UNSTEADY MOVEMENT OF RIPPLES AND DUNES RELATED TO BED-LOAD TRANSPORT

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D. B. Simons

E. V. Richardson

and

C. F. Nordin

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DARYL B. SIMONS

Professor, Civil Engineering Section
Colorado State University, Fort Collins, Colorado, U.S.A.

EVERETT V. RICHARDSON

Hydraulic Engineer, U. S. Geological Survey
Colorado State University, Fort Collins, Colorado, U.S.A.

CARL F. NORDIN, JR.

Hydraulic Engineer, U. S. Geological Survey
Colorado State University, Fort Collins, Colorado, U.S.A.

SYNOPSIS

Two methods for computing bed-load transport from dune movement are considered. An equation based on the mean forward velocity and mean height of ripples and dunes is applied to flume data, and yields satisfactory results where the average dune shapes, velocities and heights can be determined accurately, as is the case for uniform two-dimensional flow. For unsteady flow, or for three-dimensional flow, where the size, shape, and movement of dunes varies in time and space, the volume of material passing through a series of incremental widths during some total time can be determined from sonic records. Bed-load transport rates are then estimated by integrating across the width of the stream. Applications of these concepts to field conditions are considered.

RESUME

Deux méthodes de calcul du debit solide de fond a partir du mouvement des dunes sont utilisées. Une equation, basée sur la vitesse moyenne d'avancement et sur la hauteur moyenne des rides et des dunes, est appliquee a des mesures de laboratoire. Elle donne des resultats satisfaisants, lorsque les formes, vitesses et hauteurs moyennes des dunes peuvent etre déterminees avec précision, comme c'est le cas des écoulements bi-dimensionnels. Pour les écoulements instationnaires ou tri-dimensionnels, ou les dimensions, formes et mouvements des dunes varient dans l'espace comme dans le temps, le volume de materiaux, passant a travers une série d'elements de largeurs partielles pendant un certain temps, peut etre determine des enregistrements d'une sonde a ultra-sons. Le debit solide peut alors etre estimé en integrant a travers la largeur du fleuve. L'application de ces idees a des mesures in situ est discutée.

1. Introduction

The bedload in most natural sand bed channels cannot be measured economically. However, where the size and shape of bed configurations--the sand waves or ripples and dunes which form on the channel bed--and the speed of shifting of the bed forms can be determined with reasonable accuracy. Simple relations based either on the mean height and velocity of the sand waves (1), or on the volume of material passing through incremental widths in some total time can be used to determine bedload transport (2).

2. Bedload Transport

When the velocity of flow in an alluvial channel is small, the channel bed configuration is ripples or dunes, or both, and a part of the transported material moves as bedload in more or less continuous contact with the bed. The bedload moves up the face of a ripple or a dune to the crest, where the coarser particles avalanche down the slope to deposit on the downstream face of the dune or in the trough, and the finer particles may be deposited on the faces of downstream dunes or swept temporarily into suspension. The ripples and dunes move downstream due to erosion from their upstream face and deposition on their downstream face.

The differential equation of bedload transport for the ripple and dune bed configuration (see Fig. 1) may be written

$$\frac{\partial y}{\partial t} + \frac{1}{(1-\lambda)} \frac{\partial q_b}{\partial X} = 0 \qquad (1)$$

Using the transformation

$$\delta = X - V_s t \qquad (2)$$

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial \delta}\frac{\partial \delta}{\partial t} = -V_s\, dy/d\delta \qquad (3)$$

$$\frac{\partial q_b}{\partial X} = \frac{\partial q_b}{\partial \delta}\frac{\partial \delta}{\partial X} = dq_b/d\delta \qquad (4)$$

Substituting these expressions in equation 1

$$-V_s \frac{dy}{d\delta} + \frac{1}{(1-\lambda)} \frac{dq_b}{d\delta} = 0 \qquad (5)$$

Simplifying

$$dq_b = (1-\lambda)\, V_s\, dy \qquad (6)$$

From which

$$q_b = (1-\lambda)\, V_s\, y + C_1 \qquad (7)$$

Assuming that the dunes and ripples have triangular shapes, the final equation for bedload transport becomes

$$q_b = (1-\lambda)\, V_s\, h/2 + C_1 \qquad (8)$$

Values of q_b may be converted from volume to weight basis by multiplying equation 8 by the unit weight of the sediment.

(1) Simons, D. B., Richardson, E. V., and Nordin, C. F., Jr., 1965, Bedload equation for ripples and dunes: U.S. Geol. Survey Prof. Paper 462H, (in press).

(2) Richardson, E. V., Simons, D. B., and Posakony, G. J., 1961, Sonic depth sounder for laboratory and field use: U.S. Geol. Survey Circ. 450.

In the foregoing equations

q_b = volume rate of bedload transport per unit width per unit of time,

λ = porosity of the sand bed,

V_s = average velocity of the ripples or dunes in the direction of flow,

h = average amplitude of the ripples or dunes,

y = elevation of the sand bed above an arbitrary horizontal datum,

x = distance parallel to the direction of flow,

t = time.

The constant of integration, C_1, may be interpreted as that part of the bedload which does not enter into the propagation of dunes and ripples. Obviously, at the threshold of movement, $C_1 = 0$, and so long as the bed is entirely covered with ripples and dunes C_1 will remain equal to zero (assuming that bedload is defined as the material that moves in more or less continuous contact with the bed). But as the velocity of flow increases, the bed configuration changes from dunes through a transition region of multiple bed forms to a plane bed. For the plane bed condition, $h = 0$ and $C_1 = q_b$. For the transition region, C_1 is indeterminate.

Equation 8, previously given by Richardson and others (2), presents no new ideas; in fact, the concept of determining bedload from the dimension and speed of the dunes and ridges in a channel dates from the 19th century (3). However, with the recent developments of electronic equipment which is portable and reliable and which permits rapid and accurate determination of bed configuration over large areas of a channel, it is desirable to evaluate the applicability of equation 8 to the practical problem of determining bedload discharge (2) and (4).

Equation 8 was applied to 101 observations of equilibrium flows in an 8-ft wide, recirculating flume, where the average heights, h, and velocities, V_s, of ripples and dunes were recorded (5). Velocities and heights of dunes were determined by probing, by visual observation in the transparent wall of the flume, and by the use of the sonic depth sounder described by Karaki and others (4). A description of the flume and of the general procedures used in collecting data has been presented (6 and 7).

Table 1 gives a resume of the bed-material characteristics, the number of observations for each bed material, and the range of unit water and bed-material discharge considered.

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(3) Hubbell, D. W., 1964, Appratus and techniques for measuring bedload: U.S. Geol. Survey Water-Supply Paper 1748.

(4) Karaki, S., Gray, E. E., and Collins, J., 1961, Dual channel stream monitor: Am. Soc. Civil Engineers Proc., v. 87, no. HY6, p. 1-16.

(5) Simons, D. B., and Richardson, E. V , 1961, Studies of flow in alluvial channels, basic data from flume experiments: Colorado State University, CER61EVR31, Fort Collins, Colorado.

(6) Simons, D. B., and Richardson, E. V., 1962, Resistance to flow in alluvial channels: Am. Soc. Civil Engineers Trans., v. 127, Part I, p. 927-1006.

(7) Simons, D. B., Richardson, E. V., and Albertson, M. L., 1961, Flume studies using medium sand (0.45 mm): U.S. Geol. Survey Water-Supply paper 1498-A.

Table 1.--Range of variables considered

Median diameter, in mm	Number of observations	Range of unit discharge, in cubic meters per second of width	Range of bed material discharge, in grams per second per cm of width
0.19	17	0.0398 - 0.255	0.000411 - 3.363
0.28	32	0.0483 - 0.255	0.000103 - 3.140
0.47	31	0.0804 - 0.181	0.00128 - 5.208
0.93	21	0.0801 - 0.264	0.0327 - 7.053

Figure 2 shows the computed bedload transport rate from equation 8, q_b, plotted against the measured bed-material transport, q_T, for each of these observations. For the coarser material, the 0.93 mm median diameter sand; the agreement between the computed bedload and the measured total load is excellent throughout the range of transport considered. The three points which deviate most from the equal line are for observations where the bed configurations were in a transition between dunes and a plane bed. Similarly, for the 0.47 mm sand, the agreement between computed bedload transport and measured total load is reasonably good, except at the higher transport rates where the suspended load was an appreciable percentage of the total load or where the bed was in a transition stage between a dune and a plane configuration.

For the finer sizes, the computed bedload is approximately equal to the measured total load for the ripple bed configurations. However, for the dune bed, the total load is always greater than the computed bedload, indicating that a large part of the total load was in suspension. An estimate of the suspended load and of the bedload in the flume can be obtained from suspended sediment samples, and the bedload so determined agrees well with values computed from equation 8.

It is concluded that equation 8 is suitable for determining bedload transport rates for the entire range of conditions considered. Also, for the coarser material, where the suspended load was negligible, the equation may be used to estimate the total bed-material discharge, at least for relatively shallow depths. At greater flow depths, the suspended load perhaps would not be negligible.

Attempts have been made, notably among Russian investigators to develop relationships for estimating dune heights and dune velocities from the characteristics of the flow (8). Several empirical relations have been advanced. Znamenskaya (9) proposed that V_s could be evaluated from the equation

$$V_s = K_1 \frac{D}{h} (V - V_o) \qquad (9)$$

and Barekyon (10) presented the following relations:

$$V_s = K_2 \frac{V^3}{gD} \qquad (10)$$

(8) Kondratev, N. E. (editor), 1959, River flow and river channel formation: Main Admin. of the Hydrometeorological Service of the Council of Ministers of the USSR, (Selected chapters from Ruslovoi Protesses, translated by Y. Prushanoky) PST no. 471, Office of Tech. Svcs.

(9) Znamenskaya, H. S., 1962, Calculation of dimensions and speed of shifting channel formations: in Soviet Hydrology: Selected Papers, Second Issue (Amer. Geophys. Union Russian Translation), p. 111-116.

(10) Barekyon, A., Sh., 1962, Discharge of channel forming sediments elements of sand waves: in Soviet Hydrology: Selected Papers, Second Issue (Amer. Geophys. Union Russian Translations), p. 128-130.

$$h = K_3 \frac{g}{C} \left(\frac{V - V_o}{V_o}\right) \qquad (11)$$

In the above,
- D = the mean depth of flow,
- V = the mean velocity,
- V_o = the non-eroding mean velocity,
- g = the acceleration due to gravity,
- K_1, K_2, and K_3 are constants.
- C = the Chezy coefficient, $V/\sqrt{gDS}$
- S = the slope of the energy gradient.

Equation 9 is unsuited to express dune velocities for the data considered (1). However, it should be noted that equation 9 was given for the speed of shifting of "meso-forms" (large-scale bars and ridges in natural channels), so the equation probably is not applicable to the ripples and dunes formed in laboratory flumes.

When equations 8 and 9 are combined, the rate of bedload movement is found to be independent of dune height, and the transport relation of equation 8 simplifies to

$$q_b = K_4 D (V - V_o) \qquad (12)$$

when K_4 is a constant. Equation 12 is quite similar to the expression for stream capacity used by G. K. Gilbert (11).

Combining equations 10 and 11 with equation 8 yields

$$q_b = K_5 \tau_o V \left(\frac{V - V_o}{V_o}\right) \qquad (13)$$

in which
- τ_o = the shear stress at the bed, γDS
- γ = the unit weight of the fluid
- K_5 = a constant.

The product of shear stress and velocity, sometimes called "stream power", has been shown to be an important factor in sediment transport (12 and 13), but the correlation of transport to stream power is better for flow over a plane bed with antidunes and standing waves than for flow over a dune bed (14).

If values of q_b are plotted against $\tau_o V \left(\frac{V - V_o}{V_o}\right)$ for the flume data, a trend is readily apparent but the scatter of points is too great to provide confidence in equations 10 or 13, (1). It must be concluded that equation 8 is best suited for determining bedload when the average values of V_s and h can be determined quite accurately, as with the sonic depth sounder. The relations for determining dune heights and velocities from flow parameters (eqs. 9 through 11) do not appear to yield reliable results.

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(11) Gilbert, G. K., 1914, The transportation of debris by running water: U.S. Geol. Survey Prof. Paper 86, 263 p.

(12) Bagnold, R. A., 1960, Sediment discharge and stream power--a preliminary announcement: U.S. Geol. Survey Circ. 421.

(13) Cook, H. L., 1936, Outline of the energetics of stream-transportation of solids: Am. Geophys. Union Trans., Part 11, ;. 456-563.

(14) Colby, B. R., 1964, Practical computations of bed-material discharge: Am. Soc. Civil Engineers Proc., (to be published in 1964).

where

K_6 is a variable coefficient that probably is very close to unity, except possibly at times when the bedload discharge is high or when the bed relief is low.

For the entire stream width, the volume rate of bedload discharge, Q_b , is

$$Q_b = \int_0^W K_6 \frac{(abcd)}{t} dw \tag{16}$$

However, because the variations in K_6 and (abcd)/t between elemental widths are indeterminant, equation 16 must be approximated. This equation probably can best be approximated by using a K_6 that is an average for all dunes and all parts of dunes within some width interval and by summing into one total area, A , all the abcd-like areas that pass a representative point within the width interval in a reasonably long time, T . For these approximations

$$Q_b = \sum Q_b' \tag{17}$$

where

$$Q_b' = \bar{K} \frac{A}{T} \Delta w \tag{18}$$

or

$$Q_b = \bar{K} \bar{v} \bar{H} \Delta w \tag{19}$$

in which

- Δw is the width interval.
- $\bar{K}$ is the average K_6 for all dunes and all parts of dunes within Δw .
- $\bar{v}$ is the mean velocity of the part of the dunes that passes the representative point ($\bar{v} = \Sigma p/T = \bar{p} N/T$).
- $\bar{H}$ is the mean height of the part of the dunes that passes the representative point ($\bar{H} = A/\Sigma p = A/\bar{p} N$).
- $\bar{p}$ is the mean dune length.
- N is the number of dunes that passes the point in time, T .
- Q_b is the volume rate of bedload discharge in the width interval, Δw .

If the bedload discharge is expressed as a weight rate rather than a volume rate, the foregoing equations become:

$$Q_B = \sum (1 - \lambda) \gamma_s \bar{K} \frac{A}{T} \Delta w \tag{20}$$

or

$$Q_B = \sum (1 - \lambda) \gamma_s \bar{K} \bar{v} \bar{H} \Delta w \tag{21}$$

where

- Q_B is the weight rate of bedload discharge for the entire width of the stream.
- γ_s is the specific weight of solid sediment without voids.

Equations 20 and 21, which apply only if the bed is formed of ripples and dunes, can be solved if data within each width interval are available on the shape of a representative longitudinal profile and on the change in bed elevation with time at a representative point.

3. Conclusions

Two methods presented for determining bedload discharge are useful in field applications when used in conjunction with a sonic depth sounder, which provides rapid and accurate determination of bed configurations. Equation 8, which is based on mean height and mean forward velocity of ripples and dunes, assumes a triangular shaped bed form, and is most applicable for reasonably uniform flow in straight narrow channels with beds of coarse sand. The equation requires accurate determinations of dune heights and velocities, and several emperical relations for determining dune heights and velocities from flow parameters are found inadequate.

Equation 20, which is based on the total volume of material in the bed configurations passing through an incremental width in some total time, is independent of dune shape and is applicable to flows and bed configurations nonuniformly distributed across a channel or to unsteady flows where active erosion and deposition are occurring.

LIST OF FIGURES

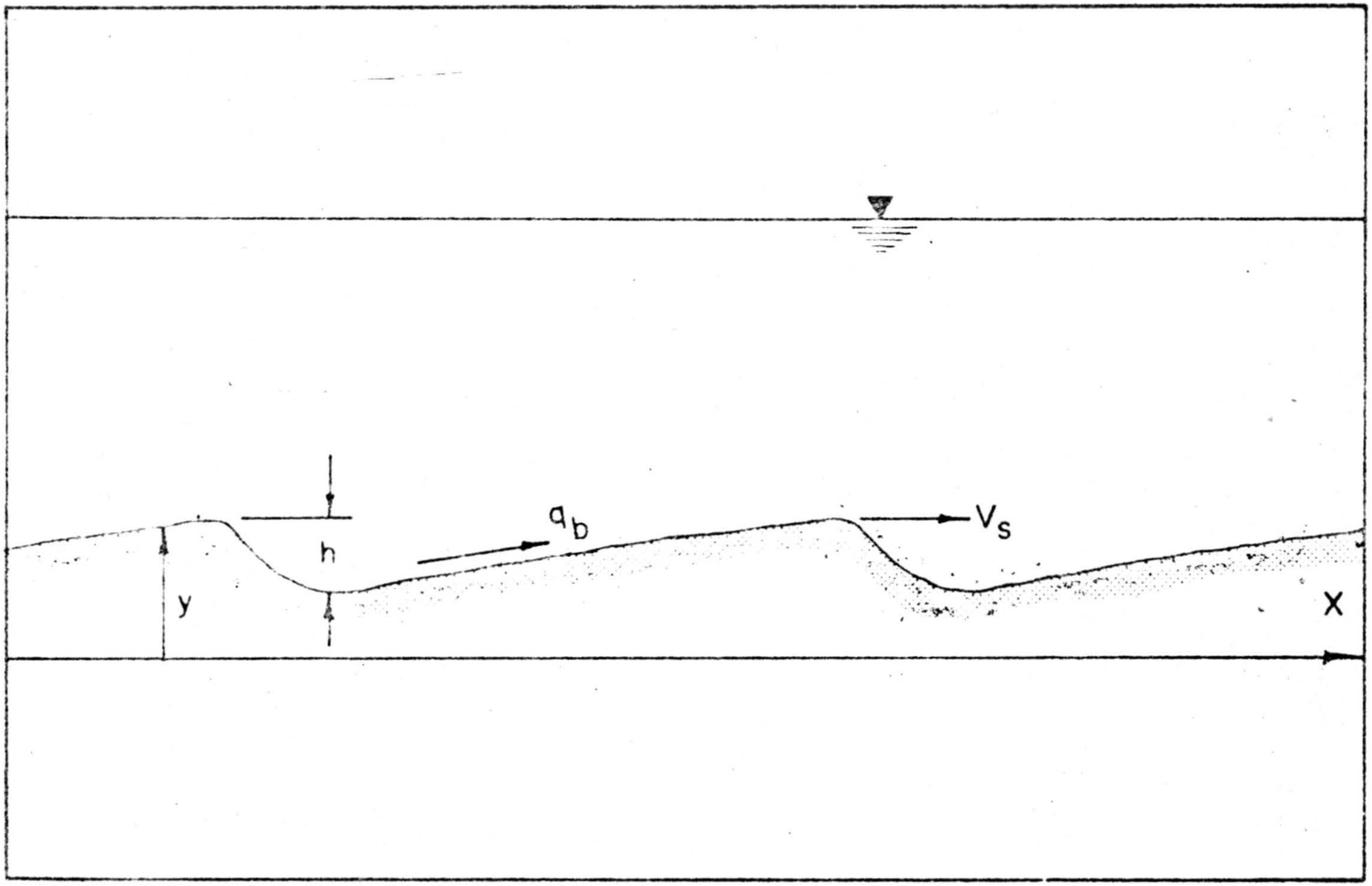
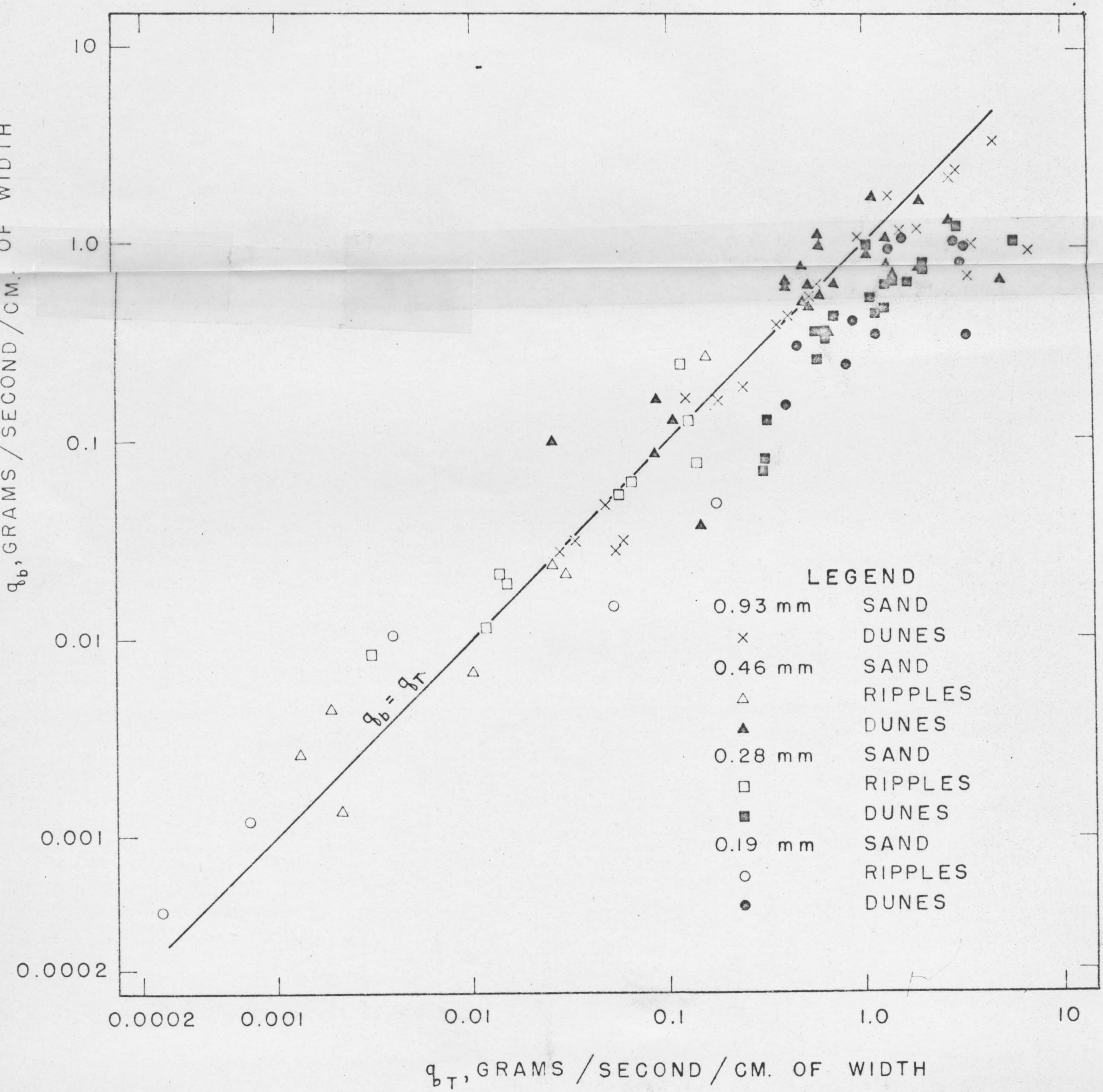


FIG. 1



10
1.0
0.1
0.01
0.001
0.0002
q_b, GRAMS / SECOND / CM. OF WIDTH
0.0002
0.001
0.01
0.1
1.0
10
q_{bT}, GRAMS / SECOND / CM. OF WIDTH
$q_b = q_T$
LEGEND
0.93 mm SAND
DUNES
0.46 mm SAND
RIPPLES
DUNES
0.28 mm SAND
RIPPLES
DUNES
0.19 mm SAND
RIPPLES
DUNES

FIG. 2

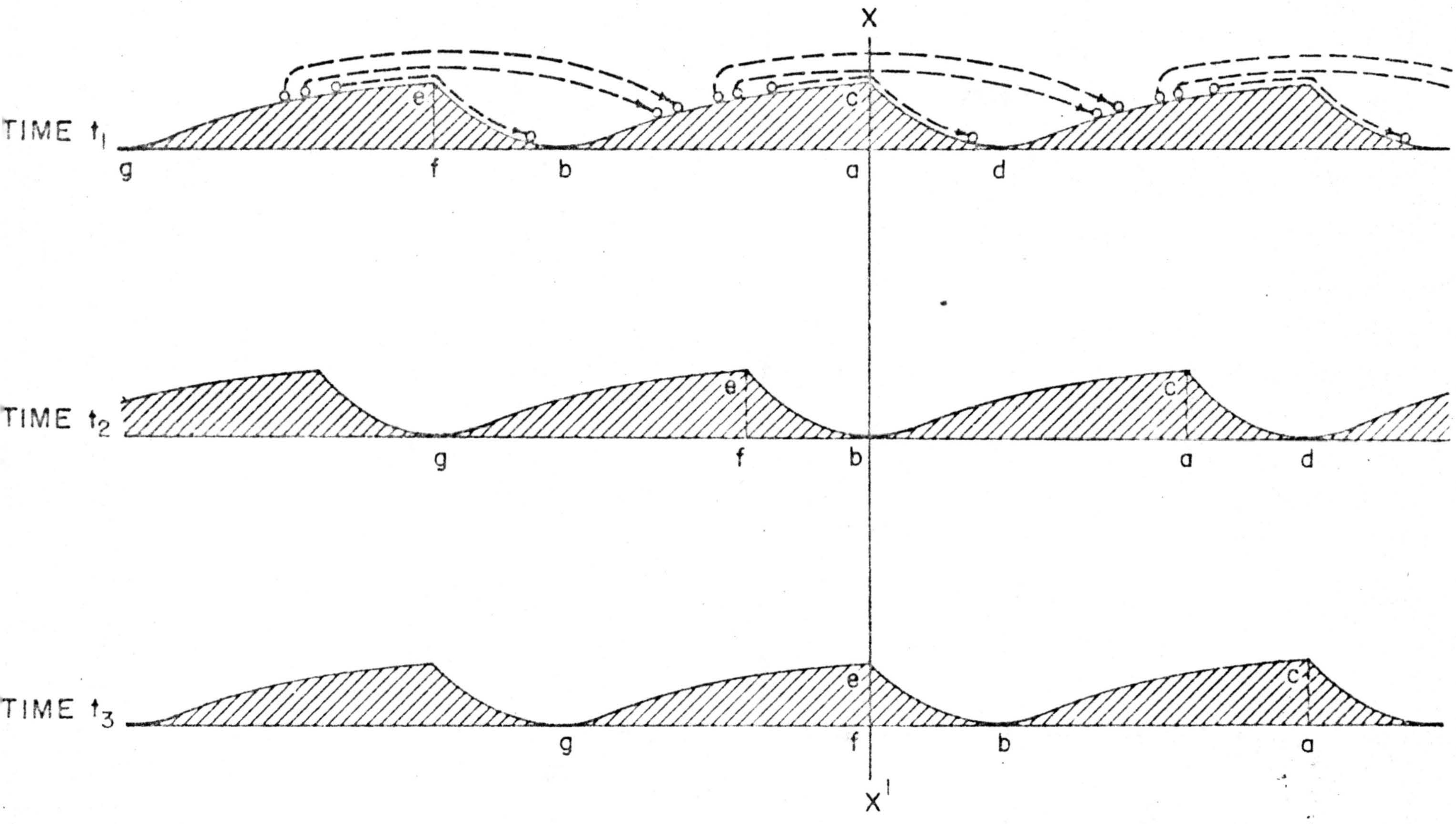


FIG. 3