Technical Note No. 2

VIRTUAL MASS OF AN OSCILLATING SPHERE WITHIN A FIXED CONCENTRIC SHELL

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ABSTRACT

The virtual mass of a sphere oscillating within a fixed concentric spherical shell filled with water was determined experimentally for seven sphere-to-shell diameter ratios between 0 and 0.865. Laboratory measurements of the virtual mass for diameter ratios less than 0.520 agreed closely with the theoretically predicted values based on a potential flow analysis. For ratios greater than 0.520 the virtual mass increased more rapidly than the potential flow theory indicated with an increase of 33% for a diameter ratio of 0.865.

INTRODUCTION

When a solid body accelerates through a fluid the inertia of the body depends upon the mass of the solid and the effect of the surrounding media. The increase in the effective mass of the solid due to the presence of the fluid is called the hydrodynamic or added mass. The mass of a solid plus its added mass is called the virtual mass. The added mass is usually expressed as a fraction or multiple of the mass of fluid displaced by the submerged body; this multiplying factor is called the added mass factor. Analysis using potential flow theory indicates that a sphere in an ideal fluid of infinite extent has an added mass factor equal to one-half; a cylinder with an infinite length-to-diameter ratio has an added mass factor equal to one. The effective inertia of a body is also dependent upon the proximity of rigid boundaries. An oscillating sphere bounded externally by a concentric shell has an effective inertia which approaches that of an infinite mass as the diameters of the sphere and shell become equal. The inertia, as predicted by potential flow theory, is further increased when the effects of viscosity are included.

Considerations of the added mass of a body are often of great practical significance. The effective inertia of a body produced by the surrounding fluid is sometimes far greater than the mass of the body itself. A bubble of air in water has an inertia hundreds of times greater than that produced by the mass of the bubble itself. The motion of ships and submarines, the opening of parachutes, the flight of balloons, and the behaviour of two phase fluid flows are only a few examples of motion significantly affected by the increase in a body's inertia due to the presence of a contiguous media.

Mathematical difficulties, however, in the theoretical analysis of a body's virtual mass, even when using ideal fluid theory, limit the number of geometric configurations and types of motion that are amenable to an exact solution. When viscous effects are included the difficulties of analysis almost always become insurmountable. Similarly, experimental efforts to measure the virtual mass of a body have been confronted with problems of considerable difficulty. The separation of effects produced by acceleration dependent or inertia forces from the velocity dependent or drag forces has been one of the major experimental complications.

In the present study the effects of a concentric boundary on the virtual mass of a sphere were measured and compared to results predicted from a potential flow analysis. This geometric configuration can be varied between two limiting conditions both of which are often approached in physical phenomenon. By varying the ratio of the diameters of the sphere and the concentric shell between zero and unity, geometric conditions are reproduced which, in their extremes, represent the case of a sphere in an infinite body of fluid and the case where the external boundary coincides with the sphere's surface. A measurement device was developed, similar to one used by Stelson and Mavis (1), which not only largely eliminated the drag effects of the supporting structure and sphere but also preserved the geometry of the system as a result of the body's motion.

APPARATUS AND TESTING PROCEDURE

Figures 1 and 2 show the apparatus used to measure the virtual mass of a sphere surrounded by a fixed concentric shell. The sphere, 3.12 inches in diameter, was attached to the end of a slender vertical rod fixed to the center of a simply supported beam. By adding weights to the vertical rod, with the sphere removed, a calibration was obtained between the natural frequency of the beam and the mass supported by the rod. The frequency was determined with an oscillograph which recorded the movement of a bar magnet fixed to the simply supported beam. When the beam was vibrated the bar magnet oscillated within the core of a solenoid which was independently supported. The voltage produced by this motion was amplified and recorded on the oscillograph. Subsequently, the sphere was suspended inside and concentric with a spherical shell filled with water. The natural frequency of the beam, supporting the rod and submerged sphere, was then measured. Using this frequency measurement the total or virtual mass of the submerged sphere was determined from the added mass-frquency calibration for the beam. The added mass was calculated by subtracting the mass of the sphere from the virtual mass.

Tests of the sphere in air with the surrounding shell removed showed an exact agreement between values of the mass obtained by the mass-frequency curve and those obtained by a direct weight measurement using a scale. In these tests the added mass due to the presence of the air was calculated as less than one-tenth of one percent of the mass of the sphere and could therefore be neglected. The effect of the viscous drag on the frequency determination was also negligible. This was concluded from the fact that the frequency response on the oscillograph record was sinusoidal in shape with no measurable distortion and that the logarithmic decrement measured from the oscillograph record was less than 8 percent in all instances.

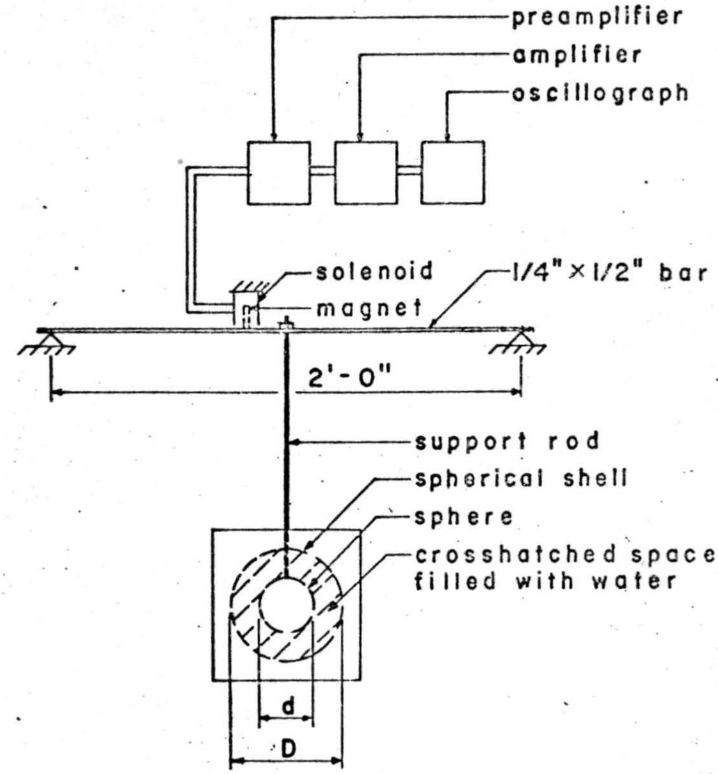

Figure 1. Schematic diagram of apparatus

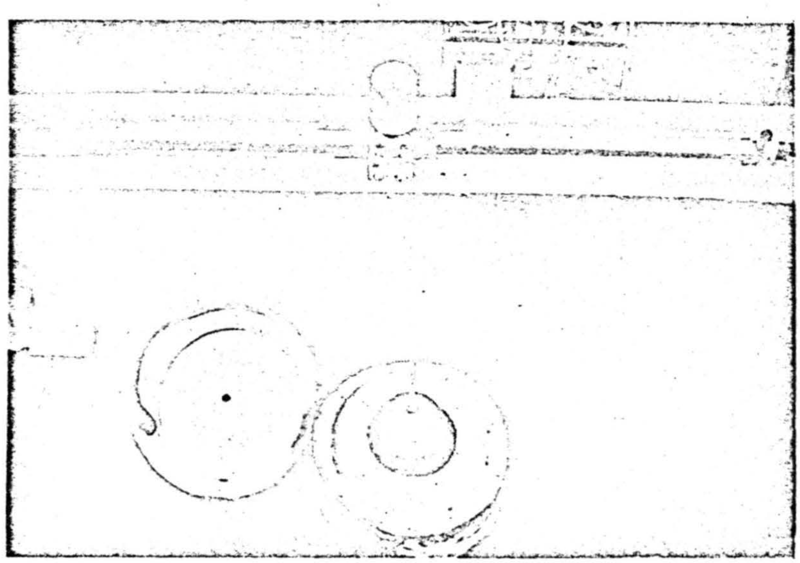

Figure 2. Apparatus: the concentric spheres, disassembled, the beam and its supports electrical transducer and oscillograph.

THEORY

A solid body of mass M moving with velocity v through an inviscid incompressible fluid has a kinetic energy equal to $1/2\ Mv^2$. The kinetic energy of the fluid surrounding the body depends upon the fluid density ρ_f and on the velocity distribution of the fluid. The latter may be described by a velocity potential ϕ. The total kinetic energy T of the system consisting of the solid and the fluid has been presented by Lamb (2) in the following form:

$$T = \frac{1}{2} Mv^2 - \frac{\rho_f}{2} \int_s \phi \frac{\partial \phi}{\partial n} dS \qquad (1)$$

where dS is a surface element of the boundary which encloses the liquid and n is the direction of its normal.

For a moving sphere concentric with a fixed spherical shell the velocity potential, as given by Lamb (3), is

$$\phi = \frac{d^3 v}{D^3 - d^3} \left(r + \frac{D^3}{16r^2}\right) \cos \theta \qquad (2)$$

where d and D are the diameters of the sphere and shell respectively; r and θ are the spherical coordinates of any point in the flow field. Employing this potential function, equation (1) becomes

$$T = \frac{1}{2} (\rho_s \forall) v^2 + (\rho_f \forall) \left[\frac{1}{4} \frac{D^3 + 2d^3}{D^3 - d^3} \right] v^2 \qquad (3)$$

where $\forall$ and ρ_s are the volume and density of the moving sphere respectively. A force F acting upon the moving sphere does work equal to the time rate of change of the system's kinetic energy and can be determined from equation (3).

$$Fv = \frac{dT}{dt} = (\rho_s \forall) v \frac{dv}{dt} + (\rho_f \forall) \left[\frac{1}{2} \frac{D^3 + 2d^3}{D^3 - d^3} \right] v \frac{dv}{dt} \qquad (4)$$

Hence the force acting on the sphere is

$$F = \left\{ \forall \rho_s + \forall \rho_f \left[\frac{1}{2} \frac{D^3 + 2d^3}{D^3 - d^3} \right] \right\} \frac{dv}{dt} \qquad (5)$$

This force is considered an inertia force because it is dependent on the acceleration. The first term of the right hand member of equation (5) is the mass of the solid and the second term is the added mass. The added mass is expressed as a factor K times the mass of fluid displaced by the moving body. For the geometric configuration used in the present investigation, equation (5) becomes

$$F = [\forall \rho_s + \forall \rho_f K] \frac{dv}{dt} \qquad (6)$$

where the added mass factor is given by the expression

$$K = \frac{1}{2} \frac{(D^3 + 2d^3)}{(D^3 - d^3)} \qquad (7)$$

If the moving body has a constant velocity it is seen from equation (6) that the force on the solid becomes zero. This conclusion, which applies to all solids moving at a constant velocity in an inviscid fluid, is known as d'Alembert's paradox. For motion in a viscous fluid, Lamb (4) found that the added mass factor for d/D = 0 is

$$K = \frac{1}{2} + \frac{9}{2d}\left(\frac{\nu}{\Pi f}\right)^{1/2} \qquad (8)$$

where f denotes the frequency of vibration and ν is the kinematic viscosity of the fluid. The writers know of no analytic solution giving the virtual mass effect in a real fluid where $d/D > 0$.

EXPERIMENTAL RESULTS

Figure 3 is a graph showing both the theoretical and experimentally determined relationships between the ratio of the diameter of the sphere to the diameter of the shell d/D and the added mass factor K. The theoretically determined curve was computed from equation (7) and the experimental results were obtained from laboratory tests. The difference between these two curves is believed to represent the effect of viscosity on the added mass factor since this variable is not included in the potential flow analysis. Table I summarizes the results of the experiments and gives the corresponding theoretically determined values.

TABLE I: Summary of Results

Diameter ratio d/D	Theoretically determined added mass factor K for sphere in		Experimentally determined added mass factor K in water.	Percentage increase of the added mass in water over that predicted for an ideal fluid.
	Ideal fluid	Water (1)		
0	0.50	0.51	0.51	2%
0.260	0.53	(2)	0.53 (3)	0%
0.445	0.64	(2)	0.65	1%
0.520	0.74	(2)	0.75	1%
0.693	1.24	(2)	1.37	10%
0.780	1.84	(2)	2.32	26%
0.865	3.22	(2)	4.28	33%

(1) Calculated for frequency of vibration and viscosity of fluid corresponding to that used in the laboratory.

(2) Theoretical solution not known.

(3) The shell used in this test run was not completely spherical; hence this result is questionable.

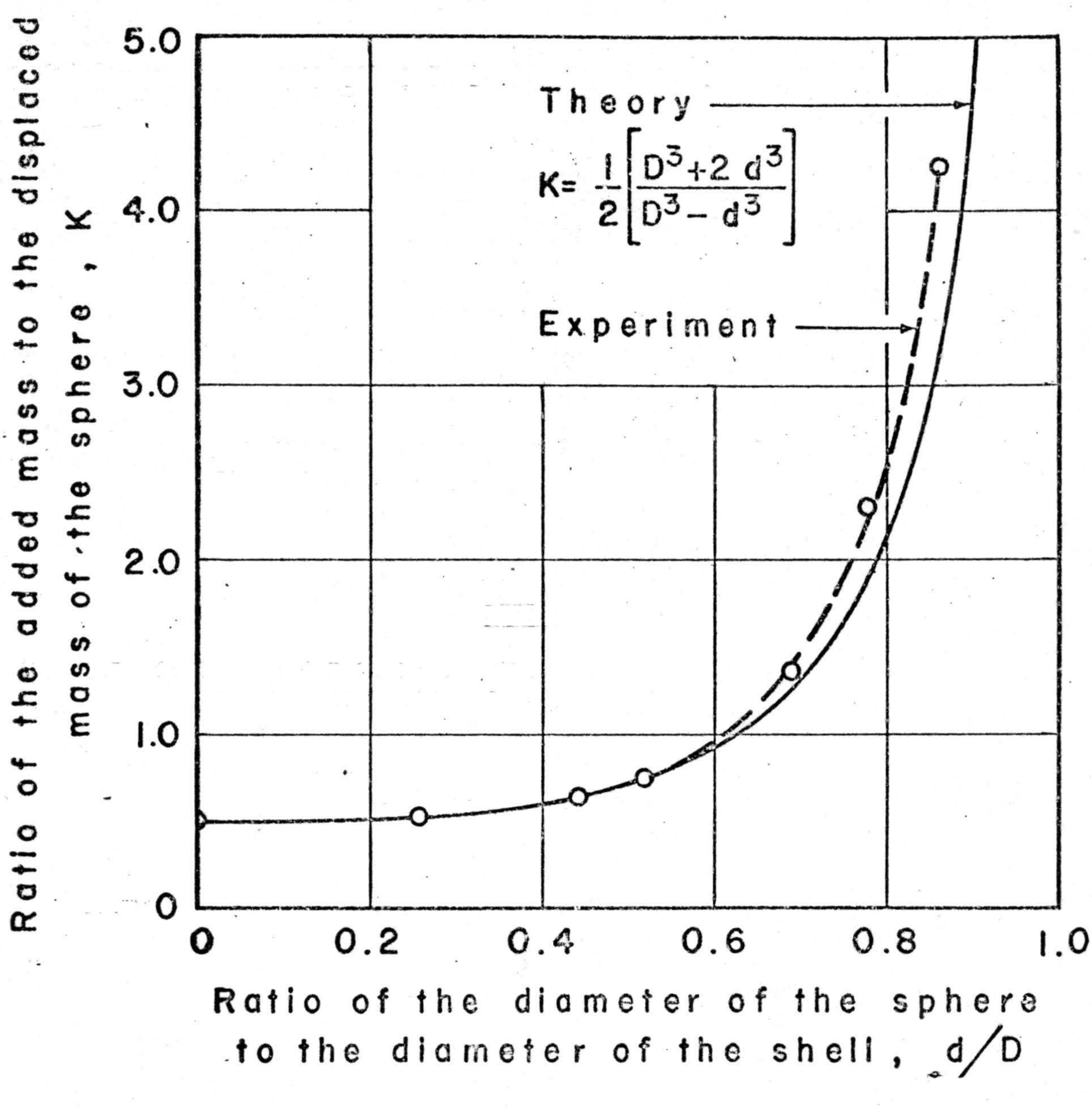

Figure 3. Relationship between diameter ratio and added mass factor K.

CONCLUSIONS

The following conclusions are based upon the results obtained from the experimental and theoretical investigations. Conclusions concerning viscous effects are based upon the test results using tap water at a temperature of approximately 80°F.

(a) The apparatus used in the investigation provides a simple and accurate method of measuring the virtual mass of a submerged body.

(b) The added mass factors predicted from a theoretical analysis based on the assumption that the fluid is inviscid and incompressible were duplicated to within 2% for d/D ratios less than 0.520. This result confirms the validity of the potential flow analysis within this range. For d/D ratios greater than 0.520 the virtual mass of the sphere increases much more rapidly than that indicated by the inviscid fluid theory.

(c) The theoretically determined viscous correction for the added mass factor when d/D = 0 was confirmed experimentally.

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(1) T.E. Stelson and F.T. Mavis, "Virtual Mass and Acceleration in Fluids", Trans. A.S.C.E., 122, 1957, 519-525.

(2) H. Lamb, Hydrodynamics, Cambridge University Press, London, 6th Edn., 1932, p.46.

(3) ibid, p.125

(4) ibid, p.644