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MODELS FOR INFERRING EVAPORATION FROM METEOROLOGICAL MEASUREMENTS

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INTRODUCTION

Methods for prediction of evaporation or evapotranspiration flux range from the early, simple formula of Dalton, in terms only of vapor pressures, to the recent empirical formulations of Van Bavel and McIlroy. Each of a wide range of methods have advantages and disadvantages and different methods will be appropriate for use in different instances. Following is an attempt to summarize most of the available methods with discussion of their applicability.

MASS TRANSPORT METHODS

The general formula for prediction of evaporation flux is attributed to Dalton (ca. 1800), that is

$$E = C(e_o - e_a) \tag{1}$$

where:

C = empirical constant containing wind speed

e_o = saturation vapor pressure at the evaporating surface

e_a = actual vapor pressure at some height above the surface (No specific level)

Several modifications of the basic Dalton equation have been made and used.

Rohwer (1931) suggested the equation

$$E = (.44 + .118u)(e_o - e_a) \tag{2}$$

where:

u = wind speed

and proposed it for use at altitudes above 1,500 m in Colorado.

Further modification of Equation (1) was done by Penman (1948). He found

$$E = .40(e_o - e_a)(1 + .17u_2) \tag{3}$$

where:

u_2 = two meter winds interpolated from the u versus z curve of Rohwer's.

A third modification of the Dalton equation was brought about by Pruitt (1963) in which:

$$E = f(u)(e_o - e_{100}) \tag{4}$$

where:

e_o = saturation vapor pressure at the temperature of the surface

e_{100} = vapor pressure at 100 cm above the surface

$f(u)$ = obtained from a plot of $ET/(e_o - e_{100})$ versus wind speed at the 100 cm level. ET is measured from a lysimeter.

AERODYNAMIC METHODS

Aerodynamic techniques assume similarity in the eddy diffusivities of momentum K_m, heat K_H, and water vapor K_v. The vertical fluxes in the form of Fick's law as a result of turbulent diffusion are

$$\tau = \rho\ K_m \frac{\partial \overline{u}}{\partial z} \qquad \text{momentum flux}$$

$$A = -\rho\ C_p\ K_H \frac{\partial \overline{T}}{\partial z} \qquad \text{sensible heat flux}$$

$$E = -\rho\ K_v \frac{\partial \overline{q}}{\partial z} \qquad \text{flux of water vapor}$$

where:

C_p = specific heat of air at constant pressure

q = specific humidity

It is assumed that

$$K_m = K_H = K_V$$

holds during neutral or non-buoyant conditions of stability.

For periods of neutral stability the most common form of the aerodynamic equations was proposed by Thornthwaite and Holzman (1942), i.e.,

$$E = \frac{\rho\ K^2\ (q_2 - q_1)(u_2 - u_1)}{[\ln\ (\frac{z_2}{z_1})]^2} \tag{5}$$

This equation assumes that $K_m = K_V$ and a log-normal wind profile.

Methods Corrected for Unstable Conditions

The basic Deacon and Swinbank (1958) equation assumes that $K_V = K_m$, yielding

$$E = C_{D_s}\ \rho\ u_s{}^2\ (\frac{q_1 - q_2}{u_2 - u_1}) \tag{6}$$

where:

$$C_{D_s} = C_D\ \frac{u_1{}^2}{u_s{}^2}$$

u_s = wind speed near ground surface

Munn (Rosenberg et al. 1968) by using developments of Monin and Obukhov (1954) derived the equations

$$E = -\rho\ u_*{}^2\ \frac{(q_1 - q_2)}{(u_2 - u_1)} \tag{7}$$

$$u_* = \frac{K(u_2 - u_1)}{\ln \frac{z_2}{z_1}} = \alpha \frac{K(z_2 - z_1)(T_2 - T_1)}{(u_2 - u_1) \ln (\frac{z_2}{z_1})} \cdot \frac{g}{T}$$

which is valid if $z < .03 |L|$

where L is Monin and Obukhov's mixing length defined by

$$L = \frac{T \rho C_p u_*^3}{g KA}$$

Williams (Rosenburg 1968) proposed the evaporation be calculated by the following equation

$$E = C \frac{f(u)(e_o - e_a)}{f(z_o)} \qquad (8)$$

where:

z_o = aerodynamic roughness parameter

The main difficulty of implementing equations of the above nature lies in sensor accuracy. Measurements of wind speed gradients and vapor pressure gradients over small incremental heights are difficult.

In order to overcome the above difficulties, an equation was developed by Pasquill (1949). That is

$$E = C \rho u_2(q_1 - q_2) \qquad (9)$$

where:

$$C = \frac{K^2 (1 - \frac{u_1}{u_2})}{\ln (\frac{z_2}{z_1})^2}$$ for a given surface roughness

The difficulty with Pasquill's equation is that C varies considerably with variations in stability from the neutral condition.

Sutton (1934), by use of turbulent transfer theory of the wind profile near the ground, showed that

$$\bar{u} = \bar{u}_1 = \left(\frac{z}{z_1}\right)^{\frac{n}{(2-n)}}$$

where:

$0 < n < 1$, and

$n = f$(turbulence, vertical temperature gradient near the ground surface, and surface roughness)

This above theory was further used as an approach to solving the evaporation prediction problem by assuming that:

$$K_m = K_v$$

it follows that

$$E \propto \bar{u}^{0.78} \tag{10}$$

where:

$\bar{u}$ = mean wind speed over the evaporating surface

Equation (10) was supported by experiments conducted with organic solvents.

The Thornthwaite-Holzman approach was evaluated by Pasquill, and he found that the approach was applicable only when dry adiabatic lapse rates prevailed. Pasquill showed that

$$E = C\, u_2(e_1 - e_2) \tag{11}$$

when the sensors were placed in the region of adiabaticity (< 50 cm).

where:

C = a complex factor involving the ratio of wind speeds at the upper and lower levels. C is also dependent upon (d), the zero plane displacement, due to the dependence upon surface roughness.

Pasquill also found that systematic errors were introduced into E when the Richardson's number (R_i) was less than .025,

$$R_i = \frac{g \frac{\partial T}{\partial z}}{T_A \left(\frac{\partial u}{\partial z}\right)^2}$$

where:

T_A = absolute temperature

thus emphasizing the influence of thermal stratification. Evaporation calculated from use of the Thornthwaite-Holzman equation was overestimated during stable periods and underestimated during unstable periods.

Pasquill (1950) later suggested that the evaporation be calculated by the following equation

$$E = \frac{\rho K^2 (u_2 - u_1)(q_1 - q_2)}{\left[\ln \left(\frac{z_2 - d}{z_1 - d}\right)\right]^2} \qquad (12)$$

He found good agreement between the above and E measured from soil containers during adiabatic conditions. He considered the above approach applicable for work with shortgrasses and also with crops if (d) is properly estimated.

Rider (1954), with use of Pasquill's equation, Deacon's wind profile equation,

$$\frac{\partial u}{\partial z} = az^{-b}$$

where:

$-b < 1$ during stable conditions

$-b = 1$ during neutral conditions

$-b > 1$ during unstable conditions

and the Thornthwaite and Holzman formula, corrected E for deviations from the log law wind profile so that:

$$E = \frac{\rho K^2 (1 - b)^2 (q_1 - q_2)(u_2 - u_1) z_o^{2(1-b)}}{[(z_2 - d)^{1-b} - (z_1 - d)^{1-b}]^2} \tag{13}$$

This method was found to underestimate E by 15% when used in an oat field.

The validity of aerodynamic approaches was questioned by other scientists. Holzman (1943) and Rossby and Montgomery (Rosenberg 1968) proposed the following modification of the original Thornthwaite-Holzman equation. That is

$$E = \frac{\rho K^2 (q_1 - q_2)(u_2 - u_1)}{\ln \frac{z_2}{z_1}} \cdot \frac{1}{\ln (\frac{z_2}{z_1})(\frac{1 - S z_1}{1 - S z_2})} \tag{14}$$

where:

$S = f(\text{wind shear})$

Pruitt (1963) has suggested that the evaporation be estimated by the following equation

$$E = -\rho (K_D)_C \frac{\partial q}{\partial z}$$

where:

$(K_D)_C = K_D^* z^2 \frac{\partial u}{\partial z}$

$(K_D)_C$ = eddy diffusivity

K_D^* = eddy diffusivity for water vapor arrived at by graphical method involving R_i

It is suggested that this equation is valid for a wide range of conditions but not conditions where R_i exceeds +.05.

Crawford (1965) has developed a method for extending the aerodynamic techniques into conditions of instability. He defines a new parameter $\overset{*}{E}$; i.e.,

$$\overset{*}{E} = \frac{E}{\rho \left(\frac{g}{T}\right)^{1/2} \left|\frac{\partial T}{\partial z}\right|^{1/2} (z - d)^2 \frac{\partial q}{\partial z}} \tag{15}$$

where:

$\overset{*}{E} = K^2 \; |R_i|^{-1/2}$ for forced convection

and

$\overset{*}{E} = h$ for free convection

Tests show free and forced convection overlap in the range of stability conditions and that $\overset{*}{E}$ is a smooth changing function of R_i.

Sheppard (1958) suggested the following aerodynamic equation for evaporation

$$E = \frac{\varepsilon \, \rho \, K \, u_* \, (e_a - e_o)}{p\left(\ln \frac{K \, u_* \, z}{D}\right)} \tag{16}$$

where:

e_a = vapor pressure at some level z

D = diffusivity of water vapor in air

This equation was tested against evaporation data from precision weighing lysimeters and found to agree within 2%. For the same conditions Bowen's Ratio agreed to within 10%.

Summary of Aerodynamic Methods

In general, these methods yield sufficiently accurate results; however, corrections for stability are necessary for most surfaces. Fetch is a necessary requirement for use in these methods; this tends to limit their application.

EDDY CORRELATION

Swinbank (Rosenberg 1968) suggested an eddy correlation method for estimation of evaporation flux:

$$H = C_p \rho T w \tag{17}$$

where:

H = vapor transport/unit area

w = vertical component of wind speed

T = air temperature

Eddy correlation seems to have a bright future; however, its application is limited due to the present state of instrumental art.

ENERGY BALANCE AND BOWEN'S RATIO

The usual form of the energy balance equation after the minor terms have been ignored is

$$R_N = S + A + LE$$

where:

R_N = net radiation

S = soil heat flow

A = sensible heat

LE = latent heat of evaporation

By use of the similarity equations to express LE and A, the Bowen's Ratio developed.

$$\beta = \frac{A}{LE} = \frac{P\ C_p\ K_H\ (\frac{\partial T}{\partial z})}{L\varepsilon\ K_v\ (\frac{\partial e}{\partial z})} = \frac{P\ C_p}{L\varepsilon}\ (\frac{\Delta T}{\Delta e}) \tag{18}$$

where:

K_H is assumed to equal K_v

Despite the questionable assumptions of Bowen's Ratio, it has been shown to yield sufficient results for practical problems, and is widely accepted. The expression does neglect the effect of advected energy and is most feasible in relatively humid climates, that is, R_N closely approximates LE. Various forms of Bowen's Ratio method have been tried (Tanner 1961, King 1961, Suomi and Tanner 1958).

In general, the authors favor the energy balance (Bowen's Ratio) over the aerodynamic approach. This is a result of less stringent fetch and instrumental requirements.

EMPIRICAL METHODS

Methods which relate ET to one or more meteorological parameters other than micrometeorological parameters are referred to as empirical methods.

The Thornthwaite Method

Thornthwaite (1948) developed an equation for estimating potential evapotranspiration from the formula

$$e = Ct^a \tag{19}$$

where:

e = monthly evapotranspiration

t = monthly mean temperature (°F)

C, a = constants

He arrived at the following general equation

$$e = 1.6 \left(10 \frac{T}{I}\right)^{a}$$

where:

$$a = 6.75 \times 10^{-7} I^{3} - 7.71 \times 10^{-5} I^{2} + 1.79 \times 10^{-2} I + .49$$

and

I = heat index derived from 12 monthly index values i

where:

$i = f$(monthly normal temperature)

i.e.,

$$i = \left(\frac{t}{5}\right)^{1.514}$$

and

e = monthly PE (in cm)

The disadvantages of this method are:

(1) The maximum annual heating is lagged by the evaporation in both the spring and fall.

(2) Its use for short-period studies results in errors due to significant temperature variations during these periods, and therefore, is not a suitable measure of incoming radiation.

Penman Method

Penman (1948) suggested the following method:

$$E_o = \frac{mH + .27E_a}{m + .27} \qquad (20)$$

where:

m = slope of the saturation vapor pressure curve

H = estimate of net radiation in evaporation equivalents of mm/day

$E_a = .35(e_a - e_d)(1 + u_2 \times 10^{-2})$ mm/day

e_d = saturation vapor pressure at dew point

e_a = saturation vapor pressure at mean T_A

u_2 = mean wind speed in miles/day at two meters

The Penman equation requires physical knowledge of vapor pressures, sunshine duration, albedo, wind speed, and mean temperature; therefore, it has a more sound physical basis.

The disadvantages are:

(1) Converting the calculated E_o (potential evaporation) to ET over vegetated surfaces.

(2) Without modification, the original equation is applicable only where optimum water supply is maintained.

Blaney-Criddle Method

Blaney-Criddle (1950) developed the following formula from the Blaney and Morin (1942) formula for consumptive use.

$$U = K_s F = \Sigma K_m f \qquad (21)$$

where:

U = consumptive use in inches during the study period

K_s = growing period consumptive use coefficient

K_m = monthly consumptive use coefficient

$f = \frac{t \cdot p}{100}$ = monthly consumptive use factor

t = monthly mean temperature (°F)

p = monthly percent of total annual daylight hours

$F = \Sigma f$ for the total period

$u = K_m f$ = monthly consumptive use (inches)

This method does have the advantages that it is easy to use, and the necessary data are available from weather records. However, several assumptions are made when implementing the above equation:

(1) that (U) varies directly with F

(2) non-limiting water supply to plants

(3) length of growing season is an index of production and consumptive water use

(4) fertility and producing powers do not vary between areas which are to be compared.

Van Bavel's Method

By use of Penman's work, Van Bavel (1966) suggested the following:

$$LE_o = \frac{\frac{-m}{\gamma H} + LB_v d_a}{\frac{m}{\gamma} + 1} \text{ cal cm}^{-2} \text{ min}^{-1} \tag{22}$$

where:

H = sum of energy inputs except for A, LE

d_a = vapor pressure deficit in mb at elevation z_a

B_v = transfer coefficient for water vapor and is equal to

$$\frac{\rho \; \varepsilon \; K^2}{p} \; \frac{u_a}{[\ln \frac{z_a}{z_o}]^2} \text{ g cm}^{-2} \text{ min}^{-1} \text{ mb}^{-1}$$

This method assumes water supply is not restricted and is usually used for hourly calculations. It is applicable for most all surface conditions. For daily calculations, Van Bavel suggests the following

$$E_o = \frac{[\frac{m}{\gamma}\frac{R_N}{L} + B_v\, d_a]}{\frac{m}{\gamma} + 1} \tag{23}$$

$$\text{with } B_v = 1.222 \times 10^{-2} \frac{u_a}{\ln\left(\frac{z_a}{z_o}\right)^2}$$

where:

u_a = wind speed at z_a in km/day

z_o = zero plane displacement

d_a = saturation vapor deficit (mb)

McIlroy Method

The McIlroy (Brooks 1966) equation has proven satisfactory for estimating latent heat flux over pastures in Australia when hourly periods are used. The McIlroy equation can be expressed as

$$LE = (R_N + S)\frac{s}{s + \gamma} + h\,(D - D_o) \tag{24}$$

where:

s = rate of change of moisture concentration of saturated air as a function of temperature

h = bulb aerodynamic transfer coefficient

D = wet bulb depression at some height above surface

D_o = wet bulb depression at crop surface

Summary of Empirical Methods

In summary the main objection to all empirical methods is that no allowance has been made for type and density of surface cover. These methods will continually be used where only simple measurements are made or are available. It appears that as more meteorological data becomes available, the more sound methods, e.g., Penman and Van Bavel, will become most popular and will be extended into new areas.

LIST OF SYMBOLS

A - Heat flux to the air

B_v - Transfer coefficient for water vapor

C_D - Drag coefficient

C_p - Specific heat of air at constant pressure

D - Diffusivity of water vapor in air

D_o - Wet bulb depression of air at crop surface

D - Wet bulb depression of air at some height above a surface

E - Flux of water vapor

ET - Flux of evapotranspiration vapor

E_o - Observed evaporation

$\overset{*}{E}$ - Non-dimensional parameter related mathematically to E

F - Σf - Consumptive use factor

H - Approximations of net radiation

I - Σi - Annual heat index

K - Evaporimeter coefficient

K_H - Transfer coefficients for heat

K_m - Transfer coefficients for momentum

K_s - Growing period consumptive use coefficients

K_v - Transfer coefficients for water vapor

K_D^* - K_v

L - Mixing length

LE - Latent heat of vaporization of water

P - Atmospheric pressure

PE - Potential evaporation

Q - Heat capacity at crop height

RH - Relative humidity

R_N - Net radiation

R_i - Richardson number

S - Soil heat flux

T - Temperature (°C)

T_A - Absolute temperature

U - Consumptive use

W - Soil moisture content

d - Zero plane displacement

d_a - Vapor pressure deficit in mb at z_a

e - Monthly evaporation

e - Vapor pressure

e_a - Vapor pressure of the air

e_d - Saturation vapor pressure at dew point

e_o - Saturation vapor pressure at surface temperature

e_s - Saturation vapor pressure

e_{100} - Vapor pressure at 100 cm above the surface

f - Functional notation

g - Gravitational constant

h - Bulk aerodynamic transfer coefficient

i - Monthly heat index values

k - Von Karmen's constant = .40

m - Slope of saturation vapor pressure versus temperature curve at mean wet bulb temperature

p - Monthly percent of annual daylight hours

q - Specific humidity

s - Rate of change of moisture concentration of saturated air as a function of temperature

t - Temperature (°F)

t - Monthly mean temperature

t - Time

u - Horizontal wind speed

u_a - Wind speed at z_a

u_s - Wind speed near ground surface

u_* - Friction velocity = $\sqrt{\frac{\tau_o}{\rho}}$

$\bar{u}$ - Mean wind speed

w - Vertical wind speed

z - Vertical distance above ground surface

z_a - Vertical distance above ground surface of a point, a, in space

z_o - Aerodynamic roughness parameter

a, b, C, h, N, n, x, 0 - Numerical constants

o, s, 1, 2, 3 - Subscripts, with reference to height at which observations are taken

α - Numerical constant taken to equal approximately 6

β - Bowen's Ratio = A/LE

γ - Psychrometric constant

ε - Ratio of molecular weight of water to air

ρ - Air density

τ - Momentum flux (shearing stress)

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