SYSTEMATIC DESIGN OF LEGAL REGULATIONS FOR OPTIMAL SURFACE — GROUNDWATER USAGE PHASE I

by

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SYSTEMATIC DESIGN OF LEGAL REGULATIONS FOR OPTIMAL SURFACE-GROUNDWATER USAGE - PHASE 1

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ABSTRACT

Even though the word Economics was left out of an already lengthy title, its importance was recognized in the study. The report, the outcome of a one-year study, is made of three parts dealing respectively with the hydrologic, legal and economic aspects of conjunctive surface-groundwater management.

Hydrologist, lawyers and economists have become increasingly and even painfully aware of the complex degree of interaction of their respective disciplines and as a result they have attempted to establish a connection. This report emphasizes the fact that the solution of the problem requires not just a mere juxtaposition of parts (a procedure acceptable as a start) but a complete integration from concepts down to coding details.

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PART I

HYDROLOGIC ASPECTS OF CONJUNCTIVE SURFACE-GROUNDWATER MANAGEMENT

A. INTRODUCTION

It is an accepted opinion that "the design of a water-resource project depends on an intimate interplay of economic and engineering considerations....To achieve the necessary close collaboration, specialists in each discipline must have a rudimentary acquaintance with the point of view and vocabulary of the other." (Maas et al., 1962, p. 88.) In his chapter (chapter 3 of Maas et al.) R. Dorfman justifies his presentation solely of economic rudiments on two grounds: "First, the essential ideas that economics has to contribute can be conveyed in much briefer compass than can the highly elaborated technology of civil engineering; accordingly it appears easier for engineers to learn the relevant economic doctrines than for economists even to approach the techniques of engineering design. Second, the common ground where economists and engineers must meet appears to be mostly on the economist's side of the borderline." (Maas et al., 1962, p. 88.)

However, encouraging these words may be to the civil engineer, yet these rudiments must be assimilated. Much time was spent indeed during this first year, Phase I, project to read the literature on economics of groundwater management [4,5,6,20,32,3,34].

In addition the legal aspects must be well understood because the law dictates the framework within which the State Engineer of Colorado must administer the river basin. The law is not without ambiguity as the following quotations and commentaries will show. For the sake of completeness and at the intention of a reader unfamiliar with the problems of water management in Colorado, a section of the Proposal to O.W.R.R. of October 1971 is reproduced verbatim in Appendix 1, Legal

Problems in Surface Groundwater Usage. In the following section only the points that bear significantly on the design of the Hydrologic Model are discussed.

B. LEGAL FRAMEWORK

In the "Water Right Determination and Administration Act of 1969" it is stated: "--it shall be the policy of this state to integrate the appropriation, use and administration of underground water tributary to a stream with the use of surface water in such a way as to maximize the beneficial use of all of the waters of this state."*

In this statement the lawmaker may not have fully realized the meaning of the "zeroth law" of Operations Research. It is not possible to optimize more than one economic objective function. It is not possible to maximize the beneficial use of surface water and to maximize the beneficial use of groundwater at the same time. It is possible, however, to maximize the beneficial use of surface water while maintaining a given level of beneficial use of groundwater, or vice versa. Or, more significantly, it is possible to maximize an overall beneficial use of groundwater and surface water. What this overall objective function should be is not precisely spelled out by the Act.

Further in the text of the Act it is stated: "No reduction of any lawful diversion because of the operation of the priority system shall be permitted unless such reduction would increase the amount of water available to and required by water rights having senior priorities."*

By starting the sentence in a negative and peremptory form, the intent of the Act is without doubt that the burden of proof that "the reduction would increase the amount of water available" must be borne by the Administration. Later the Act points out clearly that it is the

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*Underlining is author's, not legislature's.

unambiguous duty of the Administration to protect the senior water rights: "In the distribution of water, the division engineer in each division and the state engineer shall be governed by the priorities for water rights."

At this point it must be concluded that no maximization of beneficial use is possible if the strategy of the State Engineer is the strict enforcement of the priority system because in that case there are no degrees of freedom left in the operation of the system. However, despite the intent of the legislator and the consciencious effort of the Administration to design rules and regulations in accord with the law, there is no guarantee that a given decision will achieve the legal objective, due to the available but imperfect information on the state of nature (e.g. aquifer parameters) and the laws of nature (e.g. the complex chain of numerous factors that cause snowmelt runoff to be what it is on a given date). Yet the law states: "--he [the division engineer] shall also order the total or partial discontinuance of any diversion in his division to the extent the water being diverted is required by persons entitled to use water under water rights having senior priorities, but no such discontinuance shall be ordered unless the diversion is causing or will cause material injury to such water rights having senior priorities."*

On one hand the State Engineer must act to protect the senior rights in the present and anticipate the immediate need for protection into the future. On the other hand the State Engineer must substantiate

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*Underlining is author's, not legislature's.

the sound basis of his decision. Clearly he is faced with a difficult task, compounded by all the factors that must be considered: "Such factors include the current and prospective volumes in and tributary to the stream from which the diversion is made, distance and type of stream bed between the diversion points; the various velocities of this water, both surface and underground; the probable duration of the available flow; and the predictable return flow to the affected stream."*

In at least one way the legislator has given the Administration some freedom of action: "In authorizing alternate points of diversion for wells, the widest possible discretion to permit the use of wells shall prevail."*

Finally the following last quotation exemplifies the difficulty of the task of the Administration: "In administering the waters of a water course, the withdrawal of water which will lower the water table shall be permitted but not to such a degree as will prevent the water source to be recharged or replenished, under all predictable circumstances, to the extent necessary to prevent injury to senior appropriators in the order of their priorities, with due regard for daily, seasonal and longer demands on the water supply."*

It seems that the Colorado management problem is primarily legal. It would be strictly that, were it not for the caprices of the weather and the stochastic nature of rain and runoff.

It is only to the extent that there are alternatives of actions to be taken that an economic study is needed. The alternatives exist because there is an intrinsic natural variability in the hydrologic

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*Underlining is author's, not legislature's.

cycle which precludes a strictly deterministic forecast of the behavior of the system. A particular action taken to achieve a specific goal based on a given projection may fail to meet the goal as the projection is not realized. Under the circumstances a different action would have been better. Under other circumstances an altogether different action would be better. The problem of what strategy to follow in the face of uncertainty in order to act "best" in all circumstances, is equivalent to find a uniformly best estimator of an unknown population parameter. Rarely does such an estimator exist. If, on the other hand, the class of estimators (or actions) is restricted it is then possible to define a uniformly best estimator (or action). In this study if the restricted class of actions (regulations) consist of the regulations which, say, perform similarly on the average (but quite differently under specific realizations), and for which the economic consequences of their applications can be assessed, then there will exist an action (optimal) that will, say, minimize the detrimental effect of the regulations on the total gross output of the system. It is only at this stage that economics enter the picture.

C. CRITICAL REVIEW OF RELEVANT LITERATURE

The systematic design of regulations for optimal surface-groundwater usage requires the knowledge of the economic behaviour of the water user and of the hydrologic behaviour of the river basin. In the past few years Bredehoeft and Young (1970) and Young and Bredehoeft (1972) have addressed themselves to the problem of management of conjunctive groundwater and surface water systems. Based on certain assumptions they developed an economic and a hydrologic model. Their economic model "is based on the micro-economic theory of the firm and postulates a decision maker who is seeking the allocation of resources that maximizes profits within a set of technical and resource constraints" (Young and Bredehoeft, 1972, p. 542). At this stage it is assumed that the reader is familiar with the two papers of Bredehoeft and Young previously mentioned and also with the papers of Maddock (1972,1973), of Jenkins (1968), of Moulder and Jenkins (1969), and of Taylor (1971). In the negative it is strongly recommended that the reader take the time to familiarize himself with this literature.

The economic model of Young and Bredehoeft, as stated earlier, is based on the microeconomic theory of the firm. The water-user (from now on referred to as the "farmer") plans his various crop acreages at the beginning of the irrigation season based on an estimate of the availability of surface water within the priority system. In this model the farmer's attitude is rational but his attitude toward uncertainty is not explicitly spelled out, or at least let us say that Young and Bredehoeft (1972) did not discuss this point in great detail. From the sentence on page 543, "We assume the irrigator knows (1) the constraints, including the expected surface water for each month of the irrigation season,"* it can be inferred that the farmer's attitude is to accept the expected value (either in the precise statistical sense or in the practical sense of forecast, say, by the Weather Bureau) as his "certainty equivalent" (Maas et al., 1962, p. 146). In a way the farmer of Young and Bredehoeft is not overly worried by the possibility that the actual delivery of surface water may be lower than assumed in the planning stage. Such an even attitude towards uncertainty in the delivery of surface water is probably legitimate when the well capacity is large, the taxation on groundwater withdrawals is nominal, the quotas on groundwater withdrawals are liberal and the State Engineer's "capricious" interference is subdued or much tempered by Supreme Court decisions. True, Young and Bredehoeft's farmer adjusts his operations each month to the present situation (see section on monthly operating model, p. 544). At regular intervals though he does adjust his certainty equivalent to the present situation, nevertheless, he does not really guard against risk in the future. Under the previously mentioned conditions (nominal taxation, large well capacity, liberal quota or legal "laisezz-faire") this attitude may be realistic. In the simulations of Young and Bredehoeft (1972) the area operates under the priority system, with no taxation, no quota, and no interference from the State Engineer. The penalties incurred for not meeting downstream rights are not charged to the upstream users. Under these conditions a certainty-equivalent attitude on the part of the farmer is acceptable except possibly at low well capacities. For this reason the low total benefits shown on Table 6, p. 553 for low well capacities (0 to 100 cfs)

*Underlining is author's, not Young's

may reflect the bias introduced by Young and Bredehoeft in their choice of a certainty-equivalent behaviour for the farmer. More probably with low or no well capacity the farmer would naturally guard against risk by planning a given percentage below the expected values. In a statistical sense this attitude follows rationally from the fact that the slope of line B_1 on Figure 5, p. 543 is much larger than that of line B_2.

If in addition the State Engineer intervenes in the farmer's affairs by ordering wells shut, the farmer is also faced with uncertainty regarding the availability of groundwater. Even if general rules and regulations by the State Engineer are promulgated before the irrigation season, these rules do not tell precisely to the farmer whether or when his groundwater withdrawal will be limited during the season. In fact, the State Engineer does not know himself until he runs his hydrologic model (if he has one) at regular intervals, given the realization of the incoming river flow, in order to estimate what the future holds in store. Clearly the development of a State Engineer's strategy and a coherent (i.e. adapted) farmer's attitude have to be developed simultaneously. The farmer cannot shape an attitude towards this new risk, namely the State Engineer's policy, until he knows that policy and the State Engineer cannot decide on a strategy until he knows the impacts of the strategy on the regional economy. Thus the development of the State Engineer's strategy and of a farmer's attitude will require extensive simulation in a "trial and error" manner. Let us say with some amusement that the necessary character of this "trial(s) and error(s)" was well illustrated in practice in the sequence of legal cases between Mr. Kuiper, Mr. Fellhauer, Judge Carpenter and Colorado's Supreme Court (State Engineer's newsletter, Volume II, No. 2, January 1, 1972, p. 1).

Many more simulations will thus be required than the number done by Young and Bredehoeft who had only to compare a few discrete strategies (various well capacities). Yet these authors repeatedly refer to limitations on computing resources: "Owing to limitations on computing resources, formal search procedures for determining the optimum were not employed, nor were the increments in pumping capacity between the various runs as small as might be desirable," (Young and Bredehoeft, 1972, p. 549); "Since computational resources did not permit a systematic sampling of the response surface..., no global maximum can be claimed," (Bredehoeft and Young, 1970, p. 7); "The limited resources available to the project precluded any detailed field studies of the hydrologic, legal and economic relationships necessary to represent a specific area accurately." (Bredehoeft and Young, 1970, p. 12.) "Therefore it was decided that the additional precision was not worth the extra costs, and all subsequent studies were made employing the 5-year interval" (Bredehoeft and Young, 1970, p.15) and "Because of limitations of time and financial resources, it is infeasible to examine the sensitivity of the solutions to variables other than those already described." (Bredehoeft and Young, 1970, p. 19). The warning is clear. The hydrologic model must be very efficient computer-wise.

To summarize, much of the work of Bredehoeft and Young is relevant to this project and can be used, yet the objectives are so distinct that the overall system model must be completely rethought. The contrast in these objectives will be obvious by stating first what the purpose of the study is not and then what it is.

The purpose of this study is not to plan use of water for a region over the long term so as to maximize, say, total gross output, assuming that institutional barriers standing in the way of the plan will in some way be removed and that hydrologic feasibility of the plan can be automatically achieved, or at worst be achieved by a minimum of additional engineering design and construction. On the contrary the purpose of this study is to model the legal restrictions and the hydrologic constraints imposed by nature on water use as realistically as possible and to develop criteria for design of rules to enforce the law that fully utilize the available but imperfect information on the state of nature (e.g., aquifer parameters) and the laws of nature (e.g., the complex chain of numerous factors that cause snowmelt runoff to be what it is on a given date), so that the regulations will meet the goal for which they were designed.

With this purpose in mind one can proceed to the design of a hydrologic model specially conceived to answer the questions raised in the previous sections.

D. DESIGN OF THE HYDROLOGIC MODEL

First, we must ask the question: what information is the model expected to provide? For any pattern of well pumping the hydrologic model to be useful to the (hypothetical) typical farmer in the planning and operating stages must provide the drawdown in all the pumping wells. To be useful to the (real) State Engineer in defining his strategy the model must provide the stream losses to the aquifer as a response to pumping of wells and the drawdowns in the observation wells.

Immediately it is apparent that the usual finite difference models are very inefficient for this type of service. For accuracy's sake, drawdowns must be calculated at many grid points where the information has no economic interest (e.g., no wells). In addition it is even impossible at such points to verify that the calculated value is correct. If it is necessary, nevertheless, to use a finite-difference model the calculations of these intermediate values must be reduced to a minimum.

Maddock (1972) points out that an efficient method to obtain the information only where needed is to use the Green function (Kellog, 1953, p. 236) of the boundary value problem. When the boundary conditions change (i.e., pumping rates) the drawdowns in the wells can be calculated from the knowledge of the Green function. Before proceeding further with the analysis, the basic definitions needed for aquifer modeling are reviewed.

1. Basic Definitions and Equations

The basic saturated flow equation (using the Dupuit assumption and a few other traditional assumptions) describing the evolution of an isotropic water table aquifer is the Boussinesq equation:

$$\phi \frac{\partial h}{\partial t} - \frac{\partial}{\partial x} (Ke \frac{\partial h}{\partial x}) - \frac{\partial}{\partial x} (Ke \frac{\partial h}{\partial y}) = - Q_w \delta_w - q_r \delta_r \qquad (1)$$

where ϕ is the drainable (or effective) porosity, K is the saturated hydraulic conductivity (LT^{-1}), e is the saturated thickness, h is the water table level measured positive upward from a (low) horizontal datum, Q_w is the instantaneous pumping volume (L^3) of well w (chosen algebraically positive if it is an actual withdrawal rate), δ_w is a Dirac delta function singular at the point of coordinates ξ_w, η_w and τ (where ξ_w and η_w are the x,y coordinates of well w, τ is time), q_r is the aquifer instantaneous discharge volume to the river per unit reach of river and δ_r is a delta function singular along the r^{th} reach of the river. Let s be the drawdown measured positive downward from a (high) horizontal datum located at distance H above the datum for the water table elevation, and z be the elevation of the impervious bottom of the aquifer above the water-table elevation (or head) datum. Then by definition (see Figure 1):

$$s + h = H \qquad (2)$$

$$s + e + z = H \qquad (3)$$

$$e + z = h \qquad (4)$$

and Eq. (1) can be rewritten in terms of s as:

$$\phi \frac{\partial s}{\partial t} - \frac{\partial}{\partial x} (T \frac{\partial s}{\partial x}) - \frac{\partial}{\partial y} (T \frac{\partial s}{\partial y}) = Q_w \delta_w + q_r \delta_r \qquad (5)$$

where it has been assumed that the drawdowns are too small to cause a significant change on the transmissivity T (L^2T^{-1}).

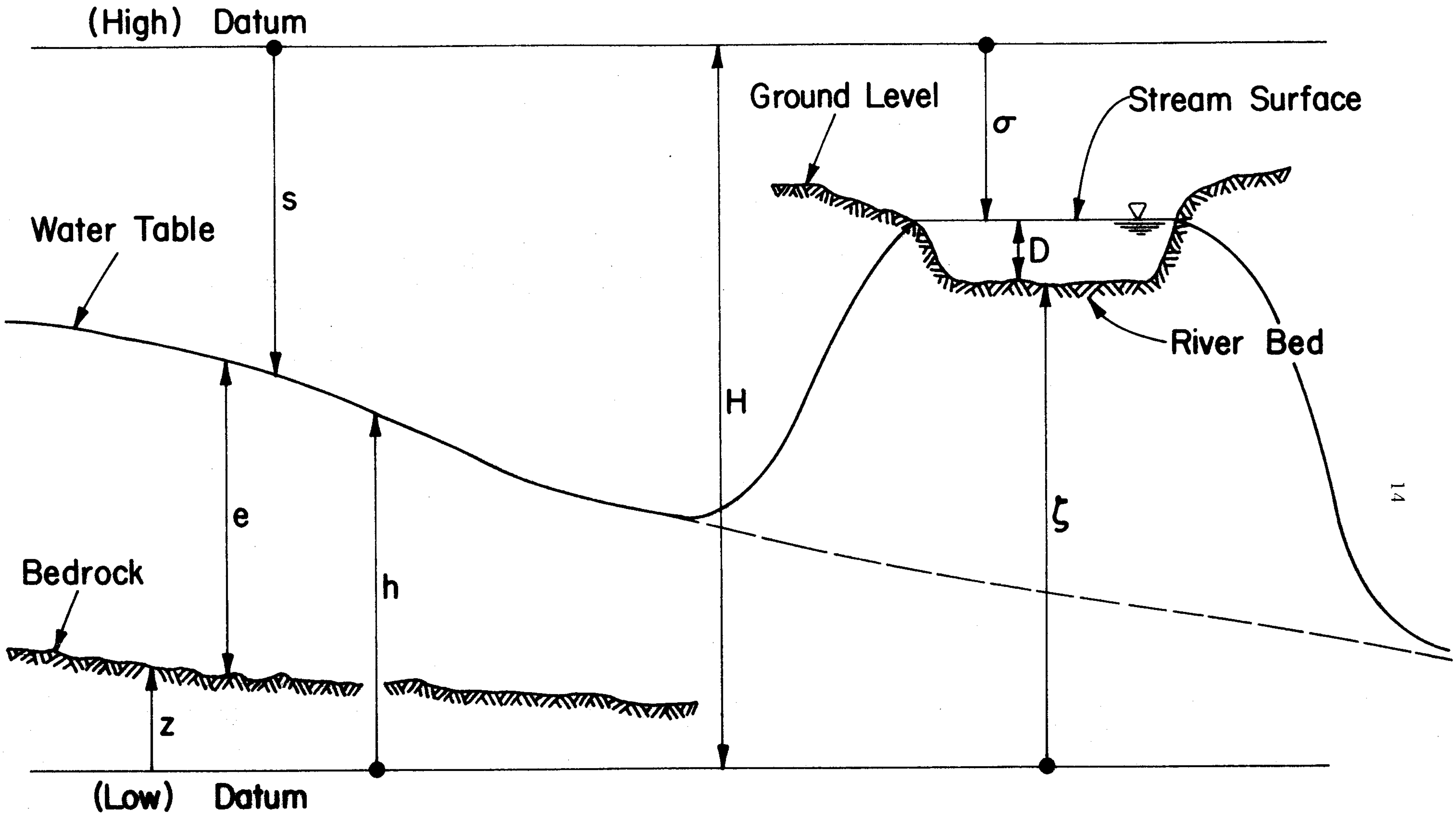
(High) Datum
Ground Level
Stream Surface
σ
s
D
Water Table
River Bed
H
e
ζ
Bedrock
h
z
(Low) Datum

Figure 1.

2. Use of Sinks for Solution of Equation (5)

If "the aquifer is assumed homogeneous and of uniform thickness, to have negligible vertical flow, and to be of infinite extent" and "assuming no previous development" (Maddock, 1972, p 130), then it can be shown (Carslaw and Jaeger, 1959, pp. 258-261) that the drawdown at well w at time t due to pumping at well p at a rate $Q(\tau)$ (L^3T^{-1}) is:

$$S_{wp}(t) = \frac{1}{4\pi T} \int_0^t Q_p(\tau)\, e^{-\frac{\phi R_{wp}^2}{4T(t-\tau)}} \frac{d\tau}{t-\tau} \tag{6}$$

where R_{wp} is the distance between well w and well p . The use of the continuous point sink instead of the instantaneous point sink (Maddock, 1972, Eq. (9), p. 130) leads more directly to the desired results.

If the pumping rate is constant during week 1 at a value $Q_p(1)$, during week 2 at a value $Q_p(2)$, etc., then the drawdown at well w at the end of week 1 is:

$$S_{wp}(1) = \frac{1}{4\pi T} Q_p(1) \int_0^1 e^{-\frac{\phi R_{wp}^2}{4T(1-\tau)}} \frac{d\tau}{1-\tau} \tag{7}$$

Let:

$$\frac{1}{4\pi T} \int_0^1 e^{-\frac{\phi R_{wp}^2}{4T(1-\tau)}} \frac{d\tau}{1-\tau} = \delta_{wp}(1) \tag{8}$$

then:

$$S_{wp}(1) = \delta_{wp}(1)\, Q_p(1) \tag{9}$$

Note that $\delta_{wp}(1)$ can be calculated once for all by a simple quadrature, and can be called an influence coefficient. Once $\delta_{wp}(1)$ is known the drawdown at well w due to pumping at well p can be readily calculated for <u>any</u> pumping rate. Thus the response of the water table to any strategy of pumping at well p for the first week is easily obtained by a simple algebraic relation. Similarly, the drawdown at well w due to pumping at well p at the end of the second week is:

$$S_{wp}(2) = \frac{Q_p(1)}{4\pi T} \int_0^1 e^{-\frac{R_{wp}^2 \phi}{4T(2-\tau)}} \frac{d\tau}{2-\tau} + \frac{Q_p(2)}{4\pi T} \int_1^2 e^{-\frac{\phi R_{wp}^2}{4T(2-\tau')}} \frac{d\tau'}{2-\tau'} \qquad (10)$$

By the change of variable $\tau' = 1 + \tau$ one easily recognizes that the coefficient of $Q_p(2)$ is $\delta_{wp}(1)$. Then letting:

$$\delta_{wp}(2) = \frac{1}{4\pi T} \int_0^1 e^{-\frac{\phi R_{wp}^2}{4T(2-\tau)}} \frac{d\tau}{2-\tau} \qquad (11)$$

one obtains:

$$S_{wp}(2) = \delta_{wp}(2)\ Q_p(1) + \delta_{wp}(1)\ Q_p(2) \qquad (12)$$

Defining:

$$\delta_{wp}(\nu) = \frac{1}{4\pi T} \int_0^1 e^{-\frac{\phi R_{wp}^2}{4T(\nu-\tau)}} \frac{d\tau}{\nu-\tau} \qquad (13)$$

and generalizing the result of Eq. (12) to 3, 4,...n weeks for a battery of P wells (method of superposition) one obtains the result:

$$s_{w}(n) = \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{wp}(\nu)\ Q_p(n-\nu+1) \tag{14}$$

which is the same as the result of Maddock (1972, p. 131, Eq. (16)) obtained in fewer lines and where $s_{w}(n)$ is the drawdown at well w at the end of the n^{th} week with all wells pumping. The advantage of Eq. (14) over the repeated solution of:

$$\phi \frac{\partial s}{\partial t} - T \left(\frac{\partial s^2}{\partial x^2} + \frac{\partial s^2}{\partial y^2}\right) = \sum_{p=1}^{P} Q_p(\tau)\ \delta_p \tag{15}$$

by a finite difference solution for all possible patterns of values for pumping, is obvious. In the case discussed above the advantage is compounded by the fact that the Green function was known analytically with little effort.

3. Heterogeneous Aquifer

Had the aquifer not been homogeneous however it would have been necessary to obtain Green's function by a finite difference technique, that is to solve numerically the partial differential equation:

$$\phi \frac{\partial G}{\partial t} - \frac{\partial}{\partial x}\left(T \frac{\partial G}{\partial x}\right) - \frac{\partial}{\partial y}\left(T \frac{\partial G}{\partial y}\right) = u_p(\tau)\ \delta_p \tag{16}$$

where $u_p(\tau)$ is the unit (withdrawal) rate at well p. Note that as the G function is evaluated numerically the $\delta_{wp}(\nu)$ coefficients are also calculated as the rates $\frac{\delta G_{wp}(\nu)}{\delta u_p}$. Note that this result is true only if during all periods the well withdrawal rate is always unity. When the aquifer is not homogeneous the Green function is not obtained analytically but the influence coefficients are calculated from a finite difference model for the Green function. For an irrigation season consisting of N weeks then the finite difference model must be run over N time-steps. Once this is done, however, the coefficients are known and the finite difference model is no longer run even if a thousand irrigation seasons were simulated. Because of this economy in computer runs one can afford to determine the $\delta_{wp}(\nu)$ coefficients quite accurately using a fine finite difference grid with more grid points than there are wells in the field.

4. Aquifer of Finite Extent

Of course the aquifer is not infinite in extent and to the equation for G defined by Eq. (16) one must add the boundary conditions $G = 0$ on the part of the boundary where the drawdown is prescribed and $\frac{\partial G}{\partial n_e} = 0$ on the part where the flux is prescribed. The procedures are unchanged; just the finite difference equations are altered due to the boundaries. However, the boundary conditions on G are not the correct boundary conditions for the drawdown unless the boundary conditions for the aquifer are conditions of no flow everywhere. This boundary condition is reasonable for an alluvial aquifer with recharge and discharge occurring essentially through the interconnection with the stream. Under these conditions Green's function with $\frac{\partial G}{\partial n_e} = 0$ everywhere on the boundary satisfies the correct boundary conditions and the coefficients $\delta_{wp}(\nu)$ are obtained.

5. Aquifer Already Developed

The most serious difficulty comes from the fact that at the beginning of the irrigation season the drawdown is not uniformly zero. Thus Green's function is not the solution of the boundary value problem. However, the solution can be obtained as the sum of G_p and another regular term v . Let:

$$s = v + Q_p G_p \tag{17}$$

The drawdown satisfies:

$$\phi \frac{\partial s}{\partial t} - \frac{\partial}{\partial x}\left(T \frac{\partial s}{\partial x}\right) - \frac{\partial}{\partial y}\left(T \frac{\partial s}{\partial y}\right) = Q_p \delta_p \tag{18}$$

the initial condition:

$$s(x,y,0) = s^\circ(x,y) \tag{19}$$

and the boundary condition:

$$\frac{\partial s}{\partial n_e} = 0 \tag{20}$$

Since $\frac{\partial G}{\partial n_e} = 0$ on the boundary, $G = 0$ at time zero, and G satisfies Eq. (16) it follows that v satisfies:

$$\phi \frac{\partial v}{\partial t} - \frac{\partial}{\partial x}\left(T \frac{\partial v}{\partial x}\right) - \frac{\partial}{\partial y}\left(T \frac{\partial v}{\partial y}\right) = 0 \tag{21}$$

the initial condition:

$$v(x,y,0) = s^\circ(x,y) \tag{22}$$

and the boundary condition:

$$\frac{\partial v}{\partial n_e} = 0 \tag{23}$$

The solution for v can be obtained by a finite difference method. Note that this solution is independent of the pumping rates. It is necessary to calculate v for each irrigation season but the study of the impact of many diverse strategies can be carried without recalculating v . In addition, if there is at least one well in each cell (see Figure 2) then the G function is evaluated at all the grid points, and of course if there is no real well in a cell an artificial one that never pumps in reality can always be introduced. From the values of the G function the values of v can be calculated by summation. Even more simply since the $\delta_{wp}(\nu)$ have been calculated one obtains the formula:

$$v_w(n) = \sum_{p=1}^{IJ} \delta_{wp}(n)\ s_p^{\circ}\ (\Delta x \Delta y)_p \tag{24}$$

where s_p° is the initial drawdown in the cell where well p is located.

We have thus obtained the result that the simulation of the behavior of the aquifer requires the calculation of at most $(IJ)^2/2$ coefficients δ_{wp} per week (or time period) with a finite difference model, or $\frac{(IJ)^2N}{2}$ coefficients if there are N weeks during the irrigation season. Then the simulation of the behavior of the aquifer is entirely modeled by the algebraic relations, Eqs. (14) and (24). Of course the number $\frac{N}{2}(IJ)^2$ can be quite large. It is comforting that it needs be calculated only once.

6. Regional and Sequential Characterization of the Influence Coefficients

For given indices w and p the behavior of $\delta_{wp}(\nu)$ as a function of ν for $\nu = 1, 2, \ldots n, \ldots N$ is regular. It is quite reasonable to expect that a few parameters curve fitting of $\delta_{wp}(\nu)$ is possible. Thus only, e.g., three values need be stored instead of say 4 x 4 = 16 assuming the irrigation season consists of four months. Similarly, when wells w and p are near the coefficients δ_{wp} are high, but

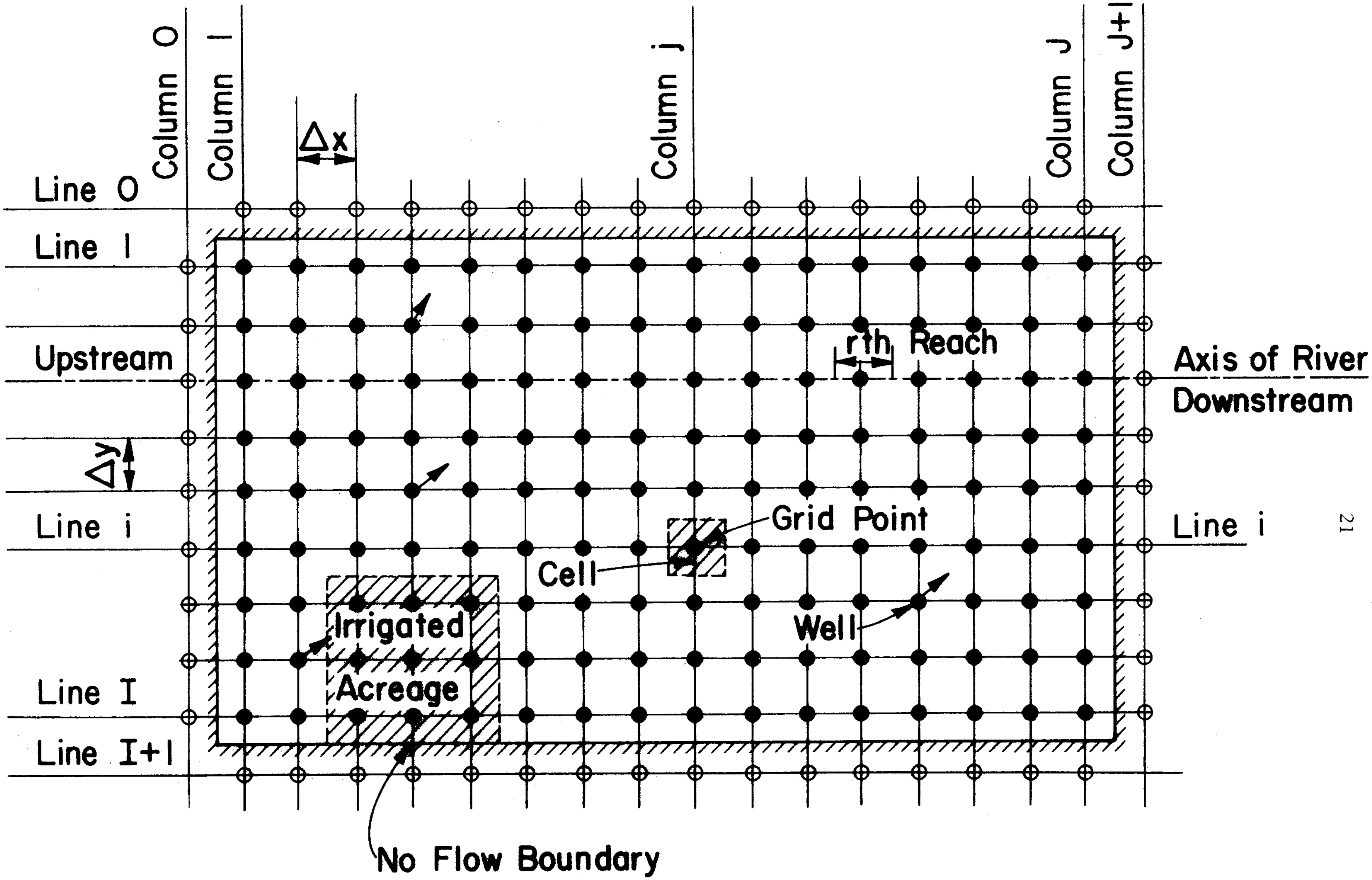
Column 0
Column 1
Column j
Column J
Column J+1
Δx
Line 0
Line 1
Upstream
rth Reach
Axis of River
Downstream
Δy
Line i
Grid Point
Line i
Cell
Irrigated
Acreage
Well
Line I
Line I+1
No Flow Boundary

Figure 2.

they are insignificant otherwise or have an effect only after the irrigation season is passed. Many of the $\delta_{wp}(\nu)$ terms in Eq. (14) can thus be dropped to study a strategy within an irrigation season. Thus only relatively few among the $\frac{(IJ)^2}{2}$ coefficients need be saved to simulate the system.

7. Initial Drawdown at Beginning of Season

Should the model simulate the long eight month winter season, or can it be assumed that sufficient observations of well levels are made at the beginning of the irrigation season? For a short-run determination of a State Engineer's strategy over one irrigation season, it seems plausible to accept the second assumption.

8. Various Forms of the Sink Terms

Implicitly the previous discussion is complete if it is understood that the sink terms do not solely represent pumping wells but also discharge from the aquifer to the river in reach r (or recharge if q_r is negative, i.e., a loss of surface water) or recharge (negative Q_p) from an irrigation plot. In the case of pumping wells the Q_p are decision variables possibly limited by pumping capacities but essentially controllable. In the case of discharge to the river (or recharge from the river) q_r is a response to a decision. This response is related to the aquifer drawdown in reach r, $s_r(n)$ and to the water depth in reach r, $D_r(n)$. Explicitly, one can write:

$$q_r(n) = \Gamma_r \{H - \zeta_r - s_r^*(n) - D_r(n)\} \tag{25}$$

where Γ_r is a coefficient of conductivity characteristic of the reach, H is the distance between the two datum (see Figure 1), ζ_r is the river bed elevation above the (low) datum, $s_r^*(n)$ is the drawdown from all causes except for the river seepage during the period n and $D_r(n)$ is the river depth. It is convenient to define the drawdown to the river surface, namely (see Figure 1):

$$\sigma_r(n) = H - \zeta_r - D_r(n) \tag{26}$$

It is of great interest to the State Engineer to know how much water is (or will be) diverted from the river by seepage into the aquifer due to the action of pumping from wells during the irrigation season. Mathematically we shall show in the next section that algebraic expressions for the $q_r(n)$ can be obtained in terms of the $\delta_{wp}(\nu)$ coefficients and the Γ_r parameters.

9. River Losses Influence Coefficients

For convenience and to avoid confusion we shall define our symbols again. Precisely, $\hat{s}_{wp}(n)$ is the drawdown at well w due to pumping at well p at the end of the n^{th} week of the irrigation season assuming a zero initial drawdown, no flow at the boundaries, no loss or gain from the river, and no recharge from irrigation. In symbols,

$$\hat{s}_{wp}(n) = \sum_{\nu=1}^{n} \delta_{wp}(n - \nu + 1)\ Q_p(\nu) \tag{27}$$

Let $v_w(n)$ be the drawdown at (observation) well w due solely to the "natural" redistribution of the water levels in the aquifer. This is the drawdown that would obtain if there was no seepage from the river and no pumping of wells. Let $s_{wp}^*(n)$ be the drawdown due to pumping from well p, due to natural redistribution and due to the seepage losses in reach r except for the seepage loss from that reach in week n (as if that reach had been sealed for that week).

Symbolically, one then has:

$$s^*_{wp}(n) = \hat{s}_{wp}(n) + v_w(n) + \sum_{\nu=1}^{n-1} \delta_{wr}(n-\nu+1)\ q_r(\nu) \tag{28}$$

It follows from that definition that:

$$q_r(n) = \Gamma_r\ [\sigma_r(n) - s^*_{rp}(n)] \tag{29}$$

The total drawdown at end of week n due to pumping at well p and seepage from reach r is:

$$s_{wp}(n) = s^*_{wp}(n) + q_r(n)\ \delta_{wr}(1) \tag{30}$$

It is possible by substitution to express $q_r(n)$ and $s_{wp}(n)$ in terms solely of the "decision" or "control" variable, $Q_p(n)$, and of the variables, $d_r(n)$, defined as:

$$d_r(n) = \sigma_r(n) - v_r(n) \tag{31}$$

Physically this means that the seepage from the river is due to the cumulative action of natural redistribution of water in the aquifer-stream system and due to the economic activity of the region. To obtain the desired result we proceed first directly and then by recurrence.

<u>Case $n = 1$</u>

In that case by definition:

$$\hat{s}_{wp}(1) = \delta_{wp}(1)\ Q_p(1) \tag{32}$$

$$s^*_{wp}(1) = \hat{s}_{wp}(1) + v_w(1) \tag{33}$$

$$q_r(1) = \Gamma_r [\sigma_r(1) - s^*_{rp}(1)] \tag{34}$$

$$s_{wp}(1) = s^*_{wp}(1) + \delta_{wr}(1)\ q_r(1) \tag{35}$$

From Eq. (33) one deduces an expression for $s^*_{rp}(1)$ (simply by setting $w = r$) and, substituting into Eq. (34), one obtains the desired relation for $q_r(1)$, namely:

$$q_r(1) = \Gamma_r\ d_r(1) - \Gamma_r\ \delta_{rp}(1)\ Q_p(1) \tag{36}$$

Substituting this last result into Eq. (35) one obtains

$$s_{wp}(1) = v_w(1) + \Gamma_r\ \delta_{wr}(1)\ d_r(1) + [\delta_{wp}(1) - \delta_{wr}(1)\ \Gamma_r\ \delta_{rp}(1)]\ Q_p(1) \tag{37}$$

Case n = 2

Following the same procedure (details are given in Appendix 3) one obtains:

$$q_r(2) = - \Gamma_r^2\ \delta_{rr}(2)\ d_r(1) + \Gamma_r\ d_r(2)$$

$$- [\Gamma_r\ \delta_{rp}(2) - \delta_{rr}(2)\ \Gamma_r^2\ \delta_{rp}(1)]\ Q_p(1) - \Gamma_r\ \delta_{rp}(1)\ Q_p(2) \tag{38}$$

and

$$s_{wp}(2) = v_w(2) + [\Gamma_r\ \delta_{wr}(2) - \delta_{wr}(1)\ \Gamma_r^2\ \delta_{rr}(2)]\ d_r(1) + \Gamma_r\ \delta_{wr}(1)\ d_r(2)$$

$$+ \{\delta_{wp}(2) - \delta_{wr}(2)\ \Gamma_r\ \delta_{rp}(1) + \delta_{wr}(1)\ [\Gamma_r\ \delta_{rp}(2) - \delta_{rr}(2)\ \Gamma_r{}^2\ \delta_{rp}(1)]\}\ Q_p(1)$$

$$+ [\delta_{wp}(1) - \delta_{wr}(1)\ \Gamma_r\ \delta_{rp}(1)]\ Q_p(2)$$

Case n = 3 (Results are given in Appendix 3)

General Recurrence Formulae

These formulae are derived in Appendix 4 in the case of P pumping wells and R distinct reaches. It is clear that the discharge from the aquifer to the reach r during period n, $q_r(n)$, can be put in the form:

$$q_r(n) = \sum_{\rho=1}^{R} \sum_{\nu=1}^{n} \gamma_{r\rho}^{n}(\nu)\, d_\rho(\nu) + \sum_{p=1}^{P} \sum_{\nu=1}^{n} \varepsilon_{rp}^{n}(\nu)\, Q_p(\nu) \tag{40}$$

and similarly:

$$s_w(n) = \sum_{\rho=1}^{R} \sum_{\nu=1}^{n} \alpha_{w\rho}^{n}(\nu)\, d_\rho(\nu) + \sum_{p=1}^{P} \sum_{\nu=1}^{n} \beta_{wp}^{n}(\nu)\, Q_p(\nu) \tag{41}$$

The computations of the coefficients γ, ε, α and β are obtained by the recurrence formulae:

$$\gamma_{r\rho}^{n}(n) = \Gamma_r \tag{42}$$

$$\gamma_{r\rho}^{n}(\nu) = -\Gamma_r \sum_{i=1}^{R} \sum_{m=\nu}^{n-1} \delta_{ri}\,(n-m+1)\, \gamma_{i\rho}^{m}(\nu) \qquad \nu < n \tag{43}$$

$$\varepsilon_{rp}^{n}(n) = -\Gamma_r\, \delta_{rp}(1) \tag{44}$$

$$\varepsilon_{rp}^{n}(\nu) = -\Gamma_r\, \delta_{rp}(n-\nu+1) - \Gamma_r \sum_{i=1}^{R} \sum_{m=\nu}^{n-1} \delta_{ri}(n-m+1)\, \varepsilon_{ip}^{m}(\nu) \qquad \nu < n \tag{45}$$

$$\alpha_{w\rho}^{n}(\nu) = \sum_{i=1}^{R} \sum_{m=\nu}^{n} \delta_{wi}(n-m+1)\, \gamma_{i\rho}^{m}(\nu) \qquad \nu \leq n \tag{46}$$

$$\beta_{wp}^{n}(\nu) = \delta_{wp}(n-\nu+1) + \sum_{\rho=1}^{R} \sum_{m=\nu}^{n} \delta_{w\rho}(n-m+1)\, \varepsilon_{\rho p}^{m}(\nu) \qquad \nu \leq n \tag{47}$$

Whereas these formulae are apparently complicated, it is important to note that these relations are linear and that the computer is ideally suited to determine the numerical values of these coefficients by a systematic procedure. Once these coefficients are generated the behavior of the stream-aquifer system is completely modeled by Eqs. (40) and (41).

10. A Plausible State Engineer's Strategy

Consider the highly simplified hypothetical stream-aquifer system of Figure 3. Let $R_0(\nu)$ be the incoming runoff for period ν. We assume that the river flow is sufficiently well regulated from the beginning of the irrigation season on to consider it as deterministically known. Let $R_1(\nu)$, $R_2(\nu)$, and $R_3(\nu)$ be the anticipated "calls placed on the river" for the various periods ν of the irrigation season as estimated by the State Engineer from recorded rights and past use. Let us suppose that zone 2 has the most senior surface water rights, then zone 3 and zone 1, all senior to the well water rights, the latter having the same seniority. If $R_0(\nu)$ is small it may happen due to heavy pumping that the surface water rights of zone 2 or 3 may not be satisfied. Legally they should be, which places the following constraints on the State Engineer's strategy:

$$R_0(\nu) - R_1(\nu) + q_1(\nu) \geq R_2(\nu) \qquad \nu = 1,2,\ldots N \tag{48}$$

$$R_0(\nu) - R_1(\nu) + q_1(\nu) - R_2(\nu) + q_2(\nu) \geq R_3(\nu) \qquad \nu = 1,2,\ldots N \tag{49}$$

Whether or not these constraints are satisfied depends on the pumping rates at the wells as is clear from Eq. (40). Note that the first summation of terms on the right-hand side of Eq. (40) is beyond control as it is (practically) only a function of the initial conditions of the

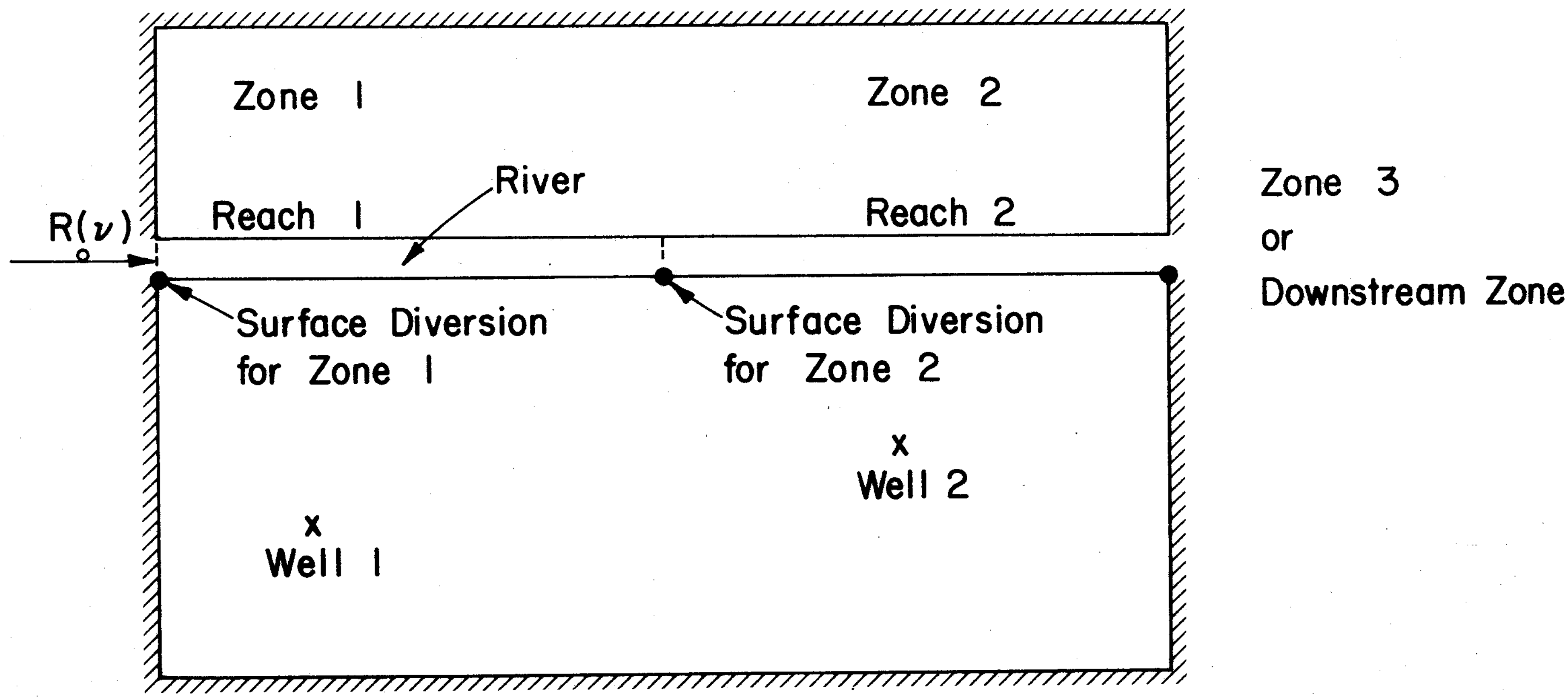
Zone 1
Zone 2
River
Reach 1
Reach 2
Zone 3
or
Downstream Zone
R(ν)
Surface Diversion
for Zone 1
Surface Diversion
for Zone 2
x
Well 2
x
Well 1

Figure 3.

aquifer at the beginning of the (irrigation) season. Since the Eqs. (42) and (49) are inequalities there are many possible strategies (i.e., choices of the values $Q_p(\nu)$) that will satisfy them. A way of choosing one among these strategies is to maximize the total volume of water pumped. A plausible strategy is obtained as the solution of a maximization problem, namely:

$$\text{Max } \{y = \sum_{p=1}^{2} \sum_{\nu=1}^{N} Q_p(\nu)\} \tag{50}$$

subject to

$$Q_p(\nu) \geq 0 \quad p = 1,2; \; \nu = 1,2,\ldots N \tag{51}$$

and subject to Eqs. (48) and (49). However, the maximization of the total availability of pumped water, regardless of time of availability, may not correspond to the optimal beneficial use. It is at this stage that agricultural economics enter into the picture. A more relevant objective function, as discussed previously by Young and Bredehoeft (1972), will be of the form:

$$y = \sum_{p=1}^{P} \sum_{\nu=1}^{N} c_p(\nu) \; Q_p(\nu) \tag{52}$$

where $c_p(\nu)$ is the mean profit (or cost) on farm p associated with use of a unit of water during the period ν. At this stage the link with the economic model is realized.

11. Illustration of the Theory on a Simple Case

We consider the case of a homogeneous infinite aquifer traversed by a creek with essentially impervious channel bed and sides except along one reach. We assume that the seepage or return flow due to the natural redistribution of the aquifer is negligible (i.e., that the variables $d_r(\nu)$ are essentially zero). There is only one well in the field and one surface diversion (ditch company) with senior right. The river is unregulated but its regime is well-known so that the (mean) runoff in any period follows an exponential decay law. The ditch company is entitled precisely to half the incoming runoff as gaged upstream of the seeping reach. The water well supplies a wealthy new development which can always use more water at a profit. The strategy of the well owner is to pump as much water as he can legally. The farmers supplied by the ditch company, unaware of the possible detrimental effect of the pumping on their water supply, are planning for the season as usual. The State Engineer who has authorized the well wants to promulgate rules and regulations at the beginning of the season which are not "capricious and arbitrary" and protect effectively the senior right holders.

From the hydrologic atlas of the area developed by the Groundwater Section of his Division of Water Resources, the State Engineer knows that the value of the transmissivity of the aquifer is $T = 10{,}000\ m^2/week$. From past work it is also known that the "seepage conductivity" for the pervious reach is $\Gamma = 4{,}000\ m^2/week$. The runoff has just been gauged by the Geological Survey at a value of $R_0 = 1{,}000\ m^3/week$, the Weather Bureau has forecast the river runoff for the irrigation season weeks according to its reliable equation:

$$R(\nu) = R_0\, e^{-\frac{\nu}{4}} \qquad \nu = 1,2,\ldots N \tag{53}$$

and the farmers plan accordingly. It is imperative to decide quickly on rules.

To exploit numerically Eq. (13) values of ϕ and R_{rp} (distance of pervious reach to pumping well) are needed: $\phi = 0.2$ and $R = 100$ m. With these values Eq. (13) becomes

$$\delta_{rp}(\nu) = \frac{1}{40{,}000 \times 3.1416} \int_0^1 e^{-\frac{0.05}{\nu-\tau}} \frac{d\tau}{\nu-\tau} \tag{54}$$

By some changes of variables this can be rewritten as

$$\delta_{rp}(\nu) = \frac{1}{40{,}000 \times 3.1416} \int_{\frac{.05}{\nu}}^{\frac{.05}{\nu-1}} \frac{e^{-u}}{u} \, du \tag{55}$$

or

$$\delta_{rp}(\nu) = \frac{1}{40{,}000 \times 3.1416} \left[\int_{\frac{.05}{\nu}}^{\infty} \frac{e^{-u}}{u} \, du - \int_{\frac{.05}{\nu-1}}^{\infty} \frac{e^{-u}}{u} \, du \right] \tag{56}$$

or

$$\delta_{rp}(\nu) = \frac{1}{125{,}700} \{-E(-\frac{.05}{\nu}) - (-E_i(-\frac{.05}{\nu-1}))\} = \frac{F(\nu)}{125{,}700} \tag{57}$$

where $-E_i(-x)$ denotes the exponential integral function, tabulated in Jahnke and Emde (1945). From these tables the values of $\delta_{rp}(\nu)$ can be obtained (Table 1). Having secured the numerical values of the $\delta_{rp}(\nu)$ coefficients it is now necessary to explicit the constraints of the problem namely:

$$-q(n) \leq \frac{1}{2} R_0 \, e^{-\frac{n}{4}} \qquad n = 1,2,\ldots N \tag{58}$$

Table I

ν	1	2	3	4	5	6	7	8
$F(\nu)$	2.47	0.67	0.40	0.28	0.22	0.18	0.15	0.13
$G(\nu)$	7.02	0.69	0.40	0.29	0.22	0.18	0.15	0.13
ν	9	10	11	12	13	14	15	16
$F(\nu)$	0.12	0.10	0.09	0.09	0.08	0.07	0.07	0.06
$G(\nu)$	0.12	0.10	0.09	0.09	0.08	0.07	0.07	0.06

by calculating the coefficients $\varepsilon^n(\nu)$ of Eq. (40). In this particular case the recurrence equations simplify to:

$$\varepsilon^n_{rp}(n) = -\Gamma \quad \delta_{rp}(1) \tag{59}$$

$$\varepsilon^n_{rp}(\nu) = -\Gamma \quad \delta_{rp}(n-\nu+1) - \Gamma \sum_{m=\nu}^{n-1} \delta_{rr}(n-m+1) \ \varepsilon^m_{rp}(\nu) \tag{60}$$

We notice that the $\delta_{rr}(\nu)$ coefficients were not calculated. By definition:

$$\delta_{rr}(\nu) = \frac{1}{4\pi T} \int_0^1 e^{-\frac{\phi b^2}{4T(\nu-\tau)}} \frac{d\tau}{\nu-\tau} \tag{61}$$

where b is the half-width of the pervious reach. For the particular case studied here then, assuming b = 10 m, we have:

$$\delta_{rr}(\nu) = \frac{1}{125,700} \{-E_i(-\frac{0.0005}{\nu}) - (-E_i(-\frac{0.0005}{\nu-1}))\} = \frac{G(\nu)}{125,700} \tag{62}$$

Since $-E_i(-x)$ increases as x decreases towards zero $G(\nu)$ will be much larger than $F(\nu)$. We note also that, as intuitively expected, $\delta_{rr}(\nu)$ attains its maximum value right at the beginning. Now the ε coefficients can be calculated:

$$\varepsilon^1(1) = -\Gamma\,\delta_{rp}(1) = -0.078$$

$$\varepsilon^2(2) = \varepsilon^1(1) = -0.078$$

$$\varepsilon^2(1) = -\Gamma\,\delta_{rp}(2) - \Gamma\,\delta_{rr}(2)\,\varepsilon^1(1) = -0.023$$

$$\varepsilon^3(3) = \varepsilon^2(2) = \varepsilon^1(1) = -0.078$$

$$\varepsilon^3(2) = -\Gamma\,\delta_{rp}(2) - \Gamma\,\delta_{rr}(2)\,\varepsilon^2(2) = \varepsilon^2(1) = -0.23$$

$$\varepsilon^3(1) = -\Gamma\,\delta_{rp}(3) - \Gamma\,\delta_{rr}(2)\,\varepsilon^2(1) - \Gamma\,\delta_{rr}(3)\,\varepsilon^1(1) = -0.014$$

etc.

Once the ε coefficients have been evaluated numerically, the constraints take the explicit form:

$$\sum_{\nu=1}^{n} \varepsilon^n(\nu)\, Q_p(\nu) \geq -\frac{R_0}{2}\, e^{-\frac{n}{4}} \quad n = 1,2\ldots N \tag{63}$$

The change of sign in the inequalities is for the convenience of having the constraints in the standard form (Morel-Seytoux, 1971):

$$\underline{A}\ \underline{x} \geq \underline{r} \tag{64}$$

where $\underline{A}$ is the matrix of coefficients, $\underline{x}$ is the (column) vector of original (or natural) variables and $\underline{r}$ is the right-hand side vector. One recognizes that the $a_{n\nu}$ elements of $\underline{A}$ are simply:

$$a_{n\nu} = \varepsilon^n(\nu) \quad \text{for} \quad \nu \leq n \tag{65}$$

$$a_{n\nu} = 0 \qquad \text{for} \quad \nu > n \tag{66}$$

Thus the matrix $\underline{A}$ is lower triangular, a fact which may be used in the optimization scheme to increase its efficiency.

To put the optimization problem in the standard form (Morel Seytoux, 1971):

$$\text{Min } \{y = \underline{c}'\underline{x} \mid A\ x \geq r\ ,\ x \geq 0 \tag{67}$$

the coefficients c^ν are simply all equal to -1 . The problem is now a standard Linear Programming (L.P.) problem. All calculations have been programmed for the CDC 3400 at CSU. Table 2 displays the elements (in absolute value) of the Matrix A. Table 3 displays the righthand side (absolute) values as well as the $F(\nu)$, $\delta_{rp}(\nu)$, $G(\nu)$ and $\delta_{rr}(\nu)$ values.

For this particular case the optimal strategy is to allow the well to be pumped only during the first week for a total volume of 3500 m^3 for that week. Even though the total amount of allowable pumped water is maximum its time distribution (i.e., 3500 m^3 the first week and none during the rest of the season) is very awkward for the developer. To supply the homes on a regular basis a large storage capacity is required. The developer would probably prefer a constant pumping rate. The State Engineer has no objection to the constant rate provided that the senior water rights are protected, that is provided that the constraint Eqs. (63) are satisfied. A mutually agreeable objective function might be:

$$y = 16\ Q \tag{68}$$

where Q is the common value of all the $Q(\nu)$. In this case the maximum value of Q is obtained very simply as:

$$Q = \underset{\nu=1,2...16}{\text{minimum of}} \left\{ \frac{r(\nu)}{\sum_{\alpha=1}^{\nu} \varepsilon^{\nu}(\alpha)} \right\} \qquad (69)$$

Since the right-hand sides of the constraint equations decrease with ν (see Table 3) and the denominator always increases with ν then the value of Q is simply:

$$Q = \frac{r(16)}{\sum_{\nu=1}^{16} \varepsilon^{16}(\nu)} = \frac{9.158}{0.179} = 51 \text{ m}^3/\text{week} \qquad (70)$$

The developer can draw at the uniform rate of 51 m^3/week during the entire season. The developer achieves regularity at the price of a large reduction of allowable total volume (only 816 m^3 instead of 3500 m^3). If the developer needs more water than that over the season he may find the price for regularity too high a price to pay. The strategy of the developer is to meet its demand (say 1600 m^3 for the season) while minimizing the size of its storage capacity. However, so to speak, this is the developer's problem not the State Engineer's.

Table 2

.0785															
.0230	.0785														
.0141	.0230	.0785													
.0103	.0141	.0230	.0785												
.0082	.0103	.0141	.0230	.0785											
.0068	.0082	.0103	.0141	.0230	.0785										
.0058	.0068	.0082	.0103	.0141	.0230	.0785									
.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785								
.0045	.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785							
.0041	.0045	.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785						
.0037	.0041	.0045	.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785					
.0034	.0037	.0041	.0045	.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785				
.0031	.0034	.0037	.0041	.0045	.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785			
.0029	.0031	.0034	.0037	.0041	.0045	.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785		
.0027	.0029	.0031	.0034	.0037	.0041	.0045	.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785	
.0026	.0027	.0029	.0031	.0034	.0037	.0041	.0045	.0051	.0058	.0068	.0082	.0103	.0141	.0230	.0785

Matrix of Coefficients of the Constraint Equations

Table 3

ν	$F(\nu)$	$10^7 \times \delta_{rp}(\nu)$	$G(\nu)$	$10^7 \times \delta_{rr}(\nu)$	$r(\nu)$
1	2.4678984	196	7.0241867	559	389.400
2	.6686099	53	.6928972	55	303.265
3	.3972180	32	.4053818	32	236.183
4	.2835456	23	.2876404	23	183.940
5	.2206576	18	.2231186	18	143.252
6	.1806625	14	.1823049	15	111.565
7	.1529648	12	.1541388	12	86.887
8	.1326415	11	.1335225	11	67.668
9	.1170906	9	.1177761	9	52.700
10	.1048064	8	.1053550	8	41.042
11	.0948567	8	.0953056	8	31.964
12	.0866334	7	.0870076	7	24.894
13	.0797228	6	.0800395	6	19.387
14	.0738338	6	.0741052	6	15.099
15	.0687552	5	.0689905	5	11.759
16	.0643305	5	.0645364	5	9.158

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the policy of this state to integrate the appropriation, use and administration of underground water tributary to a stream with the use of surface water, in such a way as to maximize the beneficial use of all of the waters of this state.

(2) Recognizing that previous and existing laws have given inadequate attention to the development and use of underground waters of the state, that the use of underground waters as an independent source or in conjunction with surface waters is necessary to the present and future welfare of the people of this state, and that the future welfare of the state depends upon a sound and flexible integrated use of all waters of the state, it is hereby declared to be the further policy of the state of Colorado that in the determination of water rights, uses and administration of water the following principles shall apply:

(a) Water rights and uses heretofore vested in any person by virtue of previous or existing laws, including an appropriation from a well, shall be protected subject to the provisions of this article.

(b) The existing use of groundwater, either independently or in conjunction with surface rights, shall be recognized to the fullest extent possible, subject to the preservation of other existing vested rights, provided, however, at his own point of diversion on a natural water course, each diverter must establish some reasonable means of effectuating his diversion. He is not entitled to command the whole flow of the stream merely to facilitate his taking the fraction of the whole flow to which he is entitled.

(c) The use of groundwater may be considered as an alternate or supplemental source of supply for surface decrees heretofore entered, taking into consideration both previous usage and the necessity to protect the vested rights of others.

(d) No reduction of any lawful diversion because of the operation of the priority system shall be permitted unless such reduction would increase the amount of water available to and required by water rights having senior priorities."

In Section 148-21-17 of the Act, responsibility of the administration and distribution of waters is placed upon the state engineer and his division engineers.

Preceding the enactment of S.B.81, and operating under 1965 groundwater laws, the state engineer ordered a number of wells shut down in the Arkansas Valley at the request of senior surface appropriators. One of the users affected by the order, Mr. Roger Fellhauer refused to comply with the order. The state engineer went to court and the court upheld the state engineer. In November, 1968, the Supreme Court reversed the decision because the order was "arbitrary and capricious." Regulations cannot be applied discriminately to some users and not others. The regulations must be clearly spelled out beforehand.

In the view of this decision and the subsequent enactment of S.B.81, the state engineer proposed to divide the irrigated areas neighboring the stream into zones (A,B,C,D,). Zone A is the closest to the river course, zone D is the most distant within two miles on either side of the river banks (see Figure 1).

A few comments are now in order. How the shutdown of a well located at the exterior limit of either zone would improve the flow in the river? More specifically how long would it take for 5 percent of the well flow rate to appear in the river after the well is shut down. Table 1 gives an estimate. It appears that shutting off wells in zone D in August when the demand is at its peak is futile because the effect would not be felt until December. By that time, the senior appropriator will be through with its need. It would seem that the order in this case violates the provisions of section 148-21-35 (2). On the other hand, assuming that wells in zones A and B are permanently shut down it can be shown that beyond anititial transient period from beginning of withdrawal, essentially the flow from any well results in an equal loss of flow in the river

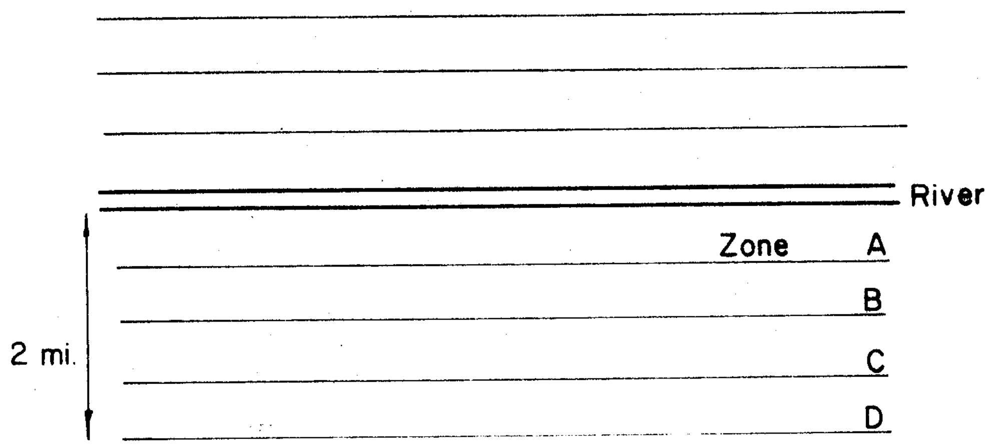

Figure 1

Zone	A	B	C	D
Days	10	30	75	150

Delay time for an Increase in River Flow Equal to 5% of the Well Flow Rate Prior to it's Shutdown.

Table 1

because of the interconnection of the river bed with the alluvial aquifer. If this is true, it would appear that according to section 142-21-17 (3) (d) "to prevent injury to senior appropriators" "under all predictable circumstances" the wells in zones C and D should also be permanently shut down. If they were, in order to fulfill this provision of the Act, one might question then whether the policy "to maximize the beneficial use of all of the waters of this state" has been fulfilled.

Again some recent experience should be reviewed. The state engineer, following the spelled out procedures of zones described earlier, ordered some wells shut down near the South Platte river in mid-1969. On July 29, 1969, an action was filed to prevent implementation of these rules. The case went to court and the judge issued an injunction against the order on August 27, 1969. Later the injunction was made permanent and the court listed 24 reasons for its action. The state engineer appealed the permanent injunction to the Supreme Court.

On October 12, 1971 (see clipping on page , the Supreme Court of Colorado handed down its decision in the case. The court overturned the lower court decisions and rebuked all 24 findings, holding the State Engineer had complied with Colorado water law (including the 1969 amendment), and that his actions were just and reasonable under the circumstances. The court emphasized the need to continually upgrade the regulation to account for technological advances in pursuit of the meximization principle of water use. Although these regulations were only effective until October 15, 1969, the case was not held moot because the problem still exists. To this end, the court recognized the need to improve the rules and regulations and made a request for guidance toward constructive progress.

4 Fort Collins Coloradoan Tuesday, Oct. 12, 1971

High court hands down major decision on well regulation

DENVER (AP) — A major decision upholding the right of the state engineer to regulate wells was handed down today by Judge Donald A. Carpenter Court.

The unanimous decision, written by Justice James K. Groves, also holds that wells have no priority until they obtain a decree from the water courts of the state.

The ruling came when the high court reversed a decision by judge Donald A. Carpenter in Water Division No. 1, located at Greeley.

Justice Groves held that state engineer Clarence J. Kuiper's set of 1969 regulations on wells along the South Platte River was valid. Judge Carpenter had issued an injunction preventing their use. The injunction was sought by the Well Owners Conservation Association and several irrigators.

In concluding his 35-page decision Justice Groves noted that the actual case is now meaningless because the regulations expire in October, 1969.

"The problem involved is not moot," he said, adding that the opinion was issued "for the guidance of the entire state and its agents that there may be constructive progress."

The justice said there must be change in the operation of wells, specifically along the South Platte River, "and courts, legislators, the state engineer and users must recognize it."

Kuiper called the decision "a land mark" in determination of rights for irrigation wells. He said his office will proceed to enact rules and regulations for the 1972 irrigating season. These, he said, will be substantially the same as the 1969 rules "with some changes."

Kuiper said that the ruling makes clear that any well which has not gone into court by next July 1 and obtained a priority date "will have no priority whatsoever."

APPENDIX 2

Derivation of the Continuous Instantaneous Point Withdrawal Solution

In the case of a homogeneous medium the groundwater drawdown equation takes the form:

$$\phi \frac{\partial s}{\partial t} = T \left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right) = T\nabla^2 s$$

or defining $\kappa = \frac{T}{\phi}$ the equation is:

$$\frac{1}{\kappa}\frac{\partial s}{\partial t} = \nabla^2 s$$

which has the same form as the heat equation whose mathematical solutions have been extensively studied by Carslaw and Jaeger.

We first verify that the equation has a solution of the type:

$$\frac{1}{t} e^{\frac{-r^2}{c\kappa t}} = \frac{1}{t} e^{(\ )}$$

$$\frac{\partial s}{\partial t} = -\frac{1}{t^2} e^{(\ )} \cdot \frac{-r^2}{c\kappa} \cdot \frac{-1}{t^2}$$

$$\frac{1}{\kappa}\frac{\partial s}{\partial t} = \frac{e^{(\ )}}{\kappa}\left[-\frac{1}{t^2} + \frac{r^2}{c\kappa t^3}\right]$$

Now, $\frac{\partial s}{\partial x} = \frac{1}{t} \cdot e^{(\ )} \cdot \frac{-1}{c\kappa t} \cdot 2r\frac{\partial r}{\partial x}$

but since $r^2 = x^2 + y^2$, $rdr = xdx + ydy$ and $r\frac{\partial r}{\partial x} = x$.

Thus $\frac{\partial s}{\partial x} = \frac{-2x\ e^{(\ )}}{c\kappa t^2}$ and

$$\frac{\partial^2 s}{\partial x^2} = \frac{-2e^{(\ )}}{c\kappa t^2} - \frac{2x\ e^{(\ )}}{c\kappa t^2} \cdot \frac{-2x}{c\kappa t} = \frac{e^{(\ )}}{\kappa}\left[\frac{-2}{ct^2} + \frac{4x^2}{\kappa c^2 t^3}\right]$$

from which we deduce by symmetry that:

$$\nabla^2 s = \frac{e^{(\ )}}{\kappa}\left[-\frac{4}{ct^2} + \frac{4r^2}{\kappa c^2 t^3}\right.$$

and by identity one finds that $\frac{1}{t} e^{\frac{-r^2}{c\kappa t}}$ is a solution if and only if $c = 4$.

It has been verified that:

$$\frac{1}{t} e^{\frac{-r^2}{4\kappa t}} \text{ is a solution of the equation.}$$

The physical significance of this particular solution will now be discussed. Note that the expression $\frac{W}{Bt} e^{\frac{-r^2}{4\kappa t}}$ is also a solution.

The total flux of water across and into the cylinder of radius r centered about the origin is:

$$\int_0^{2\pi} - Ke \frac{\partial s}{\partial r} \cdot r d\theta = -2\pi Tr \frac{\partial s}{\partial r} = -2\pi Tr \frac{W}{Bt} e^{\frac{-r^2}{4\kappa t}} \cdot \frac{-2r}{4\kappa t} = \frac{\pi Tr^2 W}{\kappa B} \frac{e^{\frac{-r^2}{4\kappa t}}}{t^2} \qquad (1)$$

We note that except for $t = 0$ the flux at $r = 0$ is zero. For $t > 0$ the flux is zero since $r = 0$ and $\frac{1}{t^2}$ is finite. However, for $t = 0$ the expression is indefinite. We calculate the total volume of water passing the cylinder over time. We obtain:

$$\int_0^{\infty} \frac{\pi TW}{\kappa B} \frac{r^2}{t^2} e^{\frac{-r^2}{4\kappa t}} dt = \frac{\pi TW}{\kappa B} \int_0^{\infty} \left(\frac{-r^2}{4\kappa t^2}\right) (-4\kappa) e^{\frac{-r^2}{4\kappa t}} dt$$

Letting $\frac{r^2}{4\kappa t} = u$, $\frac{-r^2}{4\kappa t^2} dt = du$, then the integral of Eq. (1) takes the form: $\frac{-4\pi TW}{B} \int_0^{\infty} e^{-u} du = \frac{4\pi T}{B} W$. If one chooses for the arbitrary constant B the value $4\pi T$, then W represents the total volume of water withdrawn instantaneously at time zero and at the origin.

The solution $\frac{W}{4\pi T} \frac{e^{\frac{-4^2}{4\kappa t}}}{t}$ represents the evolution of drawdown due to an instantaneous withdrawal of a volume of water W at the origin at time zero. Defining $R^2 = (x-\xi)^2 + (y-\eta)^2$ where ξ and η are the coordinates of a sink (no longer located at the origin) then

$\frac{W}{4\pi T} e^{\frac{-R^2}{4\kappa(t-\tau)}}$ represents the drawdown due to an instantaneous withdrawal of a volume W at time τ from the sink located at point of coordinates (ξ, η). If the withdrawal is taken continuously rather than instantaneously at a rate Q (L^3T^{-1}) then by superposition the expression:

$$\frac{1}{4\pi T} \int_0^t Q(\tau) \frac{e^{\frac{-\phi R^2}{4T(t-\tau)}}}{t-\tau} d\tau$$

represents the drawdown due to the continuous withdrawal from a sink at a rate Q.

APPENDIX 3

Derivation of Formulae for the Coefficients for Small Values of n

Case n = 2

In that case by definition:

$$\hat{S}_{wp}(2) = \delta_{wp}(2)\ Q_p(1) + \delta_{wp}(1)\ Q_p(2)$$

$$S^*_{wp}(2) = \hat{S}_{wp}(2) + v_w(2) + \delta_{wr}(2)\ q_r(1)$$

$$= \hat{S}_{wp}(2) + v_w(2) + \delta_{wr}(2)[\Gamma_r d_r(1) - \Gamma_r \delta_{rp}(1)\ Q_p(1)]$$

$$S^*_{rp}(2) = \hat{S}_{rp}(2) + v_r(2) + \delta_{rr}(2)[\Gamma_r d_r(1) - \Gamma_r \delta_{rp}(1)\ Q_p(1)]$$

$$q_r(2) = \Gamma_r[d_r(2) - \delta_{rr}(2)[\Gamma_r d_r(1) - \Gamma_r \delta_{rp}(1)\ Q_p(1)]$$

$$- \delta_{rp}(2)\ Q_p(1) - \delta_{rp}(1)\ Q_p(2)]$$

Thus:

$$q_r(2) = -\Gamma_r^{\ 2}\ \delta_{rr}(2)\ d_r(1) + \Gamma_r d_r(2) - [\Gamma_r \delta_{rp}(2) - \delta_{rr}(2)\ \Gamma_r^{\ 2}\ \delta_{rp}(1)]Q_p(1)$$

$$- \Gamma_r \delta_{rp}(1)\ Q_p(2)$$

Since

$$S_{wp}(2) = S^*_{wp}(2) + q_r(2)\ \delta_{wr}(1)$$

one obtains by substitution:

$$S_{wp}(2) = v_w(2) + \delta_{wp}(2)\ Q_p(1) + \delta_{wp}(1)\ Q_p(2)$$

$$+ \delta_{wr}(2)[\Gamma_r d_r(1) - \Gamma_r \delta_{rp}(1)\ Q_p(1)]$$

$$+ \delta_{wr}(1) \Big\{ -\Gamma_r^{\ 2}\delta_{rr}(2)\ d_r(1) + \Gamma_r d_r(2) - [\Gamma_r \delta_{rp}(2)$$

$$- \delta_{rr}(2)\ \Gamma_r^{\ 2}\delta_{rp}(1)]\ Q_p(1) - \Gamma_r \delta_{rp}(1)\ Q_p(2) \Big\}$$

or

$$S_{wp}(2) = v_w(2) + [\Gamma_r\delta_{rw}(2) - \delta_{wr}(1)\Gamma_r{}^2\delta_{rr}(2)]\ d_r(1) + \Gamma_r\delta_{wr}(1)\ d_r(2)$$

$$[\delta_{wp}(2) - \delta_{wr}(2)\ \Gamma_r\delta_{rp}(1) + \delta_{wr}(1)[\Gamma_r\delta_{rp}(2) - \delta_{rr}(2)\ \Gamma_r{}^2\delta_{rp}(1)]]\ Q_p(1)$$

$$[\delta_{wp}(1) - \delta_{wr}(1)\ \Gamma_r\delta_{rp}(1)]\ Q_p(2)$$

Case n = 3

By definition:

$$S^*_{wp}(3) = v_w(3) + \delta_{wp}(3)\ Q_p(1) + \delta_{wp}(2)\ Q_p(2) + \delta_{wp}(1)\ Q_p(3)$$
$$+ \delta_{wr}(3)\ q_r(1) + \delta_{wr}(2)\ q_r(2)$$

Substitution of the expressions previously derived for $q_r(1)$ and $q_r(2)$ yields:

$$S^*_{wp}(3) = v_w(3) + \delta_{wp}(3)\ Q_p(1) + \delta_{wp}(2)\ Q_p(2) + \delta_{wp}(1)\ Q_p(3)$$
$$+ \delta_{wr}(3)\ [\Gamma_r d_r(1) - \Gamma_r\delta_{rp}(1)\ Q_p(1)]$$
$$+ \delta_{wr}(2)\ \{-\Gamma_r{}^2\ \delta_{rr}(2)\ d_r(1) + \Gamma_r d_r(2) - [\Gamma_r\delta_{rp}(2)$$
$$- \delta_{rr}(2)\ \Gamma_r{}^2\delta_{rp}(1)]\ Q_p(1) - \Gamma_r\delta_{rp}(1)\ Q_p(2)\}$$

or grouping coefficients of the d_r and Q_p variables:

$$S^*_{wp}(3) = v_w(3) + \Gamma_r[\delta_{wr}(3) - \delta_{wr}(2)\ \Gamma_r\delta_{rr}(2)]\ d_r(1) + \Gamma_r\delta_{wr}(2)\ d_r(2)$$
$$+ \{\delta_{wp}(3) - \delta_{wr}(3)\ \Gamma_r\delta_{rp}(1) - \delta_{wr}(2)[\Gamma_r\delta_{rp}(2)$$
$$- \delta_{rr}(2)\ \Gamma_r{}^2\delta_{rp}(1)]\}\ Q_p(1) + \{\delta_{wp}(2) - \delta_{wr}(2)\ \Gamma_r\delta_{rp}(1)\}\ Q_p(2)$$
$$+ \delta_{wp}(1)\ Q_p(3)$$

Since $q_r(3) = \Gamma_r[\sigma_r(3) - S^*_{rp}(3)]$ one obtains:

$$q_r(3) = - \Gamma_r{}^2[\delta_{rr}(3) - \delta_{rr}(2)\ \Gamma_r\delta_{rr}(2)]\ d_r(1) - \Gamma_r{}^2\delta_{rr}(2)\ d_r(2)$$

$$+ \Gamma_r d_r(3) - \Gamma_r\{\delta_{rp}(3) - \delta_{rr}(3)\ \Gamma_r\delta_{rp}(1) - \delta_{rr}(2)[\Gamma_r\delta_{rp}(2)$$

$$- \delta_{rr}(2)\ \Gamma_r{}^2\delta_{rp}(1)]\}\ Q_p(1) - \Gamma_r\{\delta_{rp}(2) - \delta_{rr}(2)\ \Gamma_r\delta_{rp}(1)\}\ Q_p(2)$$

$$- \Gamma_r\delta_{rp}(1)\ Q_p(3)$$

Since $S_{wp}(3) = S^*_{wp}(3) + \delta_{wr}(1)\ q_r(3)$ and $q_r(3)$ is explicitly known from above, we obtain by substitution:

$$S_{wp}(3) = v_w(3) + \{\Gamma_r[\delta_{wr}(3) - \delta_{wr}(2)\ \Gamma_r\delta_{rr}(2)] - \delta_{wr}(1)\ \Gamma_r{}^2[\delta_{rr}(3)$$

$$- \delta_{rr}(2)\ \Gamma_r\delta_{rr}(2)]\}\ d_r(1) + \{\Gamma_r\delta_{wr}(2) - \delta_{wr}(1)\ \Gamma_r{}^2\delta_{rr}(2)\}\ d_r(2)$$

$$+ \delta_{wr}(1)\ \Gamma_r d_r(3) + \{\delta_{wp}(3) - \delta_{wr}(3)\ \Gamma_r\delta_{rp}(1) - \delta_{wr}(2)[\Gamma_r\delta_{rp}(2)$$

$$- \delta_{rr}(2)\ \Gamma_r{}^2\delta_{rp}(1)] - \delta_{wr}(1)\ \Gamma_r[\delta_{rp}(3) - \delta_{rr}(3)\ \Gamma_r\delta_{rp}(1)$$

$$- \delta_{rr}(2)[\Gamma_r\delta_{rp}(2) - \delta_{rr}(2)\ \Gamma_r{}^2\delta_{rp}(1)]]\}\ Q_p(1) + \{\delta_{wp}(2)$$

$$- \delta_{wr}(2)\ \Gamma_r\delta_{rp}(1) - \delta_{wr}(1)\ \Gamma_r[\delta_{rp}(2) - \delta_{rr}(2)\ \Gamma_r\delta_{rp}(1)]\}\ Q_p(2)$$

$$+ \{\delta_{wp}(1) - \delta_{wr}(1)\ \Gamma_r\delta_{rp}(1)\}\ Q_p(3)$$

which can be rewritten as:

$$S_{wp}(3) = v_w(3) + \{\Gamma_r[\delta_{wr}(3) - \delta_{wr}(1)\ \Gamma_r\delta_{rr}(3)] - \Gamma_r{}^2\delta_{rr}(2)[\delta_{wr}(2)$$

$$- \delta_{wr}(1)\ \Gamma_r\delta_{rr}(2)]\}\ d_r(1) + \Gamma_r[\delta_{wr}(2) - \delta_{wr}(1)\ \Gamma_r\delta_{rr}(2)]\ d_r(2)$$

$$+ \delta_{wr}(1)\ \Gamma_r d_r(3) + \{\delta_{wp}(3) - \delta_{wp}(1)\ \Gamma_r\delta_{rp}(3) - \Gamma_r\delta_{rp}(1)[\delta_{wr}(3)$$

$$- \delta_{wr}(1)\ \Gamma_r \delta_{rr}(3)] - [\delta_{wr}(2) - \delta_{wr}(1)\ \Gamma_r \delta_{rr}(2)][\Gamma_r \delta_{rp}(2)$$

$$- \delta_{rr}(2)\ \Gamma_r^{\,2} \delta_{rp}(1)]\}\ Q_p(1) + \{\delta_{wp}(2) - \delta_{wr}(1)\ \Gamma_r \delta_{rp}(2)$$

$$- \Gamma_r \delta_{rp}(1)[\delta_{wr}(2) - \delta_{wr}(1)\ \Gamma_r \delta_{rr}(2)]\}\ Q_p(2) + \{\delta_{wp}(1)$$

$$- \delta_{wr}(1)\ \Gamma_r \delta_{rp}(1)\}\ Q_p(3)$$

APPENDIX 4

Derivation of the General Recurrence Formulae for the Coefficients

By definition the drawdown due to all causes except the seepage from the river during the nth week is:

$$s_w^*(n) = \sum_{p=1}^{P} \sum_{\nu=1}^{m} \delta_{wp}(n-\nu+1)Q_p(\nu) + v_w(n) + \sum_{\rho=1}^{R} \sum_{\nu=1}^{n-1} \delta_{w\rho}(n-\nu+1)\ q_\rho(\nu)$$

and the seepage from the rth reach during the nth week is:

$$q_r(n) = \Gamma_r \{d_r(n) - s_r^*(n)\}$$

By substitution one obtains:

$$q_r(n) = \Gamma_r \left\{ d_r(n) - \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{rp}(n-\nu+1)\ Q_p(\nu) - \sum_{\alpha=1}^{R} \sum_{m=1}^{n-1} \delta_{r\alpha}(n-m+1)\ q_\alpha(m) \right\}$$

But the assumed form of $q_\alpha(n)$ is:

$$q_\alpha(m) = \sum_{\rho=1}^{R} \sum_{\nu=1}^{m} \gamma_{\alpha\nu\rho}^{m} d_\rho(\nu) + \sum_{p=1}^{P} \sum_{\nu=1}^{m} \varepsilon_{\alpha\nu p}^{m} Q_p(\nu)$$

Again by substitution one obtains:

$$q_r(n) = \Gamma_r \left\{ d_r(n) - \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{rp}(n-\nu+1)\ Q_p(\nu) - \sum_{\alpha=1}^{R} \sum_{m=1}^{n-1} \delta_{r\alpha}(n-m+1) \left[\sum_{\rho=1}^{R} \sum_{\nu=1}^{m} \gamma_{\alpha\nu\rho}^{m} d_\rho(\nu) + \sum_{p=1}^{P} \sum_{\nu=1}^{m} \varepsilon_{\alpha\nu p}^{m} Q_p(\nu) \right] \right\}$$

The coefficient of $d_\rho(\nu)$ is by definition $\gamma_{r\nu\rho}^{n}$. By identification of the coefficients of the variables $d_\rho(\nu)$ one obtains:

for $\nu = n$, $\gamma_{rn\rho}^{n} = \Gamma_r$ (not a function of ρ)

for $\nu < n$, $\gamma_{r\nu\rho}^{n} = -\Gamma_r \sum_{\alpha=1}^{R} \sum_{m=\nu}^{n-1} \delta_{r\alpha}(n-m+1)\gamma_{\alpha\nu\rho}^{m}$

Similarly since the coefficient of $Q_p(\nu)$ is by definition $\varepsilon^n_{r\nu p}$ then:

for $\nu = n$ $\varepsilon^n_{rnp} = -\Gamma_r \delta_{rp}(1)$

for $\nu < n$ $-\Gamma_r \delta_{rp}(n-\nu+1) - \Gamma_r \sum_{\alpha=1}^{R} \sum_{m=\nu}^{n-1} \delta_{r\alpha}(n-m+1)\ \varepsilon^m_{\alpha\nu p} = \varepsilon^n_{r\nu p}$

Now by definition:

$$s_w(n) = v_w(n) + \sum_{p=1}^{P} \sum_{\nu=1}^{m} \delta_{wp}(n-\nu+1)\ Q_p(\nu)$$

$$+ \sum_{\alpha=1}^{R} \sum_{m=1}^{n} \delta_{w\alpha}(n-m+1) q_\alpha(m)$$

or

$$s_w(n) = v_w(n) + \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{wp}(n-\nu+1)\ Q_p(\nu)$$

$$+ \sum_{\alpha=1}^{R} \sum_{m=1}^{n} \delta_{w\alpha}(n-m+1)\{ \sum_{\rho=1}^{R} \sum_{\nu=1}^{m} \gamma^m_{\alpha\nu\rho} d_\rho(\nu) + \sum_{p=1}^{P} \sum_{\nu=1}^{m} \varepsilon^m_{\alpha\nu p} Q_p(\nu)\}$$

The coefficient of $d_\rho(\nu)$ is by definition: $\alpha^n_{w\nu\rho}$. By identification of coefficients one obtains:

for $\nu = n$ $\alpha^n_{wn\rho} = \sum_{\alpha=1}^{R} \delta_{w\alpha}(1)\ \gamma^n_{\alpha n\rho}$

for $\nu < n$ $\alpha^n_{w\nu\rho} = \sum_{\alpha=1}^{R} \sum_{m=\nu}^{n} \delta_{w\alpha}(n-m+1)\ \gamma^m_{\alpha\nu\rho}$

The coefficient of $Q_p(\nu)$ is by definition: $\beta^n_{w\nu p}$. The result is:

for $\nu = n$ $\beta^n_{wnp} = \delta_{wp}(1) + \sum_{\alpha=1}^{R} \delta_{w\alpha}(1)\ \varepsilon^n_{\alpha np}$

for $\nu < n$ $\beta^n_{w\nu p} = \delta_{wp}(n-\nu+1) + \sum_{\alpha=1}^{R} \sum_{m=\nu}^{m} \delta_{w\alpha}(n-m+1)\ \varepsilon^m_{\alpha\nu p}$

PART II

LEGAL ASPECTS OF CONJUNCTIVE SURFACE-GROUNDWATER MANAGEMENT

Introduction

Background: Physical and Legal Aspects

The Physical Situation.- The principal problem of conjunctive use arises in the situation where the groundwater involved is tributary to the surface flow. Tributary groundwater is groundwater which will, if not intercepted, reach and become part of some natural stream. When a well is withdrawing water from an aquifer, or underground reservoir, a physical phenomenon called a cone of depression occurs in the alluvium of the aquifer. The cone of depression is a drained area in the shape of an inverted cone with the point of the cone at the bottom of the well pipe. This will then cause surrounding water in the aquifer to flow into the cone from all sides. If the well is in close proximity to the surface stream the effect of the withdrawal upon the visible stream will be rather immediate as part of the surface flow drains into the cone of depression. If the well is some distance from the stream the effects will not be so immediate. This lag time is affected by several factors including the distance of the well from the stream, the tansmissibility of the aquifer, the depth of the well, the time and volume of pumping and the return flow characteristics. In other words the drawdown of the water table caused by pumping wells will have variable effects on surface flows in terms of time and quantity. The effects of well pumping on surface flow is also related to the amount of time required to recharge the aquifer once the water table has been drawn down. Besides the flow or seepage of surface waters aquifers are also recharged by precipitation falling to the earth during the course of the natural hydrologic cycle or by seepage from irrigation. The water level of some underground aquifers may actually rise during the season of largest withdrawal due to the fact that the water withdrawn is used for irrigation purposes and part of it returns to the aquifer. Other aquifers may demonstrate a lowering of the water level during the pumping season and recharge during the remainder of the year. The significant point to note is that the effects of withdrawing groundwater may not be evident in surface flow for some period of time, and this time period as well as the quantity of water involved may vary greatly from location to location. This situation is one reason why the problem of conjunctive use of surface water and groundwater is so difficult to solve from a legal standpoint.

Legal Considerations.- Both legislative and judicial law often develops as a result of conflict. Either because the legislature and the courts are unable to anticipate particular problems or because these law making bodies do not have at their disposal sufficient information, many problems are solved only on an ad hoc basis as various conflicts arise. Both of these aspects seem to characterize the development of laws which seek to integrate the use of ground and surface water. State legislatures have characteristically failed to recognize the interrelationship of surface water and tributary groundwater until the time finally came when it was too late to apply any definite and immediate solutions to the problems which arose because of their reticence. In several cases the development of groundwater resources was allowed to occur independently from that of surface water resources until finally the two systems began to conflict and very complicated problems arose.

Another obstacle to the development of a realistic body of law controlling the conjunctive use of ground and surface water resources was and is the lack of an efficient technology concerning the science of hydrology. Initally the effect which wells had on surface water was simply not known, and even now there is not yet enough information available to allow a complete and efficient administration of conjunctive use. Those who first endeavored to establish a law for groundwater administration were required to do so without the benefit of much scientific knowledge on the subject of hydrology. Accordingly,legal terminology developed which did not really describe the true physical situation. The result was the establishment of a legal model before the physical model had developed. Eventually, however, the science of hydrology developed to the point of demonstrating the interconnection of surface and tributary groundwater. At this point the traditional doctrine of prior appropriation was modified to reflect this interconnection and applied to the problem of coordinating the conjunctive use of these two water resources. The ultimate goal is to maximize the beneficial utilization of surface water and tributary groundwater.

Scope and Objectives of this Section.- This portion of the report examines the legal and institutional methods and limitations concerning the conjunctive and integrated use of interrelated ground and surface water resources. The systems discussed will concern primarily the legal schemes developed by the states of Colorado and New Mexico for the purpose of administering to the conjunctive use problem, together with a cursory examination of the situations in Texas and California. More particularly the doctrine of prior appropriation will be discussed in relation to its initial development in the western states for the purpose of controlling the appropriation of surface water to its evolution and modification to control the conjunctive use of ground and surface water.

The Doctrine of Prior Appropriation

Before the development of groundwater resources became practical the western states were concerned primarily with the development of a legal system to control the allocation of surface water. This system which emerged was the doctrine of prior appropriation which provides that a person who first diverts and applies to a beneficial use the waters of a stream has a prior right thereto in relation to subsequent appropriators to the extent of his appropriation. This doctrine also traditionally recognizes what has been termed a "conditional right". Once the requirements of diverting water and applying it to a beneficial use are met, an appropriator has established his priority under the doctrine. However, many ambitious water projects may require years to complete and during such periods of completion valuable water rights could be lost due to the necessary delay in applying the water to a beneficial use. Accordingly the conditional right was created whereby a diverter may acquire a priority date which will relate back to the time the first step to secure the appropriation was taken provided the work was prosecuted with reasonable diligence. Some western states such as California had recognized riparian water rights prior to their adoption of the appropriation doctrine. Riparian water rights are incident to the ownership of land adjoining a stream and may be used only upon that land. These states found it necessary to combine the two doctrines in order to protect vested rights while other states completely abrogated riparian rights. Colorado was the first state to adopt the prior appropriation doctrine, initially through custom and ultimately through constitutional mandate. Under Colorado law all water belongs to the public but a priority right to take water is a property right, not a mere revocable privilege, which will be protected by law. Stated differently Colorado recognizes a usufructory right to water but not a property right in the water itself. The priority right to take is however, a property right. Under the Colorado Constitution the right to divert unappropriated water is absolute and although this would seem to preclude the requirement of permits to divert water recent legislation indicates a trend toward a modified permit system. The New Mexico Constitution also provides for the doctrine of prior appropriation, as well as public ownership of the streams of the state. It does not, however, guarantee the right to divert water and accordingly New Mexico has established a permit system and can deny applications to divert. New Mexico also recognizes a property interest in the right to divert and such a right may be sold and transferred.

Although the appropriation doctrine served well to control the surface allocation of water it later proved to be a principal problem to those states which were seeking methods of integrating the use of surface and groundwater. Traditionally surface water rights were acquired before any significant development of groundwater had been accomplished and, in addition, the surface supplies were usually fully appropriated. It is understandable therefore, that when groundwater resources were developed there was a resultant injurious effect on vested surface rights. Since wells initially had no administrative controls applied to them they gained a type of priority over the surface appropriators. Strict application of the priority system to protect the senior and vested surface rights would result in the shutting down of developed groundwater projects which had been established over the years at great expense. This problem is magnified by the increased recognition of the benefit which can be derived from development of groundwater resources. A strict application of the doctrine could also result in the shutting down of junior wells which are far removed from a surface stream when in fact closer senior wells are more responsible for reducing surface flow. Because of these problems it was apparent that a modified application of the appropriation doctrine would be needed to control the integrated use of ground and surface water. Before this ultimate and inevitable step was taken however, many western states continued to regulate ground and surface water as two different entities and failed to recognize their hydraulic connection.

Initial Development of Groundwater Laws

New Mexico enacted its first valid groundwater law in 1931. This law extended the doctrine of prior appropriation to underground water and declared all underground water having ascertainable boundaries to be public water. Later enactments extended public ownership to all groundwater. The State Engineer in New Mexico exerts authority over the appropriation of groundwater only in areas which he has declared underground water basins. Such a declaration is made when necessary to protect priorities and to insure beneficial use and orderly development. Outside of such areas groundwater may be appropriated without a permit subject to existing rights but within such areas a prospective appropriator must file an application with the State Engineer. If the State Engineer determines there is unappropriated water available and that it can be applied to a beneficial use without impairing existing rights he then must issue a permit to develop the water although he may attach conditions. The development of a groundwater law in Colorado was much slower. The Colorado Session Laws of 1879 established the first code for the administration of surface water but made no provision for the administration of groundwater and therefore such water remained unregulated. In spite of this fact early court decisions made attempts at regulating groundwater. The Colorado Court of Appeals in 1893 indicated a propensity toward the application of the doctrine of prior appropriation to tributary groundwater in the case of McClellan v. Hurdle, 3 Colo. App. 430, 33 P. 280. As early as 1929 a judicial attempt was also made to coordinate the use of surface and groundwater in the case of Nevius v. Smith, 86 colo. 178, 279 P. 44. In 1912 the Colorado Supreme Court decided the case of Comstock v. Ramsay, 55 Colo. 244, 133 P. 1107, wherein it was held that seepage from artificial sources that had not yet reached a stream became part of the water supply of the stream and could not be diverted to the injury of prior appropriators on the stream. Subsequent to this case the Court has been consistent in holding that tributary groundwaters belong to the stream and are subject to the appropriation doctrine. Having drawn a distinction between tributary and non-tributary groundwater it then became important for the Court to establish a method for determining the character of the water involved in each case. This method was devised in the case of DeHass v. Benesch, 116 Colo. 344, 181 P. 2d 453 (1947) wherein the presumption was established that all flowing water is tributary and the burden of proof is upon the party asserting otherwise.

This early case law proved inadequate to solve the problems of groundwater development such as the relative priorities between two well owners or between a well owner and a surface appropriator. As a result the Colorado legislature made its first attempt to regulate groundwater in 1953 (Colo. Rev. Stat. Ann. Secs. 147-18-1 et seq.). Essentially this legislation required permits for the drilling of wells and authorized studies into the problems of groundwater management. This legislation would have resulted in increased regulation if not for the fact that the State Engineer insisted it gave him no jurisdiction to regulate wells and to shut them down to protect senior surface rights. Subsequent to the 1953 Act groundwater development increased in Colorado. The surface appropriators became concerned because their senior rights were being infringed upon and the well owners were concerned lest their wells be shut down. In addition many well owners whose wells had been established for many years felt they had acquired vested property rights in them even though their rights had not been adjudicated. This situation led to the Groundwater Management Act of 1965, Colo. Rev. Stat. Ann., Sec. 148-18-1 et seq. This Act attempted to control the problem of conjunctive use by reaffirming the strict application of the appropriation doctrine to ground and surface water. In addition the State Engineer was authorized to recognize the effect a well may have upon senior surface rights and to shut down wells and deny drilling permits. In 1966 the State Engineer made an attempt to exercise this authority by ordering 39 wells to be shut down for depleting the streamflow of the Arkansas River. This action resulted in litigation which ultimately ended with the Supreme Court decision in Fellhauer v. People, 167 Colo. 320, 447 P. 2d 986 (1969). The Court upheld the State Engineer's authority to shut down wells which interfere with senior rights but only if exercised pursuant to written regulations which reasonably set out the procedure for controlling wells. The 39 wells had not been shut down persuant to such regulation and the State Engineer's conduct was therefore held to be arbitrary and capricious. The Fellhauer case also established certain requirements to which regulations must conform in order to be constitutionally acceptable. In addition the Court noted that the time had come when the existing law of vested rights had to be integrated with the doctrine of maximum utilization of the State's water. One of the aspects of this doctrine which the Court defined was the concept of "futile call" which provides that water which will do no good for a senior appropriator cannot be taken from the person using it even if that person has a junior right. Subsequent to this decision the State Engineer adopted written rules and regulations to conform to the Fellhauer mandate and to give particular deference to the maximum utilization doctrine. The principal device of the regulations was to establish a map of stream depletion factor contours along the sides of the South Platte River and to provide for well shutdowns only at a time which would cause water to be available when necessary. In this way the regulations recognized the lag time or delayed effect which well pumping has on surface flow and thereby encouraged maximum utilization. The Platte River Water Court immediately enjoined enforcement of the regulations and the State Engineer suspended statewide enforcement to await a judicial opinion from the Supreme Court. The opinion came in October of 1971 by the decision in Kuiper v. Well Owners Conservation Association, 490 P. 2d 268 wherein the Court upheld the regulations as being in conformance with the Fellhauer requirements. Regardless of the fact that the regulations had expired prior to this decision the State Engineer had a very significant victory. Armed with this victory the State Engineer has adopted new regulations which establish stream depletion factor contours for the Arkansas and South Platte Rivers similar to those upheld by the Kuiper decision. In addition all rights to appropriate groundwater tributary to these rivers for which an application for determination of the amount and priority thereof has not been filed prior to July 1, 1972 will be junior to all such claims filed prior to that date. Such junior rights may not then divert any water if curtailment is necessary to satisfy prior vested rights.

Integration of Ground and Surface Water Usage

During this same period in the history of Colorado water law the State legislature finally took important steps to solve the conjunctive use problem. The Fellhauer case had affirmed the necessity of adjudication to establish the priority of a water right. The authority of the 1965 Act to shut down wells had not been effectively used however, and therefore the need to regulate unadjudicated wells was clear. This fact led to the enactment of the Water Right Determination and Administration Act of 1969, Colo. Rev. Stat. Ann. Secs. 148-21-1 et seq., Colorado's first legislative attempt to effectively control the integrated use of ground and surface water. The legislature realized the time had long since passed to develop a completely new system of integrated use. Accordingly the new Act retained the doctrine of appropriation with its system of priorities but announced a new policy of integrating the appropriation, use and administration of tributary groundwater with the use of surface water. The substance of the Act is very close to the requirements of the Fellhauer decision and expressly adopts the concept of maximum utilization. The Act provides four principles for implementing the policy of integration: first, the protection of all vested water rights including wells; second, the fullest possible recognition of the use of groundwater; third, the use of groundwater as an alternate or supplemental source of supply for surface rights; fourth, the recognition of the futile call concept.

In order to facilitate the announced policy and principles the Act established a new procedural system. Seven water divisions were created roughly conforming to each major drainage system, and a water court in each division was given exclusive jurisdiction over water matters. Water referees were created to initially determine water cases.

The provision for using wells as alternate sources of supply is one of the more significant features of the Act in relation to maximizing the utilization of water. In this regard the Act encourages and in some cases requires persons with both surface and well diversions to utilize their wells to satisfy surface rights before seeking to assert their seniority. The Act also permits a surface appropriator to use his well as an alternate point of diversion and charge the diversion to his surface right. Once a well has been approved as an alternate means of diversion it must be used to satisfy the surface right before junior diversions will be ordered discontinued. Another significant aspect of the integration policy of the Act is that existing wells, the priorities of which have not been established, will have a priority date corresponding to the actual date of appropriation on the condition that the appropriators seek a determination of their rights prior to certain dates. This provision is not mandatory however. By 1974 and on even numbered years thereafter, each division engineer must prepare a tabulation of water rights to reflect the relative priorities. The water courts will then adjudicate these rights to legally establish their priorities. The futile call concept, as applied by the Act, is another very significant means for realizing the maximum utilization principle. The Act expressly authorizes the total or partial discontinuance of any diversion which is injuring senior rights. Such a discontinuance need not take place however, if it will not benefit a senior appropriator according to the futile call concept. This concept therefore will allow a junior well appropriator to employ the elements of lag time and aquifer recharge for his benefit since he cannot be required, by a surface appropriator, to discontinue his well diversion until the effects of his pumping are manifest. In this way the groundwater resources may be utilized to their fullest extent during the lag time period. This is a very important aspect of the 1969 Act and goes far in implementing the stated principle of maximum utilization.

This then is the basic outline of the legislation employed in Colorado to integrate the use of surface and groundwater. The Act has done nothing more than to provide the opportunity for the maximum utilization of water through integrated use; it does not provide quick solutions. The burden of accomplishing integrated use has been placed on the shoulders of the State Engineer and the water courts. This is clearly evident from the broad regulatory powers the Act gives to the State Engineer. Even more important is the grant of power to the water judiciary to allow any plan of augmentation or water management which will increase the available supply. This broad grant promotes creativity and experimentation on the part of the water courts and the appropriators and is the very essence of the Act for it seeks to establish a broad field of expertise from which to draw in solving the complexities of conjunctive use. In effect this grant delegates regulatory authority to the water courts.

New Mexico has approached the integration problem differently. The State Engineer has used his authority to declare underground water basins, discussed earlier to establish the Rio Grande Underground Water Basin. This was necessitated because the water supply of the Rio Grande River was fully appropriated and new irrigation projects were being considered which would rely on groundwater tributary to the river. It was obvious that surface rights would be adversely affected by large groundwater withdrawals, and the State Engineer therefore determined that he had the authority to issue permits for unappropriated groundwater on the condition that existing surface rights would be protected. This condition could be met only if groundwater appropriators offset the effects of their withdrawals by acquiring and retiring valid surface rights from usage. A large amount of groundwater may be removed before surface flows are affected because some of the groundwater is merely in storage in the aquifer and constitutes a large resource of unused water. In addition return flows to the Rio Grande River are quite large and accordingly it may be years before the effects of groundwater withdrawals are reflected in surface flows and new well users are not required to retire any surface rights until these effects are felt. Subsequent to the establishment of the Rio Grande Basin the authority of the State Engineer to require retirement of surface rights was attacked. The Supreme Court of New Mexico upheld the authority in the case of <u>City of Albuquerque v. Reynolds</u>, 71 N.M. 428, 379 P. 2d 73 (1962) wherein it was provided that the use, appropriation and administration of groundwater are the same as for surface water and the State Engineer has identical jurisdiction over both. The New Mexico Supreme Court has recently decided another significant case. In <u>Langenegger v. Carlsbad Irrigation District</u>, 82 N.M. 416, 483 P. 2d 297 (1971) the applicants sought to supplement their surface appropriation by diverting through wells. The application was contested on the basis that the applicants would be getting a new appropriation of water in which they did not previously have any rights. The Court treated the groundwater withdrawal as a change in the point of diversion and held that in such a case the water appropriated need not be identical to that used to satisfy the surface right. This case is significant because it allows the use of groundwater as a supplement or alternate to a surface right similar to the statutory scheme in Colorado. From the foregoing it can be seen that New Mexico like Colorado has devised a system for integrated use which protects vested rights and contemporaneously promotes maximum utilization of water resources.

The situation in California offered yet another problem due to its acceptance of both appropriation and riparian rights. California has devised the concept of average annual "safe yield" or the amount of water equivalent to the average annual recharge of an aquifer. If withdrawals from the acquifer by overlying riparian owners are less than the safe yield the difference may be taken by appropriators and exported. Overlying landowners may use all of the safe yield if they desire but have no priority system and must rely upon correlative rights. Once the safe yield has been exceeded, pumping by all wells, both appropriative and overlying, continues for at least five years. At this point a legal principle is applied; later pumpers are considered to have acquired prescriptive rights to pump because of their adverse use infringing upon the rights of prior pumpers. The prior pumpers have also gained mutual prescriptive rights, and thereafter the safe yield is achieved by proportionately reducing the withdrawals of all wells. Any overlying owners who did not withdraw water lose their rights. This method of mutal prescription was used to establish water rights in several aquifers. The basis for most of the water right adjudications in California is the presumption that all water is percolating unless proved otherwise. In one case groundwater was described as interrelated with surface water which resulted in a downstream user being entitled to share in upstream groundwater and to call on upstream appropriators to release stored water to recharge the depleted downstream aquifers. Litigation was the chief means of allocating water in California at least until a suit was filed by the Orange County Water District in 1963 seeking to adjudicate more than 2500 upstream water rights. Eventually the water users accepted four major water districts as their agents to negotiate and solved the problems by stipulation.

Conclusion

The foregoing discussion makes it obvious that Colorado and New Mexico have arrived at somewhat different solutions to the problem of integrating the use of surface and groundwater. The New Mexico system offers a more direct and immediate approach than does the Colorado system which relies more upon a statutory scheme. Also the Colorado system places more stress upon the concept of establishing some common ground of coexistence between surface and groundwater rights while the New Mexico procedure makes the existence of one dependent upon the retirement of the other, at least when the point is reached where one infringes upon the other. It is extremely significant that neither system is based upon a strict and inflexible legal scheme. In fact both systems are characterized by a singular absence of specific requirements. Instead broad regulatory powers are created and the opportunity for creativity is promoted. The failure of the legislatures to establish meaningful legislation in time to prevent or solve the problems of integrated use is the reason for the procedures now employed. California, on the other hand, has had to devise a completely different solution to the integrated use problem due to its complex dual system of water rights. The examples set by these three states will give aid to other states seeking to integrate usage of ground and surface water. Texas in particular will be faced with a highly complex situation due to its adherence to the English rule that a landowner owns the water under his land and cannot be denied the right to produce his groundwater even if it infringes upon neighboring rights. In any event western states are finally recognizing the intimate hydraulic connections of ground and surface water and the need to integrate their use. The Fellhauer case was very prophetic when it stated that this process will require the integration of the principle of vested rights with the goal of maximum utilization. Future laws which integrate water usage will, of necessity, vary from state to state as has already been demonstrated.

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PART III

ECONOMIC ASPECTS OF CONJUNCTIVE SURFACE-GROUNDWATER MANAGEMENT

Introduction

The distinctive properties of the water resource and the conditions of its availability, distribution and consumption call for special institutions (organizational, administrative and property) to control its allocation [Gaffney, 1969; Castle and Stoevener, 1970]. The most significant of these is the mobile, flowing nature of water, which makes it particularly difficult to establish and maintain the property rights which are the basis for allocation and exchange in a market economy.

Groundwater, in particular, presents a well-recognized example of a natural resource which the unregulated market economy fails to allocate so as to achieve the maximum net value of production [Ciriacy-Wantrup, 1963]. In the next section, the following theses are developed. First, many of the significant problems arising in connection with ground-water are what economists call resource allocation problems and can be properly explained and understood within theoretical economic framework. Second, these problems can be characterized as a failure of the institutions to keep pace with the growing utilization of the resources, such that the allocative institutions have become obsolete under present demands on the resource. Given these two propositions, an examination of the conceptual bases for institutional inadequacy is therefore an appropriate place to begin in formulation and evaluation of alternative allocative mechanisms.

Market Failure and Groundwater Allocation

Groundwater resources are typically utilized by a number of independent pumpers withdrawing from a common pool. Since water in unconfined aquifers ordinarily moves in response to withdrawals, the actions of any one pumper affects the conditions of production experienced by other users. Users are thus interdependent, and "external" or spillover effects may occur. The existence of substantial external effects violates at least one of the conditions required to justify the market mechanism as an optimal solution to the problem of resource allocation. In the presence of substantial external effects, the costs and gains as calculated by each of the many individual units fail to reflect their total impact on society. When "private" costs are not as large as "social" costs, the individual's incentive is to engage in more of the productive activity than is socially desirable, and a misallocation of resources is said to exist.

The type of external effect of interest here is called a "technological diseconomy"; technological because the impact is registered through a technological or physical link between production processes and "diseconomy" because the effect imposes a cost rather than a benefit upon recipient units. Thus, under conditions of heavy exploitation aquifer management becomes an issue of community concern because of technical conditions which prevent the market mechanism from properly functioning in the allocation of ground water. In the case at interest here, in the South Platte Valley in Colorado, the problem takes the form of the groundwater pumpers utilizing, in effect, water to which surface water users downstream claim a right. [Other examples of such detrimental external effects include salt water intrusion, subsidence of overlying land surface and increasing pumping costs due to lowered water tables (Young, 1970).]

The case at interest here is characteristic of river basins in arid or semi-arid regions, where surface water supplies are fully appropriated. Withdrawal of water from aquifers intimately associated with the stream may disrupt patterns of ground water flow to and from the river, thus jeopardizing the water supply of holders of junior surface water rights.

Problems of this sort arise largely because groundwater is treated in practice as an unrestricted open access resource; that is, a resource which any overlying land owner can use for whatever purpose he might chose, without charge or hindrance. Dales (1968) has observed that fugitive resources such as air and water were traditionally managed in this fashion due to physical characteristics which make their administration relatively costly. However, changing economic and technological conditions are making such institutions obsolete, a point which applies particularly to groundwater management. Groundwater has been administered as an open access resource by the public (its nominal owner) because the cost of enforcing property rights in fugitive resources under conditions of low utilization relative to supply easily may be greater than the benefits of such a policy. By its nature, groundwater flows are difficult and expensive to observe and measure. As long as withdrawals in a basin are so small that negligible effects are imposed on the group of groundwater users as a whole, it made little sense to develop elaborate systems for groundwater regulation. However, the technological changes and economic growth factors have encouraged rapid use of groundwater in the past two decades. It is becoming clear that the century-old tradition of ignoring the external consequences of groundwater withdrawal cannot persist indefinitely. Benefits from regulation have risen, in the sense that there are external costs which can be mitigated by more elaborate control mechanisms. Similarly, technological advances in fields relating to administration of groundwater resources, including monitoring and measuring devices, mathematical modeling and digital computers, suggest that costs of management may be declining. These shifts in cost and benefit suggest the possibility that some system of community management is not only feasible but optimal.

Groundwater regulation has followed a classical historical pattern consistent with the hypotheses outlined above. As detailed in the legal section of this report, groundwater use was ignored in early water statutes in Colorado. As interdependency effects became apparent, the State adopted regulations defining beneficial use and eventually placing limitations on individual use. However, there remains a strong resistance (based both on economic pressures and ideological convictions) to regulation when free access has become an established tradition.

A number of alternative solutions to market failure are suggested by economic theory (Hirshleifer, et al., 1960). These include taxation of withdrawals (to equate private and social costs), firming up property rights in water (Ciriacy-Wantrup, 1963) and centralized control. Taxation, in spite of its appeal to economists, has been adopted only as a vehicle for collecting revenue, not as a resource allocation tool. The second approach, firming up property rights is the approach followed by the present legislative trend in Colorado, in that groundwater is being brought under the appropriative doctrine, with modifications for the special characteristics of groundwater (i.e., the "doctrine of futile call").

It is further proposed that this approach be supplemented by a quasi-market system, in which the junior groundwater right-holder be able to purchase surface rights so as to permit him to use the well as an alternative source of diversion. This system has not, as of this writing, received a full practical test, since the State Engineer's groundwater regulations remain under challenge in the courts. The last general method, centralized control, also has not been tested in practice, although a variation has been shown to be desirable on narrow economic efficiency grounds in a highly simplified setting (Young and Bredehoeft, 1972).

Economic Contributions to the South Platte Optimization Model

The conceptual economic basis for selecting alternative approaches to solving conjunctive management problems has been described above. In addition to that contribution, potential additional economic aspects of the study include a) specification of an objective function, and b) predicting resource allocation behavior of water users in response to alternative regulatory practices. The latter task, in turn, requires detailed knowledge of the scale and technology of water user production activities and resource constraints which influence their production responses. During Phase I of the study, work has proceeded on both of these tasks.

Specifying an Objective Function.- As detailed in Part I of the report, the linear programming model can be specified in a form representing the maximization of the value of water to groundwater users, subject to constraints on surface water flows. A significant question arises: that of determining the value of water for irrigation at each of various periods in the irrigation season.

Markets for irrigation water rarely exist from which market prices can be used as a measure of the economic value of the resource. Hence, estimates of value must be derived by a method of indirect imputation (Young and Gray, 1972). For irrigation water, the usual imputation technique involves an estimation of the incremental net income associated with a given change in water availability. The approach is based on the economic theory of production, and utilizes the assumption that the water user allocates water and other resources so as to maximize profits within a set of technical and resource constraints. The response of the water using firm to water supply and cost depends upon the production opportunities available to it (i.e., soil, climate, adapted crops) and on the revenues and costs associated with these conditions.

Two specific problems arise in a realistic portrayal of the irrigation decision. First, for any given crop, the value of water depends on the current soil moisture status, which depends on the previous rainfall experience and irrigation practices. This sequential optimization problem has been formulated as a dynamic programming problem (Flinn and Musgrave, 1967; Hall and Butcher, 1968). Second, most irrigated farms produce a number of crops, each of which may exhibit a different value of water at a specific point in time.

One earlier approach to conjunctive surface-groundwater allocation incorporated a simulation model to estimate value of water at any point in time. A "sequential linear programming" framework was utilized to characterize both the sequential allocation aspect and the multi-crop aspect of the problem (Young and Bredehoeft, 1972). However, the approach taken in the present study will preclude this method. The value of water must be estimated prior to solution of the model, rather than determined internally as part of the simulation calculations. While this approach will necessarily sacrifice some of the realism of a sequential simulation, it opens the possibility for a more precise specification of the models of crop response to soil moisture status, upon which the marginal value estimates are to be based.

One of the graduate students employed on the project has undertaken the theoretical refinement of existing multi-stage, multi-crop models of irrigation water allocation, which would be used to estimate marginal value of water. However, his work has not yet reached a point suitable for inclusion in this report.

Empirical Data for the Model.- Coincident with the conceptual development of the "value of water" model, work has been carried forward on specifying parameters of the model which are representative of the study area. A field survey of water users was planned and commenced during the phase covered by this report. The planning stage consisted of two parts: design and pretesting of a survey research instrument and specification of sampling procedure.

The questionnaire developed for this study was adapted from one successfully used to study irrigated farmers in western Colorado. In final form, it contains twelve pages. Information requested includes resource availability (land, equiment, labor, water), crops grown, production technologies employed and respondent attitudes regarding various water management approaches. (A copy of the questionaire is available upon request from the authors.) The interviewer selected for the survey gained experience and pretested the questionnaire by interviewing twenty farmers in an area similar to,but distant from,the reach of the South Platte selected for study.

The sampling procedure undertakes to contact a representative group of water users who farm on land overlying the alluvial aquifer in the South Platte Valley between Fort Morgan and Julesburg, Colorado. Both surface water and groundwater users are to be reached. It is hoped that ten percent of this population will be contacted. The sampling process is of the type known as an "area segment" survey. Subunits of the study area (as determined on U. S. Geological Survey aquifer maps) are identified as sections (an area one mile square) in which more than half lies over the alluvial aquifer. All farmers operating on the specified section will be interviewed. Sections are numbered sequentially, within counties, in a southwest to northeast direction. Individual sections on which all farmers are to be contacted are determined by taking every tenth section on the list of four hundred and twenty. The first section was chosen from among the first ten by a random selection procedure.

The survey will occupy several months of the coming fall (1973) with one interviewer in the field most of that time. It is expected the field work portion will be completed by mid-December, with data summary and analysis to follow.

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