

Outline

- Randomness + Probability
- Investment Performance
- Different Type of Investment
- Managing Risk
- Long-Run Return
- Volatility Drag



Dice Game

Games, Gains and Gambling



Dice Investments

- V_t is the value at the end of year t
- Gross return: $R_t = \frac{V_t}{V_{t-1}}$



Gross Returns for Each Dice			
Outcome	Green	Red	White
1	0.8	0.05	0.95
2	0.9	0.2	1
3	1.1	1	1
4	1.1	3	1
5	1.2	3	1
6	1.4	3	1.1



Gross Returns

- Example: If green rolls 1, then the gross return is 0.8. i.e., each dollar invested falls to \$0.8 (a 20% drop)
- Example: If red rolls 1, the gross return is 0.05 (a 95% drop)

Round	Multiplier			Value			Gross Returns for Each Dice			
	Green	Red	White	Green	Red	White	Outcome	Green	Red	White
Start				\$1,000	\$1,000	\$1,000	1	0.8	0.05	0.95
1	0.9	3	1	900	3,000	1,000	2	0.9	0.2	1
2	1.1	0.2	1.1	990	600	1,100	3	1.1	1	1
3							4	1.1	3	1
							5	1.2	3	1
							6	1.4	3	1.1



Scenarios

- Run for 20 years
- Work in teams of 2
- One person rolls the dice
- 2nd person keeps track of the dice and reads o the values
- Which of the investments has the largest value after 20 years?
- Did you pick this investment initially?



Green: Annual Percentage Return

Probability Model for Green Dice

G	-20	-10	10	20	40
$P(G)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\mu_G = (-20) \cdot \frac{1}{6} + (-10) \cdot \frac{1}{6} + (10) \cdot \frac{1}{3} + (0) \cdot \frac{1}{6} + (40) \cdot \frac{1}{6} = 8.3$$

$$\sigma_G^2 = (-20 - 8.3)^2 \cdot \frac{1}{6} + (-10 - 8.3)^2 \cdot \frac{1}{6} + (10 - 8.3)^2 \cdot \frac{1}{3} + (0 - 8.3)^2 \cdot \frac{1}{6} + (40 - 8.3)^2 \cdot \frac{1}{6} = 380.6$$

$$\sigma_G = 19.5$$

Gross Returns for Each Dice			
Outcome	Green	Red	White
1	0.8	0.05	0.95
2	0.9	0.2	1
3	1.1	1	1
4	1.1	3	1
5	1.2	3	1
6	1.4	3	1.1



Red: Annual Percentage Return

Probability Model for Red Dice

R	-95	-80	0	200
$P(R)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

$$\mu_R = (-95) \cdot \frac{1}{6} + (-80) \cdot \frac{1}{6} + (0) \cdot \frac{1}{6} + (200) \cdot \frac{1}{2} = 70.8$$

$$\sigma_R^2 = (-95 - 70.8)^2 \cdot \frac{1}{6} + (-80 - 70.8)^2 \cdot \frac{1}{6} + (0 - 70.8)^2 \cdot \frac{1}{6} + (200 - 70.8)^2 \cdot \frac{1}{2} = 17553.5$$

$$\sigma_R = 132.5$$

Gross Returns for Each Dice			
Outcome	Green	Red	White
1	0.8	0.05	0.95
2	0.9	0.2	1
3	1.1	1	1
4	1.1	3	1
5	1.2	3	1
6	1.4	3	1.1



White: Annual Percentage Return

Probability Model for White Dice

W	-5	0	10
$P(W)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

$$\mu_W = (-5) \cdot \frac{1}{6} + (0) \cdot \frac{2}{3} + (10) \cdot \frac{1}{6} = 0.8$$

$$\sigma_W^2 = (-5 - 0.8)^2 \cdot \frac{1}{6} + (0 - 0.8)^2 \cdot \frac{2}{3} + (10 - 0.8)^2 \cdot \frac{1}{6} = 20.1$$

$$\sigma_W = 4.5$$

Gross Returns for Each Dice			
Outcome	Green	Red	White
1	0.8	0.05	0.95
2	0.9	0.2	1
3	1.1	1	1
4	1.1	3	1
5	1.2	3	1
6	1.4	3	1.1



Which Investment is the Best?

- Which of the three investments is most attractive to you?
- Comments:
 - Red is best if you only care about what happens on average
 - Average return on Red is 8.5 times that for Green
 - SD for red is 6.6 times larger than the SD for Green

Comparison

Investment	Expected Annual Percentage Change	SD of Annual Percentage Change
Green	8.3%	19.5%
Red	70.8%	132.5%
White	0.8%	4.5%



Yet Another Investment: Pink

- Consider an investment portfolio: Mix Red and White
- Calculate the multiplier by averaging the multipliers obtained for red and white

Pink

Round	Multiplier			Value			Pink	
	Green	Red	White	Green	Red	White	Multiplier	Value
Start				\$1,000	\$1,000	\$1,000		1000
1	0.9	3	1	900	3,000	1,000	2	2000
2	1.1	0.2	1.1	990	600	1,100	0.65	1300
3								

- Question: How do you expect Pink to perform? Will it be better or worse than the others?
- Calculate the multipliers and values for Pink



Pink

- Did Pink perform as well as you expected?

$$E[\text{Pink}] = E\left(\frac{R+W}{2}\right) = \frac{E[R]+E[W]}{2} = \frac{1.71+1.008}{2} = 1.36$$

$$\text{Var}(\text{Pink}) = \text{Var}\left(\frac{R+W}{2}\right) = \frac{1}{4}\text{Var}(R+W) = \frac{1}{4}(\text{Var}(R)+\text{Var}(W))$$

$$\text{Var}(\text{Pink}) = \frac{1.32^2+0.04^2}{4} = 0.436$$

If R and W are independent, $\text{Var}(R+W) = \text{Var}(R) + \text{Var}(W)$

No matter what, $E[R+W] = E[R] + E[W]$

- Were the dice rolls for red and white independent?



Pink

Comparison

Investment	Expected Annual Percentage Change	SD of Annual Percentage Change
Green	8.3%	19.5%
Red	70.8%	132.5%
White	0.8%	4.5%
Pink	35.5%	66.3%



Rebalancing

- For Pink to have this smaller variance we need to maintain the balance between Red and White.
- Example: In the first round, Red goes up by 3 and White stays the same.
- Initially our investment was \$500 in Red and \$500 in White.
- After the first year we have \$1500 in Red and \$500 in White.
- Before the second roll, we have to rebalance: \$1000 in Red and \$1000 in White
- In the second year, Red loses 80% and White goes up by 10%.
- So, we have \$200 in Red and \$1100 in White = \$1300
- If we didn't rebalance, we would have lost 80% of \$1500.



Rebalancing

- Rebalancing a portfolio means periodically dividing the total value among the components
- Rebalancing reduces risk
- Most investors forget to rebalance



Volatility

- Suppose when you graduate you earn a salary of \$100,000.
- Things go well and you get a 10% raise. Now your salary is \$110,000.
- The following year, business is bad and you take a 10% cut.
- Now your salary is \$99,000.
- Your average percentage change is zero, but your salary went down.
- Variance reduces gains and is called volatility



Volatility

- Variation in the returns on the value of an investment is called the volatility drag.
- Volatility drag = $\frac{1}{2}$ (Annual Variance)
- Long-Run Return = Expected Annual Return - Volatility Drag = Expected Annual Return - (Annual Variance)/2

Volatility Drag

Color Die	E[Return]	Variance	Long-Run Return
Green	0.083	$0.20^2 = 0.04$	$0.083 - 0.04/2 = 0.063$
Red	0.708	$1.32^2 = 1.7424$	$0.71 - 1.7424/2 = -0.161$
White	0.008	$0.04^2 = 0.0016$	$0.008 - 0.0016/2 = 0.0072$
Pink	0.035	$0.66^2 = 0.4356$	$0.355 - 0.4356/2 = 0.1372$



Comments

- Green models the gross returns of the stock market
- White models the returns on Treasuries.
- Red is similar to Start up company
- Red has potential to make a lot of money if you're lucky.
- $1000(1.71)^{20}=45,000,000$, but it's not likely.



Comments

- Pink has the largest long run return
- Pink did well because we rebalanced every year, reducing volatility
- Rolls of dice are independent, returns on two investments may not be



Dice Game by Simulation



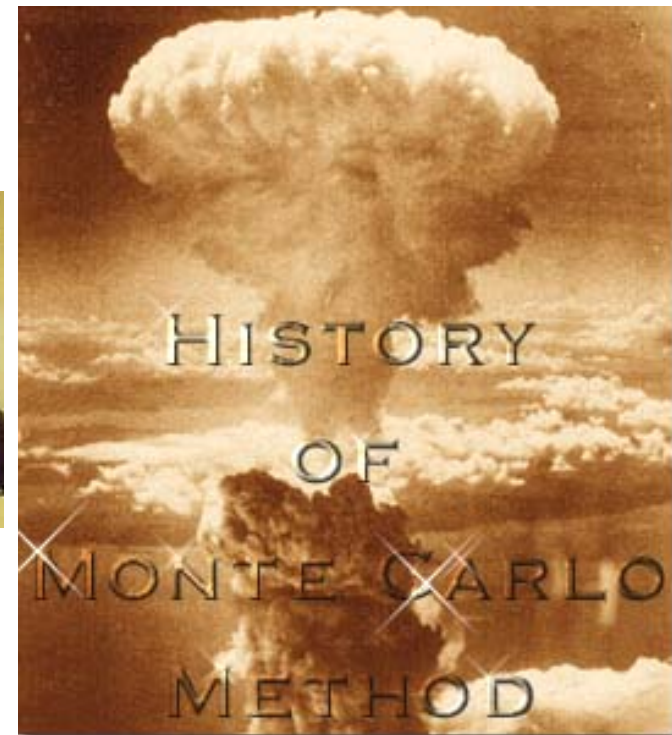
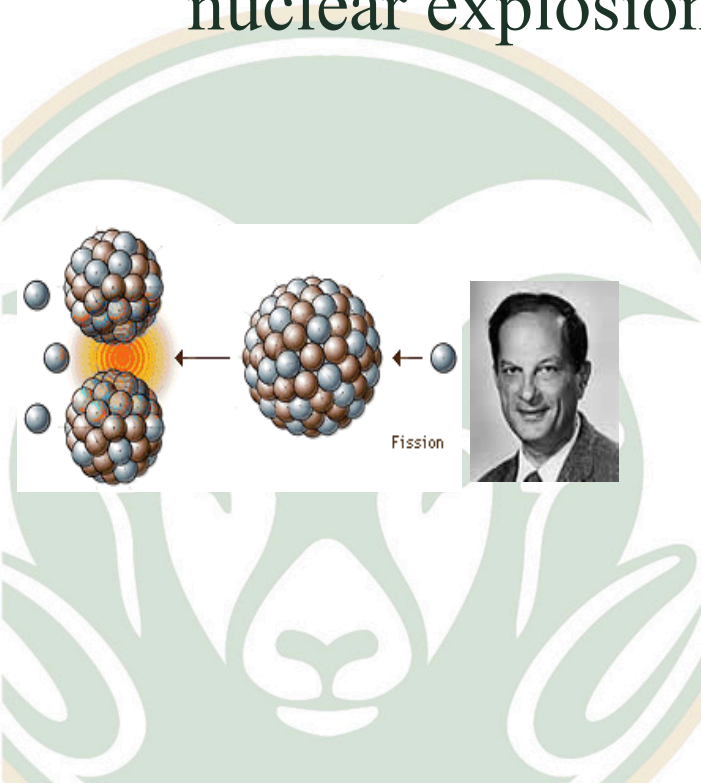
Simulation

- You Can Simulate Anything!
 - Playing poker
 - Investing in the stock market
 - Running a grocery store
- Cheap and fast experimentation: Like a lab
 - What if we use 4 checkout registers instead of 3 in the grocery store?
 - Simulation is cheaper than actually buying the equipment and see what happens.



What is Monte Carlo Simulation?

- In the late 1940s, scientists at the Manhattan Project at Los Alamos National Laboratory used Monte Carlo Simulation method to predict the range of possible nuclear explosion results.



Simulation with Excel

- Use pseudo-random numbers in place of fixed inputs in a spreadsheet
- Draw a random number many times
- Perform a **statistical analysis** of the results

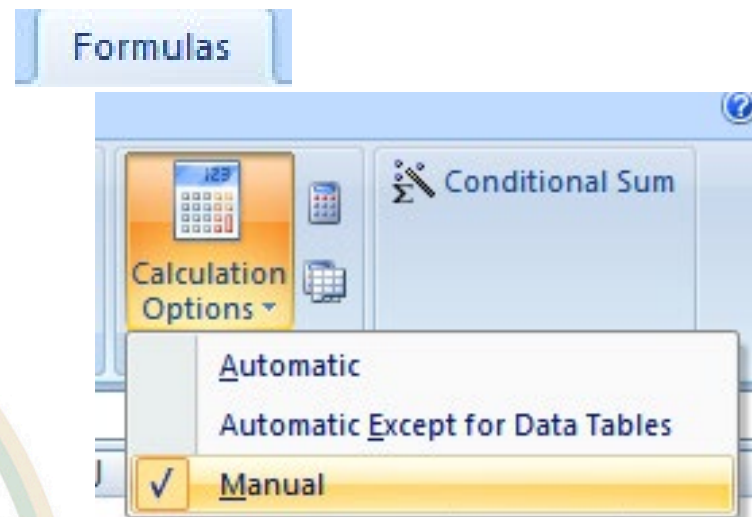
In Excel:

- RandBetween(a,b)
 - Pseudo-random number between (a,b)
- Each time the spreadsheet recalculates (e.g. whenever you calculate a new formula), new random numbers appear → new output values



Side Note: Working with Random #s

- Recalculation can be slow and annoying

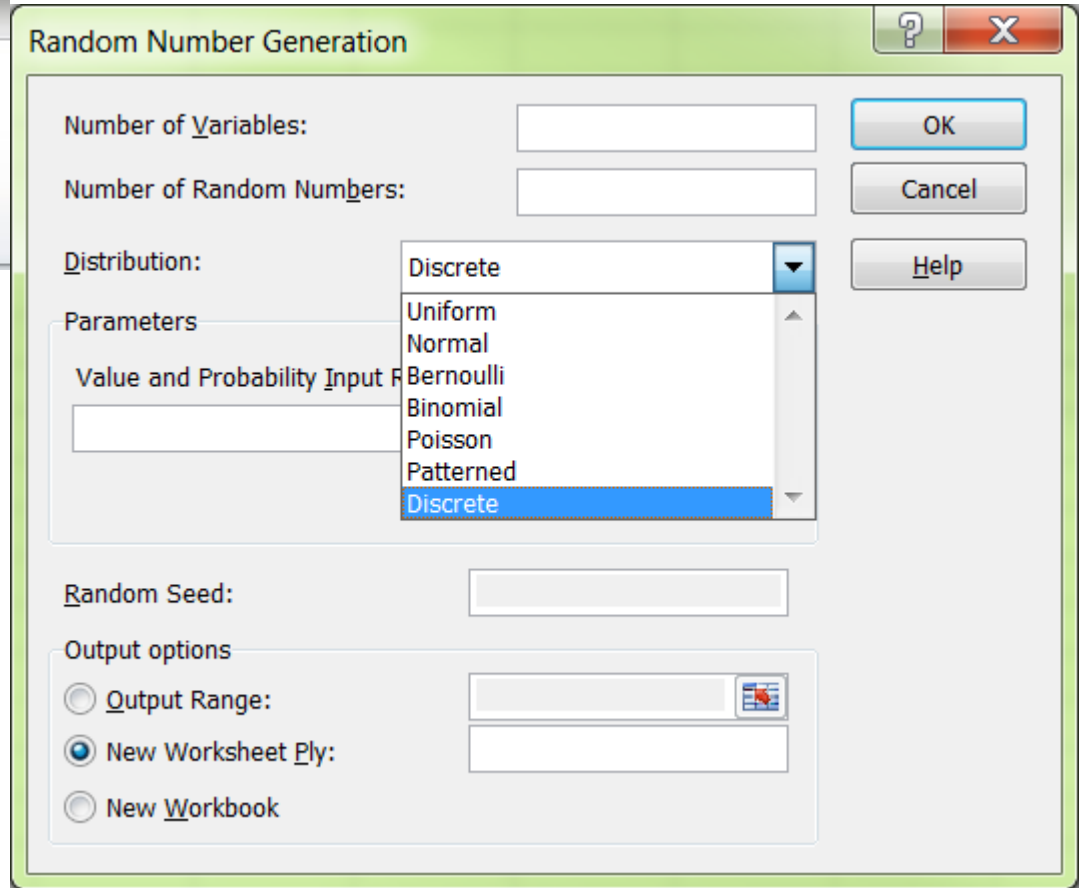
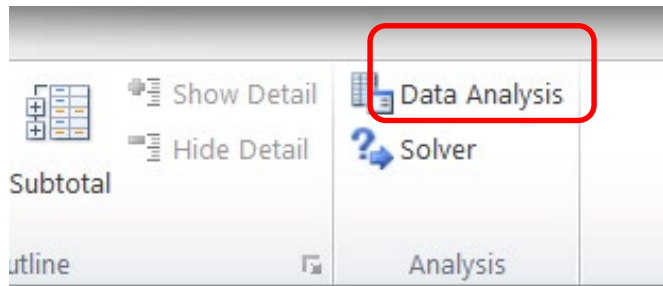


F9 to recalculate

- Or, Paste Special Values to replace random #s



Monte Carlo Simulation with Excel



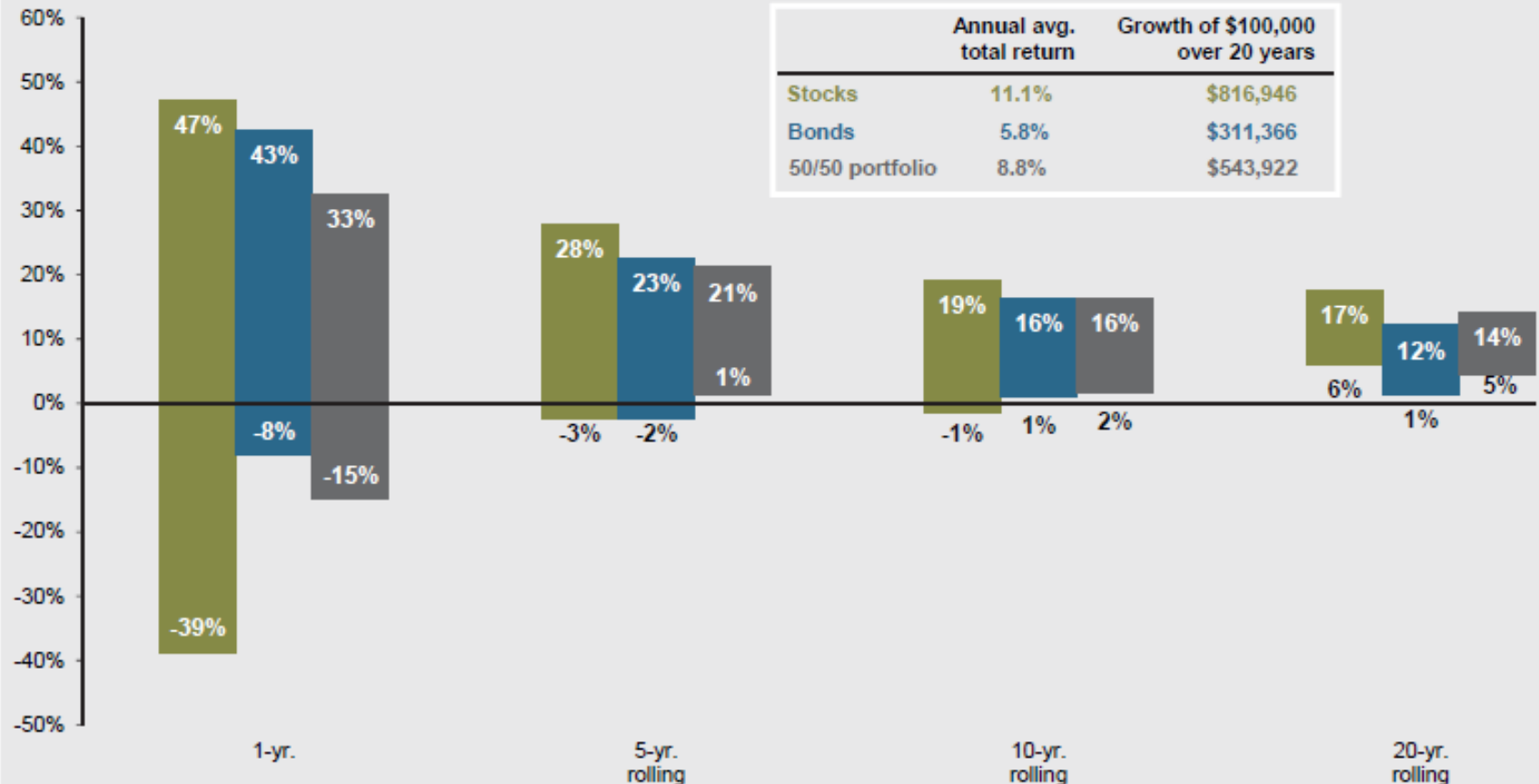
Monte Carlo Simulation with Excel

Round Start	Dice Outcome			Multiplier			Value			Pink		Gross Returns for Each Dice			
	Green	Red	White	Green	Red	White	Green	Red	White	Multiplier	Value	Outcome	Green	Red	White
							\$1,000	\$1,000	\$1,000		\$1,000				
1	2	3	6	0.9	1	1.1	\$900	\$1,000	\$1,100	1.05	\$1,050	1	0.8	0.05	0.95
2	1	2	3	0.8	0.2	1	\$720	\$200	\$1,100	0.6	\$630	2	0.9	0.2	1
3	1	1	1	0.8	0.05	0.95	\$576	\$10	\$1,045	0.5	\$315	3	1.1	1	1
4	6	3	2	1.4	1	1	\$806	\$10	\$1,045	1	\$315	4	1.1	3	1
5	5	5	2	1.2	3	1	\$968	\$30	\$1,045	2	\$630	5	1.2	3	1
6	5	3	4	1.2	1	1	\$1,161	\$30	\$1,045	1	\$630	6	1.4	3	1.1
7	6	4	1	1.4	3	0.95	\$1,626	\$90	\$993	1.975	\$1,244				
8	3	5	6	1.1	3	1.1	\$1,788	\$270	\$1,092	2.05	\$2,551				
9	6	6	3	1.4	3	1	\$2,504	\$810	\$1,092	2	\$5,101				
10	2	4	2	0.9	3	1	\$2,253	\$2,430	\$1,092	2	\$10,203				
11	5	4	5	1.2	3	1	\$2,704	\$7,290	\$1,092	2	\$20,406				
12	5	5	3	1.2	3	1	\$3,245	\$21,870	\$1,092	2	\$40,811				
13	5	3	2	1.2	1	1	\$3,894	\$21,870	\$1,092	1	\$40,811				
14	2	4	3	0.9	3	1	\$3,504	\$65,610	\$1,092	2	\$81,623				
15	6	1	5	1.4	0.05	1	\$4,906	\$3,281	\$1,092	0.525	\$42,852				
16	6	6	3	1.4	3	1	\$6,868	\$9,842	\$1,092	2	\$85,704				
17	3	2	5	1.1	0.2	1	\$7,555	\$1,968	\$1,092	0.6	\$51,422				
18	2	1	3	0.9	0.05	1	\$6,800	\$98	\$1,092	0.525	\$26,997				
19	5	2	4	1.2	0.2	1	\$8,159	\$20	\$1,092	0.6	\$16,198				
20	2	1	4	0.9	0.05	1	\$7,344	\$1	\$1,092	0.525	\$8,504				



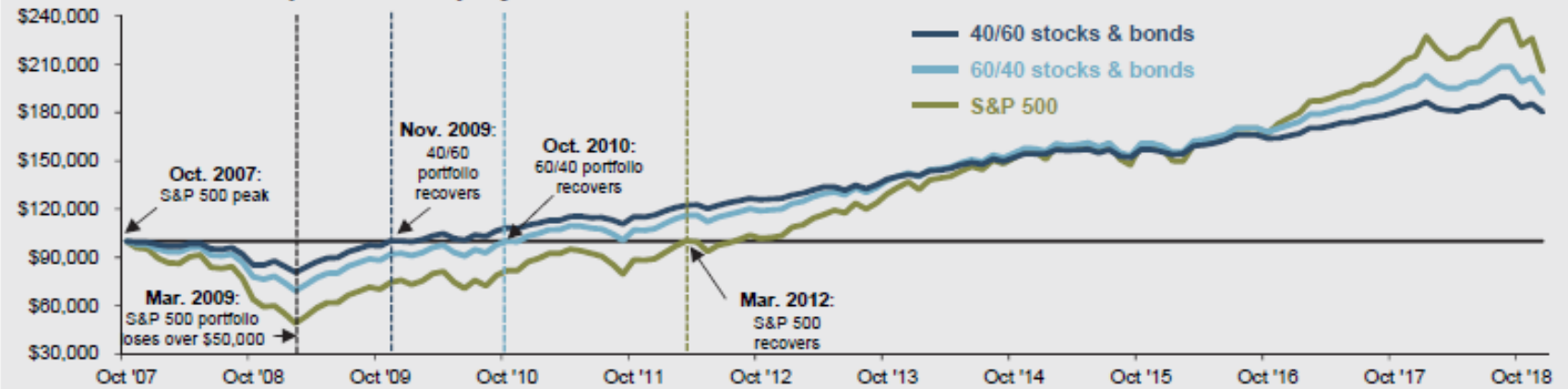
Range of stock, bond and blended total returns

Annual total returns, 1950-2018

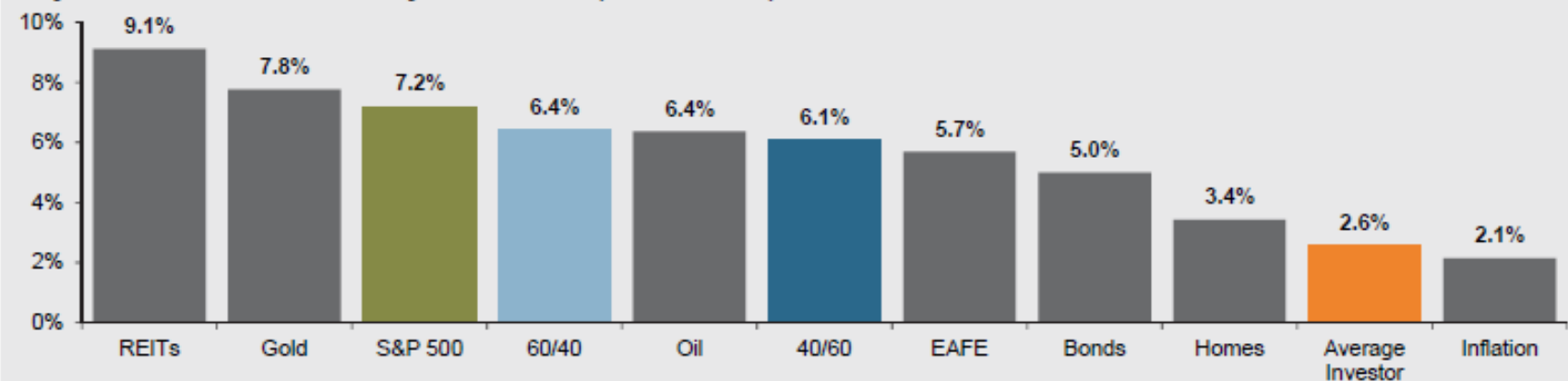


Source: Barclays, Bloomberg, FactSet, Federal Reserve, Robert Shiller, Strategas/Ibbotson, J.P. Morgan Asset Management. Returns shown are based on calendar year returns from 1950 to 2018. Stocks represent the S&P 500 Shiller Composite and Bonds represent Strategas/Ibbotson for periods from 1950 to 2010 and Bloomberg Barclays Aggregate thereafter. Growth of \$100,000 is based on annual average total returns from 1950 to 2018.
 Guide to the Markets – U.S. Data are as of December 31, 2018.

Portfolio returns: Equities vs. equity and fixed income blend



20-year annualized returns by asset class (1998 – 2017)



Source: J.P. Morgan Asset Management; (Top) Barclays, Bloomberg, FactSet, Standard & Poor's; (Bottom) Dalbar Inc. Indices used are as follows: REITs: NAREIT Equity REIT Index, EAFE: MSCI EAFE, Oil: WTI Index, Bonds: Bloomberg Barclays U.S. Aggregate Index, Homes: median sale price of existing single-family homes, Gold: USD/roy oz., Inflation: CPI. 60/40: A balanced portfolio with 60% invested in S&P 500 Index and 40% invested in high-quality U.S. fixed income, represented by the Bloomberg Barclays U.S. Aggregate Index. The portfolio is rebalanced annually. Average asset allocation investor return is based on an analysis by Dalbar Inc., which utilizes the net of aggregate mutual fund sales, redemptions and exchanges each month as a measure of investor behavior. Returns are annualized (and total return where applicable) and represent the 20-year period ending 12/31/17 to match Dalbar's most recent analysis. *Guide to the Markets - U.S.* Data are as of December 31, 2018.

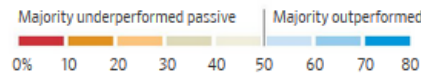
Active vs Passive Investment

- Just 38% of actively managed U.S. stock funds tracked by Morningstar outperformed their counterparts at passively managed funds last year.
- The performance of stock pickers looks even worse over the past decade, when only 24% of those funds outperformed their benchmarks, according to Morningstar.



Active vs Passive Investment

Percentage of actively managed funds that outperformed their average passive counterpart, as of Dec. 31, 2018



CATEGORY	ONE YEARS	THREE YEARS	FIVE YEARS	10 YEARS
U.S. large blend	31.1	15.5	16.4	10.9
U.S. large value	30.1	12.6	13.5	8.3
U.S. large growth	48.9	29.7	21.0	8.3
U.S. mid blend	31.4	19.3	17.9	11.9
U.S. mid value	22.0	12.5	7.5	9.4
U.S. mid growth	75.8	58.9	48.6	30.3
U.S. small blend	21.3	21.2	19.0	24.9
U.S. small value	15.9	16.8	23.6	33.3
U.S. small growth	52.1	51.4	35.1	24.3
Foreign large blend	18.5	25.4	32.8	24.5
Foreign large value	14.5	20.0	27.2	26.3
Foreign small-mid blend	21.2	19.2	16.0	70.6
World large stock	41.3	26.8	30.1	26.3
Diversified EM	20.8	33.0	51.8	54.5
Europe stock	24.0	21.7	12.5	43.3
U.S. real estate	23.9	7.2	33.3	34.2
Global real estate	72.4	9.8	51.9	14.9
Intermediate term bond	38.1	66.9	51.3	55.6
Corporate bond	28.6	59.2	53.8	66.7
High yield bond	28.8	37.8	40.2	56.3

Source: Morningstar



Autopilot



S&P 500 over past year with months highlighted

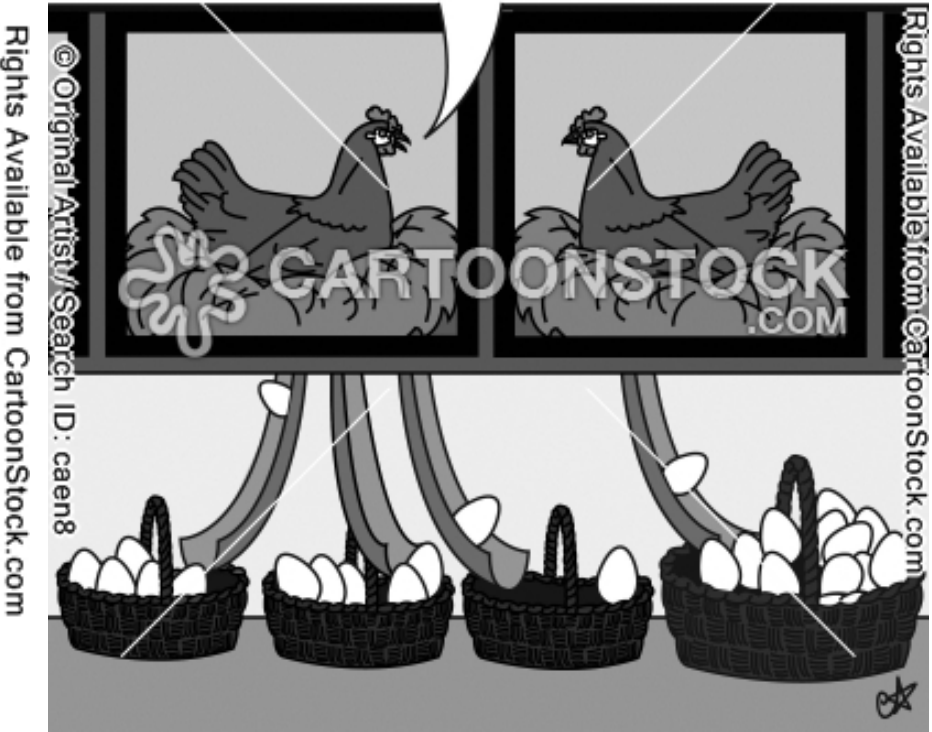
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"Diversify, diversify, diversify. Never keep all your eggs in one basket, unless it's Easter."

You may want to consider diversifying your portfolio....



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