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TECHNICAL REPORT

THE EFFECT OF THE TIME DISTRIBUTION OF RAINFALL INTENSITIES ON SMALL WATERSHED FLOODS

By

Richard Neal Downer

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ABSTRACT

THE EFFECT OF THE TIME DISTRIBUTION OF RAINFALL INTENSITIES ON SMALL WATERSHED FLOODS

The time-intensity pattern of the storm rainfall is described by taking the moments of area of the hyetograph about the intensity-axis and about the time-axis. The effect of these parameters on the shape of the hydrograph is studied by means of multiple regressions.

Three groups of hydrograph parameters are used as dependent variables in the regressions. They are: the moments of area of the hydrograph about the discharge-axis; the traditional hydrograph parameters of volume, peak and rise time; and the parameters of a mathematical function fitted to the hydrograph.

The fitted function was an incomplete gamma function of the form

$$q = q_0 \frac{t^{\gamma r} e^{-\gamma(t-r)}}{r^{\gamma r}}$$

where q is the unit discharge in inches/hour, q_0 is the peak unit discharge in inches/hour, t is the time in minutes, γ is shape parameter with the units of reciprocal minutes, r is the rise time in minutes, and e is the base of Naperian logarithms.

The fitting process was accomplished by the method of weighted least squares, whereby the squared deviations between the observed discharge and computed discharge were weighted in proportion to the observed discharge.

The results indicate that the moments of the hyetograph are objective descriptors of the time distribution of rainfall intensities. All three groups of hydrograph parameters can be predicted with nearly equal accuracy. The similarity of the relationships for the observed peak and the fitted peak, and the observed rise time and the fitted rise time attest to the methodology of fitting the three-parameter gamma function to the observed hydrograph.

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LIST OF SELECTED SYMBOLS

Symbol	Definition	Units
A	Catchment area	square miles
AP_5	Five-day antecedent precipitation	inches
β	A conversion factor	minutes/hour
C	Compactness coefficient	dimensionless
C_1	10-year 1-hour precipitation	inches
C_2	Mean annual precipitation	inches
C_3	Mean monthly temperature	degrees
D_d	Drainage density	miles/square mile
D_T	Storm duration	minutes
Δt_t	Time interval for interval index t	minutes
f	Infiltration capacity	inches/hour
F	Form factor	dimensionless
γ	A shape parameter	$(\text{minutes})^{-1}$
$\Gamma(n)$	The gamma function	dimensionless
H	Total fall	feet
i_t	Rainfall intensity for the interval index t	inches/hour
I	Inflow discharge rate	inches/hour
I_n	Maximum n-minute storm intensity	inches/hour
k	A storage coefficient	minutes
L	Length of the main stream	miles

Symbol	Definition	Units
L_c	Length to center of area	miles
L_m	Dimensionless mean travel distance	dimensionless
LR	Loss rate	inches/hour
L_s	Total length of extended streams	miles
L_t	Mean travel distance	miles
m_n'	n^{th} moment of hyetograph about its commencement	$(\text{minutes})^n$
m_n^n	n^{th} central moment of hyetograph	$(\text{minutes})^n$
M_n'	n^{th} moment of hydrograph about its commencement	$(\text{minutes})^n$
P	Catchment perimeter	miles
PI	Pattern index	per cent
P_p	Probability of rainfall	dimensionless
P_q	Probability of peak discharge	per cent
P_T	Total storm rainfall	inches
q	Discharge rate	inches/hour
q_f	Discharge rate of fitted hydrograph	inches/hour
q_i	Observed hydrograph antecedent discharge rate	inches/hour
q_0	Peak discharge rate of fitted hydrograph	inches/hour
q_p	Observed hydrograph peak discharge rate	inches/hour
q_t	Discharge rate for the index t	inches/hour
r	Fitted hydrograph rise time	minutes
R^2	Coefficient of multiple determination	dimensionless

Symbol	Definition	Units
R_1, R_2, R_3, R_4, R_5	Overland slopes	feet/mile
R_6	Relief ratio	dimensionless
s	Storage	inches
s_d	Dimensionless standard deviation of the mean travel distance	dimensionless
s_t	Standard deviation of the mean travel distance	dimensionless
S_1, S_2, S_3, S_4	Stream slopes	feet/mile
S_{ey}	Standard error of estimate, same as dependent variable	
t	Time	minutes
t_r	Observed hydrograph rise time	minutes
$\bar{t}_t$	Mean time for time interval of index t	minutes
T	Time	minutes
ν	An instantaneous inflow rate	inches/hour
V	Total volume of observed runoff	inches
W	Average catchment width	miles
y_n'	n^{th} hyetograph moment about the time-axis	$(\text{inches/hour})^n$

Chapter I

INTRODUCTION

1.1 Nature of the Problem

A flood is the overflowing of the natural or artificial banks of a stream. Man and his works have been menaced by these hydrologic phenomena since the beginning of time. The Chinese long ago learned, by trial and error, the art of river control by diking, but only in the last century has man made an effort to understand and control this troublesome part of his environment through systematic observations and the development of its regularities. In the past much attention has been given to recording floods and their effects; however, only in the last two to three decades have researchers turned their attention to a better understanding of factors that affect and determine the magnitude of floods.

The problem, then, is to provide a better understanding of the causes and effects of a flood; more particularly, to isolate and define the interactions between causes and effects. Until every cause and its resulting partial effects can be tabulated, man's understanding of this aspect of his environment cannot be considered complete.

1.2 Need for the Study

In the United States alone, floods annually cause in excess of $700 million damage [Weber, 1965, p. 60]. Far too frequently benefit-cost studies place their emphasis on the larger, more costly structures neglecting the multitude of smaller structures. At present the cost of the individual smaller structure is too small to warrant detailed individual hydrologic investigation and design. However, the need for improved general design techniques does exist.

The large watershed is, in general, more sensitive to channel storage, whereas the small watershed is normally more sensitive to both high-intensity rainfalls of short duration and to land use. Hydrologists [Horton, 1935; Horner, 1936; Horton, 1939; Brater, 1940] have recognized these differences but only recently have they turned their attentions toward research on small watershed responses. Large watershed design techniques, modified for small watersheds [Brater, 1940], have, in general, been adequate for the design of small structures. However, the rising cost of labor and materials has brought into focus the need for more precise small watershed design techniques.

Therefore, there exists a need for research on small watershed floods. The complexity of the problem indicates the necessity of limiting, dividing and subdividing until every cause and every effect can be understood.

1.3 Scope of the Study

This study is limited to small watershed floods. A small watershed is defined as one whose sensitivity to high-intensity rainfalls of short duration and to land use is not suppressed by the channel characteristics [Chow, 1964, p. 14-5]; and a flood is defined in this study as a condition of high-water whose peak discharge has a return period of two years or greater. Only summer storms are considered.

1.4 Purpose

The purpose of this study is to: (1) define a set of parameters which characterize the time distribution of the rainfall, (2) define a set of parameters which adequately characterize the shape of the hydrograph, and (3) show a relationship between the rainfall parameters and the hydrograph parameters.

Chapter II

BACKGROUND OF THE PROBLEM

2.1 Early Contributions to Runoff Prediction

Early designers of hydraulic structures relied heavily on practical experience and engineering judgment. Such methods gave satisfactory, but sometimes crude results. As an individual gained experience his observations began to show a certain regularity; and the first empirical rules were born.

2.1.1 Waterway Area Formulas

The first significant recorded contribution to runoff prediction was made by John Roe, a London Surveyor, who in 1852 prepared a "table expressing the relation between the diameter and slope of a circular outlet sewer and the size of its drainage area [Chow, 1962, p. 27]."

Numerous waterway area formulas seem to have originated in the period 1850 to 1890. Among the more famous were the Myers formula and the Talbot formula [Chow, 1962, pp. 70-71]. For the most part these formulas took the form:

$$a = b \cdot A^m ,$$

where a is the waterway area in square feet, b is a coefficient incorporating the parameters of basin slope and cover, A is the basin area in acres, and m is an exponent ranging from 0.5 to 1.0.

Although they recognized basin size, slope and cover as factors influencing the runoff process these early waterway area formulas are today considered rough engineering tools because of one's inability to correctly select the proper value of b for the basin concerned. Application of the formulas has generally been limited to small drainage basins in the regions for which they were developed.

2.1.2 Discharge Formulas

In 1889 Emil Kuichling [Kuichling, 1889] published the discharge formula which is commonly referred to as the *Rational Formula*. This simple discharge formula, which combines the rainfall intensity, drainage area and a coefficient of runoff, represents one of the early efforts to combine empirical experience with scientific principles.

The discharge was given by

$$Q = C \cdot i \cdot A ,$$

in which Q is the peak discharge in cfs; C is a coefficient embodying the ideas of a percentage of rainfall appearing as runoff, of overland flow, of channel or basin storage, and of basin cover; i is the rainfall intensity in inches per hour and is assumed to represent the average intensity over the drainage basin for a duration equal to the *time of concentration* of the basin; A is the area in acres.

Presumably because of its simplicity, the *Rational Formula* has retained wide favor among designers [Chow, 1962, p. 33]. Its simplicity, however, has also brought it under severe criticism from many hydrologists. As in the case of the waterway area formulas the determination of the proper coefficient is the weakness of the *Rational Formula*. In an effort to circumvent this difficulty Horner and Jens [Horner, 1942] developed a method which did not require the use of a coefficient. A report by Potter [Potter, 1950] pointed out the deficiencies of the Rational method and suggested the introduction of probabilities of rainfall and runoff as a way of solving the *coefficient problem*.

2.2 Hydrograph Synthesis

Not satisfied with merely predicting the peak discharge, efforts [Folse, 1929; Report, 1930; Sherman, 1932] turned toward methods of hydrograph analysis. In 1932 Sherman [Sherman, 1932] introduced the well-known unit hydrograph concept.

2.2.1 The Unit Hydrograph Method

As first proposed by Sherman the *unitgraph* (hereafter called unit hydrograph) represented the hydrograph of direct runoff resulting from an isolated event of rainfall excess occurring uniformly over the drainage area and at a uniform rate for a *unit of time*. Originally the word *unit* referred to the time of effective or excess rainfall. Normal usage, however, has altered the meaning to a "unit depth of surface runoff occurring over the basin."

Since 1932 the method has been improved and modified by the work of Bernard [Bernard, 1935], Horner and Flynt [Horner, 1936], Hoyt [Hoyt, 1936] and others [Brater, 1940]. The basic assumptions, however, remain the same:

(a) the effective rainfall is uniformly distributed in time and the duration is less than the time of concentration;

(b) the effective rainfall is uniformly distributed in space;

(c) the period of surface runoff is approximately constant regardless of the storm intensity;

(d) the ordinates of the derived hydrograph at any time are proportional to the ordinates of the unit hydrograph times the volume of effective rainfall;

(e) the observed hydrographs, from which the unit hydrograph is constructed, reflect all the combined physical characteristics of the drainage basin, including infiltration, surface detention and storage regardless of magnitude of the unit storm.

It is at once apparent that the foregoing assumptions cannot be rigidly satisfied. Of interest to this study are the limitations imposed by suppositions (a), (d) and (e).

Supposition (e) implies the invariance of basin characteristics with season, time, or the influences of man. Clearly, any sound method of hydrograph synthesis must be tolerant of changing catchment characteristics.

Supposition (d) infers that the *principle of superposition* is applicable for constructing hydrographs resulting from effective rainfalls of different intensities by simply adding or superimposing the ordinates of the individual resulting hydrographs. Implicit in (d), therefore, is the assumption of *linear watershed response*. Further ramifications of this assumption will be subsequently discussed in connection with Minshall's work.

Supposition (a) is less restrictive than the aforementioned, if one takes into account *instantaneous unit hydrograph* methods [Clark, 1945; O'Kelley, 1955; Dooge, 1955]. The *instantaneous unit hydrograph* method eliminates the need for uniform effective durations of rainfall, but because of the magnitude of the computations required to compute the *instantaneous unit hydrograph* it has not gained wide favor with practicing hydrologists.

2.2.2 The Instantaneous Unit Hydrograph

The *instantaneous unit hydrograph* is a unit hydrograph resulting from an infinitesimally short burst of effective rainfall. This ruse allows one to construct a unit hydrograph from nonuniform rainfall by subdividing the total storm into smaller, more uniform bursts. Using *linear hydrograph theory* [Dooge, 1959] the individual hydrographs are combined into a single resulting hydrograph. It should be pointed out that this modification of unit hydrograph theory does not overcome the basic restrictions of *linear basin response* and *time invariance*.

2.3 Recent Hydrograph Studies

The preceding discussion has been concerned with methods and formulas developed by practicing engineers in need of more reliable procedures for predicting storm responses. Recently investigators such as Gray, Minshall and Reich have attempted to delve deeper into the cause and effects of hydrologic responses and show the interrelationships between various parameters.

2.3.1 Gray's Method

In 1962 Gray [Gray, 1962] presented a procedure for unit hydrograph synthesis based on a method of fitting a two-parameter gamma function to empirically derived dimensionless unit hydrographs. A relationship between the hydrograph parameters and watershed characteristics was obtained. The study represents one of the more complete investigations of the interplay between various catchment topographic characteristics and parameters of the hydrograph.

2.3.2 Minshall's Work

Using data from the Agricultural Research Station at Edwardsville, Illinois, Minshall [Minshall, 1962] showed that for three very small watersheds, less than 290 acres, the base of the unit graph was not constant but actually increased as rainfall intensity decreased. For the catchments studied, late peaking storms showed a strong relationship between rainfall intensity and the peak discharge. Although a very limited study, Minshall's work does show the inherent limitations of the unit hydrograph method.

2.3.3 Reich's Work

From a study [Reich, 1962] of 47 flood events at scattered points throughout the United States Reich was able to show a relationship between the parameters of a two-parameter gamma function fitted to the observed hydrograph and a combination of catchment characteristics plus the thirty-minute maximum rainfall intensity. Reich's work can be considered a pilot study of the feasibility of fitting a mathematical function directly to the hydrograph of observed runoff rather than to the unit hydrograph.

2.4 Previous Descriptors of Rainfall Characteristics

Many workers have stated that the distribution of rainfall intensities throughout a storm has a marked effect on the hydrograph of surface runoff. However, the failure to index the temporal distribution of rainfall intensities has made it difficult to quantitatively evaluate these effects.

Schiff [Schiff, 1943] working only with data from Coshocton, Ohio, observed that rainfall excess was dependent upon the duration and order in which rates of rainfall occur and condition of the watershed, and was primarily the difference between the rainfall rate and the infiltration rate. He states, "the intensities of rainfall and their order or pattern exert a marked influence on the change in conditions of watershed during a storm as well as on antecedent conditions of watershed, and thus on the infiltration rate."

Using continuous rainfall storms, without gaps exceeding six or more hours and recording more than 0.25 inches, he grouped the storms into the following arbitrary classes:

TABLE 1. SCHIFF'S STORM CLASSIFICATIONS

Class	Rainfall Intensity
I	Uniform intensities up to and including 0.50 inches per hour, a deviation of 1 1/2 times the mean rate permitted for the maximum intensity within a storm.
II	Combination of intensities up to and including 0.50 inches per hour, with not more than 15% of the amount falling at intensities in excess of 0.50 inches per hour.
III	Combination of intensities below and above 0.50 inches per hour up to and including 1.00 inches per hour with more than 15% of the amount falling at intensities in excess of 0.50 inches per hour and less than 15% of the amount falling at intensities in excess of 1.00 inches per hour.
IV	Combination of intensities below 0.50 inches per hour and over 1.00 inches per hour, may include intensities between 0.50 and 1.00 inches per hour, with more than 15% of the amount falling at intensities below 0.50 inches per hour and more than 15% of the amount falling at intensities in excess of 1.00 inches per hour, and more than 15% may fall at intensities between 0.50 and 1.00 inches per hour.
V	Uniform and combination of intensities of 0.50 inches per hour and over, with not more than 15% of the amount falling at intensities to below 0.50 inches per hour.

Schiff, following the suggestions of Horner and Jens [Horner, 1942], set up some arbitrarily chosen intensity-patterns, referring to them as *uniform*, *advanced*, *intermediate*, and *delayed* patterns. He added two more terms, *interrupted* and *sporadic*. *Advanced* is applied to a storm having its highest intensities, representing fifteen per cent or more of the total amount of the rainfall, near the start of the storm; *intermediate*, near the center; *delayed*, near the end; and *interrupted* to a storm having high initial and final intensities separated by a period during which the intensities are lower. Schiff's classifications are helpful from a qualitative point of view but lack practical usefulness because his relationships and findings are not quantitative and therefore cannot be used universally.

Neal [Neal, 1945] observed that high runoff rates were not necessarily the results of high rainfall intensities alone, but were often dependent on antecedent moisture conditions in the watershed. Therefore, he classified rains according to their intensities and the rainfall occurring in the preceding ten days.

TABLE 2. NEAL'S STORM CLASSIFICATIONS

Class	Rainfall Intensity
I	Previous 10-day rainfall less than 1.00 inch and 30-minute intensity less than 0.50 inches per hour.
II	Previous 10-day rainfall less than 1.00 inch and 30-minute intensity 0.50 inches per hour or more.
III	Previous 10-day rainfall 1.00 inch or more, and 30-minute intensity less than 0.50 inches per hour.
IV	Previous 10-day rainfall 1.00 inch or more, and 30-minute intensity 0.50 inches per hour or more.

From his Alabama data he observed that Class III and IV rains produced the greatest amount of runoff. Neal's results like those of Schiff's are qualitative rather than quantitative and therefore lack universality.

Foster [Foster, 1950] proposed five new measures of rainfall which were in fact combinations of the frequently used 5-minute, 15-minute, 30-minute average storm intensities. He found that the best simple index of intensity was the 30-minute maximum intensity. His best compound index was that of π_1, where

$$\pi_1 = I_{15} \cdot I_{30} \cdot (bn)^{0.33} .$$

The quantity b takes on the value 1, 2, or 3 accordingly, whether the greatest amount of rain falls within the first, second, or third half-hours. For rains which last longer than three half-hours, time of fall is divided into three equal portions which follow the same convention. The factor n defines the number of peaks of the hyetograph exceeding one inch per hour, providing each peak is at least thirty minutes removed from the neighboring one. Foster's work appears to be the first to use a quantitative approach.

Hutchinson, *et al.* [Hutchinson, 1958] proposed that the rainfall intensities could be examined by dividing the hyetograph into successive intensity groups (intervals of one inch per hour were chosen), and plotting against the number of minutes during which intensities equaled or exceeded that amount.

Naturally, for the whole duration of the storm the rain fell at an intensity greater than zero inches per hour, and rain fell at the peak rate for zero minutes. These two points, therefore, provide the two end points of the curve of duration against intensity. If log duration is plotted against intensity, the relationship was approximately a straight line for each individual storm.

Shanholtz and Dickerson [Shanholtz, 1964], using point rainfall to represent the areal distribution, studied a group of rainfall characteristics with a view to defining their influence on the volume of surface runoff from watersheds of ten acres or less. No attempt was made to investigate watershed factors in this study. Rainfall characteristics examined included: (a) total amount, (b) intensity, (c) distribution, (d) pattern, (e) energy, and (f) duration. These were divided into eleven primary groups as follows:

TABLE 3. SHANHOLTZ AND DICKERSON RAINFALL PARAMETERS

Class	Characteristics
I	*Total Storm Rainfall.*
II	*Maximum Intensities for Selected Time Intervals.* Intervals used were 2, 5, 10, 15, 20, 30, 60, 90, 120, 150, and 240 minutes.
III	*Average Intensity for the Storm.* Total rainfall divided by the duration.
IV	*Average Intensity for the Rain Period.* Total rainfall divided by the effective duration (time in which rain actually occurred).
V	*Weighted Storm Intensities:* (a) The sum of the product of intensity and rainfall per time interval divided by the total rainfall. (b) The mean intensity derived by quartering the rainfall according to chronological order and computing the average intensity of each of the partitions separately.
VI	*Intensity-Amount-Distribution-Index.* The slope of Hutchinson's intensity-duration regression line.

Class	Characteristics
VII	*Pattern Index.* The area under the accumulated percentage curve of rainfall versus time. Values of 0.3, 0.5, and 0.8 represent a delayed, an intermediate, and an advanced storm pattern, respectively.
VIII	*Weighted Pattern Index.* The accumulated sum of the product of the intensity for each time interval and the area under the curve for that interval. This index has the effect of weakening the influence of very low intensities.
IX	*Total Storm Energy.* $E = 916 + 331 \cdot (\ln i)$, where i is expressed in inches per hour and E is in foot tons per acre-inch. Developed primarily to estimate soil loss, the energy may be related to the amount of runoff.
X	*Total Energy · 30-Minute Maximum Intensity.*
XI	*Storm Duration.*

From a regression analysis on the aforementioned variables plus some combinations of them it was found that the total rainfall gave the best single estimate of the runoff volume. The total energy as defined by Wischmeier, *et al.* [Wischmeier, 1958] estimated runoff with about the same accuracy as the total rainfall. The analysis indicated that no single rainfall characteristics could be used to satisfactorily estimate runoff volume. This study disclosed the need for further research in two general fields: first, conditions necessary for runoff to occur; and second, a method to determine the period which best reflects the influence of the rainfall intensity on surface runoff for all storms regardless of duration. This study seems to be the most comprehensive attempt at developing a transition from qualitative to quantitative parameters describing the rainfall input which results in runoff.

Dickinson and Ayers [Dickinson, 1965] developed two indices similar to the *Pattern Index* used by Shanholtz and Dickerson. Defining a uniform rainfall as one whose mass curve has a slope of one on a unitless plot, the area between a storm mass curve and the 45° line then becomes an index of the general uniformity of the rainfall. As this area increases, the storm becomes less uniform with time. They found, however, that this temporal distribution index was inadequate to describe the distribution for all storm types. The area under the unitless plot becomes an index of the time of occurrence of the major burst. Storms involving only one major rainfall period could be properly indexed. However, for the storms analyzed, the time distribution often varied as much from one gage to another in a particular storm as from one storm to another.

2.5 Hydrograph Parameters

The design of full-flow or storage structures requires a knowledge of the time distribution of runoff. Clearly, the selected parameters should be capable of defining the pertinent characteristics of the hydrograph, such as the peak discharge, the rise time and the volume of runoff.

2.5.1 A Mathematical Selection of Parameters

Ideally, the hydrograph shape should be reproducible from a limited number of parameters which can be obtained objectively. In 1959 Nash [Nash, 1959] suggested that if a linear relationship is assumed to exist between storm runoff and effective rainfall, a relationship should exist between the catchment characteristics and the response of the river basin to a predetermined input of effective rainfall. He called this response the *indicial response* and cited the *instantaneous unit hydrograph* as an example. Nash went on to suggest the moments of the *instantaneous unit hydrograph* as a series of response parameters and was able to show that the "first moment of the *instantaneous unit hydrograph* about the instant of effective rainfall is equal to the difference between the first moments of the storm runoff and effective rainfall about the time of beginning of effective rainfall." One sees from Figure 1 that this is equivalent to saying that $b = a + c$. The corresponding relations for higher moments was also derived.

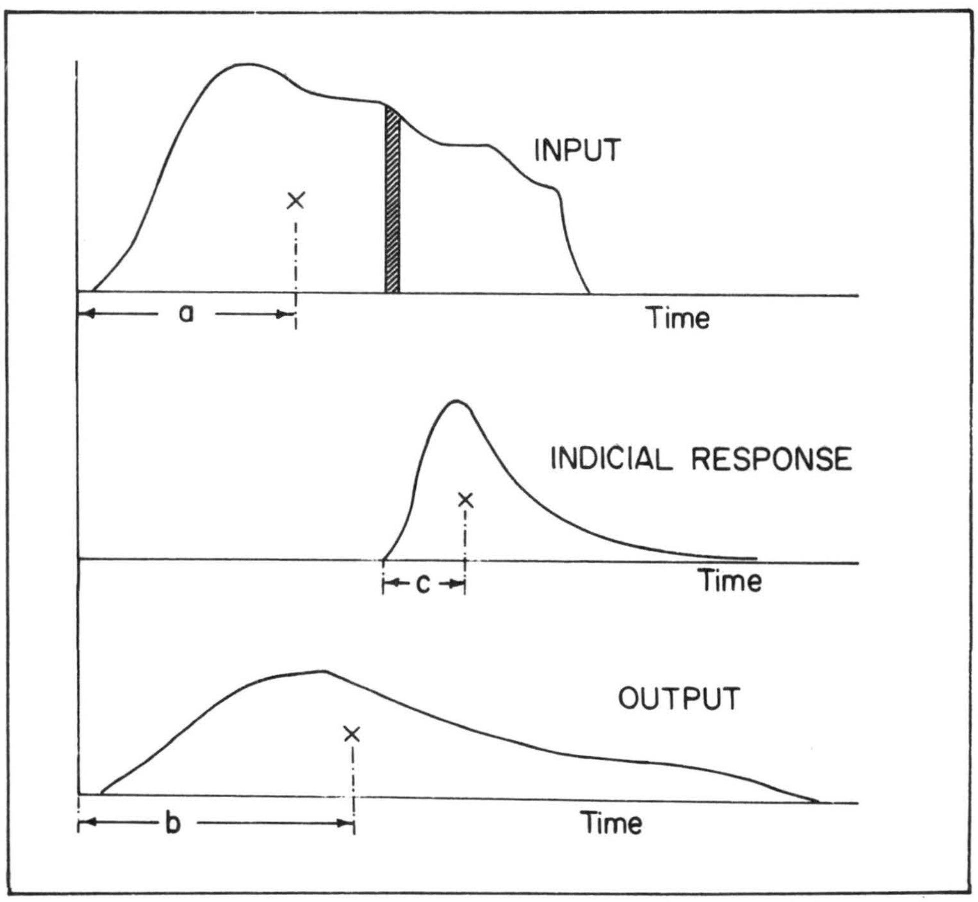


Figure 1. Nash's linear transformation

Diskin [Diskin, 1967] objecting to the restrictions of Nash's proof, which required that both the volume of effective rainfall and the volume of runoff be equal to unity, presented a more general proof of the *principle of moments* using Laplace transforms. The only restriction being that a relationship exist between the rainfall and runoff which can be given by the convolution integral.

2.5.2 Selection of a Mathematical Function to Represent the Hydrograph

Examination of the hydrograph, Figure 2, indicates it has the general shape of a skewed distribution function. On the assumption that the hydrograph can be represented by a function of the same type as a distribution function, the theoretical distribution function of best fit should have the following characteristics:

1) The function should be continuous;
2) The function should be defined for all positive values of the unit discharge, q , and the time, t ;
3) The left tail should be bounded;
4) The left tail should be tangent to the t-axis as it approaches its bound;
5) The right tail should be unbounded;
6) The right tail should be asymptotic to the t-axis for large values of time, t ;
7) The function should be unimodal;
8) The maximum or peak point should be a finite value and the first derivative should equal zero;
9) The function should be capable of assuming a large range of skewness values.

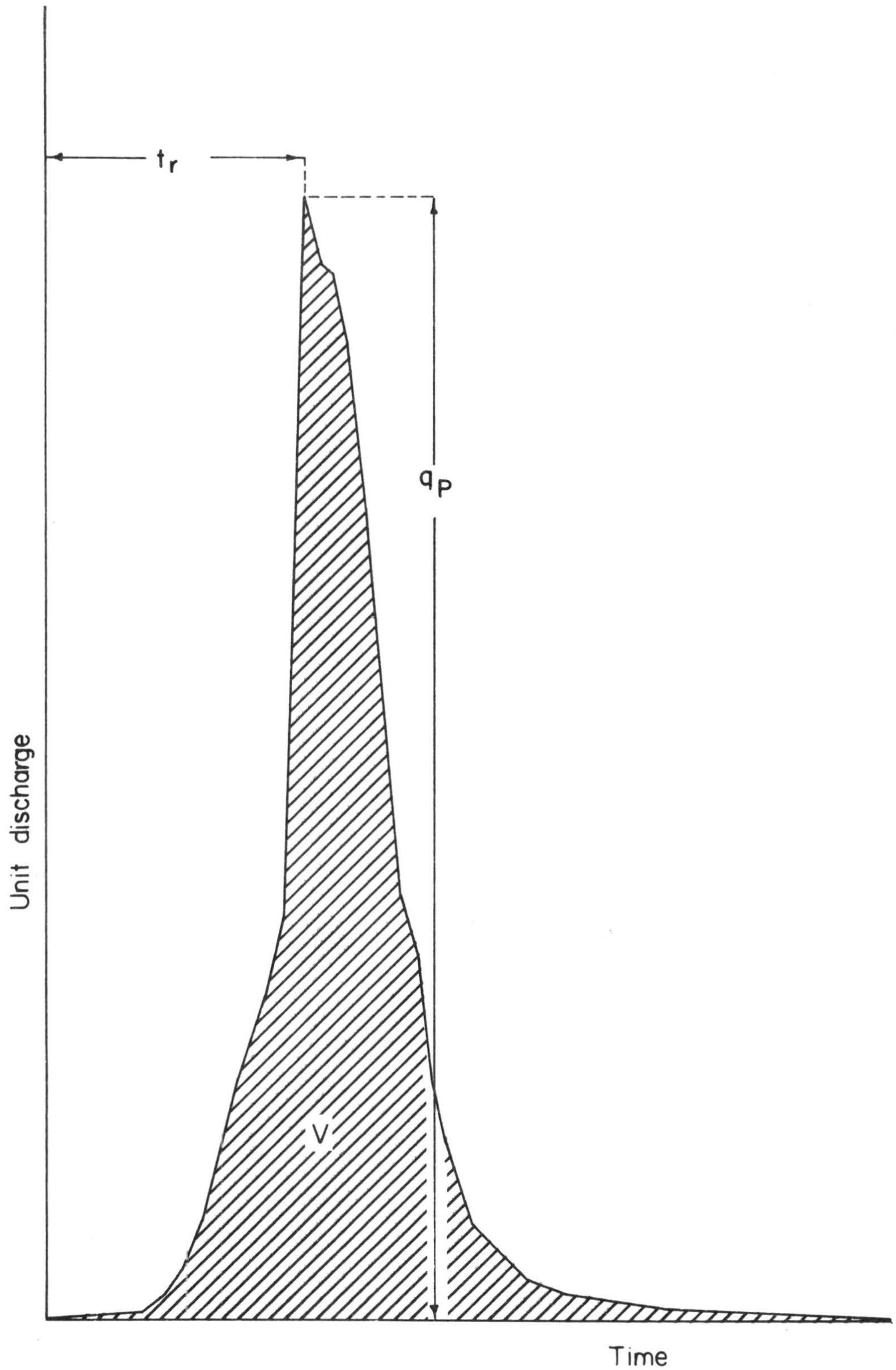


Figure 2. Definition sketch of parameters of a hydrograph

Karl Pearson [Pearson, 1956] has derived a general class of functions which satisfy the previous conditions. Of interest here is his *Type III* equation which is a skewed function bounded on the left. This function can be written in the form:

$$y = y_0 \left(1 + \frac{x}{a} \right)^{\gamma a} e^{-\gamma x} \tag{2.1}$$

in which y = an ordinate ; y_0 = a constant ; x = an abscissa ; a = the distance from the lower bound to the origin, origin is at the mode ; γ = a constant .

Using a substitution of the form, $x = x - a$, to transfer the origin to zero and then integrating:

$$\int_0^\infty y\,dx = N = \frac{y_0 e^{-\gamma a}}{a^{\gamma a}} \int_0^\infty x^{\gamma a} e^{-\gamma x}\,dx \tag{2.2}$$

From which

$$y = \frac{N\gamma^{\gamma a+1}}{\Gamma(\gamma a + 1)} x^{\gamma a} e^{-\gamma x} \tag{2.3}$$

where $\Gamma(\gamma a + 1)$ is the "gamma function" of $(\gamma a + 1)$.

$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\,dx$, $n > 0$; $\Gamma(n + 1) = n\Gamma(n) = n!$ [Hildebrand, 1963, p. 78].

As originally derived by Pearson, Equation 2.3 represented a probability density function. For the case at hand it is appropriate to consider it to be a mathematical function since the hydrograph does not represent the frequency of occurrence of a random variable, but rather the function of discharge versus time. Therefore, by analogy let

yes it is ([illegible])

$N = V$, the total volume of runoff represented by the area under the hydrograph in inches;

$y = q$, the unit discharge in inches per hour;

$x = t$, the time since the commencement of flow in minutes;

$a = r$, the time from commencement of flow to the peak in minutes;

γ = a shape parameter with dimensions of reciprocal of time;

β = conversion factor to convert inches per minute to inches per hour.

The equation of the hydrograph becomes:

$$q = \frac{\beta V \gamma^{\gamma r+1}}{\Gamma(\gamma r + 1)} \, t^{\gamma r} \, e^{-\gamma t} . \qquad (2.4)$$

Equation 2.4 is commonly referred to as the *two-parameter* or *incomplete gamma* function.

2.5.3 Further Theoretical Considerations

Edson [Edson, 1951] and Nash [Nash, 1958] relying on different underlying assumptions have developed mathematical expressions which may be reduced to the common form of Equation 2.4. Since both these developments substantiate the application of the two-parameter gamma distribution to describe the hydrograph the complete derivation as given by each author is given below.

Edson states that if isochrones could be drawn to represent the time required for each local element of effective rainfall to reach the mouth of a watershed, the cumulation of area, A, with time, t, would result in an approximate parabola

$$A \propto t^m \ , \quad m > 1 \tag{2.5}$$

so that the runoff discharge, q, might become

$$q \propto t^m \ , \quad m > 1 \ . \tag{2.6}$$

However, the time of travel required for each component is so affected by the presence of the other components that the hypothetical isochrones are invalidated. The consequent delay in delivery is generally regarded as the result of valley storage. In a sense the valley acts as a reservoir, the discharge of which is known to decrease exponentially with time.

$$q \propto e^{-\mu t} \ , \quad \mu > 0 \tag{2.7}$$

Thus the reservoir action of the valley is seen to have a dampening effect on the flow implied by proportion (2.6). Accordingly, proportion (2.6) must continue in effect indefinitely. On the other hand, since valley storage must exist for even the slightest amount of discharge, proportion (2.7) is seen to be in effect from the very inception of runoff. The combined effect, therefore becomes

$$q \propto t^{m} e^{-\mu t} \tag{2.8}$$

which possesses all the pertinent characteristics of the unit hydrograph. The fact that the falling limb of a unit hydrograph becomes approximately linear when plotted on semilogarithmic paper simply means that proportion (2.6) is dominated by proportion (2.7) some time after the peak discharge. At no time prior to the peak discharge, however, is proportion (2.7) dominated by proportion (2.6) so that proportion (2.8) cannot be developed in its cumulative form into an empirical equation by the usual methods of straight line fitting in transformed coordinates.

On the basis of proportion (2.8), the following equation was adopted

$$q = Bt^{m} e^{-\mu t} \tag{2.9}$$

By integration, the total volume, V, is obtained. To facilitate the integration let $m = \nu - 1$ and $Z = \mu t$. Then

$$V = \int_0^\infty q \, dt = \int_0^\infty Bt^{m} e^{-\mu t} \, dt = B\mu^{1-\nu} \int_0^\infty Z^{\nu-1} e^{-Z} \, dZ = B\mu^{1-\nu}\Gamma(\nu) \tag{2.10}$$

By appropriate substitution equation (2.9) becomes

$$q = \frac{V\mu^{\nu-1} t^{\nu-1} e^{-\mu t}}{\Gamma(\nu)}, \tag{2.11}$$

Again the conversion factor β is necessary to obtain the desired units.

This is Edson's equation for the unit hydrograph developed from a consideration of times of concentration and valley storage. It is easily recognizable that with the following substitutions $\mu = \gamma$ and $\nu = \gamma r + 1$, Edson's equation (2.11) and equation (2.4) are identical.

Nash assumes that any catchment may be replaced by a series of n reservoirs, each having the linear storage characteristic $s = kq$, the outflow from one reservoir becoming the inflow to the next. When the instantaneous inflow v takes place to the first reservoir its level is raised by an amount sufficient to accommodate the increased storage and the discharge rises instantaneously from zero to v/k and diminishes with time according to the equation

$$q_1 = \frac{v}{k} e^{-t/k} \tag{2.12}$$

q_1 becomes the inflow I to the second reservoir and we get

$$q_2 = \frac{e^{-t/k}}{k} \int_0^t I \, e^{t/k} \, dt \tag{2.13}$$

$$q_2 = \frac{e^{-t/k}}{k} \int_0^t \frac{ve^{-t/k} \, e^{t/k}}{k} \, dt \tag{2.14}$$

$$q_2 = \frac{ve^{-t/k} \, t}{k^2} \tag{2.15}$$

Successive routing shows that the outflow from the n^{th} reservoir is given by:

$$q_n = \frac{ve^{-t/k}}{k} \left(\frac{t}{k}\right)^{n-1} \frac{1}{(n - 1)!} \tag{2.16}$$

But $(n - 1)! = \Gamma(n)$,

$$q_n = \frac{ve^{-t/k}}{k\Gamma(n)} \left(\frac{t}{k}\right)^{n-1} \tag{2.17}$$

Nash's equation is also easily reduced to the form of Equation 2.8 with the following substitutions, $k = \frac{1}{\gamma}$, $v = V$ and $n = \gamma r + 1$.

On the basis of the characteristics of the Pearson Type III curve and the independent developments of Edson and Nash it is evident that the hydrograph can be represented by the two-parameter gamma distribution (2.8) with parameters γ and r estimated by statistical procedures from experimental data.

2.6 Losses

Losses are defined here as the difference between the total precipitation and the total surface runoff for a given storm. This difference is commonly subdivided into leakage, evaporation, and transpiration. Each of these divisions could well be the subject of a study by itself. Realizing the complexity of the problem early hydrologists used the runoff coefficient concept to account for the losses. Other methods of evaluating storm runoff developed in recent years have included short-term water balances, and various correlation or multivariate analyses. In most cases sufficient data for using these methods is not available.

The concept of loss rates provides a method of estimation which is applicable to regions of limited data. Loss rate is defined here as the average loss of rainfall on the watershed during the supply period of the storm. Tabulations of average loss rates are available for the United States [Creager, 1945] and Australia [Laurenson, 1963]. Pilgrim [Pilgrim, 1966] in a recent study has shown that loss rates can be transferred from a region of adequate data to another similar region with limited data. He states that for a given watershed loss rate is a more stable value than the runoff coefficient, and can be objectively calculated whenever data is available.

Chapter III

ANALYTICAL CONSIDERATIONS

Figure 3 shows a typical response (hydrograph) resulting from a varying input (hyetograph). It is at once obvious that no simple mathematical function could ever be fitted to the hyetograph and still retain the intensity pattern intact. The problem, therefore, is to find a quantitative descriptor of the hyetograph.

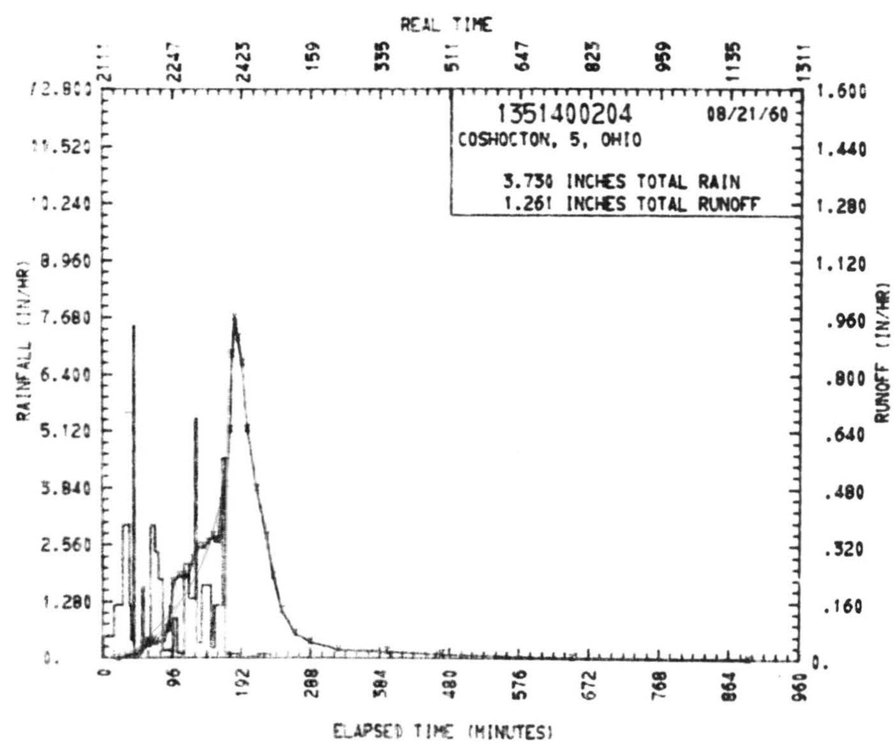


Figure 3. Typical hyetograph and hydrograph

Once this descriptor is found the problem of testing it becomes immensely more difficult. Faced with the Herculean task of measuring the causes and effects of the hydrologic cycle early investigators resigned themselves to recording only precipitation (input) and runoff (output). As a result the present day investigator finds himself in the situation of holding the ends of an invisible chain. In the one hand he has the input (hyetograph), in the other hand the response (hydrograph) and in between are the invisible links of infiltration, leakage in and out of the river basin, ~~evaporation,~~ evapotranspiration, channel storage and wave attenuation. Thus, until information about these invisible links is available for the periods preceding and during an event, investigations must be confined to *input-output* studies.

This study, in a sense, is an attempt to evaluate statistically a relationship between the input and the response. To establish this relationship by means of a statistical correlation it is necessary to define the catchment characteristics, the hydrograph, and the hyetograph by numerical parameters. Once defined, the hydrograph parameters will be considered as the dependent variables, while the hyetograph parameters and catchment characteristics will be considered as the independent variables.

Consider that the recorded hyetograph represents an unknown function $f(t^*)$. Further, consider that the recorded hydrograph is a transformation of the hyetograph and as such can be represented by the function $f(T^*)$. The transform function, $f(\tau^*)$, is also unknown, but one may ascertain some of its general properties by examining the properties of the input, $f(t^*)$, and the output, $f(T^*)$.

Clearly, the parameters chosen to represent the hyetograph and hydrograph must be numerical and easily obtainable. This leads one to consider first the *method of moments*. Although the inversion between a function and its moments is not generally unique, it is often possible to estimate the parameters of a function when its functional form is known or assumed. Such is the case here. On the basis of sections (2.5.2) and (2.5.3) it is reasonable to assume that the functional form of the hydrograph is that of the two-parameter gamma function for single-peaked hydrographs.

From an *a posteriori* knowledge of the hyetograph-hydrograph relationship and section (2.5.1) one hypothesizes that a functional relationship should exist between the parameters (moments) of these *input-output* functions. Therefore, the following moments are introduced (see Figure 4).

The weighted moments of the hyetograph are given by:

$$m_n' = \frac{\sum_{t=0}^{T} i_t \cdot \Delta t_t \cdot t_0^{\,n}}{\sum_{t=0}^{T} i_t \cdot \Delta t_t}$$

where m_n' represents the n^{th} weighted moment of the hyetograph about the commencement, i_t is the intensity occurring over the interval t, and t_0 is the distance from the commencement to the center of the interval in question. Note that $\sum_{t=0}^{T} i_t \cdot \Delta t_t$ is equal to the total rainfall occurring for the duration of the storm, T . The central moments or moments about the mean time of the hyetograph are designated by m_n .

Similarly, the weighted moments of the hydrograph are given by:

$$M_n' = \frac{\sum_{t=0}^{N} \frac{q_t + q_{t+1}}{2} \cdot \Delta t_t \cdot t_0^n}{\sum_{t=0}^{N} \frac{q_t + q_{t+1}}{2} \cdot \Delta t_t}$$

where M_n' is the n^{th} weighted moment of the hydrograph about its commencement, q_t is the discharge at time t , Δt_t is the interval between t and $t+1$, and t_0 is the distance from the commencement to the center of area of the interval in question. In this case the denominator is equal to the total observed runoff.

Having hypothesized on the relationship between the hyetograph and hydrograph moments, it remains to postulate on the effects of basin loss and storage characteristics. It is obvious that for any transform function, $f(\tau^*)$, to be meaningful, the relationship between its variables must be in keeping with the basic understanding of watershed response. For example, the response function should exhibit a tendency to decrease in value with increasing loss rates. Ordinates of the response should be inversely proportional to the catchment characteristics parameters such as slope and distance of travel which are measures of storage or delay.

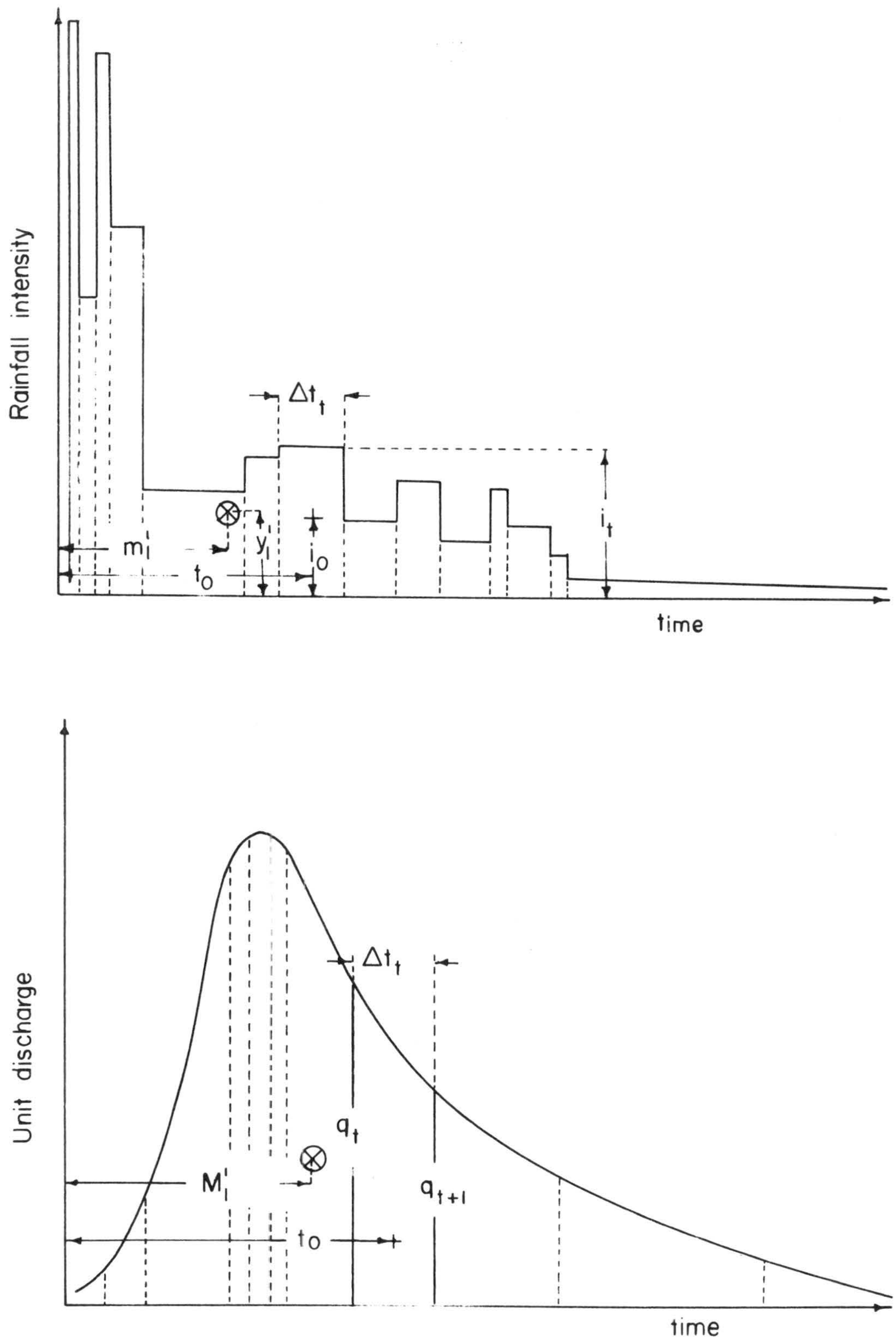


Figure 4. Definition sketch of hyetograph and hydrograph moments

Chapter IV

ANALYSIS OF DATA

4.1 The Data Available

This study is based on hydrologic data assembled under the auspices of the "Small Watershed Program" at Colorado State University [Laurenson, *et al.*, 1963; Markovic, *et al.*, May 1964; Markovic, *et al.*, July 1964; Research, 1967]. At the time of this writing the assembly consisted of 1219 watersheds scattered throughout the United States. Catchment characteristics (see section 4.2.2) had been calculated and were available on magnetic tape for 192 watersheds. Associated with these catchments were 898 separate flood events, of which 551 had been selected and reduced to digital form.

In conjunction with the assembly a study of the annual flood series was carried out. Frequency plots of the annual flood series were prepared using both the Gumbel [Gumbel, 1954] and Jenkinson [Jenkinson, 1955] methods for all annual series with two or more years of record. A great savings of time was achieved at this point by programming the computer to calculate the frequencies and plotting positions for each of the floods of the annual series for each of the catchments. Further, the cathode ray tube system attached to the computer was programmed to draw the plots. Figure 5 is an example of such a plot.

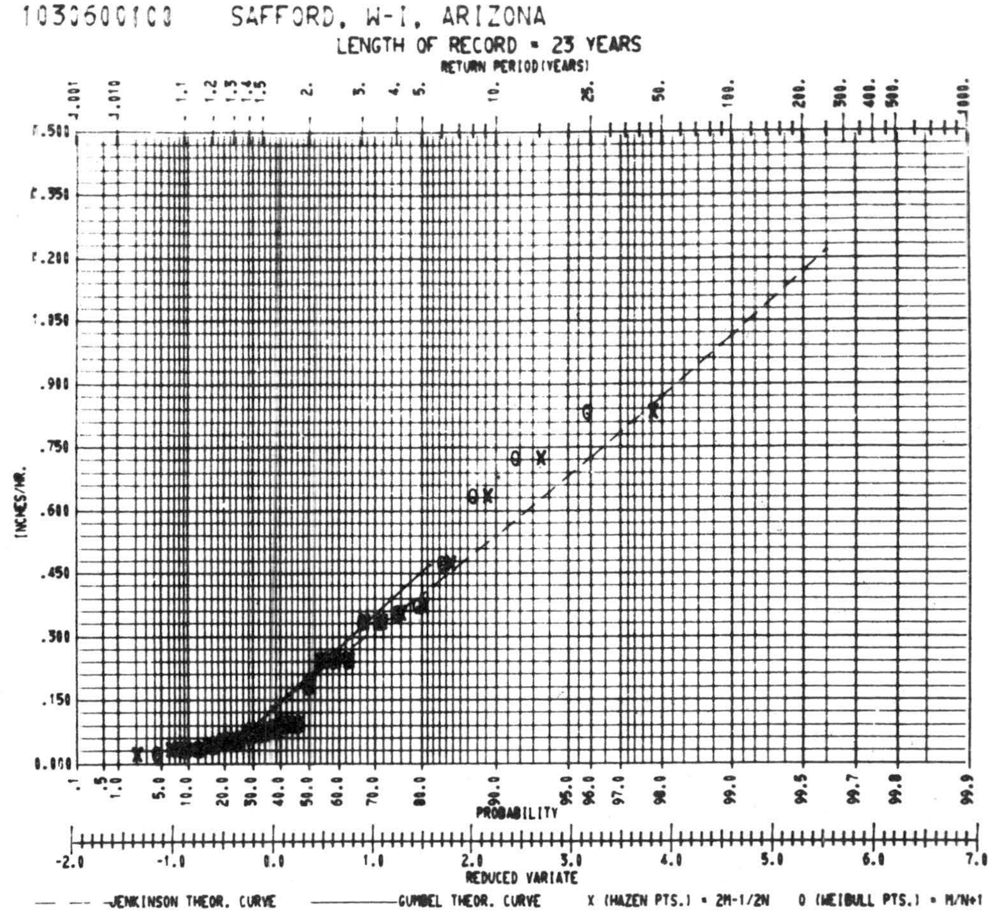


Figure 5. Frequency plot of an annual flood series

The preliminary selection of catchments and events was made using the annual series frequency plots and the following criteria:

1. In order to confine the study to floods and not merely to cases of high water only events with peak discharge return periods greater than two years were selected. Previous studies [Reich, 1962, p. 46] have shown that the peaks with lower return periods adversely affect any prediction equations which predict floods of the magnitudes required for design purposes.

2. In order to assure that the frequency plots used to select the two-year or greater return period events were sufficiently reliable, only catchments with annual series of five years or longer were considered.

Using the two criteria previously mentioned resulted in the reduction of the sample size to some 156 events. In line with the general intent to isolate the effect of rainfall on the catchment response, only flood hydrographs resulting entirely from rainfall were selected. This restriction necessarily confined the study to events resulting from summer storms.

Up to this point in the selection process no consideration was given to the geographic location, size, or climatic features of the watersheds. Neither was consideration given to the types of storms causing the floods or the shape of the resulting hydrographs. In as much as the study was intended to show a relationship between a set of rainfall parameters and a set of hydrograph parameters no selection of "textbook" type single-peaked hydrographs was made. Multi-peaked hydrographs obviously attributable to separate and distinct rainfall bursts were either separated or eliminated depending on the degree to which the hydrograph recession had developed prior to commencement of the next rise.

The distribution of watersheds used in this study according to their geographic location by state is shown in Table 4. Their size distribution is shown in Figure 6. Note that 71% of the sample have catchment areas less than 1 square mile. As stated earlier only events with peak discharge return periods of two years or greater were selected for this study. As a check of possible biasness of the sample toward certain

TABLE 4. DISTRIBUTION OF WATERSHEDS AND FLOOD EVENTS BY STATES

Number	States	Number of Watersheds	Number of Events
1	Arizona	5	14
2	Illinois	1	2
3	Iowa	1	4
4	Mississippi	3	3
5	Nebraska	3	7
6	New Mexico	5	11
7	Ohio	7	22
8	Oklahoma	3	5
9	Texas	4	9
10	Virginia	2	3
11	Wisconsin	3	10
Totals		37	90

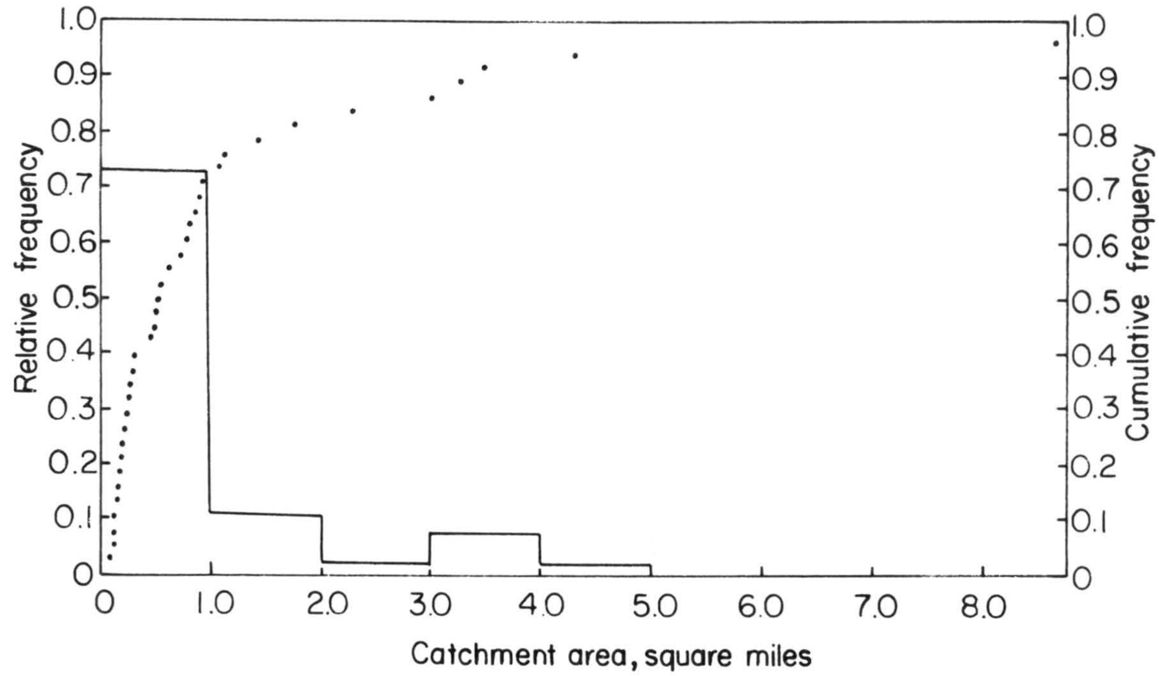


Figure 6. Distribution of catchment sizes

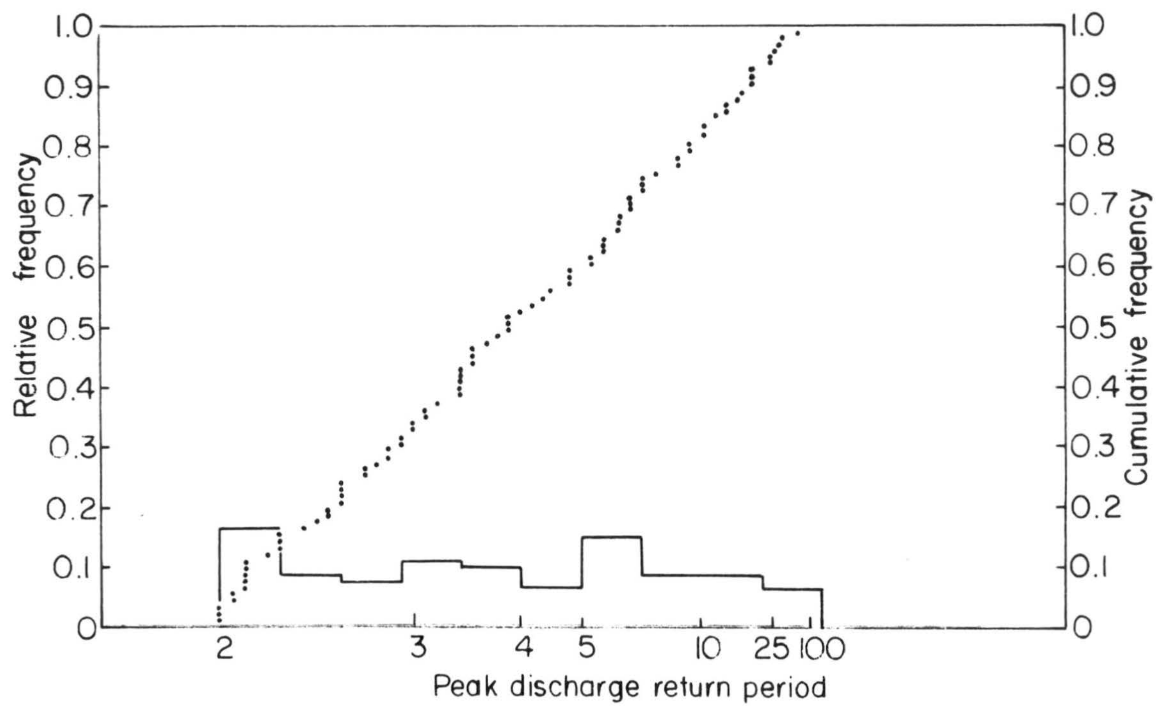


Figure 7. Distribution of peak discharge return periods

return periods Figure 7 was plotted. From the figure one sees that the sample is remarkably uniform, having events for return periods ranging from two years to fifty years. The distribution of the *Pattern Index*, PI , (for definition see Table 3) for the various storms is shown in Figure 8. Here too, the distribution is nearly uniform in the range of Pattern indices from 0.4 to 0.9. Pattern indices greater than 0.6 are generally regarded as indicating early-peaking storms. From the figure one sees that 60% of the storms selected are early-peaking, indicating that most of the floods on this sample of small watersheds are caused by early-peaking storms.

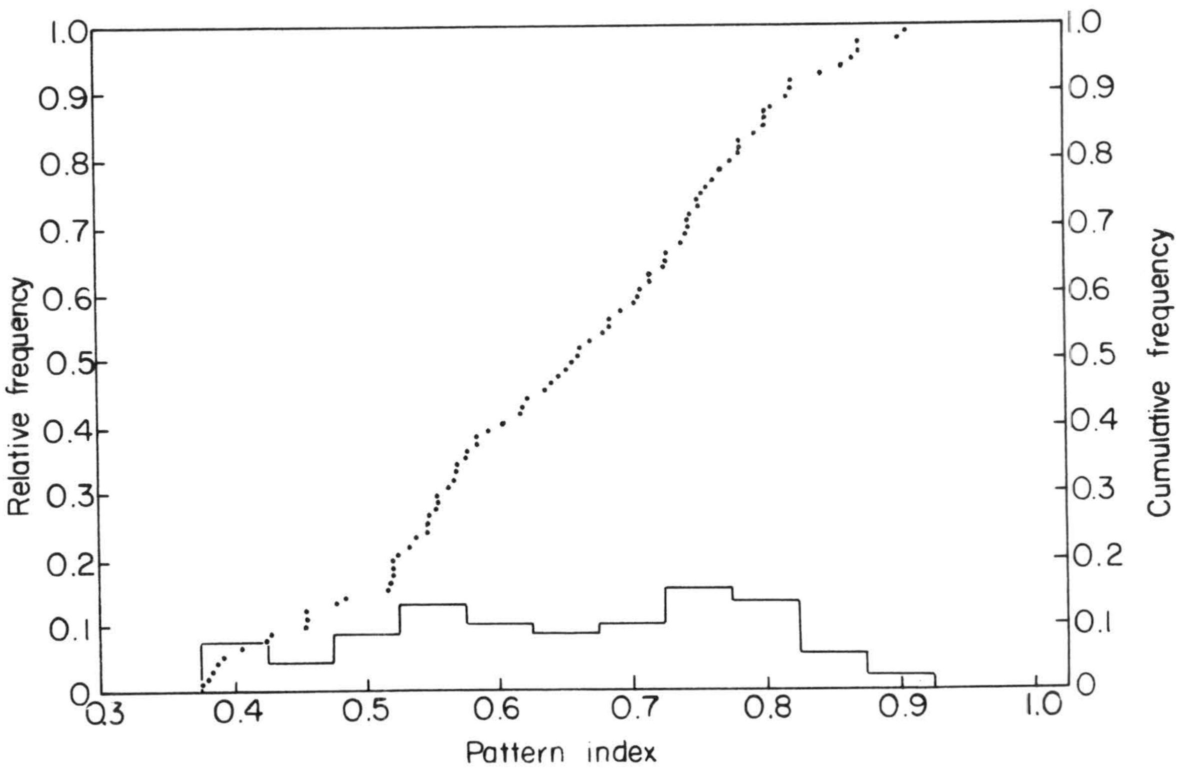


Figure 8. Distribution of Pattern Indices

4.2 Procedural Details

This section describes the details involved in choosing and calculating the various parameters for this study. The parameters considered can be divided into three major groups: (1) those parameters which describe the hyetograph, or the input, (2) those parameters which describe the influence of the watershed and its ability to modify the input, and (3) those parameters which describe the hydrograph, or the output.

4.2.1 Hyetograph Parameters

On the basis of the hypothesis of Chapter III the weighted moments of the hyetograph, m_n' , were calculated. These moments have more than mathematical significance. They, in effect, provide a description of the time-intensity pattern of the rainfall and, moreover, are objective and easy to calculate. The first two moments are analogous to the *center of area* and the *moment of inertia* so familiar to mechanics. The central moments (moments about the mean), m_n , calculated from the moments about the origin, give an indication of the dispersion of the rainfall bursts in time. The third and fourth central moments are quite sensitive to small masses at large distances from the mean and are therefore good descriptors of interrupted storms composed of smaller bursts of rainfall separated from the primary burst of rainfall.

Although the weighted moments about the vertical reflect effects of both time and intensity, they have a tendency to be unduely influenced by the timing of bursts and therefore do not always properly account for high intensities. To overcome this weakness another set of moments was introduced. The weighted moments of the hyetograph about the horizontal axis,

$$y_n' = \frac{\sum_{t=0}^{T} i_t \cdot \Delta t_t \cdot i_0}{\sum_{t=0}^{T} i_t \cdot \Delta t_t \cdot i_0} ,$$

provide an objective means of characterizing the intensity levels of the storm (see Figure 4).

Other more traditional parameters considered were the storm total, P_T , the storm duration, D_T , and the maximum storm intensities for selected time intervals of 2, 5, 10, 15, 20, 25, 30, 35, 40, 50, 60, 90, and 120 minutes. Also tabulated were the pattern index, PI , and the probability of nonoccurrence of the rainfall for the given total rainfall and duration, P_P .

In order to describe the watershed moisture conditions prior to the beginning of the event in question the following two parameters were chosen: the infiltration capacity, f , and the five-day antecedent precipitation, AP_5 . The infiltration factor was computed using the method of Hiemstra and Reich [Hiemstra, 1967] which is a modification of the method given in the *Hydrology Handbook* of the American Society of Civil Engineers [Hydrology, 1949, pp. 47-51]. A typical calculation is shown in Table 5. The five-day antecedent precipitation is the sum of all rainfall occurring prior to the storm. This included any rain falling on the same day of the storm. Continuous low intensity rainfall occurring in the early stages of the storm at rates less than the minimum infiltration rate was also considered as antecedent precipitation.

TABLE 5. EVALUATION OF f

A. Texture

1.	2.	3.	4.	5.	6.	7.	8.	9.
Sand	loamy sand	sandy loam	loam	silt loam	silt	sandy clay loam	clay loam	silty clay loam
0.200	0.150	0.100	0.080	0.050	0.020	0.018	0.006	0.004

10.	11.	12.
sandy clay	silty clay	clay
0.002	0.001	0.000

Structure

B. Strength of aggregates

1.	2.	3.	4.
structureless	weak	moderate	strong
0.030	0.005	0.002	0.001

C. Size of aggregates

1.	2.	3.	4.	5.
very coarse	coarse	medium	fine	very fine
0.020	0.008	0.004	0.002	0.001

D. Shape of aggregates

1.	2.	3.	4.	5.	6.	7.
crumbs	granular	subangular blocky	angular blocky	columnar	prismatic	platy
0.010	0.015	0.003	0.003	0.003	0.001	0.001

E. Permeability

1.	2.	3.	4.	5.	6.	7.
very rapid	rapid	moderately rapid	moderate	moderately slow	slow	very slow
0.200	0.150	0.100	0.080	0.050	0.020	0.005

F. Internal Soil Drainage

1.	2.	3.	4.	5.	6.
very rapid	rapid	medium	slow	very slow	none
0.200	0.150	0.100	0.050	0.015	0.000

G. Erosion Class

1.	2.	3.	4.	5.
few rills; up to 25% of A-hor. gone	shallow gullies, 25-75% of A-hor. lost	shallow and deep gullies 75-100% of A-hor. lost	intricate pattern of gullies soil profiles destroyed	recent alluvial and colluvial deposits
0.020	0.015	0.008	0.001	0.020-0.001

H. Land Capability

I	II	III	IV
very good for cultivation, nearly level	good for cultivation, gently sloping	moderately good for cultivation, moderate slope	fairly good cultivation, strong slope, shallow
0.010	0.008	0.005	0.003

V	VI	VII	VIII
not for cultivation, good for grazing and forestry	moderately good for grazing stony, shallow	fair grazing, steep slope	not suitable for grazing or forestry
0.001	0.001	0.000	0.000

I. Surface Drainage

1.	2.	3.	4.
excellent	good	fair	imperfect
0.001	0.002	0.003	0.005

J. Slope

1.	2.	3.	4.	5.
0 - 3%	3 - 8%	8 - 15%	15 - 25%	25% +
0.015	0.010	0.005	0.002	0.002

EXAMPLE: Safford, Arizona. A. R. S. No. 45.4.

A.	Stony, sand loam	0.180
B.	Structureless	0.030
C.	Medium size	0.004
D.	Granular, blocky shape	0.007
E.	Moderately slow permeability	0.050
F.	Slow internal drainage	0.050
G.	Erosion class 1.	0.020
H.	Land capability VI-VII	0.001
I.	Surface drainage, good	0.002
J.	Slope, 8 - 15%	0.002
	Hence, f =	0.346

4.2.2 Basin Characteristics

The following basin Characteristics were available at the time of this study on magnetic tape [Laurenson, *et al.*, 1963].

Catchment area, A

The catchment area of the stream above a gaging station is that area, expressed in square miles, which is enclosed by a drainage divide. In all cases the area was delineated on an appropriate topographic map. This afforded an opportunity to check the area by planimetering and to note any topographic peculiarities of the watershed.

Length of the main stream, L

The main stream is defined as that stream draining the greatest area. The stream was extended to the watershed boundary in accordance with the contours and its length recorded in miles. This parameter is one measure of the length of the watershed.

Total length of extended streams, L_s

Where possible all marked streams were extended to the watershed boundary in accordance with the contours. Exceptions to this rule were streams which appeared to originate in springs or swamps. The total length of all these extended streams including the main stream represents a measure of the conveyance and storage of the stream system.

Drainage density, D_d

The drainage density, expressed in miles per square miles, is the total length of extended streams, L_s , divided by the catchment area, A . The drainage density reflects the conveyance and storage characteristics on a unit area basis.

Length to center of area, L_c

This is the distance, in miles, along the main stream to a point adjacent to the center of area projected to the main stream. The writer discovered that the center of area could be found quickly and easily and with a fairly high degree of accuracy by centering over the map of the watershed a clear plastic overlay with a system of four intersecting lines drawn on it to form octants. By balancing areas and distances within each of the octants one can determine the center of area in a minute or two. This is one of several measures of the mean travel distance of a catchment.

Mean travel distance, L_t

The mean travel distance, in miles, is determined by measuring the travel distance to the outlet along the stream system from each intersection of a square grid placed over a map of the catchment and averaging these distances. The grid was always orientated in a North-South, East-West orientation and was of such a size that between 30 and 50 intersections fell within the catchment boundary.

Standard deviation of the travel distances, s_t

This self-explanatory parameter has the units of miles.

Dimensionless mean travel distance, L_m

$$L_m = \frac{L_t}{\sqrt{A}} .$$

Dimensionless standard deviation of the mean travel distance, s_d

$$s_d = \frac{s_t}{\sqrt{A}} .$$

Total fall, H

The total fall is the difference in elevation expressed in feet between the highest point on the main stream and the datum of the stream gage; sometimes considered a measure of the steepness of the watershed.

Stream slope, S_1

S_1 is the total fall, H , divided by the length of the main stream, L , and is expressed in feet per mile. This and the following three stream slopes are attempts at finding parameters which relate to the travel time of a watershed.

Stream slope, S_2

S_2 is the weighted mean slope of the main stream.

$$S_2 = \frac{2 \sum l_i z_i}{L^2}, \text{ feet per mile}$$

where l_i is the distance along the main stream between successive contours in miles, and z_i is the average elevation above the outlet for each reach in feet, l_i .

Stream slope, S_3

$$S_3 = \left(\frac{\sum l_i}{\sum \frac{l_i}{\sqrt{s_i}}} \right)^2, \text{ feet per mile}$$

where l_i is the distance along the main stream between successive contours in miles, and s_i is the slope for each reach, l_i .

Stream slope, S_4

S_4 is the difference between the elevation of the main stream at 85% of its length and the elevation at 10% of its length divided by 75% of its total length in feet per mile.

Overland, R_1

$$R_1 = \frac{c \cdot L_c}{A}, \text{ feet per mile}$$

where c is the contour interval of a topographic map of the catchment, L_c is the total length of all contour lines on the map, and A is the catchment area. This is one of six attempts at obtaining a single parameter which is representative of the overland slopes of a catchment.

Overland slope, R_2

$$R_2 = \frac{1.57cN}{L_g}, \text{ feet per mile}$$

where c is the contour interval of a topographic map of the catchment, N is the total number of intersections of the grid lines with all contour lines (see explanation of grid under Mean travel distance, L_t), and L_g is the total length of grid lines within the catchment measured in both the North-South and East-West directions.

Overland slope, R_3

$$R_3 = \frac{\sum \left(\Delta h_i \cdot \bar{L}_{c_i} \right)}{A}, \text{ feet per mile}$$

where Δh_i is the difference in elevation between two successive contours, $\bar{L}_{c_i}$ is the average length of those two contour lines, and A

is the catchment area. This is a form of weighted mean overland slope where $\bar{L}_{c_i}$ is a measure of the area between successive contours, the value of which would be exceedingly difficult to determine.

Overland slope, R_4

R_4 is the mean overland slope determined by point sampling of the catchment slopes at the intersections of the grid (see explanation of grid under Mean travel distance, L_t).

$$R_4 = \frac{1}{n} \cdot \sum \frac{c}{d_i} \text{, feet per mile}$$

where n is the number of grid intersections, c is the contour interval, and d_i is the distance between contours at each intersection of the grid.

Overland slope, R_5

R_5 is the median overland slope, in feet per mile, determined by arranging the n values comprising R_4 in descending order and finding the value which evenly splits the array.

Relief ratio, R_6

A dimensionless variable, R_6 equals the total fall, H , divided by the longest straight line dimension between any two points on the catchment boundary.

Perimeter, P

The perimeter is the length around the catchment boundary in miles and is a measure of the compactness of the watershed area.

Average catchment width, W

Expressed in miles, W equals the catchment area, A , divided by the length of the main stream, L .

Form factor, F

The form factor is a dimensionless variable which likens the shape of the catchment to a square and is obtained from the ratio of the catchment area, A , to the square of the length of the main stream, L^2 .

Compactness coefficient, C

The compactness coefficient is another dimensionless variable which is intended to describe the shape of the catchment. C is the ratio of the perimeter of the catchment, P , to the circumference of a circle having the same area as the catchment. Thus,

$$C = \frac{0.28\ P}{\sqrt{A}} .$$

The preceding parameters can be considered as possible descriptors of the fixed catchment characteristics. Included in the next group of parameters are those which may vary from flood to flood but which rightly should be associated with the basin characteristics.

Since the watersheds used in this study represent a wide variety of climatic regions, it was deemed necessary to introduce the following pseudo-climatic parameters: the 10-year 1-hour precipitation, C_1; mean annual precipitation, C_2 . A third parameter; the mean monthly temperature, C_3 ; is both a descriptor of climate and indirectly a descriptor of evaporation and transpiration. Since adequate data was not available to fully describe all possible losses it was decided to use the loss rate, LR , for this purpose.

Lastly, the probability of the hydrograph peak discharge, P_q , is introduced as a variable related to the hydrograph, but which when considered as an array of variables is properly associated with the individual catchment.

4.2.3 Hydrograph Parameters

As mentioned earlier no attempt was made in this study to separate the total runoff into surface flow and ground water flow. Experience has shown that for small watersheds of the size involved in this study the assumption of negligible ground water flow is often valid.

In most cases the tabulated data indicated that the hydrographs receded to zero flow rather rapidly. For the few hydrographs where this was not the case the recessions were extended to zero flow using the familiar recession expression,

$$q_i = q_{i-1} \cdot e^{-\Delta t/k}$$

where q_i is the flow at any instant, q_{i-1} is the flow at any previous instant, t is the time lapse between time i and time $i-1$, and k is a recession constant. k is the only unknown and can be found using the tabulated data. Zero flow was defined as a flow less than 0.00005 inches per hour, since true zero flow would occur only after a very long time.

In nearly all cases the antecedent flow was zero. For the few cases where the antecedent flow was not zero the following separation technique was used. The recession characteristics of small watersheds do not change appreciably with successive storms because of the lack of catchment storage. This assumption was borne out by the work of Ho [Ho, 1967] in his study of small watershed recessions. Similar justification for the similarity of recessions can be found in unit hydrograph theory. The separation was accomplished by transposing the hydrograph recession horizontally until it matched the antecedent flow at the time of the commencement of the rising limb. This transposed segment or pseudo-antecedent recession was then subtracted from the observed flow to give a hydrograph with a discharge rising from zero.

To find parameters which describe the hydrograph, three methods of approach were considered. First, the hypothesis of Chapter III was followed in computing the moments of the hydrograph. Second, the more traditional hydrograph parameters of total volume, V ; peak unit discharge, q_p ; and rise time, t_r ; were computed. Third, a mathematical function was fitted to the hydrograph and the parameters of this fitted function were determined. The function was a modification of the *incomplete gamma* function.

$$q_f = \frac{\beta V \gamma^{\gamma r+1}}{\Gamma(\gamma r + 1)} t^{\gamma r} e^{-\gamma t} \tag{2.4}$$

This function has all the characteristics of the hydrograph and has already been discussed and justified on a theoretical basis [Edson, 1951; Nash, 1958]. To improve its ability to fit the observed hydrograph a more elemental form (see equation 2.1) was used,

$$q_f = \frac{q_0 t^{\gamma r}}{r^{\gamma r}} e^{-\gamma(t-r)} \tag{4.1}$$

here q_f is the unit discharge at any time, t , q_0 is the peak unit discharge, γ is a shape parameter, and r is the distance from the origin to the peak.

It was decided to use a weighted least squares fitting process to estimate the three parameters: q_0 , γ and r . In general, the hydrologist is primarily concerned with the agreement of the observed and fitted hydrographs in the portion of the rising limb and the crest. By weighting the fitting process in proportion to the observed discharge a better agreement in the crest portion could be assured. Three weighting coefficients were tried: (1) the observed discharge, (2) the square root of the observed discharge, and (3) the cube root of the observed discharge. A preliminary investigation of the results obtained using each of the factors showed no appreciable difference in hydrograph fits and it was decided to use only the observed discharge.

By least squares

$$\sum(q_{observed} - q_{fitted})^2 = \Delta \tag{4.2}$$

and weighting in proportion to the observed discharge gives

$$\sum q_{ob}(q_{ob} - q_f)^2 = \Delta \ . \tag{4.3}$$

Minimizing by differentiating with respect to each of the variables and setting the results equal to zero produced three equations in three unknowns:

$$\sum_{t=0}^{T} \left(q_{ob_t}{}^2 \cdot q_{f_t} - q_{ob_t} \cdot q_{f_t}{}^2 \right) = 0 \tag{4.4}$$

$$\sum_{t=0}^{T} \left(q_{ob_t}{}^2 \cdot q_{f_t} - q_{ob_t} \cdot q_{f_t}{}^2 \right) \cdot \left(1 - \frac{1}{r} + \ln t - \ln r \right) = 0 \tag{4.5}$$

$$\sum_{t=0}^{T} \left(q_{ob_t}{}^2 \cdot q_{f_t} - q_{ob_t} \cdot q_{f_t}{}^2 \right) \cdot \left(1 - \frac{t}{r} + \ln t - \ln r \right) = 0 \tag{4.6}$$

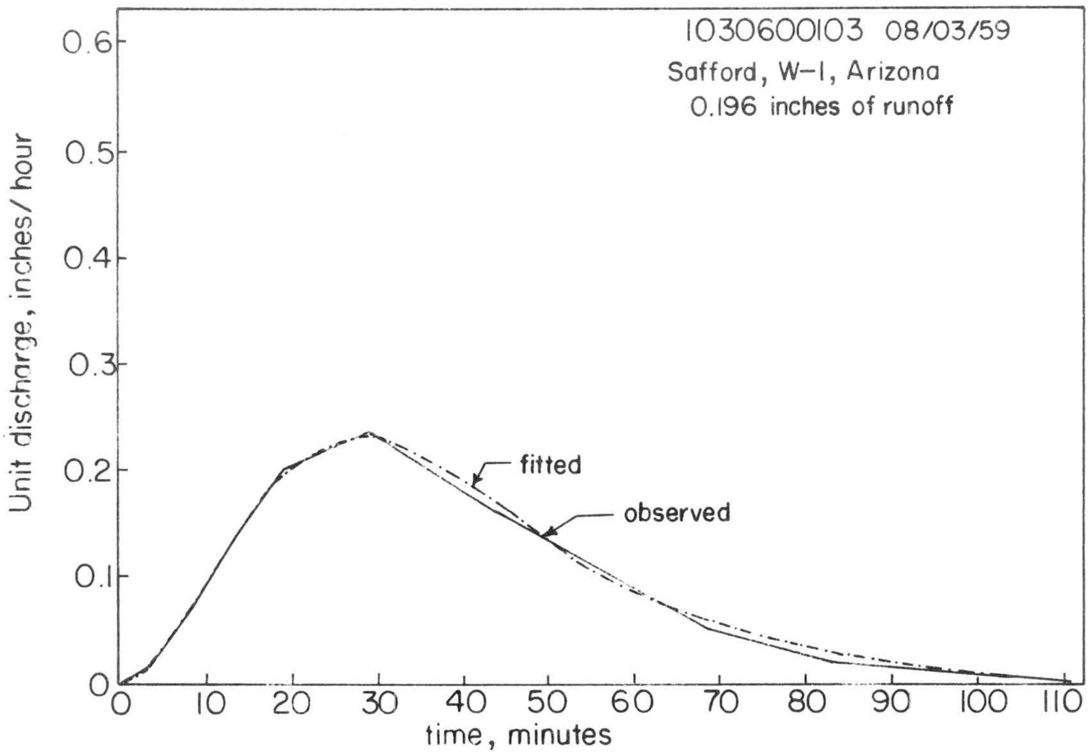
Since these could not be solved explicitly an iterative procedure was used whereby the computer closed in on the solutions after having been given a first estimate of the variables. The estimation of these three parameters: q_0 , γ , and r completed the list of parameters used to describe the hydrograph.

It is interesting to note that Equation 4.1 when fitted by the weighted least squares method gives a remarkably good fit to the hydrograph. The fitting process was carried out on the 90 selected events and Figures 9 and 10 show one of the best and one of the poorest fits, respectively. The adaptability of this iterative solution to the weighted least squares can be seen in Figure 11 where the computer has produced a sensible fit to a very difficult shaped hydrograph. Space limitations precluded showing more of these fits.

4.3 Multivariate Analysis

Multivariate analysis may be defined as the branch of statistical analysis which is concerned with the relationships of sets of dependent variables [Kendall, 1961, p. 6]. "In such analyses, a vector of means and a matrix of covariances of several variables are used instead of the simple mean and variance of a single variable. This concept allows the association of errors with more than one variable, . . . [Chow, 1964, p. 8-67]".

Numerous volumes have been written on multivariate analysis and more specifically multiple linear correlation and regression. Some of these are by Kendall [1943], Snedecor [1956], Ford [1959], Efroymsen [1960], and Chow [1964, Chapter 8, Part II]. Therefore, it is felt that it is not necessary to go deeper into multiple linear regression theory, except as it applies to this study. The concern here is to show a relationship between the parameters of the hydrograph, the hyetograph, and the basin characteristics. To this end multiple linear regression provides an approach.



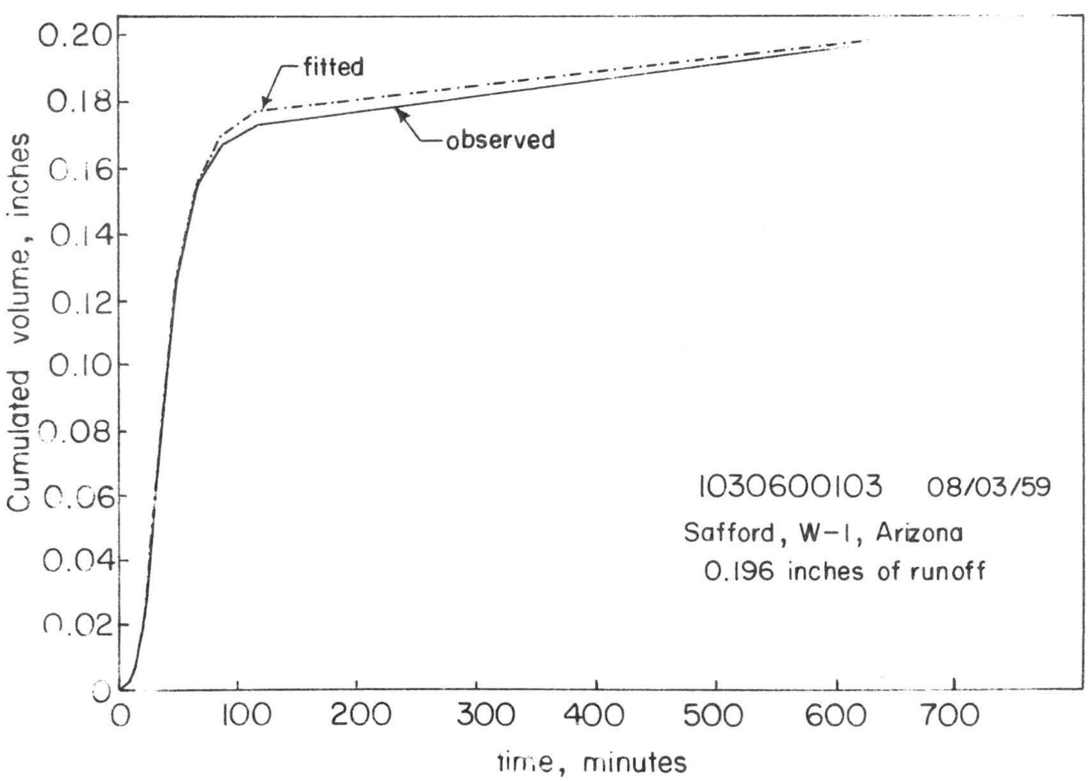
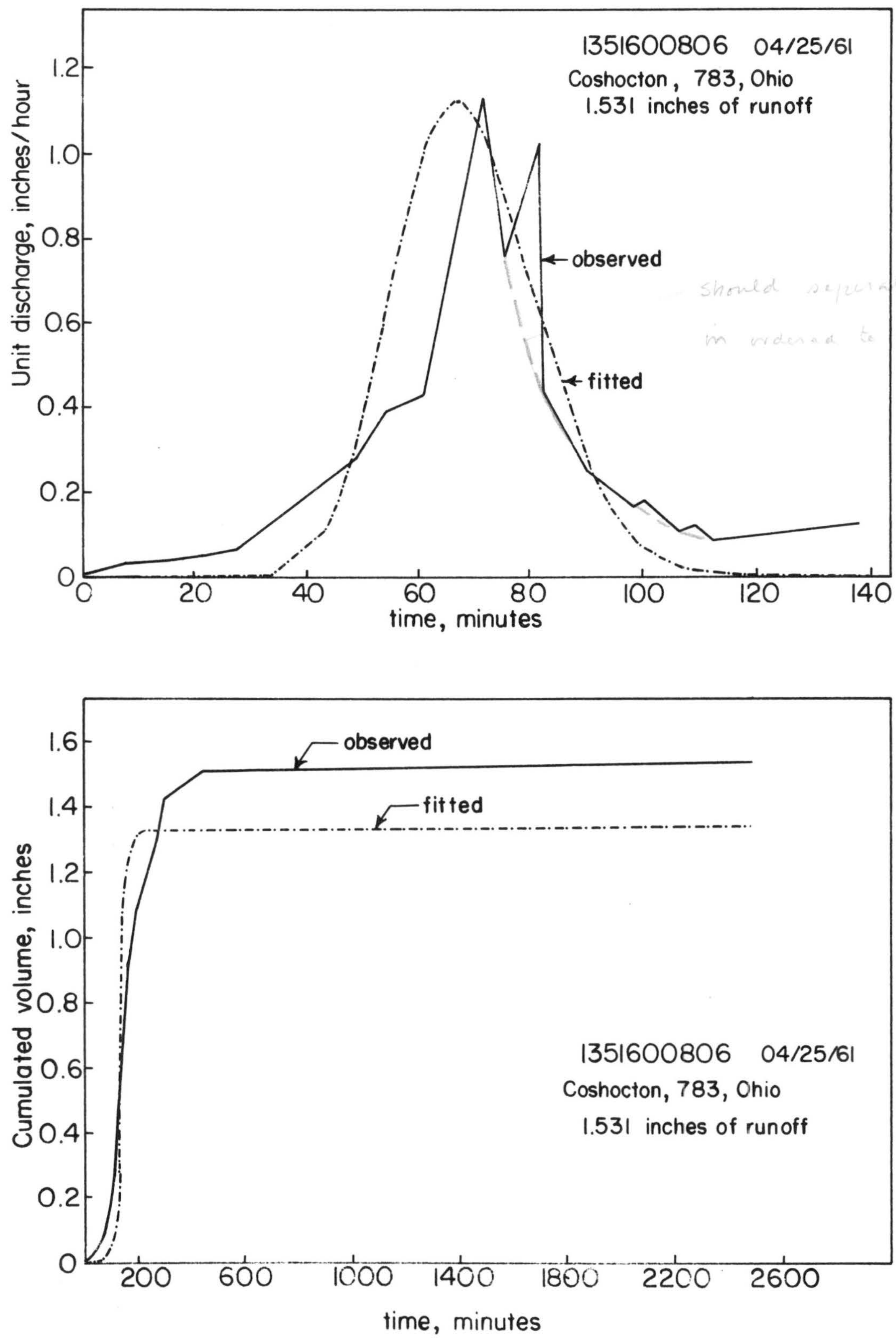


Figure 9. Plot of observed and fitted hydrographs and observed and fitted cumulative volumes (good fit)



should separated in order to get better fit

Figure 10. Plot of observed and fitted hydrographs and observed and fitted cumulative volumes (poor fit)

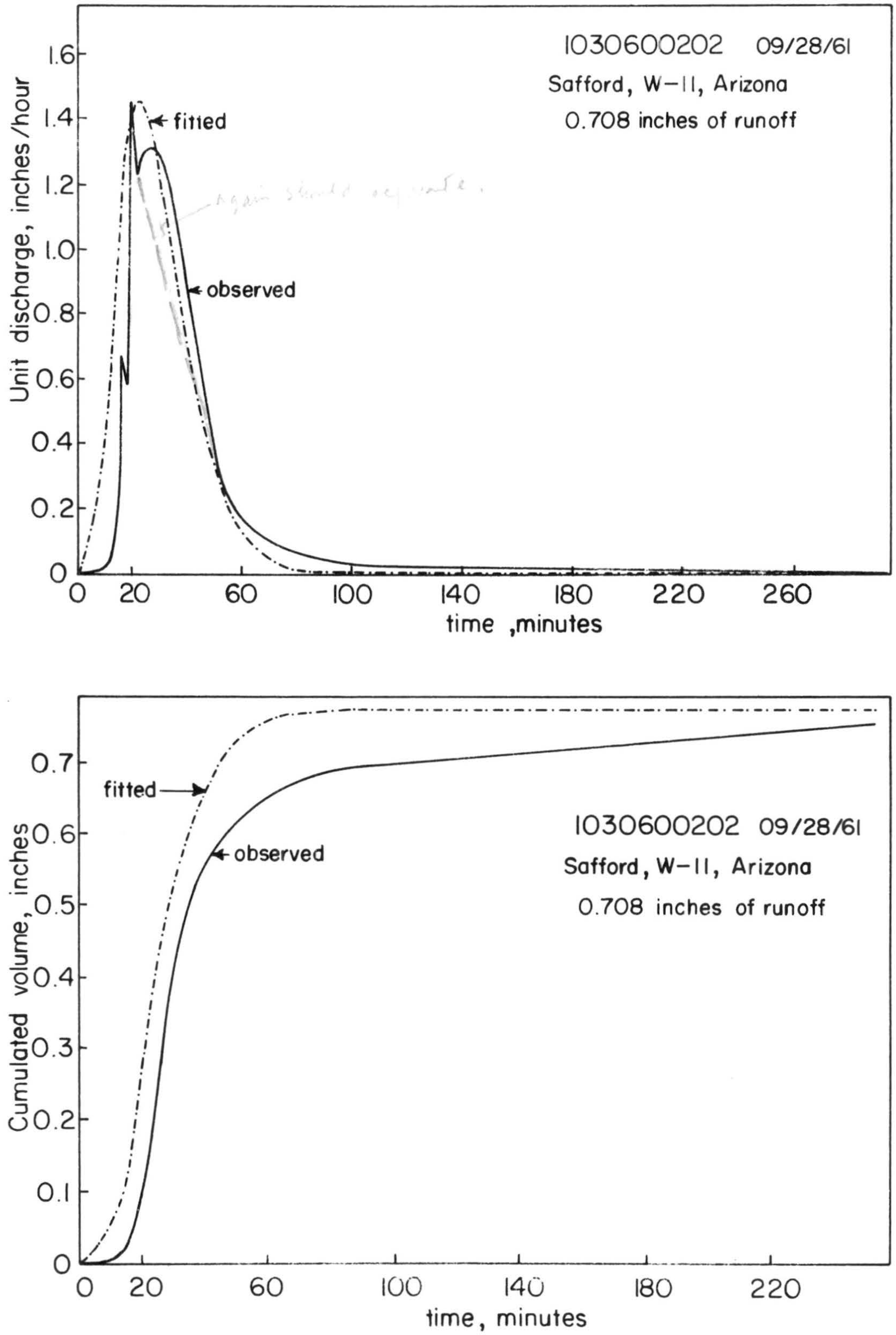


Figure 11. Plot of observed and fitted hydrographs and observed and fitted cumulative volumes (difficult fit)

This study is not concerned with the association of errors with individual variables or with estimating the independent contributions of these variables. Therefore, it is not imperative that the basic assumptions of multiple linear regression of no errors in the independent variables and no correlation between the independent variables be absolutely met.

The multiple linear regression relation takes the form

$$y = a_0 + a_1x_1 + a_2x_2 + \cdot\cdot\cdot\ a_nx_n \tag{4.6}$$

where y is the dependent variable, the x's are the independent variables, and the a's are the regression coefficients. The best functional relationship is that one which in accordance with the method of least squares produces the minimum of the squared deviations between the observed and the computed dependent variable. In practice this relation is arrived at in a stepwise fashion by adding to the relation one independent variable at a time in a manner such that the variable added has the highest partial correlation with the dependent variable partialed on the previously added independent variables.

It is conceivable that the relation between the dependent variable and independent variables may not be additive, but rather multiplicative. In this case the form may be

$$y = ax_1^{\,b}x_2^{\,c}x_3^{\,d} \cdot\cdot\cdot\ , \tag{4.7}$$

This nonlinear relation can be avoided by using a logarithmic transform to produce

ln

$$y = \ln a + b \ln x_1 + c \ln x_2 + d \ln x_3 + \cdot\cdot\cdot\ , \tag{4.8}$$

a linear relation. A computer program for performing all the requisite computations previously mentioned is available [Dixon, 1964]. The results shown in the following sections were derived using this program.

Two statistics are commonly used to measure the efficiency of a regression. They are the *standard error of estimate*, S_{ey} , and the *coefficient of multiple determination*, R^2 .

The standard error of estimate is by definition the standard deviation of the *residuals*, or differences between the estimated values and the actual values. It indicates the closeness with which new estimates may be expected to approximate the true but unknown values. Basic to this is the assumption of drawing from the same universe for both the regression variables and new variables. Its dimensions are the same as those of the original dependent variable. In the case of logarithms the the dimensions are in logarithms.

The coefficient of multiple determination is by definition the ratio of the explained variance to the total variance and shows how much of the variation in the dependent variable is accounted for by the independent variables.

The relative closeness of an estimate is best measured by R^2 , where as the absolute closeness is best measured by S_{ey} . In the case of logarithms S_{ey} represents a percentage difference rather than an absolute value.

4.4 A Preliminary Look at the Data

A preliminary look at the data involved computing the means, standard deviations and the correlation matrices for the variables and the logarithms of the variables mentioned in sections 4.2.1, 4.2.2 and 4.2.3. Table 6 shows the means and standard deviations. Tables 7 and 8 are summaries of these matrices showing only those variables whose absolute simple correlation was greater than 0.600.

Inspection of the matrix of regular variables, Table 6, showed no simple correlation of sufficient magnitude between the hydrograph moments and the hyetograph moments to suggest a linear relationship existed. It did, however, indicate a simple correlation of +0.886 between the first moment of the hyetograph, m_1' , and the hydrograph rise time, t_r . The correlations between the rise time and the other hyetograph moments were equally significant; ranging from +0.863 down to +0.556. No other hydrograph parameters showed significant simple correlations with the hyetograph parameters.

The lack of simple linear correlation between the hydrograph and the hyetograph moments is apparently due to the moment-producing effect of the hydrograph tail. As the order of the moments increased the correlation decreased. This would suggest a possible multiplicative relationship.

Inspection of Table 7, the correlation matrix of the logarithms of the variables, showed the above hypothesis to be highly probable. Logarithms of all the hydrograph moments showed significant correlations with the logarithms of the hyetograph moments; ranging from +0.706 down to +0.507. The fact that all the correlations are positive adds more credence to the hypothesis of a relation between input and output moments since a longer duration input would intuitively indicate longer response periods.

The correlations for the logarithm of the fitted rise time improved significantly, showing correlations with the logarithms of the hyetograph moments ranging from +0.795 down to +0.682.

TABLE 6. MEANS AND STANDARD DEVIATIONS OF VARIABLES

Variable	Units	Mean	Standard Deviation	Mean of Logarithm	Standard Deviation of Logarithm
LR	inches/hour	.71920	.62658	- .80217	1.19410
P_p	per cent	.64711	.30316	- .67583	.90629
PI	dimensionless	.64839	.13878	- .45779	.22785
C_3	°F	69.86111	9.05508	4.23686	.14556
f	inches/hour	.49789	.20091	- .80114	.50256
C_1	inches	2.26244	.53135	.78578	.25715
C_2	inches	27.10044	12.46747	3.15159	.60080
m_1'	minutes	42.34141	37.73534	3.44852	.76629
m_2'	minutes^2	4439.47529	10271.64004	7.26842	1.51222
m_3'	minutes^3	894661.53833	3791925.59739	11.30018	2.27353
m_4'	minutes^4	279832189.97778	1629044556.17597	15.46008	3.05790
m_2	minutes^2	1238.53260	2807.02558	5.91664	1.59241
m_3	minutes^3	78631.78313	243360.03883		
m_4	minutes^4	37965152.51301	160138726.05096	13.04201	3.48509
y_1'	inches/hour	1.35365	.60291	.19749	.48917
y_2'	$(\text{inches/hour})^2$	2.72955	2.47058	.64293	.93171
y_3'	$(\text{inches/hour})^3$	7.36846	12.00374	1.22540	1.37372
y_4'	$(\text{inches/hour})^4$	3999.90121	37706.50565	2.02422	2.12727
q_0	inches/hour	.86794	.56762	- .35096	.66579
γ	minutes^{-1}	.15527	.12354	- 2.10549	.72350
r	minutes	60.11111	61.35592	3.74334	.82674
A	square miles	1.03044	1.36754	- .58937	1.08405
L	miles	1.78989	1.48852	.29827	.73311
L_s	miles	5.33822	5.21538	1.26556	.93461
L_c	miles	.94256	.99802	- .41250	.78002
L_t	miles	.99800	.78021	- .26307	.70629
s_t	miles	.45970	.42952	- 1.10593	.76847
s_d	dimensionless	.47733	.20643	- .81890	.38643
P	miles	4.13900	2.81987	1.23800	.58119
H	feet	237.34444	250.97402	5.10548	.83342
S_1	feet/mile	162.52222	126.15562	4.81033	.77501
S_2	feet/mile	117.07778	77.56233	4.51512	.75443
S_3	feet/mile	125.25556	81.39908	4.57884	.76420
S_4	feet/mile	134.13333	99.90256	4.60473	.81122
D_d	miles/square miles	8.29422	8.45754	1.85610	.64306
W	miles	.47933	.29601	- .87638	.51425
F	dimensionless	.40111	.26389	- 1.14042	.71166
C	dimensionless	1.51678	1.50431	.27748	.43648
L_m	dimensionless	1.03522	.30502	- .00487	.27723
R_1	feet/mile	472.18889	256.14045	5.98662	.63272
R_2	feet/mile	460.16667	254.16217	5.95733	.63016
R_3	feet/mile	470.64444	251.43687	5.98419	.63903
R_4	feet/mile	415.02222	225.10003	5.85713	.62153
R_5	feet/mile	373.52222	219.97422	5.72548	.66393
R_6	dimensionless	.03210	.02332	- 3.67803	.70199
q_p	inches/hour	.87035	.56973	- .34980	.66919
P_q	per cent	.73904	.14432	- .32212	.20223
Lag	minutes	19.07778	18.02135		
t_r	minutes	61.23333	58.68102	3.78933	.79413
I_2	inches/hour	4.76567	1.96164	1.47575	.42938
I_5	inches/hour	4.16511	1.50092	1.35505	.39798
I_{10}	inches/hour	3.56711	1.25051	1.20164	.40158
I_{15}	inches/hour	3.13278	1.15864	1.06419	.42292
I_{20}	inches/hour	2.78711	1.06319	.94138	.43991
I_{25}	inches/hour	2.49967	.96984	.82648	.45860
I_{30}	inches/hour	2.27522	.89773	.72625	.47662
I_{35}	inches/hour	2.07156	.84720	.61707	.52938
I_{40}	inches/hour	1.91122	.79781	.52444	.58110
I_{50}	inches/hour	1.68556	.71474	.39396	.59742
I_{60}	inches/hour	1.49689	.65843	.26934	.61009
I_{90}	inches/hour	1.10433	.54096	- .05343	.64525
I_{120}	inches/hour	.87622	.46460	- .29444	.64592
D_T	minutes	138.22222	134.11447	4.58262	.81811
P_T	inches	1.91156	1.13883	.49624	.55864
AP_5	inches	1.52278	1.91245		
V	inches	.83344	.54126	- .43660	.76622
M_1'	minutes	149.32374	210.99595	4.51749	.91559
M_2'	minutes^2	309113.74456	1471726.31906	9.75177	2.12087
M_3'	minutes^3	2966849144.46666	18733997080.56616	15.52092	3.46977
M_4'	minutes^4	43294018897567.25000	314511513814762.00000	21.60932	4.87556

TABLE 7. SUMMARY OF CORRELATION MATRIX OF REGULAR VARIABLES WITH CORRELATION COEFFICIENTS $\geq |0.600|$

Variables	Correlation Coefficient	Variables	Correlation Coefficient	Variables	Correlation Coefficient	Variables	Correlation Coefficient	Variables	Correlation Coefficient
$f-R_2$	.609	$y_1'-I_{10}$	.860	L_t-P	.888	R_2-R_6	.657	$I_{25}-I_{40}$	.935
C_1-C_2	.773	$y_1'-I_{15}$	.814	L_t-L_m	.685	R_3-R_4	.908	$I_{25}-I_{50}$	.880
C_1-V	.647	$y_1'-I_{20}$	.761	s_t-s_d	.814	R_3-R_5	.824	$I_{25}-I_{60}$	.825
$m_1'-m_2'$	.900	$y_1'-I_{25}$	.715	s_t-P	.864	R_3-R_6	.664	$I_{25}-I_{90}$	.723
$m_1'-m_3'$	.768	$y_1'-I_{30}$	.653	s_t-L_m	.760	R_4-R_5	.940	$I_{25}-I_{120}$	.647
$m_1'-m_4'$	.697	$y_2'-y_3'$	.942	s_d-F	−.704	R_4-R_6	.653	$I_{30}-I_{35}$	.990
$m_1'-m_2$	.850	$y_2'-I_2$	.854	s_d-L_m	.954	t_r-D_T	.767	$I_{30}-I_{40}$	.975
$m_1'-m_4$	.702	$y_2'-I_5$	.801	$P-W$	.714	I_2-I_5	.870	$I_{30}-I_{50}$	.928
$m_1'-r$	.873	$y_2'-I_{10}$	.781	S_1-S_2	.903	I_2-I_{10}	.785	$I_{30}-I_{60}$	.879
$m_1'-t_r$	.889	$y_2'-I_{15}$	.711	S_1-S_3	.889	I_2-I_{15}	.737	$I_{30}-I_{90}$	.786
$m_1'-D_T$	.800	$y_2'-I_{20}$	.632	S_1-S_4	.904	I_2-I_{20}	.679	$I_{30}-I_{120}$	.712
$m_2'-m_3'$	.967	$y_3'-I_2$	.802	S_1-R_1	.714	I_2-I_{25}	.637	$I_{35}-I_{40}$	.994
$m_2'-m_4'$	.933	$y_3'-I_5$	.637	S_1-R_2	.702	I_5-I_{10}	.944	$I_{35}-I_{50}$	.958
$m_2'-m_2$	.975	$y_3'-I_{10}$	.602	S_1-R_3	.684	I_5-I_{15}	.882	$I_{35}-I_{60}$	.914
$m_2'-m_4$	.885	q_0-q_P	.997	S_1-R_4	.690	I_5-I_{20}	.820	$I_{35}-I_{90}$	.829
$m_2'-y_1'$	.874	$r-t_r$	.995	S_1-R_6	.893	I_5-I_{25}	.771	$I_{35}-I_{120}$	.756
$m_2'-t_r$	.878	$r-D_T$	.767	S_2-S_3	.971	I_5-I_{30}	.712	$I_{35}-P_T$	.617
$m_2'-D_T$	.734	$A-L$	.775	S_2-S_4	.939	I_5-I_{35}	.653	$I_{40}-I_{50}$	.979
$m_3'-m_4'$	.994	$A-L_s$	.745	S_2-R_1	.684	I_5-I_{40}	.609	$I_{40}-I_{60}$	.944
$m_3'-m_2$	.957	$A-L_c$	.683	S_2-R_2	.679	$I_{10}-I_{15}$	.961	$I_{40}-I_{90}$	.867
$m_3'-m_4$	.916	$A-L_t$	.776	S_2-R_3	.651	$I_{10}-I_{20}$	.908	$I_{40}-I_{120}$	.798
$m_3'-r$	.804	$A-s_t$	.738	S_2-R_4	.686	$I_{10}-I_{25}$	.851	$I_{40}-P_T$	.662
$m_3'-t_r$	.798	$A-P$	.924	S_2-R_6	.792	$I_{10}-I_{30}$	.797	$I_{50}-I_{60}$	.988
$m_3'-D_T$	.629	$A-W$	.839	S_3-S_4	.910	$I_{10}-I_{35}$	.741	$I_{50}-I_{90}$	.934
$m_4'-m_2$	.928	$L-L_s$	.886	S_3-R_1	.725	$I_{10}-I_{40}$	.697	$I_{50}-I_{120}$	.881
$m_4'-m_4$	.910	$L-L_c$	.958	S_3-R_2	.712	$I_{10}-I_{50}$	.622	$I_{50}-P_T$	.756
$m_4'-r$	.758	$L-L_t$	.955	S_3-R_3	.688	$I_{15}-I_{20}$	.982	$I_{60}-I_{90}$	.965
$m_4'-t_r$	.749	$L-s_t$	.934	S_3-R_4	.715	$I_{15}-I_{25}$	.943	$I_{60}-I_{120}$	.928
m_2-m_3	.655	$L-s_d$	.720	S_3-R_6	.782	$I_{15}-I_{30}$	.892	$I_{60}-P_T$	.821
m_2-m_4	.932	$L-P$	.874	S_4-R_1	.609	$I_{15}-I_{35}$	.842	$I_{90}-I_{120}$	.982
m_2-r	.869	$L-L_m$	.741	S_4-R_2	.613	$I_{15}-I_{40}$	.804	$I_{90}-P_T$	.902
m_2-t_r	.867	L_s-L_c	.851	S_4-R_4	.628	$I_{15}-I_{50}$	.736	$I_{120}-P_T$	.960
m_2-D_T	.797	L_s-L_t	.873	S_4-R_6	.786	$I_{15}-I_{60}$	.670	P_T-V	.617
m_3-m_4	.777	L_s-s_t	.773	$F-L_m$	−.806	$I_{20}-I_{25}$	.982	$M_1'-M_2'$	.882
m_3-D_T	.849	L_s-P	.776	R_1-R_2	.985	$I_{20}-I_{30}$	.947	$M_1'-M_3'$	.819
m_4-r	.784	L_c-L_t	.886	R_1-R_3	.983	$I_{20}-I_{35}$	.907	$M_1'-M_4'$	.778
m_4-t_r	.766	L_c-s_t	.870	R_1-R_4	.927	$I_{20}-I_{40}$	.876	$M_2'-M_3'$	.987
m_4-D_T	.774	L_c-s_d	.666	R_1-R_5	.845	$I_{20}-I_{50}$	.813	$M_2'-M_4'$	.964
$y_1'-y_2'$	.864	L_c-P	.821	R_1-R_6	.683	$I_{20}-I_{60}$	.750	$M_3'-M_4'$	.993
$y_1'-y_3'$	.728	L_c-L_m	.716	R_2-R_3	.966	$I_{20}-I_{90}$	.634		
$y_1'-I_2$	.834	L_t-s_t	.939	R_2-R_4	.926	$I_{25}-I_{30}$	.985		
$y_1'-I_5$	.887	L_t-s_d	.686	R_2-R_5	.857	$I_{25}-I_{35}$	.959		

TABLE 8. SUMMARY OF CORRELATION MATRIX OF LOGARITHMIC VARIABLES WITH CORRELATION COEFFICIENTS ≥ |0.600|

Variables	Correlation Coefficient	Variables	Correlation Coefficient	Variables	Correlation Coefficient	Variables	Correlation Coefficient	Variables	Correlation Coefficient
$f-R_1$	.684	m_2-r	.750	$A-L$	.911	S_3-S_4	.930	$I_{20}-I_{35}$	.879
$f-R_2$	.691	m_2-t_r	.762	$A-L_s$	.807	S_3-R_4	.601	$I_{20}-I_{40}$	.821
$f-R_3$	.655	m_2-D_T	.957	$A-L_c$	.870	S_3-R_6	.761	$I_{20}-I_{50}$	.779
C_1-C_2	.845	m_2-M_1'	.688	$A-L_t$	.918	S_4-R_6	.734	$I_{20}-I_{60}$	.745
C_1-V	.742	m_2-M_2'	.607	$A-s_t$	.889	D_d-W	−.617	$I_{20}-I_{90}$	.683
C_2-P_T	.604	m_4-r	.682	$A-P$	.966	$F-L_m$	−.952	$I_{20}-I_{120}$	.654
C_2-V	.647	m_4-t_r	.692	$A-W$	.795	R_1-R_2	.979	$I_{25}-I_{30}$	.987
C_2-M_1'	.600	m_4-D_T	.943	$L-L_s$	.830	R_1-R_3	.980	$I_{25}-I_{35}$	.941
C_2-M_3'	.608	m_4-M_1'	.665	$L-L_c$	.977	R_1-R_4	.920	$I_{25}-I_{40}$	.894
C_2-M_4'	.622	$y_1'-y_2'$	.937	$L-L_t$	.975	R_1-R_5	.859	$I_{25}-I_{50}$	.858
$m_1'-m_2'$	.988	$y_1'-y_3'$	.914	$L-s_t$	.974	R_1-R_6	.688	$I_{25}-I_{60}$	.828
$m_1'-m_3'$	.967	$y_1'-y_4'$	.899	$L-s_d$	.668	R_2-R_3	.959	$I_{25}-I_{90}$	.774
$m_1'-m_4'$	.948	$y_1'-I_2$	.845	$L-P$	.909	R_2-R_4	.921	$I_{25}-I_{120}$	.745
$m_1'-m_2$	.891	$y_1'-I_5$	.902	$L-F$	−.675	R_2-R_5	.861	$I_{25}-P_T$	.626
$m_1'-m_4$	.792	$y_1'-I_{10}$	.889	$L-L_m$	.688	R_2-R_6	.673	$I_{30}-I_{35}$	.975
$m_1'-r$	.795	$y_1'-I_{15}$	.864	L_s-L_c	.805	R_3-R_4	.897	$I_{30}-I_{40}$	.946
$m_1'-t_r$	.814	$y_1'-I_{20}$	.820	L_s-L_t	.839	R_3-R_5	.841	$I_{30}-I_{50}$	.916
$m_1'-D_T$	.844	$y_1'-I_{25}$	.778	L_s-s_t	.770	R_3-R_6	.651	$I_{30}-I_{60}$	.890
$m_1'-P_T$	.637	$y_1'-I_{30}$	.705	L_s-P	.823	R_4-R_5	.959	$I_{30}-I_{90}$	.843
$m_1'-V$	.626	$y_1'-I_{35}$	.626	L_c-L_t	.934	R_4-R_6	.666	$I_{30}-I_{120}$	.814
$m_1'-M_1'$	.699	$y_2'-y_3'$	.995	L_c-s_t	.968	$r-D_T$	.700	$I_{30}-P_T$	.676
$m_1'-M_2'$	.611	$y_2'-y_4'$	.753	L_c-s_d	.715	$r-V$	.620	$I_{35}-I_{40}$	.988
$m_2'-m_3'$	.994	$y_2'-I_2$	.861	L_c-P	.888	$r-M_1'$	.846	$I_{35}-I_{50}$	.967
$m_2'-m_4'$	.984	$y_2'-I_5$	.898	L_c-F	−.706	$r-M_2'$	.743	$I_{35}-I_{60}$	.945
$m_2'-m_2$	.948	$y_2'-I_{10}$	.889	L_c-L_m	.741	$r-M_3'$	.696	$I_{35}-I_{90}$	.905
$m_2'-m_4$	.860	$y_2'-I_{15}$	.848	L_t-s_t	.949	$r-M_4'$	.677	$I_{35}-I_{120}$	.873
$m_2'-r$	.790	$y_2'-I_{20}$	.792	L_t-s_d	.609	I_2-I_5	.930	$I_{35}-P_T$	.690
$m_2'-t_r$	.808	$y_2'-I_{25}$	.739	L_t-P	.919	I_2-I_{10}	.854	$I_{40}-I_{50}$	.990
$m_2'-D_T$	.901	$y_2'-I_{30}$	.667	L_t-F	−.614	I_2-I_{15}	.815	$I_{40}-I_{60}$	.973
$m_2'-P_T$	.628	$y_3'-y_4'$	.748	L_t-L_m	.629	I_2-I_{20}	.767	$I_{40}-I_{90}$	.941
$m_2'-V$	.625	$y_3'-I_2$	.886	s_t-s_d	.749	I_2-I_{25}	.721	$I_{40}-I_{120}$	.908
$m_2'-M_1'$	.706	$y_3'-I_5$	.905	s_t-P	.900	I_2-I_{30}	.666	$I_{40}-P_T$	.695
$m_2'-M_2'$	.620	$y_3'-I_{10}$	.884	s_t-F	−.658	I_5-I_{10}	.955	$I_{50}-I_{60}$	.995
$m_3'-m_4'$	.997	$y_3'-I_{15}$	.837	s_t-L_m	.717	I_5-I_{15}	.907	$I_{50}-I_{90}$	.974
$m_3'-m_2$	.973	$y_3'-I_{20}$	.779	s_d-F	−.885	I_5-I_{20}	.851	$I_{50}-I_{120}$	.949
$m_3'-m_4$	.900	$y_3'-I_{25}$	.726	s_d-L_m	.951	I_5-I_{25}	.797	$I_{50}-P_T$	.747
$m_3'-r$	.777	$y_3'-I_{30}$	.656	$P-W$	.725	I_5-I_{30}	.733	$I_{60}-I_{90}$	.987
$m_3'-t_r$	.793	$y_4'-I_2$	.854	$H-R_1$	.826	I_5-I_{35}	.635	$I_{60}-I_{120}$	.969
$m_3'-D_T$	.936	$y_4'-I_5$	.861	$H-R_2$	.790	$I_{10}-I_{15}$	.971	$I_{60}-P_T$	.781
$m_3'-P_T$	.618	$y_4'-I_{10}$	.796	$H-R_3$	.813	$I_{10}-I_{20}$	.924	$I_{90}-I_{120}$	.992
$m_3'-V$	.621	$y_4'-I_{15}$	.766	$H-R_4$	.718	$I_{10}-I_{25}$	.871	$I_{90}-P_T$	.830
$m_3'-M_1'$	.706	$y_4'-I_{20}$	.727	$H-R_6$	.654	$I_{10}-I_{30}$	.811	$I_{120}-P_T$	.884
$m_3'-M_2'$	.621	$y_4'-I_{25}$	.696	S_1-S_2	.963	$I_{10}-I_{35}$	.711	P_T-V	.671
$m_4'-m_2$	.982	$y_4'-I_{30}$	.634	S_1-S_3	.950	$I_{10}-I_{40}$	.635	$V-M_1'$	.634
$m_4'-m_4$	.922	q_j-q_P	.997	S_1-S_4	.935	$I_{15}-I_{20}$	.983	$M_1'-M_2'$	.973
$m_4'-r$	.764	$\gamma-M_1'$	−.656	S_1-R_1	.652	$I_{15}-I_{25}$	.946	$M_1'-M_3'$	.944
$m_4'-t_r$	.780	$\gamma-M_2'$	−.627	S_1-R_2	.662	$I_{15}-I_{30}$	.895	$M_1'-M_4'$	.926
$m_4'-D_T$	.957	$\gamma-M_3'$	−.605	S_1-R_3	.602	$I_{15}-I_{35}$	.805	$M_2'-M_3'$	.993
$m_4'-P_T$	.612	$r-D_T$	.687	S_1-R_4	.660	$I_{15}-I_{40}$	.736	$M_2'-M_4'$	.983
$m_4'-V$	.619	$r-M_1'$	.829	S_1-R_6	.822	$I_{15}-I_{50}$	.688	$M_3'-M_4'$	.998
$m_4'-M_1'$	.705	$r-M_2'$	.720	S_2-S_3	.972	$I_{15}-I_{60}$	.651		
$m_4'-M_2'$	.623	$r-M_3'$	.671	S_2-S_4	.952	$I_{20}-I_{25}$	.984		
m_2-m_4	.940	$r-M_4'$	.652	S_2-R_6	.750	$I_{20}-I_{30}$	.951		

Chapter V

RESULTS

The correlation matrices indicated some dependence between the so-called independent variables, but a critical review of the physics of the problem suggested inclusion of the somewhat interrelated variables. After preliminary runs determined the order or significance of the independent variables for each of the dependent variables, it was decided to have the computer limit the number of independent variables appearing in the output to five, since beyond this number the increase in the coefficient of multiple correlation does not generally justify the increased effort involved in collecting the additional data. Plots of R^2 versus the number of variables in the regression, Figures 12 through 21, in general support this decision.

Each group of dependent variables will be discussed individually. The relationships for each of the dependent variables for both the linear and logarithmic cases are shown in Tables 9 through 18.

Table 9 shows the regression equation at each step. Tabulated below each equation for m_1' are the coefficient of multiple correlation, R^2 ; the standard error of estimate, S_{ey} ; and the F-ratio, F . Note that for each step the coefficients of regression change. In the interest of concerving space subsequent tabulations will show only the last equation, but will indicate the R^2 , S_{ey} , and F associated with each previous step.

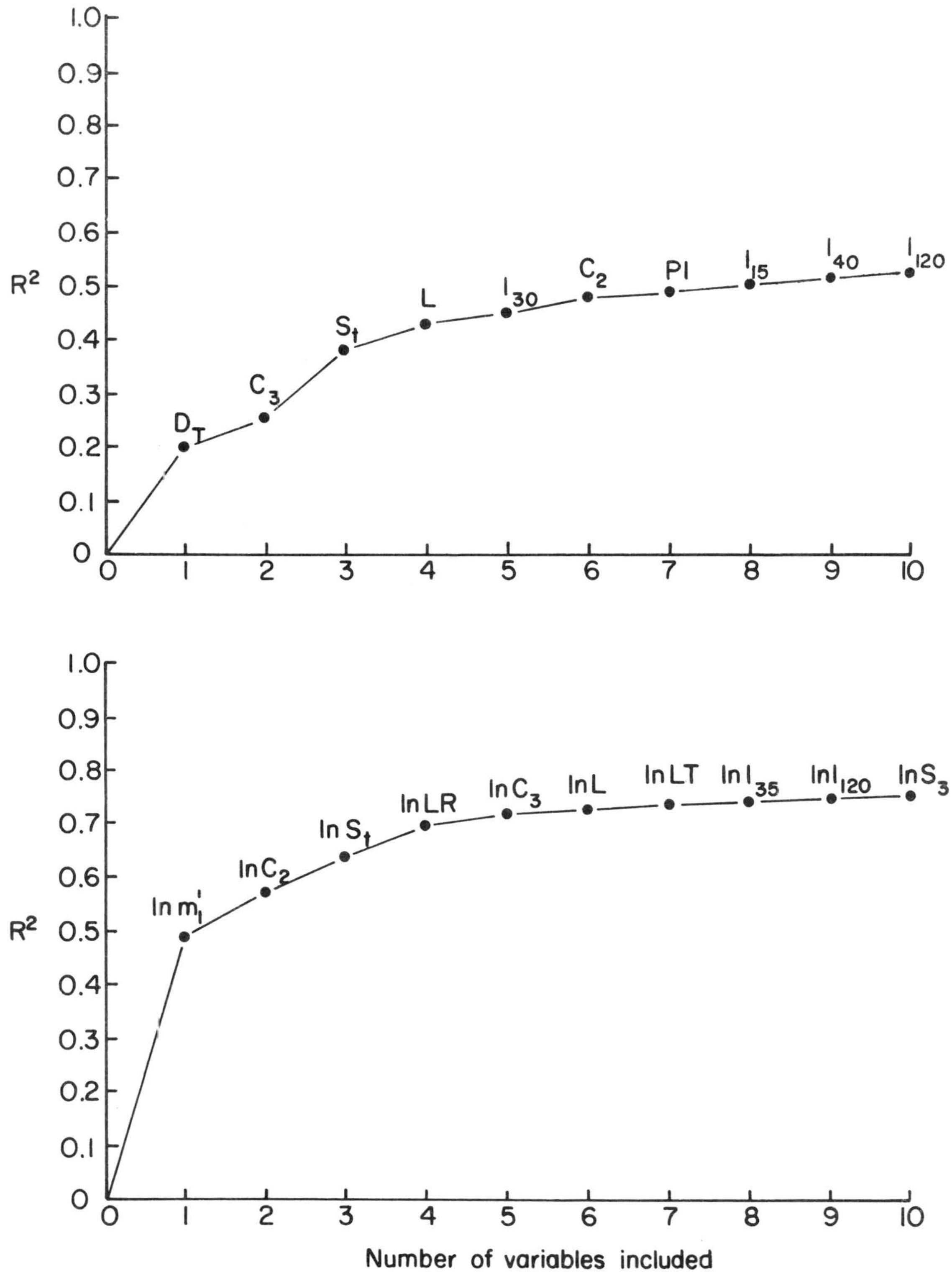


Figure 12. Variation of the coefficient of determination, R^2 , as more variables are included in the regression for M_1' , the first moment of the hydrograph

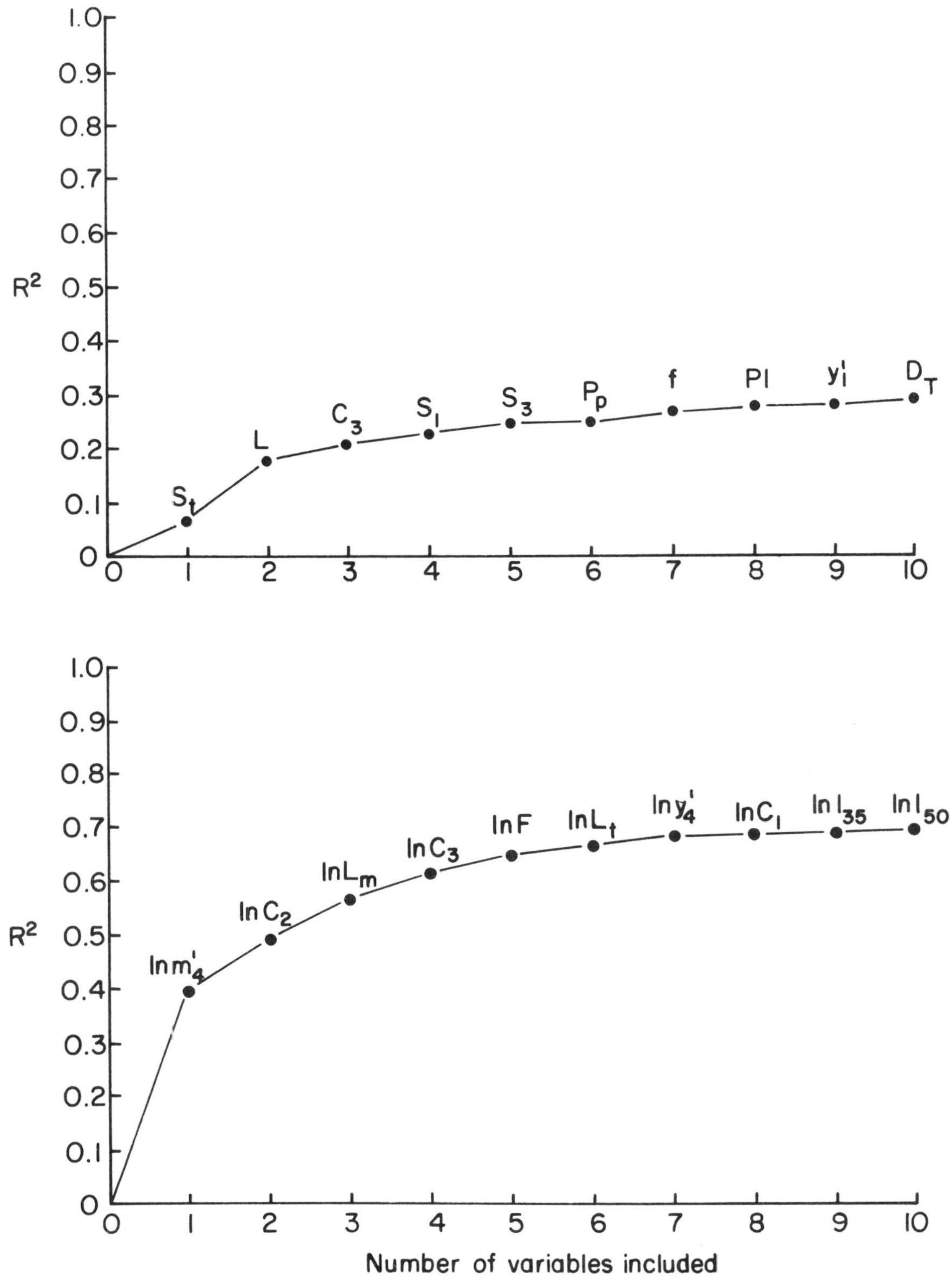


Figure 13. Variation of the coefficient of determination, R^2, as more variables are included in the regression for M_2', the second moment of the hydrograph

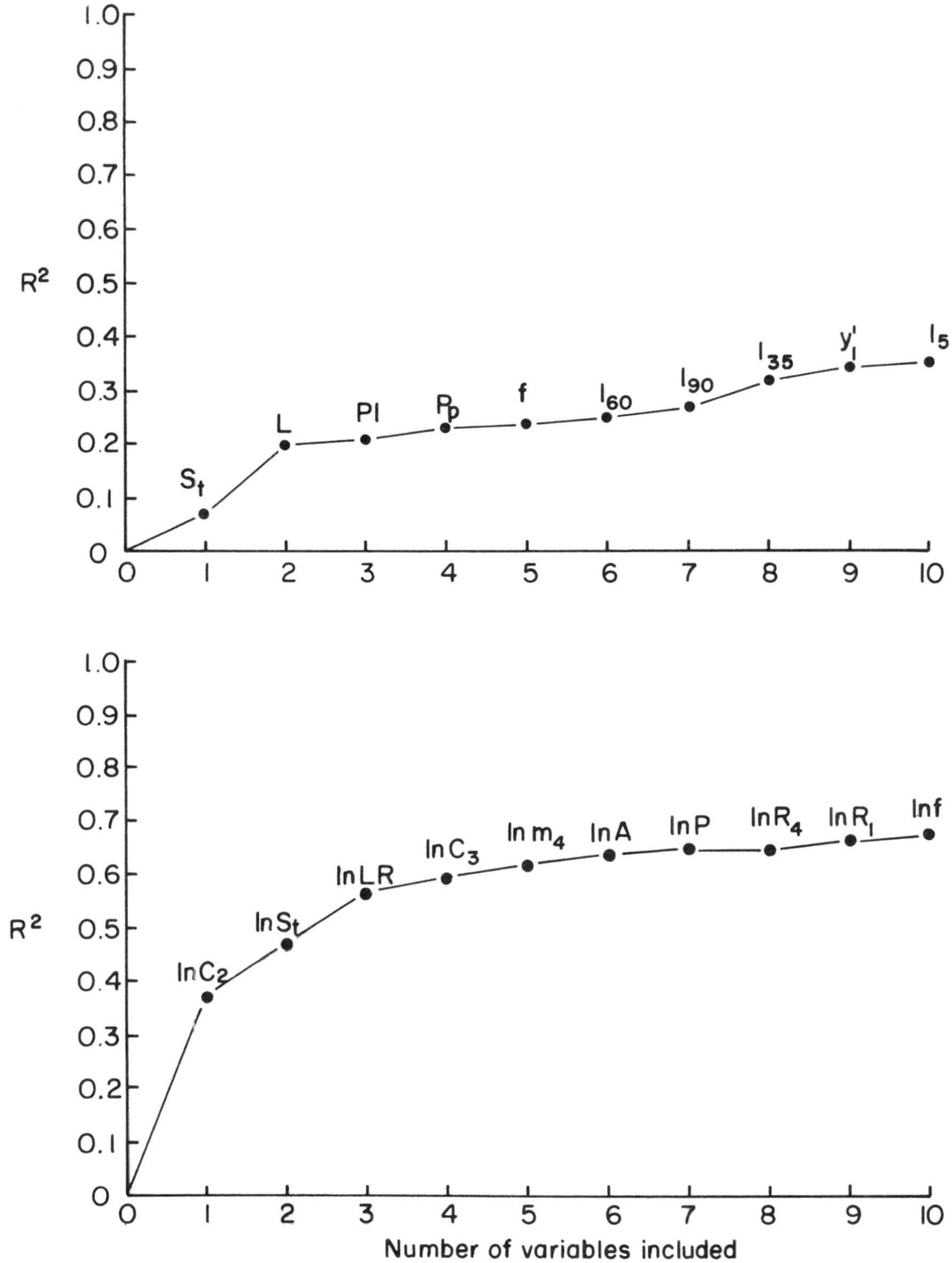


Figure 14. Variation of the coefficient of determination, R^2, as more variables are included in the regression for M_3', the third moment of the hydrograph

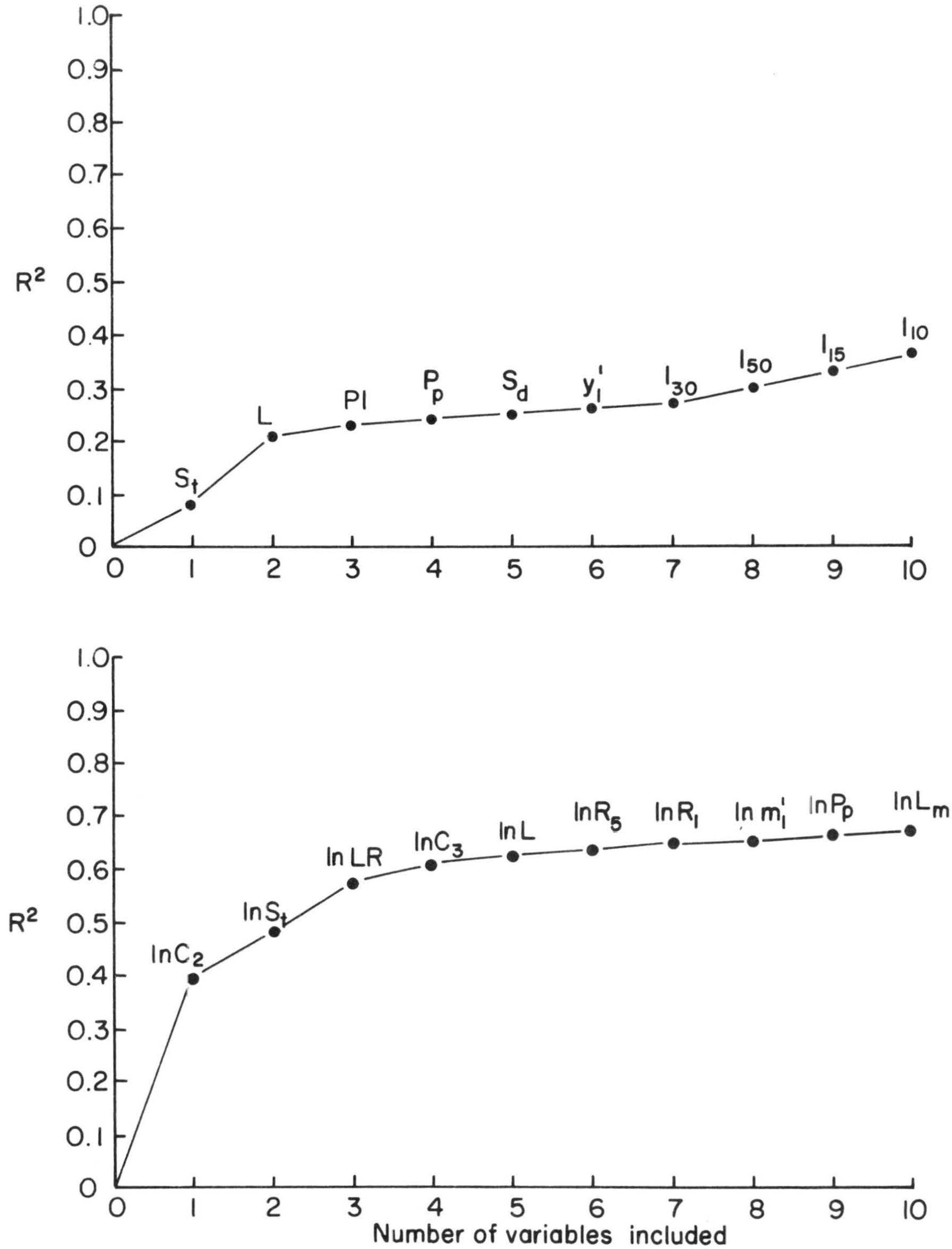


Figure 15. Variation of the coefficient of determination, R^2 , as more variables are included in the regression for M_4' , the fourth moment of the hydrograph

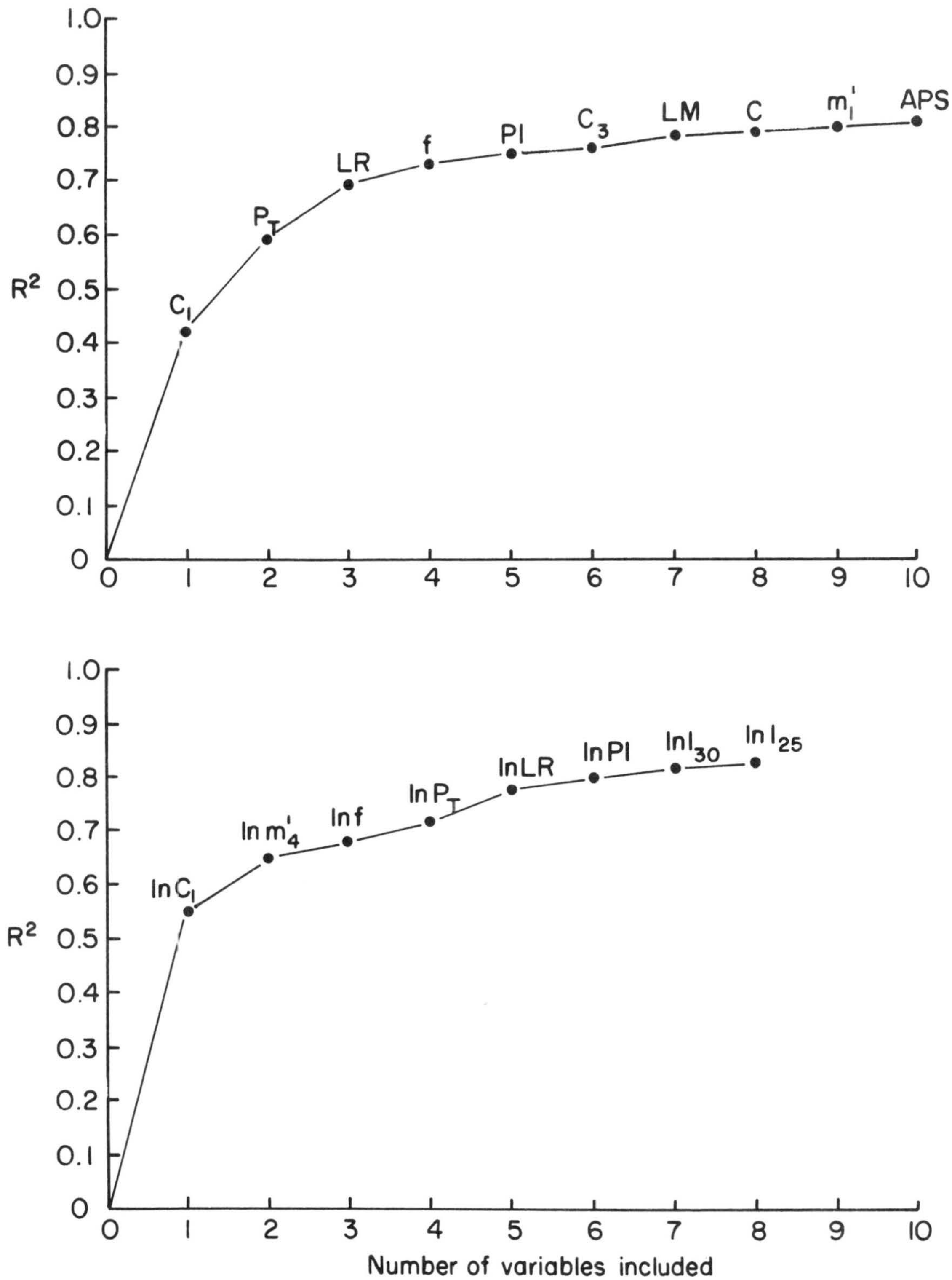


Figure 16. Variation of the coefficient of determination, R^2, as more variables are included in the regression for V, the volume of runoff

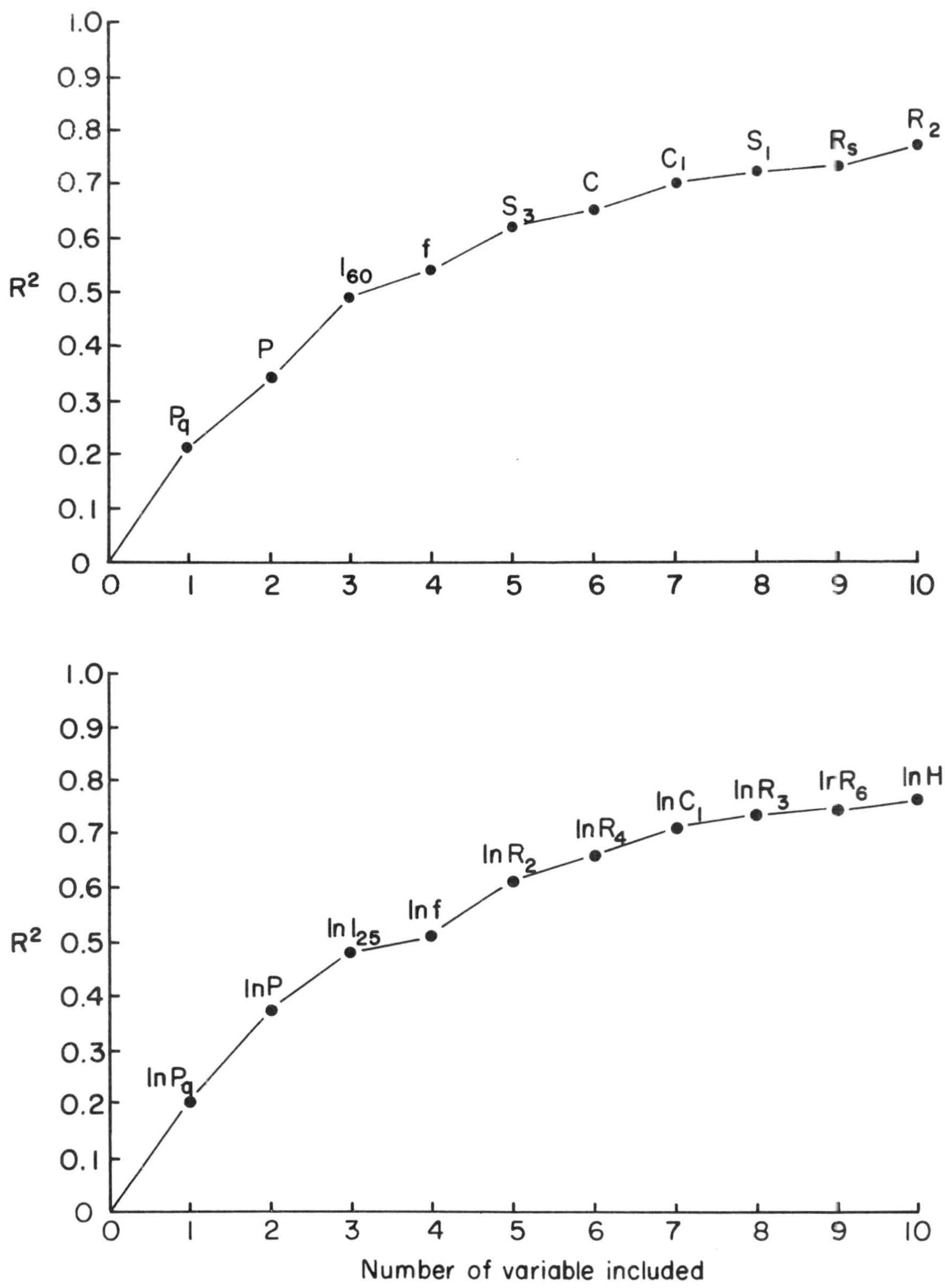


Figure 17. Variation of the coefficient of determination, R^2, as more variables are included in the regression for q_p, the observed hydrograph and peak discharge

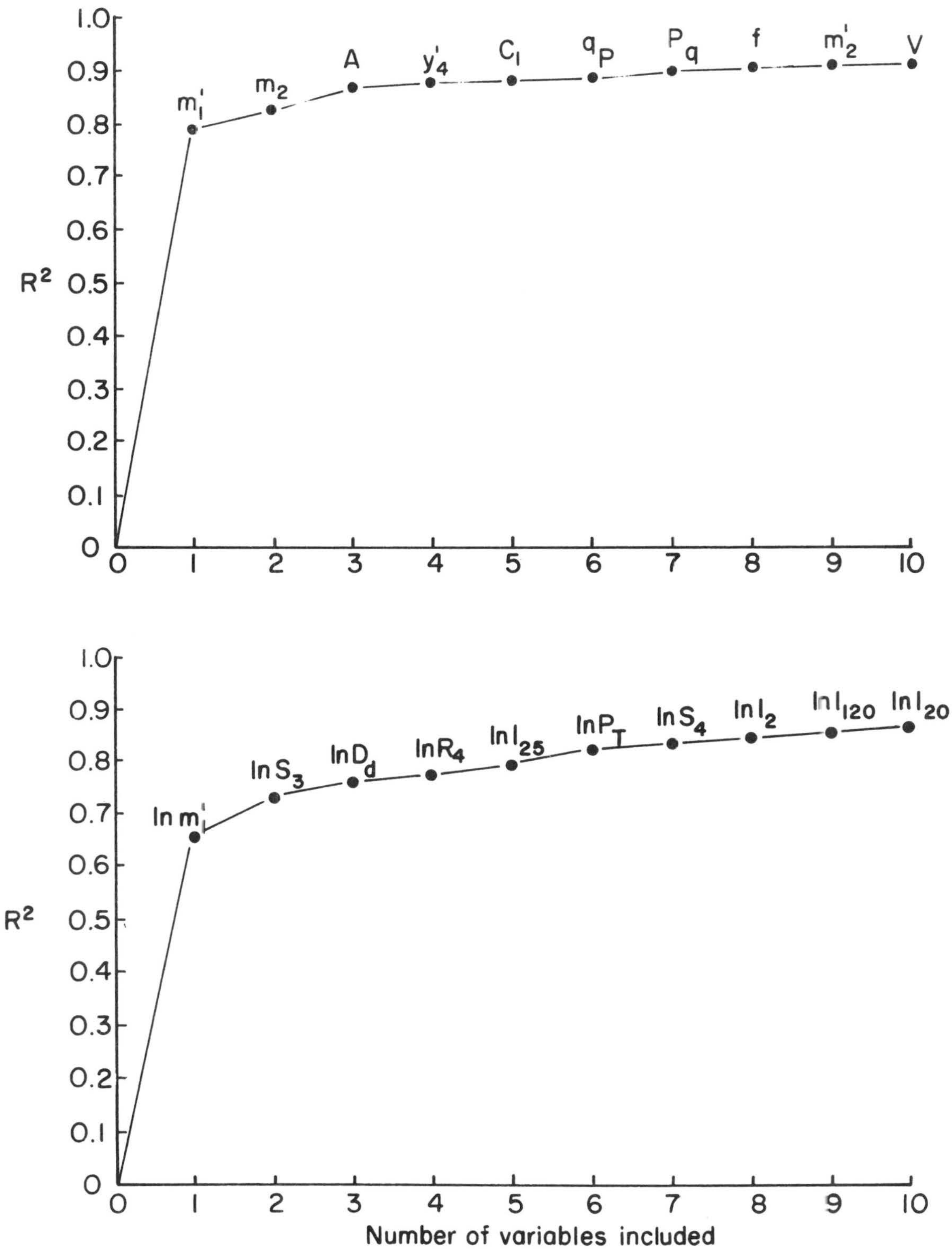


Figure 18. Variation of the coefficient of determination, R^2, as more variables are included in the regression for t_r, the observed hydrograph rise time

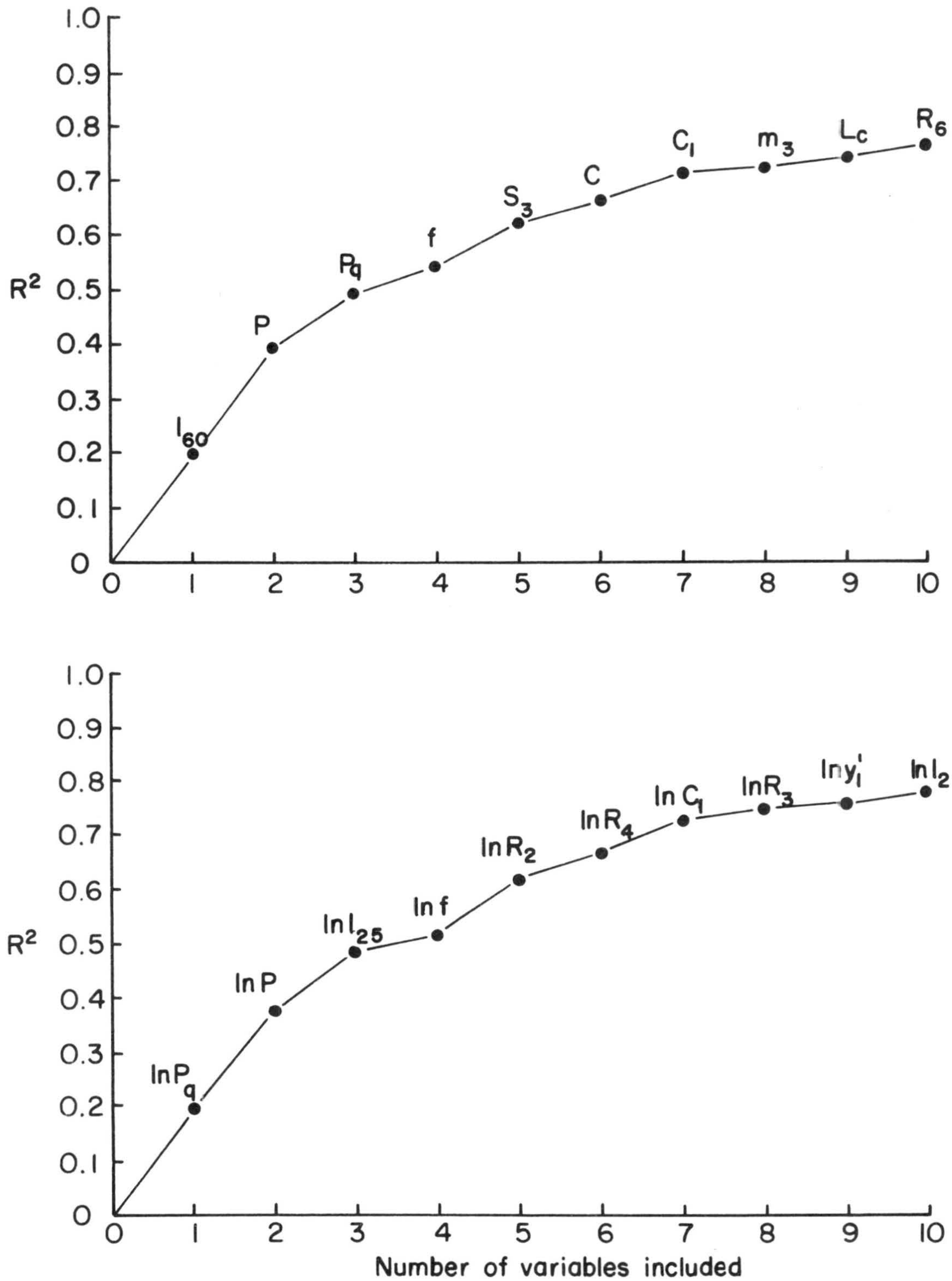


Figure 19. Variation of the coefficient of determination, R^2, as more variables are included in the regression for q_0, the fitted hydrograph peak discharge

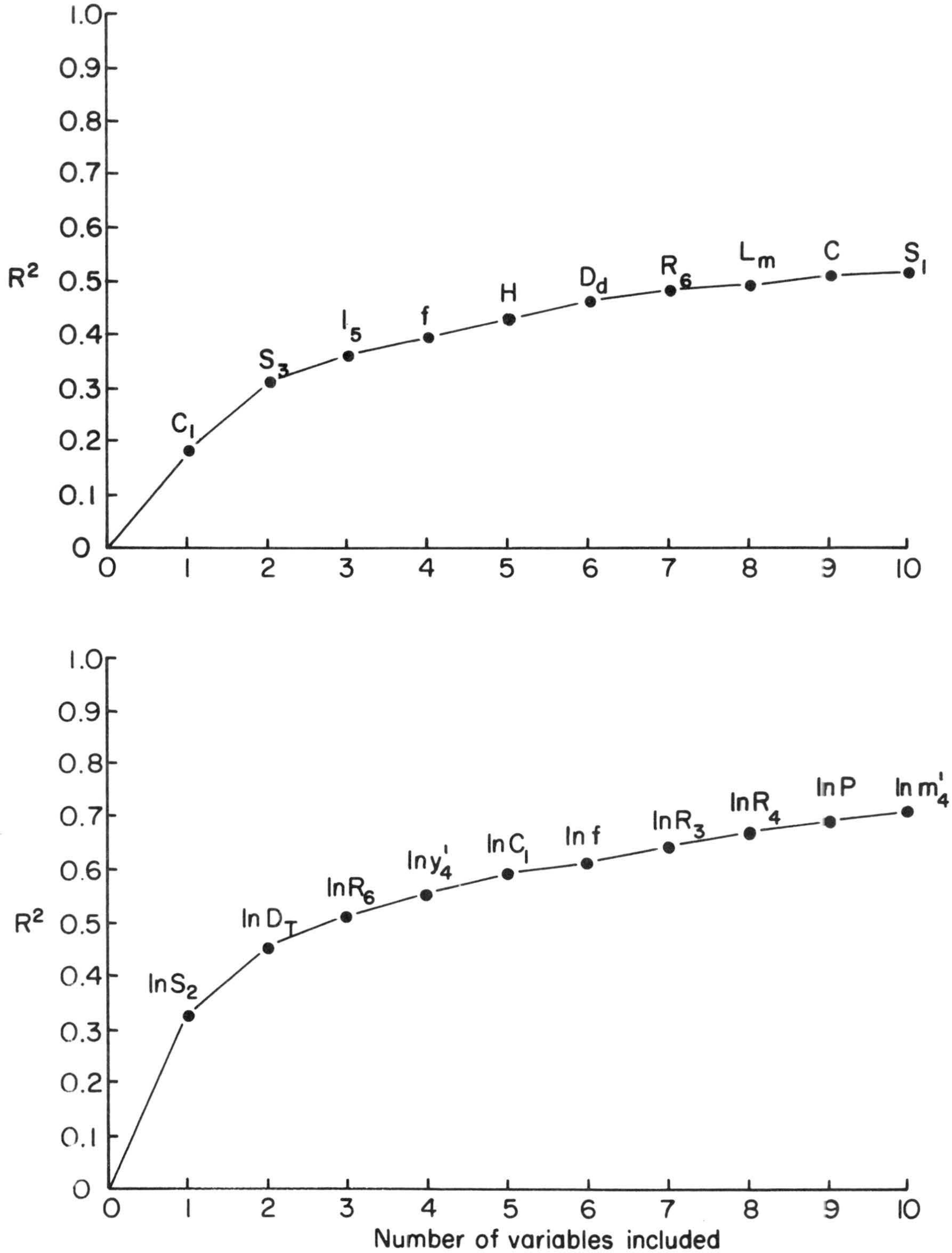


Figure 20. Variation of the coefficient of determination, R^2, as more variables are included in the regression for γ, the fitted hydrograph shape parameter

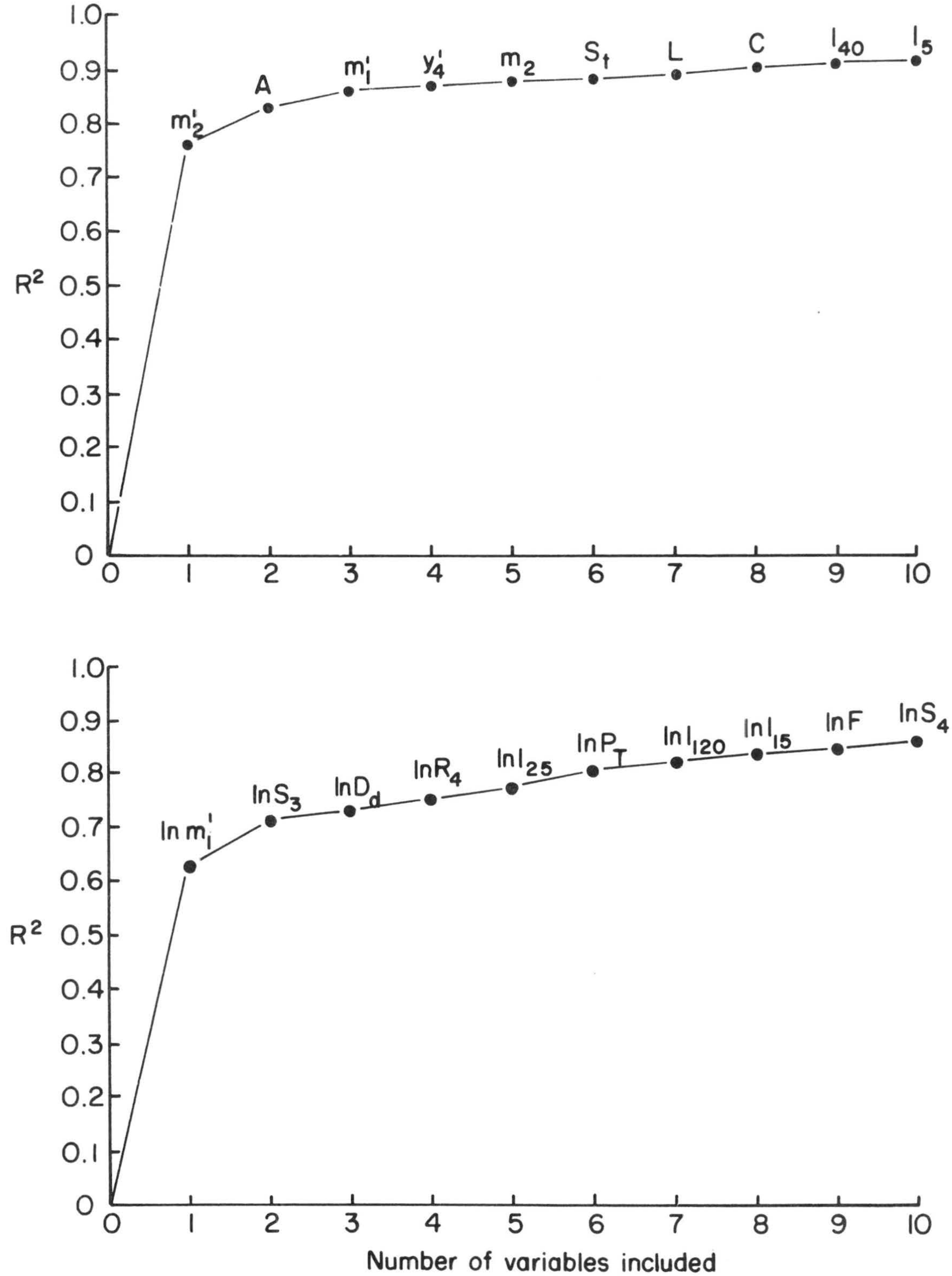


Figure 21. Variation of the coefficient of determination, R^2, as more variables are included in the regression for r, the fitted hydrograph rise time

5.1 Prediction of Hydrograph Moments

M_1' , first moment of the hydrograph

As shown in Table 9 the coefficient of multiple determination, R^2 , is only 0.45 for equation 5.1 after the inclusion of five variables. With such a low value of R^2 this equation could hardly be used for prediction. Figure 22, a plot of the observed versus estimated values of m_1' using equation 5.1, shows a wide scatter of point, thus supporting the inadequacy of this equation for prediction.

$\ln M_1'$, $\log_e$ of first moment of the hydrograph

The $\ln m_1'$ relationship (equation 5.2) indicates that 72% of the variance of $\ln m_1'$ can be explained by five variables. This is a marked improvement over the 45% for the linear relationship.

Moreover, a multicative relationship is more in keeping with basic hydrologic knowledge. A closer look at the equation indicates that the signs of the independent variables are also in keeping with the present physical understanding of the hydrologic processes. The positive sign before m_1' , the first moment of the hyetograph, indicates that as the intensity and duration of the storm increase the first moment of the hydrograph also increases. Further, not that the first variable, m_1' , alone accounts for 49% of the total variance, thus adding support for the hypothesis that the hyetograph moments are good discriptors of the hyetograph.

The annual precipitation, C_2 , is a climatic factor indicative of greater volumes of runoff and larger volumes produce larger hydrograph moments. The standard deviation of the travel distances, s_t , is a measure of the variation in the travel times of the catchment. Therefore, the positive sign is quite appropriate.

Increases in loss rate, LR , would result in less runoff and thus smaller moments. The negative sign of PI , the pattern index, arises from the fact that for a given recession characteristic a small watershed would tend to peak earlier with an early-peaking storm and thus produce less volume.

The observed versus estimated values for this relationship, equation 5.2, are plotted in Figure 23. The standard error of estimate is equal to the logarithm 0.4976. When considered in relation to the standard deviation of the dependent variable, which was the logarithm 0.9156, this is a reasonable value.

M_2' , second moment of the hydrograph

The relationship for the second moment of the hydrograph is shown in Table 10 and Figure 24. The explained variance for this equation is only 25%. This would preclude its use for predictions.

$\ln M_2'$, $\log_e$ of second moment of the hydrograph

In Chapter II it was noted that the hydrograph could be represented by the following equation:

$$q = \frac{\beta V \gamma^{\gamma r+1}}{\Gamma(\gamma r + 1)} t^{\gamma r} e^{-\gamma t} . \tag{2.4}$$

Knowing this one can show that the moment generating function is:

$$\Psi(t) = \beta\left(1 - \frac{t}{\gamma}\right)^{-(\gamma r-1)} .$$

Using this one can easily tabulate the first four moments:

1st $\quad \beta \cdot \left(\frac{\gamma r - 1}{\gamma}\right)$

2nd $\quad \beta \cdot \left(\frac{\gamma r - 1}{\gamma}\right) \cdot \left(\frac{\gamma r}{\gamma}\right)$

3rd $\quad \beta \cdot \left(\frac{\gamma r - 1}{\gamma}\right) \cdot \left(\frac{\gamma r}{\gamma}\right) \cdot \left(\frac{\gamma r + 1}{\gamma}\right)$

4th $\quad \beta \cdot \left(\frac{\gamma r - 1}{\gamma}\right) \cdot \left(\frac{\gamma r}{\gamma}\right) \cdot \left(\frac{\gamma r + 1}{\gamma}\right) \cdot \left(\frac{\gamma r + 2}{\gamma}\right)$

From the above it is at once obvious that each succeeding moment involves a product of the previous moments. Therefore, the use of a logarithmic relationship is justified for all moments higher than one.

$\ln M_3'$, $\log_e$ of third moment of the hydrograph

On the basis of the discussion of $\ln M_2'$ it is sufficient to say that $\ln M_3'$ is a function of the variables in $\ln M_1'$ and $\ln M_2'$ and that 61% of the variance can be explained by five variables. Figure 26 shows this relationship.

$\ln M_4'$, $\log_e$ of fourth moment of the hydrograph

Equation 5.8 of Table 12 indicates that 62% of the variance of $\ln M_4'$ can be explained by five variables which are similar to those used in the relationships for $\ln M_1'$, $\ln M_2'$ and $\ln M_3'$. A test of physical meaning accounts for all the variables except L , the length of the main stream. An increase in L would ordinarily suggest a decrease in $\ln M_4'$ since L is generally considered to be a measure of catchment travel times. However, for the small watershed the drainage net is usually very sparse and the main stream may represent the only dominant feature of the drainage pattern. In this case L represents a measure of channel storage and as such increases in L would result in decreases in $\ln M_4'$.

TABLE 9. REGRESSION EQUATIONS FOR THE FIRST MOMENT OF THE HYDROGRAPH

	Equation					
	$M_1' = 51.40712 + 0.70840\ D_T$					
R^2	0.2027					
S_{ey}	189.4633					
F	22.3794					
	$M_1' = 476.82714 + 0.58857\ D_T - 5.85243\ C_3$					
R^2	0.2600					
S_{ey}	183.5762					
F	15.2862					
	$M_1' = 632.20820 + 0.49452\ D_T - 9.07873\ C_3 + 180.57997\ s_t$					
R^2	0.3774					
S_{ey}	169.3606					
F	17.3794					
	$M_1' = 594.14139 + 0.50539\ D_T - 8.11767\ C_3 + 467.98813\ s_t - 90.89844\ L$					
R^2	0.4286					
S_{ey}	163.2055					
F	15.9385					
	$M_1' = 648.31472 + 0.49947\ D_T - 7.76893\ C_3 + 470.61113\ s_t - 89.97562\ L - 35.41488\ I_{30}$					(5.1)
R^2	0.4508					
S_{ey}	160.9566					
F	13.7880					
	$\ln M_1' = 6.70899 + 0.41717 \ln m_1' + 0.53905 \ln C_2 + 0.42138 \ln s_t - 0.15700 \ln LR - 1.17750 \ln C_3$					(5.2)
R^2	0.4887	0.5690	0.6389	0.7001	0.7212	
S_{ey}	0.6584	0.6079	0.5597	0.5131	0.4976	
F	04.097	57.4359	50.7137	49.6023	43.4575	

TABLE 10. REGRESSION EQUATIONS FOR THE SECOND MOMENT OF THE HYDROGRAPH

Equation

$$M_2' = 2.0985\text{x}10^6 + 3.8908\text{x}10^6 s_t - 8.6826\text{x}10^5 L - 2.9758\text{x}10^4 C_3 + 4.9761\text{x}10^3 S_1 - 6.0172\text{x}10^3 S_3 \quad (5.3)$$

R^2	0.0658	0.1780	0.2074	0.2263	0.2476
S_{ey}	$1.4306\text{x}10^6$	$1.3496\text{x}10^6$	$1.3329\text{x}10^6$	$1.3247\text{x}10^6$	$1.3140\text{x}10^6$
F	6.1944	9.4200	7.5010	6.2141	5.5296

$$\ln M_2' = 18.01299 + 0.21802 \ln m_4' + 1.76354 \ln C_2 + 6.71453 \ln L_m - 3.61391 \ln C_3 + 1.671453 \ln F \quad (5.4)$$

R^2	0.3882	0.4875	0.5585	0.6097	0.6351
S_{ey}	1.6682	1.5356	1.4336	1.3558	1.3187
F	55.8491	41.3852	36.2621	33.1995	29.2419

TABLE 11. REGRESSION EQUATIONS FOR THE THIRD MOMENT OF THE HYDROGRAPH

Equation

$$M_3' = 1.7312\text{x}10^{10} + 5.4390\text{x}10^{10} s_t - 1.3163\text{x}10^{10} L - 2.3345\text{x}10^{10} PI - 1.0262\text{x}10^{10} P_p + 1.2027\text{x}10^{10} f \quad (5.5)$$

R^2	0.0749	0.1956	0.2144	0.2282	0.2423
S_{ey}	$1.8121\text{x}10^{10}$	$1.6994\text{x}10^{10}$	$1.6892\text{x}10^{10}$	$1.6841\text{x}10^{10}$	$1.6786\text{x}10^{10}$
F	7.1244	10.5761	7.8214	6.2829	5.3718

$$\ln M_3' = 30.99064 + 2.51276 \ln C_2 + 1.66863 \ln s_t - 0.41895 \ln LR - 5.73813 \ln C_3 + 0.18648 \ln m_4 \quad (5.6)$$

R^2	0.3699	0.4667	0.5607	0.5924	0.6117
S_{ey}	2.7699	2.5628	2.3395	2.2668	2.2257
F	51.6574	38.0676	36.5877	30.8828	26.4606

TABLE 12. REGRESSION EQUATIONS FOR THE FOURTH MOMENT OF THE HYDROGRAPH

Equation						
$M_4' = 4.6109\text{x}10^{14} + 1.0859\text{x}10^{15} s_t - 2.3303\text{x}10^{14} L - 4.4796\text{x}10^{14} PI - 1.1857\text{x}10^{14} P_p - 2.7806\text{x}10^{14} s_d$						(5.7)
R^2	0.0833	0.2084	0.2324	0.2434	0.2540	
S_{ey}	$3.0284\text{x}10^{14}$	$2.8302\text{x}10^{14}$	$2.8032\text{x}10^{14}$	$2.7994\text{x}10^{14}$	$2.7962\text{x}10^{14}$	
F	7.9927	11.4554	8.6774	6.8359	5.7187	
$\ln M_4' = 45.61272 + 4.23659 \ln C_2 + 6.16779 \ln s_t - 0.95620 \ln LR - 7.10244 \ln C_3 - 4.05409 \ln L$						(5.8)
R^2	0.3873	0.4833	0.5700	0.5995	0.6180	
S_{ey}	3.8380	3.5447	3.2523	3.1571	3.1018	
F	55.6272	40.6884	38.0021	31.8126	27.1790	

$$M_1' = 648.31472 + 0.49947\, D_T - 7.76893\, C_3 + 470.61113\, S_t - 89.97562\, L - 35.41488\, I_{30}$$

D_T = storm duration, minutes

C_3 = mean monthly temperature, degrees F

S_t = standard deviation of the mean travel distance, miles

L = length of main stream, miles

I_{30} = 30-minute maximum intensity, inches/hour

R^2 = 0.4508 , S_{ey} = 160.9566 minutes , n = 90 events

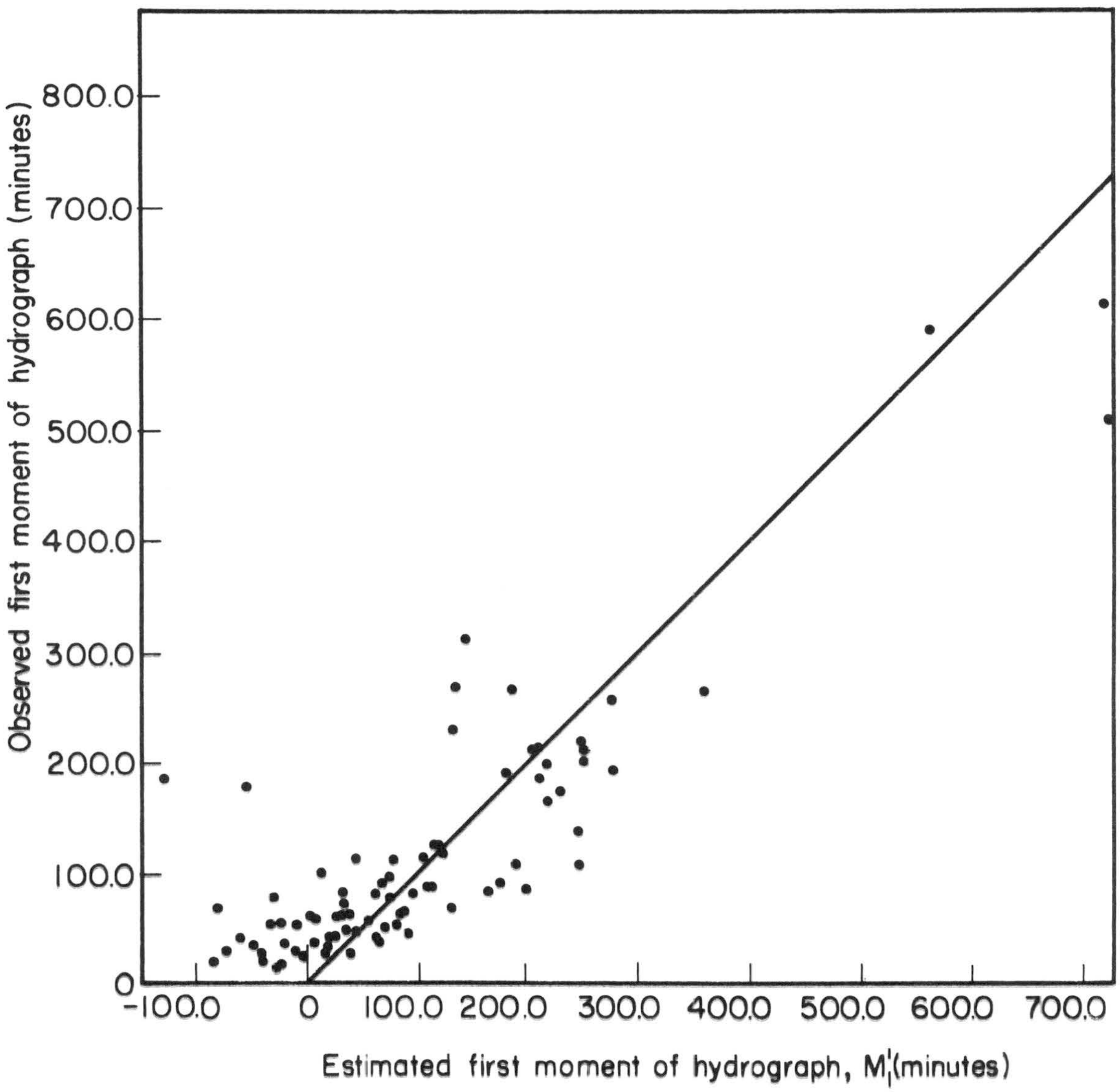


Figure 22. Correlation of observed versus estimated values of the first moment of the hydrograph for equation 5.1

$$\ln M_1' = 6.70899 + 0.41717 \ln m_1' + 0.53905 \ln C_2 + 0.42138 \ln S_t$$
$$-0.15700 \ln LR - 1.17750 \ln C_3$$

$\ln m_1'$ = $\log_e$ of first moment of hyetograph

$\ln C_2$ = $\log_e$ of mean annual precipitation

$\ln S_t$ = $\log_e$ of standard deviation of the mean travel distance

$\ln LR$ = $\log_e$ of loss rate

$\ln C_3$ = $\log_e$ of mean monthly temperature

R^2 = 0.7212, S_{ey} = 0.4976, n = 90 events

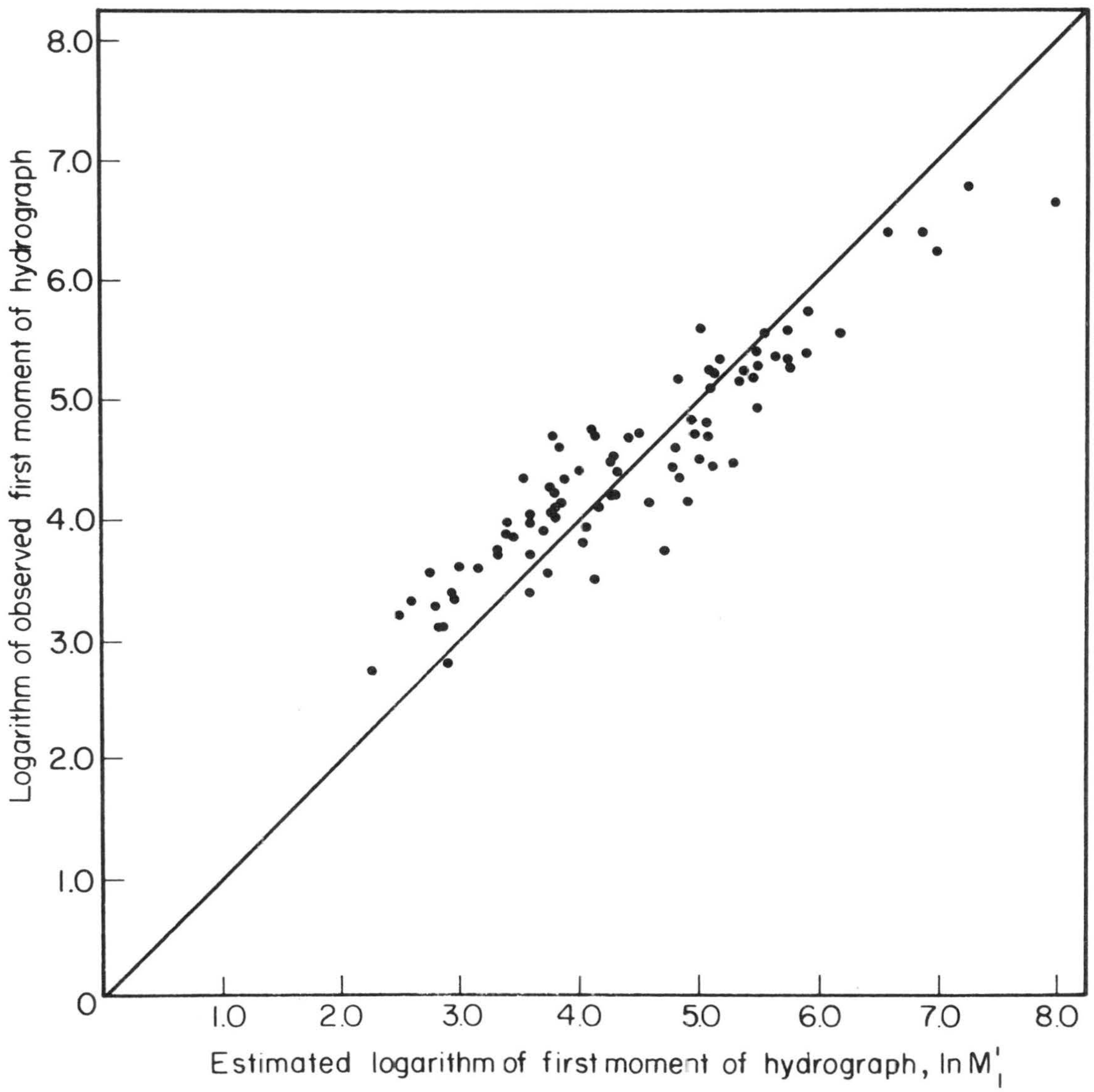


Figure 23. Correlation of observed versus estimated values of the logarithm of the first moment of the hydrograph for equation 5.2

$$M_2' = 2.09849 \times 10^6 + 3.89076 \times 10^6 S_t - 8.68264 \times 10^5 L - 2.97577 \times 10^4 C_3 + 4.97614 \times 10^3 S_1 - 6.01718 \times 10^3 S_3$$

S_t = standard deviation of travel distances, miles

L = length of main stream, miles

C_3 = mean monthly temperature, degrees F

S_1 = stream slope, feet/mile

S = stream slope, feet/mile

R^2 = 0.2476, S_{ey} = 1.3140×10^6 minutes squared, n = 90 events

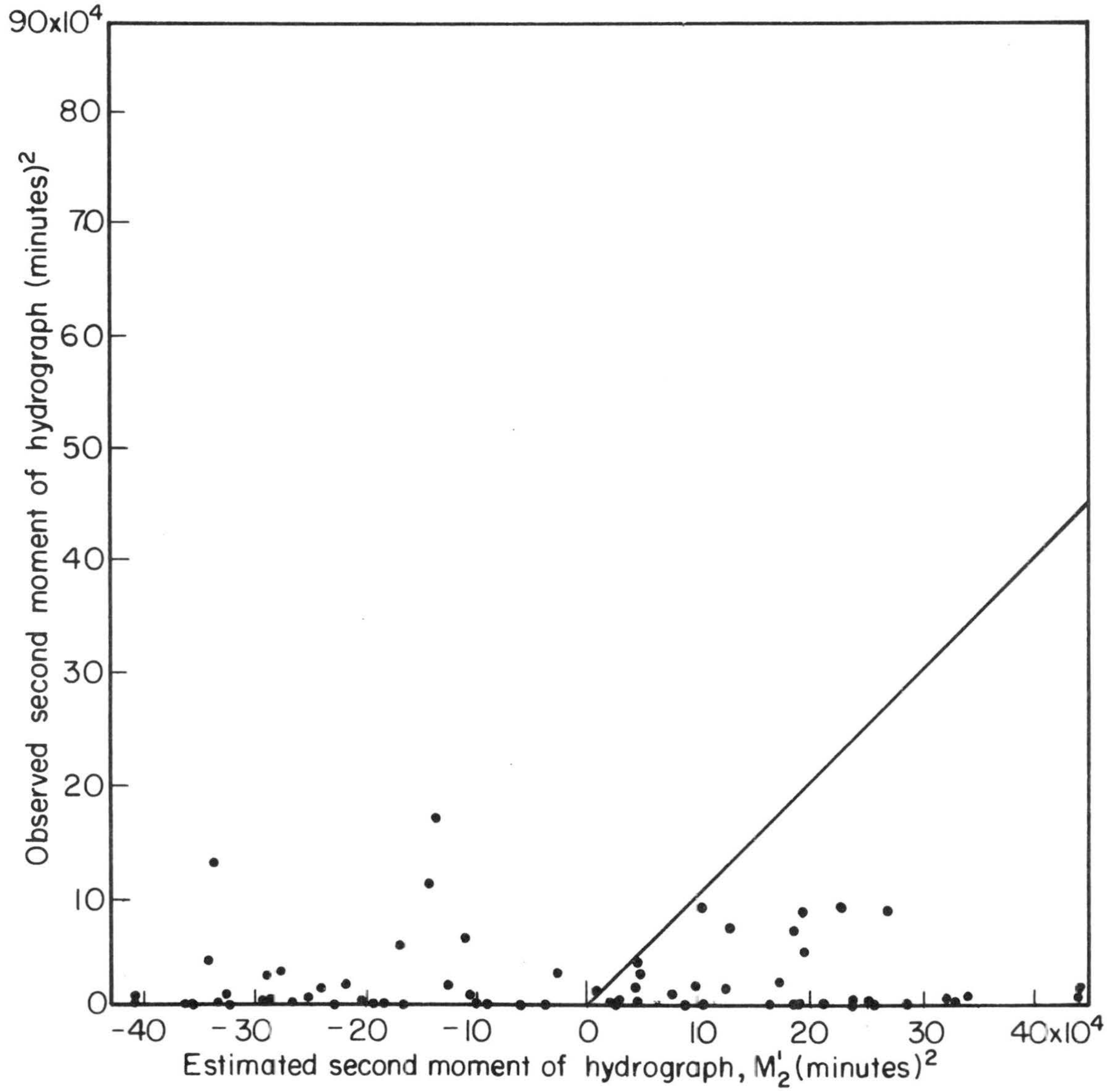


Figure 24. Correlation of observed versus estimated values of the second moment of the hydrograph for equation 5.3

$$\ln M'_2 = 18.01299 + 0.21802 \ln m'_4 + 1.76354 \ln C_2 + 6.71453 \ln L_m - 3.61391 \ln C_3 + 1.671453 \ln F$$

$\ln m'_4$ = $\log_e$ of fourth moment of hyetograph

$\ln C_2$ = $\log_e$ of mean annual precipitation

$\ln L_m$ = $\log_e$ of dimensionless mean travel distance

$\ln C_3$ = $\log_e$ of mean monthly temperature

$\ln F$ $\log_e$ of form factor

R^2 = 0.6351, S_{ey} = 1.3187, n = 90 events

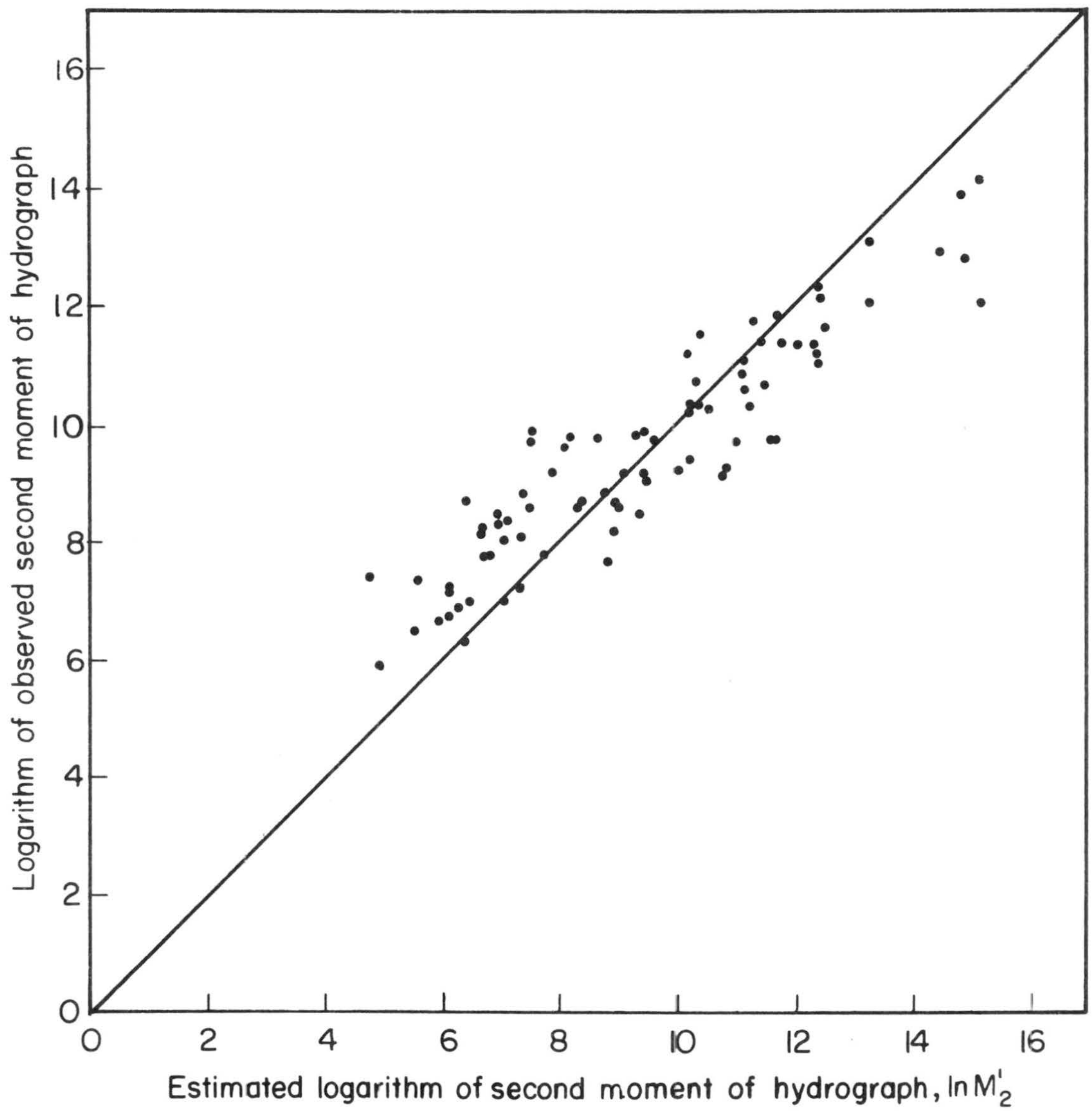


Figure 25. Correlation of observed versus estimated values of the logarithm of the second moment of the hydrograph for equation 5.4

$$\ln M_3' = 30.99064 + 2.51276 \ln C_2 + 1.66863 \ln S_t - 0.41895 \ln LR - 5.73813 \ln C_3 + 0.18648 \ln m_4$$

$\ln C_2$ = $\log_e$ of mean annual precipitation

$\ln S_t$ = $\log_e$ of standard deviation of travel distances

$\ln LR$ = $\log_e$ of loss rate

$\ln C_3$ = $\log_e$ of mean monthly temperature

$\ln m_4$ = $\log_e$ of fourth central moment of hyetograph

R^2 = 0.6117, S_{ey} = 2.2257, n = 90 events

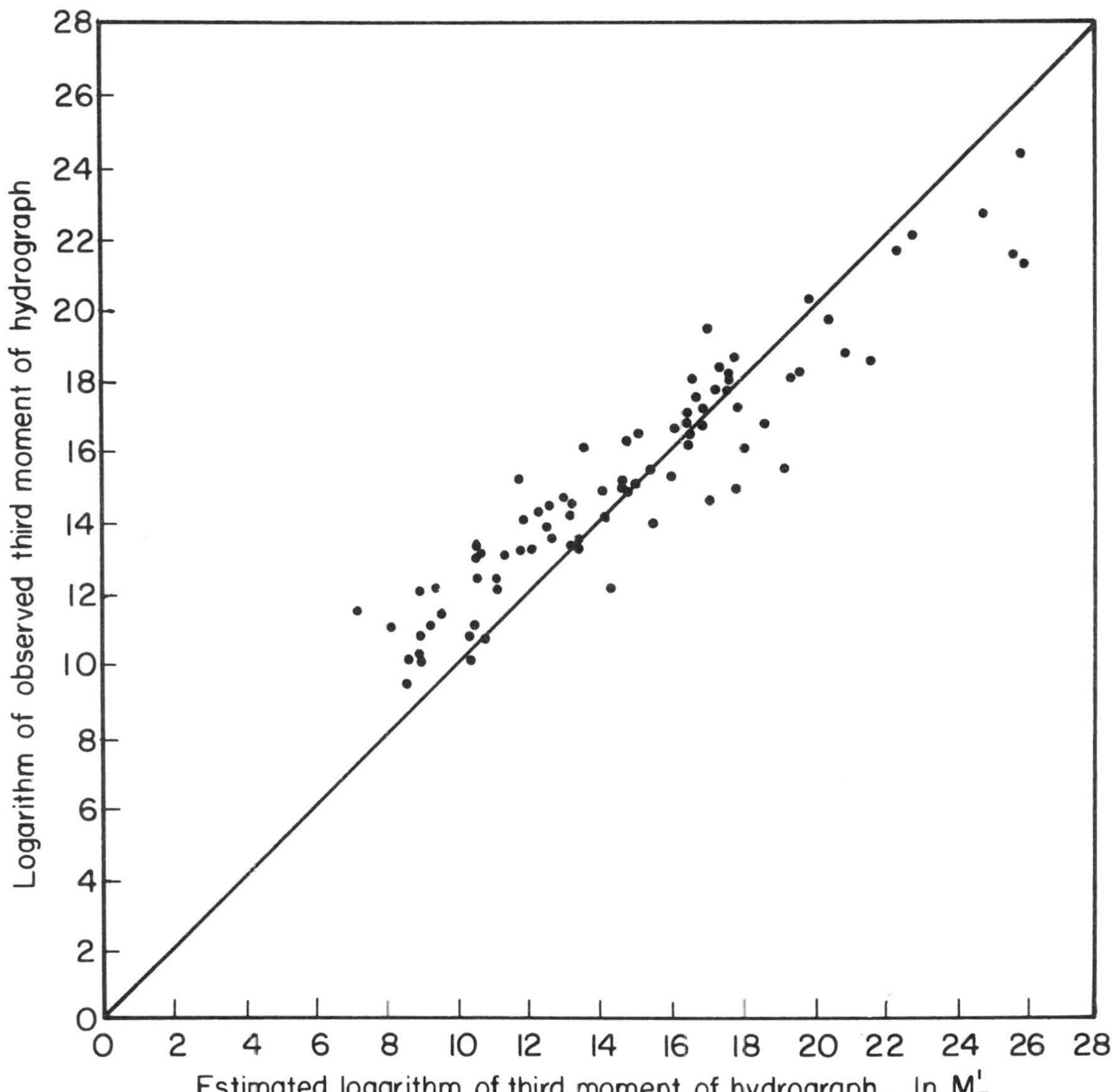


Figure 26. Correlation of observed versus estimated values of the logarithm of the third moment of the hydrograph for equation 5.6

$$\ln M_4' = 45.61272 + 4.23659 \ln C_2 + 6.16779 \ln S_t - 0.95620 \ln LR$$
$$- 7.10244 \ln C_3 - 4.05409 \ln L$$

$\ln C_2$ = $\log_e$ of mean annual precipitation

$\ln S_t$ = $\log_e$ of standard deviation of travel distances

$\ln LR$ = $\log_e$ of loss rate

$\ln C_3$ = $\log_e$ of mean monthly temperature

$\ln L$ = $\log_e$ of length of main stream

R^2 = 0.6180, S = 3.1018, n = 90 events

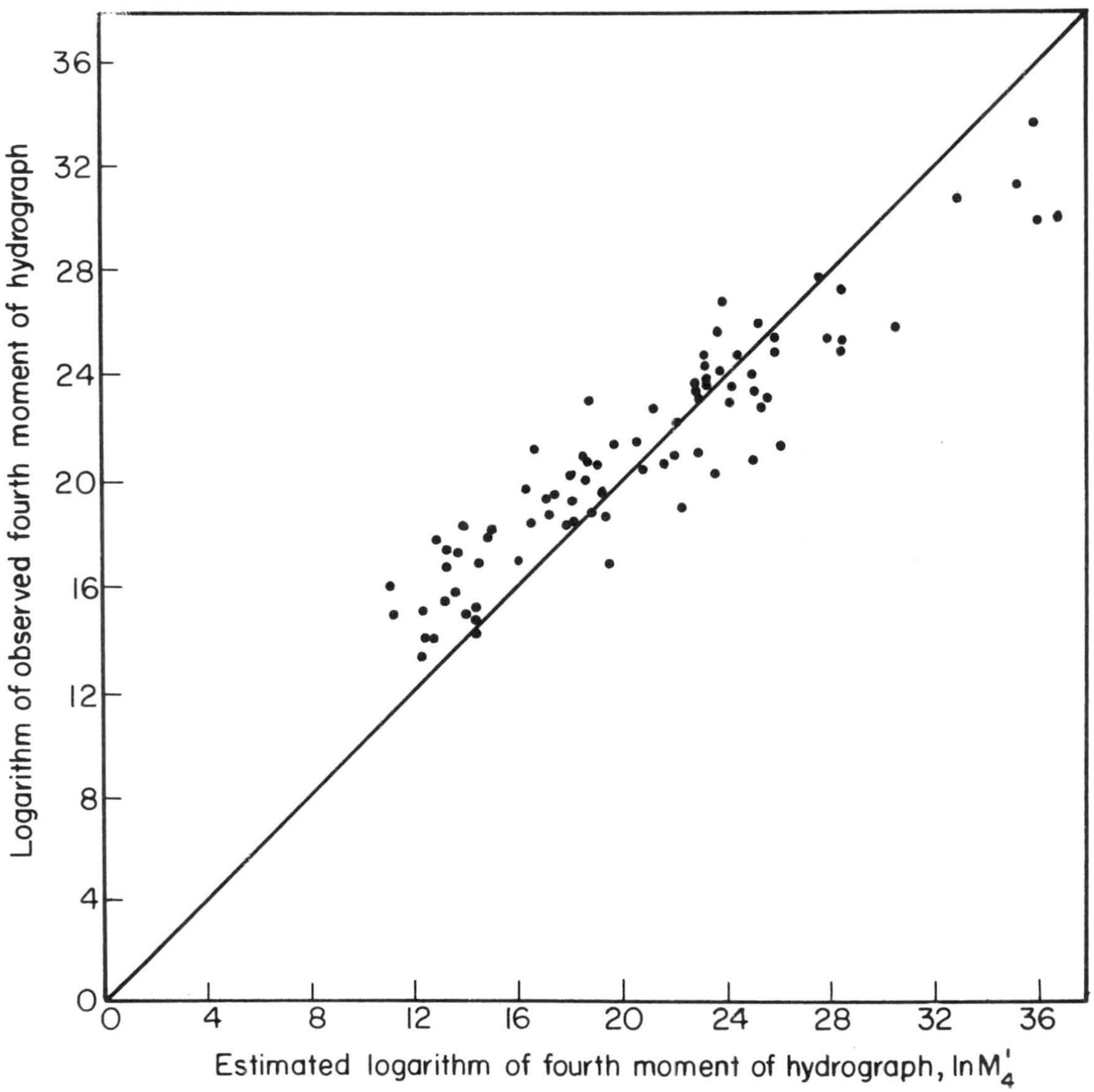


Figure 27. Correlation of observed versus estimated values of the logarithm of the fourth moment of the hydrograph for equation 5.8

5.2 Prediction of Observed Hydrograph Parameters

V , volume of runoff

The R^2 for equation 5.9 of 0.75 is acceptable. A check of the F-ratio indicates that all the variables are significant at the 1% level and the signs of the regression coefficients appear to be consistent with the expected effects of each of the variables. Regions of higher 10-year 1-hour precipitation, C_1 , are generally humid and exhibit higher volumes of runoff per unit area for a given rainfall. As expected the volume of runoff increases with the storm total, P_T . Increasing loss rates, LR , imply less effective rainfall and in turn less runoff. Similarly, increased infiltration capacities, f , also imply less runoff. As discussed in connection with equation 5.2, the negative sign before the pattern index, PI , is consistent.

In spite of its high R and consistent signs this relationship is not compatible with the multiplicative nature of quantities in hydrology. Intuitively, therefore, the logarithmic relationship should be more in accordance with the present knowledge of the physics of nature.

$\ln V$, $\log_e$ volume of runoff

Inspection of equation 5.10 reveals that physical significance can be attached to all of the variables and that the coefficient of multiple determination is 0.78, a slightly higher value than for the linear relationship of equation 5.9.

Increases in the 10-year 1-hour precipitation, C_1 ; the first moment of the hyetograph, m_1' ; and the 120-minute maximum intensity, I_{120} , all imply increases in the volume of runoff. As expected increases in the infiltration capacity, f , and the loss rate, LR , result in decreased volume of runoff. These facts coupled with the multiplicative nature of the relationship make this a very useful relationship. See Figure 29 for a plot of the observed versus the estimated values.

q_p , observed hydrograph peak discharge

From Table 14 the relation for q_p explains 62% of the variance. The positive coefficient for P_q , the probability of the peak, needs no further explanation. The catchment perimeter, P , is indicative of travel distances. Therefore, longer perimeters would suggest less compact catchments and longer supply distances which would result in lower peaks as the negative sign of P indicates. For I_{60} , the 60-minute maximum intensity, it is sufficient to say that increased intensities of rainfall produce higher rates of runoff. The infiltration capacity, f , has a negative sign as it should; and the stream slope, S_3 , has a positive sign indicating that higher slopes mean higher velocities and shorter supply periods which have the effect of increasing the peak.

$\ln q_p$, $\log_e$ observed hydrograph peak discharge

As in the case of the equations for the volume of runoff one finds little difference between the coefficients of multiple determination for the linear and logarithmic relationships. Here too, one would be inclined to choose the logarithmic relationship over the linear one since it conforms to the multiplicative concept in hydrology.

In this relationship, equation 5.12, the physical significance of the variables is identical to those in equation 5.11, except that I_{25} has replaced I_{60} and R_2 has replaced S_3 . The correlation of the observed and estimated values is shown in Figure 31.

t_r , observed hydrograph rise time

The actual time of rise is a somewhat nebulous number because of the difficulty inherent in defining the point of commencement. Many hydrographs rise slowly for some time prior to a distinct rapid rise. Therefore, it is not unreasonable to see a value of the standard error of estimate for t_r of 20 minutes. The accompanying value of R^2 is 0.88. A test of physical meaning accounts for all the variables. Note that the fourth moment of the hyetograph, y_4' , is a measure of the maximum intensities of the storm and that higher storm intensities are generally associated with shorter duration storms, the combination of which would tend to produce shorter rise times.

$\ln t_r$, $\log_e$ observed hydrograph rise time

Although one would expect the logarithmic relationship to be more meaningful than the linear one, this is not the case. Not only is the value of R^2 smaller for equation 5.14 than for equation 5.13, but one finds it extremely difficult to attach physical meaning to the stream slope, S_3 . This is not to preclude the multiplicative concept for the rise time, but rather to cast doubt on the quality of the data used in the study. The computer chooses variables on the basis of mathematical significance, not hydrologic significance. If the independent variables have large variances as many of them do (see Table 6), then the variable with the highest partial correlation with the dependent variable may not be hydrologically significant.

TABLE 13. REGRESSION EQUATIONS FOR THE VOLUME OF RUNOFF

Equation

$$V = 0.39754 + 0.35949\ C_1 + 0.26916\ P_T - 0.33684\ LR - 0.50313\ f - 0.61565\ PI \qquad (5.9)$$

R^2	0.4189	0.5907	0.6946	0.7268	0.7497
S_{ey}	0.4150	0.3502	0.3043	0.2895	0.2787
F	63.4290	62.7800	65.1949	56.5419	50.3318

$$\ln V = -\ 1.80357 + 0.82662\ \ln C_1 + 0.00230\ \ln m_1' - 0.14805\ \ln f + 0.78436\ \ln P_T - 0.25138\ \ln LR \qquad (5.10)$$

R^2	0.5511	0.6478	0.6833	0.7180	0.7797
S_{ey}	0.5163	0.4599	0.4387	0.4164	0.3701
F	108.0305	80.0102	61.8464	54.0960	59.4755

TABLE 14. REGRESSION EQUATIONS FOR THE OBSERVED PEAK

Equation

$$q_p = -\ 0.53083 + 1.69662\ P_q - 0.01414\ P + 0.36737\ I_{60} - 1.45997\ f + 0.00306\ S_3 \qquad (5.11)$$

R^2	0.2092	0.3435	0.4930	0.5398	0.6155
S_{ey}	0.5077	0.4653	0.4113	0.3941	0.3624
F	23.2764	22.7643	27.8702	24.9267	26.8971

$$\ln q_p = -\ 2.91277 + 1.59242\ \ln P_q - 0.43509\ \ln P + 0.49737\ \ln I_{25} - 0.65413\ \ln f + 0.45018\ \ln R_2 \qquad (5.12)$$

R^2	0.2031	0.3665	0.4781	0.5127	0.6056
S_{ey}	0.5993	0.5374	0.4906	0.4768	0.4315
F	22.4262	25.1702	26.2596	22.3613	25.7996

TABLE 15. REGRESSION EQUATIONS FOR THE OBSERVED RISE TIME

	Equation					
$t_r = -\ 12.45951$	$+\ 0.76878\ m_1'$	$+\ 0.00991\ m_2$	$+\ 9.16701\ A$	$-\ 0.00013\ y_4'$	$+\ 9.54705\ C_1$	(5.13)
R^2	0.7895	0.8346	0.8712	0.8784	0.8841	
S_{ey}	28.5466	25.4497	22.5865	22.0767	21.6808	
F	330.1017	219.5239	193.9560	153.5178	128.1668	
$\ln t_r = 2.52790$	$+\ 0.68346 \ln m_1'$	$-\ 0.34094 \ln S_3$	$-\ 0.27182 \ln D_d$	$+\ 0.20049 \ln R_4$	$-\ 0.2199 \ln I_{25}$	(5.14)
R^2	0.6633	0.7331	0.7567	0.7714	0.7857	
S_{ey}	0.4644	0.4158	0.3993	0.3893	0.3792	
F	173.3345	119.4597	89.1620	71.6966	61.5766	

$$V = 0.39754 + 0.35949\,C_1 + 0.26916\,P_T - 0.33684\,LR - 0.50313\,f - 0.61565\,PI$$

C_1 = 10-year 1-hour precipitation, inches

D_T = storm total, inches

LR = loss rate, inches/hour

f = infiltration capacity, inches/hour

PI = pattern index, dimensionless

$R^2 = 0.7497$, $S_{ey} = 0.2787$, $n = 90$ events

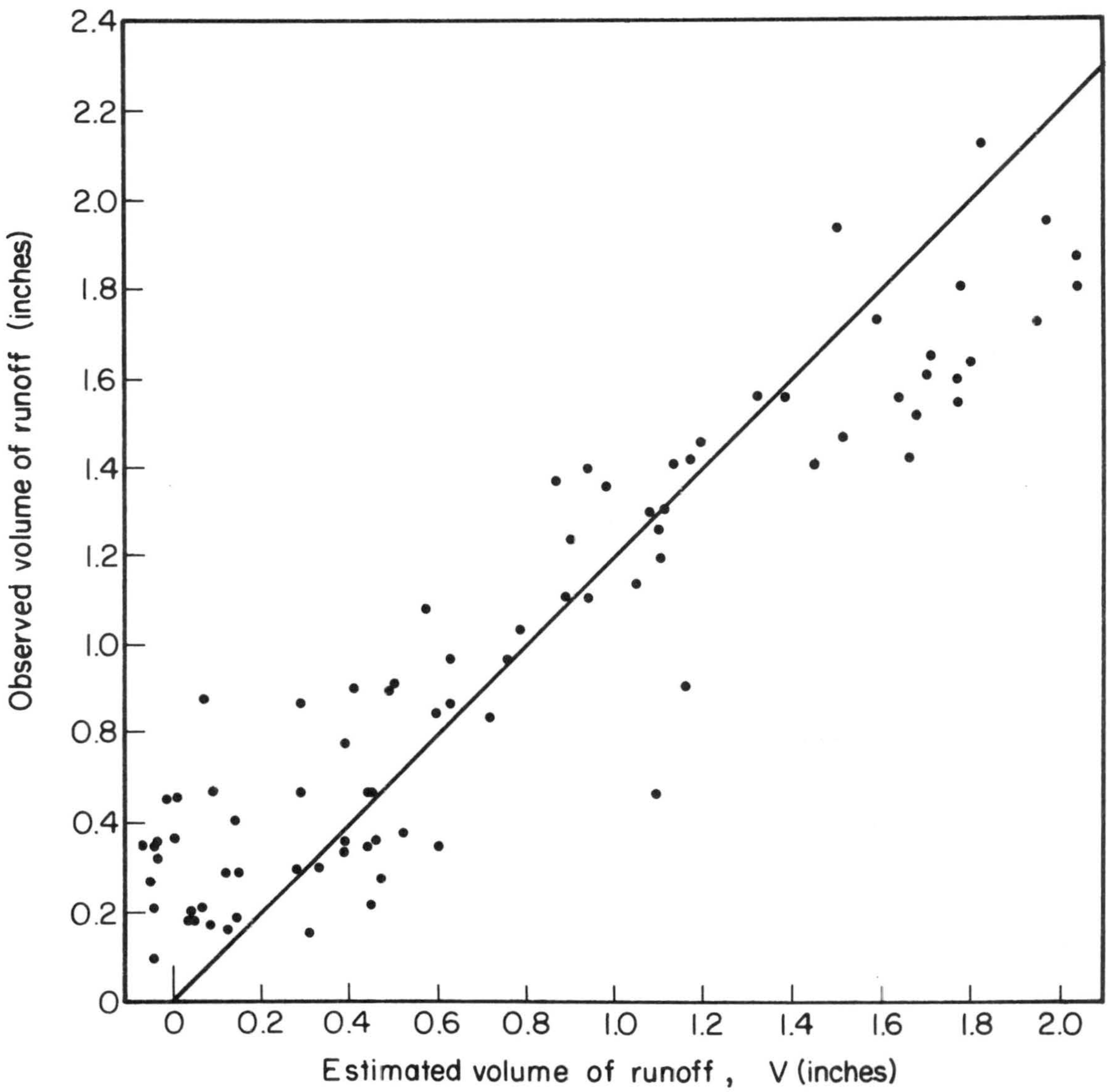


Figure 28. Correlation of observed versus estimated values of the volume of runoff for equation 5.9

$$\ln V = -1.80357 + 0.82662 \ln C_1 + 0.00230 \ln m_1' - 0.14805 \ln f + 0.78436 \ln P_T - 0.25138 \ln LR$$

$\ln C_1$ = $\log_e$ of 10-year 1-hour precipitation

$\ln m_1'$ = $\log_e$ of first moment of hyetograph

$\ln f$ = $\log_e$ of infiltration capacity

$\ln P_T$ = $\log_e$ of total storm rainfall

$\ln LR$ = $\log_e$ of loss rate

R^2 = 0.7797, S_{ey} = 0.3701, n = 90 events

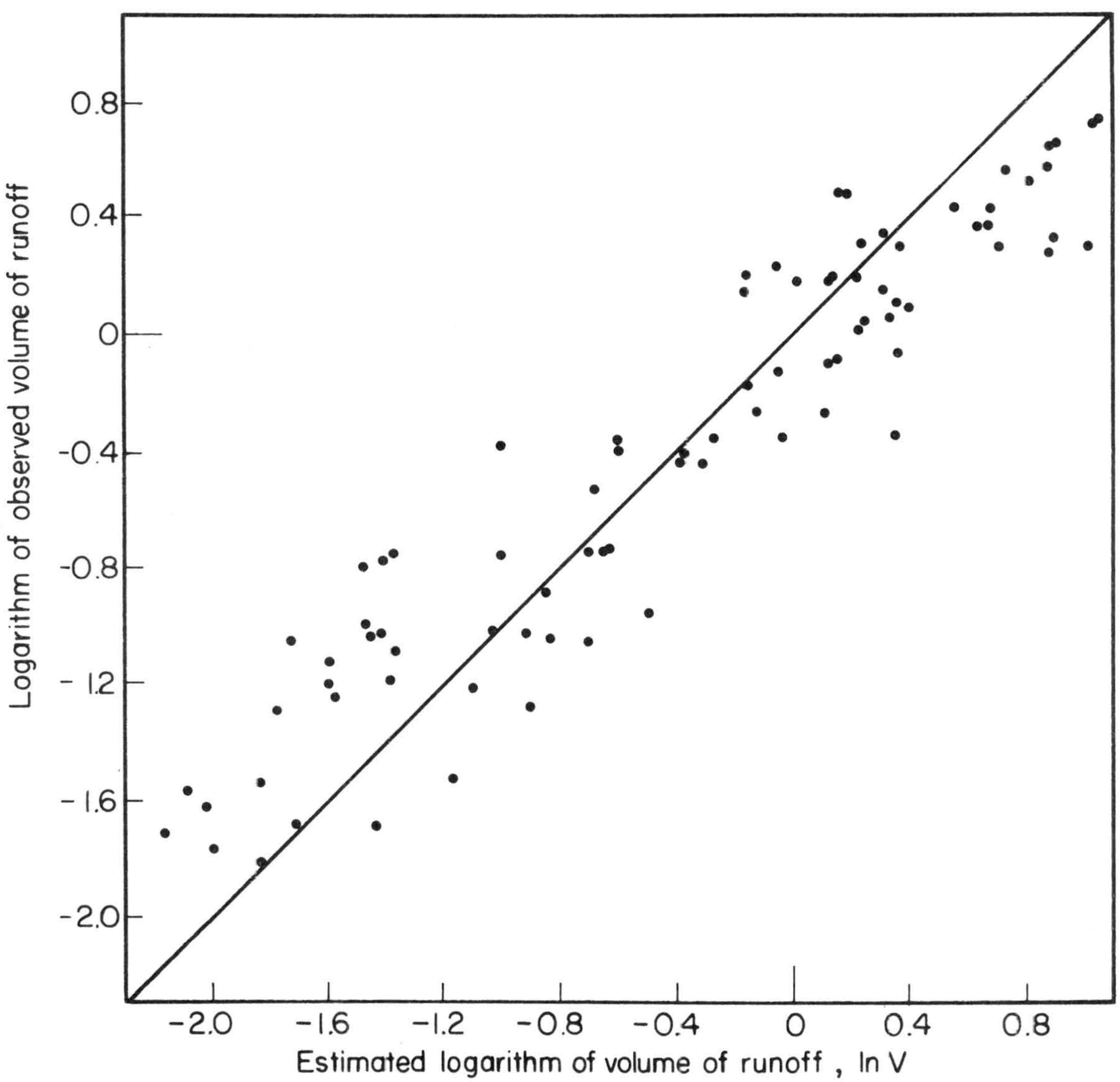


Figure 29. Correlation of observed versus estimated values of the logarithm of the volume of runoff for equation 5.10

$$q_p = -0.53083 + 1.69662\,P_q - 0.01414P + 0.36737\,I_{60} - 1.45997f + 0.00306\,S_3$$

P_q = probability of peak

P = catchment perimeter, miles

I_{60} = 60-minute maximum intensity, inches/hour

f = infiltration capacity, inches/hour

S_3 = stream slope, feet/mile

R^2 = 0.6155, S_{ey} = 0.3624, n = 90 events

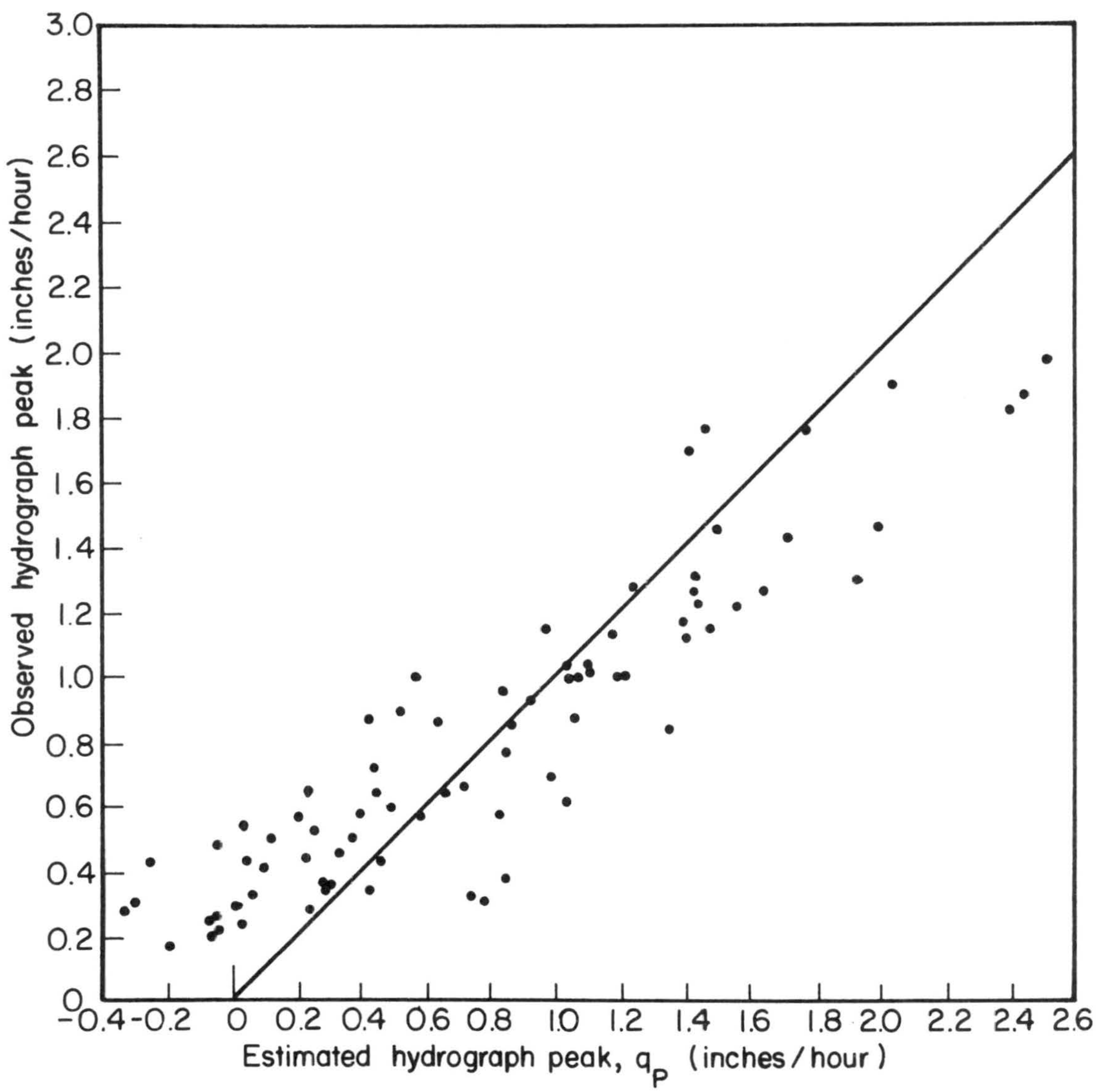


Figure 30. Correlation of observed versus estimated values of the observed hydrograph peak discharge for equation 5.11

$$\ln q_p = -2.91277 + 1.59242 \ln P_q - 0.43509 \ln P + 0.49737 \ln I_{25} - 0.65413 \ln f + 0.45018 \ln R_2$$

$\ln P_q$ = $\log_e$ of probability of peak

$\ln P$ = $\log_e$ of catchment perimeter

$\ln I_{25}$ = $\log_e$ of 25-minute maximum intensity

$\ln f$ = $\log_e$ of infiltration capacity

$\ln R_2$ = $\log_e$ of overland slope

R^2 = 0.6056, S_{ey} = 0.4315, n = 90 events

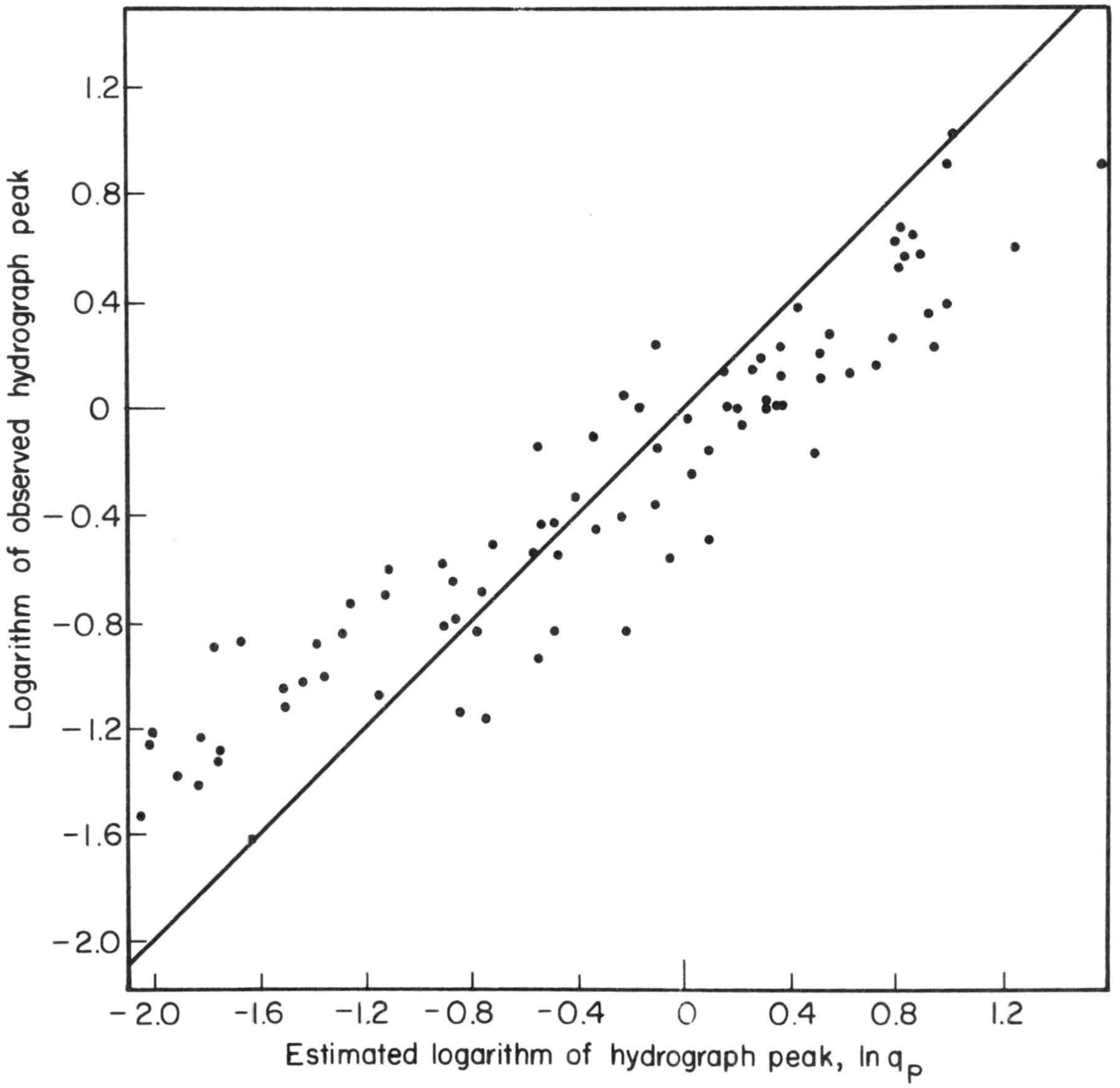


Figure 31. Correlation of observed versus estimated values of the logarithm of the observed hydrograph peak discharge for equation 5.12

$$t_r = -12.45951 + 0.76878\, m_1' + 0.00991\, m_2 + 9.16701\, A - 0.00013\, y_4' + 9.54705\, C_1$$

m_1' = first moment of hyetograph, minutes

m_2 = second central moment of hyetograph, minutes squared

A = catchment area, square miles

y_4' = fourth moment of hyetograph about t-axis, $(\text{inches/hour})^4$

C_1 = 10-year 1-hour precipitation, inches

$R^2 = 0.8841$, $S_{ey} = 21.6808$, $n = 90$ events

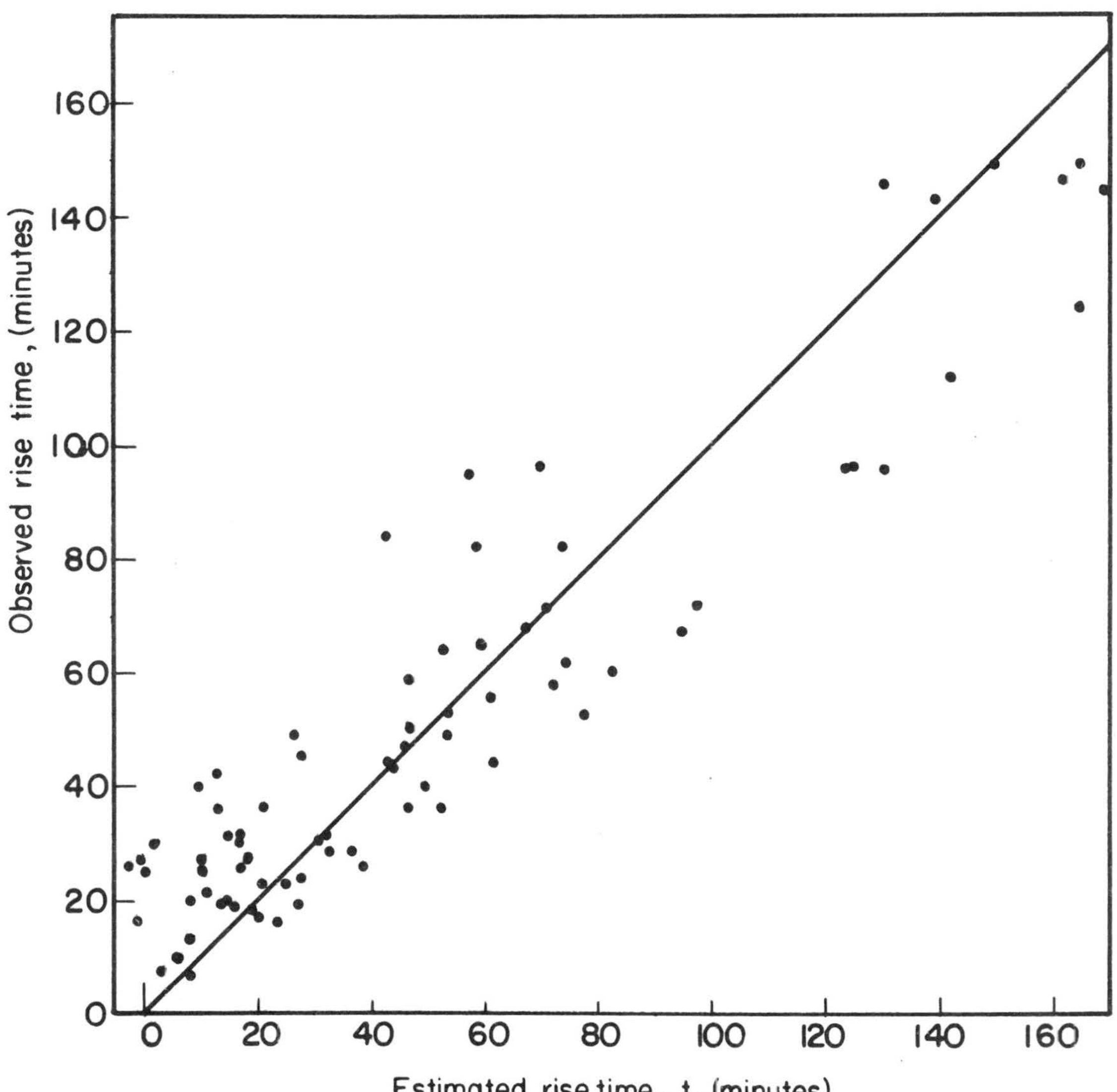


Figure 32. Correlation of observed versus estimated values of the observed hydrograph rise time for equation 5.13

$$\ln t_r = 2.52790 + 0.68346 \ln m_1' - 0.34094 \ln S_3 - 0.27182 \ln D_d + 0.20049 \ln R_4 - 0.21999 \ln I_{25}$$

$\ln m_1'$ = $\log_e$ of first moment of hyetograph

$\ln S_3$ = $\log_e$ of stream slope

$\ln D_d$ = $\log_e$ of drainage density

$\ln R_4$ = $\log_e$ of overland slope, feet/mile

$\ln I_{25}$ = $\log_e$ of 25-minute maximum intensity

R^2 = 0.7857, S_{ey} = 0.3792, n = 90 events

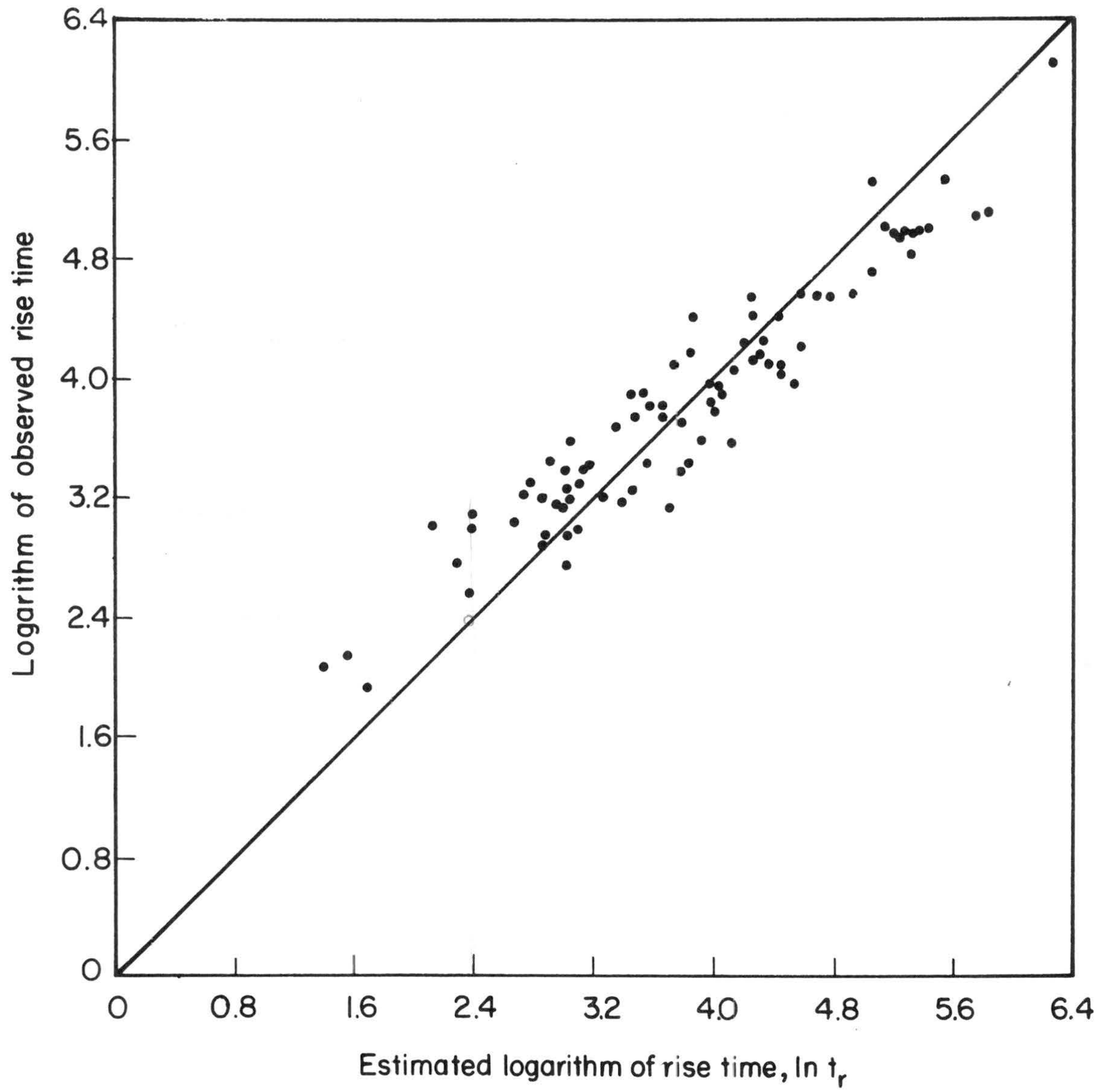


Figure 33. Correlation of observed versus estimated values of the logarithm of the observed hydrograph rise time for equation 5.14

5.3 Prediction of Fitted Hydrograph Parameters

q_0 , fitted hydrograph peak discharge

The variables included in the relation, equation 5.15, for predicting the fitted hydrograph peak, q_0 , are identical to those used to predict the peak, equation 5.11. In this case the computer found the 60-minute maximum intensity, I_{60} , to have a higher partial correlation than the probability of the peak, P_q , with the dependent variable. The R^2 of 0.62 and S_{ey} of 0.3624 inches per hour are also similar values. Even the regression coefficients are in the same orders of magnitude.

$\ln q_0$, $\log_e$ fitted hydrograph peak discharge

Since equations 5.12 and 5.16 are identical except for the regression coefficients, which if rounded off would be equal, the reader is referred to section 5.2 for a discussion of the meaning of the variables. It should be noted, however, that the similarity of these two equations lends tacit support to the applicability of the fitted function.

γ , fitted hydrograph shape parameter

In this instance, equation 5.17, Table 17, the coefficient of multiple determination is 0.43. A poor linear relationship is not unexpected since examination of the fitted function, equation 2.4, suggests a multiplicative relationship.

$\ln \gamma$, $\log_e$ fitted hydrograph shape parameter

As expected the value of R^2 is higher, 0.59, for the logarithmic relationship. But, because γ is a shape parameter incorporating the concepts of skewness and variation it is difficult to attach physical significance to the variables of the relationship. Significant though is the fact that γ is a function of S_2 , the stream slope; R_6 , the overland slope; and C_1 , the 10-year 1-hour precipitation. All of these are parameters which are associated with an individual catchment. The same concept is implicit in unit hydrograph theory since the unit hydrograph is assumed to have a constant shape for a given watershed.

r , fitted hydrograph rise time

The fitted rise time, r , can be predicted with an R^2 of 0.88 and S_{ey} of 22 minutes. The dependent variable is a function of five variables, three of which are measures of the time distribution of the rainfall. As expected, increases in the mean time of the hyetograph result in increases in the rise time. The factors m_2' and m_2 evidently entered the relation as a means of relating the dispersion of hyetograph intensities. The increase of r with A is indicative of increased travel distances. The inclusion of so many rainfall parameters in this relationship would tend to cast doubt on their physical significance, especially since they are all known to be related. For this reason the logarithmic relation is considered to be physically more sound.

$\ln r$, $\log_e$ fitted hydrograph rise time

The logarithmic function, equation 5.20, explains 77% of the total variance. Note the similarity between this equation and equation 5.14, the logarithmic relation for the observed rise time, t_r . Here is further evidence of the adequacy of the fitting methodology. The reader is referred to section 5.2 for a discussion of the meaning of the variables.

TABLE 16. REGRESSION EQUATIONS FOR THE FITTED PEAK

Equation

$$q_0 = -\ 0.50615 + 0.36978\ I_{60} - 0.01393\ P + 1.65563\ P_q - 1.48639\ f + 0.00315\ S_3 \qquad (5.15)$$

R^2	0.1991	0.3892	0.4897	0.5371	0.6172
S_{ey}	0.5109	0.4487	0.4125	0.3952	0.3615
F	21.8717	27.7194	27.5105	24.6543	27.0887

$$\ln q_0 = -\ 2.92476 + 1.56171 \ln P_q - 0.44558 \ln P + 0.49504 \ln I_{25} - 0.65738 \ln f + 0.45200 \ln R_2 \qquad (5.16)$$

R^2	0.1947	0.3669	0.4780	0.5133	0.6074
S_{ey}	0.6009	0.5358	0.4894	0.4753	0.4294
F	21.2754	25.2085	26.2527	22.4120	25.9970

TABLE 17. REGRESSION EQUATIONS FOR THE FITTED SHAPE PARAMETER

Equation

$$\gamma = 0.21921 - 0.07105\ C_1 + 0.00064\ S_3 + 0.01682\ I_5 - 0.16544\ f + 0.00012\ H \qquad (5.17)$$

R^2	0.1788	0.3092	0.3554	0.3903	0.4338
S_{ey}	0.1126	0.1039	0.1009	0.0987	0.0957
F	19.1660	19.4680	15.8046	13.6045	12.8708

$$\ln \gamma = -\ 6.10978 + 0.78830 \ln S_2 - 0.13057 \ln D_T - 0.38137 \ln R_6 + 0.07744 \ln y_4' - 0.65678 \ln C_1 \qquad (5.18)$$

R^2	0.3337	0.4542	0.5088	0.5459	0.5859
S_{ey}	0.5939	0.5406	0.5159	0.4989	0.4792
F	44.0815	36.2050	29.6882	25.5429	23.7740

TABLE 18. REGRESSION EQUATIONS FOR THE FITTED RISE TIME

Equation

$$r = 6.08576 + 0.00039\ m_2' + 10.8937\ A + 0.71942\ m_1' - 0.00015\ y_4' + 0.00906\ m_2 \quad (5.19)$$

R^2	0.7639	0.8344	0.8598	0.8694	0.8770
S_{ey}	29.9844	25.2550	23.3742	22.6910	22.1460
F	284.6603	219.1512	175.7459	141.4308	119.8295

$$\ln r = 2.46616 + 0.67864 \ln m_1' - 0.37896 \ln S_3 - 0.29465 \ln D_d + 0.24133 \ln R_4 - 0.23532 \ln I_{25} \quad (5.20)$$

R^2	0.6320	0.7068	0.7329	0.7526	0.7677
S_{ey}	0.5044	0.4528	0.4347	0.4208	0.4101
F	151.1050	104.8774	78.6440	64.6363	55.5239

$$q_0 = -0.50615 + 0.36978\, I_{60} - 0.01393P + 1.65563\, P_q - 1.48639f + 0.00315\, S_3$$

I_{60} = 60-minute maximum intensity, inches/hour

P = catchment perimeter, miles

P_q = probability of peak discharge

f = infiltration capacity, inches/hour

S_3 = stream slope, feet/mile

R^2 = 0.6172 , S_{ey} = 0.3615 inches/hour , n = 90 events

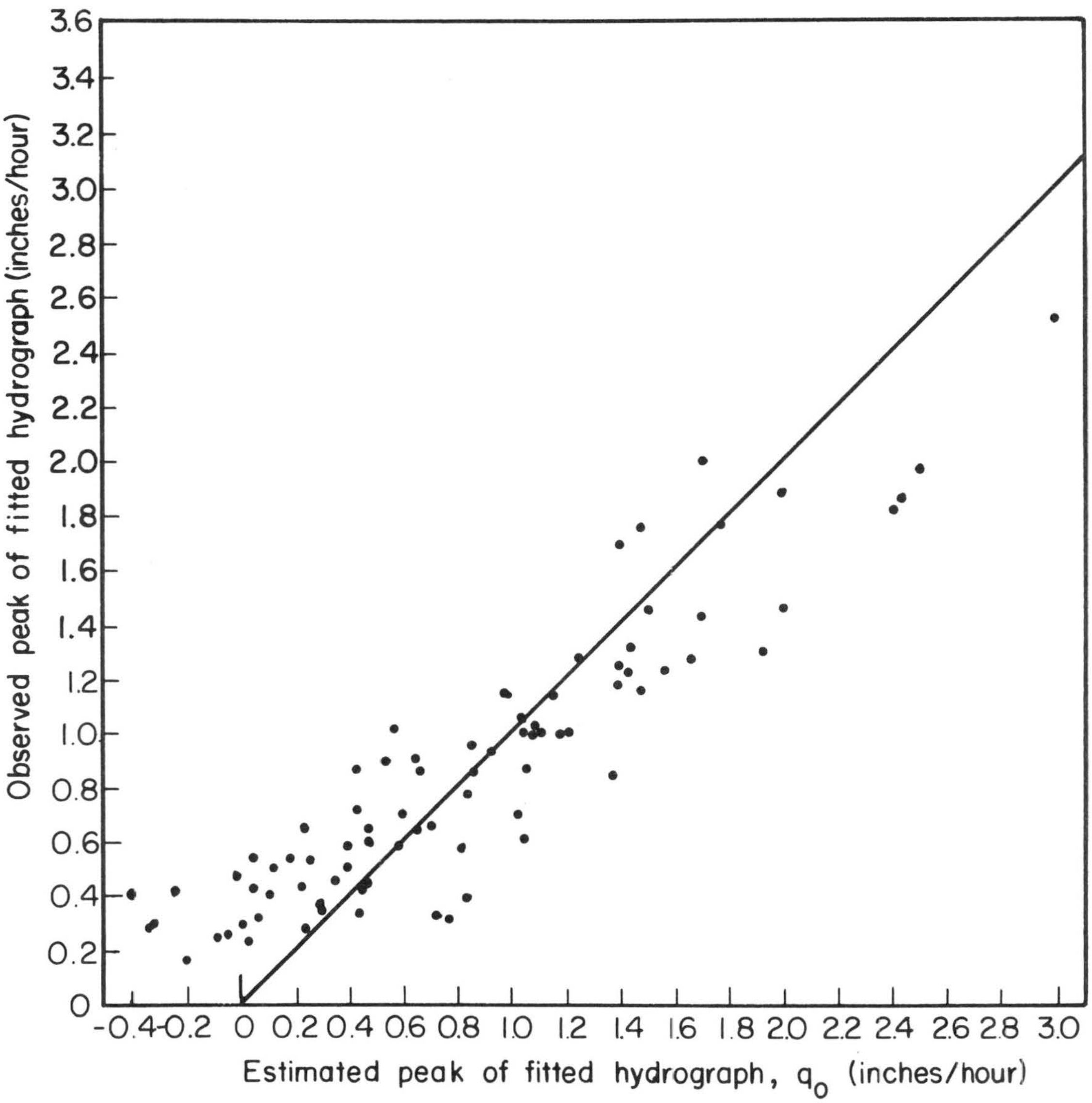


Figure 34. Correlation of observed versus estimated values of the fitted hydrograph peak discharge for equation 5.15

$$\ln q_0 = -2.92476 + 1.56171 \ln P_q - 0.44558 \ln P + 0.49504 \ln I_{25} - 0.65738 \ln f + 0.45200 R_2$$

$\ln P_q$ = $\log_e$ of probability of peak

$\ln P$ = $\log_e$ of catchment perimeter

$\ln I_{25}$ = $\log_e$ of 25-minute maximum intensity

$\ln f$ = $\log_e$ of infiltration capacity

$\ln R_2$ = $\log_e$ of overland slope

R^2 = 0.6074, S_{ey}=0.4294, n=90 events

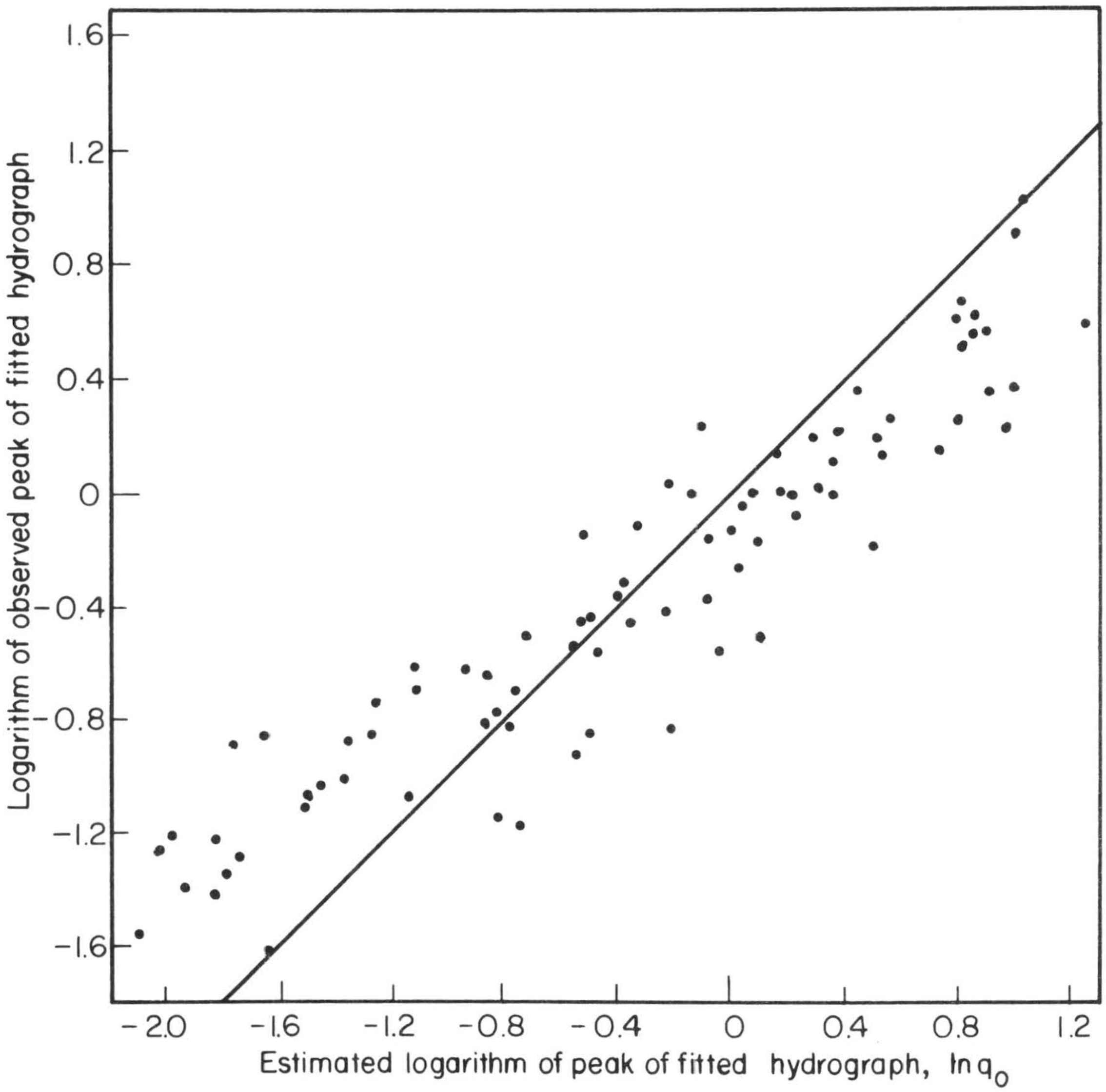


Figure 35. Correlation of observed versus estimated values of the logarithm of the fitted hydrograph peak discharge for equation 5.16

$$\gamma = 0.21920 - 0.07105 C_1 + 0.00064 S_3 + 0.01682 I_5 - 0.16544 f + 0.00012 H$$

C_1 = 10-year 1-hour precipitation, inches

S_3 = stream slope, feet/miles

I_5 = 5-minute maximum intensity, inches/hour

f = infiltration capacity, inches/hour

H = total fall, feet

R^2 = 0.4338, S_{ey} = 0.0957 inches/hour, n = 90 events

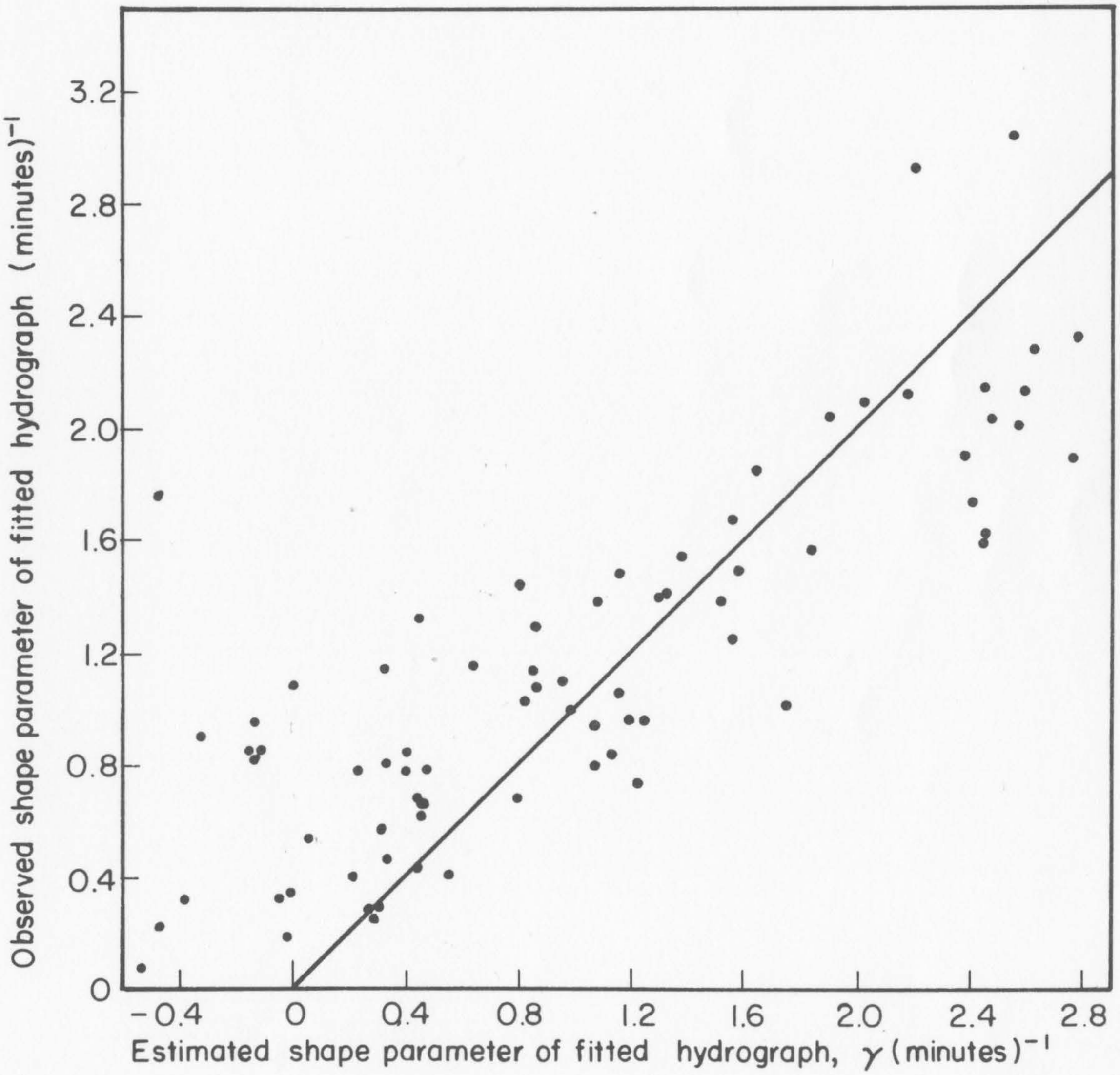


Figure 36. Correlation of observed versus estimated values of the fitted hydrograph shape parameter for equation 5.17

$$\ln\gamma = -6.10978 + 0.78830 \ln S_2 - 0.13057 \ln D_T - 0.38137 \ln R_6 + 0.07744 \ln y_4' - 0.65678 \ln C_1$$

$\ln S_2$ = $\log_e$ of stream slope

$\ln D_T$ = $\log_e$ of drainage density

$\ln R_6$ = $\log_e$ of overland slope

$\ln y_4'$ = $\log_e$ of fourth moment of hyetograph about t-axis

$\ln C_1$ = $\log_e$ of 10-year 1-hour precipitation

R^2 = 0.5859, S_{ey} = 0.4792, n = 90 events

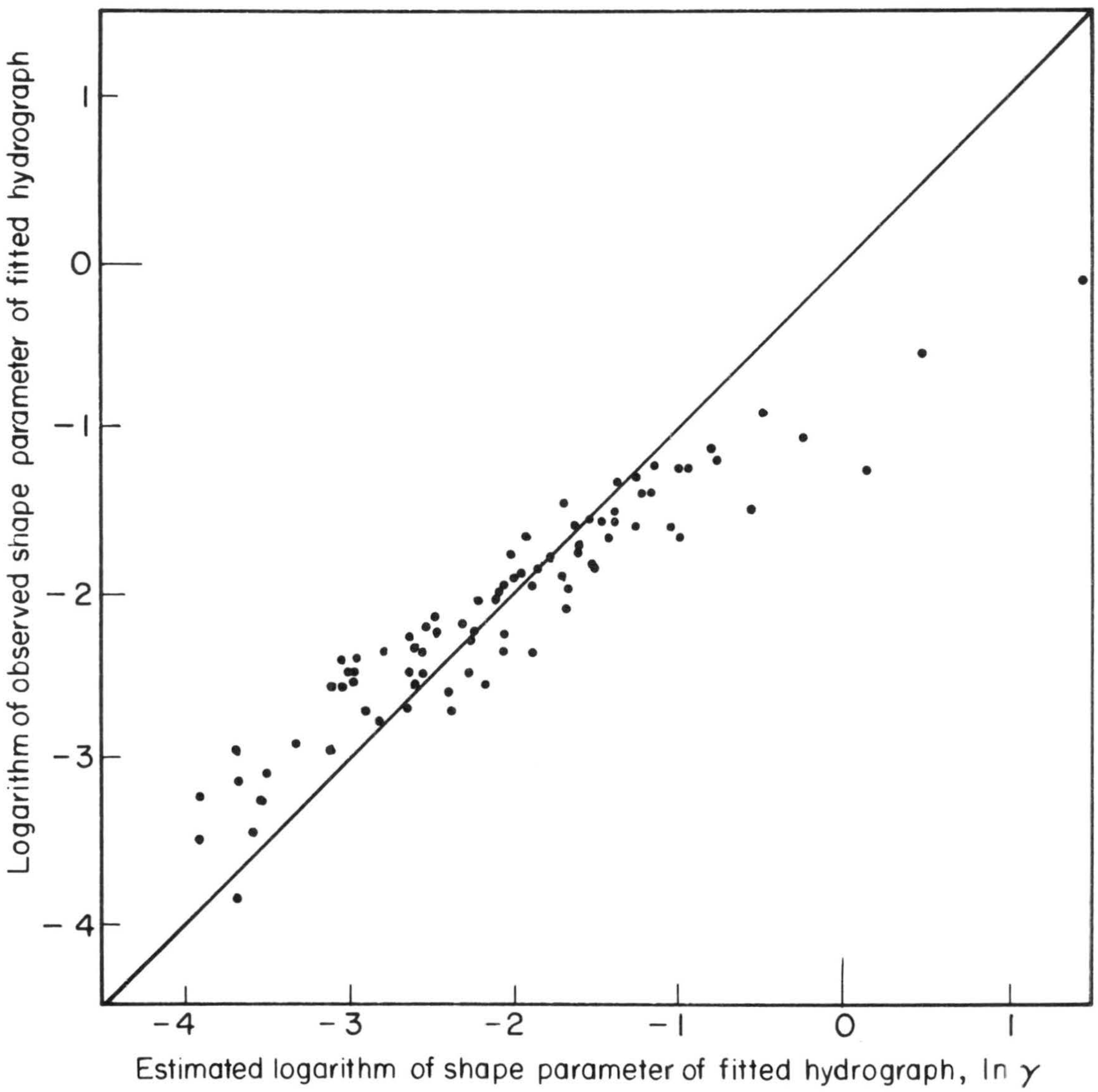


Figure 37. Correlation of observed versus estimated values of the logarithm of the fitted hydrograph shape parameter for equation 5.18

$$r = 6.08576 + 0.00039\, m_2' + 10.89370\, A + 0.71942\, m_1' - 0.00015\, y_4' + 0.00906\, m_2$$

m_2' = second moment of hyetograph, minutes squared

A = catchment area, square miles

m_1' = first moment of hyetograph, minutes

y_4' = fourth moment of hyetograph about t-axis, $(\text{inches/hour})^4$

m_2 = second central moment of hyetograph, $(\text{minutes})^2$

$R^2 = 0.8770$, $S_{ey} = 22.1460$, $n = 90$ events

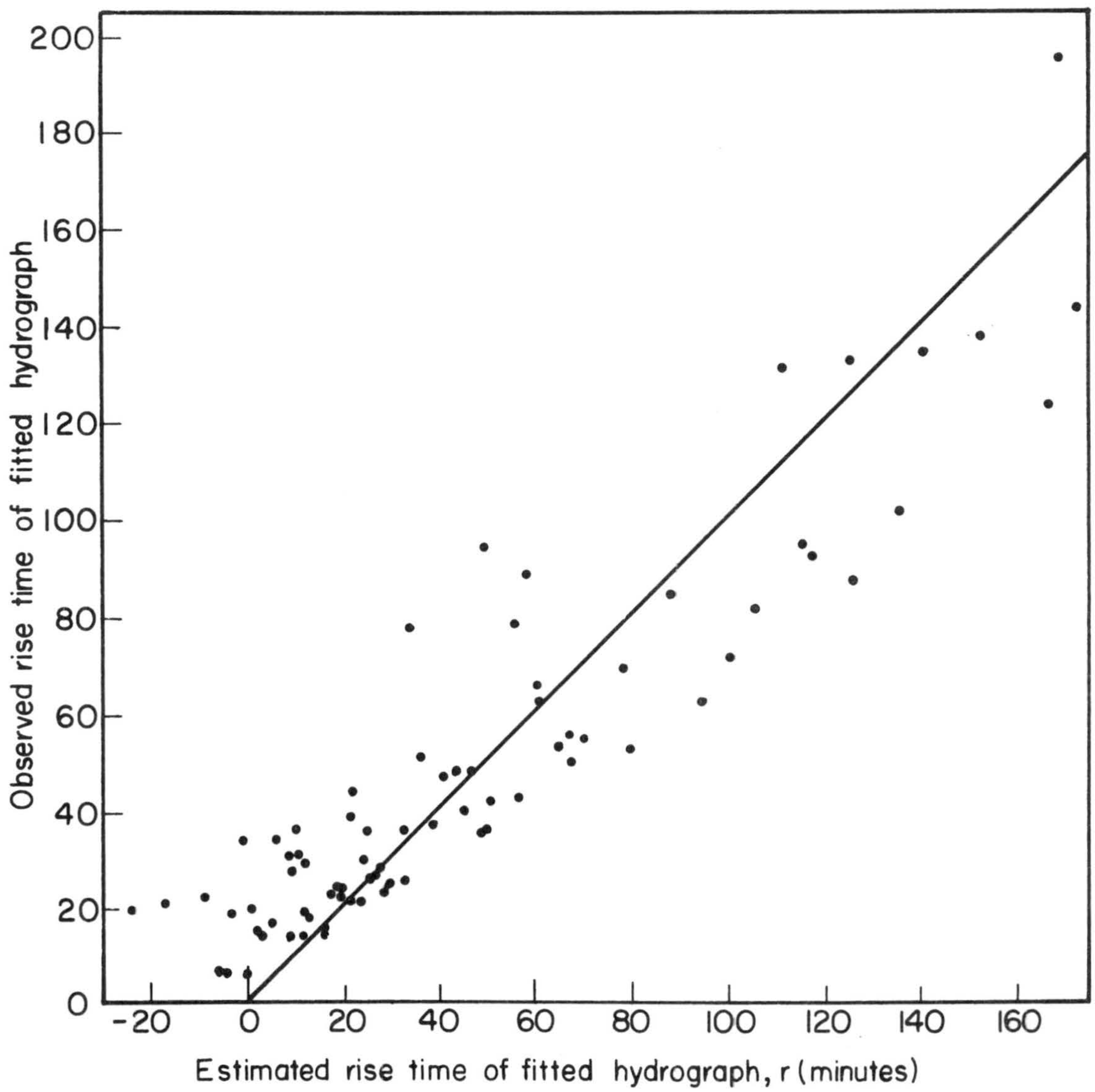


Figure 38. Correlation of observed versus estimated values of the fitted hydrograph rise time for equation 5.19

$$\ln r = 2.46616 + 0.67864 \ln m_1' - 0.37896 \ln S_3 - 0.29465 \ln D_d + 0.24133 \ln R_4 - 0.23532 \ln I_{25}$$

$\ln m_1'$ = $\log_e$ of first moment of hyetograph

$\ln S_3$ = $\log_e$ of stream slope

$\ln D_d$ = $\log_e$ of drainage density

$\ln R_4$ = $\log_e$ of overland slope

$\ln I_{25}$ = $\log_e$ of 25-minutes maximum intensity

R^2 = 0.7677 , S_{ey} = 0.4101, n = 90 events

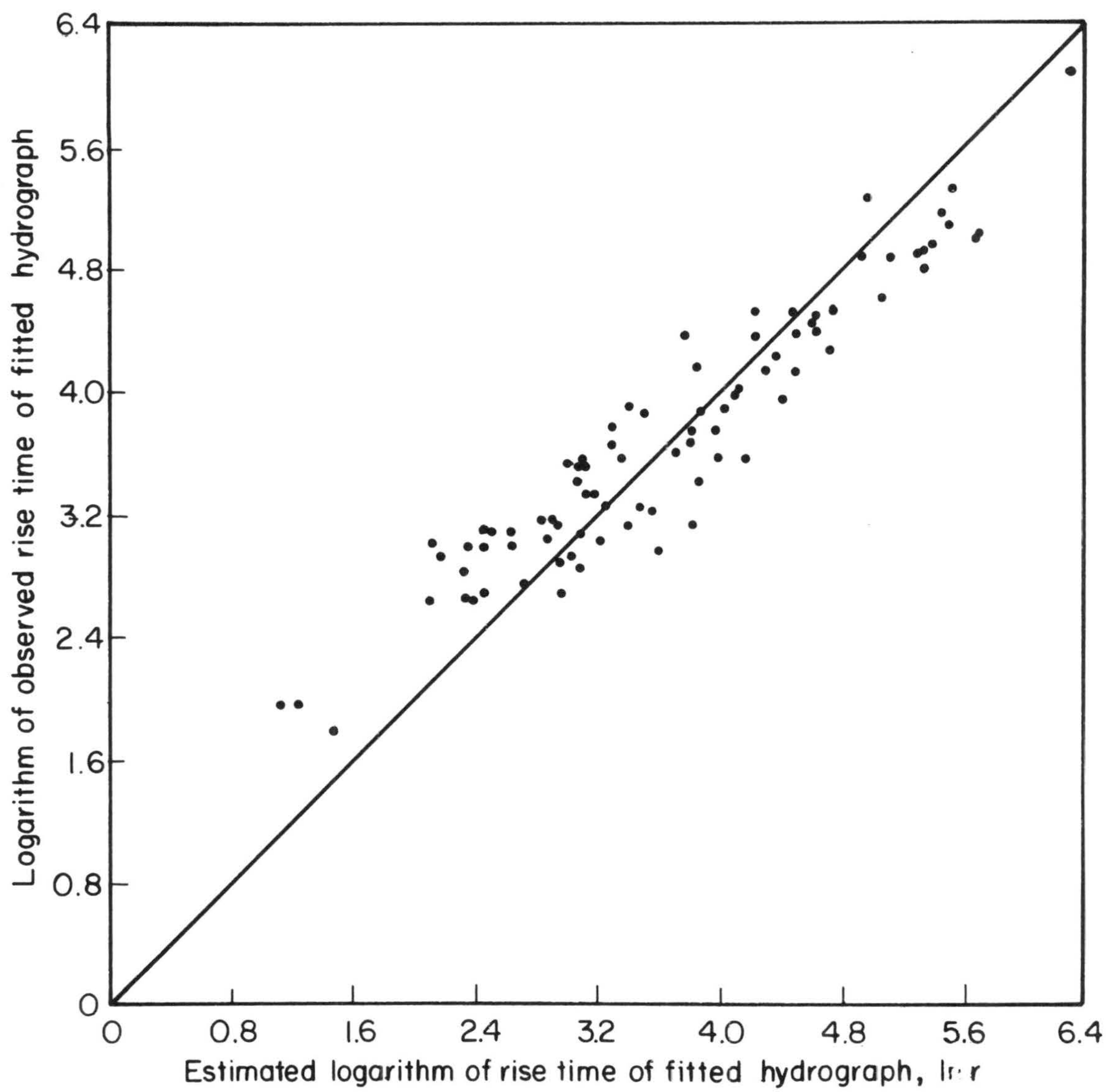


Figure 39. Correlation of observed versus estimated values of the logarithm of the fitted hydrograph rise time for equation 5.20

Chapter VI

CONCLUSIONS

This study has been exploratory and in some aspects qualitative rather than purely quantitative. However, certain points are significant and the following conclusions can be drawn:

1. For the small watershed floods considered the moments of the hydrograph are not well correlated with the rainfall parameters. This is attributed to the effect of the hydrograph tails on moments.

2. The hydrograph mode, or rise time, is a practical hydrograph parameter since it tends to be stable.

3. The similarity of the relations for the observed rise time and the fitted rise time, as well as for the observed peak and the fitted peak attest to the methodology of fitting the three-parameter gamma function to the observed hydrograph.

4. The results indicated that the traditional hydrograph parameters of volume, peak and rise time are as good if not better than the mathematical parameters for prediction purposes.

5. The moments of the hyetograph about its time of commencement and the mean intensity appear to provide new and objective descriptors of the time distribution of rainfall intensities.

Chapter VII

RECOMMENDATIONS

The feasibility of the approach has been established by the results, but because of the exploratory nature of this study further investigations are warranted.

There exists a need for increasing the sample size, particularly in regard to the number of events per watershed. For design purposes better predictions could be obtained by segregating the watersheds by climatic regions, predominant cover types or river basins.

In the present scheme the iterations of the least squares equations for the fitting process were terminated when the sums were sufficiently close to zero. This procedure always produced the best mathematical fit but not necessarily the best hydrologic fit. Further work is needed in developing criteria for goodness of fit, particularly with an eye to forcing a fit when the mathematical techniques by themselves do not lead to a good hydrologic fit.

BIBLIOGRAPHY

BIBLIOGRAPHY

Bernard, M. 1935. "An Approach to Determinate Stream Flow." American Society of Civil Engineers, Transactions, Volume 100, pp. 347-395.

Brater, E. F. 1940. "The Unit Hydrograph Principle Applied to Small Watersheds." American Society of Civil Engineers, Transactions, Volume 105, pp. 1154-1178.

Chow, V. T. 1962. "Hydrologic Determination of Waterway Area for the Design of Drainage Structures in Small Drainage Basins." University of Illinois Engineering Experiment Station Bulletin No. 462, Urbana, Illinois, 104 pp.

Chow, V. T., Editor. 1964. Handbook of Applied Hydrology. McGraw-Hill Book Company, New York.

Clark, C. D. 1945. "Storage and the Unit Hydrograph." American Society of Civil Engineers, Transactions, Volume 110, pp. 1419-1446.

Creager, W. P., J. D. Justin and J. Hinds. 1945. Engineering for Dams. Volume 1, John Wiley and Sons, Inc., New York.

Dalrymple, T. 1960. "Flow-Flow Techniques." Manual of Hydrology: Part 3, United States Geological Survey Water-Supply Paper 1543-A.

Dickinson, W. T., and H. D. Ayers. 1965. "The Effect of Storm Characteristics on the Unit Hydrograph." Engineering Institute of Canada, Transactions, Volume 8, No. A-15, December.

Diskin, M. H. 1967. "A Laplace Transform Proof of the Theorem of Moments for the Instantaneous Unit Hydrograph." Water Resources Reach, Volume 3, No. 2, pp. 385-388.

Dixon, W. J., Editor. 1965. "Biomedical Computer Programs." Health Science Computing Facility, Department of Preventive Medicine and Public Health, School of Medicine, University of California, Los Angeles, September.

Dooge, J. C. I. 1955. "Discussion of O'Kelley's Paper." Institution of Civil Engineers, Proceedings, Volume 4, No. 2, August, pp. 436-445.

Dooge, J. C. I. 1959. "A General Theory of the Unit Hydrograph." Journal of Geophysical Research, Volume 64, pp. 241-256.

Edson, Charles G. 1951. "Parameters for Relating Unit Hydrographs to Watershed Characteristics." American Geophysical Union, Transactions, Volume 32:4, August, pp. 591-595.

Elderton, W. P. 1927. Frequency Curves and Correlation, Second Edition. Charles and Edwin Layton, London.

Ezekiel, M. 1941. Methods of Correlation Analysis. John Wiley and Sons, Inc., New York, 531 pp.

Folse, J. A. 1929. "A New Method of Estimating Stream Flow Based Upon a New Evaporation Formula." Part II: A New Method of Estimating Stream Flow, Carnegie Institute of Washington, Publication No. 400, Washington, D. C.

Ford, P. 1959. "Multiple Correlation in Forecasting Seasonal Runoff." Engineering Monograph No. 2, United States Department of the Interior, Bureau of Reclamation, Denver.

Foster, A. H. 1924. "Theoretical Frequency Curves and Their Application to Engineering Problems." American Society of Civil Engineers, Transactions, Volume 87, pp. 142-173.

Foster, W. D. 1950. "A Comparison of Nine Indices of Rainfall Intensity." American Geophysical Union, Transactions, Volume 31, December, pp. 894-900.

Gray, D. M. 1962. "Derivation of Hydrographs for Small Watersheds from Measurable Physical Characteristics." Agricultural and Home Economics Experiment Station Research Bulletin No. 506, Iowa State University, Ames, Iowa, June.

Gumbel, E. J. 1954. "Statistical Theory of Extreme Values and Some Practical Applications." Applied Math Series No. 33, National Bureau of Standards.

Hershfield, D. M. 1961. "Rainfall Atlas of the United States for Durations from 30 Minutes to 24 Hours and Return Periods from 1 to 100 Years." United States Weather Bureau Technical Paper No. 40, May. (Repaged and Reprinted January, 1963.)

Hiemstra, L. A. V. and B. M. Reich. 1967. "Engineering Judgement and Small Area Flood Peaks." Hydrology Paper No. 19, Colorado State University, Fort Collins, 29 pp.

Hildebrand, F. B. 1963. Advanced Calculus for Applications. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Ho, Y. B. 1967. "Evaluation of Runoff Characteristics from Hydrograph Recessions." M.S. Thesis, Colorado State University, Fort Collins, 84 pp.

Horner, W. W. and F. L. Flynt. 1936. "Relation Between Rainfall and Runoff from Urban Areas." American Society of Civil Engineers, Transactions, Volume 101, pp. 140-206.

Horner, W. W. and S. W. Jens. 1942. "Surface-Runoff Determination from Rainfall Without Using Coefficients." American Society of Civil Engineers, Transactions, Volume 107, pp. 1039-1117.

Horton, R. E. 1935. "Surface Runoff Phenomena, Part I." Analysis of the Hydrograph, Publication No. 101, Horton Hydrological Laboratory, Voorheesville, New York, 73 pp.

Horton, R. E. 1939. "Analysis of Runoff Plat Experiments with Varying Infiltration Capacity." American Geophysical Union, Transactions, Part IV, pp. 693-711.

Hoyt, W. G., *et al.* 1936. "Studies of Relation of Rainfall and Runoff in the United States." United States Geological Survey Water-Supply Paper 772.

Hutchinson, J., H. L. Manning, and H. G. Farlrother. 1958. "On the Characterization of Tropical Rainstorms in Relation to Runoff and Percolation." Royal Meteorological Society, London, Quarterly Journal, Volume 84, pp. 250-258.

Hydrology Handbook. 1949. American Society of Civil Engineers, Manual of Engineering Practice No. 28.

Jenkinson, A. F. 1955. "The Frequency Distribution of the Annual Maximum Values of Meteorological Elements." Journal of the Royal Meteorological Society, Volume 81, No. 348, April, pp. 158-171.

Kendall, M. G. 1943. The Advanced Theory of Statistics, Volume I. Charles Griffin and Company, London, 457 pp.

Kendall, M. G. 1961. A Course in Multivariate Analysis. Hafner Publishing Company, New York.

Kuichling, E. 1889. "The Relation Between the Rainfall and the Discharge of Sewers in Populous Districts." American Society of Civil Engineers, Transactions, Volume 20, pp. 1-56.

Laurenson, E. M. and D. H. Pilgrim. 1963a. "Loss Rates for Australian Catchments and Their Significance." Journal, Institution of Engineers, Australia, Volume 35, No. 1-2, pp. 9-24.

Laurenson, E. M., E. F. Schulz and V. M. Yevdjevich. 1963b. "Research Data Assembly for Small Watershed Floods." CER63EML-EFS-VMY37, Engineering Research Center, Colorado State University, September.

Markovic, R., R. Downer, and M. Siddiqui. 1964. An Unpublished Report Entitled: "Explanation of Symbols and Method for Filling in the Watershed Card." Engineering Research Center, Colorado State University, May.

Markovic, R., R. Downer, and M. Siddiqui. 1964. An Unpublished Report Entitled: "Explanation of Symbols and Method for Filling in the Flood Event Card." Engineering Research Center, Colorado State University, July.

Minshall, N. E. 1962. "Predicting Storm Runoff on Small Experimental Watersheds." American Society of Civil Engineers, Transactions, Volume 127, pp. 625-659.

Nash, J. E. 1958. "The Form of the Instantaneous Unit Hydrograph." International Association of Scientific Hydrology, General Assembly of Toronto, September 1957, Comptes-Rendus, Number 45, Volume 3, pp. 111-121.

Nash, J. E. 1959. "Systematic Determination of Unit Hydrograph Parameters." Journal of Geophysical Research, Volume 64, No. 1, January, pp. 111-115.

National Engineering Handbook. 1957. Section 4, Supplement A, Hydrology, United States Soil Conservation Service.

Neal, J. H. 1945. "Soil and Water Losses as Affected by Rainfall Characteristics." Agricultural Engineering, Volume 26, November, pp. 463-464.

O'Kelley, J. J. 1955. "The Employment of Unit Hydrographs to Determine the Flows of Irish Arterial Drainage Channels." Institution of Civil Engineers, Proceedings, Volume 4, No. 2, August, pp. 365-445.

Om Kar, S. 1967. "Hydrograph Rise Times." M.S. Thesis, Colorado State University, Fort Collins, 124 pp.

Pearson, Karl. 1956. Early Statistical Papers. Cambridge University Press, London, 557 pp.

Pilgrim, D. H. 1966. "Storm Loss Rates for Regions with Limited Data." Journal of the Hydraulics Division, American Society of Civil Engineers, Proceedings, Volume 92, No. HY2, March, pp. 193-206.

Potter, W. D. 1950. "Surface Runoff from Agricultural Watersheds." Surface Drainage, Highway Research Board Report No. 11-B, pp. 21-35.

Reich, B. M. 1962. "Design Hydrographs for Very Small Watersheds from Rainfall." Ph.D. Dissertation, Colorado State University, Fort Collins.

Report of the Committee on Floods. 1930. Journal of the Boston Society of Civil Engineers, Volume 17, No. 7, September, pp. 285-464.

Research Data Assembly for Small Watershed Floods - Part II. 1967. Engineering Research Center, Colorado State University, Fort Collins, (In Preparation for Publication, Expected Date of Release: September, 1967).

Schiff, L. 1943. "Classes and Patterns of Rainfall with Reference to Surface Runoff." American Geophysical Union, Transactions, Volume 32, April, pp. 438-452.

Shanholtz, V. O. and W. H. Dickerson. 1964. "Influence of Selected Rainfall Characteristics on Runoff Volume." West Virginia University Agricultural Experiment Station Bulletin 497T, Morgantown, West Virginia, June.

Sherman, L. K. 1932. "Stream Flow from Rainfall by the Unit-Graph Method." Engineering News-Record, Volume 108, April 7, pp. 501-505.

Snedecor, G. W. 1964. Statistical Methods. Fifth Edition, The Iowa State University Press, Ames, 534 pp.

Weber, E. W. and W. G. Sutton. 1965. "Environmental Effects of Flood Plain Regulations." American Society of Civil Engineers, Proceedings, Hydraulics Division, Volume 91, No. HY4, July, pp. 59-70.

Wischmeier, W. H., D. D. Smith, and R. E. Uhland. 1958. "Evaluation of Factors in the Soil-Loss Equation." Agricultural Engineering, Volume 39, August, pp. 458-462.