28	0.550
Rio Puerco R., Bernardo, N.M.	M	3700	1.00	150.00	3.50	7.00	0.22	0.660
	Br	0.15	15.00	5.10	0.06	1.70	0.70	1.223
Flume, CSU, Ft. Collins Colo.	Br	0.15	16.00	5.30	0.06	1.93	0.70	1.425
Flume, CSU, Ft. Collins Colo.	Br	0.15	18.00	5.47	0.06	2.14	0.70	1.580
Flume, CSU, Ft. Collins Colo.	Br	0.15	20.00	5.63	0.05	2.40	0.70	1.820
Flume, CSU, Ft. Collins Colo.	M	0.15	2.60	1.40	0.15	0.97	0.70	0.441
Flume, CSU, Ft. Collins Colo.	M	0.15	6.40	1.84	0.15	1.27	0.70	0.588
Flume, CSU, Ft. Collins Colo.	M	0.15	7.50	2.00	0.16	1.37	0.70	0.611
Flume, CSU, Ft. Collins Colo.	M	0.15	8.50	2.20	0.16	1.6	0.70	0.701

* Anastomosed

** Fine silt-sized bed material

Shear Stress Distribution Above the Rectangular Free Overfall

Otto R. Stein

Abstract

An equation for the ratio of shear stress at a point upstream from a free overfall to that which would occur under normal flow conditions is developed from basic momentum principles. It takes into account non-uniform velocity distributions and non-hydrostatic pressure conditions which are known to exist in the area. It is shown how this equation can be simplified to generally accepted equations for shear stress ratios assuming gradually varied flow with hydrostatic pressure and uniform velocity distributions. This developed equation, and one assuming gradually varied flow are then applied to data found in the literature. Data is not detailed enough to accurately determine the necessary parameters for the developed general equation. Determined values are ofter inconsistent with physics. The gradually varied flow assumption yields results consistent in trend with measured data but not in magnitude.

Introduction

The free overfall, or the abrupt end of a long channel, is a hydraulic phenomena which has received considerable attention of past researchers. This was initially because it is a control section, theoretically in which discharge can be calculated from only geometric variables. Since early (pre 1950) studies, which assumed the free overfall was a case of flow over a weir of zero height, attention has focused on either applying the momentum equation in some particular form or measuring in detail the various parameters involved. To arrive at a meaningful conclusion, both theoretical and experimental methods must be employed because the problem is one of rapidly varied flow which has neither uniform velocity nor hydrostatic pressure distributions. Theoretical equations can be diveloped but must be modified by empirical constants. To account for gradually varied flow.

A definition sketch is shown in Figure 1. Flow approaches the free overfall, or brink, at a normal depth, y_n. If the flow is sub-critical ($F_R < 1.0$), as shown, the depth decreases as the brink is approached through the critical depth, y_c at a distance, L, above the brink to the exit depth, y_e. If the flow is supercritical ($F_R > 1.0$), y_n is less than y_c everywhere, but y decreases gradually from y_n toward y_e starting at a distance L above the brink. In either case, the distance L is approximately 4 to 10 times y_e.

Experiments and theory (6,10) show that pressure and velocity can be classically dealt with (ie. uniform velocity and hydrostatic pressure) except in the reach L. Since streamlines are curved, velocity is not

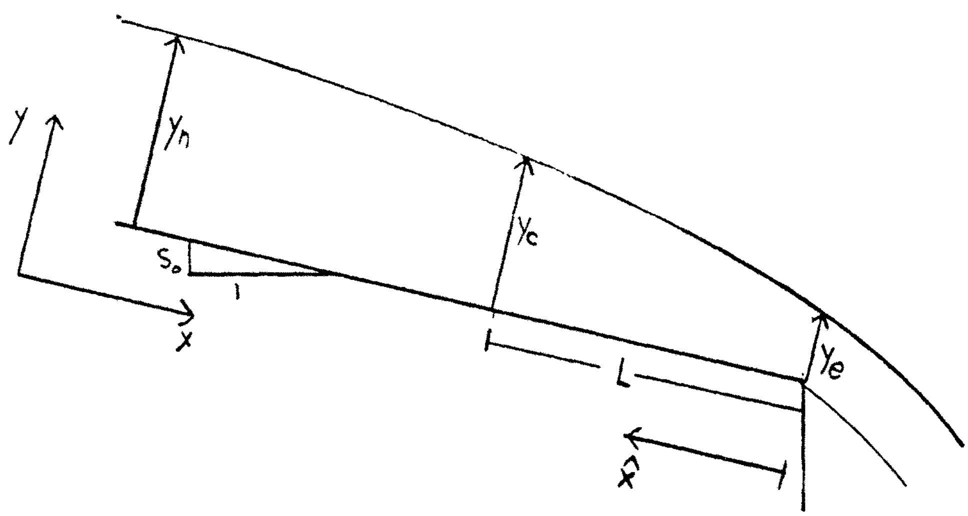

Figure 1. Definition Sketch - Mild Slope

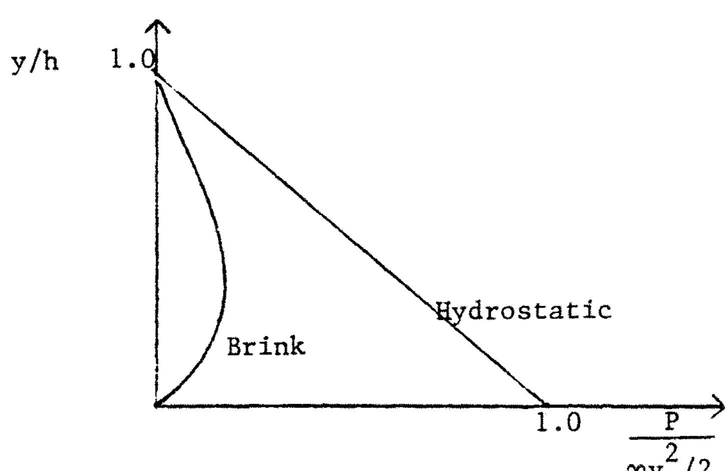

Figure 2. Pressure Distributions

uniform through this reach. It can seem that the pressure on the upper and lower streamlines must be zero at the brink because both are subjected to atmospheric conditions. Therefore, pressure at the bed varies from hydrostatic to zero through the reach L. Figure 2 shows typical pressure distributions at hydrostatic and brink conditions.

The purpose of this report is to determine how bed shear stress varies along x due to the rapidly varying flow conditions above the bed. Virtually none of the previous studies consider this, in fact most assume that the shear stress in the reach L is zero so that other variables such as y_e can be determined. Following is a short synopsis of the development of knowledge on the free overfall.

Literature Review

In one of the first publications on the topic Rouse (14) uses the basic weir formula (with weir height=0) to determine the exit depth, y_e=0.715 y_c, for horizontal channels. He is the first to show that even though brink pressures are zero at the upper and lower streamlines they are not zero through the entire brink profile, an assumption which is still often made. In a later publication Rouse (15) gives measured pressure distributions for a horizontal bed for several cross-sections within the reach L.

Southwell and Vaisey (16) use potential theory and a relaxation method to convert curvilinear lines to rectangular coordinates so that a finite difference method can be used to determine the flow profiles upstream and downstream from a free overfall. Markland (9) refines Southwell and

Vaisey's procedure, which is general, and uses it to calculate profiles for various upstream conditions. Since potential theory is used it is of no help in determining shear stress.

Fathy and Amin (6) appear to be the first to apply the momentum equation to the free overfall. From a differential form of the momentum equation, a backwater profile curve which includes both momentum flux and pressure correction factors is proposed. Their experimentally measured values for pressures at the brink are negative and the emphasis of analysis is justifying these negative pressures. Discussion by Carstens and Carter (2) shows that pressures cannot be negative and highlights other errors in analysis. Due to errors in analysis, the original paper has been largely dismissed, which is unfortunate because the initial approach is sound. Carstens (1) later provides limited data on the relations between both y_e and L against S_o, the bed slope.

Delleur et al. (4) give a more detailed analysis of the relation between y_e and S_o. They develop the momentum equation for a control volume between the brink and an upstream control section, either where $y = y_c$ for a subcritical flow or where $y = y_n$ for a supercritical flow. This corresponds to the reach, L, defined earlier. Both momentum flux and pressure correction factors are considered in the development but the first is later reasoned to be equal to unity. The weight and shear forces are assumed to cancel each other. The resulting equations give y_e in terms of K, a pressure correction factor at the brink and S_o, the bed slope. Depths are experimentally measured for various slopes and K is calculated from the fitted curves.

Rajaratnam and Muralidhar (10) report on detailed experimental measurements of many parameters of the free overfall. Measurements of flow profiles; variation in K and β, the momentum flux correction factor; bed pressures; bed shear stress; and velocity profiles throughout the reach L are discussed for several combinations of discharge and bed slopes. Results show β increases slightly, K decreases significantly from 1.0 to 0.462-0.270 and bed shear stress increases as the brink is approached. Their data is used in ensuing analysis.

Several other papers are worth noting. Diskin (5) and Rajaratnam and Muralidhar (11) discuss the trapezoidal free overfall. Replogle's (13) discussion of Diskin's paper provides more experimental results. Rajaratnam et al. (12) include roughness effects on the free overfall. More recently, Hager (7) uses energy rather than momentum principles to calculate y_e and Christodoulou (3) assumes a nonaerated free overfall where pressure is not atmospheric under the brink.

Theoretical Development

A general backwater profile equation applicable to the rapidly varied flow conditions just upstream from the free overfall can be developed from the general differential form of the momentum equation. Axis alignment is shown in Figure 1. The general momentum equation in the x direction is:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial P}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \qquad (1)$$

assume:

steady flow

two dimensional flow

$V << U$

θ small therefore $\cos\theta = 1.0$ and $\sin\theta = S_o$

using these assumptions and dividing by ρ yields:

$$u \frac{\partial u}{\partial x} = g_x - \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial y} \tag{2}$$

This equation is then integrated over the flow depth y and the unit width using the following substitutions.

$$\int_o^y g_x \, dy = gy \sin\theta = gyS_o \tag{3}$$

$$\int_o^y \frac{\partial \tau_{yx}}{\partial y} \, dy = -\gamma y \, S_f \tag{4}$$

where γ = specific weight of water

S_f = friction slope

$$\int_o^y P \, dy = K \frac{\gamma}{2} y^2 \tag{5}$$

where K is a coefficient to account for non-hydrostatic pressure

$$\int_{o}^{y} u\frac{\partial u}{\partial x}\, dy = \frac{\partial y\beta \bar{u}^2}{\partial x} \tag{6}$$

where β is the momentum correction factor

$\bar{u}$ is the mean velocity (hereafter $\bar{u} = u$)

Substituting the above integrals into Eq. 2 yields:

$$\frac{\partial y\beta u_2}{\partial x} = gyS_o - \frac{g}{2}\frac{\partial Ky^2}{\partial x} - gyS_f \tag{7}$$

Carrying out the differentiation, bringing in the continuity relation $u^2 = q^2/y^2$, definition of Froude number $F_R^{\ 2} = u^2/gy$ and simplifying yields:

$$\frac{\partial y}{\partial x} = \frac{S_o - S_f - y(\frac{1}{2}\frac{\partial K}{\partial x} + F_R^{\ 2}\frac{\partial \beta}{\partial x})}{K - \beta F_R^{\ 2}} \tag{8}$$

Equation 8 is a general backwater curve equation applicable when velocity distributions cannot be considered uniform and pressure distributions are non-hydrostatic. It is similar in form to one proposed by Carstens and Carter (2). The familiar gradually varied backwater profile equation (Eq. 9) can be obtained by using the conventional assumptions that $K=\beta=1.0$ at all points.

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - F_R^{\ 2}} \tag{9}$$

To solve for the bed shear stress, τ, distribution along x, Equation 8 is solved for S_f and multiplied by the quantity $(-\gamma y)$. The ratio of this actual shear stress, τ, to the shear stress, τ_n, which would occur at normal flow, y_n, $(S_f = S_o)$ is obtained by dividing by $\tau_n = -\gamma y_n S_o$.

$$\frac{\tau}{\tau_n} = \frac{y}{y_n} \quad 1 - \frac{y}{S_o}\left(\frac{1}{2}\ \frac{\partial K}{\partial x} + F_R^2\ \frac{\partial \beta}{\partial x}\right) - \frac{\partial y}{\partial x}\left(\frac{K - \beta F_R^2}{S_o}\right) \tag{10}$$

This is the ratio of actual shear stress to normal shear stress. If $K=\beta=1.0$ is assumed, that is gradually varied flow, Equation 10 reduces to:

$$\frac{\tau}{\tau_n} = \frac{y}{y_n}\frac{S_f}{S_o} \tag{11}$$

To further the analysis the value of S_f/S_o must be determined from one of four equations depending on flow conditions (8). They are the Chezy, Manning, Blasius and laminar flow equations. Generally, one of the first two is applied. However the Chezy, Blasius and laminar flow equations all yield the same result for S_f/S_o, namely $S_f/S_o = (y_n/y)^3$. Manning's equation yields $S_f/S_o = (y_n/y)^{10/3}$. Therefore:

$$\frac{\tau}{\tau_n} = \left(\frac{y_n}{y}\right)^2 \tag{12}$$

from the Chezy, Blasius and laminar flow equations and

$$\frac{\tau}{\tau_n} = \left(\frac{y_n}{y}\right)^{7/3} \qquad (13)$$

from Manning's equation.

Analysis

There are three levels of sophistication which can be used to determine the shear stress distribution along the channel bottom above the free overfall. The first, and most complex, is to use both momentum flux and pressure correction factors which are known to be relevant in the reach L. Under these conditions Equation 10 must be used. A simplified solution can be obtained by assuming both K and $\beta = 1.0$, therefore the flow can be considered gradually varied. Only the flow depth, y, at the point x and the normal flow depth, y_n, calculated from the relevant equation (ie. Chezy or Manning) need be known to calculate the shear stress at the point x. The governing equation is either Equation 12 or 13. The most simple, and trivial, solution is to assume $y = y_n$ for all x up to the free overfall and $\tau/\tau_n = 1.0$ by definition.

It should be noted that all prior research indicates Equation 10 need only be applied in the reach L which is defined as the distance between the overfall and the point where $y = y_c$ in the case of subcritical flow or where y is essentially equal to y_n under supercritical conditions. Outside the reach L the assumption of gradually varied flow is valid because $K \simeq \beta \simeq 1.0$.

Table 1

RUN	S_o	q	Y_c	Y_e	L	K_e	β_e
MILD	0.0005	1.582	0.427	0.307	2.417	0.340	1.085
STEEP	0.0288	1.564	0.424	0.215	0.417	0.270	1.138

q = (ft^2/s) ; Y_c, Y_e & L (ft)

From Rajaratnam and Muralidhar (11).

In order to apply Equation 10, the most general, all terms on the right hand side must be known or calculated independently from Equation 10. The best source of this data appears to be in the paper by Rajaratnam and Muralidhar (10). Detailed measurements of all the relevant parameters for a variety of slopes from adverse to steep and various discharges are given. Two cases will be assessed herein. The first is on a mild slope (S_o = 0.0005) with q = 1.582 ft^2/s and the second on a steep slope (S_o = 0.0288) with q = 1.564 ft^2/s. The relevent measurements are shown in Table 1. Graphs of y, K and τ versus x are also provided and reprinted as Figures 3, 4 and 5 respectively.

To determine if any terms in Equation 10 could be dropped the relative magnitudes were analyzed. The term containing $\partial\beta/\partial x$ was determined to be two orders of magnitude less than the rest for mild slopes and one order less for steep slopes. Clearly it can be dropped in the first case, and since no data on $\partial\beta/\partial x$ is given, will be dropped for the second as well. Note that $\partial\beta/\partial x$ is positive and

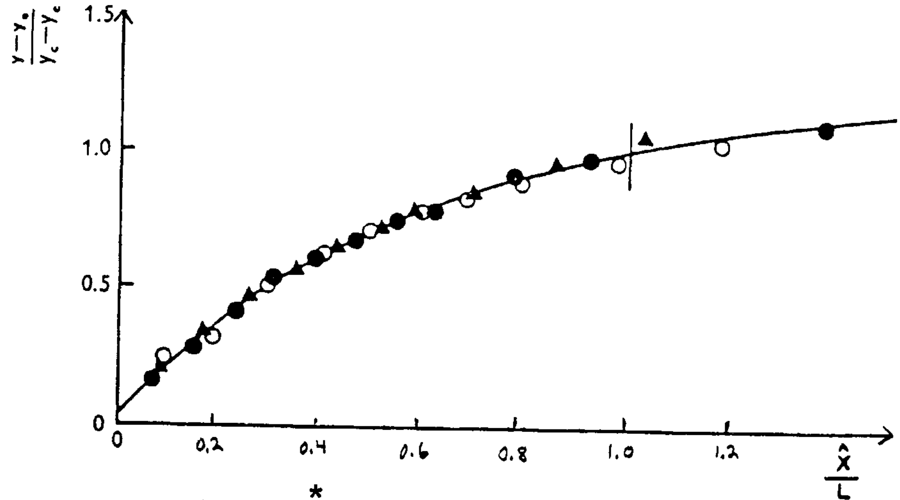

Figure 3*. y vs. x

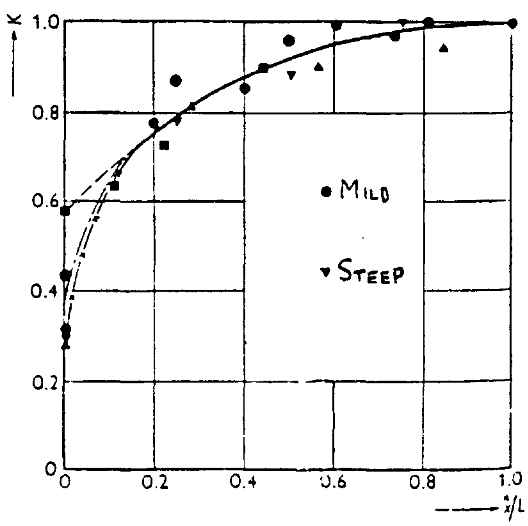

Figure 4*. K vs. x

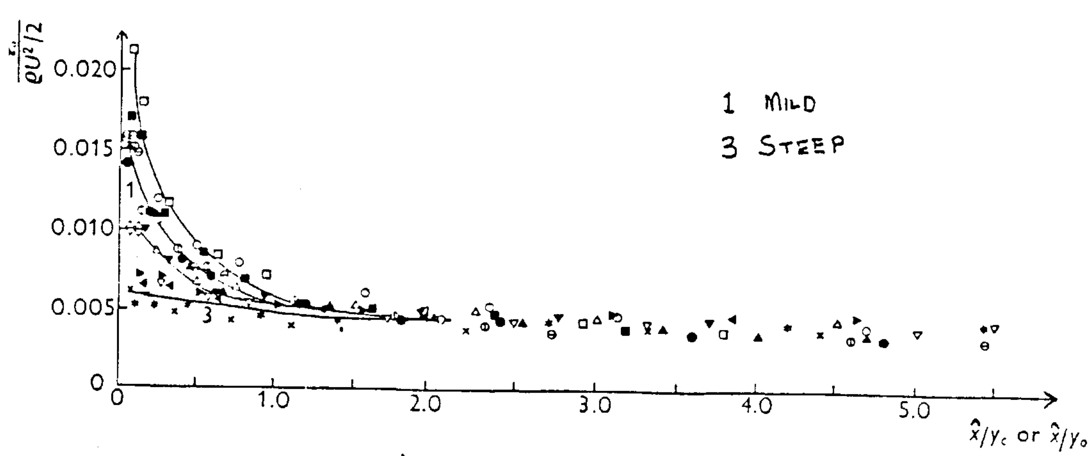

Figure 5*. τ vs. x

* From Rajaratnam and Muralidhar (1968)

therefore of opposite sign and same order of magnitude as the constant. They should approximately cancel each other. Dropping these terms and assuming $\beta = 1.0$ to eliminate it completely yields:

$$\frac{\tau}{\tau_n} = - \frac{y}{y_n S_o} \left(\frac{1}{2} \frac{\partial K}{\partial x} + (K - F_R^2) \frac{\partial y}{\partial x}\right) \qquad (14)$$

The only term not given in Table 1 or Figures 3, 4 and 5 is y_n, the normal depth of flow. For the steep slope y_n is given as 0.228 ft. but no value is given for the mild slope. If Manning's equation (Eq. 15) is assumed to be valid, y_n can be calculated if the roughness factor, n, is known.

$$q = \frac{1.486}{n} y_n^{5/3} S_o^{1/2} \qquad (15)$$

Using the steep slope values of q, y_n and S_o, n is calculated to be 0.021 for the experimental channel. Using this value, y_n equals 1.00 ft. for the mild slope. If Chezy's equation (16) is assumed valid, the same procedure yields C = 84.65 and y_n for the mild slope = 0.887 ft. These values will be assumed in further analysis.

$$q = C\, y_n^{3/2} S_o^{1/2} \qquad (16)$$

Tables 2 and 3 contain the terms needed in Equation 14 as determined from the given data. Note as defined $\hat{x} = L - x$. The y values are determined from Figure 3. The slope, $\partial y/\partial y$, and F_R^2 are determined from

Table 2. Values Determined From Data - Mild Slope

$\frac{\hat{x}}{L}$	y	$\frac{\partial y}{\partial x}$	F_R^2	K	$\frac{\partial K}{\partial x}$	τ	$\frac{\tau}{\tau_n}$
-	ft	-	-	-	ft^{-1}	lbs/ft^2	-
0.0	.307		2.69	.34		-.213	7.70
		-.099			-1.200		
0.1	.331		2.14	.63		-.086	3.11
		-.066			-.579		
0.2	.347		1.86	.77		-.073	2.64
		-.058			-.228		
0.4	.375		1.47	.88		-.060	2.17
		-.037			-.165		
0.6	.399		1.22	.96		-.060	2.17
		-.033			-.062		
0.8	.415		1.09	.99		-.053	1.92
		-.025			-.021		
1.0	.427		1.00	1.00		-.053	1.92
		-.021			-.000		
1.2	.437		0.93	1.00		-.053	1.92

Table 3. Values Determined From Data - Steep Slope

$\frac{\hat{x}}{L}$	y	$\frac{\partial y}{\partial x}$	F_R^2	K	$\frac{\partial K}{\partial x}$	τ	$\frac{\tau}{\tau_n}$	$\frac{\tau}{\tau_n^*}$
-	ft	-	-	-	ft^{-1}	lbs/ft^2	-	-
0.0	.215		7.64	.27		-.319	.78	1.6
		-.072			-8.40			
0.1	.218		7.33	.62		-.296	.72	1.4
		-.048			-3.60			
0.2	.220		7.13	.77		-.274	.67	1.3
		-.036			-1.32			
0.4	.223		6.85	.88		-.251	.61	1.2
		-.024			-.96			
0.6	.225		6.67	.96		-.228	.56	1.1
		-.024			-.36			
0.8	.227		6.49	.99		-.205	.50	1.0
		-.012			-.12			
1.0	.228		6.41	1.00		-.205	.50	1.0
		-.012			0.0			
1.2	.229		6.33	1.00		-.205	.50	1.0

the previous two columns. K is read directly from Figure 4 and $\partial K/\partial x$ is determined from the K values. τ values are obtained from Figure 5. τ_n is determined from Equation 18.

$$\tau_n = -\gamma\, y_n\, S_o \tag{17}$$

There appears to be an error in the values of shear stress on steep slopes (Table 3) as given in Figure 5. When $\hat{x}/L$ equals 1.0, $y = y_n$ and τ_n should equal -0.410 lbs/ft^2 as given by Equation 18. The value from Figure 5 is only -0.205 lbs/ft^2. If a scaling factor is assumed to be the only error, adjusting $\tau/\tau_n = 1.0$ at $\hat{x}/L = 1.0$, the value τ/τ_n^* is obtained.

Because the values of $\partial y/\partial x$ and $\partial K/\partial x$ are calculated from data, they are plotted in Figures 6 and 7 respectively to smooth out possible rounding errors. A curve is fitted to the calculated points and the values used in Eq. 14 were read from the grtaphs and appear in Tables 4 and 5. τ/τ_n is then calculated under the assumptions of rapidly varied flow (Eq. 14) and gradually varied flow (Eq. 12) and appear in Tables 4 and 5. In either case the value of y_n is determined from the Chezy equation.

It is clear that values of τ/τ_n from the rapidly varied flow assumption are inconsistent. On the mild slope the value is negative at the brink and is also negative in the center portion of the steep slope reach. These negative values indicate the direction of shear stress is reversed and have no physical meaning. On steep slopes τ/τ_n should approach 1.0 as $\hat{x}/L$ approaches 1.0 and continue at this value upstream, clearly this is not the case. However, these inconsistancies are not

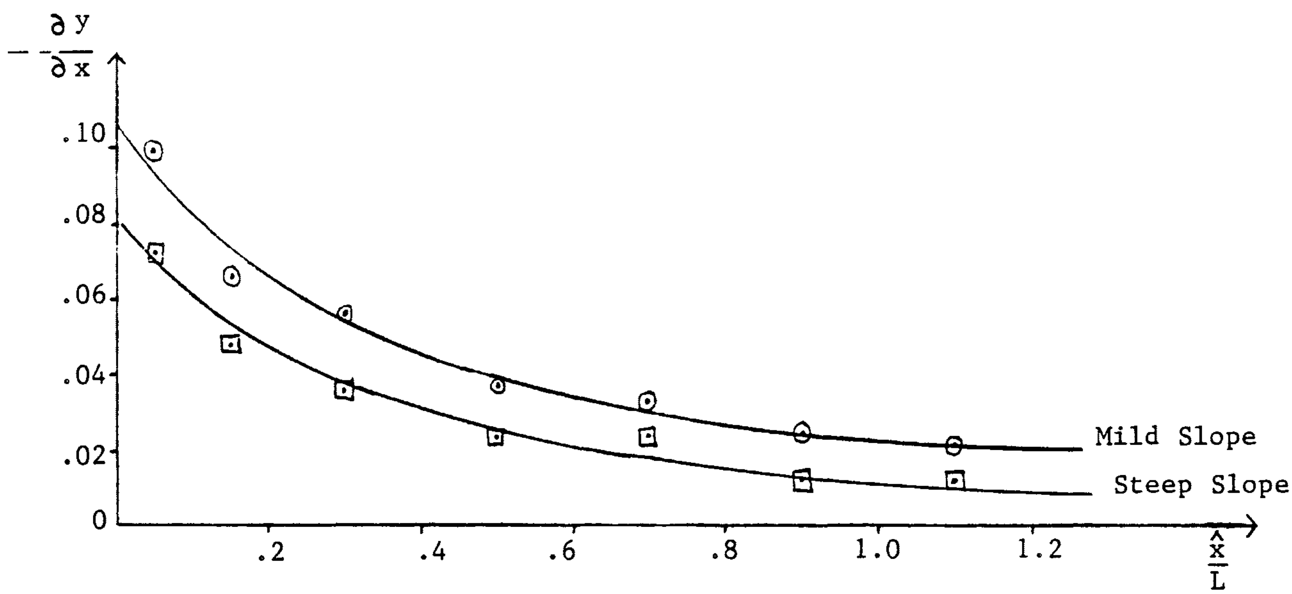

Figure 6. $\frac{\partial y}{\partial x}$ vs. $\frac{\hat{x}}{L}$

Figure 7. $\frac{\partial K}{\partial x}$ vs. $\frac{\hat{x}}{L}$

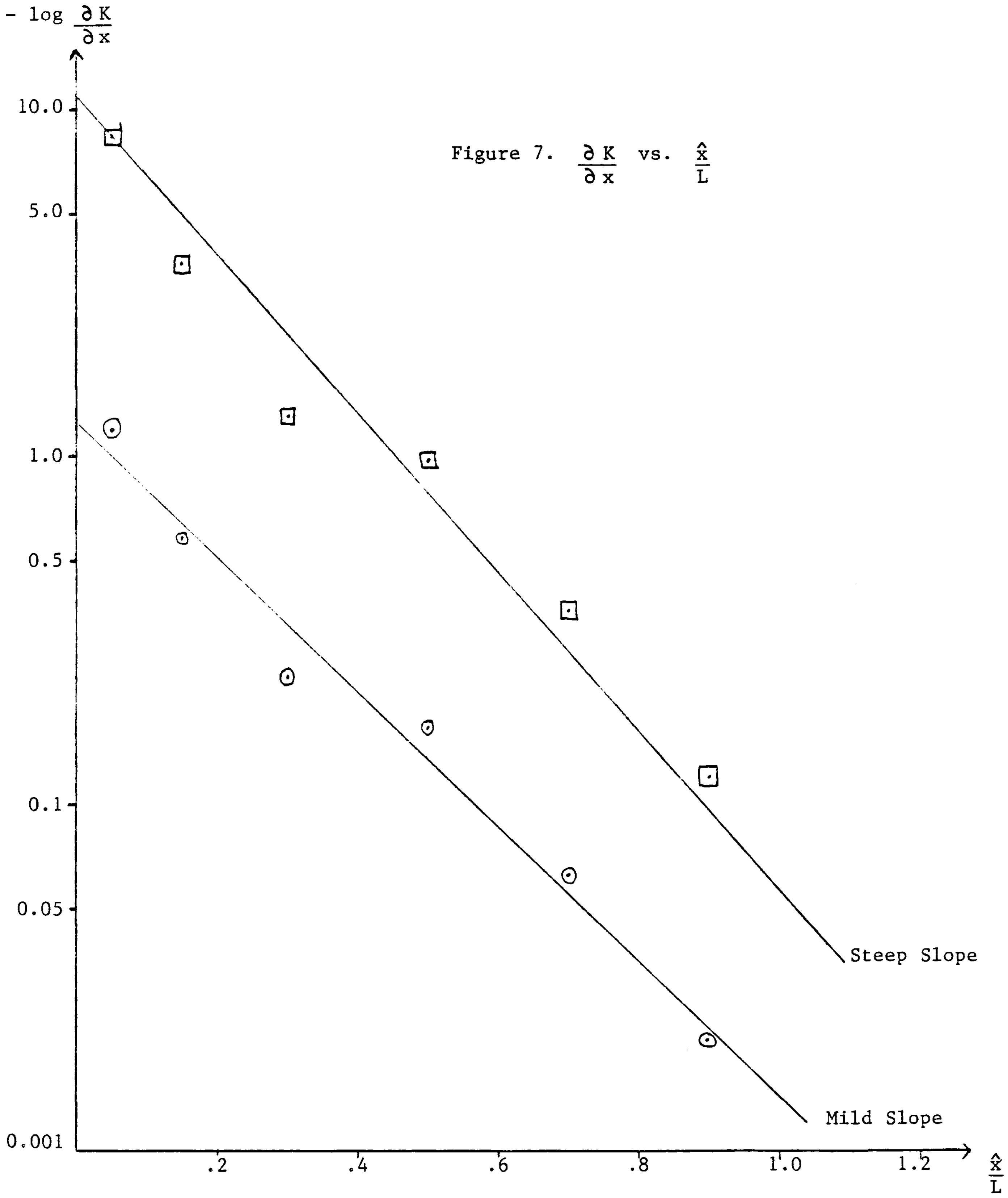

Table 4. Rapidly and Gradually Varied Shear Stress--Mild Slope

$\frac{x}{L}$	$\frac{\partial y}{\partial x}$	$\frac{\partial K}{\partial x}$	$(\frac{\tau}{\tau_n})_R$	$(\frac{\tau}{\tau_n})_G$
0.0	-.110	-.120	-51.4	8.35
0.1	-.081	-.80	7.53	7.18
0.2	-.065	-.51	13.79	6.53
0.4	-.046	-.21	10.35	5.59
0.6	-.034	-.086	7.48	4.94
0.8	-.026	-.035	4.36	4.57
1.0	-.022	-.014	2.88	4.32
1.2	-.020	-.000	1.38	4.12

G - From Gradually Varied Flow

R - From Rapidly Varied Flow

Table 5. Rapidly and Gradually Varied Shear Stress--Steep Slope

$\frac{x}{L}$	$\frac{\partial y}{\partial x}$	$\frac{\partial K}{\partial x}$	$(\frac{\tau}{\tau_\nu})_\theta$	$(\frac{\tau}{\tau_n})_G$
0.0	-.080	-10.5	17.61	1.12
0.1	-.060	- 6.5	10.16	1.09
0.2	-.047	- 3.8	3.99	1.07
0.4	-.031	- 1.3	-1.36	1.05
0.6	-.021	- 0.47	-2.30	1.03
0.8	-.015	- 0.16	-2.22	1.01
1.0	-.010	- 0.056	0.03	1.00
1.2	-.008	- 0.000	1.73	0.99

unexpected considering input data to Equation 14 is experimental. The most sensitive parameters, $\partial y/\partial x$ and $\partial K/\partial x$, are derivatives of experimental data which should always be used with extreme caution. The magnitude of these opposing values is higher than the desired result, τ/τ_n, therefore the desired "small" value is obtained by subtraction of two "large" ones. Even a small percentage error of latter can cause errors larger in magnitude than the desired value. This appears to be the case. The theoretical development of Equation 14 is sound, however calculation of the needed parameters from data is questionable.

The gradually varied flow assumption is much more consistent and realistic as values steadily decrease as $\hat{x}/L$ increases. This too can be expected because the sensitive derivative parameters are not involved. However as can be seen in Figure 3, the value of $\partial K/\partial x$ can be significant and even dominating, in the section $0.0 < \hat{x}/L < 0.2$. Only the rapidly varied flow assumption considers this.

Measured shear stress (last column Tables 2 and 3) compared to the shear stress calculated from gradually varied flow (Tables 4 and 5) shows good agreement on the steep slopes. Measured values are about one half the calculated values on mild slopes. Note however, that the scaling factor was assumed to be in error on the steep slopes.

Conclusion

It is believed that, in the reach one to two flow depths immediately upstream from the free overfall, non-hydrostatic pressure distributions greatly increase the shear stress compared to that calculated if gradually varied flow is assumed. However, it is difficult to measure the values needed to account for non-hydrostatic pressures. Use of the only data set detailed enough to determine these values lead to erranous results. Clearly an assumption of uniform flow to the brink is inaccurate. On a mild slope, measured shear stress values are 7.70 times larger than uniform flow shear. A gradually varied flow assumption yields a value 8.35 times larger. It is suggested that the gradually varied flow assumption be used to calculate shear stress while noting that this assumption might be neglecting important, if difficult to measure, parameters.

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A Comparative Analysis of

Riprap Design Methods

by Diane Hoelzer

April, 1987

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ABSTRACT

In the past sixty years numerous equations and procedures have been developed for the design of rock riprap for the control of streambank erosion. Due to lack of a full understanding of the relationship between hydraulic conditions and sediment characteristics to streambank erosion, design formulas work successfully not necessarily because they completely explain the phenomenum in mathematical terms but rather because most formulas so over design they can't help but succeed a majority of the time. By over designing higher costs may be incurred than are necessary. As a result it was of interest and the topic of this paper to compare the current riprap design methods using data where hydraulic and channel characteristics were recorded at the time of riprap failure. The seven methods addressed in this paper were those by Stevens, California Department of Public Works (Caltrans), Blodgett, University of Minnesota Study, Corps of Engineer Waterways Experiment Station (COE/WES), ASCE Task Committee on Preparation of Sedimentation Manual and Corps of Engineers (COE). These methods were analysed by comparing as a ratio, the calculated $\overline{D50}$ by each method given the hydraulics conditions at riprap failure from laboratory experiments to the actual D50 present at riprap failure. Caltrans and Stevens methods closely predicted the riprap material mean diameter (D50) at the point of failure with mean ratios ($\overline{D50}$/D50) of 1.17 and 1.13, respectively. The University of Minnesota Study and COE methods predicted with reasonable accuracy the D50 with a ratio of 1.61 and 2.51, respectively while the other methods by Blodgett, COE/WES and ASCE Task Committee generously over predicted with mean D50/D50 ratios of 6.53, 18.8 and 4.85, respectively.

INTRODUCTION

The first part of this paper will introduce the factors effecting the stability of alluvial channels and riprap design. The second part will review eight riprap design methods currently used in practice. The third part will compare seven of the riprap design methods using laboratory flume data from Steve Maynord's 1978 report:"Practical Riprap Design". Finally, the fourth part will discuss the results and present the conclusions.

FACTORS EFFECTING THE STABILITY OF ALLUVIAL CHANNELS

The first concept needing to be addressed and understood is the idea of stability: what constitutes a stable or unstable channel? According to Lane (10):

"A stable channel is an unlined earth canal for carrying water, the banks and bed of which are not scoured by the moving water and in which objectionable deposits of sediment do not occur."

Lane also defines unstable channels in three classes: 1> channels in which the banks and bed are scoured without objectionable deposits being formed, 2> channels where objectionable sediment deposits occur without scour being produced and 3> channels in which scour and objectionable deposits are both present.

There are numerous factors effecting the stability of an alluvial channel. Since the purpose of this paper is to compare different riprap design criteria, a detailed discussion on the possible effects of these factors will not be given. However, it is helpful in understanding the complexity of riprap design if one is aware of the considerable number of dependent and interdependent variables involved. For this reason the following is a list of most of the important factors:

1. discharge and velocity
2. channel side slope
3. bed channel slope
4. boundary bed material
    a. size
    b. shape
    c. mean diameter
    d. gradation
    e. weight
5. viscosity of fluid/ temperature
6. sediment transport (washload and bed load)
7. wind/wave action
8. depth of flow
9. bends and secondary circulation
10. seepage forces

11. turbulence
12. drag and lift forces
13. vegetation

(For a more detailed discussion of these factors, refer to Dr. D. B. Simons 1957 dissertation,(14).

RIPRAP DESIGN AND FACTORS EFFECTING IT

There are many ways to stabilize an unstable channel. A widely used and successful method of stabilizing an alluvial channel in an erosive condition is through the use of riprap. Riprap as defined by the Army Corps of Engineers (7), "is a layer, facing or protective mound of stones randomly placed to prevent erosion, scour or sloughing of a structure or embankment; also the stone so used."

Various empirically and theoretically derived equations have been developed for the design of riprap stone size. The ones to be addressed in this paper are methods by Stevens, Caltrans, Blodgett, Bureau of Public Roads (FHWA,HEC-11), University of Minnesota Study, COE Waterways Experiment Station, ASCE Task Committee on Preparation of Sedimentation Manual and Corps of Engineers (COE).

Many factors/variables are important and effect the successful design of riprap. These factors/variables include: specific weight, shape, size, angularity and angle of repose of riprap material, velocity, discharge and direction of flow near riprap material, side slope of channel, channel bed slope, thickness of riprap blanket, mean diameter and gradation of riprap material, roughness, shape and alignment of channel.

REVIEW OF RIPRAP DESIGN METHODS

Stevens Method

Stevens method was developed through a theoretical analysis of the forces effecting the stability of a particle on a side slope in a river channel. According to Stevens and Simons (15) the stability of a single particle is a function of the magnitude and direction of stream velocity, the depth of flow, the angle of inclined surface on which it rests and its geometric and sedimentation characteristics. Basically this method determines the stability of a particle through the use of Shields beginning of motion concept and by comparing the stabilizing forces: normal component of the particles weight with the destabilizing forces: the drag and lift forces and the tangential component of the particles weight. Through this analysis the following four inter-related equations were developed:

$$S.F. = \frac{\cos\theta \cdot \tan\phi}{\eta' \cdot \tan\phi + \sin\theta \cdot \cos\beta}$$ safety factor

$$\beta = \tan^{-1}\left[\frac{\cos\lambda}{\frac{2\cdot\sin\theta}{\eta\cdot\tan\phi} + \sin\lambda}\right]$$

$$\eta = \frac{21\cdot\tau_s}{(S_s-1)\gamma D_{50}}$$ stability factor on plane horizontal bed

$$\eta' = \eta\left[\frac{1+\sin(\lambda+\beta)}{2}\right]$$ stability factor on side slope

where:

θ = angle of side slope

ϕ = angle of repose of riprap material

β = angle between tangential component of weight along bank and resultant path particle would take due to drag force and tangential weight component

λ = angle drag force makes with the horizontal (downward:positive, upward:negative)

τ_s = (γRS) average tractive force on side slope

S_s = specific gravity of riprap material

γ = specific weight of water

D_{50} = mean diameter of riprap material

Through an iterative process the required riprap material (D50) size can be determined given the bank side slope θ, λ, average tractive force τ_s and by assuming an angle of repose ϕ and a desired safety factor. This method is applicable to a variety of channel conditions including horizontal flow on a side slope, on a plane sloping bed and on a horizontal bed. The four general equations may be modified to a simpler form in certain specific cases.

Caltrans

The Caltrans Bank and Shore Protection Manual (6) presents an equation for the determination of riprap material size taking into account stream velocity, side slope angle and the specific gravity of the riprap material. While the manual provides no explanation for the equations basis of development, it is similiar in form to the equation adopted by the ASCE Task Committee on Preparation of Sedimentation Manual and theirs is a modification of a formula proposed by Isbach (10) for the construction of dams by depositing rocks in running water. The following is the equation adopted by Caltrans:

$$W_{33} = \frac{0.00002 \cdot V^{6} \cdot Sa\ \csc^{3}(\rho - \theta)}{(Sa-1)^{3}}$$

where:

W_{33} = minimum/critical weight of outside stone for no damage (lbs.) (note: 2/3 of stone should be heavier)

V = stream velocity to which bank is exposed (fps)

Sa = specific gravity of stone

ρ = 70° for randomly place rubble

θ = bank side slope

If actual data is not available, it should be assumed the specific gravity of the stones is 2.65, face slope is 1.5 : 1, impinging velocity is 4/3 the average stream velocity and tangent velocity is 2/3 the average stream velocity.

Once the critical stone weight is obtained, reference is made to Table 3. 'Standard Classes of Rock Slope Protection for Rock Size Determination' in the manual. Due to the fact there is a vagueness and lack of information on how to interpret this table, the following method was adopted to convert the critical weight W_{33} to a mean diameter stone size D50:

$$D33 = \left(\frac{6 \cdot W_{33}}{\pi \cdot \gamma_s}\right)^{1/3}$$

where: γ_s = specific weight of stone (lbs.)

D33 = diameter of stone where 33% of stone mixture is finer

Then using table 5.2.2 Data for suggested gradation (pg.V-26) in th U.S. Department of Transportation's <u>Highways in the River Environment</u> (1987) the D33 is converted to D50:

$$D_{50} = \frac{D_{33}}{0.695}$$

Blodgett and McConaughy

The recommended "Interim" design for riprap stone size considers mean velocity only:

$$D50 = 0.0283 \cdot Va^{1.83}$$

Va = mean velocity

This equation is similiar to the riprap design equation used in the Bureau of Public Roads HEC-11 but with some modifications.

Some of the reasons this simple equation was preferred to other methods of riprap stone design were as follows:

1. The procedure presents the most reliable estimates of riprap D50 size on the basis of field data. (see fig.36,ref.3)

2. The procedure is simple and straight forward in application and uses input data that can be recognized as hydraulically reasonable.

3. Many personnel in FHWA and state highway departments are familiar with the basic concepts.

4. A conservative, or possible over design of rock riprap installation will reduce the need for repairs, and reduces the possibility of damage caused by future channel changes that induce greater hydraulic stresses at the site than originally designed for.

Bureau of Public Roads

The procedures for determining the required mean rock riprap size as outlined in HEC-11 considers the total depth of flow (d), average velocity in channel (V), velocity against the rock (Vs) and the embankment side slope (θ). Through the use of the following two figures and iterative steps, the proper rock riprap diameter size (D50) is determined.

Figure 1.

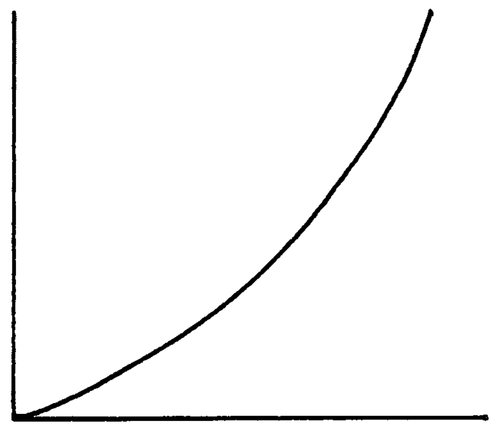
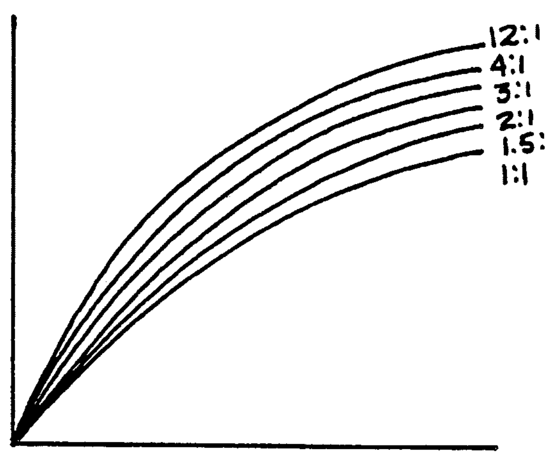


Figure 2.



Given: total depth of flow (ft), average velocity in channel (fps), embankment side slope.

Note: figure 2. assumes the specific weight of the rock is 165 lbs./cu.ft.

Step 1: assume a rock diameter size, D50(ft)

Step 2: determine the k/d ratio; k is the equivalent spherical diameter of the 50 % rock size and d is the total depth of flow during the design flood; when the total depth of flow exceeds 10 feet use d equal to 0.4 total depth of flow.

Step 3: using figure 1 and k/d, determine Vs/V

Step 4: knowing V, determine Vs

Step 5: using figure 2, knowing Vs and side slope, determine D50

Step 6: check this D50 with the assumed D50, if values are reasonably close, D50 is the design rock riprap size, if values differ substantially, assume a new D50 and repeat steps 1 - 6.

When the specific weight of the rock differs from 165 lbs/cu.ft. the size should be corrected by Creager's equation(3):

$$kw = \frac{102.5\,k}{W - 62.5}$$

k = design rock size (fig.2) (ft)
W = actual specific weight of rock
kw= adjusted design rock size (ft)

This procedure also recognizes the need for adjustment in rock size when impinging flows are involved. They suggest since data is not available for determining the rock size at the point of impingement, a factor which would vary from 1 to 2 depending upon the severity of the attack by the current, should be applied to the velocity Vs before entering figure 2.

University of Minnesota Study

This method was proposed by Anderson et al (15) and considers the stream discharge (Q), energy grade line slope (Sf), wetted perimeter (P), and the hydraulic radius (R) in determining the design rock riprap size:

$$Q = \frac{1}{118} \frac{D_{50}^{5/2}}{S_f^{13/6}} \frac{P}{R} \qquad \text{(english units)}$$

This equation is based on maximum shear stress related to rock diameter and mannings equation of flow. This method applies to triangular or trapezoidal shaped channels that are essentially straight in alignment. It is also limited to channels using small to intermediate rock sizes with flows from 6 to 1000 cfs.

COE Waterway Experiment Station

This method uses a safety factor approach and considers depth (y) as its only variable. The following equations are used to determine the design rock riprap size:

$$\mathcal{E} = 0.3\left[\frac{3.4}{\ln(12.21 y_o/k)}\right]^2 \quad (1)$$

$$S.F. = \frac{1}{2\mathcal{E}\bar{C}^2} \quad (2)$$

$\bar{C}$ = 1.2 for low levels of turbulence, COE.

y = depth of flow

k = D50 mean rock size

There are limitations for the use of these equations. The two above equations were theoretically derived and one of the equations used in the derivation was: $\rho U_r^2 = 72 \cdot \tau_s$ which is only valid for uniform flow in wide prismatic channels in which flow is fully turbulent. For purposes of riprap design this equation can be employed when flow is accelerating but should not be used in areas where the flow is decelerating or downstream of energy dissipating structures. In these areas, the shear stress is larger than would be calculated by this equation. Since this method is based in part on the above equation, the limitations on this equation must also apply in the use of the COE Waterway Experiment Station method. In addition Isbach's equation was used in the derivation and is only applicable for flow on plane beds.

ASCE Task Committee on Preparation of Sedimentation Manual Approach

The ASCE Task's method considers the stream velocity, specific weight of rock material and the embankment side slope in determining the design rock riprap size. The proposed formula is similiar to Isbach's equation (9) with a modification to take into account the slope of the bank:

$$W_{50} = \frac{4.1 \times 10^{-5} \cdot S_s \cdot V^6}{(S_s - 1)^3 \cos^3\theta}$$

where: W_{50} = weight of riprap material (lbs.)

Ss = specific gravity of rock

V = mean velocity

θ = angle of embankment side slope also:

$$W_{50} = \gamma_s \cdot \pi/6 \cdot D_{50}^3$$

where: γ_s = specific weight of riprap material

It must be remembered Isbach's equation is applicable for flow on plane flat beds.

COE Method

The COE's method considers the effects of average local *velocity ($\bar{V}$), embankment side slope (θ), angle of repose (ϕ) and specific weight of rock material (γ_s) and depth of flow (y) in determining the design riprap rock size. This method appreciates the importance of the hydrodynamic drag and lift in reducing channel stability. The primary basis for their adopted procedure is that the drag and lift forces, which are created by flow velocities are proportional to the local boundary shear. This procedure analytically determines the shear forces created by channel flow and the ability of the riprap material to resist those forces. Available laboratory data was utilized in the development of this analytical method. As part of their procedure, the computed local boundary shear is compared to the design shear. For the design D50 to be acceptable the design shear should be greater than the local boundary shear.

The design shear equation for riprap placed on an essentially level channel bottom is:

$$\tau = a(\gamma_s - \gamma) D_{50}$$

where: a = 0.04

γ_s = specific weight of riprap material

D50 = mean diameter of riprap material

The design shear equation for riprap placed on a channel side slope is:

$$\tau' = \tau \left(1 - \frac{\sin^2\theta}{\sin^2\phi}\right)^{.5}$$

where: θ = angle of side slope

ϕ = angle of repose of riprap material (usually 40)

The local boundary shear at any point on the wetted perimeter was determined using the following equation:

$$\tau_o = \frac{\gamma \cdot \bar{V}^2}{[32.6 \log(12.2\ y/D_{50})]^2}$$

where: V = average local velocity in the vertical (fps)
1.6 x (mean velocity)

Y = depth of flow (ft)

D50 = mean diameter of riprap material (ft)

This equation assumes uniform flow in a wide channel, thus Y should be the normal depth. This equation was based on the average boundary shear equation: $\tau_0 = \gamma R S_f$, the friction coefficient for hydraulically rough channels equation: $C = 32.6 \log(12.2\ R/k)$ and Chezy's equation: $S_f = \frac{V^2}{C^2 Y}$.

Riprap design method and variables considered in their equations.

METHOD	$\overline{D50}$/D50	VARIABLES
Stevens Method	1.13	$\theta\ \phi\ \lambda\ R\ S_f\ S_s\ \beta$ SF
Caltrans	1.17	$\theta\ S_s\ V$
Blodgett & McConaughy	6.53	V
Bureau of Public Roads HEC-11	x	$d\ \theta\ \gamma_s\ V_s\ V$
Univ. of Minnesota	1.61	$R\ P\ S_f\ Q$
COE-Waterway Exp Sta.	18.8	d SF $\bar{\tau}$ parameter
ASCE Task Committee	4.85	$\theta\ V\ S_s$
Corps of Engineers,COE	2.51	$\theta\ \phi\ \bar{V}\ d\ \gamma_s$ a parameter

$\overline{D50}$ = calculated mean diameter of riprap material at failure.

D50 = actual mean diameter of riprap material at failure in lab.

DATA

The data used to compare the different riprap design methods came from a report, "Practical Riprap Design", by S.T. Maynord of the Army Corps of Engineer Waterways Experiment Station in Vicksburg, Mississippi (11). In these experiments, with several different discharges, embankment slopes and riprap material sizes, the hydraulic parameters were measured and recorded at the time the riprap material failed. There were three types of failures; failure on the channel bed only, channel bed and side slope or side slope only. In each experiment, a specified discharge was held constant. The tailwater was then lowered in small increments until failure of the riprap material occurred. Failure was assumed to be the point at which the rocks began to move. The data used in this paper is shown in table 2-1. A sketch of the model test facility is provided in Figure 2-1.

ANALYSIS OF DATA

Using the data, the slope of the energy grade line was determined using the energy equation and Mannings equation. The results of the two different ways of calculating the energy grade line slope are shown on page 15. This table reveals a maximum difference of 20% and a minimum difference of 0.7% with a mean of -4.7% between the calculated slope values.

Given the mean velocity, Mannings roughness coefficient using Stricklers relationship and substituting the bed slope (S_o) for the friction slope (S_f) in Mannings equation, the normal depth was determined. A comparison was then made between the normal depth and the actual depth. On the average the normal depth was about 0.78 times the value of the actual depth (pg.16).

Finally, given the normal depth, the average normal shear stress (τ_n) was calculated and compared to the actual average shear stress(τ_o). The table on page 17 reveals the average normal shear stress is about 2.22 times larger than the actual average shear stress. At times in engineering practice a rough estimate of the average shear stress will be made using the measured depth and the bed slope (So) or the water surface slope; this is the estimated average shear stress (τ_e). The most interesting result is the comparison between the estimated average shear stress and the actual average shear stress. The table on page 17 reveals the estimated average shear stress is about 2.76 times larger than the actual average shear stress. The significance of this difference can be understood when applying the Stevens Method for riprap design. The composite effect of using a safety factor of 1.5 and an average shear stress 2.76 times the actual average shear stress would result in a design mean diameter size (D50) significantly larger than necessary.

Table 2-1

Model test results

Q (cfs)	Bottom Slope (ft/ft)	Bottom Width (ft)	Side Slope	D_{50} (ft)	Upstream Depth (ft)	Downstream Depth (ft)	Avg Depth (ft)	F	D_{50}/depth	Failtype
20.0	0.008	5.0	4	0.026	0.81	0.89	0.85	0.54	0.031	1
25.0	0.008	5.0	4	0.026	0.96	1.04	1.00	0.49	0.026	1
30.0	0.008	5.0	4	0.026	1.09	1.17	1.13	0.46	0.023	2
35.0	0.008	5.0	4	0.026	1.20	1.28	1.24	0.45	0.021	2
20.0	0.008	5.0	4	0.032	0.77	0.85	0.81	0.59	0.040	2
25.0	0.008	5.0	4	0.032	0.92	1.0	0.96	0.53	0.033	2
30.0	0.008	5.0	4	0.032	1.04	1.12	1.08	0.51	0.030	3
35.0	0.008	5.0	4	0.032	1.15	1.23	1.19	0.49	0.027	3
20.0	0.008	5.0	4	0.037	0.75	0.83	0.79	0.62	0.047	1
25.0	0.008	5.0	4	0.037	0.87	0.95	0.91	0.59	0.041	1
30.0	0.008	5.0	4	0.037	1.00	1.08	1.04	0.54	0.036	2
35.0	0.008	5.0	4	0.037	1.13	1.21	1.17	0.50	0.032	2
20.0	0.008	5.0	3	0.026	0.88	0.96	0.92	0.52	0.028	1
25.0	0.008	5.0	3	0.026	1.04	1.12	1.08	0.48	0.024	2
30.0	0.008	5.0	3	0.026	1.18	1.26	1.22	0.45	0.021	2
20.0	0.008	5.0	3	0.032	0.82	0.90	0.86	0.58	0.037	2
25.0	0.008	5.0	3	0.032	0.97	1.05	1.01	0.54	0.032	2
30.0	0.008	5.0	3	0.032	1.14	1.22	1.18	0.48	0.027	2
20.0	0.008	5.0	3	0.037	0.81	0.89	0.85	0.60	0.044	3
25.0	0.008	5.0	3	0.037	0.95	1.03	0.99	0.56	0.037	3
30.0	0.008	5.0	3	0.037	1.10	1.18	1.14	0.52	0.032	3
15.0	0.008	5.0	2	0.026	0.80	0.88	0.84	0.51	0.031	3
20.0	0.008	5.0	2	0.026	1.00	1.08	1.04	0.47	0.025	3
25.0	0.008	5.0	2	0.026	1.18	1.26	1.22	0.44	0.021	3
15.0	0.008	5.0	2	0.032	0.76	0.84	0.80	0.56	0.040	3
20.0	0.008	5.0	2	0.032	0.96	1.04	1.00	0.50	0.032	3
25.0	0.008	5.0	2	0.032	1.13	1.21	1.17	0.47	0.027	3
15.0	0.008	5.0	2	0.037	0.72	0.80	0.76	0.61	0.049	3
20.0	0.008	5.0	2	0.037	0.93	1.01	0.97	0.53	0.038	3
25.0	0.008	5.0	2	0.037	1.10	1.18	1.14	0.50	0.032	3
30.0	0.008	5.0	2	0.037	1.27	1.35	1.31	0.46	0.028	3

Failtype: 1 = bottom only; 2 = bottom and side slopes; 3 = side slopes only.

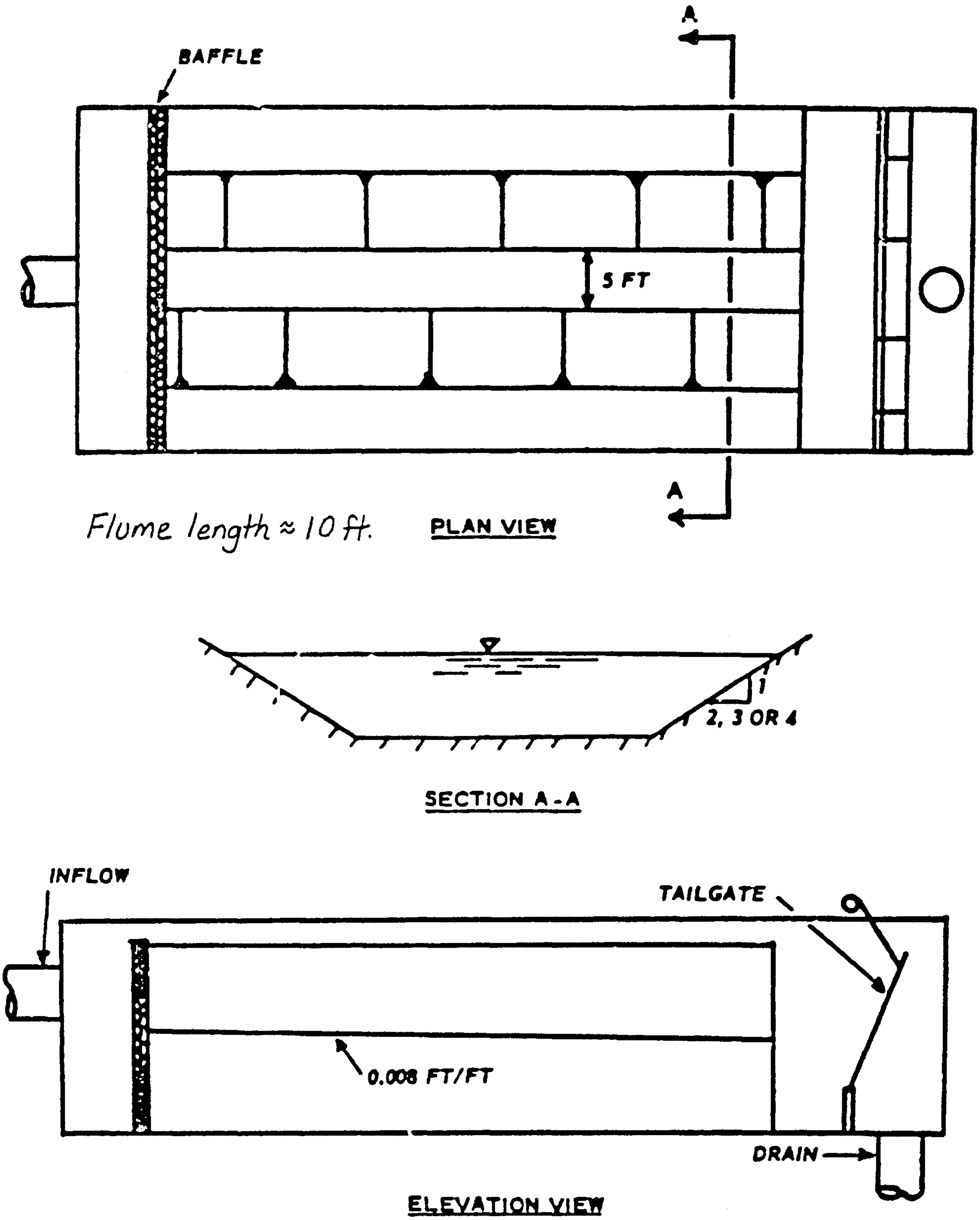


FIGURE 2-1
Model Test Facility

Comparison of the energy grade line slope (Sf)
derived by mannings equation and energy equation.

	Sfe energy equation	Sfm mannings equation	Sfe-Sfm / Sfe
	.00320	.00335	-4.7
	.00280	.00275	1.8
	.00250	.00242	3.2
	.00250	.00225	10.0
	.00380	.00427	12.4
	.00320	.00348	-8.8
	.00300	.00312	-4.0
	.00280	.00286	-2.1
	.00420	.00503	-19.8
	.00390	.00450	-15.4
	.00340	.00380	-11.8
	.00300	.003205	-6.8
	.00280	.00291	-3.9
	.00250	.002445	2.2
	.00240	.00218	9.2
	.00370	.004045	-9.3
	.00330	.003415	-3.5
	.00270	.00268	0.7
	.00380	.004425	-16.4
	.00340	.003855	-13.4
	.00300	.003205	-6.8
	.00260	.00276	-6.2
	.00230	.002245	2.4
	.00210	.001935	7.9
	.00310	.00354	-14.2
	.00260	.002785	-7.1
	.00240	.002435	-1.5
	.00380	.00446	-17.4
	.00290	.00326	-12.4
	.00260	.002805	-7.9
	.00230	.002395	-4.1
mean	.00300	.00320	-4.7

see CALC A

Comparison of normal depth (Dn) with actual depth (D)

	UPSTREAM		DOWNSTREAM		MEAN	
Dn	D	Dn/D	D	Dn/D	D	Dn/D
.67	.81	.83	.89	.75	.85	.79
.76	.96	.79	1.04	.73	1.00	.76
.83	1.09	.76	1.17	.71	1.13	.73
.90	1.20	.75	1.28	.70	1.24	.73
.69	.77	.90	.85	.81	.81	.85
.77	.92	.84	1.00	.77	.96	.80
.85	1.04	.82	1.12	.76	1.08	.79
.92	1.15	.80	1.23	.75	1.19	.77
.70	.75	.92	.83	.84	.79	.89
.78	.87	.90	.95	.82	.91	.86
.86	1.00	.86	1.08	.80	1.04	.83
.93	1.13	.82	1.21	.77	1.17	.79
.70	.88	.80	.96	.73	.92	.76
.79	1.04	.76	1.12	.71	1.08	.73
.87	1.18	.74	1.26	.69	1.22	.71
.71	.82	.87	.90	.79	.86	.83
.80	.97	.82	1.05	.76	1.01	.79
.89	1.14	.78	1.22	.73	1.18	.75
.72	.81	.89	.89	.81	.85	.85
.81	.95	.85	1.03	.79	.99	.82
.90	1.10	.82	1.18	.76	1.14	.79
.62	.80	.78	.88	.70	.84	.74
.73	1.00	.73	1.08	.68	1.04	.70
.83	1.18	.70	1.26	.66	1.22	.68
.63	.76	.83	.84	.75	.80	.79
.74	.96	.77	1.04	.71	1.00	.74
.84	1.13	.74	1.21	.69	1.17	.72
.64	.72	.89	.80	.80	.76	.84
.75	.93	.81	1.01	.74	.97	.77
.85	1.10	.77	1.18	.72	1.14	.75
.94	1.27	.74	1.35	.70	1.31	.72
mean		.81		.75		.78
high		.93		.84		.89
low		.70		.66		.68

see CALC B

Comparison of normal shear stress(τ_n) with actual shear stress(τ_o).

	UPSTREAM		DOWNSTREAM		MEAN		
τ_n	τ_o	τ_n/τ_o	τ_o	τ_n/τ_o	τ_o	τ_n/τ_o	τ_e/τ_o
.2441	.1416	1.72	.1056	2.31	.1187	2.06	2.50
.2707	.1304	2.08	.1011	2.68	.1187	2.28	2.86
.2910	.1256	2.32	.0999	2.91	.1172	2.48	3.20
.3111	.1251	2.49	.1013	3.07	.1265	2.46	3.20
.2500	.1775	1.41	.1310	1.91	.1355	1.85	2.11
.2736	.1603	1.71	.1234	2.22	.1367	2.00	2.40
.2968	.1567	1.89	.1238	2.40	.1355	2.19	2.67
.3167	.1546	2.05	.1242	2.55	.1370	2.31	2.86
.2530	.2008	1.26	.1474	1.72	.1467	1.72	1.91
.2766	.1993	1.39	.1516	1.82	.1530	1.81	2.05
.2997	.1857	1.61	.1454	2.06	.1489	2.01	2.35
.3195	.1711	1.87	.1369	2.33	.1447	2.21	2.67
.2632	.1346	1.96	.1036	2.54	.1153	2.28	2.86
.2908	.1270	2.29	.1015	2.87	.1173	2.48	3.20
.3147	.1241	2.54	.1011	3.11	.1244	2.53	3.33
.2663	.1780	1.50	.1351	1.97	.1441	1.85	2.16
.2938	.1690	1.74	.1330	2.21	.1467	2.00	2.42
.3206	.1486	2.16	.1208	2.65	.1367	2.35	2.95
.2694	.1929	1.40	.1463	1.84	.1467	1.84	2.10
.2968	.1881	1.58	.1472	2.02	.1486	2.00	2.35
.3236	.1734	1.87	.1398	2.31	.1472	2.20	2.67
.2485	.1241	2.00	.0951	2.61	.1039	2.39	3.08
.2848	.1181	2.41	.0947	3.01	.1095	2.60	3.48
.3167	.1144	2.77	.0945	3.35	.1137	2.79	3.81
.2518	.1535	1.64	.1163	2.17	.1190	2.12	2.58
.2881	.1423	2.02	.1133	2.54	.1199	2.40	3.08
.3199	.1395	2.29	.1146	2.79	.1257	2.54	3.33
.2552	.1862	1.37	.1394	1.83	.1399	1.82	2.11
.2913	.1630	1.79	.1288	2.26	.1304	2.23	2.76
.3230	.1579	2.05	.1287	2.51	.1333	2.42	3.08
.3508	.1494	2.35	.1246	2.82	.1319	2.66	3.48
mean		1.92		2.43		2.22	2.76
high		2.77		3.35		2.79	3.81
low		1.26		1.72		1.72	1.91

see CALC C

COMPARISON OF METHODS

A comparison was made between seven of the eight methods mentioned earlier. The Bureau of Public Roads method was not included in this analysis because the use of such small riprap material brought calculations to an area in figures 1. and 2. (pg.6) where interpolation became difficult. This method seems best suited for riprap design of larger materials.

A comparison was made between the different methods by looking at the ratio of the $\overline{D50}$ calculated by each method given the hydraulic conditions at riprap failure to the actual D50 present at the time of failure in the lab experiments. The results are shown on pages 20 and 21. Stevens method and Caltrans predict quite accurately what the D50 was at failure, with mean ratios ($\overline{D50}$/D50) of 1.13 and 1.18 respectively. The University of Minnesota study and the Corps of Engineers (COE) methods were acceptable with ratios of 1.61 and 2.51, respectively but Blodgett, ASCE Task Committee, and COE/Waterways Experiment Station methods generously over estimated ~~the~~ with mean ratios of 6.53, 4.85 and 18.8 respectively.

Looking at these results and the table on page 11 showing what variables are considered in each method, it is difficult to make any conclusions. The two methods showing ratios close to one consider specific weight of the riprap material and bed slope as important variables, but then so does the ASCE Task Committees' and COE methods.

In the three methods with high ratios, it is difficult to hypothesize why two of these methods so over predicted. In Blodgetts' method it is probably because only one variable was considered in the riprap design when it's obvious there are at least several other important variables involved. But an equally important reason for Blodgetts over prediction of D50 is that (as stated in his report (5)) this equation was adopted because of its simplicity in use, familiarity among personnel and because of its conservativeness in design. Looking closer at the COE/WES method, its tempting to think this method didn't accurately predict the D50 at failure because the assumptions for its use weren't valid in this particular circumstance. This could be true, however COE's method with a 1.18 ratio assumed uniform flow in a wide rectangular channel as well. Both Caltrans and the ASCE Task Committees design equations are a version of Isbach's proposed formula, however Caltrans prediction of D50 was substantially more accurate. Perhaps the ASCE Task Committee method needs to be modified further.

The COE method was looked at more closely with some interesting results. The COE procedure compares the actual local boundary shear stress (τ_o) with the allowable shear stress (τ_a). Two equations are given for allowable shear: one for the flat channel bed and one for embankments. COE determines the local boundary shear stress using $\tau_o = \dfrac{\gamma \cdot \bar{V}^2}{(32.6\log(12.2\, y/D_{50}))^2}$. This value is compared to τ_a and τ_a' . (see pg. 22) $\tau_o' = \gamma R S_f$ is another way of calculating the average local boundary shear stress. A comparison of the two different ways of calculating the local boundary shear stress with the allowable reveals $\tau_o' = \gamma R S_f$ brings the ratio closer to one than the other equation.

Comparison of calculated $\overline{D50}$ using various methods
with actual D50 at failure point

SLOPE	TYPE OF FAILURE	CALTRANS $\overline{D50}$/D50	BLODGETT $\overline{D50}$/D50	UNIV MINN $\overline{D50}$/D50	COE/ WES $\overline{D50}$/D50	ASCE TASK $\overline{D50}$/D50
4	1	1.19	7.23	1.77	18.6	5.15
4	1	1.15	7.08	1.77	21.2	5.08
4	2	1.17	7.15	1.73	23.5	5.12
4	2	1.21	7.35	1.85	25.3	5.27
4	2	1.09	6.63	1.66	14.5	4.78
4	2	1.06	6.41	1.63	17.4	4.63
4	3	1.09	6.56	1.63	18.4	4.72
4	3	1.13	6.69	1.66	19.9	4.84
4	1	1.03	6.11	1.57	12.3	4.46
4	1	1.08	6.38	1.62	13.8	4.68
4	2	1.05	6.27	1.57	15.4	4.57
4	2	1.00	6.05	1.51	17.0	4.41
3	1	1.27	7.19	1.73	20.6	5.27
3	2	1.27	7.23	1.73	23.5	5.27
3	2	1.27	7.38	1.80	26.0	5.38
3	2	1.22	6.91	1.78	15.8	5.13
3	2	1.22	6.97	1.78	18.1	5.19
3	2	1.16	6.53	1.63	20.5	4.81
3	3	1.08	6.16	1.57	13.6	4.59
3	3	1.14	6.35	1.57	15.4	4.73
3	3	1.11	6.19	1.54	17.3	4.59
2	3	1.31	6.62	1.54	20.0	4.81
2	3	1.35	6.81	1.62	23.8	5.23
2	3	1.39	6.96	1.65	27.1	5.38
2	3	1.19	6.00	1.44	15.6	4.66
2	3	1.22	6.06	1.44	18.8	4.72
2	3	1.25	6.28	1.50	21.3	4.88
2	3	1.16	5.84	1.49	12.9	4.59
2	3	1.14	5.62	1.38	15.8	4.41
2	3	1.16	5.78	1.38	18.1	4.54
2	3	1.16	5.76	1.38	20.2	4.51
	mean	1.17	6.53	1.61	18.8	4.85

Failtype: 1 = bottom only
2 = bottom and side slopes
3 = side slopes only

Comparison of calculated $\overline{D50}$ using various methods with actual D50 at failure point

SLOPE	TYPE OF FAILURE	STEVENS SF=1.0 $\overline{D50}$/D50	STEVENS SF=1.5 $\overline{D50}$/D50	C.O.E. $\overline{D50}$/D50
4	1	1.04	1.77	2.38
4	1	1.04	1.77	2.12
4	2	1.02	1.77	2.00
4	2	1.12	1.88	2.00
4	2	0.97	1.66	2.53
4	2	0.97	1.66	2.13
4	3	0.97	1.66	2.03
4	3	0.97	1.66	2.00
4	1	0.91	1.54	2.51
4	1	0.95	1.62	2.46
4	2	0.92	1.57	2.16
4	2	0.89	1.51	1.89
3	1	1.12	2.19	2.54
3	2	1.14	2.23	2.31
3	2	1.19	2.35	2.23
3	2	1.13	2.22	2.91
3	2	1.16	2.25	2.63
3	2	1.06	2.09	2.13
3	3	1.00	1.95	2.68
3	3	1.00	2.00	2.54
3	3	1.00	1.95	2.22
2	3	1.40	6.70	3.15
2	3	1.46	7.08	2.88
2	3	1.50	7.35	2.73
2	3	1.28	6.25	3.28
2	3	1.30	6.56	2.84
2	3	1.38	6.56	2.72
2	3	1.30	6.49	3.76
2	3	1.22	5.95	2.86
2	3	1.24	6.22	2.68
2	3	1.24	5.95	2.43
	mean	1.13	3.37	2.51

D50 = actual mean rock diameter size at failure.
$\overline{D50}$ = calculated mean rock diameter size at failure.

Comparison of the actual boundary shear stress with the allowable shear stress according to the C.O.E. method.

	channel bed		side slope	
	τ_o/τ_a	τ_o'/τ_a	τ_o/τ_a'	τ_o'/τ_a'
	1.61	1.09	1.74	1.18
	1.50	1.09	1.62	1.18
	1.45	1.08	1.57	1.16
	1.46	1.16	1.57	1.26
	1.64	1.01	1.77	1.09
	1.49	1.02	1.61	1.10
	1.47	1.01	1.58	1.09
	1.45	1.02	1.57	1.11
	1.61	0.95	1.74	1.02
	1.61	0.99	1.74	1.07
	1.51	0.96	1.63	1.04
	1.39	0.94	1.50	1.01
	1.57	1.06	1.80	1.22
	1.50	1.08	1.72	1.24
	1.47	1.14	1.69	1.31
	1.68	1.08	1.93	1.24
	1.60	1.10	1.84	1.26
	1.42	1.02	1.63	1.17
	1.58	0.95	1.82	1.09
	1.55	0.96	1.78	1.10
	1.44	0.95	1.65	1.09
	1.47	0.96	2.05	1.33
	1.42	1.01	1.97	1.40
	1.38	1.05	1.92	1.46
	1.48	0.89	2.06	1.24
	1.38	0.90	1.92	1.25
	1.36	0.94	1.89	1.31
	1.56	0.90	2.17	1.25
	1.37	0.84	1.91	1.17
	1.33	0.86	1.86	1.20
	1.27	0.85	1.76	1.19
mean	1.48	0.995	1.77	1.19

τ_a = allowable shear stress on bed channel
τ_a' = allowable shear stress on side slope

τ_o = actual local boundary shear stress according to COE: $\dfrac{\gamma \cdot \bar{V}^2}{(32.6 \log(12.2 \, y/D_{50}))^2}$
τ_o' = actual shear stress according to: $\gamma R S_f$

$\bar{V} = 1.6(V_{mean})$

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CONCLUSIONS

It is a well know fact that many riprap design methods are overly conservative. From a bank protection standpoint this may be acceptable, but from an economic standpoint it is not. Through a fuller understanding of the variables and factors effecting erosion, the assumptions and limitations of current riprap design equations and there application to riprap failure, perhaps an economic benefit can be realized.

This paper compared seven different riprap design methods and found Caltrans and Stevens method to most closely predict the riprap material mean diameter size at the point of failure. The University of Minnesota study and COE over estimated by about a factor of two while the other three methods; Blodgett, COE/Waterways experiment station and ASCE Task Committee generously over estimated the riprap material size at failure.

As a means for a fuller understanding of the differences and similiarities of the various riprap design methods, it is suggested similiar studies like this one be done using a different data set.

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CALC A

Energy Equation

assumes: hydrostatic pressure distribution

$$S_{fe} \cdot L = d_1 + \frac{Q^2}{2gA_1^2} + \Delta z \cdot L - d_2 - \frac{Q^2}{2gA_2^2}$$

L = channel length (10 ft)
d = depth
S_o = bed slope
Q = discharge
A = area

for slope: 4:1 $A = 5d + 4d^2$
3:1 $A = 5d + 3d^2$
2:1 $A = 5d + 2d^2$

$\Delta z = S_o \cdot L = (.008)(10) = .08$

Mannings Equation

$$S_{fm} = \left[\frac{\bar{V} \cdot n}{1.486 (A/P)^{2/3}}\right]^2$$

$n = .0395 (D_{50})^{1/6}$ strickler

P = wetted perimeter

for slope: 4:1 $P = 5 + 2\left[\frac{d}{\sin(14.036)}\right]$

3:1 $P = 5 + 2\left[\frac{d}{\sin(18.435)}\right]$

2:1 $P = 5 + 2\left[\frac{d}{\sin(26.565)}\right]$

solve for S_{fm} upstream and downstream conditions then take average of the two.

CALC B

Mannings Equation

$$Q = \frac{1.486}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}$$

at normal depth $S = S_o$

$n = .0395(D_{50})^{1/6}$

$$\frac{A^{5/3}}{P^{2/3}} = \frac{Q \cdot n}{1.486 \cdot \sqrt{S_o}} = C$$

4:1 $$C^{3/5} = \frac{(5d+4d^2)^{5/3 \cdot 3/5}}{(5+8.24635d)^{2/3 \cdot 3/5}}$$

4:1 $$C^{3/5} = \frac{(5d+4d^2)}{(5+8.24635d)^{2/5}}$$

SLOPE

4:1 $$d = \frac{C^{3/5}(5+8.24635d)^{2/5} - 4d^2}{5}$$

3:1 $$d = \frac{C^{3/5}(5+6.324538d)^{2/5} - 3d^2}{5}$$

2:1 $$d = \frac{C^{3/5}(5+4.472144d)^{2/5} - 2d^2}{5}$$

Program (HP-15C)

$\boxed{d}$

STO 2

8.24635 X — (6.324538X) — (4.472144 X) ← subst. for different slopes

5+

0.4 y^x

RCl 1

0.6 y^x

X

RCL 2

g X^2

4X — (3X) — (2X) substitute for different slopes.

CHS +

5÷

cont.

RC 2 +

2÷

g RTN

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CALC C

$\tau_n = \gamma R_n S_o$

$\tau_o = \gamma R S_{fe}$

$\tau_e = \gamma R S_o$

$S_o = 0.008$

S_f = energy slope from energy eqn.

γ = 62.4 lb/cu.ft.

R = hydraulic radius (actual depth)

R_n = hydraulic radius (normal depth)

τ_n = shear stress at normal depth

τ_o = shear stress at actual depth

τ_e = shear stress: rough estimate

SLOPE

4:1 $$\frac{A}{P} = \frac{5d + 4d^2}{5 + \left(\frac{2d}{\sin(14.036)}\right)}$$

3:1 $$\frac{A}{P} = \frac{5d + 3d^2}{5 + \left(\frac{2d}{\sin(18.435)}\right)}$$

2:1 $$\frac{A}{P} = \frac{5d + 2d^2}{5 + \left[\frac{2d}{\sin(26.565)}\right]}$$

INVESTIGATION OF RIVER BED ARMORING

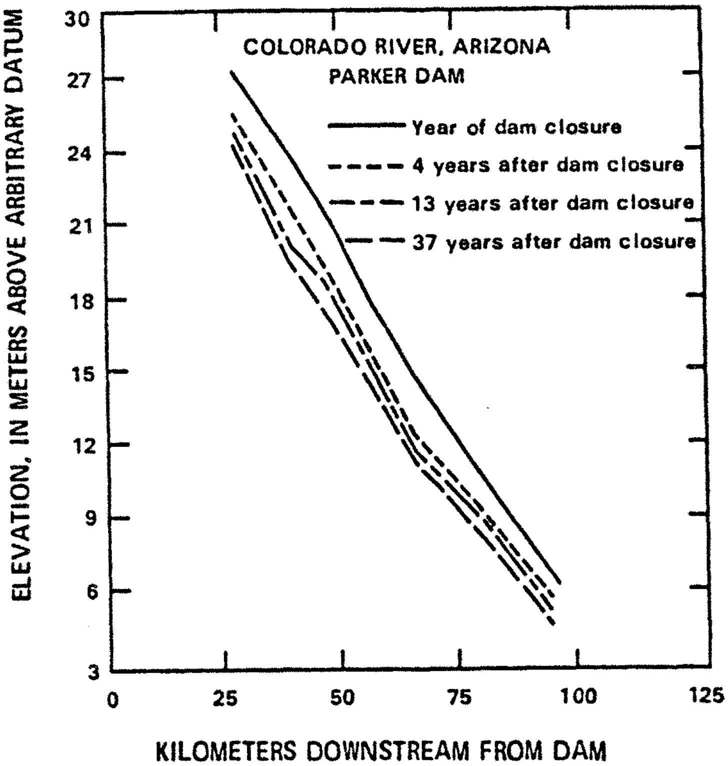
by : Mahmood Shafai-Bajestan

ABSTRACT : The process of bed armoring is investigated and 8 methods for prediction of armor coat are presented.These methods are considered into two groups :group A,including Gessler,Little & Mayer 72 and 76,Davies,and Shen & Lu's approach. Group B including Ashida et al, Bayazit and Lee et al. Wide range of data are obtained from four investigators and are used to evaluate the methods in group A. It was found that Little & Mayer's approach overestimate significantly while Shen & Lu's method is recommenden. The methods in group B are discussed based on their own results.

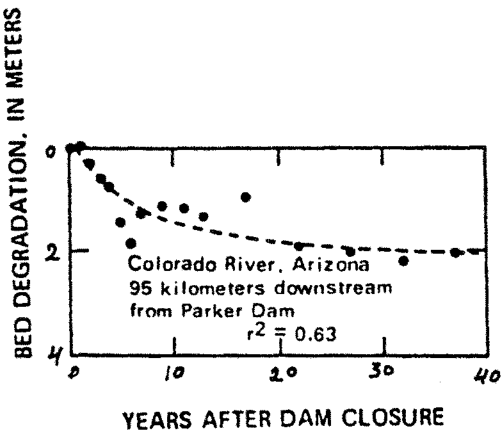
INTRODUCTION : The sediment load for flow over the bed consist of non-uniform material is finer than the bed material(Vanoni 1977),therefore as the bed degrades its bed will coarsen. When the flow condition especially the bed shear stress is such that coarser fractions of the bed material do not move , a layer of stable material called "armor coat" or "paved layer" will be formed . The term armor coat usually is used for the gradual coarsening of a river bed downstream of a dam(Gessler), and the term paved is the consequence of coarsening in the gravel bed river because of gradual reduction of its flow capacity during dry seasons(Milhous,Bray and Church)

As a result of this stable layer drastic reduction in the rate of sediment discharge ;consequently, degradation will occur. As an example degradation on the Missouri River downstream from Fort Randall Dam reported by Livesey(1965) which was expected to occur about one foot per year but after ten years the bed lowered only 3 ft because of formation of armor coat on the bed surface. Fig.1 is another example of downstream effect after closur of the Parker dam . Fig 1a and 1b show the general bed degradation downstream of the dam with time. As it can be seen the rate of degradation will decrease with time and finally stop (Fig.1b) because

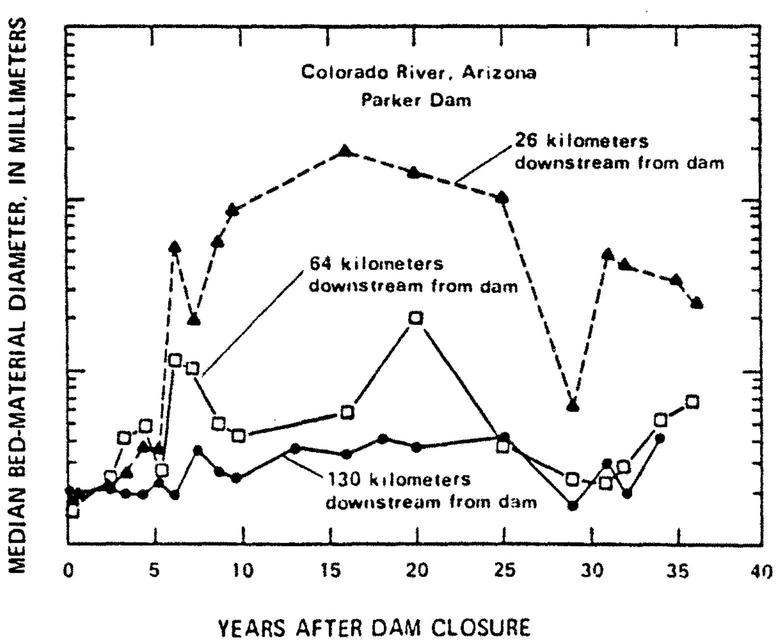
Fig. 1 : DOWNSTREAM EFFECTS OF DAMS (Williams etal 1984)



a) Degradation downstream of Parker Dam



b) Degradation at one station.



c) Variation of bed material with Time

of gradull increase in the size of bed material(Fig.1c) thereby formation of the armor coat .

The prediction of the armor layer based on the initial flow and bed material conditions is a subject which has been received considerable attention of the past three decades researchers . The result is some predictor methods which mostly are empirical . But because of the complexity of the phenomenon a lack of a general method to be in agreement with both field and flume data is obvious and needs furthur study.

Basicly these methods can be considered into two groups :

Group A : Those methods which are using the initial condition to predict the armor coat without being concern about the process of bed degradation such as : Gessler, Little & Mayer, Davies, and Shen & Lu's approach.

Group B : Those methods which simulate the armor coat and the rate of degradation simultaneously and are: Ashida & Michiue, Bayazit ,and lee & Odgaard's methods.

The primary object of this study is to present those methods in groups A and B. Secondary to compare the methods in group A using reasonable data ,While because of limitation time for this study the methods in group B will be discussed based on their own results.

Literature Review

Composition of the armor coat : Harison (1950) studied bed armoring in a flume by gradual degrading the bed . Three different bed material having logarithmic normal distribution were used . Harrison found that formation of armor coat, the thickness of one particle, increases the resistance to flow and reduce the sediment discharge to about 1% or less . The same conclusion were found by Lane and Carlson(1953) in the study of the San Luis Canals in Southern Colorado, which had been in used since the late 1800s Subsequent studies by Little and Mayer (1972),Proffitt(1980) not only confirmed the above finding but also that the armor coat is composed of all particle size in the original bed material.

Prediction of the armor coat, Group A : Gessler first in 1965 and subsequent studies in 1970,and 1971 proposed a procedure for prediction of the armor coat based on propabilistic approach . Gessler on the study of incipient motion on the bed having non-uniform material exposes to a constant mean shear, argued that transport of grains depen on the instantaneous shear exerted on the particles. Gessler found that this shear fluctuate around the mean bed shear . From his experiments Gessler found that the ratio τ_{c}/τ_{o} in which τ_{ci}=the critical shear stress (correspond to particle size d_i) and τ_o =the mean bed shear stress, can be approximated by a Guassian normal distribution with a standard deviation equal to 0.57, a value which was assumed to be constant for any flow and sediment bed size .

Gessler also found that the probability of a particle to stay (probability of grain remaining stationary),q, when τ_{c}/τ_{o}=1 is equal 0.5 . Based on the above discussion the probability of a grain to stay ($\tau_{c}/\tau_{o} \leq 1$) can be determined from Guassian equation or :

$$q\left(\frac{\tau}{\tau_c}<1\right)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{(\tau_c/\tau)-1}\exp\left(-\frac{x^2}{2\sigma^2}\right)dx \quad \text{..........} (1)$$

where σ=standard deviation (0.57), and x=dummy variable.

Gessler developed a method to simulate the grain size distribution of the final armor coat given the initial sediment size distribution and flow condition based on Eq.1 . The final equation is :

$$P_a(d_i)=\left(\int_{d_{min}}^{d} q_i\,P_o(d_i)\,dd\right)\Big/\left(\int_{d_{min}}^{d_{max}} q_i\,P_o(d_i)\,dd\right) \quad \text{.....} (2)$$

where $P_a(d_i)$=fraction in the size range of the armor coat. $P_o(d_i)$=fraction in ith size range of the initial sediment mixture, and q_i is the correspon-ding probability of stay . d_{min} and d_{max} are the minimum size and maximum size ,respectively, of the initial bed material.

Gessler also suggested that the mean value of the probabilities for the armor coat grains to stay , $\bar{q}$,may be utilized as a stability criterion of the armor coat :

$$\bar{q}=\sum_{i=Min}^{max} q_i\,P_a(d_i)=\left(\sum_{i=Min}^{max} q_i^2\,P_o(d_i)\right)\Big/\left(\sum_{i=Min}^{Max} q_i\,P_o(d_i)\right) \quad (3)$$

A value of $\bar{q}$=0.50 was suggested as the critical value above which the armor coat would be stable . For design criteria of stable irrigation canal,using factor of safety of 1.3, a value of $\bar{q}$=0.65 was recommended.

Little and Mayer(1972), conducted a series of experiments in a flume with six different bed materials . The charactrestics of material were :1) logarithmic normal distribution 2)d_{50}=1.0mm , and 3)σ_{g_o} ,geometric standard deviation of bed material ranged from 1.12 to 3.05 . The armoring process was considered to be completed when the final sediment transport rate was less than 1% of the initial transport rate . The objective of the study was to determine the effect of sediment gradation on bed armoring .

Little and Mayer argued that the armoring process is a incipient motion problem and they found the following general equation by means of dimensional analysis :

$$f\left(\frac{u_* d_{ga}}{\nu} \; , \; \frac{u_*^2}{(S_s - 1) g d_{ga}} \; , \; \frac{d_{ga}}{d_{go}\, \sigma_{go}} \right) = 0 \qquad (4)$$

Where u_*=shear velocity , ν =kinematic viscosity , S_s =specific gravity of sediment material , σ_{go}=geometric standard deviation of the original bed material, and d_{go},d_{ga} are the geometric mean size of original and armor material .

Little and Mayer found that the first and second dimensionless parametrs of the above equation is linearly related for a particular material and combined those terms, giving a new relation :

$$\frac{d_{ga}}{\sigma_{go}\, d_{go}} = f\left(\frac{u_*^3}{(S_s - 1) g \nu} \right) \qquad (5)$$

The results obtained by Little and Mayer and correlation parameters of the above equation led the following relation which can be used to predict d_{ga}.

$$\frac{d_{ga}}{\sigma_{go}\, d_{go}} = 0.908 \left[\frac{u_*^3}{(S_s - 1) g \nu} \right]^{0.353} \qquad (6)$$

To predict the geometric standard deviation of the armor coat they assumed that σ_{ga} is independent of the flow characters and is a function of only the standard deviation of the original bed material . From their experiments they proposed the following equation to predict σ_{ga}

$$\sigma_{ga} = 1.317\, \sigma_{go} - 0.2485\, \sigma_{go}^{2.0} \qquad (7)$$

Davies (1974) used data from Little and Mayer(1972),Lane and Carlson (1953),and his own experiments in an attempt to improve the Little and Mayer 's approach. The final relations which were proposed are:

$$\frac{d_{ga}}{\log(d_{go}\ \sigma_{go}^{2})} = 1.839 \left[\frac{u_*^3}{(S_s - 1)g\nu}\right]^{0.389} \quad \cdots\cdots\cdots (8)$$

$$\left(\frac{d_{go}}{\sigma_{go}}\right)^{\frac{1}{2}} \frac{d_{ga}}{\sigma_{ga}} = 0.350 \left[\frac{u_*^3}{(S_s - 1)g\nu}\right]^{0.789} \quad \cdots\cdots\cdots (9)$$

Where d_{go} and d_{ga} are in millimeteres .

Little and Mayer(1976), used their own previous data and developed the following empirical equations :

$$d_{ga}=0.530\,\sigma_{go}^{0.58}\ u_*^2 \qquad \text{(English units)} \qquad (10\text{-}1)$$

$$d_{ga}=1.740\,\sigma_{go}^{0.58}\ u_*^2 \qquad \text{(SI units)} \qquad \cdots\cdots (10\text{-}2)$$

Little and Mayer found an expresion for definning of the maximum geometric mean diameter for which armoring can occur, $d_{ga\ max}$, which is :

$$d_{ga\,max} = d_{go}\ \sigma_{go}^{1.645} \qquad \cdots\cdots (11)$$

For given flow condition (known u_*) and bed material if d_{ga} calculated from Eq. 8 is less than $d_{ga\ max}$,obtained from Eq. 10, then the bed will armor ;otherwise, because of shear velocity is such that can move even the larger particles , the armor coat cannot be expected .

Shen and Lu(1983),used Little and Mayer(1972)'s data to develope a method to simulate the size distribution of armor coat based on Gessler's method with 3 modifications .Shen and Lu argued that because of non-unifomity of the bed material the smaller particles not only shelterd by the larger ones but they are exposed to lift force difference than the lift on the larger particles because the smaller particles are in laminar sublayer of larger ones .Consequently, the effect of graded material should be considered in simulation of armor coat . Shen and Lu used the modified Einstein's hiding factor to account for non-uniformity of the material. Based on this discussion they proposed the following relation for determination of critical shear

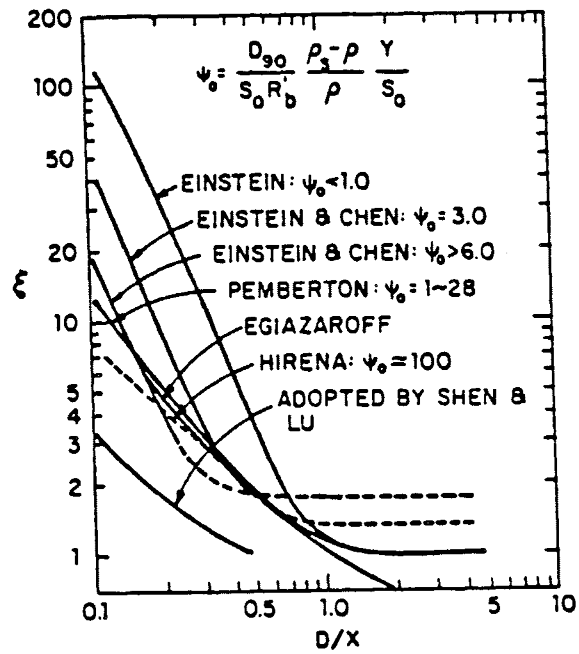


Fig. 2:
Hiding factor of sediment particles in mixture (Shen and Lu, 1983)

σ_{g_o}	1.5	2.0	2.5	3.0	3.05	3.19
σ	0.35	0.35	0.35	0.40	0.41	0.45

Table A: Variation of σ with σ_{g_o} (Shen and Lu, 1983)

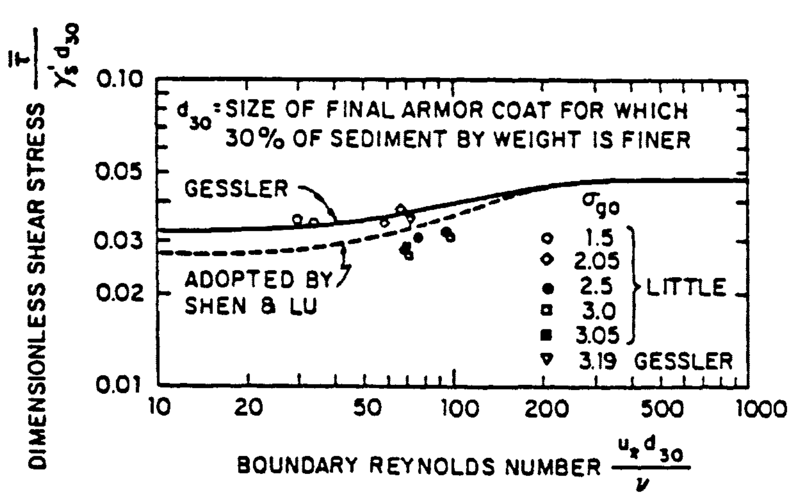


Fig 3: Modified Shields' curve (Shen and Lu, 1983)

stress of each grain size in the mixture :

$$\tau_{cn} = f\,\tau_c \qquad (12)$$

where τ_{cn}=critical shear stress of a grain size d_i in the mixture, f=the modified Einstein's hiding factor(Fig. 2), and τ_c=critical shear stress of a grain size d_i of uniform material(Fig. 3).

The second modification ,which was based on data they used, is modified Shields diagram (Fig.3). For the third modification they argued that σ in Eq.1 is not constant .Furthurmore they found that σ is a variable depending on the value of σ_{go} .Table A shows variation of σ with σ_{go} proposed by Shen and Lu .

Shen and Lu also proposed the following empirical relations which were obtained from regressional analysis :

$$\frac{d_{50a}}{d_{50o}} = 0.8525\left(\frac{\tau_o}{\tau_c}\right)^{0.4556}\left(\sigma_{go}\right)^{0.885} \qquad (13)$$

$$\frac{d_{30a}}{d_{30o}} = 0.658\left(\frac{\tau_o}{\tau_c}\right)^{-0.404}\left(\sigma_{go}\right)^{2.29} \qquad (14)$$

$$\frac{d_{84a}}{d_{84o}} = 1.189\left(\frac{\tau_o}{\tau_c}\right)^{0.710}\left(\sigma_{go}\right)^{-0.20} \qquad (15)$$

Where $_c$=is based on d_{50o} , d_{30o}, and d_{84o} .

Prediction of the armor coat, Group B: The basic equation which these methods are based on comes from the concept of an active layer in which the weight of remaining material per unit area during degradation of time interval t , is the differnce between the weight of the original and the eroded material pluss replacement of the same fraction of the material in original material from the sublayer. Infact it is assumed that the weight of particles per unit area during the process of degradation remains constant.

This concept in matematical form can be written as :

$$p_a(d_i)=P_o(d_i)-fP_e(d_i)+f\,P_o(d_i) \qquad (16)$$

or:

$$P_a(d_i)=(1+f)P_o(d_i) - fP_e(d_i) \qquad (17)$$

Where: $P_a(d_i)$, $P_o(d_i)$, and $P_e(d_i)$ are the fraction of the sizes d_i present in the partial armor at the end of time step, the original bed material ,and the eroded material respectively . f is a value related to the amount of bed degradation and its computation is differ from one method to another .

The above equation shows that as the bed degrads the composition of the active layer coarsens, until the critical shear becomes great enough that the new layer does not move .

Ashida and Michiue(1971), they used Kkikawa and Ashida's bed load equation with combination of sediment continuity equation and assuming quasi uniform flow to determine the rate of degradation and the composition of the armor layer . The value of f was determined from the rate of degradation during each time step(Δz) and the thickness of armor coat which was assumed to be equal $d_o = \dfrac{\int_{d_c}^{d_{max}} d_i\, P_o(d_i)\, dd}{\int_{d_c}^{d_{max}} P_o(d_i)\, dd}$ where d_c is the grain size in the threshold of movement . Hence the value of f will be :

$$f = \frac{\Delta z}{d_0} \qquad (18)$$

Bayazit(1975), used Einstein's bed load equation with modified of hiding factor . Bayazit proposed the following relation for f :

$$f = \frac{q_s\ \Delta t}{G\ \ L} \qquad (19)$$

where q_s is the bed material discharge per unit width of canal, Δt is the time interval , G is the total weight of the original bed material per unit are ,and L is the length of reach or a segment of that in which computation is being done .

Lee and Odgaard(1986), used the Bayazit's equation for computation of f but they argued that the total weight of the particle per unit area or G is not constant during the process of degradation and they proposed the following equation for computing G :

$$G = \gamma_s T \qquad (20)$$

where γ_s is the unit weight of the bed sediment material ,T is the thickness of the mixing layer which is T= 1/2 H (1-C), H=the height ofdunes given by Yalin's equation [H =D/2(1- $\frac{\tau_c}{\tau_o}$),D= flow depth]. The value of C is in the range of 0 to 1 and should be determined from experiments . Lee and Odgaard used the Einstein's bed load equation with modified his hiding factor to calculate q_s . From analysis of Little and Mayer's data the best value for C was found to be equal 0.30 .

Data : Data for this study were obtained from the following investigators :

Lane and Carlson(1953), They measured the hydraulic and material characteristics of San Luis Canals in an effort to develope relation for stable canal. Samples of the material in which the canal was constructed were taken from the banks and the composition of the armor coat taken from analysis of the bed material . For the present study data from the test section -6 and test section-12 were selected .

Gessler(1965), data from run 1-5 was selected to be used in this study

Little and Mayer (1972),They study the armoring process in a recirculating flume ,1.969 ft wide and 40ft long . The flow conditions(Q,u_*) were

Table 1 : Bed Material Data

Lane & Carlson Reach 6		
D_{50} = 11.5 mm, σ_{g_o} = 4.69	D_{50a} =	
PARTICLE SIZE (MM)	PERCENT FINER	
	INITIAL	ARMOURED
0.074	3.00	0.00
0.149	4.7	0.00
0.297	7.8	0.00
0.595	11.8	0.00
1.19	22.00	0.00
2.38	31.50	0.00
4.76	36.6	0.00
9.51	46.90	2.40
19.0	61.30	7.90
38.1	77.30	33.20
76.10	100.00	99.40

Lane & Carlson Reach 12		
D_{50} = 6.00mm, σ_{g_o} = 4.50	D_{50a} =	
PARTICLE SIZE (MM)	PERCENT FINER	
	INITIAL	ARMOURED
0.074	2.40	0.00
0.149	4.00	0.00
0.297	9.70	0.00
0.595	19.80	0.00
1.190	31.80	0.00
2.38	41.00	0.00
4.76	46.40	0.00
9.510	59.70	1.00
19.00	77.60	9.00
38.10	90.80	61.00
76.10	100.00	100.00

Gessler Run 1-3		
D_{50} = 1.00mm, σ_{g_o} = 3.19	D_{50a} = 3.20mm	
PARTICLE SIZE (MM)	PERCENT FINER	
	INITIAL	ARMOURED
0.25	8.00	2.00
0.40	16.00	3.00
0.50	30.00	5.00
0.80	40.00	8.00
1.00	50.00	10.00
1.50	62.00	16.00
2.00	70.00	23.00
2.40	77.00	40.00
3.10	83.00	47.00
3.500	88.0[illegible]	62.0[illegible]
4.10[illegible]	93.[illegible]	[illegible]
5.200	[illegible]	[illegible]
6.000	[illegible]	[illegible]

Ashida et al. Expereince 2		
D_{50} = 2.47 mm, σ_{g_o} = 3.73	D_{50a} =	
PARTICLE SIZE (MM)	PERCENT FINER	
	INITIAL	ARMOURED
0.200	0.00	0.00
0.300	9.50	3.30
0.40	19.50	10.00
0.60	30.00	18.00
0.80	36.00	19.00
1.00	40.00	21.00
2.00	52.00	24.00
4.00	70.00	45.00
6.00	88.00	86.00
8.00	98.00	96.00

Little & Mayer Run 3-4		
D_{50} = 1.0mm, σ_{g_o} = 2.50	D_{50a} = 3.05 mm, σ_{g_a} = 1.72	
PARTICLE SIZE (MM)	PERCENT FINER	
	INITIAL	ARMOURED
0.125	0.00	0.00
0.177	2.90	0.00
0.250	6.00	0.00
0.354	12.52	0.60
0.500	22.00	0.90
0.707	35.00	1.70
1.00	50.00	3.40
1.414	64.00	7.60
2.00	76.10	23.70
2.83	87.4	44.80
4.00	93.40	68.70
5.67	97.00	87.20
8.00	100.00	100.00

Little & Mayer Run 6-1		
D_{50} = 1.0 mm, σ_{g_o} = 3.05	D_{50a} = 3.05 mm, σ_{g_a} = 1.72	
PARTICLE SIZE (MM)	PERCENT FINER	
	INITIAL	ARMOURED
0.125	3.00	0.00
0.177	6.07	3.00
0.250	10.54	3.40
0.354	17.54	3.80
0.500	26.98	4.60
0.707	38.31	5.70
1.00	50.05	8.20
1.414	61.75	13.70
2.00	73.02	24.90
2.83	82.43	45.20
4.00	89.30	68.00
5.67	93.93	84.90
8.00	96.99	97.00

Table 2 : Proffit's Data

Run No.	Shear Velocity m/sec	Kinematic viscosity m²/sec x10⁻⁶	Original Bed Material		Armor Coat	
			d_{g_o} mm	σ_{g_o}	$d_{g_{a}}$ mm	σ_{g_a}
1-6	0.044	1.10	2.95	2.26	4.24	2.50
1-4	0.060	1.09	2.95	2.26	6.52	2.10
2-1	0.057	1.10	3.30	3.24	10.0	2.50
2-3	0.071	1.10	3.30	3.24	11.0	2.45
3-1	0.072	1.10	3.28	2.78	8.83	2.15
3-4	0.071	1.10	3.28	2.78	8.69	2.07
4-2	0.056	1.13	2.83	1.95	5.37	1.95
4-4	0.075	1.13	2.83	1.95	7.55	1.99

TABLE 3 : FLOW CONDITIONS OF DIFFERENT DATA WERE USED IN THIS STUDY.

	REFERENCE	DISCHARGE (cfs)	VELOCITY (fps)	DEPTH (ft)	WIDTH (ft)	BOTTOM SLOPE	SIDE SLOPE
C	Lane & Carlson Reach 6	159.00	4.59	1.88	15.15	0.00295	1.742
C	Lane & Carlson Reach 12	128.00	4.00	1.77	13.10	0.00240	2.300
F	Gessler Run 1-5	1.112	1.48	0.229	3.281	0.00199	0.00
F	Ashida et al. Exp. 2	1.06	1.81	0.223	2.630	0.0044	0.00
F	Little & Mayer Run 3-4	0.572	1.214	0.217	1.969	0.0019	0.00
F	Little & Mayer Run 6-1	0.448	1.236	0.184	1.969	0.0020	0.00

F= FLUME C= CHANNEL

kept constant during each run .Data from run 3-4 and run 6-1 were chosen to be used in the present study .

Ashida and Michiue(1971),carried out several laboratory experiments in a flume,2.63ft wide and 65.62ft(20m) long .The average size and the geometric standard deviation of the material were 2.47mm and 3.73 respectively .Data in Exp. 6 are used in this study.

Proffitts(1980),conducted a series of experience with four different bed materials having lognormal grading with a maximum size of 38mm. The armoring occured under constant hydraulic conditions . 8 set of data were selected for this study which are shown in table 2.

Analysis of Data : To compare the prediction with measured the geometric mean size of all data are computed using all methods in group A . To simulate the size distribution of the armor coat by Gessler and Shen etal's methods ,a computer program was written based on the procedures discussed in the previous section. The proffitts data are not used in this part beccause of lack of information which were available . The results reported by methods in group B are replotted and will be discussed. The following discussion is based largely on the results obtained in this study .

Methods of group A :

Prediction of d_{ga} : Table 5 and table 6 show the prediction of the geometric size of the armor coat . A comparison between prediction and measured shows that Little and Mayer overestimate significantly ,especially when shear velocity becomes high(Lane and Carlson's data) . Davies' method on the other hand underestimate slightly. Among these methods Gessler and Shen etal are in good agreement with the measured data .

Table 5 : prediction of mean geometric size of armor coat by various methods.

Reference	Shear velocity fps	Measured bed material			Mean geometric size of armor coat (d_{ga}) mm				
		σ_{go}	d_{go} mm	d_{ga} mm	Gessler 1971	Little&Mayer 1972	Davies 1974	Little&Mayer 1976	Shen & Lu 1983
Lane & Carlson Sec. 6	0.453	4.69	20.80	43.70	42.80	512.70	33.90	81.20	47.90
Lane & Carlson Sec. 12	0.403	4.54	13.1	36.2	33.9	276.2	27.0	63.1	40.7
Gessler Run 1-5	0.120	3.19	1.56	3.2	3.1	6.4	3.24	4.6	3.4
Ashida et al Exp. 6	0.27	3.73	2.62	5.18	3.7	29.7	10.9	2.53	4.3
Little & Mayer Run 3-4	0.11	2.5	1.49	3.39	3.2	4.3	2.4	3.33	3.9
Little & Mayer	0.11	3.05	1.68	3.36	3.9	9.9	2.9	3.73	4.4

Note: ν Kinematic viscosity for all data was assumed to be equal 1.21×10^{-5} ft²/sec

$\frac{\rho_s}{\rho}$ was assumed to be equal 2.65 for all data.

Table 6 : Predicted the mean diameter and standard deviation of armor coat

Run NO.*	Little & Mayer (1972)		Little and Mayer (1976)		Davies	
	d_{ga} mm	σ_{ga}	d_{ga} mm	σ_{ga}	d_{ga} mm	σ_{ga}
1-6	10.52	1.71	5.4	1.71	3.98	3.78
1-4	14.61	1.71	10.1	1.71	5.72	2.61
2-1	22.19	1.66	17.04	1.66	7.04	3.20
2-3	28.00	1.66	17.3	1.66	9.10	2.46
3-1	26.70	1.74	16.0	1.74	8.4	2.36
3-4	26.3	1.74	15.9	1.74	8.3	2.41
4-2	11.24	1.62	8.0	1.62	4.6	2.16
4-4	15.3	1.62	14.4	1.62	6.5	1.84

* Proffitt's data

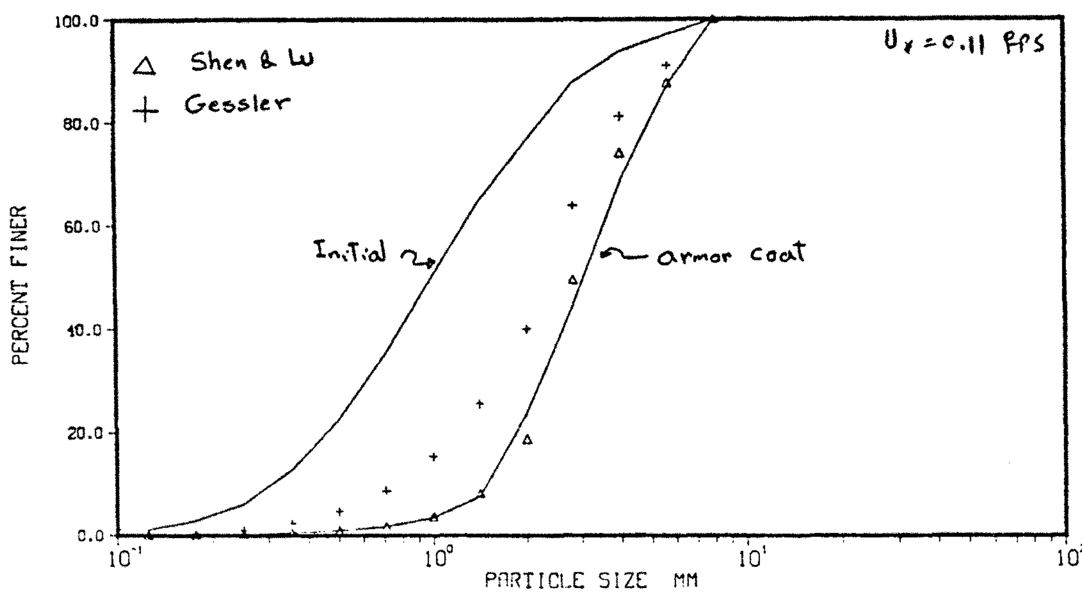
Comparison of Gessler and Shen et al's methods is shown in Fig. 4 through Fig. 6 . As it shown Gessler's theory can simulate quite well, the size distribution of the final armor coat he obtained in the laboratory flume(Fig. 5) but it predicts a much finer layer when applied for other data,Little etal(4 Fig.),Lane etal(Fig. 6), and Ashida etal(Fig.7C). Shen et al on the other hand predict quite well for all data.

One of the basic assumption in Gessler's theory is that the ratio τ_c/τ_o is normally distributed, infact for a given flow condition τ_o is constant and for given material $\tau_c \alpha T_* d_i$ where T_* is the Shields parameter . For large value of R_{e*},large particle size or shear velocity, T_* is constant and therefore This relation shows that for material having normal distribution the ratio τ_c/τ_o also can be considered to be noramlly distributed (that is why good agreement obtained from Gessler's method for Lane etal's data sec.6) . But for finer material or when u_* is small(all Little et al's data) T_* is no longer constant and is a function of R_{e*} and d_i and so the ratio τ_c/τ_o cannot be assumed to be normally distributed ;consequently, Gessler's theory fails .To improve this difficulty the ratio τ_c/τ_o should be modified so it follows the normal distribution curve especially for lower R_{e*} . Shen etal used the hiding factor coefficeint which was obtained based on the data they used .

Another assumption in Gessler's theory is that σ was assumed to be constant which later it was found it is not .Shen etal in their study found it varies with geometric standard deviation of the material, σ_{g_o},and they proposed table . The table does not contain all range of data e.g. for $\sigma_{g_o} > 3.19$. and so in this range the table should be modified. In the present study vide range of σ values were used in order to determine the best value of σ to fit

Fig. 4 :Comparison between Gessler and Shen & Lu's approaches

a) LITTLE & MAYER RUN 3-4



b) LITTLE & MAYER RUN 6-1

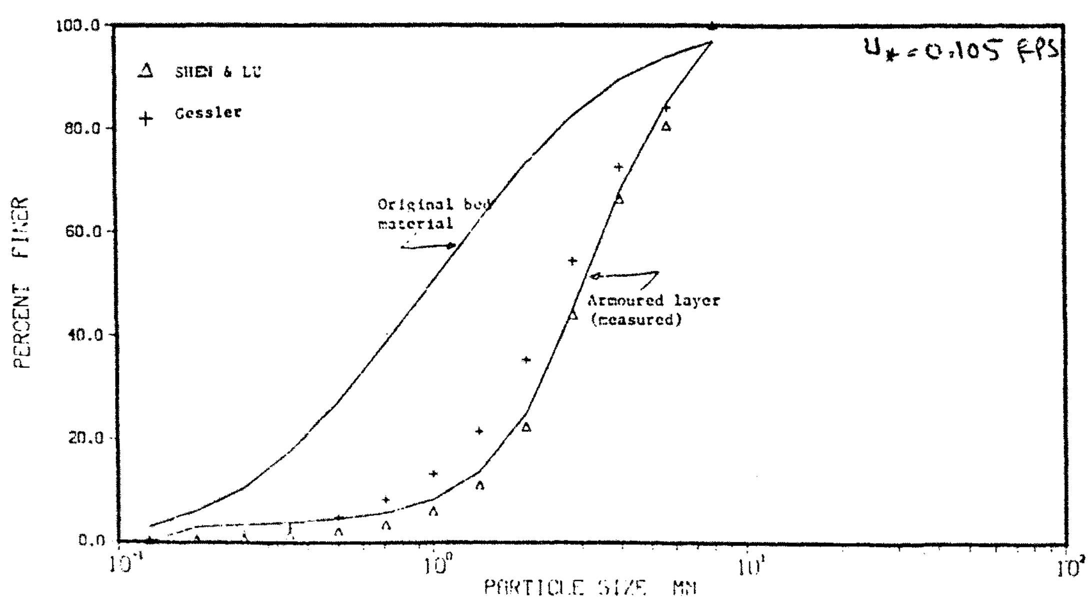
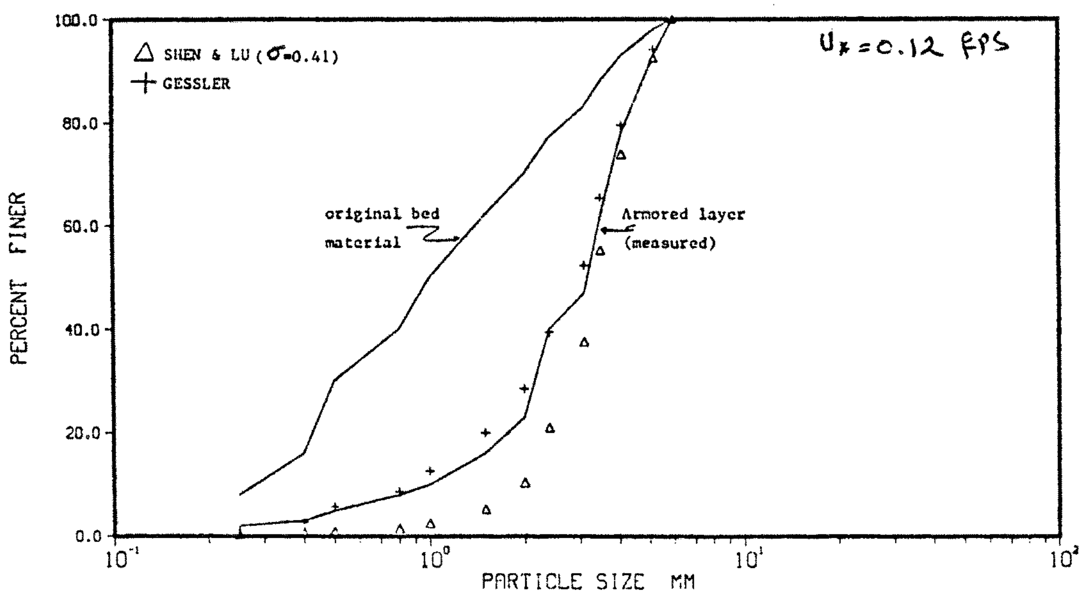
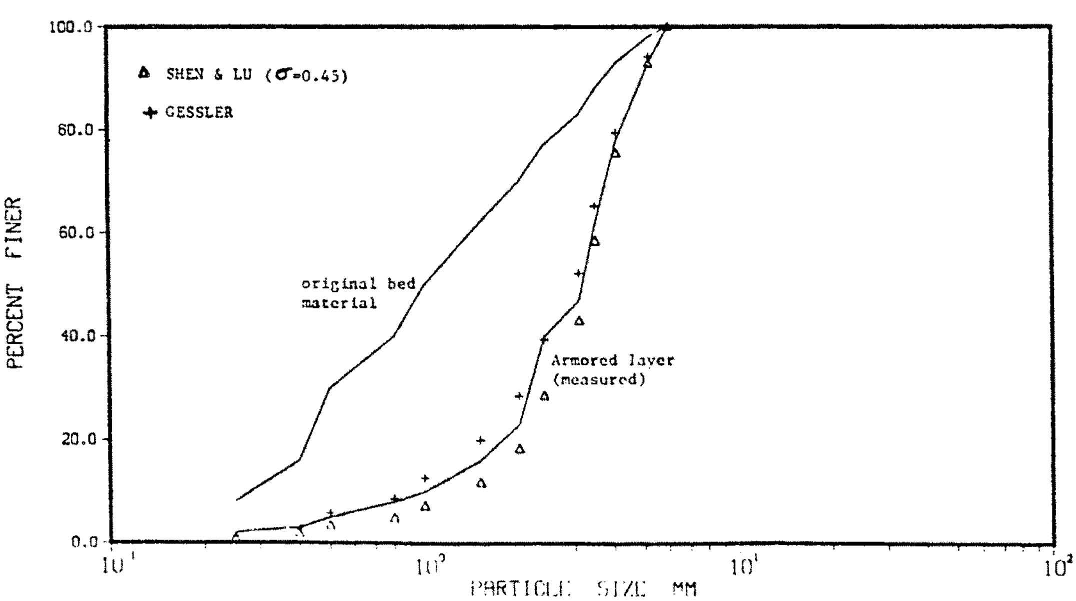


Fig. 5 : Comparison between Gessler and Shen & Lu's approaches

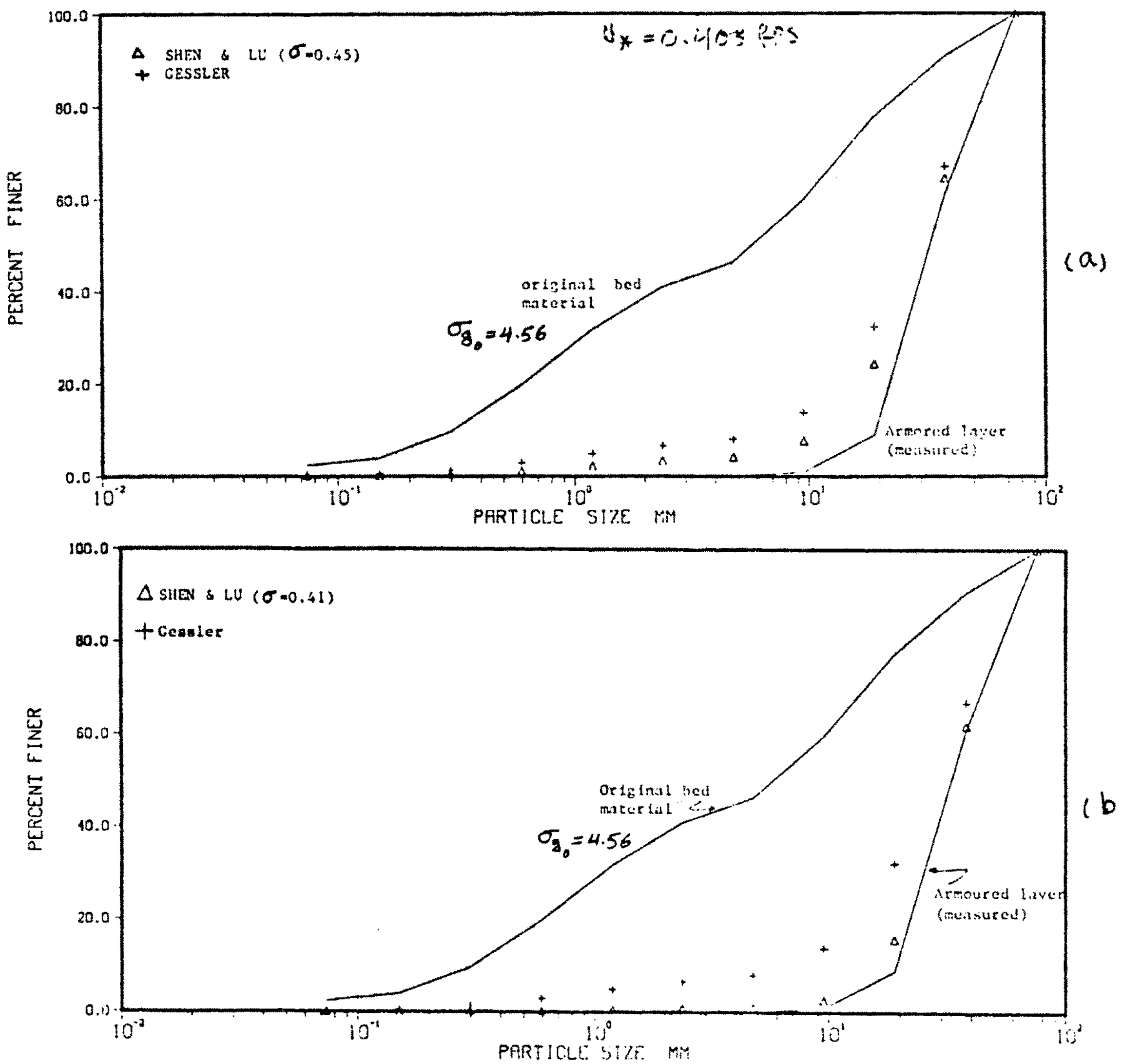
a) GESSLER'S DATA RUN 1-5



b) GESSLER'S DATA RUN 1-5



LANE & CARLSON SEC. 12



LANE & CARLSON SEC. 6

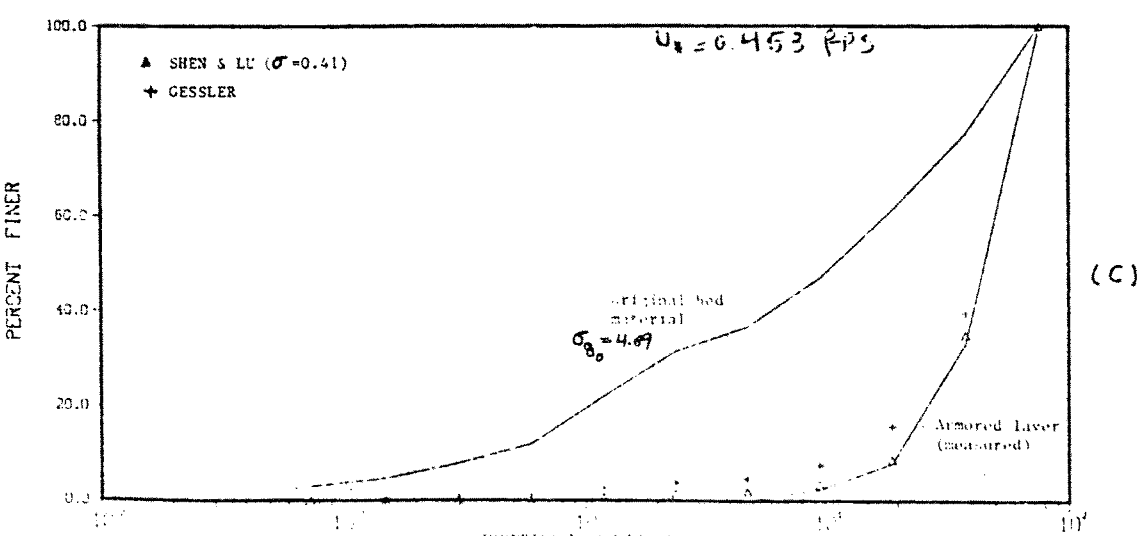


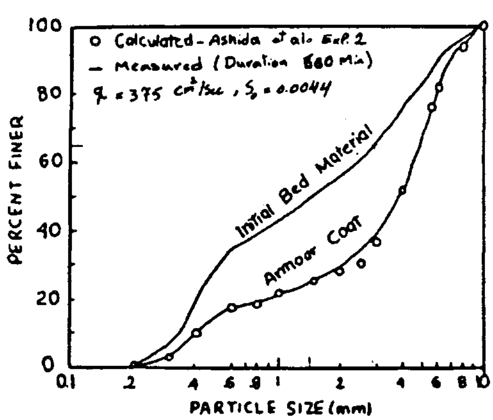
Fig. 6 : Determination of σ for Shen & Lu's method

the above range which σ was found to be equal 0.41.

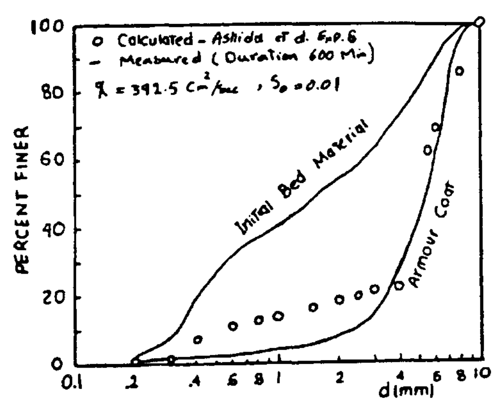
Methods of Group B :

The results reported by Ashida etal from their Exp.2 and Exp.6 are replotted and are shown in the following figures .This method predict quite well when data of Exp.2 are used but it simulate much finer material for Exp.6 with higher u_*.Fig.c shows Ashida etal and Gessler's methods have the same results while Shen etal's approach predict very well.

Fig. 7: Simulation of armor coat by Ashida et al(1971)

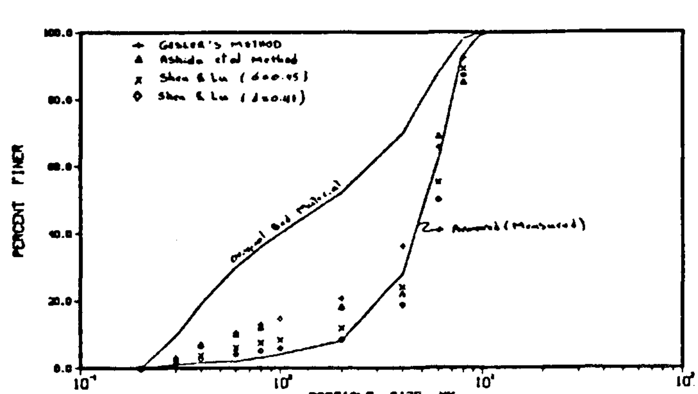


(a)



(b)

ASHIDA ET AL EXP. 6



(c) simulation of armor coat by three methods using Ashida etal Exp.6 data.

The result reported by Bayazit is shown in the following Fig. 8 .Bayazit used the Gesslre's data for run1-5 .The predicted armor coat as is shown is much coarser than the measured . Bayazit 's method requirs some modification in Einstein's hiding factor which may varies with different bed matrial that is why he reported good agreement with field data which that modification was based on.

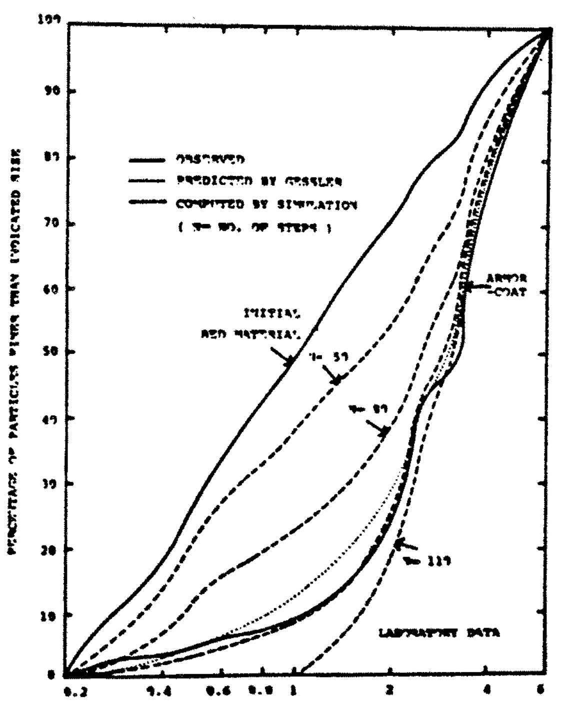


Fig. 8 Simulation of armor coat reported by Bayazit

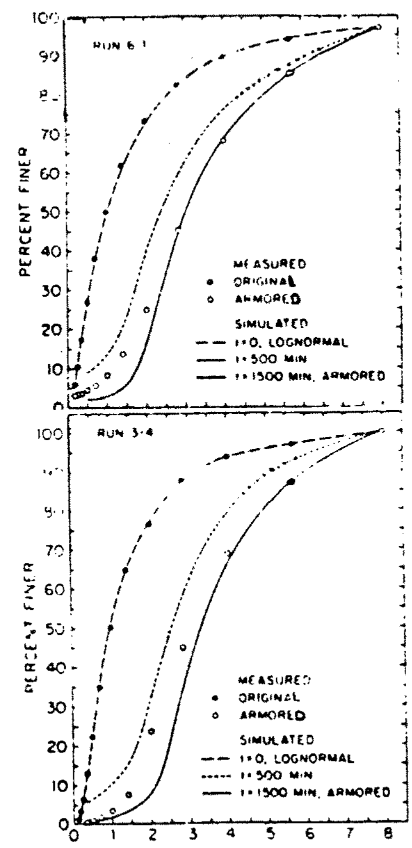


Fig. 9 Armor Coat (after Lee and Odgaard)

Lee etal used Little etal's data(Run3-4 and run 1-6) and their results are shown in Fig. 9 which also predicts coarser material . One of the assumption that Bayazit and Lee etal used is that the weight of particle remain constant while because of coarsening of the bed thereby change in its prosity the weight of particle will change. It is possible to improve these methods by computing a new value for G based on Komura's equation in which $n_t = 0.245 + \frac{0.03285}{d_{65}^{0.21}}$

where n_t is the prosity of material at time t, d_{65} is the size which 65% of the material are finer . And new G will be : $G= \gamma_s(1-n_t)$ T where Tis the thickness of the mixing layer .

Conclusion :

The process of bed armoring was investigated and wide range of data were used to predict the armor coat by several methods . From results obtained in this study the following conclusion can be outlined :

1) Because of complexity of the phenomenon a method to predict the armor coat quite well ,for vide variety of data , cannot be found in the littrature and needs furthur study .

2) Gessler's method predict reasonable value of geometric mean size while it simulate finer size distribution of the material.

3) Little and Mayer 's methods overestimate significantly and is not recommended .

4) Davies' method underestimate slightly.

5) Shen and Lu's method is the best method for the data were used in this study and is recommended.

6) The best value for σ,in Shen etal ' method, when σ_{go} has a value of 4.56 or 4.69 was found to be equal 0.41 .

7) Ashida et al predict much finer material when u_* is high .

8) A hypothesis for improving the methods in group B was proposed which needs furthur study to be verified .

Acknowledgment : The writer wishes to express his appreciation to Dr. P.Y.Julien ,professor of Hydraulic and instructor of CE717(River mechanics), for his encouragement during the preparation of this paper.

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Appendix II - Notation

The following symbols are used in this paper :

D = flow depth.

d_{84}, d_{50} = size of bed sediment which 84% and 50%, respectively, are finer .

d_{go}, d_{ga} = geometric mean size of original bed sediment and the armor coat respectively .

g = the accelaration of gravity .

G = the weight of particle per unit of bed area .

H = the height of dunes .

$Pa(d_i)$, $P_e(d_i)$ and $P_o(d_i)$ = fractions of d_i of the armor coat, eroded material and original material .

q = the probability of stay.

$\bar{q}$ = the mean probability of stay for armor coat .

q_s = the bed material discharge .

S_s = the specific gravity of the bed material .

T = thickness of mixing layer .

γ_s = the unit weight of particles.

τ_o, τ_c = mean bed shear stress and critical shear stress respectively.

T_* = Sheilds' parameter.

σ = standard deviation of the ratio τ_c/τ_o

σ_{g_o}, σ_{g_a} = the geometric standard deviation of the original and armor coat .

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