DISSERTATION

STEADY UPWARD FLOW FROM WATER TABLES

Submitted by

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Symbol	Definition	Dimensions
C	hydraulic conductivity	LT^{-1}
C_e	effective hydraulic conductivity	LT^{-1}
C_r	relative hydraulic conductivity	none
d	depth (from the surface of soil) to a water table	L
$d_.$	scaled depth to water table, $d\gamma/p_b$	none
g	acceleration due to gravity	LT^{-2}
h	hydraulic head	L
h_b	negative head on upper barrier	L
K	permeability	L^2
K_e	effective permeability	L^2
K_r	relative permeability	none
p	pressure	FL^{-2}
p_c	capillary pressure	FL^{-2}
p_e	air-entry pressure	FL^{-2}
p_b	bubbling pressure	FL^{-2}
p_{nw}	pressure in the non-wetting phase	FL^{-2}
p_w	pressure in wetting phase	FL^{-2}
$p_.$	scaled capillary pressure, p_c/p_b	none
q	volume flux rate	LT^{-1}
q_m	maximum upward flow rate	LT^{-1}
$q_.$	scaled volume flux rate, q/C	none

LIST OF SYMBOLS USED--Continued

Symbol	Definition	Dimensions
$q_{.m}$	scaled maximum upward flow rate, q_m/C	none
x	a scalar variable	various
z	elevation above water table	L
Z.	scale of elevation above water table	none
η	negative slope of k_r vs p_c/γ curve	none
γ	specific gravity of fluid	FL^{-3}
ρ	mass density of fluid	$FL^{-4}T^{-2}$
μ	dynamic viscosity of fluid	FTL^{-2}
μ^2	square micron unit of permeability equal to 10^{-8} square centimeters	L^2

CHAPTER I

INTRODUCTION

Upward flow from water tables, subsequent to evaporation from soil surfaces is an important phenomenon, expecially in irrigated areas. In seventeen western states of U.S.A. estimates are that 45 percent of the water used for irrigation is lost by evaporation from soil surfaces. Irrigation farmers sometimes find it advantageous to maintain water tables within a few feet of the soil surface because of the moisture supplied to root zones by upward flow from water tables, but in areas where alkali salts occur, the soluble salts carried by upward flow are deposited on or near the soil surface. Studies of upward flow from water tables therefore are significant in the determination of an economic depth of water table.

Problem

Many investigations have been made on actual rates of evaporation from soils in which there is a water table. There is general agreement that there are two maximum possible rates of evaporation from water tables. One is the potential evaporation rate determined by the external conditions (approximated by the rate of evaporation from a free water surface), and the other is the maximum rate at which the soil itself can transmit water upward from the water table. Evaporation rates will be limited by the lesser of these two capacities. Another important conclusion is that, except for shallow water-table depths or humid conditions, evaporation rates will be limited mostly by the capacity of soils to transmit water from water tables. This study is an investigation of the maximum upward flow rates possible under various soil-water systems in which a water table exists.

Analysis of Problem

Since hysteresis plays an important role with respect to the maximum evaporation rate [10,23,35], this study is separated into three cases as follows:

Case I. The soil-water system following a drainage cycle. This case occurs in nature when the rate of evaporation from a soil surface does not exceed the rate of upward flow, and the water table is either constant or increases in depth.

Case II. The soil water system partly follows a drainage cycle and partly imbibition. This phenomenon can be caused by a reversal of the direction of change of pressure at some region in the soil-water system. It often occurs when the potential rate (due to external evaporative conditions) is appreciably greater than the rate at which water can be transmitted from the water table to the surface. The soil near the surface being relatively dry, water is imbibed into this dry zone. A fluctuation of water-table depth may also cause this phenomenon.

Case III. The soil water system follows an imbibition cycle only. This condition probably occurs in nature only rarely, but an appreciable rise of a deep water table might produce a phenomenon close to this case.

This study therefore, attempts to answer the following questions:

1. What is the maximum rate of upward flow as a function of measurable parameters of the soil for cases in which a soil water system following a drainage cycle?

2. How does hysteresis resulting from evaporation exceeding the upward flow rate affect the maximum rate of upward flow?

3. What is the maximum upward flow rate in a soil-water system completely following an imbibition cycle?

Experimental Conditions

A. The lack of head room in the laboratory restricted the column length for which maximum rates of upward flow could be measured. Instead of water, a hydrocarbon fluid, having a density of about 0.75 gms per cubic cm. and a surface tension of about 23 dynes, was used. This reduced the column length necessary to simulate a given water-table depth to about half that which would have been necessary if water had been used. Hydrocarbon also has less tendency to cause the soil structure to change with time during the period in which the flow measurements are made.

B. Upward flow was induced by means of a siphon attached to a capillary barrier placed at the top of the soil column.

C. Soil samples used had a saturated permeability ranging approximately from 0.5 to 30 square microns.

D. Measurements were made on the drainage, imbibition-drainage, and imbibition cycles.

E. The imbibition cycle did not begin with completely dry soil, but with the capillary pressure of the soil in the column corresponding nearly to the residual saturation of the soil.

F. The water table was maintained within or below the soil column, and its location was obtained by means of Darcy's law from the capillary pressure at a point of fully saturated soil within the soil column.

G. The temperature was controlled so that thermal gradients were assumed to be negligible.

Definition of Terms Used

Wetting phase--The fluid phase which tends to be preferentially adsorbed by the solid surfaces and is at lower pressure than other fluid phases that may occupy the same porous medium.

Non-wetting phase--A fluid phase which is not preferentially adsorbed and is at higher pressure than a wetting phase occupying the same porous medium.

Permeability--The capacity of a porous medium to transmit a single fluid phase. Sometimes called "intrinsic permeability," it is a function only of the properties of the porous medium.

Effective permeability--The permeability of a porous medium to a particular fluid phase when the medium is occupied by more than one phase.

Relative permeability--The ratio of the effective permeability of a particular fluid phase to the permeability of the medium when saturated with that phase.

Hydraulic conductivity--The capacity of a porous medium to transmit a single fluid of a particular viscosity and density. It is a function of both fluid and medium properties.

Effective hydraulic conductivity--The hydraulic conductivity of a porous medium to a particular fluid when occupied by more than one fluid phase.

Relative hydraulic conductivity--The ratio of the effective hydraulic conductivity of the medium when saturated with that phase.

Capillary pressure--The pressure difference across an interface between the non-wetting and wetting phase.

Air-entry pressure--The capillary pressure at which the non-wetting phase (i.e., air) first enters the largest pores of a porous medium.

Bubbling pressure--The extraporation of the straight line relationship between $\ln p_c$ and $\ln K_r$ to the ordinate representing K_r of unity.

Saturation--The fraction of the total pore volume occupied by the wetting phase.

Residual saturation--The maximum saturation at which the wetting phase permeability is assumed to be zero.

Capillary barrier--A porous plate which sufficiently high entry pressure as to permit transmission only of the wetting phase at wetting phase pressures less than the non-wetting phase pressure.

Desaturation--Displacement of wetting phase by a non-wetting phase.

Unsaturated flow--Flow of the wetting phase when the porous medium is occupied by more than one fluid phase.

Pressure reversal--A change from a decreasing liquid pressure to an increasing pressure (or vice versa) at some point in a system during an unsteady flow process.

Saturation hysteresis--The process by which a porous solid has reached its present state of saturation, i.e., drainage or imbibition cycle or a combination of these.

Drainage cycle--A condition in which a porous medium, initially fully saturated with a wetting phase, is desaturated by a continuously increasing capillary pressure at all points in the medium.

Imbibition cycle--A condition in which a porous medium initially containing a non-wetting phase, imbibes a wetting phase as a result of a continuously decreasing capillary pressure in all parts of the medium.

Imbibition-drainage cycle--A saturation history in which the wetting phase is desaturated with a reversal of the direction of capillary pressure change at some region within the fluid system.

Water table--The locus of points where the capillary pressure is zero. This term is used here when a hydrocarbon liquid is the wetting phase.

Free water surface--A water table which is exposed to the ambient atmosphere.

Hysteresis--A change in a functional relationship between variables caused by a reversal in the direction of a process. It is used in this dissertation to designate a change in the functional relationship between permeability and capillary pressure when an increasing capillary pressure changes to a decreasing capillary pressure.

CHAPTER II

REVIEW OF THE LITERATURE

Factors influencing evaporation from soil surfaces can be separated into two general factors. One is the external evaporative factors such as wind velocity, ambient temperature, relative humidity and radiation. The other is the ability of the soil-water system to transmit water upward which is related to the depth of the water table and properties of the soil. The combination of these factors causes the process of evaporation to be divided into three stages each of which is associated with a different rate of evaporation.

Stages of Evaporation Process

Koliasev [26], probably was the first to visualize evaporation from soils as occurring in three successive stages when the external factors are constant and the water table is either absent or is lowered by the evaporation. The stages are:

> 1. a period of steady rate of evaporation . . . In this period the rate of evaporation depends exclusively on the external factors . . .
> 2. a period of fast decline of evaporation which depends on the means of flow of moisture to the evaporation surface and also on the physical condition of the water-furnishing layer and finally

3. a period of slowly declining rate of evaporation caused by the firmness with which water is tied to soil particles by means of molecular attraction.

Hide [20] published an extensive review of the work done up to 1954 on factors influencing the evaporation of soil moisture. He concluded that

> The important variables which influence the rate of evaporation of soil moisture are (1) the vapor pressure difference between the layer from which water is evaporating and that of the turbulent atmosphere and (2) the distance of vapor flow of the intervening layer.

By subdividing the upper layer of the soil surface into 1-inch sections and studying the moisture exchange between the different layers, the results indicated that evaporation can be subdivided into three major stages, namely:

1. the brief period while the soil moisture content is above field capacity and capillary flow to the surface occurs under a low tension gradient;
2. the period after the soil surface has dried and flow to the surface occurs under a high tension gradient;
3. the period when vaporization occurs principally below the surface and vapor must diffuse through a static layer of air within the pore space of dry soil.

Lemon [27] also described the evaporation process as occurring in three successive stages, i.e.,

1. rather rapid water loss . . . dependent on evaporative demand,
2. rather rapid decline in water loss . . . governed by soil factors,
3. extremely slow rates . . . governed by adsorptive forces at the soil water surface.

Influence of Water-table Depth on the Rate of Upward Flow

In irrigated areas, water tables are sometimes kept at a shallow depth so that the soil can supply water from a water table at a sufficient rate to support plant growth. Gardner [13] concluded that in general upward flow rates sufficient for supplying plants are limited to water table depths less than one meter. Keen [22] observed the fluctuation of water table depths in exposed tanks. He reported that below a depth of 35 cm. in coarse sand, 70 cm. in fine sand and 85 cm. in a heavy loam soil, fall of the water table was very slow. He then concluded that below this depth, the rate of upward flow for supplying plant growth was very limited. A field experiment was conducted by Van't Woudt [4] in Holland in which the water table was controlled below pasture land by sub-irrigation. He observed that during dry periods, sufficient water was supplied with water tables not more than 4-5' below the soil surfaces.

If the water tables are kept at shallow depths, surfaces of soils are perpetually moist, the rate of upward flow is then sufficient to cause the accumulation of salt near or on the soil surface. Gardner [13] reports that upward flow of water can present a salinity hazard from water tables as deep as seven meters. The rate of accumulation of soluble salts due to upward flow of saline ground water can be obtained by multiplying the evaporation rate by the salt concentration of ground water minus the diffusion of the salt downward. The rate of upward flow, therefore, is an important variable when water tables exist near the surface. Shaw and Smith [36] studied evaporation from soil columns in the laboratory as a function of depth of water tables. They concluded that evaporation from Yolo loam was negligible when the water table was at a depth of more than ten feet. Staley [38] studied the effect of wind velocity and water-table depth upon evaporation from a fine sand. He found that below a critical depth (which was 24 inches) the maximum evaporation rate dropped rapidly with increasing depth of water table. He associated this critical depth with the displacement pressure (presently called bubbling pressure by Brooks and Corey [3]).

Previous Investigations of Effect of Soil Factors on Upward Flow Rates From Water Tables

Parshall [30] determined rates of evaporation from soils when there is a water table. He determined the evaporation rate compared with the evaporation rate from a free-water surface. He found that at a water-table depth of six inches, evaporation rates from adobe clay were less than from the coarse soils which exceeded the rate of evaporation from the free-water surface. At a water-table depth of 12 inches, the evaporation rate from the free-water surface was higher than from any of the soils.

Sleight [37] attempted to relate a characteristic size of particle to evaporation rate and water-table depths. He measured evaporation rates from river-bed sand as samples with a range of water-table depths up to 64 inches. With a water table of three inches he found that the ratio of evaporation from five samples to evaporation from a free surface ranged from 66 to 68 percent. He attempted to relate his results with the effective size and uniformity coefficient. He noted that: "Effective size apparently offered the greatest possibilities. Uniformity coefficient was not at all applicable. The 60-percent size was found to be much better."

For a given water-table depth and "60-percent size" Sleight gave coefficients which were to be multiplied by the evaporation from a free-water surface to arrive at the evaporation from river-bed material.

The effect of the soil-water system on evaporation rates was rather a vague idea until Moore published an extensive investigation [29] in 1939 relating evaporation rates to soil characteristics. Moore used soil columns to which water was supplied at various depths and from which evaporation was permitted. By employing tensiometer cups placed throughout the soil columns, he observed functional relationships among capillary pressure, moisture content, and rate of water loss from the water table. He also obtained the capillary pressure distribution along the soil column. Moore found that when fully saturated the permeabilities of the soils used were arranged in the following order:

Sand > fine sandy loam > light clay > clay.

After increasing the capillary pressure he observed that there was little change in relative permeability with capillary pressure heads ranging from zero to 10 to 40 cm. of water. Increasing the capillary pressure beyond this resulted in a marked decrease in relative permeability, and he found the order of relative permeability to be:

Sand < fine sandy loam < light clay < clay.

This result led to the conclusion that, at higher capillary pressures (i.e., greater depths of water table), the heavier soils supported higher evaporation rates.

Gardner and Fireman [15] obtained a functional relationship between the maximum capacity of the soil to conduct water upward and soil parameters. To support their theoretical equation, they packed acrylic plastic cylinders 12 cm. in diameter and 60 to 100 cm. in length with soils. These columns were initially saturated, and allowed to drain and saturated again in order to obtain a structure as near as possible to soils in the field. The water was supplied from a reservoir through porous caps in the bottom. A range of water-table depths was simulated by varying the vertical position of the inflow bottle. The investigators found that the evaporation rates did approach a limit over a period of time. They obtained evaporation rates close to the theoretical maximum capacity of the soils to conduct water upward except for a shallow water-table depth. They concluded that there are two maximum rates of evaporation possible. One is the evaporation rate determined by the external evaporative conditions. The other is the evaporation rate controlled by the ability of the soil to transmit water upward. Except for shallow water-table depths, the evaporation rate was limited by the ability of the soil to transmit water upward.

Effect of Hysteresis

Schleusener and Corey [35] conducted evaporation studies for the purpose of establishing the effects of independent changes in temperature, humidity, air motion, radiation, and depth of water table on steady state evaporation. With three soil types in contact with a water table, the investigators discovered that the evaporation rate increased with increasing evaporativity up to a critical point beyond which an increase in evaporativity produced a decrease in evaporation from the surface. They believed that the primary cause of this phenomenon is hysteresis, and they explained that as the evaporation rate exceeds the ability of soil to transmit water upward, the soil surface starts to imbibe water from wetter soil below. The effect of hysteresis is to reduce the upward flow rate because the relative hydraulic conductivity is less on the imbibition cycle at any particular capillary pressure, than on the drainage cycle. This causes the evaporation to drop after the soil surface begins to imbibe water. The investigators also described a capillary model representing the pore space of porous media, and they showed how the restriction at the neck of a pore could cause a reversal in the direction of change in capillary pressure without a corresponding reversal in the direction of change in saturation.

In similar studies under cyclic atmospheric conditions, King and Schleusener [23] verified the findings of Schleusener and Corey.

A laboratory investigation was conducted by Duke [10] to study the relationship between measurable soil parameters and the maximum upward flow rates in soils in contact with water tables. In his study the ambient evaporative conditions were eliminated from consideration by inducing upward flow with a siphon attached to a capillary barrier at the top of test columns. He packed 10 soil samples of widely varying characteristics, saturated with a hydrocarbon fluid, and with a water table maintained near the bottom of the column. To obtain the maximum upward flow rate, he lowered the outflow barrier, until the flow rate reached a maximum. He discovered that the observed maximum flow rates were thirty to fifty percent less than those predicted from his theoretical equation. Duke also believed that this difference was due to a hysteresis effect, and explained this phenomenon as follows:

> . . . whereby the soil surface loses liquid at a rate faster than it can be supplied by the underlying soil. The result is that the surface layer transports liquid by imbibing from the soil below. At any particular capillary pressure, the soil has a lower hydraulic conductivity on the imbibition cycle than on the drainage cycle. Therefore the maximum flow rate cannot reach the rate predicted by using the hydraulic conductivity on the drainage cycle . . . however, this solution will be

useful for predicting the maximum upward flow rate when corrected by reducing the theoretical rates by about thirty percent.

The theory and results of Gardner [12], Gardner and Fireman [15], and Duke [10] are discussed in detail in the following section.

Steady Upward Flow Theory and Its Application to Maximum Rates of Upward Flow From Water Tables

In 1856, Darcy discovered experimentally that the volume flux of water transmitted through a saturated sand filter was inversely proportional to the thickness of the sand and directly proportional to the difference in head across the filter. The proportionality factor was dependent upon the properties of the sand and was called the hydraulic conductivity.

Darcy's original equation can be written as

$$q = -C \frac{\Delta H}{\Delta L} \qquad (2.1)$$

in which q is the volume flux, having dimensions of velocity, C is the hydraulic conductivity, also with dimensions of velocity and ΔH is the difference in piezometric head across the distance ΔL.

Buckingham [4] was one of the first to attempt a detailed analysis of flow through partially-saturated media. He assumed that Darcy's law is valid for unsaturated flow. He named the proportionality factor "capillary conductivity" (presently called effective hydraulic conductivity) which is not constant but is a function of capillary pressure or saturation.

By using this assumption, and the definition of capillary pressure, Staley [38] derived the differential equation for steady upward flow from a water table as follows:

$$\frac{d(p_c/\rho g)}{dz} = 1 + q/C_e \qquad (2.2)$$

$$z = \int_0^{p_c/\rho g} \frac{d(p_c/\rho g)}{1+q/C_e} \qquad (2.3)$$

In equation 2.3, q is the volume flux, C_e is the effective hydraulic conductivity, and z is the elevation above the water table.

Gardner [10] started from the diffusivity equation which was presented in detail by Klute [25], to derive the differential equation for one-dimensional upward flow. He obtained the same differential equation as shown in equation 2.3.

To solve equation 2.3, a relationship between effective hydraulic conductivity and capillary pressure must be known. Staley used an expression proposed by Corey.

$$C_e = C \quad \text{for } p_c < p_b$$

$$= C(p_b/p_c)^8 \quad \text{for } p_b > p_c \quad . \qquad (2.4)$$

Substituting equation 2.4 in equation 2.2 and rewriting the result in terms of dimensionless variables he obtained

$$\frac{d(p_c/p_b)}{d(\gamma z/p_b)} = 1 + \frac{q}{C}(p_c/p_b)^8 \quad (p_c > p_b) \quad . \tag{2.5}$$

The exact solution of equation 2.5 is extremely complex. Staley, therefore, employed a numerical method of plotting the slope $\frac{d(p_c/p_b)}{d(z\gamma/p_b)}$ for the various values of p_b/p_c. He obtained the solutions for various values of q/C as shown in Figure 1.

Gardner and Fireman [15] fitted their experimental data with a continuous functional relationship between effective hydraulic conductivity and capillary pressure by

$$C_e = \frac{a}{p_c{}^{\eta} + b} \tag{2.6}$$

where a and b are constants, C_e is effective hydraulic conductivity and η is a constant which Gardner and Fireman [15] took to be 2 or 3. Gardner [11] obtained some of his experimental data by the dynamic outflow method. This method probably gave inaccurate results when the flow rate was high, because the hydraulic head loss across the outflow membrane could not be negligible as assumed. Substituting equation 2.6 in equation 2.3 and integrating, Gardner [12] obtained the general solutions for various values of η from 1 to 4. By letting $\alpha = q/a$, $\beta = \alpha b + 1$, the resulting solutions were:

Case I: $\eta = 1$

$$z = \frac{1}{\alpha}\ln(\alpha p_c + \beta) + k \ . \tag{2.7}$$

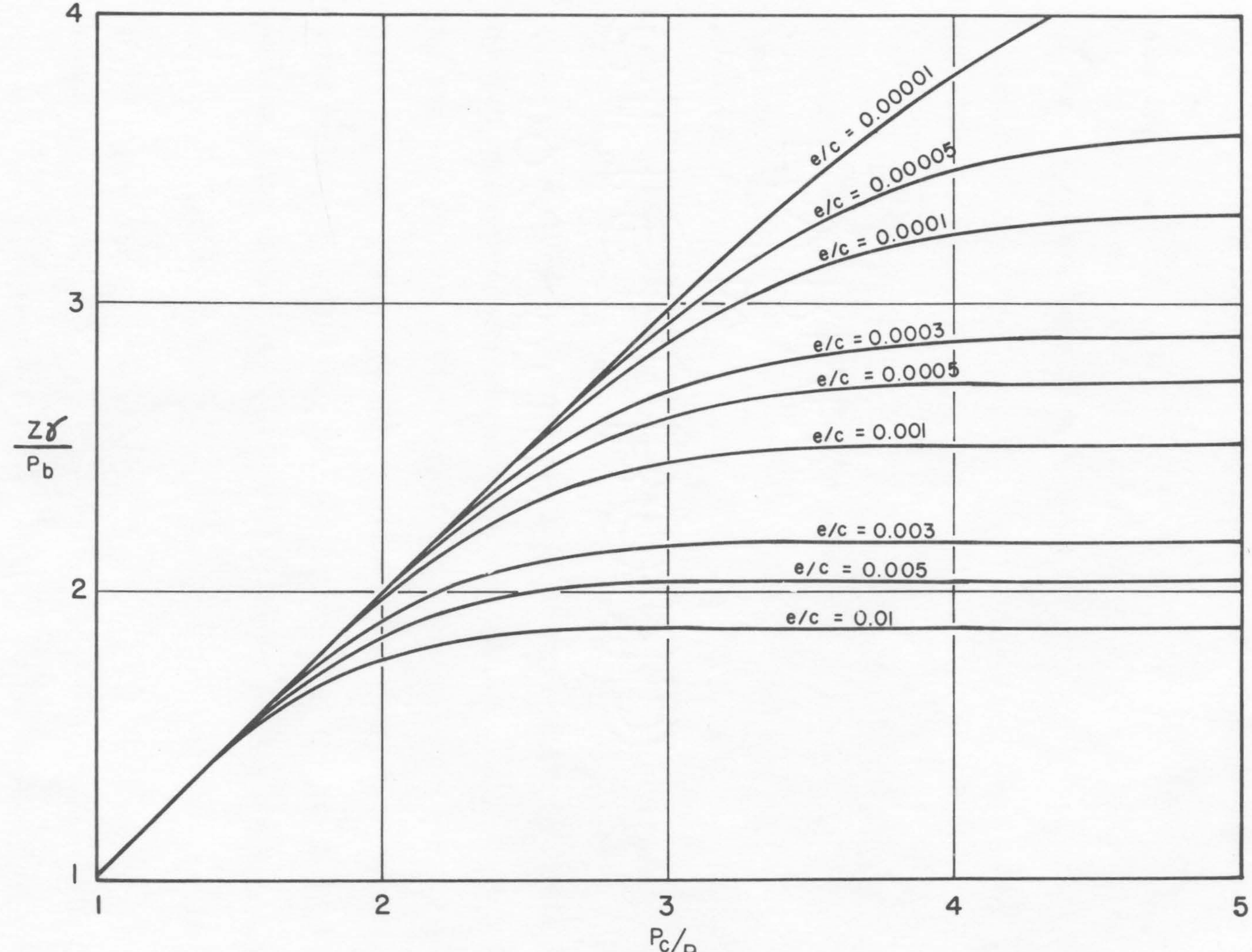

Fig. 1. -- Computed relationship between the parameters Z. and p. for various values of q.

Case II: $\eta = 3/2$

$$z = \frac{2}{\alpha}\left\{\frac{1}{6\alpha}\ln\frac{(\gamma^2-\gamma\sqrt{p_c}+p_c)}{(\gamma+\sqrt{p_c})^2} + \frac{1}{\gamma\sqrt{3}}\tan^{-1}\left(\frac{2\sqrt{3}-\gamma}{\gamma\sqrt{3}}\right)\right\} + k \qquad (2.8)$$

where $\gamma^3 = \beta/\alpha$ and k is a constant of integration.

Case III: $\eta = 2$

$$z = \frac{1}{\sqrt{\alpha\beta}}\tan^{-1}(p_c\sqrt{\alpha/\beta}) + k \;. \qquad (2.9)$$

Case IV: $\eta = 3$

$$z = \frac{1}{\alpha}\left\{\frac{1}{6\alpha}\ln\frac{(\gamma+p_c)^2}{(\gamma^2-\gamma p_c+p_c^2)} + \frac{1}{\gamma^2\sqrt{p_c}}\tan^{-1}\frac{(2p_c-\gamma)}{\gamma\sqrt{3}}\right\} + k \;. \qquad (2.10)$$

Case V: $\eta = 4$

$$z = \frac{1}{\alpha}\left\{\frac{1}{4\rho^3\sqrt{2}}\ln\frac{(p_c^2-\rho p_c\sqrt{2}+p^2)}{(p_c^2+\rho p_c\sqrt{2}+p^2)} + \frac{1}{2\rho^3\sqrt{2}}\tan^{-1}\frac{p_c\sqrt{2}}{\rho^2-p_c^2}\right\} + k \;. \qquad (2.11)$$

To obtain the maximum flow rate from a water table, Gardner employed the boundary conditions: $p_c = o$ at $z = o$, and $p_c \to \infty$ at $z = d$ where the soil surface is dry. By assuming β is equal to 1 or the volume flux q is very small compared with the constant "a" so that $q/a \to o$, he gave the maximum flow rate $q_{lim.}$, for each case as follows:

$$\eta = 3/2, q_{lim.} = 3.77ad^{-3/2} \qquad (2.12)$$

$$\eta = 2, q_{lim.} = 2.46ad^{-2} \qquad (2.13)$$

$$\eta = 3, q_{lim.} = 1.76ad^{-3} \qquad (2.14)$$

$$\eta = 4, q_{lim.} = 1.52ad^{-4} \qquad (2.15)$$

It should be noted again that these equations are applicable when q/a approaches zero. Gardner pointed out that there is no limiting rate for Case I, but no soil had been found such that Case I holds.

In a paper dealing with laboratory measurements, data were presented by Gardner and Fireman [15] comparing the rates computed from equations 2.12 and 2.14 with measured evaporation rates. The agreement was fairly good for the low rates of evaporation.

Brooks and Corey [3] employed a steady-state method to measure relative permeability as a function of capillary pressure. By maintaining steady downward flow in a column at uniform capillary pressure, they were able to measure the relative permeability for particular capillary pressures. Their method overcame the difficulties of Gardner's pressure-plate method. Since the capillary pressure gradient was measured at steady state within the soil column itself, the effect of hydraulic head loss across the capillary barrier was eliminated.

It should be pointed out that by a suitable choice of constants, Gardner's continuous functional relationship between C_e and p_c is nearly the same as the discontinuous functional relationship employed by Brooks and Corey [3]. Brooks and Corey observed that C_e is practically unchanged with p_c until $p_c = p_b$. They proposed the relationship:

$$C_e = C \quad \text{when } p_c < p_b \text{ and}$$

$$C_e = C(p_b/p_c)^n \quad \text{when } p_c > p_b \quad . \qquad (2.16)$$

Duke used equation 2.16 to determine the maximum rate of upward flow from water tables. He substituted equation 2.16 into 2.3 and re-wrote the resulting equation in terms of dimensionless variables as

$$\frac{d}{p_b/\gamma} = \frac{1}{1+q_{\circ m}/C} + \int_1^\infty \frac{d(p_c/p_b)}{1+q_{\circ m}/C\,(p_c/p_b)^\eta} \tag{2.17}$$

or

$$d_\circ = \frac{1}{1+q_{\circ m}} + \int_1^\infty \frac{dp_\circ}{1+q_{\circ m}p_\circ^{\ \eta}} \quad . \tag{2.18}$$

Duke evaluated the integral on the right of equation 2.18 from the area bounded by a plot of $\frac{1}{1+q_{\circ m}p_\circ^{\ \eta}}$ versus $p_\circ$. In order to eliminate a considerable amount of hand calculation and to insure greater accuracy, he utilized an IBM 1401 computer for the solution of equation 2.18. The solutions were compiled into a nomograph relating $q_{\circ m}$ to $d_\circ$ for each value of η. The completed values of $d_\circ$ are given in Table IX.

CHAPTER III

THEORETICAL ANALYSIS

This chapter deals with the analysis of one-dimensional steady upward flow from water tables. The differential equation employed is the same as presented by Staley [38] and Gardner [12], i.e., equation 2.3. An approximate algebraic solution was derived for the capillary pressure distribution, and the maximum rate of upward flow through homogeneous media which can be extended to the case of stratified media.

The differential equation for one-dimensional steady upward flow is shown in equation 2.2 or 2.3. Using Brooks and Corey's relationship between C_e and p_c, i.e., equation 2.16; substituting this into equation 2.3, and rewriting the resulting equation in terms of dimensionless variable gives:

$$Z_\circ = \frac{p_\circ}{1+q_\circ} \qquad \text{when } p_\circ < 1 \qquad (3.1)$$

$$Z_\circ = \frac{1}{1+q_\circ} + \int_1^{p_\circ} \frac{dp_\circ}{1+q_\circ p_\circ^{\eta}} \qquad \text{when } p_\circ > 1 \qquad (3.2)$$

where $q_\circ$ is q/C, $Z_\circ$ is $z\gamma/p_b$, and $p_\circ$ is p_c/p_b.

Equation 3.2 can be written in compact form as follows:

$$Z_\circ = \frac{1}{1+q_\circ} + \frac{1}{q_\circ^{1/\eta}} \int_1^{p_\circ} \frac{d(p_\circ q_\circ^{1/\eta})}{1+(p_\circ q_\circ^{1/\eta})^\eta}$$

or

$$Z_\circ = \frac{1}{1+q_\circ} + \frac{1}{q_\circ^{1/\eta}} \int_{q_\circ^{1/\eta}}^{p_\circ q_\circ^{1/\eta}} \frac{dx}{1+x^\eta} \qquad (3.3)$$

in which x is a new variable and represents $(p_\circ q_\circ^{1/\eta})$.

Evaluation of Equation 3.3

To solve equation (3.3) the value of the integral $\int_o^x \frac{dx}{1+x^\eta}$ must be known. Let $\int_o^x \frac{dx}{1+x^\eta}$ be denoted by I(x). The exact value of I(x) when $\eta > 4$ is extremely complex, whatever empirical expression for $C_e(p_c)$ is used. The technique employed is to expand I(x) into a convergent series. The convergent series of $\frac{1}{1+x^\eta}$ are not the same when $x < 1$ as when $x > 1$. The values of I(x), therefore, are separated into two cases:

Case I: $x < 1$

A convergent series of $\frac{1}{1+x^\eta}$ can be expressed as

$$\frac{1}{1+x^\eta} = 1-x^\eta+x^{2\eta}-x^{3\eta}+\cdots-\cdots \quad . \qquad (3.4)$$

Integrating (3.4) term by term gives the value of I(x),

$$I(x) = \int_o^x \frac{dx}{1+x^\eta}$$

$$= x - \frac{x^{\eta+1}}{\eta+1} + \frac{2\eta+1}{2\eta+1} - \frac{3\eta+1}{3\eta+1} + \cdots - \cdots$$

$$= \left\{ x - \frac{x^{\eta+1}}{\eta+1} + \frac{x^{2\eta+1}}{2\eta+2} - \frac{x^{3\eta+1}}{3\eta+3} + \cdots - \cdots \right\}$$

$$+ x^{2\eta+1}\left\{\frac{1}{2\eta+1} - \frac{1}{2\eta+2}\right\} - x^{3\eta+1}\left\{\frac{1}{3\eta+1} - \frac{1}{3\eta+3}\right\} + \cdots - \cdots$$

$$= x - \frac{x}{\eta+1}\ln(1+x^{\eta}) + \frac{x^{2\eta+1}}{(2\eta+1)(2\eta+3)} - \frac{2x^{3\eta+1}}{(3\eta+1)(3\eta+3)}$$

$$+ \cdots - \cdots \quad . \tag{3.5}$$

For values of $\eta \leq 1$ equation 3.5 does not converge, but no soil has been found for which this case will hold. When $\eta \geq 4$ the value of the fourth term is small compared to the first three, so that it can be neglected. To obtain an equation in simpler form than equation 3.5, an approximation is employed as follows:

$$I(x) \approx x - \frac{x}{\eta+1}\ln(1+x^{\eta}) + \frac{x^{2\eta}}{4(\eta+1)^2} \tag{3.6}$$

so that $I(o) = o$ and (3.7)

$$\lim_{\varepsilon \to o} I(1-\varepsilon) = 1 - \frac{1}{\eta+1}\ln 2 + \frac{1}{4(\eta+1)^2} \tag{3.8}$$

where ε is any positive number.

Case II: $x > 1$

The value of $I(x)$ must be broken into two integrals. One is an integral from o to $1 - \varepsilon$, and the other is an integral from $1 + \varepsilon$ to x where ε is any positive number. The value of $I(x)$ can be expressed as follows:

$$I(x) = \int_o^x \frac{dx}{1+x^{\eta}} = \lim_{\varepsilon \to o}\int_o^{1-\varepsilon} \frac{dx}{1+x^{\eta}} + \lim_{\varepsilon \to o}\int_{1+\varepsilon}^{x} \frac{dx}{1+x^{\eta}} \quad . \tag{3.9}$$

Substituting equation 3.7 and 3.8, in equation 3.9 for the first integrals:

$$I(x) = 1 - \frac{\ln 2}{\eta+1} + \frac{1}{4(\eta+1)^2} + \lim_{\varepsilon \to o} \int_{1+\varepsilon}^{x} \frac{dx}{1+x^{\eta}} \quad . \tag{3.10}$$

The convergent series of $\int \frac{1}{1+x^{\eta}}$ for $x > 1$ and $\eta > 1$ can be expressed as

$$\frac{1}{1+x^{\eta}} = x^{-\eta} - x^{-2\eta} + x^{-3\eta} - \cdots + \cdots \quad . \tag{3.11}$$

Integrating equation 3.11 term by term gives:

$$\int \frac{dx}{1+x^{\eta}} = -\left\{ \frac{x^{-(\eta-1)}}{\eta-1} - \frac{x^{-(2\eta-1)}}{2\eta-1} + \frac{x^{-(3\eta-1)}}{(3\eta-1)} - \cdots + \cdots \right\} + \text{constant}$$

$$= -\left\{ \frac{x^{-(\eta-1)}}{\eta-1} - \frac{x^{-(2\eta-1)}}{2\eta-2} + \frac{x^{-(3\eta-1)}}{3\eta-3} - \cdots + \cdots \right\}$$

$$+ x^{-(2\eta-1)} \left\{ \frac{1}{2\eta-1} - \frac{1}{2\eta-2} \right\} - x^{-(3\eta-1)} \left\{ \frac{1}{3\eta-1} - \frac{1}{3\eta-3} \right\}$$

$$+ \cdots - \cdots + \text{constant}$$

or

$$\int \frac{dx}{1+x^{\eta}} = \frac{x}{\eta-1} \ln(1+x^{-\eta}) - \frac{x^{-(2\eta-1)}}{(2\eta-1)(2\eta-2)} + \frac{2x^{-(3\eta-1)}}{(3\eta-1)(3\eta-3)}$$

$$- \cdots + \cdots + \text{constant.} \tag{3.12}$$

Similarly, for $\eta \geq 4$ the third term is small compared to the first two. Equation 3.12 is approximated by:

$$\int \frac{dx}{1+x^{\eta}} \approx -\frac{x}{\eta-1} \ln(1-x^{-\eta}) - \frac{x^{-2\eta}}{(2\eta-1)^2} + \text{constant.} \tag{3.13}$$

so that

$$\int_{1+\varepsilon}^{x} \frac{dx}{1+x^{\eta}} \approx - \frac{x}{\eta-1}\ln(1+x^{-\eta}) - \frac{x^{-2\eta}}{(2\eta-1)^2} + \frac{\ln 2}{\eta-1} + \frac{1}{(2\eta-1)^2} \quad (3.14)$$

Substituting equation 3.14 into equation 3.10 gives

$$I(x) \approx 1 - \left\{\frac{\ln 2}{\eta+1} + \frac{1}{4(\eta+1)^2} + \frac{\ln 2}{\eta-1} + \frac{1}{(2\eta-1)^2}\right\}$$

$$- \left\{\frac{x}{\eta-1}\ln(1+x^{-\eta}) + \frac{x^{-2\eta}}{(2\eta-1)^2}\right\}$$

$$\approx 1 + \frac{1+4\ln 2}{2(\eta^2+1)} - \frac{x}{\eta-1}\ln(1+x^{-\eta}) - \frac{x^{-2\eta}}{(2\eta-1)^2} \cdot \quad (3.15)$$

Equation 3.6 and 3.15 give the value of I(x) for $x < 1$ and $x > 1$, respectively. By means of these two equations, the capillary pressure distribution, the maximum rate of upward flow for both homogeneous and stratified soils can be determined.

Capillary Pressure Distribution

Using the value of I(x) from equation 3.6 when $x < 1$, and equation 3.15 when $x > 1$, the capillary pressure distribution which is shown in equation 3.1 and 3.3 can be obtained. The capillary pressure distribution may be expressed by:

$$z_{\circ} = \frac{p_{\circ}}{1+q_{\circ}} \quad \text{when } p_{\circ} < 1 \quad (3.16)$$

$$z_{\circ} = \frac{1}{1+q_{\circ}} + \frac{1}{q_{\circ}^{1/\eta}}\left\{I(p_{\circ}q_{\circ}^{1/\eta}) - I(q_{\circ}^{1/\eta})\right\} \text{ when } p_{\circ} > 1 \ . \quad (3.17)$$

Maximum Rate of Upward Flow Through Homogeneous Soils

For maximum rates of upward flow through homogeneous soils, one should integrate equation 3.2, using limits of Z. from o to d. (where d. is the ratio between depth to water table and P_b/γ), and p. from zero to infinity. Equation 3.17, therefore, becomes:

$$d_{\circ} = \frac{1}{1+q_{\circ m}} + \frac{1}{q_{\circ m}^{1/\eta}} \left\{ I(\infty) - I(q_{\circ m}^{1/\eta}) \right\} . \qquad (3.18)$$

I(x) gives a different value when $x < 1$ and $x > 1$. The maximum rate, therefore, must be divided into two cases:

Case I: $q_{\circ m} < 1$

Evaluating equation 3.15 for $x = \infty$, and equation 3.6 for $x = q_{\circ m}^{1/\eta}$ gives the value of $I(\infty)$ and $I(q_{\circ m}^{1/\eta})$.

$$I(\infty) \simeq 1 + \frac{1.886}{\eta^2+1} \qquad (3.19)$$

and

$$I(q_{\circ m}^{1/\eta}) \simeq \frac{1}{q_{\circ m}^{1/\eta}} - \frac{q_{\circ m}^{1/\eta}}{\eta+1} \ln(1+q_{\circ m}) + \frac{q_{\circ m}^{2}}{4(\eta+1)^2} . \qquad (3.20)$$

For values of $\eta \geq 4$ the last term of equation 3.20 is small compared to the first two, so that

$$I(q_{\circ m}^{1/\eta}) \simeq \frac{1}{q_{\circ m}^{1/\eta}} - \frac{q_{\circ m}^{1/\eta}}{\eta+1} \ln(1+q_{\circ m}) . \qquad (3.21)$$

$$d_{\circ} \simeq \frac{1}{q_{\circ m}^{1/\eta}} \left(1 + \frac{1.886}{\eta^2+1}\right) - \frac{q_{\circ m}}{1+q_{\circ m}} + \frac{\ln(1+q_{\circ m})}{\eta+1} . \qquad (3.22)$$

Case II: $q_{\circ m} > 1$

The value of $I(q_{\circ m}^{1/\eta})$, evaluated from equation 3.15 is:

$$I(q_{\circ m}^{1/\eta}) \simeq 1 + \frac{1.886}{\eta^2+1} - \frac{q_{\circ m}^{1/\eta}}{\eta-1} \ln\frac{(1+q_{\circ m})}{q_{\circ m}} - \frac{1}{(2\eta-1)^2 q_{\circ m}^2} \quad (3.23)$$

The last term of 3.23 is small compared to the first two, so that it can be neglected. Equation 3.2 then becomes

$$I(q_{\circ m}^{1/\eta}) \simeq 1 + \frac{1.886}{\eta^2+1} - \frac{q_{\circ m}^{1/\eta}}{\eta-1} \ln\frac{(1+q_{\circ m})}{q_{\circ m}} \quad . \quad (3.24)$$

Substituting equation 3.24, and 3.19 into equation 3.18 gives

$$d_{\circ} \simeq \frac{1}{1+q_{\circ m}} + \frac{1}{\eta-1} \ln\frac{(1+q_{\circ m})}{q_{\circ m}} \quad . \quad (3.25)$$

Equation 3.22 and 3.25 give the value of depth to a water table as a function of the maximum rate of upward flow for given values of soil parameters $C, P_b/\gamma$ and η for cases where $q_{\circ m} < 1$ and $q_{\circ m} > 1$ respectively.

These equations are applicable not only for the case of soil-water systems following the drainage cycle but also for the imbibition cycle, provided that the values of soil parameters $C, P_b/\gamma$ and η are given in both cases.

The application and the accuracy of equation 3.22 and 3.25 is discussed in Chapter V.

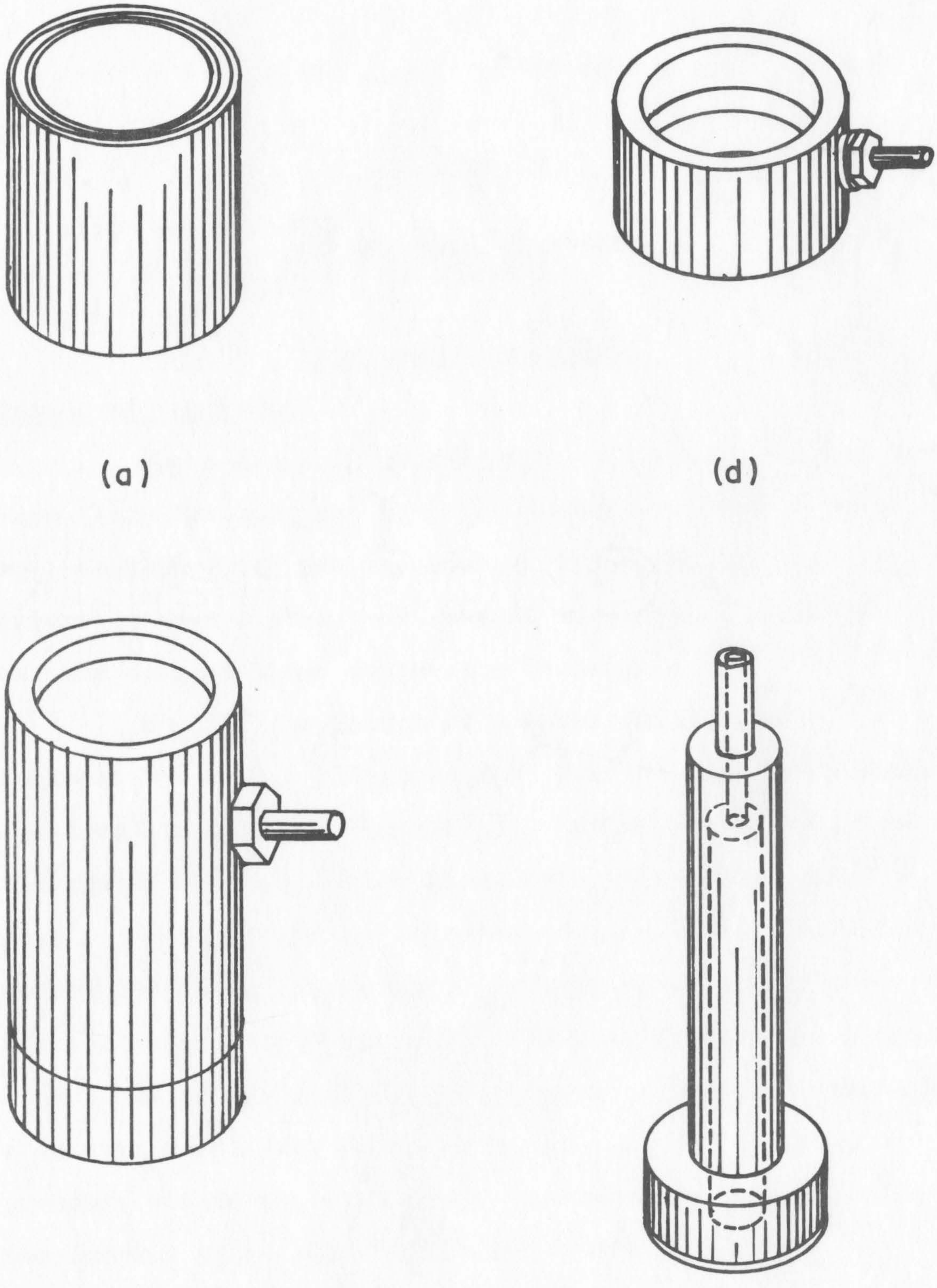

Fig. 2--Acrylic components used to contain the soil column.

CHAPTER IV

EXPERIMENTAL EQUIPMENT AND PROCEDURE

Experimental Equipment

Design of Equipment

The individual components used in this research were those designed and built by Brooks and Corey. The equipment employed was the same as that used by Duke [10]. Figure 2 shows a sketch of several components. Duke described each of these components in detail.

Most of the length of the soil columns were enclosed by acrylic cylinders with 3.02 cm. inside diameter and 4 cm. length (see Figure 2a). Inflow to the bottom of the column was supplied by means of a one-gallon glass bottle. This bottle was attached to a section of acrylic tubing closed on the bottom and fitted with a capillary barrier as shown in Figure 2b. The upward flow was induced at the top of the soil solumn by means of a siphon, through a ceramic barrier as shown in Figure 2c. The capillary pressure within the soil columns was measured by employing tensiometer rings (see Figure 2d).

The flow rate was measured by attaching the outflow tube to the top of a 5 ml burette. The flow into the burette, therefore, was continuous.

For soil materials having a very high permeability and low bubbling pressure, Porvic plastic was substituted for the capillary barrier in order to reduce the head loss through the barrier and to maintain better control of the capillary pressure.

Soil Used

Four soil samples were selected to give a wide range of C, p_b, and η. The value of η is to some extent a function of the bulk density of the medium. The same soil was packed at different bulk densities to give various values of η. Descriptions of the several soil samples are given in Table I.

TABLE I

BRIEF DESCRIPTION OF SOILS

Soil	Description
Loveland Sand	Quartz sand found in a lake bed near Loveland, Colorado.
Crab Creek Sand	A sand of volcanic origin obtained from the flood plain of lower Crab Creek west of Othello in central Washington. It is an agricultural soil
Touchet Silt Loam	An agricultural soil of low clay content having little visible aggregation but a very high content of silt and not much else. It was obtained from the Columbia river basin, Richland, Washington.
Greeley Sand	An agricultural soil from southwest of Greeley, Colorado. This material has a small amount of clay but is composed mostly of fine sands and some coarse sand.

Procedure

The column was assembled by fastening together the acrylic sections with the components which are described above. The inflow section was at the bottom, above which a tensiometer ring was placed to measure the capillary pressure for determining the water-table depth. A 4 cm. section was placed alternately with tensiometer rings until the desired length was attained. The length of column was slightly greater than half of the P_b/γ of the soil sample. The properties of the soil samples used had previously been determined by Brooks and Corey [3], but with different packing techniques. The value of bubbling pressure, however, was estimated roughly from their measurements in order to determine the column lengths.

The sections were taped together lengthwise with Magic mending tape.[1] The columns were packed by first filling the cylinders with dry soil and then vibrating them with an electric-powered device until they reached a relatively stable degree of compaction. By varying the vibrating time, various bulk densities were obtained for the same soil. The filling was accomplished by inserting a small tube (filled with soil and supplied by a funnel) and gradually withdrawing the tube as the column filled. This method avoided the necessity of dropping the soil from the top of the column

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[1]A brand of tape found to be particularly suitable when hydrocarbon liquid was used in the column.

which might have resulted in segregation of particles according to size.

After the column was packed, a disc of fiber glass mat was placed over the top of the soil surface to prevent erosion during the saturation procedure. A plastic plug containing holes to permit the escape of air held the fiber glass against the soil and retained the soil in the column. The column was then submerged in a container of hydrocarbon. The container was then evacuated with a pump until air bubbles ceased to emerge from the soil and the capillary barriers.

Permeability--Capillary Pressure Measurements

After the columns were saturated, they were removed from the container in order to measure effective permeability as a function of capillary pressure, to obtain permeability, bubbling pressure, and η. The technique employed is called the controlled-pressure method, the particular version of which was that employed by Brooks and Corey [3] which has been described in detail by Corey [8]. This method involves steady-state downward flow of fluid through soil under a hydraulic gradient of unity. The fluid entered the top of the soil column at a controlled pressure through a capillary barrier and left the bottom of the column through a similar outflow barrier at a controlled pressure. The capillary pressure was measured along the column. By controlling the inflow and outflow siphon elevations, a zone of a constant capillary pressure within the column was maintained. The effective permeability within this zone was calculated.

Some runs were made, not only on the drainage cycle, but also on the imbibition cycle. Data on the imbibition cycle were obtained by first draining the column to a very low saturation (to a capillary pressure corresponding approximately to the residual saturation) and then raising the inflow reservoir and outflow siphon in increments. The effective permeability was determined after steady state was obtained for each increment.

Measurement of Maximum Flow Rates

After the measurement of the soil parameters, the soil column was saturated again by the same method. The column was removed from the liquid container and placed in a horizontal position to prevent desaturation. The short sections of oil-filled tubing which were connected to the tensiometers were clamped to prevent air entering the capillary barriers. The column then was cut to the approximate length of one-half of the bubbling pressure head, care being taken to give a smooth soil surface. After the column was fastened to a vertical channel iron near the manometer board, the inflow bottle was immediately attached to the inflow barrier to prevent desaturation at the top of the soil column. The upper barrier which had been saturated was placed in contact with the smooth surface of the soil by means of a spring. The outflow siphon was adjusted at the level of the desired water-table depth so that fast desaturation could not occur after unclamping the outflow tube. After the outflow tube was unclamped, all manometers were connected to corresponding tensiometer rings. The inflow bottle then was gradually lowered until the lowest tensiometer indicated zero capillary pressure. The flow was very small (practically static), since the outflow tube was adjusted at an elevation slightly below the water table. Since the length of soil column was about half of P_b/γ, the soil on the top of the column remained saturated. The apparatus was then completely ready for the next operation. A schematic diagram of the apparatus is shown in Figure 3. The next operation was the determination of the maximum flow rate for the drainage, imbibition and imbibition-drainage cycles.

Drainage Cycle

After the apparatus was set up as shown in Figure 3, the outflow tube was lowered in small increments so that it did not cause a pressure reversal[1] at the top of the soil column. The flow rate was measured at each increment after steady state was reached. As the outflow tube was lowered, the lowest tensiometer no longer indicated zero pressure because of the greater flow. The inflow bottle was adjusted until the capillary pressure at the lowest

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[1]See definition of term pressure reversal in list of definitions.

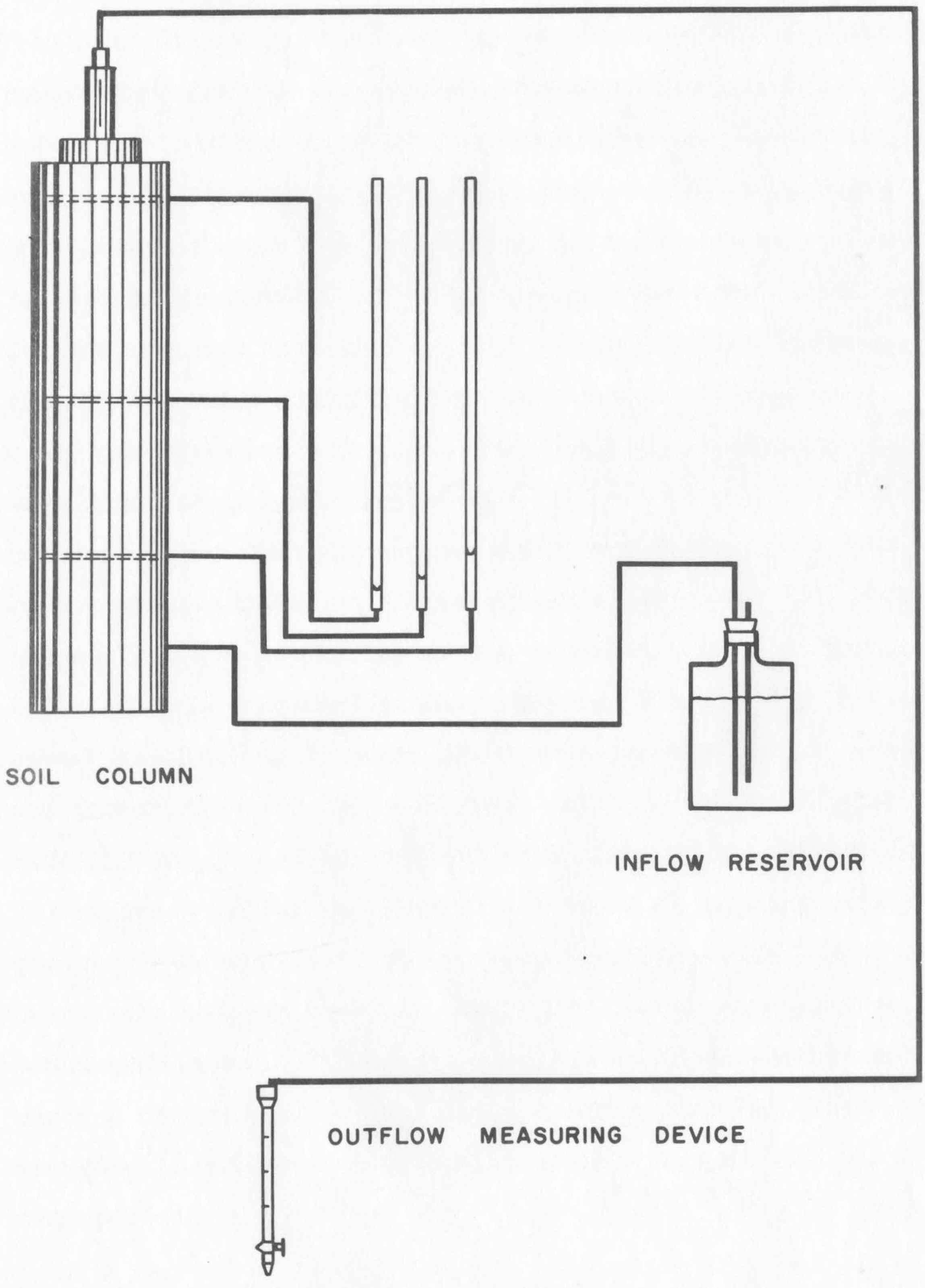

Fig. 3--Schematic flow diagram.

tensiometer was again zero. The adjusting of the water table was done very carefully so that there was no pressure reversal within the driest zone of the soil column. By carefully lowering the outflow siphon, the capillary pressure at the surface was gradually increased in small increments until the outflow appeared to approach a maximum (when the discharge remained almost constant with lowering the outflow siphon). The outflow siphon was not lowered more than this because further lowering caused a reversal of capillary pressure near the top and reduced the rate of upward flow (as shown in Figure 8).

After this the inflow bottle was lowered to simulate a greater depth to the water table until the soil near the surface was nearly dry or the capillary pressure reading from the upper tensiometer was about $1.5(P_b/\gamma)$. When the system reached steady state, the lower tensiometer was read and the outflow rate was measured. Knowing the fully saturated hydraulic conductivity of the soil, it was possible to compute an equivalent depth to a plane of zero capillary pressure (water table). The measured outflow rate represented the maximum rate of upward flow for that particular water-table depth. A series of runs were completed by repeating the process of lowering the inflow bottle. This procedure resulted in no pressure reversal at the upper tensiometer.

Imbibition Cycle

The maximum rates of upward flow for the imbibition cycle were measured in two ways. One way was by starting with a low soil saturation and, the other with a completely dry soil.

Starting with low saturation.--This procedure was started after the series of runs for the drainage cycle had been finished. The inflow bottle was lowered further so that the capillary pressures everywhere in the soil column were higher than twice the bubbling pressure of the soil. This capillary pressure produced a rather uniform saturation (close to the residual saturation). The inflow bottle was raised in increments to simulate shallower depths to the water table. At each increment the upward flow rate and capillary pressure at the lower tensiometer ring were recorded. The flow rates represented the maximum rates of upward flow and the capillary pressure at the bottom tensiometer gave the location of a water table. In this case, the flow rates were small so that the head loss across the inflow barrier was negligible. The reading of capillary pressure from the lower tensiometer, therefore, represented the location of the water table.

Starting with dry soil.--The soil was packed while all capillary barriers were saturated. To prevent the soil from imbibing liquid, a high capillary pressure was applied to each capillary barrier during the packing process. Ceramic barriers were used in this case because of their high air-entry pressure. Air, therefore, did not enter the barriers when the column was being vibrated.

The soil columns were fastened to the vertical channels; the tensiometers were connected to the manometer tube, and the outflow and inflow siphons were also connected. The initial imbibition was produced with the inflow siphon at a low elevation. The outflow siphon was placed at elevations low enough to apply capillary pressures high enough to obtain the maximum rate of upward flow. The inflow bottle then was raised to an elevation high enough to permit some outflow eventually to occur. The imbibition took place at an extremely slow rate, the system was allowed to reach equilibrium before any outflow was measured. The location of the water table was determined from the lower tensiometer reading as previously explained. The series of runs were completed by raising the inflow bottle to simulate shallower depths to the water table. The series of runs were terminated when the water table was raised to the lower joint of the column section. Higher water tables could not be employed because liquid would leak from the lower joint.

Imbibition-Drainage Cycle

The investigation of the imbibition-drainage cycle was divided according to its purposes into two procedures. First was a study of the effect of a pressure reversal (hysteresis) due to the rate of outflow exceeding the upward flow rate. This procedure was exactly the same as that used for the drainage cycle except that after the maximum rate of flow was observed, the outflow siphon was lowered several increments further until it reached the floor of the laboratory. The outflow siphon was then raised in increments for each of which the outflow rate was measured.

The second was a study of the effect of the increment of lowering the outflow siphon on the upward flow rate. The procedure was slightly different from the first. The differences were: (1) The increments of lowering the outflow siphon were uniform for each series of runs (for example, a series of runs were made for which each increment was 10 cm.), and (2) the outflow siphon was not raised after it had been lowered.

CHAPTER V

ANALYSIS OF DATA AND DISCUSSION OF RESULTS

The purpose of this study was to determine the maximum rate of upward flow from a water table for various conditions as explained in Chapter I. The factors controlling the maximum rate of flow are the liquid-soil conditions. The ambient conditions were eliminated so that they would not effect the flow rates. Upward flow was induced by lowering the outflow siphon instead of varying the ambient conditions. Theoretical equations were developed to approximate maximum rates of flow as functions of the soil parameters and depth to a water table. The flow rates were measured under various conditions to verify the theoretical equation. The soil parameters C, P_b/γ and η were measured before the upward-flow experiments were conducted, so that a convenient column length and procedure for controlling the flows could be selected.

Relative Permeability-Capillary Pressure Measurement

By controlling the outflow and inflow siphon, and assuming Darcy's law was valid both for fully saturated and unsaturated flow, the effective permeabilities were calculated in the zone where hydraulic gradients were unity.

The relative permeabilities were obtained by dividing the effective permeabilities by the permeability when fully saturated. The tabulated data and results are presented in Table VI.

It should be noted that at low capillary pressures the soil remained saturated, and a hydraulic gradient of unity was not necessary for calculation of permeability. This is because the permeabilities were practically constant in this range of capillary pressures. After the soil column was desaturated, the capillary pressures in the soil column were adjusted to be as uniform as possible. Sometimes when the outflow siphon was placed in the wrong position, it seemed impossible to obtain uniform capillary pressures. In this case hydraulic gradients from 0.8 - 1.2 were assumed acceptable and the average capillary pressure (through the zone where the hydraulic gradient was measured) was recorded.

The relative permeability-capillary pressure data were plotted on log-log paper to determine the soil parameters C, P_b/γ, and η. According to Brooks and Corey's relationship, equation 2.16, the value of η is the absolute value of the slope of the straight line portion of the curve and the bubbling pressure is the extrapolation of the straight line to the ordinate representing a relative permeability of unity.

The curves of relative permeability as a function of capillary pressure were represented very well by equation 2.16 for both the drainage and imbibition cycles as shown in Figures 4 and 5. Greeley sand was an exception, this sand being difficult to pack in a homogeneous column. Greeley sand consists of particles that fit into two distinct size groups with practically no intermediate sizes. When placed in a column the two size groups apparently tend to segregate so that the relative permeability curve contains an inflection at the capillary pressure at which the larger pores have emptied and the smaller pores have not yet started to desaturate. After smaller pores begin to desaturate the curve takes on another slope governed by the characteristics of the finer material.

As a result, it was impossible to determine a unique value of the bubbling pressure or η for the medium as a whole. In this case, the data could be approximated by Gardner's equation 2.6. The characteristics of the curve in the transition zone, however, could not be consistently reproduced for this sand.

Most materials, when packed in a homogeneous manner and when careful techniques were employed (as described by Corey [8]), yielded data that were adequately represented by equation 2.16.

Figure 5 shows the relative permeability-capillary pressure relationships for the drainage and imbibition cycles. It is apparent that the value of effective permeability at a particular value of capillary pressure on the imbibition cycle may be one or two orders of magnitude less than on the drainage cycle. The exception is for capillary

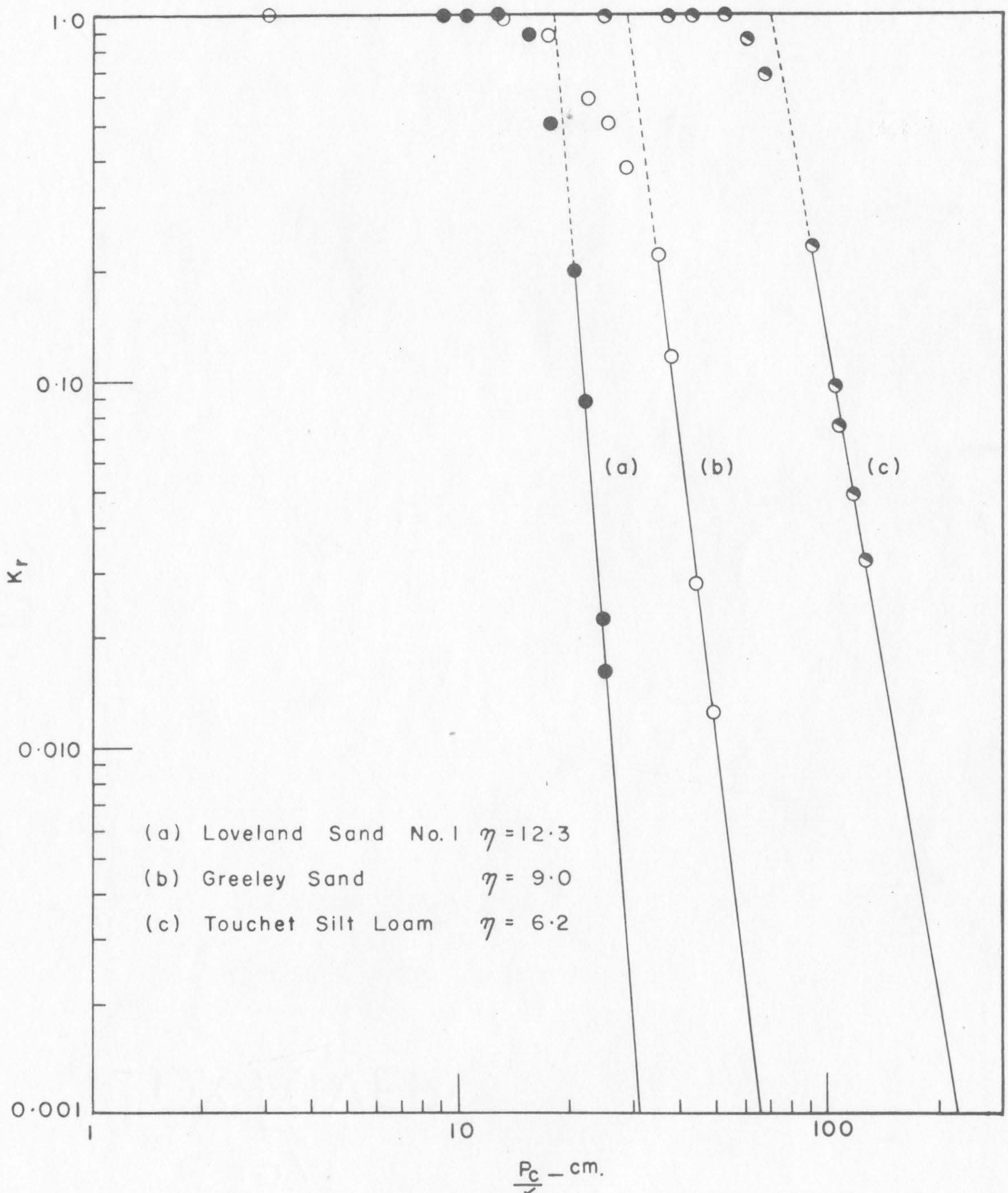

Fig. 4.--Relative permeability - capillary pressure curves of samples used.

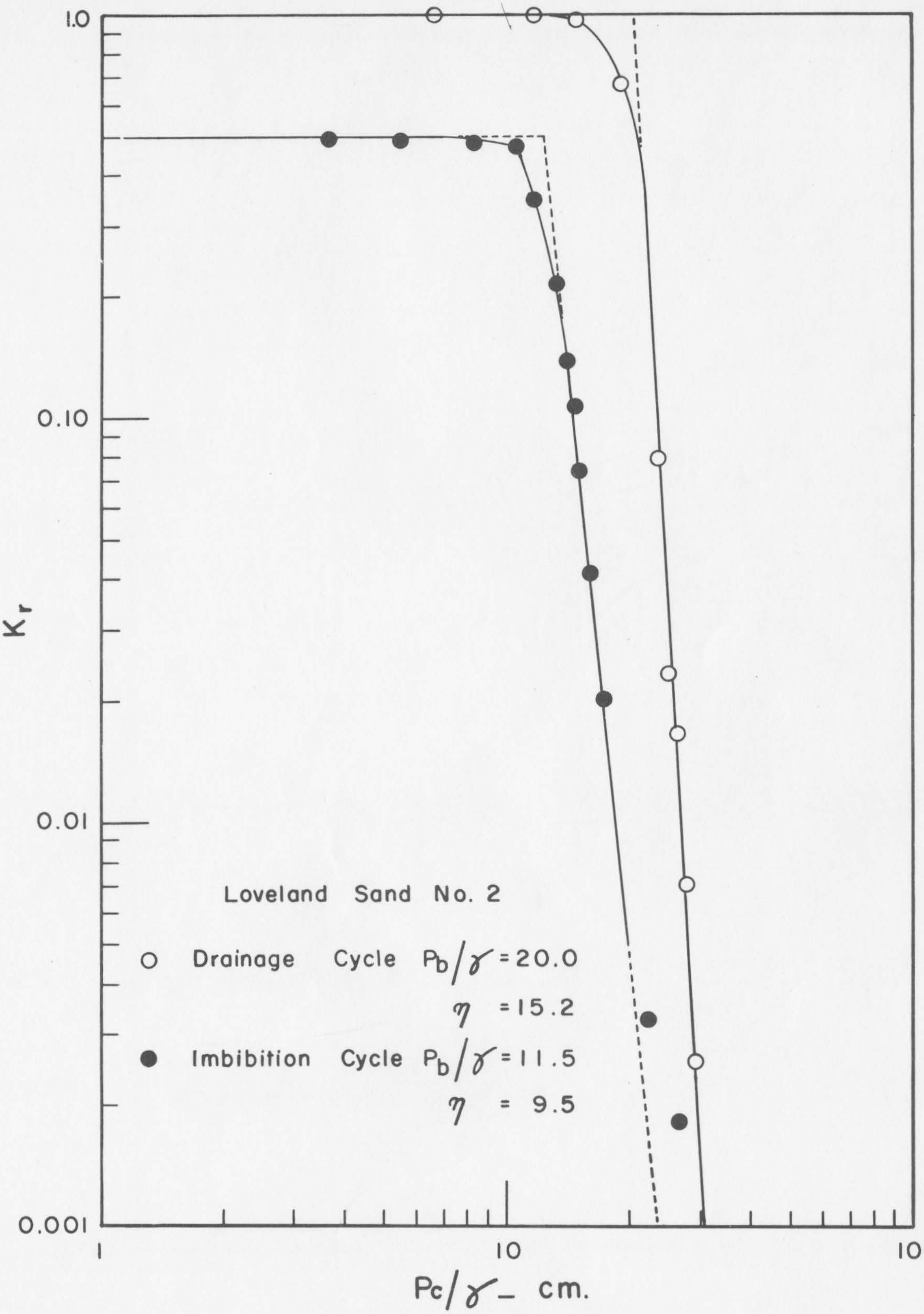

Fig. 5a. --Relative permeability-capillary pressure curves for imbibition and drainage cycles.

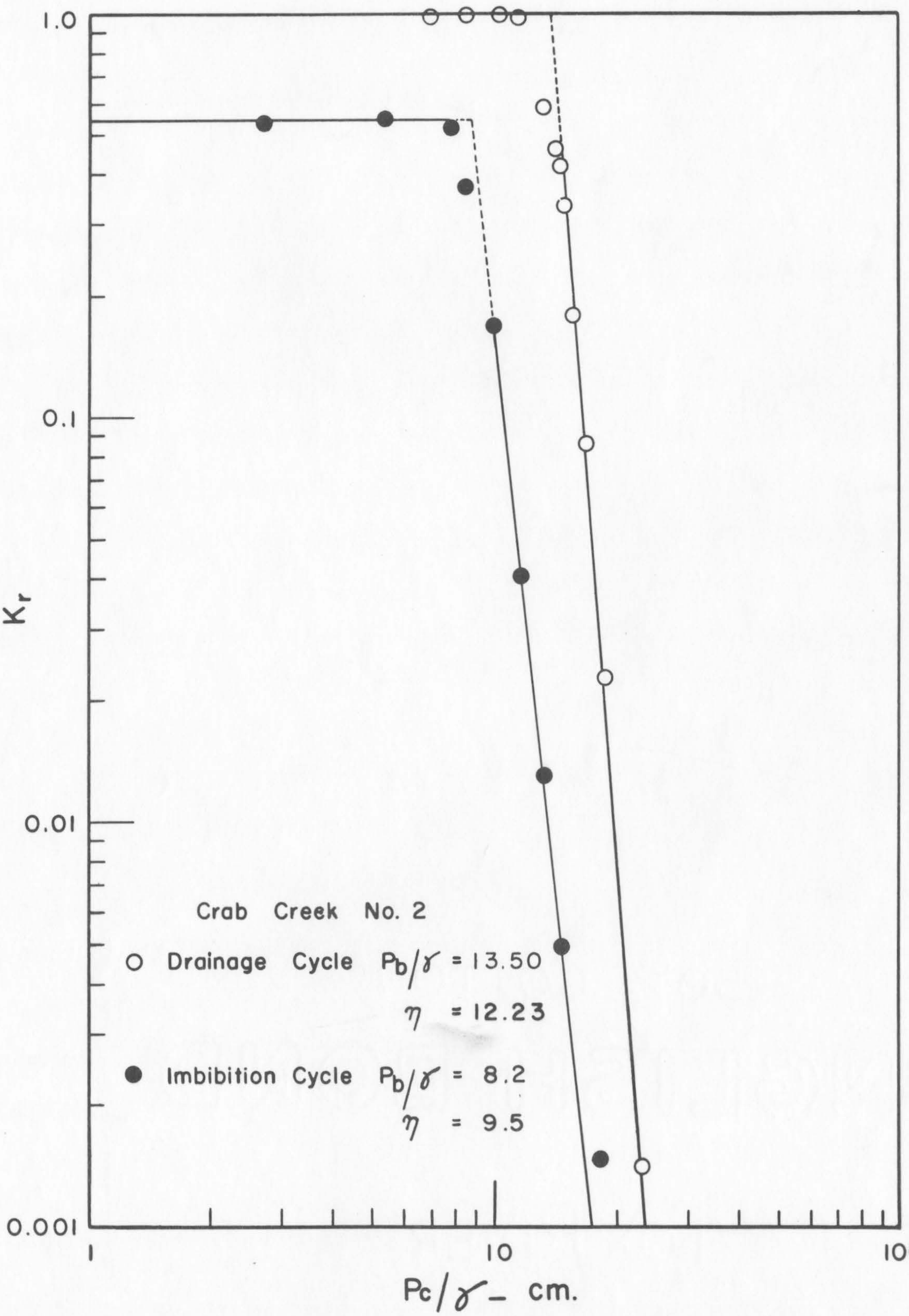

Fig. 5b. --Relative permeability-capillary pressure curves for imbibition and drainage cycles.

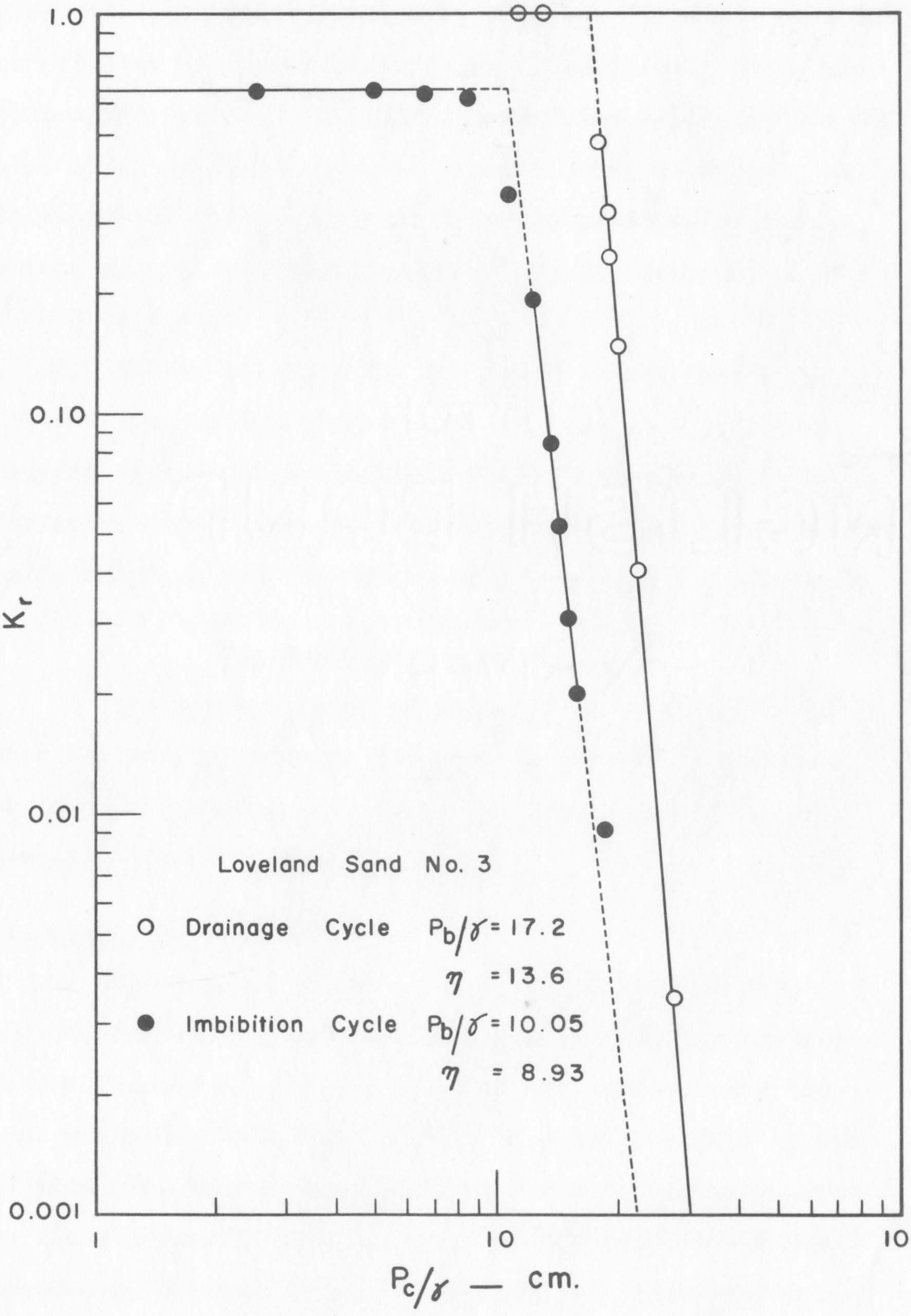

Fig. 5c. --Relative permeability-capillary pressure curves for imbibition and drainage cycle.

pressures less than the bubbling pressure, in which case the permeability on the drainage cycle is only about twice the permeability on the imbibition cycle. The value of η tended to be slightly smaller on the imbibition cycle than on the drainage cycle. The value of bubbling pressure on the imbibition cycle is about 0.6 the bubbling pressure on the drainage cycle.

It has been pointed out by Bloomsburg and Corey [2] that for capillary pressures less than the bubbling pressure on the imbibition cycle, the conductivity is a function of time since the entrapped air eventually diffuses from the system and the medium will become fully saturated.

Maximum Rates of Upward Flow

The maximum rates of upward flow were investigated, after the soil parameters had been determined. They were measured for three cases, i.e., drainage, imbibition and imbibition-drainage cycles.

Drainage Cycle

The maximum rates of upward flow were obtained by lowering the outflow and inflow siphons until the capillary pressure on the surface of the soil was sufficiently high to give almost the maximum rate. This was possible because the flow rate approaches a maximum for a finite value of capillary pressure at the surface of the soil. Capillary pressure at the top of the soil column was increased carefully so that there would be no pressure reversal near the surface of the soil, as explained in Chapter IV.

The depth to water table was calculated from the capillary-pressure readings at the lower tensiometer. When the soil at the lower tensiometer was fully saturated, a theoretical elevation of the lower tensiometer above the water table was calculated using Darcy's law and the hydraulic conductivity when fully saturated. The resulting equation was:

$$z = \frac{p_c}{1+q/C} \tag{5.1}$$

where z is the elevation of the lower tensiometer above the water table.

When the soil at the lower tensiometer was desaturated, the scaled depth d. to the water table was greater than 1.5 (since the column length was cut to the length of $0.5\ p_b/\gamma$). The value of the maximum rate of upward flow was then very small. The head loss across the inflow barrier was negligible, the height of peizometric head at the lower tensiometer then indicated the location of the water table. The tabulated data and results are shown in Table VI.

Maximum-rate experiments for the drainage cycle have already been conducted by Duke [10]. The experimental apparatus was the same as in his study. The procedures employed were significantly different in obtaining the maximum rates and varying the water-table depths. Duke obtained maximum rates by lowering the outflow siphon until the flow rate reached a maximum. To simulate shallower water-table depths, he cut the columns and maintained a water table at the lower tensiometer. The maximum rates obtained by Duke were 20 to 50 percent less than theoretical values, since his technique caused a pressure reversal near the surface of the soil. The soil-liquid system, therefore, did not completely follow a drainage cycle, but followed instead what is called an imbibition-drainage cycle in this dissertation as discussed in the following section.

It should be emphasized that by the techniques employed, as explained in Chapter III, resulted in no pressure reversal near the surface of the soil. The soil-liquid system, therefore, followed a drainage cycle and the experimental results agreed very closely with theoretical equations as Figure 6 and 7 show.

The scaled maximum rates of upward flow $q_{.m}$ were plotted against the corresponding values of scaled depth to the water table d. on log-log paper. The reason for doing this was that, theoretically, this curve should approach a straight line with the slope of η at the low values of $q_{.m}$.

It is apparent that the range of values of $q_{.m}$ and d. was not the same for all soils. This was because the values of bubbling pressure limited the range of $q_{.m}$ and d.. Touchet silt loam, for example, had a high bubbling pressure, 72 cm. The lack of head room then restricted the inflow siphon such that only shallow scaled water-table depths could be simulated. The values of $q_{.m}$ for this soil, therefore, were in the high range. The bubbling pressure of

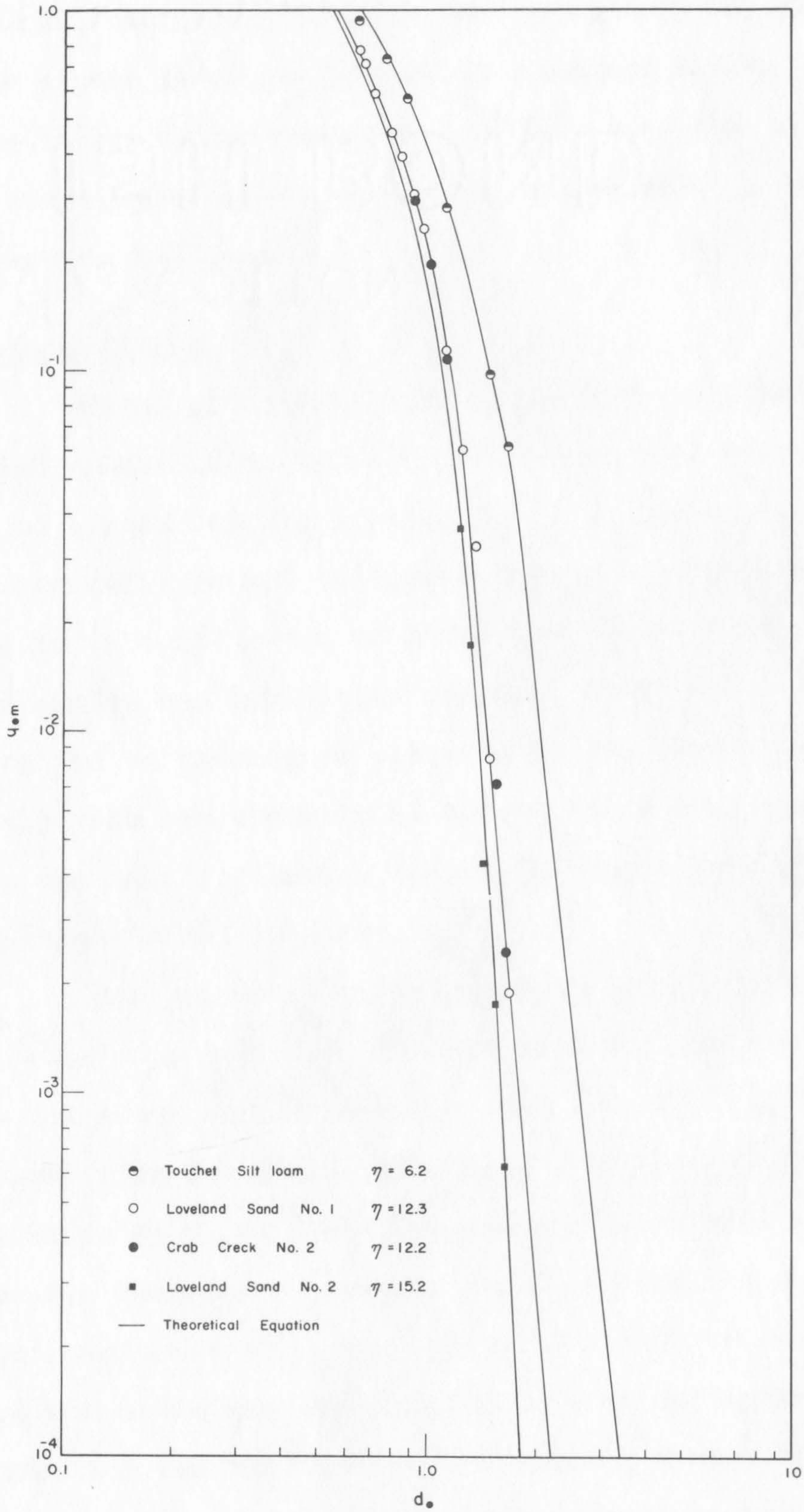

Fig. 6.--Comparison of experimental data and computed maximum rates of upward flow for drainage cycles.

Crab Creek sand, on the other hand, was only 13 cm. The inflow siphon could be lowered to simulate higher scaled depths to the water table so that soil near the surface of the column became quite dry. The values of q_m, therefore, were in the low range.

Imbibition Cycle

Three soils were used to perform experiments in which the imbibition cycle was started after draining the soil to a high capillary pressure. The inflow siphon was then gradually raised in increments. The experimental results for the first two or three increments were between the drainage and imbibition cycles. After the inflow siphon was raised to the higher elevations, the data agreed very closely with the theoretical curves which were obtained by using the soil parameters measured for the imbibition cycle, as Figures 7a and 7b show.

One series of runs were also made by starting with completely dry soil and allowing the soil to imbibe liquid from the lower inflow barrier. The initial imbibition was produced with the inflow siphon at a low elevation. The imbibition took place at an extremely slow rate and a considerable time passed before any flow from the outflow siphon was observed. Readings of the outflow rates were made, but it is extremely unlikely that sufficient time was allowed for the outflow rate to reach a steady state (or a maximum rate for the particular elevation of the inflow siphon).

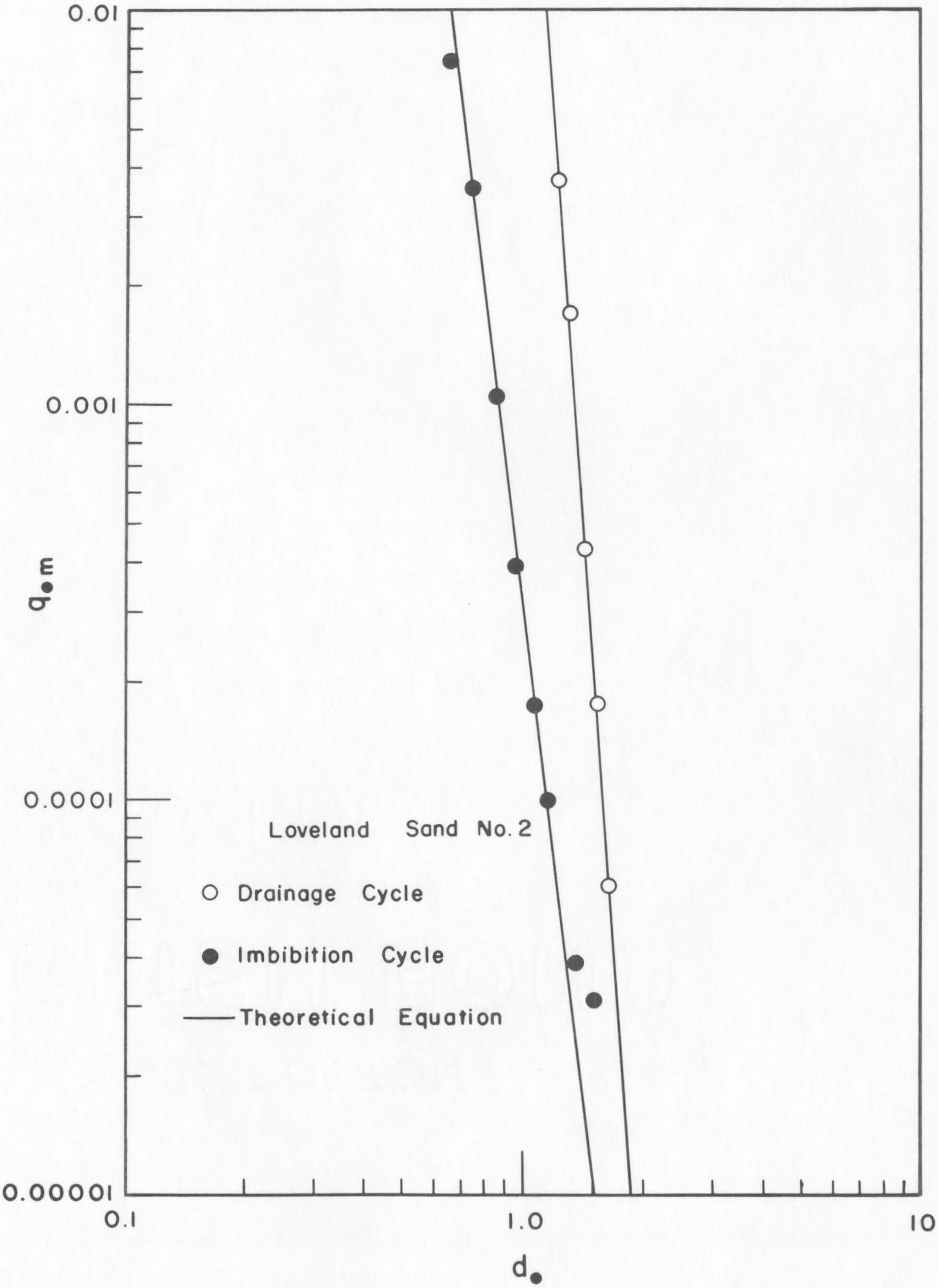

Fig. 7a. --Comparison of experimental data and computed maximum rates of upward flow for drainage and imbibition cycles.

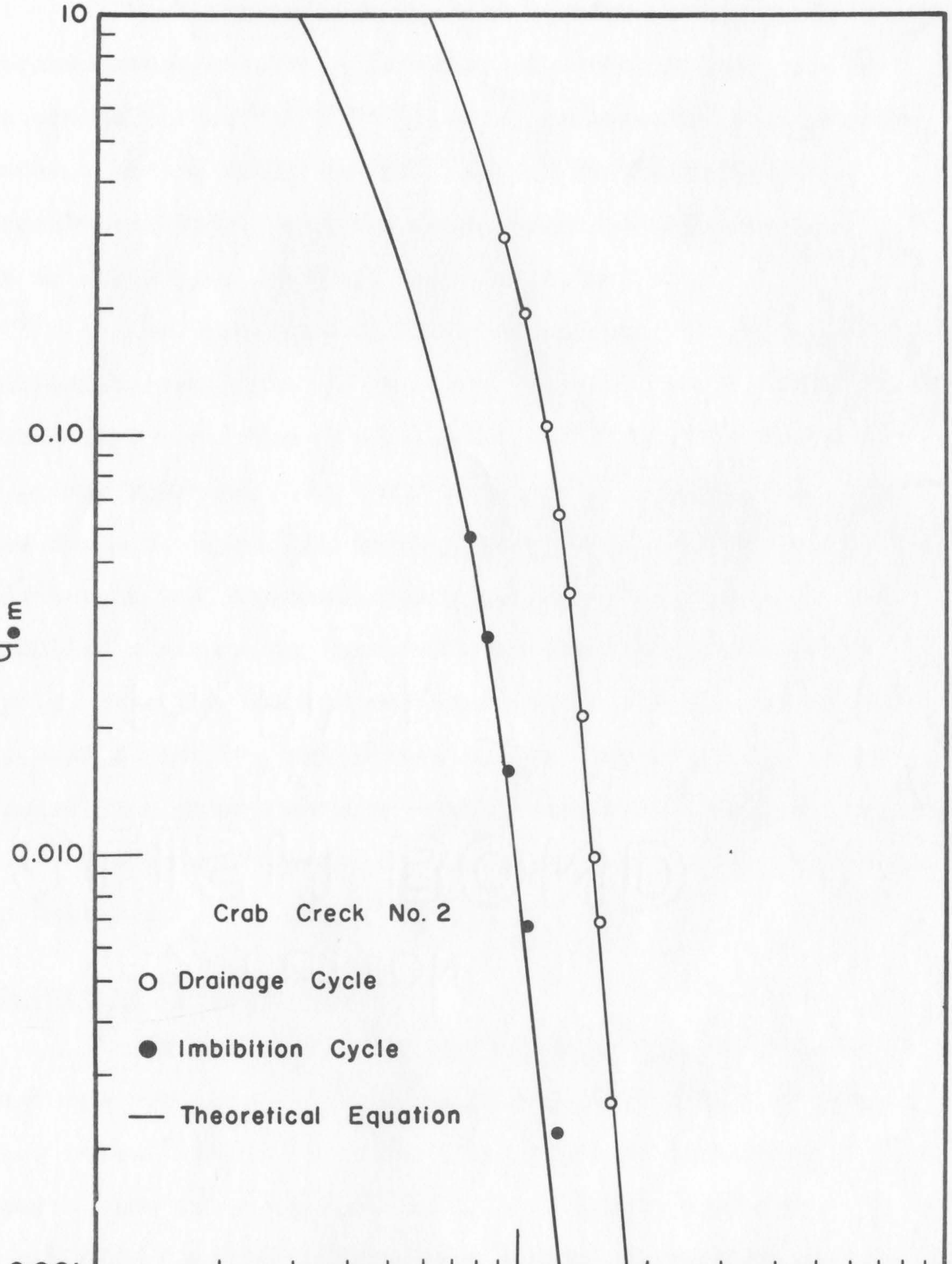

Fig. 7b. --Comparison of experimental data and computed maximum rates of upward flow for drainage and imbibition cycles.

When the inflow siphon was raised, the system reached steady state at increasingly shorter times and it is probable that the data obtained represented maximum flow rates. At any rate, the data agreed very closely with theoretical rates computed using conductivity data obtained on an imbibition cycle as shown in Figure 7c.

The theoretical relations between d. and $q_{\circ m}$ for imbibition was obtained by using the value of η of the imbibition cycle to find the value of d. for a given value of $q_{\circ m}$ from equation 3.22. Multiplying the values of $q_{\circ m}$ by the ratio of hydraulic conductivity on the imbibition cycle to that on the drainage cycle, and d. by the ratio of the bubbling pressure on imbibition to that on the drainage cycle, gave the theoretical functional relationship between $q_{\circ m}$ and d. for the imbibition cycle. A summary of maximum upward flow rates compared with theoretical values (equation 3.21) for both drainage and imbibition cycles are tabulated in Table II.

Imbibition-Drainage Cycle

One series of runs were made on the imbibition-drainage cycle to study the effect of hysteresis on upward flow rates. Figure 8 shows this effect of hysteresis. The upward flow rates were measured when steady state was reached for each increment of lowering the outflow siphon. The data are presented in Table II. The value of h_b is the elevation of the outflow siphon below the water table, the latter being kept at the elevation of the lower tensiometer.

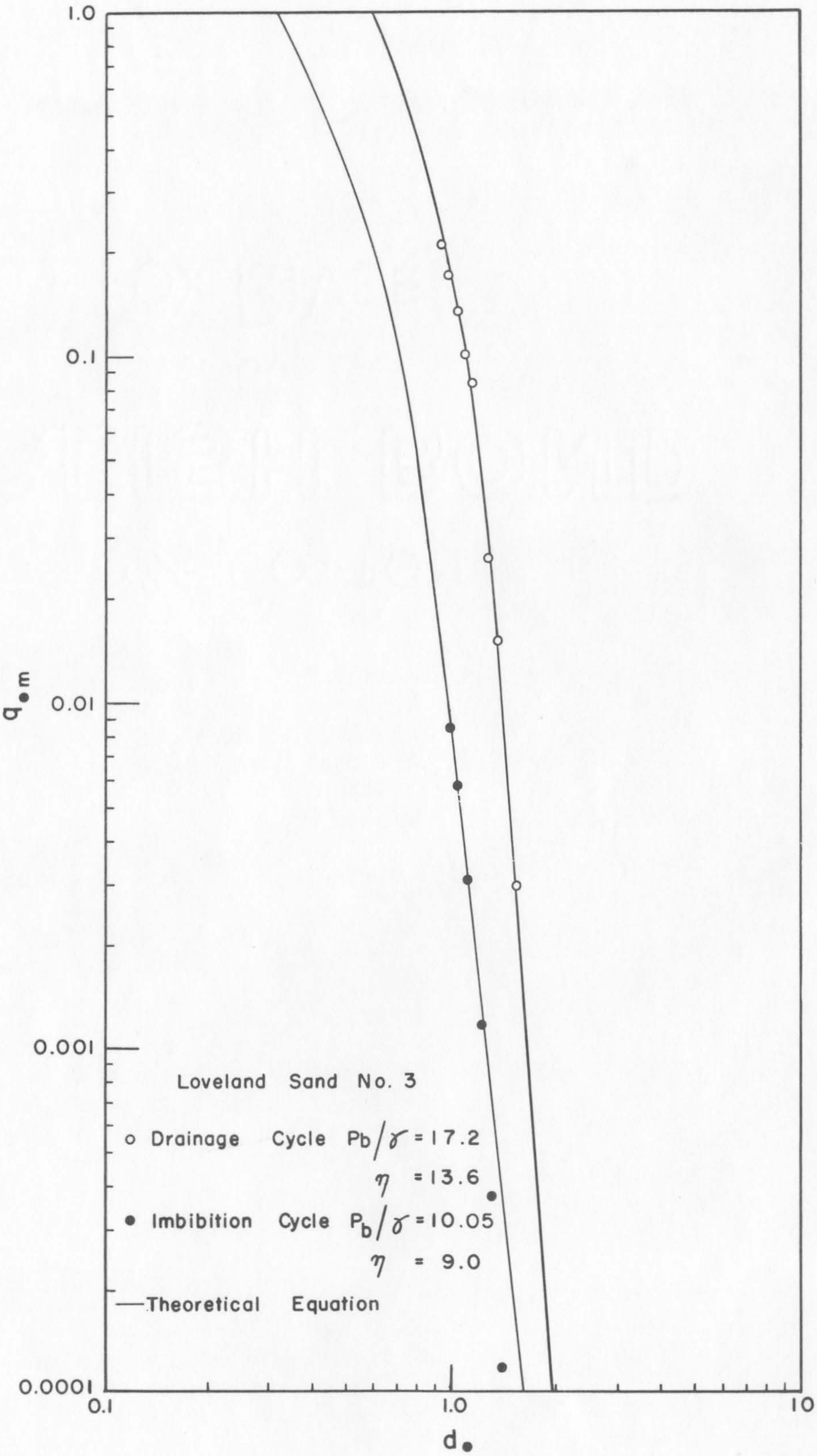

Fig. 7c. --Comparison of experimental data and computed maximum rates of upward flow for drainage and imbibition cycles.

TABLE II

SUMMARY OF MAXIMUM RATES OF UPWARD FLOW COMPARED WITH THEORETICAL VALUES (EQUATION 3.21)

Soil	$qx10^2$ cm/sec	$Cx10^2$ cm/sec	$q_{.m}$	Experimental d.	Eq.3.21 d.
Loveland Sand No. 1	0.80800	1.047	0.77200	0.647	0.639
Drainage cycle	0.74060	"	0.70700	0.676	0.667
$K = 19.64\mu^2$	0.61300	"	0.58600	0.718	0.723
$P_b/\gamma = 18.00$ cm.	0.47300	"	0.45200	0.795	0.797
$\eta = 12.3$	0.40900	"	0.39030	0.843	0.837
	0.31000	"	0.30200	0.920	0.904
	0.25200	"	0.24130	0.977	0.959
	0.20300	"	0.19400	1.034	1.008
	0.13500	"	0.12800	1.112	1.093
	0.06200	"	0.05930	1.246	1.225
	0.00870	"	0.00830	1.456	1.495
	0.00200	"	0.00190	1.772	1.710
Touchet Silt Loam					
Drainage cycle	0.02663	0.0285	0.93500	0.625	0.667
$K = 0.50\mu^2$	0.02062	"	0.72400	0.763	0.759
$P_b/\gamma = 72.2$ cm.	0.01617	"	0.50800	0.874	0.888
$\eta = 6.2$	0.00857	0.0300	0.28400	1.120	1.098
	0.00291	"	0.09600	1.573	1.453
	0.00182	"	0.06100	1.650	1.596
Loveland Sand No. 2					
Drainage cycle	0.03610	0.943	0.03820	1.140	1.210
$K = 1670\mu^2$	0.02580	"	0.03660	1.223	1.219
$P_b/\gamma = 20.0$ cm.	0.01590	"	0.01690	1.313	1.300
$\eta = 15.2$	0.00400	"	0.00428	1.407	1.436
	0.00160	"	0.00173	1.528	1.526
	0.00058	0.950	0.00061	1.625	1.637
Imbibition cycle					
$K = 8.17\mu^2$	0.00080	0.943	0.00008	1.220	1.156
$P_b/\gamma = 11.5$ cm.	0.00094	"	0.00010	1.148	1.123
$\eta = 9.5$	0.00161	"	0.00017	1.045	1.065
	0.00370	"	0.00039	0.965	0.971
	0.00970	"	0.00104	0.865	0.869
	0.03340	"	0.00355	0.739	0.738
	0.06980	"	0.00743	0.650	0.648
Crab Creek No. 2					
Drainage cycle	0.40720	1.381	0.29500	0.919	0.912
$K = 24.45\mu^2$	0.27130	"	0.19640	1.020	1.007
$P_b/\gamma = 13.50$ cm.	0.14640	"	0.10600	1.138	1.129
$\eta = 12.2$	0.09080	"	0.06520	1.216	1.211
	0.05900	1.393	0.04230	1.289	1.274
	0.03200	"	0.02297	1.377	1.360
	0.01400	1.381	0.01040	1.489	1.470
	0.00980	"	0.00709	1.542	1.513
	0.00365	"	0.00264	1.627	1.647

TABLE II (continued)

	qx10^2 cm/sec	Cx10^2 cm/sec	q_m	Experimental d.	Eq.3.21 d.
Imbibition cycle					
K = 15.89μ^2	0.00146	1.447	0.00103	1.443	1.270
p_b/γ = 8.2 cm.	0.00318	"	0.00225	1.221	1.132
η = 9.0	0.00993	"	0.00684	1.040	0.985
	0.02340	"	0.00162	0.925	0.923
	0.04870	"	0.00337	0.827	0.836
	0.08230	"	0.00569	0.754	0.770
Loveland Sand No. 3					
Drainage cycle	0.26470	1.269	0.20860	0.938	0.975
K = 21.5μ^2	0.21860	"	0.17230	0.981	1.015
p_b/γ = 17.2 cm.	0.16950	"	0.13340	1.032	1.063
η = 13.6	0.12520	1.257	0.09960	1.090	1.099
	0.10400	"	0.08270	1.157	1.139
	0.03202	1.243	0.02580	1.278	1.299
	0.01856	"	0.01490	1.358	1.363
	0.00371	"	0.00300	1.549	1.542
Imbibition cycle					
K = 13.85μ^2	0.00018	"	0.00012	1.390	1.552
p_b/γ = 10.05 cm.	0.00058	"	0.00037	1.303	1.368
η = 9.0	0.00178	"	0.00115	1.221	1.205
	0.00475	"	0.00306	1.109	1.079
	0.00889	"	0.00574	1.040	1.006
	0.01267	"	0.00818	0.985	0.962

As the outflow siphon was lowered, the capillary pressure at the soil surface immediately increased, the pressure gradient near the soil surface was then greater than that at a lower depth.

When the capillary pressure at the surface of the soil column was less than the bubbling pressure, the soil remained saturated. Increasing the capillary pressure at the top (lowering the outflow siphon), therefore, resulted in increasing the upward flow rate since hysteresis could not occur under fully-saturated conditions.

After the capillary pressure at the surface of the soil was greater than the bubbling pressure, the soil near the top of the column desaturated. Further lowering the outflow siphon caused the rate of outflow to exceed the upward flow rate. The liquid which occupied the pore space of the driest zone was removed faster than the soil near the top of the column could imbibe the liquid from the wetter soil below.

The effect of imbibition tended to decrease the upward flow rate as explained in Chapter II, although increasing capillary pressure at the surface of the soil tended to increase the upward flow. The rate of upward flow usually increased slightly as the outflow siphon was lowered (in cases where the capillary pressure at the top was slightly greater than bubbling pressure) but not at the theoretical rate.

When the capillary pressure reached the point where the upward flow rates were nearly constant with increasing capillary pressure, the upward flow rate decreased with further lowering of the outflow siphon. Instead of increasing, the capillary pressure immediately below the surface of the soil was observed to decrease as has been observed by many others [10,23,35].

The lower the outflow siphon, the greater the thickness on the zone of imbibition the smaller the measured flow rates become. The phenomenon explained above is shown in Figure 8.

The outflow siphon was raised after it had been lowered. The upward flow rates were then less than those found by lowering the outflow siphon, since the zone of imbibition became thicker.

A broken line was drawn parallel to the curve obtained when the outflow was raised, to represent the theoretical. The values of the maximum rate of upward flow represented by the broken line are equal to the value predicted from the theoretical equation.

The upward flow rates of this case were not unique for a particular soil and water-table depth, since they depend on several factors. One of the primary factors is the increment of increasing the capillary pressure at the surface of the soil. To prove this fact, a series of runs were made to study the effect of the increments of capillary pressure applied at the column surface.

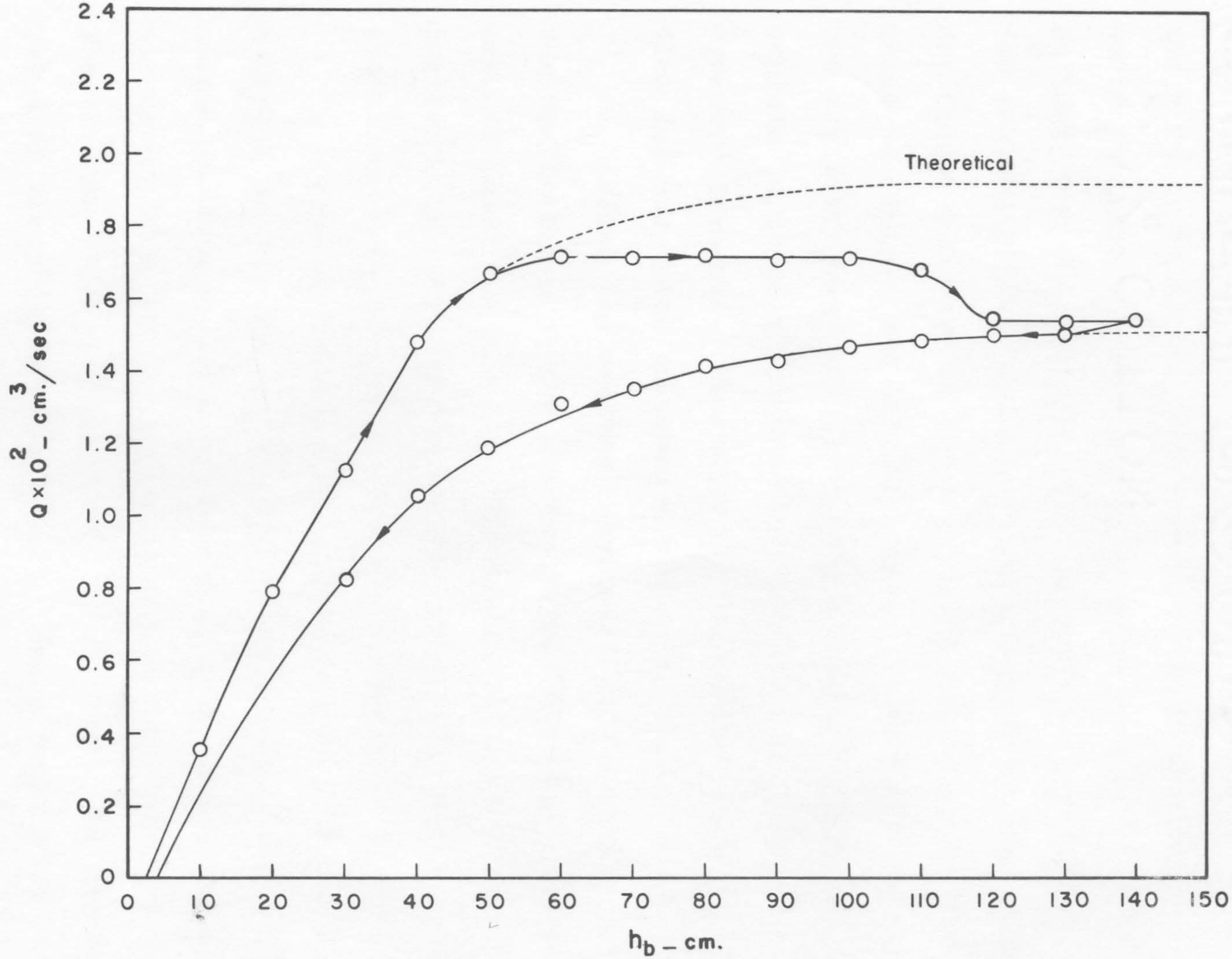

Fig. 8. --Effect of hysteresis on maximum rate of upward flow.

An experiment was made with the outflow siphon being lowered in increments of 20 cm. Similar runs were made with the siphon being lowered in increments of 10 cm. and 5 cm. The results are shown in Figure 9 where h_b represents the negative head on the outflow barrier. It is evident from this result that the maximum upward flow rates for the imbibition-drainage cycle obtained by Duke [10] were not unique but were affected by the particular increments in which he lowered his outflow siphon. Evidently, increasing the capillary pressure at the surface quickly so that the surface layer is dried quickly results in an earlier abrupt pressure reversal. The order of the maximum rates of upward flow for the three increments were: 5 cm. > 10 cm. > 20 cm.

After the maximum rates were reached however, further lowering of the outflow siphon gave results which are difficult to explain. The reduction in flow rates appeared to be associated with the number of times the outflow siphon was lowered rather than the increment of lowering.

This is insufficient evidence to explain why this behavior was observed. The most that can be definitely concluded is that the flow rates are not unique after hysteresis occurs but depend on the increment and number of times the outflow siphon is lowered. In any case, the maximum rates of upward flow were found to range from 20 to 50 percent lower than the theoretical values, as was also found by Duke [10].

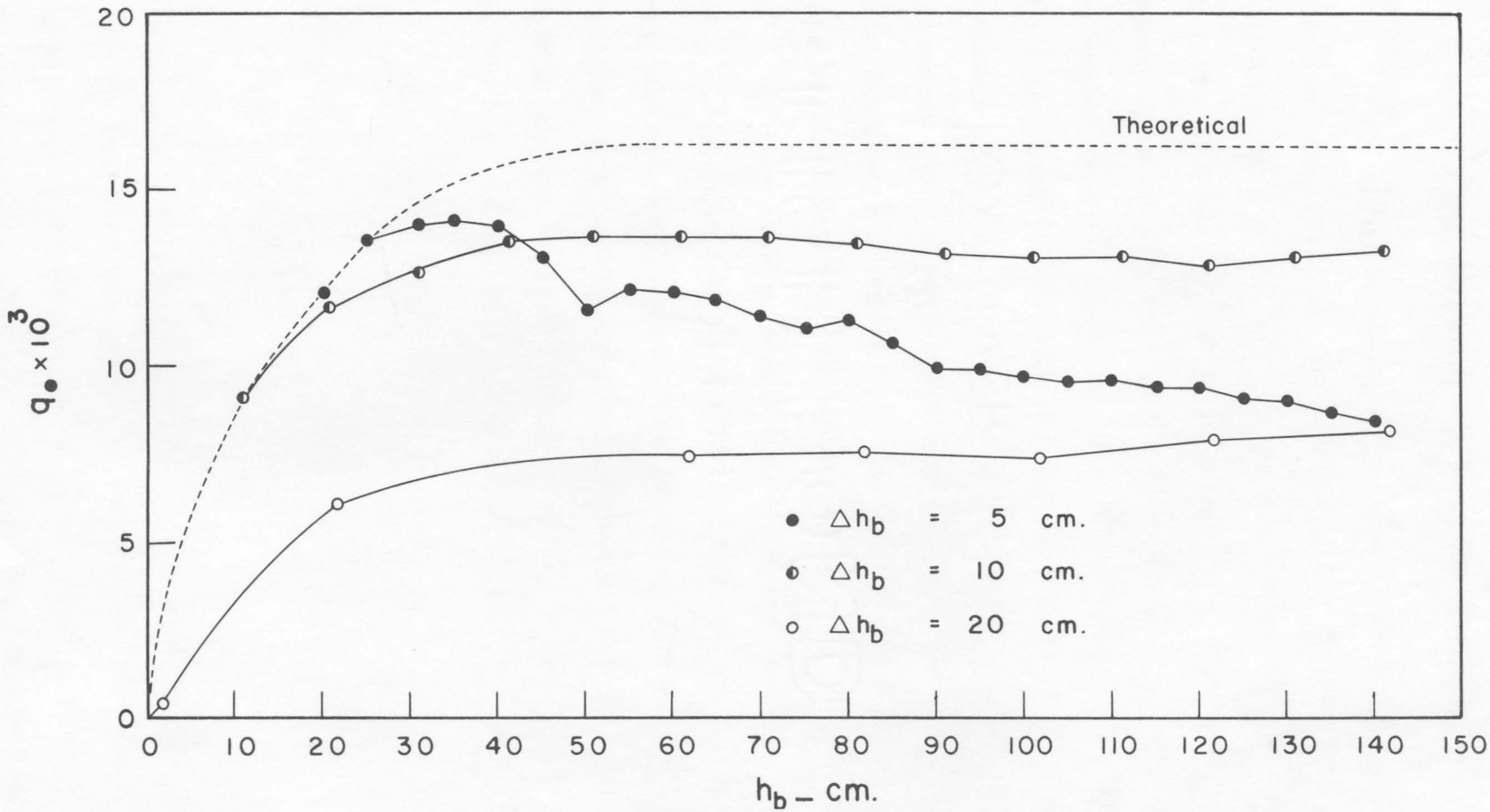

Fig. 9.--Effect of hysteresis on maximum rate of upward flow for various increments, Δh_b.

Approximate Equation for Maximum Upward Flow Rates

An algebraic equation representing an approximate solution was developed to determine the functional relationship between $q_{\circ m}$, $d_\circ$ and η. A discussion of this derivation is given in Chapter III. The resulting equations were represented by equation 3.22 (for the case of $q_{\circ m}$ less than unity) and equation 3.25 (for the case of $q_{\circ m}$ greater than unity). The significance of these two equations is that, knowing the distance (d) from the water table to the dried layer and the necessary soil parameters (also whether the soil has followed a drainage or imbibition cycle), it is possible to compute the maximum rate of upward flow.

For shallow water-table depths or for large values of $q_{\circ m}$, the maximum flow rate is controlled by external evaporative conditions. The author suspects that equation 3.25 may not be applicable to practical cases. For most actual problems in nature, the value of $q_{\circ m}$ is small and if it is less than 0.01, equation 3.22 can be approximated by:

$$d_\circ \approx \frac{1}{q_{\circ m}^{1/\eta}}\left(1+\frac{1.886}{\eta^2+1}\right) \qquad (5.2)$$

or

$$q_{\circ m} \approx \left(1+\frac{1.886}{\eta^2+1}\right)^{\eta} d_\circ^{-\eta} \quad . \qquad (5.3)$$

It should be pointed out that equation 5.3 is in a form somewhat analogous to equations 2.14-2.17 which were presented by Gardner [12] for values of η from 1 to 4 only. To obtain these equations Gardner also assumed that q_m was small so that q_m/a approached zero, "a" being a soil parameter.

The accuracy of the approximate equation of I(x) may be examined by comparing the values of $I(\infty)$ given in equation 3.19 with values of $I(\infty)$ obtained by using the contour integral method [6]. The tabulated values are presented in Table III.

TABLE III

VALUES OF $I(\infty)$ COMPUTED FROM EQUATION 3.19 COMPARED WITH VALUES COMPUTED BY THE CONTOUR INTEGRAL METHOD

η	$I(\infty)$ Contour Integral*	$I(\infty)$ Equation 3.19
2	1.5708	1.3772
4	1.1105	1.1109
6	1.0472	1.0509
8	1.0262	1.0290
10	1.0166	1.0187
12	1.0138	1.0130
∞	1.0	1.0

*The contour integral method gives $I(\infty) = \frac{\pi}{\eta} \sum_{m=1}^{\eta/2} \sin\frac{(2m-1)\pi}{\eta}$ when η is an even integer.

The value of $I(\infty)$ given by equation 3.19 is not accurate enough when $\eta < 4$. This is not a serious disadvantage since for $\eta = 2$ or 3, the exact values can easily be obtained by direct integration.

As further evidence that the values of the maximum rates of upward flow from equation 3.22 are sufficiently accurate for $\eta \geq 4$, the results from this equation were compared with the values presented by Duke [10]. He obtained these results by utilizing an IBM 1401 computer. The comparison of solutions are shown in Table IV for values of $\eta = 4, 8$, and 12.

TABLE IV

COMPARISON OF VALUES OF $d_{\circ}$ AS A FUNCTION OF $q_{\circ m}$ FROM EQUATION 3.22 WITH THE VALUES PRESENTED BY DUKE

	$\eta = 4$		$\eta = 8$		$\eta = 12$	
$q_{\circ m}$	Eq.3.22	Duke	Eq.3.22	Duke	Eq.3.22	Duke
0.001	6.253	6.24	2.439	2.43	1.801	1.80
0.002	5.253	5.25	2.236	2.23	1.701	1.70
0.004	4.419	4.41	2.046	2.04	1.601	1.60
0.008	3.708	3.71	1.874	1.87	1.507	1.50
0.016	3.111	3.11	1.712	1.71	1.415	1.41
0.032	2.601	2.60	1.555	1.55	1.312	1.32
0.064	2.161	2.16	1.398	1.39	1.22	1.22
0.128	1.768	1.77	1.230	1.23	1.10	1.10
0.256	1.403	1.40	1.042	1.04	0.948	0.947
0.512	1.057	1.06	0.826	0.822	0.764	0.762

Suggestions For Further Study

Application of this study to actual field conditions requires the value of the soil parameters. The development of techniques for measuring the bubbling pressure, permeability and η either from undisturbed samples or direct measurements in the field would be very useful.

The relative permeability-capillary pressure curves for the imbibition cycle in this study were made by starting the experiment with relatively dry soil, since the time required when starting with air dry soil is very long. A study should be made to answer the question, "How well does the imbibition cycle obtained by beginning with a low saturation represent the imbibition cycle starting with air dry soil?"

Theoretical equations developed in this study are applicable only in the case of homogeneous media. The following section discusses briefly a development which might be applicable to stratified soils but which needs experimental verification.

Maximum Rate of Upward Flow From a Water Table Through Stratified Soils

In this analysis the water is transmitted upward from a water table through a soil having parameters C_1, P_b/γ and η_1 to a distance d_1 beyond which the water is transmitted through another soil to its surface where a maximum evaporation is induced (see Figure 10). Equation 3.2 can be applied to obtain the maximum rate of upward flow in this case, by changing the limit of integrals to fit the boundary conditions which are: At the water table $z = o$, $p_c = o$; at the boundary between the two soils $z = d_1$, $p_c = p_c'$; at the

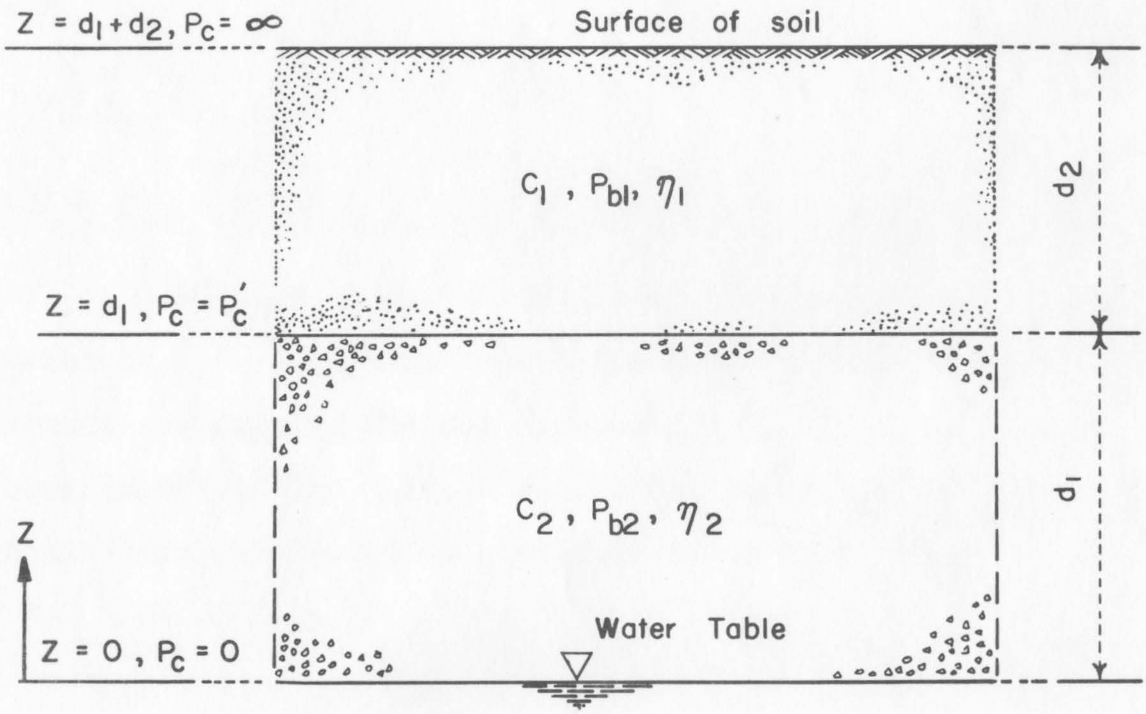

Fig. 10.--Sketch of stratified soil profile in which water flows upward at a steady rate.

surface of the soil $z = d_1 + d_2$, $p_c \to \infty$. The resulting solutions are:

$$\frac{d2}{p_{b2}/\gamma} = \frac{1-p_c'/p_b}{1+q_{.2}} + \frac{1}{q_{.2}^{1/\eta 2}} \left\{ I(\infty) - I(q_{.2}^{1/\eta 2}) \right\}$$

$$\text{for } p_c' > p_{b2} \qquad (5.4)$$

and

$$\frac{d2}{p_{b2}/\gamma} = \frac{1}{q_{.2}^{1/\eta 2}} \left\{ I(\infty) - I(p_c'/p_{b2}\, q_{.2}^{1/\eta 2}) \right\} \text{for } p_c' < p_{b1} \qquad (5.5)$$

where $q_{.2} = q/C_2$.

The solutions of equations 5.4 and 5.5 require the value of p_c at the point on the boundary between the two strata. Employing the soil parameters C_1, p_{b1}/γ and η, in equations 3.16 and 3.17, the value of p_c can be obtained by trial from the resulting equations which are:

$$\frac{d_1}{p_{b1}/\gamma} = \frac{1}{1+q_{.1}} + \frac{1}{q_{.1}^{1/\eta}} \left\{ I(p_c'/p_{b1}\, q_{.1}^{1/\eta}) - I(q_{.1}^{1/\eta}) \right\}$$

$$\text{for } p_c' < p_{b1}\,. \qquad (5.6)$$

Equations 5.4, 5.5, and 5.6 give the value of d_2 as a function of d_1 and the maximum rate of upward flow q, for given values of C_1, p_{b1}/γ, η_1, and C_2, p_{b2}/γ, η_2.

A solution in the case of 3 or more layers of soil might be obtained by a similar method.

CHAPTER VI

SUMMARY AND CONCLUSION

An investigation was conducted to study the steady upward flow from water tables. The study attempted to relate the maximum rates of upward flow to measurable soil parameters under various conditions of soil-water systems, and to determine more precisely the effect of imbibition.

Theoretical solutions were developed to determine the maximum flow rate as a function of depth of water table and the measurable soil parameters. A differential equation was obtained by assuming Darcy's law is valid when the medium is partially saturated and employing Brooks and Corey's relative permeability-capillary pressure relationship. The equation is

$$d_{\circ} = \frac{1}{1+q_{\circ m}} + \int_{1}^{\infty} \frac{d_{p_{\circ}}}{1+q_{\circ m}p_{\circ}^{\eta}}$$

A method was employed to approximate the solution of this equation. The technique was to expand the integral into a convergent series and integrate term by term. An algebraic approximation was then made to obtain the solution. The resulting equations are:

Case I: $q_{\circ m} < 1$

$$d_{\circ} \approx \frac{1}{q_{\circ m}^{1/\eta}}\left(1+\frac{1.886}{\eta^2+1}\right) - \frac{q_{\circ m}}{1+q_{\circ m}} + \frac{1}{\eta+1}\ln(1+q_{\circ m})$$

Case II: $q_{\circ m} < 0.001$

$$d_{\circ} \approx \frac{1}{q_{\circ m}^{1/\eta}}\left(1+\frac{1.886}{\eta^2+1}\right)$$

Case III: $q_{\circ m} > 1$

$$d_{\circ} \approx \frac{1}{1+q_{\circ m}} + \frac{1}{\eta-1}\ln\frac{(1+q_{\circ m})}{q_{\circ m}}$$

To verify these equations, seven soil columns were packed to give a wide range of soil parameters. The study was limited to the condition where factors controlling the maximum rates of upward flow are the liquid-soil conditions. Flow was induced at the top of the soil column by means of an outflow siphon. An inflow siphon was connected to an inflow barrier at the bottom of the column to maintain a water table within or below the column as desired. The maximum rates were measured when the outflow and inflow siphon were lowered until the soil near the top of the column was relatively dry and the system reached a steady state. The soil parameters necessary to obtain a solution were determined by employing a controlled-pressure method, before the upward flow experiments were made.

The upward flow experiments were conducted under three conditions of the soil-liquid system which were: drainage, imbibition and imbibition-drainage cycles. The first two were performed to verify the theoretical solution. The agreement between the theoretical solution and experimental results for both drainage and imbibition cycles was very good.

The imbibition-drainage cycle was studied to determine the effect of hysteresis (in cases where the outflow rate exceeded the upward flow rate). The results led to the conclusion that hysteresis caused the flow rate to drop when the capillary pressure at the surface of the soil was increased after the soil near the surface was relatively dry. The maximum flow rates obtained in this case were not unique but depended on the increments of increasing the capillary pressure at the surface of the soil. Their values, however, were about 20 to 50 percent less than theoretical values.

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A P P E N D I X

TABLE V

RELATIVE PERMEABILITY-CAPILLARY PRESSURE DATA

Soil	p_c/γ (cm.)	K_r
Loveland Sand No. 1		
Drainage cycle	10.70	1.0000
	13.00	1.0000
$K = 19.64\mu^2$	15.45	0.8960
	15.80	0.8100
$p_b/\gamma = 18.00$ cm.	17.80	0.5020
	20.30	0.2000
$\eta = 12.3$	22.00	0.0874
	24.80	0.0226
	25.0	0.016
Touchet Silt Loam		
Drainage cycle	9.65	1.0000
	24.80	1.0000
$K = 0.50\mu^2$	37.05	1.0000
	43.50	0.9770
$p_b/\gamma = 72.2$ cm.	53.10	0.9090
	60.30	0.8600
$\eta = 6.2$	68.30	0.6800
	71.50	0.6600
	91.35	0.2330
	106.90	0.09680
	107.50	0.07550
	117.50	0.04870
	127.60	0.03200
Crab Creek No. 1		
Drainage cycle	9.50	1.00000
	10.30	1.00000
$K = 29.08\mu^2$	11.10	0.98400
	14.10	0.43400
$p_b/\gamma = 13.2$ cm.	17.00	0.08190
	20.80	0.00504
$\eta = 11.0$	22.10	0.00388
Greeley Sand		
Drainage cycle	3.00	1.00000
	9.00	1.00000
$K = 15.20\mu^2$	13.00	0.98500
	17.50	0.88280
$p_b/\gamma = 23.00$ cm.	22.30	0.59050
	25.10	0.50200
$\eta = 9.0$	28.70	0.38620
	35.20	0.22100
	38.00	0.13450
	43.0	0.03070
	48.8	0.01260

TABLE V (Continued)

Soil	p_c/γ (cm.)	K_r
Loveland Sand No. 2		
Drainage cycle	6.60	1.00000
	11.30	1.00000
$K = 16.70\mu^2$	14.50	1.00000
	18.50	0.69200
$p_b/\gamma = 20.0$ cm.	23.50	0.10200
	25.10	0.00234
$\eta = 15.2$	26.10	0.00166
	27.80	0.00071
	29.30	0.000262
Imbibition cycle		
	26.20	0.00018
$K = 8.17\mu^2$	22.25	0.00033
	17.50	0.02050
$pb/\gamma = 12.2$ cm.	16.00	0.04150
	15.00	0.07514
$\eta = 9.5$	14.60	0.10920
	13.20	0.21100
	11.80	0.34870
	10.30	0.46950
	8.20	0.47700
	5.40	0.48500
	3.60	0.48960
Crab Creek No. 2		
Drainage cycle	7.00	1.00000
	8.55	1.00000
$K = 24.45\mu^2$	10.33	1.00000
	13.35	0.59000
$p_b/\gamma = 13.50$ cm.	14.25	0.45500
	14.70	0.31200
$\eta = 12.2$	15.80	0.17757
	16.70	0.08650
	18.70	0.02266
	23.00	0.00142
Imbibition cycle		
	18.10	0.00149
$K = 15.89\mu^2$	14.75	0.00489
	13.35	0.01300
$p_b/\gamma = 8.2$ cm.	11.55	0.04025
	9.80	0.16900
$\eta = 9.0$	8.45	0.37420
	7.75	0.52310
	5.37	0.54470
	2.65	0.53200

TABLE V (Continued)

Soil	p_c/γ (cm.)	K_r
Loveland Sand No. 3		
Drainage cycle	11.20	1.00000
	12.80	1.00000
$K = 21.5\mu^2$	16.65	0.58710
	17.40	0.48000
$p_b/\gamma = 17.2$ cm.	18.45	0.32000
	18.90	0.24670
$\eta = 13.6$	19.90	0.14700
	22.20	0.04090
	24.40	0.00610
	27.60	0.00344
Imbibition Cycle		
	18.15	0.00915
$K = 13.85\mu^2$	15.70	0.01994
	14.95	0.03110
$p_b/\gamma = 10.50$ cm.	14.15	0.05340
	13.45	0.08400
$\eta = 9.0$	12.05	0.19020
	10.65	0.35740
	8.35	0.62280
	6.50	0.63570
	4.90	0.64700
	2.45	0.643

TABLE VI

MAXIMUM RATES OF UPWARD FLOW DATA

	p_c/γ (cm.)	P.	qx10^2 cm/sec.	Cx10^2 cm/sec.	$q_{.m}$	Z.*	d.**
Loveland Sand No. 1							
	1.15	0.064	0.80800	1.047	0.77200	0.036	0.647
Drainage cycle	2.00	0.111	0.74060	"	0.70700	0.065	0.676
	3.10	0.172	0.61300	"	0.58600	0.108	0.718
	4.80	0.267	0.47300	"	0.45200	0.183	0.795
	5.80	0.322	0.40900	"	0.39030	0.232	0.843
	7.25	0.403	0.31000	"	0.3020	0.309	0.920
	8.20	0.455	0.25200	"	0.24130	0.366	0.977
	9.10	0.505	0.20300	"	0.19400	0.423	1.034
	10.40	0.578	0.13500	"	0.12800	0.512	1.112
	12.10	0.672	0.06200	"	0.05930	0.635	1.246
	16.40	0.910	0.00870	"	0.00830	0.840	1.456
	19.00	1.056	0.00200	"	0.00190	1.053	1.772
Touchet Silt Loam	50.30	0.699	0.02663	0.028	0.93500	0.361	0.625
	61.90	0.860	0.02062	"	0.72400	0.498	0.763
Drainage cycle	68.90	0.957	0.01617	0.030	0.56800	0.610	0.874
	78.90	1.097	0.00857	"	0.28400	0.854	1.120
	96.50	1.325	0.00291	"	0.09600	1.209	1.573
	106.40	1.478	0.00182	"	0.06100	1.392	1.650
Loveland Sand No. 2	10.90	0.540	0.03610	0.943	0.03820	0.526	1.140
	12.40	0.620	0.02580	"	0.03660	0.598	1.223
Drainage cycle	14.00	0.700	0.01590	"	0.01690	0.688	1.313
	15.70	0.785	0.00400	"	0.00428	0.782	1.407
	18.10	0.905	0.00160	0.950	0.00173	0.903	1.528
Imbibition Cycle	11.90	0.595	0.00080	0.943	0.00008	0.595	1.220
	10.50	0.525	0.00094	"	0.00010	0.525	1.148
	8.50	0.425	0.00161	"	0.00017	0.425	1.045
	6.80	0.340	0.00370	"	0.00039	0.340	0.965
	4.80	0.240	0.00970	"	0.00104	0.240	0.865
	2.30	0.115	0.03340	"	0.00355	0.115	0.739
	0.50	0.025	0.06980	"	0.00743	0.025	0.650

*Z. is the scaled distance from a water table to the lower tensiometer.

**d. = Z + (depth to lower tensiometer).

TABLE VI (Continued)

Soil	P_c/γ (cm.)	p.	qx10^2 cm/sec.	Cx10^2 cm/sec.	q.m	Z.	d.
Crab Creek No. 2	4.40	0.326	0.40720	1.381	0.29500	0.252	0.919
	5.70	0.422	0.27130	"	0.19640	0.353	1.020
Drainage cycle	7.10	0.526	0.14640	"	0.10600	0.479	1.138
	7.90	0.585	0.09080	"	0.06520	0.549	1.216
	8.75	0.648	0.05900	1.393	0.04230	0.622	1.289
	9.80	0.726	0.03200	"	0.02297	0.710	1.377
	11.20	0.830	0.01400	1.381	0.01040	0.822	1.489
	11.90	0.882	0.00980	"	0.00709	0.875	1.542
	13.00	0.963	0.00365	"	0.00264	0.960	1.627
Imbibition Cycle	10.50	0.772	0.00146	1.447	0.00103	0.776	1.443
	7.50	0.554	0.00318	"	0.00225	0.554	1.221
	5.10	0.393	0.00993	"	0.00684	0.373	1.040
	3.60	0.258	0.02340	"	0.00162	0.266	0.925
	2.30	0.170	0.04870	"	0.00337	0.160	0.824
	1.30	0.096	0.08230	"	0.00569	0.086	0.754
Loveland Sand No. 3	5.60	0.326	0.26470	1.269	0.20860	0.269	0.938
	6.30	0.366	0.21860	"	0.17230	0.312	0.981
Drainage cycle	7.10	0.413	0.16950	"	0.13340	0.364	1.037
	8.10	0.471	0.12520	1.257	0.09960	0.428	1.090
	9.10	0.529	0.10400	"	0.08270	0.488	1.157
	10.75	0.625	0.03202	1.243	0.02580	0.609	1.278
	12.05	0.701	0.01856	"	0.01490	0.690	1.358
	15.20	0.884	0.00371	"	0.00300	0.881	1.549
Imbibition	8.90	0.848	0.00018	"	0.00012	0.848	1.390
	7.40	0.705	0.00058	"	0.00037	0.705	1.303
	6.00	0.571	0.00178	"	0.00115	0.571	1.221
	4.00	0.381	0.00475	"	0.00306	0.381	1.109
	2.90	0.276	0.00889	"	0.00574	0.276	1.040
	1.95	0.186	0.01267	"	0.00818	0.186	0.985

TABLE VII

EFFECT OF HYSTERESIS ON UPWARD FLOW RATE

Lowering the Outflow Siphon		Raising the Outflow Siphon	
h_b	$qx10^2$ (cm./sec.)	h_b	$qx10^2$ (cm./sec.)
10.0	0.3567	140.0	1.5700
20.0	0.7920	130.0	1.5050
30.0	1.2100	120.0	1.5090
40.0	1.4800	110.0	1.4820
50.0	1.6670	100.0	1.4580
60.0	1.7240	90.0	1.4336
70.0	1.7204	80.0	1.4070
80.0	1.7200	70.0	1.3470
90.0	1.7060	60.0	1.3050
100.0	1.6842	50.0	1.1900
110.0	1.7090	40.0	1.0510
120.0	1.5440	30.0	0.8016
130.0	1.5440	20.0	0.7220
140.0	1.5000	10.0	0.4310
150.0	1.590		

Note:

Soil Used: Greeley Sand

$K = 15.2$

$p_b/\gamma = 23.0$

$\eta = 9.0$

Depth to water table $d = 17.0$ cm.

Theoretical value of $q_{.m} = 0.756$

TABLE VIII

EFFECT OF INCREMENTS OF LOWERING THE OUTFLOW SIPHON ON RATES OF UPWARD FLOW

5 cm. Increments		10 cm. Increments		20 cm. Increments	
h_b	$q_{\cdot} x10^2$	h_b	$q_{\cdot} x10^2$	h_b	$q_{\cdot} x10^2$
20.0	12.009	11.0	9.226	2.0	4.115
25.0	13.624	21.0	11.661	22.0	6.1769
30.0	14.147	31.0	12.655	42.0	5.701
35.0	14.147	41.0	13.452	66.0	7.425
40.0	13.923	51.0	13.452	82.0	7.488
45.0	12.989	61.0	13.626	100.0	7.271
50.0	11.572	71.0	13.626	120.0	7.816
55.0	12.207	81.0	13.450	140.0	8.028
60.0	12.041	91.0	13.220		
65.0	11.883	101.0	13.026		
70.0	11.338	110.0	13.220		
75.0	11.027	121.0	12.800		
80.0	11.251	131.0	13.123		
85.0	10.265	141.0	13.220		
90.0	9.990				
95.0	9.967				
100.0	9.728				
105.0	9.519				
110.0	9.519				
115.0	9.960				
120.0	9.439				
125.0	9.120				
130.0	9.001				
135.0	8.822				
140.0	8.454				

Note:

Soil Used: Crab Creek No. 2, $K = 24.45$

$p_b/\gamma = 13.5$

$\eta = 12.2$

Scaled depth to water table, $d. = 0.1032$

Theoretical value of $q_{\cdot m} = 0.168$

TABLE IX

COMPUTED VALUES OF Z. (from Duke)

q_o/η	2	3	4	5	6	7	8
0.001	49.67	12.09	6.24	4.25	3.31	2.77	2.43
.002	35.12	9.60	5.25	3.70	2.95	2.51	2.23
.004	24.83	7.61	4.41	3.22	2.62	2.27	2.04
.008	17.56	6.04	3.71	2.80	2.33	2.05	1.87
.016	12.41	4.79	3.11	2.43	2.07	1.85	1.71
.032	8.76	3.79	2.60	2.10	1.83	1.16	1.55
.064	6.17	2.98	2.16	1.80	1.60	1.48	1.39
.128	4.32	2.32	1.77	1.52	1.38	1.29	1.23
.256	2.98	1.76	1.40	1.24	1.14	1.08	1.04
0.512	1.99	1.27	1.06	0.951	0.890	0.850	0.822
1.024	1.26	0.861	0.733	.672	.635	.611	.594
2.048	0.754	.535	.465	.430	.409	.395	.386
4.096	.423	.308	.270	.252	.240	.233	.228
8.192	0.226	0.167	0.148	0.138	0.132	0.128	0.125

q_o/η	9	10	11	12	13	14	15
0.001	2.20	2.03	1.90	1.80	1.72	1.65	1.60
.002	2.03	1.89	1.78	1.70	1.63	1.57	1.52
.004	1.88	1.76	1.67	1.60	1.54	1.49	1.45
.008	1.74	1.64	1.56	1.50	1.46	1.42	1.38
.016	1.60	1.52	1.46	1.41	1.37	1.34	1.31
.032	1.47	1.41	1.36	1.32	1.29	1.26	1.24
.064	1.33	1.28	1.25	1.22	1.19	1.17	1.15
.128	1.18	1.15	1.12	1.10	1.08	1.06	1.05
.256	1.01	0.982	0.962	0.947	0.934	0.923	0.914
0.512	0.802	.786	.773	.762	.654	.747	.740
1.024	.581	.571	.563	.557	.552	.547	.543
2.048	.378	.373	.368	.365	.361	.359	.357
4.096	.224	.221	.218	.216	.214	.213	.212
8.192	0.123	0.122	0.120	0.119	0.118	0.118	0.117

TABLE IX (Continued)

q_o/n	16	17	18	19	20
0.001	1.55	1.51	1.47	1.44	1.42
.002	1.48	1.45	1.42	1.39	1.37
.004	1.42	1.39	1.36	1.34	1.32
.008	1.35	1.33	1.31	1.29	1.27
.016	1.29	1.27	1.25	1.23	1.22
.032	1.22	1.20	1.19	1.17	1.16
.064	1.14	1.13	1.11	1.10	1.09
.128	1.04	1.03	1.02	1.01	1.00
.256	0.905	0.898	0.892	0.887	0.882
0.512	.735	.730	.726	.722	.719
1.024	.540	.537	.534	.532	.530
2.048	.355	.353	.352	.350	.349
4.096	.211	.210	.209	.208	.208
8.192	0.116	0.116	0.116	0.115	0.115

ABSTRACT OF DISSERTATION

STEADY UPWARD FLOW FROM WATER TABLES

The rate of upward flow from water tables is an important factor in irrigated areas for determining the depth at which water tables should be maintained. This study attempted to relate the maximum rate of upward flow to measurable soil parameters under various conditions of the soil-water system, and to determine more precisely the effect of hysteresis.

Theoretical solutions were developed to determine the maximum upward flow rate as a function of depth of water table and necessary soil parameters. To verify these solutions, laboratory investigations were conducted. The experiments were conducted so that the ambient conditions did not affect the upward flow rates. Upward flow was induced at the top of the soil columns by means of an outflow siphon. An inflow siphon was connected to an inflow barrier at the bottom of the soil column to maintain the water table as desired.

Three conditions of the soil-liquid system were used, i.e., drainage, imbibition and imbibition-drainage cycles. The results gave good agreement between the theoretical solution and experimental results for both drainage

and imbibition cycles. The imbibition-drainage cycle was conducted to study the effect of hysteresis. This effect caused the maximum rates of upward flow to drop 20 to 50 percent below the theoretical values.

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