STRATUM SURVEY TECHNIQUES FOR DRAINAGE INVESTIGATION ON IRRIGATED LANDS

Colorado State University
Experiment Station
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Contents

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Stratum Survey Techniques For Drainage Investigation On Irrigated Lands

by

Norman A. Evans*

Introduction

The importance of drainage to the irrigated agriculture of the West has been recognized since about 1900. The interrelationship of irrigation and drainage had become well understood by that time because large areas of irrigated valley lands had already succumbed to the effects of an overload on their natural drainage capacity. In 1955, according to one authority (1) about eight million acres of land in the West needed drainage. Later estimates indicate that at least 20 million acres in the Western States are affected by ground water and associated problems.

Recognition of the general nature of drainage problems was a long step toward overcoming the ages-old belief that agriculture under irrigation could not be permanent, a belief due primarily to the inevitable occurrence of a drainage problem on most irrigation developments since the earliest of historical records. Indeed, there still remains some doubt about the permanency of irrigated agriculture in certain areas having complex drainage problems. In many regions, however, the solutions to drainage problems have been found which promise to be lasting.

Much accomplishment must be credited to the ingenuity of the earlier drainage engineers in arriving at workable solutions to their problems. In general, the methods of investigation have been tailored by the investigator

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in accord with his previous experience and in line with the situation facing him. The trial and error method of arriving at a workable solution has been a common procedure.

The realization has grown, however, that the complexity of drainage,—its dependence upon several physical and biological sciences — dictates a new approach. New techniques are being developed which in time will reduce the amout of "experience" or "judgment" necessary to design drainage facilities.

There is no doubt that knowledge of the drainage characteristics of the soil mantle is necessary. The zone of interest has steadily pushed deeper and deeper until today the drainage engineer is generally concerned with the entire mantle above parent rock. Stratification is one of the most important features of this mantle, and its determination has been called a "stratum survey."

The degree of detail required in any stratum survey for use in connection with drainage problems will vary from place to place, depending upon the degree to which geologic processes have sorted the soil material. Some western valleys are remarkably homogenous in their fill materials, both vertically and laterally. That is, a stratum located at one position can be counted upon to appear in the same relative position in the profile at another place a considerable distance away. On the other hand, the geologic processes in other valleys have resulted in highly heterogeneous fills. Stratum surveys necessarily must be intensive, as well as extensive, to adequately discover the important drainage characteristics of the valley.

Of the many possible methods of obtaining stratum information, the following constitute the most commonly used:

1. Borings
   - a. Percussion drilling
   - b. Rotary drilling
   - c. Auger boring
   - d. Wash boring (jetting)
   - e. Displacement boring (resistance)
   - f. Continuous samplers (drive samplers)
2. Sounding (wells)
   - a. Electrical
   - b. Nuclear
3. Geophysical Methods
   - a. Seismic
   - b. Electrical
   - c. Magnetic
   - d. Gravitational

The purposes of this bulletin are to: (1) discuss briefly the techniques and equipment for making stratum surveys by three boring methods, and (2) to discuss in detail the use of a geophysical method. The latter discussion is intended to encourage the use of this technique by drainage engineers in making substratum investigations for drainage of irrigated lands. The section on theory contains a discussion of this technique which will be of interest but not essential to the application of the method.

Boring Methods

The **auger** method is the most dependable of all, since a sample is obtained which can be visually classified. Although dependable, it is a slow, costly method and is not practical for an intensive survey. Augering to 30 or 40 feet requires two men, and some sort of derrick. Lengths of pipe stem must be repeatedly coupled and uncoupled since it is impossible to handle a string of pipe longer than about 25 feet. The time for coupling and uncoupling plus the time required for augering, and the labor involved, make the method generally unsatisfactory.

The **jetting** method utilizes a ⅜-inch pipe through which water is pumped under pressure. The pipe is forced into the soil while "jetting": the soil material being removed ahead of the pipe by the water jet. The water returns to the surface on the outside of the pipe. The rate at which such a pipe can be forced through the profile is dependent on the nature of the soil materials. Figures 1 and 2 show a jetting rig in operation.

A procedure which has been applied on some investigations has been to make a log of the resistance or rate of penetration and interpret this by comparing with a log obtained by augering. Generally only a few auger logs

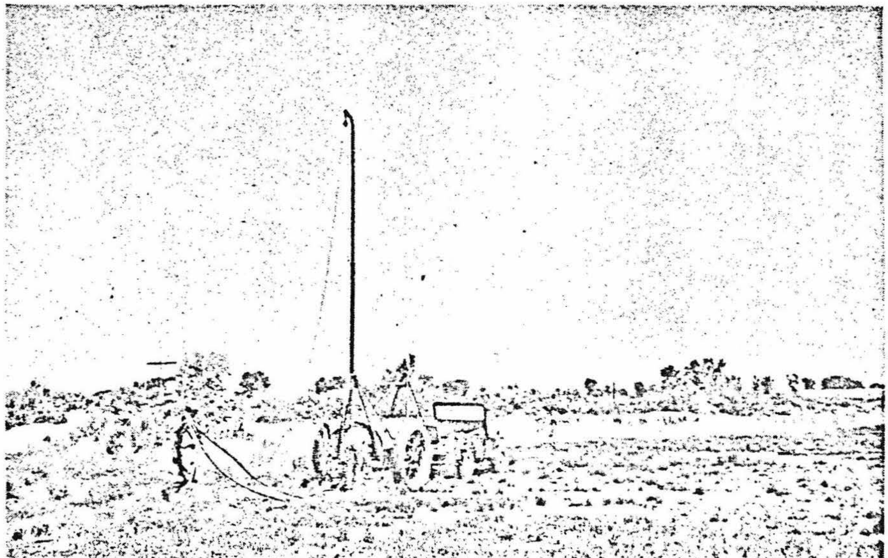


FIGURE 1. Jetting rig with 21-foot boom for handling pipe.

are necessary, and serve as "spot checks" or as indexes.

Unfortunately, the rate of penetration is not always a dependable criterion for a given stratum. A sandy stratum might be resistant at one place and easily penetrated at another depending on moisture content and degree of consolidation. Therefore a procedure is followed in which both an auger log and a jetting log are obtained at approximately one out of twelve locations. With this auger "calibration," jetting logs can be obtained in the near neighborhood, and assumed to be valid. Auger logs are taken as deemed necessary, but frequently enough to assure reliable jetting logs. Jetting pressures of 80 psi or less are found to be most satisfactory for logging.

The modified jetting method was developed as an improvement on the jetting procedure. Since a hole is created by the jetting process, it is practical to utilize the hole for obtaining samples for visual inspection. Thus it is unnecessary to auger out a complete hole and the labor saving is appreciable. A 1-inch hole can be created by jetting, and a resistance log taken at the same time. Then with the location of "unknown" strata thus obtained, the 1¼-inch auger can often be forced without turning to the point where a sample is to be taken and turned 4 to 6 turns. This collects a sample at the desired point. In some

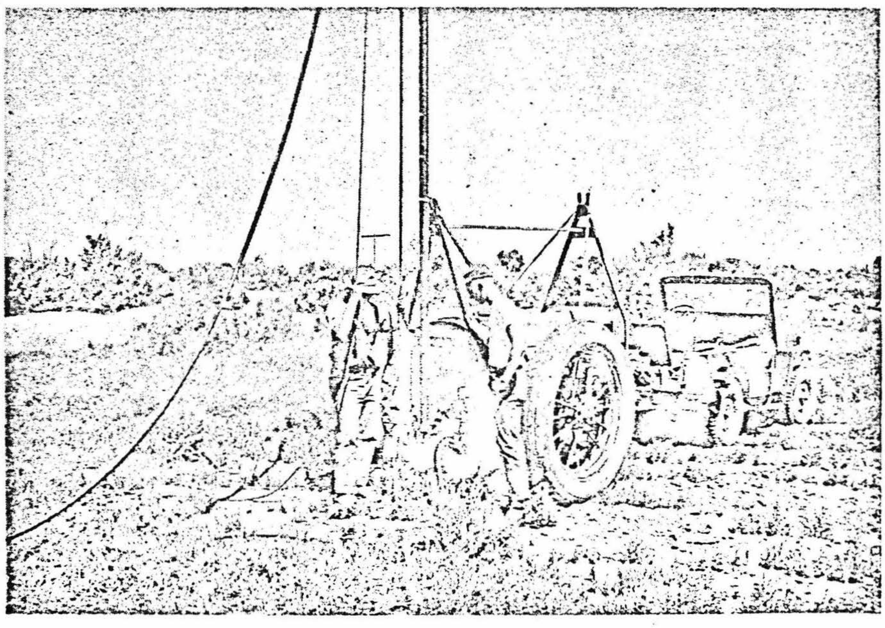


FIGURE 2. Stratum logging by jetting technique.

instances the auger needs to be turned to get it down to the necessary depth, but even so the work involved is not great.

A "window" sampler may be used to improve on the auger-jetting technique. This sampler consists of a section of 1½-inch pipe with a scoop-like door in the wall. The door is closed while the pipe is inserted into the hole to the proper depth, then the door is opened and the sampler rotated so as to take a sample from the periphery of the hole. Figure 3 shows the sampler.

Logging is much faster with this improved method and the labor requirement is considerably reduced from that required by the auger method. Such samplers can be easily fabricated.

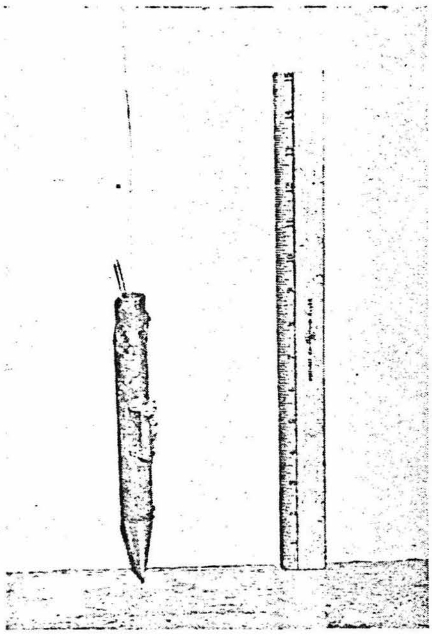


FIGURE 3. A window sampler used in conjunction with the jetting technique for logging.

Electrical Resistivity Method

Measurements made at or near the ground surface with special instruments for the purpose of obtaining subsurface geological information are known as geophysical measurements, and may supplement geological work. Geophysical measurements blend Physics and Geology, since physical measurements are made taking advantage of properties of the earth materials, such as electrical conductance, or shock wave transmission. Seldom are the measurements meaningful themselves, but they serve as a basis for inference from which interpretation can be made.

Of all the possible geophysical methods, the resistivity method offers the best possibility for a relatively cheap, rapid stratum survey. Considerable experience has been had with the method for other similar uses, such as locating gravel deposits, or ore bodies, and in geological investigations of dam sites.

Some instances are known where geophysicists have been called upon to assist in drainage investigation; but in the main, the techniques are not generally used in such work. However, since the electrical resistivity method offers much promise as a tool for the drainage engineer, it is discussed in considerable detail. The equipment discussed in the following pages was designed and built at the Colorado State University.[1] The technique has been used extensively for stratum surveys in Colorado by the writer and his associates.

Equipment and Procedure

The schematic diagram of Figure 4 shows the equipment needed to measure resistivity.

This consists of a d. c. power source for which radio "B" cells are used having 22.5 and 45 volt terminals. A milliammeter is connected into the power circuit, and 3/8-inch cold rolled iron stakes are used for current electrodes.

The potential circuit consists of two electrodes made of porous porcelain pots (3 x 6 inches) containing a saturated solution of copper sulfate and copper tubing in contact with the solution. A potentiometer in this circuit measures the potential between the electrodes. Figure 5 shows a diagram for each of these circuits.

The entire apparatus, except wires and electrodes, is contained within a single compact box weighing about 20 pounds. The set is pictured in Figures 6 and 7. This is, of course, a modest set, capable of measuring resistivity to depths of probably 150 to 200 feet. However, the instrument sensitivity is excellent and is equal to more expensive commercial equipment.

A four-man crew is ideal for extensive field work as this permits two men to move the electrodes, while one operates and reads the instrument and the

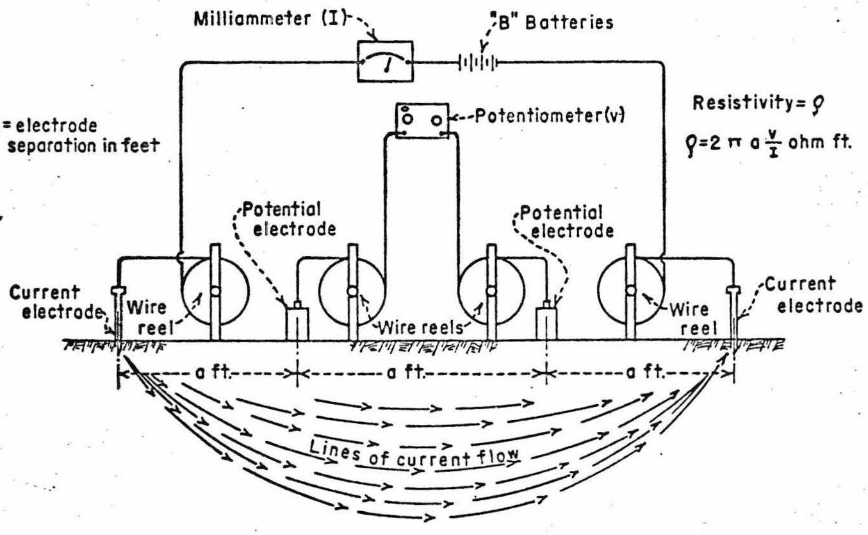
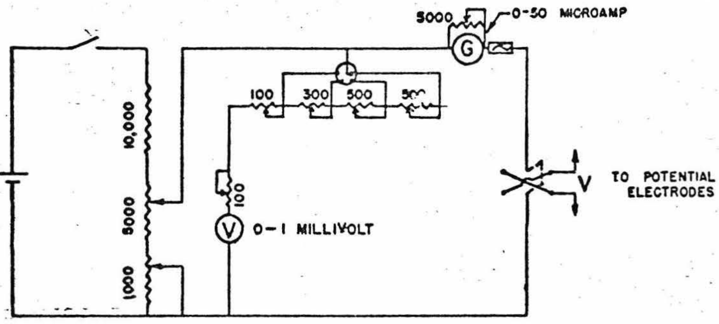


FIGURE 4. Diagrammatic arrangement of field resistivity equipment.

[1] The assistance and consultation of Dart G. Wantland, Geophysicist, U. S. Bureau of Reclamation, Denver, is gratefully acknowledged.



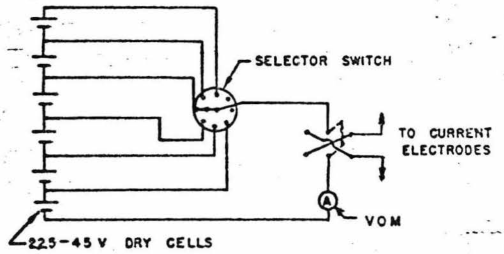


FIGURE 5. Circuit diagrams for resistivity apparatus.

fourth records data, makes computations, and plots the data. In this way, any unusual readings are noticed and can be remade immediately.

The procedure is to begin the resistivity stratum survey with electrodes at close spacing (obtaining resistivity of a shallow depth of earth.) The spacing is systematically increased until the "a" dimension corresponds with or exceeds the depth of interest. The usual procedure is to begin with an "a" spacing of 3 or 5 feet and increase by increments of 3 or 5 feet. Interpretation is facilitated if the increment is kept constant.

One man at each half of the line handles a current electrode and a potential electrode. The iron stake is moved to the new position (use of a steel or cloth tape laid along the line facilitates this) and driven one or two inches into the ground. The wire is attached with a clip. The porous pot is moved and placed on the ground surface in firm contact with the soil. Usually it is expedient to loosen and moisten the ground so that good contact can be made. The instrument man then turns on the power and applies the voltage necessary to obtain current flow sufficient to be recorded and also

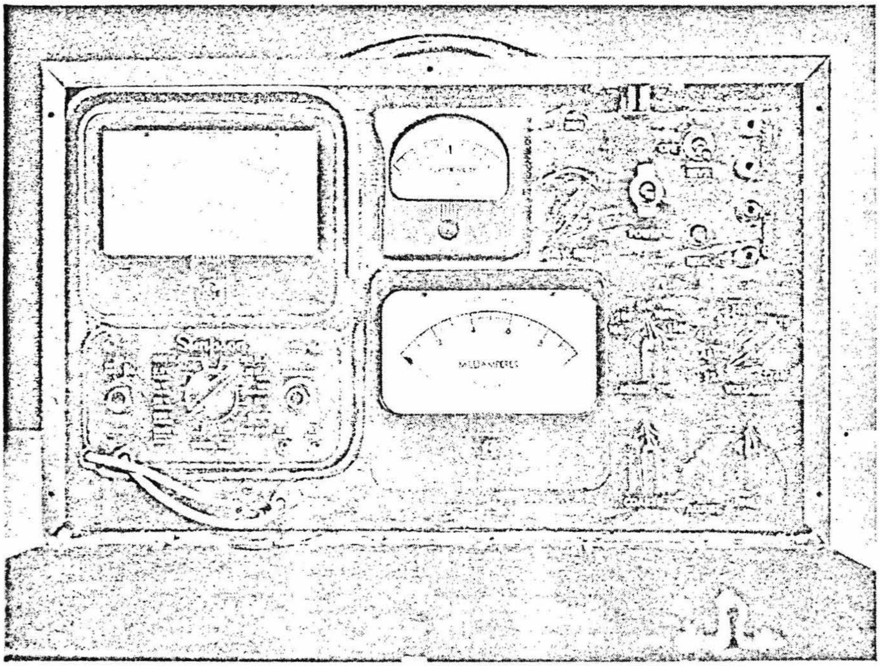


FIGURE 6. Electrical resistivity bridge (see also circuit diagram Figure 5).

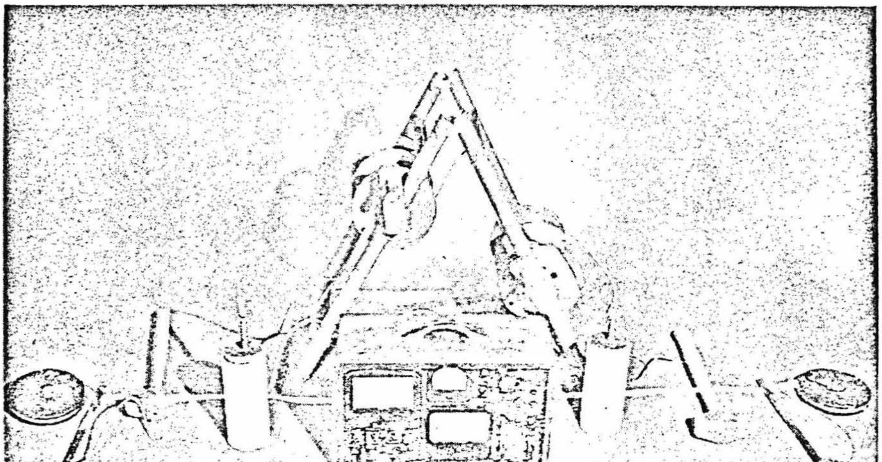


FIGURE 7. Complete resistivity set.

to obtain sufficient potential difference to be recorded. This frequently requires that rather high voltage be applied. Good contact at the electrodes is highly important. The instrument is designed to produce a maximum of 360 volts. This is generally adequate for normal drainage investigations.

Once readings of current and potential have been obtained, the direction of current flow is reversed and a second set of readings is taken. This will serve to "average out" ground currents which generally are present. It will be found that the current in one direction will be increased by ground currents so that resistivity will be calculated low, while for the other direction, the current will be reduced: and, consequently, the calculated resistivity will be high. An average of the two will remove the error due to such stray currents.

Readings at two applied voltages are useful also in eliminating errors since they serve as a cross check. The results at two different voltages should be approximately the same. Figure 8 is a sample data sheet.

Resistivity is calculated using the following equation:

$$\rho = 2 \pi a \frac{E}{I} \quad (1)$$

in which,

ρ = apparent resistivity, ohm-feet

a = electrode spacing (equivalent to depth), feet

E = potential difference between potential electrodes, millivolts

I = current applied through current electrodes, milliamperes

The section on theory contains a technical discussion of the resistivity method for the reader who is interested in the basis of the equation.

The calculated resistivity is plotted against depth, or "a" value. This gives an R-D curve illustrated in Figure 9. This curve may be of almost any shape depending upon the stratification and moisture condition. Generally, unless the top strata are very dry, the resistivity increases with depth.

Accuracy of measurement is highly important in obtaining consistent results. Meters must be sensitive, yet rugged. Of particular importance is the potentiometer circuit since it is required to detect potential differences of the order of 5 to 100 millivolts. Current readings range from less than 100 milliamperes to 1000 milliamperes.

The two potential electrodes are reasonably "non-polarizing" being made of a metal and solution of a salt of the metal. The metal is copper, and the solution is copper sulfate. The cell reaction at each electrode is reversible and unless corrosion is allowed to coat the copper pipe, the electrode will not develop a high self-potential or counter e. m. f. This is particularly true

FIELD DATA SHEET

Project : 223 Location : 23rd & Hrd

Party Chief : Evans Date : 7-10-56

[illegible]	Applied Voltage, volts	a, spacing, ft.	E, millivolts	I, millamps	E/I, ohms	mean E/I, ohms	2πa	ρ=2πa E/I ohm-feet	mean ρ̄	Σρ̄
	22½	5	16½	64½	0.256	0.282	31.42	8.84	8.64	8.64
			20	65	0.308					
	45		35½	125	0.284	0.269		8.44		
			32½	128	0.254					
	22½	10	10½	56½	0.186	0.152	62.83	9.55	9.24	17.88
			6½	54½	0.119					
	45		14½	110	0.132	0.142		8.93		
			17½	114	0.153					
	22½	15	15	148	0.101	0.125	94.23	11.78	11.68	29.56
			21	140	0.150					
	45		35½	260	0.135	0.123		11.58		
			30	270	0.111					
	22½	20	14	135	0.104	0.106	125.66	13.30	13.18	42.74
			14	130	0.108					
	45		26	245	0.106	0.104		13.05		
			26	255	0.102					
	22½	25	14½	149	0.097	0.094	157.08	14.77	14.61	57.35
			13½	148	0.091					
	45		24	265	0.091	0.092		14.45		
			26	284	0.092					
	22½	30	6½	83	0.078	0.096	188.50	18.08	17.12	74.47
			8	70	0.114					
	45		15	165	0.091	0.086		16.20		
			15	186	0.081					
	45	35	11	82	0.134	0.104	219.92	22.87	22.10	96.57
			6	80	0.075					
	90		12	145	0.109	0.097		21.33		
			17½	160	0.083					
	45	40	20	210	0.095	0.077	251.33	19.33	19.45	11602
			13	217	0.060					
	90		21	320	0.090	0.078		19.57		
			28½	315	0.066					
	45	45	12	118	0.102	0.070	282.75	19.77	20.50	13652
			4½	118	0.038					
	90		9½	180	0.053	0.075		21.20		
			18	185	0.097					
	45	50	18	200	0.090	0.072	314.16	22.62	22.62	15914
			11	204	0.054					
	90		18½	300	0.062	0.072		22.62		
			25	300	0.083					
	45	55	5½	22	0.250	0.150	345.57	51.9	50.2	20934
			5½	110	0.050					
	90		12½	207	0.060	0.140		48.4		
			5½	25	0.220					

FIGURE 8. Sample field data sheet.

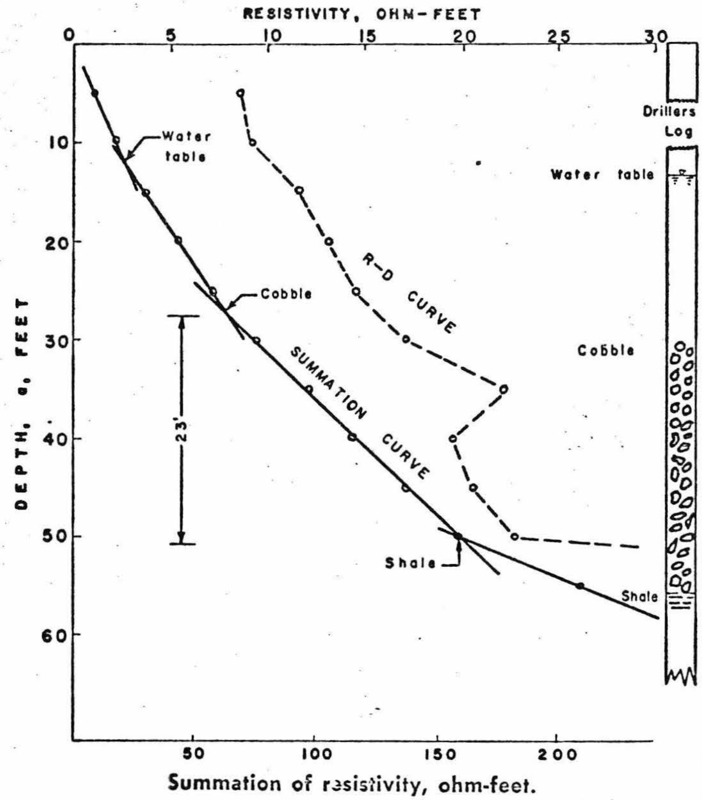


FIGURE 9. Typical resistivity data.

if current is on for only a short time and where current density is not high.

It is important that the two electrodes be of nearly the same potential with respect to each other. This is accomplished by selecting nearly identical copper electrodes and keeping the copper sulphate solution fully saturated. The electrodes should be checked daily for potential difference by setting the two pots in a common porcelain vessel in copper sulfate solution. The cell so formed should have a small potential. If this potential is too large, it will create error in readings and should be remedied. For most work 1 to 2 millivolts difference is permissible. Cleaning the copper and pots with dilute sulfuric acid is necessary occasionally; and under regular daily operation cleaning needs to be done at least once a week.

Interpretation

The resistivity-depth curve obtained directly from the data is itself not adequate for interpretation. Occasionally the changes in slope or maxima or minima are significant, but in general they are not. Consequently, an empirical method for interpretation has been adopted. This method is attributed to Moore (6).

This method is merely to plot a curve of summation of resistivity versus depth. Such a curve is illustrated in Figure 9. Theoretically, the Moore summation method should produce a curve with no straight segments, except where the R-D curve would happen to be a straight vertical line (constant resistivity). However the summation curve will often consist of segments of near constant slope which can be represented by straight lines. The intersections of the line segments have been found to correlate fairly well with significant stratum changes.

The Moore curve tends to mask the effect of single resistivity values so that accidental errors in single measurements do not seriously affect the interpretation. Its greatest utility is due to the fact that where significant changes of resistivity occur (as at major stratum planes) the slope of the summation curve distinctly changes. Straight lines fitted to segments of the curve which are essentially straight will intersect in the vicinity of major stratum changes. Fitting of the lines becomes a matter for judgment. Figure 9 shows straight lines fitted to the summation curve.

The apparent resistivity depends on pore volume, pore size distribution, degree of saturation with electrolyte, resistivity of the electrolyte, and resistivity of the mineral grains and the solution filling the pores, Hummel (4) has shown that for rock formations, the resistivity of the rock is unimportant in relation to the resistivity of the pore solution. For example, if the resistivity of the pore solution is 1/20 the resistivity of the mineral grains, the resistivity of the mass will be about 1/8 the resistivity of the mineral grains. When the pore solution is of extremely high electrolyte concentration, the resistivity measurement becomes a function of porosity and pore geometry rather than mineral materials.

Figure 9 represents the results of a resistivity log at one location in Grand Valley, Colorado. It is observed that as the resistivity measurements begin to include the aquifer, the abrupt change in porosity and pore size distribution causes apparent resistivity to change significantly. This change shows up well on the Moore summation curve analysis. Again as the apparent resistivity includes shale, another large change is observed on the summation curve.

To make this interpretation with confidence, some geologic information must be known. Logs obtained by drilling serve this purpose best. With a knowledge of what stratification to expect, and a reasonable idea of what depths are involved, the engineer can make an interpretation. It would be virtually impossible to make an interpretation without having a few reference logs available.

Limitations

Commercial resistivity equipment is available at considerable cost. However, equipment similar to that shown in Figures 6 and 7 may be fabricated by an electronics technician for a relatively small cost. This is of course, a modest instrument, capable only of measurements to depths of 150 to 200 feet. Such depths are generally adequate for most drainage investigations.

Operating costs depend on the size of the crew used, the experience of the crew, and the depth of interest. A good crew of four can make six "sets" in a day. That is, logs at six locations can be obtained to depths of up to 80 feet in one day. The cost is considerably less than for any other method which might be used for comparable logging. However, it is emphasized that the logs obtained are limited in usefulness: they are not detailed logs and must be "keyed" to known geology.

Theory of Electrical Resistivity

A conductor has resistance, R, which can be calculated by Ohms law. If resistance is expressed as the resistance per cm. of length per square cm. of cross sectional area, it is called specific resistance, or resistivity, ρ.

$$\rho = \frac{R}{l/A}$$

where,

ρ = resistivity, ohm-cm.

l = conductor length, or length path of current, cm.

A = cross sectional area of conductor, cm^2.

R = resistance of the conductor, ohm.

For a wire conductor, the resistivity can be determined easily by measuring the current and the potential drop between two ends. Application of Ohms law will yield resistance, R, from which resistivity can be calculated.

For a thin, 2-dimensional sheet or plate, the resistance can be found, but resistivity cannot be adequately calculated because the current distribution across the plate is not uniform. Current distribution depends on the shape and size of electrodes and on the homogeneity and isotropy of the sheet. Likewise for a 3-dimensional body the resistivity cannot be so simply calculated.

Resistivity of a 3-dimensional body — Resistivity of a body which is homogeneous and isotropic and of infinite extent can be determined by a method developed by Frank Wenner, in 1912 (10) (11). Searle (7) also suggested a similar method. Both are four-terminal methods. Gish and Roomey (2) in 1925 were first to make application of the

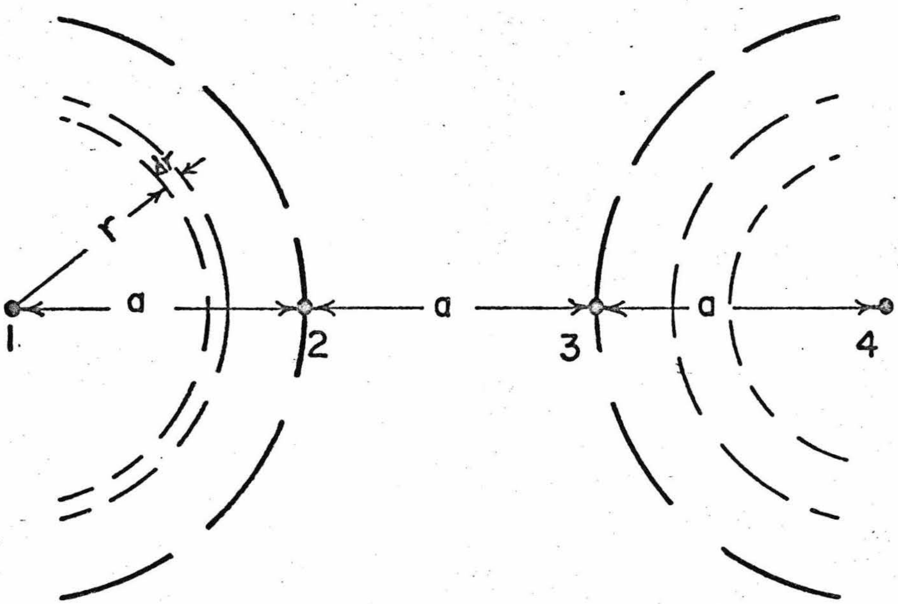


FIGURE 10. Four electrodes in a three dimensional medium of infinite extent.

method. The four-terminal network refers to a system of four electrodes, two of which are current or source electrodes, and two of which are potential electrodes.

Consider Figure 10, an arrangement of four electrodes in a homogeneous, isotropic conducting medium of infinite extent in 3-dimensions.

The potential gradient, $\frac{dE}{dr}$, at a distance, r, from the source electrode 1 is the current density times the resistivity of the conductor. For unit current entering at 1:

$$\frac{dE}{dr} = \rho \times \frac{1}{4\pi r^2} \qquad (1)$$

The potential drop between two points r_1 and r_2 from 1 can be found by integrating Eq. (1)

$$-\int_{E_1}^{E_2} dE = \frac{(1)\rho}{4\pi} \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$E_1 - E_2 = \frac{(1)\rho}{4\pi} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] \qquad (2)$$

Then $E_{[2,3]}$ the potential difference between electrodes 2 and 3 at distances a and 2a from electrode 1 due to unit current flowing *from* 1 is,

$$E_{[2,3]} = \frac{(1)\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{2a}\right]$$

$$= \frac{(1)\rho}{4\pi}\left[\frac{1}{2a}\right] \qquad (3)$$

Also $E'_{[2,3]}$ the potential change between 2 and 3 due to a unit current leaving at 4 can be obtained from Eq. (1).

$$-\int_{E_2}^{E_3} dE = \frac{(1)\rho}{4\pi}\int_{r_2}^{r_3} \frac{dr}{r^2}$$

$$-(E_3 - E_2) = \frac{(1)\rho}{4\pi}\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

thus,

$$E'_{[2,3]} = \frac{(1)\rho}{4\pi}\left[\frac{1}{r_3} - \frac{1}{r_2}\right]$$

$$= \frac{(1)\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{2a}\right] = \frac{(1)\rho}{4\pi}\left[\frac{1}{2a}\right] \qquad (4)$$

The difference in potential between 2 and 3 due to the unit current entering at 1 and leaving at 4 is the algebraic sum of the Eqs. (3) and (4).

$$E_{1\text{-}4} = E_{[2,3]} + E'_{[2,3]} = \frac{\rho}{8\pi a} + \frac{\rho}{8\pi a} = \frac{\rho}{4\pi a} \qquad (5)$$

Since unit current is flowing, the Eq. (5) also gives the resistance, R, of the earth between the equipotential surfaces on which the potential electrodes are located.

$$R = E/1 = \frac{\rho}{4\pi a} \qquad (6)$$

Semi-infinite conductor—Equation (6) is valid only for a conductor of infinite extent. For a conductor of semi-infinite extent, however, an equation can be developed using the method of images.

First consider **four** current electrodes (1, 2, 3, and 4) and two potential electrodes (2 and 3) in a conductor of infinite extent in three dimensions shown in Figure 11.

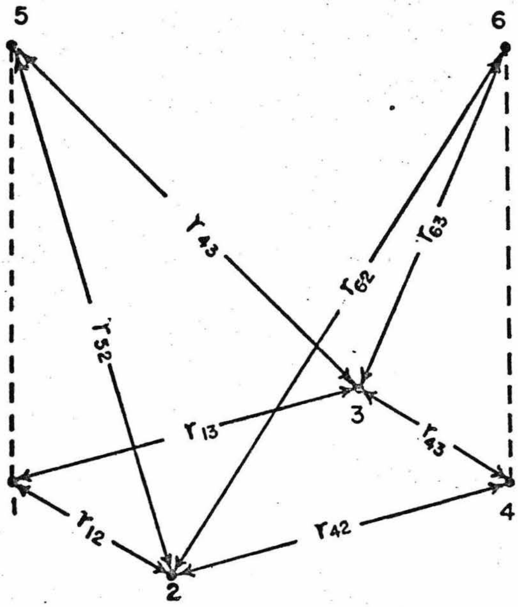


FIGURE 11. Six electrodes in three dimensional medium of infinite extent.

Writing an equation equivalent to Eq. (5) for the potential drop between electrodes 2 and 3 due to unit current entering at 1 and unit current leaving at 4, we have:

$$E_{1\text{-}4} = \frac{\rho}{4\pi}\left[\frac{1}{r_{12}} - \frac{1}{r_{13}}\right] + \frac{\rho}{4\pi}\left[\frac{1}{r_{43}} - \frac{1}{r_{42}}\right]$$

$$= \frac{\rho}{4\pi}\left[\frac{1}{r_{12}} - \frac{1}{r_{13}} + \frac{1}{r_{43}} - \frac{1}{r_{42}}\right] \qquad (7)$$

For unit current entering at 5 and leaving at 6, we have,

$$E_{5\text{-}6} = \frac{\rho}{4\pi}\left[\frac{1}{r_{52}} - \frac{1}{r_{53}}\right] + \frac{\rho}{4\pi}\left[\frac{1}{r_{62}} - \frac{1}{r_{63}}\right]$$

$$= \frac{\rho}{4\pi}\left[\frac{1}{r_{52}} - \frac{1}{r_{53}} + \frac{1}{r_{62}} - \frac{1}{r_{63}}\right] \qquad (8)$$

The combined effect of the four current electrodes on the potential drop between 2 and 3 is the algebraic sum of the Eqs. (7) and (8),

$$E_{2\text{-}3} = \frac{\rho}{4\pi}\left[\frac{1}{r_{12}} - \frac{1}{r_{13}} + \frac{1}{r_{43}} - \frac{1}{r_{42}} + \frac{1}{r_{52}} - \frac{1}{r_{53}} + \frac{1}{r_{63}} - \frac{1}{r_{62}}\right] \qquad (9)$$

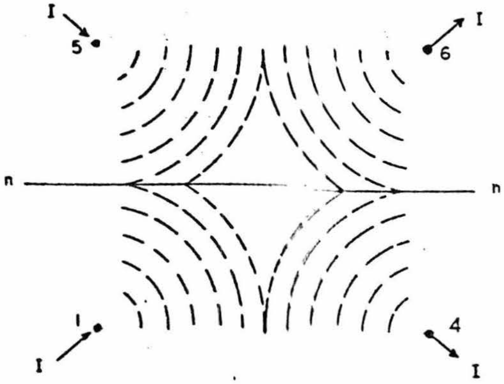


FIGURE 12. Equipotential lines in a system of four electrodes.

Now suppose a plane, n-n, can be found as shown in Figure 12 which is normal to the lines connecting electrodes 1 to 5 and 4 to 6 and which also is a bisector of these lines. The plane then could represent a mirror and the points 5 and 6 would be the mirror images of 1 and 4. Consider the pattern of equipotential lines in this system, as shown in Figure 12.

It is apparent that no current passes across the plane n-n on account of the symmetry of the system. Therefore the upper half can be removed without changing the distribution of equipotentials in the lower half.

We may thus use Eq. (9) for the semi-infinite conductor if we take into consideration that images would exist and consider the distances of the images from the points 2 and 3 between which the potential difference is measured. This does not require that the points 2 and 3 be in the same vertical plane.

Equation (9) gives the potential difference between two electrodes located some distance into the ground. If the electrodes are all at uniform depth, b, and equally spaced, a, in a straight line, then

$$r_{12} = a, \quad r_{13} = 2a, \quad r_{43} = a, \quad r_{12} = 2a$$

$$r_{52} = \sqrt{4b^2 + a^2}, \quad r_{53} = \sqrt{4b^2 + 4a^2}, \quad r_{62} = \sqrt{4b^2 + 4a^2}, \quad r_{63} = \sqrt{4b^2 + a^2}$$

Therefore by Eq. (9) for unit current

$$E_{2\text{-}3} = \frac{(1)\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{2a} + \frac{1}{a} - \frac{1}{2a} + \frac{1}{\sqrt{4b^2 + a^2}} - \frac{1}{\sqrt{4b^2 + 4a^2}} + \frac{1}{\sqrt{4b^2 + a^2}} - \frac{1}{\sqrt{4b^2 + 4a^2}}\right]$$

$$= \frac{(1)\rho}{4\pi}\left[\frac{2}{a} - \frac{2}{2a} + \frac{2}{\sqrt{4b^2 + a^2}} - \frac{2}{\sqrt{4b^2 + 4a^2}}\right]$$

$$= \frac{(1)\rho}{4\pi}\left[\frac{1}{a} + \frac{2}{\sqrt{4b^2 + a^2}} - \frac{2}{\sqrt{4b^2 + 4a^2}}\right] \qquad (10)$$

Equation (10) was derived for unit current. Since E/I = R, Eq. (10) can be written.

$$R = \frac{\rho}{4\pi}\left[\frac{1}{a} + \frac{2}{\sqrt{4b^2 + a^2}} - \frac{2}{\sqrt{4b^2 + 4a^2}}\right] \qquad (11)$$

or

$$\rho = \frac{4\pi R}{\frac{1}{a} + \frac{2}{\sqrt{4b^2 + a^2}} - \frac{2}{\sqrt{4b^2 + 4a^2}}} \qquad (12)$$

If the electrodes are on the ground surface so that b = 0, Eq. (12) becomes:

$$\rho = \frac{4\pi R}{\left[\frac{1}{a} + \frac{2}{a} - \frac{2}{2a}\right]} = 2\pi a R \qquad (13)$$

Equation (13) gives, then, the resistivity of a mass of earth between two equipotential hemi-spheres on which the potential electrodes are positioned. The assumption of uniform resistivity was made for this mass of earth. The dimensions of this earth mass are indefinite, but it is assumed that the earth below a depth equal to a,—the electrode spacing—does not carry a significant amount of the current.

With this assumption, we have an empirical means for obtaining the "apparent resistivity" of a mass of soil included between the equipotential hemi-spheres and of depth equal to a, the electrode spacing. Figure 13 illustrates such a mass of earth.

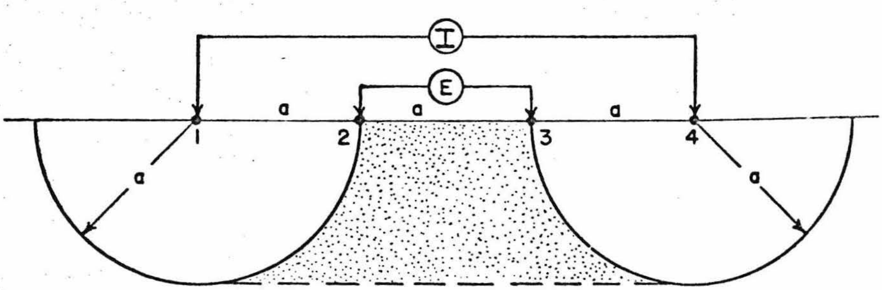


FIGURE 13. Resistivity is measured for shaded mass of earth.

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