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HYDRAULIC PROPERTIES OF SOILS AS FACTORS IN DRAINAGE DESIGN

by

R. H. Brooks

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HYDRAULIC PROPERTIES OF SOILS AS FACTORS IN DRAINAGE DESIGN [1/]

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R. H. Brooks [2/]

Introduction

Considerable effort has been spent in arriving at equations or formulas which predict the position of the water table for steady and non-steady conditions. Several of these formulas seem to provide reasonably accurate results provided the boundary conditions and various hydraulic properties of the soil needed in the equations can be determined. These properties and conditions are usually considered to be permeability, specific yield, the depth to an impermeable barrier and the depth of the tile. Some of the design factors which must usually be arrived at by either a formula or rule of thumb include, spacing between drains, and drain tile size. All of these factors are used to produce essentially one result, i.e., a favorable moisture environment for plant growth.

1/ Contribution from Northern Plains Branch, Soil and Water Conservation Research Division, Agricultural Research Service, U. S. Department of Agriculture, Fort Collins, Colorado, Colorado Agricultural Experiment Station cooperating.

2/ Agricultural Engineer, Northern Plains Branch, Soil and Water Conservation Research Division, Agricultural Research Service, U. S. Department of Agriculture, Fort Collins, Colorado.

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Almost without exception, the position of the water table has been used to indicate when the moisture conditions are favorable. When the water table is lowered, the moisture content in the region near the soil surface may decrease. As a result, air penetrates into the soil more easily and thus may produce favorable aeration conditions in which plant roots can develop. By increasing the aeration of the soil, however, the water becomes less available to plants. This decrease in availability is a result of a decrease in water content and in a decrease in the permeability. In the arid west, the problem of maintaining a favorable moisture environment for plant roots is further complicated by water quality and soil salinity.

If the primary objective of agricultural drainage engineer is to provide a favorable moisture environment for plant growth by controlling the water table, then he must take into consideration some other hydraulic properties of the soil when considering the proper depth of the water table.

The objective of this paper is to consider some hydraulic properties of soils and their relation to the depth of the water table. Some equations relating the various variables involved in an air-water soil system will be discussed. No attempt will be made to incorporate these additional hydraulic properties into drainage formulas, but rather, the reader's attention will be focused at additional factors which should be considered in a rational design.

Water Permeability - Capillary Pressure - Water Content Relations

For purposes of this discussion, it will be necessary to define some variables and say something about the derivation of the equations used.

To begin with, all of the expressions given herein imply that the following relation is valid, i. e. ,

$$S_e = \left(\frac{P_b}{P_c} \right)^{\gamma} \quad , \quad P_c \geq P_b \quad . \tag{1}$$

Capillary Pressure P_c is the difference in pressure between the non-wetting fluid (air) and the wetting fluid (water). For an air-water system, P_{nw} is assumed to be the pressure of the atmosphere or zero gage pressure. The effective saturation S_e has been defined as

$$S_e = \frac{S - S_r}{1 - S_r} \tag{2}$$

where the saturation S is the ratio of the volume of water contained in the pores at a given capillary pressure to the total pore volume. The terms P_b and S_r are called break-through capillary pressure and residual saturation respectively. They are assumed to be constants for a given porous medium and their significance will be discussed later. It should be pointed out in equation 2 that $1 - S_r$ represents approximately the pore volume at a capillary pressure near field capacity.

Equation 1 has been examined for a number of soils and sands and the results are favorable. Data for two samples are shown in figure 1.

Let us examine under static conditions how capillary pressure, permeability and water content vary with elevation above the water table. Darcy's Law can be written as

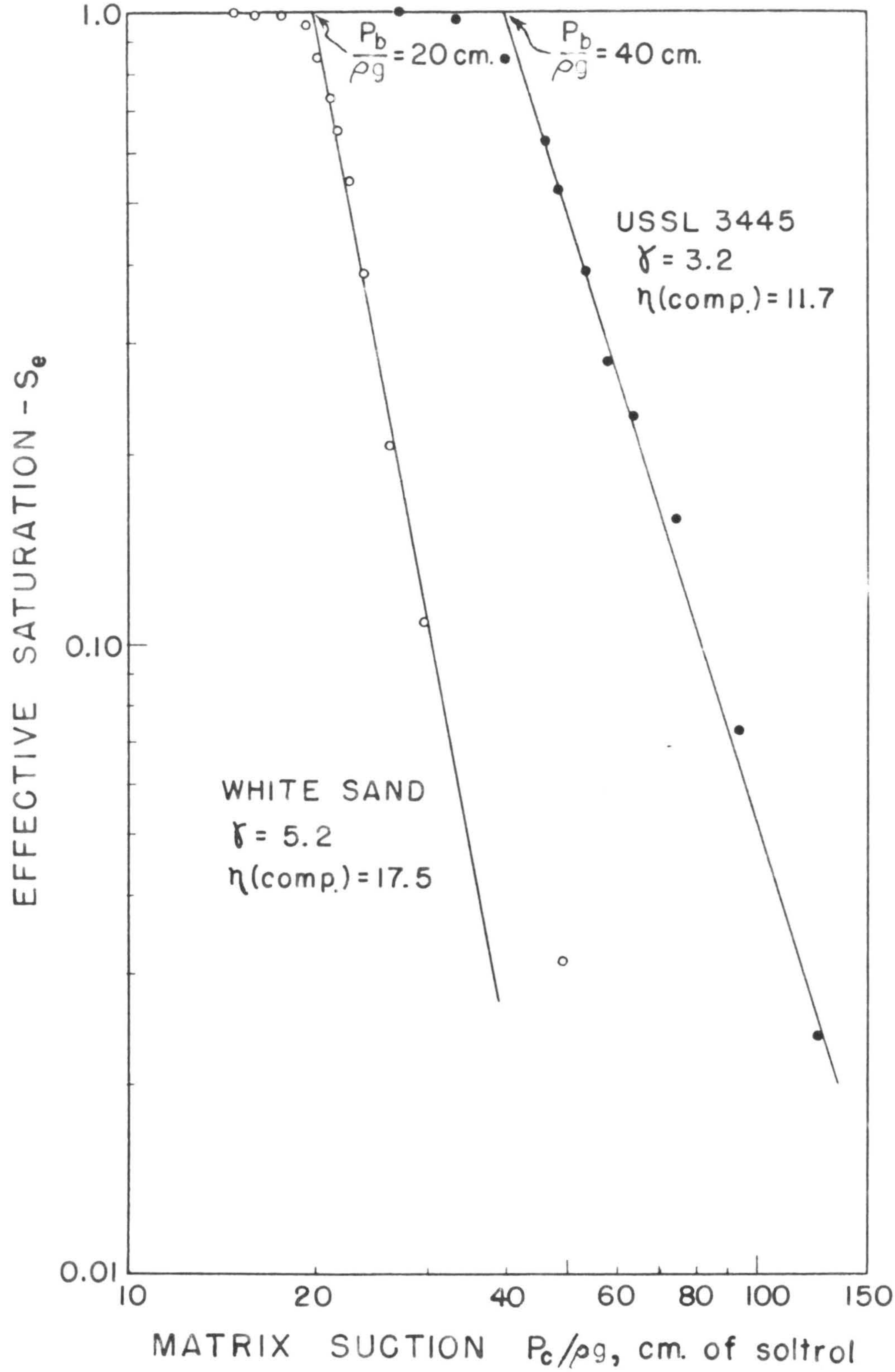

Figure 1 - Effective saturation S_e as a function of matrix suction $P_c/\rho g$ for two sands

$$q = -\frac{K_e}{\mu}\left(\frac{\partial P}{\partial r} + \rho g \operatorname{Sin}\theta\right) \tag{3}$$

in which θ is the angle of r with respect to the horizontal, K_e is the effective permeability, P is the pressure of the fluid, ρ and μ are the density and viscosity of the fluid, g is the gravitational constant and q is the volume flux of fluid per unit time in the direction r. Solving for $\frac{\partial P}{\partial r}$ and substituting its equivalent in terms of capillary pressure we have

$$\frac{\partial P_c}{\partial r} = \frac{q\mu}{K_e} + \rho g \operatorname{Sin}\theta \quad . \tag{4}$$

To determine P_c as a function of elevation Z above the water table for static conditions, we substitute $q = o$, and $r = Z$. This substitution reduces equation 3 to

$$\frac{\partial P_c}{\partial Z} = \rho g$$

or

$$P_c = \rho g Z.$$

The pressure head or matrix suction is simply equal to the elevation above the water table, i. e.,

$$\frac{P_c}{\rho g} = Z \tag{5}$$

where $\frac{P_c}{\rho g}$ is defined as matrix suction. In order to determine how permeability varies with elevation or matrix suction, we can derive an expression involving permeability and capillary pressure by using equation 1 in connection with some other assumptions, the details of which will not be given. For purposes of our discussion, it is sufficient to say that

$$\frac{K_e}{K} = \left(\frac{P_b}{P_c}\right)^{\eta} \qquad P_c \geq P_b \tag{6}$$

is a good approximation for many soils. The term $\frac{K_e}{K}$ is defined as relative permeability K_{rw} of the ratio or effective permeability to the saturated permeability. The term η is defined as a pore size distribution index which is related to γ . Substituting equation 5 into 6 gives the relationship between elevation and permeability as

$$K_{rw} = \left(\frac{P_b/\rho g}{Z}\right)^{\eta} \quad \text{for } Z \geq \frac{P_b}{\rho g} \quad . \tag{7}$$

To obtain an expression relating water content or saturation to the elevation above the water table, we substitute equation 5 into equation 1 and the result is

$$S_e = \left(\frac{P_b/\rho g}{Z}\right)^{\gamma} \quad \text{for } Z \geq P_b/\rho g \, . \tag{8}$$

The parameters η and γ are related as follows:

$$\gamma = \frac{\eta - 2}{3} \quad , \tag{9}$$

or

$$\eta = 2 + 3\gamma \quad . \tag{10}$$

Two sets of curves plotted from equations 6 and 7 are shown in figures 2 and 3. These figures show how the value of the pore size distribution index η affects the permeability and the water content in the root zone above a water table. A large value of η indicates there is a very marked change in the permeability and saturation for small changes in elevation above the water table; on the other hand, a small value of η indicates a gradual change in the water content above the water table. The values of η also indicate the importance of the position of the water table in regard to aeration of the upper soil layers.

It should be noted that for downward flow the water content and the corresponding permeability values will always be higher than the conditions indicated by the static case.

Air Permeability - Capillary Pressure - Water Content Relations

Let us now consider air permeability as a function of saturation percentage and capillary pressure.

The equations relating air permeability to saturation and capillary pressure are given below. As before, no attempt will be made to derive these expressions as our discussion will be confined to the parameters and the equations as they are related to conditions above the water table. The air permeability equations are

$$K_{ra} = (1 - S_e)^2 (1 - S_e^{\frac{2 + \gamma}{\gamma}}) \tag{11}$$

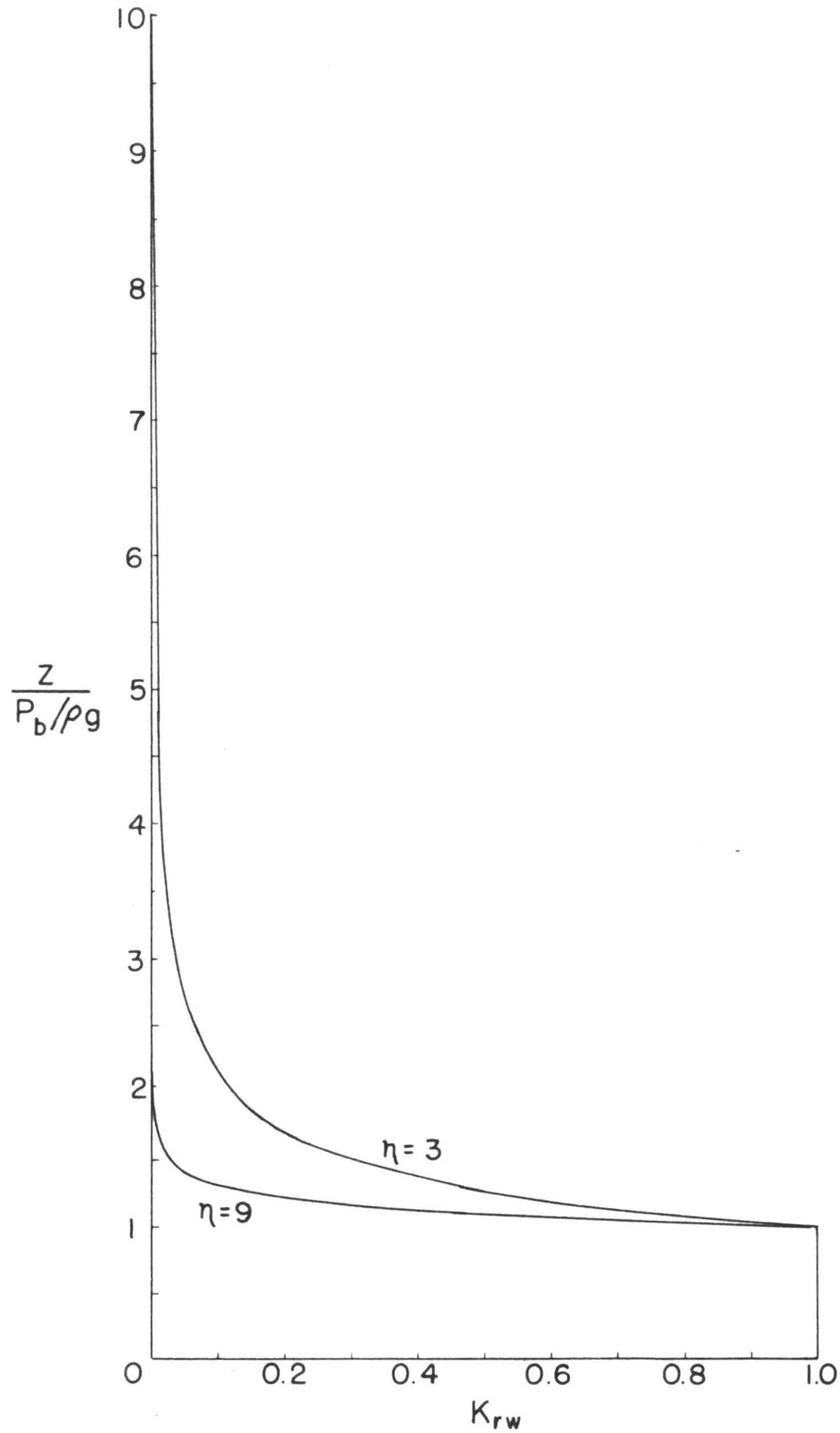

Figure 2 - Theoretical curves of relative water permeability K_{rw} as a function of relative elevation $Z/P_b/\rho g$ above the water table for two soil pore size distributions η assuming **q** = o in **the** direction of Z .

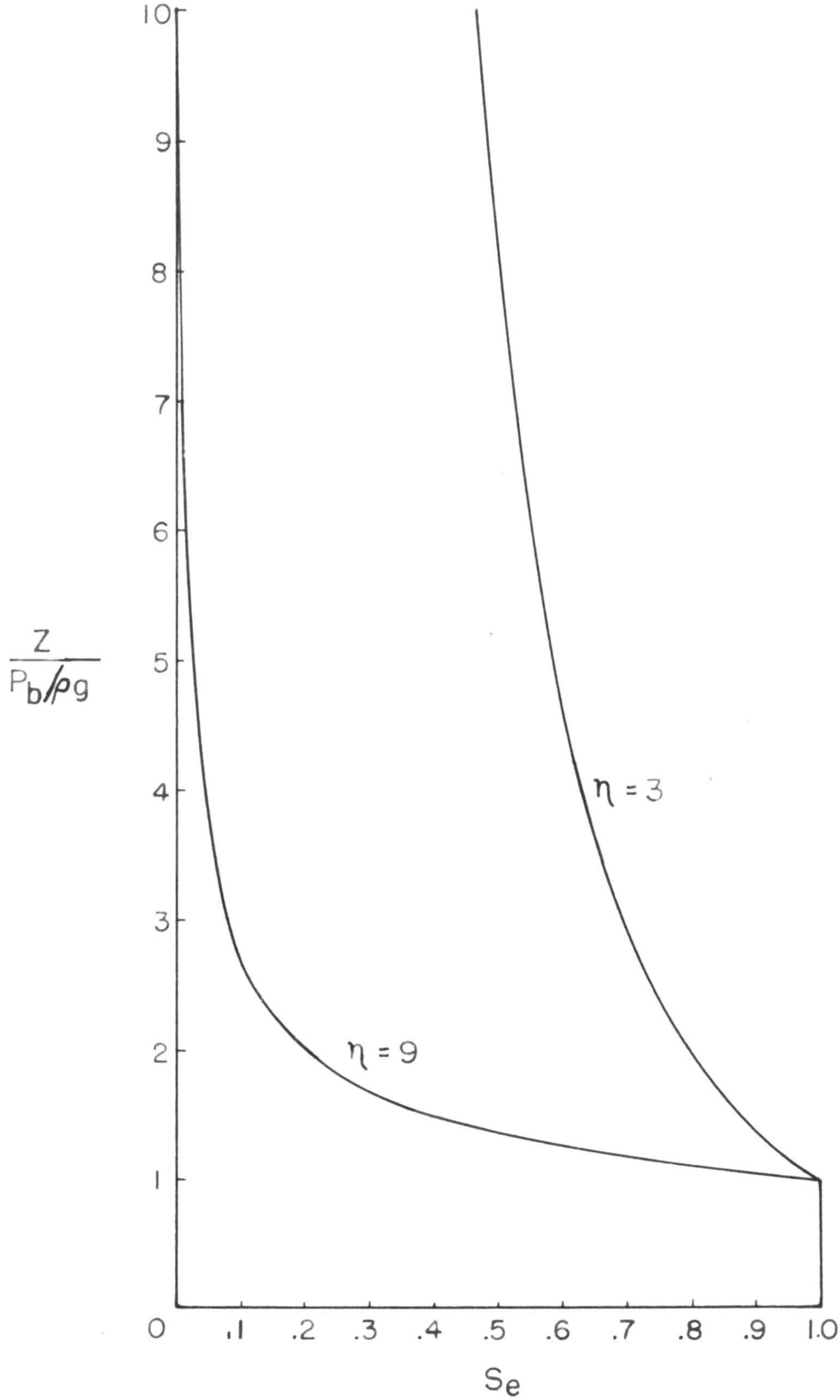

Figure 3 - Theoretical curves of effective saturation S_e as a function of relative elevation $Z/P_b/\rho g$ above the water table for two soil pore size distributions η assuming $q = o$ in the direction of Z .

and

$$K_{ra} = \left[1 - \left(\frac{P_b}{P_c}\right)^{\gamma}\right]^2 \left[1 - \left(\frac{P_b}{P_c}\right)^{2+\gamma}\right] \tag{12}$$

where K_{ra} is relative air permeability and the exponent γ is related to the pore size distribution index in equation 8. These equations are plotted in figures 4 and 5 along with some experimental data. The experimental data are shown as points while the equations appear as solid lines.

Residual saturation is artibrarily chosen by extrapolating the air permeability curve to $K_{ra} = 1.0$, as shown in figure 4. In most instances, the air-permeability nearly reaches its maximum value at the residual saturation. Also, air does not usually begin to flow until the saturation percentage has been reduced to approximately 90 percent.

Let us return momentarily to equation 6 which relates relative water permeability to capillary pressure. If permeability and capillary pressure are measured in a homogeneous isotropic media, we find the data can be plotted as shown in figure 6. The data is represented as a straight line on log-log paper over a wide range of capillary pressure. The extrapolation of the straight line to the capillary pressure where the relative permeability is unity is defined as the break-through capillary pressure. This particular capillary pressure appears to have significance in the air-permeability measurements.

In figure 5, air-permeability is shown as a function of capillary pressure. The permeability is absolutely zero up until

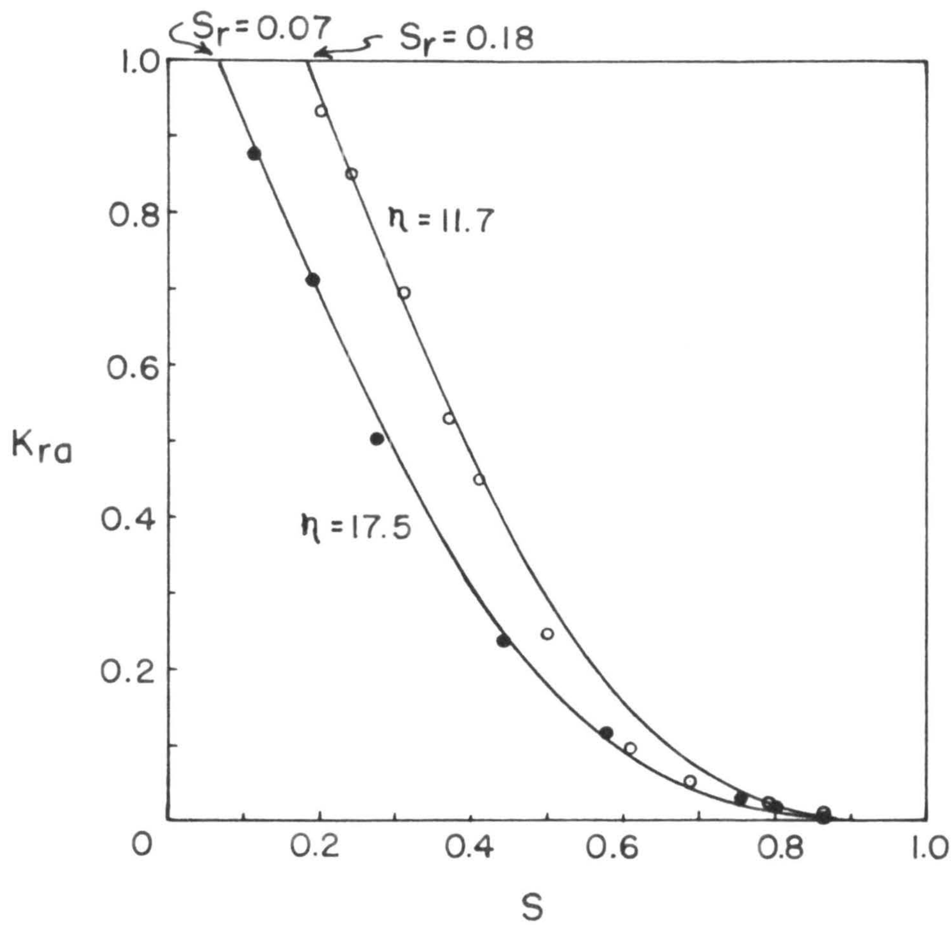

Figure 4 - Relative air permeability K_{ra} as a function of Saturation S for two pore size distributions η .

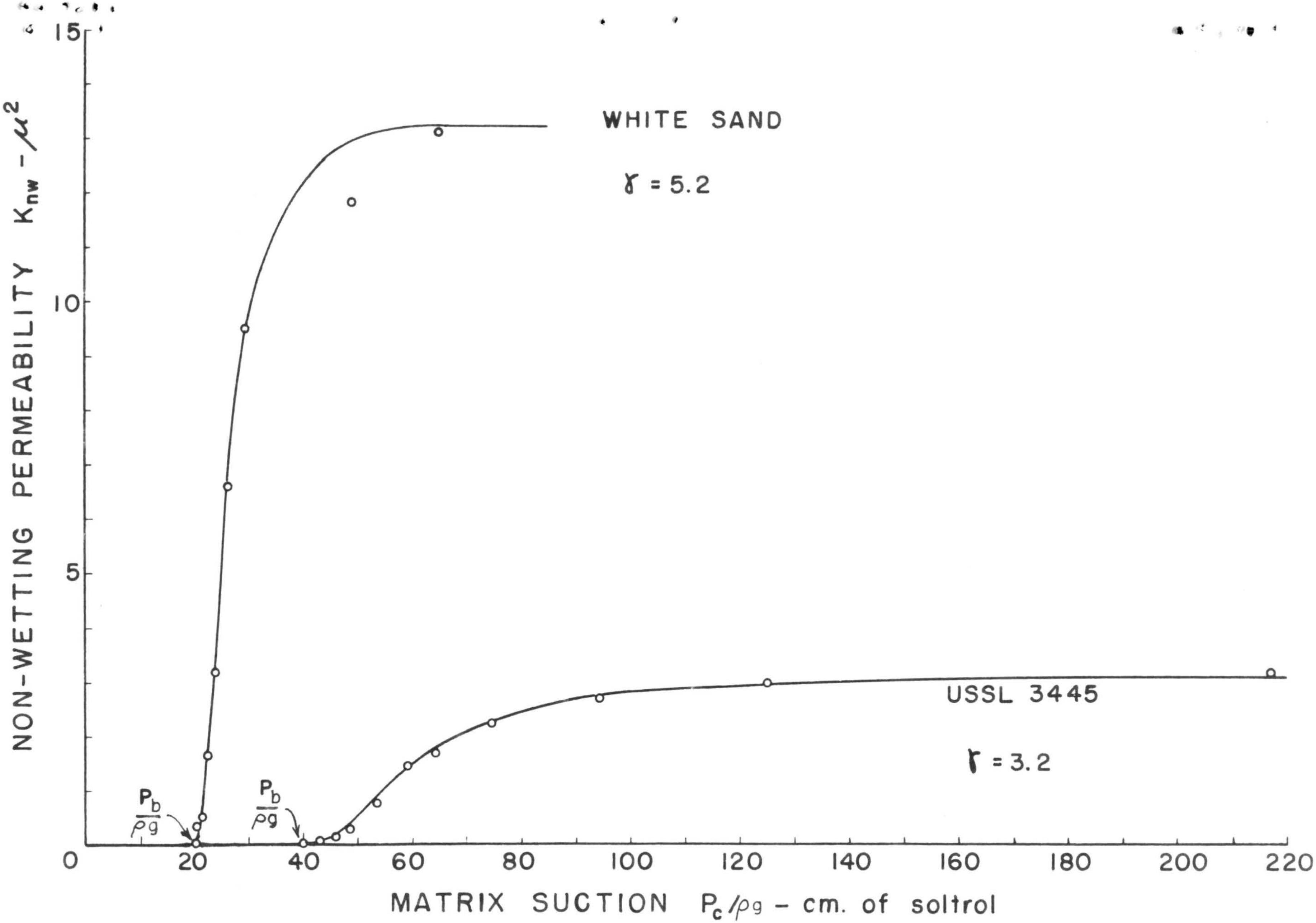

Figure 5 - Non-wetting permeability K_{nw} as a function of matrix suction $P_c/\rho g$ for two sands.

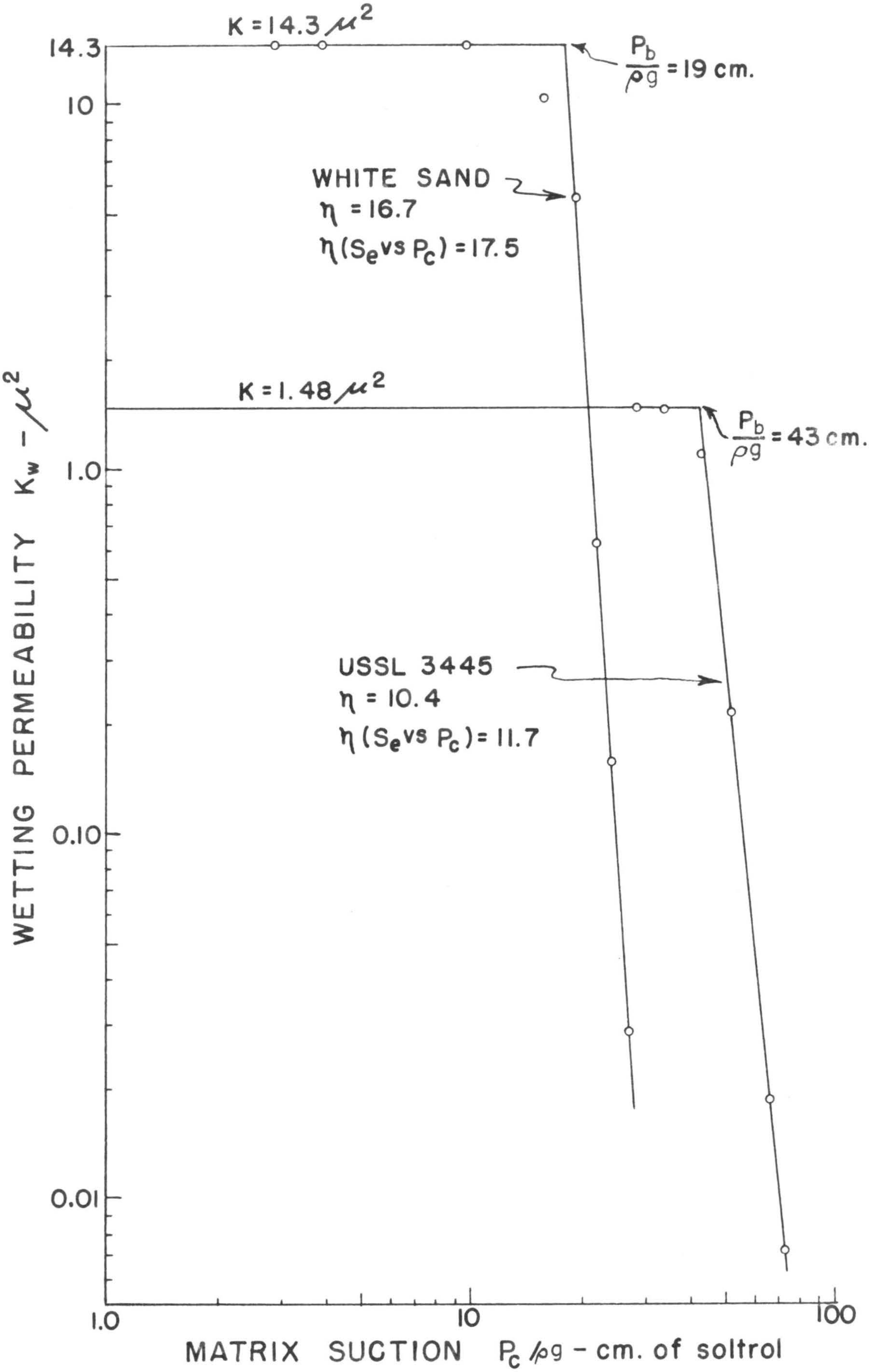

Figure 6 - Wetting permeability K_w as a function of matrix suction $P_c/\rho g$ for two sands

the break-through capillary pressure P_b is reached. At this point, the air becomes continuous throughout the medium and a finite permeability value can be measured. The capillary pressure at which the air becomes continuous corresponds very closely with the P_b value obtained from the water permeability-capillary pressure curve (figure 6). For the porous media studied by this author to date, there is excellent agreement between the P_b values obtained from the air and water permeability-capillary pressure curves. If this phenomena holds in general, the break-through capillary pressure should have considerable importance on drainage design. For example, let us consider again the distribution of water in the soil above the datum of the water table, assuming static conditions, air cannot be continuous below the elevation where $Z = \frac{P_b}{\rho g}$. The affect of the pore size distribution index η on the distribution of air above the water table is shown in figure 7. For small values of η, the water table may have to be held at a depth equal to several times the value of $\frac{P_b}{\rho g}$. The break-through capillary pressure P_b for some soils may be in excess of 100 mb.

Conclusions

No effort has been made to discuss the theory upon which most of the equations presented herein are based. There is some experimental evidence to substantiate the equations and the inherent assumptions. Considerable study is further needed to determine the range of soils over which the theory can be used with confidence. Based upon the above relations, the capillary pressure P_b at which the air phase is continuous, the pore size distribution index η, the intrinsic permeability K, and the residual saturation

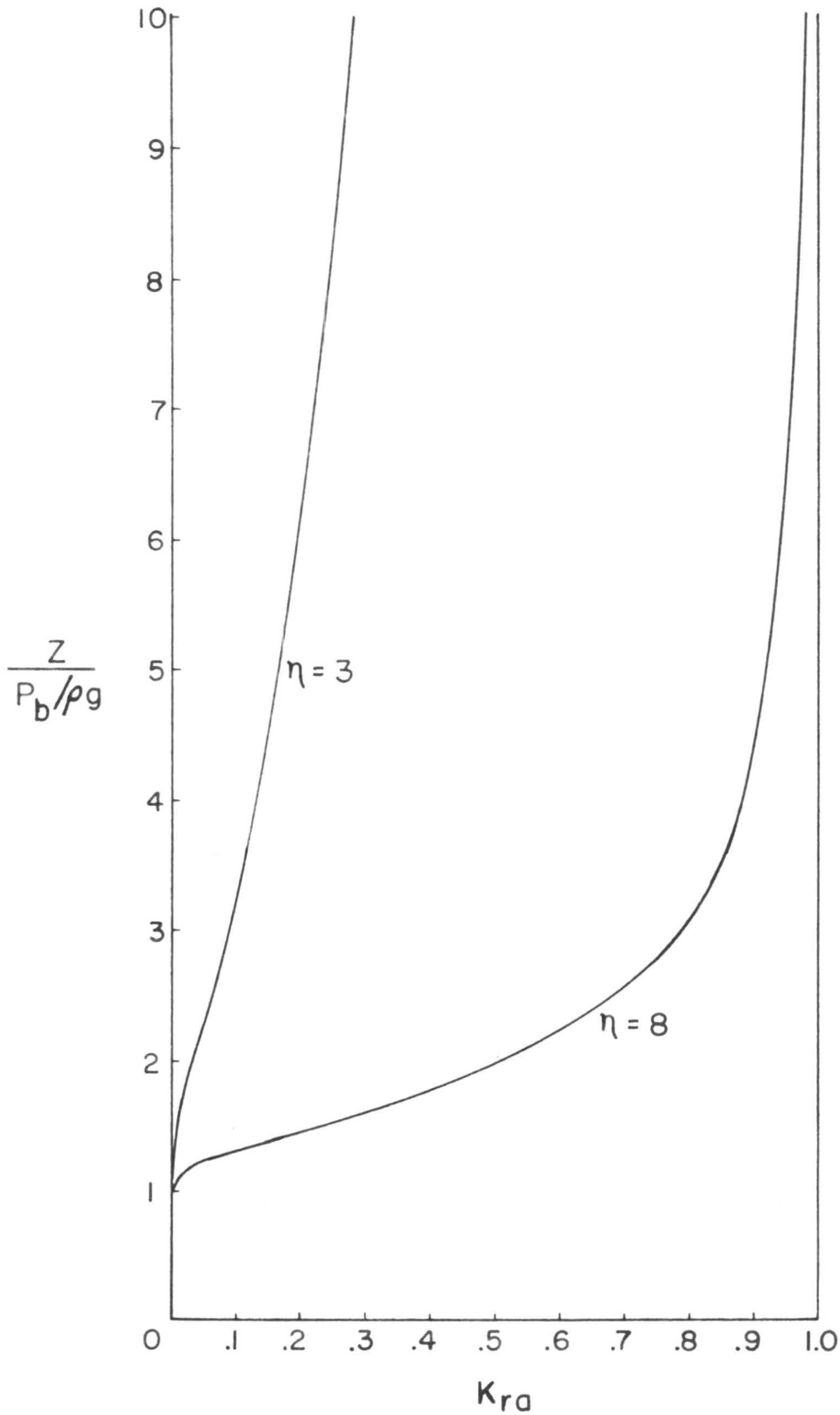

Figure 7 - Theoretical curves of relative air permeability as a function of relative elevation $Z/P_b/\rho g$ above the water table for two pore size distributions η assuming $q = o$ in the direction of Z .

percentage are hydraulic properties of the porous media which should be considered in drainage design. No attempt has been made to incorporate these hydraulic properties into drainage equations mostly because non-steady state relations are needed. Methods are being devised to obtain these hydraulic properties by simple means in the laboratory.

If the drainage engineer understands how the variables are related to each other and something about the order of magnitude of the hydraulic properties of the soil, he will be better equipped to make a rational drainage design.