

THESIS

APPLIED MATHEMATICS THROUGH AN EMBODIED STORYTELLING LENS

Submitted by

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In partial fulfillment of the requirements

For the Degree of Master of Science

Colorado State University

Fort Collins, Colorado

Spring 2025

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## ABSTRACT

### APPLIED MATHEMATICS THROUGH AN EMBODIED STORYTELLING LENS

In this research, I investigated how an applied mathematics professor and her students organized and presented information that aligns with elements found in traditional storytelling. The participants of this study were involved in an applied mathematics laboratory that researched applications of inverse problems and met once a week to discuss the progress they had made with their respective contributions to the lab. I also investigated how the participants communicated information through an embodied lens. Using my originally constructed storytelling framework, along with the embodied cognition framework of Nathan (2021), I analyzed audio- and video-recordings of the laboratory meetings. My findings suggest that the participants of the laboratory organically organized and presented information that aligned with traditional storytelling elements. Furthermore, my findings also suggest that the participants utilized embodiment to communicate the applied mathematical content within each storytelling element. I conclude by offering the potential research and teaching applications of this work, along with potential future research endeavors inspired by this study.

## ACKNOWLEDGEMENTS

I would like to express my sincerest gratitude to my advisor, Dr. Hortensia Soto. Her guidance, feedback, support, extensive knowledge, and experience were instrumental in the completion of this thesis. I would also like to sincerely thank the participants of this study for allowing me to attend their laboratory meetings. Finally, I would like to thank each Professor that I had the opportunity and privilege to learn from over the course of my graduate school experience.

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## INTRODUCTION

In higher education, undergraduate mathematics courses are primarily taught using lecture-based teaching methods (Cuoco, 2001; Thurston, 1994). However, the use of these methods has made it challenging for students to develop a deeper understanding of mathematics (Davis & Hersh, 1981; Dreyfus, 1991). Reflection upon how to better structure an undergraduate mathematics course can help instructors decide how to engage students in effective ways, beyond traditional lecture methods, which may lead to a deepened understanding of the material.

Unfortunately, determining how to structure undergraduate mathematics courses in rich and meaningful ways beyond traditional methods is difficult. Furthermore, the current literature that investigates the structure of undergraduate mathematics courses is limited to pure mathematics (Fukawa-Connelly, 2012; Lee et al., 2009; Stewart et al., 2019; Weber, 2004; Weinberg et al., 2015). For example, Fukawa-Connelly examined an instructor's lecture-based teaching on proof in an abstract algebra course via case-study. Fukawa-Connelly documented how the instructor presented an alternative to traditional lecture-based teaching strategies for writing proofs. In another case study, Weber investigated the instruction of an undergraduate real analysis course. Weber detailed how the professor utilized, and improved upon, the definition-theorem-proof (DTP) method (Thurston, 1995), which is the traditional method of teaching pure mathematics courses.

Some of the aforementioned studies (Stewart et al., 2019; Weinberg et al., 2015) adopted an embodiment lens to investigate how instructors facilitate their course while teaching pure mathematics. Nathan (2021) defines embodied learning to be, "the process of connecting unfamiliar and abstract ideas and representations to one's lived and felt experiences, including

body-based interactions and perceptions.” (p. xi). For example, Stewart et al. tracked the movement of an instructor through Tall’s three worlds of mathematical thinking in a linear algebra course (Tall, 2013). Weinberg et al. utilized Arzarello’s (2006) idea of semiotic bundle as a framework to analyze a case study of an instructor’s gestures in an undergraduate abstract algebra course. Clearly, there are rich studies that investigated the teaching of pure mathematics, yet there is little to no literature that exists related to the teaching of applied mathematics. I believe that applied mathematics is inherently embodied because although there exists a level of abstraction in applied mathematics, there exists the opportunity to connect that abstraction to a student’s lived and felt experiences. Applied mathematics has the potential to create a learning environment that provides students with experiences that relate abstract concepts with their real-world applications. Hence, this research aims to extend the current literature of instruction in pure mathematics to applied mathematics through an embodied lens.

Although embodied cognition provides the opportunity to study how applied mathematics concepts can be presented beyond traditional lecture methods, I have chosen to add an additional component that aims to address how applied mathematics concepts can be structured and organized. This organizational component is the notion of “storytelling”. Glover (2023) defines storytelling as, “the process by which a story is shared or told; an ancient art form that involves the presentation and delivery of a narrative, revealing the elements, images and events of a story whilst encouraging the imagination of the audience” (p. 1). Rather than having the characters of our story be wizards or superheroes, our characters are mathematical concepts such as the chain rule or  $u$ -substitution. Instead of our stories detailing the journey of a hobbit that takes a ring to Mordor, our stories detail the mathematical process of derivation or integration. If we consider teaching mathematics like telling a metaphorical story, then we can leverage the power that

storytelling has to help us organize, remember, and share mathematical knowledge. It is with this notion of storytelling, along with the power of embodied cognition that I will attempt to answer the following two research questions: (1) How does an instructor and her students organize and present concepts of applied mathematics that align with the elements and structure that is associated with “storytelling”? (2) In what ways does an instructor and her students engage in embodiment as they discuss aspects of applied mathematics? To accomplish this, I observed Dr. Julia Miles (pseudonym), an applied mathematics professor at a research institution in the Rocky Mountain area, as she conducted laboratory meetings during the Fall 2023 semester. In the next chapter, I provide the details of my theoretical framework that guided and shaped this research. Chapter 3 contains relevant literature in undergraduate mathematics education and the integration of intentional embodiment in mathematical environments. In Chapter 4, I detail my methods and provide a sample overlay of my storytelling framework on existing literature. Finally, in Chapter 5 I elaborate on my findings indicating how the participants of this study organized and presented applied mathematics concepts in a way that aligns with the elements and structure in traditional storytelling, as well as highlight the participants' use of embodiment as they present each storytelling component.

## THEORETICAL FRAMEWORK

In this section, I present the two components of my theoretical framework: embodiment and storytelling. The purpose of the embodiment subsection is to describe how I plan to incorporate Nathan's (2021) embodied cognition framework, along with Streeck's (2009) definition of gesture, and Nemirovsky and Ferrara's (2009) notion of "utterances" to give a robust interpretation of my data. The purpose of the storytelling subsection is to illustrate how I relate the teaching and learning of applied mathematics to components found in traditional storytelling. Storytelling functions on the macro-level of my analysis and aids in partitioning my data into larger emergent themes, whereas embodiment operates on the micro-level to zoom into the finer details of my data. I conclude by stating how these two components overlap and inform one another.

### **Embodiment**

Nathan (2021) claims embodied learning occurs when, "The meaning of what is learned is grounded in body movement and perception" (p. 59). Nathan articulates that when embodied learning is deliberately integrated into the design structure of education, students can experience much greater gains in overall performance. For my research, I chose Nathan's definition of embodiment because in his synthesis of the literature he identified four types of embodiments which I leverage for my work. The four types are body form and movement, simulation, materialist epistemology, and gesture. Nathan argues that our perception of the world is heavily integrated with our movement, which makes body form and movement the most primary of human experiences. In this context, he defines simulations as "mental events conducted offline, away from the place of their original intent" (p. 90). Simulations arise from our imagination, to

extend our cognitive processes beyond what is presently available to what is possible or unfamiliar. Materialist epistemology includes the ways in which physical objects and materials are used to perform operations that would have been previously limited to occur only within the mind. Lastly, Nathan defines gesture to be special actions of the hands, arms and other body parts that occur during communication, which serve to contribute to the experience of the listener, as well as reflect the thoughts of the speaker. Although Nathan's definition of gesture is frequently adopted in the literature, I adopted Streeck's (2009) definition of gesture: "a constantly evolving set of largely improvised, heterogenous, partly conventional, partly idiosyncratic, and partly culture specific, partly universal practices of using the hands to produce saturated understandings." (p. 5). This gesture definition better aligns with my study because it encapsulates all of Nathan's previously mentioned components of embodiment. Thus, with Streeck's definition of gesture, I can identify not only gestures themselves, but also how the other three components of embodiment are prevalent within gesture.

Alongside Nathan's (2021) embodiment framework, and Streeck's (2009) definition of gesture, I found it beneficial to also adopt Nemirovsky and Ferrara's (2009) notion of "utterances." The authors define utterances as "all types of bodily activity that play a part in a given conversational turn or transaction" (p. 162). This definition allowed me to identify the potential pieces of my data that required further analysis using Nathan's embodiment framework, and Streeck's (2009) definition of gesture to determine how the applied mathematician presented concepts and ideas in an embodied way to the graduate students. Moreover, I also utilized the notion of "storytelling" to describe the overarching structure and organization of the content presented in the lab meetings.

## Storytelling

Storytelling has been used throughout human history to share knowledge between individuals. Because of this rich history, storytelling has the potential to become a powerful tool to better understand the structure and organization of complex mathematical concepts and ideas. Before summarizing how I adopted storytelling in my research it is important to summarize how it has been used in previous literature (Patton, 2015) that is relevant to my study. A prevalent method of storytelling consists of how stories are used to convey messages utilizing past accounts of individuals. In this rendition of storytelling, past accounts of individuals are used to motivate the recipients of these stories by outlining what is possible for their own lives and experiences. Another example of storytelling is that of *practice stories*, which are utilized to detail how events occur and why they occur. For example, parents who have working-class careers are more likely to encourage their children to also seek working-class careers. Stories that are meant to narrate reality are a third method of storytelling and generally account for an individual's past experiences. This type of storytelling is prevalent in qualitative research, because it allows researchers to collect data about their participants' personal experiences. Although these examples of storytelling are powerful, they do not emphasize the metaphorical approach to storytelling that I adopted.

In an effort to bridge the gap between how storytelling has been used in previous literature, I drew upon the work of de Freitas and Sinclair (2014). Utilizing Chalet's (2000/1993) notion of *actualizing the virtual*, de Freitas and Sinclair discuss that it is up to us to *animate* concepts of mathematics. However, the process of animating mathematical concepts requires an intimate understanding of how the concepts fit into a given context; thus, de Freitas and Sinclair

highlight Lockhart's (2009) metaphor of story to help understand mathematical context.

Lockhart claims:

Mathematical structures, useful or not, are invented and developed within a problem context, and derive their meaning from that context... There are no facts per se; everything is relative and relational. It is the story that matters, not just the ending. (p. 17).

de Freitas and Sinclair further elaborate on Lockhart's metaphor of story with reference to the work of Dietiker (2012), who agrees that the mathematical story is the most important part, not the ending or mathematical fact at which one arrives. Thus, Dietiker seems to indicate that he values the process rather than the end product. In fact, Dietiker mentions that in any mathematics problem, there are multiple ways to navigate it, just as there are multiple ways to tell the same story. For my research framework, I adopted this metaphor of mathematics presented as a story because of the natural relationship between context and story. Complementing the work of de Freitas and Sinclair (2014), Sfard (2013) utilizes the metaphor of storytelling that best aligns with my adaptation. Sfard states, "those who tell stories of numbers or functions cannot experience the heroes of their story directly" (p. 226). The term "heroes" refers to numbers or functions, in this case, and the "story" is the mathematical context of these numbers or functions. The notion of considering purely mathematical entities as metaphorical components of a story is precisely what my framework of storytelling aims to accomplish.

In summary, I built on the metaphorical storytelling approach of Lockhart (2009), Dietiker (2012), and Sfard (2013) by relating components that appeared in my participant's teaching to storytelling components such as: characters, plot, setting, etc. This new storytelling framework served as the overarching piece of my framework that allowed me to divide the data

into segments that I mapped onto their equivalent storytelling component as described in the Methods section. From this zoomed out perspective, I shed light on how mathematical concepts and ideas take on a material dimension that aligns with the structure of storytelling. After the data were divided and mapped onto these metaphorical story segments, I used Nathan's (2021) embodied framework, Steeck's (2009) definition of gesture, and Nemirovsky and Ferrara's (2009) notion of utterances to describe how the applied mathematician presented the content of the lab meetings within each storytelling component. In the Methods Chapter, I describe how I operationalized the various components of these different pieces. In the next Chapter I synthesize relevant literature at the intersection of embodiment and mathematics that guided my research.

## LITERATURE REVIEW

In this chapter, I present literature that highlights current research in teaching undergraduate pure mathematics via lecture, embodied cognition in K-12 mathematics, and embodied cognition in undergraduate mathematics. In the first subsection, I discuss articles in undergraduate mathematics that investigated traditional lecture-based instruction, specific to pure mathematics courses. The first two articles display how effective a case study can be at narrating an instructor's implementation of lectures. I then present literature that attempts to address problems with traditional lecture-based teaching methods. I include these articles because they showcase some of the difficulties students have with traditional lectures, and the researchers offer potential teaching methods that can help students overcome these difficulties. In the second subsection I highlight literature related to embodied cognition and the teaching and learning of mathematics in the K-12 setting. Specifically, I synthesize articles that emphasize each component of Nathan's (2021) framework of embodiment. The purpose of this subsection is to illustrate how the various components of Nathan's (2021) framework can take shape in a mathematical environment. In the final subsection, I present research specific to undergraduate mathematics education that adopted an embodied cognition lens. The purpose of the final subsection is to justify my choice of using an embodied cognition lens to discover how an applied mathematics professor and her students discuss content, as well as to connect the work in undergraduate mathematics education with work in embodied cognition.

### **Lecture and Pure Mathematics Research**

To better understand where an embodied cognition lens fits within the context of undergraduate mathematics education research, I first highlight work regarding the study of

traditional lecture-based instruction in pure mathematics courses. I have chosen to present literature focused on proof-based mathematics courses because these courses are typically taught with traditional lecture-based methods which can intimidate students and mislead them about the processes used in mathematical thinking (Cuoco, 2001; Thurston, 1994). There is a perception that teaching advanced mathematics in this traditional way can hide the processes that students need to build robust mathematical reasoning and intuition (Davis & Hersh, 1981; Dreyfus, 1991). To better understand this phenomenon, Weber (2004) employed a case study of an undergraduate mathematics professor, Dr. T, as he taught a real-analysis course. During this observation, Weber noticed that Dr. T did little to emphasize student participation and instead utilized the traditional definition-theorem-proof (DTP) framework of Davis and Hersh (1981). However, Dr. T appeared to expand this framework by dividing up lectures into three different components: logico-structural, procedural, and semantic. The logico-structural lectures were designed to build students' understanding of logic, before asking them to prove a meaningful mathematical theorem. From here, Dr. T built upon students' logical understanding and moved into more procedural lectures. It was during these procedural lectures that Dr. T outlined proofs in a way that illustrated their general structure: the assumption, followed by the conclusion of the proof. Dr. T's goal was to fill in the rest of the proof, giving students detailed descriptions of each step, often accompanied by side work. This extension of the traditional DTP method of teaching a proof-based course was designed to allow students access to mathematical reasoning that is necessary to build a robust understanding of proof writing.

Fukawa-Connelly (2012) expanded upon the work of Weber (2004) by employing his own case study of an undergraduate mathematics professor, Dr. Tripp, during her instruction of an abstract algebra course. One critical aspect that differed between the work of Weber and

Fukawa-Connelly, was the difference of the two instructors' emphasis on student participation. In the work of Weber, Dr. T appeared to neglect student participation, whereas student participation was a key component of Dr. Tripp's lecture methods. Dr. Tripp appeared to delegate some responsibility of proof production to the students in her class, by *funneling questions* (Steinberg, 1989). Dr. Tripp began with a broad rhetorical question or statement, followed by a sequence of questions intended to focus students' thinking on one aspect of the proof, which ultimately led to a question that was expected to be answered by the students.

Along the same lines as the previous two studies mentioned, Fukawa-Connelly et al. (2017) continued to analyze traditional lecture-based instruction in undergraduate mathematics. In this study, the researchers focused on student comprehension of the main ideas presented in lecture and the content the students deemed noteworthy. Fukawa-Connelly et al. observed lectures from 11 different instructors. The courses included: set theory, real analysis, number theory, linear algebra, abstract algebra, geometry, and differential geometry. The researchers recorded audio and video data of the instructors, as well as took pictures of student notes after each lecture. The purpose of this study was to determine how instructors presented informal versus formal mathematics, and if students understood the main idea of the lecture. Fukawa-Connelly et al. found that instructors typically only presented informal thinking orally, whereas formal mathematics (i.e. definitions, theorems, proofs etc.) were explicitly written on the board. The researchers discovered that students only recorded the information presented on the board and did not record oral presentations. Fukawa-Connelly et al. observed that this discrepancy between the entirety of the content presented by the instructors, and the information recorded by the students, resulted in a misunderstanding of the main ideas of the lecture. This work further

emphasized the difficulty students have in attaining the mathematical thinking necessary to develop a deeper understanding of the content within traditional lecture-based instruction.

In a more recent study, Fuedel and Panse (2021) explored the benefits that guided notes can have on students' mathematical understanding. Guided notes in this context are defined as "a modified version of the instructor's notes with certain blanks that students have to fill in during the lecture" (p. 8). Fuedel and Panse hypothesized that guided notes could allow students to spend their time in lecture more efficiently. Instead of students spending most of their time and effort diligently copying down every piece of information written on the board, the use of guided notes, in theory, allows students the time to absorb the information presented by the instructor. In practice, the researchers discovered that the use of guided notes improved students' overall perceptions of the lecture. The students in this study mentioned that guided notes allowed them the time to critically think about the provided information. Students also reported that they were able to write down a larger amount of the informal information that the instructor presented orally, leading to positive student perception with the use of guided notes in undergraduate mathematics courses. Given this work centered on students' perceptions, the authors intended to extend their research by evaluating the effect of guided notes on student performance. Given my research interest, I now discuss some of the literature pertaining to embodied cognition, and the positive impact it has on the teaching and learning of mathematics.

### **Embodied Cognition and the Teaching and Learning of Mathematics**

The purpose of this section is to show the nuanced approaches taken in embodied cognition that facilitate mathematical understanding in the K-12 setting. I separate this section into Nathan's (2021) four components of embodied cognition: gesture, simulation, body form

and movement, and materialist epistemology. This separation allows me to isolate the specific contributions each component makes to the teaching and learning of mathematics.

### *Gesture*

There is an abundance of literature on how K-12 students use gestures to convey mathematical reasoning (e.g. Alibali et al., 2014; Bucciarelli et al., 2016; Chu & Kita, 2011; Gerofsky, 2010). Hostetter and Alibali (2019) utilized an embodied lens to observe teachers' use of gesture to support students' contributions in classroom discourse during the analysis of three separate cases of previously collected data. For the sake of clarity, I highlight the third case that focused on a pre-engineering class as part of a lesson relating Boolean Logic to digital circuit design. The purpose of this lesson was to link the students' constructed circuits to their symbolic truth table representation. The authors found that during one student's explanation of their circuit, the gestures made by the teacher were performed in silence along with the student's explanation. The purpose of the teacher's gestures was to highlight the current row of the truth table as a student explained it. By pointing to the row of the truth table that corresponded to the student's explanation, the teacher was able to link the student's actions and speech to the truth table representation, allowing for a shared understanding amongst the students that were observing the explanation.

### *Simulation*

Connecting the work regarding gesture to that of simulation, I have chosen to highlight the work of Alibali and Nathan (2012) because it shows how a simulated action can take the form of a gesture. The authors argue that "mathematics is embodied in two key senses: it is based in perception and action, and it is grounded in the physical environment" (p. 247). The foundation for this claim is based on evidence of students' and teachers' spontaneously produced

gestures within the classroom. Alibali and Nathan define three types of gestures: pointing, representational, and metaphorical. “Pointing gestures reflect the grounding of cognition in the physical environment, representational gestures manifest mental simulations of action and perception, [and] metaphoric gestures reflect body-based conceptual metaphors” (p. 247). For the purpose of discussing simulation, I focus on representational gestures. Alibali and Nathan discuss the *Gesture as Simulated Action* framework of Hostetter and Alibali (2008) and how simulating actions (such as gestures made with the hands while speaking) involves activating the same areas of the brain that control planning actions and the perception of using objects. This type of simulation is believed to manifest itself as representational gestures. The authors provided evidence for this claim using data from a tutoring session between a teacher and a student discussing the change in slope of a line. In this example the teacher represented the slope of a line changing from two to four by placing her elbow on the table and changing the angle of her forearm with the table to simulate the slope of the line changing. Alibali and Nathan claim that this type of simulated action is produced when “the motor activation involved in simulating actions or perceptual experiences, in conjunction with the motor activation in speaking, sometimes exceeds the speaker’s threshold for producing overt action, so the speaker produces a gesture” (p. 267). This claim hypothesizes that a threshold exists where the use of only words is not enough to convey understanding, and thus, one spontaneously creates a simulated action (representational gesture) to complement and abstract the concept one is trying to convey. The work of Alibali and Nathan showcased how mathematical knowledge is grounded in embodiment, and how the expression of that knowledge can manifest itself as simulated actions through representational gestures.

### *Body Form and Movement*

Beyond the use of gesture and simulation, embodied cognition also acknowledges how one's entire body can lead to mathematical understanding through the body form and movement. Extending mathematics beyond its typical isolated sedentary environment is what led Kelton and Ma (2018) to investigate how learners operate in an environment that promotes whole-body engagement and collaboration. In this research, the student participants had their gym floor transformed into a number line made of blue tape and yellow hash marks. The students were familiar with number lines drawn on paper and were familiar with the gym's usual function. However, now the gym with a number-line took on a new meaning that allowed for full-body engagement. During the activity, students selected a hashmark and made it their "home" (p. 183). Once each student selected a place to call their own, they were then asked to perform different tasks such as doubling or tripling their distance away from another student. Rather than students marking down varying distances on paper, their bodies became the vehicle in which their thinking traveled. This type of full-body engagement also provided its own set of unique challenges such as bumping into one another when all the students simultaneously attempted to switch places with the student opposite of them. Such challenges introduced the need for social interaction and collaboration to ensure the safety of the participants as they navigated the tasks.

Similar to Kelton and Ma's (2018) work, Nathan et al. (2021) explored the role that body-form and movement plays in the context of geometric reasoning. In this work, the researchers observed the changes in participants' ability to produce geometric proofs after playing the interactive game *The Hidden Village* (THV). Nathan et al. were interested in the impact that an embodied intervention has on the production of geometric proofs. The intervention utilized, in this case, were movements that the participants were required to mimic

during gameplay of THV, known as *directed actions*. Directed actions, as defined by Nathan et al., are movements that were chosen from prior sessions that “demonstrated successful mathematical reasoning” (p. 241). The participants were required to repeat the directed action five times, with no mathematical context whatsoever. After the fifth successful performance of the directed action, the researchers gave the participants a geometric conjecture, asked them to determine if it is true or false, and to support their reasoning. To determine if these directed actions contributed to successful geometric reasoning, Nathan et al. also varied the *cognitive relevance* of the movements.

Cognitive relevance describes the effectiveness of how directed actions influence mathematical insight. By varying the cognitive relevance of the movements, Nathan et al. (2021) gained insight into the effectiveness of this embodied intervention. Based on the results of this study, Nathan et al. found that participants who were given relevant actions far outperformed, in terms of geometric reasoning, students who were given irrelevant actions. Furthermore, the students who were given relevant actions were far more likely to include spontaneous gestures during their explanation. These results highlight how mathematical reasoning is not necessarily localized within the mind, but rather, it is possible that this type of reasoning is also informed by the body’s movement in the world. Nathan et al. claims that proving mathematical conjectures often requires abstract thought that exists outside of first-hand experience. This study suggests that there is a potential reciprocal relationship between one’s body movement and mathematical reasoning, where movement (such as directed action) can inform the mind, and the mind can express reasoning through the body (such as gesture).

### *Materialist Epistemology*

Often the body and its movement throughout the world garners the most focus within the realm of embodied cognition. However, sometimes learning mathematics extends to materials

beyond the body. This material extension is covered under the umbrella of materialist epistemology. To better understand how materials play a significant role in learning mathematics, I have chosen to highlight the work of Nemirovsky et al. (2013) because it highlights how a material can progress from an inanimate object to a means of developing and expressing our thinking. Nemirovsky et al. sought to understand the role that *mathematical instruments* play in learning mathematics, by way of understanding what they term *perceptuomotor integration*. The authors define mathematical instruments as

a material and semiotic device together with a set of embodied practices that enable the user to produce, transform, or elaborate on expressive forms (e.g. graphs, equations, diagrams, or mathematical talk) that are acknowledged within the culture of mathematics (p. 376).

Nemirovsky et al. also define perceptuomotor integration as the merging of the perceptual and motor aspects that relate to the achievement of tool fluency. In this context, tool fluency is achieved when the “use of a mathematical instrument allows for culturally recognizable creation in mathematics domains” (p. 373).

To understand a learner’s gradual ascent into perceptuomotor integration, the researchers investigated how participants engaged in a non-classroom-based activity known as *Math Core for Museums*. In this activity, two participants were each given access to a handle that, when moved, produced a drawing on a graph that was displayed on a screen in front of them. One participant controlled the “x-direction” movements of the graph, while the other controlled the “y-movements.” The participants were unfamiliar with this activity and were given a series of tasks to complete, such as collaboratively drawing a line with a slope of one. Initially, the participants’ perceptual and motor aspects of this task were separated from one another but

merged as the participants became more familiar with the instruments and their use. Nemirovsky et al. (2013) documented that the participants began to exhibit a temporal flow of perceptuomotor activity that allowed the instrument to express the mathematical thought required to construct a graph with specific requirements. The mathematical instruments in this study were no longer merely physical objects, but rather, an extension of the participants' mathematical thoughts. Nemirovsky et al. mention that an experienced piano player does not think about each individual key they press or note they read, but rather they blend all the necessary notes and keys to create a piece of music. A similar blending of motor and perception occurs in mathematics with the use of a mathematical instrument. When tool fluency is achieved, the use of a mathematical instrument is no longer an isolated instance of motor use and perception, but rather, the temporal flow of perceptuomotor activity is "infused with past and future." (p. 378). Once an individual achieves this level of tool fluency, the mathematical instrument is transformed into an extension of the individual's mathematical thoughts and expressions. Although each component of Nathan's (2021) framework in isolation provides significant insight into the learning of mathematics, the true power of this framework lies with the interplay between each component.

### **Undergraduate Mathematics Research Through an Embodied Lens**

In this section I synthesize relevant research in undergraduate mathematics that utilizes an embodied lens. In the work of Weinberg et al. (2015), the researchers observed the instruction of an undergraduate abstract algebra course to understand the specific role gesture plays in a proof-based mathematics course. To describe the role of gesture, Weinberg et al. drew upon the work of Arzarello et al. (2009), who defined the dynamic structure of a *semiotic bundle* to allow the framing of gesture production in mathematical activities. This *semiotic bundle* encompasses

the relationship between the teacher's and students' use of speech, gesture, and inscriptions during mathematical reasoning. Weinberg et al. adopted Arzarello's framework to describe the instructor's use of these *semiotic bundles* to service the "main idea" of the lecture. For example, when the instructor described properties of an equivalence relation, the *semiotic bundle* used included speaking about concepts such as "reflexive" and "symmetric," coupled with a gesture of an index finger pivoting back and forth. The pairing of formal language with gesture allowed the instructor to convey the "main idea" of the abstract concept of an equivalence relation.

Another example of undergraduate mathematics education research through an embodied cognition lens can be seen in the work of Stewart et al. (2019). The focus of this study was David, an instructor in an inquiry-oriented linear algebra course. David provided the researchers with self-reflections of his instructional decisions, so that they could better understand his methods of instruction. Stewart et al. adopted the Tall's (2013) "Three World's" framework to analyze David's self-reflections. In this framework, mathematical reasoning can be categorized into three worlds: embodied, symbolic, and formal. Stewart et al. used this framework to identify how David's instruction methods shifted between Tall's three worlds, which helped the researchers better understand David's curriculum decisions. David's self-reflection gave him a platform to justify shifting between one world to another. Initially, the researchers hypothesized that the purpose of David's shifting was to support students' ability to think symbolically, because he believed this world was the most difficult for students to comprehend. However, when students did not make necessary connections within the symbolic world, it seemed to motivate David to shift into the embodied world to help build students' intuition. David then used this new intuition to shift back into the symbolic world.

In another study, Rasmussen et al. (2004) examined the function of gesture in the collective development of knowledge in an undergraduate mathematics classroom. Through the re-analysis of data collected by Stephan and Rasmussen (2002), who observed the mathematical practices of a course in differential equations, Rasmussen et al. explored the role gesture plays in the development of mathematical practices. Rasmussen et al. observed that gesture is semiotically tied to the inscriptions and tools students use when solving differential equations. The authors defined gesture/argumentation dyads to be instances of classroom talk where a mathematical idea is expressed along with gesture. With the notion of gesture/argumentation dyad, Rasmussen et al. analyzed the role gesture played when students debated their mathematical understanding of different conjectures. The authors claimed that “gesturing and symbolizing are semantically tied to discursive activity in that their meanings become taken-as-shared in argumentation” (p. 311). The idea that gestures become taken-as-shared meaning allowed Rasmussen et al. to notice that gestures not only became a way for students to communicate their own understanding but were also used for the purpose of argument. During small group discussions, gestures often emerged from students as they communicated their own mathematical understanding. Later during a larger group discussion, a similar gesture was adopted by different students in the class while arguing their own understanding. Rasmussen et al. mentioned that although these gestures were not a representation of formal symbolic mathematics, they served to communicate knowledge with the added dimension of taken-as-shared-meaning amongst the class.

The last article regarding undergraduate mathematics education through an embodied cognition lens that I highlight is the work of Nemirovsky et al. (2021). In this work, the researchers investigated how students interacted with a water wheel, self-drawn diagrams, and

computer-generated graphs, to deepen their understanding of oscillatory motion. The water wheel was designed to allow students to see the motion of the wheel digitally graphed in real-time while they physically manipulate it. Nemirovsky et al. conducted eight interviews with three undergraduate students. The interviews consisted of open-ended tasks (guided by a researcher) that were designed to allow the students to explore their mathematical understanding of oscillatory motion. Nemirovsky et al. highlighted the experiences of one student, Jake, as he interacted with the water wheel. These interactions included Jake's "conversations" with the water wheel, self-drawn diagrams, and computer-generated graphs.

Conversations, in this context, refer to the notion of *plural speech* (Blanchot, 1992), which is the idea that one can have conversations with materials and inscriptions based on information received by interacting with or making changes to the materials or inscriptions. As Jake navigated the tasks, the development of his mathematical understanding was illustrated by the "conversations" he had with the water wheel, diagrams, and graphs. The "conversations" between Jake and the materials replicated a typical conversation, however, no words were exchanged between the actors of the conversation. In this context, "talking" occurred when Jake produced diagrams on a white board or when the computer created digital graphs of the water wheel's motion. "Listening" occurred when Jake felt the water wheel as it moved, or when Jake was puzzled at certain digital graphs. The exchange of information that occurred during Jake's "conversations" with these materials, allowed for a deepened understanding of oscillatory motion that would not have been possible without access to the materials in this study.

## **Summary**

The literature suggests that undergraduate mathematics is primarily taught using lecture (e.g. Fuedel & Panse, 2021; Fukawa-Connelly, 2012; Fukawa-Connelly et al., 2017; Weber,

2004) and that there is a shift to acknowledge students' use of embodiment at both the K-12 level (e.g. Kelton & Ma, 2018; Nathan et al., 2021; Nemirovsky et al., 2013) and the undergraduate level (e.g. Nemirovsky, 2021; Rasmussen et al., 2004) as students engage with mathematics. Moreover, researchers have investigated K-12 mathematics teachers' use of embodiments (e.g. Alibali & Nathan, 2012; Hostetter & Alibali, 2019) in the classroom as well as the use of embodiment by undergraduate mathematics professors (e.g. Stewart et al., 2019; Weinberg et al., 2015) as they teach pure mathematics courses. To date, there are a limited number of studies (e.g., Rasmussen et al., 2004) that investigate professors' use of embodiment as they teach applied mathematics. My research will contribute to this gap. In the next chapter, I will detail my research methods to gather, analyze, and interpret my data.

## METHODS

In this section, I will detail the methods I used in this study. The first subsection provides justification for using a qualitative methodology. The next subsection contains information about the participants and setting for this study. I follow this with a section on data collection and analysis including the steps I took to ensure the validity of my data analysis.

### **Qualitative Method, Research Design and Rationale**

Using an exploratory qualitative research method approach (Creswell & Creswell, 2018), I conducted a case-study, that aligns with previous research (Fukawa-Connelly, 2012; Weber, 2004) to explore how an applied mathematics professor that conducts research on the application of inverse problems and her student's integrated embodiment in their lab discussions. Based on similar studies regarding pure mathematics research (Stewart et al., 2019; Weinberg et al., 2015), I also adopted an embodied cognition lens for my study. Lastly, I used the "storytelling" component of my framework to describe how the professor and her students organized and structured the mathematical content. According to Cresswell and Cresswell, a case study is best suited to understand this phenomenon because it allows me to provide a detailed analysis of the themes that I observed in my data through an embodied cognition storytelling lens.

### *Participants and Setting*

I recruited graduate and undergraduate mathematics students enrolled in an applied mathematics laboratory and the course instructor, Julia Miles, PhD (pseudonym). The laboratory I observed is located at a research institute in the Rocky Mountain region; the laboratory focused on research in electrical impedance tomography (EIT). EIT is a medical imaging technique used to image the body in real-time, utilizing algorithms to render an image. The goal of this

laboratory is to leverage the solutions to inverse problems to refine these reconstruction algorithms that manufacture body images. The rationale for selecting this laboratory was based on the potential to observe the instruction and learning of applied mathematics in a non-traditional classroom setting.

### *Data Collection*

Given this research was exploratory, the primary method for data collection was observation, audio- and videorecording, and field notes of the weekly one-hour laboratory meetings during the fall 2023 semester. Before obtaining IRB approval (see Appendix I), I strictly took field notes during the entirety of the first three laboratory meetings of the semester. After obtaining IRB approval, and written consent of the participants, I audio- and video-recorded the next three laboratory meetings. The recordings were primarily of Dr. Miles because she was the focus of this study, and they allowed me to capture how Dr. Miles engaged the laboratory participants in the learning of applied mathematics and organized the mathematical content. Although Dr. Miles was the primary focus of this research, the contributions made by the students in the laboratory were also captured to recognize their engagement in learning applied mathematics and to convey a complete story of my findings.

### *Data Analysis*

Regarding my data analysis, I utilized the embodied cognition and storytelling components of my framework to guide how I coded my data, as well as to justify the pieces of data I chose to showcase. I used software to transcribe my data verbatim which gave me the ability to use direct quotes within my results section as needed. After transcribing the data, I began my first round of coding. During this first round of coding, I used the storytelling component (Glover, 2023), shown in Table 1, as the overarching piece of my framework to

winnow the data into episodic segments that best showcased mathematical concepts presented as metaphorical components found in traditional storytelling. Once these storytelling segments were selected, I then determined which segments most closely aligned with one related “storyline.” The purpose of choosing one “storyline” was to highlight one complete story from beginning to end, as opposed to disconnected, non-linear story segments.

**Table 1.**

Storytelling components from Glover (2023) that are used to code data.

<b>Storytelling Component</b>	<b>Role in Traditional Storytelling</b>	<b>Metaphorical Role in Mathematics</b>
<b>Message</b>	The purpose or general idea that one wants to get across to the audience	The solution to the problem one is solving
<b>Plot</b>	The sequence of events that the character goes through to achieve their goal	How theorems/definitions are connected and used to solve a problem.
<b>Character</b>	The individuals that drive the plot forward	Individual theorems/definitions that are used while solving the problem
<b>Setting</b>	Where the story takes place	Mathematics course that the problems take place in, as well as the mathematical background of the instructor/students
<b>Conflict/Resolution</b>	How the character(s) struggle to achieve their goal	The struggles that one faces with their understanding when solving a problem and how these struggles are overcome to achieve a solution
<b>Story Arc</b>	The way the story is structured and shaped	The way the results of a problem are presented

After this, I mapped the metaphorical storytelling components of my selected data to the traditional storytelling component that it is most closely related to, along with justification for the selection. The purpose of this “storytelling” round of coding was to attempt to answer my

first research question: How does an instructor and her students organize and present concepts of applied mathematics that align with the elements and structure that is associated with storytelling? Upon completion of the storytelling portion of coding, I then subjected the selected data to the embodiment component of my framework. During this round of coding, I used Nemirovsky and Ferrara's (2009) notion of utterances, to locate every possible example of embodiment that I believed was present within each storytelling segment. I then used Nathan's (2021) framework to determine which form of embodiment was expressed: gesture, body form and movement, simulation, or material epistemology. After identifying each instance of Nathan's type of embodiment, I provided a rich and detailed description of how Dr. Miles and the students showed evidence of embodiment. In my results I provide images and direct quotes from the data that best highlight the embodiment present when deemed necessary. The purpose of these rounds of coding was to attempt to answer my second research question: In what ways do the instructor and her students engage in embodiment as they discuss aspects of applied mathematics?

Appendix II has a sample transcription with respective storytelling and embodiment coding. A transcription, as defined by Fuson and Abrahamson (2005), is "made by transcribing the videotapes and pasting in clips from the videotape to accompany the transcriptions." (p. 224). A transcription also functions to "enable the research team and readers to experience classroom teaching and learning data in a richer way than is usual." (p. 232). In this research, I used transcriptions to present the embodiment present in each storytelling component found in my data, as well as provide the reader with a more immersive experience of the laboratory meetings.

To assure the validity of my data analysis, I conducted member checking with the participants (Creswell & Creswell, 2018). Member checking ensured that I did not misrepresent the thoughts and intentions of my research participants. I also conducted peer debriefing during

my data analysis (Creswell & Creswell). Peer debriefing helped to increase the validity of the results I have reported, because the educated opinions of my peers helped to confirm or deny my interpretation of the data. Both member checking and peer debriefing assisted in my ability to attempt to overcome my inherent biases and help guide my data analysis to the most accurate interpretation possible. The precautions that I used during my data analysis also allowed me to accurately attempt to answer my two research questions with as much validity and reliability as possible. To further increase the validity of my storytelling framework, the next section contains a reinterpretation of previously published literature in mathematics education that embraces embodied cognition. I overlay my storytelling framework on two articles, one of which was previously mentioned in the literature review (e.g. Nemirovsky et al., 2021), while the other was not (e.g. Kelton & Ma, 2020).

### **Storytelling Framework on Existing Literature**

I apply my storytelling framework to the work of Kelton and Ma (2020) and Nemirovsky et al. (2021). Because the work of Nemirovsky et al. has been previously summarized in the literature review of this paper, I only summarize the work of Kelton and Ma in this section and then describe how one can overlay the storytelling framework to each paper while maintaining an embodied cognition lens.

In the work of Kelton and Ma (2020), the researchers were interested in how individuals developed an understanding of unfamiliar, complex geometric objects by walking through large versions of the objects that were constructed for a museum exhibit. The focus of this research is the experience of individuals as they navigated through a torus. Given that a torus was unfamiliar to the participants, Kelton and Ma analyzed the walking patterns of the participants to gain insight into how they began to make sense of the complex object. Under the storytelling

framework, I consider the torus to be a “character” of this “story” as outlined in Table 2. The walking patterns of the participants can be interpreted as their attempt to understand the “message” this “character” tried to deliver. Kelton and Ma noticed that some participants walked the same path multiple times to gain understanding about the torus. Aligning with the storytelling framework, this can be interpreted as the participants’ attempts to “re-read passages” of this “story,” to better understand the “message” of the torus “character.” After the participants left the immersive exhibit, they were introduced to tabletop versions of the torus. This new interaction could be viewed as reading about the same “character,” but from a different “perspective.” For example, this is akin to a traditional story where one reads passages that are written from the perspective of a narrator vs. the perspective of a character. Such different perspectives illuminate different information about the same “plot.” The participants in Kelton and Ma “read” about the torus from one “perspective,” i.e. while they were immersed inside a torus. They then “read” about the same “character” from a different “perspective,” i.e. the tabletop experience. This combination of “perspectives” gave the participants the ability to compare what they understood from “reading” one “perspective” vs the other, which ultimately led to a deeper understanding of the “message” of the torus “character.”

**Table 2.**

Aspects of Kelton and Ma’s (2020) work overlaid with story-telling framework.

<b>Segment</b>	<b>Kelton and Ma (2020)</b>	<b>Forms of embodiment present</b>	<b>Story-telling framework</b>
<b>Segment 1</b>	Walking patterns of participants observed as they attempted to gain insight about the torus	Body form, materialist epistemology	Participants attempted to determine the “message” about the “character”
<b>Segment 1</b>	Participants walked the same paths multiple times to	Body form, materialist epistemology	Participants “re-read passages” to better

	increase understanding about the torus		understand the “plot” of the “character”
<b>Segment 3</b>	Participants moving from the immersive experience of the torus to the tabletop experience	Simulation, materialist epistemology	Participants “read” about the “character” from a different “perspective”
<b>Segment 3</b>	Participants compare what they observed in the immersive experience with the tabletop experience	Simulation, materialist epistemology	Participants compared what they “read” from the different “perspectives” they had access to

I now highlight aspects of the work of Nemirovsky et al. (2021), and how it could potentially be reanalyzed using the storytelling framework as summarized in Table 3. I focus on the episode where Jake produced a diagram regarding velocity vs. time. Nemirovsky et al. discussed how the production of this diagram allowed Jake to have a conversation with it. Instead of considering this interaction with a self-constructed diagram to be a conversation, I consider this to be Jake’s rendering of a “story” involving velocity vs. time, utilizing the storytelling framework. After Jake constructed the diagram, he began to make claims about what he noticed, which could be viewed as Jake “reading” his newly constructed “story.” However, Jake made an incorrect claim about his diagram, specifically regarding where he believed acceleration was zero. Jake’s realization of his mistake, under the storytelling framework, could be considered an instance where Jake misunderstood the “message” of his own “story.” When Jake reanalyzed his diagram, he arrived at the correct notion that acceleration would be zero at relative minimums and maximums on the graph of velocity. This reanalyzing or, according to Nemirovsky et al., conversation, could be viewed as Jake “re-reading” his story to determine the aspects of the ‘plot” he had misinterpreted. This resembles when one re-reads the passage of a book that one did not fully comprehend the first time through.

**Table 3.**

Aspects of Nemirovsky et al.’s (2021) work overlaid with story-telling framework.

<b>Episode</b>	<b>Nemirovsky et al. (2021)</b>	<b>Forms of embodiment present</b>	<b>Story-telling framework</b>
<b>Episode 1</b>	Jake creates and converses with hand-drawn diagram	Materialist epistemology	Jake renders his “story” involving velocity vs time
<b>Episode 1</b>	Jake makes claims about what his self-drawn diagram is telling him	Gesture, body form, simulation	Jake “reads” the “story” he just created, centered around the computer-generated diagram “character”
<b>Episode 1</b>	Jake misinterprets where velocity would be zero	Gesture, body form, simulation	Jake misunderstands the “message” of the “story”
<b>Episode 1</b>	Jake reanalyzes the diagram and comes to the correct notion of when velocity is zero	Gesture, body form, simulation	Jake “re-reads” his “story” to determine aspects of the “plot” he misinterpreted
<b>Episode 4</b>	Jake notices an unexpected “dimple” in the computer-generated diagram of acceleration vs velocity	Gesture, body form, simulation, materialist epistemology	Jake encountered a “conflict” in his understanding of the “story” of acceleration vs velocity
<b>Episode 4</b>	Jake feels the wheel with his hand as the motion is computer-generated on a graph	Gesture, body form, simulation, materialist epistemology	Jake continues to “read” the “story” of acceleration vs velocity to arrive at a “resolution”

In their results, Nemirovsky et al. (2021) point out another critical moment of Jake’s conversation; this is when he and the interviewer added marbles to one part of the water wheel. With the newly added marbles, Jake noticed an unexpected “dimple” in the computer-generated graph of acceleration vs. velocity when the wheel began to turn. This unexpected observation resembles a “conflict” one encounters in a traditional story. Like the “conflict” of a traditional story, where one continues reading in hopes of discovering a “resolution” to this “conflict”, a

parallel situation occurred with Jake, when he decided to feel the wheel with his hand as its motion was graphed on the computer. This new interaction can be thought of as Jake continuing to “read” past the “conflict” of his “story” of interest. As Jake continued to “read,” he was able to conclude these dimples occurred at the three and nine o’clock positions, or where acceleration was the greatest. Thus, Jake’s perseverance through the story allowed him to achieve a “resolution” to this puzzling “conflict.” These are just a very few examples of how the storytelling framework with an embodied lens can be adapted to the work of Nemirovsky et al. (2021). In the next section, I will provide the results of my data analysis.

## RESULTS

In this chapter I present the results of my analysis, illustrating how two distinct storylines emerged. The data I chose to present occurred on the first day I audio- and video-recorded the class. Prior to this day, Dr. Miles gave a brief overview of where the class was headed, and what image reconstruction techniques she wanted the students to focus on. On the day that I recorded, Keith began the class with a presentation that detailed the improvements he made to his image reconstructions. After Keith's presentation concluded, Dr. Miles checked the progress of each participant, as they related to their class contributions. During this meeting, two distinct storylines emerged from two of the class's most experienced students: Keith and Nick. The first storyline occurred during Keith's presentation. I refer to this storyline as Keith's story. The second storyline occurred when Nick described a problem he had with his own data. I refer to this storyline as Nick's story. There are two reasons why I chose to present these storylines. The first reason is that these storylines received the most attention during the first laboratory meeting that I audio- and video-recorded. The second reason is because these two storylines demonstrate different levels of involvement by Dr. Miles. During Keith's story, Dr. Miles allowed Keith to progress through most of his presentation uninterrupted. However, part of the way through his presentation, Dr. Miles began to consistently influence and contribute to the information that Keith shared with the class. Conversely, during Nick's story, Dr. Miles' contributions were much less consistent. Instead, Dr. Miles allowed Nick to guide the direction of the story and only contributed when she needed clarification or had an idea to help overcome a dilemma that Nick expressed.

Recall my first research question: How does an instructor and her students organize and present concepts of applied mathematics that align with the elements and structure that is associated with storytelling? In an effort to answer this question, I will highlight the most significant portions of each storyline. I draw upon my theoretical framework as an outline, and I provide examples of the components that I saw in both Keith and Nick's stories. It is important to note that the order I have chosen to present each component of my framework in this section differs from the order listed in my methods section. The reason for this is because I am presenting these components in the order that they would typically be presented in traditional storytelling (Glover, 2023). Furthermore, my second research question was: In what ways do the instructor and her students engage in embodiment as they discuss aspects of applied mathematics? To attempt to answer this second question, I also discuss which form(s) of embodiment the participants utilized as they discussed each storytelling component. In the next section, I give an outline of the nature of Keith's story. In an effort to document verbiage and embodiment that occurred simultaneously, I sometimes have the description of the embodiment in brackets following the associated verbiage in bold.

### **Keith's Story**

On the first day I audio- and video-recorded, the class meeting began with Keith's presentation of the improvements he had made to his image reconstructions. Keith began his presentation with a projection of a diagram of a machine learning technique, known as a U-Net. After Keith introduced this character, he displayed his first image reconstruction on the screen. This image prompted Nick to ask how Keith produced the image. In response to Nick's question, Dr. Miles began to develop the plot of the displayed image for Nick. This plot included the mention of a new character, the NOSER character. NOSER stands for Newton's One-Step Error

Reconstructor and is a common reconstruction algorithm used in EIT (Cheney et al., 1990). After Nick acquired the important plotlines he was missing, Keith continued with his presentation and showcased the improvements he had made to the reconstructed images. Keith highlighted the story arc of his work when he showed how he could even reproduce image reconstructions that had strange features such as “weird lungs.” Keith concluded by delivering the overall message of his work when he informed Dr. Miles of the details he would include in the paper the class planned to publish. Table 4 outlines the storytelling components of Keith’s story that I have mentioned and will elaborate on further in the coming sections. The table also includes the forms of embodiment that were used to convey understanding within each component.

**Table 4.**

Outline of storytelling components and embodiment present in Keith’s story.

<b>Storytelling Component</b>	<b>Context</b>	<b>Embodiment Present</b>
Characters	Keith details the U-Net character	Materialist Epistemology, Simulation, Gesture
Plot	Dr. Miles informed Nick of the plot for the NOSER character	Gesture, Simulation, Body Form
Story Arc/Message	Keith showed the improved image reconstructions, and discussed plans for the published paper with Dr. Miles	Simulation

**Nick’s Story**

After Keith’s presentation concluded, Dr. Miles went around to each participant and asked how their contributions to the class were going. Most participants gave a brief overview of their progress, except for Nick. Nick took this as an opportunity to ask Dr. Miles how to overcome a problem that he faced in his work. While doing so, Nick introduced the harmonic

character. In Nick’s story, the harmonic character represented the intensity of airflow in a patient. After Nick introduced this character, he described the corresponding setting. He mentioned that there were multiple harmonics in his data but was troubled that some of them were displayed in the wrong place. The presence of this conflict in Nick’s story prompted another student, Timothy, to offer up a potential resolution. Timothy described the approach he thought might work, and the two agreed to meet later to discuss this resolution in more detail. Table 5 outlines the storytelling components of Nick’s story, as well as the forms of embodiment that were present. In the next section I elaborate on the character development that occurred in Keith and Nick’s stories.

**Table 5.**

Outline of storytelling components and embodiment present in Nick’s story.

<b>Storytelling Component</b>	<b>Context</b>	<b>Embodiment Present</b>
Character	Nick introduced the harmonic character	Simulation, Gesture
Setting	Nick described the setting the harmonic character resides in	Simulation, Gesture
Conflict/Resolution	Nick described the problems he had in his data, and Timothy offers up a potential solution	Simulation, Gesture

**Characters**

In the context of my research, the introduction and development of characters are instances where a participant discussed a theorem, or a definition used while making contributions to the class. Just as characters are the individual building blocks that drive a story forward, the same is true with definitions and theorems in mathematics. Furthermore, just as characters need to be introduced and developed, theorems and definitions also necessitate such

introductions and developments. The participants of this study introduced or discussed theorems/definitions by briefly mentioning them or by providing a detailed explanation. For example, during Nick’s story he discussed the current state of his contributions to the class, when he referenced the emergence of the “harmonic” character that he saw in his data. Nick did not give a detailed background for this character, but rather briefly mentioned it while discussing his work. When Nick (third student from left) discussed this character with the rest of the class, he performed a simulated gesture where he brought his hands together, then moved his right hand away from his left as he said “harmonic” as seen in Figure 1.

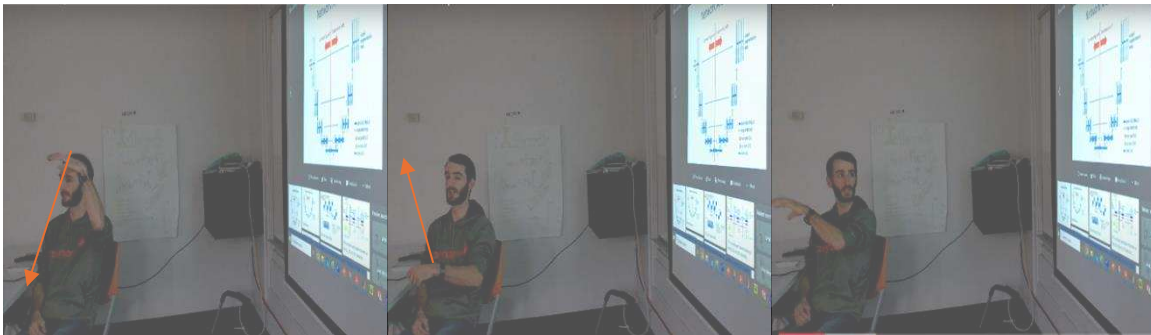


**Figure 1.**

*Nick talks about the harmonics he saw in his data.*

Beyond briefly mentioning characters, the participants also offered detailed character developments, similar to a detailed explanation of a theorem a teacher regularly gives in a lecture. An example of this type of character development was seen in Keith’s story. As Keith talked about the different strategies he used to improve the quality of his image reconstructions; he introduced the “U-Net” character. Rather than briefly mentioning this character, Keith developed this character in depth by pulling up a picture in the form of a diagram of it on the screen. With this diagram visible to his classmates, Keith explained, “There is an **encoding arm** [moved the computer mouse over the encoding arm label], and a **decoding arm** [moved the computer mouse over the decoding arm label].” Keith’s use of the computer mouse functioned to

capture the components of the diagram that he wanted his classmates to focus on. Keith continued to develop the U-Net character when he illustrated how this character differed from similar characters in the context of this class. This explanation was accompanied by Keith's use of a simulated gesture, as seen in Figure 2. When Keith said, "The key thing about a U-Net that differs from other architectures where **it just** [brought left hand up] **compresses** [moved left hand down] and **decompresses** [moved left hand back up]." Keith's simulated gesture was spontaneously created along with his verbal description of a U-Net, to provide a detailed description of this character to the class.



**Figure 2.**

*Keith explains how a U-Net differs from similar characters.*

In both storylines, the introduction of characters was necessary for Keith and Nick to elaborate on other details of their respective stories. I now detail how the participants introduced the setting characteristic of the storyline aspect of my theoretical framework.

## **Setting**

Beyond the emergence of characters, another fundamental aspect to storytelling is the development of the setting for the story. The setting provides context for the story and lets the reader or listener of the story know where it takes place (Glover, 2023). In my data, I looked for instances where the participants described background contextual information that was relevant

to the mathematics discussed. One such instance occurred during Keith’s story when Nick had a question for Dr. Miles, based on the image that Keith presented on the screen. Nick’s question may have indicated that he did not completely understand the storyline setting, which prompted Dr. Miles to provide Nick with an embodied explanation of the setting.

*Nick:* So, are the NOSER reconstructions the inputs?

*Dr. Miles:* So, what was done **here** [both hands held up towards the screen] was there were a whole bunch of training sets that were created using **simulated data** [hands brought back together in front of her] (Figure 3).



**Figure 3.**

*Dr. Miles’ use of gesture to describe the setting to Nick.*

Dr. Miles continued to develop the setting for Nick by stating, “From those, EIT data was created using 16 electrodes just like we would on a **patient in the hospital** [indicated on her torso where the electrodes would typically go on a patient in the hospital], and those reconstructions were completed using NOSER.” (Figure 4). Dr. Miles concluded the development of the setting for Nick by gesturing with her index finger towards the displayed image and stated, “So this was an example using NOSER.”



**Figure 4.**

*Dr. Miles' use of body-form to indicate the position of electrodes on a patient.*

Another instance when the participants discussed the setting occurred during Nick's story, when he described the setting, where the harmonic character resided. Nick indicated that when he displayed his data, there were three harmonics that were graphed. Nick detailed that, "For the first harmonic, you can see **where the lungs are** [held hands in front of him, fingers bent], but for the second and third harmonics, the changes are in **different locations** [held left hand still and moved right hand to different places]." (Figure 5).



**Figure 5.**

*Nick described the setting the harmonic character resides in.*

The setting, in this context, was an important component for each story, because it provided the details of the specific areas that Keith and Nick work on regarding EIT. Although each participant in this class worked in EIT, their respective settings had important differences. When Keith and Nick provided the setting, it functioned to highlight these differences for the rest

of the class. After the characters were introduced, and the setting was established, Keith and Nick began to discuss the plot of their respective stories. The most complete plot development was illustrated by Dr. Miles during Keith’s story; thus, I will highlight this development in the next section.

## Plot

In traditional storytelling, the plot is the sequence of events that characters go through to achieve their goal (Glover, 2023). Similar to characters in traditional stories, the characters of mathematics (theorems or definitions) go through a sequence of events. A difference is that in this context, the events a theorem/definition experiences involve their implementation into a mathematics problem. After Dr. Miles established the setting for Nick when he questioned the source of the image Keith had presented on the screen, she detailed the plot of the character known as a NOSER reconstruction. This plot development began when Dr. Miles produced a framing gesture with both hands towards the screen, and stated “Okay, so we took **this stuff** [made framing gesture with both hands held towards the screen] and then we mapped it to the 64x64 grid.” (Figure 6).



**Figure 6.**

*Dr. Miles illustrated two holding gesture instances.*

Dr. Miles continued to develop the plot for the NOSER character when she uttered, “Okay, so all of **those** [brought hands up in front of her] **NOSER reconstructions** [brought hands down in front of her] (Figure 7) and all their **true conductivities** [raised left hand up then

brought back down] (Figure 8) were used to form the training set for **this** [pointed at the displayed image behind Keith] post-processing machine learning.” The plot development by Dr. Miles showcased the sequence of events the NOSER reconstruction character experienced to produce the image Keith had presented on the screen. This was important for the rest of the class, because with only the presented image available to them, the students in the class would not have understood how this image came to be. In traditional storytelling, once the plot has been developed there is the potential for a conflict to arise. The presence of a conflict motivates the reader/listener of the story to continue, in hopes of arriving at a satisfying resolution. One such instance of conflict/resolution occurred during Nick’s story, which I will detail in the next section.



**Figure 7.**

*Dr. Miles’ simulated hands up then moved down gesture.*



**Figure 8.**

*Dr. Miles moved left hand up, then back down.*

## Conflict/Resolution

Within any great story, there is always the presence of a conflict, and with any conflict there is typically a resolution that ensues. In traditional storytelling, this takes the form of a character's struggle to achieve their goal (Glover, 2023). In my study, conflict/resolution took the form of a challenge the participants faced with their understanding of the application of the mathematics, and the strategies they attempted to overcome these challenges. One key example of conflict/resolution occurred during Nick's story. Nick described how when a patient breathes, the harmonics that represent the airflow intensity of the patient are in different locations other than the lungs. This explanation was accompanied by the same simulated gesture that Nick produced when he illustrated the setting for the harmonic characters, with his hands held in front of him (third from left in Figure 9). Nick then elaborated on this simulated gesture when he moved his right hand around his left and said, "When they are breathing, there are some other harmonics there so, I want to figure out how to quantify spatially that's **related to the pathology** [hands moved down onto table]" (Figure 10).



**Figure 9.**

*Nick's repeated simulated gesture.*



**Figure 10.**

*Nick moved right hand around left, then moved both hands onto the table.*

Nick’s description of his conflict with the location of the harmonics prompted Timothy to offer a potential resolution. Timothy stated, “You could do a prediction from the **mean to the variance** [pressed hands together then pulled apart]” (second from right Figure 11). Based on the **structure**, [held out hands in front of him] (Figure 12) you can do a truncation after a **certain degree of change** [spread hands apart from each other] (Figure 12)”. Nick seemed optimistic that Timothy’s suggested resolution might help him overcome the conflict he faced in his work. Timothy then offered to get together with Nick, to explore how they might attempt to overcome this conflict in more detail.



**Figure 11.**

*Timothy’s simulated gestures.*



**Figure 12.**

*Timothy's simulated gesture of hands held in front of him, and hands spread apart.*

Each component that I have illustrated thus far are important pieces that contribute to the overall goal of the story, and function to develop the story arc and message which are the final two components of my storytelling framework. I will finish up the results of this study by discussing the presence of these components in Keith's story.

### **Story Arc and Message**

I chose to group these two components together because they are closely related, as they pertain to my storytelling framework. The message in traditional storytelling is the purpose or general idea that one wants to convey to the audience (Glover, 2023); whereas the story arc is the way the story is structured and shaped (Glover). In my data, the message took the form of the participants' final solution to the mathematical problem. A story arc took the form of how the participants planned to present the results of their work in the paper they wanted to publish. Regarding message, the most prominent example of this occurred during Keith's story, when he began to show how well his model performed. Keith pulled up a reconstructed image that showed how his model was able to handle a "weird case" where the patient was "on a tilt and had weird lungs" (Figure 13). Keith followed up this reconstructed image with another

case where the left lung was not visible, and explained how his model was able to capture this (Figure 14). This prompted Dr. Miles to exclaim, “Wow, this model is killing it!”. Keith replied, “Yes, this model is doing fantastic!” This indicated that Keith and Dr. Miles achieved the message that they had hoped to convey.



**Figure 13.**

*Reconstructed image where patient was tilted, and lungs were “weird”.*



**Figure 14.**

*Case where model captured the missing left lung.*

Finally, regarding the story arc of these data, Keith ended his presentation and described how he planned to present these results in the future paper. Keith explained that in the paper he would, “make sure that the images are the correct shape.” Dr. Miles replied, “Yes, the image shape is extremely important.” Keith added, “I will also add a montage of images to the paper.” This conversation showcased how the participants of the class wanted their message and story

arc to take shape in the paper they planned to publish. The storytelling components of my framework were prominent in both Keith and Nick's stories. Furthermore, when the participants illustrated these components, they often relied on the use of embodiment to share knowledge with the rest of the class. In the next chapter, I will provide a discussion of the results of my research including implications for teaching and research, and limitations of my work.

## DISCUSSION

In this section, I will discuss how my results contribute to mathematics education research and provide teaching implications. I also delineate limitations and offer future research endeavors. Based on my data analysis, my results suggest that the instructor and her students organically presented applied mathematics concepts in ways that align with the elements and structure found in traditional storytelling. I discovered that each storytelling component of my framework was present in the data. Furthermore, my results also suggest that the participants in this study exhibited embodiment as they presented applied mathematical concepts within each storytelling component. As part of my discussion, I will synthesize how my work contributes to literature related to embodied cognition, specifically how it extends the scope from pure mathematics to applied mathematics. I will also present how this work relates to implications for teaching, and finally I will consider the limitations of this research.

### **Research Implications**

Many of us have experienced story problems in mathematics, where the math is interwoven into the story. However, if we consider mathematics itself to be the actual story, there is the potential to explore new ways of structuring mathematical concepts in a different and engaging way. Furthermore, if we leverage the power that embodiment possesses to engage learners and deepen conceptual understanding, then we potentially have the means to make our mathematical stories come alive, especially when they are about humans. In this research, I discovered that the participants naturally presented information about applied mathematics that aligned with elements found in traditional storytelling. Although the participants did not plan their stories ahead of time, the stories naturally emerged. A novel discovery in this research was

the participants' use of embodiment as they presented their stories. What made this discovery surprising was that in this specific area of applied mathematics, the participants were working with data generated from other real humans. It has been documented in previous literature regarding embodied cognition (e.g. Kelton & Ma, 2018; Nathan et al., 2021; Nemirovsky et al., 2013) that embodiment happens subconsciously when dealing with abstract mathematics, however in this case the participants subconsciously displayed embodiment that represented other human beings. It appeared as though the participants would visualize the individuals behind the data, and generated forms of embodiment based on that visualization. In the next section, I will discuss the contributions this research makes to the literature on storytelling.

### *Storytelling Contributions*

One purpose of this study was to explore how an applied mathematician presented concepts in an applied mathematics class that align with the elements and structure associated with “storytelling”. My results indicate that, although the participants did not plan to present applied mathematics in the form of a story, two distinct stories naturally emerged: Keith’s and Nick’s. Throughout the course of this study, I noticed that the participants consistently presented information that aligned with elements found in traditional storytelling, starting with the introduction and development of characters. Characters are what drive a story forward, and they are what make the listener/reader care about and invest in the story. The importance of character development was also evident in my results. For example, in Keith’s story he introduced and developed the “U-Net” character. He took the time to describe how this character functioned, and the role it played in the model he had created. While doing so, Keith highlighted the important components of this character, when he projected a diagram of it to the class. Similar to the work of Fukawa-Connelly (2012) when Dr. Tripp made use of funneling questions (Steinberg, 1989)

to narrow her students' focus on important aspects of a proof, Keith narrowed his classmates' focus on important aspects of the U-Net character. With the use of the computer mouse, Keith drew the attention of the class to the pieces of the U-Net character he described. Since this seemed to be a relatively unknown character to the rest of the class, the development of this character appeared to be important to deepen the overall understanding for the class, regarding Keith's model, and allowed him to progress through his story.

The setting is another key component found in storytelling, because it functions to give the listener/reader important context for the story. The same seemed to be true for the participants in this study. Only in this instance, the setting did not include time or location, but rather, the relevant mathematical context. One such example occurred during Keith's story when he presented a reconstructed image on the screen. Nick seemed to lack an understanding of the mathematical context that this image was created in, because he asked Dr. Miles, "So, are the NOSER reconstructions the inputs?" Dr. Miles then proceeded to paint the scene of the reconstruction for Nick. This gave him the necessary context to understand the setting for the reconstructed image. Without the guidance of Dr. Miles through this setting, it seemed as though Nick would have struggled to completely understand the message and story arc of Keith's story.

With the characters and setting in place, one can then proceed with the plot of the story. In traditional storytelling, this takes the form of the sequence of events the characters go through to achieve their goal. In the context of this study, the plot takes the form of the events a definition/theorem goes through pertaining to their implementation into a mathematical problem. After Dr. Miles gave Nick the necessary contextual information to understand the image presented on the screen, she was then able to elaborate upon the plot of the NOSER character. During this explanation, Dr. Miles maneuvered between the formal and informal mathematics

that were involved. Similar to the work of Fukawa-Connelly et al. (2017), Dr. Miles primarily presented the informal mathematics with the use of speech and gesture, whereas she presented the formal mathematics while referencing the images on the screen. It appeared that even though this plot development was improvised by Dr. Miles, there was a clear navigation between the formal and informal mathematics involved and seemed to give a deeper meaning to the events the NOSER character went through to create the reconstructed image. The natural emergence of each storytelling component highlighted the relationship between telling a well-crafted story and presenting applied mathematics concepts. In the next section, I will present the contributions this research makes to the literature regarding embodiment.

### *Embodiment Contributions*

Beyond the notion of storytelling, this research also aimed to extend the existing literature regarding embodied cognition in undergraduate pure mathematics, to that of applied mathematics. Alibali and Nathan (2012) argued that “mathematics is embodied in two key senses: it is based in perception and action, and it is grounded in the physical environment” (p. 247). Since work in EIT is concerned with imaging the body, it seemed as though it was natural for the participants to use their bodies to express and share knowledge. For example, when Dr. Miles described the setting for Nick, she used her own torso to highlight where the electrodes would typically go on a patient in the hospital. Alibali and Nathan also claim that there exists a threshold where the use of words is not enough to convey understanding, and the speaker then spontaneously creates a simulated action in the form of a representational gesture. There was evidence of this spontaneous gesture creation throughout this study. One such example occurred when Nick introduced the harmonic character. It appeared that Nick passed through the threshold where words alone were not enough to convey understanding, and he spontaneously produced a

representational gesture that was coordinated with his utterance of “harmonic.” Furthermore, Nathan et al. (2021) suggested that there is a potential reciprocal relationship between one’s body movement and mathematical reasoning, and the mind can express this reasoning through the body. The results of this study provide evidence of this type of embodied expression, such as when Nick discussed the conflict in his work and Timothy suggested a potential resolution. Nick expressed the conflict the harmonic character faced when he utilized the gesture he had established for “lungs,” and then expanded upon it when he moved his left hand around his right, to indicate the location of the harmonic. Timothy expressed his potential resolution when he coordinated his utterance of “mean to the variance” with the representational gesture of his hands pressed together, then pulled apart. It seemed as though the participants of this study consistently passed through the gesture threshold, described by Alibali and Nathan, and expressed mathematical reasoning through their bodies.

Nemirovsky et al. (2013) highlighted the notion of plural speech, where one can have conversations with materials and inscriptions based on the information received by interacting with or making changes to the materials or inscriptions. I observed the concept of plural speech in my results, only in this context, the materials took the form of the reconstructed EIT images that were generated from the torsos of actual human beings. When Keith presented the images his model reconstructed, he described how well his model had performed. Dr. Miles observed the images as well and echoed the sentiment that the model had performed well. The images were able to “tell” both Keith and Dr. Miles that Keith’s model was working correctly. In other words, the real patient’s torso communicated the truth to Dr. Miles and Keith. Keith and Dr. Miles’ ability to have this conversation with the images relates back to Nemirovsky et al.’s (2013) notion of tool fluency with a mathematical instrument. Reconstructed EIT images depict a cross-

section of an individual's torso, and display simplified features such as the lungs, heart, spine, and ribcage. To the untrained eye, it may not appear that the images Keith presented were exceptional reconstructions. However, Dr. Miles and Keith have extensive experience with these images. They have developed the tool fluency necessary to recognize when the image is well-reconstructed, versus when it is poorly reconstructed. With this tool fluency, Dr. Miles and Keith can "listen" to what EIT images can "tell" them and decide if they would like to "say" something different (change aspects of the image) or if the conversation is over. Since these EIT images were generated by actual humans, it appeared as if Dr. Miles and Keith were not only "listening" to the images but what the patients themselves were "saying". The results of this study illustrated the participants' use of embodiment as they presented the applied mathematical content within each storytelling component. In the next section, I will provide potential teaching implications for this research.

### **Teaching Implications**

My study suggests that applied mathematics concepts have the potential to be organized and structured like a story and that they can be taught/understood using embodiment. Furthermore, my work suggests that there is a connection between embodiment and storytelling: embodiment can play a role in presenting each component within the mathematical story. Although the participants in this study did not plan to tell a story, they did so without knowing it. Only in this context, they did not tell a linear story from start to finish. Instead, they utilized the different storytelling components as needed. The benefit of having a particular story in mind could potentially allow instructors a different way to organize their lectures. A well-crafted story brings one in; it can engage and enthrall the learner. With the use of a well-crafted story, there is the potential to provide an engaging presentation of mathematics to students. Furthermore, with

the idea of storytelling in place, it can motivate instructors to determine what kind of story they want to tell. Whether it be a re-telling (perhaps a review session) or a mystery (have students discover the answer on their own), the structure of each story can change dramatically, based on the overall narrative goal. However, even the best stories can seem uninteresting and unengaging when they are presented in an uninspiring fashion. That is where the concept of embodiment comes into play. Embodiment has the potential to make each piece of the story come alive. The appropriate and effective use of embodiment may not only make each mathematical “story” more engaging, but it can also deepen students’ conceptual understanding of mathematics. In the final section, I will discuss the limitations of this research as well as provide potential future research endeavors to explore.

### **Limitations and Future Work**

I recognize that this study was limited to a non-traditional applied mathematics environment because it took place in a lab. One can adapt this research to investigate the notion of storytelling in a traditional classroom setting, perhaps comparing the notion of storytelling to traditional lecture methods as it pertains to student engagement. Furthermore, applied mathematics is similar to pure mathematics in the sense that both disciplines promote the understanding of abstract concepts. This correlation between applied and pure mathematics may lead one to investigate the use of storytelling in a pure mathematics classroom as well. Moreover, the participants of this study seemed to only tell their stories from an outside perspective, which may lead one to investigate the role perspective plays when telling a mathematical story. Another line of inquiry that could be followed is to investigate the role that storytelling plays in promoting conceptual understanding. Although storytelling possesses the

potential to deepen conceptual understanding, this research cannot claim that storytelling deepens conceptual understanding for mathematics.

## REFERENCES

- Alibali, M. W., Boncoddò, R., & Hostetter, A. B. (2014). The role of gesture in reasoning and problem solving. In L. Shapiro (Ed.), *Routledge handbook of embodied cognition* (pp. 150–159). New York, NY: Routledge.
- Alibali, M. W., & Nathan, M. J. (2012). Embodiment in Mathematics Teaching and Learning: Evidence from Learners' and Teachers' Gestures. *The Journal of the Learning Sciences*, 21(2), 247–286. <http://www.jstor.org/stable/23266406>
- Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics*, 70, 97–109.
- Bucciarelli, M., Mackiewicz, R., Khemlani, S. S., & Johnson-Laird, P. N. (2016). Children's creation of algorithms: Simulations and gestures. *Journal of Cognitive Psychology*, 28, 297–318. <https://doi.org/10.1080/20445911.2015.11.34541>
- Châtelet, G. (2000/1993). *Les enjeux du mobile*. Paris: Seuil. (Engl. transl., by R. Shore & M. Zagha: *Figuring space: Philosophy, Mathematics and Physics*). Dordrecht: Kluwer Academy Press, 2000.
- Cheney, M., Isaacson, D., Newell, J. C., Simske, S., & Goble, J. (1990). NOSER: An Algorithm for Solving the Inverse Conductivity Problem. *International journal of imaging systems and technology*, 2(2), 66–75. <https://doi.org/10.1002/ima.1850020203>
- Chu, M., & Kita, S. (2011). The nature of gestures' beneficial role in spatial problem solving. *Journal of Experimental Psychology: General*, 140, 102–116. <https://doi.org/10.1037/a0021790>

- Cuoco, A. (2001). Mathematics for teaching. *Notices of the American Mathematical Society*, 48, 168–174
- Davis, P. J., & Hersh, R. (1981). *The mathematical experience*. New York: Viking Penguin Inc.
- de Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge University Press.
- Dietiker, L. (2015). Mathematical story: a metaphor for mathematics curriculum. *Educ Stud Math* 90, 285–302. <https://doi.org/10.1007/s10649-015-9627-x>
- Dreyfus, T. (1991). Advanced mathematical thinking processes. In: D. Tall (Ed.), *Advanced mathematical thinking* (pp. 25–41). Dordrecht: Kluwer
- Feudel, F., & Panse, A. (2022). Can Guided Notes Support Students' Note-taking in Mathematics Lectures? *International Journal of Research in Undergraduate Mathematics Education*, 8(1), 8–35. <https://doi.org/10.1007/s40753-021-00146-9>
- Fukawa-Connelly, T. (2012). A case study of one instructor's lecture-based teaching of proof in abstract algebra: making sense of her pedagogical moves. *Educational Studies in Mathematics*, 81(3), 325–345. <https://doi.org/10.1007/s10649-012-9407-9>
- Fukawa-Connelly, T., Weber, K., & Mejía-Ramos, J.P. (2017). Informal Content and Student Note-Taking in Advanced Mathematics Classes. *Journal for Research in Mathematics Education*, 48(5), 567–579. <https://doi.org/10.5951/jresmetheduc.48.5.0567>
- Fuson, K. C., & Abrahamson, D. (2005). Understanding ratio and proportion as an example of the Apprehending Zone and Conceptual-Phase problem-solving models. In *The Handbook of Mathematical Cognition* (pp. 213-234). Psychology Press.

- Gerofsky, S. (2010). Mathematical learning and gesture: Character viewpoint and observer viewpoint in students' gestured graphs of functions. *Gesture*, 10, 321–343.  
<https://doi.org/10.1075/gest.10.2-3>. 10ger
- Glover, J. (2023). The Essential Elements of Storytelling. *Sketches & Storytelling - Art, Illustration, and Storytelling*.  
<https://sketchesandstorytelling.com/2025/03/07/the-essential-elements-of-storytelling/>
- Hostetter, A. B., & Alibali, M. W. (2008). Visible embodiment: Gestures as simulated action. *Psychonomic Bulletin & Review*, 15, 495-514.
- Hostetter, A. B., & Alibali, M. W. (2019). Gesture as simulated action: Revisiting the framework. *Psychonomic Bulletin & Review*, 26(3), 721–752.  
<https://doi.org/10.3758/s13423-018-1548-0>
- Kelton, M. L., & Ma, J. Y. (2018). Reconfiguring mathematical settings and activity through multi-party, whole-body collaboration. *Educational Studies in Mathematics*, 98(2), 177–196. <https://doi.org/10.1007/s10649-018-9805-8>
- Lockhart, P. (2009). *A Mathematician's Lament*. United States: Bellevue Literary Press.
- Nathan, M. J. (2021). *Foundations of Embodied Learning: A Paradigm for Education*. Routledge. <https://doi.org/10.4324/9780429329098>
- Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies of Mathematics*, 70(2), 159–174
- Nemirovsky, R., Kelton, M. L., & Rhodehamel, B. (2013). Playing Mathematical Instruments: Emerging Perceptuomotor Integration With an Interactive Mathematics Exhibit. *Journal for Research in Mathematics Education*, 44(2), 372–415.  
<https://doi.org/10.5951/jresematheduc.44.2.0372>

- Patton, M. (2015). *Qualitative Research and Evaluation Methods*. 4th Edition, Sage Publications, Thousand Oaks.
- Rasmussen, C., Stephan, M., & Allen, K. (2004). Classroom mathematical practices and gesturing. *The Journal of Mathematical Behavior*, 23(3), 301–323.  
<https://doi.org/10.1016/j.jmathb.2004.06.003>
- Sfard, A. (2018). On the need for theory of mathematics learning and the promise of “commognition.” In P. Ernest (Ed.), *The philosophy of mathematics education today* (Vol. 13, pp. 219–228). essay, Springer International Publishing.
- Steinbring, H. (1989). Routine and meaning in the mathematics classroom. *For the Learning of Mathematics*, 9(1), 24–33.
- Stephan, M., & Rasmussen, C. (2002). Classroom mathematical practices in differential equations. *Journal of Mathematical Behavior*, 21, 459–490.
- Stewart, S., Troup, J., & Plaxco, D. (2019). Reflection on teaching linear algebra: Examining one instructor’s movements between the three worlds of mathematical thinking. *ZDM*, 51, 1253–1266. <https://doi.org/10.1007/s11858-019-01086-0>
- Streeck, J. (2009). *Gesturecraft: The manu-facture of meaning*. John Benjamins Publishing Company. <https://doi.org/10.1075/gs.2>
- Tall, D. O. (2013). *How humans learn to think mathematically: Exploring the three worlds of mathematics*. Cambridge: Cambridge University Press.
- Thurston, W. P. (1994). On proof and progress in mathematics. *Bulletin of the American Mathematical Society*, 30(2), 161–177. doi:10.1090/S0273-0979-1994-00502-6.  
 arXiv:math/9404236.

Weber. (2004). Traditional instruction in advanced mathematics courses: a case study of one professor's lectures and proofs in an introductory real analysis course. *The Journal of Mathematical Behavior*, 23(2), 115–133. <https://doi.org/10.1016/j.jmathb.2004.03.001>

Weinberg, A., Fukawa-Connelly, T., & Wiesner, E. (2015). Characterizing instructor gestures in a lecture in a proof-based mathematics class. *Educational Studies in Mathematics*, 90(3), 233–258. doi: 10.1007/s10649-015-9623-1

# APPENDIX I

## IRB Approval

12/2/24, 12:51 PM

Protocols

PROTOCOLS

**kuali**



**COLORADO STATE  
UNIVERSITY**

**The protocol listed below has been approved by the CSU IRB Determinations Fort Collins on Tuesday, October 17th 2023.**

PI: Soto, Hortensia

Submission Type and ID: Initial 5010

Title: Developing Curriculum for Applied Mathematics Courses Through an Embodied Lens

Approval Date: Tuesday, October 17th 2023

Expiration Date: Tuesday, October 17th 2028

The CSU IRB (FWA0000647) has completed its review of protocol 5010 Developing Curriculum for Applied Mathematics Courses Through an Embodied Lens. In accordance with federal and state requirements, and policies established by the CSU IRB, the committee has approved this protocol under Exempt review.

Any additional comments regarding this approval are included below. If you have additional questions about this please contact [IRB Staff](#).

The CSU IRB (FWA0000647) has completed its review of the above-referenced protocol. In accordance with federal and state requirements, and policies established by the CSU IRB, the designated reviewer has determined your project is exempt under 45CFR46.104(d). Although this research is considered exempt, CSU expects that researchers continue to follow moral and ethical obligations of research conduct, by honoring the principles in the Belmont Report.

Exempt Category 2: Research that only includes interactions involving educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures, or observation of public behavior (including visual or auditory recording) if at least one of the following criteria is met:

(ii) Any disclosure of the human subjects' responses outside the research would not reasonably place

<https://colostate.kuali.co/protocols/protocols/651b42a36f16a800260e8919/correspondence/652eeb1863b2410029708598>

1/2

the subjects at risk of criminal or civil liability or be damaging to the subjects' financial standing, employability, educational advancement, or reputation.

(iii) The information obtained is recorded by the investigator in such a manner that the identity of the human subjects can readily be ascertained, directly or through identifiers linked to the subjects, and an IRB conducts a limited IRB review to make the determination required by §46.111(a)(7).

Please note, that any changes to the proposed research should be reported to the IRB via an amendment prior to implementation.

If you have any questions, please contact the CSU IRB.

Thank you,  
CSU\_IRB@colostate.edu

**Please note:**

Any additional changes to this approved protocol must be obtained prior to implementation of those changes, by submitting an amendment request to the CSU IRB for review/approval.




Good luck in your research endeavors!

## Attachments

Consent	PhotoVideoReleaseForm.docx	Audio and Video Release Form
Methodology Section	Research Methodology.pdf	Research Methodology
Recruitment Materials	Email Recruitment Script (2).docx	Email Recruitment Script
Consent	IRB_SignedConsent.docx	Participant Signed Consent
Consent	IRB_SignedConsent_Dr_Mueller.docx	Signed consent for Jennifer Mueller, PhD
Other	Group_2_Training.pdf	Tyler Stephens - Group 2 Training

## APPENDIX II

### Sample Transcription

Snapshot	Dialogue	Embodiment
 <p>A video frame showing a student sitting in a chair, pointing his right hand upwards towards a large screen displaying a diagram. An orange arrow is overlaid on the image, pointing from the student's hand to the top of the screen.</p>	<p>Student K: <b>And the key thing about a U-Net that is different from other architectures where it just...</b></p>	<p>Gesture/Simulation: brings hand up.  (1)</p>
 <p>A video frame showing the same student pointing his right hand downwards towards the screen. An orange arrow is overlaid, pointing from the student's hand to the bottom of the screen.</p>	<p><b>...compresses...</b></p>	<p>Gesture/Simulation: moves hand down.  (2)</p>
 <p>A video frame showing the student pointing his right hand in a different direction, towards the left side of the screen. An orange arrow is overlaid, pointing from the student's hand to the left edge of the screen.</p>	<p><b>... and decompresses.</b></p>	<p>Gesture/Simulation: moves hand back up in different direction.  (3)</p>