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FINAL REPORT

LAMINAR AND TURBULENT FLOW OF WATER THROUGH POROUS MEDIA

Principal Investigator
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Colorado State University

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I. SUMMARY

Two quadratic equations, one for compressible and the other for incompressible fluids flowing through porous media in the linear and nonlinear flow regimes were developed theoretically and verified experimentally. The nonlinear flow regime was found to be the result of the effects of convective acceleration and not turbulence. Since convective acceleration is dependent on boundary conditions in the porous medium, relationships similar to the Carman-Kozeny equation were investigated in order to evaluate the geometrical properties of the porous medium.

One of the most important physical properties of the porous medium affecting flow is the specific surface. A semi-empirical correlation for obtaining the value of specific surface from hydraulic measurement was developed and verified. The method is fast and relatively accurate (within $\pm$ 10% of the value obtained from statistical analysis of cut-sections), thus allowing it uses in industrial applications. Examples of applications would be processes involving rate of chemical reaction of particles which depend upon surface area such as cement, ion adsorption and leaching of soils.

Since convective acceleration is the primary cause of the nonlinear flow behavior of fluid in porous media, a study of longitudinal dispersion was performed. Previous studies in literature indicated that the longitudinal dispersion coefficient in the nonlinear flow region should be a function of velocity raised to a power greater than one. This was postulated on the basis that the effects of turbulence, which increase mixing of two miscible fluids,

was the predominant flow characteristic. However, the longitudinal dispersion coefficient, because of convective acceleration terms, was found to be a function of the velocity raised to a power less than one. An application of this result would be the design and operation of column chromatography in the nonlinear flow regime in order to decrease the time of operation (normally hours) without a significant increase in the mixing of the various constituents.

Miniature probes of 1.8 mm diameter and less were developed to measure water velocities in range between 1 cm/sec to 15 cm/sec. Probes were designed to measure absolute magnitude and directions of the fluid velocity without disturbing the flow field. To date, these probes can only be used under laboratory conditions which would restrict their use to research applications.

II. DISCUSSION

The results of the work accomplished under grant GK-1162 are given in detail in accompanying thesis, dissertation and paper. The purpose of the following discussion is to briefly summarize the work contained in the theses and dissertations.

A) Ahmed, N., Physical Properties of Porous Medium Affecting Laminar and Turbulent Flow of Water, Ph.D. Dissertation, June 1967.

Discussion

An equation relating the hydraulic gradient to the flow velocity was obtained from analysis of the Navier-Stokes equation. This equation, valid for laminar and nonlinear flow of incompressible fluids through a given porous medium, is

$$\phi = \frac{\mu}{\rho g k} q + \frac{1}{g\sqrt{ck}} q^2 \tag{1}$$

where μ = viscosity (FT/L^2), ρ = density (FT^2/L^4), g = acceleration of gravity (L/T^2), c = dimensionless constant dependent upon the geometry of the medium, k = permeability (L^2), q = velocity (L/T) and ϕ = hydraulic gradient.

In Equation 1, the constants c and k are related by

$$k = cd^2$$

where d is the characteristic length of the medium. Equation 1 was verified from experimental results of 18 different media. The maximum standard error of estimate was 6.5% which is within experimental error.

Equation 1 can be rearranged and written as:

$$gcd \frac{\phi}{q^2} = \frac{\mu}{\rho q d} + 1$$

or

$$H = \frac{1}{N} + 1 \qquad (2)$$

where

$$H = gcd \frac{\phi}{q^2} \quad ; \quad \text{(unitless)}$$

and

$$N = \frac{\rho q d}{\mu} \quad ; \quad \text{(unitless)} \quad .$$

A graphical representation of Equation 2 shows that a unique relationship exists between H and N for the 18 porous media studied in the investigation. This relationship is shown in Figure 1.

B) Liu, H. C., Hydraulic Measurement for Specific Surface, M.S. Thesis, December 1968.

Discussion

An equation which enables one to obtain the specific surface of a porous medium from hydraulic measurements has been developed theoretically and experimentally verified for porous medium having particles of relatively uniform size. The equation may be expressed as

$$S = \frac{\varepsilon}{d} \left(\frac{\varepsilon}{\xi' c} \right)^{\frac{1}{2}} \quad . \qquad (3)$$

where S is the specific surface, ε the porosity, d the characteristic length, c a constant reflecting the effects of geometry on the flow and ξ' as defined by

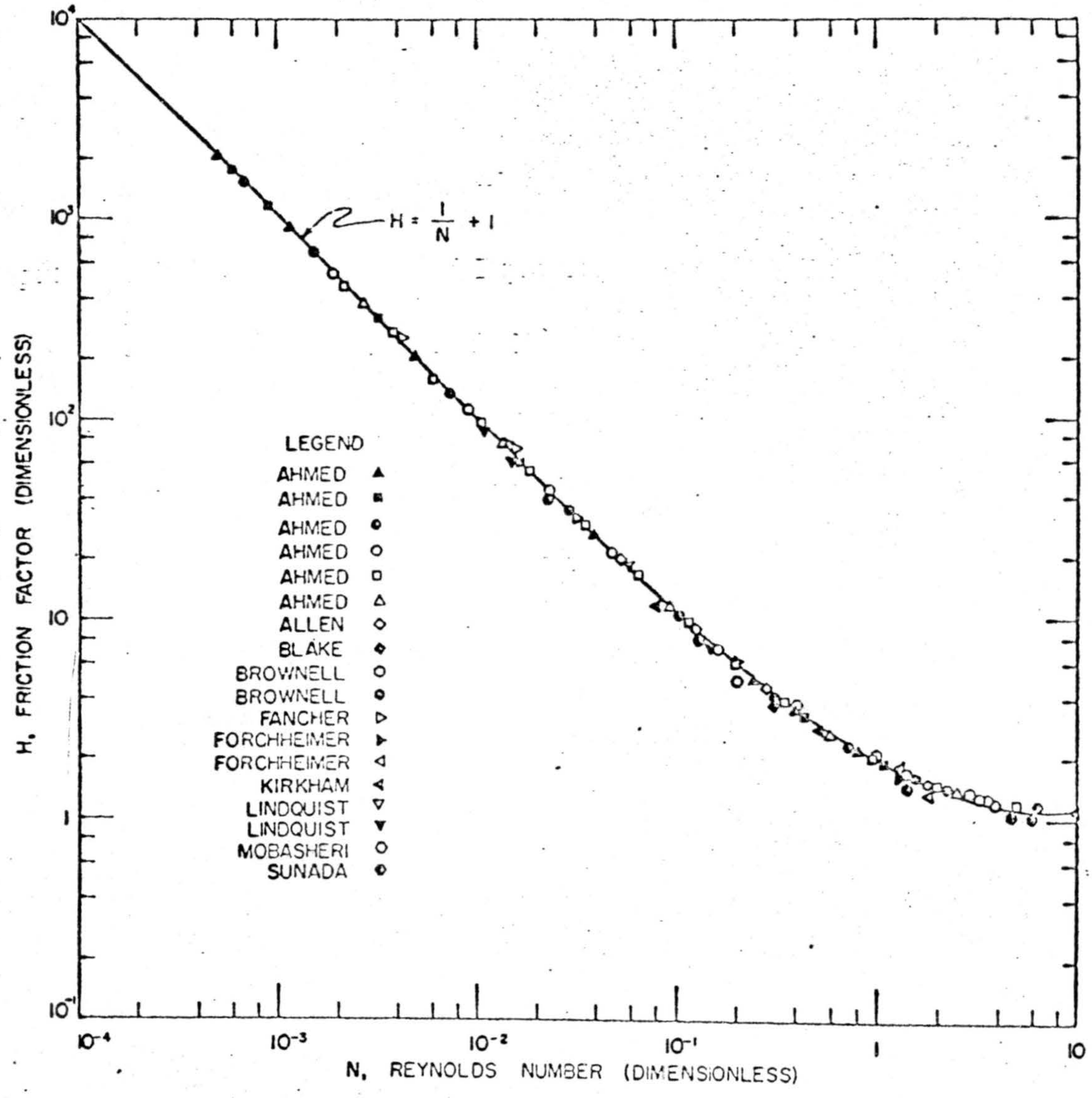


Figure 1. Graphical correlation of friction factor versus Reynolds number

$$\xi' = \frac{\varepsilon}{cd^2} (\overline{R})^2 \quad . \tag{4}$$

In Equation 2, $\overline{R}$ is the average hydraulic radius of the medium. All parameters in this equation can be determined from hydraulic measurements.

In this study, the values of ξ' lie between 5.5 and 9.9 demonstrating that ξ' is not a single constant for all porous media. The value of ξ' for a given medium depends upon porosity, particle size, permeability, pore-size distribution and particle shape.

The pore sizes in the medium were shown to be normally distributed. Thus, the Kozeny-Carman constant can be obtained from a probability theory approach. It was shown, from statistical analysis of cut-sections, that

$$\xi = \frac{\varepsilon}{cd^2} \left[\frac{1}{2} (\sigma + \nu)^2 + \left(\sqrt{2/\pi} - 1 \right) \nu\sigma \right] \quad . \tag{5}$$

where σ is the standard deviation of the distribution of pore size, ν is mean and π is 3.14. The values of ξ computed by Equation 5 lie between 5.7 and 9.1 which compares favorably with ξ' in Equation (4).

The specific surfaces obtained from the method of hydraulic measurements (Equation 4) have been compared with the values obtained from the statistical method (Equation 5) and have been shown to differ by a maximum of 10 percent for the six porous media studied in this investigation.

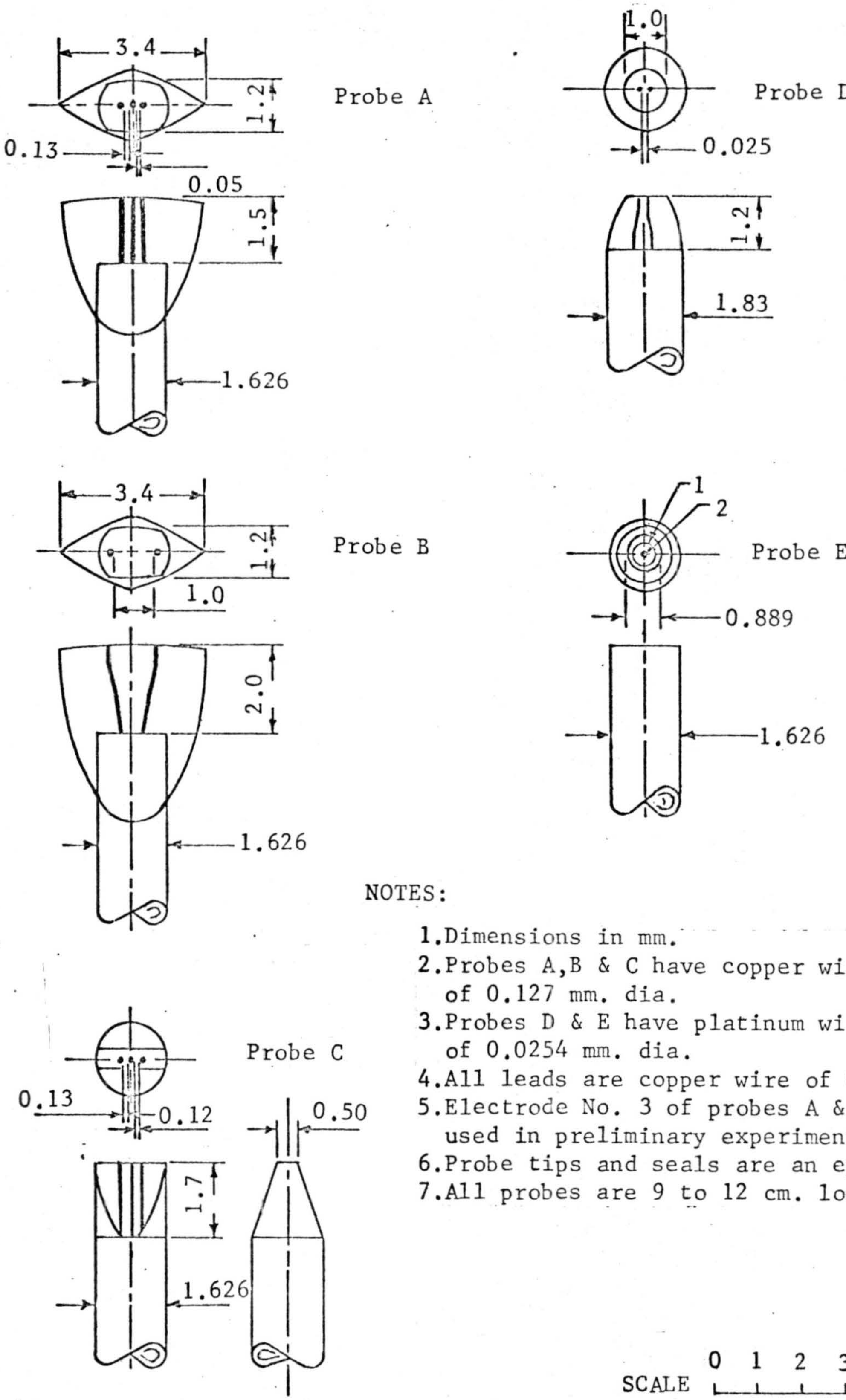
C) Groetsch, S., Micromeasurement of Water Velocity, M.S. Thesis, June 1968.

Discussion

An electrical method for measuring water velocity with a minimum of disturbance of the flow regime was studied in this investigation. Miniature probes (diameters ranging from 1.63 mm to 1.83 mm) were used to support the electrodes. Various probe geometries shown in Figure 2 were used to study the effects of geometry upon the flow regime. A positive voltage source (with respect to ground) is supplied at one electrode and a current flow is measured from a second electrode. The variation in current flow for a given chemical and physical property of the fluid is a function of water velocity. A typical electrical current-water velocity relationship is shown in Figure 3. A range of velocities from 1 cm/sec to 15 cm/sec could be accurately measured. As can be seen, however, each run for a given fluid must be calibrated as ion concentration has a marked influence upon the conductivity of the fluid.

Figure 4 presents a typical relationship showing the directional sensitivity of the probe under steady one-directional flow condition. If directional sensitivity is not desired, a probe having one wire at the center of a circular electrode can be used. This type of probe eliminates directional properties and only absolute magnitude of the velocity can be obtained.

This study demonstrates that water velocities in the range of 1 cm/sec to 15 cm/sec can be easily measured with very little disturbance of the flow regime. However, because of the changing chemical properties of the fluid, the probes would be primarily limited to measurements of water velocity under laboratory conditions.



NOTES:

1. Dimensions in mm.
2. Probes A,B & C have copper wire electrodes of 0.127 mm. dia.
3. Probes D & E have platinum wire electrodes of 0.0254 mm. dia.
4. All leads are copper wire of 0.4049 mm. dia.
5. Electrode No. 3 of probes A & C was only used in preliminary experiments.
6. Probe tips and seals are an epoxy resin.
7. All probes are 9 to 12 cm. long.

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Fig. 2 Probe Geometries

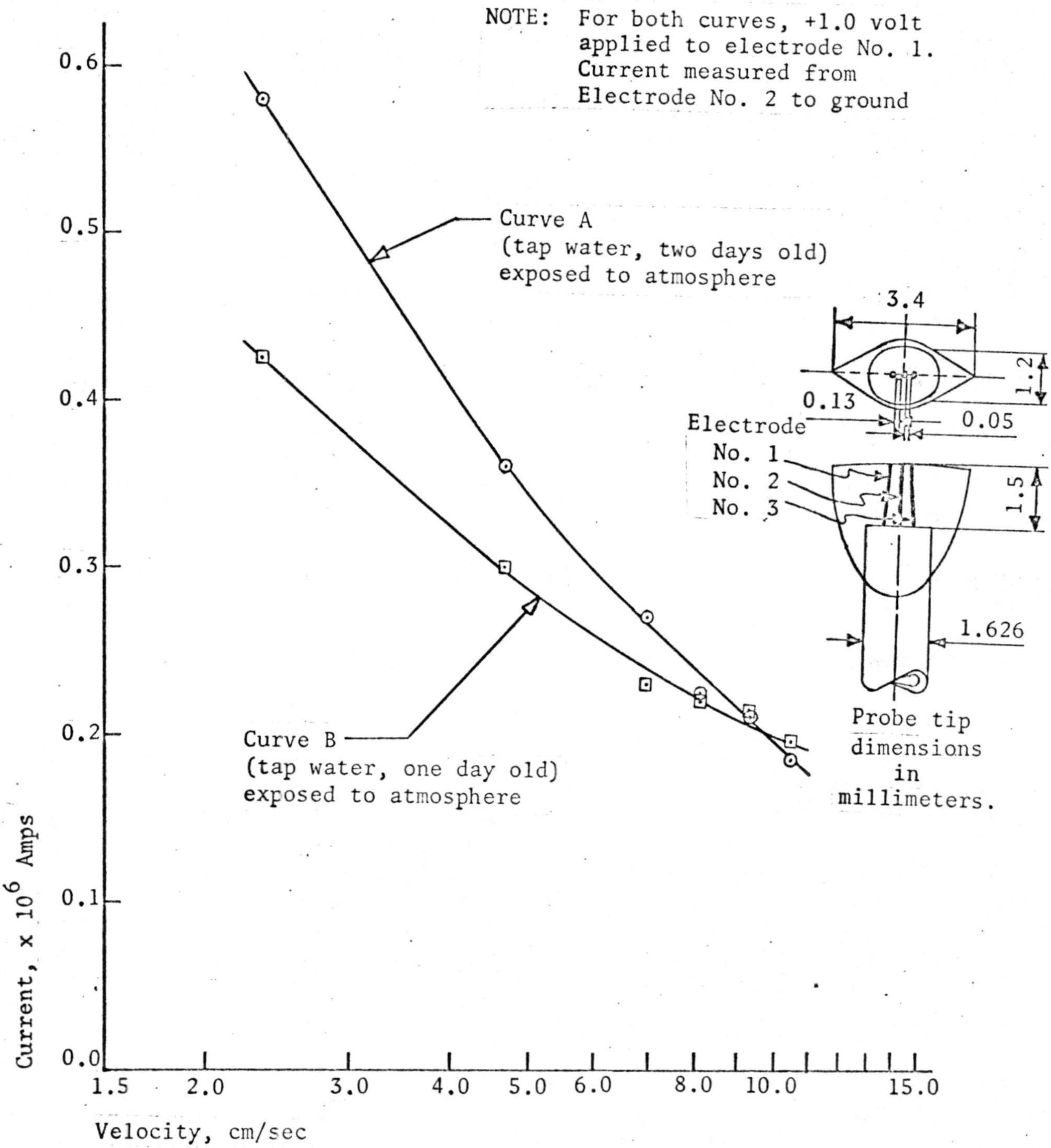

Fig. 3 Velocity vs. Current

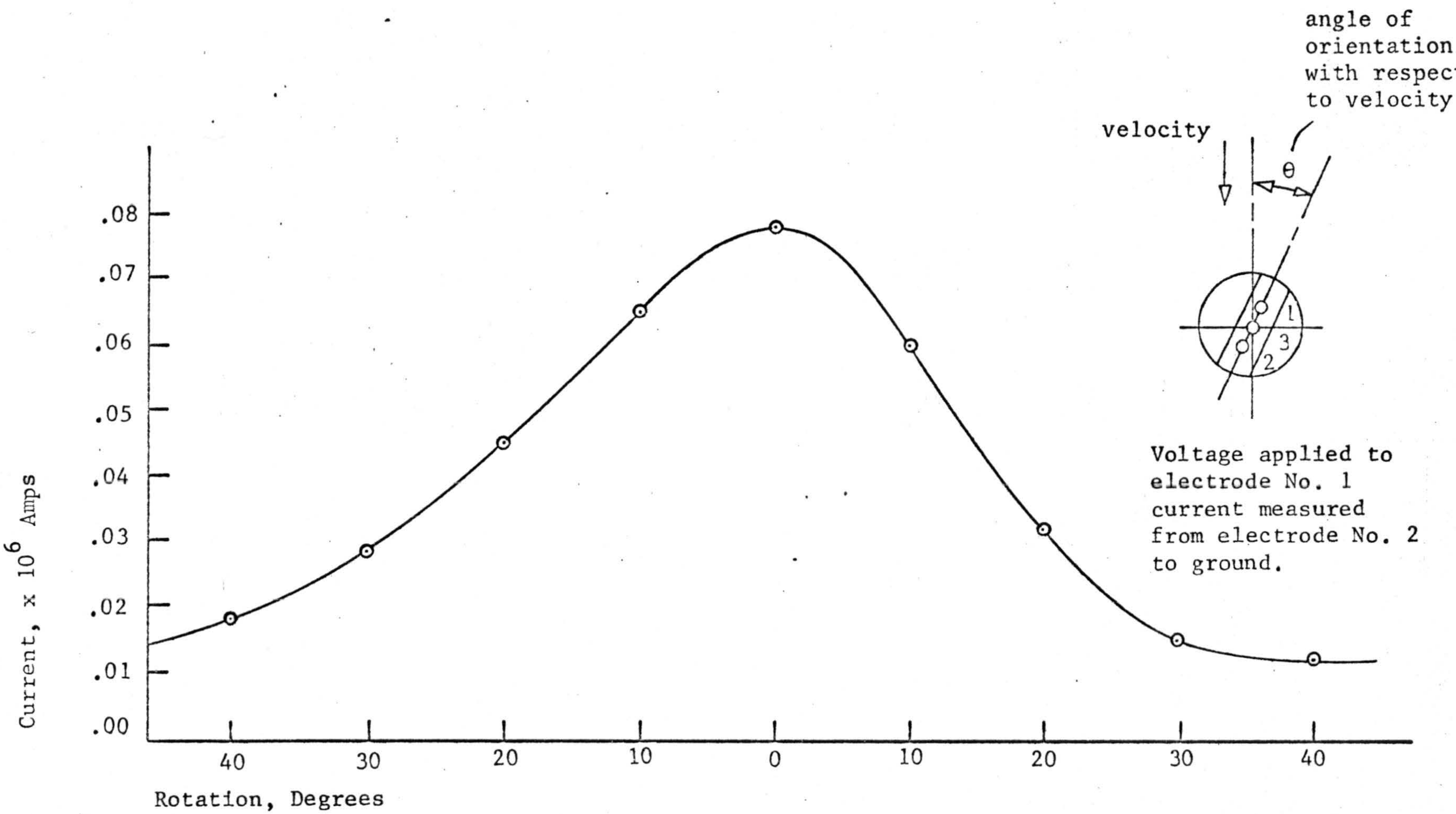

Fig. 4 Typical Directional Sensitivity of Probes

D) Simpson, H., Laminar and Turbulent Dispersion of Miscible Fluids in Porous Media, June 1969.

Discussion

Many investigators have reported that the longitudinal dispersion coefficient for flow in porous medium may be given by the relationship

$$D_L = C\, w^n \tag{6}$$

where D_L is the longitudinal dispersion coefficient, C is a constant for a particular medium, w the flow velocity and n an exponent determined from experimental data, to be between the range of one to two. Equation 6 has been verified experimentally for flows occurring in the laminar regime.

In this study it was found that the exponent n had a value less than one for flows in the nonlinear or convective flow regime. This is due primarily to the fact that convective acceleration effects are predominant over turbulent effects. Figure 5 presents the relationship between longitudinal dispersion coefficient and flow velocity for two porous media. As can be seen from this figure, the coefficient of dispersion in the laminar flow regime behaves as predicted by other investigators. However, when the flow is in the completely nonlinear regime, the exponent n in the dispersion equation decreases from 1.4 to 0.6.

Figure 6 presents the data in terms of Reynolds number. One additional set of data by Ebach and White (reference may be found in the accompanying thesis) supports the result obtained in this study. Unfortunately, there are no theoretical developments to predict the

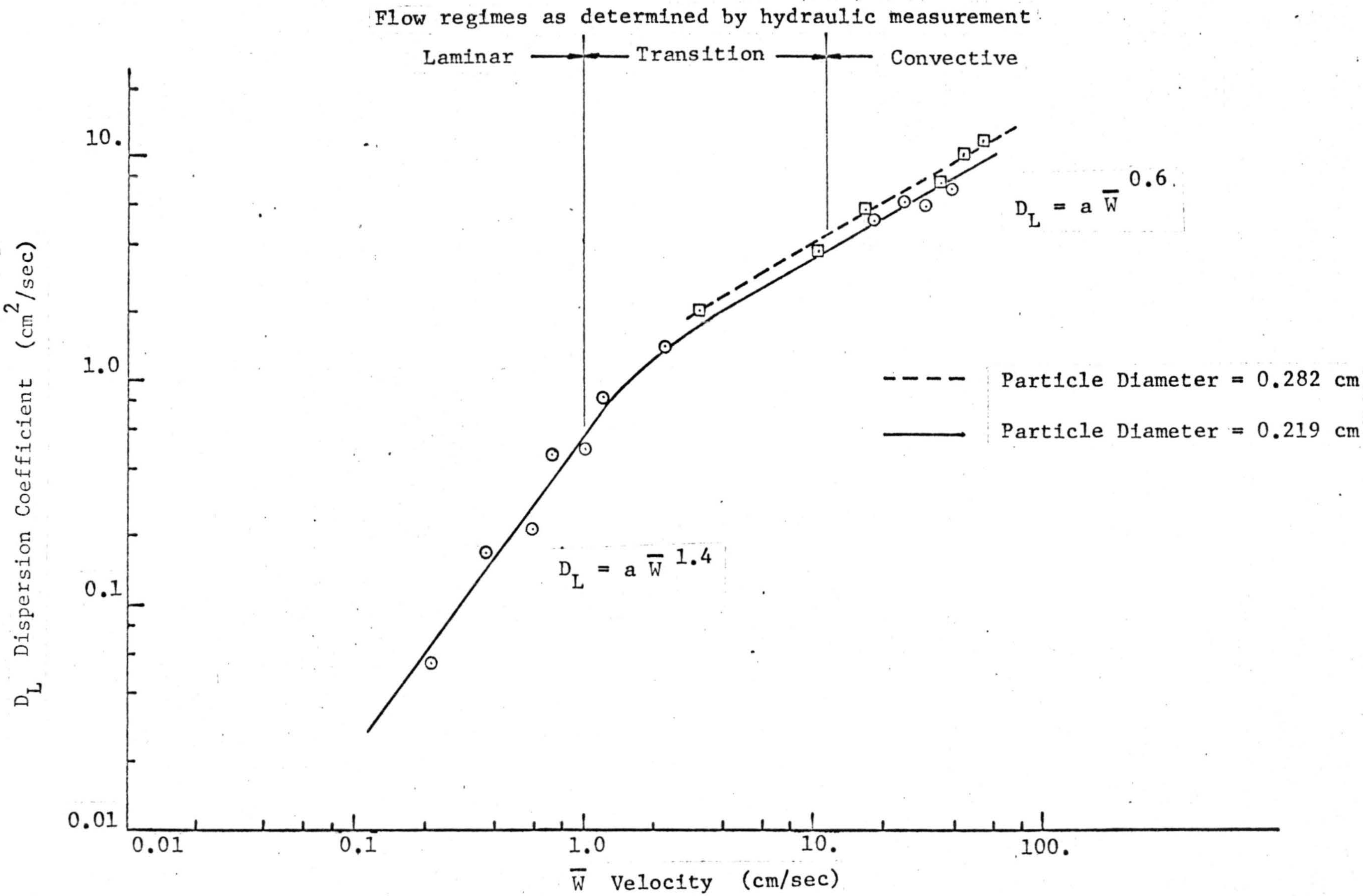

Figure 5 Longitudinal dispersion versus average velocity

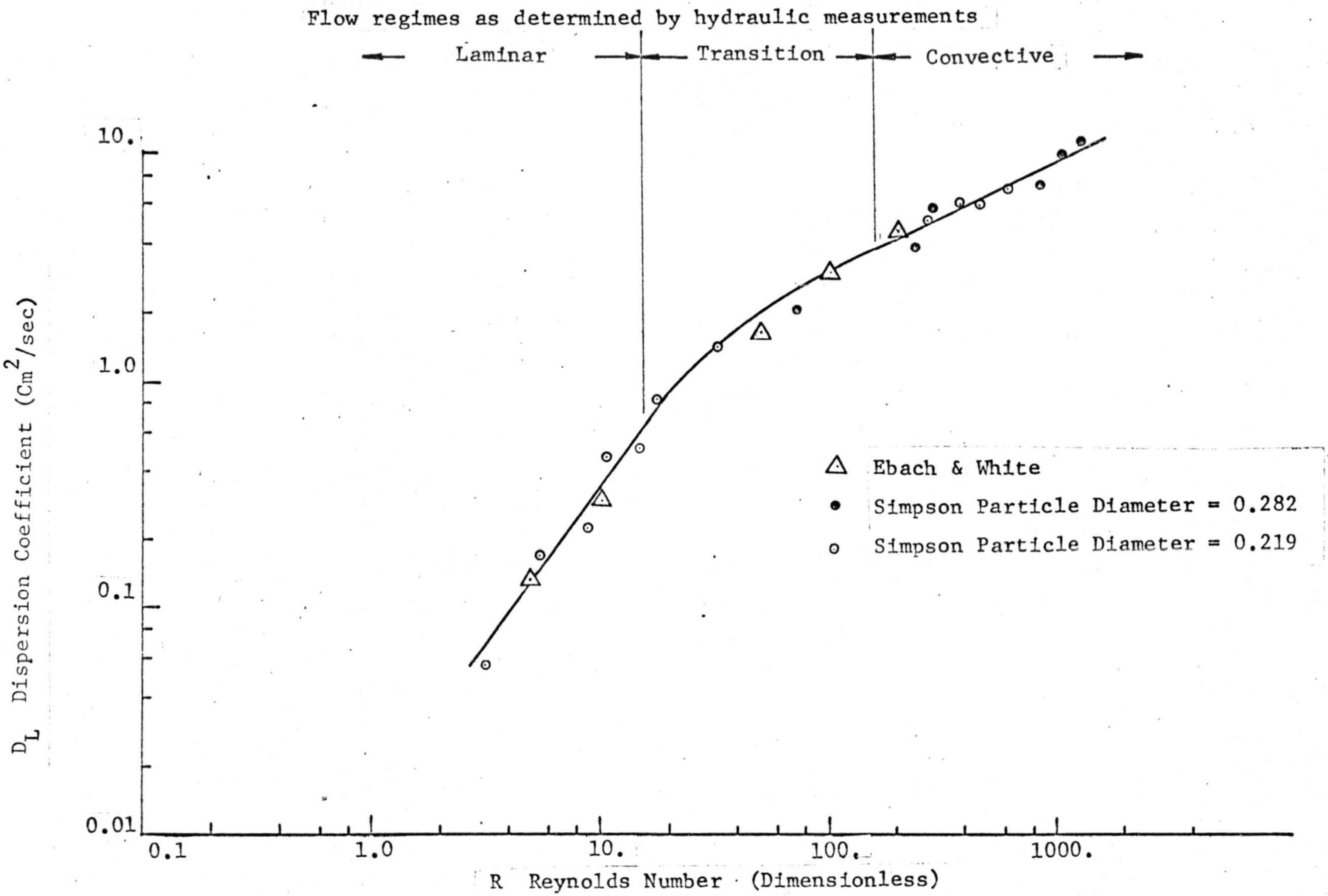

Figure 6 Longitudinal dispersion versus Reynolds Number

dispersion coefficient in the nonlinear flow regime and more study is needed in this area.

E) Hibbert, W. H., Compressible Fluid Flow Through Porous Media, M.S. Thesis, June 1969.

Discussion

An equation describing the flow of compressible fluids through porous medium was developed from the Navier-Stokes equation. For gas flow the equation

$$\frac{M}{2Z\bar{R}T_a} \frac{d(\bar{P}^2)}{dx} = \frac{\mu}{k} G_s + \frac{G_s^{\ 2}}{\sqrt{ck}} \tag{7}$$

is applicable for steady, horizontal and isothermal flow. In this equation, M is the molecular weight of the gas, $\bar{R}$ the universal gas constant, Z is the compressibility factor, T_a is the absolute temperature, P the absolute pressure, μ the viscosity, k the permeability of the medium, c a constant reflecting boundary condition, and G_s the mass flow. Experimental data obtained in this study supported the validity of Equation 7.

Equation 1 may be put in dimensionless terms as

$$L = \frac{1}{N} + 1 \tag{8}$$

where

$$L = \frac{M\sqrt{ck}}{Z\bar{R}T_a G_s^{\ 2}} \frac{d(\bar{P}^2)}{dx}$$

and

$$N = \frac{G_s}{\mu} \sqrt{k/c} \quad .$$

Figure 7 presents the experimental data along with the theoretical curve predicted by Equation 8. The standard error of estimate for the data is 9.5% which supports the validity of the theoretical development.

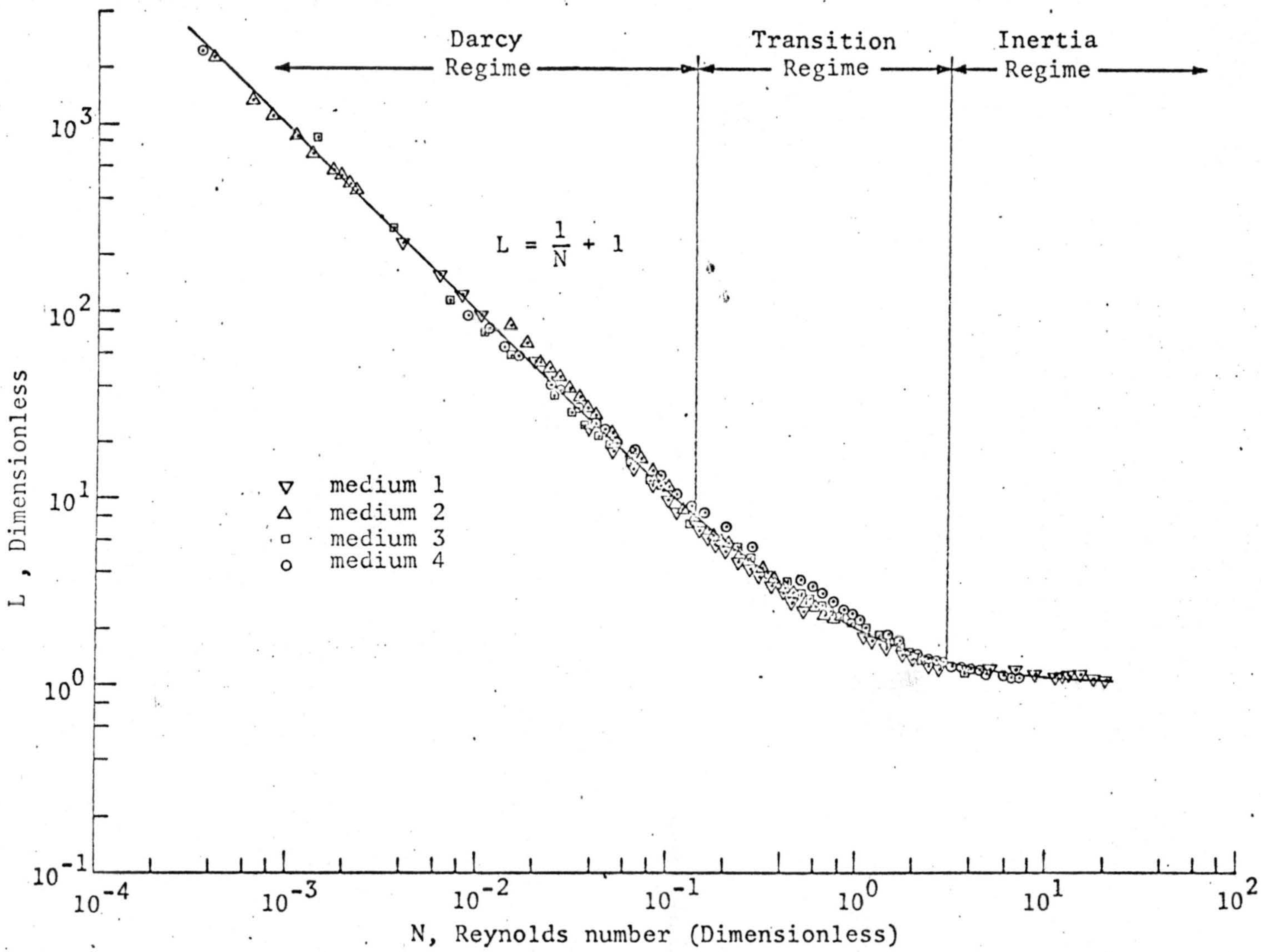

Figure 7. Graphical correlation of L versus Reynolds number

III. GRANT ACTIVITIES

1) Dissertations and theses (two copies of each enclosed)

   A) Ph.D. Dissertation by N. Ahmed, "Physical Properties of Porous Medium Affecting Laminar and Turbulent Flow of Water," June 1967.

   B) M.S. Thesis by H. C. Liu, "Hydraulic Measurements for Specific Surface," December 1968.

   C) M.S. Thesis by S. F. Groetsch, "Micromeasurements of Water Velocity," June 1968.

   D) M.S. Thesis by H. Simpson, "Laminar and Turbulent Dispersion of Miscible Fluids in Porous Media," June 1969.

   E) M.S. Thesis by W. H. Hibbert, "Compressible Fluid Flow Through Porous Media," June 1969.

2) Publications:

Nonlinear Flow in Porous Media, by N. Ahmed and D. K. Sunada, accepted for publication in Hydraulic Division ASCE (editors' copy enclosed).

3) Students supported by grant:

   A) N. Ahmed, Ph.D. (completed all requirements)

   B) S. F. Groetsch, M.S. (completed all requirements)

   C) H. C. Liu, M.S. (completed all requirements)

   D) H. Simpson, M.S. (completed all requirements)

   E) W. H. Hibbert, M.S. (completed all requirements)

   F) A. V. Sundaran, Ph.D. (temporary)

   G) J. Chitwood, Ph.D. (temporary)

4) Professional Staff supported by grant:

A) D. K. Sunada, Assistant Professor

B) H. R. Duke, Jr., Civil Engineer

5) Professional Staff not supported by grant:

A) R. A. Longenbaugh, Assistant Professor

B) V. A. Sandborn, Professor

6) Related activities:

Attendance to summer institute at M.I.T. on Porous Media Flow.