THE HYDRAULICS OF SMALL, ROUGH IRRIGATION CHANNELS
(L'HYDRAUIQUE DES PETITS CANAUX D'IRRIGATION RUGUEUX)

by

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Paper submitted to the 5th Congress of

International Commission on Irrigation and Drainage

Tokyo, Japan

May 1963

CER62EGK44

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INTRODUCTION

The design of surface irrigation systems is currently based on the experience of the designer and a few empirical equations. These equations do not give adequate consideration to variations in soil factors such as infiltration, roughness, and erodibility. In order to improve design, knowledge of the effects of these factors on irrigation flows are needed. Furthermore, these factors must be described in such a way that they can be evaluated for a given field before the irrigation system is installed. Methods are already available for evaluating retardance, infiltration and erosion for existing systems.

The study reported in this paper deals with only one of the factors listed above, namely, the resistance to flow in small irrigation channels. Many efforts have been made in the past to predict flow resistance in conduits by measurement of the roughness. However, the only successful method of determining resistance to turbulent flow in channels has been to make trials in a given channel, after which the results can be

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extrapolated to a range of slopes and discharges. Predicting flow resistance in small irrigation channels is further complicated by the possibility that completely turbulent flow may not exist in the channels and that channel shape effects may be present.

This study was conducted cooperatively by the Agricultural Research Service and the Agricultural Engineering Section, Colorado State University. It is part of the regional cooperative study of irrigation hydraulics being conducted by Universitites and Federal Agencies in the Western United States.

The study was designed with three objectives in mind: (1) to determine whether laminar as well as turbulent flows will occur under conditions similar to those in surface irrigation systems, and to seek criteria for defining the point at which flow changes from laminar to turbulent; (2) to determine which physical measurements of a random natural boundary roughness can be related to resistance to flow in a channel having this roughness; and (3) to determine the effect of cross-sectional shape of a small open channel on resistance to flow in the channel.

Review and Analysis

The efficiency of surface irrigation is dependent on the rate of advance of the irrigation water in the furrows or borders and the relation of the rate of advance to soil intake rate and length of run. Therefore, if rate of advance of water can be predicted before the design of an irrigation system, a better design will result.

Hall (3) has developed a method for predicting rate of advance of water in an irrigation border. Shull (10) and Davis (2) have developed similar methods for predicting rate of advance in irrigation furrows. All of these methods are based on a type of continuity relation where the total volume of water entering the channel is equated to the infiltrated volume plus the volume in surface storage at any given time. To use these methods, therefore, it is necessary to have information available on the water surface profile of the advancing stream so that the volume of water in surface storage can be computed.

Tinney and Bassett (11) have developed an equation for the surface profile of an advancing two-dimensional stream over impermeable boundaries. To use the equation, values of Manning's "n" for the flow boundary must be known. This paper presents values of resistance coefficients such as Manning's "n" for irrigation channels on the basis of physical measurements of surface roughness and channel shape.

The formulas that define resistance to flow in conduits have different forms, depending on whether flow is laminar or turbulent. Classical experiments on circular pipes have shown that flow changes from laminar to turbulent at a value of Reynolds number equal to 2,000. Expressed in terms of hydraulic radius this is equal to the following criteria:

$$\text{Critical Reynolds number, } \frac{VR}{\nu} = 500$$

where:

V = mean flow velocity

R = hydraulic radius

ν = kinematic viscosity of fluid.

By analogy it would be expected that the critical Reynolds number for open channels would be near this value.

For turbulent flows the resistance equations with the most rational basis are developed from the Prandtl - von Karman universal velocity distribution law. An example of this type of equation is that of Keulegan (5) who integrated the Prandtl- von Karman law over the cross-sections of trapezoidal channels. The constant of integration for this equation depended on whether the boundary was smooth, wavy, or rough. In his original integration, Keulegan considered the effects of free surface resistance and of non-uniform boundary shear due to variations in channel shape. For hydraulically rough boundaries, the equation has the form:

$$\frac{V}{V_*}(1+\bar{\epsilon}) = A_r + 5.75 \log \frac{R}{k} \tag{1}$$

where:

$V_* = \sqrt{gRS}$ = shear velocity

S = slope of energy line

k = height of roughness elements

$\bar{\epsilon}$ and A_r express channel shape and other effects.

However, comparing the derived equation to the experimental data of Bazin (1), Keulegan decided that free-surface effects and shear variations caused relatively little resistance and could be safely ignored. He also found that the effect of channel shape contained in A_r could be given an average constant value for a variety of channel shapes and sizes. Keulegan's rough boundary equation is then given as:

$$\frac{V}{V_*} = a_r - 2.25 + 5.75 \log \frac{R}{k} \tag{2}$$

where:

a_r is constant for each boundary roughness.

For smooth and wavy boundaries, the theoretical equation of Keulegan is:

$$\frac{V}{V_*} = A + 5.75 \log \frac{RV_*}{\nu} \tag{3}$$

where:

ν is the kinematic viscosity.

A has the value 2.6 for smooth boundaries and decreases with increasing boundary waviness.

Sayre and Albertson (9) presented a resistance parameter for open channels similar to the equivalent sand grain roughness for pipes. The parameter is defined by an equation very similar in form to the Keulegan equation for rough boundaries:

$$\frac{C}{\sqrt{g}} = 6.06 \log \frac{y_n}{\chi} \qquad (4)$$

where:

C is the resistance coefficient defined by the Chezy equation, $V = C\sqrt{RS}$ and $C/\sqrt{g} = V/V_*$.

y_n is the normal depth of flow.

χ is a resistance parameter.

Sayre and Albertson's studies were made in a very wide channel so that normal depth was essentially equal to hydraulic radius. When equation 1 is compared with equation 4 it is apparent that χ includes effects of both roughness dimensions and channel shape. Attempts have been made to relate χ to the height and density of spacing of artificial roughness elements but the relationship is not well defined.

In order to know whether equation 1 or equation 3 will be applicable to a given flow the condition of the boundary must be known. The boundary condition is determined by the relationship of the laminar sub-layer thickness to the height of the roughness elements. When the roughness height is much greater than the sub-layer thickness the boundary is termed hydraulically rough. When the sub-layer thickness is greater than the roughness height the boundary is hydraulically smooth. The effects of viscosity are pronounced for hydraulically smooth boundaries, whereas resistance is independent of viscosity for rough boundaries.

Nikuradse (7) presented criteria for determining the characteristics of the sand grain covered boundary. The following relationships between thickness of the laminar sub-layer and height of the roughness elements specify what the boundary effects will be:

$$\frac{11.6\nu}{V_*} > 4k \qquad \text{smooth boundary}$$

$$4k > \frac{11.6\nu}{V_*} > \frac{k}{6} \qquad \text{transition boundary} \qquad (5)$$

$$\frac{11.6\nu}{V_*} < \frac{k}{6} \qquad \text{rough boundary}$$

where:

k is the sand grain diameter.

After examining Bazin's open channel data, Keulegan specified that the transition between wavy and rough boundaries occurred when $11.6\,\nu/V_* = k_s/3.64$, where k_s is the equivalent sand grain diameter of the boundary roughness.

The work of Nikuradse and Keulegan has suggested that the height of the roughness elements is the most important factor determining resistance to flow over rough boundaries. In a natural soil channel each roughness element has a different height. It is then necessary to use some statistical measurement of roughness height. The standard deviation of the bed elevation measurements, σ, about the mean bed elevation would be expected to describe the roughness height.

Morris (6) concluded from his analysis of channel flow resistance that the longitudinal spacing of the roughness elements would be a more important factor in determining flow resistance than the height of the roughness elements. For natural soil channels roughness spacing is even more difficult to measure than roughness height. Spacing can be measured in two ways: (1) by determining the average spacing between the elements that project more than one standard deviation above the mean elevation of the boundary, and (2) by determining the mean longitudinal spacing of all roughness crests regardless of their height.

Keulegan, analyzing Bazin's data, concluded that the effect of the usual range of open channel shapes on flow resistance could be neglected in comparison to the effect of channel roughness. Powell (8), also found that the shape of smooth channels was not important in determining flow resistance. However, shape was investigated in these studies because of the possibility that it would have considerable effect for small flows.

The method that appears most likely to succeed in the analysis of the laboratory data is then as follows. First, from dye stream observations, separate the laminar flows from the turbulent flows, noting the Reynolds number at which the transition occurs. Then plot the data for turbulent flows in the form $V/V_* - 6.06 \log R$ vs $\log V_*/\nu$. For those channels having a rough boundary for all runs, such a plot will yield horizontal lines from which the value of χ, the resistance parameter, can be determined by comparison with equation 4 if R is substituted for y_n in that equation. Then χ can be related empirically to such measures of the soil roughness and channel dimensions as can be practically obtained. For those channels having a boundary in the transition range, the intercept in equation 3 should be determined as a function of the height and spacing of the roughness elements.

Procedure and Equipment

The study was conducted in the hydraulics laboratory of the Civil Engineering Department, Colorado State University. The soil channels in which flow resistance was studied were constructed in a 60-foot tilting flume. The slope of the flume could be varied from 0.01 to 0.5 percent. Discharges were measured by a 90^0 V-notch weir.

Channel and water surface elevations were measured with a point gage mounted upon a carriage which travelled along the flume on rails parallel to the bed. The mean bed elevation was determined by equally spaced measurements taken across the flume at each 5-ft station.

A very fine sand was used to form the channel boundaries. When the desired boundary roughness had been formed, the surface was stabilized by a spray application of chemicals similar to the method described by Vanoni and Brooks (12), so that no erosion of the surface roughness would occur during a series of tests. Two types of channels were studied. The first was rectangular in cross section with a flat soil bottom and smooth side walls. Depths were never greater than 1/5 the width. The results obtained should therefore be essentially that for two-dimensional flow and would apply to irrigation borders. The second type of channel was parabolic in cross section. The parabolic cross sections were found to conform closely to the most common shape of natural furrows. The average cross section of the parabolic channels can be expressed by an equation of the form:

$$y = ax^2$$

where y is the elevation of the channel boundary at a distance x from the center line. In these studies "a" was given values of 0.40, 0.65, and 0.90.

After the soil channels had been prepared, experimental runs were started by introducing the smallest discharge through the channel that would just cover the roughness elements. The tailgate was adjusted so that flow was uniform through the channel. Measurements of discharge, water surface elevations and temperature were recorded. Discharge was then increased in increments until the channel capacity was reached or until erosion of the boundary seemed imminent. Slope of the channel was then varied and the sequence repeated.

After completing a series of runs, plaster casts were made of the channel boundary to provide a permanent record of the roughness. Values of the roughness height and spacing were determined from measurements of these casts as shown in Figure 1.

Figure 1. Sketch of plaster cast of boundary roughness. σ and λ are measured roughness dimensions.

Results

Many of the laboratory runs at low discharges were in the laminar flow range. These were detected by observations of dye streams in the flow and by comparing plottings of resistance coefficient versus Reynolds number with the theoretical relations for laminar flow. Dye was injected both near the water surface and near the flow boundary. At Reynolds numbers, VR/ν[1/], less than 400, all runs were laminar, although the dye streams near the boundary indicated some eddies forming at separation points on the crests of the roughness elements. This turbulence did not extend to the surface of the flow. For the smoothest boundaries, flow remained laminar for Reynolds numbers up to 700. At higher Reynolds numbers, of course, the turbulence extended throughout the flow.

[1/] In all calculations of data for the rectangular channels the normal depth was assumed equal to the hydraulic radius and used in the calculations.

To illustrate flow conditions at the transition from laminar to turbulent flows, the following data from representative rectangular and parabolic channels are given. Each example represents flow near the critical Reynolds number. For one rectangular channel at a Reynolds number of 533 the depth of flow was 0.066 ft and the discharge was 3.25 gallons per minute per foot of width. For one of the parabolic channels at a Reynolds number of 466, the depth of flow at the center of the channel was 0.071 ft and the discharge was 2.0 gallons per minute. Since flows this small will not occur extensively in surface irrigation systems, the remainder of this discussion will be concerned only with flows with Reynolds numbers greater than 500. The resistance to flow over a rough boundary at low Reynolds numbers has been considered by Huntley (4).

The flows with Reynolds numbers greater than 500 were assumed to be completely turbulent. These Reynolds numbers ranged from 500 to 45,000. At Reynolds numbers of 500 to 1,000, the development of turbulence probably was not always complete. This could not be determined from observation of the dye streams because some eddying could be observed, but the intensity could only be judged qualitatively. Therefore, all runs having a Reynolds number greater than 500 were analyzed by assuming that completely turbulent flow existed.

When considering the data for turbulent flows, it is necessary to determine whether the boundary is rough, smooth, or in the transition region in order that the proper type of resistance formula may be applied. The function $V/V_* - 6.06 \log R$ plotted against $\log V_*/\nu$ should indicate the boundaries wherein viscosity affected resistance and the boundaries for which it had no effect. Such a plot then provides a method of determinating the boundary effects. Figure 2, 2a and 2b, indicate that

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Figure 2a. Data for turbulent flows in rectangular channels. Horizontal lines indicate hydraulically rough boundaries.

Figure 2b. Data for turbulent flows in rectangular channels. Horizontal lines indicate hydraulically rough boundaries.

the data for nine of the rectangular channels can be represented by a constant value of $V/V_{*} - 6.06 \log R$. This indicates that resistance was independent of viscosity and therefore the boundary was completely rough. The data for bed C are plotted as an inclined line and indicate that the boundary was in a transition between rough and smooth. The form of bed C could be considered wavy owing to the manner of its formation. It is to be expected that data for this channel would plot as an inclined line similar to the wavy channels analyzed by Keulegan.

Figure 3 shows a similar plot of the data for the parabolic channels

Figure 3. Data for turbulent flows in parabolic channels. Horizontal lines indicate hydraulically rough boundaries.

where all boundaries were rough.

A criteria for the transition from wavy to rough boundaries can be established by considering Figures 2 and 3 and the calculated values of the laminar sub-layer thickness for each run. The standard deviation of the bed elevation measurements, σ, was used to represent the effect of roughness height. For a normal distribution of bed elevation measurements 4σ should include about 95 percent of the values or all but the extremely high and low elevations. Therefore, 4σ should roughly approximate the diameter of uniform sand grain roughness. The roughness elevation measurements were approximately normally distributed. Using this approximation, the following criteria for determining the roughness conditions of the boundary were established:

$$\frac{4\sigma V_*}{\nu} < 25 \quad \text{wavy boundary}$$

$$\frac{4\sigma V_*}{\nu} > 25 \quad \text{rough boundary} \qquad (6)$$

These criteria are similar to those presented by Keulegan (5).

The resistance to flow in rough pipes is sometimes represented by a graph where the resistance coefficient is plotted against the relative roughness. Such a diagram is shown for data from one of the rectangular channels in Figure 4. Values of σ/R are used to represent the

Figure 4. Relationship of resistance coefficient to relative roughness for one rectangular channel.

relative roughness. The equation for the straight line defined by the data in Figure 4 is:

$$\frac{C}{\sqrt{g}} = 0.53 - 6.68 \log \frac{\sigma}{R} \qquad (7)$$

The coefficient of the logarithmic term, 6.68, is considerably larger than the value 5.75 which is commonly accepted for turbulent flow and which corresponds to a value of the Karman turbulent constant of 0.40.

A similar analysis of data for other channels indicated that the slope of $C/\sqrt{g}$ versus σ/R plots were different for each one. Since most of the data for these plots were taken at high Reynolds numbers where flow should be turbulent, the reason for the variable coefficient was investigated. It was found that if the value 5.75 was used as the coefficient of the logarithmic term, a depth correction was necessary for each of the channels. The amount of the correction varied for each channel. Furthermore, the correction could not be correlated with the height of roughness as characterized by σ or any other measured roughness dimensions. Therefore, it does not seem possible to predict the depth correction for a given channel until some flow resistance data have been taken.

The fact that the experimentally determined value of the Karman constant is different for each bed when the mean bed elevation is used to calculate flow depth explains the scatter in Figures 2 and 3. This scatter can be reduced considerably by using as the coefficient in the resistance function the slope of the $C/\sqrt{g}$ versus σ/R line that best fits the data for the channel being considered.

The values of the resistance parameter, χ, can now be calculated for each of the channels for which the boundary is rough. Reference to equation 4 indicates that the mean value of the resistance function $V/V_* - 6.06 \log R$ for each bed is equal to $- 6.06 \log \chi$.

To predict flow resistance from measurements of surface roughness these measurements must be related to χ. Since height of roughness is conceded by most investigators to be the most important characteristic in causing flow resistance, a correlation of χ and σ was the first attempted. A plot of corresponding χ and σ values is shown in Figure 5.

Figure 5. Relationship of the resistance parameter, χ, to σ, a measure of roughness height.

This plot contains data for both rectangular and parabolic channels. The best fit line for the points for all channels has the equation:

$$\chi = 12.9\,\sigma^{1.66} \qquad (8)$$

It should be possible to improve the estimate of χ by the use of other roughness dimensions. The longitudinal roughness spacing was used in an attempt to reduce the scatter in Figure 5. No correlation was evident between spacing and the divergence of the points from the best fit line.

The shapes of the individual roughness elements could not be measured or expressed numerically. However, there does seem to be some correlation between the shape or rugosity of the individual roughness elements and the resistance to flow. Those points lying below the line in Figure 5 represent channels where the roughness elements had been smoothed off by flooding or running low flows of water over them before they were stabilized. The data above the line represent the channels that were stabilized after the soil was tilled and the surface was therefore very irregular and cloddy.

For wavy boundaries Keulegan concluded from dimensional considerations that the coefficient A of equation 3 will be a function of the ratio of roughness height to spacing (relative waviness). Not enough data were obtained for channels with wavy boundaries to determine the relationship between A and relative waviness. However, the channel with the wavy boundary was smoother than any channel likely to be encountered in the field; therefore, this boundary condition does not have much practical significance in irrigation flows.

The value of χ, as indicated in the previous section, theoretically expresses the effects of both roughness dimensions and channel shape. Values of χ for parabolic channels with three different shapes are plotted in Figure 5. The divergence in the data from the least squares χ vs σ line for these channels is less than that for the rectangular channels. It must therefore be concluded that the effect of the shape of these channels on flow resistance was small compared to the effect of surface roughness and was given adequate consideration by use of the hydraulic radius.

Figure 5 indicates that there may be considerable error in the estimated value of χ using equation 4 and a value of σ measured from the bed roughness. For instance, the greatest error in estimating χ for the channels shown is about 210 percent. However, if this estimated χ value is used to predict depth of flow for given values of slope and discharge, much smaller errors in depth result. For an error in χ of 210 percent, the corresponding error in R (from equation 4) is 27 percent at the lowest turbulent discharge. For higher discharges the error is further reduced. For most channels, of course, the error is much smaller.

The error involved in the calculation of the desired flow variables by the foregoing method may seem large. However, the method is much more accurate than the common practice of assuming a single value of Manning's "n" for a given channel and using this "n" value for resistance determinations at all discharges. One rectangular channel produced values of Manning's "n" ranging from 0.016 to 0.040 for turbulent flows. Similarly the "n" values for one of the parabolic channels ranged from 0.020 to 0.052. For smoother channels, the variation in "n" was not as extreme. It is apparent, then, that if the mean "n" values is selected to represent channels such as these, the error involved in its use will be large at high or low discharges.

Practical Application

The first step in applying the resistance equations of this paper to a given open channel is to determine σ. The bed elevation should be measured at 0.025-foot increments over sections 1 foot long, parallel to the direction of flow. Measurement should be made over five to ten randomly selected sections to assure obtaining a representative average value of σ. The value of σ should then be calculated for each set of measurements using the following formulas for standard deviation of individual measurements about a least squares regression line.

$$\sigma = \left[\frac{\Sigma(Y-a-bX)^2}{n-2}\right]^{1/2}$$

where:

$$b = \frac{\Sigma XY - \frac{\Sigma X \Sigma Y}{n}}{\Sigma X^2 - (\Sigma X)^2/n}, \quad a = \overline{Y} - b\overline{X}$$

Y = measured elevation at distance X from some arbitrary datum.

n = number of elevation measurements.

The estimated value of χ can then be computed using equation 8.

The value of $V/V_* - 6.06 \log R$ is obtained using equation 4. This can be written $q/\sqrt{gR^3S} - 6.06 \log R$ for very wide channels and solved for R by trial and error if q and S are known. The trial and error solution for parabolic channels is slightly more involved, requiring knowledge of the hydraulic radius -- area relationship for the channel.

Summary and Conclusions

Laboratory studies of flow resistance in small, rough channels similar to irrigation furrows and borders have shown:

1. That the transition from laminar to turbulent flows occurs at Reynolds numbers, RV/ν, between 400 and 700 depending on channel roughness. Laminar flows will be of limited importance in surface irrigation systems.
2. The standard deviation of the boundary elevation measurements used as an expression of the height of the roughness

elements is sufficient to predict the flow resistance coefficient. However, if roughness shape and spacing could be measured adequately the resistance estimate could be improved.

3. The effect of channel shape, within the range of shapes of natural irrigation furrows, exerts a negligible influence on flow resistance as compared to the effect of the boundary roughness.

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