DRAINAGE OF A SALINE-WATER AQUIFER RECHARGED BY FRESH WATER

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INTRODUCTION

Large areas in the arid regions of the world are underlain by aquifers containing saline water. When these areas are brought under irrigation, the water table will begin to rise. To prevent water-logging of the terrain, some type of artificial drainage must be installed. The drainage system may consist of drainage wells, tile drains, open ditches, or a combination thereof.

The present paper is concerned with a tile drain system installed in a saline-water aquifer which is recharged with relatively fresh water. When the fresh water reaches the watertable from above, an interface is formed between the fresh and the salt water, due to their difference in density. The water table rises, and the drains will start running. The drain effluent is composed of water from both above and below the salt-fresh water interface. The

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effluent initially may contain considerable amounts of salt. In time, the interface will move downward and the proportion of fresh water entering the drain will increase. Therefore, the salinity of the drain effluent will decrease with time.

When, as a result of a new irrigation project large quantities of saline drainage effluent are returned to a stream, the effect on the quality of the water in the stream may be considerable. Especially, when the amount of return flow from the project is large compared to the flow in the stream.

The purpose of the study reported herein was to predict the quality of drain effluent in time, as influenced by such factors as permeability and saturated thickness of the aquifer, drain spacing, and recharge rates.

Notation. - The symbols adopted for use in this paper are defined where they first appear.

EXPERIMENTAL PROCEDURE

Experiments for this study were carried out with a viscous flow analogy model, commonly known as a Hele-Shaw[3, 4] model,

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[3] Hele-Shaw, H. S., "Experiments on the Nature of Surface Resistance of Water and of Stream Line Motion under Certain Experimental Conditions", Trans. Inst. Naval Architects, vol. 40, p. 21-40, 1898.

[4] Hele-Shaw, H. S., "Streamline Motion of a Viscous Film," Report, 68th Meeting British Assoc. Advancement of Science, p. 136-142, 1899.

consisting in its simplest form of two parallel plates, separated by a narrow interspace.

A sketch of the Hele-Shaw model used is shown in Figure 1. A constant rate of recharge was obtained by connecting the recharge supply tube (G, Fig. 1) to a Mariotte siphon in a reservoir of recharge liquid. The model was used in such a manner as to simulate only half the distance between drain tiles, assuming no flow across a vertical plane midway between parallel drains. For the runs reported herein all holes (E, Fig. 1) except two were sealed. One hole near the top of the model served as a drain. On the opposite end of the model the bottom hole was used to fill and drain the interspace at the beginning and end of each run.

A mixture consisting of 60% glycerine and 40% water was used in the model. Appropriate amounts of salt were added so as to simulate initial salinity within the aquifer of 10, 000, 15, 000, and 30, 000 ppm of total dissolved solids. Food coloring was used to color the saline liquid. (No salt or coloring was added to the recharge liquid, which contained less than 50 ppm TDS.) The effluent from the drain was funneled through a glass flow cell for measuring electrical conductivity. The cell was connected to a self-balancing Wheatstone bridge, and recorder, and a continuous record of conductivity with time was obtained.

The drain outflow rate was measured at suitable time intervals. Since the viscosity of a glycerine-water mixture is dependent on the temperature, periodic temperature measurements were taken. During the course of any one run, the temperature of the liquid varied less than two, and normally less than one, degree centigrade.

The following systems were studied in this investigation:

1) Parallel drains in an aquifer of uniform permeability
   a) Initial concentration of ground water 10,000 ppm TDS
   b) Initial concentration of ground water 15,000 ppm TDS
   c) Initial concentration of ground water 30,000 ppm TDS
2) Parallel drains in a stratified aquifer (permeability of the upper half of the aquifer approximately twelve times the permeability of the lower half)
   a) Initial concentration of ground water 15,000 ppm TDS
   b) Initial concentration of ground water 30,000 ppm TDS
3) Parallel drains in a stratified aquifer (permeability of the lower half of the aquifer approximately six times the permeability of the upper half)
   a) Initial concentration of ground water 15,000 ppm TDS
   b) Initial concentration of ground water 30,000 ppm TDS

For each arrangement of the model a series of runs was performed covering a range of values of model discharge. A total of 80 uniform recharge runs were performed with the seven model setups described above.

SCALES OF THE VISCOUS FLOW MODEL

The principle of the Hele-Shaw model is based on the similarity of the equations describing flow of groundwater (Darcy's law) and the flow of a viscous liquid between two parallel plates, spaced closely together,[5] i. e. for ground water flow

$$v = K \frac{d\phi}{d\ell} \qquad (1)$$

where v = velocity ϕ = piezometric head, and ℓ = length,

and for flow between parallel plates

$$v = \frac{1}{12} \frac{g\,b^2}{\nu} \frac{d\phi}{d\ell} \qquad (2)$$

where g = acceleration of gravity, b = spacing between the plates and ν = kinematic viscosity.

Combining Eqs. (1) and (2) with the continuity equation gives

$$\frac{\partial u_{ave}}{\partial x} + \frac{\partial w_{ave}}{\partial z} = 0 \qquad (3)$$

where u_{ave}, and w_{ave} are the velocity components in the horizontal (x) and vertical (z) direction respectively, resulting in the well-known Laplace Equation, which is thus valid for both types of flow.

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5 De Wiest, Roger J. M., "Geohydrology", New York, John Wiley and Sons, 1965, 366 p.

Bear[6] has shown that the equation for the water table in an unconfined aquifer with recharge is·

$$n \frac{\partial \phi}{\partial t} = K_x \left(\frac{\partial \phi}{\partial x} \right)^2 + K_z \left(\frac{\partial \phi}{\partial z} \right)^2 - K_z \frac{\partial \phi}{\partial z} + h \left(\frac{\partial \phi}{\partial z} - 1 \right) \quad (4)$$

where n = specific yield, t = time, K_x and K_z are the permeabilities in the x and z directions, and h is the recharge rate (say, meters/ years). Equation 4 is equally valid for prototype and model. Therefore, in order to adequately simulate prototype conditions in a model the following relations between scaling ratios must hold[6]:

$$\frac{t_r}{n_r \phi_r} = \frac{x_r^2}{K_{xr} \phi_r^2} = \frac{Z_r^2}{K_{zr} \phi_r^2} = \frac{Z_r}{K_{zr} \phi_r} = \frac{Z_r}{\phi_r \, r} = \frac{1}{h_r} \quad (5)$$

where the subscript r indicates the scale ratio, i. e. $t_r = t_m / t_p$.

From Eq. 5, it follows that

$$\phi_r = Z_r \quad (6)$$

$$t_r = \frac{n_r x_r^2}{K_{xr} Z_r} \quad (7)$$

$$\frac{K_{xr}}{K_{zr}} = \frac{x_r^2}{Z_r^2} \quad (8)$$

and

$$K_{zr} = h_r \quad (9)$$

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6 Bear, Jacob, "Scales of viscous analogy models for ground water studies", Jour. of Hydraulics Division, ASCE, vol 86, No. HY2, Feb., 1960, p. 11-23.

In order to obtain the discharge scale relation, Darcy's law may be used, thus:

$$Q_r = \frac{K_{xr} b_r z_r^2}{x_r} \tag{10}$$

Writing Eqs. 7 and 10 in terms of model and prototype values results in:

$$t_m = \frac{12}{g} \frac{\nu}{n_p} \frac{x_m^2}{x_p^2} \frac{Z_p}{Z_m} \frac{K_{xp}}{b^2} tp \tag{11}$$

and

$$Q_m = \frac{g}{12} \frac{b^3}{\nu} \frac{h}{K_{xp}} \frac{Z_m^2}{Z_p^2} \frac{x_p^2}{x_m} \tag{12}$$

In part of this study it was desired to model a layered aquifer system with permeability in one half of the aquifer approximately 10 times the other half. The discharge scale for both parts of the aquifer must be unique. Denoting the two layers of different permeability by subscripts 1 and 2, we obtain using Eq. 10:

$$\left(\frac{g}{12} \frac{b^3}{\nu} \frac{1}{K_{xp} b_p} \frac{Z_r^2}{x_r} \right)_1 = \left(\frac{g}{12} \frac{b^3}{\nu} \frac{1}{K_{xp} b_p} \frac{Z_r^2}{x_r} \right)_2 \tag{13}$$

Since ν, g, and b_p are constant and Z_r and x_r are fixed throughout the model, Eq. 13 may be written as:

$$\left(\frac{b^3}{K_{xp}} \right)_1 = \left(\frac{b^3}{K_{xp}} \right)_2 \tag{14}$$

To obtain layer permeability natios of 1 to 10, b_2 must be approximately 2.16 b_1. In order to satisfy both Eq. 14 and the requirement of a unique time scale (Eq. 11) the ratio b_m/n_p must be constant. Therefore, changes in interspace width in the model and changes in effective porosity in the prototype can not be made independently.

EXPERIMENTAL RESULTS

The results obtained from each run were plotted in terms of effluent concentrations versus model time for particular values of drain discharge or rate of application. An example of one of these graphs is shown in Figure 2. A sufficient number of points to define the curves was taken off the recorder charts. The curves were all similar in appearance. Initially, the drop in salinity is quite large, but gradually the rate of change decreases.

Rewriting Eqs. 11 and 12 as follows:

$$\frac{g}{12} \; \frac{z_m}{\nu} \; \frac{b^2}{x_m^2} \, t_m = \frac{z_p K_{xp} t_p}{n_p x_p^2} \qquad (15)$$

and

$$\frac{12}{g} \; \frac{\nu}{b^3} \; \frac{x_m}{z_m^2} \, Q_m = \frac{h}{K_{xp}} \; \frac{x_p^2}{z_p^2} \qquad (16)$$

one notices that both sides of Eqs. 15 and 16 are dimensionless. Moreover, the magnitude of the factors on the left side of the equations are known for each run. Therefore, dimensionless plots of

$(z_p K_{xp} t_p)/(n_p x_p^2)$ versus $(h x_p^2)/(K_{xp} z_p^2)$ may be obtained by determining the time, t_m, required to reach effluent salt concentrations of 90, 80, 70, etc, per cent of the initial concentrations. These charts are shown in Figure 3 through 9. The figures are composite in the sense that they show the results of a series of runs in one graph. To reduce scatter, several points indicated on the chart were obtained to averaging two adjacent points in both directions.

Figures 3, 4, and 5 are graphs obtained from the results of the experiments with a uniform aquifer and original salinities of 10, 000, 15, 000 and 30, 000 ppm respectively. The value of z_p represents the total thickness of the prototype aquifer. Results of runs for the 12 to 1 stratified aquifer are summarized in Figures 6 and 7 for initial concentrations of 15, 000 and 30, 000 ppm respectively. The value of z_{p1} refers, in this case, to the thickness in the prototype of the high permeability layer. The saturated thickness ratio of upper part to lower part was 1 to 0. 95.

The composite graphs resulting from the runs with the 1 to 6 layered aquifer (bottom half six times as permeable as the top half) are shown in Figures 8 and 9 for initial concentrations of 15, 000 ppm and 30, 000 ppm respectively. Again z_{p1} represents the thickness of the upper member of the aquifer. The saturated thickness ratio of the upper part of the model to the lower was 0. 91 to 1.

EFFECT OF VARIABLES ON DEGREE OF AQUIFER CLEANING

At the end of each run, the position of the water table and the fresh-salt water interface was recorded and plotted. By means of a planimeter, the areas between the water table and fresh-salt water interface, and between the water table and the bottom of the aquifer were determined. The ratio of the two areas is the fraction of the aquifer volume flushed free of saline water.

Figure 10 is a plot of the dimensionless parameter $(h\, x_p^2) / (K_{xp}\, z_p^2)$ versus the average percentage of aquifer thickness cleaned free of saline water for the uniform aquifer. The curve was drawn through points obtained using an initial concentration of 30, 000 ppm. The points in this figure show a considerable amount of scatter because many runs were discontinued before a true steady-state had been reached. The terminal concentration of the effluent was not the same in each run; thus, some runs were closer to a steady-state than others.

Nevertheless, Figure 10 shows some interesting features. The curve rises steeply initially with an increase of the dimensionless parameter, and then approaches 100 per cent asymptotically. This shows that for particular values of permeability and saturated thickness, the per cent of aquifer cleaned is a function of the product of the square of the drain spacing and the recharge rate. Thus, if salinity of the effluent is of concern, tile drains spaced at closer intervals than required for adequate drainage may be desirable.

The depth of cleaning results agree with findings by Bouwer[7] who distinguishes between an active zone and a passive zone. The active zone is the portion of the aquifer in which flow takes place. In the passive zone, no flow occurs, thus the saline water is removed from it. The Dupuit-Forchheimer theory, commonly used for describing flow in unconfined aquifer, assumes that flow occurs through the entire thickness of the aquifer. From Figure 10, it is evident that such does not occur for every combination of drain spacing, aquifer thickness, permeability and rate of recharge.

Another interesting feature of Figure 10 is the location of the points denoting the three initial concentrations used. In order for the three sets of points to be truly comparable, the circled points (representing initial concentration of 30,000 ppm) should be lower in the figure than indicated and the squared points (representing initial concentration of 10,000 ppm) should be higher than indicated. The reason for this is that the runs of high initial salinity generally were closer to a steady state condition than those runs having a lower initial concentration. Figure 10 shows that even without this adjustment the per cent of aquifer thickness cleaned was inversely related to the initial concentration. The probable reason for this is the difference in density between fresh and salt water. In order for two flow situations to reach the same terminal state, the hydraulic gradient must

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[7] Bouwer, Herman, "Analyzing Ground Water Mounds by Resistance Network", Journ. of Irrigation & Drainage Division, ASCE, vol. 88, No. IR 3, 1962, pp. 15-36.

be larger for a more dense liquid than for a less dense liquid. Or, alternatively, for the same piezometric head, an aquifier containing a less dense (lower salinity) liquid would be cleaned to a greater depth than one containing a more dense liquid.

EFFECT OF VARIABLES ON CONCENTRATION OF EFFLUENT

The dimensionless charts (Figures 3 through 9) make it possible to predict the quality of effluent from drains under various conditions. By substituting values of the prototype aquifer characteristics (permeability and porosity), aquifer dimensions (drain spacing and thickness of aquifer), and recharge rate, one can predict the concentration of effluent at any time. Since K_x, x, and z occur in the variables of both abscissa and ordinate it is difficult to make an immediate judgment on how their variation changes the quality of effluent in time. Therefore, a few quantitative examples are illustrated in Figure 11.

Curve A in Figure 11 shows the change of effluent concentration for the following conditions: (1) horizontal permeability 4000 cm/year (2) recharge rate of 90 cm/year, (3) aquifer thickness of 24m, (4) half distance between drains of 240m and (5) specific yield of 0. 2. Under these conditions approximately 13. 5 years are required for the effluent concentration to reach one tenth the original concentration of 10, 000 ppm. The effect of a twofold increase of recharge rate (180 cm/year) is shown by curve B. The effluent concentration at any time is considerably lower than in the first case and the time required to reach the 0. 1 level is decreased to 6. 5 years. The efficiency of flushing was in both cases approximately the same. In case A, 2920 m^3 of recharge water (0. 9 x 13. 5 x 240) would be required to remove approximately 1150 m^3 of saline water or a ratio of 2. 54 to 1. In case B, these figures amount to 2820 and 1150 m^3 width respectively, or a ratio of 2. 45 to 1.

Curve C differs from curve A only in that it represents a half spacing between drains increased by a factor $\sqrt{2}$ to 345 m. The effect of this increase on the salinity of the effluent is small.

Finally, curve D in Figure 11 shows the decrease in concentration with time for the same situation as A, except the aquifer thickness is reduced by a factor $\sqrt{2}$ to 17. 5 m. The smaller aquifer thickness results in a more rapid reduction in effluent concentration.

A final comment is in order concerning the comparisions made of the effects of drain spacing. Equation 5 shows that prototype anisotropy is implied, by distortion of the model, according to the relation

$$\frac{K_{xr}}{K_{xr}} = \frac{x_r^2}{z_r^2} \qquad (17)$$

Since $K_{xm} = K_{zm}$, and x_m/z_m is constant, one may write Eq. (17) as

$$\frac{K_{zp}}{K_{xp}} = C \frac{z_p^2}{x_p^2} \tag{18}$$

This expression shows that, for example, curves A, C, and D, in Figure 11 are not strictly comparable since the degree of anisotropy for these aquifer and spacing conditions is not the same. In the field the x dimension is generally large compared to the z dimension, thus in most of the aquifer flow is in the horizontal direction. In addition, the horizontal permeability is often larger than the vertical permeability under field conditions. Therefore, the anisotropy simulated by allowing vertical distortion (i. e. $z_r > x_r$) is often in the proper direction, and usually does not introduce error.

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