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ENGINEERING RESERVE

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AN OUTLINE OF
RECENT DEVELOPMENTS IN THE THEORIES OF SEDIMENT TRANSPORTATION

by

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A number of advancements have been made in the theoretical aspects of sediment transportation. These advancements have been stimulated primarily by a need for new theories and for greater refinement of already existing theories. The purpose of this brief paper is not to give a comprehensive coverage to all the developments in the theories of sediment transport, but rather to highlight certain recent developments with which the writer is acquainted. An excellent reference is the recent ASCE Separate No. 565 by Ning Chien entitled "The Present Status of Research on Sediment Transport".

The basic equation for transport of suspended sediment remains unchanged, but the understanding of coefficients, variables, and terms that were previously assumed to be constant in this equation has experienced considerable evolution.

Whereas most of the considerations of suspended load theory stem from a single basic equation, the developments in expressions for bed-load transport stem from different basic assumptions but have shown to be somewhat similar when analysed in detail.

The total load generally has been considered to be the sum of the suspended load and the bed load (the saltation load being relatively small). However, each is related inextricably to the same flow conditions, so it should be possible also to relate the total load to these flow conditions. Some work has been done along this line.

Types of Flow

In a discussion of sediment transportation in streams it is well to state clearly the meaning of the various types of flow involved.

Steady flow -- at a given point the velocity is constant with respect to time. Example: usual flow in a channel where there is no appreciable change of stage with time.

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Unsteady flow -- at a given point the velocity varies with time. Example: flow during the passing of a flood wave or when a control gate is being opened or closed.

Uniform flow -- at a given time the velocity does not vary (either in magnitude or direction) along the direction of flow. Example: flow in very long and straight reaches of channels having uniform cross-section, roughness, and slope throughout. (Perfectly uniform channels seldom occur, although some artificial channels and an occasional natural channel closely approach uniformity.)

Non-uniform flow -- at a given time the velocity varies (either in magnitude or direction or both) along the direction of flow. Examples: flow around a bend, through a contraction, through a change in channel slope, or backwater reach immediately upstream from a reservoir or immediately upstream from a dam.

Gradually varied flow -- unsteady and/or non-uniform flow in which the velocity changes so slowly with respect to time or distance that the dynamic effects (forces of acceleration) are negligible. Examples: the usual passing of floods (not flash floods on small drainage areas) and the usual backwater curves associated with natural channels.

Types of Sediment Load

Although the various terms describing the types of sediment load are well known, the sediment engineer tends to use them rather loosely, which sometimes results in confused thinking and lack of exact communication of ideas. They may be grouped according to type of movement, source of material, and method of measurement, as follows; and the total load is the sum of any one of the three groups:

Type of Movement

Suspended Load
Saltation Load
Bed Load

Source of Material

Wash Load
Bed-Material Load

Method of Measurement

Measured Load
Unmeasured Load

Saturation

Another concept, which is well known but sometimes neglected, is that of saturation. If the flow in an alluvial stream is carrying bed material to its full carrying capacity, then it is saturated with

bed material. For a given discharge this will vary with a number of factors, but the sediment transport and flow are not in equilibrium unless the flow is saturated.

In a canal where the flow is both steady and uniform, the concentration at saturation does not vary from section to section along the flow. However, if the flow is unsteady (at a point) or non-uniform (along the flow), such as in most natural streams, then the concentration will vary with time at a point or with distance along the channel.

The foregoing facts no doubt contribute materially to the difficulties that have been encountered in analysing field data where- in steady and uniform flow are only approximated.

Bed Load

In his excellent summary of the bed load theories, Chien¹ points out that the tractive force concept introduced by DuBoys and modified by O'Brien, Kalinske, and others is not greatly different from the Einstein bed-load function and the Meyer-Peter formula.

Perhaps the bed-load equation most frequently used in the United States is that of Einstein's. However, because of difficulty in evaluating the slope directly, the Geological Survey and the Bureau of Reclamation have developed a method of determining the slope directly from the mean bed shear and the logarithmic distribution of velocity.

For purposes of comparison, a tabulation is shown of the various bed load equations.

Suspended Load

Chien¹ has also given a summary of the developments in the theory of suspended load. He brings out the original development together with the various steps in refinement of the theory.

The suspended load theory is summarized herein by means of a table giving each of the basic equations involved in the developments. Listed together with the equations are the assumptions and limitations involved when they are applied to sediment transport. By this means one may see immediately the possible causes for discrepancies which have been encountered between theory and experimental

¹ ASCE Separate No. 565 Dec. 1954.



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FORMULAS FOR RATE OF BED-LOAD TRANSPORTATION

Investigator	Formula	Type of Sediment
DuBoys	$G = \psi \tau_0 (\tau_0 - \tau_c)$	
U. S. Waterways Experiment Station	$G = \frac{K}{n} (\tau_0 - \tau_c)^m$	Sand mixtures
Chang	$G = Kn \tau_0 (\tau_0 - \tau_c)$	Uniform sand
Kalinske	$G = f \left(\frac{\tau_c}{\tau_0} \right) D \gamma_s \sqrt{\tau_0 / \rho}$	Sand mixtures
O'Brien	$G = A \left(\frac{V}{R^{V_3}} \right)^m$	Sand mixtures
MacDougall	$G = AS^m (S q - S q_c)$	Sand mixtures
Schoklitsch	$G = \frac{A}{\sqrt{D}} S^{3/2} (q - q_c)$	Uniform sand
Meyer-Peter	$G = (B S q_1^{2/3} - B_1 D)^{3/2}$	Uniform sand large size
Einstein	$\phi = f(\psi)$	Sand and gravel

- G = Rate of sediment movement in lbs/sec/ft of width
 n = Mannings roughness coefficient
 τ_0 = Tractive force at bed in lbs/sq ft
 τ_c = Critical tractive force at bed in lbs/sq ft
 γ_s = Specific weight of sediment
 D = Diameter of sand in mm
 S = Slope of the energy gradient in feet per foot
 q = rate of flow per unit width in cu ft per ft of width
 q_c = Critical value of flow per unit width or flow when general movement of the bed material first starts - cu ft/ft of width
 q_1 = rate of flow per unit width in lb per sec per ft of width
 $K, A,$ and B = empirical constants
 m = empirical exponent

$$\phi = \frac{G}{\rho_f g} \left(\frac{\rho_f}{\rho_s - \rho_f} \frac{1}{g D^3} \right)^{1/2}$$

$$\psi = \frac{\rho_s - \rho_f}{\rho_f} \frac{D}{R_b S}$$

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SUSPENDED LOAD EQUATIONS

Equations

Assumptions
(including steady-uniform flow)

1. $\tau = \epsilon_m \frac{d(\rho v)}{dy} = \rho \epsilon_m \frac{dv}{dy}$

1. ρ independent of y ; v_m influence insignificant; τ uniform across the flow

2. $q_s = -\epsilon_s \frac{dc}{dy}$

3. $\tau = \gamma S(D-y)$

3. γ independent of y ; τ uniform across the flow.

4. $\frac{dv}{dy} = \frac{1}{ky} \sqrt{\tau_0/\rho}$ (Karman)

4. $v \sim \log y$;
 k is Karman's universal constant usually assumed to be 0.40;
 v uniform across the flow.

5. $wC + k \sqrt{\tau_0/\rho} \frac{(D-y)}{D} \gamma \frac{dc}{dy} = 0$

5. $\epsilon_m = \epsilon_s$ for all values of y .

6. $C/C_a = \left[\left(\frac{D-y}{y} \right) \left(\frac{a}{D-a} \right) \right]^Z$

6. All of above assumptions.
Net result:

where

$$Z = \frac{W}{k \sqrt{\tau_0/\rho}} = \frac{W}{k \sqrt{g D s}}$$

$$\beta = Z_1/Z$$

- a. k is not a constant
- b. computed and measured Z may differ by as much as 50%.
- c. β may be greater or less than unity depending on boundary roughness and concentration and distribution of sediment.

Lane and Kalinske

7. $c/c_a = e^{-\frac{15W}{D \sqrt{\tau_0/\rho}} (y-a)}$

7. Average ϵ_s in vertical section.

8. $G = c_a v_m D e^{\frac{15Wa}{D \sqrt{\tau_0/\rho}}} P \left(\frac{W}{\sqrt{\tau_0/\rho}}, \frac{R}{D^{1/4}} \right)$

8. Total suspended load.

$K = 2.3 \frac{U_*}{v_m}$

data. Unfortunately, it is easy to apply various equations for a homogeneous fluid to a non-homogeneous fluid (such as a suspension of sediment in water), make simplifying assumptions in the development, and then neglect to consider these assumptions when searching for an explanation of the discrepancies.

Eq 1, for example, is for the turbulent transfer of momentum as reflected by the shear in a homogeneous fluid wherein the density ρ is a constant with respect to y . However, when sediment is added to the flow the density may vary considerably with y . Likewise, in the derivation of Eq 3, the specific weight γ is assumed to be constant with respect to y .

In some flow in open channels there is considerable secondary circulation, which has a marked influence on the distribution of the bed shear τ_0 across the bottom of the channel. Therefore, it is possible that in some cases appreciable error may result from the assumption of no variation of shear transversely. This is illustrated in Figs. 1, 2, and 3 by Simons² where the bed shear has been computed by different methods. The secondary circulation is believed to be influenced materially by the width-depth ratio, the side or bank roughness relative to the bed roughness, and upstream flow conditions such as bends.

In Eq 4 the assumption is made that the Karman universal coefficient $k = 0.4$ is a constant for all flow conditions. However, several investigators have demonstrated that it varies throughout the flow. Hence, another possible source of error, which at times is appreciable.

Eq 5 is developed by assuming that the coefficient for turbulent diffusion of momentum ϵ_m is equal to the corresponding coefficient for diffusion of sediment ϵ_s . The error involved in this assumption probably is not important except for heavy sediment loads where the variation of sediment concentration with respect to y is large.

Both Z and Z_1 in Eq 6 involve all of the assumption and limitations of the previous equations. Hence, it is only to be expected that β should vary considerably from point to point in the flow -- approaching 1.0 only for small values of Z .

² Simons, Daryl, Data taken in stable irrigation canals by Simons and Bender for theses, which are in preparation. Cooperative field study with USGS, Corps of Engineers, USBR and Colorado A and M College.

The factors that influence k and β (and the quantitative extent of this influence) are the subject of considerable investigation at the present time. Figs. 4 and 5 by Bender³ show the marked difference between Z and Z_1 and their variation across the flow. Fig. 6 shows the variation Bender found for very small values of Z .

Total Load

In most cases, it is the total load that is desired for design purposes. Hence, information is needed regarding the relation between total load, suspended load, bed load, and measured load. At one extreme flow condition the suspended load represents a rather high percentage of the total load, while at another extreme the total load may be entirely bed load.

The equipment for measuring the sediment load in natural streams is limited almost completely to suspended sediment samplers, which admittedly sample only a portion of the flow since the nozzle does not reach the bed of the stream. The error involved in this procedure was investigated by Sadar, who sampled throughout the flow in a laboratory flume 70 ft long and 4 ft wide. Fig. 7 shows that under some conditions sampling 90% of the depth ($a/D = 0.1$) will obtain as much as 80% of the total load while under other conditions only 20% of the total load is sampled. Simons has made a plot, Fig. 8, of measured load versus total load in irrigation canals.

Obviously this situation is one of real concern, and total-load programs such as the ones on the Middle Loup, the Snake, and the Niobrara Rivers should be expanded to cover a still wider variety of conditions. Furthermore, laboratory investigations should be continued to extend and to refine the work of Sadar.

Unfortunately, one must say in summary that the present knowledge of total load is very meager.

Roughness in Alluvial Channels

In alluvial channels, the bed roughness is a complex problem, but it is nonetheless very closely related to sediment transport and a study of the latter will be simplified by first investigating the roughness problem.

³ Bender, Donald, Data taken in stable irrigation canals by Bender and Simons for theses, which are in preparation. Cooperative field study with USGS, Corps of Engineers, USBR and Colorado A and M College.

By use of a free body diagram, the shear at the bed may be shown to be

$$\tau_0 = R \gamma \sin \alpha \quad (1)$$

where R is the hydraulic radius, γ is the specific weight of water and α is the angle of slope of the stream. Furthermore, in alluvial channels the shear is logically a function of the following variables.

$$\tau_0 = \phi_1(sf, V, R, \rho, \rho_s, \mu, \gamma, D, \sigma) \quad (2)$$

where

- V = mean velocity of flow in the channel
- R = hydraulic radius
- sf = factor expressing the shape of the channel
- D = mean diameter of bed material
- ρ = density of water
- ρ_s = density of bed material
- μ = viscosity of the water
- γ = specific weight of water
- σ = standard deviation of diameter of bed material.

Choosing V , R , and ρ as repeating variables, dimensional analysis yields:

$$\tau_0 = \rho V^2 \phi_2(sf, R/D, D/\sigma, \rho_s/\rho, V/\sqrt{(\gamma/\rho)R}, VR/\nu) \quad (3)$$

in which $V/\sqrt{(\gamma/\rho)R}$ is a Froude number Fr , and VR/ν is a Reynolds number Re .

Equating Eq 3 to Eq 1 yields

$$\gamma RS = \rho V^2 \phi_2(sf, D/R, D/\sigma, \rho_s/\rho, Fr, Re) \quad (4)$$

or

$$V = \sqrt{g} \phi_3(sf, R/D, D/\sigma, \rho_s/\rho, Fr, Re) \sqrt{RS}. \quad (5)$$

Comparing this equation with that of Chezy's, which is

$$V = C \sqrt{RS} = \frac{1}{C'} \sqrt{gRS}$$

and rearranging,

$$C' = \sqrt{g}/C = \phi_3(sf, R/D, D/\sigma, \rho_s/\rho, Fr, Re). \quad (6)$$

For flow of water in wide alluvial channels, sf and ρ_s/ρ may be considered constant. Also, the variation of the relative standard deviation D/σ would be small. The Froude number will be considered unimportant because conditions of uniform flow and no surface waves will be assumed. Eq 6 therefore simplifies as a first approximation to:

$$\sqrt{g}/C = C' = \phi_4(R/D, Re). \quad (7)$$

Under the supervision of the writer, Ali⁴ prepared a plot of Eq 7 using all of the data available at the time from the laboratory and the field, see Fig. 9. This plot shows that for any given relative size of bed material the resistance coefficient varies appreciably with Reynolds number. Although the data from both the laboratory and field agree amazingly well, they are not sufficiently complete to define the variation for a given D/R at the lower Reynolds numbers.

As the Reynolds number increases, the configuration of the alluvial bed changes from dunes to bars to a plane bed, as shown by the photographs of Barton and Lin⁵, Figs. 10, 11, and 12. In these

⁴ Ali, S.M. Some aspects of roughness in alluvial channels. Colorado A and M College. Master's thesis. August 1953.

⁵ Barton, J.R. and Lin, P.N. A study of the sediment transport in alluvial channels. Colorado A and M College, Civil Engineering Dept. Report No. 55JRB2. March 1955.

photographs it is quite obvious that the bed is getting progressively less rough. Furthermore, the dunes are many times rougher than the plane bed, which corresponds rather well to the Nikuradse roughness.

The curves and data of Fig. 8 were also plotted in the transition function of Fig. 13. Here it is particularly significant that the curves for constant D/R in Fig. 9 plot on a single line in Fig. 13 -- thereby demonstrating that the transition function is sufficient to express the variation of the resistance coefficient with the higher values of Reynolds number for a given relative size of bed material.

Recent Investigations in Mechanics of Sediment Transport

Dimensional analysis may be used to obtain

$$\sqrt{g}/C = C^{\gamma} = \phi(Ri, w/U_{*}) \quad (8)$$

where:

$$Ri = \frac{wc}{SU_{*}}$$

$$U_{*} = \sqrt{\tau_{0}/\rho}$$

c = Concentration of sediment by dry weight in per cent of sample weight

S = Slope of the energy line

w = Fall velocity of median sediment size

A plot of this function, Fig. 14, has been prepared by Barton and Lin⁵ and extended by Simons², see Fig. 15. These plots show rather clearly that two distinct functions exist, depending not so much upon w/U_{*} as upon the nature of the bed -- that is, whether or not there are dunes. Evidently, the Richardson number Ri is important only in the region of about 0.5 to 5.0. Unfortunately, the water surface is so irregular with bars on the bed that accurate depth measurements could not be made.

The variation of C/\sqrt{g} with K/D is shown in Fig. 16 by Barton and Lin. Here the roughness K is determined as the average height of the dunes or bars.

In connection with formation of dunes and other bed configuration, it is well to study the following table by Barton and Lin⁵ which shows some values of the Manning coefficient obtained by keeping the discharge constant while varying the slope and depth. The table shows clearly that for a given discharge, the roughness of an alluvial channel decreases with a decrease in the mean depth or with an increase in the average velocity. It is significant that for a given discharge a bed with sandbars has smaller effective roughness than a bed with dunes. The plane bed would be expected to have the smallest effective roughness. Needless to say, the values of the different variables pertaining to the conditions of sandbars are only approximate, owing to the fact that uniform flow does not exist when sandbars occur. Nevertheless, the general order of magnitudes should be correct.

<u>Discharge</u> Q (cfs)	<u>Mean</u> <u>Depth</u> D (ft)	<u>Average</u> <u>Velocity</u> $U_m = Q/A$ (ft/sec)	<u>Manning</u> n	<u>Bed</u> <u>Condition</u>
5.8	0.94	1.54	0.0216	Dunes
5.8	0.85	1.70	0.0204	Dunes
5.8	0.79	1.84	0.0183	Sandbars
5.8	0.66	2.20	0.0154	Sandbars
5.8	0.61	2.38	0.0137	Plane

The variation of the Karman coefficient k with the Richardson number is shown in Fig. 17 by Barton and Lin⁵ for both the plane bed and the bed with dunes. Again Richardson number is seen to be of primary importance above 0.5. Variation of the resistance coefficient C/\sqrt{g} with the concentration of total sediment load is shown in Fig. 18. Evidently concentration has an important influence only for values greater than $q_t/q \gamma = 0.0002$.

As a part of the study of the mechanics of sediment transport, the influence of particle shape on fall velocity was studied by Schulz and Wilde⁶ and Corey⁷. A plot of the results, Fig. 19, shows that the Corey shape factor is very important in determining the fall velocity of sediment particles where the Reynolds number is greater than about 100. In fact the special drag coefficient may

⁶ Schulz, B.F., Wilde, R.H., and Albertson, M.L. Influence of shape on the fall velocity of sedimentary particles. U. S. Corps of Engineers, Missouri River Division. Sediment Series No. 5. July 1954.

⁷ Corey, A.T., Influence of shape on the fall velocity of sand grains. December 1949. Colorado A and M College, Civil Engineering Department, Master's thesis.

be increased over that for a sphere by as much as 500 per cent in the case of a rather flat particle. A useful plot in determining the ratio of the sieve diameter to the sedimentation diameter is Fig. 20, which again shows that Reynolds number is of importance primarily at values greater than 100.

In connection with the design of stable channels, Simons² has prepared a plot of the allowable shear for a given diameter of bed material, see Fig. 21, using data from the Bureau of Reclamation and data collected by himself and Bender as mentioned previously.

Suggestions for Future Research

Additional investigations in both laboratory and field should be conducted and correlated simultaneously. The laboratory investigations will be far less expensive, and a much greater number and wider range of data can be obtained, than for field investigations. But the laboratory data are idealized to such an extent that experimental field data are necessary in the final analysis. Therefore, laboratory investigations should be used to help in establishing the fundamental principles and in guiding the direction of field investigations.

Laboratory and field investigations of the USGS should be aimed at helping to evaluate past records and helping to improve the quality and quantity of present and future records as well as reducing the unit cost.

Specific ways of accomplishing these objectives are:

1. Develop methods of evaluating more accurately the roughness and resistance coefficients (such as Manning's n and Chezy's C) for alluvial channels. That is, improve and extend:
 - (a) the C/\sqrt{g} vs Re plot
 - (b) the C/\sqrt{g} vs Ri plot
 - (c) the C/\sqrt{g} vs K/D plot
 - (d) evaluate more accurately the sequence of dune formation

2. Develop methods of sampling, analysing and estimating more accurately the size and concentration of sediment. That is, improve and extend:
 - (a) the sampling time and location investigations
 - (b) the sampling efficiency relationships

- (c) the causes for variation of the Karman coefficient (effect of roughness and effect of sediment concentration)
- (d) the plot of C/\sqrt{g} vs $q_t/q \sqrt{\gamma}$,
- (e) the plot of Z vs Z_1 ,
- (f) the influence of wash load
- (g) the relationship between bed material size and the size (at various depths) of material transported
- (h) the chemical composition of the wash load

3. Miscellaneous items of study

- (a) unsteady flow superposed on a steady-state system in equilibrium -- flood hydrograph passed through laboratory flume to study changes in concentration, dune formation, depth of flow, nature of flow patterns, and changes in bed elevation.
- (b) expand studies on meanders
- (c) develop techniques for visually observing a stream to determine qualitative answers to many questions listed herein.
- (d) investigate possibility of using the velocity distribution equations of Reichardt and Sutton
- (e) establish routine (frequency, location, and method) for taking bed-material samples
- (f) continue to take temperature measurements (if possible record on same chart as stage)
- (g) study importance of secondary circulation in open channels
- (h) use sonic probe to measure dune formations.

$K = \frac{2.3 \times 2.7}{m}$

Farmer's Canal

$S = 0.000132$
 Small dunes (0.3 ft)
 $n = .0196$
 $Q = 1000$ cfs
 $V = 2.54$ fps

$\tau = \rho K (V_2 - V_1)^2 : \triangle \text{---} \triangle$
 $\tau = \gamma d S : \square \text{---} \square$
 Shear scale = 1 unit = .01 #/sq' measured normal to channel bottom

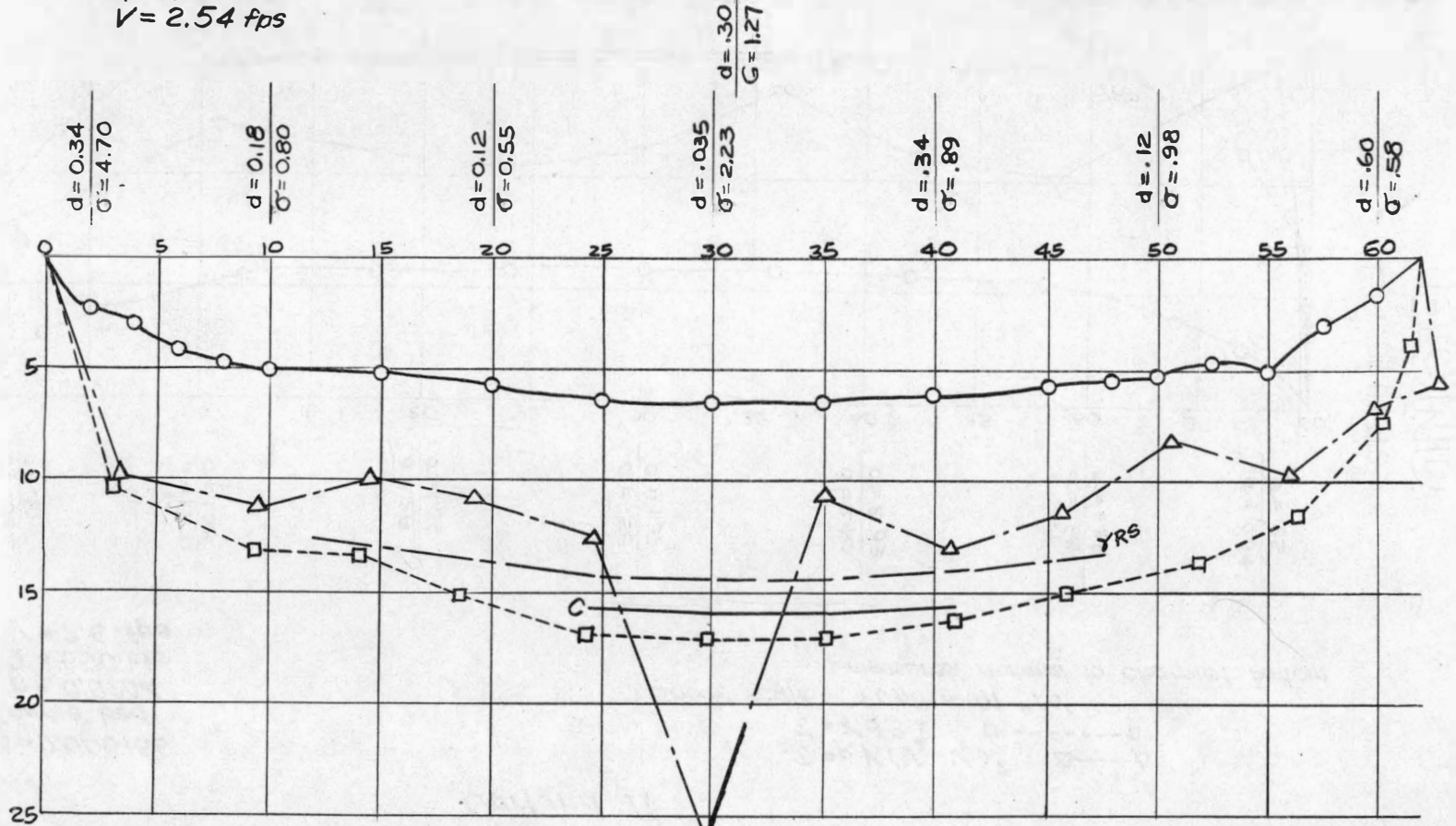


Fig. 1

Garland II

$S = 0.000166$
 Coarse bed
 $n = 0.0204$
 $Q = 550$ cfs
 $V = 2.6$ fps

$\tau = \rho K (V_2 - V_1)^2$: $\Delta \cdots \Delta$
 $\tau = \gamma d S$: $\square \cdots \square$
 Shear scale = 1 unit = .01 #/sq'

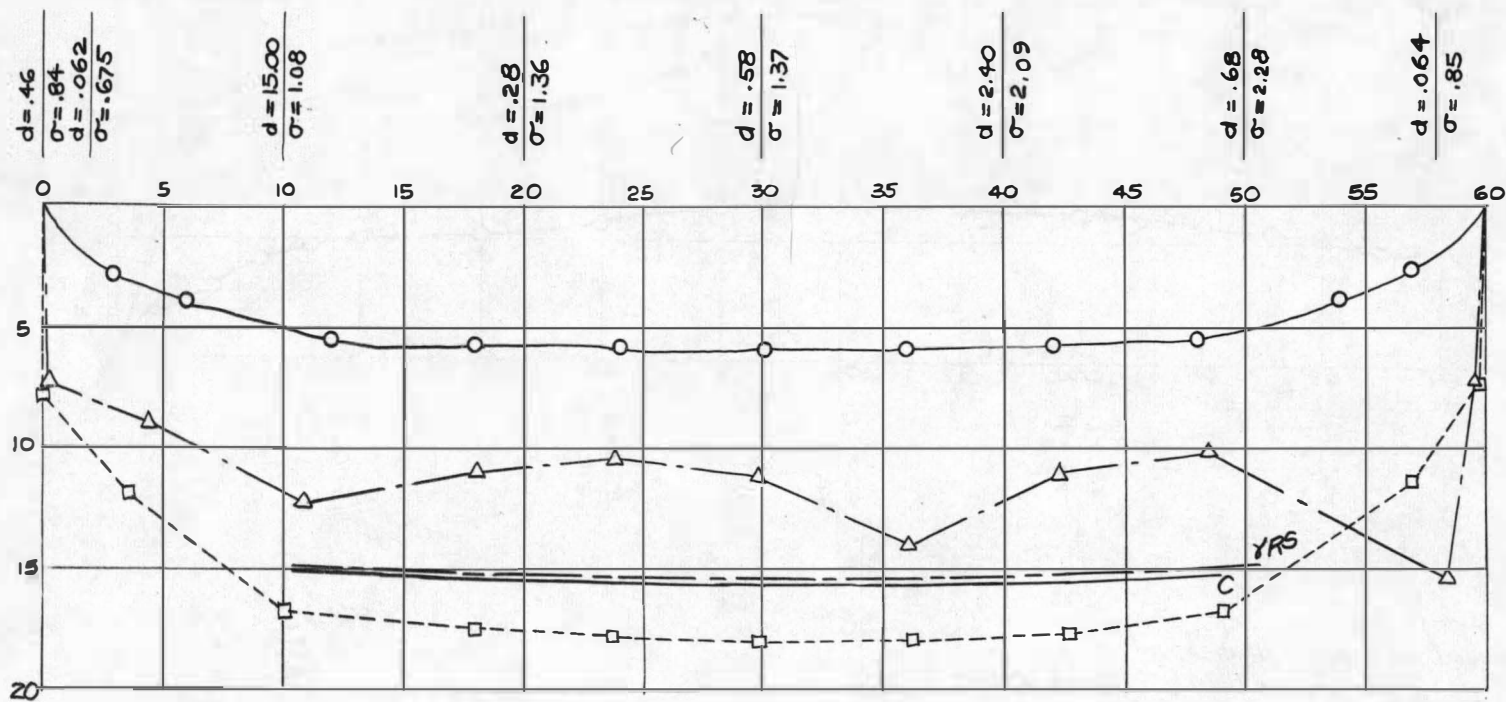


Fig. 2

Bijou Canal 1954

$S = 0.00030175$
 Plane bed - Sand
 $n = 0.018$
 $Q = 200$ cfs
 $V = 2.42$ fps

$\tau = \rho K (V_2 - V_1)^2 : \triangle \text{---} \triangle$
 $\tau = \gamma d S : \square \text{---} \square$
 Shear scale: 1 unit = .01 #/ft²
 measured normal to channel bottom

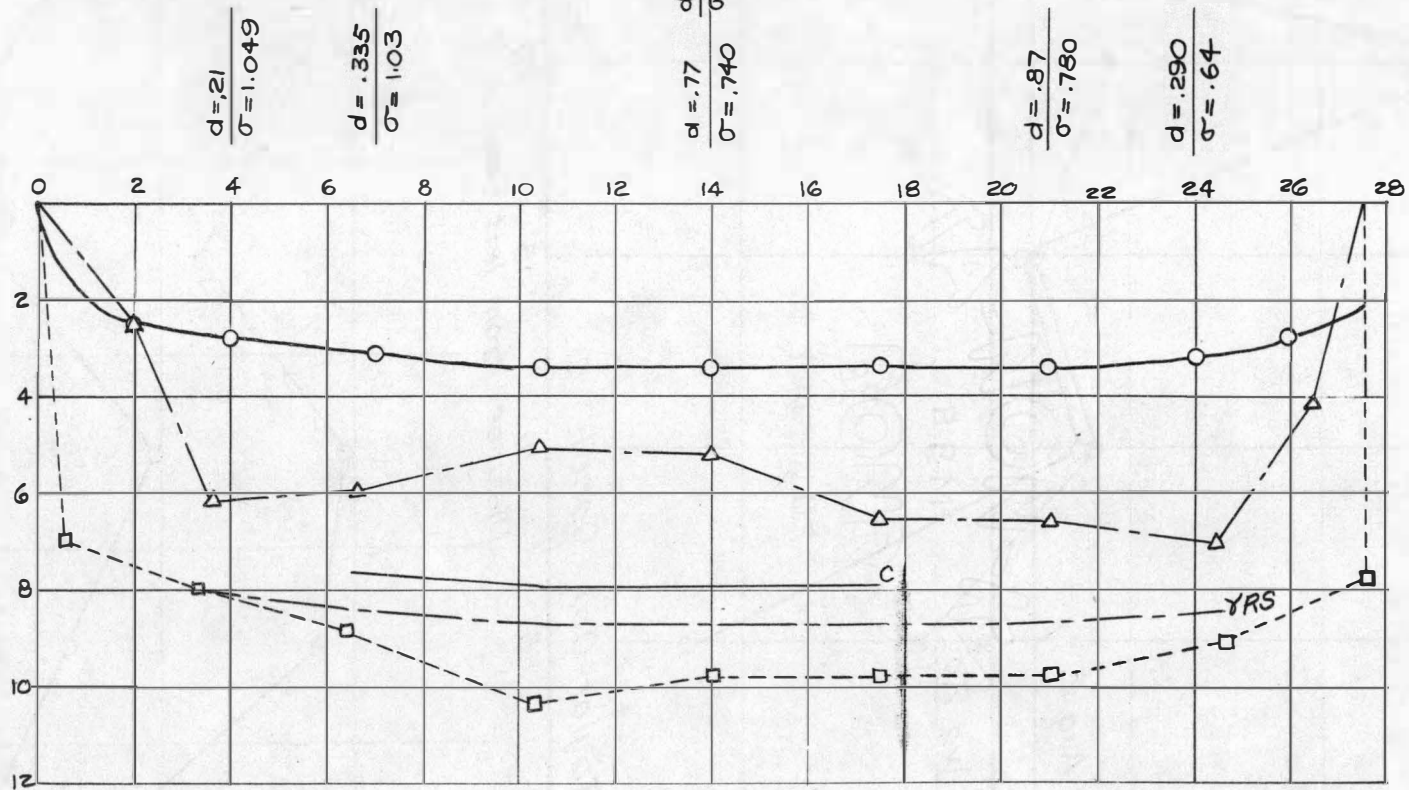


Fig. 3

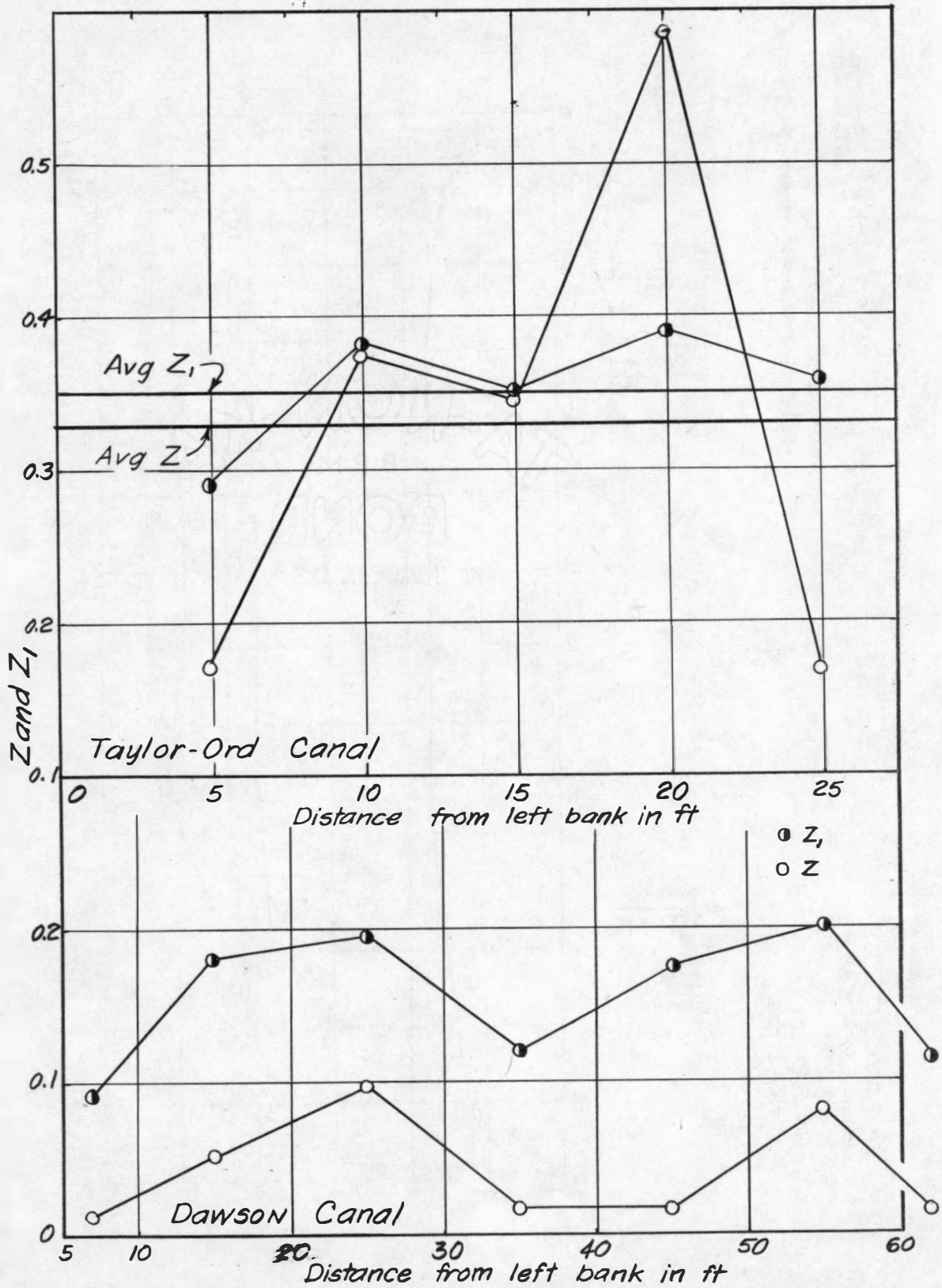


Fig. 4

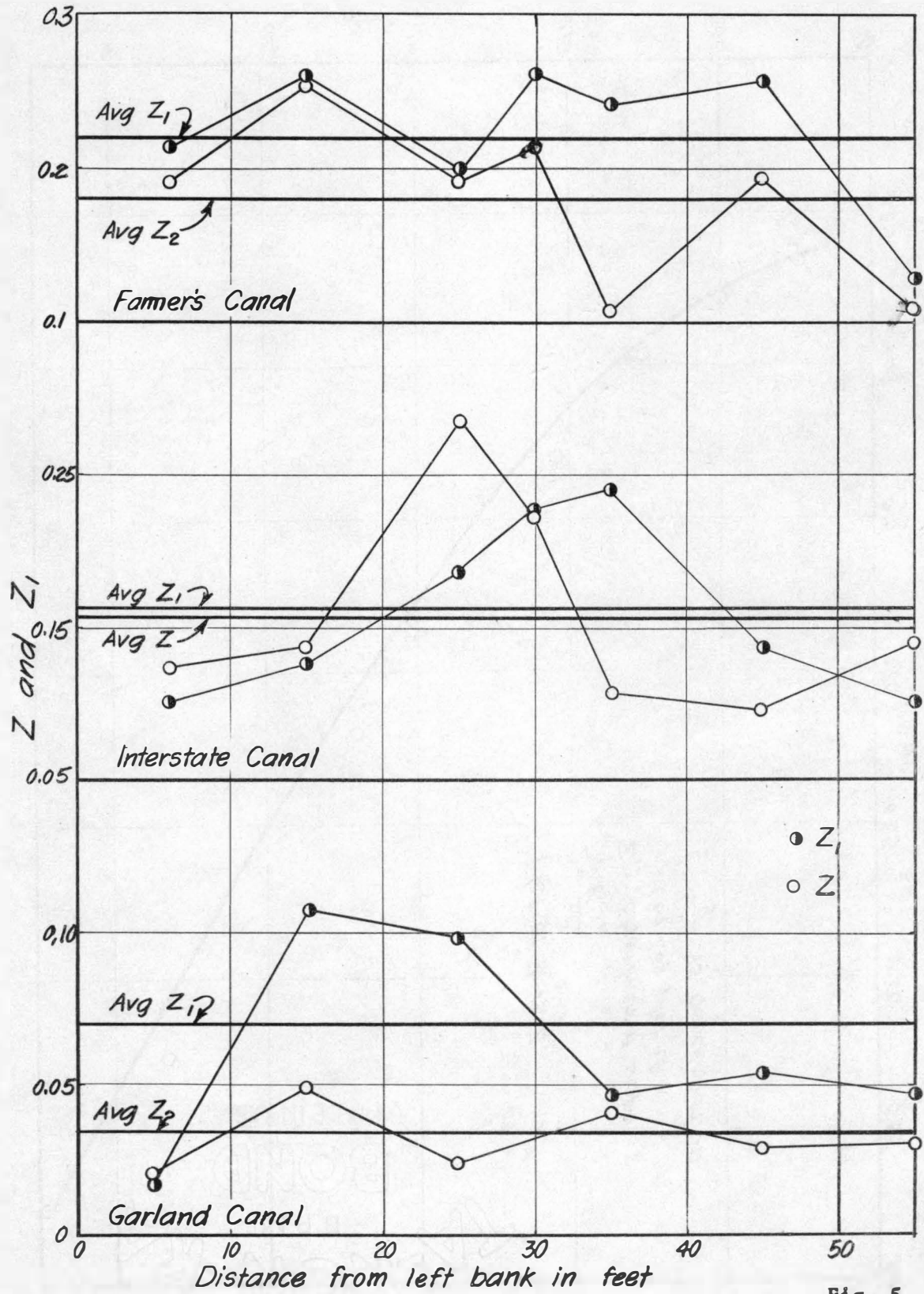


Fig. 5

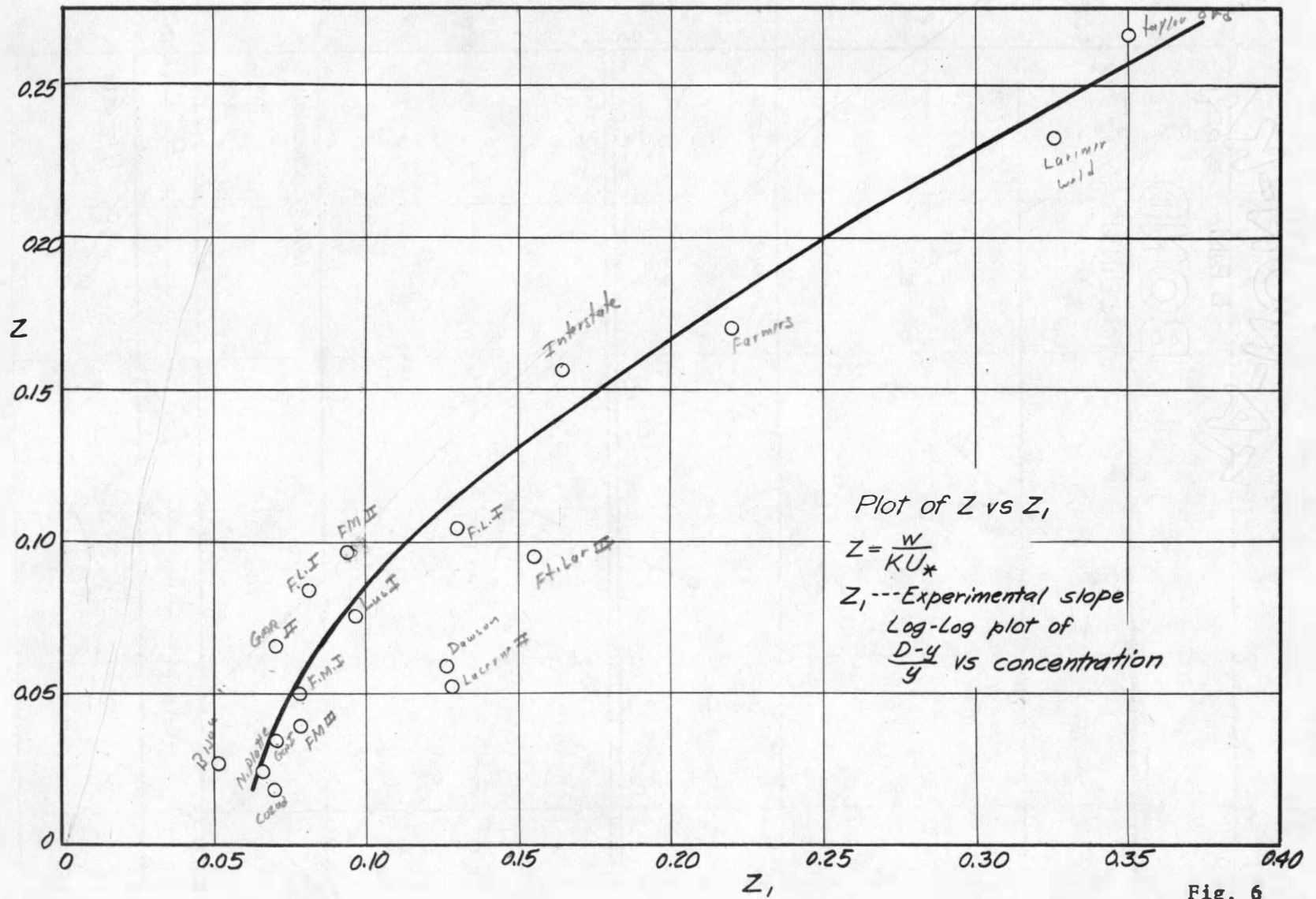


Fig. 6

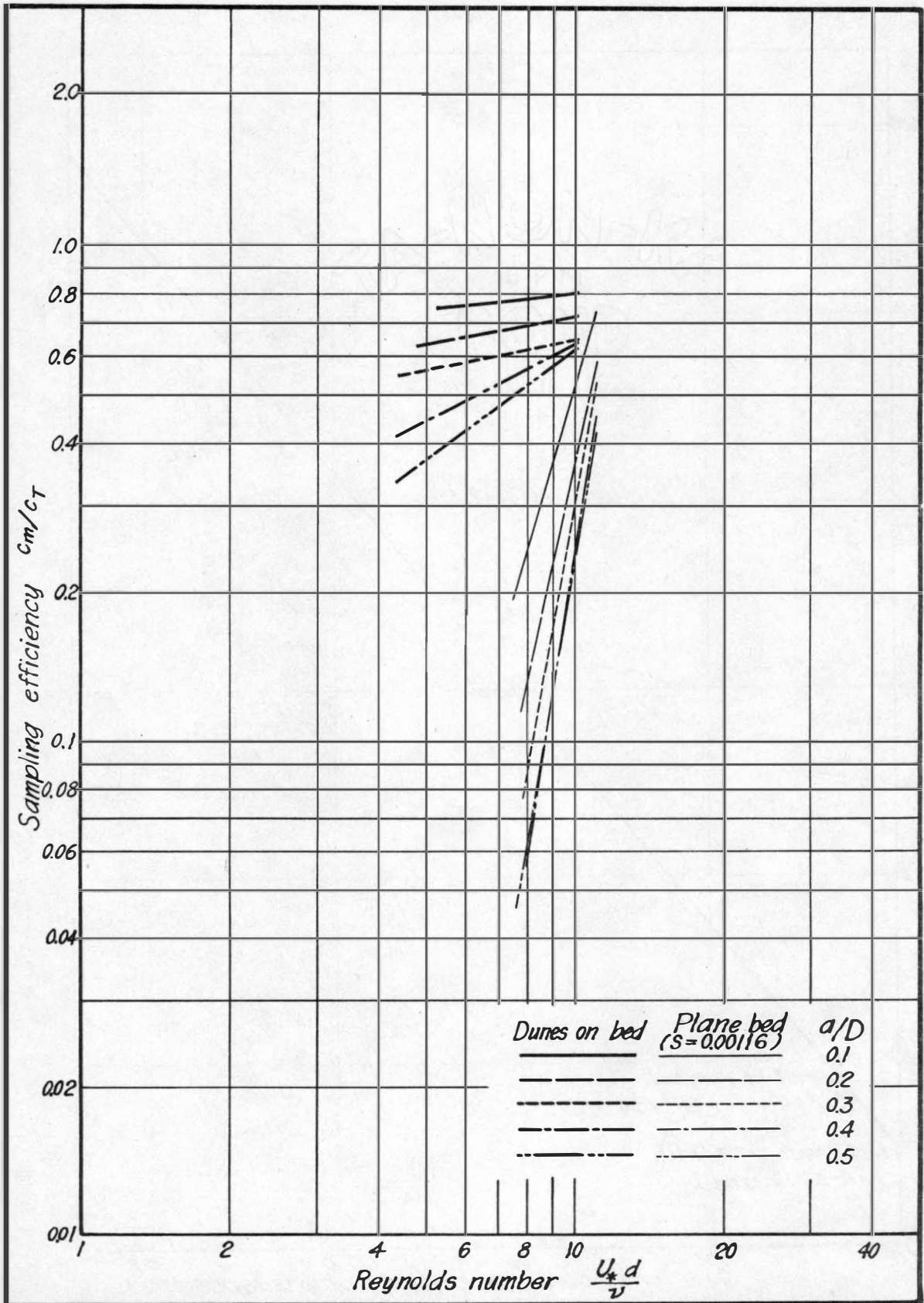


Fig. 7 Variation of sampling efficiency with Reynolds number

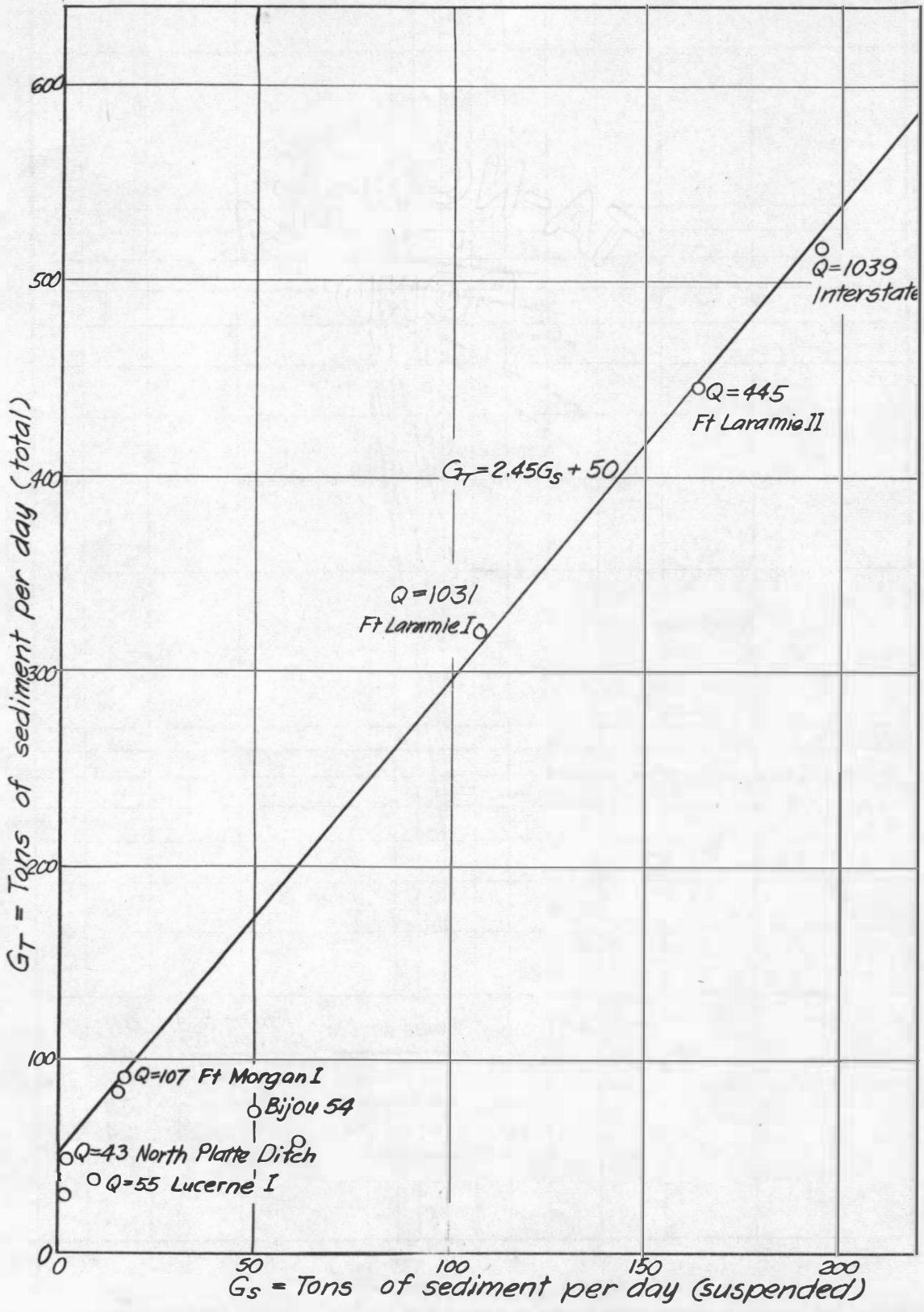
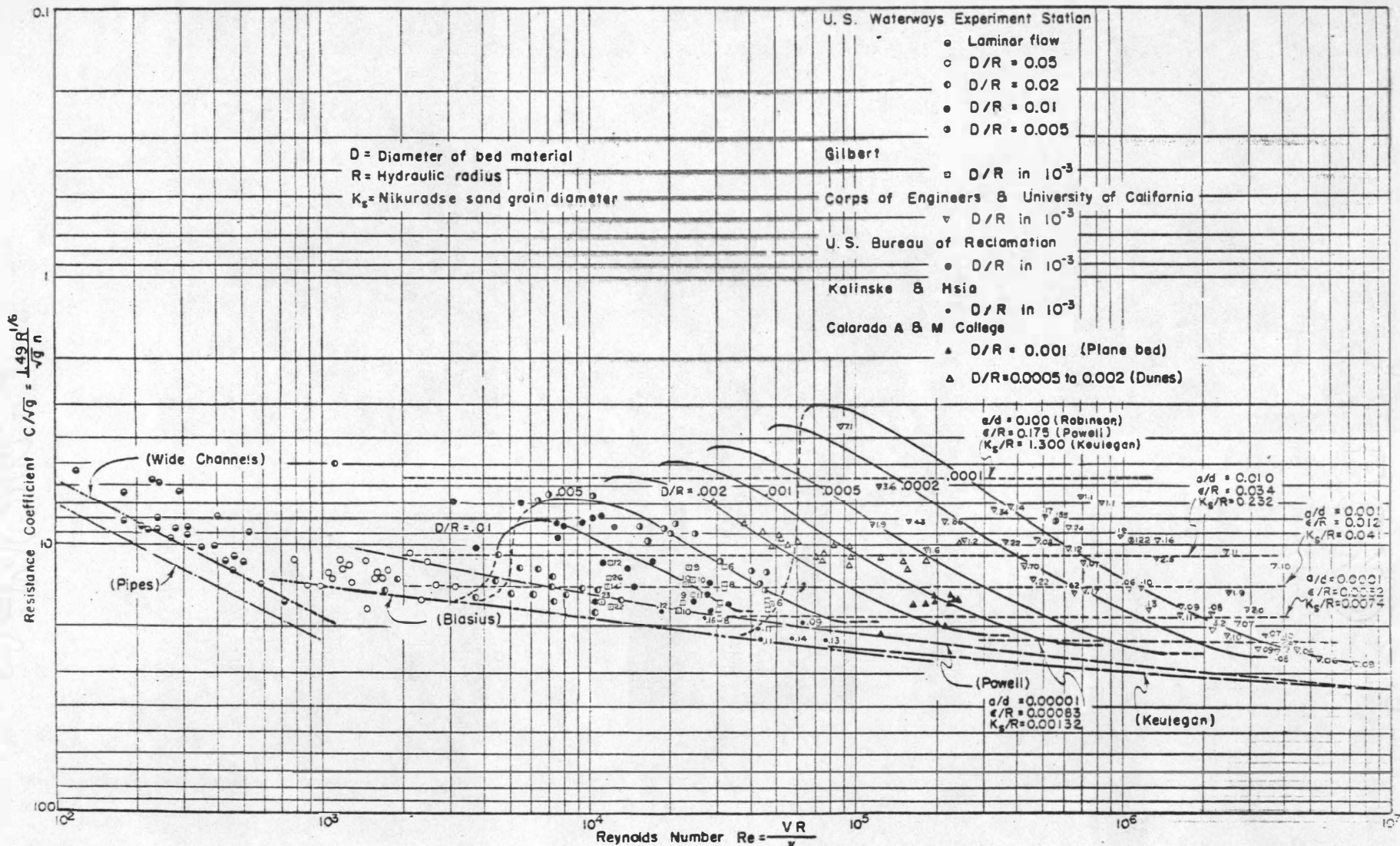
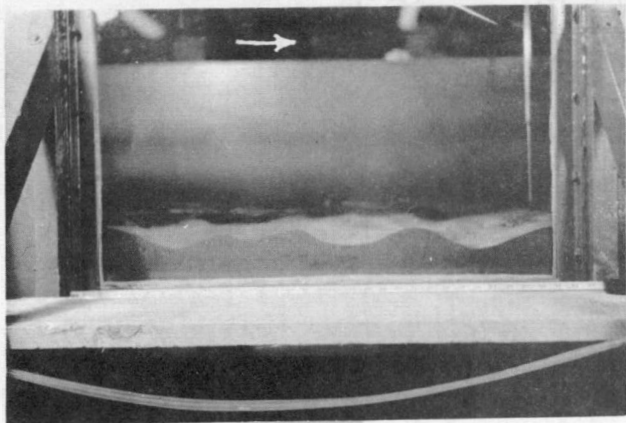


Fig. 8

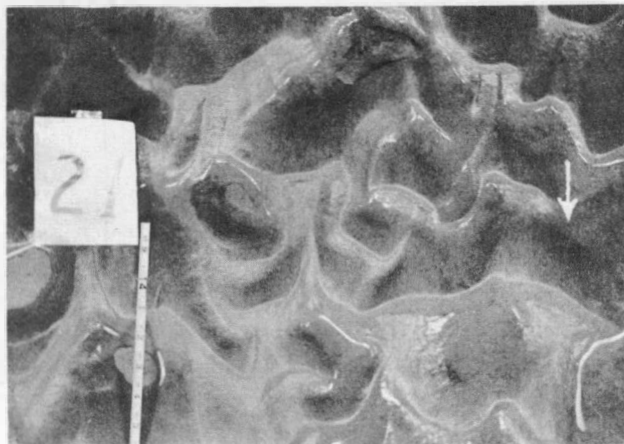


Variation of Resistance Coefficient with Reynolds Number —
 Relative Roughness as Third Variable

Fig. 9



Profile



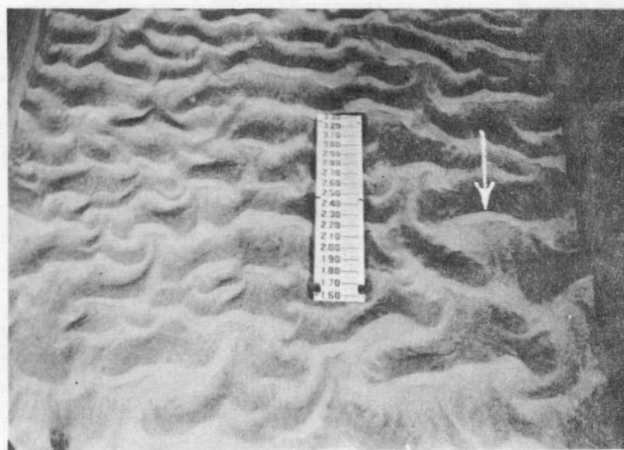
Run 21



Run 27

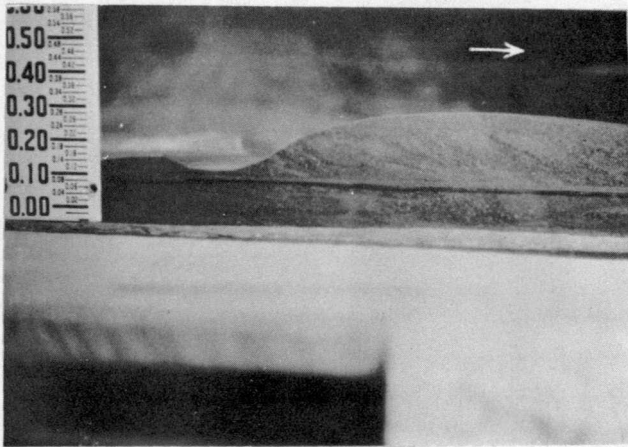


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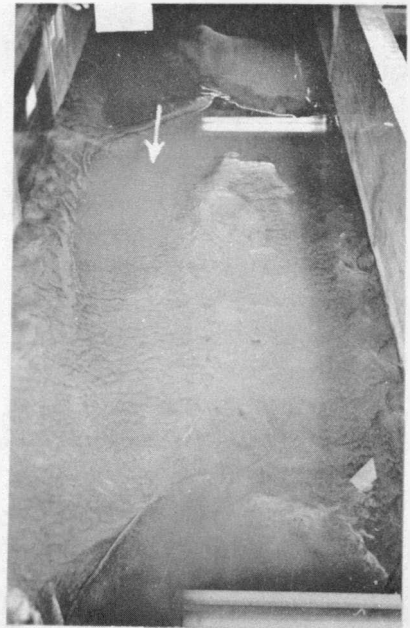


Run 37

Fig. 10 Views of typical dune patterns



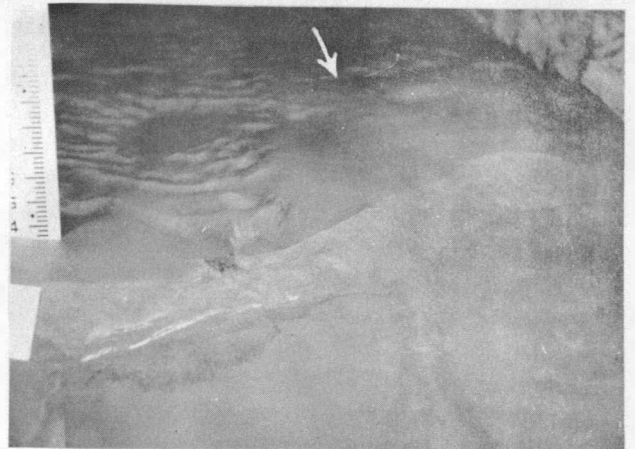
Profile



Run 15



Run 15

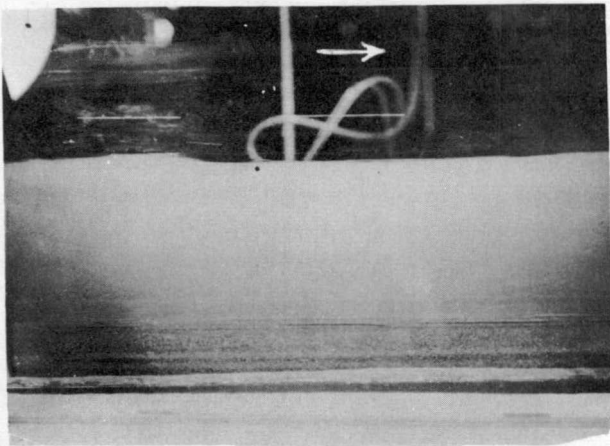


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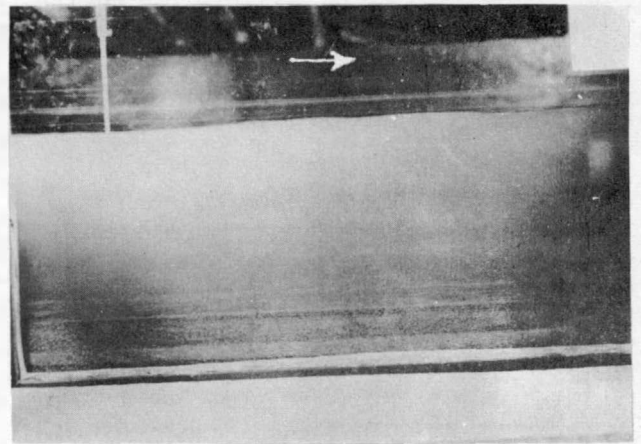
Fig. 11 Views of typical sandbar patterns



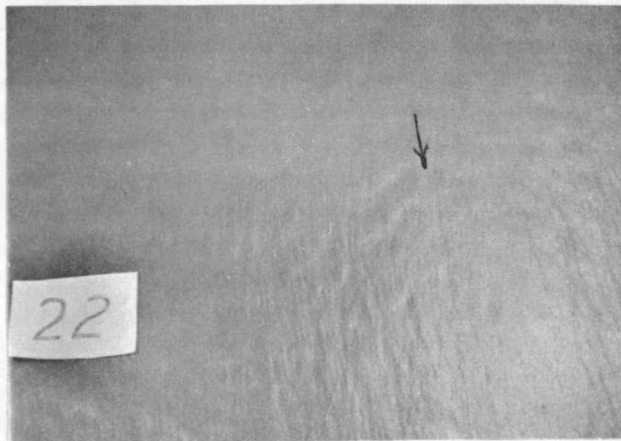
Run 18



Profile



Profile



Run 22

Fig. 12 Views of a typical plane bed

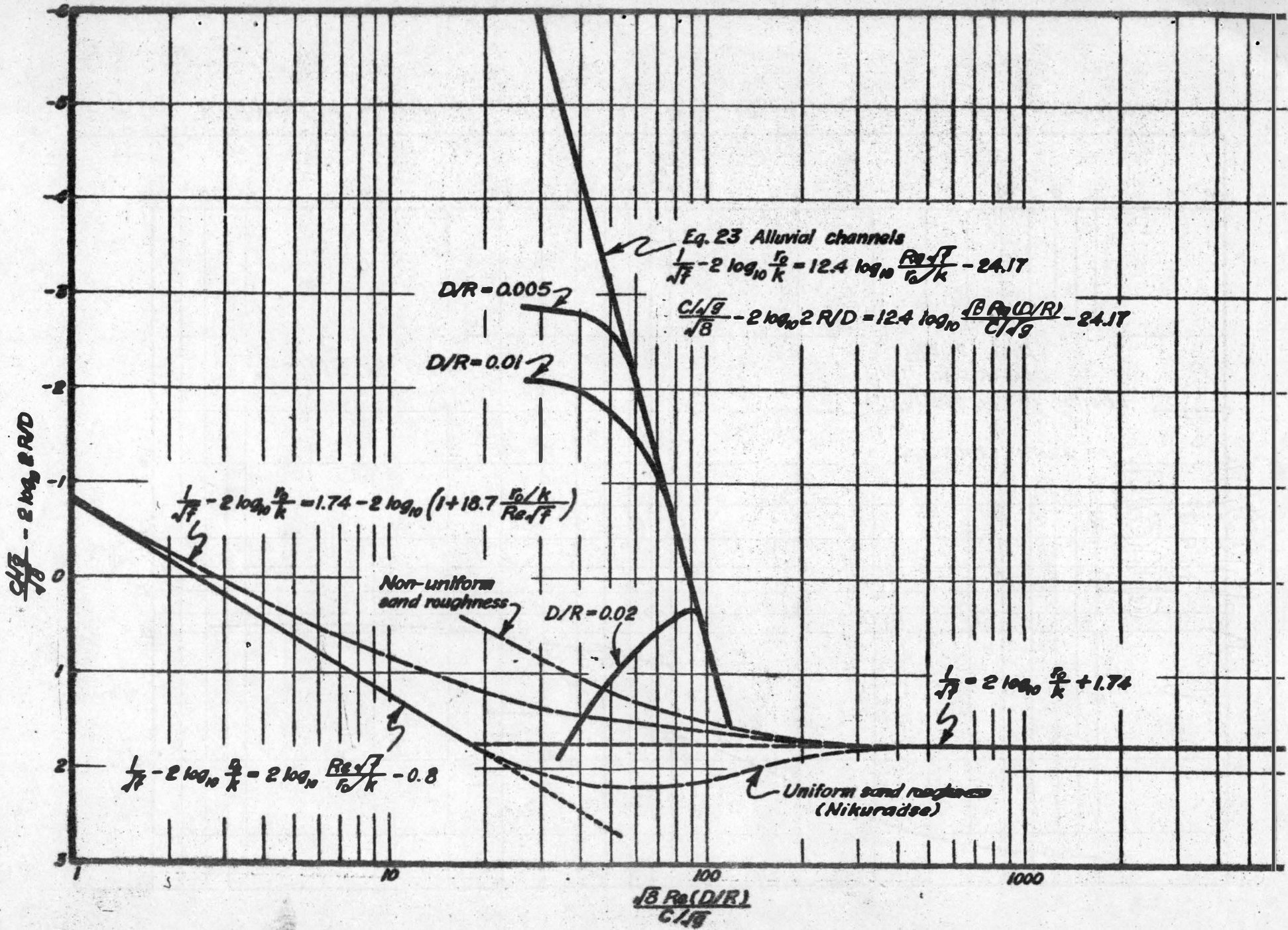


Fig. 13 Transition from smooth to rough boundary for wide alluvial channels

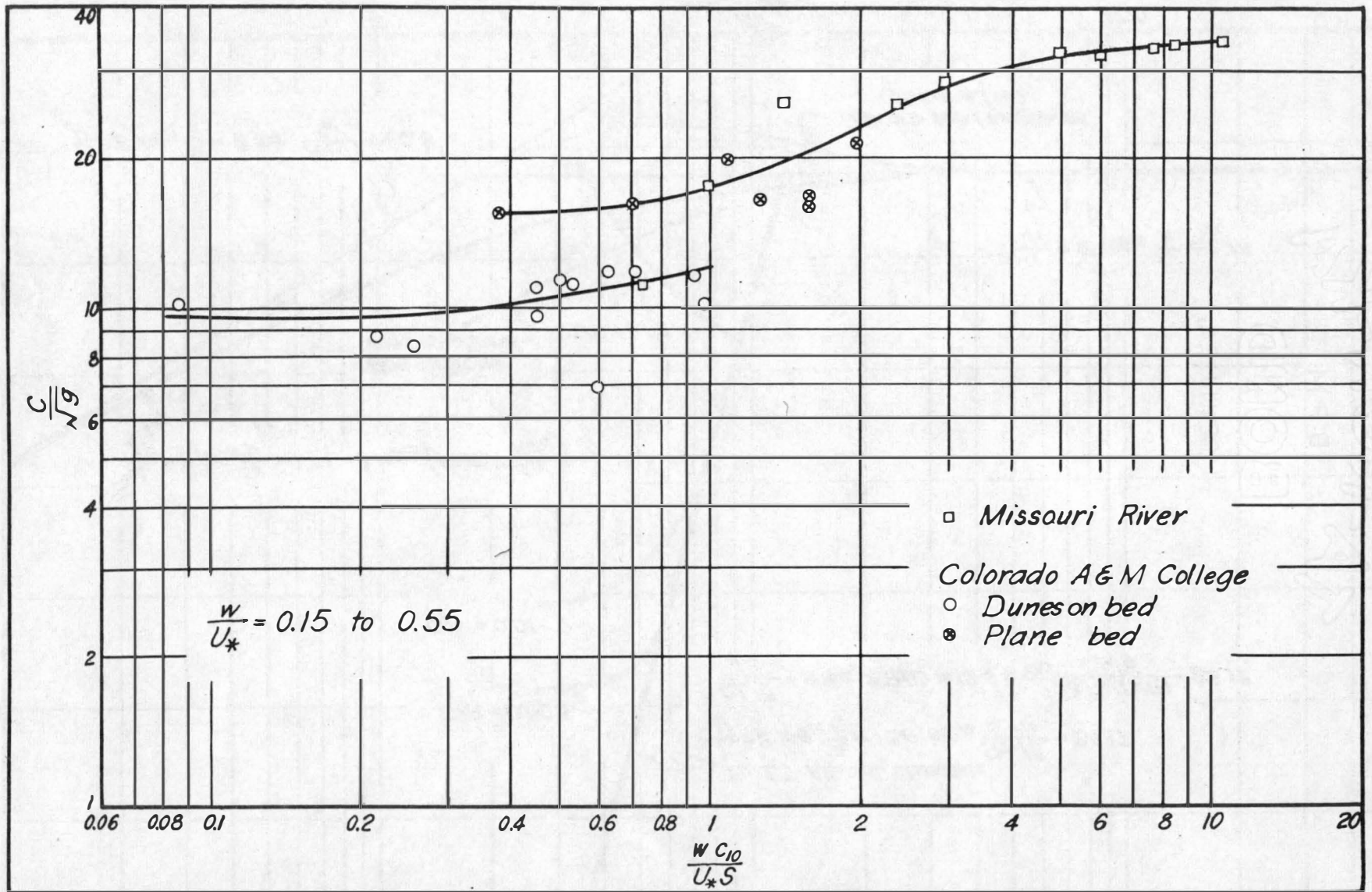


Fig. 14 Variation of resistance coefficient with Richardson number and bed conditions

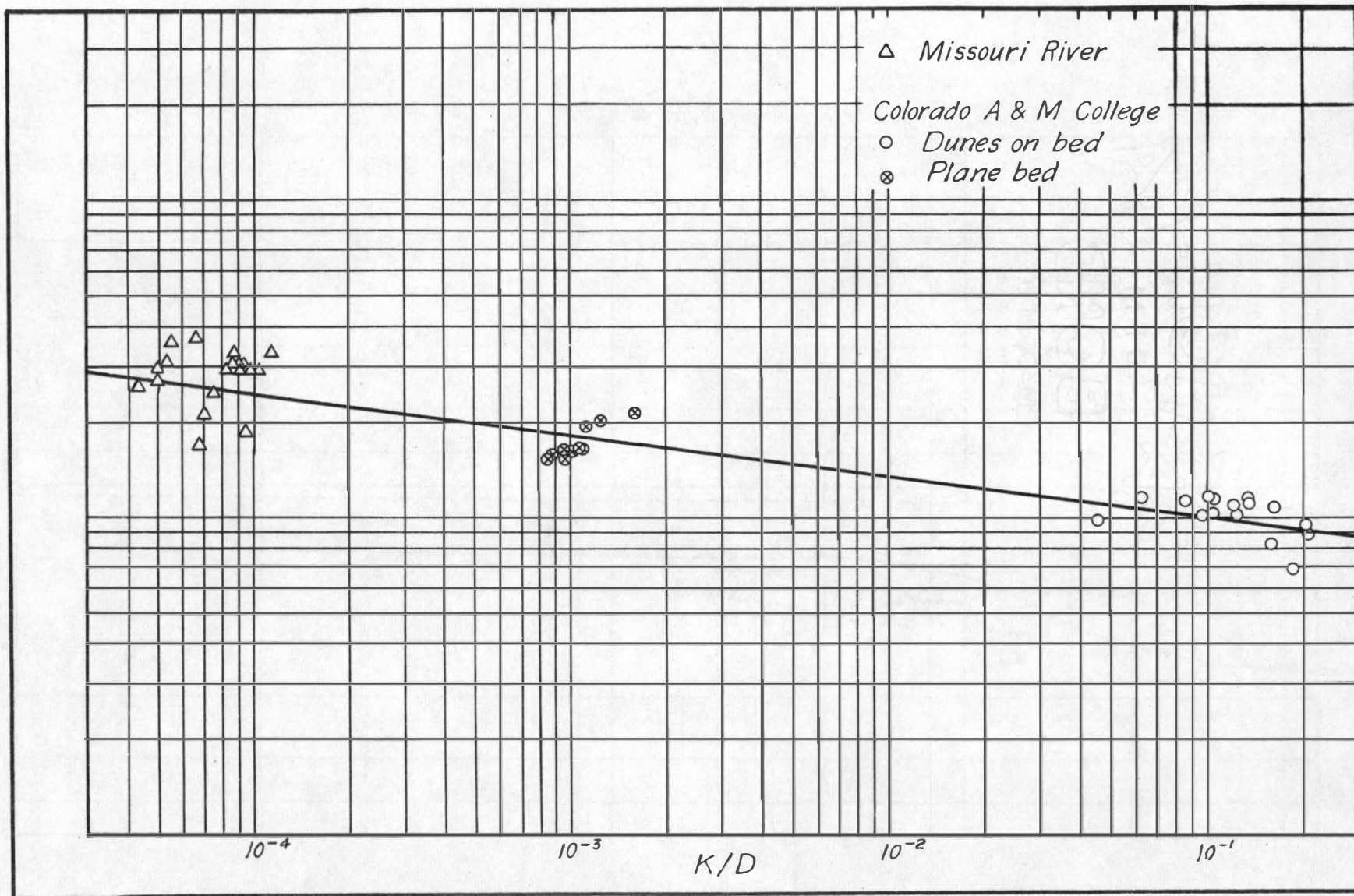


Fig.16 Variation of resistance with relative roughness

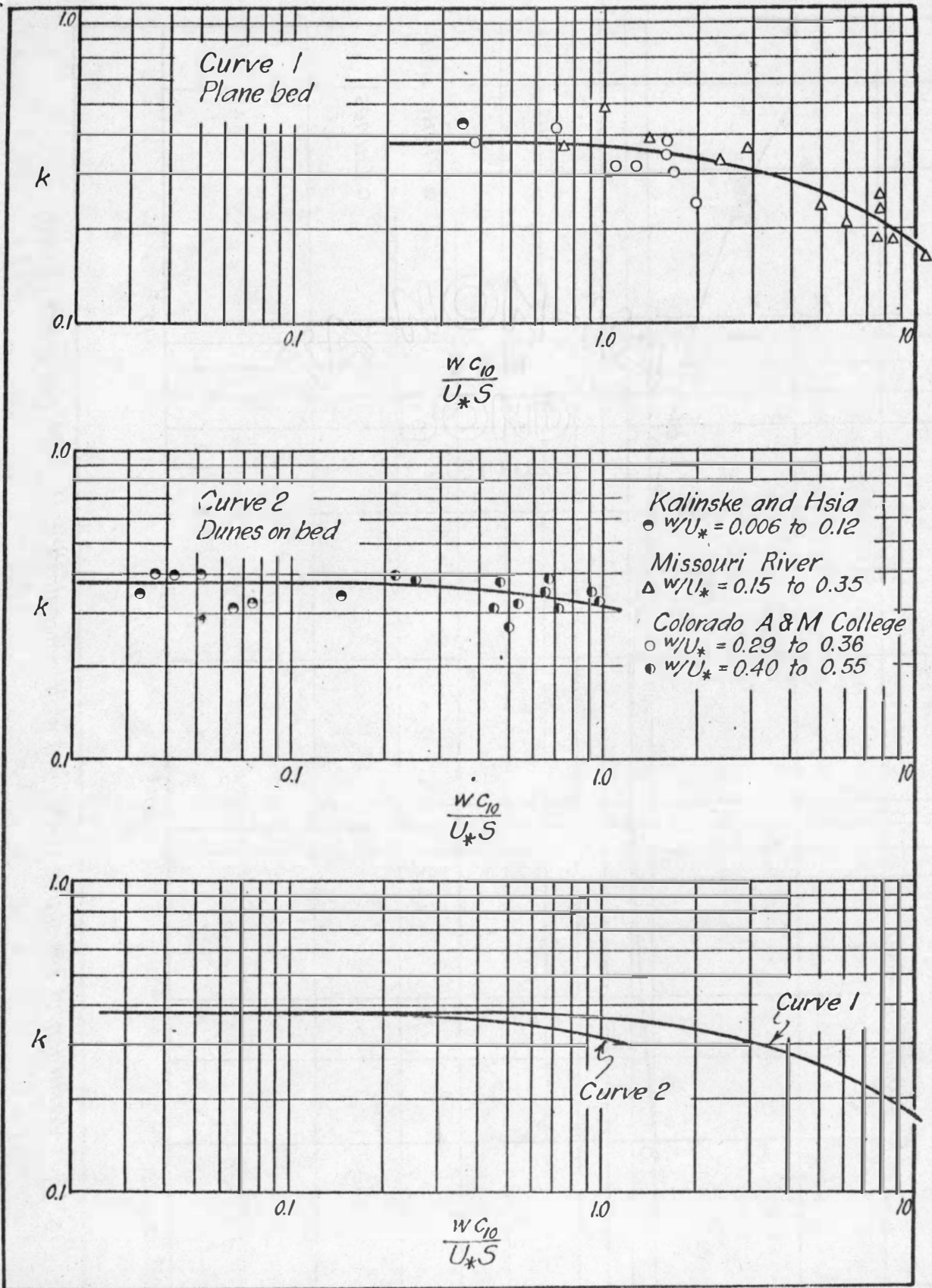


Fig.17 Variation of Kármán constant with Richardson number and relative fall velocity

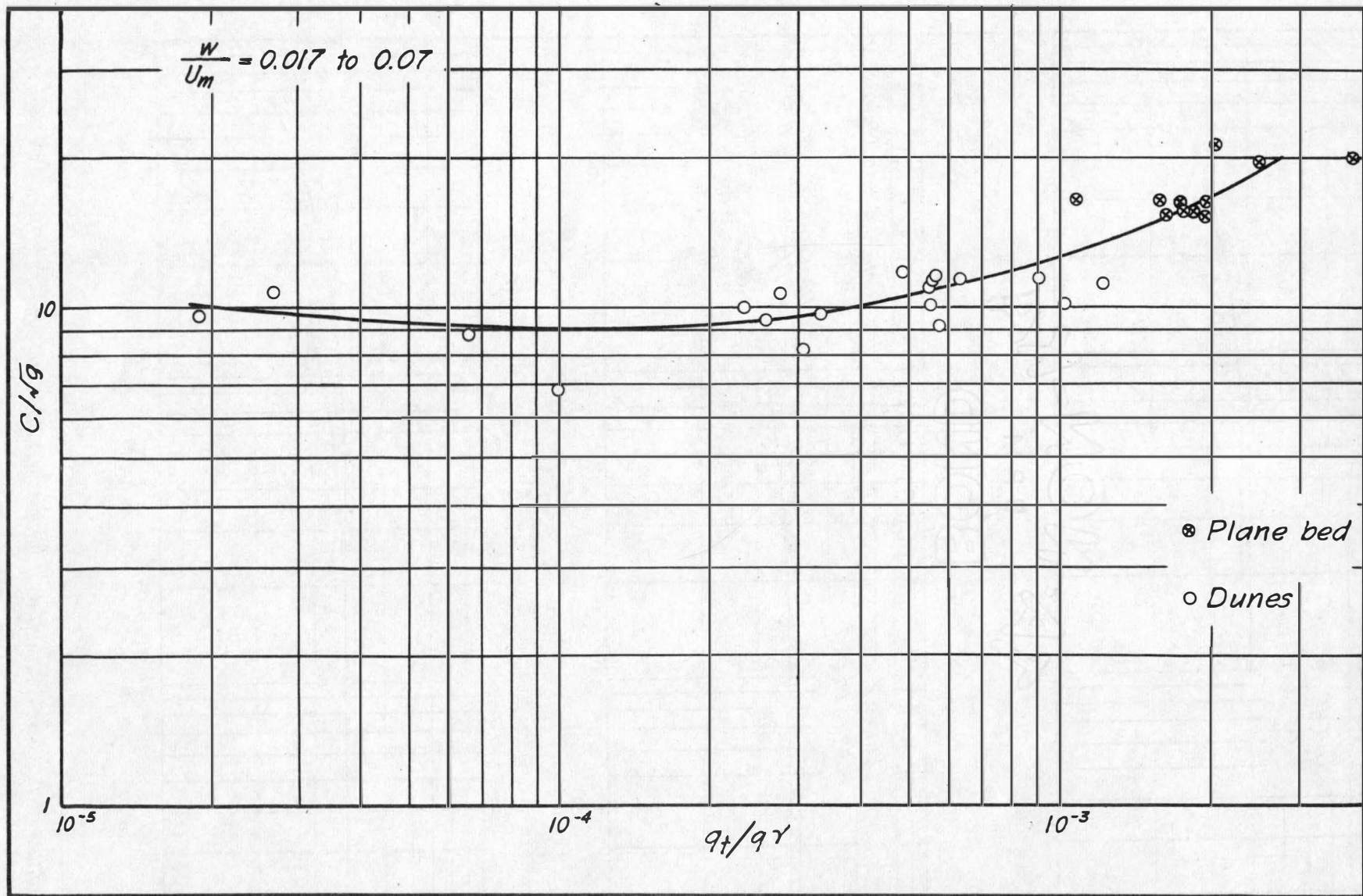
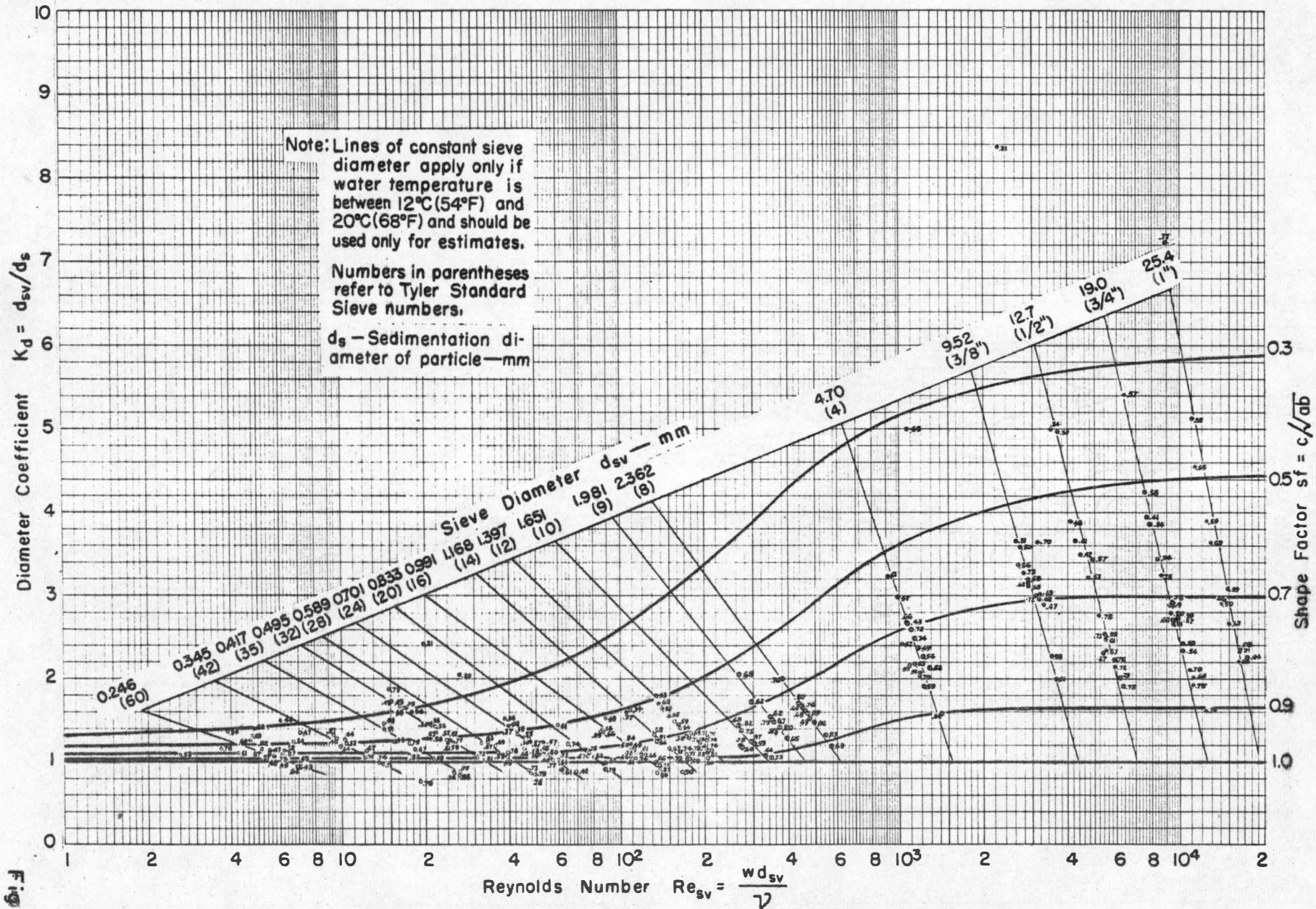


Fig. 18 Variation of resistance coefficient with concentration of total sediment load



Variation of Diameter Coefficient with Reynolds Number

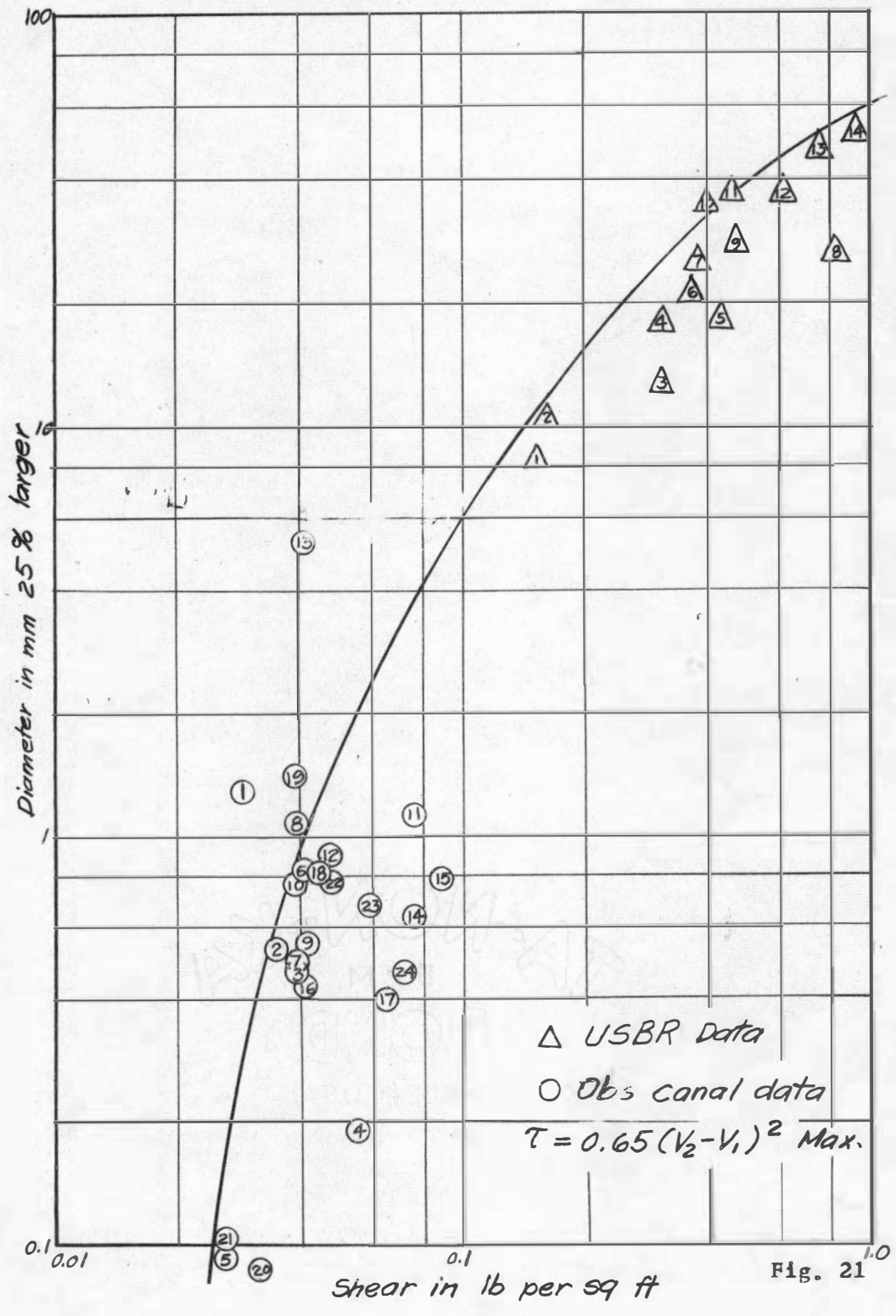


Fig. 21