# REGIONAL DISCRIMINATION OF CHANGE

## IN RUNOFF

by

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## RELATION OF HYDROLOGY PAPER NO. 37 TO RESEARCH PROGRAM: "HYDROLOGY OF WEATHER MODIFICATION"

The present study is part of a more comprehensive project which has as one of its objectives the development of methods of evaluation of atmospheric water resources programs. Correlatively the application of the methods to a variety of basins forms a basis for selection of suitable watersheds, basins or regions.

Several approaches were pursued. This report discusses one of them. Several other approaches were previously described in Hydrology Papers 22, 34, and 36 (see back inside cover for complete reference).

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ABSTRACT

The object of this study is to find answers to the following questions:

What is the appropriate statistical test for a regional target-control technique of evaluation?

What is a suitable method for reduction of an originally large number of variables?

Which of the Upper Basin of the Colorado River or the San Juan Mountains is a more suitable area of operations, if the effectiveness of precipitation management is to be detected as quickly as possible?

The results of this research study show:

1. The $T^2$-test is the appropriate test for multiple target-control technique of evaluation.

2. The canonical analysis is the suitable method for the reduction of a large number of original variables.

3. The Upper Basin of the Colorado River is preferable under the assumption of an equal percentage of increase in runoff. However, if the percentage increase in the southern area is at least 1.2 times as large as in the northern area (and recent publications suggest that this ratio is probably around 3) then the southern area is preferable.

Based on the $T^2$-test, the minimum number of years for detecting an increase of 10 percent in spring runoff means are three years in the Upper Basin of the Colorado River, and four years in the San Juan Mountains.

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Viboon Nimmannit* and Hubert J. Morel-Seytoux**

Chapter I

## INTRODUCTION

1.1 Motivation of study. As interference with nature is accelerating [1,2,3] there is a need for early detection of direct or side effects of man's actions. Because of the rapid pace of development [3, 4,5] it is important to develop techniques that will display the effect of any given practice on water resources availability and distribution at the earliest possible time. For large scale field research, the availability of an efficient and regionally representative test would reduce the duration of experiments required to attain conclusive results and therefore costs, and provide a basis for managerial decision at an earlier stage, without additional observations. The decision may be to stop a project earlier when it becomes apparent, based on real time analysis of data, that the objectives cannot be achieved in the planned time. Better, pre-experiment data simulation would permit to assess the chances of being in that unfortunate situation as a function of a range of values of the suspected or hoped for change. Useful charts can be drawn in terms of the parameters, (magnitude of change, basin characteristics, etc.) for first stage planning.

The techniques which are described in this paper could be used for detection of the effects of watershed management of any origin upon water supply. They could be used to determine the effect of urbanization on the local hydrology, to detect when such urbanization has created a significant change that calls for reappraisal of the protective designs, e.g., flood control, etc. In other words, they are quite general. To a certain degree the techniques will indeed be discussed in a general abstract form, but their practical applicability will be demonstrated with a very special and very important application in mind.

The Bureau of Reclamation will most probably initiate in the fall of 1970 a pilot project of massive cloud seeding operations, covering some 4000 square miles within the state of Colorado. It will be the primary purpose of this paper to establish as accurately as possible how long it will take to detect a regional hydrologic change and to attribute it with little risk of error to the cloud seeding operations. To understand this practical illustration of the technique some knowledge of the geographic and hydrologic features of the region, of the water situation and of the plans of the Bureau of Reclamation is a prerequisite. The purpose of the following sections is to provide this background information.

1.2 Geographic and hydrologic setting. The Colorado River begins high in the snow-capped Rocky Mountains of north central Colorado, flows nearly 1,400 miles southwest, and empties into the Gulf of California in Mexico far to the south. It drains a vast area of 244,000 square miles, 242,000 square miles in the United States--one-twelfth of the area of Continental United States--and 2,000 square miles in northern Mexico. The basin from Wyoming to below the Mexican border is some 900 miles long and varies in width from about 300 miles in the upper section to 500 miles in the lower section. It is bounded on the north and east by the Continental Divide in the Rocky Mountains, on the west by the Wasatch Range, and on the southwest by the San Jacinto Mountains, a range of the Sierra Nevada Mountains. The area, larger than the states of New York, Pennsylvania, and New Jersey combined, above Lee Ferry, Arizona, is known as the Upper Colorado River Basin (Fig. 1). This area is the source of the greatest part of water reaching the Colorado River. The upper portion of this basin in Wyoming and Colorado is a mountainous plateau, 5,000 to 8,000 feet in altitude, marked by broad rolling valleys, deep canyons, and intersecting mountain ranges. Climatologically, the Colorado River Basin has heavy precipitation on the high peaks of the Rockies and truly desert conditions with little rain in the southern area around Yuma, Arizona. Extremes of temperatures in the basin range from 50° below zero to 130° above zero degree Fahrenheit. Development and utilization of resources in this arid land depend on the availability of water. Crops must be irrigated; cattle on the vast ranges must be partially fed from hay produced on irrigated land; towns and cities must be located within distance of dependable domestic and municipal water supplies, and mining and many other industries depend, to an extent, on the availability of hydroelectric power [1].

1.3 The water resources outlook. The U.S. Geological Survey estimates total water demand in the United States was 280 billion gallons per day (314 million acre-feet per year) in 1960. As a point of comparison let us note that the average annual flow of the biggest river in the United States, the Mississippi, is 440 maf and that of the Upper Colorado is about 14 maf. The U.S.G.S. estimates the total water demand for the U.S. will be 600 billion gallon per day (672 million acre-feet per year) by 1980. In 1960 the demand in the Western States alone was estimated at 125 billion gallons per day (140 million

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Fig. 1  The Upper Colorado River Basin (after Upper
        Colorado River Commission)

The most favorable conditions for cloud seeding
are in regions where moist winds blow more or less
constantly up the slopes of the mountains.  Cloud
seeding involves artificial introduction of tiny par-
ticles into clouds so that moisture can depose around
each of the nuclei to form a crystal heavy enough
to fall to the ground.  Among nuclei that have been
used experimentally in cloud seeding operations are
solid carbon dioxide, silver iodide, water spray, and
carbon black.  To date, the greatest number of cloud-
seeding attempts have been made by using silver
iodide generators operated on the ground.  However,
seeding operations using aircraft flown directly over
cloud layers have demonstrated that this technique
may be more effective [6].

In 1968, the Bureau of Reclamation adopted a
plan to start pilot programs for weather modification
operations in the Upper Colorado River Basin (Fig. 1),
and two regions were selected for this purpose [7].
The first was the Upper Basin of the Colorado River*,
which will for brevity be referred to in this study
as the Northern Project area (Fig. 2).  The second
area was the San Juan Mountains region referred to as
the Southern Project area (Fig. 3).  Since the initia-
tion of this study, the plans of the Bureau were modi-
fied.  Currently [8] only one area is considered:
the Southern area.  Nevertheless, because they had
already been calculated, the results for the Northern
area are also reported.

1.5  <u>Objective of study and approach</u>.  The pri-
mary objective was to develop an appropriate and
efficient methodology that can be used to demonstrate
the effectiveness of cloud seeding in each project
region.  In order to achieve this, a multivariate
analysis of geographically well distributed stations
in each region is carried out.  These stations are
referred to as targets.  Variables used in this study
are spring runoffs.  The spring runoff of a station
is defined here as the average flow, in cubic feet
per second, of that station during the spring months.
Because this flow is substantially contributed by
winter snow, it can be regarded as an indirect mea-
sure of the effect of weather modification.  However,
because of the lack of a precise date for the start
of snow melting, two different time intervals will be
used for spring months.  The first interval will be
composed of four months:  April, May, June and July;
the second of six months:  March, April, May, June,
July and August.

Because the use of controls, which are the
stations free from the effect of weather modification,
is a well proven means of making tests more effective,
(9), it also will be utilized in this study.  An area
between the Northern and Southern Project areas has
been selected (Fig. 4) to serve as the control area.

acre-feet per year) and for 1980 at 190 billion gallons
per day (213 million acre-feet per year).  The lower
percentage of demand growth for the Western States
reflects different demands of industry in the East and
agriculture in the West.  Because rainfall is low in
the Western States, the conservation use must be
greater than in the East and Midwest.  Municipal or
domestic use has first priority in the West, with
irrigation second.  It is estimated the 44,000,000
population of the Western States in 1960 will expand
to more than 100,000,000 by the year 2000 [2].

From the population figures given above, it is
obvious much more water will be needed in the near
future.  So, the question one must answer is, "What
can be used as sources for additional water to alle-
viate the shortages?"  Several agencies, such as the
Bureau of Reclamation [3], the Upper Colorado River
Commission [4], and the Committee on Water of the
National Research Council [5], feel cloud seeding, to
augment the precipitation amount in the Upper Colorado
River Basin, may become a partial solution to the
recurrent water shortage.

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*The reader is warned for possible confusion.  In this paper the expression "Upper Colorado River Basin" refers
to the Colorado Basin above Lee's Ferry.  On the other hand, the expression "Upper Basin of the Colorado River"
refers to a much smaller drainage basin including the main stem of the Colorado close to its source and a few
tributaries.  The limits of that basin are shown on Fig. 2.
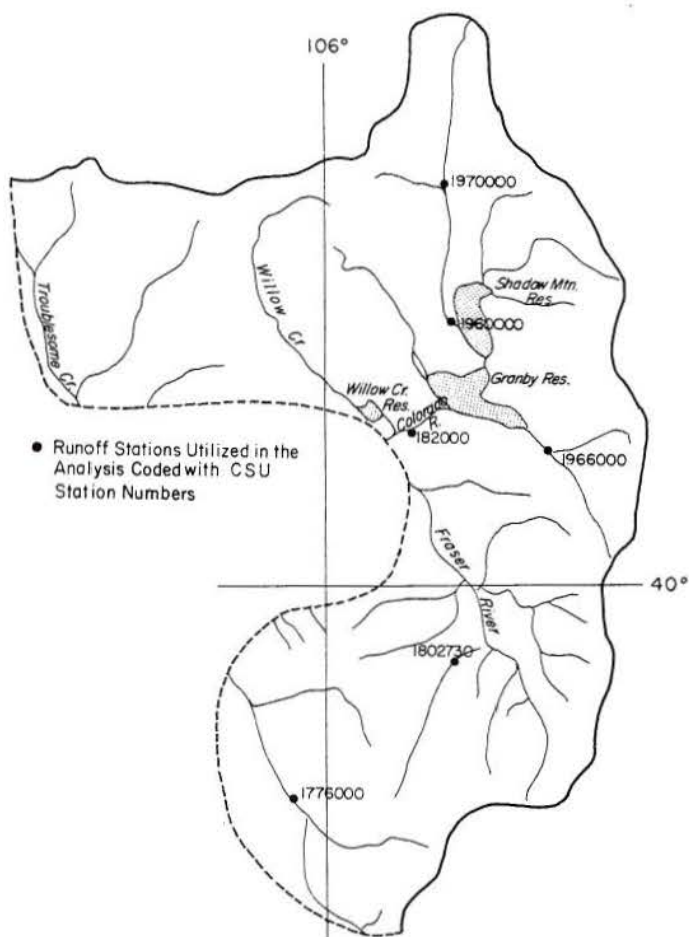
Fig. 2 General configuration of and location of gages within the Upper Basin of the Colorado River
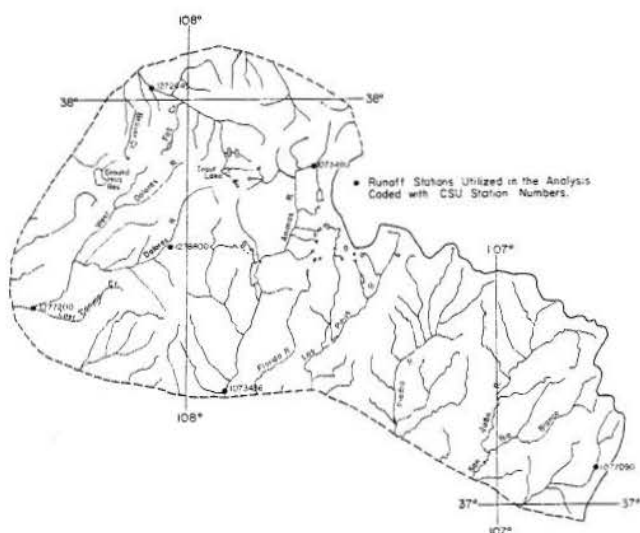


Fig. 3 General configuration of and location of gages within the Colorado River Basin Pilot Project area
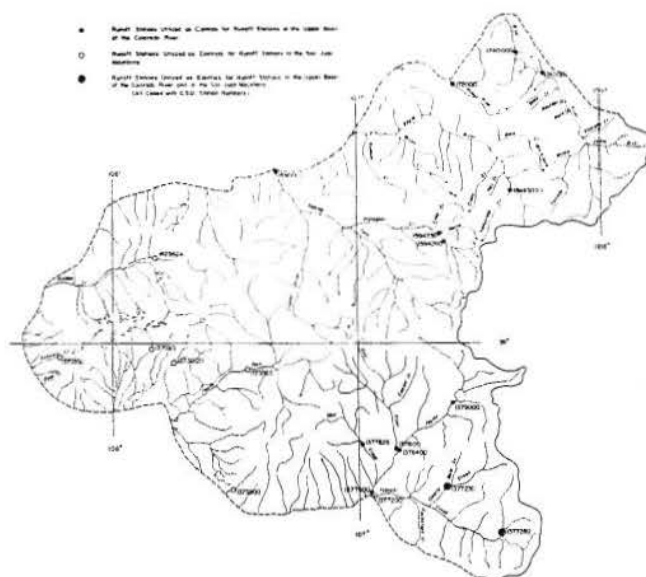


Fig. 4 General configuration of and location of gages within the Colorado River Basin Pilot Project control area

For brevity, the following symbols will be employed:

| | |
|---|---|
| N-4: | 4-month runoff series in the northern target region, |
| N-6: | 6-month runoff series in the northern target region, |
| CN-4: | 4-month runoff series in the northern control region, |
| CN-6: | 6-month runoff series in the northern control region, |
| S-4: | 4-month runoff series in the southern target region, |
| S-6: | 6-month runoff series in the southern target region, |
| CS-4: | 4-month runoff series in the southern control region, |
| CS-6: | 6-month runoff series in the southern control region. |
| N-CN-4: | the combination of N-4 and CN-4, |
| N-CN-6: | the combination of N-6 and CN-6, |
| S-CS-4: | the combination of S-4 and CS-4, |
| S-CS-6: | the combination of S-6 and CS-6. |

In applying theories of statistics to an engineering problem, it is necessary to assume certain properties of the variables. The assumptions made in this study are:

a) The observations of runoff follow a multivariate normal distribution.

b) The estimated means in both target and control areas from the period before seeding are essentially equal to the population values.

c) After seeding the means in the target areas will change but the means in the control areas will remain unchanged.

d) The covariance matrix of the target and control variables is the same for both periods before and after seeding.

The above assumptions are required in this study because of the difficulty in developing the theoretical distribution of the test criterion otherwise. In dealing with more than two variables, the knowledge of distributions, except that of the normal distribution, are not sufficiently developed [10]. So, even though it is rather obvious the assumptions made here will be violated to some degree in reality, they are practically as good as one can make with the present state of statistical knowledge.

From the work of Ref. [9], it is found that the $\chi^2$-test which is based on the population values, and the conditional Student's t-test which is based on the sample values, give very closely the same results for sample sizes around 30. Thus, for convenience in handling the mathematics, the population values are assumed to be known here and this assumption appears justified. Also, all the observations of runoff station used in this study have been plotted on normal probability paper. If the runoff were exactly distributed as a normal variate, all the observations would fall exactly on a straight line. The actual observations did not in any case deviate appreciably from a straight line. The assumption of normality may therefore be entertained for these data.

Based on the above assumptions, a $T^2$-statistic is obtained [11,12]. The minimum number of years, N*, to detect the expected increase can be obtained [11] from the formula,

$$N^* = \frac{\tau^2}{\underline{\mu}'\underline{V}^{-1}\underline{\mu}} \quad , \tag{1}$$

where $\tau^2$ is the noncentrality parameter (it is a measure of the amount of deviation from being central which is the case when the variables under study have means zero),

$\underline{\mu} = \underline{\mu}^* - \underline{\mu}_o$ , $\underline{\mu}^*$ is the runoff mean vector for the seeded period, and $\underline{\mu}_o$ is the runoff mean vector for the non-seeded period,

$\underline{\mu}'$ is the transpose of $\underline{\mu}$ , and

$\underline{V}^{-1}$ is the inverse of the covariance matrix of runoff variables, $\underline{V}$ .

In Chapter II, most approaches used to detect the effectiveness of weather modification by other investigators are summarized. The theoretical concepts of the principal component analysis, the canonical analysis, and the $T^2$-statistic are the main subjects of Chapter III. Chapters IV and V deal with data assembly, analysis of data, and results.

The study led to two major conclusions, one of general theoretical interest and the second of practical significance for the plans of the Bureau:

a) Canonical analysis coupled with the multivariate $T^2$-test provides an effective technique of detection of a suspected regional hydrologic change and,

b) Assuming a 10% uniform increase in runoff by precipitation management 3 and 4 years only are required for significant evaluation for the Upper Basin of the Colorado and the San Juan Mountains, respectively.

## REVIEW OF PREVIOUSLY USED TESTS

The statistical content of this chapter is not new. The material here is provided for the sake of convenience to a reader whose statistical background is that of the average engineer. A statistician can bypass this chapter without detrimental effect to the continuity and understanding of this paper.

In this chapter the statistical tests, which have been employed by other investigators for detecting the effectiveness of weather modification, will be presented. The literature is further discussed in Ref. 12. Because all tests are concerned with the expected increase in the means of either runoff or precipitation during the seeded period, the hypotheses for all tests can be stated as:

$H_o$ (null hypothesis) - there is no increase in the mean of the hydrologic variable during the seeded period,

$H_a$ (alternate hypothesis) - there is an increase in the mean.

2.1 Target sample u-test. Let $q_{11}$, $q_{12}$, $\ldots$, $q_{1n_1}$, be $n_1$ observations of a hydrologic variable for the nonseeded period, and $q_{21}$, $q_{22}$, $\ldots$, $q_{2n_2}$ be $n_2$ observations for the seeded period of a target watershed. When $n_1$ is large the mean and variance of the series $q_{11}$, $q_{12}$, $\ldots$, $q_{1n_1}$ can be considered to be the population mean and population variance. Assuming the variance of the seeded period is the same as the non-seeded period, the test statistic is [13]

$$u_o = \frac{\overline{q}_2 - \mu_1}{\sigma_1 / \sqrt{n_2}} \quad ,$$

where $u_o$ is normally distributed with mean o and variance 1

$$\overline{q}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} q_{2i}$$

$$\mu_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} q_{1i}$$

$$\sigma_1^2 = \frac{1}{n_1} \sum_{i=1}^{n_1} (q_{1i} - \mu_1)^2 \quad .$$

The null hypothesis, $H_o$, will be accepted at a 5% level of significance if $u_o$ has a value less than 1.645. That is, there is no increase in the mean. On the contrary, if $u_o$ is greater than 1.645

the alternative hypothesis, $H_a$, will be accepted at a 5% level of significance. The use of this test can be found in References [9] and [14]. South Fork San Joaquin, California, was the target basin for the study in Reference [9]. There were 15 years of seeded record, and 29 years of non-seeded record. The apparent percentage increase in the mean of the seasonal runoff for the seeded period was about 10%. By the use of the target sample u-test it was found that $u_o = 1.20$. This shows that the target sample u-test was not powerful enough to detect the increase in mean value in the order of 10% of the old mean.

2.2 Target two-sample t-test. This test does not require knowledge of population parameters. Let $q_{11}$, $q_{12}$, $\ldots$, $q_{1n_1}$ and $q_{21}$, $q_{22}$, $\ldots$, $q_{2n_2}$ be $n_1$ and $n_2$ observations for the non-seeded and seeded periods of a target watershed.

Assuming the variances of the non-seeded and seeded periods are equal, the test statistic [15]

$$t_o = \frac{\overline{q}_2 - \overline{q}_1}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

is distributed as t-distribution with $n_1 + n_2 - 2$ degrees of freedom, where:

$$\overline{q}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} q_{1i}$$

$$\overline{q}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} q_{2i}$$

$$s^2 = \frac{\sum_{i=1}^{n_1} (q_{1i} - \overline{q}_1)^2 + \sum_{i=1}^{n_2} (q_{2i} - \overline{q}_2)^2}{(n_1 - 1) + (n_2 - 1)} \quad .$$

The use of this test can be found in References [8], [14], [16], [17], [18], [19], [20], [21], [22], [23], and [24]. The value of the t-statistic was also computed for South Fork San Joaquin [9] from the same set of data used in computing the target sample $u_o$. The computed t-statistic has the value of 0.89. So, again no significant increase was concluded. The target two-sample t-test, and the target sample u-test therefore can be considered to be insufficiently powerful tests for studies of this nature.

2.3 Target-control $\chi^2$-test. The detectability of the test can be improved by the use of a control [9]. This can be done by comparing sets of hydrologic data of non-seeded and seeded periods for the target watershed with those for an unseeded control watershed located in the vicinity of the target area.

Let $q_{11}$, $q_{12}$, ..., $q_{1n_1}$ and $q'_{11}$, $q'_{12}$, ..., $q'_{1n_1}$ be $n_1$ observations for the period prior to seeding of the target and control watersheds respectively. Also, let $n_2$ observations for the seeded period in the target be denoted by $q_{21}$, $q_{22}$, ...,$q_{2n_2}$, and those in the control by $q'_{21}$, $q'_{22}$, ..., $q'_{2n_2}$.

When the length of record before seeding is long enough, the estimated statistics of the target and control can be assumed to be the population values. Assuming the variables in the target and control are bivariate normally distributed, then the test statistic [14]:

$$\chi_o^2 = \frac{n_2}{1-\rho^2}\left\{\left(\frac{\overline{q}_2 - \mu_1}{\sigma}\right)^2 - 2\rho\,\frac{(\overline{q}_2-\mu_1)(\overline{q}'_2-\mu'_1)}{\sigma\,\sigma'} + \left(\frac{\overline{q}'_2-\mu'_1}{\sigma'}\right)^2\right\}$$

is distributed as Chi-square distribution with two degrees of freedom, where

$\rho$ is the population coefficient of correlation between the target and control for the non-seeded period, given by

$$\rho = \frac{\sum_{i=1}^{n_1}(q_{1i}-\mu_1)(q'_{1i}-\mu'_1)}{\left[\sum_{i=1}^{n_1}(q_{1i}-\mu_1)^2\sum_{i=1}^{n_1}(q'_{1i}-\mu'_1)^2\right]^{\frac{1}{2}}}$$

$$\mu_1 = \frac{1}{n_1}\sum_{i=1}^{n_1}q_{1i}$$

$$\mu'_1 = \frac{1}{n_1}\sum_{i=1}^{n_1}q'_{1i}$$

$$\overline{q}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2}q_{2i}$$

$$\overline{q}'_2 = \frac{1}{n_2}\sum_{i=1}^{n_2}q'_{2i}$$

$$\sigma = \sqrt{\frac{1}{n_1}\sum_{i=1}^{n_1}(q_{1i}-\mu_1)^2}$$

$$\sigma' = \sqrt{\frac{1}{n_1}\sum_{i=1}^{n_1}(q'_{1i}-\mu'_1)^2} \quad.$$

This test has been used in References [9] and [14].

With the use of Merced River at Pohono Bridge as a control runoff station for the target, South Fork San Joaquin, the observed $\chi_o^2$-statistic was found to be [9] 22.2. The value of $\chi^2$ for significance at 99% level of confidence is 9.2. Therefore, a significant increase was detected by the use of the target-control $\chi^2$-test. This shows that for the same set of data for the target basin, the target-control $\chi^2$-test is overwhelmingly more discriminating than the target two-sample t-test and the target two-sample u-test.

2.4 Target-control conditional Student's t-test. In this test population parameters are not known. What is tested is the normality or abnormality of the target, given the behavior of the control, normal or otherwise [9].

Let $q_{11}$, $q_{12}$, ..., $q_{1n_1}$ and $q_{21}$, $q_{22}$, ..., $q_{2n_2}$ be the $n_1$ and $n_2$ observations of a hydrologic variable in the target watershed before and during seeded periods respectively. Let $q'_{11}$, $q'_{12}$, ..., $q'_{1n_1}$ and $q'_{21}$, $q'_{22}$, ..., $q'_{2n_2}$ be the corresponding observations in the control watershed.

By application of the maximum-likelihood ratio method [25], the test statistic:

$$t_o = \frac{\sqrt{n_1+n_2-3}\left[(\overline{q}_2-\overline{q}_1)-(\overline{q}'_2-\overline{q}'_1)\left\{\sum_{i=1}^{n_1}a_i(\Delta q_{1i})+\sum_{i=1}^{n_2}b_i(\Delta q_{2i})\right\}\right]}{\left[\frac{1}{n_1}+\frac{1}{n_2}+\left(\frac{\overline{q}'_2-\overline{q}'_1}{\Delta}\right)^2\right]^{\frac{1}{2}}\left[\sum_{i=1}^{n_1}(\Delta q_{1i})^2+\sum_{i=1}^{n_2}(\Delta q_{2i})^2-\left\{\sum_{i=1}^{n_1}a_i(\Delta q_{1i})+\sum_{i=1}^{n_2}b_i(\Delta q_{2i})\right\}^2\right]^{\frac{1}{2}}}$$

is obtained and it is distributed as Student's t-distribution with $n_1 + n_2 - 3$ degrees of freedom, where

$$\overline{q}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1}q_{1i}$$

$$\overline{q}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2}q_{2i}$$

$$\overline{q}'_1 = \frac{1}{n_1}\sum_{i=1}^{n_1}q'_{1i}$$

$$\overline{q}'_2 = \frac{1}{n_2}\sum_{i=1}^{n_2}q'_{2i}$$

$$(\Delta q_{1i}) = q_{1i} - \overline{q}_1$$

$$(\Delta q_{2i}) = q_{2i} - \overline{q}_2$$

$$(\Delta q'_{1i}) = q'_{1i} - \overline{q}'_1$$

$$(\Delta q'_{2i}) = q'_{2i} - \overline{q}'_2$$

$$\Delta^2 = \sum_{i=1}^{n_1}(\Delta q'_{1i})^2 + \sum_{i=1}^{n_2}(\Delta q'_{2i})^2$$

$$a_i = \frac{(\Delta q'_{1i})}{\Delta}$$

$$b_i = \frac{(\Delta q'_{2i})}{\Delta} \quad .$$

The use of this test can be found in References [9] and [14].

In Reference [9], the application of the target-control conditional Student's t-test was made for the target, South Fork San Joaquin, and the control, Merced River at Pohono Bridge. The observed $t_0$-statistic by this method was 3.80. The value of $t$ for significance at 99% was 2.71. Therefore, a significant increase was the result of this test. Comparison of the results of the above mentioned statistic tests show that the target-control $\chi^2$-test and the target-control conditional Student's t-test are better tests than the target two-sample t-test and the target sample u-test. Also note that for runoff data from high elevation watersheds the outcomes of the two tests are essentially the same for a sample size around 30. However, it should be noted that all these tests are applicable only when single target or single target-control technique is used. None of these tests can be applied without modification when the number of variables in the study is greater than two, which is the usual case.

2.5 <u>Rank test</u>. Let $q_{11}$, $q_{12}$, $\ldots$, $q_{1n_1}$ and $q_{21}$, $q_{22}$, $\ldots$, $q_{2n_2}$ be $n_1$ and $n_2$ observations of a hydrologic variable for the non-seeded and seeded periods respectively.

Arrange the observations in a common sequence of increasing magnitude,

$$q_{11}, q_{12}, q_{21}, q_{22}, q_{13}, q_{14}, q_{15}, q_{23}, q_{16}, \cdots \quad .$$

Assign ranks from 1 to n , where $n = n_1 + n_2$ , to the above sequence so that rank 1 is given to the smallest observation and $n$ to the largest.

The test statistic is now [26]:

$$Z = \frac{T_s - \overline{T}}{\sigma} \quad ,$$

where Z is approximately a standard normal variate, $T_s$ is the sum of ranks for seeded observations, $\overline{T}$ is the expected mean value of $T_s$ , given by

$$\overline{T} = \frac{n_2(n_2 + n_1 + 1)}{2}$$

$$= \frac{n_2(n + 1)}{2} \quad ,$$

and $\quad \sigma = \sqrt{\frac{n_2 n_1 (n + 1)}{12}} \quad .$

If Z is greater than 1.645, then, one rejects the null hypothesis and concludes that at the 5% level of significance weather modification was effective.

This test has been used in References [27] and [28]. From the data in the Necaxa Watershed, Mexico,

it was found that [27] the value of Z was 2.64, which is a value significant beyond the 99% level. The numbers of observations were 45 seeded days and 29 unseeded days. However, the apparent increase in the mean of the seeded period here was large. The seeded mean was about 26 percent larger than the unseeded mean. So, the use of rank test in Reference [27] does not tell much about the efficiency of the test at all. In fact, with the amount of increase of this order, one can find with any statistical test that the cloud seeding is effective. For example, when the u-test is applied the approximate number of observations needed to detect the 26 percent increase in the mean is obtained from:

$$N^* = \frac{4\sigma^2}{h^2 \mu^2}$$

where $N^*$ is the approximate number of observations required to detect a certain amount of increase in the mean,
$\sigma^2$ is the variance of the hydrologic variable for the unseeded period,
$\mu$ is the mean of the hydrologic variable for the unseeded period, and
$h$ is the fractional increase in mean.

Upon substituting the values of $\sigma^2$ , $\mu$ , $h$ from the data of Reference [27], it was found that

$$N^* = \frac{4 \times 600.17}{(.26)^2 (88.14)^2} \approx 5 \quad .$$

Thus, it is clear that the required number of observations to detect a 26 percent increase in the mean is much smaller than 45 which is the actual number of observations. So, with this large amount of increase any statistical test will always give the positive result.

2.6 <u>Median test</u>. The median of a distribution is that value which divides the distribution halfway, i.e., half the distribution have lower and half have higher values. The median test determines primarily if the medians of the populations from which the samples come are well separated or not.

Let $q_{11}$, $q_{12}$, $\ldots$, $q_{1n_1}$ and $q_{21}$, $q_{22}$, $\ldots$, $q_{2n_2}$ be $n_1$ and $n_2$ observations of a hydrologic variable for the non-seeded and seeded periods respectively. Arrange the observations in a common sequence of increasing magnitude, e.g.,

$$q_{11}, q_{12}, q_{21}, q_{22}, q_{23}, q_{13}, q_{14}, q_{15}, q_{16}, q_{24} \cdots \quad .$$

If the total number of observations is even, the median is taken to be halfway between the two middle observations. If this total number is odd, the median observation is removed since it does not contribute any information to the question of whether the distribution of that sample has its median above or below the joint sample median. The case then reduces to the even case.

Let the numbers of $q_{1i}$'s above and below the median of the common sequence be $n_{1a}$ and $n_{1b}$, and

the numbers of $q_{2j}$'s above and below the same common sample median be $n_{2a}$ and $n_{2b}$. Under the null hypothesis that the two samples come from identical distributions, the proportion of each sample lying below any point should be the same.

If the test function [29]

$$M = (|2n_{1a}-(n_{1a}+n_{1b})|-1)^2/n_1 + (|2n_{2a}-(n_{2a}+n_{2b})|-1)^2/n_2$$

is greater than $\chi^2_{0.95}$ with one degree of freedom, then, one rejects, at the 95% level, the hypothesis that the samples have the same median.

This test has been used in Reference [20]. The data used in Reference [20] were obtained from an experiment on artificial stimulation of rain in three climatologically similar regions, Delhi, Agra and Jaipur in northwest India. The net increase in rainfall obtained over all three regions was 41.9%. Thus, it was found that there was a highly significant increase in the amount of rainfall. The observations were made from 1957 to 1965 (excluding 1962) in Delhi, from 1960 to 1965 in Agra, and from 1960 to 1963 in Jaipur. There was, however, no observed statistic given in this report.

2.7 The Mann-Whitney U test. Let $q_{11}$, $q_{12}$, ..., $q_{1n_1}$ and $q_{21}$, $q_{22}$, ..., $q_{2n_2}$ be $n_1$ and $n_2$ observations of a hydrologic variable for the non-seeded and seeded periods respectively. Arrange the observations in a common sequence of increasing magnitude, e.g.,

$$q_{11}, q_{12}, q_{13}, q_{21}, q_{14}, q_{15}, q_{22}, q_{23}, q_{24}, \cdots .$$

The statistic U is defined as the number of times a $q_{2j}$ precedes a $q_{1i}$. This test was used to test the null hypothesis

$H_o$ - the $q_{1i}$ and $q_{2j}$ values have the same distribution against the alternative hypothesis,

$H_a$ - the location parameter of $q_{2j}$ is larger than the location parameter of $q_{1i}$, i.e., the bulk of the distribution of $q_{2j}$'s is to the right of the bulk of the distribution of $q_{1i}$'s.

If $H_a$ is true, one expects U to be small. Mann and Whitney [30] computed tables that give probabilities associated with small (lower tail) values of U, and Auble [31] gives tables of critical values of U for significant levels of 0.001, 0.01, 0.025, and 0.05 for a one-sided test. For the one-sided alternative hypothesis that the location parameter of $q_{2j}$ is smaller than the parameter of $q_{1i}$, one computes the statistic U', defined to be the number of times a $q_{1i}$ precedes a $q_{2j}$, and uses Auble's tables to test $H_o$.

The relationship between U and the sum of ranks for seeded observations, $T_s$, in the rank test can be expressed as (Wine [32]):

$$U = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - T_s .$$

The U statistic is usually computed by the above equation, since it is tedious to compute from the definition of U when $n_1$ and $n_2$ become fairly large.

The test statistic is

$$W = \frac{U-\overline{U}}{\sigma} ,$$

where W is approximately a standard normal variate, $\overline{U}$ is the expected value of U, given by

$$\overline{U} = \frac{n_1 n_2}{2} ,$$

and

$$\sigma = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} .$$

If W is greater than 1.65, then the null hypothesis is rejected and one can conclude the location of $q_{2j}$ is larger than that of $q_{1i}$. This test has been used by many authors - [20], [21], [28], [33], and [34].

In Reference [21], the data used were collected from a five-year period experiment (1960 through 1964) in Missouri. On comparing the average rainfall (inches/hour) of the seeded days with that of the non-seeded days, it was found there was, on the average, a decrease of 67.9%. The values of W ranged from smaller than 0.01 to 0.88. Thus, it was concluded that no evidence of increases in precipitation because of cloud seeding was achieved.

2.8 Run test. Let $q_{11}$, $q_{12}$, ..., $q_{1n_1}$ and $q_{21}$, $q_{22}$, ..., $q_{2n_2}$ be $n_1$ and $n_2$ observations of a hydrologic variable for the non-seeded and seeded periods respectively.

Arrange the observations in a common sequence of increasing magnitude, e.g.,

$$q_{11}, q_{12}, q_{21}, q_{13}, q_{14}, q_{22}, q_{23}, \cdots .$$

A run is defined as an unbroken sequence of elements of the same type, i.e., a sequence of $q_{1i}$'s or a sequence of $q_{2j}$'s. Let the number of runs be denoted by $\eta$. If two samples are from the same population, the non-seeded and seeded observations will be well mixed and the number of runs, $\eta$, will be large.

The test statistic is now [14]

$$U = \frac{\eta - \overline{\eta}}{\sigma} ,$$

where U is a standard normal variate, $\overline{\eta}$ is the expected value of $\eta$, given by

$$\bar{n} = \frac{2n_1 n_2}{n_1 + n_2}$$

$$\sigma = \sqrt{\frac{2n_1 n_2 (2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}} \quad .$$

If $U$ is greater than 1.65, then the null hypothesis is rejected and the alternative hypothesis is accepted. This test has been used in Reference [35].

In Reference [35], the data of the King River at Piedra, California was analyzed. The observations were the annual flows from 1917 to 1954 for the non-seeded period, and 1955 to 1966 for the seeded period There was a decrease of about 3.3% in mean annual

flows for the seeded period. The number of runs, $\dot{n}$, was found to be 17, $\bar{n} = 19.240$, and $\sigma = 2.533$. From the above values, $U$ was obtained as -0.88. Therefore, no significant increase in the mean annual flow was concluded.

Of all the tests stated above, it is found that none of them can be applied for testing the increase in runoff means when the number of runoff variables is greater than two. In the evaluation of weather modification effectiveness based on a multiple target-control concept the number of runoff variables involved is large. So, it is necessary to find an approach to detect the increase in means of these runoff variables.

In Chapter III, the principal components, canonical analysis, and the $T^2$-statistic are discussed.

Chapter III

## PRINCIPAL, CANONICAL COMPONENTS AND THE $T^2$-STATISTIC

For small scale operations the method of evaluation of a significant change in hydrologic characteristics based on the single target-control concept is adequate. For large regions this procedure would not be very representative. Besides if the test were performed for many pairs of target and control it is not clear how one should treat the ensemble of the outcomes. On the other hand, there is no problem of interpretation when a single test is performed even though the tested statistic may itself be a complicated combination of many observations from many targets and controls. For representativity the station runoff variables should be geographically well distributed over the large area of interest. This results in a selection of a large number of variables that are usually not independent variables. Sometimes the number of variables involved may be so large that any study can hardly be made economically. In fact, this is one of the difficulties in this study since there are three big areas under investigation. It is, therefore, also an object of this study to find a suitable method for reducing the number of variables involved in the analysis.

There are several ways to reduce the number of variables. However, two methods are used here before the statistical test is carried out. One is the principal components analysis, the other the canonical analysis.

3.1 Principal component analysis. The principal components are linear combinations of random variables, which have special properties in terms of variances. Usually, the linear combination with the maximum variance is referred to as the first principal component; the second component is the one that is uncorrelated with the first and has the second largest variance, and so on. The idea of this analysis was discussed thoroughly by Hotelling [36] in 1933.

From the hydrologic point of view, these principal components can be considered as new transformed runoff variables though lacking simple physical meaning. These transformed variables have, in total, the same amount of fluctuation or variation as do the original runoff variables. But the number of the transformed variables can be smaller than that of the original variables. Also these transformed variables are independent while the original variables are not.

A priori what can be expected from the principal components analysis for the purpose of evaluation? Suppose the principal components analysis is carried for all the targets and all the controls. The first principal component for each group will be the most statistically representative single combination of targets and controls, respectively, because that combination will account for the largest fraction of the total variation. If the percentage is high (say 95%) all the other principal components can be dropped. Then the originally multivariate test reduces again to a familiar single target control t-test, even

though the target variable and the control variable are each a combination of many target and control ones. The procedure will be simple and effective if the target first principal component and the control one are highly correlated. However, this need not happen because the targets and controls are treated separately and the procedure does not attempt to maximize the correlation between the two components (which canonical analysis does). It can be concluded that principal components analysis can provide the basis for a simple and highly representative test but it will not be, by far, a minimal time evaluation one. (The procedure for the actual computation of the principal components is summarized in Chapter V, Section 1).

3.2 The canonical analysis. Canonical analysis is a technique to maximize the correlations between two groups of random variables. This analysis gives new sets of transformed variables as linear combinations of the original runoff variables. The first linear combination of each group will have the highest correlation, and each is uncorrelated with the other linear combinations in its group. The second linear combinations will have the second highest correlation, the third linear combinations will have the third highest correlation and so on. These linear combinations are referred to as canonical variables or components.

In this study the first group is the group of runoff stations in the target region and the second group is made of stations in the control region. This analysis is particularly advantageous for evaluation purposes. The canonical analysis yields a smaller number of variables for the final test, and most importantly it also guarantees high correlations between the variables of the target and control regions.

3.3 Computation of canonical variables. The steps for computing the canonical variables are now described:

Step 1) Compute the covariance matrix, $\hat{V}$, of the runoff variables of the two sets (target and control). For $p_1$ runoff stations in the target region and $p_2$ in the control region, then

$$\hat{V} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \cdots \cdots \sigma_{1p_1} & \sigma_{1(p_1+1)} \cdots \cdots \cdots \sigma_{1(p_1+p_2)} \\ \sigma_{21} & \sigma_{22} \cdots \cdots \sigma_{2p_1} & \sigma_{2(p_1+1)} \cdots \cdots \cdots \sigma_{2(p_1+p_2)} \\ \vdots & \vdots \quad \vdots & \vdots \quad \vdots \\ \sigma_{p_11} & \sigma_{p_12} \quad \sigma_{p_1p_1} & \sigma_{p_1(p_1+1)} \cdots \cdots \cdots \sigma_{p_1(p_1+p_2)} \\ \sigma_{(p_1+1)1} & \sigma_{(p_1+1)2} \cdots \cdots \sigma_{(p_1+1)p_1} & \sigma_{(p_1+1)(p_1+1)} \cdots \cdots \sigma_{(p_1+1)(p_1+p_2)} \\ \vdots & \vdots \quad \vdots & \vdots \quad \vdots \\ \sigma_{(p_1+p_2)1} & \sigma_{(p_1+p_2)2} \cdots \sigma_{(p_1+p_2)p_1} & \sigma_{(p_1+p_2)(p_1+1)} \cdots \cdots \sigma_{(p_1+p_2)(p_1+p_2)} \end{bmatrix} \qquad (2)$$

The subscripts of $\sigma$ are the ordering numbers of the stations. The numbers 1 to $p_1$ are for the $p_1$ stations in the target region. The numbers $p_1+1$ to $p_1+p_2$ are for the $p_2$ stations in the control region. For example, the subscript 1 will refer to the first station in the target region, while the subscript $p_1+1$ will refer to the first station in the control region and the subscript $p_1+2$ the second station in the control region, etc.

$\sigma_{ii}$ is the variance of the runoff series for station i, defined as,

$$\sigma_{ii} = \frac{1}{N} \sum_{s=1}^{N} (q_{is} - \bar{q}_i)^2 \quad , \tag{3}$$

where N is the number of years of recorded runoff data,
$q_{is}$ is the $s^{th}$ recorded runoff of station i, and
$\bar{q}_i$ is the mean of the recorded runoff of station i.

$\sigma_{ij}$ is the covariance of stations i and j, defined as,

$$\sigma_{ij} = \frac{1}{N} \sum_{s=1}^{N} (q_{is} - \bar{q}_i)(q_{js} - \bar{q}_j) \tag{4}$$

$$\sigma_{ij} = \sigma_{ji} \quad .$$

Step 2) Partition the covariance matrix, $\hat{V}$, such that,

$$\hat{V} = \begin{bmatrix} \hat{V}_{11} & \hat{V}_{12} \\ \hat{V}_{21} & \hat{V}_{22} \end{bmatrix} \quad , \tag{5}$$

where $\hat{V}_{11}$ is a $p_1 \times p_1$ matrix,

$$\hat{V}_{11} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots\cdots\sigma_{1p_1} \\ \sigma_{21} & \sigma_{22} & \cdots\cdots\sigma_{2p_1} \\ \vdots & \vdots & \vdots \\ \sigma_{p_1 1} & \sigma_{p_1 2} & \cdots\cdots\sigma_{p_1 p_1} \end{bmatrix} \tag{6}$$

$$\hat{V}_{12} = \begin{bmatrix} \sigma_{1(p_1+1)} & \sigma_{1(p_1+2)} & \cdots\cdots\sigma_{1(p_1+p_2)} \\ \sigma_{2(p_1+1)} & \sigma_{2(p_1+2)} & \cdots\cdots\sigma_{2(p_1+p_2)} \\ \vdots & \vdots & \vdots \\ \sigma_{p_1(p_1+1)} & \sigma_{p_1(p_1+2)} & \cdots\cdots\sigma_{p_1(p_1+p_2)} \end{bmatrix} \tag{7}$$

$$\hat{V}_{12}' = \hat{V}_{21} \tag{8}$$

$$\hat{V}_{22} = \begin{bmatrix} \sigma_{(p_1+1)(p_1+1)} & \sigma_{(p_1+1)(p_1+2)} & \cdots\cdots\sigma_{(p_1+1)(p_1+p_2)} \\ \sigma_{(p_1+2)(p_1+1)} & \sigma_{(p_1+2)(p_1+2)} & \cdots\cdots\sigma_{(p_1+2)(p_1+p_2)} \\ \vdots & \vdots & \vdots \\ \sigma_{(p_1+p_2)(p_1+1)} & \sigma_{(p_1+p_2)(p_1+2)} & \cdots\cdots\sigma_{(p_1+p_2)(p_1+p_2)} \end{bmatrix} \tag{9}$$

Step 3) Obtain the values of canonical correlations by solving the system,

$$\begin{vmatrix} -\theta\hat{V}_{11} & \hat{V}_{12} \\ \hat{V}_{21} & -\theta\hat{V}_{22} \end{vmatrix} = 0 \quad . \tag{10}$$

The values of $\theta$ are the canonical correlations.

Step 4) Let $\underline{\alpha}$ and $\underline{\gamma}$ be the column vectors of coefficients for the canonical variables of the target and control regions respectively. Then, for a given value $\theta_i$, the vectors $\underline{\alpha}_i$ and $\underline{\gamma}_i$ can be obtained by solving the system,

$$\begin{bmatrix} -\theta_i\hat{V}_{11} & \hat{V}_{12} \\ \hat{V}_{21} & -\theta_i\hat{V}_{22} \end{bmatrix} \begin{bmatrix} \underline{\alpha}_i \\ \underline{\gamma}_i \end{bmatrix} = \underline{0} \tag{11}$$

subject to the standardization conditions:

$$\underline{\alpha}_i' \hat{V}_{11} \underline{\alpha}_i = 1 \tag{12}$$

$$\underline{\gamma}_i' \hat{V}_{22} \underline{\gamma}_i = 1 \quad ; \tag{13}$$

$\underline{\alpha}_i'$ and $\underline{\gamma}_i'$ are the transposes of $\underline{\alpha}_i$ and $\underline{\gamma}_i$ respectively.

Once the $\underline{\alpha}_i$ and $\underline{\gamma}_i$ are obtained, the canonical variables for the target region are obtained from the relations:

$$\zeta_i = \underline{\alpha}_i' \underline{Q}_1 \tag{14}$$

where $\zeta_i$ is the $i^{th}$ canonical variable in the target region

$$\underline{\alpha}_i' = (\alpha_{i1}, \alpha_{i2}, \cdots, \alpha_{ip_1}) \tag{15}$$

$$\underline{Q}_1 = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{p_1} \end{bmatrix} \tag{16}$$

$Q_1, Q_2, \ldots, Q_{p_1}$ are runoff variables in the target region.

Similarly, $\varepsilon_i$ is the $i^{th}$ canonical variable in the control region defined by the relation:

$$\varepsilon_i = \underline{\gamma}_i' \, \underline{Q}_2 \, , \tag{17}$$

where $\quad \underline{\gamma}_i' = (\gamma_{i(p_1+1)} \gamma_{i(p_1+2)} \cdots \gamma_{i(p_1+p_2)}) \tag{18}$

$$\underline{Q}_2 = \begin{bmatrix} Q_{p_1+1} \\ Q_{p_1+2} \\ \vdots \\ Q_{p_1+p_2} \end{bmatrix} \tag{19}$$

$Q_{p_1+1}$, $Q_{p_1+2}$, ..., $Q_{p_1+p_2}$ are runoff variables in the control region.

3.4 The minimum number of years for detecting an increase in runoff means. In the previous sections two techniques to transform the original runoff variables were described and in the case of canonical analysis even the basic steps of the procedure were described. However, the multivariate $T^2$ test applies just as well for the set of original variables. The principal and canonical transformations will either simplify some of the calculations or improve the outcome of the test. Again, the transformations are not necessary to apply the test. Nevertheless in this study the test was only performed for the transformed variables.

Assuming the values of the population mean vector $\underline{\mu}^*$ and covariance matrix $\underline{V}$ for the seeded period are known, the minimum number of observations, $N^*$, that one needs in order to be able to reject the hypothesis $\underline{\mu}^* = \underline{\mu}_o$, where $\underline{\mu}_o$ is a given vector, is given by

$$N^* = \frac{\tau^2}{(\underline{\mu}^* - \underline{\mu}_o)' \, \underline{V}^{-1} \, (\underline{\mu}^* - \underline{\mu}_o)} \, , \tag{20}$$

where $\tau^2$ is the noncentrality parameter with degrees of freedom $k$ and $N-k$,
$k$ is the total number of runoff variables, and
$N$ is the number of observations for the nonseeded period.

Select values of $\tau^2$ as given by Tang [37] and Lehmer [38] are shown for convenience in Table 1.

TABLE 1 - VALUE OF $\tau^2$

Level of significance, $\alpha = 0.05$; power $\beta = 0.50$

| Degrees of freedom | | |
|---|---|---|
| k | N-k | $\tau^2$ |
| 2 | 28 | 5.468 |
| 4 | 26 | 7.640 |
| 5 | 25 | 8.640 |
| 6 | 24 | 9.646 |
| 8 | 22 | 11.655 |

In this study the value of $\underline{\mu}_o$ is assumed to be the mean vector of target and control runoff variables for the period before seeding. $\underline{\mu}^*$ is similar to $\underline{\mu}_o$ except that the means of the target runoff variables are 1.1 times greater than those in $\underline{\mu}_o$. In other words, it is assumed in this study that the effect of precipitation management over the target areas will be to increase the runoff uniformly throughout the target areas by 10%. The covariance matrix $\underline{V}$ is assumed to be the same as that of the nonseeded period.

When the principal components (or the canonical variables) are used for computing $N^*$, then $\underline{\mu}^*$ and $\underline{\mu}_o$ are the mean vectors of the principal components (or the canonical variables) for the seeded and nonseeded periods respectively, and $\underline{V}$ is the covariance matrix of the principal components (or the canonical variables) for the non-seeded period. The original runoff variables can also be used in computing $N^*$. However, because of the large number of the original runoff variables, they are not used in this study.

It should be noted here that the use of principal components in equation (20) will yield approximately the same results as the use of the original runoff variables. This is due to the fact that the amount of variation accounted for by the principal components is practically the same as the variation of the original runoff variables. Thus, the principal component analysis will merely reduce the number of original variables, but will not improve the final outcome of the test.

However, if the number of variables can be reduced to one component then the principal component analysis will be very useful because one can apply a bivariate test, such as the conditional Student's t-test which is less restrictive in its assumptions than the $T^2$-test. Unfortunately, this usefulness will not be known until one has completed the analysis.

In the next chapter the collection of data in the Upper Basin of the Colorado River, the San Juan Mountains area, and the Maroon Peak and Grand Mesa region is discussed.

Chapter IV

RESEARCH DATA ASSEMBLY

The data used in this study are the records of the runoff from three regions in the Colorado River Basin. These are:

1. The Upper Basin of the Colorado River,

2. The San Juan Mountains area,

3. The Maroon Peak and Grand Mesa region.

The first two areas were originally [7] proposed as sites for extensive cloud seeding operation. They are called northern and southern target regions (Figs. 2 and 3), while the third is called the control region (Fig. 4). Currently [8] only one area is considered: the southern area. The selection of the control stations is done primarily on the basis of the high correlations with those in the target regions.

It is virgin flow, which is the flow free from any man-made intervention, that is necessary for this study. So, corrections must be made for the records

of runoff. The records of runoff were obtained from U.S. Geological Survey Water Supply Papers. However, only the corrections due to transmountain, transbasin diversions, and regulation can be made. The diversion for irrigation cannot be made because there is no record for the amount of water diverted for this purpose. Thus, it is assumed after making the corrections above, that virgin flows are obtained.

Out of a large number of stations, seven stations are chosen for the final analysis in the northern target region, and six stations in the southern region. There are fourteen stations used as controls for the northern region, and nine stations as controls for the southern region. These stations and their descriptions are listed in Table 2. The correlations for these stations computed from all the corresponding actually available records are shown in Tables 3, 4, 5 and 6. There are two stations used as controls for both the northern and southern regions.

TABLE 2 - DESCRIPTION OF STATIONS

| Types | Seq. No. | CSU Sta. No. | USGS Sta. No. | Names | Lat. (o) | ( ' ) | ( " ) | Long. (o) | ( ' ) | ( " ) | Area (Sq. Mi.) | Elevation (ft.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Target-stations in Northern Project | 1 | 1970000 | 9.0105 | Colorado River below Baker Gulch, near Grand Lake, Colorado. | 40 | 19 | 33 | 105 | 51 | 22 | 53 | 8750 |
| | 2 | 1960000 | 9.0110 | Colorado River near Grand Lake, Colo. | 40 | 13 | 08 | 105 | 51 | 25 | 103 | 8380 |
| | 3 | 1866000 | 9.0163 | Arapaho Creek at Monarch Lake outlet, Colo. | 40 | 06 | 45 | 105 | 44 | 57 | 47.1 | 8310 |
| | 4 | 1830000 | 9.0190 | Colorado River below Lake Granby, Colo. | 40 | 08 | 39 | 105 | 52 | 00 | 311 | 8050 |
| | 5 | 1820000 | 9.0195 | Colorado River near Granby, Colo. | 40 | 07 | 15 | 105 | 54 | 00 | 322 | 7960 |
| | 6 | 1802730 | 9.0265 | St. Louis Creek near Fraser, Colo. | 39 | 54 | 30 | 105 | 52 | 45 | 32.8 | 8980 |
| | 7 | 1776000 | 9.0360 | Williams Fork near Leal, Colo. | 39 | 49 | 55 | 106 | 03 | 20 | 89.5 | 8790 |
| Control-stations for Northern Project | 1 | 1742100 | 9.0535 | Blue River above Green Mountain Reservoir, Colo. | 39 | 49 | 55 | 106 | 13 | 20 | 514 | 7947 |
| | 2 | 1740000 | 9.0575 | Blue River below Green Mountain Reservoir, Colo. | 39 | 52 | 50 | 106 | 20 | 00 | 599 | 7683 |
| | 3 | 1720000 | 9.0595 | Piney River near State Bridge, Colo. | 39 | 48 | 00 | 106 | 35 | 00 | 82.6 | 7272 |
| | 4 | 1666300 | 9.0645 | Homestake Creek near Red Cliff, Colo. | 39 | 28 | 25 | 106 | 22 | 00 | 58.9 | 8783 |
| | 5 | 1594260 | 9.0780 | Fryingpan River at Norrie, Colo. | 39 | 19 | 50 | 106 | 39 | 30 | 89.5 | 8410 |
| | 6 | 1594236 | 9.0785 | North Fork Fryingpan River near Norrie, Colo. | 39 | 20 | 40 | 106 | 39 | 50 | 41.2 | 8400 |
| | 7 | 1590000 | 9.0850 | Roaring Fork River at Glenwood Springs, Colo. | 39 | 32 | 50 | 107 | 19 | 50 | 1460 | 5721 |
| | 8 | 1379000 | 9.1090 | Taylor River below Taylor Park Reservoir, Colo. | 38 | 48 | 50 | 106 | 36 | 40 | 254 | 9170 |
| | 9 | 1378400 | 9.1100 | Taylor River at Almont, Colo. | 38 | 40 | 00 | 106 | 51 | 00 | 477 | 8011 |
| | 10 | 1378100 | 9.1125 | East River at Almont, Colo. | 38 | 40 | 00 | 106 | 51 | 00 | 295 | 8006 |
| | 11 | 1377825 | 9.1135 | Ohio Creek near Baldwin, Colo. | 38 | 42 | 00 | 107 | 00 | 00 | 124 | 8180 |
| | 12 | 1377500 | 9.1145 | Gunnison River near Gunnison, Colo. | 38 | 32 | 50 | 106 | 57 | 00 | 1010 | 7670 |
| | 13 | 1377280 | 9.1155 | Tomichi Creek at Sargents, Colo. | 38 | 24 | 00 | 106 | 25 | 00 | 155 | 8420 |
| | 14 | 1377230 | 9.1180 | Quartz Creek near Ohio City, Colo. | 38 | 33 | 35 | 106 | 38 | 10 | 106 | 8430 |
| Target-stations in Southern Project | 1 | 1278800 | 9.1650 | Dolores River below Rico, Colo. | 37 | 38 | 20 | 108 | 03 | 35 | 105 | 8422 |
| | 2 | 1278050 | 9.1665 | Dolores River at Dolores, Colo. | 37 | 28 | 00 | 108 | 30 | 00 | 556 | 6919 |
| | 3 | 1272445 | 9.1725 | San Miguel River near Placerville, Colo. | 38 | 02 | 05 | 108 | 07 | 15 | 305 | 7056 |
| | 4 | 1077090 | 9.3440 | Navajo River at Banded Peak Ranch, near Chromo, Colo. | 37 | 05 | 07 | 106 | 41 | 20 | 69.8 | 7941 |
| | 5 | 1073480 | 9.3575 | Animas River at Howardsville, Colo. | 37 | 50 | 00 | 107 | 36 | 00 | 55.9 | 9617 |
| | 6 | 1073436 | 9.3615 | Animas River at Durango, Colo. | 37 | 16 | 45 | 107 | 52 | 47 | 692 | 6502 |
| Control-stations for Southern Project | 1 | 1425623 | 9.0975 | Buzzard Creek near Collbran, Colo. | 39 | 16 | 20 | 107 | 51 | 00 | 139 | 6955 |
| | 2 | 1377280 | 9.1155 | Tomichi Creek at Sargents, Colo. | 38 | 24 | 00 | 106 | 25 | 00 | 155 | 8420 |
| | 3 | 1377230 | 9.1180 | Quartz Creek near Ohio City, Colo. | 38 | 33 | 35 | 106 | 38 | 10 | 106 | 8430 |
| | 4 | 1377500 | 9.1190 | Tomichi Creek at Gunnison, Colo. | 38 | 31 | 20 | 106 | 56 | 25 | 1020 | 7629 |
| | 5 | 1373900 | 9.1275 | Crystal Creek near Maher, Colo. | 38 | 33 | 05 | 107 | 30 | 20 | 42.2 | 8070 |
| | 6 | 1373055 | 9.1325 | North Fork Gunnison River near Somerset, Colo. | 38 | 55 | 45 | 107 | 26 | 55 | 521 | 6039 |
| | 7 | 1373020 | 9.1345 | Leroux Creek near Cedaredge, Colo. | 38 | 55 | 35 | 107 | 47 | 35 | 35.1 | 7160 |
| | 8 | 1371815 | 9.1430 | Surface Creek near Cedaredge, Colo. | 38 | 59 | 00 | 107 | 51 | 00 | 26.7 | 8180 |
| | 9 | 1370300 | 9.1520 | Kannah Creek near Whitewater, Colo. | 38 | 59 | 00 | 108 | 14 | 00 | 61.9 | — |

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Table 3 - CORRELATION MATRIX BETWEEN N-4 AND CN-4 (as computed from all available data)

| CSU STA. No. | USGS STA. No. | 1970000 9.0105 | 1960000 9.0105 | 1866000 9.0110 | 1830000 9.0165 | 1820000 9.0290 | 1802730 9.0265 | 1776000 9.0360 |
|---|---|---|---|---|---|---|---|---|
| | CN-4 | N-4 | | | | | | |
| 1742100 | 9.0535 | .8625 | .6365 | .8375 | .8234 | .6475 | .7779 | .9342 |
| 1740000 | 9.0575 | .6055 | .7277 | .6970 | .7077 | .4634 | .8427 | .8357 |
| 1720000 | 9.0595 | .9164 | .9003 | .8322 | .8476 | .7171 | .6076 | .9470 |
| 1666300 | 9.0645 | .6781 | .7548 | .8147 | .8304 | .6023 | .6515 | .8033 |
| 1594260 | 9.0780 | .8952 | .8514 | .4854 | .8919 | .7218 | .6618 | .9319 |
| 1594236 | 9.0785 | .8608 | .8567 | .9187 | .9089 | .7975 | .6291 | .8647 |
| 1590000 | 9.0850 | .8723 | .8776 | .8382 | .8770 | .7701 | .6381 | .8717 |
| 1379000 | 9.0190 | .8364 | .8174 | .7846 | .8541 | .6999 | .4699 | .9080 |
| 1378400 | 9.1100 | .8474 | .8434 | .7942 | .8473 | .7329 | .5012 | .7744 |
| 1378100 | 9.1125 | .8635 | .8151 | .7971 | .8301 | .6581 | .6456 | .7896 |
| 1377825 | 9.1135 | .8741 | .6554 | .5844 | .7306 | .5222 | .6190 | .7672 |
| 1377500 | 9.1145 | .8714 | .6338 | .7996 | .8434 | .6851 | .5337 | .8012 |
| 1377280 | 9.1155 | .8026 | .6197 | .7937 | .8009 | .6634 | .5672 | .7082 |
| 1377230 | 9.1180 | .8644 | .6436 | .7113 | .7675 | .6274 | .5090 | .7536 |

Table 4 - CORRELATION MATRIX BETWEEN N-6 AND CN-6 (as computed from all available data)

| CSU STA. No. | USGS STA. No. | 1970000 9.0105 | 1960000 9.0110 | 1866000 9.0165 | 1830000 9.0190 | 1820000 9.0195 | 1802730 9.0265 | 1776000 9.0360 |
|---|---|---|---|---|---|---|---|---|
| | CN-6 | N-6 | | | | | | |
| 1742100 | 9.0535 | .6648 | .9578 | .9124 | .9243 | .9937 | .5921 | .9155 |
| 1740000 | 9.0575 | .7233 | .4359 | .8704 | .8640 | .9038 | .4789 | .8409 |
| 1720000 | 9.0595 | .6944 | .3230 | .5386 | .6146 | .3806 | .3611 | .3126 |
| 1666300 | 9.0645 | .7348 | .6093 | .5738 | .5336 | .3514 | .3718 | .6702 |
| 1594236 | 9.0780 | .4923 | .5923 | .6371 | .7567 | .8406 | .5299 | .7359 |
| 1590000 | 9.0785 | .7153 | .5548 | .8017 | .6039 | .2842 | .3247 | .4008 |
| 1590000 | 9.0850 | .7877 | .3323 | .6076 | .6012 | .7616 | .5203 | .5695 |
| 1379000 | 9.0190 | .6576 | .5072 | .6912 | .2488 | .4766 | .5284 | .7748 |
| 1378400 | 9.1100 | .7135 | .4373 | .5538 | .7051 | .3055 | .5282 | .7908 |
| 1378100 | 9.1125 | .3529 | .4010 | .7701 | .1470 | .4582 | .4277 | .4510 |
| 1377825 | 9.1135 | .6645 | .6396 | .7202 | .0136 | .7097 | .4439 | |
| 1377500 | 9.1145 | .7503 | .4247 | .5132 | .5766 | .5478 | .4155 | .8483 |
| 1377280 | 9.1155 | .7354 | .5034 | .6163 | .5225 | .4351 | .4590 | .8899 |
| 1377230 | 9.1180 | .8004 | .5576 | .1133 | .6640 | .6830 | .6961 | .4868 |

Table 5 - CORRELATION MATRIX BETWEEN S-4 AND CS-4 (as computed from all available data)

| CSU STA. NO. | USGS STA. NO. | 1278800 9.1650 | 1278050 9.1665 | 1272445 9.1725 | 1077090 9.3440 | 1073480 9.3440 | 1073436 9.3615 |
|---|---|---|---|---|---|---|---|
| | CS-4 | S-4 | | | | | |
| 1425625 | 9.0975 | .9004 | .8519 | .8872 | .7978 | .8466 | .8258 |
| 1377280 | 9.1155 | .9020 | .7565 | .8040 | .7529 | .8295 | .7353 |
| 1377230 | 9.1180 | .9108 | .7289 | .5841 | .6336 | .7553 | .6964 |
| 1377200 | 9.1190 | .9865 | .8587 | .8428 | .7859 | .8895 | .8423 |
| 1373900 | 9.1275 | .8879 | .8710 | .9059 | .7988 | .8578 | .8549 |
| 1373055 | 9.1325 | .8900 | .8599 | .7981 | .7835 | .9582 | .8216 |
| 1373020 | 9.1345 | .8335 | .8608 | .7064 | .8226 | .8118 | .8069 |
| 1371815 | 9.1430 | .8961 | .8993 | .8021 | .8490 | .2168 | .4315 |
| 1370300 | 9.1520 | .9299 | .8058 | .8909 | .8576 | .8276 | .7837 |

Table 6 - CORRELATION MATRIX BETWEEN S-6 AND CS-6 (as computed from all available data)

| CSU STA. NO. | USGS STA. NO. | 1278800 9.1650 | 1278050 9.1665 | 1272445 9.1725 | 1077090 9.3440 | 1073480 9.3575 | 1073436 9.3615 |
|---|---|---|---|---|---|---|---|
| | CS-6 | S-6 | | | | | |
| 1425625 | 9.0975 | .8217 | .6427 | .7111 | .6302 | .8128 | .3267 |
| 1377280 | 9.1155 | .9310 | .9100 | .7033 | .3573 | .9009 | .9126 |
| 1377230 | 9.1180 | .8008 | .8536 | .7309 | .7921 | .7754 | .8297 |
| 1377200 | 9.1190 | .8864 | .7361 | .8601 | .9381 | .9729 | .7719 |
| 1373900 | 9.1275 | .9406 | .7605 | .8115 | .7576 | .7964 | .7121 |
| 1373055 | 9.1325 | .9368 | .7148 | .8990 | .8423 | .8881 | .9498 |
| 1373020 | 9.1345 | .8947 | .6922 | .8129 | .5556 | .8410 | .6934 |
| 1371815 | 9.1430 | .8844 | .8071 | .7831 | .7217 | .7546 | .7877 |
| 1370300 | 9.1520 | .7872 | .7429 | .6865 | .7034 | .8869 | .7467 |

The major part of the spring runoff will occur because of the melting of the winter snow, which is subject to the effect of seeding during winter time. So, it is reasonable to consider whatever changes in the value of the spring runoff as an indirect indicator of the effect of cloud seeding. This is equivalent to saying a larger amount of snowfall in winter will produce a larger amount of runoff in spring. Because of the uncertainty of the start of snow melting, both the runoff during the four months of April, May, June and July, and during the six months of March, April, May, June, July and August are used. These four-month runoff and six-month runoff periods are treated separately in this analysis.

The number of years of record for all stations is fixed at 30, starting from 1938 up to 1967. To assure that these stations are still in operation, the selection has been made in such a way that only stations that have records available for 1967 are considered. It is not likely that the operation of these stations will be discontinued in the near future.

The characteristics of the data used in this study are shown in Tables 7, 8, 9, 10, 11, 12, 13, and 14. There are some data missing in the runoff record of the stations selected but they are filled in by the regression method [39] with the random component superimposed. These stations with missing data are shown in Table 15. Also shown in Table 15 are the stations used in evaluating the missing data. Graphical representations of the data used are shown in Figs. 5, 6, 7, 8, 9, 10, 11, and 12 according to the regions. The means and standard deviations computed from the year 1938 up to 1967 data are shown in Table 16; and the correlations between N-4 and CN-4, N-6 and CN-6, S-4 and CS-4, and S-6 and CS-6 are shown in Tables 17, 18, 19, and 20, respectively.

In Chapter V, the analysis of the data and the results are presented.

TABLE 7 - N-4 SERIES (CFS)

| CSU | 1970000 | 1960000 | 1866000 | 1830000 | 1820000 | 1802730 | 1776000 |
|---|---|---|---|---|---|---|---|
| USGS | 9.0105 | 9.0110 | 9.0165 | 9.0190 | 9.0195 | 9.0265 | 9.0360 |
| Year | | | | | | | |
| 1938 | 180.35 | 366.27 | 244.61 | 1179.06 | 1136.39 | 102.10 | 379.77 |
| 1939 | 216.18 | 245.75 | 129.61 | 754.27 | 763.81 | 72.41 | 247.57 |
| 1940 | 139.53 | 193.02 | 168.37 | 686.57 | 673.55 | 53.67 | 175.50 |
| 1941 | 163.66 | 251.27 | 318.69 | 994.25 | 805.08 | 70.50 | 224.29 |
| 1942 | 216.65 | 219.05 | 233.59 | 1192.95 | 850.85 | 83.66 | 282.53 |
| 1943 | 180.00 | 230.67 | 145.06 | 585.11 | 807.90 | 93.99 | 245.73 |
| 1944 | 191.53 | 224.75 | 170.58 | 661.96 | 726.79 | 73.77 | 218.40 |
| 1945 | 217.40 | 251.49 | 196.92 | 1060.47 | 837.71 | 72.80 | 333.42 |
| 1946 | 195.31 | 179.06 | 201.58 | 662.49 | 641.57 | 73.00 | 226.37 |
| 1947 | 200.72 | 317.40 | 246.16 | 1556.30 | 1035.63 | 98.51 | 317.49 |
| 1948 | 57.75 | 198.77 | 161.58 | 1023.12 | 682.34 | 74.92 | 231.42 |
| 1949 | 302.87 | 306.21 | 244.28 | 1069.03 | 1057.22 | 84.06 | 268.34 |
| 1950 | 170.28 | 180.74 | 155.98 | 923.67 | 246.07 | 72.07 | 227.35 |
| 1951 | 252.28 | 305.83 | 245.06 | 1000.46 | 992.64 | 107.37 | 301.62 |
| 1952 | 244.13 | 352.67 | 285.75 | 1188.94 | 1188.71 | 107.45 | 334.81 |
| 1953 | 235.87 | 207.48 | 186.89 | 740.89 | 737.48 | 75.53 | 225.97 |
| 1954 | 108.66 | 117.48 | 129.00 | 529.31 | 486.97 | 54.57 | 108.27 |
| 1955 | 145.96 | 159.63 | 171.09 | 749.74 | 438.18 | 52.00 | 163.32 |
| 1956 | 206.35 | 249.24 | 215.13 | 1046.18 | 874.92 | 42.58 | 219.86 |
| 1957 | 274.13 | 368.69 | 320.64 | 1625.95 | 1252.69 | 84.12 | 306.48 |
| 1958 | 226.14 | 249.54 | 220.52 | 929.26 | 995.11 | 80.52 | 247.84 |
| 1959 | 171.07 | 206.87 | 204.50 | 853.75 | 578.18 | 25.05 | 201.92 |
| 1960 | 212.75 | 280.40 | 217.77 | 1122.92 | 1384.95 | 43.85 | 252.19 |
| 1961 | 181.57 | 202.94 | 171.29 | 836.40 | 823.74 | 23.22 | 178.74 |
| 1962 | 268.49 | 373.30 | 229.44 | 1211.50 | 1108.41 | 48.16 | 313.24 |
| 1963 | 150.16 | 151.40 | 156.76 | 655.72 | 636.17 | 21.13 | 100.42 |
| 1964 | 199.19 | 198.73 | 177.40 | 826.43 | 810.51 | 22.48 | 177.83 |
| 1965 | 271.36 | 311.11 | 264.29 | 1193.47 | 1180.51 | 39.82 | 275.53 |
| 1966 | 141.71 | 125.82 | 99.35 | 288.08 | 272.07 | 20.50 | 142.67 |
| 1967 | 235.42 | 243.04 | 193.79 | 798.78 | 788.61 | 36.67 | 211.29 |

TABLE 8 - N-6 SERIES (CFS)

| CSU | 1970000 | 1960000 | 1866000 | 1830000 | 1820000 | 180273? | 1776000 |
|---|---|---|---|---|---|---|---|
| USGS | 9.0105 | 9.0110 | 9.0165 | 9.0190 | 9.0195 | 9.0265 | 9.0360 |
| Year | | | | | | | |
| 1938 | 126.75 | 258.12 | 171.72 | 806.46 | 857.32 | 75.55 | 269.28 |
| 1939 | 152.42 | 172.51 | 91.96 | 499.87 | 539.45 | 53.70 | 176.43 |
| 1940 | 97.59 | 155.77 | 121.40 | 456.37 | 479.85 | 40.21 | 126.53 |
| 1941 | 114.87 | 176.89 | 228.05 | 683.26 | 571.12 | 53.16 | 162.34 |
| 1942 | 152.78 | 153.63 | 166.17 | 837.27 | 597.26 | 62.61 | 202.05 |
| 1943 | 127.65 | 164.24 | 104.99 | 379.79 | 578.21 | 69.92 | 180.20 |
| 1944 | 133.58 | 155.78 | 121.51 | 426.80 | 511.21 | 55.45 | 157.48 |
| 1945 | 150.34 | 188.16 | 149.80 | 773.27 | 637.78 | 61.19 | 183.51 |
| 1946 | 139.16 | 128.63 | 143.48 | 447.68 | 466.07 | 55.12 | 164.47 |
| 1947 | 143.83 | 228.14 | 176.76 | 1116.29 | 752.17 | 76.19 | 237.03 |
| 1948 | 39.11 | 140.56 | 114.31 | 714.13 | 482.15 | 55.95 | 166.26 |
| 1949 | 215.43 | 216.55 | 172.61 | 735.79 | 731.75 | 63.60 | 195.86 |
| 1950 | 119.74 | 127.12 | 109.07 | 638.61 | 178.45 | 53.53 | 163.16 |
| 1951 | 185.38 | 226.48 | 177.25 | 723.93 | 718.66 | 80.43 | 221.28 |
| 1952 | 175.29 | 253.57 | 205.26 | 855.98 | 854.73 | 81.65 | 244.36 |
| 1953 | 171.42 | 154.66 | 136.06 | 543.19 | 542.88 | 58.32 | 169.86 |
| 1954 | 77.05 | 82.52 | 91.46 | 377.96 | 359.63 | 26.67 | 80.90 |
| 1955 | 105.61 | 112.60 | 125.39 | 560.94 | 319.30 | 41.40 | 125.25 |
| 1956 | 144.56 | 277.11 | 152.30 | 739.35 | 621.87 | 33.12 | 159.25 |
| 1957 | 201.49 | 273.95 | 234.88 | 1176.83 | 904.12 | 59.48 | 229.80 |
| 1958 | 157.49 | 175.79 | 154.87 | 644.97 | 710.91 | 57.91 | 177.54 |
| 1959 | 123.44 | 151.96 | 148.11 | 618.79 | 422.89 | 20.29 | 147.68 |
| 1960 | 150.52 | 199.93 | 153.26 | 780.54 | 995.41 | 31.58 | 180.31 |
| 1961 | 130.37 | 148.48 | 124.20 | 599.67 | 589.04 | 19.44 | 133.18 |
| 1962 | 193.05 | 269.76 | 164.31 | 851.17 | 762.64 | 35.21 | 225.39 |
| 1963 | 112.60 | 119.22 | 120.65 | 493.81 | 477.88 | 16.44 | 79.82 |
| 1964 | 141.16 | 146.24 | 128.08 | 589.81 | 575.56 | 18.43 | 130.85 |
| 1965 | 196.22 | 227.93 | 193.31 | 760.75 | 749.88 | 33.52 | 209.79 |
| 1966 | 101.65 | 93.69 | 74.06 | 109.61 | 95.96 | 17.30 | 106.26 |
| 1967 | 166.64 | 175.06 | 135.65 | 409.44 | 399.60 | 27.09 | 153.62 |

TABLE 9 - CN-4 SERIES (CFS)

| CSU | 1742100 | 1740000 | 1720000 | 1666300 | 1594260 | 1594236 | 1590500 | 1379000 | 1378400 | 1378100 | 1377825 | 1377500 | 1377280 | 1377230 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| USGS | 9.0535 | 9.0575 | 9.0595 | 9.0645 | 9.0780 | 9.0785 | 9.0850 | 9.1090 | 9.1100 | 9.1125 | 9.1135 | 9.1145 | 9.1155 | 9.1180 |
| Year | | | | | | | | | | | | | | |
| 1938 | 1553.47 | 1425.54 | 266.05 | 201.44 | 535.55 | 67.04 | 3970.41 | 431.36 | 787.47 | 909.49 | 196.74 | 1527.83 | 142.26 | 147.15 |
| 1939 | 684.33 | 1141.80 | 226.68 | 203.33 | 354.16 | 142.06 | 2339.75 | 557.54 | 544.07 | 574.20 | 177.54 | 1514.84 | 111.77 | 92.95 |
| 1940 | 766.92 | 809.29 | 2.00 | 149.42 | 260.23 | 106.91 | 1747.83 | 213.23 | 302.13 | 427.80 | 162.24 | 1026.65 | 55.15 | 48.25 |
| 1941 | 1025.03 | 1020.70 | 184.60 | 178.80 | 233.90 | 145.17 | 2740.32 | 382.29 | 592.46 | 822.97 | 252.74 | 21.26 | 157.12 | 129.47 |
| 1942 | 1398.12 | 1191.44 | 100.61 | 197.32 | 243.19 | 114.37 | 3125.42 | 470.89 | 764.47 | 754.52 | 229.56 | 3676.62 | 211.14 | 193.06 |
| 1943 | 619.19 | 1234.40 | 61.42 | 209.79 | 306.11 | 164.67 | 2943.42 | 451.96 | 720.99 | 816.08 | 272.66 | 1167.75 | 148.61 | 142.62 |
| 1944 | 807.17 | 973.33 | 230.99 | 199.29 | 277.45 | 164.86 | 2897.58 | 372.48 | 587.09 | 774.26 | 274.26 | 1709.32 | 147.20 | 90.74 |
| 1945 | 860.43 | 1050.51 | 194.62 | 193.80 | 239.87 | 139.00 | 2763.15 | 335.35 | 490.27 | 700.26 | 220.31 | 1337.85 | 112.58 | 88.16 |
| 1946 | 867.13 | 983.65 | 109.00 | 199.61 | 342.19 | 121.38 | 2476.55 | 326.24 | 490.30 | 632.29 | 157.63 | 1143.95 | 60.34 | 61.08 |
| 1947 | 1322.76 | 1516.34 | 246.24 | 256.92 | 408.32 | 139.61 | 3720.75 | 501.64 | 755.42 | 902.67 | 259.66 | 1846.62 | 147.52 | 136.91 |
| 1948 | 1076.73 | 1238.84 | 218.43 | 222.78 | 340.15 | 130.56 | 3399.91 | 525.59 | 802.91 | 941.12 | 513.44 | 2022.60 | 153.99 | 128.68 |
| 1949 | 1133.94 | 1322.54 | 209.23 | 246.43 | 362.99 | 157.57 | 3093.04 | 472.44 | 758.16 | 824.58 | 265.90 | 1782.76 | 177.18 | 139.93 |
| 1950 | 926.02 | 1124.04 | 156.12 | 202.52 | 277.07 | 103.50 | 2489.03 | 354.73 | 343.50 | 729.29 | 216.72 | 1350.09 | 73.22 | 84.37 |
| 1951 | 1377.80 | 1558.42 | 200.49 | 265.24 | 363.02 | 148.72 | 2777.03 | 397.45 | 609.77 | 781.07 | 210.68 | 1436.51 | 120.06 | 86.09 |
| 1952 | 1298.34 | 1349.32 | 270.03 | 269.57 | 392.33 | 176.25 | 4112.90 | 629.16 | 1056.48 | 1228.04 | 381.88 | 2580.67 | 223.94 | 178.77 |
| 1953 | 982.09 | 1156.54 | 156.10 | 207.93 | 285.07 | 138.78 | 2476.33 | 378.85 | 609.77 | 648.70 | 110.59 | 1345.06 | 160.10 | 143.88 |
| 1954 | 435.33 | 516.35 | 68.95 | 108.61 | 156.27 | 60.74 | 1268.37 | 286.50 | 415.92 | 371.21 | 160.29 | 712.17 | 33.35 | 69.57 |
| 1955 | 595.77 | 726.05 | 153.83 | 157.44 | 233.58 | 93.63 | 1950.25 | 321.17 | 471.66 | 538.06 | 122.31 | 1027.53 | 53.45 | 100.22 |
| 1956 | 1019.09 | 1226.45 | 170.76 | 230.74 | 283.36 | 133.35 | 2384.36 | 412.39 | 620.21 | 696.76 | 218.34 | 1402.35 | 98.03 | 101.33 |
| 1957 | 1339.02 | 1586.86 | 290.43 | 323.31 | 477.71 | 246.11 | 5117.86 | 765.81 | 1308.94 | 1415.11 | 554.87 | 3221.64 | 278.21 | 216.46 |
| 1958 | 968.24 | 1167.64 | 229.26 | 200.47 | 298.62 | 131.64 | 2725.42 | 365.70 | 668.05 | 861.16 | 266.45 | 1704.05 | 189.82 | 141.25 |
| 1959 | 805.33 | 975.27 | 169.04 | 189.39 | 289.58 | 109.71 | 2287.30 | 290.53 | 419.22 | 492.07 | 144.33 | 940.06 | 74.47 | 93.10 |
| 1960 | 865.45 | 1029.95 | 182.68 | 200.25 | 285.94 | 130.83 | 2406.42 | 418.06 | 635.63 | 585.98 | 177.17 | 1278.18 | 121.16 | 103.95 |
| 1961 | 599.61 | 727.52 | 111.20 | 139.84 | 209.11 | 83.65 | 1768.37 | 294.09 | 436.30 | 477.43 | 129.08 | 913.05 | 90.66 | 68.80 |
| 1962 | 1084.42 | 1346.70 | 291.43 | 241.40 | 374.65 | 162.43 | 5686.80 | 597.80 | 982.49 | 1048.02 | 506.37 | 2545.79 | 163.62 | 148.87 |
| 1963 | 422.88 | 535.25 | 84.07 | 151.78 | 178.74 | 85.67 | 1413.14 | 253.52 | 355.39 | 417.41 | 81.39 | 791.08 | 57.07 | 53.86 |
| 1964 | 577.11 | 396.76 | 138.68 | 171.43 | 256.81 | 100.13 | 3340.17 | 298.35 | 473.43 | 563.00 | 136.51 | 1108.08 | 123.24 | 86.33 |
| 1965 | 1259.69 | 653.70 | 261.83 | 286.15 | 399.81 | 173.93 | 3707.54 | 598.89 | 1000.70 | 918.40 | 350.96 | 2574.95 | 193.54 | 168.81 |
| 1966 | 427.33 | 520.84 | 121.10 | 110.80 | 202.57 | 77.59 | 1833.54 | 317.30 | 483.67 | 524.86 | 134.20 | 1034.07 | 66.46 | 74.56 |
| 1967 | 624.37 | 588.06 | 162.84 | 59.72 | 263.00 | 107.42 | 2320.36 | 381.22 | 562.37 | 708.26 | 154.24 | 1308.58 | 44.29 | 78.78 |

TABLE 10 - CN-6 SERIES (CFS)

| CSU | 1742100 | 1740000 | 1720000 | 1666300 | 1594260 | 1594236 | 1590500 | 1379000 | 1378400 | 1378100 | 1377825 | 1377500 | 1377280 | 1377230 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| USGS | 9.0535 | 9.0575 | 9.0595 | 9.0645 | 9.0780 | 9.0785 | 9.0850 | 9.1090 | 9.1100 | 9.1125 | 9.1135 | 9.1145 | 9.1155 | 9.1180 |
| Year | | | | | | | | | | | | | | |
| 1938 | 1156.86 | 1054.07 | 184.23 | 140.66 | 238.33 | 44.61 | 2979.64 | 316.79 | 583.73 | 655.26 | 139.63 | 963.01 | 103.19 | 108.20 |
| 1939 | 512.72 | 830.52 | 156.10 | 141.35 | 254.00 | 98.26 | 1609.53 | 264.92 | 408.91 | 416.61 | 127.55 | 1112.28 | 82.40 | 71.45 |
| 1940 | 576.41 | 604.49 | .46 | 103.25 | 186.31 | 74.01 | 1275.78 | 164.25 | 236.45 | 310.62 | 74.91 | 769.56 | 43.81 | 38.15 |
| 1941 | 766.62 | 757.80 | 127.32 | 124.66 | 164.67 | 99.70 | 1998.73 | 283.64 | 440.50 | 588.59 | 176.72 | 15.10 | 113.88 | 96.58 |
| 1942 | 1039.35 | 876.05 | 67.08 | 136.77 | 171.51 | 77.86 | 2253.84 | 350.01 | 565.74 | 539.18 | 161.07 | 2665.26 | 150.62 | 140.25 |
| 1943 | 492.25 | 933.16 | 47.19 | 151.81 | 225.05 | 136.98 | 2244.66 | 347.56 | 556.40 | 599.26 | 200.14 | 902.68 | 109.85 | 108.00 |
| 1944 | 595.34 | 711.86 | 172.50 | 138.46 | 198.77 | 114.46 | 2098.35 | 277.19 | 441.90 | 555.34 | 192.79 | 1248.25 | 106.73 | 68.86 |
| 1945 | 726.18 | 856.90 | 138.97 | 148.88 | 179.62 | 100.35 | 2157.63 | 262.68 | 397.07 | 521.22 | 164.32 | 1046.98 | 85.38 | 72.17 |
| 1946 | 660.28 | 735.90 | 119.43 | 138.42 | 245.65 | 83.88 | 1831.32 | 242.16 | 371.93 | 456.40 | 112.69 | 858.04 | 48.21 | 48.99 |
| 1947 | 1007.11 | 1145.85 | 171.30 | 182.06 | 295.19 | 97.25 | 2777.15 | 373.55 | 568.91 | 654.97 | 187.60 | 1366.27 | 110.36 | 101.76 |
| 1948 | 789.57 | 898.78 | 151.99 | 153.36 | 240.23 | 89.65 | 2488.47 | 387.42 | 591.17 | 669.06 | 218.91 | 1454.97 | 112.83 | 93.41 |
| 1949 | 847.36 | 958.77 | 146.09 | 171.20 | 259.18 | 108.31 | 2266.56 | 349.87 | 562.02 | 592.65 | 187.88 | 1296.83 | 130.97 | 104.72 |
| 1950 | 679.63 | 824.37 | 108.06 | 138.51 | 194.34 | 70.00 | 1810.28 | 261.24 | 401.21 | 317.90 | 181.53 | 977.46 | 55.06 | 63.45 |
| 1951 | 1051.02 | 1184.87 | 140.74 | 188.25 | 261.82 | 102.98 | 2090.26 | 300.69 | 465.77 | 574.60 | 153.15 | 1083.94 | 91.09 | 67.33 |
| 1952 | 972.44 | 1164.23 | 187.30 | 192.30 | 284.64 | 123.11 | 3045.42 | 466.85 | 778.99 | 879.92 | 268.12 | 1874.68 | 161.78 | 151.00 |
| 1953 | 744.67 | 883.46 | 111.51 | 147.93 | 212.46 | 99.23 | 1868.67 | 287.46 | 464.68 | 475.36 | 82.38 | 1009.12 | 117.86 | 107.21 |
| 1954 | 340.53 | 406.60 | 48.77 | 75.41 | 113.46 | 42.56 | 968.70 | 209.69 | 318.18 | 276.42 | 137.02 | 549.98 | 29.41 | 55.67 |
| 1955 | 492.35 | 592.03 | 94.67 | 113.92 | 171.63 | 66.29 | 1487.11 | 249.92 | 372.09 | 399.65 | 91.53 | 797.95 | 42.97 | 76.91 |
| 1956 | 753.20 | 898.64 | 118.91 | 158.60 | 199.60 | 91.55 | 1725.41 | 305.53 | 459.60 | 492.50 | 152.71 | 1021.19 | 72.53 | 76.05 |
| 1957 | 1046.28 | 1233.56 | 206.67 | 253.52 | 354.67 | 172.16 | 3812.55 | 589.23 | 999.55 | 1051.31 | 398.44 | 2421.41 | 209.08 | 162.65 |
| 1958 | 705.37 | 848.10 | 156.58 | 137.47 | 211.06 | 89.96 | 1973.10 | 275.83 | 501.10 | 611.32 | 186.40 | 1233.51 | 135.40 | 103.21 |
| 1959 | 617.20 | 743.62 | 118.89 | 130.26 | 203.00 | 75.55 | 1993.25 | 220.54 | 324.16 | 361.54 | 104.17 | 713.27 | 59.03 | 72.53 |
| 1960 | 647.29 | 771.17 | 126.52 | 139.80 | 202.69 | 90.29 | 1771.79 | 310.01 | 475.38 | 423.15 | 125.97 | 944.26 | 92.17 | 77.19 |
| 1961 | 489.68 | 591.51 | 79.11 | 98.23 | 151.59 | 57.94 | 1320.01 | 218.30 | 336.60 | 345.45 | 92.44 | 684.65 | 69.14 | 54.36 |
| 1962 | 804.98 | 997.67 | 202.09 | 169.55 | 265.87 | 112.78 | 2692.93 | 437.68 | 725.61 | 747.02 | 213.08 | 1694.99 | 118.03 | 108.55 |
| 1963 | 363.95 | 459.21 | 61.81 | 111.42 | 136.12 | 61.80 | 1123.25 | 202.44 | 285.55 | 307.42 | 62.05 | 630.50 | 48.55 | 44.90 |
| 1964 | 463.43 | 354.04 | 97.06 | 123.74 | 192.34 | 71.12 | 1994.92 | 232.22 | 370.13 | 410.33 | 99.95 | 858.04 | 81.40 | 66.98 |
| 1965 | 981.95 | 605.49 | 188.31 | 211.24 | 298.97 | 125.11 | 2797.98 | 463.25 | 769.46 | 684.62 | 251.77 | 1925.98 | 143.61 | 131.49 |
| 1966 | 332.45 | 410.52 | 86.49 | 81.82 | 149.03 | 55.27 | 1390.00 | 243.24 | 376.24 | 383.67 | 97.05 | 782.29 | 53.50 | 59.98 |
| 1967 | 455.68 | 467.94 | 114.39 | 48.23 | 188.83 | 75.57 | 1736.02 | 285.94 | 432.59 | 519.56 | 116.15 | 996.10 | 36.49 | 61.74 |

TABLE 11 - S-4 SERIES (CFS)

| CSU | 1278800 | 1278050 | 1272445 | 1077090 | 1073480 | 1073436 |
|---|---|---|---|---|---|---|
| USGS | 9.1650 | 9.1665 | 9.1725 | 9.3440 | 9.3575 | 9.3615 |
| Year | | | | | | |
| 1938 | 576.81 | 1562.90 | 575.59 | 325.62 | 327.16 | 2336.76 |
| 1939 | 58.94 | 606.58 | 425.55 | 182.92 | 209.50 | 1098.61 |
| 1940 | 84.90 | 745.99 | 298.12 | 164.90 | 197.06 | 1028.62 |
| 1941 | 632.30 | 1851.80 | 1088.38 | 478.13 | 336.10 | 3077.72 |
| 1942 | 316.67 | 1715.27 | 530.48 | 338.20 | 304.86 | 2230.25 |
| 1943 | 449.01 | 1137.48 | 422.13 | 213.89 | 224.83 | 1595.65 |
| 1944 | 424.04 | 1639.32 | 759.53 | 311.61 | 308.98 | 2621.76 |
| 1945 | 540.66 | 1162.65 | 509.43 | 289.46 | 232.80 | 1706.87 |
| 1946 | 110.87 | 714.89 | 408.29 | 127.24 | 215.96 | 1217.53 |
| 1947 | 329.00 | 1042.36 | 517.55 | 176.18 | 291.08 | 1816.22 |
| 1948 | 358.02 | 1347.54 | 726.76 | 260.75 | 337.26 | 2426.52 |
| 1949 | 456.36 | 1380.07 | 638.53 | 290.50 | 340.17 | 2677.44 |
| 1950 | 92.95 | 793.92 | 334.92 | 148.56 | 180.39 | 1206.87 |
| 1951 | 203.16 | 462.49 | 260.02 | 127.78 | 185.28 | 928.70 |
| 1952 | 563.69 | 1859.43 | 697.05 | 380.26 | 565.02 | 2825.00 |
| 1953 | 241.09 | 647.71 | 397.84 | 169.55 | 180.92 | 1152.80 |
| 1954 | 194.39 | 494.48 | 258.19 | 144.24 | 162.09 | 1055.70 |
| 1955 | 232.12 | 638.44 | 349.38 | 149.37 | 164.89 | 1118.89 |
| 1956 | 248.65 | 648.34 | 347.71 | 161.12 | 205.02 | 1146.48 |
| 1957 | 568.05 | 1769.26 | 765.01 | 379.73 | 563.13 | 2551.41 |
| 1958 | 506.66 | 1494.09 | 888.34 | 310.55 | 285.13 | 2401.89 |
| 1959 | 143.14 | 352.93 | 282.65 | 123.73 | 176.30 | 861.30 |
| 1960 | 337.76 | 1020.05 | 316.91 | 253.53 | 248.82 | 1735.15 |
| 1961 | 265.36 | 770.40 | 482.98 | 187.66 | 208.59 | 1372.06 |
| 1962 | 561.31 | 1059.16 | 511.90 | 246.10 | 273.25 | 1855.92 |
| 1963 | 190.51 | 522.54 | 270.80 | 129.95 | 165.65 | 957.37 |
| 1964 | 230.50 | 630.49 | 398.27 | 142.47 | 175.87 | 1013.54 |
| 1965 | 503.80 | 1387.75 | 669.28 | 325.56 | 323.32 | 2487.34 |
| 1966 | 269.79 | 852.00 | 364.29 | 219.23 | 216.47 | 1419.96 |
| 1967 | 188.41 | 510.44 | 285.76 | 170.36 | 165.16 | 973.56 |

TABLE 12 - S-6 SERIES (CFS)

| CSU | 1278800 | 1278050 | 1272445 | 1077090 | 1073480 | 1073436 |
|---|---|---|---|---|---|---|
| USGS | 9.1650 | 9.1665 | 9.1725 | 9.3440 | 8.3575 | 8.3615 |
| Year | | | | | | |
| 1938 | 264.70 | 1087.32 | 428.99 | 233.04 | 234.72 | 1679.61 |
| 1939 | 32.79 | 441.02 | 323.29 | 136.78 | 149.07 | 832.04 |
| 1940 | 64.40 | 535.57 | 252.45 | 126.39 | 138.39 | 760.03 |
| 1941 | 438.72 | 1294.88 | 799.02 | 344.08 | 247.28 | 2228.01 |
| 1942 | 228.65 | 1207.55 | 401.54 | 243.35 | 220.95 | 1618.76 |
| 1943 | 323.53 | 820.58 | 351.01 | 158.26 | 172.10 | 1237.58 |
| 1944 | 298.37 | 1142.23 | 555.73 | 221.03 | 222.45 | 1864.03 |
| 1945 | 385.68 | 826.26 | 402.87 | 208.82 | 171.12 | 1270.55 |
| 1946 | 85.90 | 524.96 | 320.06 | 97.90 | 134.90 | 915.27 |
| 1947 | 243.36 | 767.28 | 410.77 | 137.72 | 215.76 | 1396.48 |
| 1948 | 254.48 | 948.10 | 530.04 | 185.66 | 241.83 | 1749.94 |
| 1949 | 323.65 | 972.30 | 469.79 | 211.16 | 242.48 | 1905.66 |
| 1950 | 70.94 | 571.55 | 265.88 | 109.55 | 128.96 | 886.80 |
| 1951 | 145.70 | 336.17 | 201.96 | 96.99 | 134.88 | 707.76 |
| 1952 | 389.84 | 1288.06 | 520.53 | 272.82 | 264.68 | 2010.24 |
| 1953 | 175.03 | 479.38 | 304.84 | 126.15 | 131.97 | 869.32 |
| 1954 | 136.63 | 361.00 | 204.30 | 108.44 | 117.40 | 788.60 |
| 1955 | 172.42 | 485.31 | 270.27 | 114.10 | 121.38 | 857.79 |
| 1956 | 176.90 | 494.15 | 258.79 | 123.56 | 144.73 | 853.34 |
| 1957 | 422.62 | 1291.87 | 587.51 | 285.37 | 273.42 | 1939.02 |
| 1958 | 349.47 | 1046.41 | 628.61 | 221.47 | 203.19 | 1701.05 |
| 1959 | 111.50 | 268.50 | 255.53 | 95.02 | 129.51 | 671.83 |
| 1960 | 237.14 | 723.01 | 376.56 | 182.67 | 175.81 | 1265.89 |
| 1961 | 189.47 | 555.96 | 361.68 | 141.88 | 149.11 | 1024.97 |
| 1962 | 261.22 | 754.19 | 378.03 | 176.30 | 196.04 | 1348.15 |
| 1963 | 144.45 | 405.77 | 129.42 | 103.45 | 122.82 | 764.80 |
| 1964 | 172.04 | 474.99 | 305.47 | 114.14 | 127.19 | 777.66 |
| 1965 | 367.10 | 1007.67 | 517.49 | 234.53 | 241.37 | 1863.86 |
| 1966 | 194.47 | 633.65 | 277.44 | 163.20 | 154.73 | 1080.99 |
| 1967 | 143.22 | 406.01 | 235.85 | 140.37 | 121.59 | 771.56 |

TABLE 13 - CS-4 SERIES (CFS)

| CSU | 1425625 | 1377280 | 1377230 | 1377200 | 1373900 | 1373055 | 1373020 | 1371815 | 1370300 |
|---|---|---|---|---|---|---|---|---|---|
| USGS | 9.0975 | 9.1155 | 9.1180 | 9.1190 | 9.1275 | 9.1325 | 9.1345 | 9.1430 | 9.1520 |
| Year | | | | | | | | | |
| 1938 | 211.98 | 142.26 | 147.15 | 405.19 | 114.53 | 1642.21 | 205.27 | 90.56 | 111.74 |
| 1939 | 74.54 | 111.77 | 92.95 | 262.73 | 72.35 | 894.18 | 100.73 | 65.04 | 67.78 |
| 1940 | 65.77 | 55.15 | 48.25 | 89.25 | 105.32 | 841.89 | 98.93 | 75.51 | 97.24 |
| 1941 | 182.76 | 157.12 | 129.47 | 416.78 | 86.48 | 1395.74 | 175.09 | 132.19 | 148.55 |
| 1942 | 289.53 | 211.14 | 193.06 | 647.95 | 152.50 | 1579.42 | 172.69 | 125.79 | 143.21 |
| 1943 | 73.88 | 148.61 | 142.62 | 363.90 | 75.68 | 1059.44 | 111.46 | 75.26 | 60.63 |
| 1944 | 185.98 | 147.20 | 90.74 | 412.16 | 120.69 | 1480.44 | 139.55 | 116.49 | 132.25 |
| 1945 | 130.83 | 112.58 | 38.16 | 361.20 | 52.12 | 1249.94 | 126.28 | 101.08 | 87.60 |
| 1946 | 92.40 | 60.34 | 61.08 | 124.30 | 60.75 | 847.02 | 67.73 | 55.91 | 56.20 |
| 1947 | 136.23 | 147.52 | 136.91 | 401.64 | 75.77 | 1194.08 | 136.04 | 113.53 | 81.28 |
| 1948 | 170.25 | 153.99 | 128.68 | 485.11 | 115.13 | 1392.71 | 118.72 | 97.64 | 92.22 |
| 1949 | 134.23 | 177.18 | 130.93 | 182.28 | 86.42 | 1239.87 | 142.84 | 95.66 | 96.21 |
| 1950 | 80.95 | 73.22 | 84.37 | 189.60 | 57.45 | 1222.23 | 119.98 | 84.06 | 53.78 |
| 1951 | 42.20 | 120.06 | 86.09 | 239.52 | 48.28 | 900.74 | 83.58 | 62.22 | 44.54 |
| 1952 | 181.21 | 223.94 | 178.77 | 636.87 | 120.89 | 1760.47 | 195.21 | 166.94 | 112.23 |
| 1953 | 74.08 | 160.10 | 143.88 | 330.64 | 77.19 | 857.61 | 74.72 | 64.32 | 56.70 |
| 1954 | 34.92 | 33.55 | 69.57 | 68.52 | 21.66 | 445.60 | 70.88 | 57.12 | 43.29 |
| 1955 | 59.96 | 53.45 | 100.22 | 114.93 | 39.45 | 619.78 | 103.11 | 85.36 | 61.89 |
| 1956 | 47.87 | 98.93 | 101.55 | 227.28 | 51.96 | 864.61 | 58.79 | 61.85 | 42.92 |
| 1957 | 221.73 | 278.21 | 216.48 | 800.97 | 147.08 | 2200.99 | 259.23 | 128.75 | 128.55 |
| 1958 | 185.21 | 189.82 | 141.25 | 496.77 | 139.64 | 1297.41 | 136.86 | 132.44 | 125.35 |
| 1959 | 45.48 | 74.47 | 93.10 | 130.29 | 66.34 | 660.52 | 129.36 | 52.46 | 42.69 |
| 1960 | 86.97 | 122.16 | 103.95 | 271.42 | 45.57 | 882.82 | 172.63 | 71.83 | 66.45 |
| 1961 | 65.26 | 90.66 | 68.80 | 150.39 | 46.03 | 651.57 | 66.94 | 69.55 | 43.75 |
| 1962 | 190.55 | 163.62 | 148.87 | 407.72 | 95.34 | 1651.73 | 161.40 | 111.98 | 76.97 |
| 1963 | 23.34 | 57.07 | 55.86 | 107.67 | 30.68 | 558.43 | 66.95 | 49.42 | 19.60 |
| 1964 | 106.35 | 123.24 | 86.53 | 270.83 | 77.64 | 989.99 | 95.81 | 61.17 | 49.85 |
| 1965 | 142.90 | 195.54 | 368.81 | 524.61 | 112.12 | 1573.51 | 131.16 | 97.77 | 82.81 |
| 1966 | 62.55 | 66.46 | 74.56 | 156.68 | 56.62 | 739.28 | 85.61 | 84.05 | 45.48 |
| 1967 | 41.83 | 44.29 | 78.78 | 104.93 | 30.83 | 829.28 | 105.95 | 71.72 | 29.71 |

TABLE 14 - CS-6 SERIES (CFS)

| CSU | 1425625 | 1377280 | 1377230 | 1377200 | 1373900 | 1373055 | 1373020 | 1371815 | 1370300 |
|---|---|---|---|---|---|---|---|---|---|
| USGS | 9.0975 | 9.1155 | 9.1180 | 9.1190 | 9.1275 | 9.1325 | 9.1345 | 9.1430 | 9.1520 |
| Year | | | | | | | | | |
| 1938 | 144.14 | 105.19 | 108.20 | 382.53 | 78.66 | 1136.71 | 142.02 | 69.48 | 80.24 |
| 1939 | 53.15 | 82.40 | 71.45 | 238.90 | 52.07 | 636.67 | 70.95 | 50.37 | 48.77 |
| 1940 | 46.75 | 45.61 | 39.15 | 91.35 | 72.57 | 582.49 | 69.85 | 56.37 | 60.68 |
| 1941 | 123.01 | 113.88 | 96.58 | 326.14 | 60.40 | 968.91 | 122.12 | 97.97 | 105.32 |
| 1942 | 194.53 | 150.62 | 140.25 | 474.42 | 103.08 | 1090.52 | 119.28 | 92.60 | 101.63 |
| 1943 | 54.77 | 209.85 | 108.00 | 291.91 | 53.81 | 758.22 | 78.86 | 59.76 | 45.54 |
| 1944 | 124.48 | 206.75 | 68.86 | 308.07 | 81.75 | 1018.14 | 97.68 | 89.37 | 96.08 |
| 1945 | 90.12 | 85.38 | 72.17 | 229.69 | 24.91 | 884.45 | 90.13 | 80.43 | 64.59 |
| 1946 | 64.12 | 48.21 | 48.99 | 123.56 | 43.47 | 600.70 | 49.37 | 45.81 | 42.73 |
| 1947 | 92.65 | 110.36 | 101.76 | 320.93 | 55.40 | 846.48 | 96.55 | 85.87 | 60.44 |
| 1948 | 115.45 | 112.35 | 95.41 | 365.62 | 79.24 | 966.05 | 84.57 | 73.55 | 66.79 |
| 1949 | 80.92 | 130.97 | 104.72 | 435.31 | 60.49 | 660.60 | 101.68 | 75.39 | 71.48 |
| 1950 | 55.62 | 55.06 | 63.45 | 152.94 | 40.33 | 843.82 | 86.45 | 67.46 | 40.53 |
| 1951 | 29.41 | 91.08 | 67.35 | 106.46 | 34.44 | 634.29 | 61.60 | 50.05 | 34.28 |
| 1952 | 122.63 | 161.78 | 131.00 | 468.72 | 82.59 | 1320.74 | 136.59 | 122.21 | 80.65 |
| 1953 | 51.39 | 117.86 | 107.21 | 271.75 | 54.44 | 613.11 | 55.97 | 50.46 | 42.77 |
| 1954 | 24.46 | 29.41 | 55.67 | 71.27 | 15.99 | 523.08 | 52.32 | 45.07 | 33.25 |
| 1955 | 40.93 | 42.97 | 76.91 | 109.68 | 28.76 | 584.86 | 74.74 | 66.55 | 47.04 |
| 1956 | 33.63 | 72.53 | 76.05 | 174.49 | 37.67 | 615.69 | 64.30 | 48.27 | 33.49 |
| 1957 | 149.42 | 209.08 | 162.35 | 619.51 | 100.67 | 1553.36 | 181.89 | 100.35 | 92.67 |
| 1958 | 126.36 | 135.40 | 103.21 | 365.13 | 94.29 | 900.00 | 82.60 | 99.83 | 90.44 |
| 1959 | 31.24 | 59.03 | 72.53 | 123.35 | 46.51 | 468.01 | 91.14 | 42.57 | 33.60 |
| 1960 | 60.51 | 92.17 | 77.19 | 219.38 | 33.19 | 623.26 | 120.63 | 57.48 | 49.85 |
| 1961 | 37.06 | 69.14 | 54.38 | 133.32 | 32.17 | 460.72 | 50.14 | 47.68 | 30.89 |
| 1962 | 129.10 | 118.03 | 108.33 | 304.76 | 65.28 | 1146.48 | 114.67 | 85.67 | 54.38 |
| 1963 | 20.40 | 48.55 | 44.90 | 114.68 | 28.22 | 405.58 | 48.99 | 38.41 | 15.95 |
| 1964 | 72.26 | 91.40 | 66.98 | 217.55 | 53.85 | 693.73 | 68.87 | 49.22 | 35.18 |
| 1965 | 96.84 | 143.61 | 131.49 | 419.56 | 77.91 | 1102.29 | 94.71 | 76.68 | 59.73 |
| 1966 | 46.73 | 53.50 | 59.98 | 143.56 | 41.21 | 538.26 | 62.46 | 65.36 | 32.40 |
| 1967 | 51.96 | 56.49 | 61.74 | 101.06 | 23.33 | 615.85 | 76.43 | 58.01 | 21.54 |

TABLE 15 - STATIONS WITH MISSING DATA

| Station with missing data | | Filling in of missing data is made with station | | Year of missing data |
|---|---|---|---|---|
| CSU Sta. No. | USGS Sta. No. | CSU Sta. No. | USGS Sta. No. | |
| 1820000 | 9.0195 | 1830000 | 9.0190 | 54-60 |
| 1830000 | 9.0190 | 1960000 | 9.0110 | 38-50 |
| 1970000 | 9.0105 | 1960000 | 9.0110 | 38-53 |
| 1272445 | 9.1725 | 1277200 | 9.1665 | 38-42 |
| 1278800 | 9.1650 | 1277200 | 9.1665 | 38-51 |
| 1371815 | 9.1430 | 1370300 | 9.1520 | 38-39 |
| 1373020 | 9.1345 | 1373055 | 9.1325 | 57-60 |
| 1373900 | 9.1275 | 1373360 | 9.1285 | 38-45; 55-60 |
| 1377230 | 9.1180 | 1377280 | 9.1155 | 51-60 |
| 1377825 | 9.1135 | 1378100 | 9.1125 | 38-40; 51-58 |
| 1594236 | 9.0785 | 1378400 | 9.1100 | 38-47 |
| 1720000 | 9.0595 | 1590000 | 9.0850 | 38-44 |
| 1377500 | 9.1145 | 1378400 | 9.1100 | 38-44 |
| 1379000 | 9.1090 | 1378400 | 9.1100 | 38 |
| 1594260 | 9.0780 | 1378400 | 9.1100 | 38-47 |

Fig. 5  N-4 series



N-4  CSU STA. 1970000  USGS STA. 9.0105
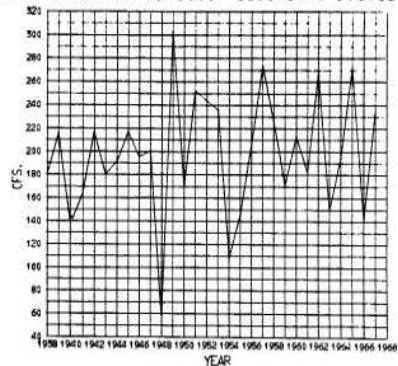


N-4  CSU STA. 1960000  USGS STA. 9.0110
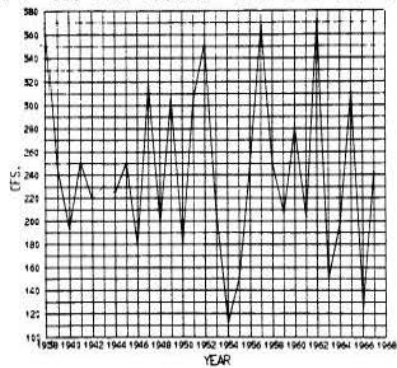


N-4  CSU STA. 1866000  USGS STA. 9.0165
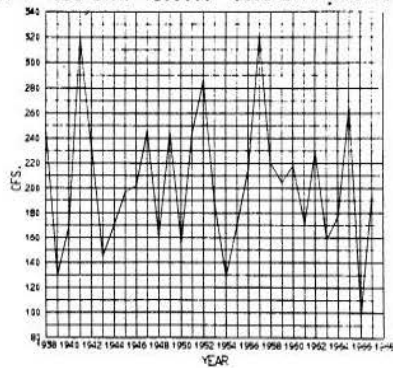


N-4  CSU STA. 1830000  USGS STA. 9.0190
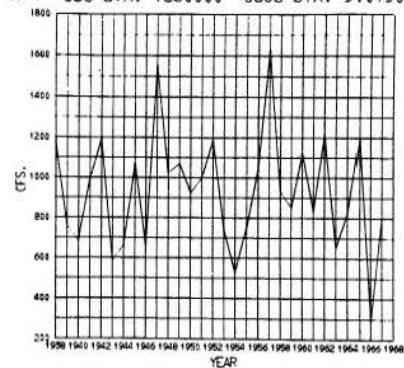


N-4  CSU STA. 1820000  USGS STA. 9.0195
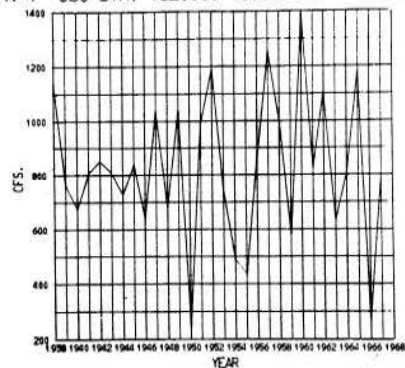


N-4  CSU STA. 1802730  USGS STA. 9.0265
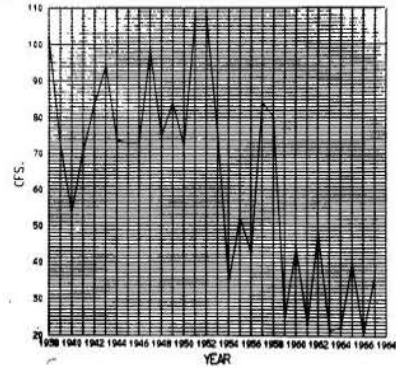


N-4  CSU STA. 1776000  USGS STA. 9.0360

17
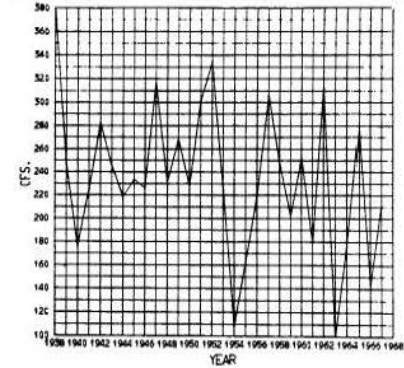
Fig. 6 N-6 series

N-6 CSU STA. 1970000 USGS STA. 9.0105
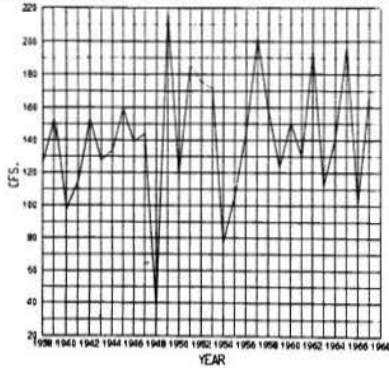
N-6 CSU STA. 1960000 USGS STA. 9.0110
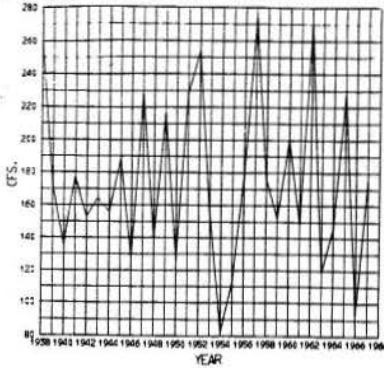
N-6 CSU STA. 1866000 USGS STA. 9.0165
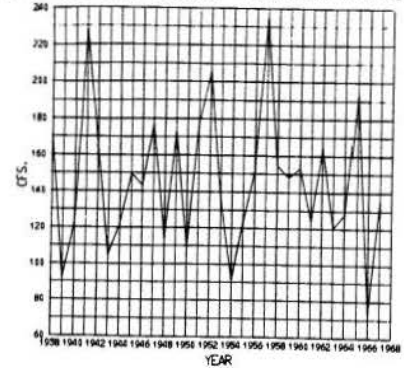
N-6 CSU STA. 1830000 USGS STA. 9.0190
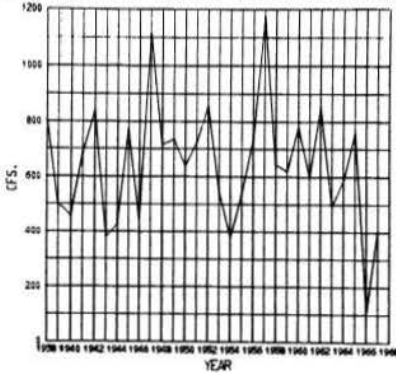
N-6 CSU STA. 1820000 USGS STA. 9.0195
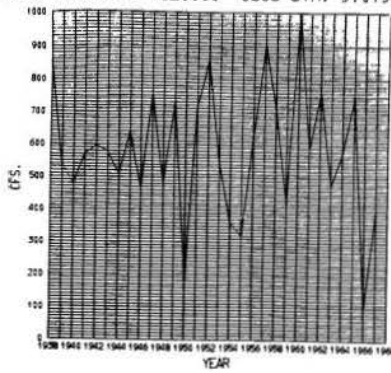
N-6 CSU STA. 1802730 USGS STA. 9.0265
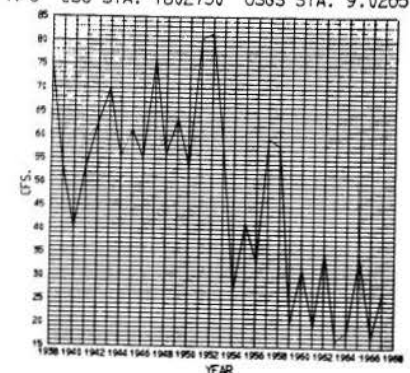
N-6 CSU STA. 1776000 USGS STA. 9.0360
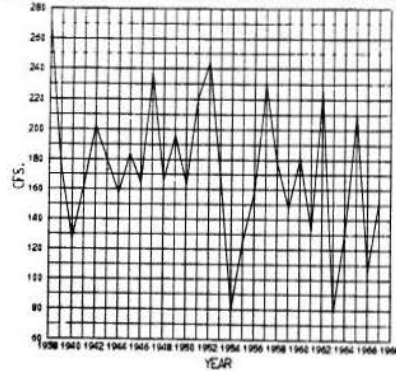
Fig. 7 CN-4 series

CN-4 CSU STA. 1742100 USGS STA. 9.0535
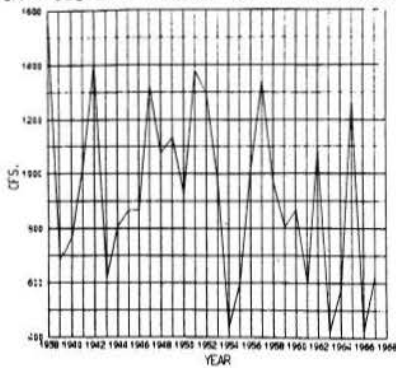
CN-4 CSU STA. 1740000 USGS STA. 9.0575
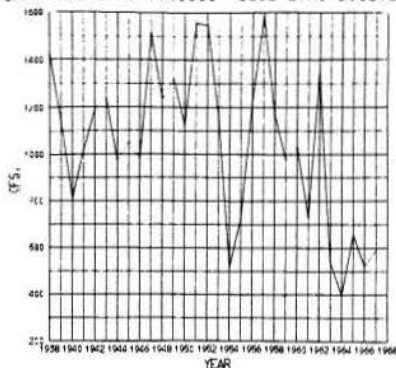
CN-4 CSU STA. 1720000 USGS STA. 9.0595

18
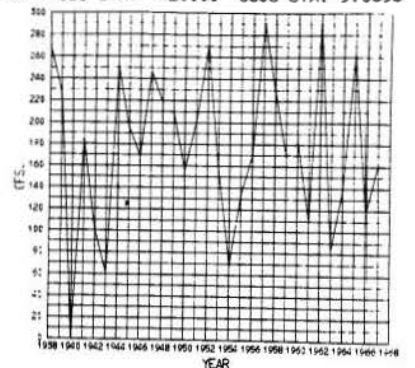
Fig. 7  CN-4 series - Continued

CN-4 CSU STA. 1666300  USGS STA. 9.0645
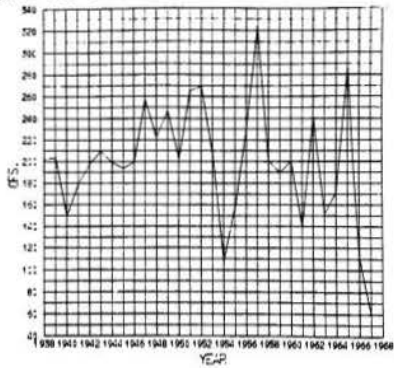
CN-4 CSU STA. 1594260  USGS STA. 9.0780
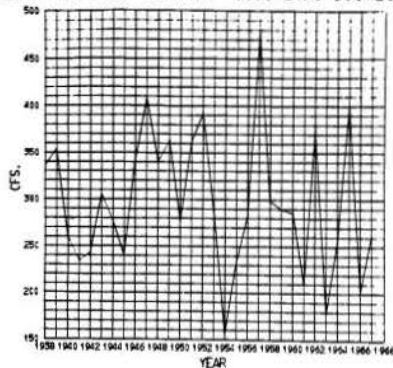
CN-4 CSU STA. 1594236  USGS STA. 9.0785
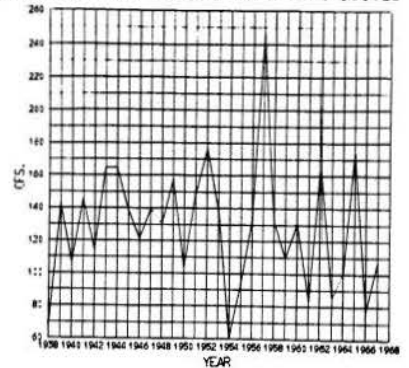
CN-4 CSU STA. 1590000  USGS STA. 9.0850
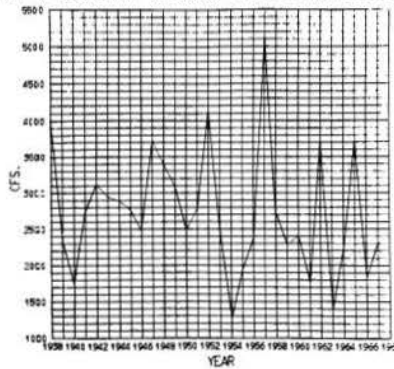
CN-4 CSU STA. 1379000  USGS STA. 9.1090
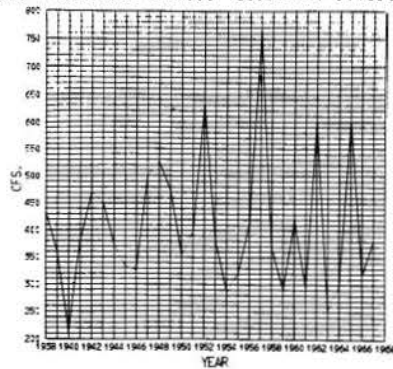
CN-4 CSU STA. 1579400  USGS STA. 9.1100
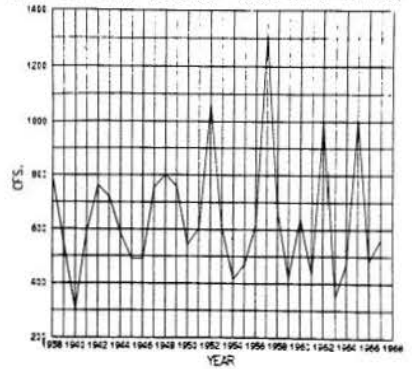
CN-4 CSU STA. 1378100  USGS STA. 9.1125
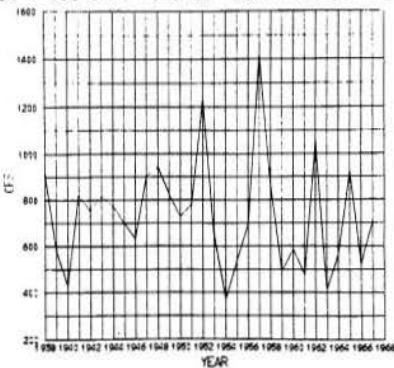
CN-4 CSU STA. 1377825  USGS STA. 9.1135
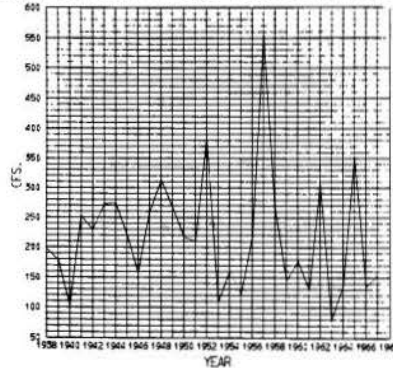
CN-4 CSU STA. 1377500  USGS STA. 9.1145
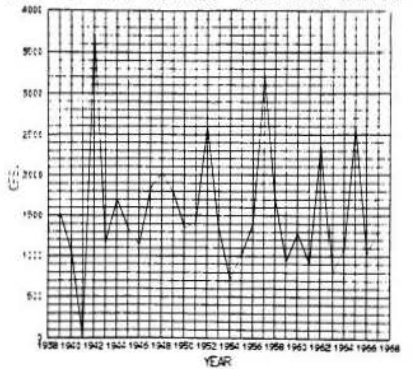
CN-4 CSU STA. 1377280  USGS STA. 9.1155
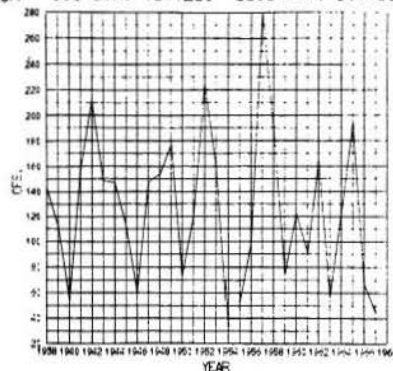
CN-4 CSU STA. 1377230  USGS STA. 9.1180

19
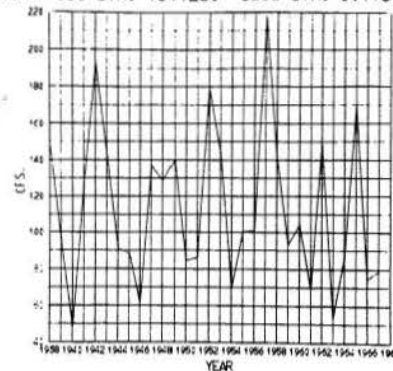
Fig. 8  CN-6 series

CN-6 CSU STA. 1742100  USGS STA. 9.0535
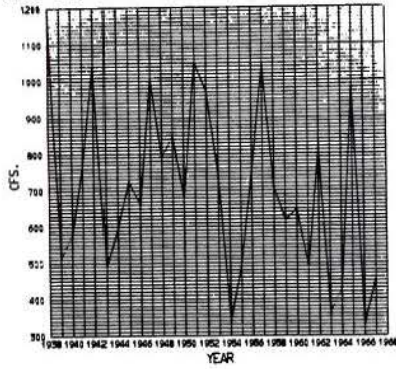
CN-6 CSU STA. 1740000  USGS STA. 9.0575
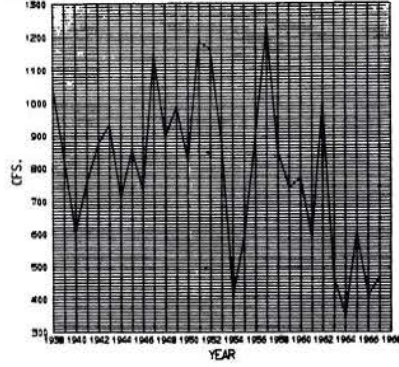
CN-6 CSU STA. 1720000  USGS STA. 9.0595
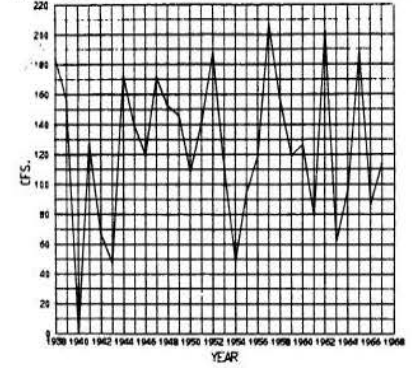
CN-6 CSU STA. 1666300  USGS STA. 9.0645
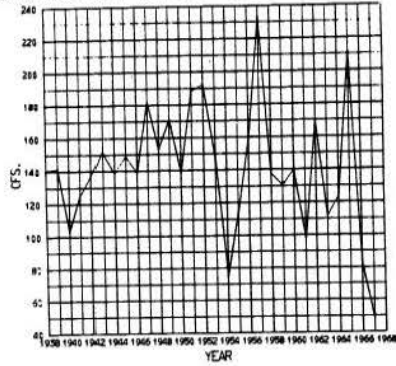
CN-6 CSU STA. 1594260  USGS STA. 9.0780
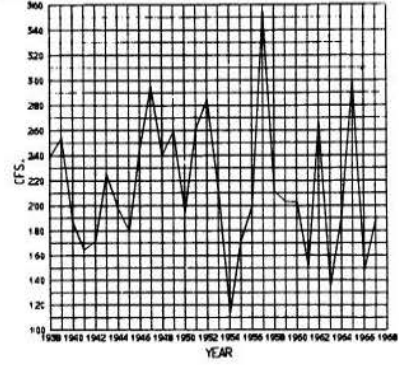
CN-6 CSU STA. 1594236  USGS STA. 9.0785
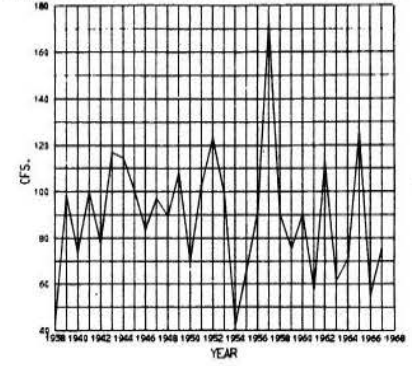
CN-6 CSU STA. 1590000  USGS STA. 9.0850
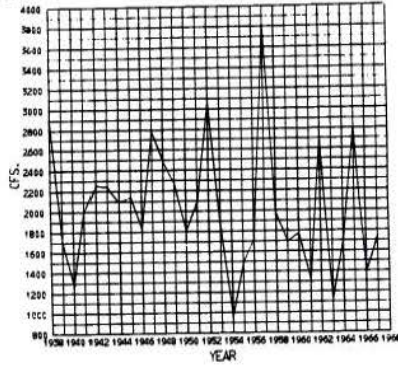
CN-6 CSU STA. 1379000  USGS STA. 9.1090
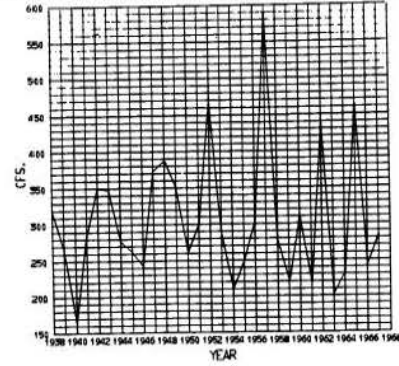
CN-6 CSU STA. 1378400  USGS STA. 9.1100
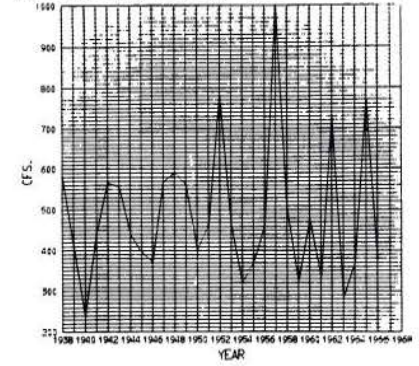
CN-6 CSU STA. 1378100  USGS STA. 9.1125
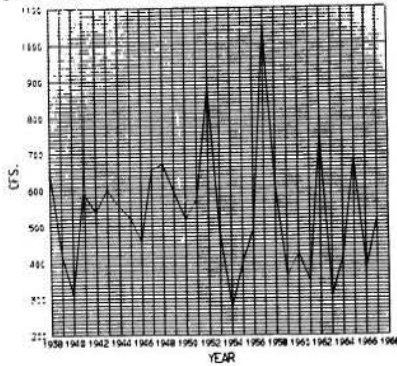
CN-6 CSU STA. 1377825  USGS STA. 9.1135
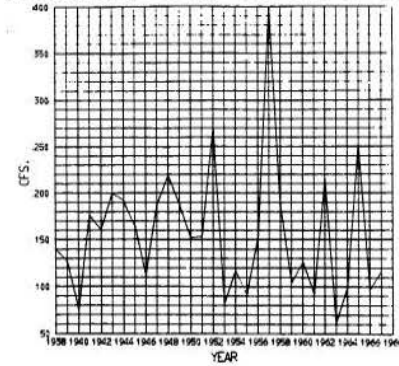
CN-6 CSU STA. 1377500  USGS STA. 9.1145

20
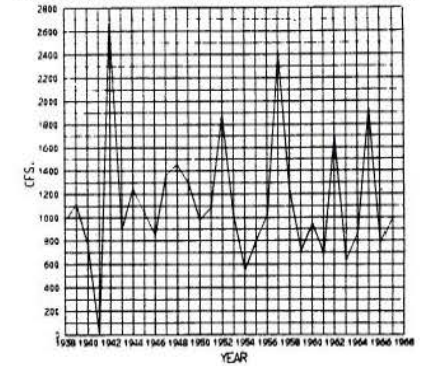
Fig. 8  CN-6 series - Continued

CN-6 CSU STA. 1377280  USGS STA. 9.1155
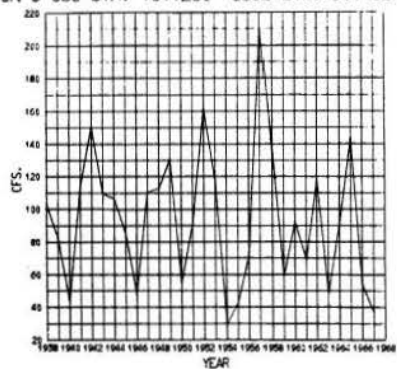
CN-6 CSU STA. 1377230  USGS STA. 9.1180
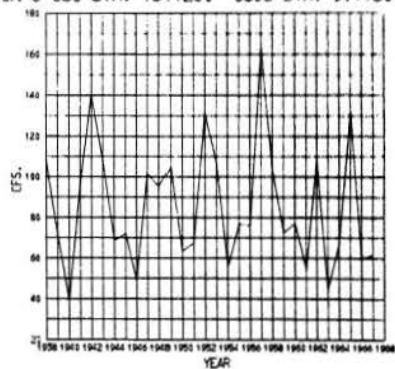
Fig. 9  S-4 series

S-4  CSU STA. 1278800  USGS STA. 9.1650
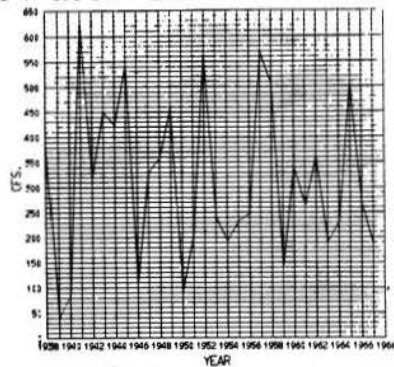
S-4  CSU STA. 1278050  USGS STA. 9.1665
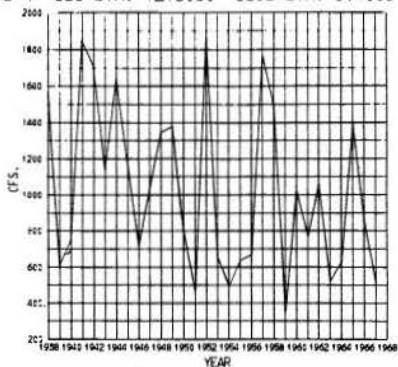
S-4  CSU STA. 1272445  USGS STA. 9.1725
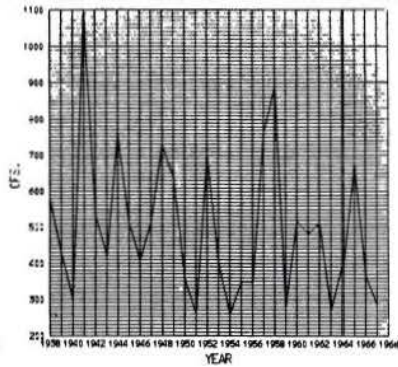
S-4  CSU STA. 1077090  USGS STA. 9.3440
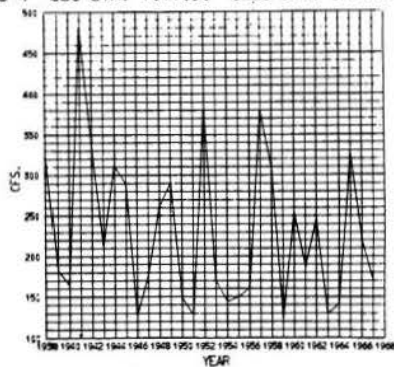
S-4  CSU STA. 1073480  USGS STA. 9.3575
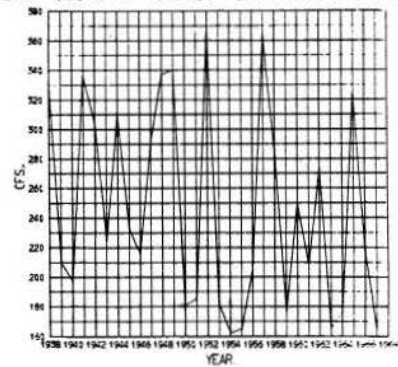
S-4  CSU STA. 1073436  USGS STA. 9.3615
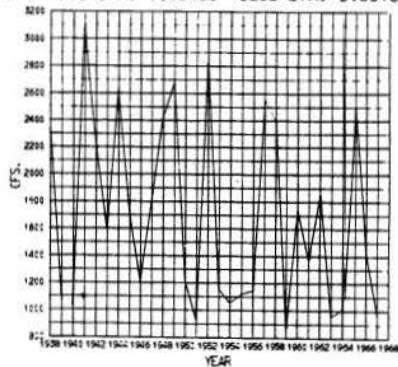
Fig. 10  S-6 series

S-6  CSU STA. 1278800  USGS STA. 9.1650
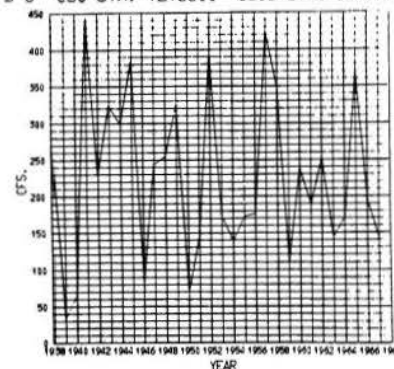
S-6  CSU STA. 1278050  USGS STA. 9.1665
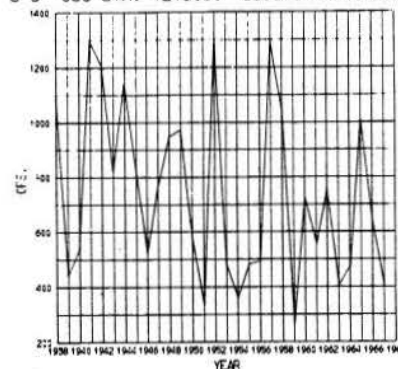
S-6  CSU STA. 1272445  USGS STA. 9.1725

21
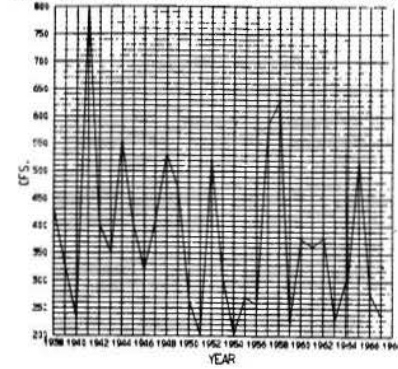
Fig. 10 S-6 series - Continued

S-6 CSU STA. 1077090 USGS STA. 9.3440
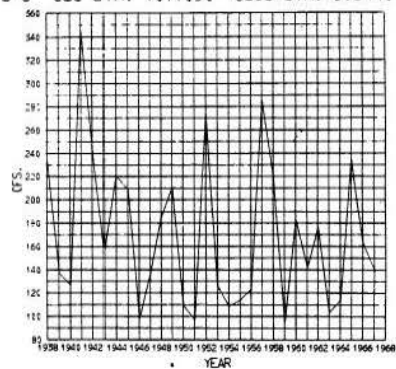
S-6 CSU STA. 1073480 USGS STA. 9.3575
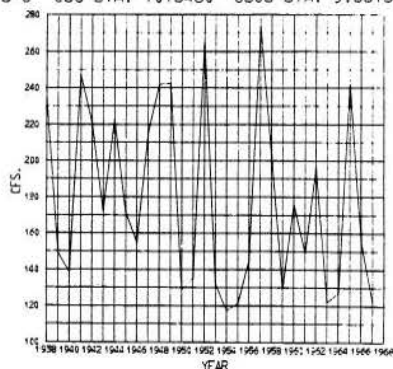
S-6 CSU STA. 1073436 USGS STA. 9.3615
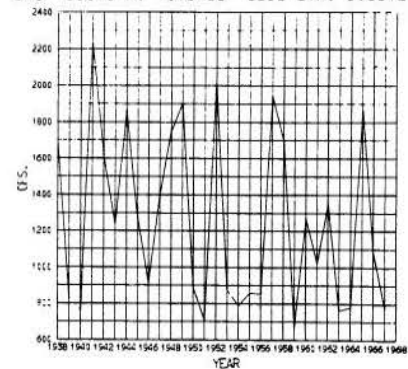
Fig. 11 CS-4 series

CS-4 CSU STA. 1425625 USGS STA. 9.0975
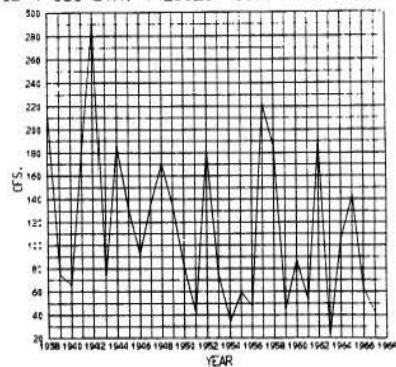
CS-4 CSU STA. 1377280 USGS STA. 9.1155
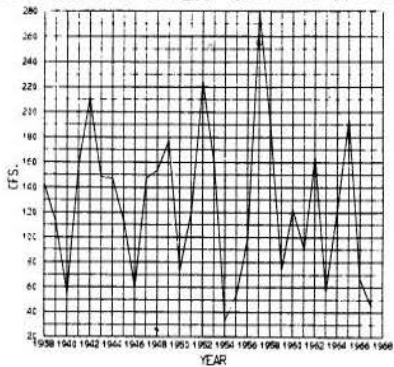
CS-4 CSU STA. 1377230 USGS STA. 9.1180
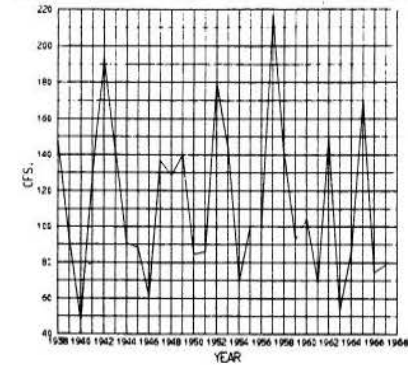
CS-4 CSU STA. 1377200 USGS STA. 9.1190
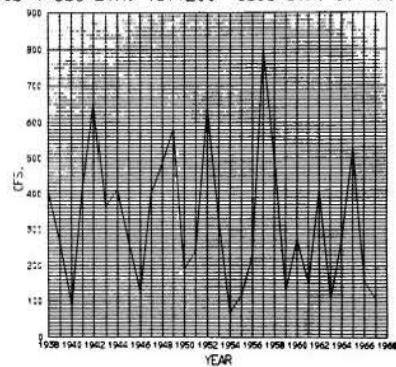
CS-4 CSU STA. 1373900 USGS STA. 9.1275
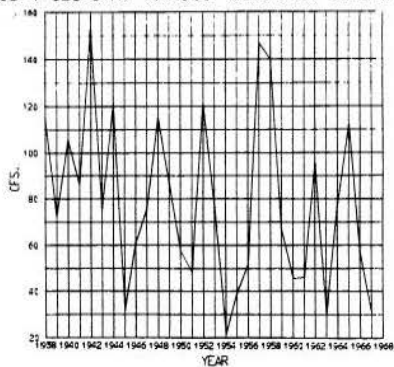
CS-4 CSU STA. 1373055 USGS STA. 9.1325
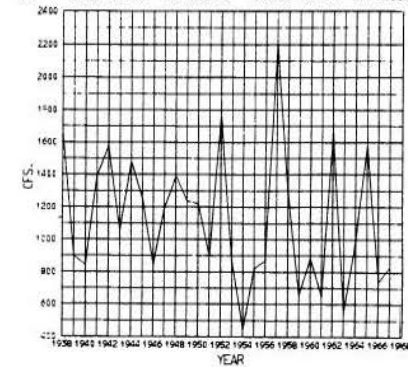
CS-4 CSU STA. 1373020 USGS STA. 9.1345
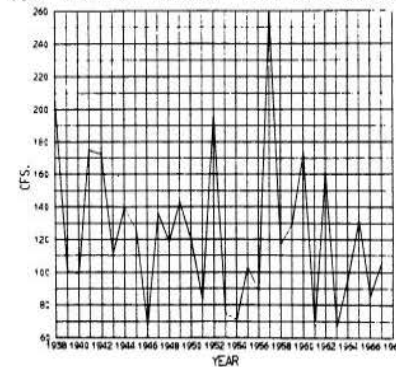
CS-4 CSU STA. 1371815 USGS STA. 9.1430
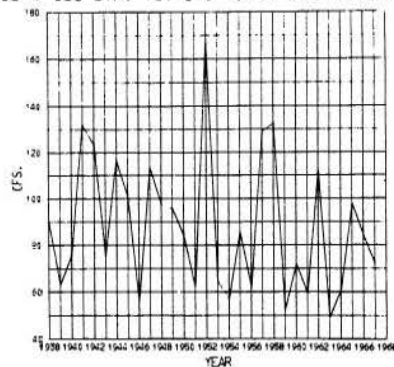
CS-4 CSU STA. 1370300 USGS STA. 9.1520

22
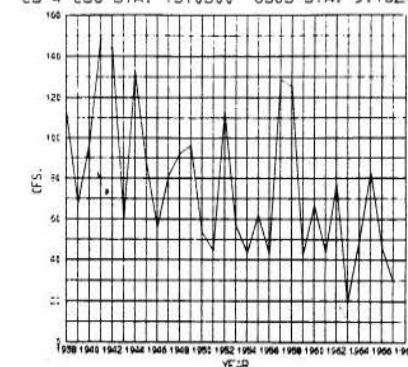
Fig. 12  CS-6 series

CS-6 CSU STA. 1425625  USGS STA. 9.0975
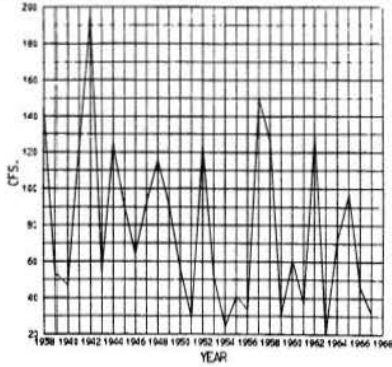
CS-6 CSU STA. 1377280  USGS STA. 9.1155
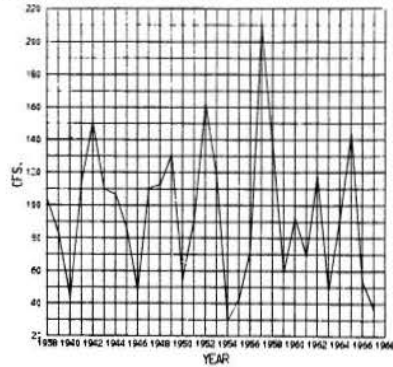
CS-6 CSU STA. 1377230  USGS STA. 9.1180
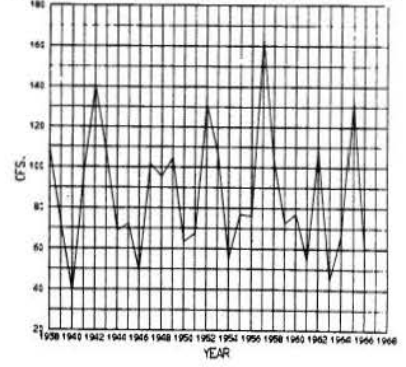
CS-6 CSU STA. 1377200  USGS STA. 9.1190
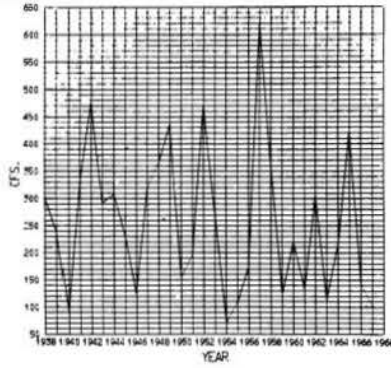
CS-6 CSU STA. 1373900  USGS STA. 9.1275
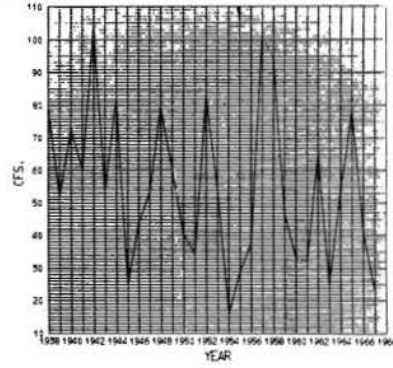
CS-6 CSU STA. 1373055  USGS STA. 9.1325
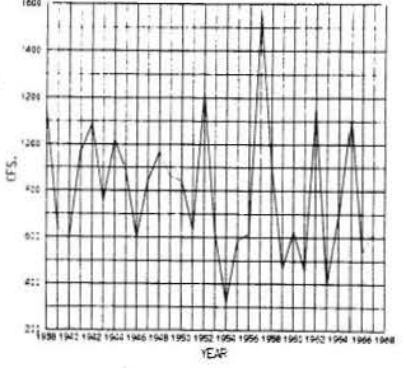
CS-6 CSU STA. 1373020  USGS STA. 9.1345
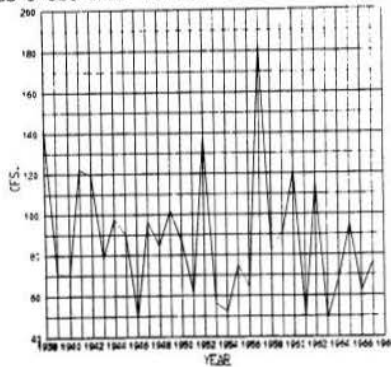
CS-6 CSU STA. 1371815  USGS STA. 9.1430
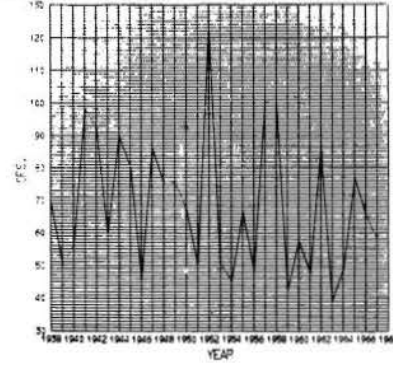
CS-6 CSU STA. 1370300  USGS STA. 9.1520

23
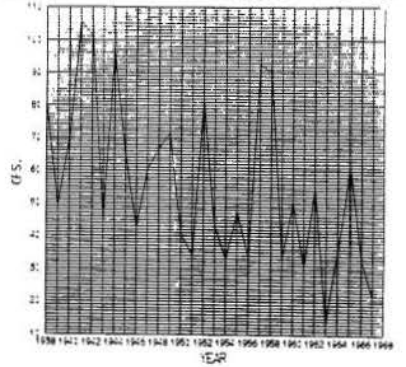
TABLE 16 - MEANS AND STANDARD DEVIATIONS OF 30 YEAR RAW DATA

| CSU Sta. No. | USGS Sta. No. | Mean of 4-month averages (cfs) | Std. Dev. of 4-month averages (cfs) | Mean of 6-month averages (cfs) | Std. Dev. of 6-month averages (cfs) |
|---|---|---|---|---|---|
| 1970000 | 9.0105 | 198.449 | 55.552 | 141.865 | 38.338 |
| 1960000 | 9.0110 | 241.821 | 71.196 | 174.569 | 51.065 |
| 1866000 | 9.0165 | 203.590 | 53.612 | 146.297 | 38.545 |
| 1830000 | 9.0190 | 931.757 | 290.050 | 644.206 | 220.359 |
| 1820000 | 9.0195 | 826.556 | 274.385 | 582.724 | 203.842 |
| 1802730 | 9.0265 | 63.007 | 27.307 | 47.814 | 20.085 |
| 1776000 | 9.0360 | 234.679 | 64.188 | 171.954 | 45.884 |
| 1742100 | 9.0535 | 924.237 | 316.280 | 702.783 | 233.361 |
| 1740000 | 9.0575 | 1043.263 | 341.708 | 794.292 | 241.819 |
| 1720000 | 9.0595 | 177.674 | 72.281 | 124.358 | 50.072 |
| 1666300 | 9.0645 | 199.184 | 55.142 | 141.030 | 39.140 |
| 1594260 | 9.0780 | 297.711 | 73.851 | 215.045 | 53.847 |
| 1594236 | 9.0785 | 128.576 | 38.515 | 89.611 | 27.138 |
| 1590000 | 9.0850 | 2739.444 | 854.102 | 2031.847 | 624.926 |
| 137900 | 9.1090 | 406.685 | 122.728 | 306.039 | 91.987 |
| 1378400 | 9.1100 | 641.932 | 224.886 | 486.119 | 165.435 |
| 1378100 | 9.1125 | 736.162 | 236.020 | 534.129 | 170.870 |
| 1377825 | 9.1135 | 219.301 | 98.861 | 157.011 | 69.485 |
| 1377500 | 9.1145 | 1521.405 | 754.366 | 1129.644 | 546.188 |
| 1377280 | 9.1155 | 126.388 | 59.684 | 94.178 | 42.221 |
| 1377230 | 9.1180 | 113.268 | 42.698 | 85.892 | 30.376 |
| 1278800 | 9.1650 | 314.930 | 159.674 | 226.413 | 111.344 |
| 1278050 | 9.1665 | 1028.025 | 467.197 | 738.323 | 319.454 |
| 1272445 | 9.1725 | 500.048 | 204.184 | 379.190 | 144.622 |
| 1077090 | 9.3440 | 230.964 | 93.315 | 170.480 | 64.553 |
| 1073480 | 9.3575 | 245.702 | 68.310 | 178.304 | 50.207 |
| 1073436 | 9.3615 | 1696.563 | 688.607 | 1254.713 | 481.412 |
| 1425625 | 9.0975 | 114.324 | 68.267 | 78.467 | 45.159 |
| 1377280 | 9.1155 | 126.388 | 59.684 | 94.178 | 42.221 |
| 1377230 | 9.1180 | 113.268 | 42.698 | 85.892 | 30.376 |
| 1377200 | 9.1190 | 322.737 | 192.349 | 257.047 | 136.341 |
| 1373900 | 9.1275 | 78.756 | 36.701 | 55.056 | 24.150 |
| 1373055 | 9.1325 | 1124.250 | 410.613 | 790.065 | 280.861 |
| 1373020 | 9.1345 | 124.166 | 46.158 | 88.247 | 31.349 |
| 1371815 | 9.1430 | 88.115 | 29.509 | 68.390 | 21.124 |
| 1370300 | 9.1520 | 76.695 | 35.297 | 56.029 | 24.863 |

TABLE 17 - CORRELATION MATRIX BETWEEN N-4 AND CN-4 (computed from 30-year data)

| | | N-4 | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | CSU STA. NO. | 1970000 | 1960000 | 1866000 | 1830000 | 1820000 | 1802730 | 1776000 |
| CSU STA. NO. | USGS STA. NO. | 9.0105 | 9.0110 | 9.0165 | 9.0190 | 9.0195 | 9.0265 | 9.0360 |
| 1742100 | 9.0535 | .477 | .785 | .778 | .815 | .641 | .710 | .894 |
| 1740000 | 9.0575 | .411 | .728 | .566 | .669 | .531 | .843 | .836 |
| 1720000 | 9.0595 | .524 | .771 | .592 | .660 | .593 | .421 | .730 |
| 1666300 | 9.0645 | .535 | .717 | .640 | .728 | .619 | .592~ | .721 |
| 1594260 | 9.0780 | .582 | .802 | .597 | .702 | .640 | .585 | .805 |
| 1594236 | 9.0785 | .621 | .652 | .602 | .565 | .584 | .477 | .549 |
| CN-4 1590000 | 9.0850 | .502 | .845 | .722 | .806 | .673 | .641 | .870 |
| 1379000 | 9.1090 | .517 | .767 | .655 | .754 | .662 | .469 | .730 |
| 1378400 | 9.1100 | .541 | .789 | .671 | .743 | .685 | .489 | .757 |
| 1378100 | 9.1125 | .497 | .801 | .714 | .740 | .621 | .627 | .791 |
| 1377825 | 9.1135 | .426 | .680 | .639 | .686 | .564 | .498 | .627 |
| 1377500 | 9.1145 | .497 | .533 | .419 | .632 | .491 | .395 | .602 |
| 1377280 | 9.1155 | .553 | .710 | .692 | .694 | .694 | .567 | .708 |
| 1377230 | 9.1180 | .490 | .674 | .674 | .704 | .612 | .537 | .719 |

TABLE 18 - CORRELATION MATRIX BETWEEN N-6 AND CN-6 (computed from 30-year data)

| | | N-6 | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | CSU STA. NO. | 1970000 | 1960000 | 1866000 | 1830000 | 1820000 | 1802730 | 1776000 |
| CSU STA. NO. | USGS STA. NO. | 9.0105 | 9.0110 | 9.0165 | 9.0190 | 9.0195 | 9.0265 | 9.0360 |
| 1742100 | 9.0535 | .486 | .783 | .780 | .813 | .671 | .706 | .896 |
| 1740000 | 9.0575 | .457 | .756 | .598 | .731 | .631 | .825 | .855 |
| 1720000 | 9.0595 | .529 | .770 | .587 | .620 | .569 | .408 | .736 |
| 1666300 | 9.0645 | .570 | .740 | .659 | .750 | .668 | .578 | .745 |
| 1594260 | 9.0780 | .598 | .798 | .592 | .659 | .621 | .555 | .803 |
| 1594236 | 9.0785 | .627 | .650 | .603 | .529 | .552 | .448 | .562 |
| CN-6 1590000 | 9.0850 | .509 | .840 | .720 | .764 | .659 | .623 | .876 |
| 1379000 | 9.1090 | .529 | .766 | .658 | .704 | .613 | .438 | .734 |
| 1378400 | 9.1100 | .551 | .788 | .669 | .693 | .639 | .456 | .761 |
| 1378100 | 9.1125 | .507 | .802 | .716 | .700 | .597 | .599 | .799 |
| 1377825 | 9.1135 | .433 | .681 | .640 | .650 | .541 | .477 | .644 |
| 1377500 | 9.1145 | .503 | .532 | .420 | .596 | .450 | .373 | .604 |
| 1377280 | 9.1155 | .555 | .708 | .701 | .689 | .695 | .542 | .715 |
| 1377230 | 9.1180 | .497 | .668 | .673 | .674 | .593 | .509 | .723 |

TABLE 19 - CORRELATION MATRIX BETWEEN S-4 AND CS-4 (computed from 30-year data)

| | | S-4 | | | | | |
|---|---|---|---|---|---|---|---|
| | CSU STA. NO. | 1278800 | 1278050 | 1272445 | 1077090 | 1073480 | 1073436 |
| CSU STA. NO. | USGS STA. NO. | 9.1650 | 9.1665 | 9.1725 | 9.3440 | 9.3575 | 9.3615 |
| 1425625 | 9.0975 | .656 | .890 | .752 | .811 | .849 | .830 |
| 1377280 | 9.1155 | .748 | .807 | .718 | .753 | .830 | .783 |
| 1377230 | 9.1180 | .682 | .742 | .593 | .695 | .760 | .709 |
| CS-4 1377200 | 9.1190 | .748 | .859 | .737 | .786 | .889 | .842 |
| 1373900 | 9.1275 | .491 | .776 | .661 | .663 | .770 | .714 |
| 1373055 | 9.1325 | .707 | .861 | .711 | .792 | .858 | .810 |
| 1373020 | 9.1345 | .643 | .776 | .612 | .773 | .777 | .723 |
| 1371815 | 9.1430 | .772 | .804 | .784 | .836 | .819 | .850 |
| 1370300 | 9.1520 | .675 | .908 | .835 | .866 | .828 | .856 |

TABLE 20 - CORRELATION MATRIX BETWEEN S-6 AND CS-6 (computed from 30-year data)

| | | S-6 | | | | | |
|---|---|---|---|---|---|---|---|
| | CSU STA. NO. | 1278800 | 1278050 | 1272445 | 1077090 | 1073480 | 1073436 |
| CSU STA. NO. | USGS STA. NO. | 9.1650 | 9.1665 | 9.1725 | 9.3440 | 9.3575 | 9.3615 |
| 1425625 | 9.0975 | .653 | .892 | .754 | .804 | .846 | .826 |
| 1377280 | 9.1155 | .753 | .805 | .722 | .747 | .840 | .786 |
| 1377230 | 9.1180 | .702 | .747 | .608 | .699 | .779 | .722 |
| CS-6 1377200 | 9.1190 | .753 | .854 | .746 | .783 | .894 | .841 |
| 1373900 | 9.1275 | .492 | .778 | .659 | .660 | .770 | .712 |
| 1373055 | 9.1325 | .716 | .863 | .720 | .791 | .866 | .813 |
| 1373020 | 9.1345 | .656 | .779 | .624 | .780 | .787 | .733 |
| 1371815 | 9.1430 | .777 | .888 | .793 | .837 | .828 | .854 |
| 1370300 | 9.1520 | .666 | .896 | .829 | .848 | .821 | .845 |

## Chapter V

### DATA ANALYSIS AND RESULTS

In this chapter the data described in Chapter IV are analyzed according to the procedures discussed in Chapter III. The approaches used for reducing the number of runoff variables are the principal component analysis and the canonical analysis. The minimum numbers of years to detect the increase in the runoff means are obtained by application of equation (1).

In the principal component analysis and the canonical analysis, the coefficients for the principal components and the canonical variables are obtained basically from the analysis of the covariance matrix. Therefore, because the covariance matrix is assumed to be the same for both periods, it follows that the coefficients obtained for the non-seeded period apply for the seeded period as well. The suspected change in the means of the runoff leave the coefficients of the components invariant.

5.1 The application of principal component analysis. The numerical procedures for the reduction of the number of runoff variables by the principal components method were executed separately in each region on the CDC 6400 digital computer of Colorado State University. The program BMD01M from the University of California Press was modified to accommodate nonstandardized variables. The zero mean is not desirable here because a certain percent increase in the mean will be postulated later.

The steps in obtaining the principal components in each region may be summarized as follows:

1) Compute the covariance matrix of the runoff variables in that region, $\underline{\hat{V}}$ , as defined in equation (2).

2) Solve the system,

$$|\underline{\hat{V}} - \lambda I| = 0 \quad , \tag{21}$$

to obtain $\lambda_1, \lambda_2, ..., \lambda_p$ , the characteristic roots, which are the amounts of variances of components 1, 2, ..., p .

3) Solve the system,

$$(\underline{\hat{V}} - \lambda_i I)\underline{\beta}_i = \underline{0} \tag{22}$$

subject to the normalization condition,

$$\underline{\beta}_i' \, \underline{\beta}_i = 1 \tag{23}$$

to obtain $\underline{\beta}_i$ which is the vector of the coefficients for the $i^{th}$ component in that region.

For example, when N-4, which is the four-month runoff of the northern region, is used the coefficients for the first principal component are found to be (Table 21),

$$\beta_{1,1} = 0.0859$$

$$\beta_{1,2} = 0.1679$$

$$\beta_{1,3} = 0.1151$$

$$\beta_{1,4} = 0.7065$$

$$\beta_{1,5} = 0.6576$$

$$\beta_{1,6} = 0.0332$$

$$\beta_{1,7} = 0.1359$$

where the first subscript of $\beta$ indicates the ordering number of the principal component, the second one indicates the sequential number of the station as shown in Table 2.

Let $\xi_i$ be the $i^{th}$ principal component in the target region before seeding, then for N-4,

$$\xi_1 = \sum_{j=1}^{7} \beta_{1,j} \, Q_j$$

$$= 0.0859Q_1 + 0.1679Q_2 + 0.1151Q_3 + 0.7065Q_4$$

$$+ 0.6576Q_5 + 0.0332Q_6 + 0.1359Q_7$$

where $Q_1$, $Q_2$, $Q_3$, $Q_4$, $Q_5$, $Q_6$ and $Q_7$ are runoff variables listed in order corresponding to the numbers in the 'Seq. No.' column in Table 2. This first principal component will account for the largest percentage of the total variation in this whole region based on the four-month spring runoff.

The coefficients for the principal components in N-4, N-6, CN-4, CN-6, S-4, S-6, CS-4 and CS-6 are shown in Tables 21, 22, 23, 24, 25, 26, 27, and 28, respectively. The cumulative percentages of total variation accounted for by the principal components in each region are shown in Table 29. A 99 cumulative percentage was used to limit the number of the principal components to be retained for the study, because it was found that beyond this percentage of total variation, the rate of increase of the cumulative percentage was very slow.

After the coefficients of the principal components in each region have been found, then the series of the principal components can be simply obtained from the original series [12].

<table>
<tr><td colspan="6" align="center">TABLE 21 - COEFFICIENTS FOR THE PRINCIPAL<br>COMPONENTS OF N-4</td></tr>
<tr><th>CSU<br>Sta.<br>No.</th><th>USGS<br>Sta.<br>No.</th><th>1st<br>Comp.</th><th>2nd<br>Comp.</th><th>3rd<br>Comp.</th><th>4th<br>Comp.</th></tr>
<tr><td>1970000</td><td>9.0105</td><td>.0859</td><td>-.0894</td><td>-.4339</td><td>-.8081</td></tr>
<tr><td>1960000</td><td>9.0110</td><td>.1679</td><td>-.0529</td><td>-.4719</td><td>.0637</td></tr>
<tr><td>1866000</td><td>9.0165</td><td>.1151</td><td>.0334</td><td>-.1221</td><td>-.2757</td></tr>
<tr><td>1830000</td><td>9.0190</td><td>.7065</td><td>.6848</td><td>.1407</td><td>-.0308</td></tr>
<tr><td>1820000</td><td>9.0195</td><td>.6576</td><td>-.7201</td><td>.1966</td><td>.0688</td></tr>
<tr><td>1802730</td><td>9.0265</td><td>.0332</td><td>.0191</td><td>-.3072</td><td>.2822</td></tr>
<tr><td>1776000</td><td>9.0360</td><td>.1359</td><td>.0132</td><td>-.6491</td><td>.4262</td></tr>
</table>

<table>
<tr><td colspan="6" align="center">TABLE 22 - COEFFICIENTS FOR THE PRINCIPAL<br>COMPONENTS OF N-6</td></tr>
<tr><th>CSU<br>Sta.<br>No.</th><th>USGS<br>Sta.<br>No.</th><th>1st<br>Comp.</th><th>2nd<br>Comp.</th><th>3rd<br>Comp.</th><th>4th<br>Comp.</th></tr>
<tr><td>1970000</td><td>9.0105</td><td>.0767</td><td>-.0806</td><td>-.5604</td><td>-.6494</td></tr>
<tr><td>1960000</td><td>9.0110</td><td>.1549</td><td>-.0680</td><td>-.5037</td><td>.0808</td></tr>
<tr><td>1866000</td><td>9.0165</td><td>.1084</td><td>.0256</td><td>-.2105</td><td>-.2926</td></tr>
<tr><td>1830000</td><td>9.0190</td><td>.7191</td><td>.6784</td><td>.1135</td><td>-.0377</td></tr>
<tr><td>1820000</td><td>9.0195</td><td>.6510</td><td>-.7266</td><td>.0154</td><td></td></tr>
<tr><td>1802730</td><td>9.0265</td><td>.0339</td><td>.0048</td><td>-.1892</td><td>.4046</td></tr>
<tr><td>1776000</td><td>9.0360</td><td>.1279</td><td>-.0079</td><td>-.5424</td><td>.5664</td></tr>
</table>

<table>
<tr><td colspan="6" align="center">TABLE 23 - COEFFICIENTS FOR THE PRINCIPAL<br>COMPONENTS OF CN-4</td></tr>
<tr><th>CSU<br>Sta.<br>No.</th><th>USGS<br>Sta.<br>No.</th><th>1st<br>Comp.</th><th>2nd<br>Comp.</th><th>3rd<br>Comp.</th><th>4th<br>Comp.</th></tr>
<tr><td>1742100</td><td>9.0535</td><td>.2250</td><td>-.1496</td><td>-.4782</td><td>-.7580</td></tr>
<tr><td>1740000</td><td>9.0575</td><td>.2155</td><td>-.2714</td><td>-.7505</td><td>.5419</td></tr>
<tr><td>1720000</td><td>9.0599</td><td>.0444</td><td>-.0508</td><td>.0329</td><td>-.0134</td></tr>
<tr><td>1666300</td><td>9.0645</td><td>.0378</td><td>-.0229</td><td>-.0434</td><td>.0232</td></tr>
<tr><td>1594260</td><td>9.0780</td><td>.0524</td><td>-.0388</td><td>-.0108</td><td>.0455</td></tr>
<tr><td>1594236</td><td>9.0785</td><td>.0240</td><td>-.0114</td><td>.0178</td><td>.0891</td></tr>
<tr><td>1590000</td><td>9.0850</td><td>.7025</td><td>-.4654</td><td>.3580</td><td>-.0949</td></tr>
<tr><td>1379000</td><td>9.1090</td><td>.0971</td><td>-.0158</td><td>.0912</td><td>.1170</td></tr>
<tr><td>1378400</td><td>9.1100</td><td>.1803</td><td>-.0355</td><td>.1781</td><td>.1584</td></tr>
<tr><td>1378100</td><td>9.1125</td><td>.1862</td><td>-.1309</td><td>.1338</td><td>.2226</td></tr>
<tr><td>1377825</td><td>9.1135</td><td>.0733</td><td>-.0277</td><td>.0926</td><td>.1475</td></tr>
<tr><td>1377500</td><td>9.1145</td><td>.5637</td><td>.8144</td><td>-.0985</td><td>.0404</td></tr>
<tr><td>1377280</td><td>9.1155</td><td>.0444</td><td>-.0050</td><td>.0128</td><td>.0178</td></tr>
<tr><td>1377230</td><td>9.1180</td><td>.0320</td><td>-.0011</td><td>.0114</td><td>.0005</td></tr>
</table>

<table>
<tr><td colspan="6" align="center">TABLE 24 - COEFFICIENTS FOR THE PRINCIPAL<br>COMPONENTS OF CN-6</td></tr>
<tr><th>CSU<br>Sta.<br>No.</th><th>USGS<br>Sta.<br>No.</th><th>1st<br>Comp.</th><th>2nd<br>Comp.</th><th>3rd<br>Comp.</th><th>4th<br>Comp.</th></tr>
<tr><td>1742100</td><td>9.0535</td><td>.2268</td><td>-.1513</td><td>-.5634</td><td>-.7013</td></tr>
<tr><td>1740000</td><td>9.0575</td><td>.2156</td><td>-.2566</td><td>-.6879</td><td>.6239</td></tr>
<tr><td>1720000</td><td>9.0595</td><td>.0422</td><td>-.0478</td><td>.0304</td><td>-.0128</td></tr>
<tr><td>166300</td><td>9.0645</td><td>.0375</td><td>-.0220</td><td>-.0413</td><td>.0318</td></tr>
<tr><td>1594260</td><td>9.0780</td><td>.0528</td><td>-.0362</td><td>-.0035</td><td>.0583</td></tr>
<tr><td>1594236</td><td>9.0785</td><td>.0233</td><td>-.0100</td><td>.0220</td><td>.0898</td></tr>
<tr><td>1590000</td><td>9.0850</td><td>.7062</td><td>-.4663</td><td>.3442</td><td>-.1245</td></tr>
<tr><td>1379000</td><td>9.1090</td><td>.1005</td><td>-.0147</td><td>.1038</td><td>.1050</td></tr>
<tr><td>1378400</td><td>9.1100</td><td>.1825</td><td>-.0337</td><td>.1947</td><td>.1356</td></tr>
<tr><td>1378100</td><td>9.1125</td><td>.1858</td><td>-.1261</td><td>.1495</td><td>.2038</td></tr>
<tr><td>1377825</td><td>9.1135</td><td>.0710</td><td>-.0262</td><td>.0944</td><td>.1331</td></tr>
<tr><td>1377500</td><td>9.1145</td><td>.5577</td><td>.8196</td><td>-.0869</td><td>.0420</td></tr>
<tr><td>1377280</td><td>9.1155</td><td>.0433</td><td>-.0047</td><td>.0120</td><td>.0192</td></tr>
<tr><td>1377230</td><td>9.1180</td><td>.0313</td><td>-.0008</td><td>.0125</td><td>-.0051</td></tr>
</table>

<table>
<tr><td colspan="5" align="center">TABLE 25 - COEFFICIENTS FOR THE PRINCIPAL<br>COMPONENTS OF S-4</td></tr>
<tr><th>CSU<br>Sta.<br>No.</th><th>USGS<br>Sta.<br>No.</th><th>1st<br>Comp.</th><th>2nd<br>Comp.</th><th>3rd<br>Comp.</th></tr>
<tr><td>1278800</td><td>9.1650</td><td>-.1608</td><td>-.0738</td><td>-.8889</td></tr>
<tr><td>1278050</td><td>9.1665</td><td>-.5304</td><td>.8066</td><td>-.0525</td></tr>
<tr><td>1272445</td><td>9.1725</td><td>-.2180</td><td>-.4039</td><td>-.2817</td></tr>
<tr><td>1077090</td><td>9.3440</td><td>-.1027</td><td>.0634</td><td>-.1532</td></tr>
<tr><td>1073480</td><td>9.3575</td><td>-.0754</td><td>-.0045</td><td>.1153</td></tr>
<tr><td>1073436</td><td>9.3615</td><td>-.7931</td><td>-.4205</td><td>.3017</td></tr>
</table>

<table>
<tr><td colspan="5" align="center">TABLE 26 - COEFFICIENTS FOR THE PRINCIPAL<br>COMPONENTS OF S-6</td></tr>
<tr><th>CSU<br>Sta.<br>No.</th><th>USGS<br>Sta.<br>No.</th><th>1st<br>Comp.</th><th>2nd<br>Comp.</th><th>3rd<br>Comp.</th></tr>
<tr><td>1278800</td><td>9.1650</td><td>-.1622</td><td>-.1421</td><td>-.8496</td></tr>
<tr><td>1278050</td><td>9.1665</td><td>-.5207</td><td>.8186</td><td>-.1102</td></tr>
<tr><td>1272445</td><td>9.1725</td><td>-.2240</td><td>-.3730</td><td>-.3252</td></tr>
<tr><td>1077090</td><td>9.3440</td><td>-.1013</td><td>.0618</td><td>-.1660</td></tr>
<tr><td>1073480</td><td>9.3575</td><td>-.0802</td><td>-.0038</td><td>.1155</td></tr>
<tr><td>1073436</td><td>9.3615</td><td>-.7973</td><td>-.4084</td><td>.3456</td></tr>
</table>

<table>
<tr><td colspan="4" align="center">TABLE 27 - COEFFICIENTS FOR THE PRINCIPAL<br>COMPONENTS OF CS-4</td></tr>
<tr><th>CSU<br>Sta.<br>No.</th><th>USGS<br>Sta.<br>No.</th><th>1st<br>Comp.</th><th>2nd<br>Comp.</th></tr>
<tr><td>1425625</td><td>9.0975</td><td>-.1341</td><td>.0453</td></tr>
<tr><td>1377280</td><td>9.1155</td><td>-.1167</td><td>.2714</td></tr>
<tr><td>1377230</td><td>9.1180</td><td>-.0799</td><td>.1717</td></tr>
<tr><td>1377200</td><td>9.1190</td><td>-.3879</td><td>.8378</td></tr>
<tr><td>1373900</td><td>9.1275</td><td>-.0658</td><td>.0819</td></tr>
<tr><td>1373055</td><td>9.1325</td><td>-.8906</td><td>-.4286</td></tr>
<tr><td>1373020</td><td>9.1345</td><td>-.0859</td><td>-.0353</td></tr>
<tr><td>1371815</td><td>9.1430</td><td>-.0537</td><td>.0138</td></tr>
<tr><td>1370300</td><td>9.1520</td><td>-.0616</td><td>.0331</td></tr>
</table>

<table>
<tr><td colspan="4" align="center">TABLE 28 - COEFFICIENTS FOR THE PRINCIPAL<br>COMPONENTS OF CS-6</td></tr>
<tr><th>CSU<br>Sta.<br>No.</th><th>USGS<br>Sta.<br>No.</th><th>1st<br>Comp.</th><th>2nd<br>Comp.</th></tr>
<tr><td>1425625</td><td>9.0975</td><td>-.1278</td><td>-.0209</td></tr>
<tr><td>1377280</td><td>9.1155</td><td>-.1201</td><td>-.2711</td></tr>
<tr><td>1377230</td><td>9.1180</td><td>-.0831</td><td>-.1661</td></tr>
<tr><td>1377200</td><td>9.1190</td><td>-.3994</td><td>-.8374</td></tr>
<tr><td>1373900</td><td>9.1275</td><td>-.0625</td><td>-.0688</td></tr>
<tr><td>1373055</td><td>9.1325</td><td>-.8857</td><td>.4363</td></tr>
<tr><td>1373020</td><td>9.1345</td><td>-.0853</td><td>.0376</td></tr>
<tr><td>1371815</td><td>9.1430</td><td>-.0567</td><td>-.0003</td></tr>
<tr><td>1370300</td><td>9.1520</td><td>-.0621</td><td>-.0283</td></tr>
</table>

TABLE 29 – CUMULATIVE PERCENTAGE OF TOTAL VARIATION
ACCOUNTED FOR BY THE PRINCIPAL COMPONENTS

| Type | Principal component | Cumulative percentage of total variation accounted for |
|------|---------------------|--------------------------------------------------------|
| N-4 series | $\xi_1$ | 85 |
| | $\xi_1$ and $\xi_2$ | 97 |
| | $\xi_1, \xi_2$ and $\xi_3$ | 98 |
| | $\xi_1, \xi_2, \xi_3$ and $\xi_4$ | 99 |
| N-6 series | $\xi_1$ | 85 |
| | $\xi_1$ and $\xi_2$ | 97 |
| | $\xi_1, \xi_2$ and $\xi_3$ | 98 |
| | $\xi_1, \xi_2, \xi_3$ and $\xi_4$ | 99 |
| CN-4 series | $\eta_1$ | 82 |
| | $\eta_1$ and $\eta_2$ | 94 |
| | $\eta_1, \eta_2$ and $\eta_3$ | 98 |
| | $\eta_1, \eta_2, \eta_3$ and $\eta_4$ | 99 |
| CN-6 series | $\eta_1$ | 83 |
| | $\eta_1$ and $\eta_2$ | 95 |
| | $\eta_1, \eta_2$ and $\eta_3$ | 98 |
| | $\eta_1, \eta_2, \eta_3$ and $\eta_4$ | 99 |
| S-4 series | $\xi_1$ | 97 |
| | $\xi_1$ and $\xi_2$ | 98 |
| | $\xi_1, \xi_2$ and $\xi_3$ | 99 |
| S-6 series | $\xi_1$ | 97 |
| | $\xi_1$ and $\xi_2$ | 98 |
| | $\xi_1, \xi_2$ and $\xi_3$ | 99 |
| CS-4 series | $\eta_1$ | 95 |
| | $\eta_1$ and $\eta_2$ | 99 |
| CS-6 series | $\eta_1$ | 95 |
| | $\eta_1$ and $\eta_2$ | 99 |

TABLE 30 – MEANS AND STANDARD DEVIATIONS
OF THE PRINCIPAL COMPONENTS

| Type | Principal component | Mean (cfs) | Std. Dev. (cfs) |
|------|---------------------|------------|-----------------|
| N-4 series | $\xi_1$ | 1316.896 | 385.728 |
| | $\xi_2$ | 23.431 | 144.526 |
| | $\xi_3$ | -103.167 | 50.427 |
| | $\xi_4$ | -55.122 | 37.622 |
| N-6 series | $\xi_1$ | 919.996 | 289.498 |
| | $\xi_2$ | -7.066 | 106.279 |
| | $\xi_3$ | -103.770 | 39.834 |
| | $\xi_4$ | -19.400 | 27.483 |
| CN-4 series | $\eta_1$ | 3566.570 | 1171.757 |
| | $\eta_2$ | -616.325 | 446.487 |
| | $\eta_3$ | -124.669 | 238.558 |
| | $\eta_4$ | 41.293 | 146.087 |
| CN-6 series | $\eta_1$ | 2656.142 | 853.439 |
| | $\eta_2$ | -442.328 | 326.332 |
| | $\eta_3$ | -118.678 | 169.248 |
| | $\eta_4$ | 49.830 | 101.846 |
| S-4 series | $\xi_1$ | -2092.706 | 865.153 |
| | $\xi_2$ | -95.873 | 108.688 |
| | $\xi_3$ | 30.022 | 81.462 |
| S-6 series | $\xi_1$ | -1538.060 | 601.913 |
| | $\xi_2$ | -71.786 | 74.436 |
| | $\xi_3$ | 28.886 | 55.548 |
| CS-4 series | $\eta_1$ | -1190.877 | 459.172 |
| | $\eta_2$ | -146.711 | 86.254 |
| CS-6 series | $\eta_1$ | -849.227 | 315.594 |
| | $\eta_2$ | 85.939 | 61.805 |

For the control region, it is obvious that following the assumption that the means of the runoff stations in the control region remain unchanged,

$$E\{\eta_i^*\} = E\{\eta_i\}$$

where $\eta_i^*$ is the $i^{th}$ principal component of the control region during the seeded period.

After the principal components in each separate region have been obtained, they are gathered into four major target-control groups as N-4 and CN-4, N-6 and CN-6, S-4 and CS-4, and S-6 and CS-6. For brevity, after the principal components in the target are combined with those in the control, the following symbols will be used:

N-CN-4 - the combination of N-4 and CN-4

N-CN-6 - the combination of N-6 and CN-6

S-CS-4 - the combination of S-4 and CS-4

S-CS-6 - the combination of S-6 and CS-6.

Since it is the principal components that will be utilized in the final test, the computations of the covariance matrices are carried out for these principal components. These are as shown in Tables 31, 32, 33, and 34; also shown are the correlations matrices in Tables 35, 36, 37, and 38.

The means and standard deviations of the series of the principal components for N-4, N-6, CN-4, CN-6, S-4, S-6, CS-4 and CS-6 are given in Table 30.

It is simply proven [12] that if all the means in the target areas during the seeded period have been increased by a certain fraction of the old means, say h , that is, the increase of $Q_1$ is $hQ_1$ , of $Q_2$ is $hQ_2$ , and so on, then the increase in the means of the principal components will also be h . If h is assigned a value of 0.10 , then

$$E\{\xi_i^*\} = 1.1 \, E\{\xi_i\} \quad ,$$

where $E\{\}$ denotes the expected value of $\{\}$ , which is the cloud seeding effect assumed in this study.

## TABLE 31 - COVARIANCE MATRIX OF N-CN-4 PRINCIPAL COMPONENT SERIES

|  |  | N-4 | | | | CN-4 | | | |
|---|---|---|---|---|---|---|---|---|---|
|  |  | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
| N-4 | $\xi_1$ | 148794.103 | -5.194 | -1.641 | -4.525 | 362585.644 | -32549.149 | -7106.856 | -8545.409 |
|  | $\xi_2$ | -5.194 | 20887.838 | -.361 | -.075 | 31294.727 | 1427.066 | -4668.399 | -2250.743 |
|  | $\xi_3$ | -1.641 | -.361 | 2542.946 | .155 | -18240.198 | 5629.908 | 2877.776 | -679.743 |
|  | $\xi_4$ | -4.525 | -.075 | .155 | 1415.475 | 4402.075 | -4938.812 | -1612.864 | 149.454 |
| CN-4 | $\eta_1$ | 362585.644 | 31294.727 | -18240.198 | 4402.075 | 1373015.745 | -71.56 | 57.238 | -49.204 |
|  | $\eta_2$ | -32549.149 | 1427.066 | 5629.908 | -4938.812 | -71.561 | 199350.870 | .050 | -3.719 |
|  | $\eta_3$ | -7106.856 | -4668.399 | 2877.776 | -1612.864 | 57.238 | .050 | 56920.290 | 2.404 |
|  | $\eta_4$ | -8545.409 | -2250.743 | -679.743 | 149.454 | -49.204 | -3.719 | 2.404 | 21341.511 |

## TABLE 32 - COVARIANCE MATRIX OF N-CN-6 PRINCIPAL COMPONENT SERIES

|  |  | N-6 | | | | CN-6 | | | |
|---|---|---|---|---|---|---|---|---|---|
|  |  | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
| N-6 | $\xi_1$ | 83809.330 | 3.376 | 15.820 | 1.313 | 191098.859 | -20577.629 | -10540.052 | -2678.418 |
|  | $\xi_2$ | 3.376 | 11295.421 | 2.644 | .066 | 13712.454 | 3162.736 | -2003.334 | -963.144 |
|  | $\xi_3$ | 15.820 | 2.644 | 1586.754 | .068 | -13182.329 | 1678.320 | 24.742 | -126.575 |
|  | $\xi_4$ | 1.313 | .066 | .068 | 755.354 | 3884.894 | -2718.278 | -993.925 | 121.930 |
| CN-6 | $\eta_1$ | 191098.859 | 13712.454 | -13182.329 | 3884.894 | 728334.925 | 20.301 | 20.187 | -14.259 |
|  | $\eta_2$ | -20577.629 | 3162.736 | 1678.320 | -2718.278 | 20.301 | 104493.191 | -5.314 | -.917 |
|  | $\eta_3$ | -10540.052 | -2003.334 | 24.742 | -993.925 | 20.187 | -5.314 | 28645.205 | -.067 |
|  | $\eta_4$ | -2678.418 | -963.144 | -126.575 | 121.930 | -14.259 | -.917 | -.067 | 10372.757 |

## TABLE 33 - COVARIANCE MATRIX OF S-CS-4 PRINCIPAL COMPONENT SERIES

|  |  | S-4 | | | CS-4 | |
|---|---|---|---|---|---|---|
|  |  | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\eta_1$ | $\eta_2$ |
| S-4 | $\xi_1$ | 748491.385 | -45.282 | 41.666 | 338072.405 | -12524.935 |
|  | $\xi_2$ | -45.282 | 11813.224 | -.485 | -11808.907 | -1953.006 |
|  | $\xi_3$ | -41.666 | -.485 | 6636.209 | -636.724 | -167.057 |
| CS-4 | $\eta_1$ | 338072.405 | -11808.907 | -636.724 | 210839.108 | 7.238 |
|  | $\eta_2$ | -12524.935 | -1953.006 | -167.057 | 7.238 | 7439.863 |

## TABLE 34 - COVARIANCE MATRIX OF S-CS-6 PRINCIPAL COMPONENT SERIES

|  |  | S-6 | | | CS-6 | |
|---|---|---|---|---|---|---|
|  |  | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\eta_1$ | $\eta_2$ |
| S-6 | $\xi_1$ | 362299.490 | 9.116 | 18.085 | 162180.856 | 5816.690 |
|  | $\xi_2$ | 9.116 | 5540.858 | -.073 | -5460.831 | 1113.406 |
|  | $\xi_3$ | 18.085 | -.073 | 3085.627 | 10.105 | 11.309 |
| CS-6 | $\eta_1$ | 162180.856 | -5460.831 | 10.105 | 99600.481 | -.854 |
|  | $\eta_2$ | 5816.690 | 1113.406 | 11.309 | -.854 | 3819.884 |

## TABLE 35 - CORRELATION MATRIX OF N-CN-4 PRINCIPAL COMPONENT SERIES

|  |  | N-4 | | | | CN-4 | | | |
|---|---|---|---|---|---|---|---|---|---|
|  |  | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
| N-4 | $\xi_1$ | 1.000 | -.000 | -.000 | -.000 | .800 | -.189 | -.077 | -.152 |
|  | $\xi_2$ | -.000 | 1.000 | -.000 | -.000 | .185 | .022 | -.135 | -.107 |
|  | $\xi_3$ | -.000 | -.000 | 1.000 | .000 | -.309 | .250 | .239 | -.092 |
|  | $\xi_4$ | -.000 | -.000 | .000 | 1.000 | .100 | -.294 | -.180 | .027 |
| CN-4 | $\eta_1$ | .800 | .185 | -.309 | .100 | 1.000 | -.000 | .000 | -.000 |
|  | $\eta_2$ | -.189 | .022 | .250 | -.294 | -.000 | 1.000 | .000 | -.000 |
|  | $\eta_3$ | -.077 | -.135 | .239 | -.180 | .000 | .000 | 1.000 | .000 |
|  | $\eta_4$ | -.152 | -.107 | -.092 | .027 | -.000 | -.000 | .000 | 1.000 |

## TABLE 36 - CORRELATION MATRIX OF N-CN-6 PRINCIPAL COMPONENT SERIES

|  |  | N-6 | | | | CN-6 | | | |
|---|---|---|---|---|---|---|---|---|---|
|  |  | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
| N-6 | $\xi_1$ | 1.000 | .000 | .001 | .000 | .773 | -.218 | -.215 | -.091 |
|  | $\xi_2$ | .000 | 1.000 | .001 | .000 | .151 | .091 | -.111 | -.089 |
|  | $\xi_3$ | .001 | .001 | 1.000 | .000 | -.358 | .129 | .004 | -.031 |
|  | $\xi_4$ | .000 | .000 | .000 | 1.000 | .166 | -.303 | -.214 | .044 |
| CN-6 | $\eta_1$ | .773 | .151 | -.358 | .166 | 1.000 | .000 | .000 | -.000 |
|  | $\eta_2$ | -.218 | .091 | .129 | -.303 | .000 | 1.000 | -.000 | -.000 |
|  | $\eta_3$ | -.215 | -.111 | .004 | -.214 | .000 | -.000 | 1.000 | -.000 |
|  | $\eta_4$ | -.091 | -.089 | -.031 | .044 | -.000 | -.000 | -.000 | 1.000 |

## TABLE 37 - CORRELATION MATRIX OF S-CS-4 PRINCIPAL COMPONENT SERIES

|  |  | S-4 | | | CS-4 | |
|---|---|---|---|---|---|---|
|  |  | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\eta_1$ | $\eta_2$ |
| S-4 | $\xi_1$ | 1.000 | -.000 | -.001 | .851 | -.168 |
|  | $\xi_2$ | -.000 | 1.00 | -.000 | -.237 | -.208 |
|  | $\xi_3$ | -.001 | -.000 | 1.000 | -.017 | -.024 |
| CS-4 | $\eta_1$ | .851 | -.237 | -.017 | 1.000 | .000 |
|  | $\eta_2$ | -.168 | -.208 | -.024 | .000 | 1.000 |

## TABLE 38 - CORRELATION MATRIX OF S-CS-6 PRINCIPAL COMPONENT SERIES

|  |  | S-6 | | | CS-6 | |
|---|---|---|---|---|---|---|
|  |  | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\eta_1$ | $\eta_2$ |
| S-6 | $\xi_1$ | 1.000 | .000 | .001 | .854 | .156 |
|  | $\xi_2$ | .000 | 1.000 | -.000 | -.232 | .242 |
|  | $\xi_3$ | .001 | -.000 | 1.000 | .001 | .003 |
| CS-6 | $\eta_1$ | .854 | -.232 | .001 | 1.000 | -.000 |
|  | $\eta_2$ | .156 | .242 | .003 | -.000 | 1.000 |

5.2 The minimum number of years needed to detect a 10% increase in runoff based on the principal components. The minimum number of years, N* , for detecting the increase of one-tenth of the old runoff means can be computed from equation (1) again,

$$N^* = \frac{\tau^2}{\underline{\mu}' \underline{V}^{-1} \underline{\mu}} \qquad (24)$$

where $\tau^2$ = the noncentrality parameter,

$\underline{\mu}$ = $\underline{\mu}^* - \underline{\mu}_o$ ,

$\underline{\mu}^*$ = the mean vector of the runoff variables for the seeded period,

$\underline{\mu}_o$ = the mean vector of the runoff variables for the period before seeding, and

$\underline{V}^{-1}$ = the inverse of covariance matrix $\underline{V}$ .

The values of $\tau^2$ are given in Table 1.

With this table the number of years needed to detect the increase can be computed easily. The values of N* are shown in Table 39.

5.3 The application of canonical analysis. In this analysis the set of the runoff variables in the target region is first combined with the set of those in the control region. As for the principal component analysis, the computation of the canonical variables were performed on the CDC 6400 digital computer of the University Computer Center at Colorado State University. The steps in finding the coefficients for the canonical variables were described in Chapter III Section 3.

After the coefficients of the canonical variables for N-4, N-6, CN-4, CN-6, S-4, S-6, CS-4 and CS-6 are all computed and tabulated in Tables 40-47, the canonical series of each region are easily calculated [12].

TABLE 39 - MINIMUM NUMBER OF YEARS TO DETECT THE INCREASE OF
10 PERCENT IN RUNOFF MEAN USING PRINCIPAL COMPONENTS

| Type | No. of principal components in target | No. of principal components in control | Value of $\underline{\mu}'\underline{V}^{-1}\underline{\mu}$ | $\tau^2$ | Minimum number of years to detect the increase, N* | Remarks |
|------|------|------|------|------|------|------|
| N-CN-4 | 4 | 4 | 1.066 | 11.655 | 11 | The minimum value of N* |
| N-CN-6 | 4 | 4 | 0.813 | 11.655 | 15 | is obtained from the |
| S-CS-4 | 3 | 2 | 0.243 | 8.640 | 36 | larger of $N^* = \tau^2/\underline{\mu}'\underline{V}^{-1}\underline{\mu}$ |
| S-CS-6 | 3 | 2 | 0.273 | 8.640 | 32 | or $N^* = k + 1$ where k |

is the total number of
components in both tar-
get and control regions

TABLE 40 - COEFFICIENTS FOR THE CANONICAL VARIABLES OF N-4

| CSU Sta. No. | USGS Sta. No. | 1st Variable | 2nd Variable | 3rd Variable | 4th Variable |
|------|------|------|------|------|------|
| 1970000 | 9.0105 | -.003956 | -.006628 | -.003592 | .018543 |
| 1960000 | 9.0110 | .003128 | -.009783 | .011935 | -.042535 |
| 1866000 | 9.0165 | .005767 | -.004685 | .009310 | |
| 1830000 | 9.0190 | .000796 | .003972 | -.002342 | .002199 |
| 1820000 | 9.0195 | -.001320 | -.002450 | -.001804 | .001937 |
| 1802730 | 9.0265 | .008752 | .008348 | .024461 | -.012694 |
| 1776000 | 9.0360 | .008385 | .002618 | -.023413 | .019209 |

TABLE 41 - COEFFICIENTS FOR THE CANONICAL VARIABLES OF N-6

| CSU Sta. No. | USGS Sta. No. | 1st Variable | 2nd Variable | 3rd Variable | 4th Variable |
|------|------|------|------|------|------|
| 1970000 | 9.0105 | -.006033 | -.009805 | -.005114 | .032451 |
| 1960000 | 9.0110 | .004802 | -.007516 | .011799 | -.033462 |
| 1866000 | 9.0165 | .009597 | .005297 | .041092 | -.001293 |
| 1830000 | 9.0190 | .001003 | .003991 | -.004721 | .002069 |
| 1820000 | 9.0195 | -.001910 | -.003885 | -.001016 | .002457 |
| 1802730 | 9.0265 | .013825 | .025201 | .014417 | .035857 |
| 1776000 | 9.0360 | .010705 | -.008078 | -.021553 | -.008330 |

TABLE 42 - COEFFICIENTS FOR THE CANONICAL VARIABLES OF CN-4

| CSU Sta. No. | USGS Sta. No. | 1st Variable | 2nd Variable | 3rd Variable | 4th Variable |
|------|------|------|------|------|------|
| 1742100 | 9.0535 | .001564 | .001900 | .003294 | .003207 |
| 1740000 | 9.0575 | .000620 | -.000087 | -.002110 | -.002931 |
| 1720000 | 9.0595 | .000086 | -.001640 | -.004363 | .002621 |
| 1666300 | 9.0645 | -.001139 | .015690 | .001530 | .004480 |
| 1594260 | 9.0780 | .001374 | .001985 | -.002575 | .003694 |
| 1594236 | 9.0785 | -.003596 | -.040136 | .019047 | .013573 |
| 1590000 | 9.0850 | .000525 | .000354 | -.001849 | -.001949 |
| 1379000 | 9.1090 | .002959 | .029446 | -.003503 | .007790 |
| 1378400 | 9.1100 | -.004647 | -.030526 | .005096 | -.010398 |
| 1378100 | 9.1125 | .001847 | .006202 | .009424 | .002551 |
| 1377825 | 9.1135 | .001334 | .011723 | .005682 | -.002092 |
| 1377500 | 9.1145 | -.000174 | .000685 | -.001047 | .000878 |
| 1377280 | 9.1155 | -.003380 | -.010015 | -.008777 | -.003425 |
| 1377230 | 9.1180 | .008358 | .033933 | .021453 | .033986 |

TABLE 43 - COEFFICIENTS FOR THE CANONICAL VARIABLES OF CN-6

| CSU Sta. No. | USGS Sta. No. | 1st Variable | 2nd Variable | 3rd Variable | 4th Variable |
|------|------|------|------|------|------|
| 1742100 | 9.0535 | .002365 | -.001262 | .006282 | .000515 |
| 1740000 | 9.0575 | .000555 | -.000926 | -.004814 | -.002931 |
| 1720000 | 9.0595 | .000399 | -.003421 | -.005800 | -.000807 |
| 1666300 | 9.0645 | -.002081 | .019407 | -.009901 | .012097 |
| 1594260 | 9.0780 | .002055 | .004230 | -.001362 | .003426 |
| 1594236 | 9.0785 | -.003831 | -.049581 | .040428 | .029201 |
| 1590000 | 9.0850 | .000478 | -.000866 | -.002470 | -.002815 |
| 1379000 | 9.1090 | .006095 | .041344 | -.006513 | -.013108 |
| 1378400 | 9.1100 | -.008394 | -.045125 | .001749 | -.003420 |
| 1378100 | 9.1125 | .004031 | .013690 | .010848 | .005814 |
| 1377825 | 9.1135 | .001566 | .017428 | -.000181 | -.005258 |
| 1377500 | 9.1145 | -.000219 | .000494 | -.001734 | .001375 |
| 1377280 | 9.1155 | -.005293 | -.011148 | -.003944 | .002038 |
| 1377230 | 9.1180 | .013811 | .053074 | .011299 | .031200 |

TABLE 44 - COEFFICIENTS FOR THE CANONICAL VARIABLES OF S-4

| CSU Sta. No. | USGS Sta. No. | 1st Variable | 2nd Variable | 3rd Variable | 4th Variable |
|------|------|------|------|------|------|
| 1278800 | 9.1650 | -.000949 | .010797 | -.004734 | .000415 |
| 1278050 | 9.1665 | .002273 | -.002086 | .003651 | -.002148 |
| 1272445 | 9.1725 | .000895 | .002056 | .008422 | -.004012 |
| 1077090 | 9.3440 | .000256 | .009180 | .003705 | -.009945 |
| 1073480 | 9.3575 | .007460 | .009551 | -.023825 | -.047496 |
| 1073436 | 9.3615 | -.003435 | -.003435 | -.002076 | .008598 |

TABLE 45 - COEFFICIENTS FOR THE CANONICAL VARIABLES OF S-6

| CSU Sta. No. | USGS Sta. No. | 1st Variable | 2nd Variable | 3rd Variable | 4th Variable |
|------|------|------|------|------|------|
| 1278800 | 9.1650 | -.001790 | .017228 | .001080 | -.003766 |
| 1278050 | 9.1665 | .003301 | -.004374 | .004401 | -.006282 |
| 1272445 | 9.1725 | .001264 | -.000937 | .010721 | -.014541 |
| 1077090 | 9.3440 | .000707 | .007274 | -.011061 | .011509 |
| 1073480 | 9.3575 | .014087 | .018233 | -.052315 | -.056559 |
| 1073436 | 9.3615 | -.001675 | -.002813 | .000808 | .013618 |

TABLE 46.- COEFFICIENTS FOR THE CANONICAL VARIABLES OF CS-4

| CSU Sta. No. | USGS Sta. No. | 1st Variable | 2nd Variable | 3rd Variable | 4th Variable |
|------|------|------|------|------|------|
| 1425625 | 9.0975 | .001734 | -.004611 | .004333 | -.005155 |
| 1377280 | 9.1155 | -.003347 | .055553 | .035314 | -.034470 |
| 1377230 | 9.1180 | -.005054 | .005968 | .000726 | .005480 |
| 1377200 | 9.1190 | .003365 | -.014545 | -.017608 | .013770 |
| 1373900 | 9.1275 | .000054 | -.003457 | .002488 | -.02948 |
| 1373055 | 9.1325 | .000225 | .000372 | -.000186 | -.000378 |
| 1373020 | 9.1345 | .002328 | .007410 | -.007410 | -.022485 |
| 1371815 | 9.1430 | .004076 | .010501 | .000507 | .023824 |
| 1370300 | 9.1520 | .010696 | .012852 | .036629 | .024040 |

TABLE 47.- COEFFICIENTS FOR THE CANONICAL VARIABLES OF CS-6

| CSU Sta. No. | USGS Sta. No. | 1st Variable | 2nd Variable | 3rd Variable | 4th Variable |
|------|------|------|------|------|------|
| 1425625 | 9.0975 | .003565 | -.010123 | .004390 | -.000819 |
| 1377280 | 9.1155 | -.007498 | .056512 | .054781 | -.081378 |
| 1377230 | 9.1180 | -.006890 | .014796 | -.000303 | .016373 |
| 1377200 | 9.1190 | .005324 | -.012051 | -.020296 | .025777 |
| 1373900 | 9.1275 | .004037 | -.053190 | -.028107 | -.030019 |
| 1373055 | 9.1325 | .000329 | .001040 | .000236 | -.003660 |
| 1373020 | 9.1345 | .004450 | -.000957 | -.031094 | .005098 |
| 1371815 | 9.1430 | .005299 | .012212 | .024251 | .036120 |
| 1370300 | 9.1520 | .010325 | .000287 | .053491 | .006772 |

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The series of the canonical variables are tabulated in Tables 48-55 for N-4, N-6, CN-4, CN-6, S-4, S-6, CS-4, and CS-6, respectively. The means and standard deviations of the canonical series are shown in Table 56.

TABLE 48 - N-4 CANONICAL SERIES (cfs)

| Year | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ |
|------|------|------|------|------|
| 1938 | 5.356 | -2.195 | -1.045 | .827 |
| 1939 | 2.963 | -2.067 | -1.608 | 1.737 |
| 1940 | 2.621 | -1.618 | .602 | 1.452 |
| 1941 | 4.203 | -1.884 | 3.477 | 2.474 |
| 1942 | 4.103 | - .582 | - .923 | 5.512 |
| 1943 | 3.128 | -2.357 | - .364 | 1.256 |
| 1944 | 2.971 | -2.235 | .303 | 1.703 |
| 1945 | 3.402 | -1.408 | - .306 | 2.703 |
| 1946 | 3.167 | -1.729 | .509 | 4.003 |
| 1947 | 5.015 | - .291 | -1.002 | 2.790 |
| 1948 | 3.835 | .538 | - .802 | 1.186 |
| 1949 | 3.636 | -3.039 | .383 | 3.314 |
| 1950 | 3.739 | .635 | - .523 | 2.882 |
| 1951 | 4.328 | -2.584 | .609 | 2.509 |
| 1952 | 4.910 | -2.824 | .700 | 2.171 |
| 1953 | 2.966 | -2.111 | .030 | 3.728 |
| 1954 | 1.655 | - .943 | .533 | 2.180 |
| 1955 | 2.723 | - .476 | .674 | 2.865 |
| 1956 | 3.098 | -1.871 | - .248 | 2.906 |
| 1957 | 4.865 | -2.033 | .654 | 3.208 |
| 1958 | 3.365 | -2.401 | .150 | 3.345 |
| 1959 | 2.979 | -1.403 | .070 | 2.835 |
| 1960 | 2.855 | -3.081 | -1.656 | 3.486 |
| 1961 | 2.184 | -2.026 | - .791 | 2.903 |
| 1962 | 3.978 | -3.188 | -1.473 | 1.454 |
| 1963 | 1.504 | -1.735 | .921 | 2.158 |
| 1964 | 2.148 | -2.119 | - .679 | 3.336 |
| 1965 | 3.473 | -3.180 | - .720 | 3.962 |
| 1966 | 1.651 | -1.613 | - .401 | 1.842 |
| 1967 | 2.634 | -2.746 | - .196 | 2.709 |

TABLE 49 - N-6 CANONICAL SERIES (cfs)

| Year | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ |
|------|------|------|------|------|
| 1938 | 5.220 | -2.656 | .059 | - .505 |
| 1939 | 2.893 | -2.476 | - .902 | 1.870 |
| 1940 | 2.679 | -1.385 | 1.301 | .978 |
| 1941 | 4.411 | - .711 | 4.330 | .885 |
| 1942 | 4.138 | - .805 | - .152 | 3.364 |
| 1943 | 3.198 | -2.354 | .342 | 1.723 |
| 1944 | 3.012 | -1.995 | 1.019 | 1.780 |
| 1945 | 3.747 | -1.514 | .188 | 2.513 |
| 1946 | 3.236 | -1.534 | 1.364 | 2.704 |
| 1947 | 5.197 | - .650 | - .825 | 1.721 |
| 1948 | 3.884 | .209 | - .482 | - .298 |
| 1949 | 3.712 | -2.711 | 1.024 | 3.491 |
| 1950 | 3.721 | .334 | - .572 | 1.811 |
| 1951 | 4.513 | -2.259 | 1.273 | 2.446 |
| 1952 | 5.098 | -2.365 | 1.539 | 1.697 |
| 1953 | 3.143 | -1.963 | .609 | 3.348 |
| 1954 | 1.736 | - .761 | .828 | 1.569 |
| 1955 | 2.933 | - .278 | .877 | 1.845 |
| 1956 | 3.152 | -1.874 | .550 | 1.478 |
| 1957 | 5.089 | -1.963 | 1.283 | 1.943 |
| 1958 | 3.370 | -2.207 | .873 | 2.707 |
| 1959 | 3.080 | -1.422 | 1.006 | .546 |
| 1960 | 2.774 | -3.571 | - .238 | 1.683 |
| 1961 | 2.289 | -2.217 | .168 | 1.377 |
| 1962 | 4.004 | -3.549 | - .196 | .046 |
| 1963 | 1.706 | -1.470 | 1.505 | 1.636 |
| 1964 | 2.227 | -2.279 | .342 | 1.726 |
| 1965 | 3.805 | -3.339 | 1.237 | 1.362 |
| 1966 | 1.851 | -1.664 | .974 | .259 |
| 1967 | 2.803 | -2.707 | 1.528 | .894 |

TABLE 50 - CN-4 CANONICAL SERIES (cfs)

| Year | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
|------|------|------|------|------|
| 1938 | 5.491 | .115 | -1.004 | - .420 |
| 1939 | 2.734 | - .116 | -1.548 | 1.356 |
| 1940 | 2.615 | .354 | .971 | 1.517 |
| 1941 | 4.068 | .150 | 3.984 | 1.335 |
| 1942 | 4.074 | 2.026 | - .968 | 4.727 |
| 1943 | 3.220 | .655 | .902 | - .005 |
| 1944 | 3.106 | - .109 | .309 | 1.172 |
| 1945 | 3.502 | 1.417 | .684 | 1.071 |
| 1946 | 3.261 | .466 | - .008 | .919 |
| 1947 | 5.064 | 2.293 | - .718 | 1.325 |
| 1948 | 4.185 | 2.310 | - .353 | .906 |
| 1949 | 3.901 | - .062 | .090 | 1.921 |
| 1950 | 3.628 | 2.314 | .803 | .889 |
| 1951 | 4.410 | .050 | .907 | 1.639 |
| 1952 | 4.792 | .306 | .297 | 1.187 |
| 1953 | 3.284 | -1.026 | .337 | 2.767 |
| 1954 | 1.706 | 1.475 | .351 | .867 |
| 1955 | 2.617 | 1.851 | .501 | 2.010 |
| 1956 | 3.454 | .852 | .812 | 1.785 |
| 1957 | 5.042 | - .562 | .234 | 1.013 |
| 1958 | 3.567 | -0. | 1.252 | 1.950 |
| 1959 | 3.244 | 1.619 | - .080 | 1.708 |
| 1960 | 2.788 | -1.017 | - .883 | 1.251 |
| 1961 | 2.161 | .360 | - .346 | .701 |
| 1962 | 3.934 | - .921 | -1.099 | .704 |
| 1963 | 1.605 | .767 | .207 | 1.260 |
| 1964 | 2.151 | .491 | .053 | 1.798 |
| 1965 | 3.539 | - .812 | .163 | 3.907 |
| 1966 | 1.896 | .599 | - .422 | .600 |
| 1967 | 2.791 | - .149 | .551 | 1.160 |

TABLE 51 - CN-6 CANONICAL SERIES (cfs)

| Year | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
|------|------|------|------|------|
| 1938 | 5.435 | - .743 | - .753 | -1.522 |
| 1939 | 2.761 | - .672 | -1.612 | 2.013 |
| 1940 | 2.720 | .376 | .907 | 1.868 |
| 1941 | 4.347 | 1.186 | 3.955 | .553 |
| 1942 | 4.185 | 1.426 | -1.182 | 3.186 |
| 1943 | 3.364 | .413 | .095 | 1.183 |
| 1944 | 3.191 | - .404 | .263 | 1.457 |
| 1945 | 3.889 | .973 | - .041 | 1.240 |
| 1946 | 3.395 | - .020 | - .033 | .823 |
| 1947 | 5.229 | 1.489 | -1.339 | .433 |
| 1948 | 4.290 | 1.883 | - .778 | - .139 |
| 1949 | 4.025 | - .170 | - .034 | 1.808 |
| 1950 | 3.739 | 2.145 | - .209 | .809 |
| 1951 | 4.659 | .076 | .437 | 2.087 |
| 1952 | 5.008 | .190 | - .012 | 1.158 |
| 1953 | 3.472 | -1.173 | .497 | 2.826 |
| 1954 | 1.825 | 1.054 | .052 | - .140 |
| 1955 | 2.919 | 1.474 | - .004 | 1.060 |
| 1956 | 3.554 | .719 | .244 | 1.561 |
| 1957 | 5.326 | - .878 | .242 | 1.018 |
| 1958 | 3.617 | .154 | .726 | 2.551 |
| 1959 | 3.364 | 1.098 | - .704 | 1.214 |
| 1960 | 2.811 | -1.515 | - .516 | .645 |
| 1961 | 2.313 | - .274 | - .274 | .450 |
| 1962 | 3.998 | -1.686 | - .719 | .351 |
| 1963 | 1.888 | .542 | - .128 | 1.010 |
| 1964 | 2.322 | .302 | .169 | .812 |
| 1965 | 3.921 | -1.187 | 1.097 | 1.002 |
| 1966 | 2.035 | .174 | - .273 | - .195 |
| 1967 | 3.040 | - .127 | 1.315 | - .522 |

## TABLE 52 - S-4 CANONICAL SERIES (cfs)

| Year | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ |
|------|-------|--------|--------|--------|
| 1938 | 4.095 | .364 | -2.671 | -4.194 |
| 1939 | 2.327 | .148 | - .980 | -5.317 |
| 1940 | 2.453 | - .015 | -1.387 | -4.918 |
| 1941 | 4.406 | 2.663 | .354 | -2.341 |
| 1942 | 4.391 | - .427 | -1.419 | -4.347 |
| 1943 | 2.809 | 2.182 | -2.294 | -3.036 |
| 1944 | 3.989 | - .096 | -1.276 | -1.624 |
| 1945 | 2.834 | 3.730 | -2.042 | -3.576 |
| 1946 | 2.414 | - .203 | -1.678 | -4.181 |
| 1947 | 3.075 | .861 | -3.444 | -4.148 |
| 1948 | 3.735 | .191 | -2.762 | -3.410 |
| 1949 | 3.437 | .397 | -4.331 | -1.361 |
| 1950 | 2.313 | - .805 | - .805 | -2.759 |
| 1951 | 1.656 | 1.645 | -2.952 | -4.037 |
| 1952 | 4.550 | 1.260 | -3.162 | -3.385 |
| 1953 | 1.937 | 1.592 | -1.501 | -3.254 |
| 1954 | 1.450 | .972 | -2.460 | -2.073 |
| 1955 | 1.786 | 1.169 | -1.523 | -2.373 |
| 1956 | 2.116 | 1.677 | -2.476 | -4.210 |
| 1957 | 4.638 | 2.586 | -2.328 | -5.720 |
| 1958 | 3.719 | 1.944 | - .092 | -2.539 |
| 1959 | 1.478 | 1.392 | -2.539 | -4.031 |
| 1960 | 2.794 | 1.582 | -2.113 | -3.544 |
| 1961 | 2.280 | 1.493 | -1.499 | -3.458 |
| 1962 | 2.926 | 1.492 | -2.984 | -3.647 |
| 1963 | 1.642 | 1.144 | -2.166 | -3.056 |
| 1964 | 1.992 | 1.696 | -1.201 | -3.911 |
| 1965 | 3.494 | 1.786 | -3.343 | -2.664 |
| 1966 | 2.378 | 1.268 | -2.392 | -3.432 |
| 1967 | 1.625 | 1.464 | -1.932 | -3.333 |

## TABLE 53 - S-6 CANONICAL SERIES (cfs)

| Year | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ |
|------|-------|--------|--------|--------|
| 1938 | 4.315 | .650 | -3.827 | -1.785 |
| 1939 | 2.608 | - .295 | -3.195 | -3.121 |
| 1940 | 2.712 | - .147 | -3.104 | -3.009 |
| 1941 | 4.493 | 1.887 | - .200 | -1.089 |
| 1942 | 4.659 | - .473 | -3.052 | -1.967 |
| 1943 | 3.037 | 2.460 | -2.033 | -2.540 |
| 1944 | 4.106 | .041 | -1.266 | -1.033 |
| 1945 | 2.979 | 3.682 | -1.863 | -2.466 |
| 1946 | 2.701 | - .155 | -2.611 | -3.445 |
| 1947 | 3.413 | 1.457 | -3.637 | -3.310 |
| 1948 | 3.950 | .576 | -3.158 | -2.331 |
| 1949 | 3.596 | 1.477 | -3.813 | .509 |
| 1950 | 2.501 | - .872 | -1.819 | -1.651 |
| 1951 | 1.887 | 2.023 | -3.754 | -2.471 |
| 1952 | 4.765 | 1.748 | -3.567 | -1.583 |
| 1953 | 2.149 | 1.476 | -2.031 | -2.269 |
| 1954 | 1.611 | 1.328 | -2.774 | - .413 |
| 1955 | 1.982 | 1.232 | -1.707 | -1.486 |
| 1956 | 2.338 | 1.780 | -3.107 | -2.676 |
| 1957 | 5.055 | 2.684 | -3.450 | -4.024 |
| 1958 | 3.792 | 1.383 | .019 | -2.808 |
| 1959 | 1.737 | 1.697 | -3.562 | -2.468 |
| 1960 | 2.923 | 1.542 | -2.718 | -1.512 |
| 1961 | 2.436 | 1.360 | -2.011 | -2.307 |
| 1962 | 3.145 | 1.739 | -3.473 | -1.878 |
| 1963 | 1.881 | 1.324 | -2.507 | -1.724 |
| 1964 | 2.215 | 1.561 | -1.735 | -3.363 |
| 1965 | 3.766 | 2.294 | -3.333 | - .807 |
| 1966 | 2.578 | 1.285 | -3.051 | - .899 |
| 1967 | 1.901 | 1.538 | -2.819 | -1.276 |

## TABLE 54 - CS-4 CANONICAL SERIES (cfs)

| Year | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
|------|-------|-------|-------|--------|
| 1938 | 2.930 | 1.215 | 1.510 | -3.400 |
| 1939 | 1.591 | 2.091 | 1.492 | -1.727 |
| 1940 | 1.759 | .584 | 3.668 | -2.275 |
| 1941 | 3.394 | 3.652 | 3.261 | - .218 |
| 1942 | 3.803 | .792 | 1.554 | -1.400 |
| 1943 | 1.592 | 2.963 | .687 | -1.611 |
| 1944 | 3.313 | 1.341 | 2.705 | -1.182 |
| 1945 | 2.203 | 4.083 | 2.156 | - .240 |
| 1946 | 1.248 | 1.090 | 1.964 | -1.471 |
| 1947 | 2.324 | 2.780 | .819 | - .605 |
| 1948 | 2.743 | .375 | .301 | - .867 |
| 1949 | 2.928 | 1.488 | - .815 | .334 |
| 1950 | 1.582 | 1.640 | .697 | -1.436 |
| 1951 | 1.172 | 3.505 | 1.263 | -1.684 |
| 1952 | 3.543 | 3.331 | .329 | - .868 |
| 1953 | 1.218 | 3.747 | 1.844 | -1.956 |
| 1954 | .790 | 1.778 | 1.233 | - .007 |
| 1955 | 1.242 | 2.389 | 1.687 | - .296 |
| 1956 | 1.121 | 2.417 | .685 | -1.332 |
| 1957 | 4.062 | 3.149 | - .353 | -3.382 |
| 1958 | 3.096 | 2.103 | 2.758 | - .973 |
| 1959 | .921 | 1.808 | 1.273 | -3.346 |
| 1960 | 1.737 | 3.652 | 1.126 | -2.610 |
| 1961 | .962 | 2.936 | 1.963 | -1.602 |
| 1962 | 2.434 | 2.865 | 1.149 | -2.768 |
| 1963 | .634 | 1.824 | .478 | -1.287 |
| 1964 | 1.480 | 2.019 | 1.262 | -2.769 |
| 1965 | 2.462 | 2.433 | .434 | -1.807 |
| 1966 | 1.234 | 1.464 | .990 | - .835 |
| 1967 | .924 | 1.732 | .216 | -1.053 |

## TABLE 55 - CS-6 CANONICAL SERIES (cfs)

| Year | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
|------|-------|--------|--------|--------|
| 1938 | 3.125 | .666 | - .268 | -1.690 |
| 1939 | 1.867 | 1.119 | .241 | - .796 |
| 1940 | 1.871 | - .579 | 1.762 | -2.090 |
| 1941 | 3.368 | 2.396 | 2.874 | .131 |
| 1942 | 3.969 | .358 | .766 | - .335 |
| 1943 | 1.786 | 2.652 | .400 | -1.213 |
| 1944 | 3.375 | .546 | 2.335 | -1.515 |
| 1945 | 2.292 | 3.207 | 2.501 | - .102 |
| 1946 | 1.463 | .475 | 1.185 | -1.301 |
| 1947 | 2.513 | 2.392 | .927 | .182 |
| 1948 | 2.948 | .382 | .056 | - .645 |
| 1949 | 3.056 | 1.832 | - .294 | .970 |
| 1950 | 1.764 | 1.431 | .319 | - .692 |
| 1951 | 1.245 | 3.109 | 1.424 | -2.270 |
| 1952 | 3.640 | 3.039 | .844 | - .329 |
| 1953 | 1.387 | 3.068 | 1.520 | -2.353 |
| 1954 | .848 | 1.630 | 1.126 | .792 |
| 1955 | 1.357 | 2.024 | 1.420 | .655 |
| 1956 | 1.223 | 2.196 | .605 | -1.273 |
| 1957 | 4.357 | 3.251 | -1.240 | -2.029 |
| 1958 | 3.165 | 1.284 | 2.821 | -1.556 |
| 1959 | 1.145 | 1.309 | - .355 | -1.342 |
| 1960 | 1.855 | 2.945 | .362 | - .880 |
| 1961 | 1.025 | 2.263 | 1.682 | -1.794 |
| 1962 | 2.618 | 2.358 | .671 | -2.190 |
| 1963 | .816 | 1.432 | - .026 | - .750 |
| 1964 | 1.645 | 1.454 | .473 | -2.581 |
| 1965 | 2.718 | 2.316 | - .082 | -1.514 |
| 1966 | 1.418 | 1.063 | .549 | - .018 |
| 1967 | 1.119 | 1.632 | - .258 | .303 |

TABLE 56 - MEANS AND STANDARD DEVIATIONS
OF CANONICAL VARIABLES

| Type | Canonical Variable | Mean (cfs) | Std. Dev. (cfs) |
|---|---|---|---|
| N-4 series | $\zeta_1$ | 3.315 | 1.000 |
| | $\zeta_2$ | -1.819 | 1.000 |
| | $\zeta_3$ | -0.104 | 1.000 |
| | $\zeta_4$ | 2.648 | 1.000 |
| N-6 series | $\zeta_1$ | 3.421 | 1.000 |
| | $\zeta_2$ | -1.804 | 1.000 |
| | $\zeta_3$ | .695 | 1.000 |
| | $\zeta_4$ | 1.620 | 1.000 |
| CN-4 series | $\varepsilon_1$ | 3.394 | 1.000 |
| | $\varepsilon_2$ | 0.523 | 1.000 |
| | $\varepsilon_3$ | 0.199 | 1.000 |
| | $\varepsilon_4$ | 1.434 | 1.000 |
| CN-6 series | $\varepsilon_1$ | 3.555 | 1.000 |
| | $\varepsilon_2$ | .227 | 1.000 |
| | $\varepsilon_3$ | .046 | 1.000 |
| | $\varepsilon_4$ | 1.020 | 1.000 |
| S-4 series | $\zeta_1$ | 2.825 | 1.000 |
| | $\zeta_2$ | 1.172 | 1.000 |
| | $\zeta_3$ | -2.047 | 1.000 |
| | $\zeta_4$ | -3.463 | 1.000 |
| S-6 series | $\zeta_1$ | 3.041 | 1.000 |
| | $\zeta_2$ | 1.276 | 1.000 |
| | $\zeta_3$ | -2.639 | 1.000 |
| | $\zeta_4$ | -2.040 | 1.000 |
| CS-4 series | $\varepsilon_1$ | 2.015 | 1.000 |
| | $\varepsilon_2$ | 2.241 | 1.000 |
| | $\varepsilon_3$ | 1.278 | 1.000 |
| | $\varepsilon_4$ | -1.489 | 1.000 |
| CS-6 series | $\varepsilon_1$ | 2.166 | 1.000 |
| | $\varepsilon_2$ | 1.775 | 1.000 |
| | $\varepsilon_3$ | .811 | 1.000 |
| | $\varepsilon_4$ | -0.941 | 1.000 |

Similar to the principal component analysis, it is clear now that,

$$E\{\zeta_i^*\} = (1+h) E\{\zeta_i\}$$

where 100h is the percent increase of the runoff means in the target region. If h = 0.10 , then,

$$E\{\zeta_i^*\} = 1.1 E\{\zeta_i\}$$

and

$$E\{\varepsilon_i^*\} = E\{\varepsilon_i\}$$

where $\varepsilon_i^*$ is the $i^{th}$ canonical variable of the control region for the seeded period.

The covariance matrices of N-CN-4, N-CN-6, S-CS-4, and S-CS-6 are shown in Tables 57-60, respectively. In this analysis the correlation matrices are the same as the covariance matrices since all the canonical variables have unit variances.

5.4 The minimum number of years needed to detect a 10% increase in runoff based on the canonical variables. As discussed before in Section 5.2, the minimum number of years needed to detect the increase can be obtained with the use of Table 1, which gives the value of $\tau^2$ . After the canonical analysis has been performed because the high corre-

TABLE 57 - COVARIANCE MATRIX OF N-CN-4 CANONICAL SERIES

| | | N-4 | | | | CN-4 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
| N-4 | $\zeta_1$ | 1.000 | 0. | 0. | 0. | .989 | 0. | 0. | 0. |
| | $\zeta_2$ | 0. | 1.000 | 0. | 0. | 0. | .890 | 0. | 0. |
| | $\zeta_3$ | 0. | 0. | 1.000 | 0. | 0. | 0. | .847 | 0. |
| | $\zeta_4$ | 0. | 0. | 0. | 1.000 | 0. | 0. | 0. | .767 |
| CN-4 | $\varepsilon_1$ | .989 | 0. | 0. | 0. | 1.000 | 0. | 0. | 0. |
| | $\varepsilon_2$ | 0. | .890 | 0. | 0. | 0. | 1.000 | 0. | 0. |
| | $\varepsilon_3$ | 0. | 0. | .847 | 0. | 0. | 0. | 1.000 | 0. |
| | $\varepsilon_4$ | 0. | 0. | 0. | .767 | 0. | 0. | 0. | 1.000 |

TABLE 58 - COVARIANCE MATRIX OF N-CN-6 CANONICAL SERIES

| | | N-6 | | | | CN-4 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
| N-6 | $\zeta_1$ | 1.000 | 0. | 0. | 0. | .990 | 0. | 0. | 0. |
| | $\zeta_2$ | 0. | 1.000 | 0. | 0. | 0. | .894 | 0. | 0. |
| | $\zeta_3$ | 0. | 0. | 1.000 | 0. | 0. | 0. | .869 | 0. |
| | $\zeta_4$ | 0. | 0. | 0. | 1.000 | 0. | 0. | 0. | .768 |
| CN-6 | $\varepsilon_1$ | .990 | 0. | 0. | 0. | 1.000 | 0. | 0. | 0. |
| | $\varepsilon_2$ | 0. | .894 | 0. | 0. | 0. | 1.000 | 0. | 0. |
| | $\varepsilon_3$ | 0. | 0. | .869 | 0. | 0. | 0. | 1.000 | 0. |
| | $\varepsilon_4$ | 0. | 0. | 0. | .768 | 0. | 0. | 0. | 1.000 |

TABLE 59 - COVARIANCE MATRIX OF S-CS-4 CANONICAL SERIES

| | | S-4 | | | | CS-4 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
| S-4 | $\zeta_1$ | 1.000 | 0. | 0. | 0. | .968 | 0. | 0. | 0. |
| | $\zeta_2$ | 0. | 1.000 | 0. | 0. | 0. | .771 | 0. | 0. |
| | $\zeta_3$ | 0. | 0. | 1.000 | 0. | 0. | 0. | .703 | 0. |
| | $\zeta_4$ | 0. | 0. | 0. | 1.000 | 0. | 0. | 0. | .617 |
| CS-4 | $\varepsilon_1$ | .968 | 0. | 0. | 0. | 1.000 | 0. | 0. | 0. |
| | $\varepsilon_2$ | 0. | .771 | 0. | 0. | 0. | 1.000 | 0. | 0. |
| | $\varepsilon_3$ | 0. | 0. | .703 | 0. | 0. | 0. | 1.000 | 0. |
| | $\varepsilon_4$ | 0. | 0. | 0. | .617 | 0. | 0. | 0. | 1.000 |

TABLE 60 - COVARIANCE MATRIX OF S-CS-6 CANONICAL SERIES

| | | S-6 | | | | CS-6 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
| S-6 | $\zeta_1$ | 1.000 | 0. | 0. | 0. | .969 | 0. | 0. | 0. |
| | $\zeta_2$ | 0. | 1.000 | 0. | 0. | 0. | .777 | 0. | 0. |
| | $\zeta_3$ | 0. | 0. | 1.000 | 0. | 0. | 0. | .696 | 0. |
| | $\zeta_4$ | 0. | 0. | 0. | 1.000 | 0. | 0. | 0. | .568 |
| CS-6 | $\varepsilon_1$ | .969 | 0. | 0. | 0. | 1.000 | 0. | 0. | 0. |
| | $\varepsilon_2$ | 0. | .777 | 0. | 0. | 0. | 1.000 | 0. | 0. |
| | $\varepsilon_3$ | 0. | 0. | .696 | 0. | 0. | 0. | 1.000 | 0. |
| | $\varepsilon_4$ | 0. | 0. | 0. | .568 | 0. | 0. | 0. | 1.000 |

lation between target and control variables are desirable here, only the highly correlated canonical variables will be retained for further study.

For example, consider the case of S-CS-4. The correlation between the first canonical variable in S-4 and the first canonical variable in CS-4 is found to be 0.968, which is the maximum of all the correlations between the canonical variables for S-CS-4. If it is decided to use only these two canonical variables in the test, then all one needs to do is the following. From Table 56, obtain

$$\underline{\mu}_o = \begin{bmatrix} 2.825 \\ 2.015 \end{bmatrix} .$$

Assuming that there is an increase of 10% in the means of the target region and the means in the control region remain unchanged, then, the mean vector for the seeding period can be obtained as

$$\underline{\mu}^* = \begin{bmatrix} 3.107 \\ 2.015 \end{bmatrix} .$$

Now $\underline{\mu} = (\underline{\mu}^* - \underline{\mu}_0)$, that is,

$$\underline{\mu} = \begin{bmatrix} 3.107 \\ 2.015 \end{bmatrix} - \begin{bmatrix} 2.825 \\ 2.015 \end{bmatrix}$$

$$\underline{\mu} = \begin{bmatrix} 0.282 \\ 0.0 \end{bmatrix} .$$

Compute the inverse of the covariance matrix of the first canonical variables in the target and control regions, $\underline{V}^{-1}$. In this case,

$$\underline{V}^{-1} = \begin{bmatrix} 15.879 & -15.371 \\ -15.371 & 15.879 \end{bmatrix}$$

and then compute,

$$\underline{\mu}'\underline{V}^{-1}\underline{\mu} = [0.282 \quad 0.0] \begin{bmatrix} 15.879 & -15.371 \\ -15.371 & 15.879 \end{bmatrix} \begin{bmatrix} 0.282 \\ 0.0 \end{bmatrix}$$

$$= 1.271 .$$

TABLE 61 - INVERSE OF COVARIANCE MATRIX OF N-CN-4 CANONICAL SERIES

| | | N-4 | | | | CN-4 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
| N-4 | $\zeta_1$ | 45.706 | 0. | 0. | 0. | -45.203 | 0. | 0. | 0. |
| | $\zeta_2$ | 0. | 4.810 | 0. | 0. | 0. | -4.281 | 0. | 0. |
| | $\zeta_3$ | 0. | 0. | 3.539 | 0. | 0. | 0. | -2.997 | 0. |
| | $\zeta_4$ | 0. | 0. | 0. | 2.429 | 0. | 0. | 0. | -1.863 |
| CN-4 | $\varepsilon_1$ | -45.203 | 0. | 0. | 0. | 45.706 | 0. | 0. | 0. |
| | $\varepsilon_2$ | 0. | -4.281 | 0. | 0. | 0. | 4.810 | 0. | 0. |
| | $\varepsilon_3$ | 0. | 0. | -2.997 | 0. | 0. | 0. | 3.539 | 0. |
| | $\varepsilon_4$ | 0. | 0. | 0. | -1.863 | 0. | 0. | 0. | 2.429 |

TABLE 62 - INVERSE OF COVARIANCE MATRIX OF N-CN-6 CANONICAL SERIES

| | | N-6 | | | | CN-6 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
| N-6 | $\zeta_1$ | 50.251 | 0. | 0. | 0. | -49.749 | 0. | 0. | 0. |
| | $\zeta_2$ | 0. | 4.981 | 0. | 0. | 0. | -4.453 | 0. | 0. |
| | $\zeta_3$ | 0. | 0. | 4.084 | 0. | 0. | 0. | -3.549 | 0. |
| | $\zeta_4$ | 0. | 0. | 0. | 2.438 | 0. | 0. | 0. | -1.872 |
| CN-6 | $\varepsilon_1$ | -49.749 | 0. | 0. | 0. | 50.251 | 0. | 0. | 0. |
| | $\varepsilon_2$ | 0. | -4.453 | 0. | 0. | 0. | 4.981 | 0. | 0. |
| | $\varepsilon_3$ | 0. | 0. | -3.549 | 0. | 0. | 0. | 4.084 | 0. |
| | $\varepsilon_4$ | 0. | 0. | 0. | -1.872 | 0. | 0. | 0. | 2.438 |

TABLE 63 - INVERSE OF COVARIANCE MATRIX OF S-CS-4 CANONICAL SERIES

| | | S-4 | | | | CS-4 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
| S-4 | $\zeta_1$ | 15.879 | 0. | 0. | 0. | -15.371 | 0. | 0. | 0. |
| | $\zeta_2$ | 0. | 2.466 | 0. | 0. | 0. | -1.901 | 0. | 0. |
| | $\zeta_3$ | 0. | 0. | 1.977 | 0. | 0. | 0. | -1.390 | 0. |
| | $\zeta_4$ | 0. | 0. | 0. | 1.615 | 0. | 0. | 0. | -.996 |
| CS-4 | $\varepsilon_1$ | -15.371 | 0. | 0. | 0. | 15.879 | 0. | 0. | 0. |
| | $\varepsilon_2$ | 0. | -1.901 | 0. | 0. | 0. | 2.466 | 0. | 0. |
| | $\varepsilon_3$ | 0. | 0. | -1.390 | 0. | 0. | 0. | 1.977 | 0. |
| | $\varepsilon_4$ | 0. | 0. | 0. | -.996 | 0. | 0. | 0. | 1.615 |

TABLE 64 - INVERSE OF COVARIANCE MATRIX OF S-CS-6 CANONICAL SERIES

| | | S-6 | | | | CS-6 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ |
| S-6 | $\zeta_1$ | 16.383 | 0. | 0. | 0. | -15.875 | 0. | 0. | 0. |
| | $\zeta_2$ | 0. | 2.524 | 0. | 0. | 0. | -1.961 | 0. | 0. |
| | $\zeta_3$ | 0. | 0. | 1.940 | 0. | 0. | 0. | -1.350 | 0. |
| | $\zeta_4$ | 0. | 0. | 0. | 1.476 | 0. | 0. | 0. | -.839 |
| CS-6 | $\varepsilon_1$ | -15.875 | 0. | 0. | 0. | 16.383 | 0. | 0. | 0. |
| | $\varepsilon_2$ | 0. | -1.961 | 0. | 0. | 0. | 2.524 | 0. | 0. |
| | $\varepsilon_3$ | 0. | 0. | -1.350 | 0. | 0. | 0. | 1.940 | 0. |
| | $\varepsilon_4$ | 0. | 0. | 0. | -.839 | 0. | 0. | 0. | 1.476 |

The degrees of freedom here are 2 and 28, which are the number of canonical variables and the number of observations less the number of canonical variables, respectively. With these degrees of freedom, the value of $\tau^2$ is found to be 5.468, at the level of significance $\alpha = 0.05$ and power $\beta = 0.50$. Now from

$$N^* = \frac{\tau^2}{\underline{\mu}'\underline{V}^{-1}\underline{\mu}}$$

the value of $N^*$ is obtained as

$$N^* = \frac{5.468}{1.271} = 4.3 = 5 \text{ years} ,$$

since $N^*$ must be an integer. These values of $N^*$ are shown in Table 65.

The previous results are based on the assumption that the sample mean is the same as the population mean during the non-seeded period. Now consider what effect a violation of this assumption would have on the results.

Suppose the true population mean is not equal to the sample mean. Instead it lies at the upper extremity of the 50% confidence interval established for the sample mean of the non-seeded period. Then a 10% increase in the true population mean results in a larger absolute increase than does a 10% increase in the assumed population mean (simply because the actual population mean is larger than the assumed population mean).

In the northern region, an actual 10% increase in the true population mean yields a 14.2% increase in the assumed population mean. This results in a reduction in the number of observations required to detect a change. The number of observations would be reduced to 50% of the previously determined number of observations. Similarly, in the southern region an

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TABLE 65 - MINIMUM NUMBER OF YEARS TO DETECT THE INCREASE OF
10 PERCENT IN RUNOFF MEANS USING CANONICAL VARIABLES

| Type | No. of canonical variables in target | No. of canonical variables in control | Value of $\underline{\mu}'\underline{V}^{-1}\underline{\mu}$ | $\tau^2$ | Minimum number of years to detect the increase, N* | Remarks |
|---|---|---|---|---|---|---|
| N-CN-4 | 1 | 1 | 5.037 | 5.468 | 3 | The minimum value of N* |
| | 2 | 2 | 5.197 | 7.640 | 5 | is obtained from the |
| | 3 | 3 | 5.198 | 9.646 | 7 | larger of $N* = \tau^2/\underline{\mu}'\underline{V}^{-1}\underline{\mu}$ |
| | 4 | 4 | 5.368 | 11.655 | 9 | |
| N-CN-6 | 1 | 1 | 5.877 | 5.468 | 3 | or $N* = k + 1$ where k |
| | 2 | 2 | 6.040 | 7.640 | 5 | is the total number of |
| | 3 | 3 | 6.060 | 9.646 | 7 | variables in both target |
| | 4 | 4 | 6.124 | 11.655 | 9 | and control |
| S-CS-4 | 1 | 1 | 1.271 | 5.468 | 5 | |
| | 2 | 2 | 1.305 | 7.640 | 6 | |
| | 3 | 3 | 1.388 | 9.646 | 7 | |
| | 4 | 4 | 1.581 | 11.655 | 9 | |
| S-CS-6 | 1 | 1 | 1.423 | 5.468 | 4 | |
| | 2 | 2 | 1.465 | 7.640 | 6 | |
| | 3 | 3 | 1.690 | 9.646 | 7 | |
| | 4 | 4 | 1.752 | 11.655 | 9 | |

actual 10% increase in the true population mean yields
a 15.6% increase in the assumed population mean, and
a corresponding reduction in the required number of
observations by 60 percent.

Now, suppose that the true population mean lies
at the lower end of the 50% confidence interval.
Then a 10% increase in the true population mean re-
sults in a smaller absolute increase than does a 10%
increase in the assumed population mean.

In the northern region, an actual 10% increase
in the true population mean yields a 5.8% increase in
the assumed population mean. This results in an

increase in the number of observations required to
detect a change. The number of observations would be
increased by a factor of three. Similarly, in the
southern region an actual 10% increase in the true
population mean yields a 4.4% increase in the assumed
population mean, and the number of observations re-
quired would be increased by a factor of 5.2.

In view of the above discussion, it is seen
that if the number of observations is calculated by
assuming different values for the population mean a
distribution is obtained. The median number of
observations will be the same as that number obtained
by using the sample mean of the non-seeded period.

Chapter VI

CONCLUSIONS

It was the objective of this study to develop a technique for detection of a geographically widespread change in a minimum amount of time.

It was found that a combination of techniques, namely canonical analysis and multivariate $T^2$ test was the most effective means to provide positive results in the least time. Assuming a 10% increase in runoff, 3 and 4 years are the minimum number of years needed for significance in the Upper Basin of the Colorado and the San Juan Mountains, respectively.

A word of caution is needed at this point. If the effect of precipitation management is to produce exactly a uniform 10% increase in runoff the use of only one set of canonical components is very efficient. However, if the increase is not uniform, it is safer to use several canonical components. With more canonical components, however, the number of years needed for significance increases.

It is apparent that there exists a trade-off between power of the test and representativity of the tested variables. This is well illustrated by the combined use of principal components analysis and the $T^2$ test. The first three or four principal components account for 99% of the total variation in the target regions. These sets of components so to speak, are 99% representative. The number of years calculated from the $T^2$ test is much higher than the corresponding figure for the same number of canonical components. This number of years could be decreased by using only one principal component, which already accounts on the average for 90% of the total variation. (This number was not actually calculated but the validity of the statement can be inferred from examination of the covariance matrices).

Note that when the $\chi^2$-test is applied to each target station with the best correlated control station, the lowest minimum number of years is found to be seven in both northern and southern regions. Again, a single station is, of course, poorly representative of the entire region. The technique (canonical components - $T^2$ test) improves both the power of the test and the regional representativity of the tested variable, over what it would have been even with the best single target control pair.

The results from the use of four-months or six-months spring runoff are very similar. Nevertheless, better results are obtained with the six-months runoff series, particularly in the southern area.

## LIST OF SYMBOLS

| Symbol | Meaning |
|---|---|
| $Q_i$ | Runoff at station i (i is the number in the 'Seq. No.' column in Table 2) |
| $q_i$ | Observation of $Q_i$ |
| $\bar{q}_i$ | The mean of $Q_i$ |
| $q_{i,m}$ | The $m^{th}$ observation of $Q_i$ |
| $\underline{Q}$ | Column vector of runoff at all stations |
| $\underline{q}_i$ | Column vector of the $i^{th}$ observation of $\underline{Q}$ |
| $\bar{\underline{q}}$ | Mean vector of observations of $\underline{Q}$ |
| N | Number of observations of non-seeded period |
| N* | Minimum number of years for detecting a 10% increase in the runoff means of seeded period |
| N-4 | Four-month runoff series in the northern target region (the 4 months are: April, May, June, and July) |
| N-6 | Six-month runoff series in the northern target region (the 6 months are: March, April, May, June, July and August) |
| CN-4 | Four-month runoff series in the northern control region |
| CN-6 | Six-month runoff series in the northern control region |
| S-4 | Four-month runoff series in the southern target region |
| S-6 | Six-month runoff series in the southern target region |
| CS-4 | Four-month runoff series in the southern control region |
| CS-6 | Six-month runoff series in the southern control region |
| N-CN-4 | The combination of N-4 and CN-4 |
| N-CN-6 | The combination of N-6 and CN-6 |
| S-CS-4 | The combination of S-4 and CS-4 |
| S-CS-6 | The combination of S-6 and CS-6 |
| k | Total number of runoff variables, i.e., the number of all target and control variables |
| h | The fractional increase in the runoff mean |
| E{} | The expected value of {} |
| p | The number of runoff variables in target (or control) region in the principal component analysis |
| $p_1$ | The number of runoff variables in target region |
| $p_2$ | The number of runoff variables in control region |
| $\underline{\beta}_i$ | Column vector of coefficients for computing the $i^{th}$ principal component |

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| Symbol | Meaning |
|--------|---------|
| $\beta_{ij}$ | Coefficient of runoff at station j in the computation of the $i^{th}$ principal component |
| $I$ | Identity matrix |
| $\underline{V}$ | Covariance matrix of runoff variables |
| $\underline{V}^{-1}$ | Inverse of $\underline{V}$ |
| $\underline{W}$ | $\underline{V}^{-1}$ |
| $\xi_i$ | The $i^{th}$ principal component of target region before seeding |
| $\xi_i^*$ | The $i^{th}$ principal component of target region for seeded period |
| $\eta_i$ | The $i^{th}$ principal component of control region before seeding |
| $\eta_i^*$ | The $i^{th}$ principal component of control region for seeded period |
| $\xi_{i,m}$ | The $m^{th}$ data point of $\xi_i$ |
| $\eta_{i,m}$ | The $m^{th}$ data point of $\eta_i$ |
| $\lambda_i$ | The amount of variance accounted for by the $i^{th}$ principal component |
| $\zeta_i$ | The $i^{th}$ canonical variable of target region before seeding |
| $\zeta_i^*$ | The $i^{th}$ canonical variable of target region for seeded period |
| $\varepsilon_i$ | The $i^{th}$ canonical variable of control region before seeding |
| $\varepsilon_i^*$ | The $i^{th}$ canonical variable of control region for the seeded period |
| $\zeta_{i,m}$ | The $m^{th}$ data point of $\zeta_i$ |
| $\varepsilon_{i,m}$ | The $m^{th}$ data point of $\varepsilon_i$ |
| $\theta_i$ | Correlation between $\zeta_i$ and $\varepsilon_i$ |
| $\underline{\alpha}_i$ | Vector of coefficients for computing $\zeta_i$ |
| $\underline{\gamma}_i$ | Vector of coefficients for computing $\varepsilon_i$ |
| $\alpha_{i,j}$ | Coefficient of runoff at station j (target region) in the computation of $\zeta_i$ |
| $\gamma_{i,j}$ | Coefficient of runoff at station j (control region) in the computation of $\varepsilon_i$ |
| $\underline{\mu}^*$ | Runoff mean vector for the seeded period |
| $\underline{\mu}_o$ | Runoff mean vector for the non-seeded period |
| $\underline{\mu}$ | $\underline{\mu}^* - \underline{\mu}_o$ |
| $\underline{\mu}'$ | Transpose of $\underline{\mu}$ |
| $\sum\limits_{i=1}^{N}$ | Summation from $i=1$ to $i=N$ |
| $\prod\limits_{i=1}^{N}$ | Product from $i=1$ to $i=N$ |
| $\tau^2$ | Noncentrality parameter |
| $\hat{\phantom{a}}$ | Estimated value |

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LIST OF SYMBOLS - Continued

| Symbol | Meaning |
|--------|---------|
| ' | Transpose of a matrix |
| $\sigma_{ii}$ | Variance of runoff variable $Q_i$ |
| $\sigma_{ij}$ | Covariance of runoff variables $Q_i$ and $Q_j$ |
| * | Of seeded period |
| cfs | Cubic feet per second |

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Abstract: The object of this study is to find answers to the following questions: What is the appropriate statistical test for a regional target-control technique of evaluation? What is a suitable method for reduction of an originally large number of variables? Which of the Upper Basin of the Colorado River or the San Juan Mountains is a more suitable area of operations, if the effectiveness of precipitation management is to be detected as quickly as possible? The results of this research study show: 1. The $T^2$-test is the appropriate test for multiple target-control technique of evaluation. 2. The canonical anlysis is the suitable method for the reduction of a large number of original variables. 3. The Upper Basin of the Colorado River is preferable under the assumption of an equal percentage of increase in runoff. However, if the percentage increase in the southern area is at least 1.2 times as large as in the northern area (and recent publications suggest that this ratio is probably around 3) then the southern area is preferable. Based on the $T^2$-test, the minimum number of years for detecting an increase of 10 percent in spring runoff means are three years in the Upper Basin of the Colorado River, and four years in the San Juan Mountains.

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