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HYDRAULIC PROPERTIES OF POROUS MEDIA AND THEIR RELATIONSHIP TO DRAINAGE DESIGN

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HYDRAULIC PROPERTIES OF POROUS MEDIA AND THEIR RELATIONSHIP TO DRAINAGE DESIGN[1]

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INTRODUCTION

Almost without exception, drainage systems are designed such that the water table is maintained at a certain depth below the soil surface to produce a favorable environment for plant roots. Most of the existing design equations are based upon the assumption that a "free surface" exists at the water table; i.e., below the water table the media is fully saturated and above the water table the media is unsaturated. The "specific yield" or drainage porosity and permeability when fully saturated are two properties often used to describe porous media in these equations.

There are other properties of porous media, however, which affect the moisture and aeration status of the root zone above the water table. The objective of this paper is to describe some hydraulic properties of soil that are pertinent to drainage problems involving both the partially and completely saturated regions of the soil profile. No attempt is made here to incorporate these additional properties into drainage formulas, but equations relating the various variables important in drainage design are presented. These equations relate moisture content, air and water permeability to capillary pressure head for several soils. The parameters in these relations are shown to be characteristic properties of soil.

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POROUS MEDIA PROPERTIES AND THEORY

Capillary pressure P_c is defined as the difference in pressure between the non-wetting fluid and the wetting fluid that occupy the pores of the media. In the case of a soil-water system,

$$P_c = P_a - P_w \tag{1}$$

where P_a is the pressure of the air and P_w is the pressure of the water. If P_a is assumed to be zero gage pressure, then

$$P_c = - P_w \ . \tag{2}$$

Brooks and Corey (1) have shown the expression

$$S_e = \left(\frac{P_b}{P_c}\right)^{\lambda} \quad \text{for } P_c \geq P_b \tag{3}$$

is a good approximation for many isotropic soils. The effective saturation S_e has been defined by Corey (3) as

$$S_e = \frac{S - S_r}{1 - S_r} \ , \quad 1.0 \geq S \geq S_r \tag{4}$$

where the saturation S is the ratio of the pore volume occupied by the water to the total pore volume. The residual saturation S_r is the saturation at which the effective permeability of water is assumed to approach zero. The terms P_b and λ are called bubbling pressure and pore size distribution index respectively. The significance of the constants P_b , λ and S_r are discussed later.

Schleusener and Corey (4) and Scott and Corey (5) have used the equation

$$K_e = K \left(\frac{P_b}{P_c} \right)^{\eta} \quad \text{for } P_c \geq P_b \tag{5}$$

as an approximation for a wide range of permeability values. The permeability when the saturation is 1.0 is K and the effective permeability at lesser saturations is K_e . Brooks and Corey (1) have derived equation 5 from equation 3 using Burdine's (2) equation. According to this theory, the exponent η is related to λ by the equation

$$\eta = 3\lambda + 2 \ . \tag{6}$$

The value of λ may range from 0 to ∞ depending upon the pore size distribution; consequently, η ranges from 2 to ∞ . Small values of λ indicate the pore sizes are distributed over a wide range, while large values of λ indicate the range of pore sizes is small.

When capillary pressure is plotted as a function of effective saturation on log-log paper from experimental data, a straight line may be drawn through most of the experimental points having a slope λ . If the straight line is extrapolated to $S_e = 1.0$, the capillary pressure at this point is called the bubbling pressure P_b . The method of determining the residual saturation is discussed in the section to follow on experimental data.

In some recent work by Brooks and Corey (1) on isotropic media, it was found that the bubbling pressure P_b as obtained from the capillary pressure - effective saturation curve was related to air permeability. In their work, studies were made to determine how air permeability was related to capillary pressure and saturation. These studies showed air permeability was zero until the capillary pressure exceeded the bubbling pressure P_b . The shape of the air permeability - capillary pressure curve was found to be related to the pore size distribution λ . The air permeability equation of Brooks and Corey is

$$Kr_a = \left[1 - \left(\frac{P_b}{P_c}\right)^{\lambda}\right]^2 \left[1 - \left(\frac{P_b}{P_c}\right)^{2+\lambda}\right] \text{for } P_c \geq P_b \qquad (7)$$

where Kr_a is the ratio of the air permeability of the partially saturated soil to the air permeability of the dry soil. This ratio is called relative air permeability.

POROUS MEDIA PROPERTIES AND EXPERIMENTAL DATA

If the flow of water in a partially saturated region is parallel to the water table or if the flow is zero in all directions (static conditions), then capillary pressure is a linear function of elevation above the water table, i.e.,

$$\frac{P_c}{\gamma} = Z \ . \qquad (8)$$

If there is downward flow, the capillary pressure head P_c/γ is less than for static conditions.

In Figure 1, capillary pressure head is plotted as a function of percent saturation from experimental data with a smooth curve connecting the points. Curve number 1 is for a clean sand with well-rounded particles. Similar sands often have been used in the laboratory as material for porous media models. Curve 2 represents a fine-textured volcanic sand containing porous aggregates, curve 3 a silty fine sand and curve 4 a silt loam. These same data are plotted on log-log scales in Figure 2 using effective saturation instead of saturation.

The residual saturation is determined to a first approximation by estimating the position of a vertical asymptote to the curve in Figure 1. Using this approximate value of residual saturation, effective saturation is computed and plotted as a function of capillary pressure on log-log paper. A straight line is drawn through most of the points by ignoring saturation values near both ends of the curve. These data point at higher values of P_c/γ are then fitted on the straight line by computing a new residual saturation value. The effective saturation is recomputed as a function of capillary pressure using the second approximation of residual saturation. The process is repeated until all the points are fitted onto the straight line, with the exception of saturation values near unity. An example of this computation is given in reference (1).

In Figure 2, the slope of the curve λ and the bubbling pressure head P_b/γ is given for each medium. The η value as computed from λ using equation 6 is also shown for each medium.

Relative air permeability Kr_a as a function of capillary pressure head is shown in Figure 3. The solid lines in Figure 3 represent theoretical curves while the points are experimental data. These curves are shown on a relative permeability scale in order to show how the pore-size distribution index affects the distribution of air permeabilities above the water table.

DISCUSSION AND CONCLUSIONS

If the pore-size distribution index is small, the air permeability gradually increases above the capillary fringe. Conversely, if the pore size distribution index is large there is an abrupt change in the air permeability above the capillary fringe. This contrast is shown in Figure 3 by observing curve 1 and curve 4. The height of the capillary fringe for static conditions is defined here as P_b/γ .

The air permeability is zero during desaturation until the bubbling pressure of the soil is exceeded. Although oxygen reaches plant roots by diffusion (not by bulk flow) a zero value of air permeability indicates a discontinuous gas phase and a negligible rate of diffusion. As shown in Figure 3 the bubbling pressure can be exceedingly large for fine textured soils if the soil is initially saturated.

Drains should be placed at a depth and spacing to provide the desired moisture or aeration conditions during and after drainage. If the bubbling pressure is large, drains should be placed at a depth to maintain the zone of negligible aeration at an adequate depth below the soil surface. Likewise, if the pore size distribution index is small the drains should probably be placed at a greater depth than for soils with a higher index.

Curve 1 in the figures mentioned above represents data on a clean sand having a relatively small bubbling pressure and a relatively high permeability. With respect to these two properties, this sand is very suitable for use in many porous media models. The pore size distribution index λ , however, is very large. As noted by Scott and Corey (5), an important requirement of similitude between the porous media model and the proto-type is

$$\eta_m = \eta_p$$

where the subscripts m and p denote model and proto-type respectively; obviously the same relationship must be true for λ . As soils have η values which in general range from 3 to 8, the sand represented by curve 1 is not suitable as material for modeling irrigation and drainage problems. For example, if the proto-type consisted of the silt loam represented by curve 4, similarity can not be achieved by using this sand (curve 1). Sands in general have large values of η and do not meet all the similitude requirements as material for modeling soils. The smaller values of η are found only for media possessing secondary as well as primary porosity; i.e., structure.

Residual saturation is somewhat related to the field capacity or moisture equivalent of isotropic soils. Water permeabilities at this saturation are very small. The higher the clay content of the soil, the higher the residual saturation.

It should be noted that all the hydraulic properties of soils mentioned above may be obtained from a single capillary pressure desaturation curve. Further, once these properties are determined for each soil layer, the distribution of air, moisture and permeability above the water table for static conditions may be computed. It is essential for this purpose, however, to have precise capillary pressure-desaturation data from complete saturation to field capacity. As noted by Brooks and Corey (1) the methods usually used by soil physicists are inadequate for this purpose, at least in the range of high saturations.

SUMMARY

Experimental data and theoretical curves relating air and water permeability and saturation to capillary pressure are shown. The capillary pressure-desaturation curve is shown to yield hydraulic properties that are important when dealing with the soil as a whole, i.e., above and below the water table. These properties are: (1) bubbling pressure, (2) pore size distribution index, and (3) residual saturation. When used with the theoretical equations they can be useful for predicting the moisture and aeration status of soils for static conditions above the water table. No attempt has been made to incorporate these hydraulic properties into drainage equations.

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