THESIS

BIOMASS INVENTORY OPTIMIZATION FOR WILLOW BASED PELLET PRODUCTION INTEGRATING SEASONAL SUPPLY AND STOCHASTIC DEMAND

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ABSTRACT

BIOMASS INVENTORY OPTIMIZATION FOR WILLOW BASED PELLET PRODUCTION INTEGRATING SEASONAL SUPPLY AND STOCHASTIC DEMAND

Renewable biomass energy is a critical component in the global transition to sustainable energy systems. Its contribution to reducing greenhouse gas emissions and fossil fuel dependence has become increasingly vital. However, biomass supply chains face unique challenges due to material characteristics such as bulk, low energy density, and seasonality, making effective inventory management critical to their economic viability.

This study develops a Mixed Integer Linear Programming (MILP) model to optimize inventory management policies for a willow-based wood pellet production facility. Built upon an (*s, S*) inventory control system, the model integrates internal operational decisions like inventory adjustment frequencies, while incorporating external uncertainties such as seasonal biomass harvesting capacity constraints and stochastic pellet demand patterns.

Results demonstrate that the stochastic approach could maintain zero stockout probability when the lost-sale-cost is assumed to be higher across our modeled stochastic demand scenarios while achieving lower overall inventory management costs compared to solutions from deterministic methods. Critically, the stochastic framework provides substantially lower cost variability (over 55% reduction in coefficient of variation) and greater operational robustness under demand uncertainty, with the performance advantage potentially increasing as uncertainty levels rise. Weekly inventory policy adjustments provide a balance between efficiency and complexity, with total inventory management costs of approximately 2.30 million USD with

72,000 tons annual production capacity. The model successfully accounts for seasonal supply constraints and stochastic market demand and facilitates a multi-feedstock strategy that offers additional supply chain resilience and associated cost reduction. The framework's computational efficiency and broad applicability make it suitable for adoption by diverse biomass industries with variability in their supply chains.

Keywords: biomass supply chain; wood pellet; inventory management; two-stage stochastic programming

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1. INTRODUCTION

Biomass, defined as organic material that stems from plants through the conversion of sunlight into plant material via photosynthesis (McKendry, 2002), is a vital component of renewable energy and plays a significant role in reducing greenhouse gas emissions and promoting sustainable energy practices (Demirbas, 2005). Dedicated woody biomass energy crops, particularly short-rotation woody crops (SRWCs), including Populus (cottonwoods, poplars, aspens), Salix (willows), Pinus (southern pines), and Eucalyptus (eucalypts), have been widely studied in the United States as renewable feedstocks for bioenergy, biofuels and bioproducts (Zalesny et al., 2011). As one of the SRWCs, willow is being commercialized in the Northeast region of the United States (Langholtz et al., 2016). While wood pellets are conventionally produced from either mill residues or processed roundwood, willow presents a promising alternative feedstock due to its high-density growth, efficient harvesting, and processing characteristics similar to forest-derived biomass (Stachowicz and Stolarski, 2023). Willow has also emerged as an option for keeping marginal agricultural lands in productive use without competing with food production (Stolarski et al., 2011). A stochastic techno-economic analysis demonstrated that shrub willow production can achieve cost-competitive biomass with an 82% probability of meeting federal bioenergy targets, providing quantified assessments that support commercial deployment decisions (Frank et al., 2018).

However, biomass supply chains are typically long and complex. Biomass raw material supply often has seasonal cycles and features low mass, energy density, and bulk density. Harvesting, transportation, storage and processing of biomass through the supply chain impact both the overall product cost and energy use. The uncertainty in biomass product demands can also greatly impact biomass supply chain resiliency and efficiency. Due to these factors, the biomass supply chains require careful management to ensure raw material availability when needed and that they meet the demand from end consumers.

Inventory control is a crucial aspect of supply chain management that focuses on maintaining optimal stock levels to meet production requirement and customer demand while minimizing costs (Chen and Simchi-Levi, 2004). Keeping a larger inventory of raw material on site can ensure the continuous production of a facility to better meet potential increases in associated production demand, but storage of large amounts of biomass is infeasible on many manufacturing sites and can be costly and risky. Simply implementing a fixed raw material inventory policy through a year may run the risk of stockout, or not meeting the potentially fluctuating customer demands for end product, especially when biomass harvest has seasonal constraints. Effective and flexible biomass inventory management helps facilities to maintain a sustainable and resilient supply chain with lower cost to meet customer demands.

Despite the known advantages of inventory management and control methods in many industries, these techniques have been relatively underexplored in the biomass-to-bioenergy domain. We believe by utilizing modern inventory control techniques, stakeholders can design more reliable and robust supply chain structures.

In our research, a Mixed Integer Linear Programming (MILP) model is built to optimize the inventory control of a hypothetically proposed willow-based wood pellet facility in Schenectady County, New York. It is a two-stage stochastic programming model that provides optimal inventory control across multiple simulated stochastic pellet demand scenarios using the so called (*s, S*) inventory control policy.

This study contributes to the biomass energy literature by demonstrating how stochastic optimization can address the unique challenges of seasonal biomass supply chains. The research provides quantified evidence that stochastic approaches achieve key contributions including: (1) development of a comprehensive MILP framework integrating seasonal supply constraints with stochastic pellet demand, (2) demonstration and comparison of different inventory policy adjustment frequencies for practical implementation, and (3) availability of multi-feedstock strategies for enhanced supply chain resilience. The findings offer actionable insights for industry entities on optimal inventory policies and strategic decision making in biomass supply chain management.

2. LITERATURE REVIEW

2.1 Inventory Control Theory Foundations

Inventory control and inventory modeling have been major discussion points in operations research and industrial engineering for more than 100 years, and focus on answering two main questions: How much to order and when to order (Bushuev et al., 2015). Based on the approach for obtaining optimal inventory control parameters, inventory models can be classified as analytical and non-analytical (Jackson et al., 2020). Analytical approaches use mathematical formulas and proofs to derive the optimal inventory parameters and result in economic order quantity (EOQ) models and its variations. Non-analytical approaches includes methods like dynamic programming, linear programming, integer programming, simulation optimization, and metamodeling to search for optimal inventory policies (Jackson et al., 2020).

Introduced by Harris in 1913, the EOQ model is the first formal inventory model that helps determine size and timing of inventory replenishment (Harris, 1990). The EOQ model makes strict assumptions such as backlogs are not permitted, and replenishment lead time equals zero. It is a basic model but describes the trade-off between fixed order costs and holding costs, and provide foundations to many other inventory models (Nahmias and Olsen, 2015). The extension of EOQ models lead to deterministic models built on some relaxed assumptions such as non-zero replenishment lead time, quantity discount and finite production rate (Nahmias and Olsen, 2015). These deterministic extensions were followed by the study of models that integrate uncertainties in demand and/or lead time (Arrow et al., 1951; Dvoretzky et al., 1952a, 1952b), which falls into the realm of stochastic inventory theory, as noted in recent literature reviews (Bushuev et al., 2015).

Within stochastic inventory theory, several key approaches are described in the literature (Axsäter, 2015): (*s, S*), (*R, Q*), (*R, S*), and (*R, s, S*) (Dillon et al., 2017). Here, those parameters represent the periodicity of inventory review (*R*), the fixed-order quantity (*Q*), the reorder point (*s*), and the target stock level (*S*), respectively (Dillon et al., 2017). The (*R, Q*), (*R, S*) and (*R, s, S*) policies are periodical review policies that either order a fixed amount (*Q*) of items, or a variable quantity sufficient to raise the stock level to the target (*S*). For the (*R, s, S*) system an order is only placed when the stock position is at or below position *s*. Conversely, both the (*s, Q*) and (*s, S*) inventory control policies assume continuous inventory review, and whenever the stock position is less than or equal to *s*, an order of *Q* units or a variable quantity sufficient to raise the stock level to the position *S* is placed.

Compared to other inventory policies, (*s, S*) provides more flexible and in-time inventory management, as it implements continuous inventory review and variable replenishment quantities. Previous research shows that variable replenishment inventory policy often has lower cost than fixed replenishment policy (Akhtari et al., 2019). Studies also find advantages in implementing (*s, S*) policy based inventory management under stochastic situation when a fixed ordering cost exists (Perera and Sethi, 2023). Its optimality can be achieved in some specific cases like discrete time (Benkherouf, 2008; Perera and Sethi, 2023) and instant replenishment, with no lead time (Huh and Janakiraman, 2008).

The (*s, S*) policy has been widely used across various disciplines and industries, including hospital pharmacy departments for managing medicines and medical supplies under intermittent demand patterns (Pham et al., 2022), and chemical manufacturing for optimizing inventory levels of both fast-moving and slow-moving chemical products (Fukkwarmdee, 2023), etc. It is also used as the inventory policy for retailers in a multi-echelon system supply chain analysis

(Bakthavachalam et al., 2012), and as a comparison with periodical review in construction supply chains (Xue et al., 2011).

2.2 Biomass Supply Chain Importance and Challenges

Biomass has been considered as one of the major sources of renewable energy. The biomass supply chain is a process from cultivation to field preparation to biomass utilization at the production station, and is responsible for supplying energy conversion plants with the required quantities of biomass, at the appropriate time and with certain quality specifications (Nunes et al., 2020; Rentizelas, 2013; Suurs, 2002). Biomass supply chains create opportunities for rural economic development, renewable energy generation, and reducing dependence on fossil fuels (Rentizelas, 2013). However, the biomass supply chain also faces many challenges from raw material harvesting to end products (Mafakheri and Nasiri, 2014). Past studies categorized biomass supply chain challenges into two primary dimensions: intrinsic challenges stemming from biomass material properties and extrinsic challenges arising from external operational environments. The inherent physical and chemical characteristics of biomass create fundamental intrinsic supply chain challenges. Biomass raw materials are non-uniform, bulky, and characterized by high moisture content (Gautam et al., 2017), making transportation, storage, and processing throughout the supply expensive and energy intensive (Rentizelas et al., 2009). These characteristics directly contribute to significant technical and economic challenges through the whole biomass supply chain operation. Harvesting, transporting and processing biomass requires specialized equipment and adaptive processing technologies that differ from conventional industrial material systems (Mafakheri and Nasiri, 2014). The seasonal nature of biomass production represents another intrinsic challenge. Unlike many manufactured goods that can be produced year round, biomass harvesting is inherently tied to growth cycles and following certain seasonal patterns (Lim et al., 2019).

Many external factors also create complexity that may amplify biomass supply chain challenges. For example, market uncertainties such as demand and price fluctuations represent primary extrinsic challenges (Castillo-Villar et al., 2017), and the issue is exacerbated by uncertainty and seasonality in both demand and supply (Gautam et al., 2017). Because biomass itself is a renewable energy source, managing the biomass source to achieve both sustainable productions and ecosystem services presents another type of challenge. For example, without a careful plan and sustainable operations, biomass utilization could lead to biodiversity loss, habitat degradation, and soil overexploitation (Mafakheri and Nasiri, 2014). Carbon emissions from transportation and processing activities may also offset renewable energy benefits (Yue et al., 2014).

Other extrinsic challenges in biomass utilization include financial barriers from capital investment requirements for biomass infrastructure, potential conflicts between biomass and food supply chains, policy uncertainties regarding government incentives and regulations, and institutional challenges arising from diverse ownership structures across biomass supply chain participants (Mafakheri and Nasiri, 2014).

2.3 Stochastic Optimization in Biomass Supply Chains

Compared with deterministic optimization models implemented in biomass supply chain analysis (Atashbar et al., 2016; Eriksson and Bjorheden, 1989; Freppaz et al., 2004; Keirstead et al., 2012; Nagel, 2000), stochastic optimization models have advantages to address the challenges of biomass supply chain uncertainties (Ba et al., 2016). Stochastic inventory models incorporate uncertain demand and other parameters in biomass supply chain and their dynamics over time (Vidal, 2023) when determining the timing of placing replenishment orders and the corresponding order quantities (Ma et al., 2019). The literature suggests many different approaches for optimization under uncertainty (Sahinidis, 2004). Stochastic programming supports decision-making informed by probabilistic distributions of uncertain future scenarios. Two-stage stochastic programming models includes both initial-stage (e.g., inventory control policy) and recourse stages (e.g., specific ordering and other facility operation decisions) that allow corrective actions after uncertainty is revealed (Birge and Louveaux, 2011). Robust stochastic programming seeks solutions that perform well under worst-case scenarios (Ahmed and Sahinidis, 1998; Bertsimas and Thiele, 2006). Fuzzy programming addresses uncertainty through fuzzy set theory by allowing imprecise objectives and constraints to be modelled, enabling flexible constraint satisfaction where violations are acceptable to varying degrees rather than requiring strict feasibility (Zimmermann, 2010). Stochastic dynamic programming optimizes sequential decision-making processes where future states depend on current decisions and random events (Bitran et al., 1998; Cheng et al., 2003). Among these approaches, stochastic programming has emerged as a promising framework for woody biomass supply chain optimization and has been widely adopted in biofuel production systems to cope with uncertainty, particularly through two-stage stochastic programming models (Nguyen and Chen, 2018).

Kim et al. (Kim et al., 2011) developed a two-stage stochastic programming model for a biofuel supply chain, in which uncertainty of final product price, conversion yield ratios, biomass availability, and maximum demands are included. Kostin et al. (Kostin et al., 2012) proposed a two-stage stochastic MILP model to include uncertainty in the demand of a bioethanol-sugar supply chain. A MIP (mixed integer program) model was developed by Giarola et al. (Giarola et al., 2013) for a multi-period multi-echelon ethanol supply chain, and in this research, uncertainty in feedstock cost and carbon cost was considered using a two-stage stochastic model. A two-stage stochastic programming model was developed to include uncertainty of a bioethanol plant supply chain design (Osmani and Zhang, 2013). The same author also studied the problem of grid design, in which uncertainties of wind speed and electricity sale price were considered in a two-stage stochastic programming model (Osmani and Zhang, 2014). Various methods exist in the literature for solving a two-stage stochastic programming model, such as benders decomposition or L-shaped algorithm (Awudu and Zhang, 2013; Marufuzzaman et al., 2014), and a progressive hedging algorithm (Chen and Fan, 2012).

Most of the existing stochastic programming models are for biofuel production, with a relatively small amount of them focused on forest-based biomass supply chains, like the study by Shabini and Sowlati (Shabani and Sowlati, 2016). To the best of our knowledge, no previous work has focused on willow biomass inventory management using two-stage stochastic programming models.

2.4 The North American Wood Pellet Industry

The North American wood pellet industry has experienced substantial growth, largely driven by international demand for renewable energy sources, especially from Europe and Asia (Atasoy et al., 2023; Guo et al., 2023). The United States and Canada are the primary producers in this region, with the global wood pellet market valued at an estimated USD 8.4 billion in 2022, projected to grow significantly with a forecasted compound annual growth rate (CAGR) of 5.8% till 2030 (Vantage Market Research, 2023).

Manufacturing methods of wood pellets range from large, centralized industrial plants to smaller, decentralized operations. Centralized production typically involves log chipping, milling, drying, pelletizing, and packaging (Quinteiro et al., 2020), while decentralized production at sawmills or local manufacturers offers environmental and economic benefits by utilizing wood by-products and residues, such as chips, sawdust and shavings from nearby sources (Quinteiro et al., 2020). Many centralized facilities also procure wood residues for pellets from sawmills and other primary processors, in addition to using logs, depending on market conditions. The geographic landscape is concentrated in the southeastern United States, which has grown as a global production hotspot due to abundant biomass resources, established infrastructure, and proximity to ports (Aguilar et al., 2020; Dale et al., 2017). Wood pellet mill capacity in the U.S. South increased from 3.8 million to 7.7 million tons between 2012 and 2017 (Brandeis and Abt, 2019).

North America functions primarily as a net exporter in the global wood pellet trade, with about 85% of U.S. wood pellet production being exported, mainly to Europe, Japan, and South Korea (Atasoy et al., 2023). This export orientation is driven by the European Union's Renewable Energy Directive, which incentivizes biomass use to displace coal in power generation (Aguilar et al., 2020; Dale et al., 2017). U.S. wood pellet exports grew from essentially zero in 2008 to 4.6 million metric tons in 2015 (Dale et al., 2017). North American residential and commercial use of wood pellets is primarily for fueling small heating appliances and sometimes larger, automated heating systems, with growing use in the Northeast United States (Buchholz et al., 2017).

The industry relies on diverse raw materials including sawmill residues, logging residues, and low-grade pulpwood (Brandeis and Abt, 2019; Parajuli, 2021). Wood pellets are categorized into industrial-grade pellets for large-scale power generation and residential-grade pellets for heating applications, governed by international standards such as ENplus and PFI that specify properties like mechanical durability, ash content, and moisture content (Boukherroub et al., 2017). The industry comprises interconnected stakeholders including pellet producers ranging from large export-oriented facilities to smaller sawmill operations, brokers facilitating international trade, equipment manufacturers supplying specialized production technology, and bulk shippers managing global distribution (Boukherroub et al., 2017; Gilvari et al., 2021; Thrän et al., 2019).

Transportation and storage show significant logistical challenges due to the low energy density of raw biomass and the fragile nature of finished pellets, which are prone to physical degradation and pose safety risks including dust explosions and off-gassing during handling and storage (Dafnomilis et al., 2018; Gilvari et al., 2021; Vitale et al., 2022).

The industry's pursuit of consistent pellet quality has triggered interest in dedicated energy crops, which can provide more uniform feedstock characteristics compared to traditional raw materials such as sawmill residues and logging materials that exhibit variability in composition and properties (Lee et al., 2024).

2.5 Willow as Biomass Feedstock for Pellet Production

Willow is an important short-rotation woody crop being commercialized to supply biomass, especially in the Northeast United States (Langholtz et al., 2016). Fast-growing willow (Salix spp.) species like Salix viminalis have demonstrated effectiveness in short rotation biomass cropping systems (Liu et al., 2017; Wang et al., 2006; Wu et al., 2012), particularly on degraded sites, partly because they are facultative nitrogen fixers with high productivity even in low-nutrient soils. Various trials have been conducted with willow at multiple locations and on larger scales (Volk et al., 2018), demonstrating commercial viability and scalability. Using purpose-grown, coppicing willow for pellet feedstock is currently unconventional, as most pellets utilize mill residues like sawdust or processed roundwood that is debarked and processed through hammer mills. However, willow is considered advantageous for wood pellet production because it can be sustainably produced from short rotation woody crop plantations, representing a renewable and sustainable biomass source. Willow is also grown on sites that have been remediated following heavy mining and industrial use (Rodzkin and Volk, 2017), which provides additional benefits associated with environmental restoration and pollution control. When mixed with forest-derived biomass or used independently, willow contributes to efficient and environmentally friendly wood pellets due to its high-density growth characteristics, efficient harvesting potential, and processing properties similar to forest-derived biomass.

A critical operational characteristic of willow biomass is its seasonal harvesting requirement. Existing guidelines for shrub willow mainly suggest harvests should occur during the dormant season (November to March) when leaves are off, which minimizes disruption to growth cycles and reduces biomass loss (Abbas et al., 2011; *Energy from Willow*, 2000). In practice, operators may expand the harvesting window as machine access might be limited on poorly drained marginal land during dormant season (Eisenbies et al., 2020). These seasonal supply constraints create unique challenges for willow-based pellet facilities from other biomass sources, requiring strategic planning to bridge supply gaps while managing storage costs and meeting uncertain market demand.

2.6 Research Gaps and Study Contributions

The identified gaps above motivate our development of a stochastic optimization framework for willow-based wood pellet production, especially focusing on the inventory management component of a specific pellet production facility. Two-stage stochastic programming is effective in modeling biomass supply chain uncertainties, as reviewed in Section

2.3. Therefore, we adopt this approach to address challenges such as seasonal willow supply and stochastic pellet demand. Our framework uses a two-stage stochastic MILP model with recourse. In the first stage, inventory control policies, specifically the (*s, S*) policy parameters, are set before demand uncertainty is revealed (Birge and Louveaux, 2011). The second stage allows for corrective operational actions (Birge and Louveaux, 2011). This research contributes to inventory control under seasonal biomass feedstock and demand uncertainty. It advances the identified gaps and provides a reference for industry facilities and supply chain managers.

3. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

3.1 Problem Illustration

Consider a wood pellet production facility that must manage inventory under multiple operational constraints including supply constraints, processing constraints, storage constraints, transportation constraints and demand uncertainty, aiming at minimal inventory management cost.

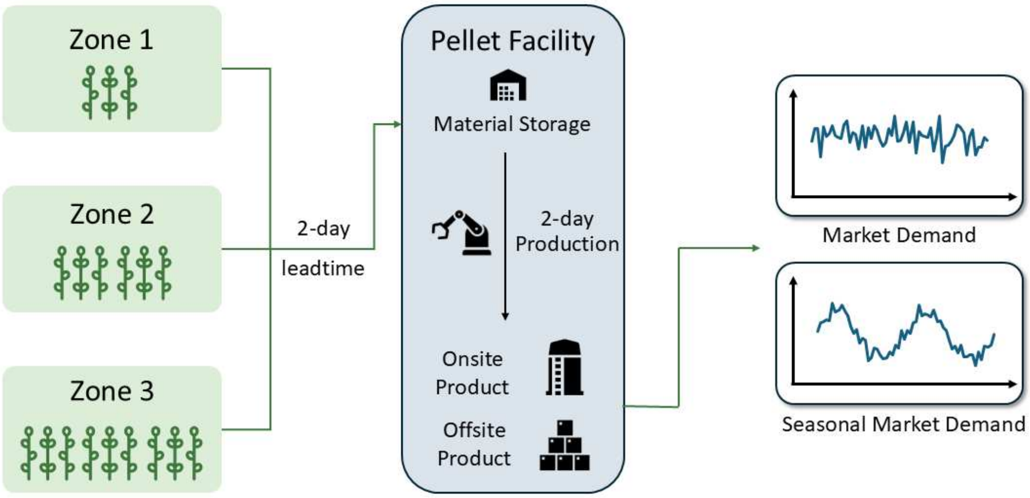


Figure 1: A hypothetical willow-based wood pellet supply chain structure. We assume that there are three distinct willow raw materials supply zones, one wood pellet facility and fluctuation of pellet demand between seasons and days.

As shown in figure 1, the willow-based pellet production facility operates as a central node in a biomass energy supply chain. Raw willow biomass is harvested from three supply zones and transported to the facility, where it goes through processing procedures to produce wood pellets for energy markets. The supply of willow raw materials fluctuates seasonally; the demand for wood pellets often displays both seasonal trends and random fluctuations.

3.2 Mathematical Model

3.2.1 Two-Stage Stochastic Programming and (s, S) Inventory Control

The model uses a two-stage stochastic programming framework (Birge and Louveaux, 2011) that separates the general policy decisions from detailed facilities operational decisions, an approach that has proven effective in biomass supply chain optimization (Aranguren et al., 2021). To be specific, the first stage determines strategic inventory control parameters before uncertainty is revealed, while the second stage handles operational decisions (daily ordering, transportation, production and storage etc.) after stochastic pellets demand scenarios are observed. This decision-making structure enables the model to establish robust policies that perform well across multiple stochastic demand scenarios.

Building on this framework, we implement an (*s, S*) inventory policy for modeling willow-based pellet production supply chain. We assume managers would review the willow raw material inventory at the facility daily. Whether a replenishment action would be taken is determined by the model calculated upper (maximum or *S*) and lower (minimum or *s*) inventory levels at the start of each day. Material will be reordered if the facility's willow raw material inventory is below the minimum required level *s*; the order quantity is calculated by subtracting the current inventory from the maximum level *S* based on the inventory review results. Inventory policy parameters are allowed to be adjusted at different frequencies (e.g., daily, weekly, or monthly). As established in Section 2.1, the (*s, S*) policy has demonstrated superior cost performance and practical implementation advantages in industries. The basic (*s, S*) inventory policy provides the foundational framework for our model, with extensions to address seasonal biomass supply constraints that will be detailed in the mathematical formulation below.

3.2.2 Model Notation

The mathematical model uses the notation summarized in Tables 1 and 2. Table 1 presents the indices and parameters that define the problem structure, while Table 2 lists the decision variables organized by their role in the two-stage stochastic programming framework.

Table 1: Model indices and parameters.

Category	Symbol	Description
Indices	m	Index of inventory policy adjustment periods
	t or τ	Index of days within a year
	ω	Index of modeled demand scenarios; Ω is the total number of scenarios modeled
	i	Index of willow raw material supply zones
	k	Remaining days for material arrival since the ordering day
	u	Remaining days for production completion since the beginning day of production;
Cost Parameters		
	c_i	Variable cost of ordering each ton of willow from zone i
	f_i	Fixed cost from each time ordering willow from zone i
	s	Raw material daily storage cost measured by dollar per ton
	w_{inner}	On-site finished product daily storage cost per ton
	w_{outer}	Off-site finished product daily storage cost per ton
	o	Lost sales penalty by dollar per ton of pellets
Capacity Parameters		
	$capStore$	Raw material storage capacity in the facility by tons
	$capInter$	Processing capacity by the facility by tons
	$capInner$	On-site finished product storage capacity by tons
	$capProc$	Daily Procurement capacity by tons
Supply Parameters		
	$yield_i$	Annual supply capacity from zone i measured by tons
	$a_{t,i}$	Cumulative raw material supply capacity in zone i, up to day t since the beginning of each year, see figure 3

	r	Biomass-to-pellet weight-based conversion ratio, apply to both willow and forest residue
Demand Parameters		
	$d_{\omega,t}$	Pellet demand (scenario ω, day t), see figure 4
Other		
	h_0	Initial raw material inventory level at the beginning of each year
	l_0	Initial finished pellet raw material inventory level at the beginning of each year
	M	A large positive number

Table 2: Decision variables.

Stage	**Symbol**	**Type**	**Description**
First Stage			
	US_m	Continuous	Upper inventory threshold (period m)
	LS_m	Continuous	Lower inventory threshold (period m)
Second Stage			
	$N_{\omega,t}$	Binary	Daily reorder decision (scenario ω, day t)
	$R_{\omega,t,i}$	Binary	Daily order decision from zone i (scenario ω, day t)
	$Y_{\omega,t,i}$	Continuous	Daily order quantity from zone i (scenario ω, day t)
	$WH_{\omega,m,i}$	Continuous	Total period order quantity from zone i (scenario ω, period m)
	$H_{\omega,t}$	Continuous	Daily raw material inventory in a facility (scenario ω, day t)
	$L_{\omega,t}^{inner}$	Continuous	Daily on-site finished product inventory (scenario ω, day t)
	$L_{\omega,t}^{outer}$	Continuous	Daily off-site finished product inventory (scenario ω, day t)
	$L_{\omega,t}$	Continuous	Daily total finished product inventory (scenario ω, day t)
	$B_{\omega,t}$	Continuous	Daily lost sales of pellet (scenario ω, day t)
	$I_{\omega,t,k}$	Continuous	Daily in-transit raw material inventory before reaching the facility (scenario ω, day t, k days remaining)
	$P_{\omega,t}$	Continuous	Total daily raw material procurement (scenario ω, day t) from all supply zones
	$Q_{\omega,t,u}$	Continuous	Daily in-process raw material before final product (scenario ω, day t, u days remaining)

Note: “ton” or “tons/day” is the common measurement unit for continuous variables

3.2.3 Mathematical Formulation for Single Feedstock (Willow-Only Model)

Objective function:

$$min\frac{1}{\Omega}\sum_{\omega}\left\{\sum_{t}\left[\sum_{i}\left(c_iY_{\omega,t,i}+f_iR_{\omega,t,i}\right)+sH_{\omega,t}+w_{inner}L^{inner}_{\omega,t}+w_{outer}L^{outer}_{\omega,t}+oB_{\omega,t}\right]\right\}$$

Subject to:

Group A: First Stage Decisions and Constraints

$$US_m \geq LS_m \qquad \forall m \tag{1}$$

$$US_m \leq capStore \qquad \forall m \tag{2}$$

Group B: Second Stage Decisions and Constraints

$$M \times R_{\omega,t,i} - Y_{\omega,t,i} \geq 0 \qquad \forall \omega, t, i \tag{1.1}$$

$$\sum_{i} Y_{\omega,t,i} - P_{\omega,t} = 0 \qquad \forall \omega, t \tag{1.2}$$

$$\sum_{t\in m} Y_{\omega,t,i} = WH_{\omega,m,i} \qquad \forall \omega, m, i \tag{1.3}$$

$$\sum_{\tau=0}^{t} Y_{\omega,\tau,i} \leq a_{t,i} \qquad \forall \omega, t, i \tag{2.1}$$

$$Y_{\omega,t',i} = 0 \qquad \forall \omega, t' \in \text{during off season}, i \tag{2.2}$$

$$M \times N_{\omega,t} \geq LS_m - H_{\omega,t} \qquad \forall \omega, t, m \tag{3.1}$$

$$M(1 - N_{\omega,t}) \geq H_{\omega,t} - LS_m \qquad \forall \omega, t, m \tag{3.2}$$

$$P_{\omega,t} \geq US_m - H_{\omega,t} - M(1 - N_{\omega,t}) \qquad \forall \omega, t, m \tag{3.3}$$

$$P_{\omega,t} \leq US_m - H_{\omega,t} + M(1 - N_{\omega,t}) \qquad \forall \omega, t, m \tag{3.4}$$

$$P_{\omega,t} \leq M \times N_{\omega,t} \qquad \forall \omega, t \tag{3.5}$$

$$P_{\omega,t} \leq capProc \qquad \forall \omega, t \tag{3.6}$$

$$I_{\omega,t,k_{max}} = P_{\omega,t} \qquad \forall \omega, t \tag{4.1}$$

$$I_{\omega,t,k} = 0 \qquad \forall \omega, t \leq k_{max} - 1, k \leq k_{max} - t \tag{4.2}$$

$$I_{\omega,t,k} = I_{\omega,t-1,k+1} \qquad \forall \omega, t \geq 2, k \leq k_{max} - 1 \tag{4.3}$$

$$H_{\omega,1} = h_0 - Q_{\omega,1,u_{max}} \qquad \forall \omega \tag{5.1}$$

$$H_{\omega,t} = H_{\omega,t-1} - Q_{\omega,t,u_{max}} + I_{\omega,t,1} \qquad \forall \omega, t \geq 2 \tag{5.2}$$

$$Q_{\omega,t,u} = 0 \qquad \forall\omega, t \le u_{max} - 1, u \le u_{max} - t \qquad (6.1)$$

$$Q_{\omega,t,u} = Q_{\omega,t-1,u+1} \qquad \forall\omega, t \ge 2, u \le u_{max} - t \qquad (6.2)$$

$$L_{\omega,1} = l_0 + B_{\omega,1} - d_{\omega,1} \qquad \forall\omega \qquad (7.1)$$

$$L_{\omega,t} = L_{\omega,t-1} + B_{\omega,t} + rQ_{\omega,t,1} - d_{\omega,t} \qquad \forall\omega, t \ge 2 \qquad (7.2)$$

$$L_{\omega,t} = L_{\omega,t}^{inner} + L_{\omega,t}^{outer} \qquad \forall\omega, t \qquad (7.3)$$

$$\sum_{u} Q_{\omega,t,u} \le capInter \qquad \forall\omega, t \qquad (8.1)$$

$$L_{\omega,t}^{inner} \le capInner \qquad \forall\omega, t \qquad (8.2)$$

$$L_{\omega,t}^{outer} \ge L_{\omega,t} - capInner \qquad \forall\omega, t \qquad (8.3)$$

$$H_{\omega,t_{max}} \ge h_0 \qquad \forall\omega \qquad (9.1)$$

$$L_{\omega,t_{max}} \ge l_0 \qquad \forall\omega \qquad (9.2)$$

$$\text{All decision variables} \ge 0 \qquad (10.1)$$

The decisions and constraints are organized into two groups: Group A contains first-stage policy constraints that determine the inventory control parameters US_m and LS_m, while Group B contains all second-stage operational constraints that govern daily operations under each stochastic pellet demand scenario.

The objective function minimizes the expected total annual inventory management cost of the willow-based pellet supply chain to meet the demands across all modeled scenarios ω. It aggregates, for every day t and supply zone i, for each scenario ω, the variable procurement and transport expense $c_i Y_{\omega,t,i}$, the fixed ordering charge $f_i R_{\omega,t,i}$ that is incurred only when the binary variable $R_{\omega,t,i} = 1$, the daily cost of storing raw willow materials within the facility $sH_{\omega,t}$, the costs of storing finished pellets on-site $w_{inner} L_{\omega,t}^{inner}$ and off-site $w_{outer} L_{\omega,t}^{outer}$, and finally the penalty $oB_{\omega,t}$ associated with any lost sales $B_{\omega,t}$ that arise when daily demand exceeds available pellet inventory. Because the summation of cost and penalty spans across all scenarios, we select a formulation reflecting the risk neutral decisions and seeking the minimum expected total modeled cost and penalty. In the further explanations of this model, we may omit the ω unless explicitly stated for simplification.

Group A constraints enforce $US_m \geq LS_m$, and limits the inventory policy upper bound level by the facility's total raw material storage capacity $capStore$.

In Group B, constraints (1.1) to (1.3) describe the daily procurement logic for each scenario. A big M in (1.1) sets up the logic between the continuous order quantity $Y_{\omega,t,i}$ and the binary decision $R_{\omega,t,i}$; any order amount greater than zero will set the value of $R_{\omega,t,i}$ to be one. Constraint (1.2) tracks the total willow raw material order quantity on day t as $P_{\omega,t}$, and (1.3) accumulates daily purchases into a per period m variable $WH_{\omega,m,i}$, which is used later for willow's periodic harvest capacity checks.

Constraint (2.1) makes sure the accumulative biomass ordered up to day t from a supply zone i is no more than the accumulative willow raw material production up to day t from that same zone. To test the impact of willow supply seasonality, we create constraint (2.2) to make sure willow biomass is not available to be ordered beyond the willow harvesting season.

The (*s, S*) inventory control policy is executed through constraints (3.1) to (3.5). When on-hand raw material stock $H_{\omega,t}$ falls below the lower bound LS_m, binary variable $N_{\omega,t}$ is set to be one by (3.1) and (3.2) triggers reordering. If $N_{\omega,t} = 1$, constraints (3.3) and (3.4) set the replenishment $P_{\omega,t}$ amount to raise the raw material inventory level exactly to the upper bound US_m; otherwise, constraint (3.5) sets the order amount to be zero when $N_{\omega,t} = 0$. Constraint (3.6) ensures the daily raw material purchase amount does not surpass the total accessible transportation capacity by the facility.

Transportation lead time is modeled through constraints (4.1) to (4.3). Constraint (4.1) inserts the freshly ordered material into the k_{max} day queue, and (4.3) advances each shipment one day closer to the arrival date to the facility. Constraint (4.2) makes sure raw material inventory in transit can only come from the initial order amount. Constraint (5.1) initializes the willow raw material stock at the beginning of each year. Once material reaches the facility, Equation (5.2) updates $H_{\omega,t}$ each day by subtracting the amount of raw material transferred to production from storage and adding the raw materials arrivals from transit due to reordering.

The chipping and pelletizing production line imposes a u_{max} day delay that is modelled by $Q_{\omega,t,u}$ and advanced through (6.1) to (6.2). Finished pellet product inventory balance at the start of the year is calculated by (7.1). Constraint (7.2) tracks final product inventory at the start of day $t-1$ plus newly completed pellets minus demand $d_{\omega,t}$ equals the inventory stock at the start of the next day t, with any shortfall captured as lost sales $B_{\omega,t}$. Equation (7.3) partitions that final product inventory between on-site storage $L_{\omega,t}^{inner}$ and off-site storage $L_{\omega,t}^{outer}$.

Constraint (8.1) limits the total in-process raw material quantity to be no more than the total facility processing capability $capInter$. Constraint (8.2) ensures that the onsite finished product inventory never exceeds the onsite product storage capacity $capInner$. Offsite finished product storage rules in (8.3) guarantee that external storage is used only when onsite capacity is filled.

Sustainable operation is enforced by constraint (9.1) to (9.2): raw material and finished product inventories on the last day of the year must at least equal to their opening levels h_0 and l_0 at the start of each year. Finally, constraint (10.1) declares non-negativity for all decision variables.

Collectively, these constraints ensure that the model delivers a cost-minimal and operationally sustainable inventory policy that is robust across a range of stochastic demand scenarios considered.

3.2.4 An Extension to Multi-Feedstock Model (Willow + Forest Residues)

We also extend the base model with additional decision variables and constraints to incorporate forest residues as an alternative feedstock, while maintaining the core (*s, S*) inventory policy framework. The extended model includes three willow supply zones and one forest residual supply area (see Table 3).

Table 3: Additional variables and parameters for multi-feedstock extension.

Category	Symbol	Description	Range/Values
Extended Indices			
	num_willow_zones	Number of willow supply zones	3
	num_forest_zones	Number of forest residue supply zones	1
	i	Total number of supply zones	4
Feedstock Classification			
	i	Supply zone number as feedstock type identifier	1, 2, 3 for willow; 4 for forest residue
Extended Cost Parameters			
	c_i (for $i = 4$)	Variable ordering cost for forest residue zones	\$70/ton
	f_i (for $i = 4$)	Fixed ordering cost for forest residue	\$100/order

4. CASE STUDY

4.1 Study Context and Location

We select one of the proposed facility locations from a previous study (Wang et al., 2020) as a hypothetical wood pellet facility location. This site is in Schenectady County, New York and surrounded by abundant willow biomass production sites. County-level estimates of annual willow biomass availability were obtained from the US DOE Billion-Ton Report (Langholtz et al., 2016).

To roughly capture the economic trade-offs between transportation costs and supply availability, we systematically delineate three concentric willow raw material supply zones around the facility (see figure 2): Zone 1 (0-20 miles from the facility), Zone 2 (20-50 miles), and Zone 3 (50-100 miles). The annual willow availability is estimated at 7,469 tons in Zone 1, 42,590 tons in Zone 2, and 103,294 tons in Zone 3. Transportation costs are roughly estimated by zones and increase with distance, resulting in delivered prices of \$43.60, \$46.00 and \$50.00 per ton for the three zones respectively.

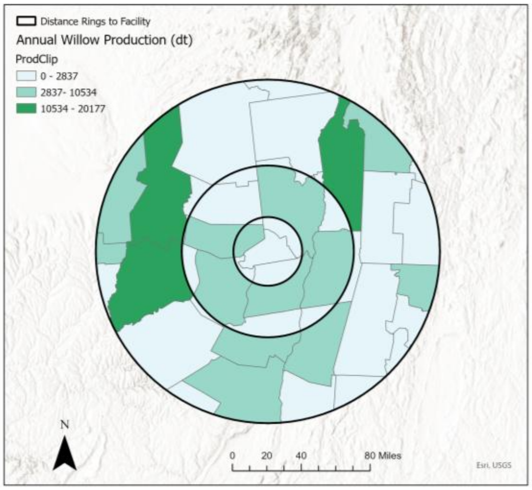


Figure 2: Willow biomass availability by distance zone from the proposed Schenectady County willow pellet production facility.

4.2 Assumptions and Parameter Estimation

4.2.1 Supply Parameters

Potential critical operational constraints include willow's harvesting seasonality and operational capacity limitations. Willow biomass supply to the facility may be constrained by harvesting capacity including labor availability, equipment limitations, and logistical coordination. Ideal willow harvesting should be planned during the dormant season when leaves are off to minimize disruption to the natural coppicing cycle and reduce biomass quality degradation. Some research suggests that harvesting willow during the leaf-on season may result in biomass loss. However, if sustainable production is preferred by a pellet facility, there might be benefits to harvesting a certain amount of willow during leaf-on seasons. To reflect this assumption, we allow willow harvesting in suboptimal months, but we adjust the harvesting capacity rate to account for the decrease in willow production when harvested during sub-optimal leaf-on months. To be specific, we assume different willow harvesting capacity at different months throughout the year: full capacity during the dormant season (November to March), zero capacity during the critical growth period (April to May), and reduced 80% of full capacity during the late growing season (June to October). We calculated the daily accumulative willow raw material supply amount across a year, as shown in figure 3, based on the harvesting capacity assumptions. This accumulated amount in each day represents the maximum amount of willow raw material supply up to that day that can be used by the facility to produce wood pellets.

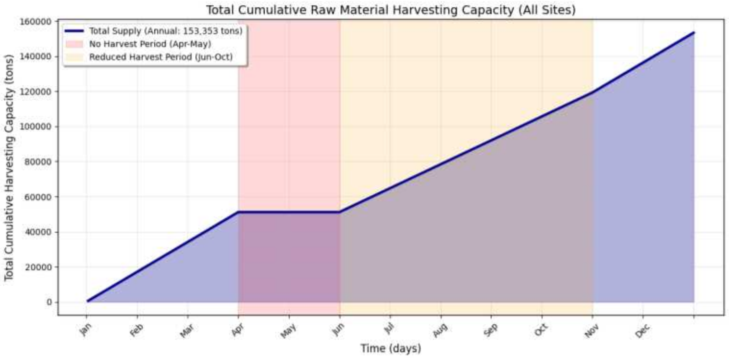


Figure 3: Cumulative willow harvesting capacity through a year in the study site. It represents the maximum achievable cumulative harvest given operational constraints (labor, equipment, logistics).

4.2.2 Demand Characterization

We model daily demand for wood pellets produced by the proposed facility as stochastically varying. It is important to note that true market demand for a single facility's products is currently unavailable. Therefore, we use sales data as a proxy for demand in this study.

Normal distribution is widely adopted in inventory management literature and textbooks for modeling continuous demand with known mean and variance parameters, though its appropriateness depends on demand characteristics (Ramaekers and Janssens, 2008). To empirically validate the use of normal distribution for our application, we analyzed 9 years (2016-2024) of monthly densified biomass fuel sales data from the U.S. Energy Information Administration (U.S. Energy Information Administration, 2025). Testing within-month demand variability, 92% of observations in the East region (where our case study is located) passed Shapiro-Wilk normality tests ($p > 0.05$), with 100% pass rate when considering the U.S. total. These results support the normal distribution assumption for modeling demand in this market context (see detailed results in Appendix).

For our base case, we assume daily demand follows a normal distribution with a mean of 100 tons/day and a standard deviation of 10 tons/day, using a coefficient of variation (CV) of 10%. We generate multiple demand scenarios using this distribution as input to incorporate stochasticity and improve the robustness of the facility's willow raw material inventory control policies.

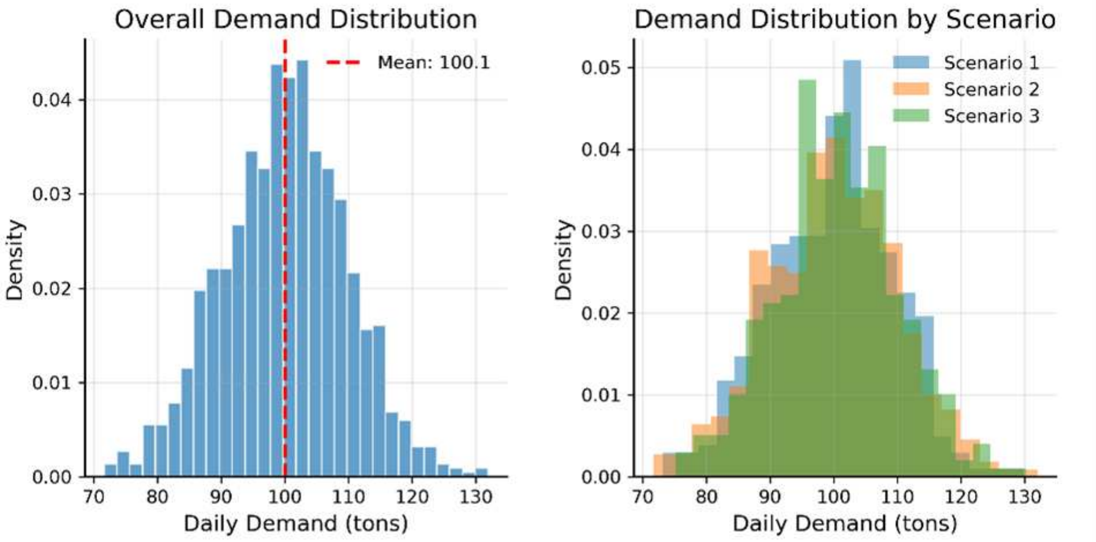
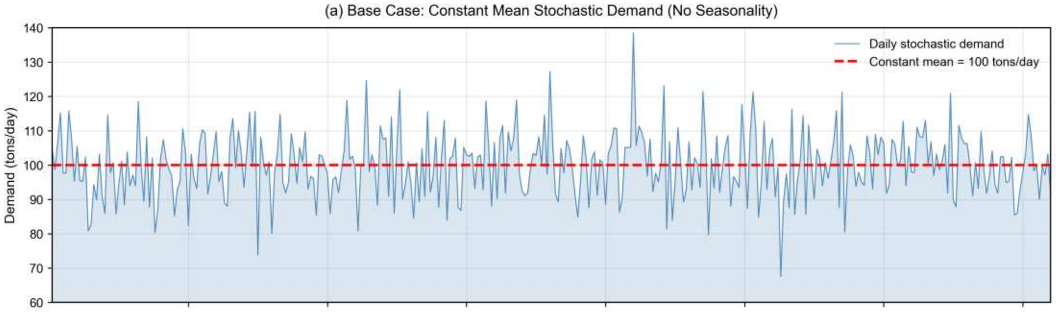


Figure 4: Demand scenarios distribution across a modeled year. Left panel shows the total overall demand for the facility's pellet products following a normal distribution; the right panel shows the three specific demand scenarios created by doing random draw from the normal distribution.

To evaluate model performance under more realistic market conditions, we further extend the constant-mean normal distribution demand (Figure 4) to incorporate seasonal variations. Using monthly sales data from the Eastern U.S. densified biomass fuel market in 2024 (U.S. Energy Information Administration, 2025) as a proxy for wood pellet demand, we calculate seasonal demand factors by dividing each month's sales volume by the annual average monthly sales. To convert these 12 monthly factors into continuous daily values for our 360-day model, we apply cubic interpolation to these monthly factors, using mid-month points (days 15, 45, etc.)

as reference nodes to generate daily seasonal factors for all 360 days. These interpolated daily seasonal factors are then multiplied with the base stochastic demand distribution (mean = 100 tons/day, standard deviation = 10 tons/day) to generate seasonally adjusted daily pellet demand scenarios that preserve both empirical market trends and stochastic variability.



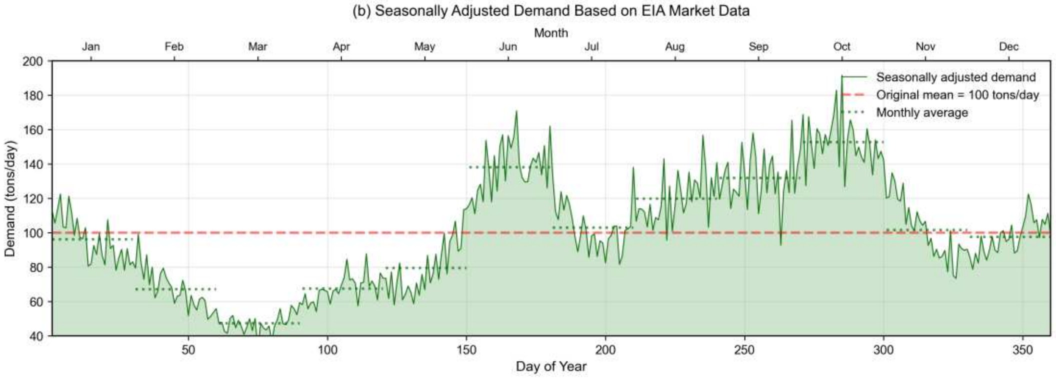


Figure 5: Comparison of a random scenario generated based on constant-mean stochastic demand (panel a) and seasonally adjusted demand based on 2024 EIA market data (panel b).

4.2.3 Operational Assumptions and Parameters

We assume the pellets production facility operates 360 days per year as an operation cycle. The use of 360-day annual cycle instead of 365 days is a modeling convenience that facilitates clean division of the cycle into different inventory policy adjustment periods for fair comparison (e.g. daily, 6-day weekly, or 30-day monthly) and simplifies computational implementation. It does not imply that a real-world facility would operate continuously for 360 days of the year. The detailed operation parameters are listed below.

Table 4: Operational parameters for the willow-based pellet production facility.

Parameter	Value	Unit
Transportation lead time	2	days
Production process time	2	days
Conversion ratio	0.8	pellet/biomass weight ratio
On-site raw material storage cost	1.00	\$/ton/day
On-site product storage cost	0.50	\$/ton/day
Off-site product storage cost	1.50	\$/ton/day
Raw material storage capacity	2000	tons
Processing capacity	200	tons/day
On-site product storage capacity	2000	tons
Daily raw material procurement	800	tons/day
Lost sales penalty	235	\$/ton
Fixed ordering cost	100	\$/order
Variable ordering costs (Zone 1-3)	43.6, 46, 50	\$/ton

4.2.4 Parameter Estimation and Justification

Due to limited commercial-scale willow-based wood pellet production data, some parameters were adopted from conventional wood pellet operations with adjustments.

Transportation and processing parameters. Two days of transportation lead time from supply sites to wood pellet facility is set, considering the maximum 100-mile radius transportation distance in our case study and corresponding administrative activities for each raw material order. Two days of processing time from raw material to end product is set, considering pre-treatment, pelleting, cooling and packaging (Póliska et al., 2014). The biomass-to-pellet conversion ratio varies, as the affecting factors, including feedstock moisture content, preprocessing requirements, and dry matter losses, may vary. For example, the dry matter losses of willow biomass storage could range from 6.3% to 33.6% (Therasme et al., 2020). For model simplification, we adopt a uniform conversion ratio of 0.8 (meaning 1 ton of biomass yields 0.8 tons of pellets) for both willow and forest residue feedstocks, acknowledging previous studies and actual conversion rates may vary between feedstock types.

Cost parameters. Lost sales penalty is considered as the price of purchasing market wood pellet to fill lost sales, and $235 per ton is taken as it is the average densified biomass fuel price for east U.S. region in 2024 (U.S. Energy Information Administration, 2025). A base willow feedstock price of $42 per ton is set according to EIA 2024 average feedstock costs (U.S. Energy Information Administration, 2025). Transportation costs are estimated using a rate of $0.10 per ton-kilometer, with average Euclidean distances of 10, 25 and 50 miles (16, 40, and 81 kilometers respectively) from supply zone 1, 2, and 3 to the facility. This results in transportation costs of $1.60, $4.00 and $8.00(rounded) per ton for the three zones, yielding total variable ordering costs of $43.60, $46.00, and $50.00 per ton respectively. Fixed ordering costs of $100 per order apply across all supply zones, representing administrative and coordination expenses of procurement activities. For the multi-feedstock extension, forest residue variable cost was set at $50/ton with $100 fixed cost per order, reflecting the higher procurement costs associated with spot contracting and alternative supply chain arrangements outside the primary willow-based system infrastructure.

Capacity parameters. Daily production capacity was set at 200 tons, based on the eastern U.S. regional average facility capacity of 189 tons per day in 2024 (U.S. Energy Information Administration, 2025) (equivalent to approximately 68,000 tons annually), yielding an annual production capacity of 72,000 tons under our assumed 360-day operation cycle. On-site raw material and product storage capacity are both set to 2000 tons, providing a ten-fold buffer relative to daily production capacity. Off-site end-product storage capacity has no limit, representing external rental facilities. Daily procurement limit is 800 tons based on assumed maximum truck loading capacity per facility per day. Storage costs were established considering their proportion within total wood pellet supply chain costs reported in previous studies (Visser et al., 2020). Raw material storage cost of \$1.00/ton/day. On-site finished product storage cost of \$0.50/ton/day, lower due to the higher density and stability of pelletized material. Off-site storage cost is \$1.50/ton/day, accounting additional handling, transportation to external facilities, and premium rental rates for third-party storage.

4.3 Test Cases Design and Implementation

To comprehensively evaluate our stochastic optimization framework, we design six test cases that progressively examine different aspects of the model's performance. These tests are structured to: (1) establish baseline performance metrics, (2) evaluate key operational parameters, (3) validate the stochastic approach against deterministic methods, and (4) assess computation time requirements.

To assess the quality of optimization solutions, we implement an out-of-sample evaluation framework. After obtaining optimal inventory policies from our two-stage stochastic programming model using the predefined parameters, we extract these first-stage decisions and apply them to independently generated demand scenarios that were not used during optimization. These out-of-sample scenarios follow the same statistical properties but represent previously unseen demand realizations. We then simulate the operational performance under these new samples and compare key performance metrics. This out-of-sample evaluation approach ensures unbiased performance assessment across different parameters.

Table 5: Test cases design.

Category	Test Case	Objective	Key Parameters	Figure Reference
Baseline	Base Test (BT)	Establish baseline performance	Seasonal supply; 3 demand scenarios; Monthly inventory policy adjustment.	Fig 6, 7

Operational Analysis	Policy Adjustment Frequency Test (PAFT)	Test various adjustment frequency	Daily/Weekly/Monthly; Out-of-sample method; 50 runs per frequency.	Fig 8
Method Validation	Multiple Scenario Test (MST)	Stochastic vs. deterministic	1-5 demand scenarios; Out-of-sample method; 50 runs per scenario setting.	Fig 9, 10
Application	Seasonal Demand Test (SDT)	Market seasonality integration	Adjusted demand scenarios	Fig 11-13
	Multi-Feedstock Test (MFT)	Multi-feedstock benefits	Willow and forest residue supply	Fig 14, 15
	Computational Performance Test (CPT)	Computational efficiency	2 demand scenarios; MIP gaps 0.11%-1%.	Fig 16

Note: other tests will use same parameters with BT unless otherwise specified

Base test (BT) is for establishing baseline performance. Seasonal willow raw material supply, three demand scenarios and monthly inventory policy adjustment frequency are used as parameters.

Policy Adjustment Frequency Test (PAFT) compares the solution quality of daily, weekly, and monthly adjustment frequencies using out-of-sample evaluation approach. For each policy adjustment frequency, 50 separated evaluating runs are conducted with each run using one randomly generated out-of-sample demand scenario. Objective values from the 50 evaluating runs are aggregated to establish statistical significance of cost differences across inventory policies generated from different adjustment frequencies.

The Multiple Scenario Test (MST) employs the same out-of-sample approach to evaluate stochastic optimization quality. Separate optimization runs are conducted using 1 to 5 randomly generated demand scenarios to derive different inventory policies. The single-scenario case (ω=1) represents the deterministic approach. For each inventory policy obtained, we conduct 50 independent single-demand-scenarios to generate statistical distributions of objective values.

This design enables fair comparison between deterministic (optimized for the single demand scenario) and stochastic (optimized for 2 to 5 demand scenarios) approaches by testing all policies under identical evaluation conditions.

Seasonal Demand Test (SDT) evaluates model performance under realistic market conditions by incorporating empirical seasonal patterns (see an example in Figure 5). This preserves both seasonal market trends and demand uncertainty, enabling assessment of inventory optimization under real-world seasonal consumption patterns.

Multi-Feedstock Test (MFT) extends the baseline model to evaluate raw biomass material supply diversification benefits using the framework from Section 3.2.4. Forest residues are incorporated as an alternative feedstock with year-round availability, priced at \$70/ton with \$100 fixed ordering cost. This enables optimization across two feedstock types with different cost structures and availability patterns, examining whether multi-feedstock flexibility can reduce costs and improve supply chain resilience.

For the Computational Performance Test (CPT), convergence times are recorded for each MIP gap tolerance level run, while all other parameters remain controlled to be the same as the BT cases.

All MILP models in our test are implemented and solved using CPLEX optimization solver (version 22.1.0) within an Anaconda environment. All tests are conducted on a Windows 11 Enterprise system with Intel Xeon Gold 5217 CPU and 64GB RAM. Default CPLEX parameters are used except for MIP gap tolerance parameter.

5. RESULTS AND ANALYSIS

5.1 Base Test Performance (BT Results)

Following the BT parameters defined in Section 4.3, the model reported optimal monthly inventory policy demonstrates distinct seasonal patterns. The upper stock level (US) peaks at 750 tons in month 3 of each year, with the corresponding lower stock level (LS) at 250 tons, representing strategic preparation for the April-May no harvesting period. During the other stable willow supply months (1, 4-11), the policy maintains moderate levels with US around 135 tons and LS around 30 tons, indicating efficient just-in-time operations.

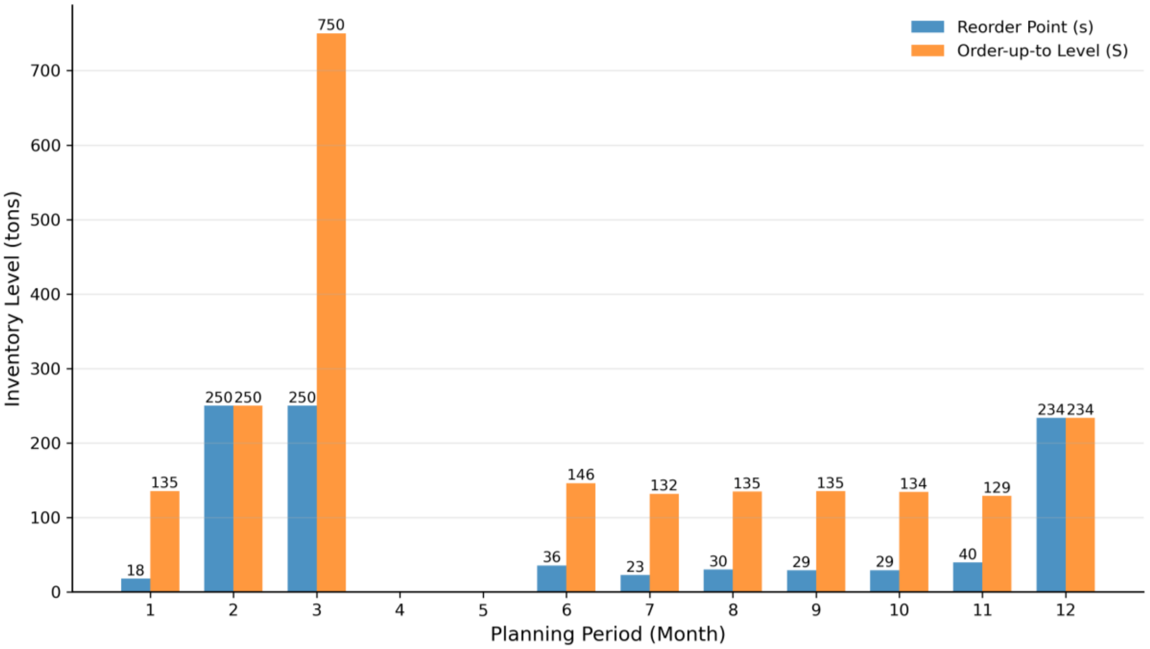


Figure 6: The optimal monthly inventory policy for base test.

The raw material inventory buildup in month 3 reflects the model's anticipation of seasonal willow supply disruption. Increasing the daily raw material storage level up to 750 tons (Figure 6) also helps increase pellet production and storage to meet the pellet demand from months 4 and 5 when willow cannot be harvested. The convergence of US and LS levels to 234 tons in month 12 results from the sustainable operation constraint (see Equation 9.1 and 9.2), which requires the end-of-year willow inventory levels to be no less than the initial willow inventory levels at the beginning of a year to ensure the pellet mill can operate sustainably between two consecutive years.

The finished wood pellet product inventory also follows a clear annual cycle, peaking at approximately 5000 tons by the end of the spring accumulation period in both internal and external storages (Figure 7). When entering the no harvesting season in month 4 and 5, the pellet mill will utilize both the raw willow material storage and the final pellet product storage to meet the pellet demand before it could acquire new raw willow materials at month 6. Benefit from those extra accumulated storages, we found there is no stockout across all pellet demand scenarios when the mill needs to meet the assumed pellet demands.

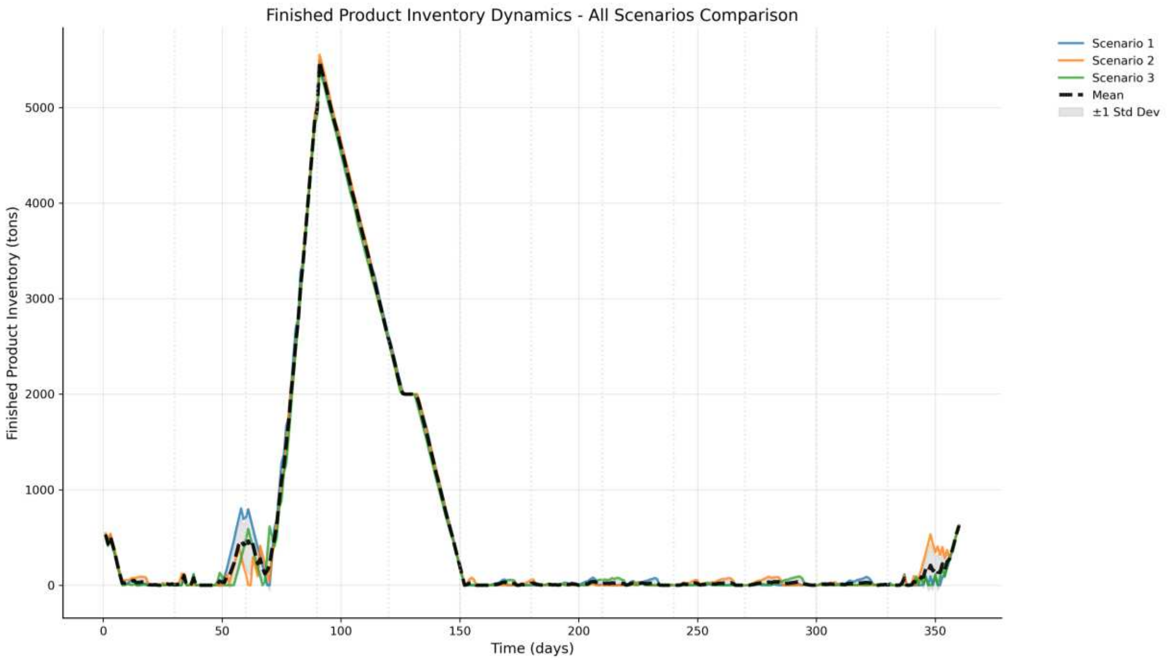


Figure 7: The finished product inventory for base test.

5.2 Inventory Policy Adjustment Frequency Analysis (PAFT Results)

The Inventory Policy Adjustment Frequency Analysis (PAFT) results, aggregated across 50 independent runs as described in Section 4.3, reveal significant cost variations across different inventory policy adjustment frequencies. Allowing the most frequent daily adjustments achieves the lowest mean total inventory management costs at 2.28 million USD per year, while monthly adjustments result in the highest costs at 2.31 million USD. Weekly adjustments provide an intermediate total annual cost of 2.30 million USD.

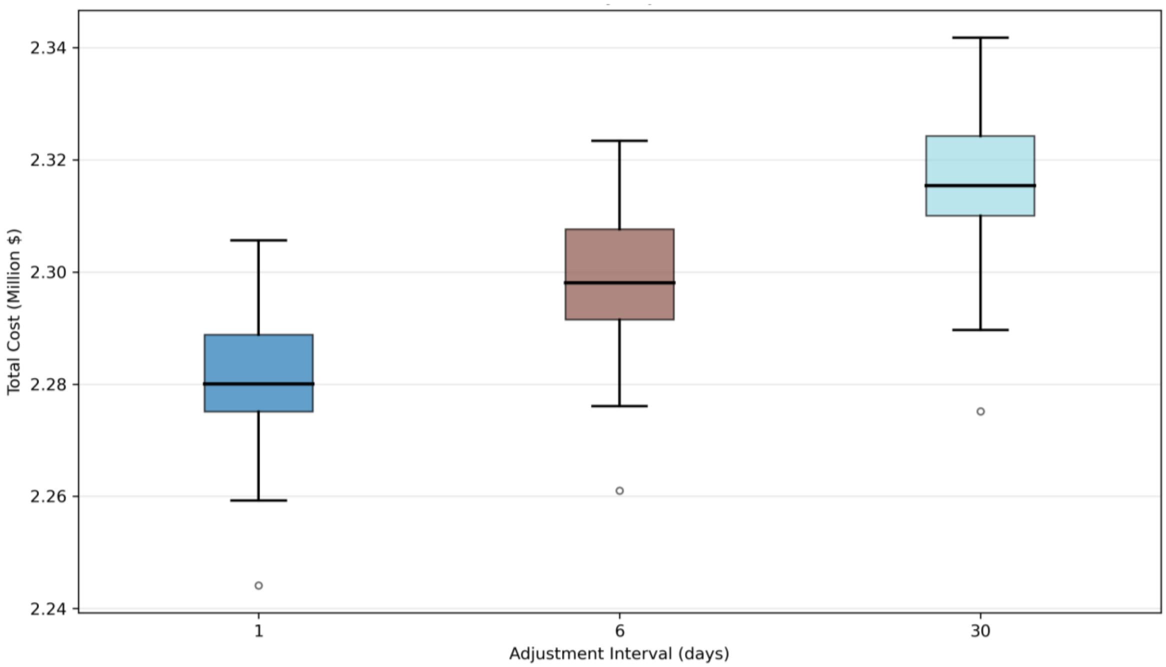


Figure 8: Objective value distribution across different inventory adjustment intervals.

5.3 Stochastic vs Deterministic Comparison (MST Results)

Following the two-phase evaluation framework described in Section 4.3, we test and compare the performance of stochastic optimization approaches versus the deterministic approach in configuring monthly inventory policies. The box plot distribution compares the total annual costs by implementing inventory policies derived from different sized training samples to reflect the stochastic fluctuation of pellet demands (1-5) on top of the assumed 100 tons/day constant demand. Those policies were evaluated and compared on independent testing scenarios that were not used during the policy training and optimization process.

The deterministic approach (ω=1) shows the highest mean objective value at approximately 2.32 million USD with considerable variance, indicating higher total cost and poorer robustness to demand uncertainty. As the number of training scenarios increases from 2 to 5, mean costs decrease. Policies trained on 3-5 scenarios achieve mean costs around 2.29-2.30 million USD, representing approximately 1% cost decrease through stochastic modeling approach over the deterministic approach.

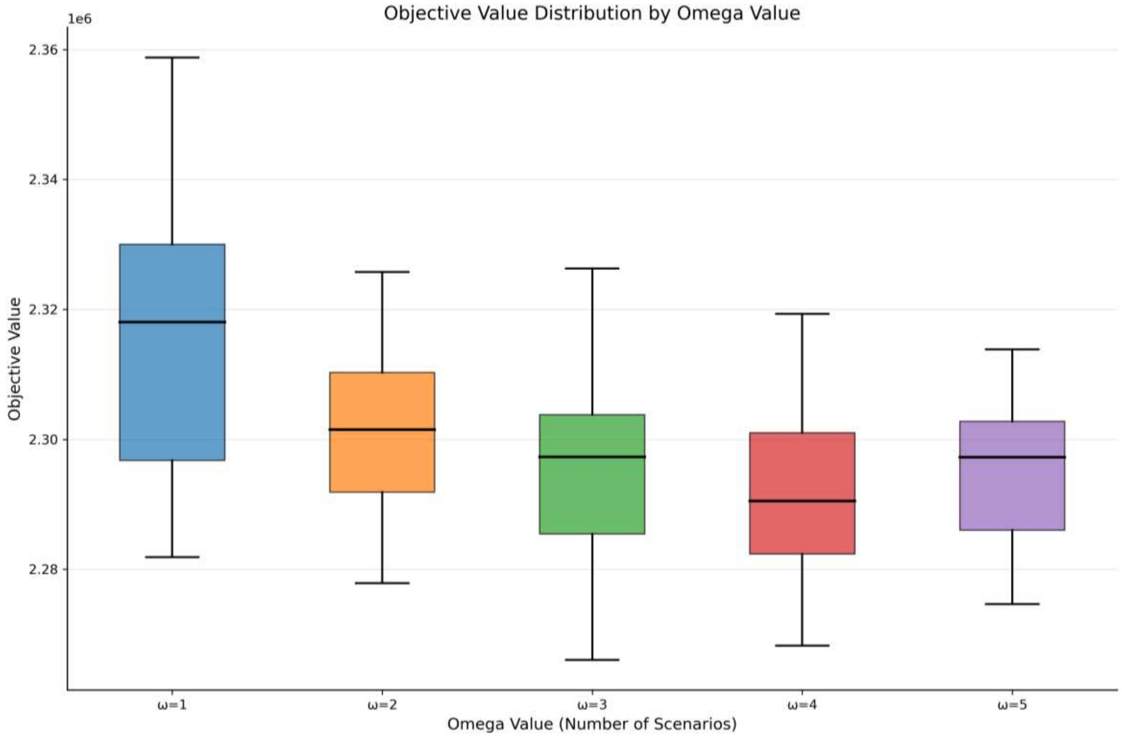


Figure 9: Objective value distribution across different demand scenarios.

Beyond mean cost comparison, the stochastic approach demonstrates substantial advantages in operational stability. The deterministic policy (ω=1) exhibits a standard deviation of \$39,892, while stochastic policies (ω=3-5) maintain standard deviations around \$17,000, representing 55% reduction in coefficient of variation. This reduced variance translates directly to more predictable operations and lower financial risk for facility managers.

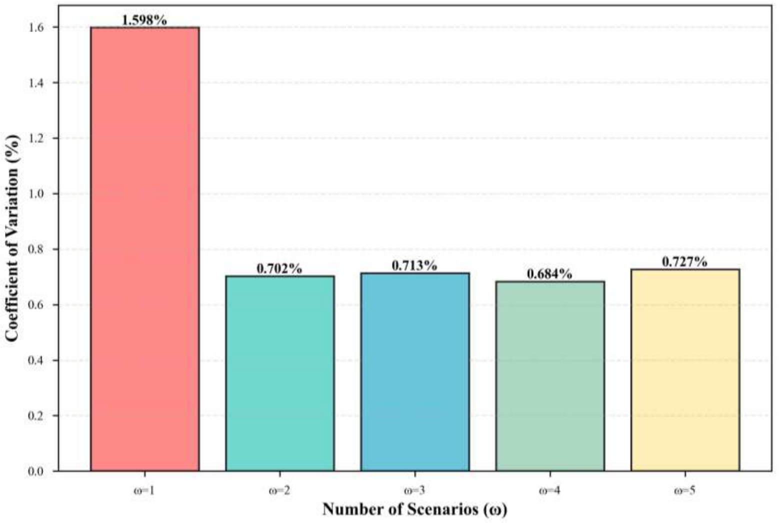


Figure 10: Coefficient of variation by scenario count.

5.4 Seasonal Market Integration (SDT Results)

Wood pellet demand can be classified into industrial (mainly electricity generation) and non-industrial (mainly residential heating) markets (Flinkman et al., 2018; U.S. Energy Information Administration, 2025). Since specific wood pellet seasonal demand and price data were not available for our study, we used monthly densified biomass fuel sales data from the EIA as a proxy, given that wood pellets constitute the primary component of densified biomass fuel products (U.S. Energy Information Administration, 2025).

Under the empirical market conditions with seasonal demand variations shown in Figure 5, the optimal inventory policy (Figure 12) demonstrates adaptive behavior throughout the year. The policy shows high inventory targets during month 3 (US=800 tons) in preparation for the April-May harvesting restriction, similar to the base test. However, months 6-11 exhibit more dynamic adjustments compared to the base test, with inventory levels responding to the seasonal demand patterns. Notably, months 8-10 show elevated inventory levels (US ranging from 319-350 tons) corresponding to the pre-winter demand surge identified in the market data.

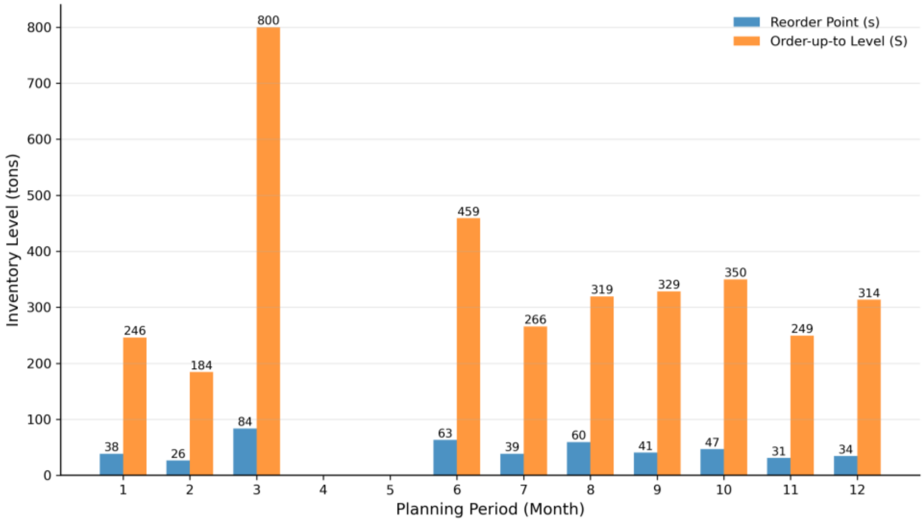


Figure 11: Optimal monthly inventory policy under real world demand and price trend.

The finished product inventory (Figure 13) fluctuates much more than the base test because of the fluctuation in demand. Despite these fluctuations, it maintains a similar annual cycle as the base test, achieving zero stockouts across all 12 months of production, demonstrating the robustness of the optimized inventory policy under more realistic market conditions.

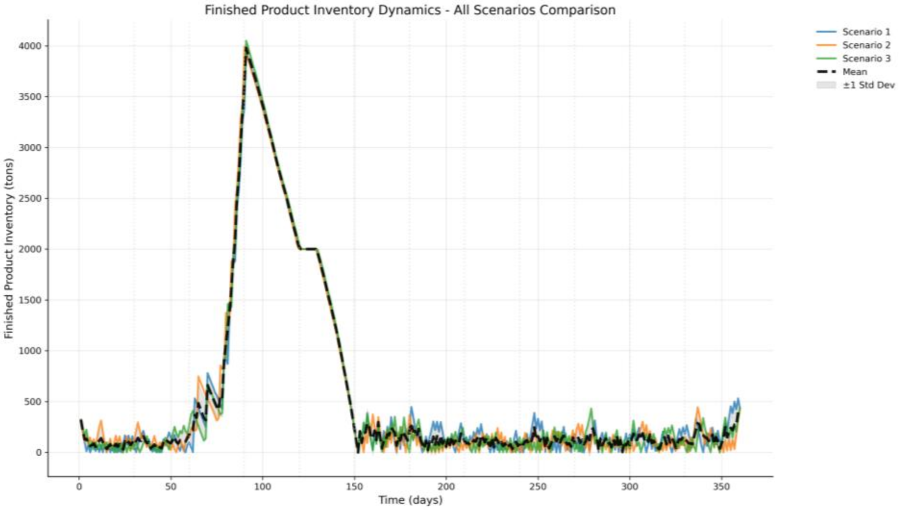


Figure 12: The finished product inventory under real world demand and price trend.

5.5 Multi Feedstock Benefits (MFT Results)

By using the MFT parameters defined in Section 4.3, we conducted multi-feedstock analysis, which demonstrates certain benefit in biomass supply diversification. Figure 14 shows optimal procurement strategy utilizing both willow and forest residue, with forest residue (priced at \$70/ton with \$100 fixed cost) serving as a supplementary source during willow's no-harvesting periods (months 4-5). Because we set the forest residual price to be higher than the willow raw material price in our tests, we can observe that the mill will only use forest residual for pellet production during the non-willow-harvesting season.

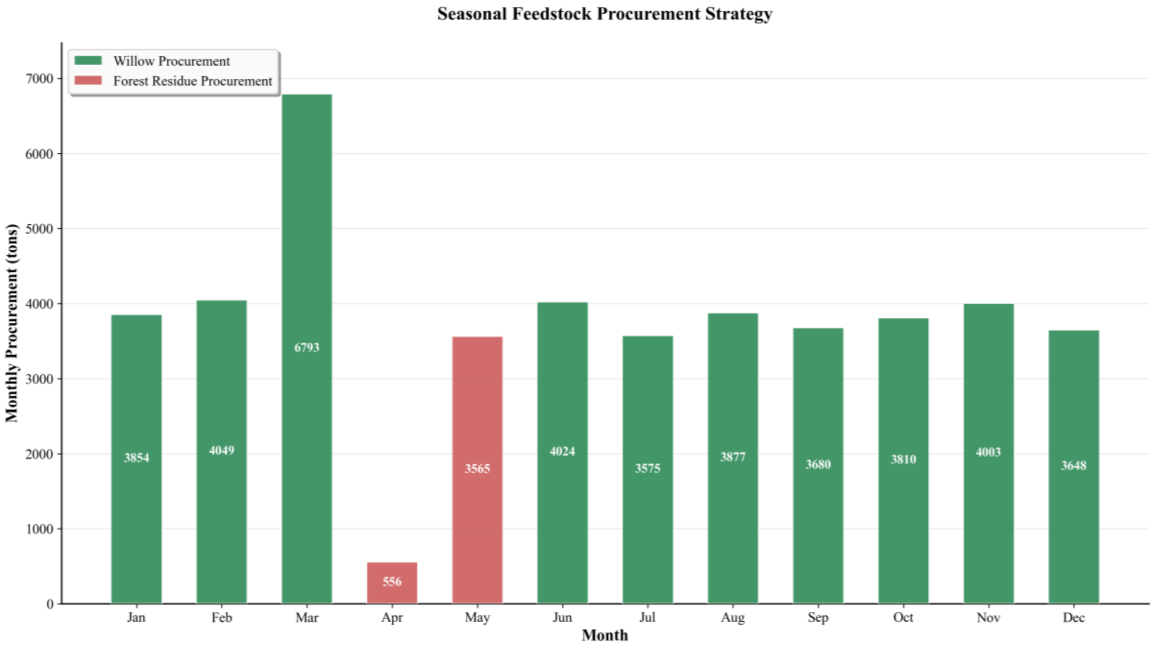


Figure 13: Monthly biomass procurement under multi-feedstock.

The corresponding optimal inventory policy (Figure 15) exhibits more moderate seasonal variations compared to single willow-based feedstock operations, as supply diversification reduces extreme inventory requirements before willow's non-harvesting season.

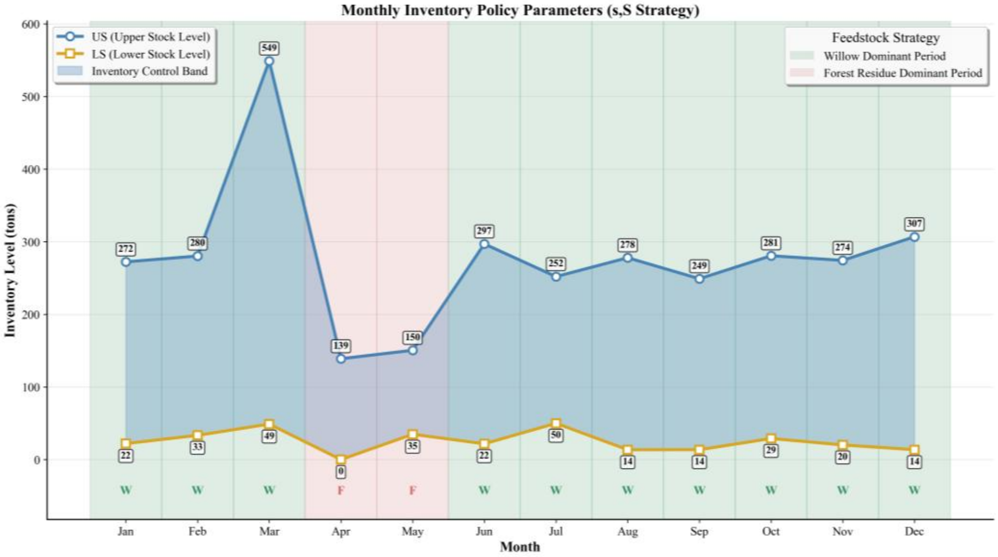


Figure 14: The optimal inventory policy for multi-feedstock.

Note: The upper stock level and lower stock level are calculated based on the total biomass amount of willow and forest residual.

5.6 Computational Performance Analysis (CPT Results)

We test the trade-off between the MILP’s solution accuracy reported by computing time and the relative gap between best solution found and the best possible solution (Figure 16) using the CPT settings in Section 4.3. The analysis demonstrates substantial increase in computational time as the MIP gap tolerance tightens. If we set the MIP gaps at 0.20% or higher, solution times remain at approximately 100-200 seconds by integrating two stochastic demand scenarios into the second stage decision. An inflection point can be found at 0.15% MIP gap, where solution time is approximately 260 seconds. However, tightening the gap further to 0.11% results in solution times exceeding 30,000 seconds, representing more than a 100-fold increase for minor MIP accuracy improvements. Based on the CPT analysis, a MIP gap tolerance of 0.15% for our model with three stochastic demand-scenarios provides a good balance for practical implementation, enabling reasonable solution times while maintaining adequate precision for operational decision-making in willow-based pellet facility management.

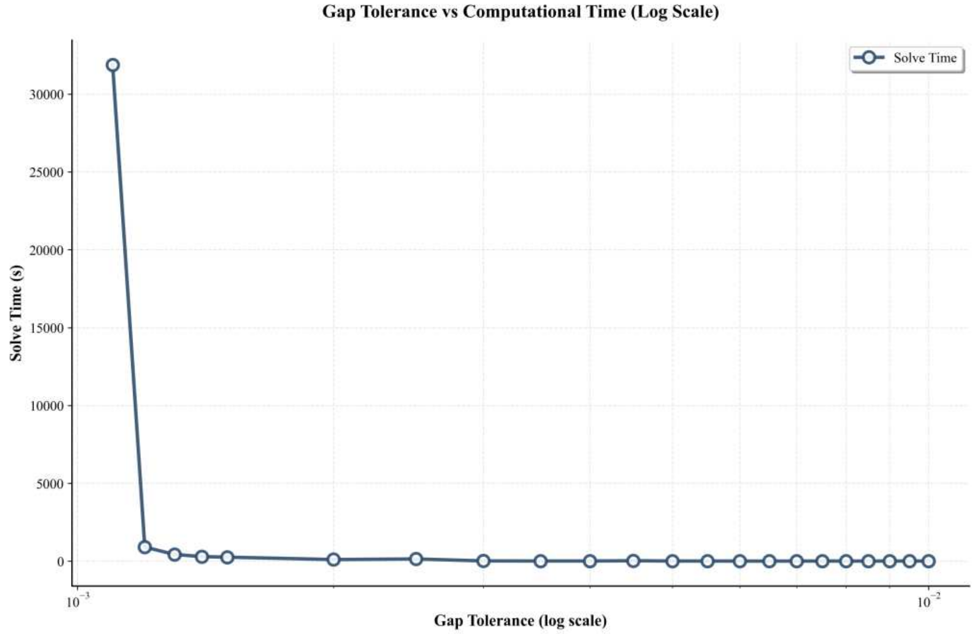


Figure 15: Solve time across different gap tolerances.

6. DISCUSSION

This study aims to demonstrate that stochastic optimization approaches such as two-stage MILP models could better address the unique challenges faced by pellet producers due to seasonal fluctuations of raw material supplies and uncertain pellet demand through proactive planning of raw biomass material inventory management. We created multiple hypothetical test cases by referencing published willow harvesting and supply research. Our test cases approximate pellet facilities' storage and production processes and utilizes the available pellet market demand and price data published for the eastern US. Instead of modeling an entire woody biomass pellet supply chain, we select an important segment of the supply chain, e.g., raw material acquisition, transportation, storage, pellet production and product storage to test our inventory management model. It is worth noting that our model assumes sustained market demand with moderate fluctuation, though the developing North American wood pellet market is substantially influenced by export policies such as EU renewable energy directives, and minimum demand thresholds for economic viability remain an important consideration for future market entry analysis.

Inventory management is one of many important decisions a pellet production facility needs to make to support continuous facility operation and meet the end market pellet demand. In general, keeping a higher level of raw material inventory would help a facility better adapt to seasonal raw material supply and the fluctuation of pellet demands and pellet market price. However, maintaining a higher inventory level would also require more storage space and increased storage costs along with the other risk of raw material quality degrading with longer storage time. An intuitive strategy to overcome the off-season willow supply shortage is to increase both the willow raw material inventory level and the end pellet product storage level before the start of the non-harvesting season and higher pellet demand seasons. Our model results configured a set of upper and lower bound raw willow material inventory levels for each predefined adjustment period (day, week, month). Those results not only are consistent with intuitive and logical solutions but also provide quantified and time specific management suggestions. However, the choice of adjustment frequency involves practical trade-offs beyond pure cost optimization. While daily adjustments achieve the lowest inventory management costs in our model ($2.28 million annually), the practical challenge of implementing daily policy adjustments may outweigh theoretical cost saving compared to longer adjustment periods ($2.30 million for weekly adjustment). Real world implementation must account for supplier coordination costs, management capacity constraints, and transaction overhead not captured in our current optimization framework. Facility managers may need to balance between cost efficiency and operational complexity when deciding the inventory policy adjustment frequency.

Deterministic models are ideal in finding the optimal inventory management decisions when a supply chain (or a section of a supply chain) possesses a higher level of certainty. However, this is not the situation for most biomass supply chains. Designing stochastic models to capture both the patterns and uncertainties in willow-based pellet supply chain helps managers gain insights and confidence in dealing with those potential uncertainties. Our stochastic modeling framework demonstrates its ability to find a set of inventory management policies to prevent pellet stockout while reducing cost variability compared to deterministic methods. Under moderate demand uncertainty (CV=10%), the stochastic approach achieved approximately 3% mean cost improvement, but more significantly, delivered over 55% reduction in coefficient of variation, which reduces cost uncertainty range from $148,391 (deterministic) to approximately $70,000 (stochastic). This variance reduction provides tangible value through more predictable budgeting and lower financial risk exposure. Our tests highlight the value of explicitly incorporating uncertainties into inventory planning. In real-world management conditions, we anticipate there will be many different types of intermixed uncertainties influencing the facility's inventory management and other operations, including machine maintenance, breakdown and repair, raw material price or transportation cost variations, pellet price change, the variation of the products' quality, and other sources of uncertainty. Similar modeling methods could be developed to incorporate the influence of such uncertainties into the decision-making process.

Quantifying the joint distributions of those uncertainties and integrating them into a stochastic programming model is a challenging task and requires future studies. It is important to note that our observed benefits, cost improvement and variance reduction, are based on hypothetical moderate demand uncertainty conditions. Real-world pellet markets may encounter higher volatility driven by weather extremes affecting heating demand, customer variability, market dynamics and regulatory changes. The stochastic optimization demonstrates that performance benefits would likely scale with uncertainty levels, suggesting that facilities facing greater real-world volatility would likely experience larger advantages from adopting stochastic inventory optimization frameworks.

Seasonality commonly exists in many biomass raw materials' supply, including willow, switchgrass, and many agricultural products. Different types of storage facilities could be built or rented to store raw materials to prepare for the non-harvesting season. This study only assumed in-facility storage for willow biomass and both in-facility and out-facility storage for pellet products for demonstration purposes. Inventory management policy is only implemented for in-facility raw material storage in this model. The seasonal inventory patterns revealed by the model provide insights for facility managers facing similar operational constraints. Our results highlight that seasonal biomass availability represents a critical bottleneck in this supply chain.

Because of the positive externalities associated with the willow biomass supply chain, such as carbon sequestration, improved soil productivity, and the restoration of abandoned sites etc., government subsidies or tax incentives are commonly allocated to this industry. Our model can help identify potential financial bottlenecks within the willow biomass supply chain and guide the more effective allocation of these incentives. Targeted policy incentives addressing specific constraints, such as storage infrastructure financing or harvest season labor support, may be more effective than broad-based subsidies in supporting biomass industry development. However, we acknowledge the real-world situation would likely to be more complicated than our model currently considered. Expanding our model to cover longer supply chains with more "nodes" and "arcs" would be important for future modeling research in this field.

The willow supply availability patterns used in this study employ county level data, while more detailed research of New York State land availability and biomass production potential for willow has been conducted recently (Hossain and Volk, 2024). Integration with such spatially explicit data could enable more granular inventory management optimization that accounts for detailed feedstock land availability. However, expanding from our current model framework to account for hundreds of individual supply locations would significantly increase model complexity, likely requiring decomposition algorithms or heuristic approaches to maintain computational tractability. Another limitation of the current model from the willow supply side is that it does not explicitly consider supply agreements between growers and producers or between producers and consumers, which could significantly alter risk distribution and optimal inventory strategies in practice.

Our fixed biomass-to-pellet conversion ratio represents a necessary simplification, as recent work has shown that conversion ratios can be complex and variable, and affected by many factors like dry matter loss during storage (Therasme et al., 2020). Additionally, our economic optimization does not quantify environmental externalities such as carbon sequestration and ecosystem services from willow cultivation, which could be valued through future life cycle assessments. While these modeling choices enable computational efficiency, they point toward areas where future research could enhance realism.

The two-feedstock tests, using both willow and forest residues, demonstrate the benefits from using forest residues to substitute willow during the non-harvesting season of willow, which includes lowering the cost of storing a large amount of willow raw material before the non-harvesting season, and potentially decreased total operational cost of the mill. This multi-feedstock strategy also enhances supply chain resilience by diversifying biomass sources and buffering against supply disruptions or demand volatility. However, multi-feedstock operation is more complicated and may create more challenges in the mill’s quality control. For example, different feedstock types may require separate processing procedures to meet industry standards such as ENplus certification requirements discussed in Section 2.4. Additionally, multi-feedstock operations may require more sophisticated processing equipment capable of handling varying feedstocks, with their varying moisture contents, particle sizes, and material flow characteristics, potentially increasing both capital investment and maintenance costs compared to single-feedstock operations.

7. CONCLUSION

This research contributes a stochastic MILP framework for optimizing inventory management in willow-based pellet production facilities, addressing the gap in current biomass supply chain literature. The two-stage stochastic programming approach integrates seasonal supply constraints with demand uncertainty, providing inventory policies that outperform traditional deterministic methods.

The results provide operational indication for the biomass energy sector. Compared to deterministic methods, the stochastic framework reduced cost variability by over 55% under moderate demand uncertainty, with variance reduction providing more operational value than the mean cost improvement. These benefits are expected to scale upward under higher real-world uncertainty conditions. The strategic inventory accumulation patterns for offsetting seasonal supply gaps highlight the high requirement of willow-based pellet system operation, while the multi-feedstock analysis demonstrates the advantage and necessity of supply diversification, as it reduced both costs and stockout risks. The framework’s ability to maintain low stockout levels while minimizing costs across varied demand scenarios validates the value of stochastic optimization for this application domain, subject to changing conditions and operational realities.

Key practical insights include adjusting the frequency of inventory policy to balance efficiency and complexity, and the substantial benefits through strategic feedstock diversification. This approach maintained zero stockouts across all scenarios under the set of assumptions we adopted, which proves the framework’s potential effectiveness for real-world supply chain operation.

For businesses in the pellet industry, this research provides insights into inventory policy design and strategic planning. For researchers, it establishes the foundation of using stochastic methods in a willow-based pellet production system that can be extended to other seasonal biomass systems. As the biomass energy sector continues to grow in importance for renewable energy transitions, optimization frameworks that address the unique challenges of its supply chain could become more valuable. Future research directions include real-world implementation through industry partnerships, integration of AI driven demand forecasting and dynamic pricing, and development of user-friendly decision support tools.

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9. APPENDIX

Appendix A: Normality validation for demand distribution

To validate the normal distribution assumption for wood pellet demand in our stochastic optimization model (Section 4.2.2), we analyzed 9 years (2016-2024) of monthly densified biomass fuel sales data from the U.S. Energy Information Administration. We tested whether inter-annual variation within each calendar month follows a normal distribution using the Shapiro-Wilk test ($\alpha = 0.05$).

Table A.1: Summary of Shapiro-Wilk Normality Test Results within each calendar month by region.

Region	Quantity Tests Passed	Quantity Testes Pass Rate(rounded)
East	11/12	91.7%
South	11/12	91.7%
West	12/12	100%
U.S. Total	12/12	100%
Overall	46/48	95.8%

Table A.2: Detailed monthly normality test results for East Region (2016-2024).

Month	p-value	Mean (tons)	CV (%)
Jan	0.6572	82,236	36
Feb	0.9181	60,412	27.8
Mar	0.9791	49,434	32.5
Apr	0.5119	52,241	25.2
May	0.3379	63,867	36.7
Jun	0.5597	81,643	25.4
Jul	0.0913	83,596	19.1
Aug	0.0460	116,324	10.8
Sep	0.2101	119,195	10.2
Oct	0.6908	130,792	13.9
Nov	0.2353	110,449	17.4
Dec	0.6651	93,581	9