

**DISSERTATION**  
**FRACTIONAL TRANSPORT OF BED-MATERIAL LOAD**  
**IN SAND-BED CHANNELS**

Submitted by

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WE HEREBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER OUR SUPERVISION BY **BAOSHENG WU** ENTITLED **FRACTIONAL TRANSPORT OF BED-MATERIAL LOAD IN SAND-BED CHANNELS** BE ACCEPTED AS FULFILLING IN PART REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

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**ABSTRACT OF DISSERTATION**  
**FRACTIONAL TRANSPORT OF BED-MATERIAL LOAD**  
**IN SAND-BED CHANNELS**

This dissertation presents a new method for predicting fractional transport rates of bed-material load in sand-bed channels. The proposed method is developed based on the concept of the transport capacity fraction (TCF) approach. The bed-material concentration for a given size fraction is obtained by weighting the bed-material concentration,  $C_i$ , with a transport capacity distribution function,  $P_{ci}$ . The procedure and a detailed example problem showing the use of the proposed method are provided.

Two transport capacity distribution functions are developed. The first function is in terms of relative fall velocity. This function is derived from the unit stream power theory and the concepts of the TCF approach and the bed material fraction (BMF) approach. The second function is in terms of relative diameter. It is derived from the Engelund and Hansen's transport relations and the concepts of the TCF approach and the BMF approach. The sheltering and exposure effects are considered in both functions. The coefficients in both functions were calibrated using 118 sets of flume and field data (891 data points) falling in sand sizes. The formulations using relative diameter is suggested for practical applications because of its simplicity (no need for relative fall velocity computations).

For the computation of bed-material concentrations, the effect of size gradations on

the transport of sediment mixtures is investigated in detail. First, a new relationship is proposed for predicting the median diameter,  $D_{50b}$ , of bed-material load. This equation is developed based on the 118 sets of data used for the development of transport capacity distribution functions plus 280 sets of CSU flume data. Then, the effect of size gradation on the transport of sediment mixtures is demonstrated by the use of Engelund and Hansen's transport function and Yang's unit stream power function. To account for size gradation effects, the newly developed expression for the median diameter,  $D_{50b}$ , is proposed for use as the representative size in bed-material load computations. For the existing bed-material load equations, an equivalent diameter,  $D_e$ , is proposed. This equivalent diameter, which is related to  $D_{50b}$ , is incorporated into the Engelund and Hansen, Ackers and White, and Yang formulas for the computation of bed-material concentrations.

The proposed method is compared with various existing fractional transport methods using 118 sets of measurements (891 data points) and verified using 48 sets of independent data (327 data points). Comparison and verification indicate that the proposed method provides better predictions for fractional bed-material concentrations and size fractions of sediment in transport.

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## LIST OF SYMBOLS

The following symbols are used in the text:

- A = coefficient;
- AGD = average geometric deviation between computed and measured values;
- B = coefficient;
- C, C<sub>1</sub>, C<sub>2</sub> = coefficients;
- C<sub>T</sub> = concentration of total sediment load;
- C<sub>t</sub> = concentration of bed-material load;
- C<sub>i</sub> = concentration of bed-material load corresponding to the size fraction i;
- C<sub>pi</sub> = potential concentration of bed-material load corresponding to the size fraction i, using D<sub>i</sub> as if it exists alone;
- C<sub>tc</sub>, C<sub>tm</sub> = computed and measured bed-material concentrations, respectively;
- C<sub>tcj</sub>, C<sub>tmj</sub> = computed and measured bed-material concentrations, respectively, corresponding to the size fraction i;
- D = diameter of bed material;
- D<sub>A</sub> = scaling size defined by White and Day;
- D<sub>a</sub> = arithmetic mean diameter of sediment mixtures;
- D<sub>w</sub> = wash load limit diameter;
- D<sub>e</sub> = equivalent diameter defined by Eq. (4.6);
- D<sub>gr</sub> = dimensionless grain diameter defined by Ackers and White;
- D<sub>gri</sub> = dimensionless grain diameter corresponding the size fraction i;
- D<sub>i</sub> = representative diameter (geometric mean) of bed material corresponding to the size fraction i;

- $D_m$  = mean diameter of bed material;  
 $D_n$  = scaling size;  
 $D_p$  = particle sizes for which p percent of bed material is finer by dry weight;  
 $D_s$  = representative diameter for suspended load computations;  
 $D_{50t}$  = median diameter of bed-material load;  
 $d$  = average flow depth;  
 $F_{gri}$  = mobility number corresponding to the size fraction i;  
 $F_r$  = Froude number;  
 $f$  = general function;  
 $f'$  = friction factor defined by Engelund and Hansen;  
 $G$  = size gradation coefficient of bed material =  $(D_{84}/D_{50} + D_{50}/D_{16})/2$ ;  
 $g$  = gravitational acceleration;  
 $I$  = coefficient;  
 $I_1, I_2$  = integrals of Einstein's form of the suspended sediment equation;  
 $i$  = size fraction number or data point number in a data set;  
 $J$  = coefficient;  
 $JN$  = total number of data points;  
 $K_d$  = coefficient;  
 $K_e$  = coefficient;  
 $M$  = Kramer's uniformity coefficient for sediment mixtures;  
 $MNE$  = mean normalized error;  
 $m$  = coefficient;  
 $N$  = number of size fractions present in a sediment mixture;  
 $n$  = coefficient;  
 $P_{ai}$  = areal function of bed material defined by Karim;  
 $P_{bi}$  = fraction of bed material, by dry weight, corresponding to the size fraction i;

- $P_{ci}$  = fraction of bed-material transport capacity, by dry weight, corresponding to the size fraction  $i$ ;
- $P_{cci}$  = percentage of computed bed-material load, by dry weight, corresponding to the size fraction  $i$ ;
- $P_{cmi}$  = percentage of measured bed-material load, by dry weight, corresponding to the size fraction  $i$ ;
- $P_{si}$  = fraction of suspended load, by dry weight, corresponding to the size fraction  $i$ ;
- $q_t$  = bed-material load per unit width by dry weight;
- $q_{ti}, q_{bi}, q_{si}$  = bed-material load, bedload, and suspended load, respectively, per unit width by dry weight corresponding to the size fraction  $i$ ;
- $R, R'$  = hydraulic radius and hydraulic radius associated with grain roughness, respectively;
- $R$  = discrepancy ratio between computed and measured bed-material concentrations;
- $R_i$  = discrepancy ratio between computed and measured fractional bed-material concentrations corresponding to the size fraction  $i$ ;
- $S$  = energy slope;
- $s_g$  = specific gravity =  $\gamma_s / \gamma$ ;
- $T$  = transport stage parameter;
- $V$  = flow velocity;
- $V_{cr}$  = critical velocity at incipient motion;
- $V_*$  = shear velocity =  $\sqrt{\tau / \rho} = \sqrt{gRS}$ ;
- $W_i$  = hiding factor introduced by Karim and Kennedy;
- $\alpha, \beta$  = coefficients;
- $\gamma, \gamma_s$  = specific weight of water and sediment, respectively;
- $\Delta$  = apparent roughness of the bed surface;
- $\delta'$  = laminar sublayer thickness due to grain roughness;

$\epsilon_{bi}, \epsilon_{bi}, \epsilon_{si}$  = correction factors for critical shear stresses to account for the sheltering and exposure effect for bed-material load, bedload, and suspended load, respectively;

$\zeta$  = coefficient;

$\eta$  = sheltering parameter;

$\theta$  = dimensionless shear parameter =  $\tau / [(\gamma_s - \gamma)D]$ ;

$\theta_c$  = critical dimensionless shear parameter =  $\tau_c / [(\gamma_s - \gamma)D]$ ;

$\theta_{ci}$  = critical dimensionless shear parameter corresponding to the size fraction  $i$   
=  $\tau_c / [(\gamma_s - \gamma)D_i]$ ;

$\theta_e$  = dimensionless shear parameter using equivalent diameter,  $D_e$ ;

$\theta_i$  = dimensionless shear parameter corresponding to the size fraction  $i$   
=  $\tau / [(\gamma_s - \gamma)D_i]$ ;

$\kappa$  = von Kàrmàn constant;

$\nu$  = kinematic viscosity;

$\xi_i$  = Einstein's sheltering parameter;

$\xi_{bi}, \xi_{bi}, \xi_{si}$  = correction factors for effective shear stresses to account for the sheltering and exposure effect for bed-material load, bedload, and suspended load, respectively;

$\sigma_g$  = geometric standard deviation of bed material =  $\sqrt{D_{84} / D_{16}}$ ;

$\tau, \tau_0$  = shear stress along the bed =  $\gamma R S \approx \gamma d S$ ;

$\tau_c$  = critical shear stress;

$\tau_{ci}$  = critical shear stress corresponding to the size fraction  $i$ ;

$\tau'$  = grain shear stress =  $\gamma R' S$ ;

$\Phi_t$  = dimensionless sediment transport function;

$\Phi_e$  = dimensionless sediment transport function using equivalent diameter,  $D_e$ .

$\Phi_{bi}, \Phi_{bi}, \Phi_{si}$  = dimensionless sediment transport function corresponding to the size fraction  $i$  for bed-material load, bedload, and suspended load, respectively;

- $\phi_i$  = weighting function for  $i$ th fraction of a sediment mixture;
- $\bar{X}$  = scaling size of the sediment mixture defined by Einstein;
- $\omega_e$  = equivalent fall velocity corresponding to the equivalent diameter of  $D_e$ ;
- $\omega_n$  = scaling fall velocity corresponding to the scaling size of  $D_n$ ;
- $\omega_i$  = fall velocity of sediment corresponding to the particle size of  $D_i$ ; and
- $\omega_{50}$  = fall velocity of sediment corresponding to the particle size of  $D_{50}$ ;

# CHAPTER 1

## INTRODUCTION

### 1.1 BACKGROUND

The transport of nonuniform sediment mixtures is more complicated than the transport of uniform sediment because both the condition for initiation of motion of a given size of sediment and its transport rate are affected by the presence of other sizes in the mixtures. The coarser particles on the bed are exposed more to the action of flow past them and act like isolated roughness elements. On the other hand, the finer fractions are sheltered in the wakes of the coarser fractions. These mechanisms cause the entrainment and transport of different size fractions, in the case of nonuniform mixtures, to deviate from the behavior of uniform sediments. Generally the finer fractions are transported at a relatively lower rate than if they were in a uniform sediment bed, and the coarser particles consequently are transported at a higher rate.

Transport of uniform sediments in open channels has been extensively investigated for decades and is reasonably well understood at present. However, the subject of fractional transport of nonuniform sediment mixtures is still very challenging because of its complexity. Starting from Einstein (1950), many attempts including field measurements, laboratory studies, empirical and theoretical analysis, and numerical simulations have been made in the past to understand the mechanisms of nonuniform sediment transport and to predict the bed-

material transport rates by size fractions. However, prediction of sediment transport rates by size fractions has not been accomplished following a purely analytical method. All existing fractional sediment transport methods have been established relying on calibration using limited flume and field data collected under so-called steady uniform flow conditions. When different methods were applied to a given river, computed results of fractional transport rates could vary drastically from each other and from actual measurements.

In alluvial river simulation models, computation of sediment transport rates for individual size fractions is one of the key elements in the case of nonuniform sediment mixtures (Wu and Molinas, 1996). Especially for those models involved in the simulation of the change of bed material composition, sediment sorting processes, and bed armoring, accurate prediction of fractional transport rates is essential for their successful implementation in natural rivers. Unfortunately, none of the existing methods satisfactorily predicts fractional transport rates of nonuniform sediment mixtures in open channels. It is of practical importance to develop a reliable prediction method of fractional transport rate for the implementation of numerical models in more sophisticated problems encountered in natural rivers.

## **1.2 OBJECTIVES**

The primary objective of this study is to investigate the mechanics of fractional transport and to present a procedure for predicting the fractional transport rate of bed-material sediments in sand-bed channels. The proposed methodology should be theoretically sound and practically applicable. The study also includes a comprehensive analysis of the differences and relationships between the transported sediment size composition and the bed material size

composition, and an intensive investigation of the effect of size gradations on the bed-material transport of sediment mixtures.

### 1.3 SCOPE OF WORK

#### *Bed-Material Load Concept*

There are two common classifications of total sediment load in a stream, as shown in Fig. 1.1. By the type of movement, the total sediment load can be divided into bedload and suspended load; by the source of sediment, the total sediment load is separated into the supply-limited wash load and the capacity-limited bed-material load (Julien, 1995). Bedload refers to the transport of sediment particles that frequently maintain contact with the bed. Suspended load, by definition, moves in suspension. Wash load refers to the finest portion of sediment not found with a significant amount in the bed, for which sediment transport is limited by the upstream supply of fine particles and is generally not correlated with the hydraulic

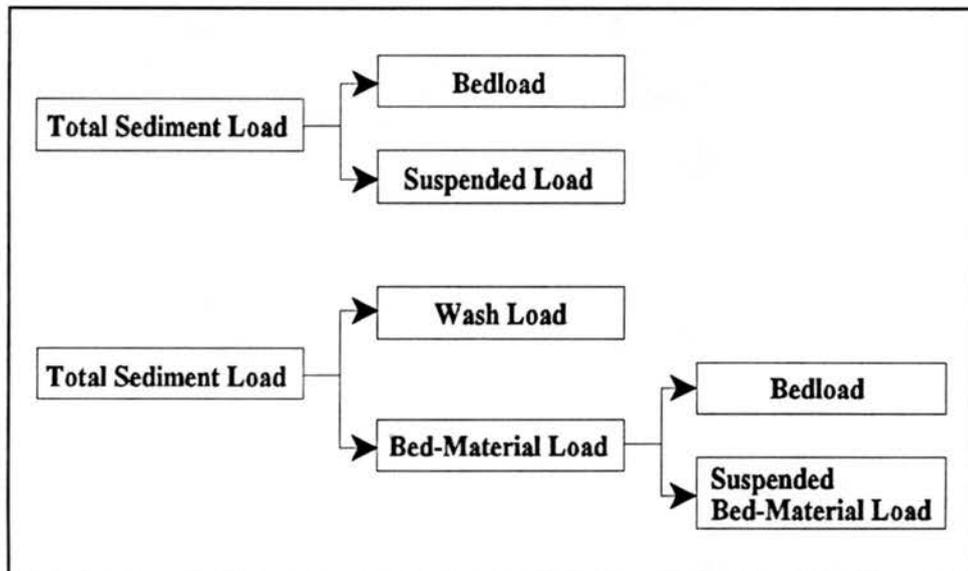


Fig. 1.1. Classification of Sediment Transport in Open Channels.

characteristics of a river. The bed-material load consists of particles generally found in the bed and can be predicted by the transport capacity of the stream. Bed-material load can also be defined as the sum of bedload and suspended bed-material load.

In flume experiments (except in special cases), wash load is almost invariably absent, and total sediment load would be the bed-material load. On the other hand, in natural rivers, wash load is invariably present, and the total sediment load is the summation of the bed-material load and the wash load. In these cases, wash load should be subtracted from the measurements for the analysis and comparison of bed-material load.

A sediment particle may be transported as bedload at one time and as suspended load at another time or location. Considering the dynamics of sediment movement, the process of suspension may be visualized as an advanced stage of traction along the bed; therefore the total sediment transport rate should be related primarily to the shear parameter, and no distinction needs to be made between bedload and suspended load (Garde and Ranga Raju, 1985). Raudkivi (1990) argued that once the suspension phase of transport has developed, the distinction is less meaningful, although there would still be particles which roll and slide on the bed as bedload. In the case of nonuniform sediments, the finer sizes of the bed material may move predominantly in suspension, while the coarser fractions of the bed material may move mostly as bedload. Distinction between bedload and suspended load becomes more difficult and unnecessary for nonuniform sediment mixtures. In practice, we are interested in the bed-material load, not in how much is transported in which mode. Therefore, the bed-material load, without dividing it into bedload and suspended bed-material load, will be considered for the determination of sediment transport capacity in natural rivers.

### *Sediment Size*

The size of sediment particles is one of the most important physical properties in transportation. Sediment sizes, along with the flow conditions, determine the manner of sediment movement (such as the fall velocity, cohesion, initiation of motion of particles, mode of sediment movement, and so on) in a stream. Generally in mountain streams with gravel-bed, sediments are mostly transported as bedload, while in alluvial rivers with sand-bed, sediments are mainly transported as suspended load. This study focuses on the fractional transport of sediment mixtures in the sand range without cohesive effect. Most of the nonuniform sediment transport experiments which included the measurement of both bed material size distribution and bed-material load size distribution were conducted in sand-bed flumes. These experimental data are necessary for the analysis and development of a transport equation in the sand range.

### *Transport capacity*

The sediment transport capacity is defined as the amount of sediment that is transported by a stream in equilibrium conditions for given conditions of flow and sediment. Equilibrium conditions refer to a flow for which neither erosion nor deposition occurs along the channel. Equilibrium of sediment transport is a result of the balance between the transport capacity of a flow and the sediment load carried by the flow, which can be only achieved under constant sediment supply and uniform flow or gradually-varied flow conditions. Sediment transport rate in a river is not necessarily equal to the transport capacity of the flow. If the supply of sediment is larger than the transport capacity, aggradation occurs. Conversely, if in a sand-bed river the supply of sediment is less than the transport capacity, degradation and associated

fluvial processes will alter the channel until a new equilibrium condition is achieved. Throughout this study, we will consider the sediment transport capacity or sediment transport rate in equilibrium conditions, not the sediment transport during the processes of aggradation or degradation.

#### **1.4 DISSERTATION OUTLINE**

This dissertation includes six chapters. Chapter 1 presents an introduction to the dissertation, including the background, objectives, and scope of work. Chapter 2 comprises a comprehensive review of the available literature addressing the problem and theories under investigation.

Chapter 3 presents the development of new transport capacity distribution functions, including their theoretical derivation, physical consideration, and calibration.

Chapter 4 discusses the variation of sediment sizes in transport, the effect of size gradation on the transport of sediment mixtures, and the use of a representative diameter in bed-material load predictions.

Chapter 5 tests the proposed fractional bed-material load computation methodology through comparison with other fractional load computation methods based on flume and field data.

Finally, Chapter 6 summarizes the main results and conclusions drawn out of the study coupled with recommendations.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 GENERAL

In order to predict the response of mixed-size sediment to flow (initiation of motion, hydraulic sorting, bed armoring, evolution of stream bed), it is necessary to predict the transport rates of the individual size fractions in the mixture, which is essential in numerical models such as HEC-6 (U.S. Corps of Engineers, 1977; Thomas, 1982), GSTARS (Molinas and Yang, 1986), BRI-STARS (Molinas, 1990, Molinas and Trent, 1991), IALLUVIAL (Karim and Kennedy, 1982; Dorrough, Holly, and Wei, 1988), CHARIMA and SEDICOUP (Holly, 1988), and other sediment transport models (Han, 1973, 1980; Ribberink, 1987; Wu, 1992b; Zhang and Wu, 1993; and Qu et al., 1994). Recently, research on the fractional transport of sediment mixtures has become very active due to its practical significance in numerical models to simulate the change of bed composition, hydraulic sorting, and bed armoring of rivers with nonuniform sediment mixtures. In 1991, a seminar on grain sorting was held at the Center Stefano Franscini on Monte Verità in Ascona, Switzerland (Vischer, 1992), which presented the progress and further needs in research concerning transport process of individual size fractions and the hiding and exposure effect between different sizes.

Fractional transport of nonuniform sediment mixtures has intrigued scientists for decades. Einstein (1950) and Einstein and Chien (1953) started the study of fractional

transport of nonuniform sediment mixture in the early fifties. Following Einstein, many attempts including field observations, laboratory studies, empirical and theoretical analysis, and numerical simulations have been made to attempt to understand the mechanisms of transport process for sediment mixtures and to predict the transport rates for individual size fractions. The research has covered the fractional transport processes of both sand- and gravel-bed materials. Even though the current research of fractional load transport concentrates on the sand-bed materials, relevant works on gravel-bed materials will be also discussed in this literature review.

## 2.2 CLASSIFICATION

From the theoretical point of view, and based on the treatment in formulations and the physical considerations in the development, the extensive literature on fractional sediment transport can be classified into four categories (Wu and Molinas, 1996):

- Direct computation by the size fraction approach;
- Shear stress correction approach;
- Bed Material Fraction approach (BMF); and
- Transport Capacity Fraction approach (TCF).

### *Direct computation by the size fraction approach*

Direct computation by the size fraction approach aims at computing sediment transport rates for each size fraction present in nonuniform mixtures. After the computation of transport capacities corresponding to each size group, the bed-material load is calculated by the summation of fractional sediment transport rates from

$$q_t = \sum_{i=1}^N q_{ti} \quad \text{or} \quad C_t = \sum_{i=1}^N C_{ti} \quad (2.1)$$

in which  $q_t$  = bed-material load per unit width by dry weight;  $C_t$  = concentration of bed-material load;  $N$  = number of size fractions present in the sediment mixture; and subscript  $i$  denotes the size fraction number in a mixture.

### *Shear stress correction approach*

The shear stress correction approach focuses on extending a uniform sediment transport formula or a bed-material transport rate formula to fractional transport rate for nonuniform sediment mixtures. In doing so, the actual shear stresses acting on each size fraction or the critical shear stresses for each size fraction are corrected by introducing a correction factor. This approach may be written as

$$\Phi_{ti} = P_{bi} f(\xi_{ti} \theta_i), \quad q_t = \sum_{i=1}^N q_{ti} \quad (2.2)$$

or

$$\Phi_{ti} = P_{bi} f(\theta_i - \varepsilon_{ti} \theta_{ci}), \quad q_t = \sum_{i=1}^N q_{ti} \quad (2.3)$$

in which  $f$  = general functional relationship;  $P_{bi}$  = fraction of bed material by dry weight corresponding to size fraction  $i$ ;  $\Phi_{ti}$  = dimensionless sediment transport function corresponding to size fraction  $i$ ;  $\theta_i$  = dimensionless shear stress corresponding to size fraction  $i$ ;  $\theta_{ci}$  = critical dimensionless shear stress corresponding to size fraction  $i$ ; and  $\xi_{ti}$ ,  $\varepsilon_{ti}$  = correction factors accounting for the sheltering and exposure effect. Parameters of  $\Phi_{ti}$ ,

$\theta_i$ , and  $\theta_{ci}$  are expressed as follows, respectively

$$\Phi_{ii} = \frac{q_{ii}}{\gamma_s \sqrt{\frac{\gamma_s - \gamma}{\gamma} g D_i^3}} \quad (2.4)$$

$$\theta_i = \frac{\tau}{(\gamma_s - \gamma) D_i} \quad (2.5)$$

$$\theta_{ci} = \frac{\tau_{ci}}{(\gamma_s - \gamma) D_i} \quad (2.6)$$

in which  $D_i$  = the representative diameter of bed material corresponding to size fraction  $i$ ;  $g$  = gravitational acceleration;  $\gamma$ ,  $\gamma_s$  = specific weight of water and sediment, respectively;  $\tau$  = shear stress along the bed; and  $\tau_{ci}$  = critical shear stress along the bed corresponding to size fraction  $i$ . Note that the dimensionless sediment transport functions for bedload,  $\Phi_{bi}$ , and suspended load,  $\Phi_{si}$ , are obtained by replacing  $q_{ii}$  in Eq. (2.4) with  $q_{bi}$  or  $q_{si}$ , respectively. The corresponding correction factors for bedload and suspended load are referred to as  $\xi_{bi}$  and  $\xi_{si}$  in Eq. (2.2) and  $\epsilon_{bi}$  and  $\epsilon_{si}$  in Eq. (2.3), respectively.

The product  $\xi_{ii}\theta_i$  in Eq. (2.2) is believed to be the actual or effective dimensionless shear stress acting on the particles of size fraction  $i$  in a mixture, while  $\epsilon_{ii}\theta_{ci}$  in Eq. (2.3) may be regarded as the effective critical shear parameter. Correction with  $\xi_{ii}$  is to reduce the value of shear stress for the finer fractions and to increase the value for the coarse fractions. Conversely, correction with  $\epsilon_{ii}$  is to increase the value of critical shear stress for the finer fractions and to reduce the value for the coarse fractions. These corrections result in a similar effect, viz. a reduction of the transport rate of the smaller sizes and increase of the transport rate of the larger sizes. There is no reason to suppose  $\xi_{ii} = \epsilon_{ii}$ .

### *The BMF approach*

The BMF approach relates the fractional transport rates directly to the size distribution of bed material. It assumes that a channel bed can be considered as a hypothetical mixture of sediment particles; the mixture can be formed into class intervals by size, and a potential transport capacity can be calculated for each class interval, whether or not particles are physically present. Subsequently, particle availability can be evaluated and expressed as  $P_{bi}$ . Availability and potential transport capacity can then be combined to give transport capacity as follows

$$C_{ti} = P_{bi} C_{pi} , \quad C_t = \sum_{i=1}^N C_{ti} \quad (2.7)$$

in which  $C_{pi}$  = potential concentration for size fraction  $i$  in the case of uniform sediment in identical hydraulic conditions. In using the BMF approach, the potential concentration,  $C_{pi}$ , for a given size fraction  $i$ , is computed with a bed-material load formula by replacing the representative size with the average (or geometric mean) diameter,  $D_i$ , of the corresponding size fraction of the bed material. Conceptually, a stream bed can be considered as a hypothetical mixture of sediment particles, and the mixture can be formed into size groups. If it is assumed that individual size fractions have no influence on each other, then a potential transport rate can be computed for each size fraction whether or not particles are physically present on the bed surface. Consequently,  $P_{bi}$  can be visualized as the availability of sediment particles on the bed surface.

### *The TCF approach*

The TCF approach relates the fractional transport rate to the bed-material transport rate and

the transport capacity distribution function. First, the bed-material sediment concentration is computed by the use of a bed-material load equation. Then, the computed bed-material concentration is broken into fractional concentrations by a transport capacity distribution function. This concept is expressed as

$$C_{ti} = P_{ci} C_t, \quad \sum_{i=1}^N P_{ci} = 1 \quad (2.8)$$

in which  $P_{ci}$  = transport capacity distribution function corresponding to size fraction  $i$ . The TCF approach comprises two components: the computation of bed-material transport capacity,  $C_t$ , and the computation of its fraction,  $P_{ci}$ . The bed-material sediment concentration  $C_t$  can be determined by using any appropriate bed-material transport relationships available in the literature. In essence, the transport capacity distribution function  $P_{ci}$  is the size distribution of the transported sediments and does not necessarily resemble the size distribution of bed material. The availability concept and the sheltering and exposure effect are included in the consideration of  $P_{ci}$  by relating it to both hydraulic conditions and sediment properties.

### 2.3 DIRECT COMPUTATION BY THE SIZE FRACTION APPROACH

Direct computation by the size fraction approach includes methods of Einstein (1950), Laursen (1958), and Toffaleti (1968, 1969), which were originally developed to compute the sediment transport rates by size fractions for nonuniform sediment mixtures. Einstein (1950) presented the most extensive analysis on sediment transport of nonuniform mixtures based on fluid mechanics and probability. The sediment transport computations were made for the individual size fraction that has a representative grain size equal to the geometric mean grain

diameter of each fraction. Einstein recognized the effect of the presence of one size on the transport rate of another in case of nonuniform sediment, and he proposed to account for this effect by introducing a hiding factor. Some fundamental concepts in sediment transport introduced by Einstein were later modified or simplified by others for the computation of sediment transport rate.

In Einstein's approach, the unit bed-material discharge for a given size fraction,  $q_{bi}$ , was expressed as the summation of unit bedload discharge,  $q_{bi}$ , and unit suspended load discharge,  $q_{si}$ , that is,

$$q_{ti} = q_{bi} + q_{si} = q_{bi} (1 + P_E I_1 + I_2) \quad (2.9)$$

in which  $P_E = 2.303 \log(30.2d/\Delta)$  is the transport parameter; and  $I_1$  and  $I_2 =$  integrals of Einstein's form of the suspended sediment equation.

This equation relates the bedload transport to suspended load transport for all size fractions. The effects of other size fractions on the transport rate of a given size are accounted for through the treatments in bedload computation. Einstein's bedload function relates the dimensionless transport function,  $\Phi_{*i}$ , to the flow intensity function,  $\psi_{*i}$ , by the following relationship (Fig. 2.1)

$$1 - \frac{1}{\sqrt{\pi}} \int_{-(1/7)\psi_{*i}^{-2}}^{(1/7)\psi_{*i}^{-2}} e^{-t^2} dt = \frac{43.5 \Phi_{*i}}{1 + 43.5 \Phi_{*i}} \quad (2.10)$$

where

$$\Phi_{*i} = \frac{q_{bi}}{P_{bi} \gamma_s \sqrt{\frac{\gamma_s - \gamma}{\gamma} g D_i^3}} \quad (2.11)$$

$$\psi_{*i} = \xi_i Y \left[ \frac{\log(10.6)}{\log(10.6 \bar{X}/\Delta)} \right]^2 \psi_i \quad (2.12)$$

and

$$\psi_i = \frac{(\gamma_s - \gamma) D_i}{\gamma R' S} \quad (2.13)$$

in which  $D_{65}$  = the bed material size by which 65 percent is finer;  $R'$  = the hydraulic radius associated with grain roughness;  $S$  = channel slope;  $\xi_i$  = correction factor defined by Einstein and given as a function of  $D_i/\bar{X}$  (Fig. 2.2);  $Y$  = the correction factor for the lift coefficient given as a function of  $D_{65}/\delta'$  (Fig. 2.3);  $\bar{X}$  = the characteristic grain size of the mixture, which is given by

$$\bar{X} = \begin{cases} 0.77\Delta & \text{when } \Delta/\delta' > 1.8 \\ 1.39\delta' & \text{when } \Delta/\delta' < 1.8 \end{cases} \quad (2.14)$$

in which  $\Delta$  = the apparent roughness of the bed surface, which equals to  $D_{65}/X$ ;  $\delta'$  = the laminar sublayer thickness due to grain roughness; and  $X$  = a correction factor that accounts for the variation in flow regime in the logarithmic velocity distribution (Fig. 2.4).

Most of the concern has centered around the sheltering function  $\xi$  or hiding factor. In principle, it is intended to account for the difference in mobility of the various grain sizes in the mixture compared to their mobility in beds of respective uniform sized grains. Fig. 2.5 shows a comparison by Misri et al. of the Einstein and Hayashi et al. (1980) functions, and the function proposed by Pemberton (1972), where  $D_a$  is the average grain size of mixture. Misri's experiments on coarse sediments highlighted several important limitations of Einstein's method, including the inadequacy of his  $\xi_i \sim D_i/\bar{X}$ . In general, Einstein's method overpredicts the transport rates of finer sizes and underpredicts the transport rates of coarser

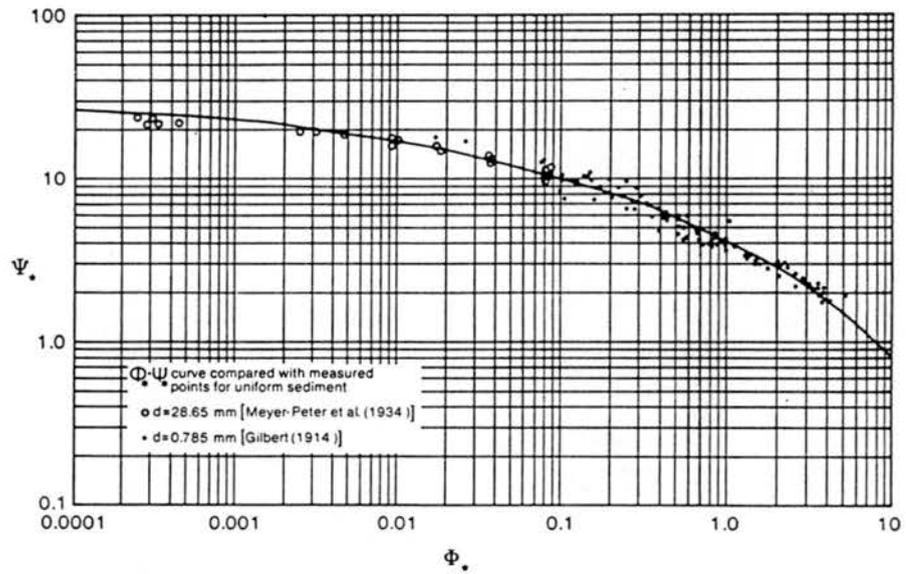


Fig.2.1. Relationship between  $\Phi_*$  and  $\Psi_*$  for Einstein's Bed-Load Function (Einstein, 1950).

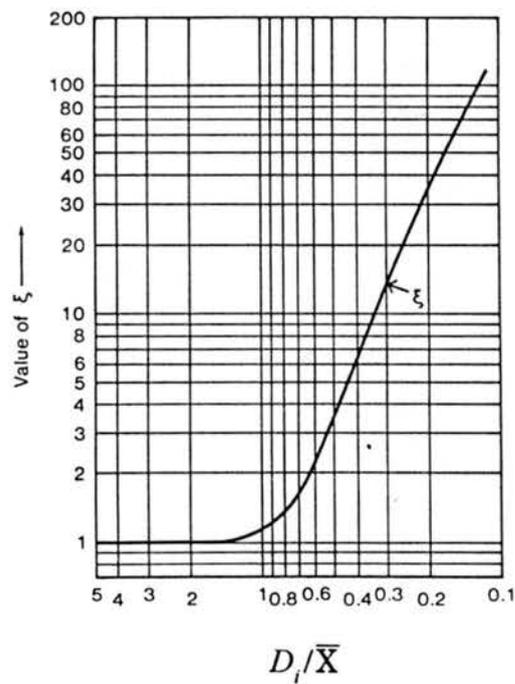


Fig. 2.2. Hiding Factor in Einstein's Bed-Load Function (Einstein, 1950).

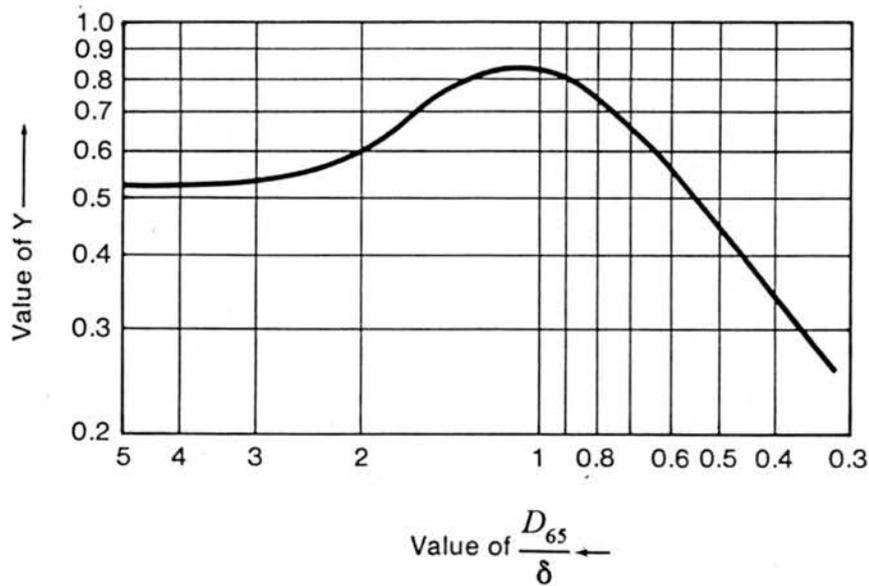


Fig. 2.3. Lifting Correction Factor (Einstein, 1950).

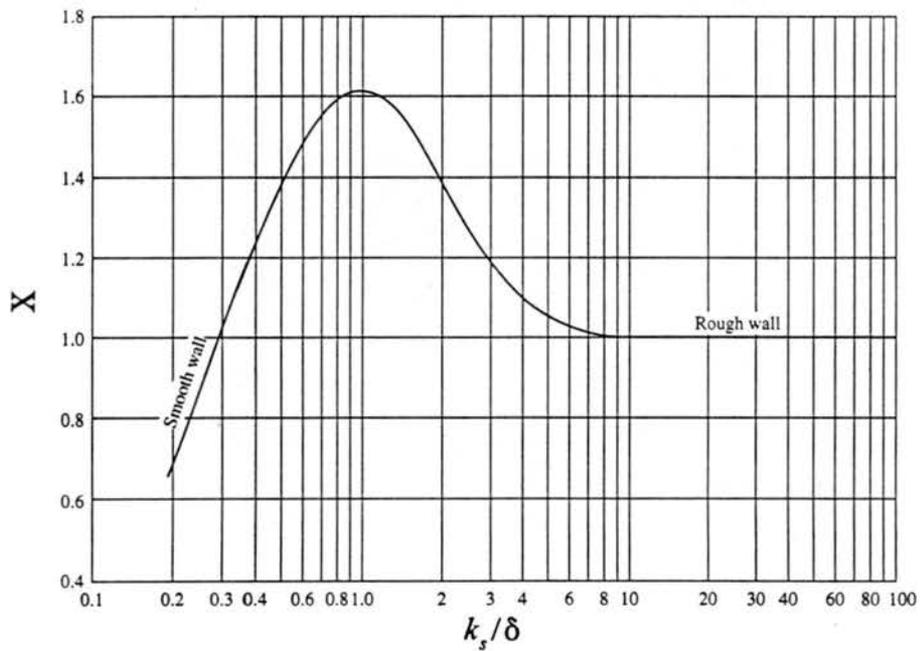


Fig. 2.4. Correction Factor in the Logarithmic Velocity Profile (Einstein, 1950).

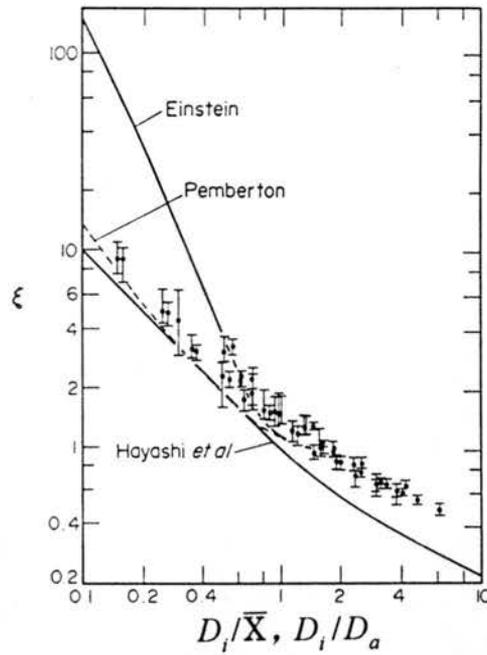


Fig. 2.5. The Sheltering Function  $\xi$  according to Misri et al. (1984).

fractions. Misri et al. also found that there was a very poor agreement with the relation for an additional correction factor,  $\theta$ , introduced by Einstein and Chien (1953). These conclusions were confirmed later by the experiments and verification of Samaga el al. (1986a, b).

Laursen (1958) proposed a bed-material sediment transport formula based on his flume experimental data. His bed-material sediment concentration formula for a given size fraction may be expressed as (ASCE Task Committee, 1971)

$$C_{ti} = 0.01\gamma P_{bi} \left( \frac{D_i}{d} \right)^{7/6} \left( \frac{\tau_0'}{\tau_{ci}} - 1 \right) f \left( \frac{V_*}{\omega_i} \right) \quad (2.15)$$

in which  $C_{ti}$  = bed-material sediment concentration by weight of size fraction  $i$ ;  $V_*$  = shear velocity;  $\tau_0'$  = bed shear stress due to grain resistance;  $\tau_{ci}$  = critical shear stress for grain size of  $D_i$  as given by the Shields diagram;  $\omega_i$  = fall velocity of particle of size  $D_i$ ; and

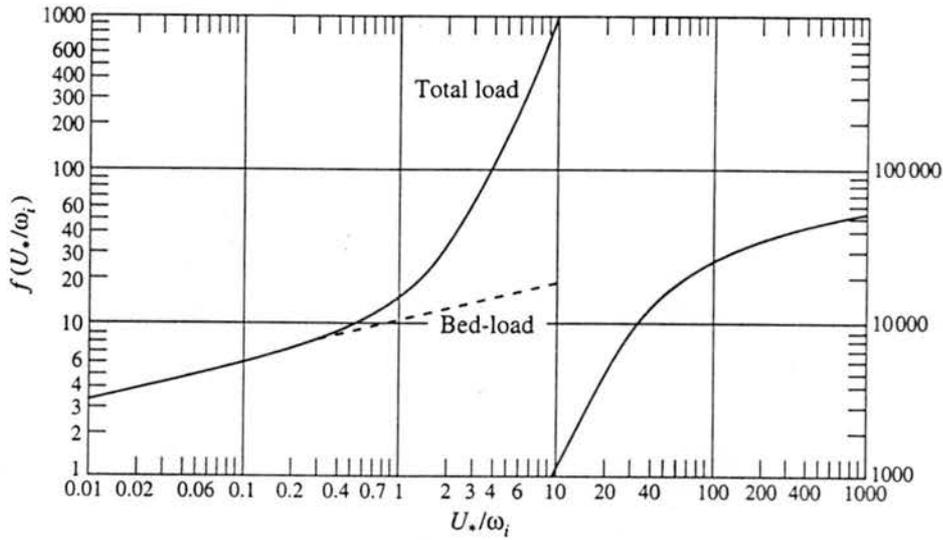


Fig. 2.6. Laursen's Sediment Transport Function (Laursen, 1958).

$f(V/\omega_i)$  = functional relation given in graphical form (Fig. 2.6). Laursen's shear stress due to grain resistance resulting from the use of Manning-Strickler equations is

$$\tau'_0 = \frac{\rho V^2}{58} \left[ \frac{D_{50}}{d} \right]^{1/3} \quad (2.16)$$

Toffaletti (1968, 1969) developed a procedure for the computation of sediment transport discharge based on the concept of Einstein (1950) and Einstein and Chien (1953). In his method, the total depth of flow is divided into four zones (Fig. 2.7), and the unit sediment discharge for each size fraction in each zone is determined individually. Then the unit bed-material load discharge for a sediment of size  $D_i$  is given by

$$q_{ti} = q_{bi} + q_{sui} + q_{smi} + q_{sli} \quad (2.17)$$

in which  $q_{sui}$ ,  $q_{smi}$ , and  $q_{sli}$  = suspended load discharges per unit width in upper, middle, and

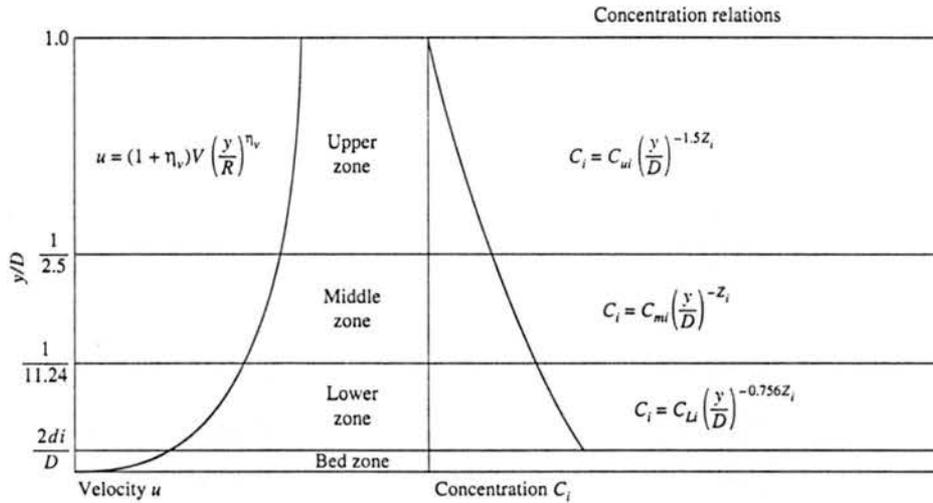


Fig. 2.7. Toffaleti's Velocity, Concentration, and Sediment Discharge Relations (Toffaleti, 1969).

lower zones, respectively, for sediment of size  $D_i$ .

Generally speaking, this group of approaches was found unsatisfactory in predicting transport rate by size fractions (Misri et al., 1984; Samaga et al., 1986b). This is due to the complexity of transport of sediment mixtures and the lack of knowledge concerning the motion of individual size and its effect on other sizes.

## 2.4 SHEAR STRESS CORRECTION APPROACH

Relevant contributions following the shear stress correction approach include those of the Ashida and Michiue (1973), Parker et al. (1982), White and Day (1982), Profitt and Sutherland (1983), Misri et al. (1984), Samaga et al. (1986a, 1986b), Diplas (1987), Bridge and Bennett (1992), Patel and Ranga Raju (1996), Wilcock and McArdell (1997), and Wilcock (1997).

Ashida and Michiue (1973) developed a bedload transport equation for nonuniform sediment mixtures by applying their bedload transport equation for uniform sediment. In

doing so, the critical shear stress for uniform sediment was replaced by the critical shear stress for each size fraction. Ashida and Michiue found that the physical meaning for the hiding and the exposure effects of the nonuniform sediment transportation is due to the discrepancy between the critical shear stress for uniform sediment and that for nonuniform sediments. They adopted Egiazaroff's (1965) expression for critical shear stress of each individual size fraction in their bedload transport equation for nonuniform sediment mixtures. Based on a number of laboratory experiments, they also presented an empirical correction to Egiazaroff's expression in the range  $D_i/D_m < 0.4$ . The correction can be translated to a correction factor  $\epsilon_{bi}$  for the critical shear stress as follows

$$\epsilon_{bi} = \begin{cases} \left[ \frac{\log(19)}{\log(19D_i/D_m)} \right]^2 & \text{for } \frac{D_i}{D_m} \geq 0.4 \\ 0.85 \left( \frac{D_i}{D_m} \right)^{-1} & \text{for } \frac{D_i}{D_m} < 0.4 \end{cases} \quad (2.18)$$

in which  $D_m$  = mean diameter of bed material.

Parker et al. (1982) and Parker (1990) developed an empirical gravel transport relationship based on the equal mobility concept and the similarity transformation concept. Parker et al.'s equal mobility hypothesis states that the existence of a bed pavement regulates the entrainment of particles by stream, resulting in their mobility being approximately equal. That is, all grain sizes are entrained at about the same flow discharge and are transported at rates in proportion to their presence in the bed material. The similarity hypothesis transformation concept assumes that by choosing the proper parameter, different curves pertaining to different size fractions collapse into a single universal curve. Based on the

analysis of field data from Oak Creek, the relationships between dimensionless bedload transport function  $W_i^*$  and dimensionless shear stress parameter  $\tau_{ri}^*$  were given in a graphic form. These parameters are defined as

$$W_i^* = \frac{\gamma_s - \gamma}{\gamma} \frac{q_{bi}}{P_{bi} (gdS)^{1/2} dS} \quad (2.19)$$

$$\phi_i = \frac{dS}{(\gamma_s / \gamma - 1) D_i \tau_{ri}^*} \quad (2.20)$$

$$\tau_{ri}^* = 0.0876 \frac{D_{50}}{D_i} \quad (2.21)$$

Because of the approximate equal mobility of all sizes, only one grain size, namely the subpavement size  $D_{50}$ , is used to characterize bedload discharge as a function of the dimensionless shear stress

$$W^* = \begin{cases} 0.0025 \exp[14.2 (\phi_{50} - 1) - 9.28 (\phi_{50} - 1)^2] & \text{for } 0.95 < \phi_{50} < 1.65 \\ 11.2 \left(1 - \frac{0.822}{\phi_{50}}\right)^{4.5} & \text{for } 1.65 < \phi_{50} \end{cases} \quad (2.22)$$

in which  $\phi_{50} = \tau_{50}^* / \tau_{r50}^*$ ;  $\tau_{50}^*$  = Shields stress for median diameter of subpavement;  $\tau_{r50}^*$  = reference value of  $\tau_{50}^* = 0.0876$ .

Noting that Parker et al.'s approach constitutes only a first-order approximation of reality, Diplas (1987) analyzed the same data used by Parker et al. and indicated that the hiding function dependent on the average shear stress in addition to its dependence on grain size,  $D_i / D_{50}$ . An empirical expression for reduced hiding function based on Oak Creek data was proposed as follows

$$\xi_{bi} = \left[ \phi_{50}^{(D_i/D_{50})^{0.3412} - 1} \right] \left[ \left( \frac{D_i}{D_{50}} \right)^{-0.057} \right]^{(D_i/D_{50})^{0.3214}} \quad (2.23)$$

White and Day (1982) investigated threshold conditions for individual grain sizes in a mixture in a recirculating flume. The prediction of transport rates was made by using Ackers and White's (1973) formulations by determining the initial motion parameter for each size fraction. The results led to a hiding function which is related to  $D_i/D_A$ , where  $D_A$  is the scaling size in a mixture defined by White and Day. White and Day's correction factor for mobility number can be transformed into a correction factor for effective shear stress

$$\xi_{bi} = \left( \frac{0.4}{(D_i/D_A)^{1/2}} + 0.6 \right)^2 \quad (2.24)$$

and

$$\frac{D_A}{D_{50}} = 1.6 \left( \frac{D_{84}}{D_{16}} \right)^{-0.28} \quad (2.25)$$

Proffitt and Sutherland (1983) studied the effect of sediment nonuniformity by comparing the transport rate of individual fractions in a mixture with that of uniform material of the same size. In the analysis they considered Paintal's (1971) and Ackers and White's (1973) transport functions as the basis for studying the effect of nonuniformity and suggested corrections to be applied to these two functions. By analyzing experimental data, they defined the following expression of  $\xi_{bi}$  for Paintal's function

$$\xi_{bi} = \begin{cases} 1.0 \left( \frac{D_i}{D_u} \right)^{0.51} & \text{for } 0.60 < \frac{D_i}{D_u} < 10.0 \\ 1.16 \left( \frac{D_i}{D_u} \right)^{0.81} & \text{for } \frac{D_i}{D_u} < 0.60 \end{cases} \quad (2.26)$$

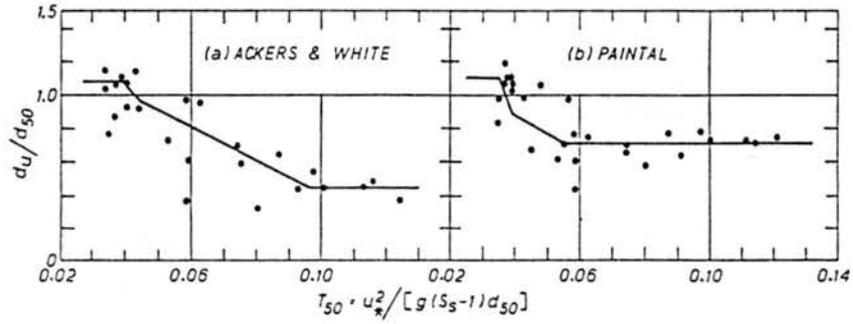


Fig. 2.8. Proffitt and Sutherland's Scaling Size (Proffitt and Sutherland, 1983).

in which  $D_u$  = scaling size which determines the roughness of the bed (Fig. 2.8).

A similar expression of  $\xi_{bi}$  was proposed by Proffitt and Sutherland for the correction of Ackers and White's function

$$\xi_{bi} = \begin{cases} 1.30 & \text{for } 3.70 < \frac{D_i}{D_u} \\ 0.53 \log \left( \frac{D_i}{D_u} \right) + 1.0 & \text{for } 0.075 < \frac{D_i}{D_u} < 3.70 \\ 0.40 & \text{for } \frac{D_i}{D_u} < 0.075 \end{cases} \quad (2.27)$$

Proffitt and Sutherland's correction factor does not take into account all the relevant parameters, and the data used in the development of their relation for  $\xi_{bi}$  covers a relatively narrow range of flow conditions.

Misri et al. (1984) modified the Einstein type transport relationships and obtained a function for uniform sediment as follows by fitting experimental data

$$\Phi_b = \begin{cases} 4.6 \times 10^7 (\theta')^8 & \text{for } \theta' \leq 0.065 \\ \frac{8.5 (\theta')^{1.8}}{[1 + 5.95 \times 10^{-6} / (\theta')^{4.7}]^{1.43}} & \text{for } \theta' > 0.065 \end{cases} \quad (2.28)$$

where

$$\theta' = \frac{\tau'}{(\gamma_s - \gamma) D} \quad (2.29)$$

in which  $D$  = sediment size;  $\tau'$  = grain shear stress.

For nonuniform sediment, Misri et al. introduced a factor  $\xi_{bi}$  into Eq. (2.28) to account for the sheltering and exposure effect. Similar to Proffitt and Sutherland's method, Misri et al. conceptually related the coefficient  $\xi_{bi}$  to  $D_i / D_a$ . Then, based on dimensionless analysis, they defined the following functional relationship for  $\xi_{bi}$

$$\xi_{bi} = \frac{D_i}{D_a} = \frac{\tau_e}{\tau'} = f\left(\theta'_i, \frac{\tau'}{\tau_c}, M\right) \quad (2.30)$$

where

$$\theta'_i = \frac{\tau'}{(\gamma_s - \gamma) D_i} \quad (2.31)$$

in which  $\tau_e$  = effective shear stress for transport of size fraction  $D_i$  as bedload;  $\tau_c$  = critical shear stress for size  $D_a$  based on Shields' criterion;  $D_a$  = arithmetic mean diameter of sediment mixtures;  $M$  = the Kramer's uniformity coefficient for the mixture, which was defined as

$$M = \frac{\sum_0^{50} D_i \Delta P_{bi}}{\sum_{50}^{100} D_i \Delta P_{bi}} \quad (2.32)$$

in which  $D_i$  = geometric mean of size group  $i$ ; and  $\Delta P_i$  = percentage weight corresponding to size group  $i$ .

Through extensive analysis of their experimental data, the correction factor was given as

$$\xi_{bi} = \frac{0.038 K_2 (\tau' / \tau_c)^{0.75}}{(\theta')^{1.2} [(1 + 0.0031 (\theta')^{-2.1})^{1/3}]^{1/3}} \quad (2.33)$$

in which  $K_2$  = coefficient.

Samaga et al. (1986a, 1986b) proposed a correction factor  $\xi_{bi}$  through modifying Misri's model to calculate bedload transport rate of individual fractions for sediment mixtures. Following a line of analysis similar to that for bedload transport, Samaga et al. also defined a coefficient  $\xi_{si}$  for suspended load. The functional relationship for  $\xi_{si}$  was expressed as

$$\xi_{si} = \frac{\tau_e}{\tau'} = f\left(\theta'_i, \frac{\tau'}{\tau_c}, M\right) \quad (2.34)$$

The coefficients  $\xi_{bi}$  and  $\xi_{si}$  were expressed in graphical forms. Samaga et al. referred to  $\xi_{bi}$  and  $\xi_{si}$  as the sheltering-cum-exposure coefficient and sheltering-cum-exposure-cum-interference coefficient, respectively.

Samaga et al. used the coefficients defined by

$$(K_b, K_s) = f(\tau'_0 / \tau_{0c}) \quad (2.35)$$

$$(L_b, L_s) = f(M) \quad (2.36)$$

and

$$(K_b L_b \xi_{bi}), (K_s L_s \xi_{si}) = f(\theta') \quad (2.37)$$

The relationship between  $L_B$  and  $M$  is given in a table. The coefficients  $K_b$  and  $(K_b L_b \xi_{bi})$  are given in graphical forms. With known  $\xi_{bi}$   $\theta'_i$  the value of dimensionless bedload  $\Phi_{bi}$  can be read from a graphical relation. Following the same procedure,  $\Phi_{si}$  can be obtained.

Bridge and Bennett (1992) developed a model for the bedload transport of sediment grains of different sizes, in which the shape and density were also considered.

Recently, Patel and Ranga Raju (1996) checked the performance of the relationships for the methods of Ashida and Michiue(1973), Proffitt and Sutherland (1983), and Bridge and Bennett (1992) based on a large number of bedload data from flume and fields, and indicated that none of these methods give satisfactory results. Patel and Ranga Raju (1996) thus proposed an empirical relationship for fractional bedload transport which they calibrated using both flume and field data. The exposure-cum-sheltering correction factor proposed by Patel and Ranga Raju was as follows

$$\xi_{bi} = 0.0713 (C_s \theta'_i)^{-0.75144} / C_m \quad (2.38)$$

where

$$\begin{aligned} \log(C_s) = & -0.1957 \\ & - 0.9571 \left[ \log \left( \frac{\tau'}{\tau_c} \right) \right] - 0.1949 \left[ \log \left( \frac{\tau'}{\tau_c} \right) \right]^2 + 0.0644 \left[ \log \left( \frac{\tau'}{\tau_c} \right) \right]^3 \end{aligned} \quad (2.39)$$

and

$$C_m = \begin{cases} 0.7092 \log(M) + 1.293 & \text{for } 0.05 < M < 0.38 \\ 1.0 & \text{for } M > 0.38 \end{cases} \quad (2.40)$$

This new method is only applicable for fractional bed-material load.

More recently, Wilcock and McArdell (1997) and Wilcock (1997) studied partial transport and fractional transport. They expressed the fractional transport rate as the product of the spatial entrainment, displacement frequency, and displacement length. In their fractional transport expression, the partial transport of individual size fractions was emphasized. But this common feature in gravel-bed rivers is not a major concern in sand-bed rivers.

The shear stress correction approach is most commonly used for the computation of fractional bedload transport in nonuniform mixtures. Some of the concepts used in this approach, which relate the shear stress correction factor to relative diameter ( $D_i/D_{50}$ ,  $D_i/D_A$ ,  $D_i/D_w$ ,  $D_i/D_a$ , etc.), bed material size gradation (uniformity coefficient,  $M$ ), and flow intensity (average shear stress), can be expanded to the study of the transport of sediment mixtures.

## 2.5 BED MATERIAL FRACTION APPROACH

Among the four categories of fractional sediment transport rate computation methods, the BMF approach is the most commonly used method in numerical models, even though the shortcomings of using this approach in nonuniform sediment transport models have been pointed out in the literature (Hsu and Holly 1992). The BMF approach does not account for the interactions between different size particles present in sediment mixtures. Hsu and Holly (1992) point out that one important disadvantage of this approach is the difficulty in obtaining

the correct gradation and total transport amount, and another minor disadvantage is its sensitivity to the number and distribution of class intervals. However, due to its simplicity and the fact that it may be acceptable under certain transport conditions, the BMF approach is still widely in use in numerical models such as HEC-6 (U. S. Corps of Engineers 1977; Thomas, 1982), GSTARS (Molinas and Yang, 1986), CARICHAR (Rahuel et al., 1989), and BRISTARS (Molinas, 1990; Molinas and Trent, 1991; Molinas 1993).

Karim (1985) introduced a so-called hiding factor,  $W_i$ , into the BMF approach to reflect the influence of other sediment particles in the mixture on the transport of given size fraction  $i$

$$C_{ti} = W_i P_{bi} C_{pi} \quad (2.41)$$

Eq. (2.41) can be visualized as a modified BMF method. They expressed  $W_i$  as a simple power function of  $D_i/D_{50}$

$$W_i = C_1 \left( \frac{D_i}{D_{50}} \right)^{C_2} \quad (2.42)$$

in which  $C_1$  and  $C_2$  = coefficients, which were determined to be 1.0 and 0.8, respectively, using typical Missouri River bed material size distributions and flow data and a trial and error procedure.

Following the same procedure, Karim (1998) developed new relations for predicting sediment discharge for each size fraction

$$C_{ti} = \phi_i C_{pi} \quad (2.43)$$

in which  $\phi_i$  = weighing function for  $i$ th fraction:

$$\phi_i = P_{ai} \eta \quad (2.44)$$

in which  $P_{ai}$  = areal function of bed material in  $i$ th size fraction; and  $\eta$  = sheltering factor.

The areal function and sheltering factor proposed by Karim are

$$P_{ai} = \frac{\frac{P_{bi}}{D_i}}{\sum_{i=1}^N \frac{P_{bi}}{D_i}} \quad (2.45)$$

$$\eta = C_1 \left( \frac{D_i}{D_{50}} \right)^{C_2} \quad (2.46)$$

in which  $C_1, C_2$  = coefficients determined based on experimental data from Einstein and Chien (1953):

$$C_1 = 1.15 \left( \frac{\omega_{50}}{V_*} \right), \quad C_2 = 0.60 \left( \frac{\omega_{50}}{V_*} \right) \quad (2.47)$$

Similar to Karim and Kennedy's modification [Eq. (2.41)], Wang and Zhang (1990), Wang et al. (1995), and Wang et al. (1998) introduced a modifying coefficient  $K_D$  into the BMF concept for fractional load computation

$$q_{ti} = K_D (q_{bi} + q_{si}) \quad (2.48)$$

Based on data analysis and heuristic reasoning, they expressed  $K_D$  as follows

$$K_D = D_k^m, \quad D_k = \frac{D_i}{D_p} \quad (2.49)$$

in which

$$D_p = D_m \theta^{0.5}, \quad D_m = \sum P_{bi} D_i \quad (2.50)$$

and

$$m = 0.06 + 2.55D_k - 0.863D_k^2 + 0.1088D_k^3 - 0.0046D_k^4 \quad (2.51)$$

It needs to be pointed out that all modification factors ( $W_i$ ,  $\phi_i$ , and  $K_D$ ) in the use of the BMF concept were calibrated along with a specific transport equation used to compute the potential transport rate in their development. Therefore, these modification factors may not be used in conjunction with other transport equations to predict fractional transport rate.

## 2.6 TRANSPORT CAPACITY FRACTION APPROACH

The TCF approach comprises two components, the computation of bed-material transport capacity,  $C_b$ , and the computation of its fraction,  $P_{ci}$ . It is assumed that these two components can be treated separately in the development and application of the TCF concept. The bed-material sediment concentration,  $C_b$ , can be determined by using the available bed-material transport formulas in the literature. The key problem in the application of the TCF approach is the proper determination of the sediment transport capacity distribution function,  $P_{ci}$ .

For suspended load, a simple method was suggested by Dou et al. (1987) by relating to  $P_{si} / \omega_i$

$$P_{csi} = \frac{(P_{si} / \omega_i)^\alpha}{\sum_{i=1}^N (P_{si} / \omega_i)^\alpha} \quad (2.52)$$

where  $P_{csi}$  = the fraction of suspended sediment transport capacity, by dry weight, corresponding to size fraction  $i$ ;  $P_{si}$  = the fraction of suspended load, by dry weight,

corresponding to size fraction  $i$ , and  $\omega_i$  = the fall velocity of sediment corresponding to particle size  $D_i$ . The deficiency of this method is that it is basically developed for suspended load and it does not take into account the influence of the bed material size and the hydraulic conditions.

From statistical theory, Li (1988) derived a method which related to  $P_{bi}$ ,  $\omega_i$ , and  $V_*$ .

$$P_{csi} = \frac{P_{bi} \frac{1-A_i}{\omega_i} (1 - e^{-6\omega_i/\kappa V_*})}{\sum_{i=1}^N P_{bi} \frac{1-A_i}{\omega_i} (1 - e^{-6\omega_i/\kappa V_*})} \quad (2.53)$$

where

$$A_i = \frac{\omega_i}{\frac{\sigma_v}{\sqrt{2\pi}} e^{-\omega_i^2/2\sigma_v^2} + \omega_i \phi\left(\frac{\omega_i}{\sigma_v}\right)} \quad (2.54)$$

where  $\kappa$  = the universal constant of von Kármán;  $\sigma_v$  = the turbulence coefficient, and it was assumed that  $\sigma_v = V_*$ ;  $V_*$  is the shear velocity; and  $\phi$  = the standardized normal distribution frequency function. Li's method was basically developed for the computation of fractions of suspended bed-material sediment transport capacity.

Karim and Kennedy (1981) proposed an equation for  $P_{ci}$  as follows

$$P_{ci} = \frac{P_{bi} \left(\frac{D_{50}}{D_i}\right)^x}{\sum_{i=1}^N P_{bi} \left(\frac{D_{50}}{D_i}\right)^x} \quad (2.55)$$

where

$$x = 0.0316 \left(\frac{d}{D_{50}}\right)^{0.5} \quad (2.56)$$

As indicated by Holly and Karim (1986), this method has no theoretical basis, and it was developed through heuristic reasoning supported by data analysis of measured suspended sediment size distribution of the Missouri, the Niobrara, and the Middle Loup Rivers. Karim and Kennedy's conceptual model was also used by Holly and Rahuel (1990) for the fractional bedload transport computation in their general framework for mobile-bed modeling.

Based on observation of sediment-mixture experiments, Hsu and Holly (1992) postulated that the fraction of each size class in transported material is proportional to the joint probability of the relative mobility ( $P_{mo_i}$ ) of each particle size and the availability ( $P_{bi}$ ) of each size class on the bed surface. Therefore, they expressed the transported distribution as

$$P_{ci} = \frac{P_{mo_i} P_{bi}}{\sum_{i=1}^N (P_{mo_i} P_{bi})} \quad (2.57)$$

where

$$\begin{aligned} P_{mo_i} &= \frac{1}{\sigma\sqrt{2\pi}} \int_{(V_{ci}/V)-1}^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \\ &= 0.5 - 0.5 \operatorname{erf}\left(\frac{\frac{V_{ci}}{V} - 1}{\sigma\sqrt{2}}\right) \end{aligned} \quad (2.58)$$

in which  $\operatorname{erf}(z)$  = the error function;  $V_{ci}$  = the incipient velocity for a particular size class  $i$  in a mixture;  $V'$  = the absolute fluctuations of velocity; and  $\sigma$  = the standard deviation of  $V'/V$  distribution. Unfortunately, Hsu and Holly's method is limited to the transport of bedload and has not been verified with direct measurements of fractional transport data.

## CHAPTER 3

### TRANSPORT CAPACITY DISTRIBUTION FUNCTION

#### 3.1 GENERAL

Presently, none of the four groups of approaches gives a satisfactory prediction of fractional transport rates of nonuniform sediment mixtures in open channels. The approach using direct computation by size fractions results in the worst prediction of fractional transport rates and is rarely used in numerical models. The shear stress correction approach is commonly used for the computation of fractional bedload transport. This approach is more suitable for flows with gravel-bed materials since the sheltering and exposure effects are more pronounced in these cases. The physical considerations and the parameters used in the shear correction approach can be incorporated into research on fractional transport of sand-bed materials. The BMF approach is widely used in numerical models due to its simplicity. But both the shear stress correction approach and the BMF concept fail to estimate the correct gradation and total amount of bed-material transport rate. The TCF concept has its advantages in avoiding additional errors in estimating the bed-material concentration by the summation of transport capacities of individual size fractions and may limit the discrepancies in computing concentrations for individual size fractions. The sheltering and exposure effect considered in the shear stress correction approach for nonuniform mixtures can be incorporated in the determination of the

transport capacity distribution function,  $P_{ci}$ . Currently, most research utilizing the TCF concept is limited to fractional load transport of suspended sediments. It is desirable to develop a new method for the computation of bed-material load transport capacity by size fractions based on the TCF approach and using the concept of sheltering and exposure corrections.

The TCF approach comprises two components, the computation of bed-material transport capacity,  $C_b$ , and the computation of transport capacity distribution function,  $P_{ci}$ . It is assumed in the present study that these two components can be treated separately in development and application of the TCF concept. In this section, we focus on the computation of  $P_{ci}$ , which is the crucial component in the successful implementation of the TCF concept.

### 3.2 DATA SOURCES

The processes and mechanisms of nonuniform sediment transport are still not well understood in our knowledge due to their complexity. Prediction of sediment transport rates by size fractions has not been accomplished following a purely analytical method. It is a common practice to develop a fractional sediment transport procedure following semi-empirical derivation and relying on calibration using flume and field data. Therefore it is very important to collect a complete and reliable set of sediment transport data for the analysis, development, calibration, and comparison of fractional sediment transport methods. It is required that the data include measurements of size distribution for both bed material and the sediments in transport.

The flume data of Einstein (1978), Einstein and Chien (1953), and Samaga et al.

(1986a, 1986b), and field data from the Niobrara River near Cody, Nebraska (Colby and Hembree, 1955), and the Middle Loup River at Dunning, Nebraska (Hubbell and Matejka, 1959) were used to analyze the fractional transport rate of sediment mixtures. A summary of this data base is given in Table 3.1. It incorporates 118 data sets containing a total of 891 data points. These data are limited to sand sizes with median diameter in the range of 0.10 to 0.90 mm, geometric standard deviation in the range of 1.30 to 3.0, flow discharge in the range of 0.0056 to 16.06 m<sup>3</sup>/s, velocity in the range of 0.49 to 1.41 m/s, flow depth in the range of 0.056 to 0.58 m, and slope in the range of 0.00093 to 0.013.

For the laboratory data, the bed-material concentrations were measured directly to eliminate the uncertainty of unmeasured load near the bed surface. The bed-material concentrations for the Niobrara River near Cody, Nebraska are measured as suspended bed-material concentrations at a contracted section and are based on depth integrated samples. The bed-material concentrations reported for the Middle Loup River at Dunning, Nebraska are measured suspended bed-material concentrations with a turbulence flume and are also based on depth integrated samples. Data with unmeasured load near the bed surface determined by the use of indirect methods are not included in this study.

Detailed sediment transport data are given in Tables 3.2 - 3.6. The sediment transport data contains a complete record for flow and sediment information pertaining to each measurement. This information for each record includes:

- Flow properties,
- Bed material properties,
- Transported sediment properties, and
- Size distributions of bed material and transported sediments.

Table 3.1. Summary of Laboratory and River Data

Data Source (1)	Average Discharge (m <sup>3</sup> /s) (2)	Average Velocity (m/s) (3)	Average Depth (m) (4)	Slope (m/m) (5)	Temperature (°C) (6)	Median Diameter (mm) (7)	Geometric Standard Deviation of Bed Size (8)	Bed-Material Conc. (PPM) (9)	No. of Data Sets (10)	No. of Size Groups (11)	No. of Data Points (12)
(a) Laboratory Data											
Einstein (IRTCES, 1978)	0.019 -0.042	0.54 -1.41	0.099 -0.139	0.00262 -0.0127	16.0 -27.2	0.108 -0.903	1.245 -2.158	1325 -39560	29	13	289
Einstein and Chien (1953)	0.043 -0.066	0.73 -1.12	0.177 -0.237	0.00157 -0.00489	8.9 -27.8	0.104 -0.381	1.414 -2.968	2115 -57970	22	15	218
Samaga et al. (1986a, b)	0.0056 -0.015	0.49 -0.78	0.056 -0.101	0.00449 -0.00693	14.0 -26.5	0.212 -0.404	1.480 -2.460	3392 -10260	33	10	258
(b) River Data											
Colby and Hembree (1955)	5.86 -16.06	0.62 -1.27	0.421 -0.576	0.00114 -0.00180	0.56 -28.3	0.215 -0.349	1.514 -2.345	257 -1600	19	8	59
Hubell and Matejka (1959)	9.34 -12.54	0.63 -1.11	0.250 -0.370	0.000928 -0.00146	1.10 -31.1	0.219 -0.424	1.651 -2.403	411 -1831	15	8	67
(c) Total of Laboratory and River Data											
Total	0.0056 -16.06	0.49 -1.41	0.056 -0.576	0.000928 -0.0127	0.56 -31.1	0.104 -0.903	1.245 -2.968	257 -57970	118		891

Table 3.2. Laboratory Data of Einstein (IRTCES, 1978)

Data No. (1)	Run ID (2)	Flow Properties					Bed Material				Transported Sediment			
		Q (m <sup>3</sup> /s) (3)	W (m) (4)	d (m) (5)	S (m/m) (6)	T (°C) (7)	D <sub>50</sub> * (mm) (8)	D <sub>65</sub> * (mm) (9)	σ <sub>g</sub> * (10)	s <sub>g</sub> (11)	C <sub>T</sub> (kg/m <sup>3</sup> ) (12)	C <sub>T</sub> (PPM) (13)	D <sub>50t</sub> * (mm) (14)	σ <sub>gt</sub> * (15)
1	10	0.0329	0.2667	0.1049	0.010330	21.0	0.150	0.173	1.529	2.65	41.33	40290.77	0.122	1.510
2	11	0.0260	0.2667	0.1390	0.002622	20.1	0.150	0.173	1.529	2.65	2.00	2001.52	0.161	1.586
3	12	0.0300	0.2667	0.1356	0.003867	21.8	0.150	0.173	1.529	2.65	3.86	3854.92	0.162	1.595
4	13	0.0343	0.2667	0.1329	0.005542	22.1	0.150	0.173	1.529	2.65	8.26	8216.38	0.148	1.637
5	14	0.0385	0.2667	0.1289	0.008750	22.7	0.150	0.173	1.529	2.65	21.84	21542.65	0.123	1.510
6	15	0.0425	0.2667	0.1277	0.011789	23.6	0.150	0.173	1.529	2.65	43.79	42623.29	0.121	1.450
7	16	0.0306	0.2667	0.0994	0.008988	25.6	0.223	0.247	1.367	2.65	16.81	16631.27	0.212	1.352
8	17	0.0271	0.2667	0.1003	0.006411	26.0	0.223	0.247	1.367	2.65	8.61	8560.20	0.219	1.406
9	18	0.0239	0.2667	0.1058	0.003975	25.9	0.223	0.247	1.367	2.65	3.55	3544.13	0.232	1.350
10	19	0.0203	0.2667	0.1052	0.002840	23.4	0.223	0.247	1.367	2.65	1.92	1916.13	0.231	1.385
11	20	0.0269	0.2667	0.1273	0.006373	20.0	0.595	0.658	1.245	2.65	3.16	3149.08	0.604	1.241
12	21	0.0220	0.2667	0.1148	0.005493	19.1	0.595	0.658	1.245	2.65	2.31	2311.38	0.593	1.229
13	22	0.0345	0.2667	0.1134	0.007631	17.4	0.595	0.658	1.245	2.65	5.65	5633.37	0.589	1.229
14	23	0.0233	0.2667	0.1350	0.003318	16.0	0.595	0.658	1.245	2.65	1.46	1456.70	0.580	1.208
15	24	0.0230	0.2667	0.1332	0.003510	18.9	0.903	1.037	1.383	2.65	1.33	1327.94	0.879	1.372
16	25	0.0226	0.2667	0.1274	0.004820	18.5	0.903	1.037	1.383	2.65	1.57	1567.34	0.891	1.379
17	26	0.0326	0.2667	0.1204	0.009433	16.4	0.903	1.037	1.383	2.65	6.01	5990.52	0.871	1.399
18	27	0.0404	0.2667	0.1075	0.012380	19.5	0.609	0.845	2.158	2.65	11.14	11058.46	0.692	1.995
19	28	0.0357	0.2667	0.1029	0.010460	21.0	0.609	0.845	2.158	2.65	9.31	9257.81	0.749	1.792
20	29	0.0357	0.2667	0.1036	0.010630	21.3	0.609	0.845	2.158	2.65	7.12	7092.39	0.756	1.775
21	31	0.0389	0.2667	0.1043	0.012720	19.2	0.609	0.845	2.158	2.65	9.72	9657.18	0.647	1.743
22	32	0.0308	0.2667	0.1014	0.008709	24.8	0.609	0.845	2.158	2.65	6.47	6444.42	0.714	1.977
23	33	0.0189	0.2667	0.1317	0.002846	26.5	0.121	0.135	1.329	2.65	2.42	2420.98	0.101	1.290
24	34	0.0227	0.2667	0.1302	0.003087	27.2	0.119	0.131	1.307	2.65	2.12	2114.97	0.109	1.326
25	35	0.0262	0.2667	0.1298	0.004418	26.3	0.119	0.131	1.320	2.65	3.91	3902.89	0.116	1.327
26	35	0.0261	0.2667	0.1307	0.004544	25.0	0.110	0.119	1.308	2.65	5.36	5338.11	0.109	1.317
27	35	0.0260	0.2667	0.1308	0.004447	22.5	0.108	0.117	1.306	2.65	5.98	5961.74	0.110	1.314
28	36	0.0299	0.2667	0.1298	0.006282	26.1	0.116	0.128	1.340	2.65	7.15	7114.17	0.120	1.311
29	37	0.0333	0.2667	0.1309	0.008513	25.5	0.114	0.123	1.323	2.65	15.94	15787.15	0.111	1.312

Note: \* — Values computed from adjusted bed material size distribution (wash load materials excluded).

Table 3.2. Laboratory Data of Einstein (IRTCES, 1978) (continued)

Data No. (1)	Run ID (2)	D <sub>c</sub> (mm) (16)	Size distribution of bed material, finer than indicated diameters												
			Grp1 (%) (17)	Grp2 (%) (18)	Grp3 (%) (19)	Grp4 (%) (20)	Grp5 (%) (21)	Grp6 (%) (22)	Grp7 (%) (23)	Grp8 (%) (24)	Grp9 (%) (25)	Grp10 (%) (26)	Grp11 (%) (27)	Grp12 (%) (28)	Grp13 (%) (29)
			<b>0.061</b>	<b>0.074</b>	<b>0.088</b>	<b>0.104</b>	<b>0.124</b>	<b>0.147</b>	<b>0.175</b>	<b>0.208</b>	<b>0.246</b>	<b>0.295</b>	<b>0.351</b>	<b>0.417</b>	<b>0.589mm</b>
1	10	0.061	3.49	6.33	13.78	20.66	35.99	49.94	67.69	77.18	88.04	94.17	97.60	98.87	100.00
2	11	0.061	3.49	6.33	13.78	20.66	35.99	49.94	67.69	77.18	88.04	94.17	97.60	98.87	100.00
3	12	0.061	3.49	6.33	13.78	20.66	35.99	49.94	67.69	77.18	88.04	94.17	97.60	98.87	100.00
4	13	0.061	3.49	6.33	13.78	20.66	35.99	49.94	67.69	77.18	88.04	94.17	97.60	98.87	100.00
5	14	0.061	3.49	6.33	13.78	20.66	35.99	49.94	67.69	77.18	88.04	94.17	97.60	98.87	100.00
6	15	0.061	3.49	6.33	13.78	20.66	35.99	49.94	67.69	77.18	88.04	94.17	97.60	98.87	100.00
7	16	0.061	0.03	0.04	0.08	0.14	0.75	4.13	21.56	39.79	64.72	81.27	91.13	95.51	100.00
8	17	0.061	0.03	0.04	0.08	0.14	0.75	4.13	21.56	39.79	64.72	81.27	91.13	95.51	100.00
9	18	0.061	0.03	0.04	0.08	0.14	0.75	4.13	21.56	39.79	64.72	81.27	91.13	95.51	100.00
10	19	0.061	0.03	0.04	0.08	0.14	0.75	4.13	21.56	39.79	64.72	81.27	91.13	95.51	100.00
			<b>0.246</b>	<b>0.295</b>	<b>0.351</b>	<b>0.417</b>	<b>0.489</b>	<b>0.589</b>	<b>0.701</b>	<b>0.833</b>	<b>0.991</b>	<b>1.168</b>	<b>1.397</b>	<b>1.981mm</b>	
11	20	0.417	0.01	0.09	0.71	4.29	16.63	50.61	75.68	92.15	98.30	99.80	100.00	100.00	
12	21	0.417	0.01	0.09	0.71	4.29	16.63	50.61	75.68	92.15	98.30	99.80	100.00	100.00	
13	22	0.417	0.01	0.09	0.71	4.29	16.63	50.61	75.68	92.15	98.30	99.80	100.00	100.00	
14	23	0.417	0.01	0.09	0.71	4.29	16.63	50.61	75.68	92.15	98.30	99.80	100.00	100.00	
15	24	0.417	0.08	0.13	0.25	0.46	1.34	7.19	22.30	41.40	60.37	77.75	93.92	100.00	
16	25	0.417	0.08	0.13	0.25	0.46	1.34	7.19	22.30	41.40	60.37	77.75	93.92	100.00	
17	26	0.417	0.08	0.13	0.25	0.46	1.34	7.19	22.30	41.40	60.37	77.75	93.92	100.00	
			<b>0.089</b>	<b>0.125</b>	<b>0.175</b>	<b>0.246</b>	<b>0.350</b>	<b>0.495</b>	<b>0.700</b>	<b>0.991</b>	<b>1.397</b>	<b>1.981</b>	<b>2.790</b>	<b>3.100mm</b>	
18	27	0.175	1.08	2.16	5.22	12.29	25.52	42.86	59.22	73.27	84.47	91.95	98.87	100.00	
19	28	0.175	1.08	2.16	5.22	12.29	25.52	42.86	59.22	73.27	84.47	91.95	98.87	100.00	
20	29	0.175	1.08	2.16	5.22	12.29	25.52	42.86	59.22	73.27	84.47	91.95	98.87	100.00	
21	31	0.175	1.08	2.16	5.22	12.29	25.52	42.86	59.22	73.27	84.47	91.95	98.87	100.00	
22	32	0.175	1.08	2.16	5.22	12.29	25.52	42.86	59.22	73.27	84.47	91.97	98.87	100.00	
			<b>0.061</b>	<b>0.074</b>	<b>0.088</b>	<b>0.104</b>	<b>0.124</b>	<b>0.147</b>	<b>0.175</b>	<b>0.208</b>	<b>0.246</b>	<b>0.351</b>	<b>0.495</b>	<b>0.701</b>	<b>0.991mm</b>
23	33	0.061	3.36	5.48	18.28	30.27	55.17	78.21	95.80	98.74	99.40	99.88	99.96	99.98	100.00
24	34	0.061	2.94	5.15	17.61	29.89	58.16	81.38	96.93	98.83	99.45	99.91	99.97	99.98	100.00
25	35	0.061	1.65	5.00	17.28	28.39	58.20	80.24	96.00	98.66	99.42	99.92	99.98	99.99	100.00
26	35	0.061	2.80	8.30	27.62	40.30	73.00	90.51	98.50	99.70	99.91	99.99	100.00	100.00	100.00
27	35	0.061	3.44	9.93	31.36	44.68	76.40	91.68	98.75	99.75	99.91	99.99	100.00	100.00	100.00
28	36	0.061	1.80	6.35	21.09	31.13	62.04	81.93	95.85	98.75	99.45	99.93	99.99	100.00	100.00
29	37	0.061	2.08	6.91	23.64	34.67	66.65	85.95	97.20	99.20	99.67	99.96	99.99	100.00	100.00

Table 3.2. Laboratory Data of Einstein (IRTCEs, 1978) (continued)

Data No. (1)	Run ID (2)	Size distribution of sediment load, finer than indicated diameters												
		Grp1 (%) (30)	Grp2 (%) (31)	Grp3 (%) (32)	Grp4 (%) (33)	Grp5 (%) (34)	Grp6 (%) (35)	Grp7 (%) (36)	Grp8 (%) (37)	Grp9 (%) (38)	Grp10 (%) (39)	Grp11 (%) (40)	Grp12 (%) (41)	Grp13 (%) (42)
		<b>0.061</b>	<b>0.074</b>	<b>0.088</b>	<b>0.104</b>	<b>0.124</b>	<b>0.147</b>	<b>0.175</b>	<b>0.208</b>	<b>0.246</b>	<b>0.295</b>	<b>0.351</b>	<b>0.417</b>	<b>0.589mm</b>
1	10	8.49	14.38	27.21	37.33	56.01	69.21	82.54	88.53	93.90	97.14	98.85	99.45	100.00
2	11	2.71	5.70	11.41	18.07	31.39	43.39	58.77	68.84	81.49	89.79	96.05	97.82	100.00
3	12	3.05	5.34	10.64	17.88	31.21	43.03	57.93	68.03	80.36	89.46	95.98	97.99	100.00
4	13	4.40	8.09	15.76	25.06	40.18	51.81	65.49	73.90	83.51	91.05	96.11	98.02	100.00
5	14	7.37	12.77	25.57	36.07	54.92	69.44	82.16	87.86	93.40	96.64	98.55	99.31	100.00
6	15	7.37	13.41	20.77	37.47	56.61	69.89	82.91	88.57	93.98	97.20	98.89	99.50	100.00
7	16	0.50	0.70	0.90	1.20	2.60	8.00	28.10	47.40	72.10	85.60	93.40	96.60	100.00
8	17	1.00	1.20	1.50	1.80	3.10	7.80	26.70	43.70	65.50	80.40	90.00	94.40	100.00
9	18	0.17	0.17	0.23	0.40	0.68	2.55	16.55	33.73	58.50	78.74	91.27	95.52	100.00
10	19	0.20	0.30	0.70	1.10	2.00	4.40	19.50	36.40	58.40	77.40	90.20	95.10	100.00
		<b>0.246</b>	<b>0.295</b>	<b>0.351</b>	<b>0.417</b>	<b>0.489</b>	<b>0.589</b>	<b>0.701</b>	<b>0.833</b>	<b>0.991</b>	<b>1.168</b>	<b>1.397</b>	<b>1.981mm</b>	
11	20	0.06	0.24	0.93	3.30	14.87	47.83	74.83	92.77	98.83	99.88	100.00	100.00	
12	21	0.00	0.16	1.12	4.14	16.87	51.02	78.74	94.27	99.17	99.96	100.00	100.00	
13	22	0.07	0.12	0.54	2.74	15.54	51.38	78.18	94.28	99.08	99.95	100.00	100.00	
14	23	0.07	0.20	0.93	3.93	18.85	55.05	83.15	96.20	99.53	100.00	100.00	100.00	
15	24	0.00	0.10	0.20	0.30	1.10	7.30	22.90	44.10	63.80	80.00	95.00	100.00	
16	25	0.00	0.20	0.50	0.90	1.80	8.60	22.70	42.80	62.40	78.80	95.20	100.00	
17	26	0.00	0.40	1.10	1.80	3.40	11.60	26.80	46.30	64.40	79.60	95.00	100.00	
		<b>0.089</b>	<b>0.125</b>	<b>0.175</b>	<b>0.246</b>	<b>0.350</b>	<b>0.495</b>	<b>0.700</b>	<b>0.991</b>	<b>1.397</b>	<b>1.981</b>	<b>2.790</b>	<b>3.100mm</b>	
18	27	0.24	0.29	0.46	1.63	9.09	27.39	50.97	70.63	76.37	96.28	100.00	100.00	
19	28	0.06	0.11	0.21	0.90	6.40	23.78	45.93	67.33	85.21	96.76	100.00	100.00	
20	29	0.05	0.07	0.18	0.95	6.77	22.56	44.91	68.19	85.59	97.19	100.00	100.00	
21	31	0.02	0.04	0.10	1.01	8.75	29.64	56.09	76.41	89.97	97.76	100.00	100.00	
22	32	0.30	0.74	2.14	6.45	16.55	31.85	49.99	68.45	84.12	94.83	100.00	100.00	
		<b>0.061</b>	<b>0.074</b>	<b>0.088</b>	<b>0.104</b>	<b>0.124</b>	<b>0.147</b>	<b>0.175</b>	<b>0.208</b>	<b>0.246</b>	<b>0.351</b>	<b>0.495</b>	<b>0.701</b>	<b>0.991mm</b>
23	33	16.72	20.36	44.81	61.15	82.39	94.10	98.93	99.65	99.83	99.95	99.98	100.00	100.00
24	34	7.81	11.76	32.10	46.79	72.05	88.05	97.97	99.09	99.49	99.94	99.97	99.99	100.00
25	35	2.17	6.46	21.33	31.62	62.58	83.49	96.62	98.91	99.49	99.92	99.98	99.99	100.00
26	35	3.64	9.44	30.48	43.24	73.08	90.39	98.29	99.61	99.84	99.98	100.00	100.00	100.00
27	35	3.35	8.63	30.02	41.48	73.00	90.51	98.57	99.63	99.83	99.98	100.00	100.00	100.00
28	36	1.48	4.66	17.04	26.23	56.93	81.37	96.68	98.92	99.47	99.91	99.98	99.99	100.00
29	37	3.06	8.01	29.23	40.21	71.13	91.23	98.74	99.69	99.88	99.98	100.00	100.00	100.00

Table 3.3. Laboratory Data of Einstein and Chien (1953)

Data No. (1)	Run ID (2)	Flow Properties					Bed Material				Transported Sediment			
		Q (m <sup>3</sup> /s) (3)	W (m) (4)	d (m) (5)	S (m/m) (6)	T (°C) (7)	D <sub>50</sub> <sup>*</sup> (mm) (8)	D <sub>65</sub> <sup>*</sup> (mm) (9)	σ <sub>g</sub> <sup>*</sup> (10)	s <sub>g</sub> (11)	C <sub>T</sub> (kg/m <sup>3</sup> ) (12)	C <sub>T</sub> (PPM) (13)	D <sub>50t</sub> <sup>*</sup> (mm) (14)	σ <sub>gt</sub> <sup>*</sup> (15)
1	A-1	0.0623	0.3048	0.1981	0.003752	18.3	0.193	0.271	2.065	2.65	19.51	19277.42	0.096	1.431
2	A-2	0.0628	0.3048	0.2003	0.004894	24.4	0.366	0.412	1.413	2.65	2.14	2136.61	0.328	1.417
3	A-3	0.0627	0.3048	0.1996	0.003520	22.2	0.187	0.254	2.071	2.65	17.86	17664.50	0.096	1.411
4	A-4	0.0629	0.3048	0.1966	0.003760	16.7	0.279	0.371	2.191	2.65	15.87	15717.37	0.095	1.462
5	B-1	0.0629	0.3048	0.1878	0.003948	12.8	0.218	0.315	2.138	2.65	12.91	12811.59	0.103	1.574
6	B-2	0.0629	0.3048	0.2009	0.004188	14.4	0.173	0.243	2.090	2.65	13.56	13450.35	0.096	1.449
7	B-3	0.0626	0.3048	0.1996	0.003328	8.9	0.145	0.197	2.030	2.65	16.38	16214.02	0.096	1.442
8	B-4	0.0629	0.3048	0.1984	0.003578	11.1	0.141	0.195	2.094	2.65	17.24	17060.36	0.093	1.420
9	B-5	0.0629	0.3048	0.1996	0.003661	10.6	0.129	0.174	2.100	2.65	18.76	18542.08	0.092	1.406
10	C-1	0.0629	0.3048	0.1932	0.003050	16.1	0.158	0.228	2.417	2.65	80.08	76281.29	0.054	1.597
11	C-2	0.0623	0.3048	0.2030	0.003931	20.6	0.151	0.218	2.491	2.65	117.94	109874.57	0.050	1.564
12	C-3	0.0623	0.3048	0.1954	0.003680	14.4	0.145	0.208	2.479	2.65	148.67	136071.28	0.048	1.572
13	C-4	0.0623	0.3048	0.1945	0.003460	15.6	0.189	0.265	2.293	2.65	35.31	34550.59	0.059	1.782
14	C-5	0.0623	0.3048	0.1942	0.003945	13.9	0.233	0.313	2.071	2.65	12.52	12425.50	0.109	1.782
15	D-1	0.0629	0.3048	0.2140	0.004448	17.8	0.161	0.217	2.163	2.65	36.15	35350.86	0.057	1.636
16	D-4	0.0430	0.3048	0.1774	0.003063	17.8	0.140	0.211	2.658	2.65	79.77	75994.43	0.045	1.445
17	D-5	0.0439	0.3048	0.1978	0.003015	22.8	0.270	0.357	2.952	2.65	78.53	74873.71	0.045	1.335
18	D-6	0.0657	0.3048	0.1945	0.003865	26.1	0.351	0.490	2.324	2.65	28.03	27548.95	0.094	1.609
19	D-7	0.0430	0.3048	0.1850	0.003000	20.0	0.307	0.438	2.573	2.65	19.71	19466.29	0.054	1.607
20	D-8	0.0620	0.3048	0.1847	0.003655	26.1	0.135	0.174	1.971	2.65	42.05	40980.71	0.060	1.850
21	D-9	0.0428	0.3048	0.1871	0.001570	27.8	0.104	0.136	2.001	2.65	30.02	29467.84	0.046	1.550
22	E-1	0.0629	0.3048	0.1847	0.003485	21.1	0.203	0.278	2.420	2.65	52.49	50827.05	0.054	1.696

Note: \* — Values computed from adjusted bed material size distribution (wash load materials excluded).

Table 3.3. Laboratory Data of Einstein and Chien (1953) (continued)

Data No. (1)	Run ID (2)	D <sub>c</sub> (mm) (16)	Size distribution of bed material, finer than indicated diameters														
			Grp1 (%) (17)	Grp2 (%) (18)	Grp3 (%) (19)	Grp4 (%) (20)	Grp5 (%) (21)	Grp6 (%) (22)	Grp7 (%) (23)	Grp8 (%) (24)	Grp9 (%) (25)	Grp10 (%) (26)	Grp11 (%) (27)	Grp12 (%) (28)	Grp13 (%) (29)	Grp14 (%) (30)	Grp15 (%) (31)
			<b>0.061</b>	<b>0.074</b>	<b>0.104</b>	<b>0.147</b>	<b>0.208</b>	<b>0.295</b>	<b>0.417</b>	<b>0.589</b>	<b>0.833</b>	<b>1.168mm</b>					
1	A-1	0.061	2.51	8.28	18.45	38.30	54.82	69.22	82.43	95.15	99.34	99.80					
2	A-2	0.061	0.00	0.04	0.14	0.25	2.30	23.31	66.14	93.45	99.38	99.65					
3	A-3	0.061	3.11	9.64	21.14	40.66	56.00	73.08	84.44	94.45	99.06	99.58					
4	A-4	0.061	1.69	5.33	11.75	23.60	36.31	52.90	71.00	83.52	93.42	98.89					
5	B-1	0.061	0.00	5.07	14.44	31.69	47.92	61.74	78.00	90.21	97.31	99.66					
6	B-2	0.061	0.00	6.17	21.04	41.88	59.08	71.86	84.95	93.56	98.11	99.73					
7	B-3	0.061	0.00	9.55	28.04	51.01	67.44	78.99	89.89	95.86	98.73	99.71					
8	B-4	0.061	0.00	10.67	30.68	52.41	67.59	78.25	88.66	95.43	98.90	99.79					
9	B-5	0.061	0.00	12.22	35.17	58.81	71.36	79.84	88.71	94.85	98.59	99.83					
			<b>0.008</b>	<b>0.013</b>	<b>0.02</b>	<b>0.03</b>	<b>0.045</b>	<b>0.061</b>	<b>0.074</b>	<b>0.104</b>	<b>0.147</b>	<b>0.208</b>	<b>0.295</b>	<b>0.417</b>	<b>0.589</b>	<b>0.833</b>	<b>1.168mm</b>
10	C-1	0.03	0.46	1.17	2.60	5.30	11.00	18.00	23.60	36.00	49.50	63.00	76.00	86.70	93.40	97.50	99.40
11	C-2	0.03	0.52	1.32	3.00	6.70	13.40	21.00	26.50	38.20	52.00	65.20	77.00	86.60	93.20	97.20	99.20
12	C-3	0.03	0.87	2.40	5.10	10.00	17.00	24.00	30.00	42.00	55.00	68.00	78.40	87.50	93.50	97.20	99.20
13	C-4	0.03	0.10	0.41	1.25	3.05	7.00	11.00	15.20	25.50	39.00	56.00	70.00	83.00	92.00	97.10	99.42
14	C-5	0.061	0.00	0.00	0.00	0.55	2.00	4.80	7.70	16.00	29.00	46.00	63.00	78.00	89.00	95.30	98.30
15	D-1	0.03	0.13	0.40	1.00	2.60	6.20	12.00	17.00	30.00	46.50	64.00	78.50	89.00	95.70	98.75	99.82
16	D-4	0.03	0.62	1.50	3.50	7.80	16.00	24.50	30.50	42.50	55.00	66.50	77.00	85.20	91.50	96.20	98.70
17	D-5	0.03	0.30	0.90	2.50	6.30	14.00	22.00	26.00	31.50	35.50	43.00	56.00	75.50	90.00	96.70	99.22
18	D-6	0.061	0.00	0.00	0.03	0.18	0.90	2.50	4.40	10.00	19.00	29.00	42.00	54.50	70.50	84.00	94.00
19	D-7	0.03	0.40	0.80	1.58	3.00	5.50	8.50	11.00	17.00	25.00	35.00	46.00	59.00	72.00	84.00	92.00
20	D-8	0.03	0.13	0.56	1.80	4.70	10.50	18.00	24.00	38.50	57.00	76.70	91.30	97.70	99.60	100.00	100.00
21	D-9	0.03	0.83	2.60	6.10	12.20	22.00	33.00	40.50	56.00	73.00	87.00	95.50	98.90	99.81	100.00	100.00
22	E-1	0.03	0.00	0.90	2.20	4.60	8.70	13.40	17.20	26.00	38.00	52.40	68.00	79.50	89.00	94.80	98.00

Table 3.3. Laboratory Data of Einstein and Chien (1953) (continued)

Data No.	Run ID	Size distribution of transported sediment, finer than indicated diameters														
		Grp1 (%) (32)	Grp2 (%) (33)	Grp3 (%) (34)	Grp4 (%) (35)	Grp5 (%) (36)	Grp6 (%) (37)	Grp7 (%) (38)	Grp8 (%) (39)	Grp9 (%) (40)	Grp10 (%) (41)	Grp11 (%) (42)	Grp12 (%) (43)	Grp13 (%) (44)	Grp14 (%) (45)	Grp15 (%) (46)
		<b>0.061</b>	<b>0.074</b>	<b>0.104</b>	<b>0.147</b>	<b>0.208</b>	<b>0.295</b>	<b>0.417</b>	<b>0.589</b>	<b>0.833</b>	<b>1.168mm</b>					
1	A-1	17.21	34.81	65.81	87.78	93.91	96.28	97.99	99.09	99.77	99.93					
2	A-2	0.91	1.45	2.40	3.04	7.30	38.25	78.29	96.65	99.69	99.85					
3	A-3	15.25	33.42	65.43	89.66	95.63	97.40	98.45	99.17	99.81	99.92					
4	A-4	22.60	39.44	69.10	87.21	92.83	95.45	97.33	98.58	99.50	99.93					
5	B-1	8.91	25.55	54.87	80.17	89.25	92.96	95.85	97.71	99.13	99.86					
6	B-2	17.26	35.04	66.37	86.55	92.84	95.29	97.00	98.35	99.33	99.86					
7	B-3	13.95	32.42	64.95	86.36	93.38	96.06	97.83	98.89	99.58	99.93					
8	B-4	14.62	34.23	68.77	88.78	94.49	96.68	98.13	99.06	99.63	99.90					
9	B-5	15.54	35.54	70.78	90.17	95.05	96.80	98.05	98.95	99.58	99.92					
		<b>0.008</b>	<b>0.013</b>	<b>0.02</b>	<b>0.03</b>	<b>0.045</b>	<b>0.061</b>	<b>0.074</b>	<b>0.104</b>	<b>0.147</b>	<b>0.208</b>	<b>0.295</b>	<b>0.417</b>	<b>0.589</b>	<b>0.833</b>	<b>1.168mm</b>
10	C-1	17.40	26.00	37.50	51.70	68.00	81.00	87.60	94.60	97.70	99.10	99.60	99.80	99.89	99.94	99.97
11	C-2	21.00	32.00	45.00	60.00	76.50	87.00	91.50	96.20	98.20	99.15	99.50	99.65	99.78	99.87	99.94
12	C-3	18.80	29.00	43.00	59.50	77.70	87.50	91.50	96.10	98.40	99.26	99.61	99.73	99.82	99.90	99.95
13	C-4	13.00	20.00	29.20	42.00	58.50	72.50	79.50	88.50	93.50	96.45	97.60	98.45	99.04	99.46	99.76
14	C-5	1.80	4.70	10.00	19.00	35.00	50.00	58.50	73.50	84.00	90.50	94.20	96.50	98.10	99.05	99.59
15	D-1	15.00	25.00	37.00	50.80	66.00	78.50	85.00	93.30	97.10	98.70	99.30	99.55	99.75	99.91	99.97
16	D-4	23.50	40.00	56.00	72.50	86.50	93.80	96.00	98.15	99.25	99.56	99.72	99.80	99.86	99.91	99.95
17	D-5	17.00	29.00	44.00	61.00	80.00	93.70	97.40	99.00	99.38	99.49	99.54	99.63	99.75	99.87	99.95
18	D-6	0.00	8.49	20.60	32.00	48.00	64.00	73.20	85.20	91.90	95.10	96.60	97.35	98.05	98.72	99.30
19	D-7	13.00	22.00	33.60	48.00	66.20	79.20	86.50	94.00	97.50	98.60	99.10	99.40	99.60	99.78	99.90
20	D-8	16.50	28.00	40.20	53.00	67.00	77.00	82.50	89.70	94.40	97.00	98.55	99.36	99.73	100.00	100.00
21	D-9	20.40	34.50	50.60	69.00	84.00	91.00	94.00	97.30	98.80	99.44	99.74	99.87	99.93	100.00	100.00
22	E-1	16.50	28.00	42.00	56.50	72.00	82.00	87.00	93.00	96.20	98.10	98.92	99.32	99.56	99.74	99.85

Table 3.4. Laboratory Data of Samaga et al. (1986a, b)

Data No. (1)	Run ID (2)	Flow Properties					Bed Material				Transported Sediment			
		Q (m <sup>3</sup> /s) (3)	W (m) (4)	d (m) (5)	S (m/m) (6)	T (°C) (7)	D <sub>50</sub> * (mm) (8)	D <sub>65</sub> * (mm) (9)	σ <sub>g</sub> * (10)	s <sub>g</sub> (11)	C <sub>T</sub> (kg/m <sup>3</sup> ) (12)	C <sub>T</sub> (PPM) (13)	D <sub>50t</sub> * (mm) (14)	σ <sub>gt</sub> * (15)
1	M1-1	0.0068	0.200	0.057	0.005760	24.5	0.403	0.639	2.458	2.65	4.6321	4618.81	0.659	2.511
2	M1-2	0.0104	0.200	0.077	0.005760	23.0	0.403	0.639	2.458	2.65	4.9910	4975.52	0.489	2.843
3	M1-3	0.0133	0.200	0.092	0.005760	22.0	0.403	0.639	2.458	2.65	5.7284	5708.03	0.639	2.559
4	M1-4	0.0081	0.200	0.064	0.006870	18.5	0.403	0.639	2.458	2.65	6.7757	6747.28	0.649	2.457
5	M1-5	0.0109	0.200	0.080	0.006870	19.5	0.403	0.639	2.458	2.65	6.9021	6872.53	0.655	2.446
6	M1-6	0.0133	0.200	0.091	0.006870	20.2	0.403	0.639	2.458	2.65	7.0541	7023.30	0.709	2.386
7	M1-7	0.0056	0.200	0.057	0.005047	20.5	0.403	0.639	2.458	2.65	3.4479	3440.48	0.551	2.443
8	M1-8	0.0114	0.200	0.089	0.005047	17.5	0.403	0.639	2.458	2.65	6.4285	6402.85	0.629	2.551
9	M1-9	0.0146	0.200	0.101	0.005047	17.0	0.403	0.639	2.458	2.65	6.1268	6103.51	0.745	2.263
10	M2-1	0.0075	0.200	0.072	0.004960	16.0	0.316	0.514	2.373	2.65	3.7577	3748.93	0.315	2.301
11	M2-2	0.0108	0.200	0.080	0.004960	16.0	0.316	0.514	2.373	2.65	5.0442	5028.39	0.552	2.830
12	M2-3	0.0124	0.200	0.092	0.004960	14.5	0.316	0.514	2.373	2.65	6.0750	6052.15	0.606	2.752
13	M2-4	0.0083	0.200	0.069	0.006048	14.5	0.316	0.514	2.373	2.65	7.2608	7228.17	0.596	2.652
14	M2-5	0.0116	0.200	0.083	0.006048	14.5	0.316	0.514	2.373	2.65	7.4719	7437.30	0.552	2.826
15	M2-6	0.0144	0.200	0.100	0.006048	15.3	0.316	0.514	2.373	2.65	8.3955	8351.81	0.654	2.747
16	M2-7	0.0091	0.200	0.070	0.006926	14.0	0.316	0.514	2.373	2.65	7.4258	7391.61	0.692	2.454
17	M2-8	0.0118	0.200	0.084	0.006926	16.3	0.316	0.514	2.373	2.65	9.2762	9222.98	0.722	2.433
18	M2-9	0.0143	0.200	0.093	0.006926	16.5	0.316	0.514	2.373	2.65	9.1903	9138.02	0.659	2.709
19	M3-1	0.0109	0.200	0.078	0.006926	18.5	0.281	0.439	2.196	2.65	10.3286	10262.65	0.597	2.266
20	M3-2	0.0129	0.200	0.087	0.006926	23.0	0.281	0.439	2.196	2.65	9.0970	9045.75	0.591	2.260
21	M3-3	0.0088	0.200	0.068	0.006926	23.5	0.281	0.439	2.196	2.65	8.3844	8340.87	0.605	2.197
22	M3-4	0.0075	0.200	0.066	0.005415	24.5	0.281	0.439	2.196	2.65	7.3982	7364.32	0.409	2.310
23	M3-5	0.0101	0.200	0.077	0.005415	22.1	0.281	0.439	2.196	2.65	7.1173	7085.94	0.568	2.245
24	M3-6	0.0134	0.200	0.091	0.005415	25.0	0.281	0.439	2.196	2.65	5.8694	5847.99	0.598	2.203
25	M3-7	0.0081	0.200	0.064	0.006097	23.5	0.281	0.439	2.196	2.65	7.5946	7558.90	0.560	2.239
26	M3-8	0.0063	0.200	0.056	0.006097	25.0	0.281	0.439	2.196	2.65	6.1781	6154.41	0.373	2.212
27	M3-9	0.0129	0.200	0.086	0.006097	24.5	0.281	0.439	2.196	2.65	7.7749	7737.44	0.566	2.201
28	M4-1	0.0081	0.200	0.062	0.006097	24.5	0.212	0.249	1.481	2.65	5.8615	5840.15	0.301	1.771
29	M4-2	0.0121	0.200	0.080	0.006097	25.5	0.212	0.249	1.481	2.65	7.5698	7534.28	0.320	1.772
30	M4-3	0.0083	0.200	0.068	0.005268	26.0	0.212	0.249	1.481	2.65	5.6843	5664.25	0.304	1.983
31	M4-4	0.0132	0.200	0.087	0.005268	26.0	0.212	0.249	1.481	2.65	6.4086	6383.09	0.296	1.950
32	M4-5	0.0085	0.200	0.068	0.004487	26.0	0.212	0.249	1.481	2.65	4.3627	4350.91	0.304	1.888
33	M4-6	0.0135	0.200	0.091	0.004487	26.5	0.212	0.249	1.481	2.65	4.5958	4582.71	0.298	1.902

Note: \* — Values computed from adjusted bed material size distribution (wash load materials excluded).

Table 3.4. Laboratory Data of Samaga et al. (1986a, b) (continued)

Data No. (1)	Run ID (2)	D <sub>c</sub> (mm) (16)	Size distribution of bed material, finer than indicated diameters									
			Grp1 (%) (17)	Grp2 (%) (18)	Grp3 (%) (19)	Grp4 (%) (20)	Grp5 (%) (21)	Grp6 (%) (22)	Grp7 (%) (23)	Grp8 (%) (24)	Grp9 (%) (25)	Grp10 (%) (26)
			<b>0.0849</b>	<b>0.158</b>	<b>0.170</b>	<b>0.265</b>	<b>0.329</b>	<b>0.539</b>	<b>0.927</b>	<b>1.502</b>	<b>1.856</b>	<b>3.000 mm</b>
1	M1-1	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
2	M1-2	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
3	M1-3	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
4	M1-4	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
5	M1-5	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
6	M1-6	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
7	M1-7	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
8	M1-8	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
9	M1-9	0.158	1.00	6.20	10.80	39.20	47.21	61.45	79.58	89.38	94.76	100.00
10	M2-1	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
11	M2-2	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
12	M2-3	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
13	M2-4	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
14	M2-5	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
15	M2-6	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
16	M2-7	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
17	M2-8	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
18	M2-9	0.158	0.50	6.30	14.60	45.34	54.90	68.51	82.48	91.86	96.00	100.00
19	M3-1	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
20	M3-2	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
21	M3-3	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
22	M3-4	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
23	M3-5	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
24	M3-6	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
25	M3-7	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
26	M3-8	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
27	M3-9	0.0849	2.60	12.40	19.30	48.66	58.20	71.42	88.70	99.31	100.00	100.00
28	M4-1	0.0849	4.80	20.50	32.90	72.37	84.02	95.50	99.89	100.00	100.00	100.00
29	M4-2	0.0849	4.80	20.50	32.90	72.37	84.02	95.50	99.89	100.00	100.00	100.00
30	M4-3	0.0849	4.80	20.50	32.90	72.37	84.02	95.50	99.89	100.00	100.00	100.00
31	M4-4	0.0849	4.80	20.50	32.90	72.37	84.02	95.50	99.89	100.00	100.00	100.00
32	M4-5	0.0849	4.80	20.50	32.90	72.37	84.02	95.50	99.89	100.00	100.00	100.00
33	M4-6	0.0849	4.80	20.50	32.90	72.37	84.02	95.50	99.89	100.00	100.00	100.00

Table 3.4. Laboratory Data of Samaga et al. (1986a, b) (continued)

Data No. (1)	Run ID (2)	Size distribution of sediment load, finer than indicated diameters									
		Grp1 (%) (27)	Grp2 (%) (28)	Grp3 (%) (29)	Grp4 (%) (30)	Grp5 (%) (31)	Grp6 (%) (32)	Grp7 (%) (33)	Grp8 (%) (34)	Grp9 (%) (35)	Grp10 (%) (36)
		<b>0.0849</b>	<b>0.158</b>	<b>0.170</b>	<b>0.265</b>	<b>0.329</b>	<b>0.539</b>	<b>0.927</b>	<b>1.502</b>	<b>1.856</b>	<b>3.000 mm</b>
1	M1-1	0.95	3.69	8.60	20.17	30.90	39.79	72.31	78.93	97.57	100.00
2	M1-2	1.34	2.25	13.46	32.61	44.87	52.70	78.37	83.45	98.23	100.00
3	M1-3	1.12	2.27	8.34	20.09	30.86	41.81	71.58	78.42	98.43	100.00
4	M1-4	0.89	1.91	3.61	18.24	28.79	39.36	73.22	80.50	98.57	100.00
5	M1-5	0.93	2.49	7.87	17.92	31.23	40.57	70.32	80.26	95.45	100.00
6	M1-6	0.82	1.45	5.46	14.44	22.56	33.66	67.32	75.98	98.33	100.00
7	M1-7	1.14	1.40	4.24	19.17	33.75	49.48	79.87	84.27	98.04	100.00
8	M1-8	0.92	1.64	7.03	20.13	32.30	41.97	73.04	80.25	98.97	100.00
9	M1-9	0.71	1.41	5.70	9.44	18.42	29.88	64.70	75.59	96.89	100.00
10	M2-1	0.72	3.01	13.31	40.38	54.20	62.04	83.89	88.85	98.29	100.00
11	M2-2	0.91	3.97	12.74	32.63	46.32	51.03	74.56	80.64	97.59	100.00
12	M2-3	0.62	2.73	10.18	26.64	37.75	45.82	71.42	78.98	96.81	100.00
13	M2-4	0.31	2.15	7.71	25.88	37.07	45.07	77.35	83.24	97.05	100.00
14	M2-5	0.49	2.32	11.40	30.85	42.09	50.11	74.12	80.85	96.33	100.00
15	M2-6	0.37	1.90	9.31	24.37	32.11	40.73	69.52	76.42	95.84	100.00
16	M2-7	0.21	1.47	5.35	15.93	26.04	35.87	68.13	76.34	95.07	100.00
17	M2-8	0.20	0.91	5.34	15.47	24.25	32.69	65.57	76.66	97.79	100.00
18	M2-9	0.70	1.86	11.38	23.34	31.64	39.45	70.50	80.64	97.71	100.00
19	M3-1	2.44	5.84	13.70	25.38	35.27	44.54	80.08	100.00	100.00	100.00
20	M3-2	3.33	6.39	13.74	26.25	37.31	45.91	79.70	100.00	100.00	100.00
21	M3-3	2.65	5.08	12.68	23.62	34.55	43.57	80.08	100.00	100.00	100.00
22	M3-4	3.63	6.71	19.88	37.48	48.07	56.53	86.03	100.00	100.00	100.00
23	M3-5	3.58	6.41	14.16	27.71	38.99	48.55	81.47	100.00	100.00	100.00
24	M3-6	2.90	5.60	13.54	24.76	34.81	44.27	81.54	100.00	100.00	100.00
25	M3-7	3.32	6.06	16.55	28.67	39.76	49.07	84.61	100.00	100.00	100.00
26	M3-8	1.99	3.80	15.42	34.69	48.49	58.37	87.35	100.00	100.00	100.00
27	M3-9	3.45	6.63	14.46	26.61	38.23	48.61	82.99	100.00	100.00	100.00
28	M4-1	3.44	9.41	18.11	38.58	60.76	84.39	100.00	100.00	100.00	100.00
29	M4-2	3.15	9.09	12.71	31.87	54.55	79.71	100.00	100.00	100.00	100.00
30	M4-3	2.47	6.34	19.55	38.68	58.72	74.83	100.00	100.00	100.00	100.00
31	M4-4	3.55	8.77	21.45	42.27	60.89	77.68	100.00	100.00	100.00	100.00
32	M4-5	3.55	8.32	17.93	36.70	60.20	79.07	100.00	100.00	100.00	100.00
33	M4-6	3.71	8.67	17.76	39.31	62.50	78.16	100.00	100.00	100.00	100.00

Table 3.5. Niobrara River Data of Colby and Hembree (1955)

Data No. (1)	Survey Date (yymmdd) (2)	Flow Properties					Bed Material				Transported Sediment			
		Q (m <sup>3</sup> /s) (3)	W (m) (4)	d (m) (5)	S (m/m) (6)	T (°C) (7)	D <sub>50</sub> * (mm) (8)	D <sub>65</sub> * (mm) (9)	σ <sub>g</sub> * (10)	s <sub>g</sub> (11)	C <sub>T</sub> (kg/m <sup>3</sup> ) (12)	C <sub>T</sub> (PPM) (13)	D <sub>50t</sub> * (mm) (14)	σ <sub>gt</sub> * (15)
1	490713	7.56	21.49	0.47	0.001345	23.9	0.304	0.363	1.619	2.65	0.9706	970.00	0.248	1.679
2	500303	11.35	21.34	0.49	0.001705	5.0	0.307	0.411	2.345	2.65	1.8922	1890.00	0.209	1.618
3	500414	11.72	21.64	0.49	0.001705	6.7	0.286	0.348	1.643	2.65	2.0025	2000.00	0.242	1.645
4	500511	16.05	21.95	0.58	0.001799	11.7	0.215	0.256	1.573	2.65	2.2231	2220.00	0.227	1.650
5	500607	7.64	21.34	0.48	0.001269	18.3	0.286	0.355	1.687	2.65	0.7804	780.00	0.237	1.649
6	500613	6.65	21.49	0.44	0.001288	23.3	0.292	0.347	1.594	2.65	0.7904	790.00	0.240	1.647
7	500709	7.16	21.18	0.46	0.001288	22.2	0.337	0.403	1.660	2.65	0.9105	910.00	0.237	1.652
8	500802	7.22	21.34	0.44	0.001174	17.2	0.306	0.366	1.625	2.65	1.0006	1000.00	0.230	1.646
9	500830	9.74	21.34	0.47	0.001420	15.6	0.262	0.327	1.653	2.65	1.7820	1780.00	0.263	1.657
10	500920	9.42	21.03	0.48	0.001402	16.1	0.327	0.380	1.514	2.65	1.4914	1490.00	0.223	1.655
11	510427	12.88	21.64	0.53	0.001686	14.4	0.314	0.382	1.669	2.65	1.9023	1900.00	0.236	1.670
12	520619	6.62	21.34	0.47	0.001250	20.6	0.285	0.350	1.663	2.65	0.7544	754.00	0.223	1.625
13	520704	7.78	21.34	0.49	0.001288	22.8	0.314	0.374	1.609	2.65	0.9345	934.00	0.268	1.650
14	520720	6.57	21.34	0.43	0.001136	24.4	0.280	0.341	1.638	2.65	0.5032	503.00	0.230	1.632
15	520731	5.91	21.03	0.42	0.001250	28.3	0.320	0.384	1.631	2.65	0.3921	392.00	0.235	1.636
16	520816	7.53	21.34	0.49	0.001155	22.2	0.299	0.378	1.824	2.65	0.8204	820.00	0.238	1.625
17	520829	5.86	21.34	0.44	0.001212	22.8	0.349	0.428	1.956	2.65	0.4291	429.00	0.219	1.596
18	520926	6.65	21.18	0.47	0.001136	16.1	0.300	0.362	1.638	2.65	0.7363	736.00	0.218	1.603
19	521211	9.40	21.34	0.43	0.001610	0.6	0.334	0.395	1.575	2.65	1.2209	1220.00	0.223	1.611

Note: \* — Values computed from adjusted bed material size distribution (wash load materials excluded).

Table 3.5. Niobrara River Data of Colby and Hembree (1955) (continued)

Data No. (1)	Survey Date (yymmdd) (2)	D <sub>c</sub> (mm) (16)	Size distribution of bed material, finer than indicated diameters							
			Grp1 (%) (17)	Grp2 (%) (18)	Grp3 (%) (19)	Grp4 (%) (20)	Grp5 (%) (21)	Grp6 (%) (22)	Grp7 (%) (23)	Grp8 (%) (24)
			<b>0.062</b>	<b>0.125</b>	<b>0.25</b>	<b>0.5</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>8 mm</b>
1	490713	0.125	0.0	2.0	35.0	92.0	98.0	99.0	100.0	100.0
2	500303	0.125	0.0	4.0	42.0	76.0	86.0	92.0	98.0	100.0
3	500414	0.125	0.0	4.0	42.0	93.0	99.0	100.0	100.0	100.0
4	500511	0.125	0.0	6.0	66.0	100.0	100.0	100.0	100.0	100.0
5	500607	0.125	0.0	2.0	42.0	89.0	95.0	98.0	100.0	100.0
6	500613	0.125	0.0	1.0	37.0	97.0	99.0	100.0	100.0	100.0
7	500709	0.125	0.0	1.0	26.0	83.0	95.0	98.0	99.0	100.0
8	500802	0.125	0.0	1.0	34.0	91.0	97.0	99.0	100.0	100.0
9	500830	0.125	0.0	4.0	49.0	94.0	99.0	100.0	100.0	100.0
10	500920	0.125	0.0	23.0	41.0	94.0	98.0	99.0	100.0	100.0
11	510427	0.125	0.0	2.0	34.0	86.0	96.0	99.0	100.0	100.0
12	520619	0.125	0.0	1.0	41.0	91.0	99.0	100.0	100.0	100.0
13	520704	0.125	0.0	1.0	31.0	90.0	97.0	99.0	100.0	100.0
14	520720	0.125	0.0	1.0	42.0	94.0	98.0	99.0	100.0	100.0
15	520731	0.125	0.0	2.0	31.0	87.0	97.0	99.0	100.0	100.0
16	520816	0.125	0.0	2.0	40.0	83.0	92.0	95.0	99.0	100.0
17	520829	0.125	1.0	2.0	27.0	77.0	90.0	96.0	99.0	100.0
18	520926	0.125	0.0	1.0	36.0	91.0	99.0	100.0	100.0	100.0
19	521211	0.125	0.0	1.0	25.0	86.0	96.0	98.0	99.0	100.0

Table 3.5. Niobrara River Data of Colby and Hembree (1955) (continued)

Data No. (1)	Survey Date (yymmdd) (2)	Size distribution of sediment load, finer than indicated diameters							
		Grp1 (%) (25)	Grp2 (%) (26)	Grp3 (%) (27)	Grp4 (%) (28)	Grp5 (%) (29)	Grp6 (%) (30)	Grp7 (%) (31)	Grp8 (%) (32)
		<b>0.062</b>	<b>0.125</b>	<b>0.25</b>	<b>0.5</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>8 mm</b>
1	490713	7.0	15.0	58.0	93.0	99.0	100.0	100.0	100.0
2	500303	13.0	33.0	78.0	96.0	97.0	100.0	100.0	100.0
3	500414	7.0	20.0	62.0	96.0	100.0	100.0	100.0	100.0
4	500511	15.0	33.0	72.0	96.0	100.0	100.0	100.0	100.0
5	500607	10.0	26.0	66.0	96.0	100.0	100.0	100.0	100.0
6	500613	8.0	23.0	64.0	96.0	100.0	100.0	100.0	100.0
7	500709	14.0	30.0	68.0	96.0	100.0	100.0	100.0	100.0
8	500802	10.0	26.0	68.0	96.0	100.0	100.0	100.0	100.0
9	500830	27.0	40.0	68.0	96.0	100.0	100.0	100.0	100.0
10	500920	9.0	25.0	70.0	95.0	100.0	100.0	100.0	100.0
11	510427	12.0	32.0	69.0	95.0	100.0	100.0	100.0	100.0
12	520619	11.0	25.0	70.0	97.0	100.0	100.0	100.0	100.0
13	520704	9.0	20.0	56.0	95.0	100.0	100.0	100.0	100.0
14	520720	12.0	26.0	68.0	97.0	100.0	100.0	100.0	100.0
15	520731	13.0	29.0	68.0	97.0	100.0	100.0	100.0	100.0
16	520816	14.0	33.0	69.0	98.0	100.0	100.0	100.0	100.0
17	520829	23.0	40.0	77.0	99.0	100.0	100.0	100.0	100.0
18	520926	9.0	23.0	71.0	98.0	100.0	100.0	100.0	100.0
19	521211	9.0	23.0	69.0	98.0	100.0	100.0	100.0	100.0

Table 3.6. Middle Loup River Data of Hubbell and Matejka (1959)

Data No. (1)	Survey Date (yymmdd) (2)	Flow Properties					Bed Material				Transported Sediment			
		Q (m <sup>3</sup> /s) (3)	W (m) (4)	d (m) (5)	S (m/m) (6)	T (°C) (7)	D <sub>50</sub> <sup>*</sup> (mm) (8)	D <sub>65</sub> <sup>*</sup> (mm) (9)	σ <sub>g</sub> <sup>*</sup> (10)	s <sub>g</sub> (11)	C <sub>T</sub> (kg/m <sup>3</sup> ) (12)	C <sub>T</sub> (PPM) (13)	D <sub>50t</sub> <sup>*</sup> (mm) (14)	σ <sub>gt</sub> <sup>*</sup> (15)
1	500301	12.23	43.89	0.30	0.000928	3.9	0.278	0.347	1.684	2.65	2.3133	2310.00	0.241	1.769
2	500321	11.61	42.98	0.29	0.001439	4.4	0.292	0.364	1.707	2.65	2.4437	2440.00	0.248	1.709
3	500425	11.72	44.20	0.30	0.001023	10.6	0.335	0.420	1.976	2.65	1.4814	1480.00	0.223	1.691
4	500509	11.30	44.20	0.34	0.001250	16.1	0.382	0.471	1.926	2.65	1.3411	1340.00	0.223	1.772
5	500606	10.31	43.89	0.32	0.001458	24.4	0.317	0.399	1.980	2.65	0.6322	632.00	0.245	1.698
6	500706	10.45	43.28	0.36	0.001250	21.7	0.424	0.586	2.403	2.65	0.6873	687.00	0.250	1.862
7	501108	12.54	45.11	0.25	0.001345	2.8	0.219	0.268	1.651	2.65	1.4213	1420.00	0.230	1.721
8	510330	10.22	44.20	0.33	0.001345	10.0	0.339	0.416	1.849	2.65	1.4112	1410.00	0.237	1.758
9	510726	10.39	44.81	0.37	0.001288	31.1	0.383	0.476	2.301	2.65	0.5482	548.00	0.259	1.721
10	511031	12.09	46.33	0.31	0.001307	3.9	0.351	0.428	1.877	2.65	1.6417	1640.00	0.272	2.176
11	511205	11.30	37.49	0.33	0.001174	1.1	0.351	0.435	1.952	2.65	2.0225	2020.00	0.240	1.805
12	520718	9.34	44.81	0.33	0.001420	30.6	0.282	0.365	1.808	2.65	0.8525	852.00	0.278	2.048
13	520813	10.93	45.11	0.33	0.001326	26.1	0.334	0.424	2.095	2.65	0.6863	686.00	0.277	1.869
14	520911	9.46	45.42	0.31	0.001288	20.0	0.351	0.440	2.153	2.65	1.0407	1040.00	0.350	2.237
15	520924	10.36	45.11	0.33	0.001307	18.3	0.274	0.344	1.687	2.65	1.0206	1020.00	0.302	1.961

Note: \* — Values computed from adjusted bed material size distribution (wash load materials excluded).

Table 3.6. Middle Loup River Data of Hubbell and Matejka (1959) (continued)

Data No. (1)	Survey Date (yymmdd) (2)	D <sub>c</sub> (mm) (16)	Size distribution of bed material, finer than indicated diameters							
			Grp1 (%) (17)	Grp2 (%) (18)	Grp3 (%) (19)	Grp4 (%) (20)	Grp5 (%) (21)	Grp6 (%) (22)	Grp7 (%) (23)	Grp8 (%) (24)
			<b>0.062</b>	<b>0.125</b>	<b>0.25</b>	<b>0.5</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>8 mm</b>
1	500301	0.125	0.0	2.0	44.0	90.0	97.0	99.0	99.0	100.0
2	500321	0.125	0.0	4.0	42.0	87.0	96.0	98.0	99.0	100.0
3	500425	0.125	0.0	2.0	32.0	77.0	92.0	96.0	98.0	100.0
4	500509	0.125	0.0	2.0	21.0	70.0	90.0	94.0	96.0	100.0
5	500606	0.125	0.0	2.0	36.0	80.0	90.0	95.0	99.0	100.0
6	500706	0.125	0.0	1.0	17.0	61.0	80.0	88.0	94.0	100.0
7	501108	0.125	1.0	11.0	66.0	94.0	97.0	98.0	100.0	100.0
8	510330	0.125	0.0	2.0	29.0	79.0	94.0	98.0	99.0	100.0
9	510726	0.125	1.0	2.0	22.0	69.0	83.0	90.0	96.0	100.0
10	511031	0.125	0.0	1.0	25.0	77.0	92.0	96.0	99.0	100.0
11	511205	0.125	0.0	1.0	27.0	75.0	92.0	97.0	99.0	100.0
12	520718	0.125	0.0	5.0	46.0	84.0	94.0	97.0	98.0	100.0
13	520813	0.125	0.0	2.0	33.0	76.0	89.0	94.0	97.0	100.0
14	520911	0.125	0.0	2.0	29.0	74.0	87.0	94.0	98.0	100.0
15	520924	0.125	0.0	2.0	45.0	90.0	96.0	98.0	100.0	100.0

Table 3.6. Middle Loup River Data of Hubbell and Matejka (1959) (continued)

Data No. (1)	Survey Date (yymmdd) (2)	Size distribution of sediment load, finer than indicated diameters							
		Grp1 (%) (25)	Grp2 (%) (26)	Grp3 (%) (27)	Grp4 (%) (28)	Grp5 (%) (29)	Grp6 (%) (30)	Grp7 (%) (31)	Grp8 (%) (32)
		<b>0.062</b>	<b>0.125</b>	<b>0.25</b>	<b>0.5</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>8 mm</b>
1	500301	14.0	30.0	67.0	90.0	96.0	99.0	100.0	100.0
2	500321	7.0	25.0	63.0	92.0	98.0	100.0	100.0	100.0
3	500425	7.0	25.0	70.0	93.0	98.0	100.0	100.0	100.0
4	500509	9.0	35.0	74.0	91.0	98.0	100.0	100.0	100.0
5	500606	10.0	28.0	65.0	93.0	98.0	100.0	100.0	100.0
6	500706	10.0	30.0	65.0	88.0	95.0	98.0	100.0	100.0
7	501108	8.0	28.0	69.0	92.0	99.0	100.0	100.0	100.0
8	510330	9.0	26.0	66.0	90.0	97.0	100.0	100.0	100.0
9	510726	12.0	25.0	61.0	91.0	98.0	100.0	100.0	100.0
10	511031	4.0	18.0	56.0	81.0	91.0	97.0	100.0	100.0
11	511205	4.0	19.0	62.0	87.0	95.0	99.0	100.0	100.0
12	520718	10.0	24.0	58.0	84.0	93.0	98.0	100.0	100.0
13	520813	12.0	28.0	60.0	87.0	96.0	100.0	100.0	100.0
14	520911	5.0	15.0	40.0	76.0	88.0	97.0	100.0	100.0
15	520924	6.0	20.0	51.0	84.0	93.0	98.0	100.0	100.0

Flow properties for each record include the measurements of flow discharge, channel width, flow depth, water surface/energy slope, and water temperature. Bed material properties include the measured value of median diameter and  $D_{65}$ , size gradation coefficient, and specific gravity of sediment particles. Transported sediment properties include measured values of sediment concentration, median diameter, and size gradation coefficient of sediment in transport. Size distribution data include both the bed material size distribution and the transported sediment size distribution.

### 3.3 TRANSPORT CAPACITY DISTRIBUTION FUNCTION BASED ON RELATIVE FALL VELOCITY

The fraction of sediment transport capacity,  $P_{ci}$ , is analogous to the bed material size fraction,  $P_{bi}$ . However, rather than the size distribution of bed material it denotes the size distribution of transported material. The fraction,  $P_{bi}$ , of bed material may be visualized as the availability of size class  $i$  on the bed surface. It is implied in the BMF concept that the fractional transport rates are directly proportional to the availability of sediment particles on the bed surface. This is also the case for the shear stress correction approach given by Eqs. (2.2) and (2.3). The critical shear stress for incipient motion, the particle entrainment, and transport mechanism for a single size fraction are all affected by the other sizes existing in a sediment mixture. The effective shear stress acting on a particle or the unit stream power expenditure for the entrainment of a given size particle is different for a nonuniform sediment mixture than for uniform material. The resulting fractional transport rates are greatly affected by the so-called sheltering and exposure effect.

Since the fractional transport rates are directly proportional to the availability of

sediment particles on the bed surface, the concept of the BMF approach may be adopted as a first approximation in the development of a method to predict the transport capacity distribution function. Further development may introduce appropriate modification factors or terms accounting for the sheltering and exposure effects. In doing so, Yang's dimensionless unit stream power equation may be simplified into (Yang and Molinas 1982)

$$C_t = I \left( \frac{VS}{\omega_{50}} \right)^J \quad (3.1)$$

in which I, J = coefficients, which are related to flow and sediment properties, and  $\omega_{50}$  = fall velocity of sediment corresponding to particle size  $D_{50}$ . Applying the BMF concept [Eq. (2.6)], the bed-material concentration of size fraction i can be expressed as

$$C_{ti} = P_{bi} \left[ I \left( \frac{VS}{\omega_i} \right)^J \right] \quad (3.2)$$

Using the conceptual equation of the TCF approach [ Eq. (2.7)], the bed-material concentration,  $C_{ti}$ , can be also expressed as

$$C_{ti} = P_{ci} \left[ I \left( \frac{VS}{\omega_{50}} \right)^J \right] \quad (3.3)$$

From Eqs. (3.2) and (3.3), the following relation is obtained

$$\frac{P_{ci}}{P_{bi}} = \left( \frac{\omega_i}{\omega_{50}} \right)^{-J} \quad (3.4)$$

This equation is similar to Dou et al.'s equation (1987) for suspended load transport capacity. It is a first approximation for the transport capacity fraction in the case of nonuniform sediment mixtures. Figs. 3.1 and 3.2 show the variation of  $P_{c_{mi}}/P_{b_i}$  with  $D_i/D_{50}$  and the variation of  $P_{c_{mi}}/P_{b_i}$  with  $\omega_i/\omega_{50}$ , respectively, where  $P_{c_{mi}}$  is the measured size fraction of bed-material load for size group  $i$ . The data shown in Figs. 3.1 and 3.2 include the 118 sets of measurements (891 points) given in Table 3.1. Except for the Samaga et al. data which approximated the surface layer composition with the original mixture composition, it can be seen that a trend exists between  $P_{c_{mi}}/P_{b_i}$  and  $D_i/D_{50}$  or between  $P_{c_{mi}}/P_{b_i}$  and  $\omega_i/\omega_{50}$ . It should be noted that the relationship between  $P_{c_{mi}}/P_{b_i}$  and  $D_i/D_{50}$  or between  $P_{c_{mi}}/P_{b_i}$  and  $\omega_i/\omega_{50}$  is not a simple power relationship. The smaller particles are sheltered by the larger ones and are therefore transported at a relatively smaller rate. On the other hand, the larger particles experience larger fluid dynamic forces than they would if they were in a uniform sediment bed and are consequently transported at a higher rate. At low stream powers and low shear stresses the coarse fractions may not move at all, resulting in a state of partial transport.

The theoretical analysis presented earlier in this study indicates that the effect of sheltering on smaller sizes and the effect of exposure on coarse sizes are primarily dependent on the relative diameters (Einstein, 1950; Karim and Kennedy, 1981; White and Day, 1982; Proffitt and Sutherland, 1983; Misri et al., 1984; Wang and Zhang, 1990; Wang et al. 1995; Karim, 1998), such as  $D_i/X$ ,  $D_i/D_{50}$ ,  $D_i/D_A$ ,  $D_i/D_w$ ,  $D_i/D_a$ , and  $D_i/D_p$ , of the bed material. Instead of using relative diameters, it is assumed that the ratio  $\omega_i/\omega_{50}$  can also be used to express the sheltering and exposure effect on a size fraction due to the presence of other fractions in sediment mixtures. Retaining the basic form of Eq. (3.4) but introducing a second term to accommodate the sheltering and exposure effect results in

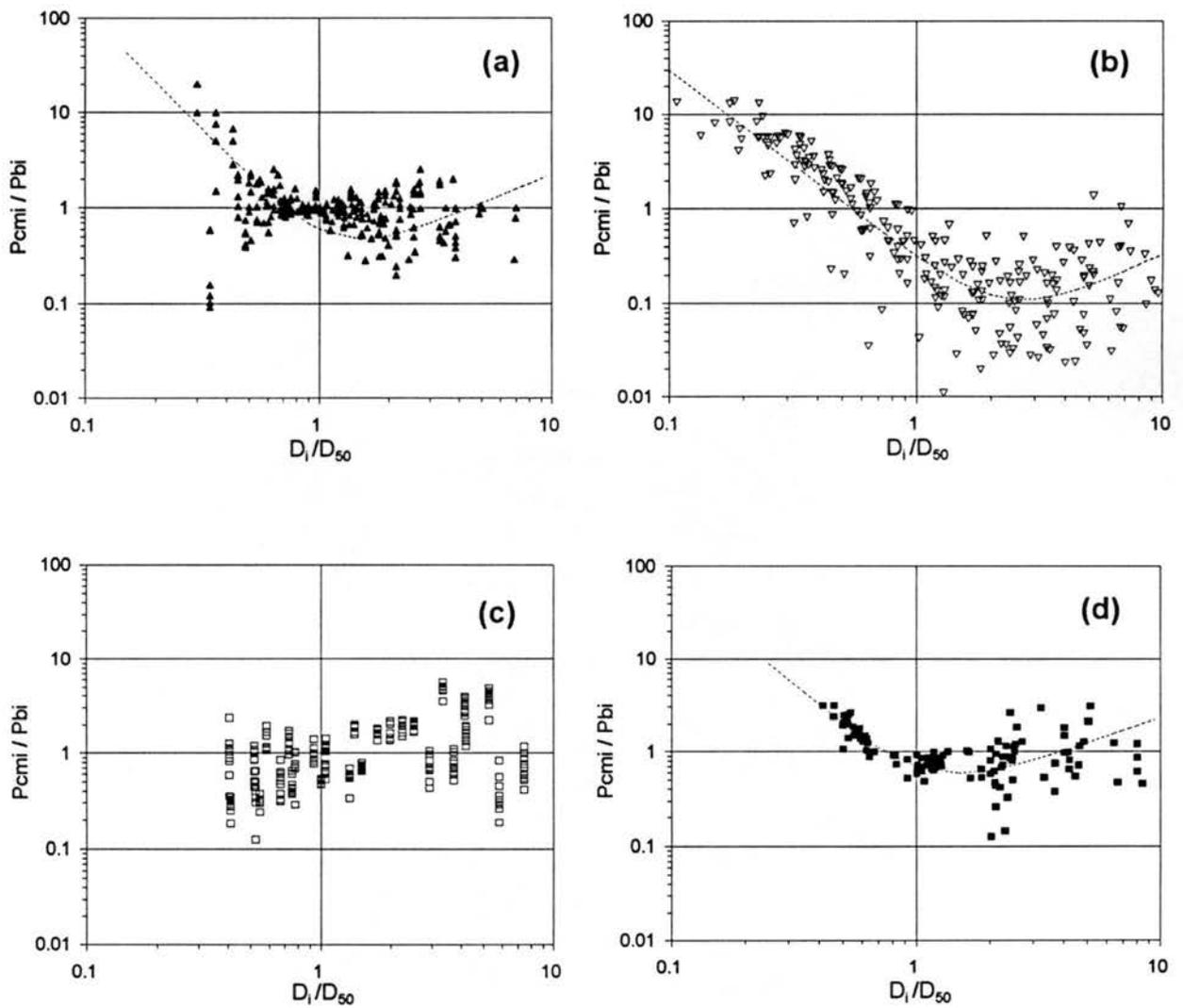


Fig. 3.1 Variation of  $P_{c_{mi}} / P_{b_i}$  with  $D_i / D_{50}$ : (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

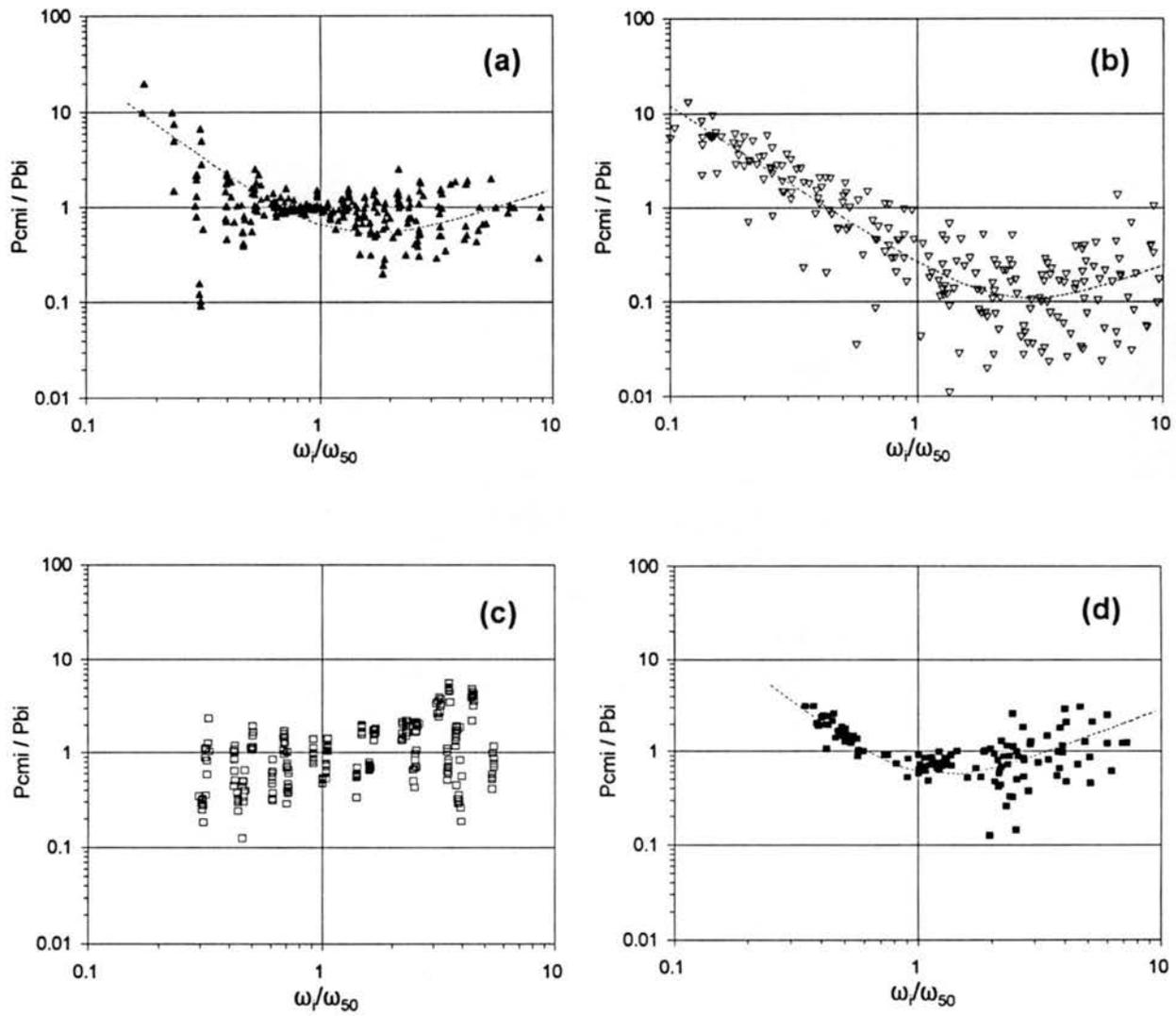


Fig. 3.2 Variation of  $P_{cmi} / P_{bi}$  with  $\omega_i / \omega_{50}$ : (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

$$\frac{P_{ci}}{P_{bi}} = C_1 \left( \frac{\omega_i}{\omega_{50}} \right)^\alpha + C_2 \left( \frac{\omega_i}{\omega_{50}} \right)^\beta \quad (3.5)$$

in which  $C_1$ ,  $C_2$ ,  $\alpha$ ,  $\beta$  = coefficients which will be analyzed in the following.

In Eq. (3.5), the fall velocity  $\omega_{50}$  may be regarded as a scaling or normalizing factor. Particles having a fall velocity smaller than this scaling factor (size) experience sheltering effects, and those particles having a fall velocity larger than this size experience exposure effects. In more general terms, Eq. (3.5) may be expressed as

$$\frac{P_{ci}}{P_{bi}} = C_1 \left( \frac{\omega_i}{\omega_n} \right)^\alpha + C_2 \left( \frac{\omega_i}{\omega_n} \right)^\beta \quad (3.6)$$

or

$$P_{ci} = P_{bi} \left[ C_1 \left( \frac{\omega_i}{\omega_n} \right)^\alpha + C_2 \left( \frac{\omega_i}{\omega_n} \right)^\beta \right] \quad (3.7)$$

in which  $\omega_n$  = fall velocity corresponding to a scaling size of bed material. According to Eq. (2.7), the summation of  $P_{ci}$  should be equal to 1. Applying this as a constraint for Eq. (3.7) gives the following relation

$$1 = \sum_{i=1}^N P_{bi} \left[ C_1 \left( \frac{\omega_i}{\omega_n} \right)^\alpha + C_2 \left( \frac{\omega_i}{\omega_n} \right)^\beta \right] \quad (3.8)$$

Dividing Eq. (3.7) by Eq. (3.8) results in the following basic form of equation for the determination of  $P_{ci}$

$$P_{ci} = \frac{P_{bi} \left[ \left( \frac{\omega_i}{\omega_n} \right)^\alpha + \zeta \left( \frac{\omega_i}{\omega_n} \right)^\beta \right]}{\sum_{i=1}^N P_{bi} \left[ \left( \frac{\omega_i}{\omega_n} \right)^\alpha + \zeta \left( \frac{\omega_i}{\omega_n} \right)^\beta \right]} \quad (3.9)$$

in which  $\zeta = C_2/C_1$ . In this study,  $D_{50}$  is taken as the scaling size of bed material. Correspondingly, the scaling fall velocity is expressed as

$$\omega_n = \omega_{50} = f(D_{50}) \quad (3.10)$$

The sheltering and exposure effect experienced by the particles on the bed is dependent upon the amplitude and speed of bed form, and on the different size ranges present on the bed. As the strength of the flow increases, the sheltering and exposure effect becomes secondary since the intensity of turbulence becomes more dominant. This indicates that the coefficients in Eq. (3.9) are not constant but are related to flow and sediment properties and can be defined by the following general function

$$\alpha, \beta, \zeta = f \left( F_r, \frac{V}{V_*}, \frac{d}{D_{50}}, \sigma_g, \dots \right) \quad (3.11)$$

in which  $f$  = general function;  $d$  = average flow depth;  $V$  = average flow velocity; and  $F_r$  = Froude number. Detailed procedure to determine the three coefficients in Eq. (3.9) is given in the following:

Step 1. Analyze qualitatively the variability of each coefficient in Eq. (3.9) from its theoretical derivation and physical considerations. For example,  $\alpha$  is mainly a representative of  $(-J)$  appeared in Eq. (3.4) by comparing Eq. (3.5) with Eq. (3.4).

Therefore,  $\alpha$  must be a negative value since  $J$  is always a positive value. The order of magnitude of  $\alpha$  should be roughly between 0-3 because  $J$  has a value between 0.5-2.0 for bed-material transport in sand-bed channels.

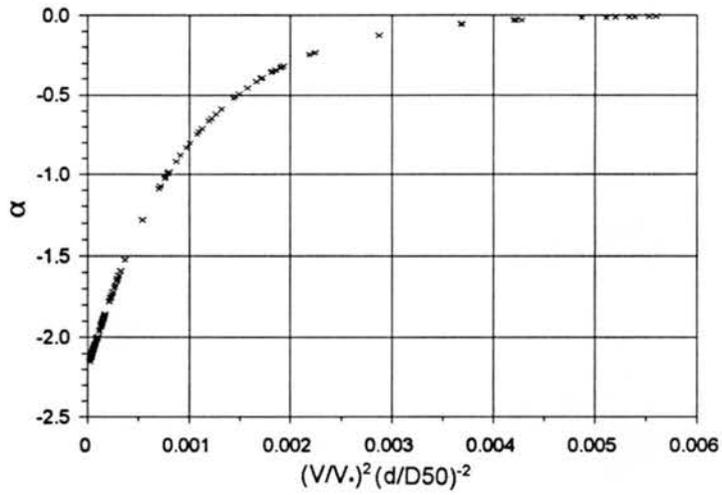
- Step 2. Choose relevant flow and sediment parameters based on previous studies, data analysis, and general understanding of the problem.
- Step 3. Choose a functional relationship for each coefficient, including linear and nonlinear relations based on previous studies and use parsimony.
- Step 4. Determine the constant values contained in the functional relationship chosen for each coefficient based on the data analysis.
- Step 5. Optimize the constant values in the functional relationship for each coefficient by comparing the goodness-of-fit between the computed and measured size fractions of bed-material load.
- Step 6. Repeat the trial and error process from step 2 to step 5 until a satisfactory set of coefficients is obtained.

Following the procedure outlined above and through linear and nonlinear analysis of 118 sets of data given in Table 3.1, the expressions of  $\alpha$ ,  $\beta$ , and  $\zeta$  are determined as

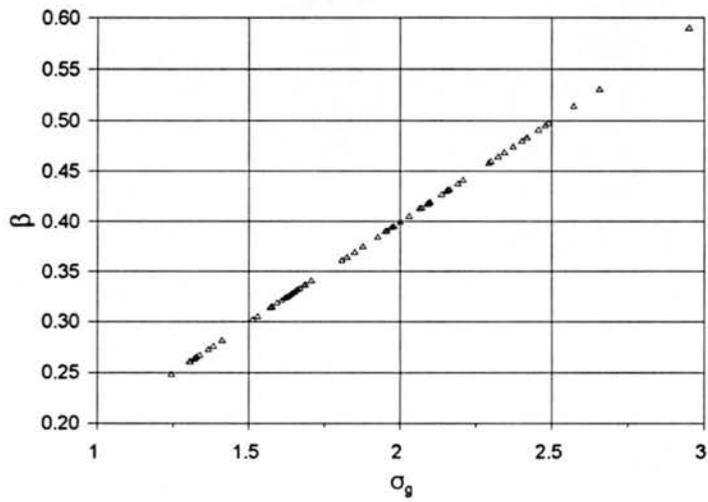
$$\alpha = -2.2 \exp \left[ -1000 \left( \frac{V}{V_*} \right)^2 \left( \frac{d}{D_{50}} \right)^{-2} \right] \quad (3.12)$$

$$\beta = 0.2 \sigma_g \quad (3.13)$$

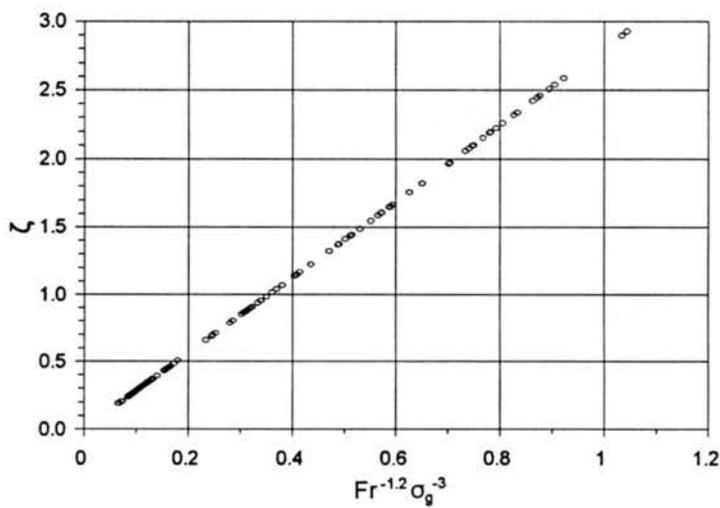
$$\zeta = 2.8 F_r^{-1.2} \sigma_g^{-3} \quad (3.14)$$



(a)



(b)



(c)

Fig. 3.3. Variations of the Coefficients Used in Eq. (3.9): (a)  $\alpha$  from Eq. (3.12); (b)  $\beta$  from Eq. (3.13); and (c)  $\zeta$  from Eq. (3.14).

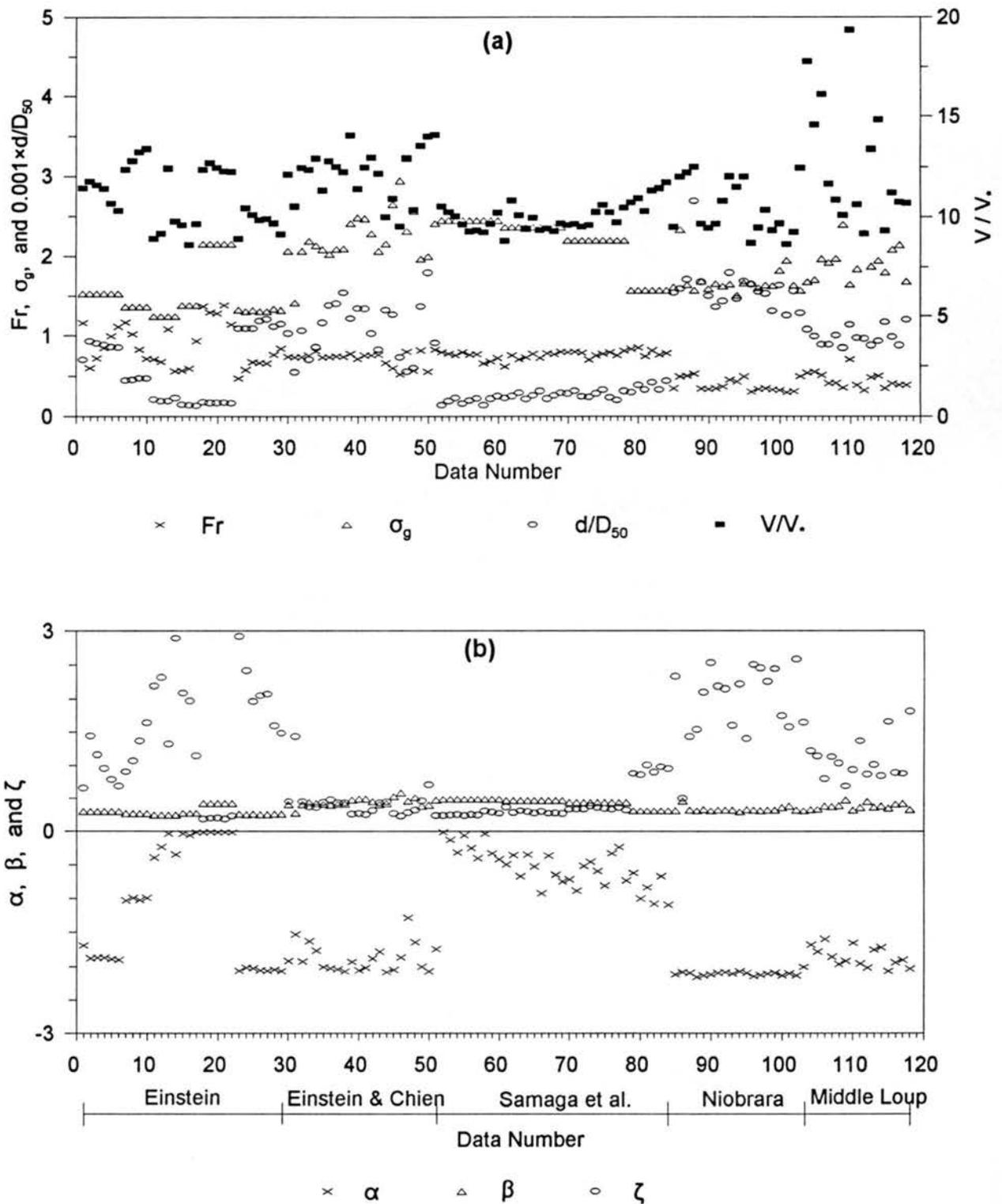


Fig. 3.4. Variability of the Coefficients Used in Eq. (3.9): (a) Variations of the Variables Used in the Computation of  $\alpha$ ,  $\beta$ , and  $\zeta$ ; (b) Variations of  $\alpha$ ,  $\beta$ , and  $\zeta$ .

Corresponding to the above set of coefficients for Eq. (3.9), the fall velocities of the sediment particles are determined from Fig. 2 presented in the U. S. Inter-Agency Committee on Water Resources, Subcommittee on Sedimentation (1957).

Variations of the coefficients of  $\alpha$ ,  $\beta$ , and  $\zeta$  expressed by Eqs. (3.12)-(3.14) are plotted in Figs. 3.3 and 3.4. Fig. 3.3 shows the variation of the coefficients with corresponding parameters, while Fig. 3.4 shows the variation with data number or data sources. Values of  $\alpha$ ,  $\beta$ , and  $\zeta$  for the 118 sets of data given in Table 3.1 are in the ranges of  $\alpha = -2.153 \sim -0.008$ ,  $\beta = 0.249 \sim 0.590$ , and  $\zeta = 0.189 \sim 2.926$ .

### 3.4 TRANSPORT CAPACITY DISTRIBUTION FUNCTION BASED ON RELATIVE DIAMETER

Following the same procedure,  $P_{ci}$  can be expressed in terms of  $D_i/D_{50}$  starting from Engelund and Hansen's function (1967). An energy approach was used by Engelund and Hansen for bed-material load over a dune bed. The moving sediment particle is lifted over the height of the dune; thus, energy is required. The relationship obtained is

$$f'/\Phi_t = 0.1 \theta^{2.5} \quad (3.15)$$

where

$$\Phi_t = \frac{q_t}{\gamma_s \sqrt{\frac{\gamma_s - \gamma}{\gamma} g D_{50}^3}} \quad (3.16)$$

$$\theta = \frac{\tau}{(\gamma_s - \gamma) D_{50}} \quad (3.17)$$

$$f' = \frac{2 g S d}{V^2} \quad (3.18)$$

Applying the conceptual equation of BMF and TCF approaches, the unit bed-material load of size fraction  $i$  can be expressed as

$$q_{ti} = P_{bi} \frac{\theta_i^{2.5}}{\gamma_s \sqrt{\frac{\gamma_s - \gamma}{\gamma} g D_i^3}} \quad (3.19)$$

and

$$q_{ti} = P_{ci} \frac{\theta_i^{2.5}}{\gamma_s \sqrt{\frac{\gamma_s - \gamma}{\gamma} g D_{50}^3}} \quad (3.20)$$

Eqs. (3.19) and (3.20) yields

$$\frac{P_{ci}}{P_{bi}} = \frac{\frac{\theta_i^{2.5}}{\gamma_s \sqrt{\frac{\gamma_s - \gamma}{\gamma} g D_i^3}}}{\frac{\theta_i^{2.5}}{\gamma_s \sqrt{\frac{\gamma_s - \gamma}{\gamma} g D_{50}^3}}} = \left( \frac{D_i}{D_{50}} \right) \quad (3.21)$$

Similar to Eq. (3.5), introducing a second term to accommodate the sheltering and exposure effect results in

$$\frac{P_{ci}}{P_{bi}} = C_1 \left( \frac{D_i}{D_{50}} \right) + C_2 \left( \frac{D_i}{D_{50}} \right) \quad (3.22)$$

Eq. (3.22) can be written as the following general form of equation

$$\frac{P_{ci}}{P_{bi}} = C_1 \left( \frac{D_i}{D_{50}} \right)^\alpha + C_2 \left( \frac{D_i}{D_{50}} \right)^\beta \quad (3.23)$$

As indicated earlier, the scaling size  $D_n$  is chosen to  $D_{50}$  of the bed material in this study.

Repeating the procedure outlined by Eqs. (3.6)-(3.9) results in

$$P_{ci} = \frac{P_{bi} \left[ \left( \frac{D_i}{D_{50}} \right)^\alpha + \zeta \left( \frac{D_i}{D_{50}} \right)^\beta \right]}{\sum_{i=1}^N P_{bi} \left[ \left( \frac{D_i}{D_{50}} \right)^\alpha + \zeta \left( \frac{D_i}{D_{50}} \right)^\beta \right]} \quad (3.24)$$

Using the data sets given in Table 3.1, the coefficients in Eq. (3.24) are determined as

$$\alpha = -2.9 \exp \left[ -1000 \left( \frac{V}{V_*} \right)^2 \left( \frac{d}{D_{50}} \right)^{-2} \right] \quad (3.27)$$

$$\beta = 0.2 \sigma_g \quad (3.25)$$

$$\zeta = 2.8 F_r^{-1.2} \sigma_g^{-3} \quad (3.26)$$

Variations of the coefficients of  $\alpha$ ,  $\beta$ , and  $\zeta$  expressed by Eqs. (3.25)-(3.27) are plotted in Figs. 3.5 and 3.6. Fig. 3.5 shows the variation of the coefficients with corresponding parameters, while Fig. 3.6 shows the variation with data number or data sources. Values of  $\alpha$ ,  $\beta$ , and  $\zeta$  for the 118 sets of data given in Table 3.1 are in the ranges of  $\alpha = -2.839 \sim -0.011$ ,  $\beta = 0.249 \sim 0.590$ , and  $\zeta = 0.189 \sim 2.926$ .

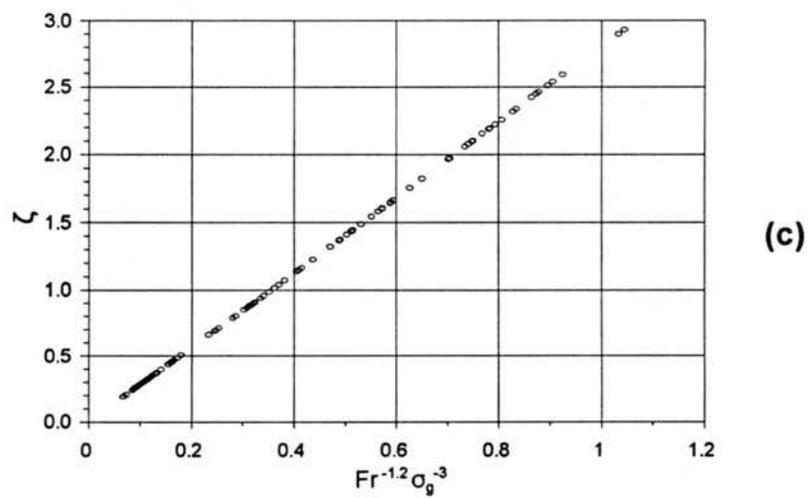
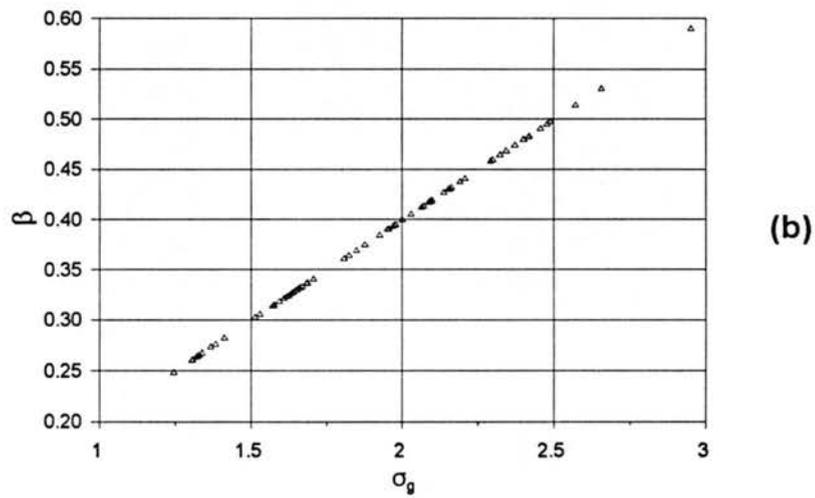
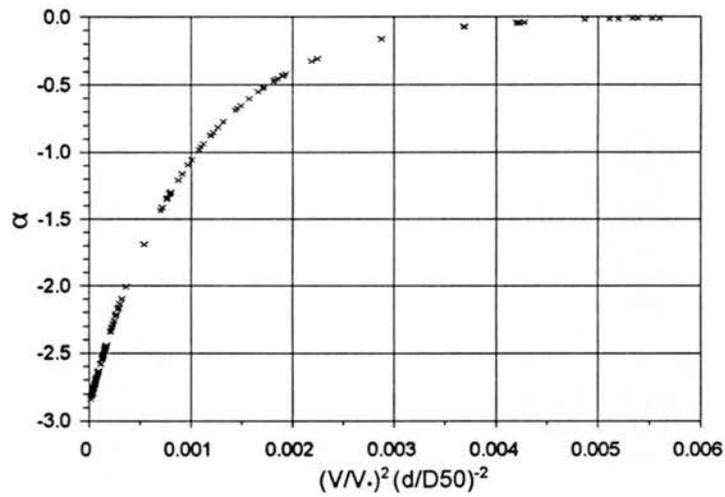


Fig. 3.5. Variations of the Coefficients Used in Eq. (3.24): (a)  $\alpha$  from Eq. (3.25); (b)  $\beta$  from Eq. (3.26); and (c)  $\zeta$  from Eq. (3.27).

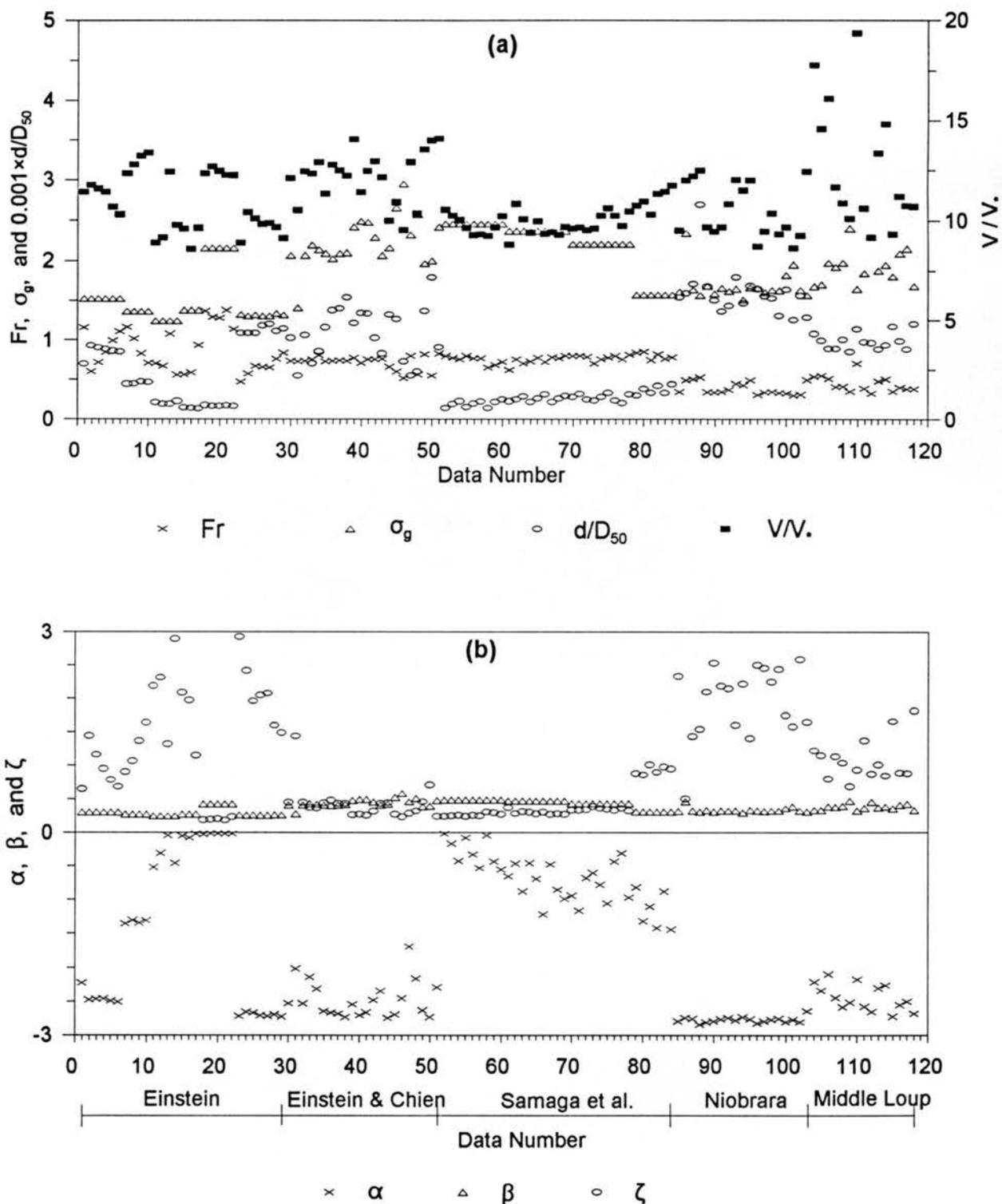


Fig. 3.6. Variability of the Coefficients Used in Eq. (3.24): (a) Variations of the Variables Used in the Computation of  $\alpha$ ,  $\beta$ , and  $\zeta$ ; (b) Variations of  $\alpha$ ,  $\beta$ , and  $\zeta$ .

### 3.5 EVALUATION OF THE NEW TRANSPORT CAPACITY DISTRIBUTION FUNCTIONS

The size fractions of bed-material transport capacity are computed by using both Eqs. (3.9) and (3.24) with the 118 sets of data. The correlation coefficients between the computed and measured size fractions is 0.856 for Eq. (3.9), and 0.852 for Eq. (3.24). In Figs. 3.7 and 3.8, the ratio of computed size fractions to the measured size fractions of bed-material load are plotted against  $D_i/D_{50}$  for Eqs. (3.9) and (3.24), respectively. In these figures, values of  $P_{cci}/P_{cmi}$  equal to 1 indicate the perfect agreement of computed fractions to measured transport capacity fractions. It is seen that most of the points fall near the perfect agreement line of  $P_{cci}/P_{cmi}$  equal to 1, especially for data points around  $D_i/D_{50}$  equal to 1. However, some scatter can be observed for both finer and coarser fractions (data points with smaller and larger values of  $D_i/D_{50}$ ). This is understandable because the movement of finer and coarser particles in a sediment mixture has higher uncertainty, and is greatly affected by the sheltering and exposure effects.

Figs. 3.9 and 3.10 show comparisons between computed and measured size fractions of bed-material load for Eqs. (3.9) and (3.24), respectively. A close agreement is obtained at the whole range of  $P_{cmi}$ , especially at larger values, by the use of Eqs. (3.9) and (3.24).

A earlier version of Eq. (3.24) was presented by Wu and Molinas (1996) and Molinas and Wu (1997). Chapter 5 presents more detailed comparison and an independent test of the transport capacity distribution functions proposed in this Chapter [ Eqs. (3.9) and (3.24)]. The fractional bed-material load predictions using different methods will be conducted, and the computed results will be evaluated.

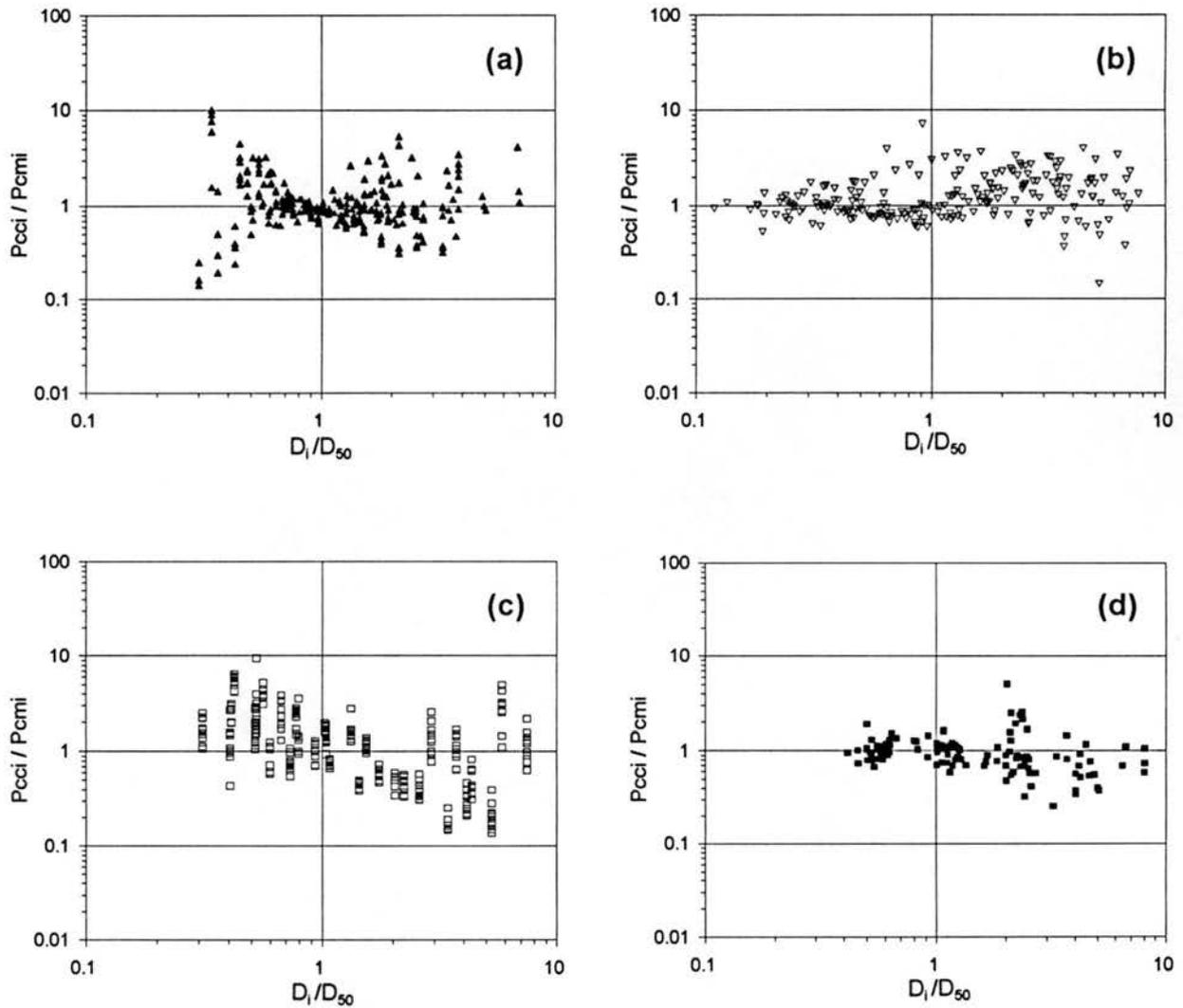


Fig. 3.7. Variation of  $P_{cci} / P_{cmi}$  with  $D_i / D_{50}$  by the Use of the Transport Capacity Distribution Function of Eq. (3.9) Derived from the TCF Concept: (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

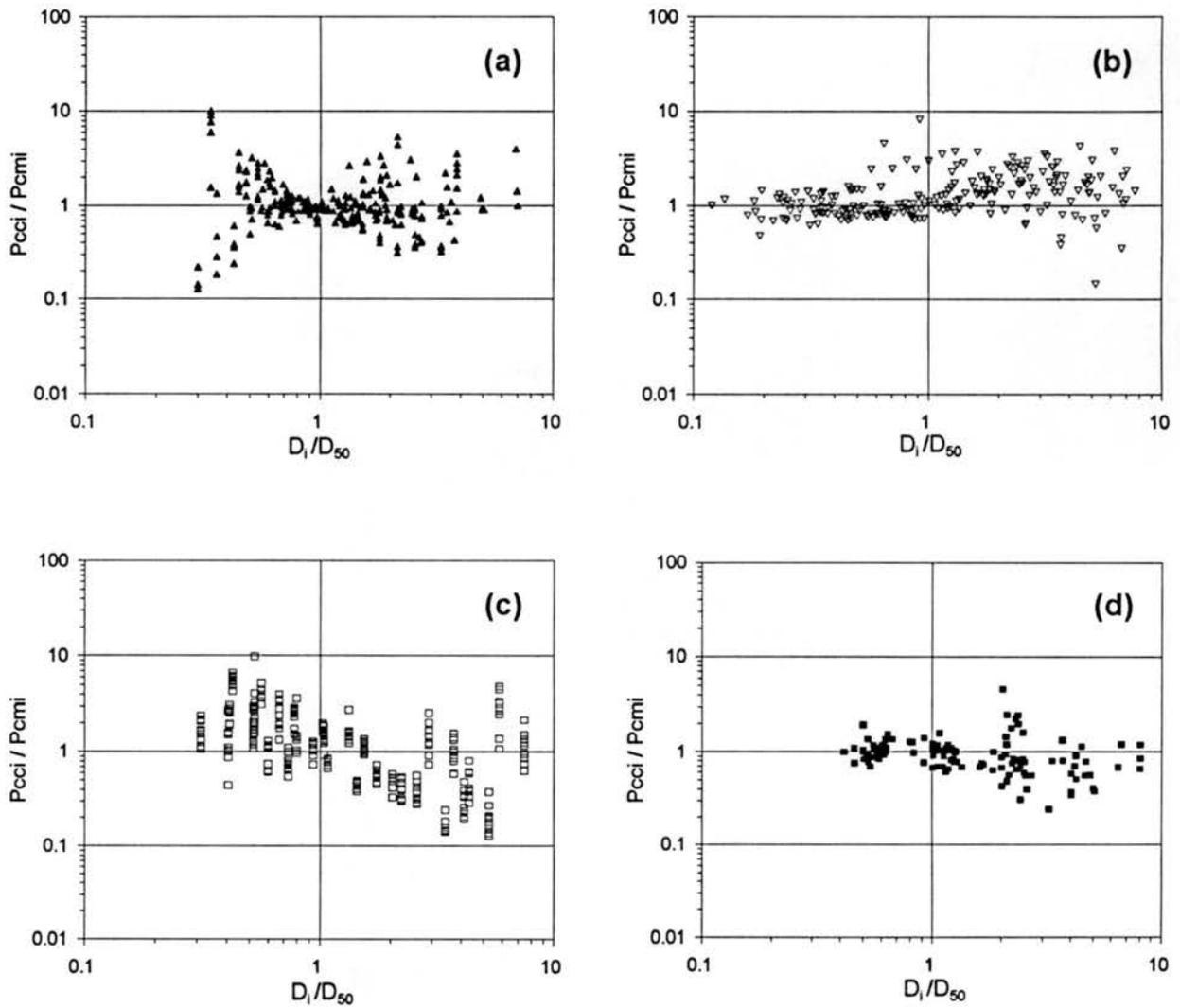


Fig. 3.8. Variation of  $P_{cci} / P_{cmi}$  with  $D_i / D_{50}$  by the Use of the Transport Capacity Distribution Function of Eq. (3.24) Derived from the TCF Concept: (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

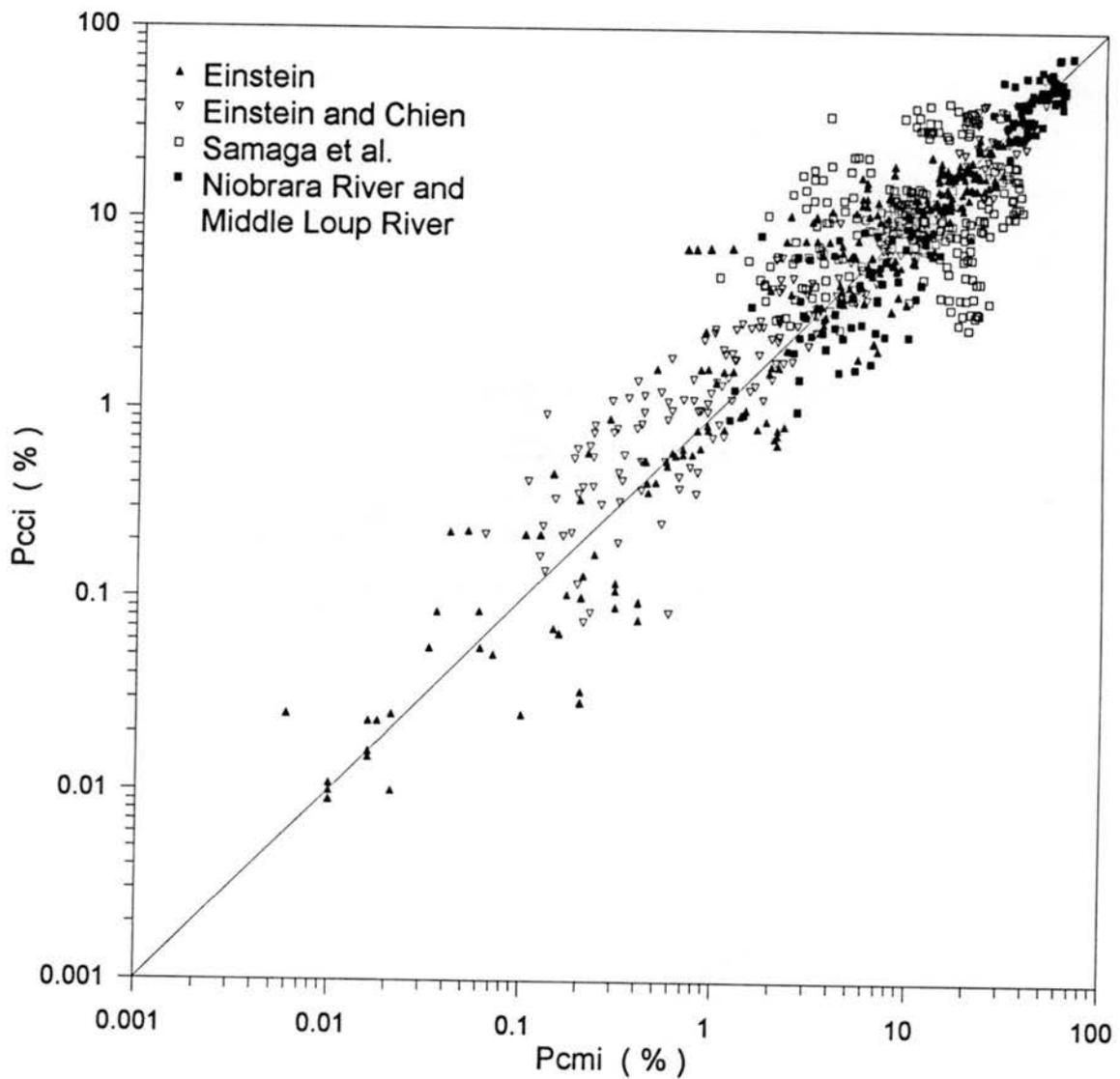


Fig. 3.9. Comparison between Percentages of Computed and Measured Bed-Material Concentrations for Individual Size Fractions in Sediment Mixtures by the Use of the Transport Capacity Distribution Function of Eq. (3.9) Derived from the TCF Concept.

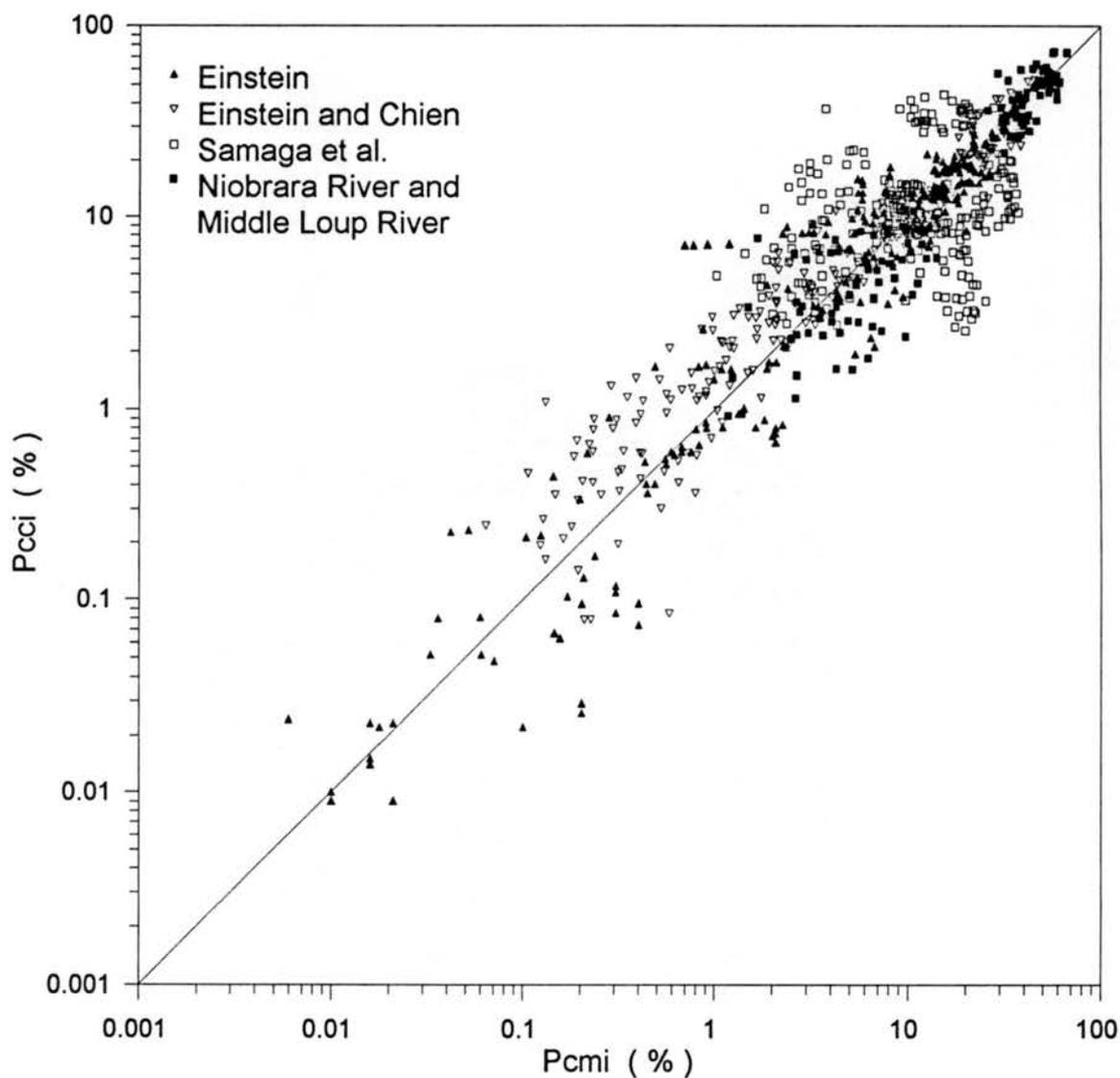


Fig. 3.10. Comparison between Percentages of Computed and Measured Bed-Material Concentrations for Individual Size Fractions in Sediment Mixtures by the Use of the Transport Capacity Distribution Function of Eq. (3.24) Derived from the TCF Concept.

## CHAPTER 4

### BED-MATERIAL TRANSPORT RATE

#### 4.1 GENERAL

According to the concept of the TCF approach,  $C_t$  can be determined using any appropriate bed-material load equations available in the literature. However, the performance of a particular equation selected for the computation of  $C_t$  affects the absolute magnitude of fractional transport rates. One should be aware of the limitations and the applicability of those equations developed based on uniform sediments, and choose reliable equations, possibly those considering the effects of the size gradation.

Of the commonly used sediment transport relations, only Einstein (1950), Laursen (1958), and Toffaleti (1969) seek to take the grading of sediment into account by directly computing sediment transport rates for each size fraction. The bed-material load is obtained from the summation of transport rates for each size fraction. Other relations use an "equivalent, effective, or significant" particle size which may be  $D_{35}$ ,  $D_{50}$ , or another characteristic size. It is assumed that the chosen equivalent size will produce the correct sediment transport rate for the whole mixture when used with the equations derived from uniform sediments.

In principle, those relations which directly compute transport rate for each size fraction are supposed to produce a more reliable and accurate prediction of total sediment

transport rate for sediment mixtures. Unfortunately, it is not the case in practice. Even though the bed-material load formulas of Engelund and Hansen (1967), Ackers and White (1973), and Yang (1973) were developed based on a single fixed representative bed material size ( $D_{35}$  or  $D_{50}$ ), they have gained increasing acceptance. However, even when they are applied to nonuniform sediment mixtures, these methods yield a relatively large scatter of computed results.

Various representative sizes of bed material have been used for the computation of transport rate for sediment mixtures. Einstein (1944) proposed  $D_{35}$  as the representative diameter in transport computations. Meyer-Peter and Müller (1948) suggested that the weighted mean diameter of bed material should be used as the representative size for nonuniform sediment mixtures. For graded sediment mixtures, Ackers and White (1973) suggested the use of  $D_{35}$  in their equation. Han (1973) proposed a weighted average fall velocity for nonuniform sediment mixtures. In addition to  $D_{50}$ , a size distribution parameter,  $D_{90}/D_{30}$ , was used by Smart and Jaeggi (1983) to develop their sediment transport equation for steep-slope rivers. Smart and Jaeggi compensated the effect due to sediment gradation by a weak power function of  $D_{90}/D_{30}$ . In applying the Engelund and Hansen equation to sediment mixtures, Nordin (1989) stated that a weighted representative diameter of bed material should be used. The gradation coefficient,  $G$ , defined as the arithmetic mean of  $D_{84}/D_{50}$  and  $D_{50}/D_{16}$ , was used by Shen and Rao (1991) to compute transport rate in sediment mixtures. In their regression equation for sediment transport, Shen and Rao showed the improvement due to the inclusion of  $G$ .

Different investigators suggesting different representative sizes of bed material for nonuniform sediment mixtures points out the fact that a single fixed size, such as  $D_{50}$ , is

inadequate in representing the various sizes present in sediment mixtures. In computing the transport rate for mixtures, the representative sediment size should reflect the different bed material size distributions. This is clearly of significance since grading curves with differing shapes will have different effects on the transport process. Considering the physical processes affecting the sediment transport in the flow,  $D_{90}/D_{30}$ ,  $G$ , or other factors representing the gradation of mixtures are all believed to be significant in the transport of sediment mixtures. These parameters aim to represent the range of particle sizes which are significantly present in the bed material. For a given flow condition, even if  $D_{50}$  of bed material remains the same, the resistance to flow, incipient motion, sand wave movement, and transport of sediment mixtures are different for different size distributions.

Instead of using a single fixed size or a single fixed size with a size gradation parameter as the representative property of bed material, van Rijn (1984) and Hsu and Holly (1992) suggested the use of variable representative sizes for the computation of sediment transport rate for nonuniform mixtures. The variable representative size is analogous to the median or mean size of sediments in transport. It is believed that the variable representative size is a better representation of the sediment load than a fixed particle diameter such as  $D_{35}$ ,  $D_{50}$ , or  $D_{65}$  of the bed material.

In the development of the suspended load transport equation, van Rijn (1984) proposed an empirical equation to estimate the representative diameter,  $D_s$ , for suspended sediment load. The equation that gave the same value for the suspended load as that computed with Einstein's method was determined by trial and error. This equation was expressed as

$$\frac{D_s}{D_{50}} = 1 + 0.011 (G - 1) (T - 25) \quad (4.1)$$

in which  $T$  = transport stage parameter. van Rijn stated that the  $D_s$  is a better representation of the suspended load than a fixed particle diameter such as  $D_{35}$ ,  $D_{50}$ , or  $D_{65}$  of the bed material. Fig. 4.1 is a plot showing a comparison of Eq. (4.1) with some experiment data given by Guy et al. (1966).

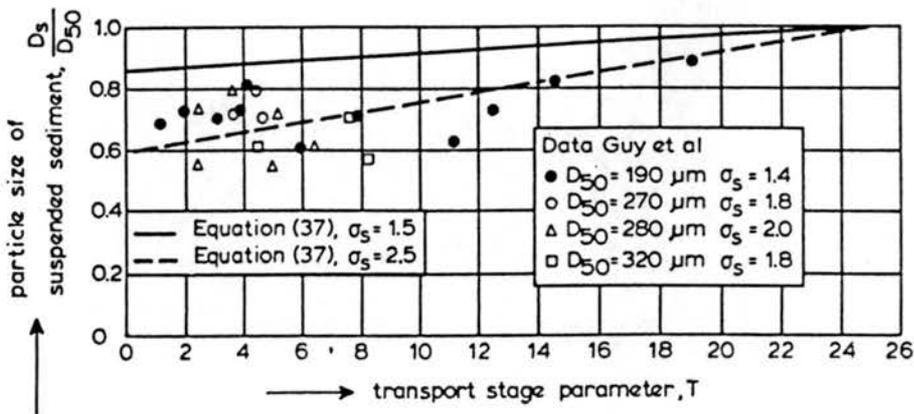


Fig. 4.1. Representative Particle Diameter of Suspended Sediment (after van Rijn, 1984).

Hsu and Holly (1992) suggested the use of a mean size of transported material (bedload),  $D_{mt}$ , as the representative diameter in their bedload computations. First they proposed a method to compute the size distribution of the transport material [see Eqs. (2.57)-(2.58)]. Then they determined the mean size,  $D_{mt}$ , from the computed size distribution. Hsu and Holly argued that if  $D_{mt}$  was visualized as the representative property of a uniform sediment, the bedload discharge could be evaluated using any appropriate bedload equations.

The use of a variable representative size is an interesting attempt for the prediction of sediment transport rate. Unfortunately, the representative size of Eq. (4.1) is developed based

on the results computed with Einstein's method; and it is limited to suspended load. The representative size proposed by Hsu and Holly is limited to bedload; and it is not verified with measurements. In this chapter, the variation of median diameter,  $D_{50t}$ , of bed-material sediments in transport is studied. An equation for the prediction of  $D_{50t}$  is proposed. The use of  $D_{50t}$  as representative size for the computation of bed-material load is presented.

#### 4.2 MEDIAN DIAMETER OF SEDIMENT IN TRANSPORT

The size distribution of sediment in transport is directly related to the size distribution of bed material and to the effective shear stress acting on each size group and does not necessarily resemble the gradation of bed material. Hsu and Holly (1992) pointed out that one must distinguish not only between parent-bed material composition and bed-surface material composition (or active-layer material) but also between bed-surface material composition and transported-material composition. The size gradation of transported material is generally different from the gradation of bed-surface material and should be treated as a new unknown variable.

Fig. 4.2 shows the relationship between the median diameter of transported material (bed-material load),  $D_{50t}$ , and the median diameter of bed material,  $D_{50}$ . The data shown in Fig. 4.2 include 85 sets of flume and field data given in Table 3.1 and 280 sets of Colorado State University flume data (Guy, Simons, and Richardson, 1966). Those 33 sets of flume data from Samaga et al. (1986a, b) listed in Table 3.1 are not plotted in Fig. 4.2 because the bed material composition data reported by Samaga et al. are composition of parent-bed material. As indicated earlier, the composition of bed-surface layer is different from the composition of parent-bed material. The composition of sediment in transport is directly

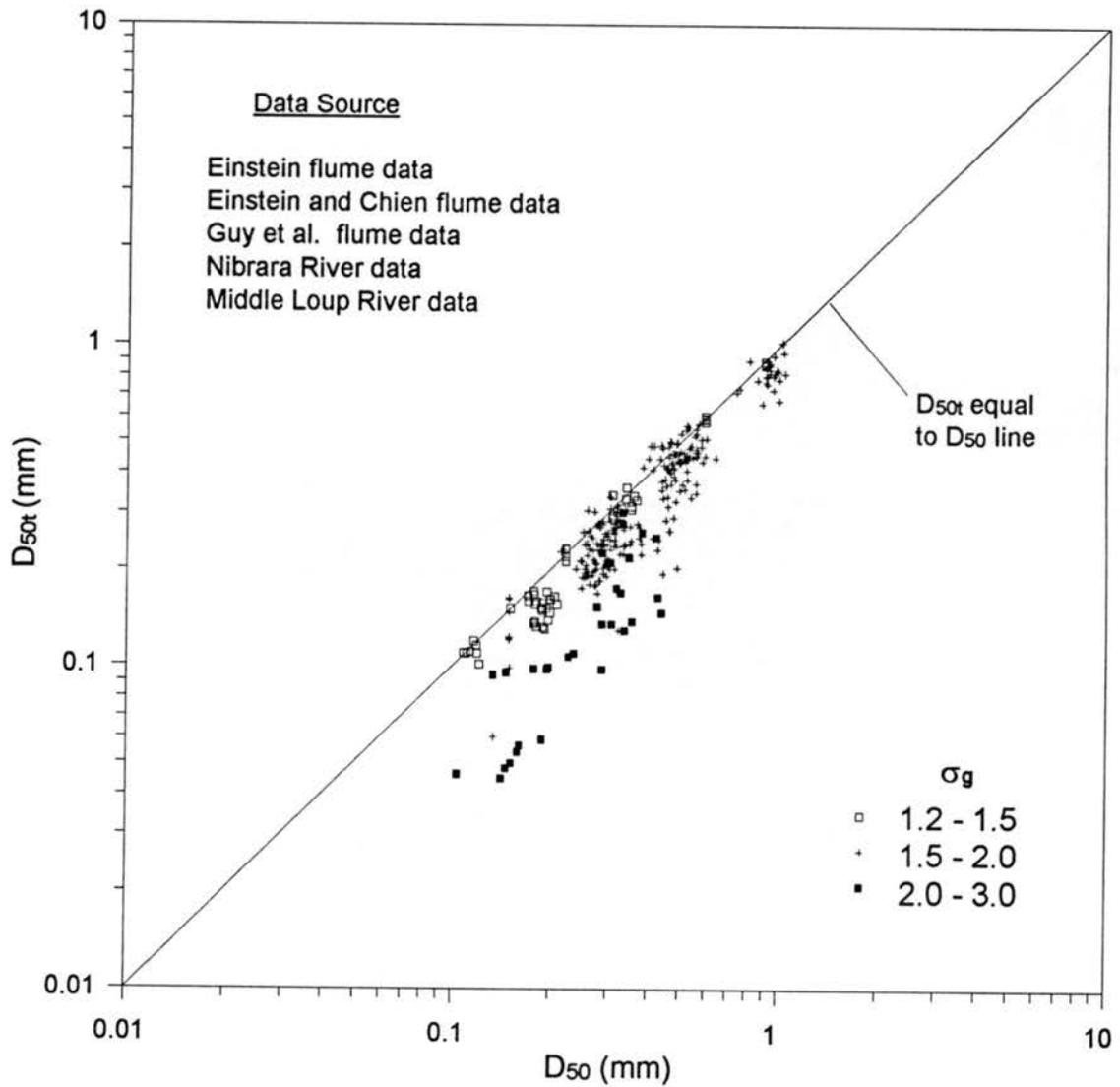


Fig. 4.2. Relationship between the Median Diameter of Bed-Material Load Sediment in Transport,  $D_{50t}$ , and the Median Diameter of Bed Material,  $D_{50}$ .

related to the composition of bed-surface layer, not the parent-bed material. From Fig. 4.2 it can be seen that, for the majority of the data, the  $D_{50t}$  corresponding to the sediment in transport is finer than the  $D_{50}$  corresponding to the bed material. This is due to the fact that the finer sizes in sediment mixtures are more readily transported by flow, which is commonly referred as the selective transport of grains by flow or hydraulic sorting. It can also be seen that for a given bed material size, for larger gradations the size of sediment in transport is finer (i.e., the larger the  $\sigma_g$ , the finer the sediment in transport).

The median diameter of bed-material load sediment in transport,  $D_{50t}$ , may be related to bed (bed-surface) material composition through a functional relationship of the form

$$D_{50t} = f(D_{50}, \sigma_g, V_*/\omega_{50}, \dots) \quad (4.2)$$

In order to reflect the physical processes related to the variation of sediment size in transport, as  $\sigma_g$  increases,  $D_{50t}$  should decrease. Following extensive comparisons of linear and nonlinear functional relationships, the following expression was determined for  $D_{50t}$

$$D_{50t} = \frac{D_{50}}{1 + B(V_*/\omega_{50})^m (\sigma_g - 1)^n} \quad (4.3)$$

in which B, m, and n = coefficients, which are equal to 0.8, 0.1, and 2.2, respectively.

Fig. 4.3 shows a comparison between the computed median diameter,  $D_{50t}$ , of bed-material load sediment in transport using Eq. (4.3) and the measured values. It is seen that the computed median diameters,  $D_{50t}$ , are in good agreement with the measured values.

Since the value of exponent m in Eq. (4.3) is very small (0.1), the resulting value of  $(V_*/\omega_{50})^{0.1}$  is close to unity. Therefore, the flow intensity term,  $V_*/\omega_{50}$ , may be neglected. This results

in the following simplified equation

$$\frac{D_{50t}}{D_{50}} = \frac{1}{1 + 0.8 (\sigma_g - 1)^{2.2}} \quad (4.4)$$

which represents the variation of relative size of sediment in transport with bed size gradation.

The value of relative median diameter,  $D_{50t}/D_{50}$ , is plotted against the geometric standard deviation,  $\sigma_g$ , of bed material in Fig. 4.4 for the same data shown in Fig. 4.2. It is clearly demonstrated that as  $\sigma_g$  increases, the value of  $D_{50t}/D_{50}$  decreases, and it generally follows the equation line given by Eq. (4.4). The variation of  $D_{50t}/D_{50}$  versus  $\sigma_g$  results from the selective transport of nonuniform sediment mixtures, which is a significant phenomenon in the transport process of nonuniform sediments.

Fig. 4.5 shows the variation of  $D_{50t}/D_{50}$  versus  $\sigma_g$  for another 124 sets of flume and field data, including flume data of Nonicos (Vanoni and Brooks, 1957; 12 sets), Taylor and Vanoni (1972, 6 sets), Vanoni and Brooks (1957, 15 sets), Vanoni and Huang (1967, 16 sets), and Wang and Zhang (1990, 27 sets), field data from the Platte River (Kircher, 1983, 20 sets), Rio Grande Conveyance Canal (Culbertson, Scott, and Bennett, 1972; 9 sets), and Yellow River at Tuchengzi (Long and Liang, 1994; 19 sets). These data cover wider ranges of variations of flow and sediment conditions with median diameter of 0.055-2.10 mm, geometric standard deviation of 1.25-4.06, discharge of 0.0037-3980, velocity of 0.19-2.81, depth of 0.062-1.91, and slope of 0.000078-0.0039. Since these 124 sets of data were not used in the development of Eq. (4.3) and its simplified relationship of Eq. (4.4), Fig. 4.5 is an independent test for the relationship given by Eq. (4.4).

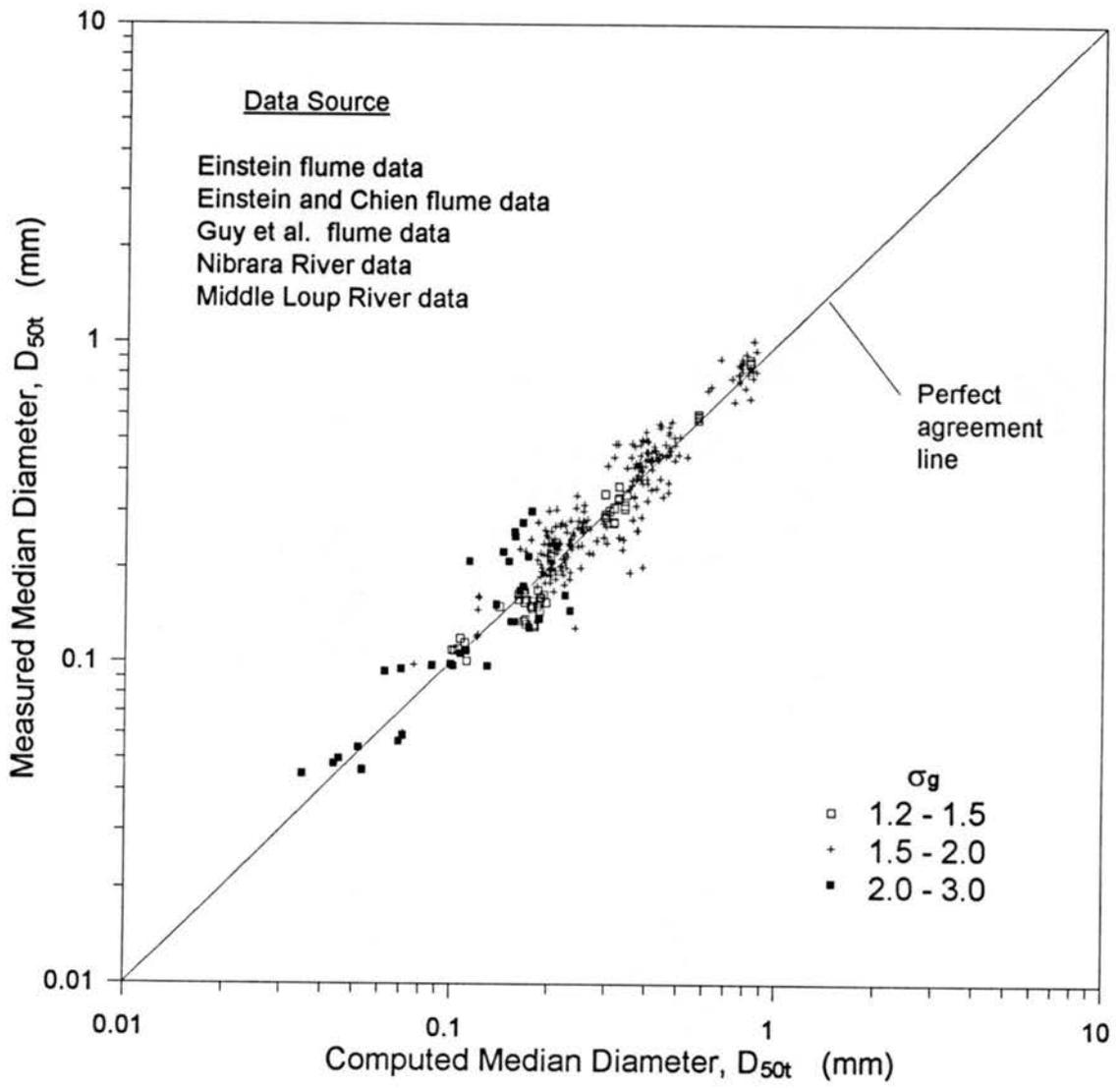


Fig. 4.3. Comparison between the Computed and Measured Median Diameter of Bed-Material Load.

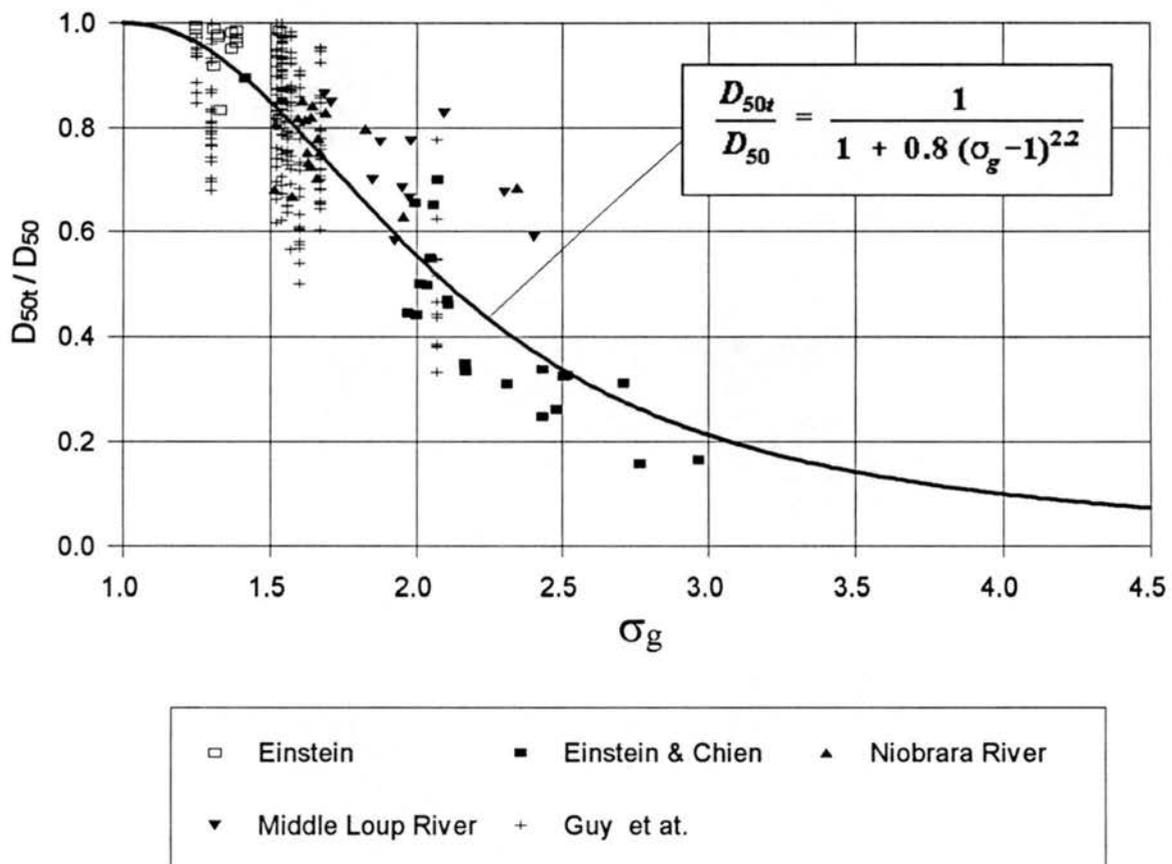


Fig. 4.4. Relationship between Relative Diameter,  $D_{50t}/D_{50}$ , and Geometric Standard Deviation,  $\sigma_g$ , for the 85 Sets of Flume and Field Data Derived from Table 3.1 and the 280 Sets of Data from Guy et al. (1966).

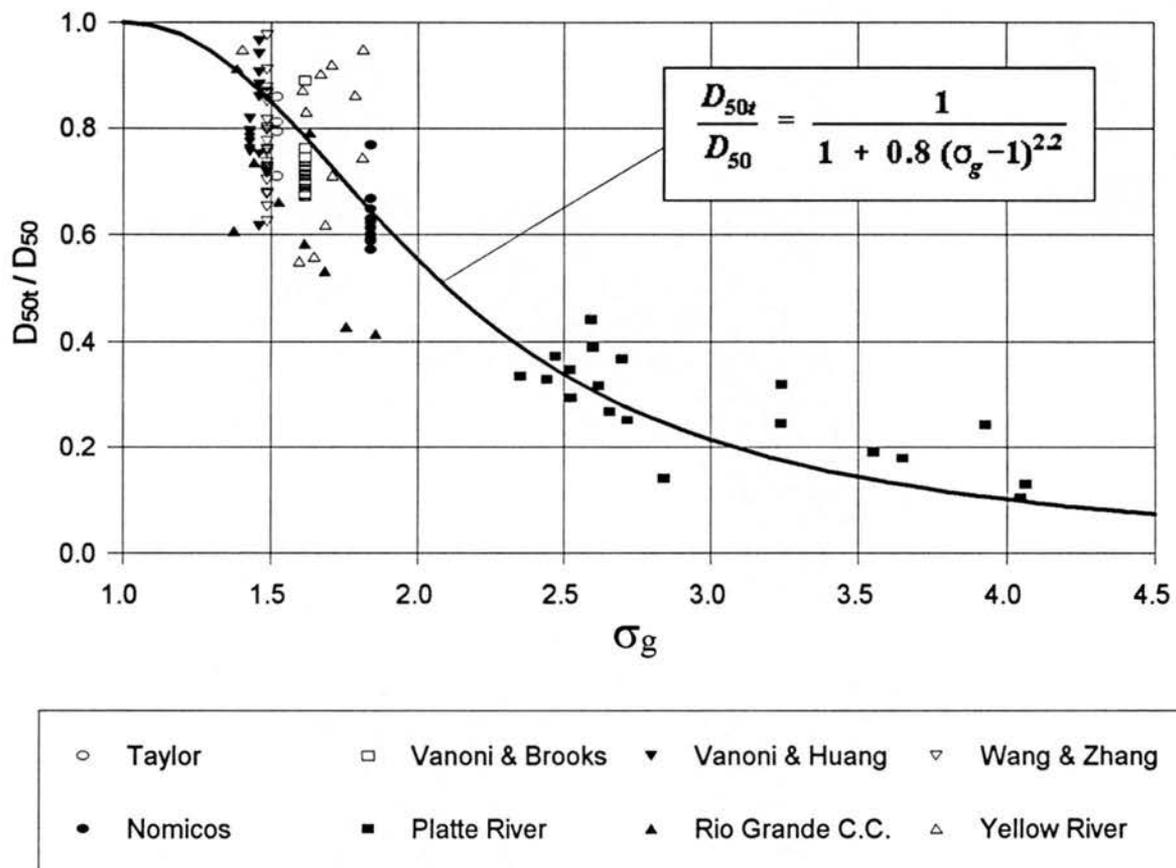


Fig. 4.5. Relationship between Relative Diameter,  $D_{50t}/D_{50}$ , and Geometric Standard Deviation,  $\sigma_g$ , for Another 124 Sets of Flume and Field Data (Independent Verification Data).

### 4.3 EFFECT OF SIZE GRADATION ON TRANSPORT OF SEDIMENT MIXTURES

The function of Engelund and Hansen (1967) can be used to demonstrate the effect of size gradation on the transport of sediment mixtures. This equation is given as

$$f'/\Phi_t = 0.1\theta^{2.5} \quad (4.5)$$

in which  $\Phi_t$  = dimensionless sediment transport function; and  $\theta$  = dimensionless shear parameter. Their definitions are given by Eqs. (3.16) and (3.17), respectively.

For the 118 sets of data given in Table 3.1,  $f'/\Phi_t$  versus  $\theta$  is plotted in Fig. 4.6. The Engelund and Hansen equation given by Eq. (4.5) is shown by a solid line, and the experimental data are sorted by sediment size ranges. Engelund and Hansen's function represents the transport phenomenon adequately, on the average, and  $\Phi_t$  is inversely proportional to  $D_{50}$ . However, for a given flow condition and sediment size, a considerable scatter exists around the equation line.

The functional relationship of Engelund and Hansen given by Eq. (4.5) is replotted in Fig. 4.7 for three ranges of  $\sigma_g$  values using the same data. This shows that sediment transport is significantly affected by  $\sigma_g$ . For a given  $D_{50}$ , the dimensionless sediment transport function,  $\Phi_t$ , is larger for larger  $\sigma_g$  values. This indicates that the combination of  $D_{50}$  and  $\sigma_g$  is a more effective measure for quantifying the effect of bed material size on the transport of sediment mixtures than  $D_{50}$  alone. From Fig. 4.7 it can be seen that the scatter in the relation is also related to the  $\theta$  value. This implies that the adjustment is not fixed but varies with the flow conditions as well as with  $\sigma_g$ .

Similar to those representative diameters of sediment in transport used by van Rijn (1984) and Hsu and Holly (1992), the median diameter estimated by Eq. (4.3) can be used

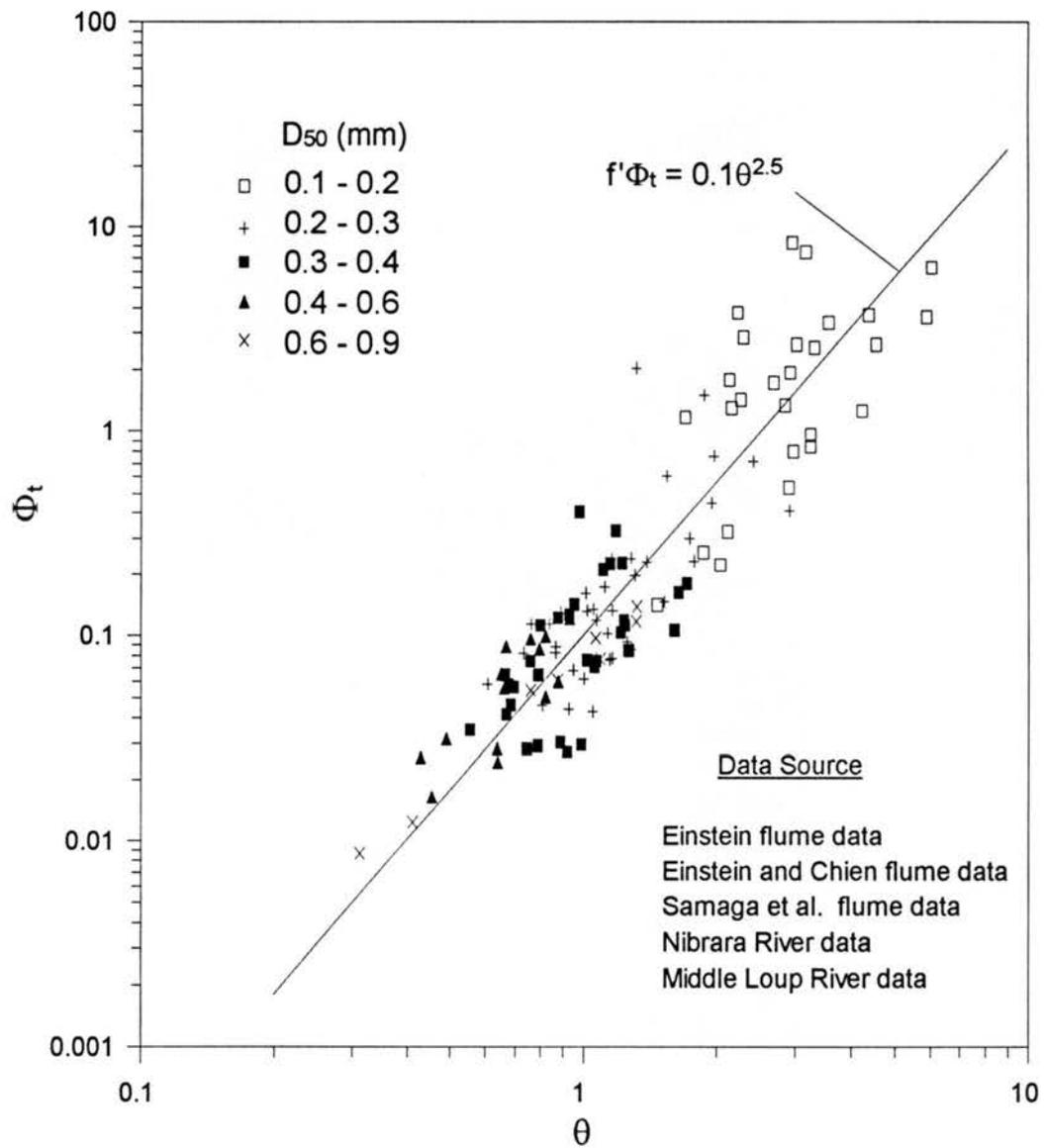


Fig. 4.6. Relationship between  $f'\Phi_t$  and  $\theta$  Sorted by Bed Material Size.

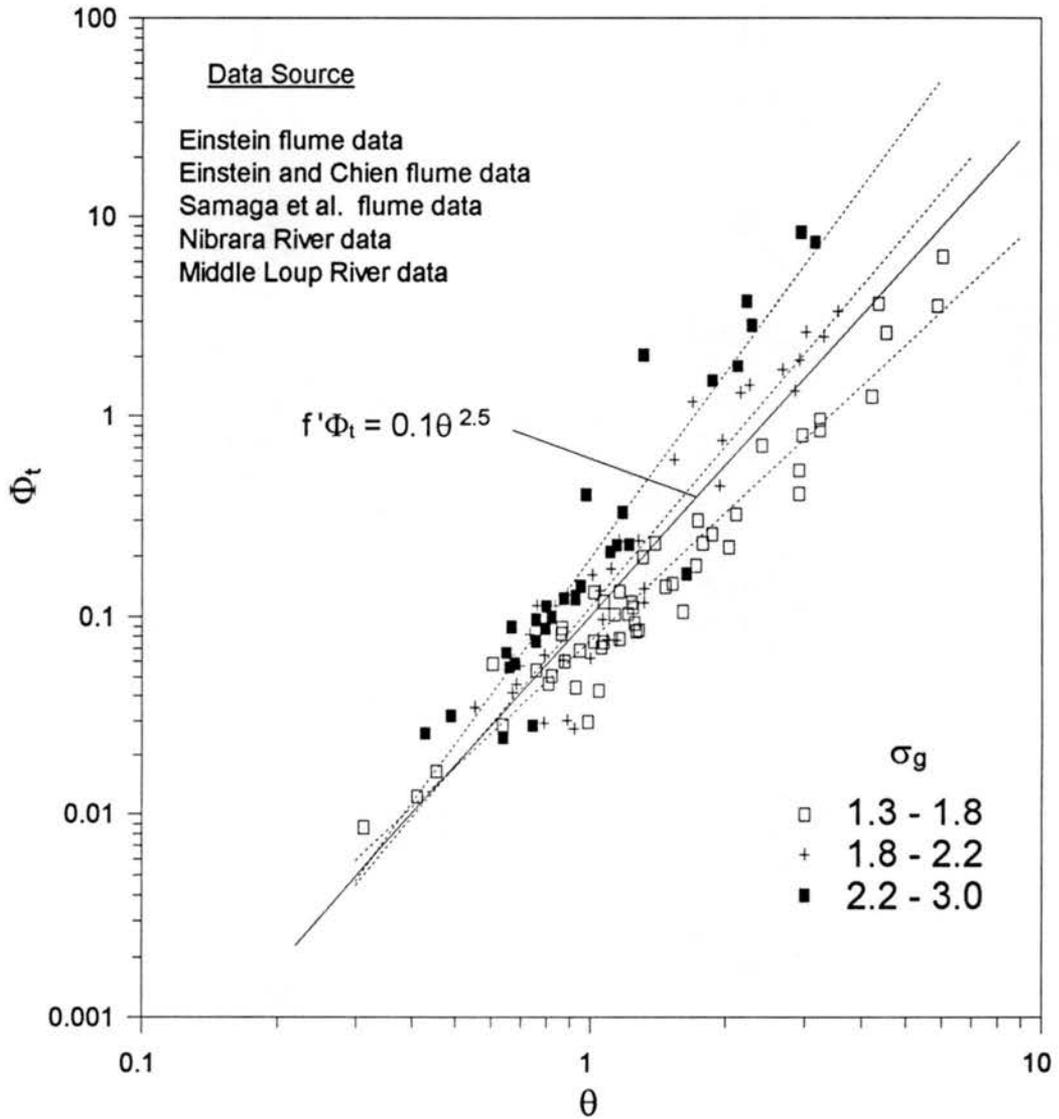


Fig. 4.7. Relationship between  $f'\Phi_t$  and  $\theta$  Sorted by Size Gradation ( $\sigma_g$  values).

as representative size in estimating the transport discharge for nonuniform sediment mixtures. It is assumed that this representative diameter will produce the correct sediment transport rate for the whole mixture when used with the equations derived from uniform sediments. As discussed earlier,  $D_{50t}$  decreases as  $\sigma_g$  increases. A smaller representative diameter is thus corresponding to a larger computed transport rate. This is consistent with previous results (e.g., Einstein, 1944; Ackers and White, 1973).

Considering that the existing transport equations were calibrated with data including nonuniform sediment mixtures, the coefficients in these equations implicitly include certain gradation effects. As a result, they overestimate sediment loads for uniform material and underestimate the transport rate for highly graded mixtures. To apply the representative diameter defined by Eq. (4.3), an equivalent diameter,  $D_e$ , is defined as follows

$$D_e = K_e D_{50t} = \frac{K_e D_{50}}{1 + B(V_* / \omega_{50})^m (\sigma_g - 1)^n} \quad (4.6)$$

in which  $K_e$  = a coefficient to compensate the bias in the existing transport equations, which is determined to be 1.8 in this study. The use of  $D_e$  to compensate the nonuniformity effect was recently presented in a study by Molinas and Wu (1998).

By replacing the representative size,  $D_{50}$ , in Eq. (4.5) with  $D_e$ , Engelund and Hansen's function can be transformed into

$$f' \Phi_e = 0.1 \theta_e^{2.5} \quad (4.7)$$

where

$$\Phi_e = \frac{q_t}{\gamma_s \sqrt{\frac{\gamma_s - \gamma}{\gamma} g D_e^3}} \quad (4.8)$$

$$\theta_e = \frac{\tau}{(\gamma_s - \gamma)D_e} \quad (4.9)$$

The use of an equivalent diameter determined from Eq. (4.6) is to compensate for size gradation in existing formulas. It should not be confused with a representative size for the transported sediments even though the two are related. For example, for uniform mixtures ( $\sigma_g$  equal to 1), the  $D_e$  obtained from Eq. (4.6) is equal to  $1.8D_{50}$ , which results in smaller transport values than the values obtained using  $D_{50}$ . On the other hand, for highly graded sediment mixtures, the value of  $D_e$  is smaller than the  $D_{50}$  size of the mixture (e.g., for  $\theta = 3.0$  and  $\sigma_g = 3$ ,  $D_e = 0.35D_{50}$ ). The effect is to increase the computed sediment transport rate.

The data shown in Fig. 4.7 are replotted in Fig. 4.8 utilizing the functional relationship give by Eq.(4.7). The relationship between  $f' \Phi_e$  and  $\theta_e$  shows a definite improvement; the use of  $D_e$  reduces scatters. The largest effects of the correction are observed on larger concentration values for high gradation factors. At small concentrations, and therefore lower flow intensities, the correction effects are minor.

Similar to Engelund and Hansen's function, the dimensionless unit stream power ( $VS/\omega_{50}$ ) relationship for bed-material concentration developed by Yang (1973), and Yang and Molinas (1982) may also be used to demonstrate the effect of size gradation on the transport of sediment mixtures. Yang's dimensionless unit stream power relationship can be simply expressed as

$$C_t = I \left( \frac{VS}{\omega_{50}} \right)^J \quad (4.10)$$

in which I and J are coefficients which are related to flow and sediment properties.

$C_t$  versus the dimensionless unit stream power is plotted in Fig. 4.9. It can be seen

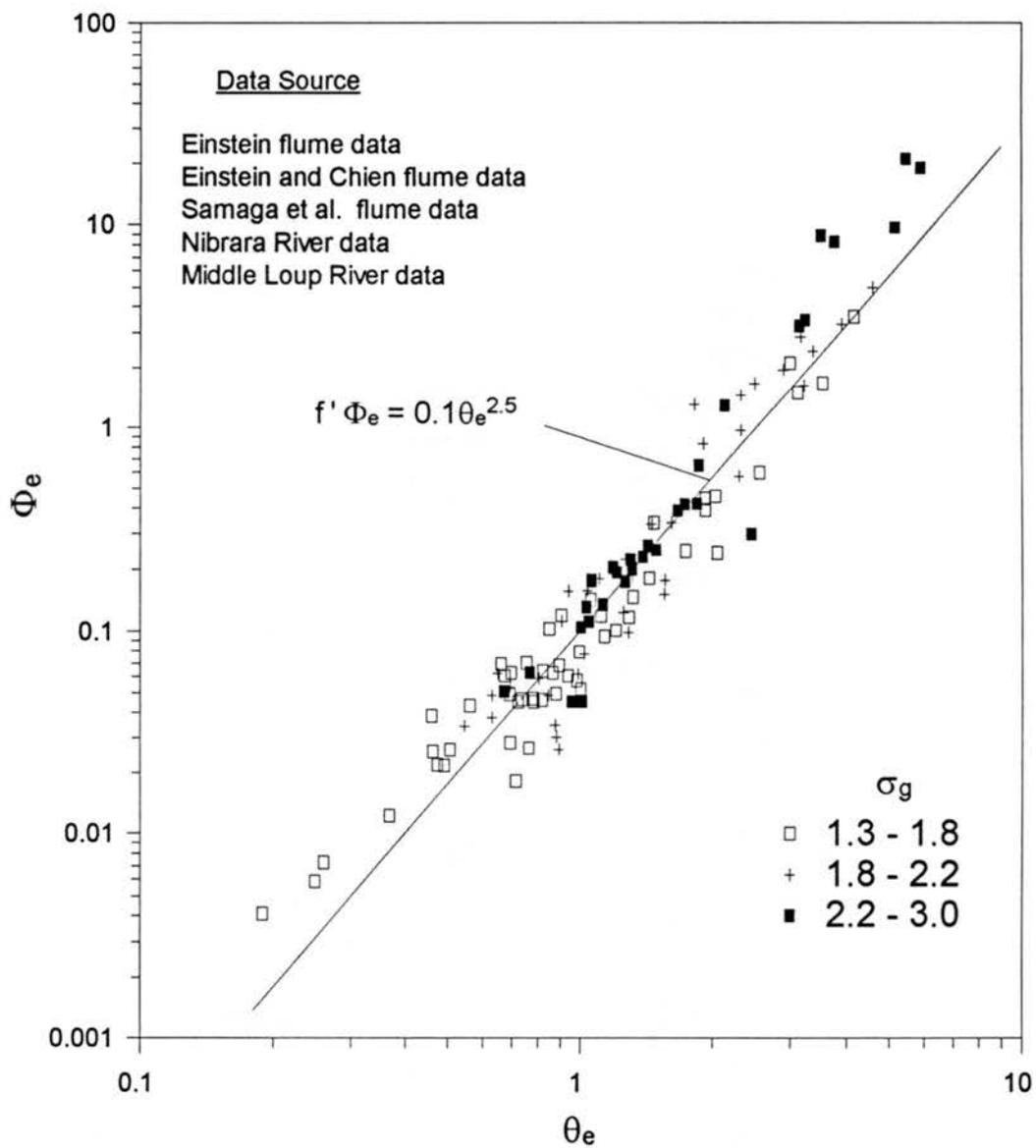


Fig. 4.8. Relationship between  $f' \Phi_e$  and  $\theta_e$  Using the Equivalent Representative Diameter,  $D_e$ .

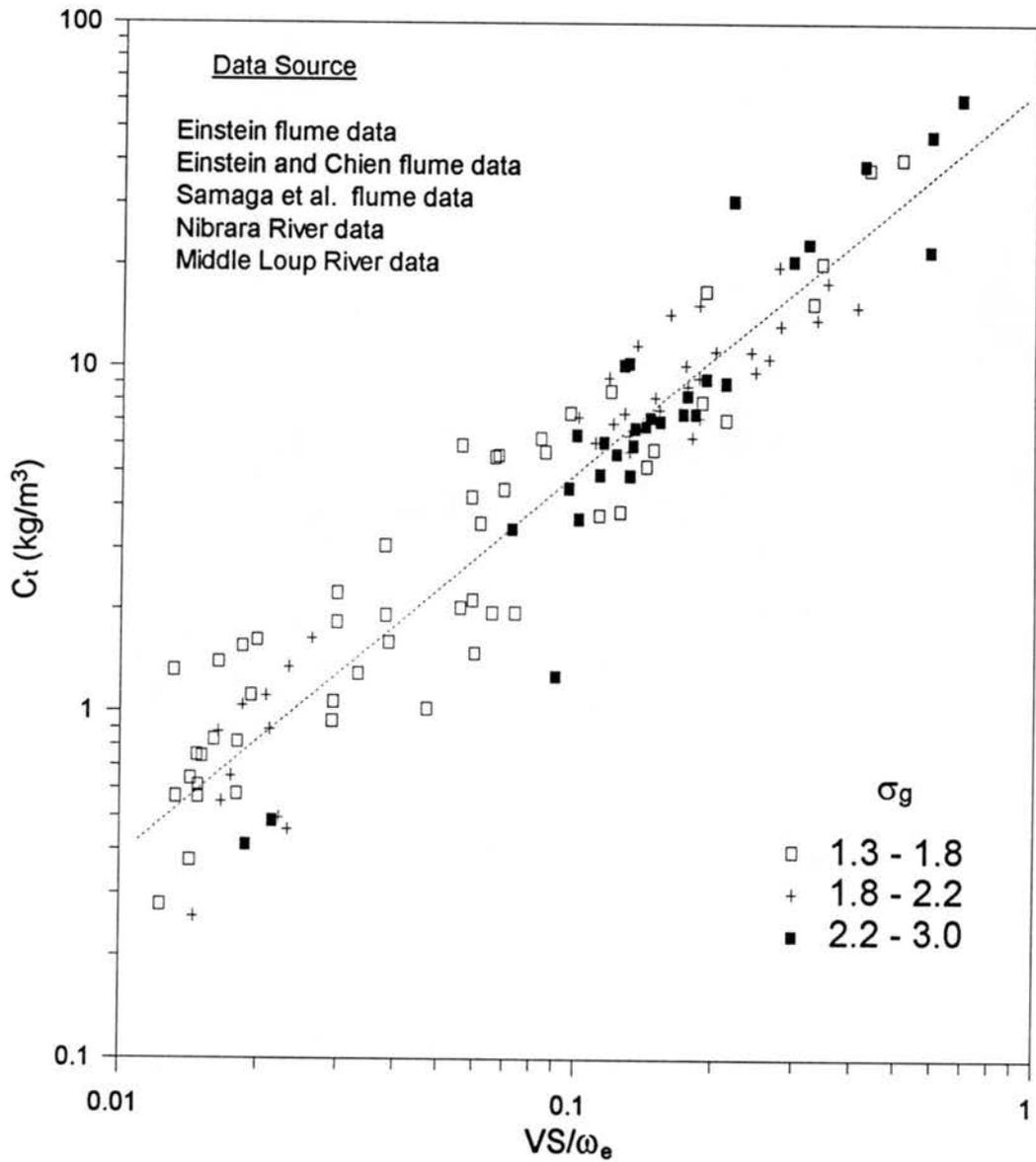


Fig. 4.11. Relationship between the Bed-Material Concentration,  $C_t$ , and the Dimensionless Unit Stream Power,  $VS/\omega_e$ , Using the Equivalent Representative Diameter,  $D_e$ .

that on the average  $C_t$  is correlated with  $VS/\omega_{50}$  and the size gradation effect on the transport cannot be observed. When the same data are replotted in Fig. 10 for three ranges of  $\sigma_g$  values, it indicates that the transport relationship is clearly a function of size gradation.

Using the equivalent diameter,  $D_e$ , defined by Eq. (4.6), the corresponding fall velocity can be expressed as

$$\omega_e = f(D_e) \quad (4.11)$$

Replacing  $\omega_{50}$  in Eq. (4.10) with  $\omega_e$  given by Eq. (4.11) results in

$$C_t = I \left( \frac{VS}{\omega_e} \right)^J \quad (4.12)$$

Fig. 4.11 shows the functional relationship given Eq. (4.12) using  $D_e$  ( $\omega_e$ ) as the representative size. It demonstrates that using  $D_e$ , the series of curves (for different  $\sigma_g$  values) are reduced to a single relationship.

#### 4.4 BED-MATERIAL TRANSPORT RATE COMPUTATION

The equivalent diameter,  $D_e$ , defined in Eq. (4.6) is introduced into the Engelund and Hansen, Ackers and White, and Yang formulas to account for the effects of nonuniformity of bed material size in sediment mixtures. First, the bed-material concentrations for the 118 sets of laboratory and river data given in Table 3.1 are computed in the normal manner. Then, the representative size is replaced by the equivalent diameter defined in Eq. (4.6). In the Ackers and White formula, the term  $D$  used to define the dimensionless grain diameter,  $D_{gr}$ , is not replaced by  $D_e$ , because  $D_{gr}$  was specifically defined for sediment mixtures. All other occurrences of the sediment size are replaced to be  $D_e$ .

The comparison between computed bed-material concentrations using the Engelund

and Hansen, Ackers and White, and Yang formulas and the measured values is summarized in Table 4.1 and in Figs.4.12-14. In Table 4.1, three different statistical methods are used to indicate the goodness of fit between the computed and measured results:

(i) the discrepancy ratio

$$R = \frac{C_{tc}}{C_{tm}} \quad (4.13)$$

in which  $C_{tc}$ ,  $C_{tm}$  = computed and measured bed-material concentrations, respectively. For a perfect fit,  $R = 1$ .

(ii) the Average Geometric Deviation between computed and measured bed material concentrations

$$AGD = \left( \prod_{j=1}^J RR_j \right)^{1/J}, \quad RR_j = \begin{cases} C_{tc} / C_{tm} & \text{for } C_{tc} \geq C_{tm} \\ C_{tm} / C_{tc} & \text{for } C_{tc} < C_{tm} \end{cases} \quad (4.14)$$

in which  $j$  = data set number,  $j = 1, 2, \dots, J$ ; and  $J$  = total number of data sets. For a perfect fit,  $AGD = 1$ .

(iii) the Mean Normalized Error

$$MNE = \frac{100}{J} \sum_{j=1}^J \left| \frac{C_{tc} - C_{cm}}{C_{cm}} \right| \quad (4.15)$$

for a perfect fit,  $MNE = 0$ .

Table 4.1 and Figs. 4.12-14 show that, on the average, selected formulas predict the sediment transport adequately. However, without using  $D_e$ , considerable scatter between computed and measured sediment concentrations exists.

Use of  $D_e$  reduced the average geometric deviation between computed and measured

Table 4.1. Summary of Comparison between Computed and Measured Total Bed-Material Concentrations

Author of Formula (1)	Representative Size of Bed Material (3)	Data in Range of Discrepancy Ratio, R (%)				Average Geometric Deviation, AGD (8)	Mean Normalized Error, MNE (%) (9)	Number of Data Sets (10)
		0.75-1.25 (4)	0.5-1.5 (5)	0.25-1.75 (6)	0.5-2.0 (7)			
Engelund and Hansen (1967)	$D_{50}$	22	57	73	74	1.74	59.6	118
	$D_e$	37	77	89	86	1.47	40.4	118
Ackers and White (1973)	$D_{35}$	34	64	74	80	1.61	59.9	118
	$D_e$	44	71	82	87	1.45	43.8	118
Yang (1973)	$D_{50} (\omega_{50})$	29	64	83	72	1.68	51.0	118
	$D_e (\omega_e)$	48	84	98	94	1.37	29.7	118

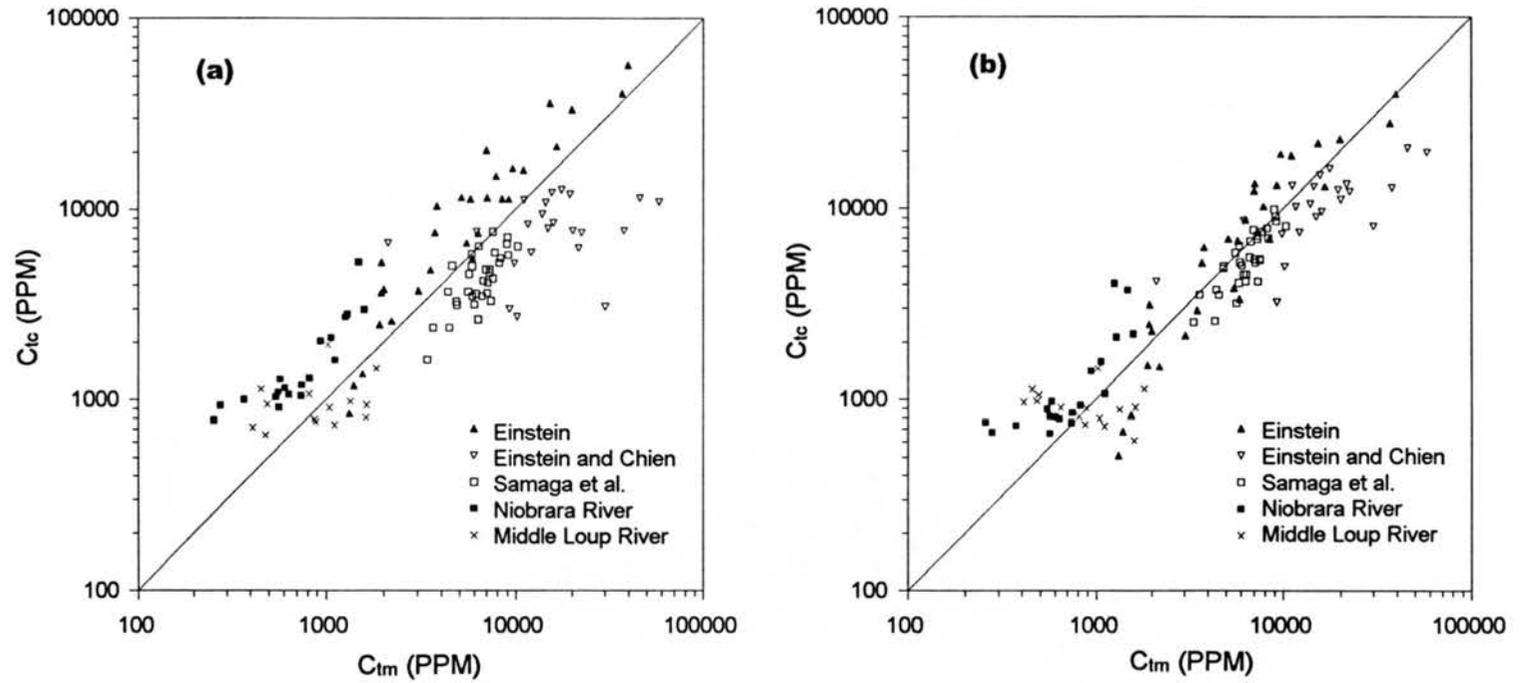


Fig. 4.12. Comparison between Computed and Measured Bed-Material Concentrations for the Engelund and Hansen Formula: (a) Using  $D_{50}$  as Representative Size; (b) Using  $D_6$  as Representative Size.

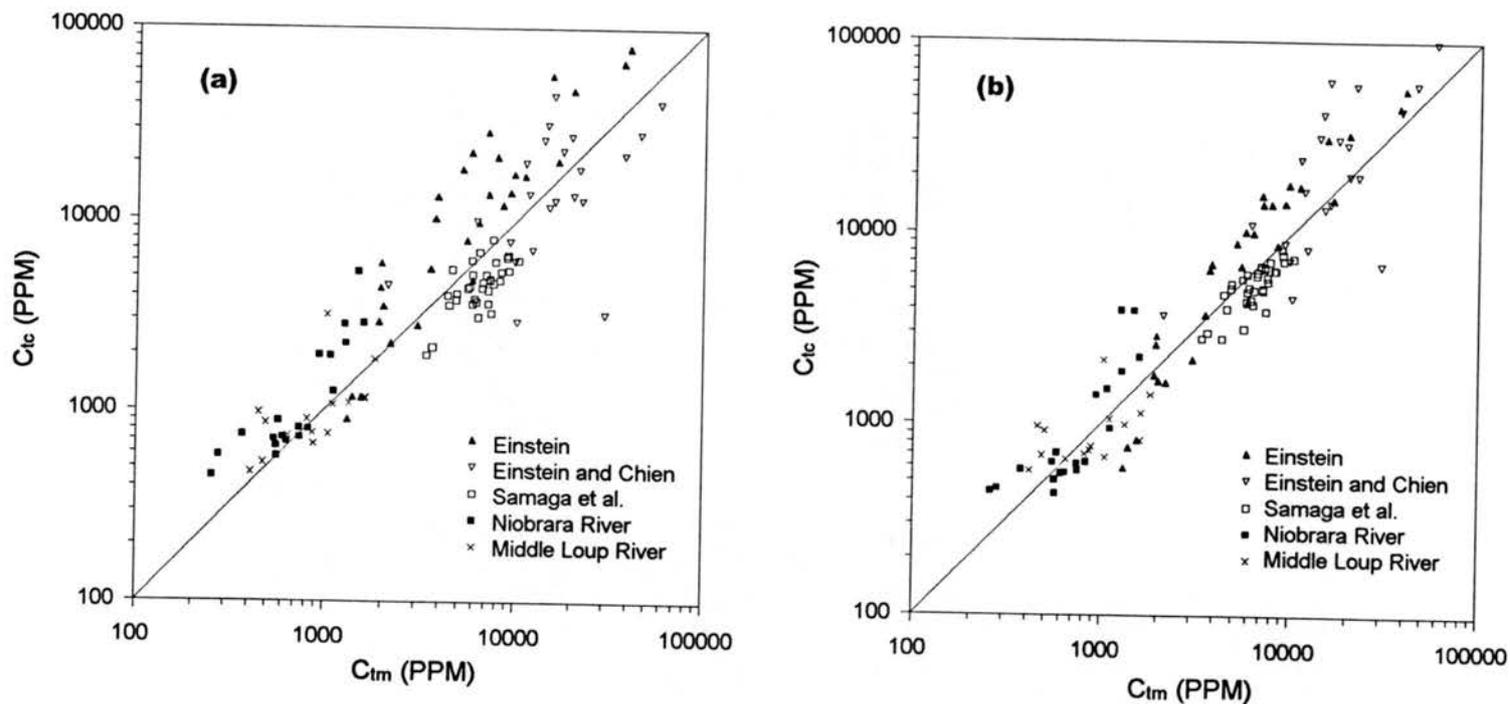


Fig. 4.13. Comparison between Computed and Measured Bed-Material Concentrations for the Akers and White Formula: (a) Using  $D_{35}$  as Representative Size; (b) Using  $D_e$  as Representative Size.

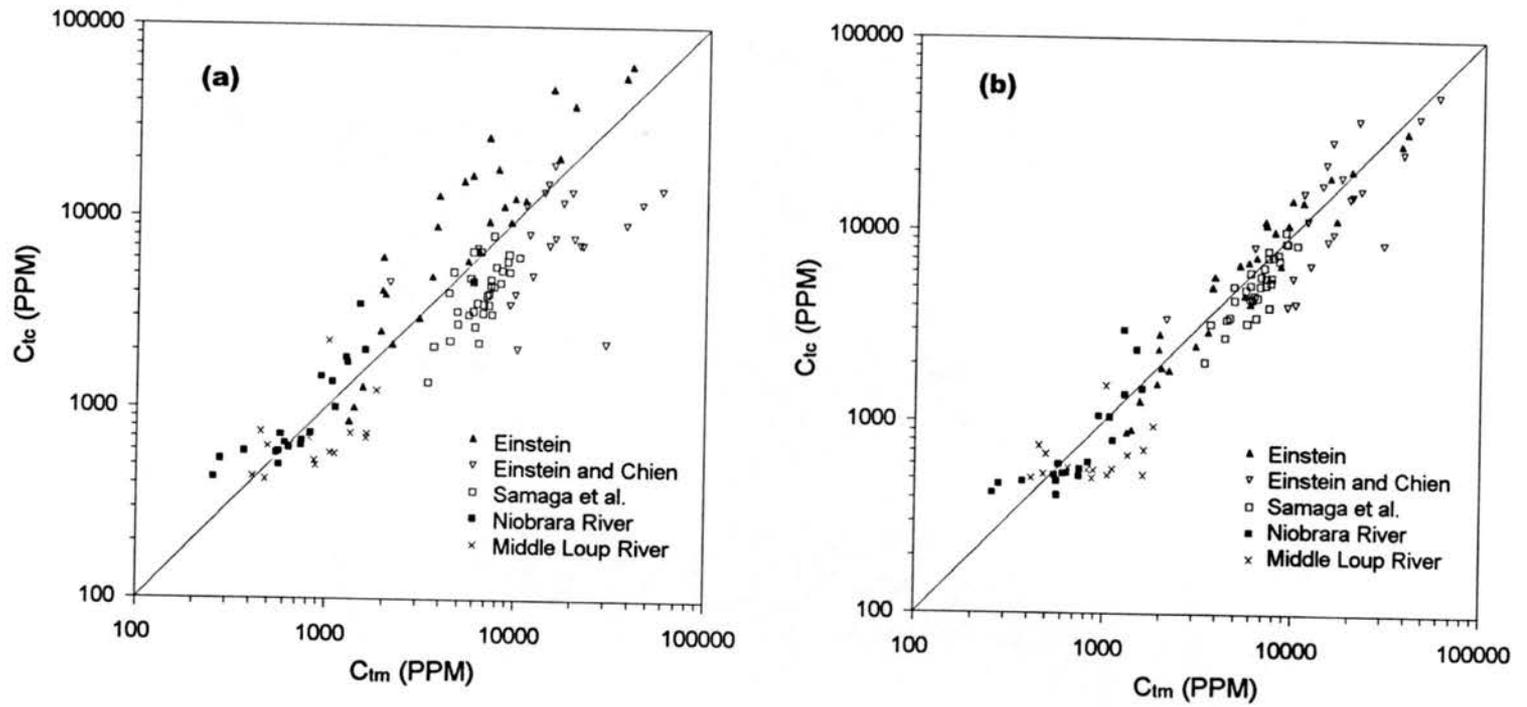


Fig. 4.14. Comparison between Computed and Measured Bed-Material Concentrations for the Yang Formula: (a) Using  $D_{50}$  as Representative Size; (b) Using  $D_e$  as Representative Size.

bed-material concentrations for the Engelund and Hansen, Ackers and White, and Yang formulas significantly from 1.74, 1.61, and 1.68 to 1.47, 1.45, and 1.37, respectively. The mean normalized error was also significantly reduced from 59.6%, 59.9%, and 51.0% to 40.4%, 43.8%, and 29.7%, respectively. The discrepancy ratios given in Table 4.1 also reflect these findings. The improvement in the range  $\pm 50\%$  was from 57% to 77% for the Engelund and Hansen formula, from 64% to 71% for the Ackers and White formula, and from 64% to 84% for the Yang formula. Through the use of  $D_e$ , 89% of the data could be accounted for within the range  $\pm 75\%$  (up from 73%) for the Engelund and Hansen formula, 82% (up from 74%) for the Ackers and White formula, and 98% (up from 83%) for the Yang formula.

For the Ackers and White formula, since the  $D_{35}$  of bed material was already used by the authors to accommodate the effects of size gradation on the transport of sediment mixtures, the additional improvement made by using  $D_e$  is not as significant as the improvement in the other two formulas.

#### 4.5 VERIFICATION OF THE USE OF EQUIVALENT DIAMETER, $D_e$ .

A group of 54 sets of flume data from Colorado State University (Guy et al., 1966) was chosen to verify the use of  $D_e$  for computing bed-material load. These data were chosen since they specifically include three different gradations ( $\sigma_g = 1.25, 1.57, \text{ and } 2.07$ ) for the same median sediment size of 0.33 mm.

The statistical results for the computed sediment concentrations using  $D_{50}$  and  $D_e$  as representative size are given in Table 4.2. The comparisons between computed and measured bed-material concentrations are also shown in Figs. 4.15-4.17 for the Engelund and Hansen,

Table 4.2. Summary of Comparison between Computed and Measured Total Bed-Material Concentrations for the CSU Flume Data with Particle Size of 0.33mm

Author of Formula (1)	Representative Size of Bed Material (3)	Data in Range of Discrepancy Ratio, R (%)				Average Geometric Deviation, AGD (8)	Mean Normalized Error, MNE (%) (9)	Number of Data Sets (10)
		0.75-1.25 (4)	0.5-1.5 (5)	0.25-1.75 (6)	0.5-2.0 (7)			
Engelund and Hansen (1967)	$D_{50}$	18.5	44.4	57.4	64.8	1.92	136.8	54
	$D_e$	29.6	61.1	81.5	74.1	1.65	88.1	54
Ackers and White (1973)	$D_{35}$	22.2	46.3	61.1	79.6	1.64	70.7	54
	$D_e$	40.7	68.5	83.3	85.2	1.49	43.1	54
Yang (1973)	$D_{50} (\omega_{50})$	33.3	63.0	77.8	81.5	1.58	61.2	54
	$D_e (\omega_e)$	48.2	66.7	92.6	79.6	1.51	48.8	54

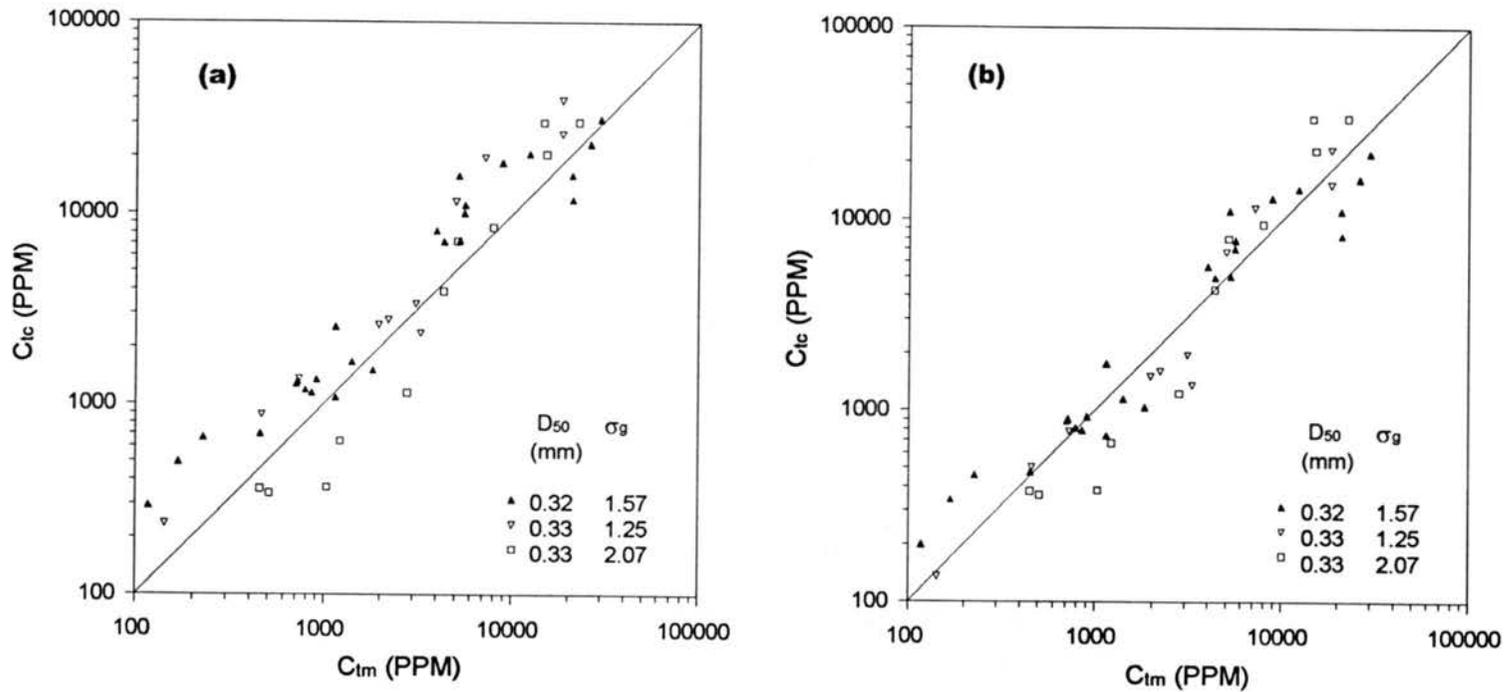


Fig. 4.15. Comparison between Computed and Measured Bed-Material Concentrations for the CSU Data with Particle Size of 0.33mm by the Engelund and Hansen Formula: (a) Using  $D_{50}$  as Representative Size; (b) Using  $D_c$  as Representative Size.

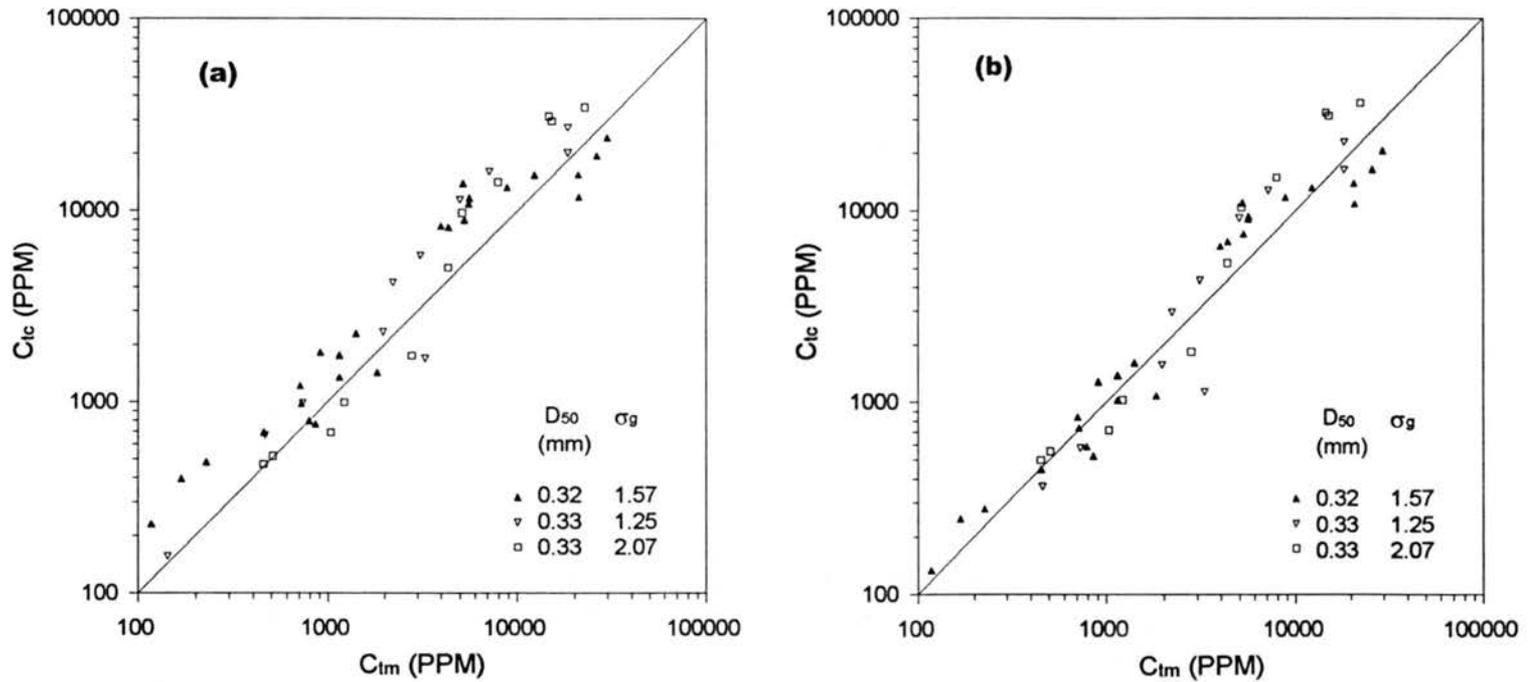


Fig. 4.16. Comparison between Computed and Measured Bed-Material Concentrations for the CSU Data with Particle Size of 0.33mm by the Ackers and White Formula: (a) Using  $D_{50}$  as Representative Size; (b) Using  $D_c$  as Representative Size.

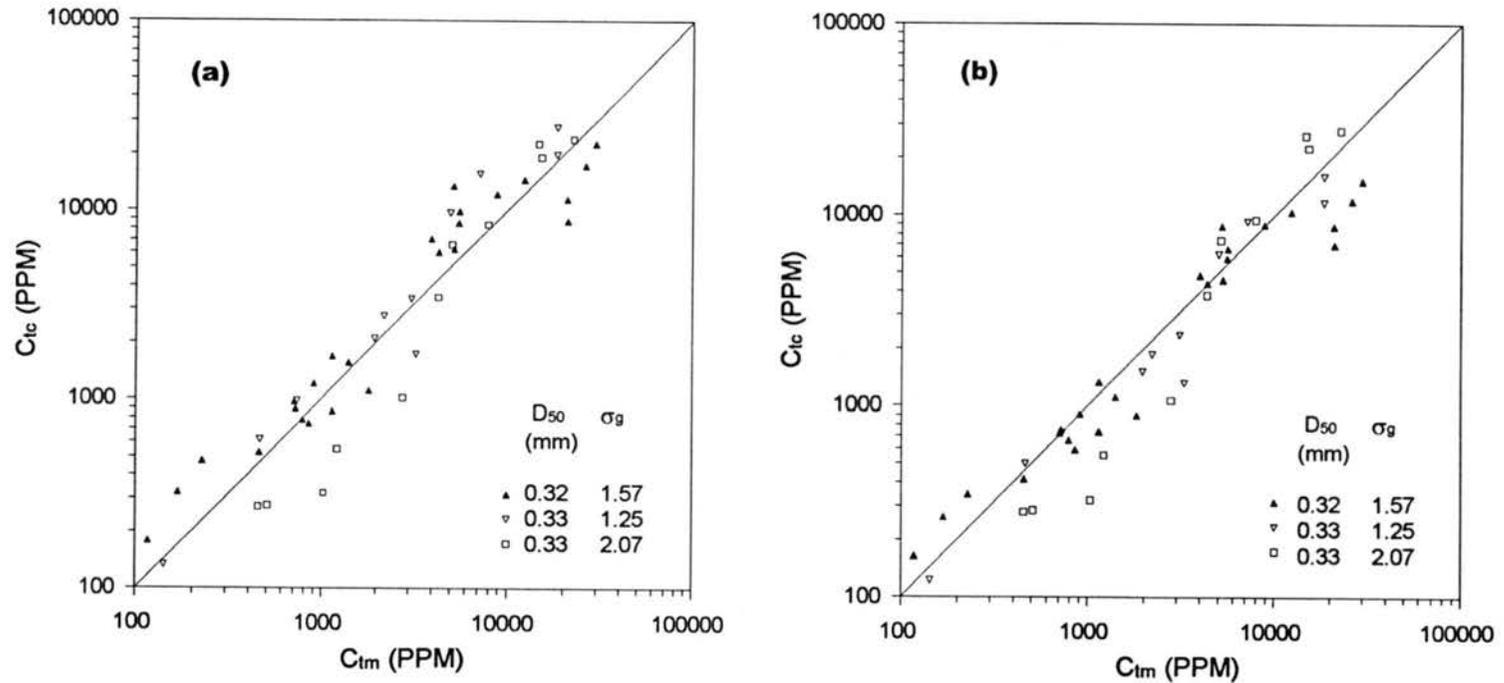


Fig. 4.17. Comparison between Computed and Measured Bed-Material Concentrations for the CSU Data with Particle Size of 0.33mm by the Yang Formula: (a) Using  $D_{50}$  as Representative Size; (b) Using  $D_e$  as Representative Size.

Ackers and White, and Yang formulas, respectively. The Engelund and Hansen formula predicts the transport rate adequately on the average. By using  $D_e$ , even though the range of  $\sigma_g$  variation is small, the scatter around the mean is reduced. Similar results were obtained also for the Ackers and White formula and Yang formula. The improvement in the range  $\pm 50\%$  was from 44.4% to 61.1% for the Engelund and Hansen formula, from 46.3% to 68.57% for the Ackers and White formula, and from 63.0% to 66.7% for the Yang formula. Also 81.5% of data fell in the range  $\pm 75\%$  (up from 57.4%) for the Engelund and Hansen formula, 83.3% (up from 61.1%) for the Ackers and White formula, and 92.6% (up from 77.8%) for the Yang formula. The average geometric deviation was reduced from 1.92 to 1.65 for the Engelund and Hansen formula, from 1.64 to 1.49 for the Ackers and White formula, and from 1.58 to 1.51 for the Yang formula. The mean normalized error also reduced from 136.8% to 88.8% for the Engelund and Hansen formula, from 70.7% to 43.1% for the Ackers and White formula, and from 61.2% to 48.8% for the Yang formula.

#### 4.6 SUMMARY

The variation of sediment sizes in transport and the effect of size gradation on the transport of sediment mixtures were studied extensively. The data used are limited to the sand size range, and to standard deviations,  $\sigma_g$ , in the range of 1.30 to 3.0. The findings in this chapter can be summarized as follows

1. The size composition of sediment in transport is different from the size composition of bed-surface material. The median diameter of sediment in transport is generally finer than the median diameter of bed material, which is resulting from selective transport of grains by flow. Eq. (4.3) is developed to estimate the median size of

bed-material load. This equation relates  $D_{50t}$  to median size of bed material, the size gradation of bed material, and flow intensity. The relative median size of sediment in transport,  $D_{50t}/D_{50}$ , decreases as size gradation increases, and the relationship between them can be presented by Eq. (4.4).

2. The effects of bed material size gradation on transport of sediment mixtures cannot be reflected appropriately by a single fixed size, such as  $D_{35}$  or  $D_{50}$ . Bed-material load formulas such as those developed by Engelund and Hansen, Ackers and White, and Yang are based on a single representative size of bed material and may generate considerable scatter when applied to nonuniform sediment mixtures.
3. Considering the physical processes governing the transport of sediment mixtures, the geometric standard deviation,  $\sigma_g$ , which represents the range of particle sizes present in the bed material, is found to be a significant additional parameter. For the same flow condition and the same  $D_{50}$ , the sediment size in transport and the transport rate of sediment mixtures are different for different sediment size gradations. For a given flow condition and median bed-material size, as the size gradation increases, the size of sediment in transport decreases resulting in higher sediment transport rates.
4. The median diameter,  $D_{50t}$ , predicted using Eq. (4.3) (equivalent diameter,  $D_e$ , for the existing bed-material load formulas), is a better indicator for nonuniform bed material. The median diameter,  $D_{50t}$ , is a function of the geometric standard deviation,  $\sigma_g$ , of the bed material and the flow conditions, in conjunction with the  $D_{50}$  of the bed material. Using  $D_{50t}$  ( $D_e$ ) will produce a more accurate prediction of bed-material sediment transport discharge for nonuniform sediment mixtures.
5. By incorporating  $D_e$  in the Engelund and Hansen, Ackers and White, and Yang

formulas, the accuracy of these functions is improved significantly. The equivalent diameter,  $D_e$ , does not change the overall functional behavior of the existing sediment transport functions, but it reduces the scatter due to the secondary effects of size gradations.

## CHAPTER 5

### APPLICATION OF THE PROPOSED METHOD

In this chapter, a general procedure for the computation of fractional bed-material concentrations using the proposed method is given. This procedure is illustrated through a detailed example problem.

#### 5.1 GENERAL PROCEDURE

The general procedure for the computation of fractional be-material concentrations using the proposed method can be summarized as:

**a) Adjustment of the size distributions of the bed material and the sediment in transport**

Measured Sediment concentrations are the total sediment concentrations and may include the wash load. As pointed out earlier in Chapter 1, for the analysis and comparison of bed-material load, the wash load portion of total load should be excluded from the measurements. Correspondingly, the size distributions of the bed material and the sediment in transport need to be adjusted.

**b) Computation of bed-material concentration,  $C_t$**

For the computation of bed-material concentration, an appropriate bed-material transport

equation should be selected for the problem. The Engelund and Hansen, Ackers and White, and Yang formulas are commonly used for sand-bed channels. In using these equations, the equivalent diameter,  $D_e$ , proposed in Chapter 4 should be used for nonuniform sediment mixtures to compensate for the nonuniformity effect. First, the median diameter,  $D_{50t}$ , of bed-material sediment in transport is computed by

$$D_{50t} = \frac{D_{50}}{1 + B(V_* / \omega_{50})^m (\sigma_g - 1)^n} \quad (4.3)$$

in which  $B$ ,  $m$ , and  $n = 0.8$ ,  $0.1$ , and  $2.2$ , respectively.

Then, the equivalent diameter is determined by

$$D_e = K_e D_{50t} \quad (4.6)$$

in which  $K_e = 1.8$ .

### c) Computation of transport capacity distribution function, $P_{ci}$

The transport capacity distribution function,  $P_{ci}$ , can be computed by either Eq. (3.9) or Eq. (3.24) which were derived in Chapter 3. From practical consideration, the use of Eq. (3.24) is suggested since it does not require the computation of relative fall velocity, and since it provides the same accuracy as Eq. (3.9). This equation is expressed as follows

$$P_{ci} = \frac{P_{bi} \left[ \left( \frac{D_i}{D_{50}} \right)^\alpha + \zeta \left( \frac{D_i}{D_{50}} \right)^\beta \right]}{\sum_{i=1}^N P_{bi} \left[ \left( \frac{D_i}{D_{50}} \right)^\alpha + \zeta \left( \frac{D_i}{D_{50}} \right)^\beta \right]} \quad (3.24)$$

The coefficients are given by

$$\alpha = -2.9 \exp \left[ -1000 \left( \frac{V}{V_*} \right)^2 \left( \frac{d}{D_{50}} \right)^{-2} \right] \quad (3.25)$$

$$\beta = 0.2 \sigma_g \quad (3.26)$$

$$\zeta = 2.8 F_r^{-1.2} \sigma_g^{-3} \quad (3.27)$$

**d) Computation of fractional bed-material concentrations,  $C_{ui}$**

After the bed-material concentration,  $C_b$ , and the transport capacity distribution function,  $P_{ci}$ , are determined, the fractional bed-material concentrations,  $C_{ui}$ , can be computed according to the TCF concept, i.e.

$$C_{ui} = P_{ci} C_b \quad (2.8)$$

## 5.2 EXAMPLE PROBLEM

An example problem showing the detailed steps in using the proposed method for the computation of fractional bed-material concentrations is presented. This example problem is derived from the measurements on July 7, 1949 at the Niobrara River (see Table 3.5). The measured flow and sediment properties are as follows:

$Q = 7.56 \text{ m}^3/\text{s}$	$S = 0.001345$
$V = 0.7485 \text{ m/s}$	$T = 23.9 \text{ }^\circ\text{C}$ ( $\nu = 9.28 \times 10^{-7} \text{ m}^2/\text{s}$ )
$W = 21.49 \text{ m}$	$s_g = 2.65$
$d = 0.47 \text{ m}$	$C_T = 970.0 \text{ PPM}$

The detailed size distribution of bed material (including the sizes corresponding to wash load) and the size distribution of sediment in transport (total sediment load) are given

in the following tabulation:

k	1	2	3	4	5	6	7	8
$D'_k$ (mm)	0.062	0.125	0.25	0.5	1	2	4	8
$P'_{bk}$ (%)	0.0	2.0	35.0	92.0	98.0	99.0	100.0	100.0
$P'_{tk}$ (%)	7.0	15.0	58.0	93.0	99.0	100.0	100.0	100.0

In the above tabulation for size distributions,  $k$  is the size fraction number;  $D'_k$  is the upper bound diameter;  $P'_{bk}$  is the percentage of bed material finer than the indicated sizes by weight; and  $P'_{tk}$  is the percentage of total sediment load finer than the indicated sizes by weight.

The detailed steps for the computation of fractional bed-material concentration for the given problem are as follows:

**a) Adjustment of bed material size distribution**

*Step 1. Wash load limit diameter,  $D_w$*

There are various methods to determine the wash load limit diameter in the literature. For simplicity, the wash load limit diameter for the given problem is determined to be

$$D_w = 0.125 \text{ (mm)} \quad (5.1)$$

since there is no significant quantity (2%) of bed material finer than this size. The corresponding size fraction number,  $k_w$ , is

$$k_w = 2 \quad (5.2)$$

The corresponding percentages of the bed material and the total sediment load finer than  $D_w$  are

$$P'_b(D_w) = 2.0(\%) \quad (5.3)$$

and

$$P'_i(D_w) = 15.0(\%) \quad (5.4)$$

*Step 2. Adjusted size distributions*

The size distributions of bed material and the sediment in transport are adjusted by subtracting sizes corresponding to the wash load. The computed results are given in

Table 5.1 Adjusted Size Distributions of Bed Material and the Sediment in Transport

i	1	2	3	4	5	6	7
$D_i''$ (mm)	0.125	0.25	0.5	1	2	4	8
$D_i$ (mm)	0.177	0.354	0.707	1.414	2.828	5.657	
$P_{bi}''$ (%)	0	33.67	91.84	97.96	98.98	100.0	100.0
$P_{bi}$	0.3367	0.5817	0.0612	0.0102	0.0102	0	
$P_{ti}''$ (%)	0	50.59	91.97	98.82	100.0	100.0	100.0
$P_{ti}$	0.5059	0.4117	0.0706	0.0118	0	0	

Table 5.2 Computations of Size Fractions of the Bed-Material Transport Capacity

i	1	2	3	4	5	6	$\Sigma$
$D_i$ (mm)	0.177	0.354	0.707	1.414	2.828	5.657	
$P_{tempi}$	2.336	1.797	0.193	0.039	0.049	0	4.4152
$P_{ci}$	0.529	0.407	0.044	0.009	0.011	0	1.000
$C_{ti}$ (PPM)	493.0	379.3	40.7	8.3	10.4	0	931.7
$C_{tmi}$ (PPM)	417.1	339.4	58.2	9.7	0	0	824.4

Table 5.1. The variables used in this table are determined by the following relations

$$D_i'' = D_{i+k_w-1}' \quad (i = 1, 2, \dots, 7) \quad (5.5)$$

$$D_i = \sqrt{D_i'' D_{i+1}''} \quad (i = 1, 2, \dots, 6) \quad (5.6)$$

$$P_{bi}'' = \frac{P_{bi+k_w-1}' - P_b'(D_w)}{100 - P_b'(D_w)} \times 100 \quad (i = 1, 2, \dots, 7) \quad (5.7)$$

$$P_{ti}'' = \frac{P_{ti+k_w-1}' - P_t'(D_w)}{100 - P_t'(D_w)} \times 100 \quad (i = 1, 2, \dots, 7) \quad (5.8)$$

$$P_{bi} = \frac{P_{bi+1}'' - P_{bi}''}{100} \quad (i = 1, 2, \dots, 6) \quad (5.9)$$

$$P_{ti} = \frac{P_{ti+1}'' - P_{ti}''}{100} \quad (i = 1, 2, \dots, 6) \quad (5.10)$$

### Step 3. Characteristic sizes of bed material

Assuming that the adjusted bed material size follows a logarithmic normal distribution, the characteristic diameters can be interpolated as follows

$$D_{16} = 10^{\left( \log 0.125 + \frac{\log 0.25 - \log 0.125}{33.67 - 0.0} (16.0 - 0.0) \right)} = 0.1738 \text{ (mm)} \quad (5.11)$$

$$D_{50} = 10^{\left( \log 0.25 + \frac{\log 0.5 - \log 0.25}{91.84 - 33.67} (50.0 - 33.67) \right)} = 0.3037 \text{ (mm)} \quad (5.12)$$

$$D_{84} = 10^{\left(\log 0.25 + \frac{\log 0.5 - \log 0.25}{91.84 - 33.67} (84.0 - 33.67)\right)} = 0.4554 \text{ (mm)} \quad (5.13)$$

and

$$\sigma_g = \sqrt{D_{84}/D_{16}} = \sqrt{0.4554/0.1738} = 1.619 \quad (5.14)$$

**b) Computation of bed-material concentration,  $C_t$**

*Step 4. Shear velocity and fall velocity of  $D_{50}$*

Assuming  $R \approx d$ :

$$V_* = \sqrt{gdS} = \sqrt{9.81 \times 0.47 \times 0.001345} = 0.0787 \text{ (m/s)} \quad (5.15)$$

The fall velocity is determined using the method presented by the U. S. Inter-Agency Committee on Water Resources, Subcommittee on Sedimentation (1957), which gives

$$\omega_{50} = 0.0432 \text{ (m/s)} \quad (5.16)$$

*Step 5.  $D_{50t}$  from Eq. (4.3) and  $D_e$  from Eq. (4.6)*

$$\begin{aligned} D_{50t} &= \frac{D_{50}}{1 + B(V_*/\omega_{50})^m (\sigma_g - 1)^n} \\ &= \frac{0.3037}{1 + 0.8(0.0787/0.0432)^{0.1} (1.619 - 1)^{2.2}} \\ &= 0.2344 \text{ (mm)} \end{aligned} \quad (5.17)$$

$$D_e = K_e D_{50t} = 1.8 \times 0.2344 = 0.4219 \text{ (mm)} \quad (5.18)$$

*Step 6. Bed-material concentration by Engelund and Hansen formula*

The Engelund and Hansen (1967) formula for bed-material concentration can be expressed as

$$C_v = \frac{0.05 V d^{1/2} S^{3/2}}{(s_g - 1)^2 g^{1/2} D_{50}} \quad (5.19)$$

in which  $C_v$  = concentration of bed-material load by volume. Using  $D_e$  as the representative diameter gives

$$\begin{aligned} C_v &= \frac{0.05 V d^{1/2} S^{3/2}}{(s_g - 1)^2 g^{1/2} D_e} \\ &= \frac{0.05 \times 0.7485 \times 0.47^{1/2} \times 0.001345^{3/2}}{(2.65 - 1)^2 \times 9.81^{1/2} \times 0.0004219} = 3.5179 \times 10^{-4} \end{aligned} \quad (5.20)$$

The concentration by volume can be transferred into concentration in the unit of PPM through

$$C_t = \frac{10^6 s_g C_v}{1 + (s_g - 1) C_v} = \frac{10^6 \times 2.65 \times C_v}{1 + (2.65 - 1) \times C_v} = 931.7 \text{ (PPM)} \quad (5.21)$$

**c) Computation of transport capacity distribution function,  $P_{cl}$**

*Step 7. Coefficients in Eq. (3.24)*

$$\begin{aligned} \alpha &= -2.9 \exp \left[ -1000 \left( \frac{V}{V_*} \right)^2 \left( \frac{d}{D_{50}} \right)^{-2} \right] \\ &= -2.9 \exp \left[ -1000 \left( \frac{0.7485}{0.0787} \right)^2 \left( \frac{0.47}{0.0003037} \right)^{-2} \right] \\ &= -2.7925 \end{aligned} \quad (5.22)$$

$$\beta = 0.2 \sigma_g = 0.2 \times 1.619 = 0.3238 \quad (5.23)$$

$$\begin{aligned} \zeta &= 2.8 F_r^{-1.2} \sigma_g^{-3} \\ &= 2.8 \times \left( \frac{0.7485}{\sqrt{9.81 \times 0.47}} \right)^{-1.2} \times 1.619^{-3} \\ &= 2.3369 \end{aligned} \quad (5.24)$$

*Step 8. Size fractions of the bed-material transport capacity*

The computed size fractions of the bed-material transport capacity are given in Table 5.2, which are computed using Eq. (3.24), i.e.

$$\begin{aligned} P_{ci} &= \frac{P_{bi} \left[ \left( \frac{D_i}{D_{50}} \right)^\alpha + \zeta \left( \frac{D_i}{D_{50}} \right)^\beta \right]}{\sum_{i=1}^N P_{bi} \left[ \left( \frac{D_i}{D_{50}} \right)^\alpha + \zeta \left( \frac{D_i}{D_{50}} \right)^\beta \right]} \\ &= \frac{P_{bi} \left[ \left( \frac{D_i}{0.3037} \right)^{-2.7925} + 2.3369 \left( \frac{D_i}{0.3037} \right)^{0.3238} \right]}{\sum_{i=1}^6 P_{bi} \left[ \left( \frac{D_i}{0.3037} \right)^{-2.7925} + 2.3369 \left( \frac{D_i}{0.3037} \right)^{0.3238} \right]} \end{aligned} \quad (5.25)$$

in which  $D_i$  is expressed in mm; and  $P_{bi}$  is given in Table 5.1. The numerator in Eq. (5.25) is denoted as  $P_{tempi}$ , i.e.

$$P_{tempi} = P_{bi} \left[ \left( \frac{D_i}{0.3037} \right)^{-2.7925} + 2.3369 \left( \frac{D_i}{0.3037} \right)^{0.3238} \right] \quad (5.26)$$

First, the values of  $P_{tempi}$  corresponding to each size fraction are computed for various size groups, and the summation of  $P_{tempi}$  are obtained. Then, the capacity fractions for each size group are obtained by dividing  $P_{tempi}$  with  $\sum P_{tempi}$  according to Eq. (5.25).

**d) Computation of fractional bed-material concentrations,  $C_{ii}$**

*Step 9. Fractional bed-material concentrations*

The computed fractional bed-material concentrations are given in Table 5.2. The value of  $C_{ii}$  is computed according to the TCF concept, i.e.

$$C_{ii} = P_{ci} C_t = P_{ci} \times 931.7 \text{ (PPM)} \quad (5.27)$$

*Step 10. Measured fractional bed-material concentrations*

$$C_{tmi} = P_{ii} C_{tm} \text{ (PPM)} \quad (5.28)$$

in which  $P_{ii}$  = given in Table 5.1; and  $C_{tm}$  = the measured bed-material concentration, which is computed by

$$C_{tm} = [100 - P'_t(D_w)] \times C_T = (100 - 15) / 100 \times 970.0 = 824.0 \text{ (PPM)} \quad (5.29)$$

The measured fractional bed-material concentrations presented in Table 5.2 can be used for comparison with computed results.

## CHAPTER 6

### COMPARISON AND EVALUATION

#### 6.1 GENERAL

Comparison and evaluation of existing transport functions generally limit the analyses to comparisons of bed-material sediment transport rate in the literature (American Society of Civil Engineering, 1982; White et al., 1975; Alonso, 1980; Brownlie, 1981; Yang and Molinas, 1982; Vetter, 1989; Yang and Wan, 1991; Wu, 1992a; and Wu and Long, 1992, 1993). Along with the application of the transport capacity distribution functions proposed in Chapter 3, an extensive evaluation on the fractional load computations is conducted using flume and field data. This evaluation is necessary to qualitatively and quantitatively demonstrate the performance of the newly developed fractional load computation method, and to show the limitations and variations of different methods.

In the four groups of fractional load computation methods, those of direct computation by the size fraction approach, the BMF approach, and the TCF approach will be evaluated in this chapter. Since most of the methods derived following the shear stress correction approach are only applicable for the predictions of fractional transport rates of gravel bed materials, methods in this group are excluded from this comparison.

For the computation of fractional load using different methods, Hydrau-Tech, Inc.'s SedWin (Visually Interactive Sediment Transport Computation Model for Windows 95/98)

(Wu and Molinas, 1998) is applied in this study. This program is developed for the computations of sediment transport rate by selected transport equations. Both bed-material transport rate and fractional bed-material transport rate can be computed. The new transport capacity distribution functions,  $P_{ci}$ , developed in Chapter 3 and the use of the equivalent diameter,  $D_e$ , proposed in Chapter 4 for the computation of bed-material load are incorporated in the program.

## **6.2 FRACTIONAL LOAD COMPUTATIONS**

### **6.2.1 Fractional Load Using the Direct Computation by Size Fraction Approach**

Even though the equations of Einstein (1950), Laursen (1958), and Toffaleti (1968, 1969) were developed based on the computations of sediment transport rates for each individual size fraction of sediment mixtures, accuracy of their predictions as to the distribution by different sediment size fractions is not known.

The bed-material concentrations of individual size fractions are computed for these three methods using the SedWin program. Eqs. (2.9), (2.15), and (2.17) are the basic transport relations of Einstein, Laursen, and Toffaleti, respectively. Detailed computation procedures for these methods can be found in Stevens and Yang (1989), Raudkivi (1990), Simons and Sentürk (1992), Julien (1995), and Yang (1996).

### **6.2.2. Fractional Load Using the Bed Material Fraction Approach**

In the four groups of methods for the computation of fractional sediment transport rates, the BMF approach is still the most commonly used one in numerical models, even though the

shortcomings of using this approach in nonuniform sediment transport models have been recognized in the literature (Karim and Kennedy, 1982; Hsu and Holly, 1992). This fact indicates that the basic concept of the BMF approach may be acceptable to some extent. According to the concept of the BMF approach, the potential concentration,  $C_{pi}$ , for a given size fraction,  $i$ , is computed by replacing the representative size used in an equation with the average diameter,  $D_i$ . Then the fractional transport rates are ready to be determined as the product of  $P_{bi}$  and  $C_{pi}$ . In the present study, the potential concentration is obtained by applying the transport formulas developed by Engelund and Hansen (1967), Ackers and White (1973), and Yang (1973) to laboratory and river data.

*Engelund and Hansen's Method.* Engelund and Hansen (1967) used the similarity principle to obtain the sediment transport function [see Eq. (3.15)]. In using the BMF approach to compute the bed-material concentrations of individual size fractions for sediment mixtures, the Engelund and Hansen formula can be transferred into

$$C_{pi} = \frac{0.05 V d^{1/2} S^{3/2}}{\left(\frac{\gamma_s - \gamma}{\gamma}\right)^2 g^{1/2} D_i} \quad (6.1)$$

in which  $C_{pi}$  = potential concentration of bed-material load by volume corresponding to the size fraction  $i$ .

*Ackers and White's Method.* Ackers and White (1973) developed their transport functions based on the mobility theory. In using the BMF approach, their transport function can be transferred into

$$C_{pi} = C \frac{\gamma_s}{\gamma} \frac{D_i}{d} \left( \frac{V}{V_*} \right)^n \left( \frac{F_{gri}}{A} - 1 \right)^m \quad (6.2)$$

where

$$F_{gri} = \frac{V_*^n}{\sqrt{gD_i \left( \frac{\gamma_s - \gamma}{\gamma} \right)}} \left[ \frac{V}{\sqrt{32} \log \left( \frac{\alpha d}{D_i} \right)} \right]^{1-n} \quad (6.3)$$

$$D_{gri} = D_i \left[ \frac{g \left( \frac{\gamma_s - \gamma}{\gamma} \right)}{v^2} \right]^{1/3} \quad (6.4)$$

in which  $A$ ,  $C$ ,  $m$ , and  $n$  = parameters as function of  $D_{gri}$ ;  $D_{gri}$  = dimensionless grain diameter corresponding to the size fraction,  $i$ ;  $F_{gri}$  = mobility number for sediment corresponding to the size fraction,  $i$ ;  $\alpha$  = coefficient; and  $v$  = kinematic viscosity. In the use of Ackers and White equation, the modified coefficients of  $m$  and  $C_A$  (White and Wang<sup>1)</sup> are used:

$$m = 6.38 / D_{gr} + 1.67 \quad (6.5)$$

$$C_A = 2.79 / \log(D_{gr}) - 0.97 [\log(D_{gr})]^2 - 3.46 \quad (6.6)$$

*Yang's Method.* Yang (1973) sediment transport formula is based on unit stream power theory. In using the BMF approach, his dimensionless unit stream power formula can be transferred into

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<sup>1)</sup> Private communication.

$$\log(C_{pi}) = 5.435 - 0.286 \log\left(\frac{\omega_i D_i}{v}\right) - 0.457 \log\left(\frac{V_*}{\omega_i}\right) + \left[ 1.799 - 0.409 \log\left(\frac{\omega_i D_i}{v}\right) - 0.314 \log\left(\frac{V_*}{\omega_i}\right) \right] \log\left(\frac{VS}{\omega_i} - \frac{V_{cr} S}{\omega_i}\right) \quad (6.7)$$

where

$$\frac{V_{cr}}{\omega_i} = \begin{cases} \frac{2.05}{\log\left(\frac{V_* D_i}{v}\right) - 0.06} & \text{for } 1.2 < \frac{V_* D_i}{v} < 70 \\ 2.05 & \text{for } 70 \leq \frac{V_* D_i}{v} \end{cases} \quad (6.8)$$

in which  $C_{pi}$  = potential concentration of bed-material load in PPM, by weight, corresponding to the size fraction  $i$ ; and  $V_{cr}$  = critical velocity at incipient motion.

*Karim's Modified BMF Method.* Recently, Karim (1998) developed a new method for the prediction of fractional loads, which can be classified as the modified BMF method. This relation is proposed for sand-bed flows and can be expressed as

$$q_{ti} = 0.00139 \sqrt{g(s_g - 1)D_i^3} \left( \frac{V}{\sqrt{g(s_g - 1)D_i}} \right)^{2.97} \left( \frac{V_*}{\omega_i} \right)^{1.47} \phi_i \quad (6.9)$$

in which  $q_{ti}$  = volumetric sediment discharge per unit width for  $i$ th fraction; and  $\phi_i$  = weighing function for  $i$ th fraction, which is the function of an areal function ( $P_{ai}$ ) and a sheltering factor ( $\eta$ ). Formulations for the weighing function were given in Chapter 2 [Eqs. (2.44)-(2.47)].

### 6.2.3 Fractional Load Using the Transport Capacity Fraction Approach

To obtain transport capacities for individual size fractions based on the TCF approach, both the bed-material concentration,  $C_b$ , and the fractions of bed-material transport capacity,  $P_{ci}$ , are needed. As pointed out earlier,  $C_b$  can be determined using any appropriate bed-material load equations. For this purpose, the Yang (1973) formula, along with the equivalent diameter,  $D_e$ , proposed in Chapter 4, can be used to determine the bed-material transport rate. For the computation of  $P_{ci}$ , both Karim and Kennedy's (1981) method and the new transport capacity distribution functions proposed in Chapter 3 can be used.

*Karim and Kennedy's Function.* The transport capacity distribution function proposed by Karim and Kennedy (1981) is applicable to flows in which total sediment discharge mainly consists of suspended load. This function was used in their numerical river simulation model IALLUVIAL (Karim and Kennedy, 1982; and Karim, 1985). Formulations for this method are given by Eqs. (2.55) and (2.56) in Chapter 2.

*Li's Function.* The transport capacity distribution function proposed by Li (1988) was developed for suspended load. Thus, it is applicable to flows in which total sediment discharges are composed mainly of suspended load. Formulations for Li's method are given by Eqs. (2.53) and (2.54) in Chapter 2.

*Proposed Functions.* The transport capacity distribution functions proposed in Chapter 3 were derived from the basic concept of the BMF approach in conjunction with the consideration of the sheltering and exposure effect introduced in the shear stress correction

approach. The formulation of Eq. (3.9) is a function of relative fall velocity, while the formulation of Eq. (3.24) is a function of relative diameter. Both equations are used as weighting functions to calculate fractional load for sediment mixtures, and their performance will be compared with other fractional load methods. The general procedure and a detailed example problem showing how to use the proposed method are given in Chapter 5.

### 6.3 COMPARISON OF COMPUTED RESULTS

The fractional bed-material concentrations computed using various methods are compared in this section. Sediment transport data used in this comparison are those 112 sets of flume and field data given in Table 3.1, which contain a total of 891 data points. Methods used in comparisons, and their classifications are summarized as follows:

- a) Direct computation by size fraction approach
  - 1) Einstein's equation (1950);
  - 2) Laursen's equation (1958);
  - 3) Toffaleti's equation (1968);
- b) BMF approach using:
  - 4) Engelund and Hansen's equation (1967);
  - 5) Ackers and White's equation (1973);
  - 6) Yang's equation (1973);
  - 7) Karim's modified BMF method (1998);
- c) TCF approach using the Yang (1973) equation with  $D_e$  and
  - 8) Transport capacity distribution function of Karim and Kennedy (1981);
  - 9) Transport capacity distribution function of Li (1988);

- 10) Proposed transport capacity distribution function of Eq. (3.9); and
- 11) Proposed transport capacity distribution function of Eq. (3.24).

For fractional transport of sediment mixtures, not only the absolute values of transport sediment concentration need to be predicted accurately, but also the size compositions need to be estimated correctly. From computed fractional bed-material concentrations, the size fraction of bed-material transport capacity (in percent) for a given size group can be obtained by

$$P_{cci} = 100 \frac{C_{tci}}{C_{tc}} = 100 \frac{C_{tci}}{\sum_{i=1}^N C_{tci}} \quad (6.10)$$

### 6.3.1 Statistical Analysis

In the statistical analysis, three different statistical methods are adopted to indicate the goodness of fit between the computed and measured results. These statistical methods are similar to those used in Chapter 4. However, the statistics conducted in this section are for the fractional bed-material concentrations in a sediment mixture, rather than the bed-material concentrations. These statistical methods are as follows:

- 1) the discrepancy ratio,  $R_i$

$$R_i = \frac{C_{tci}}{C_{tmi}} \quad (6.11)$$

in which  $C_{tci}$ ,  $C_{tmi}$  = computed and measured bed-material concentrations corresponding to size fraction  $i$ , respectively; and  $i$  = size fraction number or data point number in a data set. For a perfect fit,  $R_i = 1$ .

- 2) the Average Geometric Deviation between computed and measured fractional bed-material concentrations, AGD

$$AGD = \left( \prod_{j=1}^J \prod_{i=1}^{N_j} RR_i \right)^{1/JN}, \quad RR_i = \begin{cases} C_{tci} / C_{tmi} & \text{for } C_{tci} \geq C_{tmi} \\ C_{tmi} / C_{tci} & \text{for } C_{tci} < C_{tmi} \end{cases} \quad (6.12)$$

in which  $j$  = data set number,  $j = 1, 2, \dots, J$ ;  $J$  = total number of data sets;  $N_j$  = number of points in a given data set; and  $JN$  = the total number of data points. For a perfect fit,  $AGD = 1$ .

- 3) the Mean Normalized Error, MNE

$$MNE = \frac{100}{JN} \sum_{j=1}^J \sum_{i=1}^{N_j} \left| \frac{C_{tci} - C_{tmi}}{C_{tmi}} \right| \quad (6.13)$$

for a perfect fit,  $MNE = 0$ .

Detailed statistical results for the comparison between the computed and measured bed-material concentrations of individual size fractions are given in Table 6.1. It can be seen that the mean normalized errors for the direct computation by size fraction approach of Einstein, Laursen, and Toffaleti were in the range of 84.0-156.1%; for the BMF approach using Engelund and Hansen, Ackers and White, and Yang were in the range of 110.3-222.9%; and for the TCF approach using the Yang equation with  $D_e$  and the transport distribution functions of Karim and Kennedy, and Li were in the range of 89.9-135.0%.

By using the Yang equation with  $D_e$  and the newly proposed transport capacity distribution functions of Eqs. (3.9) and (3.24), the mean normalized errors were significantly reduced to 65.6% and 68.5% (up from range of 84.0-222.9% for other methods), respectively. The average geometric deviations were also considerably reduced to 1.80 and

1.81 (down from 2.15-8.76 for other methods) by Eqs. (3.9) and (3.24), respectively.

The percentages of data falling within the range of discrepancy ratios between 0.25 and 1.75 were in the range of 36.0-62.2% for the direct computation by the size fraction approach of Einstein, Laursen, and Toffaleti. They were in the range of 44.2-57.8% for the BMF approach using Engelund and Hansen, Ackers and White, and Yang. Also they were in the range of 44.7-68.5% by using the Yang equation with  $D_e$  and the transport capacity distribution functions of Karim and Kennedy and Li. By using the Yang equation with  $D_e$  and the newly proposed transport capacity distribution functions of Eqs. (3.9) and (3.24), the percentages of data falling within the range of discrepancy ratios between 0.25 and 1.75 were increased to 77.7% and 77.0% (up from the range of 36.0-68.5% for other methods), respectively.

Statistical results for the comparison between the computed and measured size fractions of bed-material load sediment in transport are given in Table 6.2. The mean normalized errors were reduced significantly to 60.8% and 61.8% (down from the range of 96.1-252.0% for other methods) by using the newly proposed transport capacity distribution functions of Eqs. (3.9) and (3.24), respectively. The average geometric deviations were also considerably reduced to 1.64 and 1.65 (down from the range of 2.04-8.26 for other methods) by Eqs. (3.9) and (3.24), respectively. The percentage of data falling within the range of discrepancy ratios between 0.25 and 1.75 were improved to 78.7% and 78.7% (up from the range of 46.1-70.2% for other methods) by Eqs. (3.9) and (3.24), respectively.

Table 6.1 Comparison between Computed and Measured Fractional Bed-Material Concentrations for the 118 Sets of Flume and Field Data Given in Table 3.1

Fractional Bed-Material Load Computation Method (1)	Data in Range of Discrepancy Ratio, $R_i$ (%)				Mean Normalized Error, MNE (%) (6)	Average Geometric Deviation, AGD (7)	No. of Data Points (8)
	0.75-1.25 (2)	0.5-1.5 (3)	0.25-1.75 (4)	0.5-2.0 (5)			
<b>a) Direct computation by size fraction approach</b>							
Einstein's equation (1950)	19.4	43.3	62.2	55.3	84.0	2.71	891
Laursen's equation (1958)	14.6	30.2	52.9	36.8	156.1	4.01	891
Toffaletti's equation (1968)	13.9	23.2	36.0	30.8	120.7	5.59	891
<b>b) BMF approach using</b>							
Engelund and Hansen's equation (1967)	19.0	38.2	55.9	48.4	111.8	2.45	891
Ackers and White's equation (1973)	20.0	39.8	57.8	51.9	222.9	2.69	891
Yang's equation (1973)	19.3	40.6	57.7	52.8	138.8	2.41	891
Karim's modified BMF method (1998)	12.1	24.8	44.2	29.1	110.3	6.07	891
<b>c) TCF approach using Yang eq. (1973) with <math>D_s</math> and</b>							
Function of Karim and Kennedy (1981)	24.1	49.5	68.5	58.5	89.9	2.15	891
Function of Li (1988)	14.4	27.8	44.7	35.1	135.0	8.76	891
Proposed function of Eq. (3.9)	31.3	59.2	77.7	68.4	65.6	1.80	891
Proposed function of Eq. (3.24)	29.7	58.6	77.0	68.5	66.2	1.81	891

Table 6.2 Comparison between Computed and Measured Size Fractions of Sediment in Transport (*Bed-Material Load*) for the 118 Sets of Flume and Field Data Given in Table 3.1

Fractional Bed-Material Load Computation Method (1)	Data in Range of Discrepancy Ratio, $R_i$ (%)				Mean Normalized Error, MNE (%) (6)	Average Geometric Deviation, AGD (7)	No. of Data Points (8)
	0.75-1.25 (2)	0.5-1.5 (3)	0.25-1.75 (4)	0.5-2.0 (5)			
<b>a) Direct computation by size fraction approach</b>							
Einstein's equation (1950)	29.9	46.1	59.7	55.2	252.0	2.55	891
Laursen's equation (1958)	22.2	39.0	55.2	43.2	123.0	3.75	891
Toffaletti's equation (1968)	18.6	33.1	49.7	40.4	134.1	4.19	891
<b>b) BMF approach using</b>							
Engelund and Hansen's equation (1967)	28.5	48.0	65.1	55.2	117.8	2.27	891
Ackers and White's equation (1973)	21.7	39.3	58.8	44.1	99.2	2.88	891
Yang's equation (1973)	32.7	53.1	70.2	58.4	96.1	2.19	891
Karim's modified BMF method (1998)	17.0	33.3	48.5	39.1	131.2	4.68	891
<b>c) TCF approach using</b>							
Function of Karim and Kennedy (1981)	32.3	52.5	70.2	60.6	105.3	2.04	891
Function of Li (1988)	16.5	30.3	46.1	37.3	139.5	8.26	891
Proposed function of Eq. (3.9)	42.3	65.1	78.8	74.2	60.8	1.64	891
Proposed function of Eq. (3.24)	41.1	64.5	78.7	73.7	61.8	1.65	891

### 6.3.2 Graphical Comparison

Comparisons between computed and measured fractional bed-material concentrations and size fractions are graphically displayed in four different types of plots for the fractional transport methods discussed above. These plots are intended to show the agreement between predicted and measured values.

1) *Plots showing the comparison between computed and measured fractional bed-material concentrations*

Figs. 6.1-6.9 show the comparisons between computed and measured fractional bed-material concentrations for various fractional load methods. A large scatter of computed results from the measured values can be noticed for the direct computation by size fraction approach of Einstein, Laursen, and Toffaleti; the BMF approach using the Engelund and Hansen, Ackers and White, and Yang equations; and the TCF approach using the Yang equation with  $D_e$  and the transport distribution functions of Karim and Kennedy, and Li. The scatter is mostly in the range of two logarithmic scales for these 9 methods.

It can also be seen that the Einstein method predicts his own data very well, but it fails to predict the fractional transport rate for data from other sources. The TCF approach using the Yang equation with  $D_e$  and the transport distribution functions of Karim and Kennedy gives slightly better results, even though Karim and Kennedy treated their transport capacity distribution function in a very simple manner.

Figs. 6.10-6.11 show the results computed from the TCF approach using the Yang equation with  $D_e$  and the newly proposed transport distribution functions of Eqs. (3.9) and (3.24), respectively. A close agreement between the computed fractional bed-material

concentrations and measured values can be observed.

2) *Plots showing discrepancy ratio distribution*

The discrepancy ratio distributions of computed fractional bed-material concentrations are plotted in Figure 6.12-6.22. It can be seen that there are 32.8% of data with discrepancy ratio  $R_i < 1/7.5$  for Toffaleti, 30.8% for the Karim method, and 30.1% for the Li method, respectively. This indicates that these three methods greatly underestimate the fractional transport rates. There are also large percentages (11% and 17%) of data with a discrepancy ratio  $R_i < 1/7.5$  for Einstein and Laursen methods.

The discrepancy ratio distributions are close to normal for the BMF approach using the Engelund and Hansen, Ackers and White, and Yang equations, even though the predictions are not highly concentrated around perfect agreement ( $C_{tci}/C_{tmi}=1$ ). Overall, the discrepancy ratios resulting from the Karim and Kennedy method are normally distributed and are more concentrated around the perfect agreement.

The discrepancy ratios for the predictions using the newly developed transport capacity distribution functions of Eqs. (3.9) and (3.24) are normally distributed and have higher density close to the perfect agreement than all other methods. Predictions with extreme discrepancy ratios ( $R_i < 1/7.5$  or  $R_i > 7.5$ ) are very limited.

3) *Plots showing the comparison between computed and measured size fractions of bed-material load*

Figs. 6.23-6.31 and Figs. 3.9-3.10 show the comparison between computed and measured size fractions of bed-material load. In general, Figs. 6.23-6.31 show that the

computed size fractions of bed-material load sediment in transport deviate significantly from actual measurements for the direct computation by the size fraction approach of Einstein, Laursen, and Toffaleti; the BMF approach using the Engelund and Hansen, Ackers and White, and Yang equations; and the TCF approach using the Yang equation with  $D_c$  and the transport distribution functions of Karim and Kennedy, and Li. As a contrast, Figs. 3.9-3.10 show that a close agreement between computed and measured values is obtained at the whole range of  $P_{c_{mi}}$  and especially at larger values of  $P_{c_{mi}}$  by the use of the proposed transport capacity distribution functions of Eqs. (3.9) and (3.24).

4) *Plots showing variations of the ratio of computed to measured size fractions of bed-material load with relative diameter of bed material size*

The ratio of computed size fractions of bed-material load to the measured fraction of bed-material load against the relative diameter of the bed material are shown in Fig. 6.32-6.40 and Figs. 3.7-3.8. Values of  $P_{cci}/P_{c_{mi}}$  equal to 1 indicate perfect agreement. It can be seen from Figs. 3.7-3.8. that most of the points fall near the perfect agreement line of  $P_{cci}/P_{c_{mi}}$  equal to 1 for the newly developed transport capacity distribution functions. The plots shown in Fig. 6.32-6.40 for other methods indicate that the values of  $P_{cci}/P_{c_{mi}}$  are generally near perfect agreement at values of  $D_i/D_{50}$  around 1; and the values of  $P_{cci}/P_{c_{mi}}$  diverge from the perfect agreement at smallest and largest values of  $D_i/D_{50}$ . Overall, the Einstein method underpredicts the transport rate for finer sizes and overpredicts for the coarser sizes, while the other methods overestimate the finer fractions and underestimate the coarser fractions.

In the modified BMF method, the weighting function of Eq. (2.44) proposed by Karim (1998) may be expressed as

$$\phi_i = P_{ai} \eta = \left[ \frac{C_1 \left( \frac{1}{D_{50}} \right)^{C_2}}{\sum_{i=1}^N \frac{P_{bi}}{D_i}} \right] \left[ \frac{P_{bi}}{D_i^{1-C_2}} \right] \quad (6.14)$$

It can be seen that the first term in Eq. (6.14) is a constant value for a given data set, and the weighting function varies only with  $P_{bi}$  and  $D_i$  for different size fractions. In general, due to the sheltering and exposure effect in a sediment mixture, the finer fractions are transported at a relatively lower rate than they would be if they were in a uniform sediment bed. The coarser particles consequently are transported faster. To reflect this phenomenon, the exponent  $(1-C_2)$  in Eq. (6.14) should be a negative value. A positive value of  $(1-C_2)$  will result in unrealistic results by increasing the transport rate for finer fractions and decreasing the transport rate for coarser sizes.

As an example, the flume data of Einstein and Chien (1953) can be used to evaluate the variation of  $C_2$ . Since the values of  $\omega_{50} / V_*$  are in the range of 0.13-0.61 for Einstein and Chien's data, the value of exponent  $C_2$  [Eq. (2.47)] will vary as

$$C_2 = 0.60 \left( \frac{\omega_{50}}{V_*} \right) = 0.60 (0.13 \sim 0.61) = 0.078 \sim 0.366 \quad (6.15)$$

and the value of exponent  $(1-C_2)$  will be 0.634-0.922. As a result, this produced the unreasonable results for the data of Einstein and Chien shown in Fig. 6.38. Similar results can be seen for data derived from other sources.

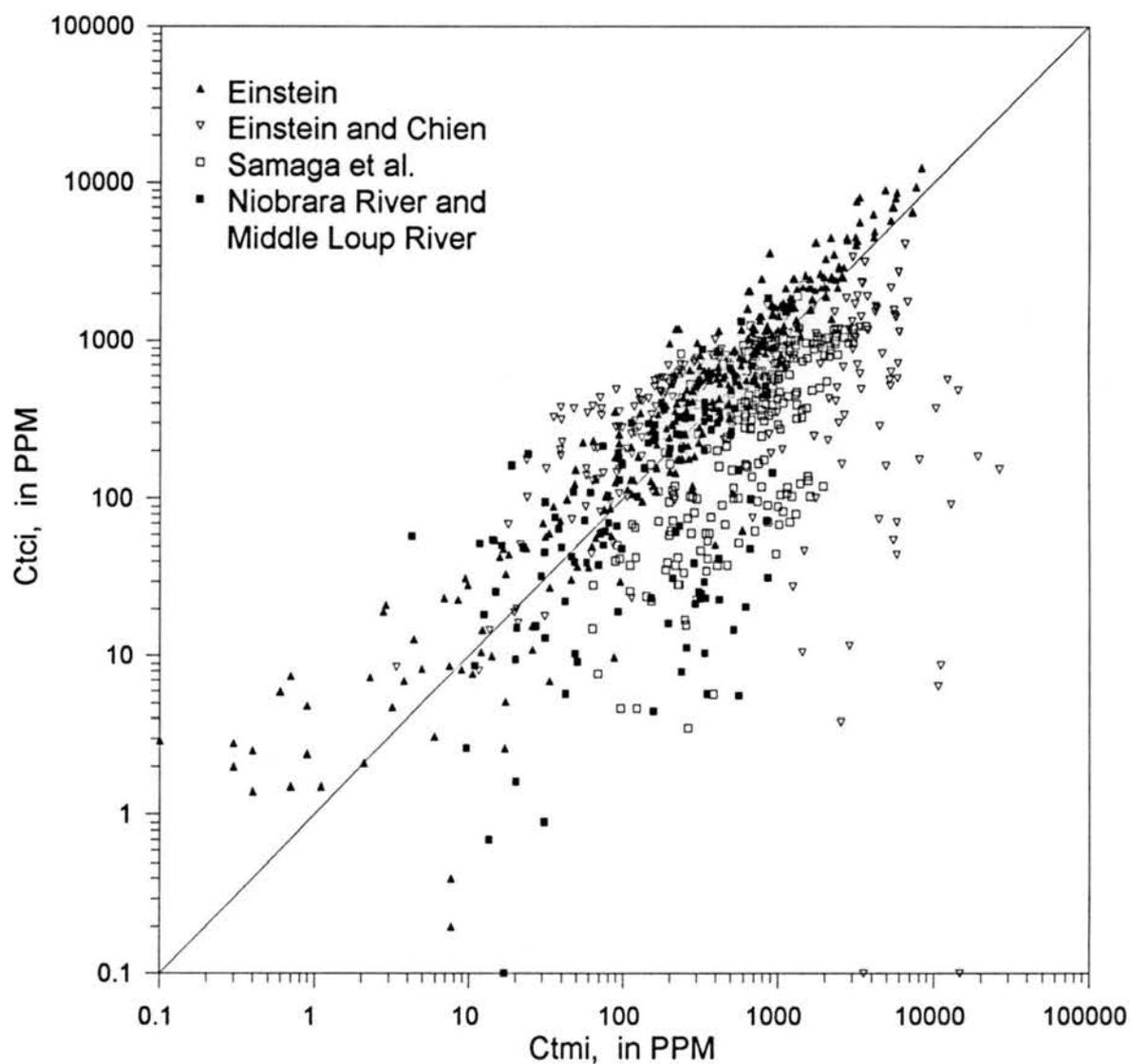


Fig. 6.1. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the Einstein Equation (1950).

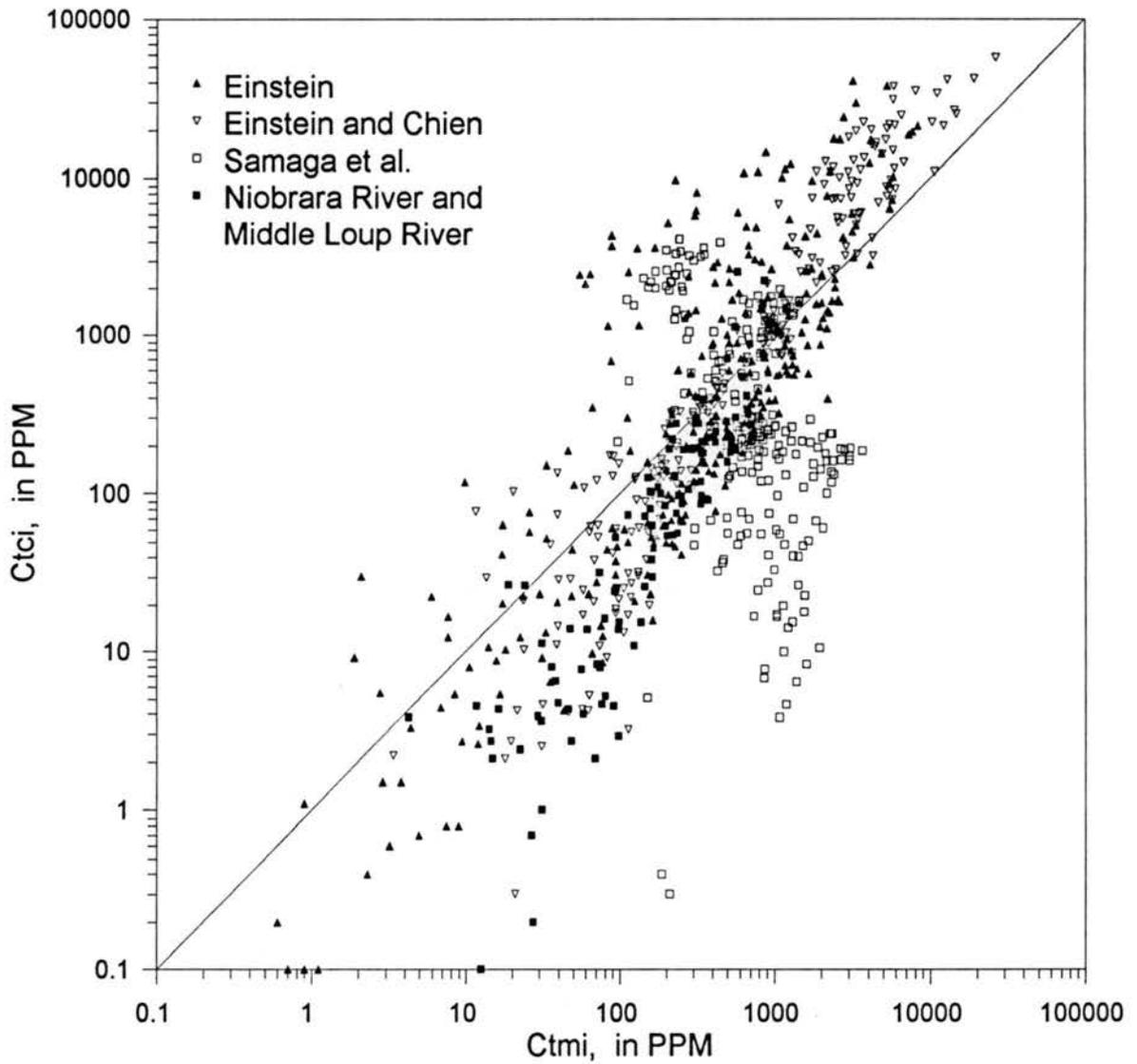


Fig. 6.2. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the Laursen Equation (1958).

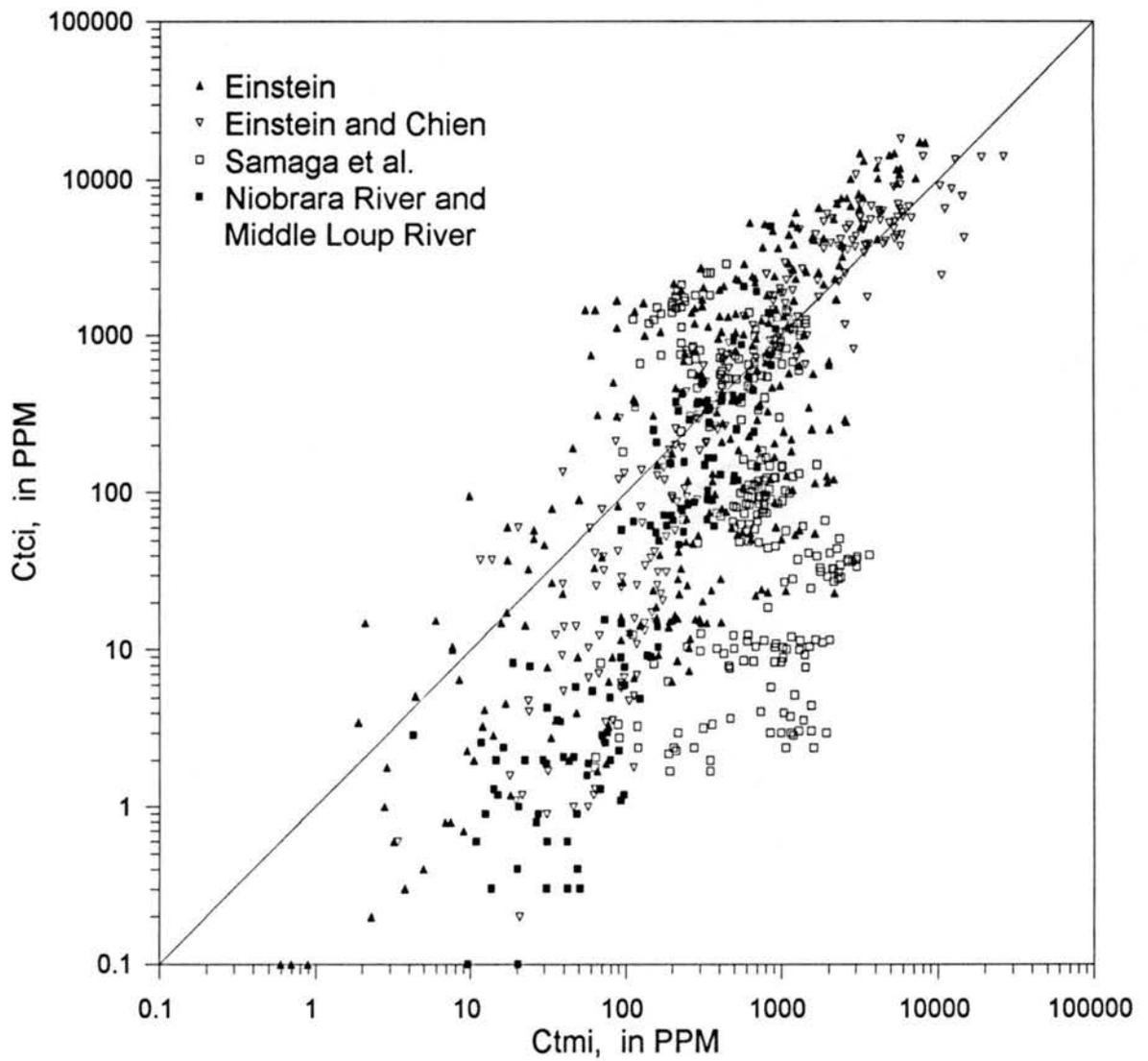


Fig. 6.3. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the Toffaleti Equation (1968).

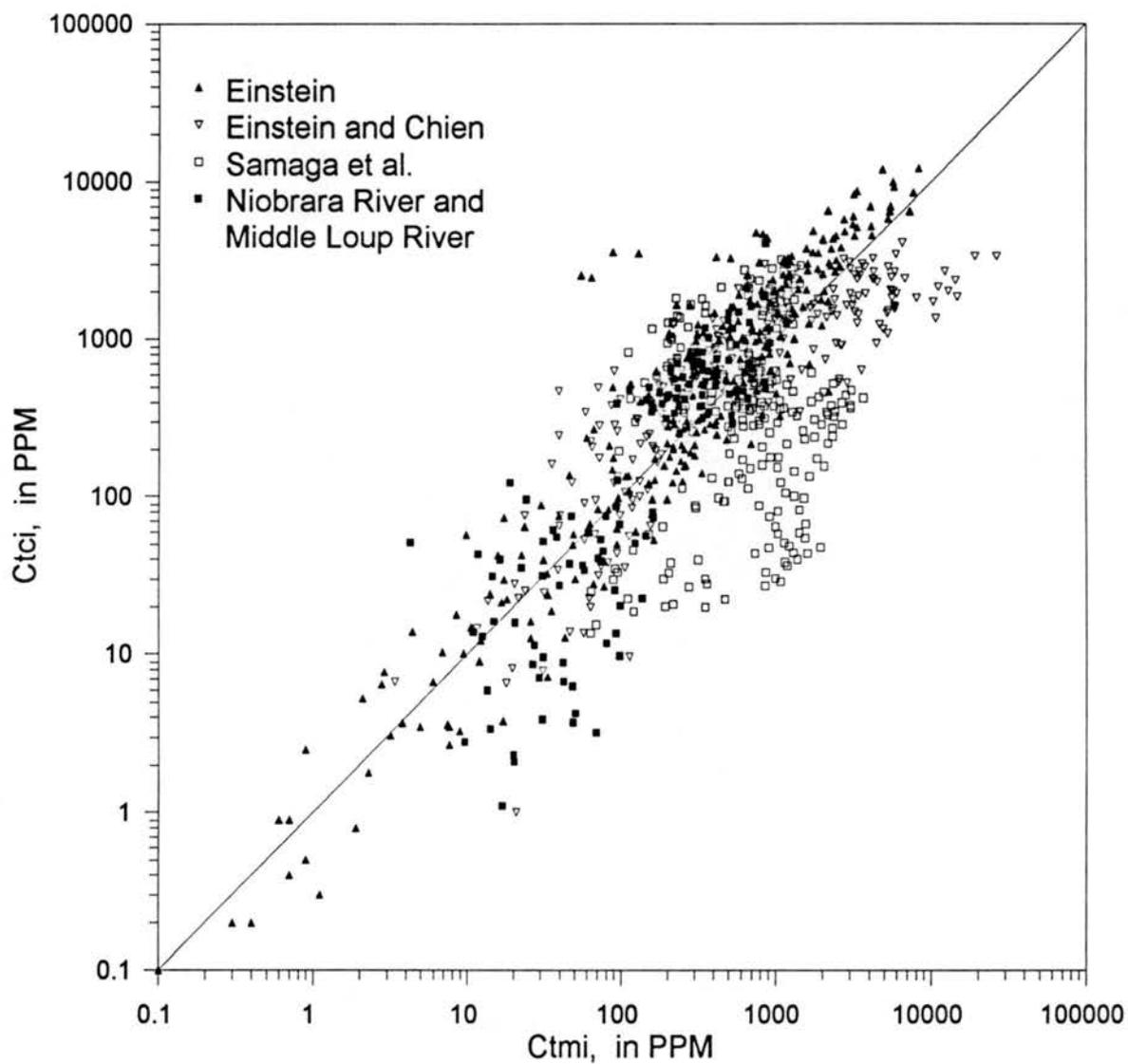


Fig. 6.4. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the BMF Approach Using the Engelund and Hansen Transport Equation (1967).

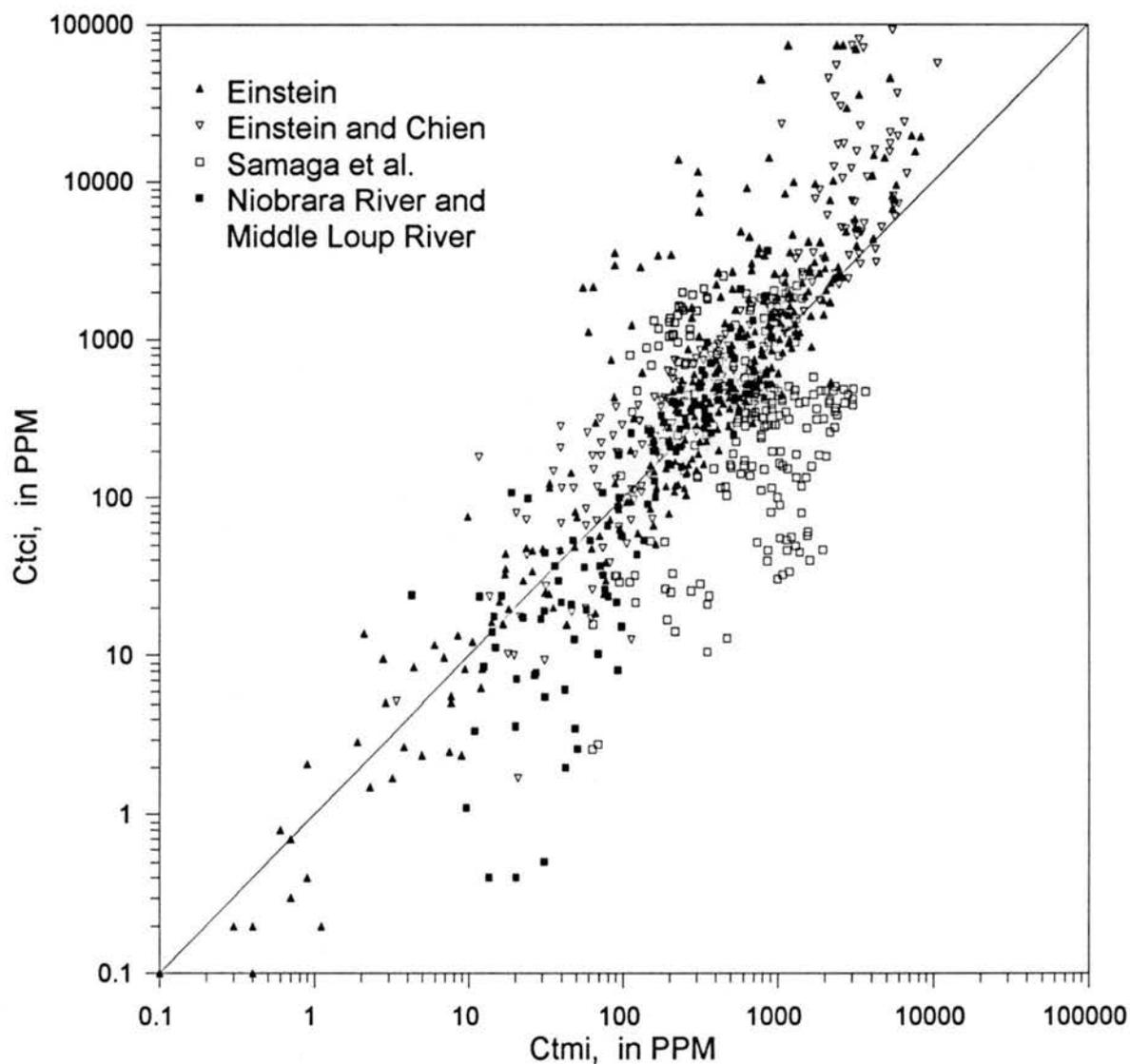


Fig. 6.5. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the BMF Approach Using the Ackers and White Transport Equation (1973).

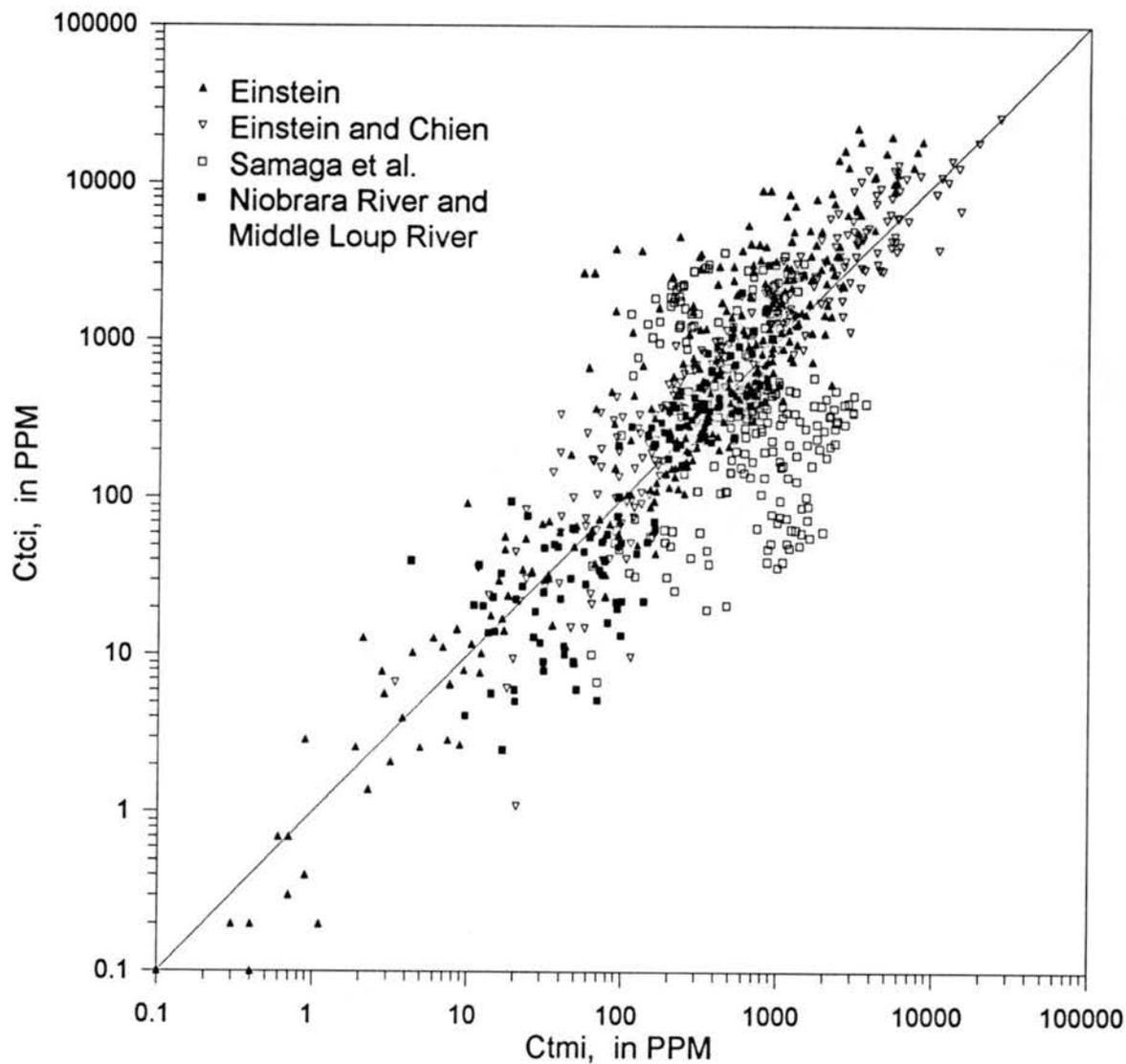


Fig. 6.6. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the BMF Approach Using the Yang Transport Equation (1973).

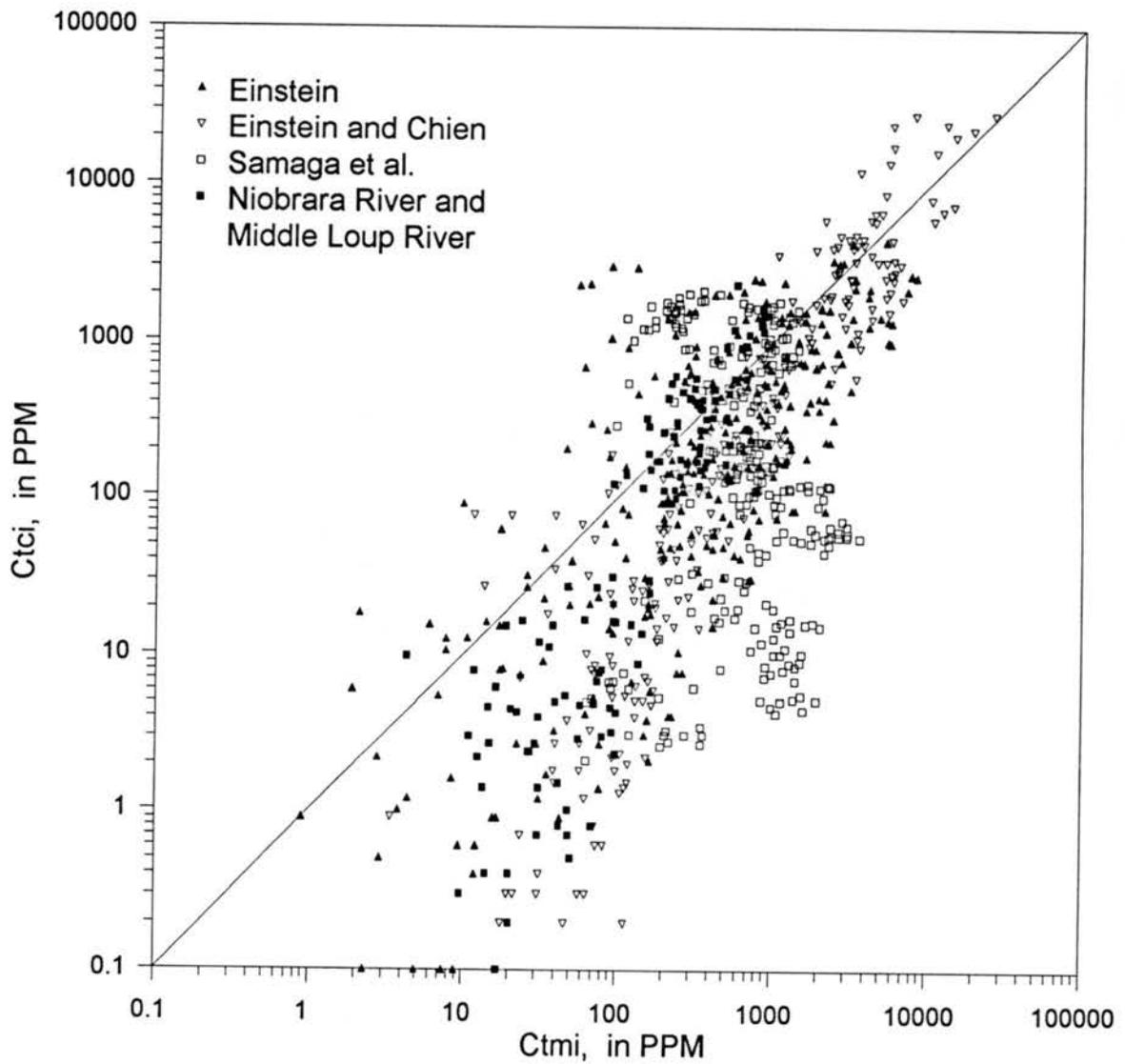


Fig. 6.7. Comparison between Computed and Measured Fractional Bed-Material Concentrations for Karim's Modified BMF Approach (1998).

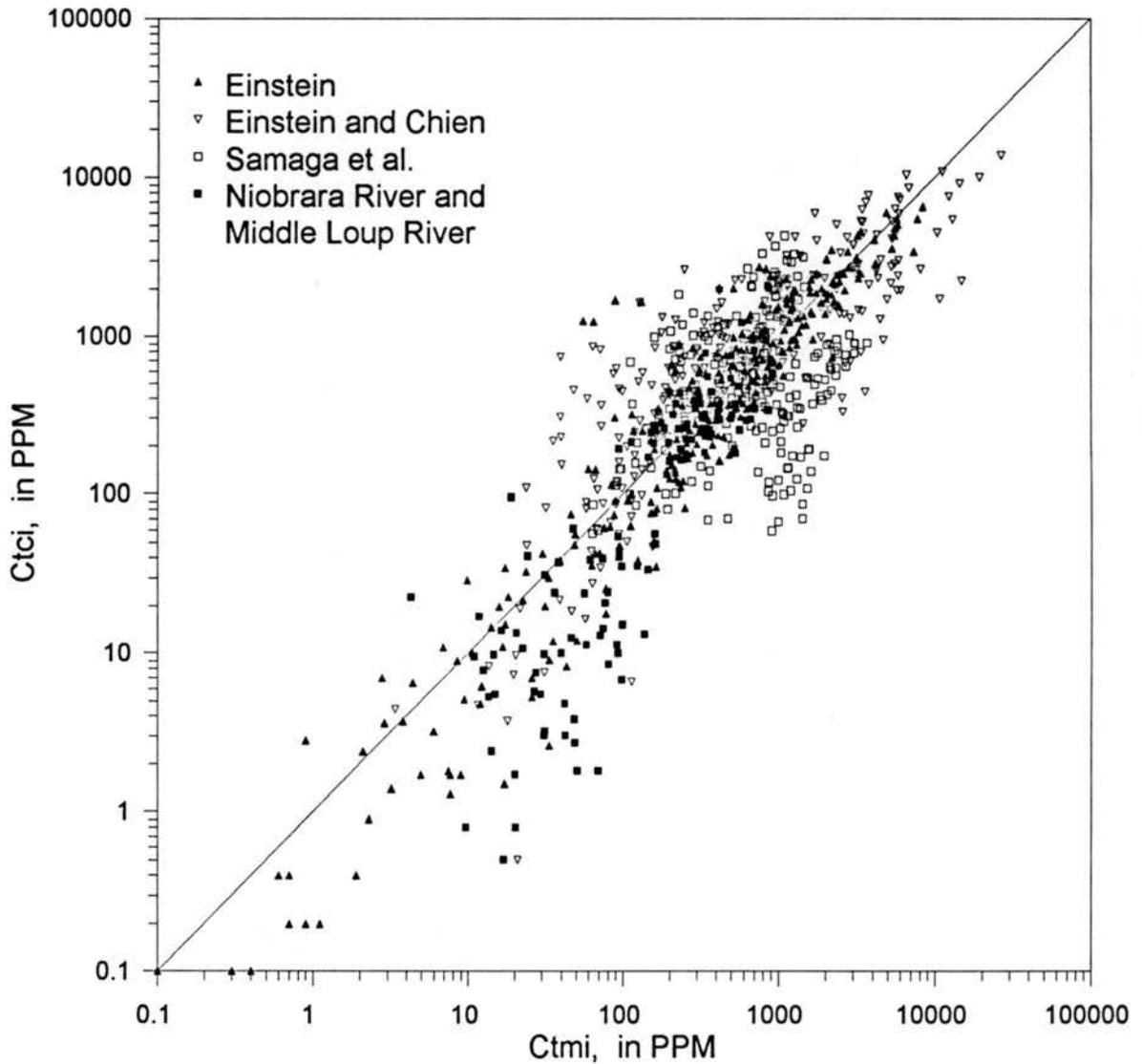


Fig. 6.8. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the TCF Approach Using the Yang Equation (1973) with  $D_c$  and the Transport Capacity Distribution Function of Karim and Kennedy (1981).

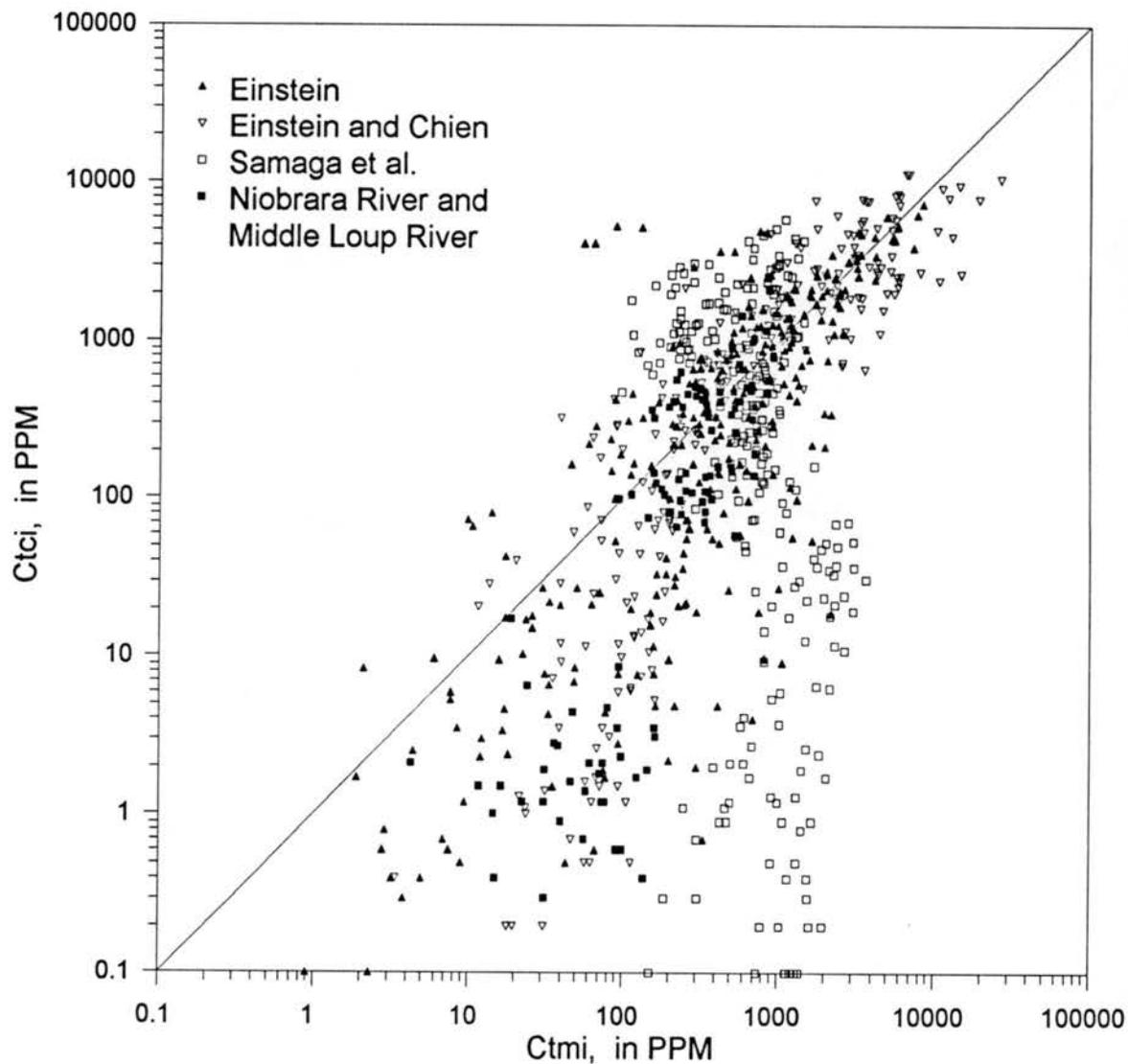


Fig. 6.9. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the TCF Approach Using the Yang Equation (1973) with  $D_e$  and the Transport Capacity Distribution Function of Li (1988).

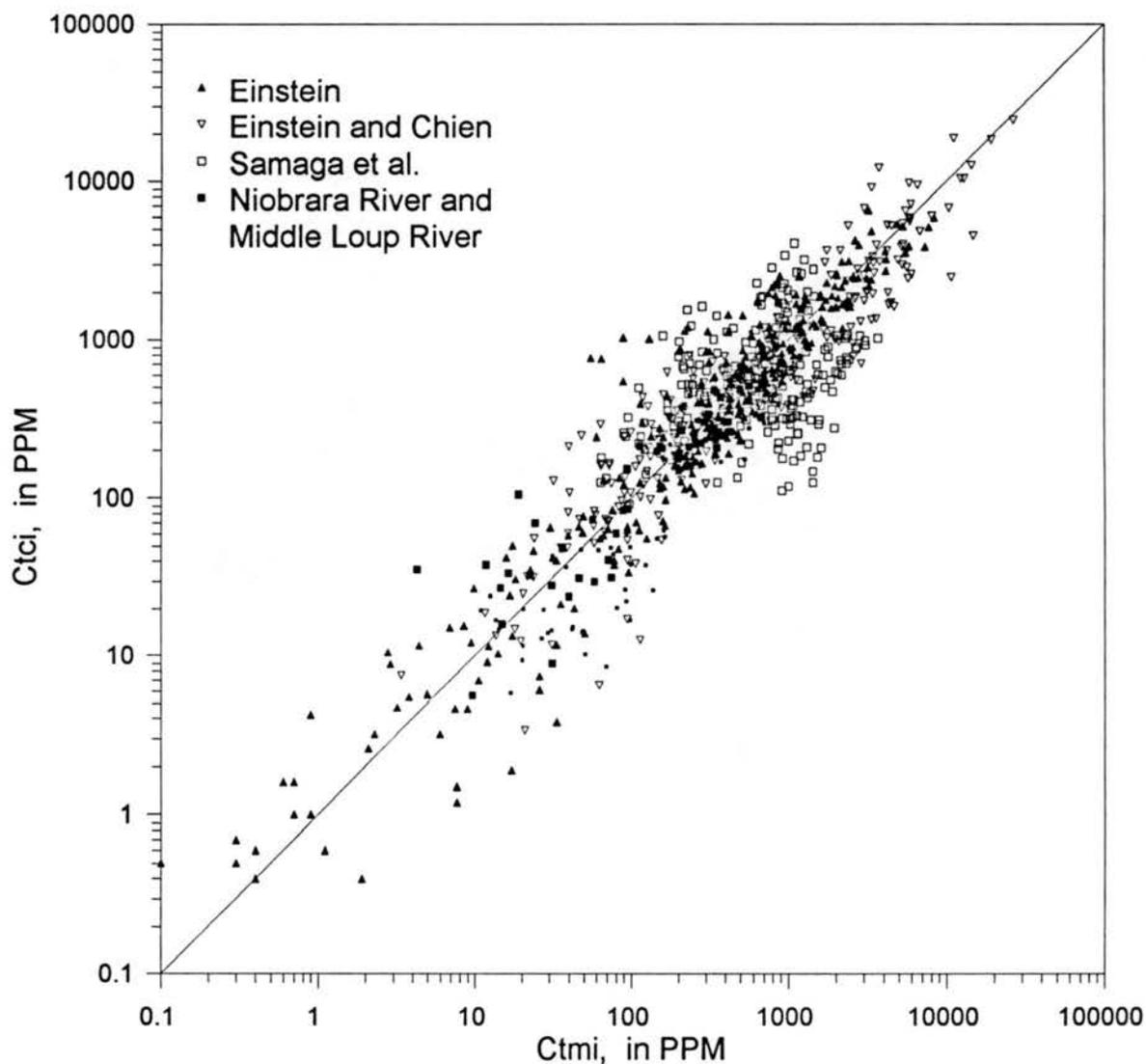


Fig. 6.10. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the TCF Approach Using the Yang Equation (1973) with  $D_e$  and the Proposed Transport Capacity Distribution Function of Eq. (3.9).

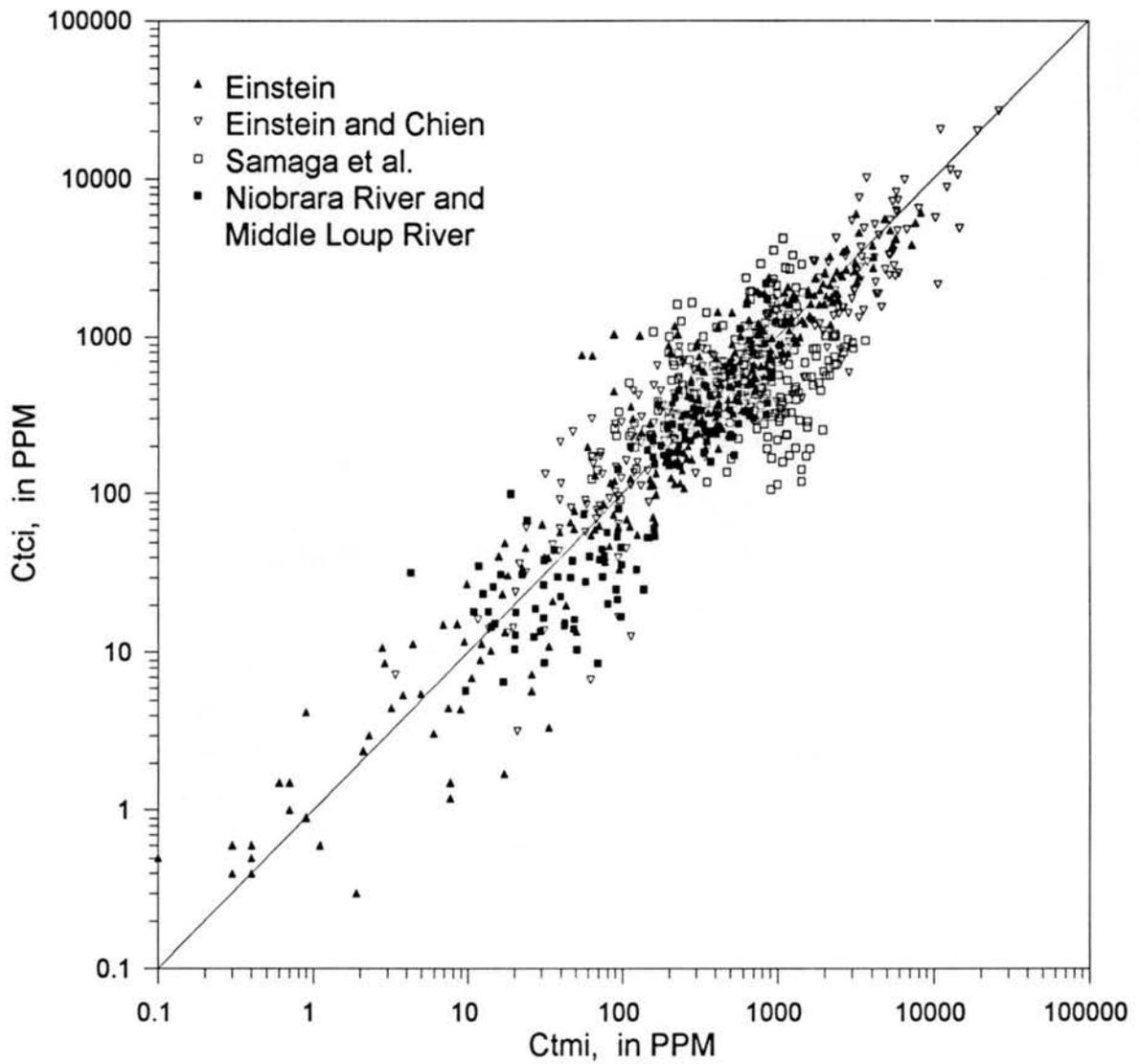


Fig. 6.11. Comparison between Computed and Measured Fractional Bed-Material Concentrations for the TCF Approach Using the Yang Equation (1973) with  $D_e$  and the Proposed Transport Capacity Distribution Function of Eq. (3.24).

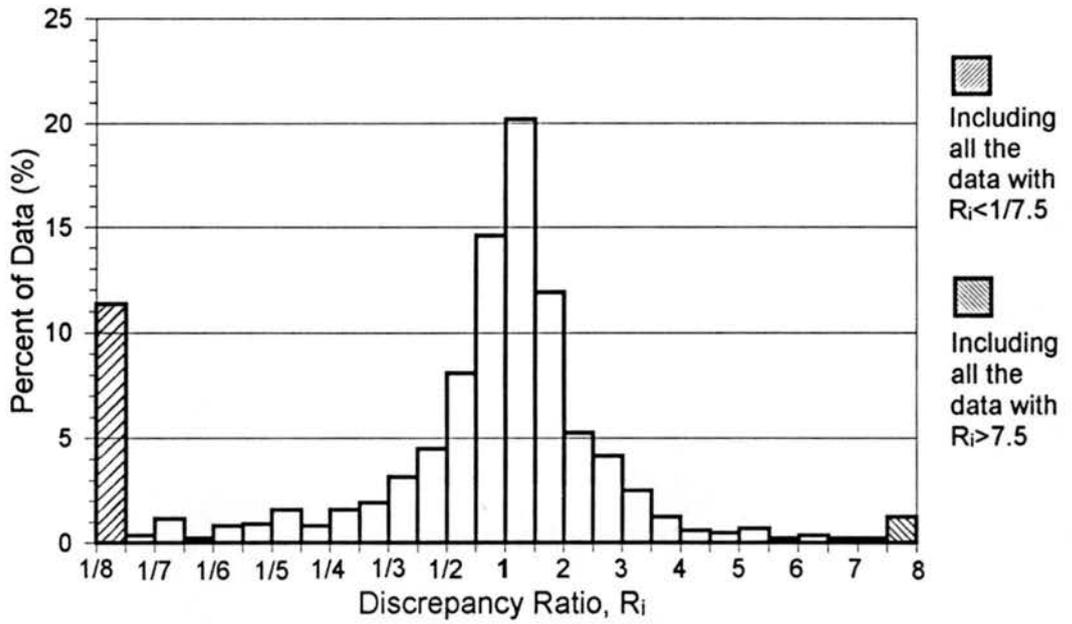


Fig. 6.12. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the Einstein Equation (1950).

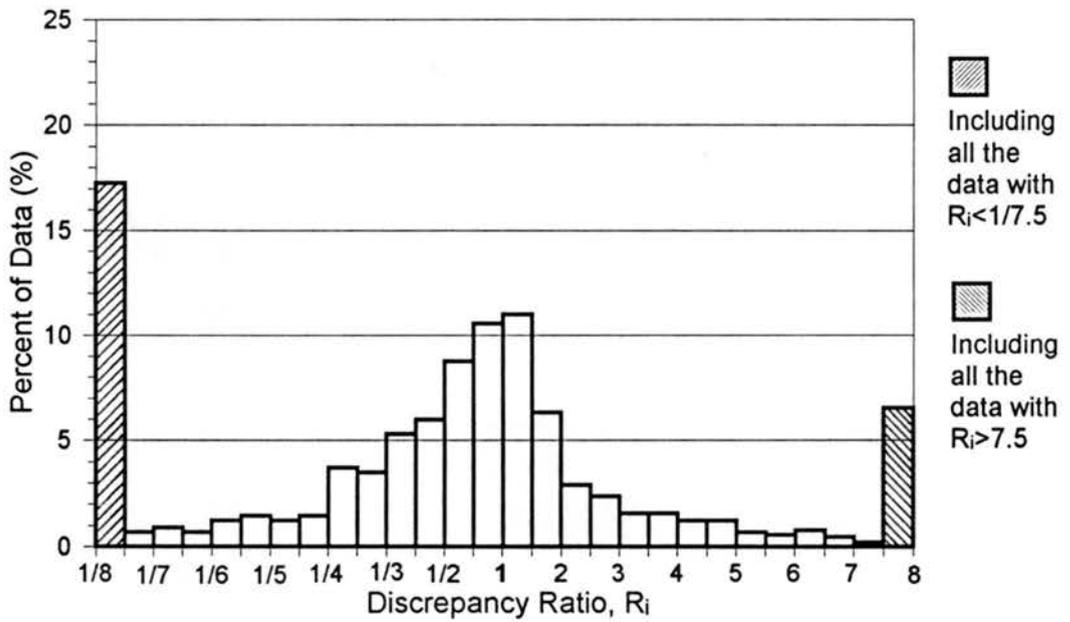


Fig. 6.13. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the Laursen Equation (1958).

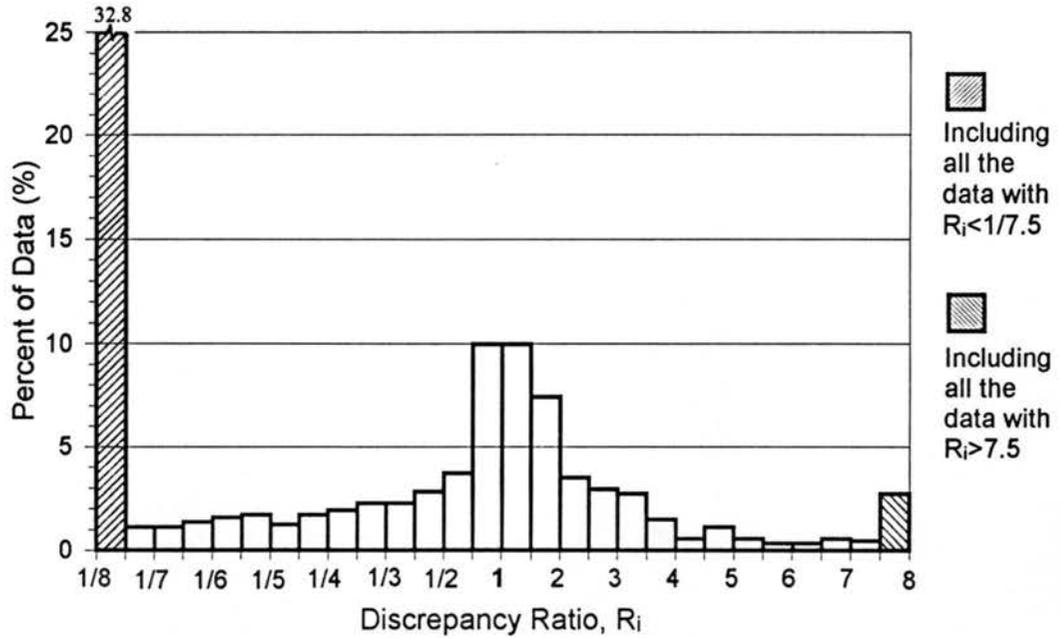


Fig. 6.14. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the Toffaleti Equation (1968).

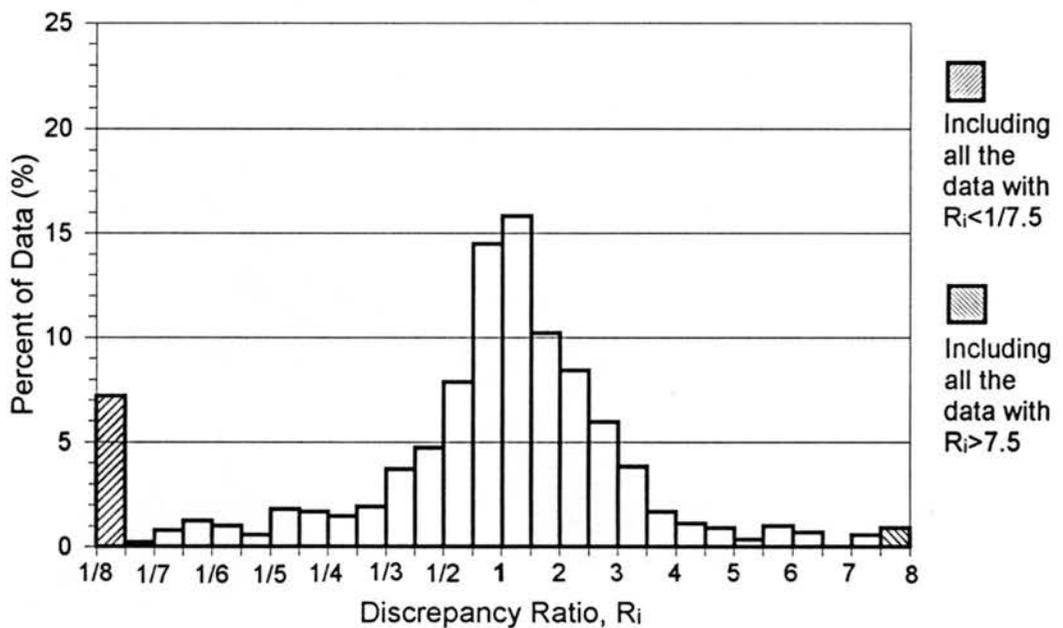


Fig. 6.15. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the BMF Approach Using the Engelund and Hansen Equation (1967).

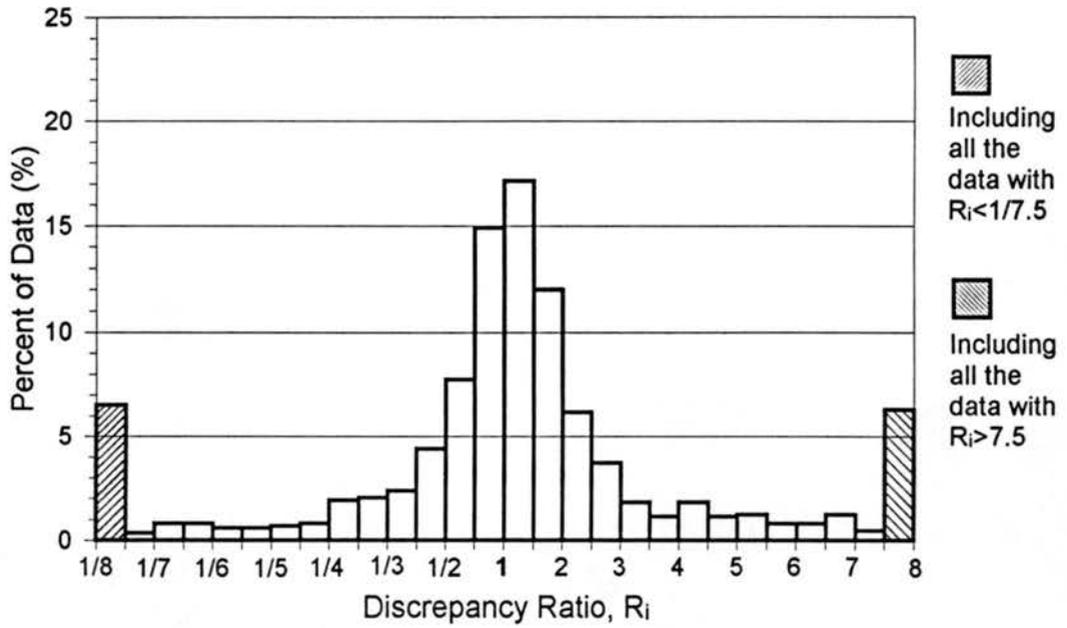


Fig. 6.16. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the BMF Approach Using the Ackers and White Equation (1973).

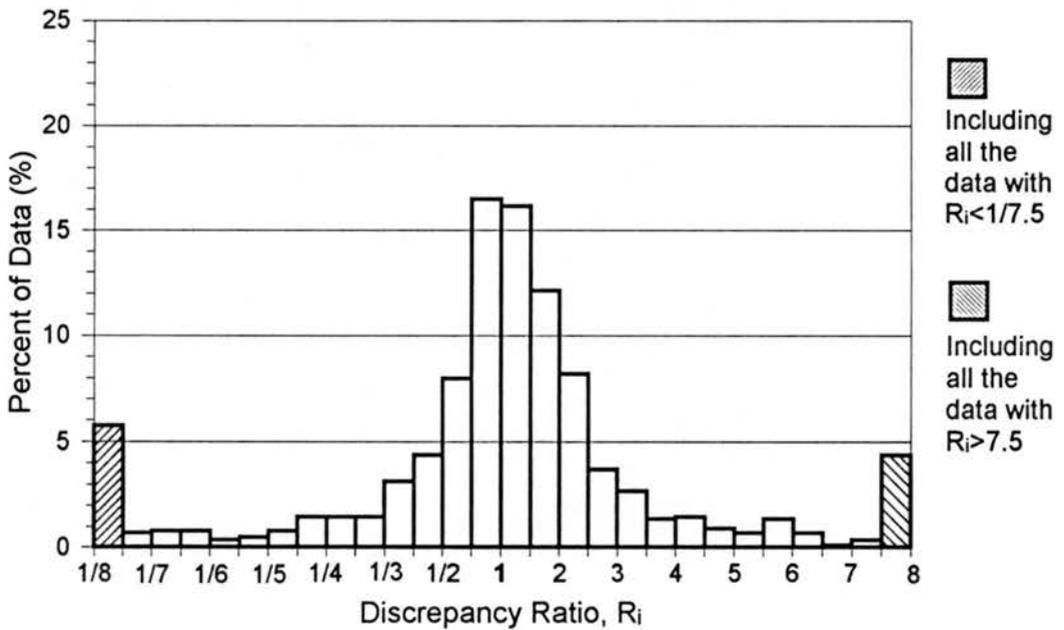


Fig. 6.17. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the BMF Approach Using the Yang Sand Equation (1973).

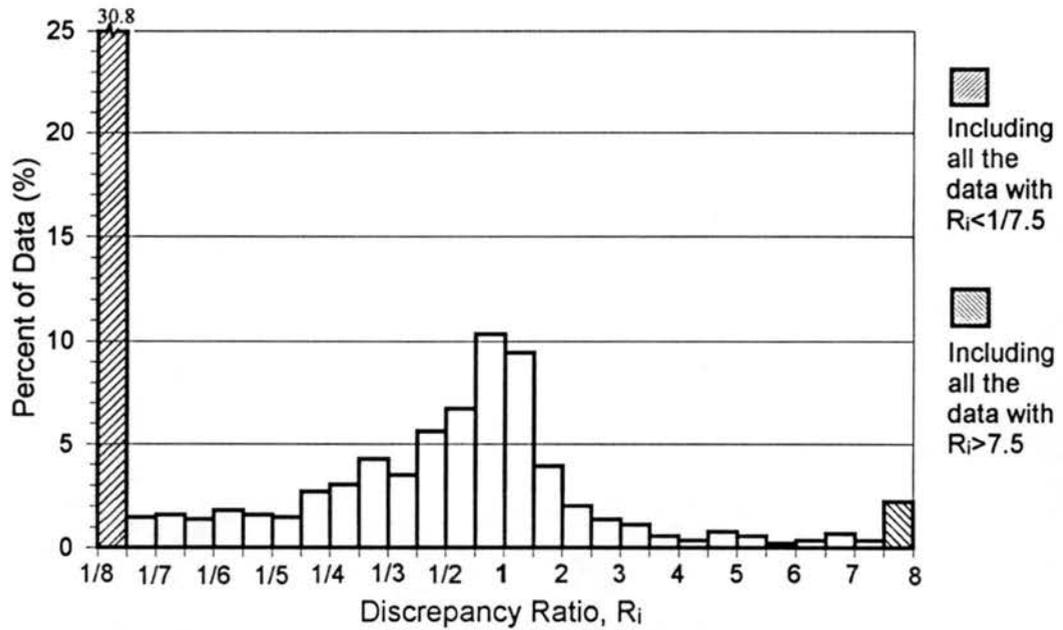


Fig. 6.18. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for Karim's Modified BMF Method (1998).

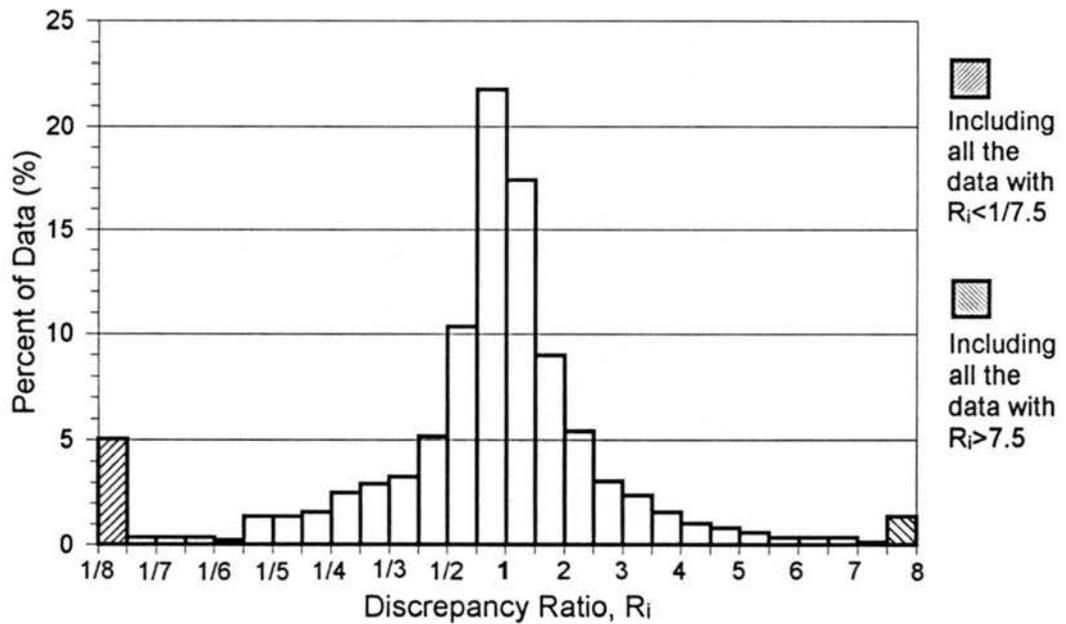


Fig. 6.19. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the TCF Approach Using the Yang Equation (1973) with  $D_c$  and the Transport Capacity Distribution Function of Karim and Kennedy (1981).

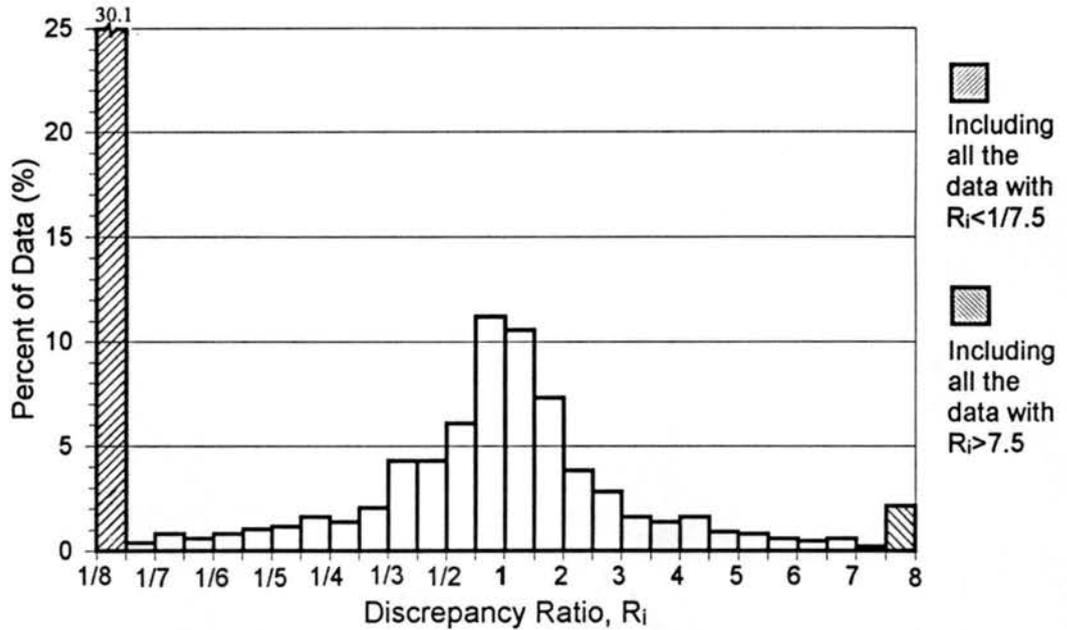


Fig. 6.20. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the TCF Approach Using the Yang Equation (1973) with  $D_e$  and the Transport Capacity Distribution Function of Li (1988).

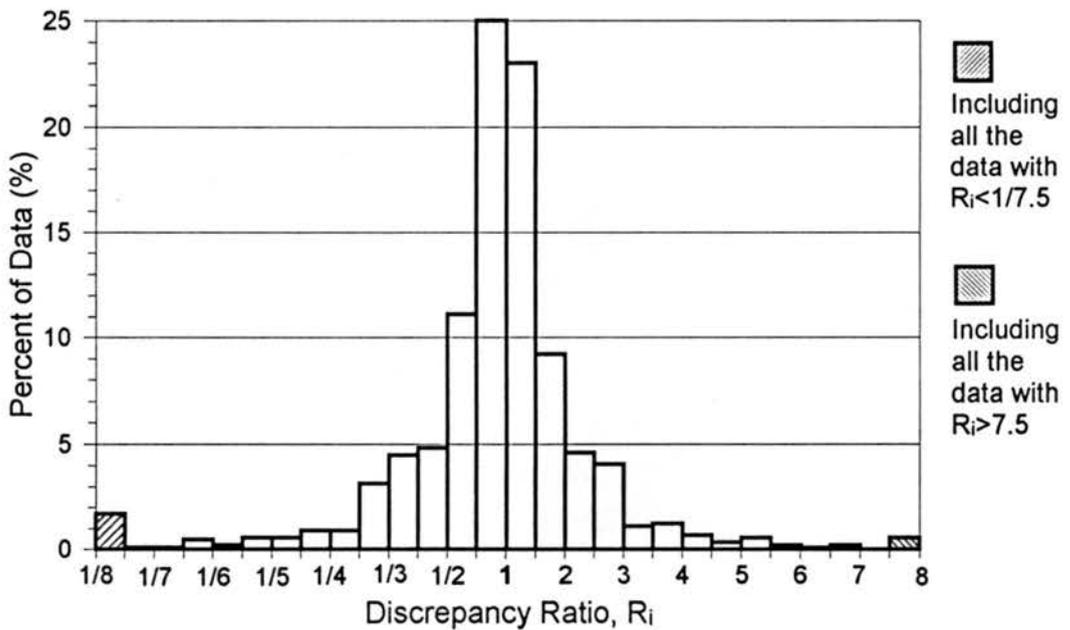


Fig. 6.21. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the TCF Approach Using the Yang Equation (1973) with  $D_e$  and the Proposed Transport Capacity Distribution Function of Eq.(3.9).

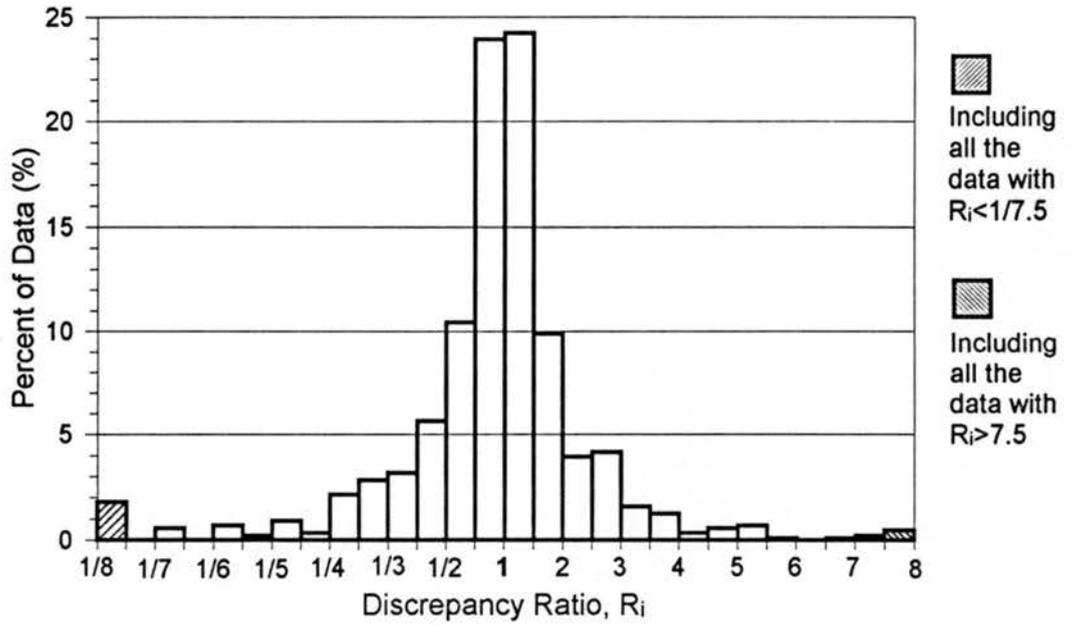


Fig. 6.22. Discrepancy Ratio Distribution of Fractional Bed-Material Concentrations for the TCF Approach Using the Yang Equation (1973) with  $D_e$  and the Proposed Transport Capacity Distribution Function of Eq.(3.24).

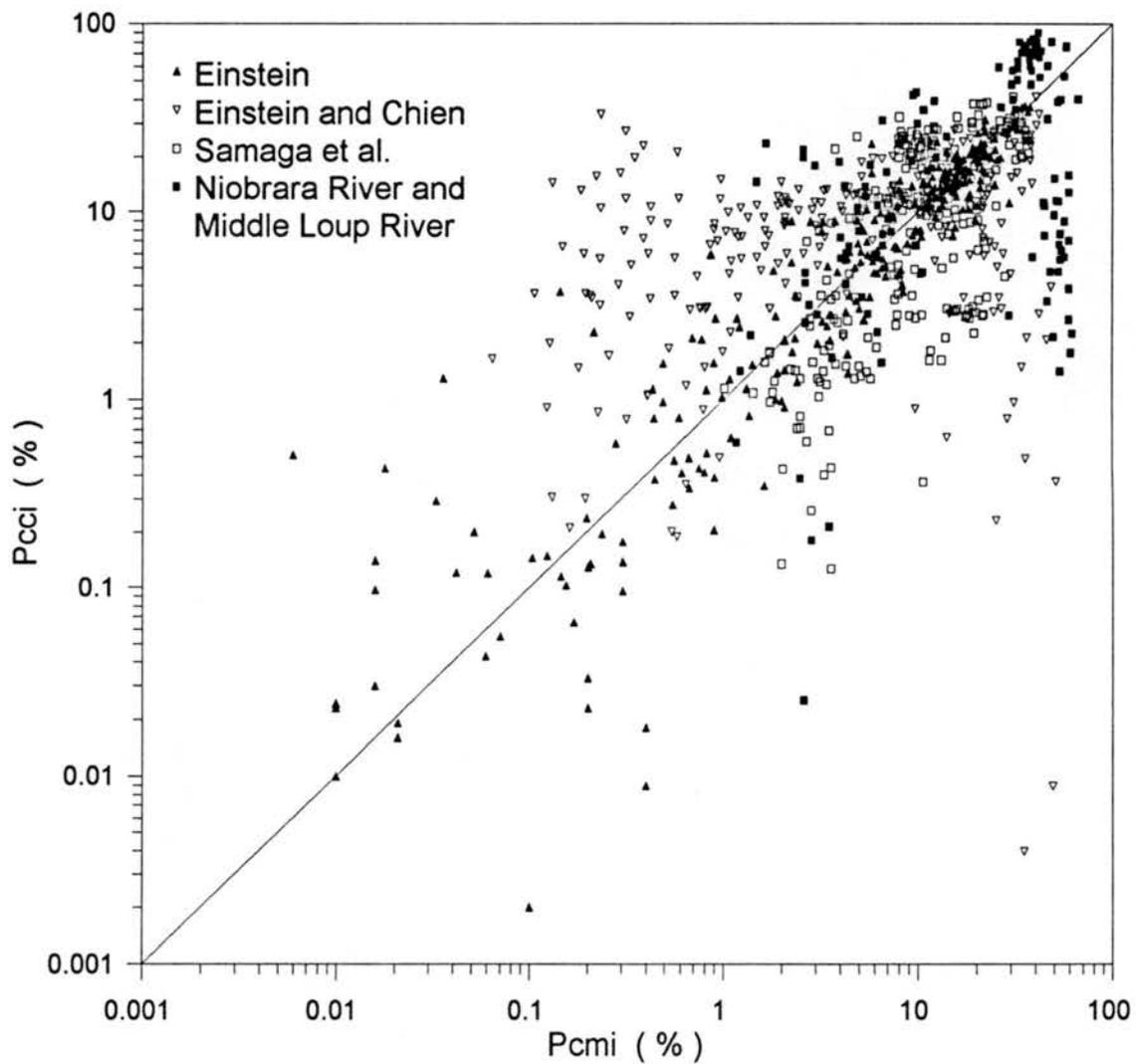


Fig. 6.23. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for the Einstein Equation (1950).

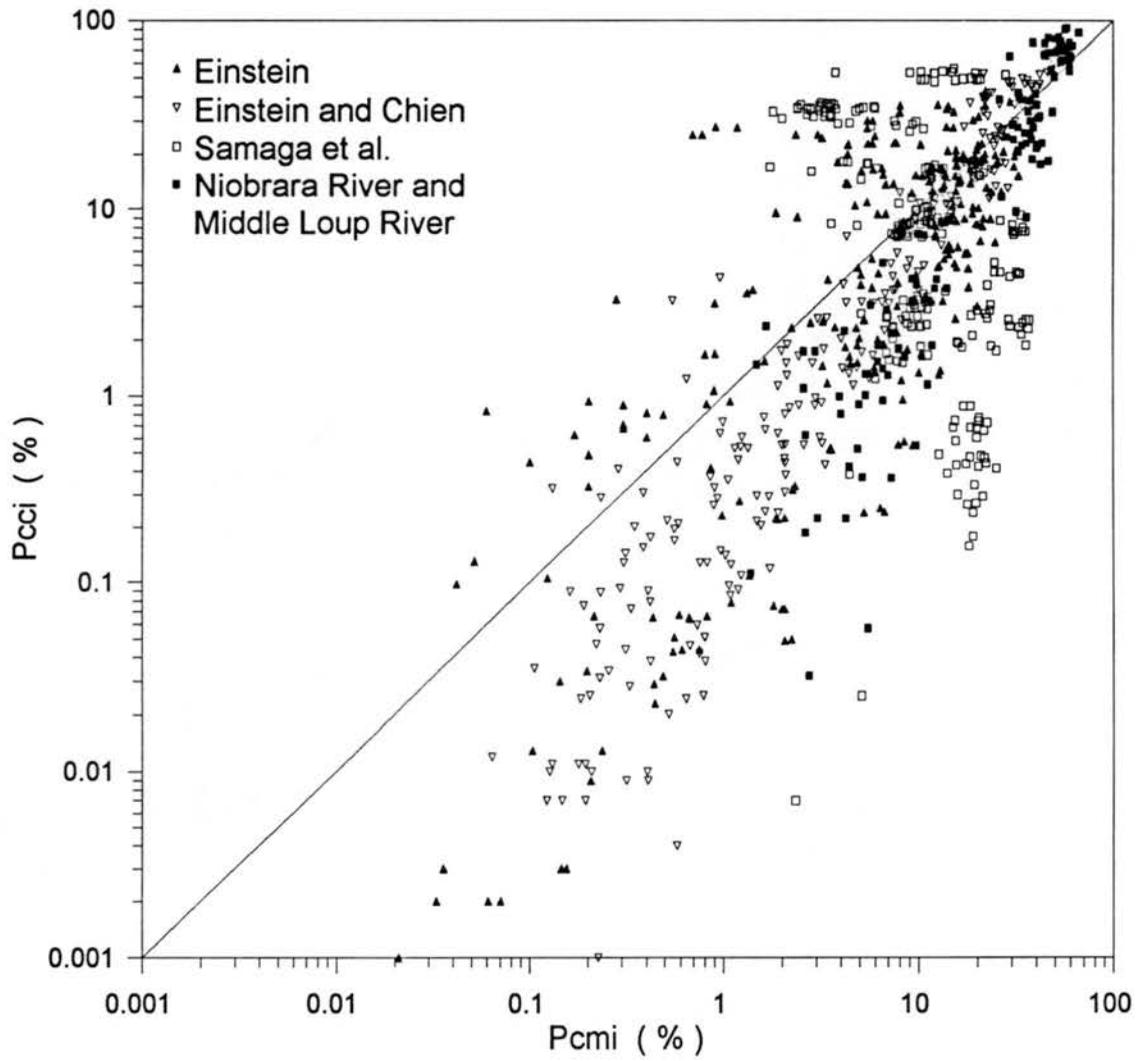


Fig. 6.24. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for the Laursen Equation (1958).

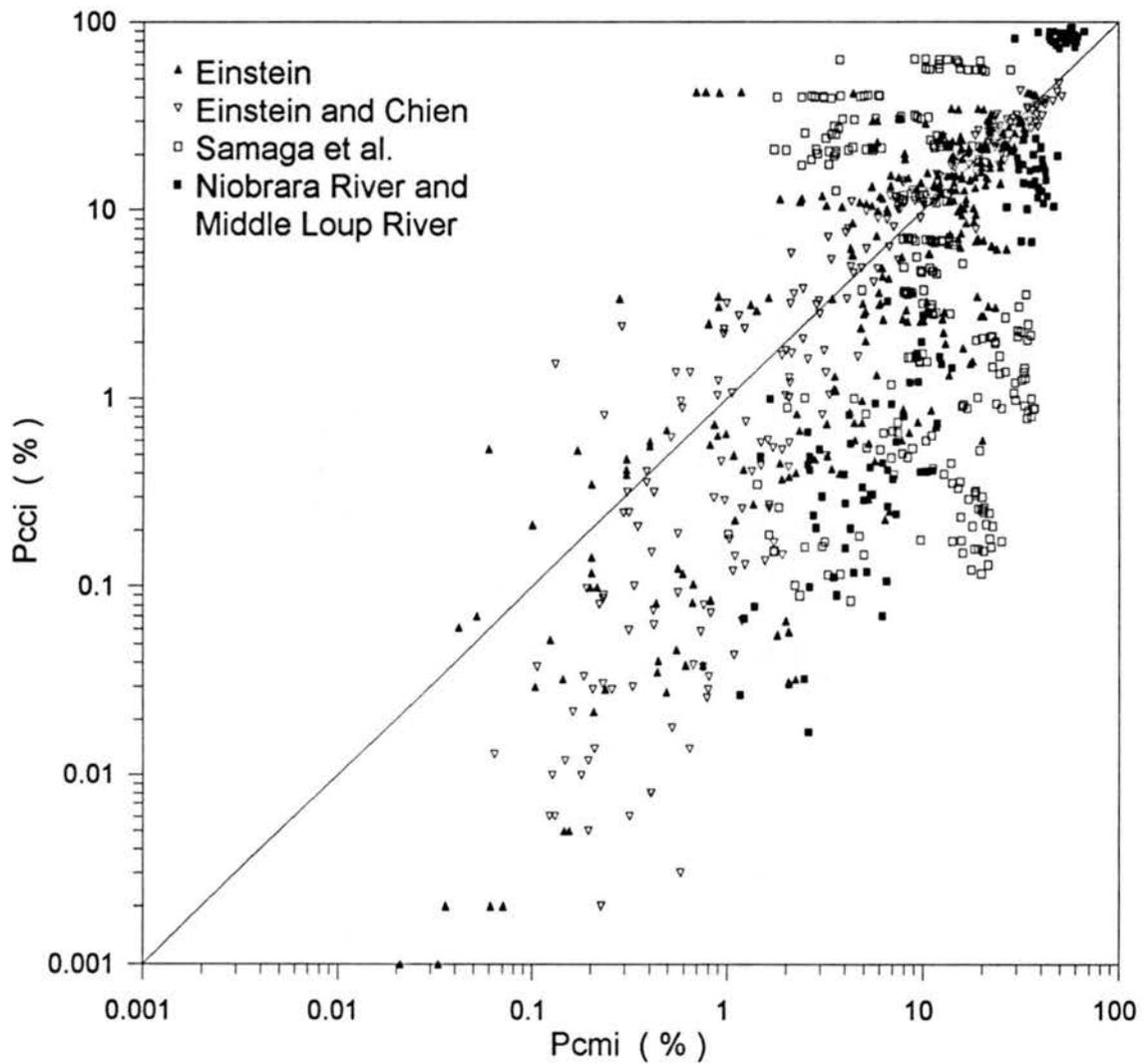


Fig. 6.25. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for the Toffaleti Equation (1968).

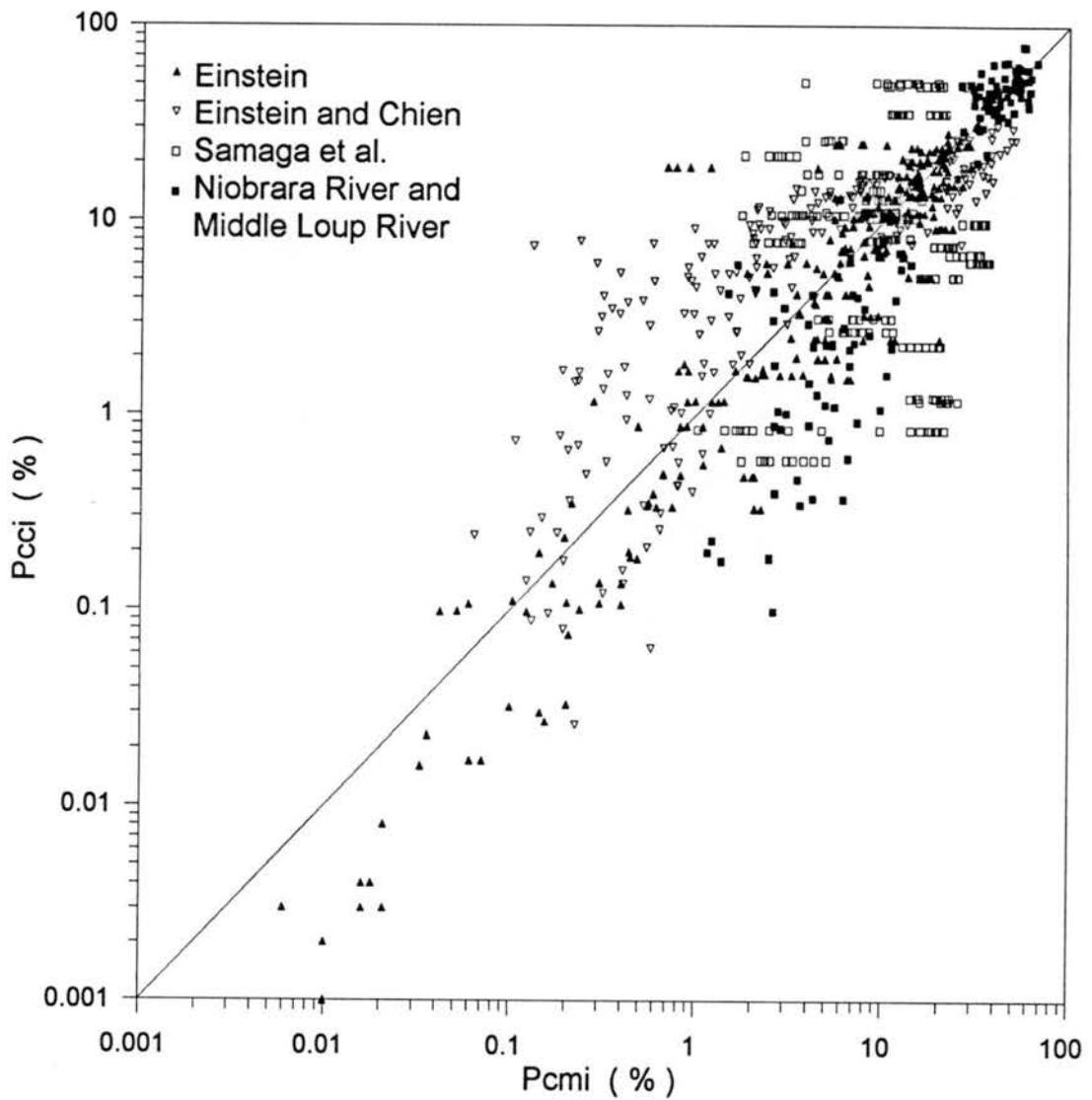


Fig. 6.26. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for the BMF Approach Using the Engelund and Hansen Transport Equation (1967).

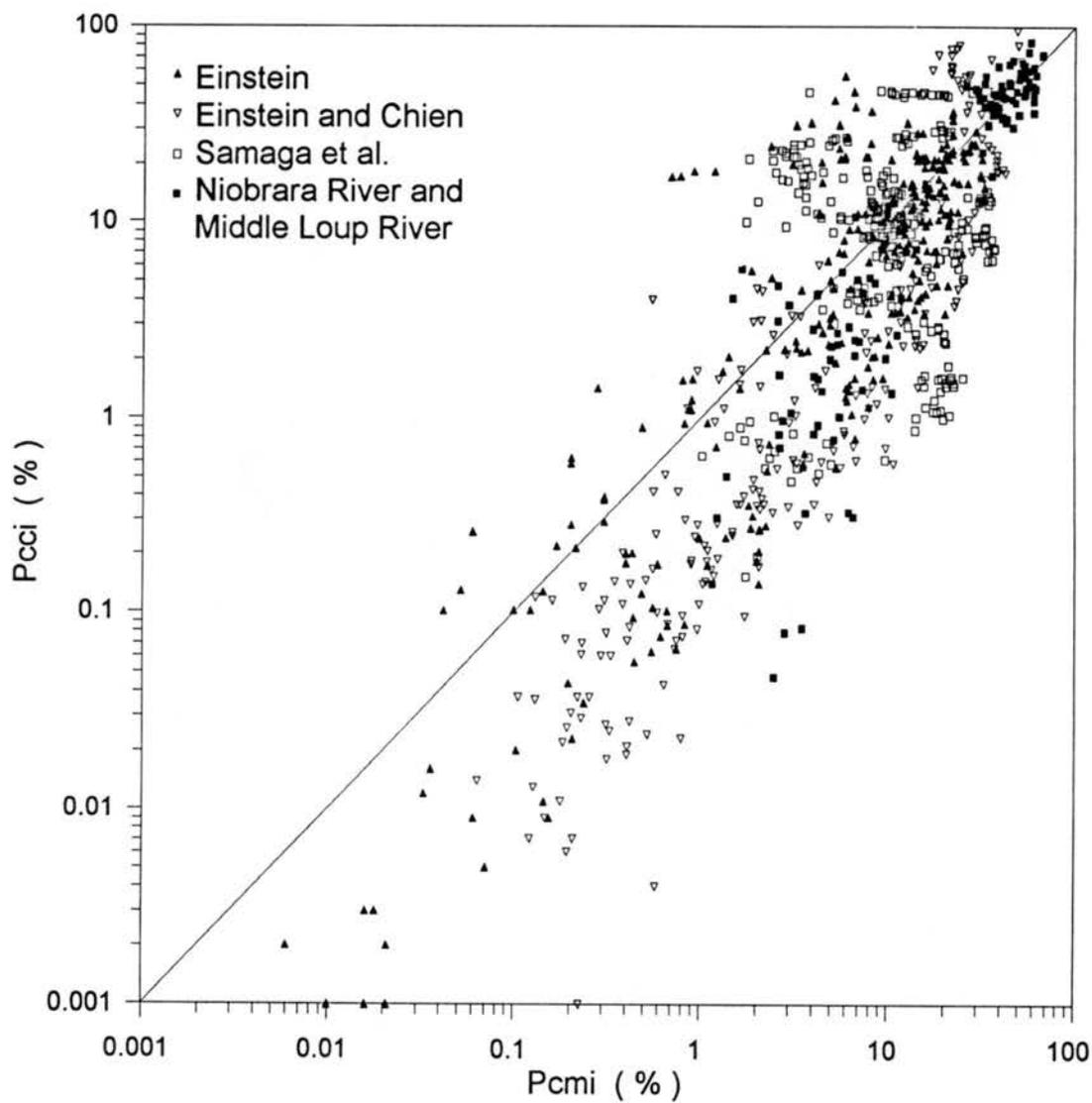


Fig. 6.27. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for the BMF Approach Using the Ackers and White Transport Equation (1973).

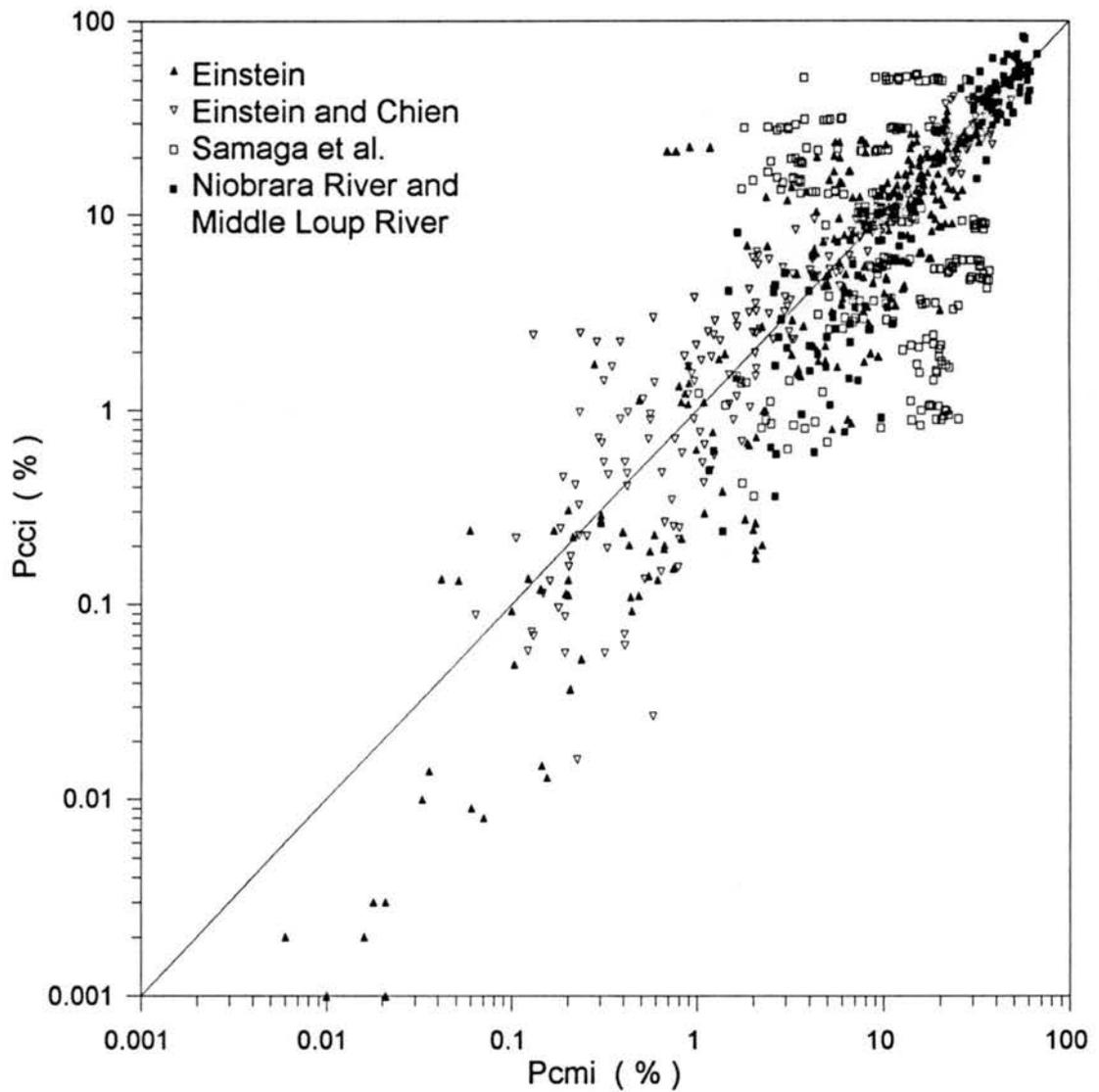


Fig. 6.28. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for the BMF Approach Using the Yang Transport Equation (1973).

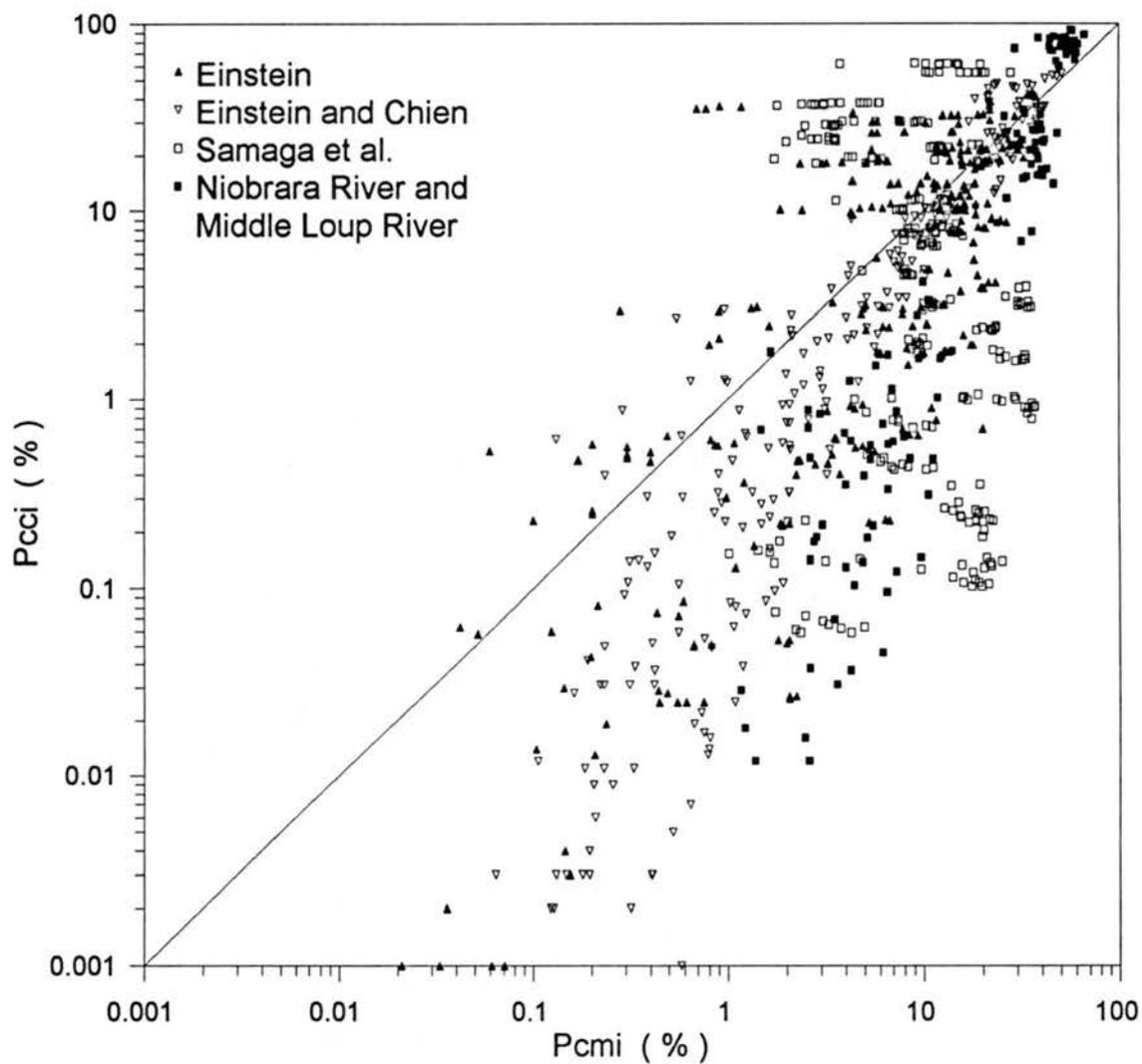


Fig. 6.29. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for Karim's Modified BMF Approach (1998).

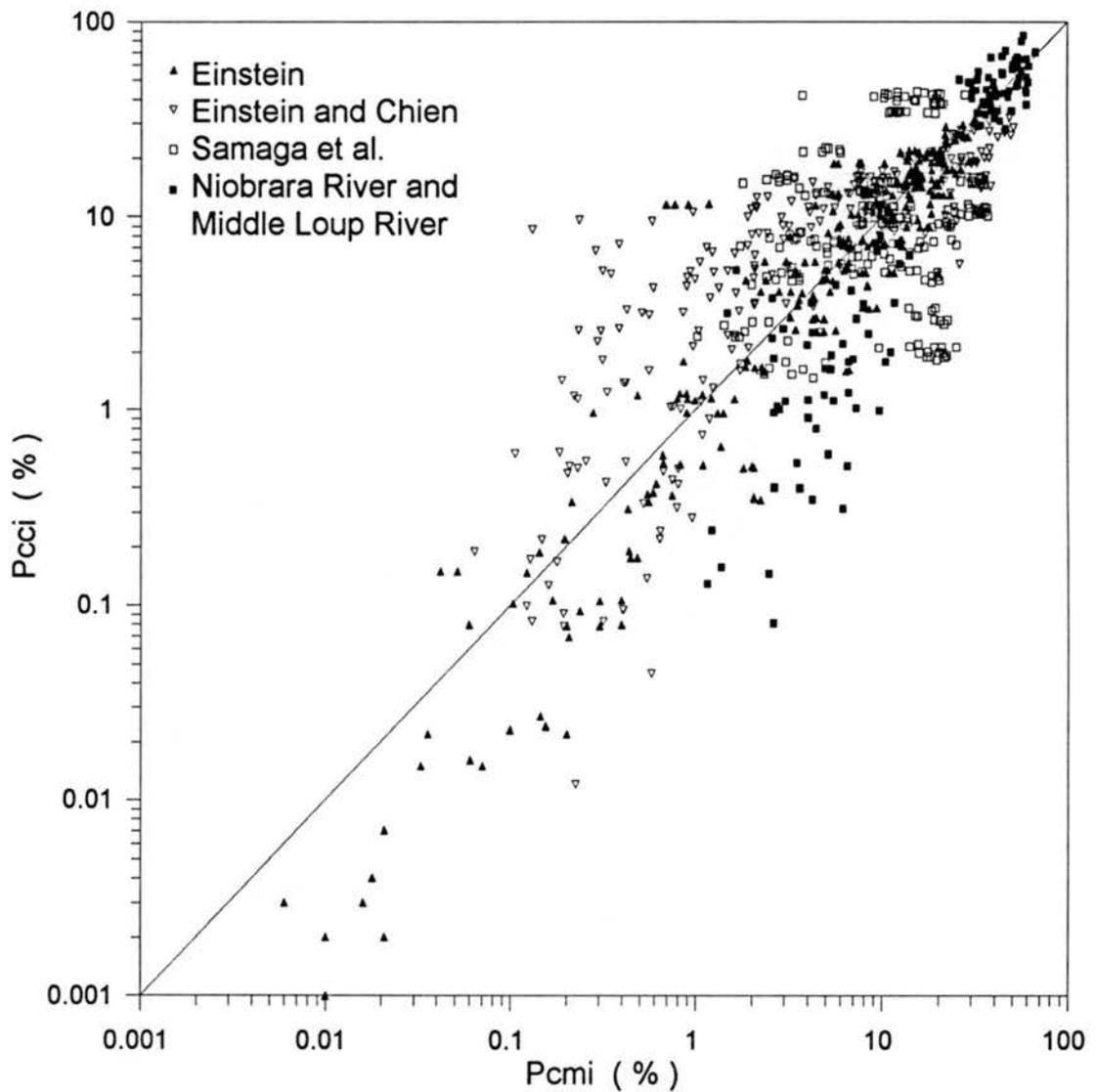


Fig. 6.30. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for the TCF Approach Using the Transport Capacity Distribution Function of Karim and Kennedy (1981).

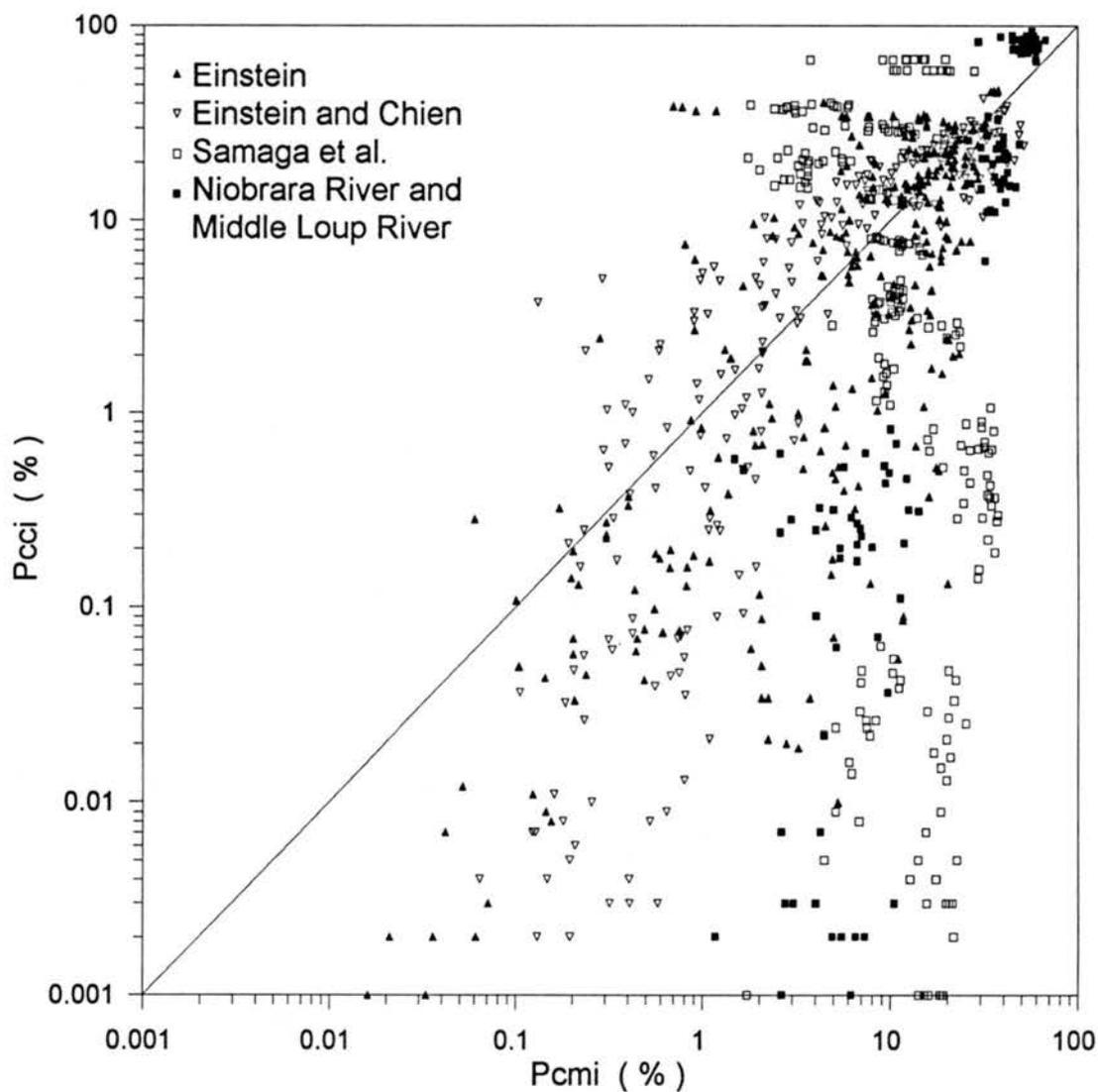


Fig. 6.31. Comparison between Computed and Measured Size Fractions of Bed-Material Load Sediment in Transport for the TCF Approach Using the Transport Capacity Distribution Function of Li (1988).

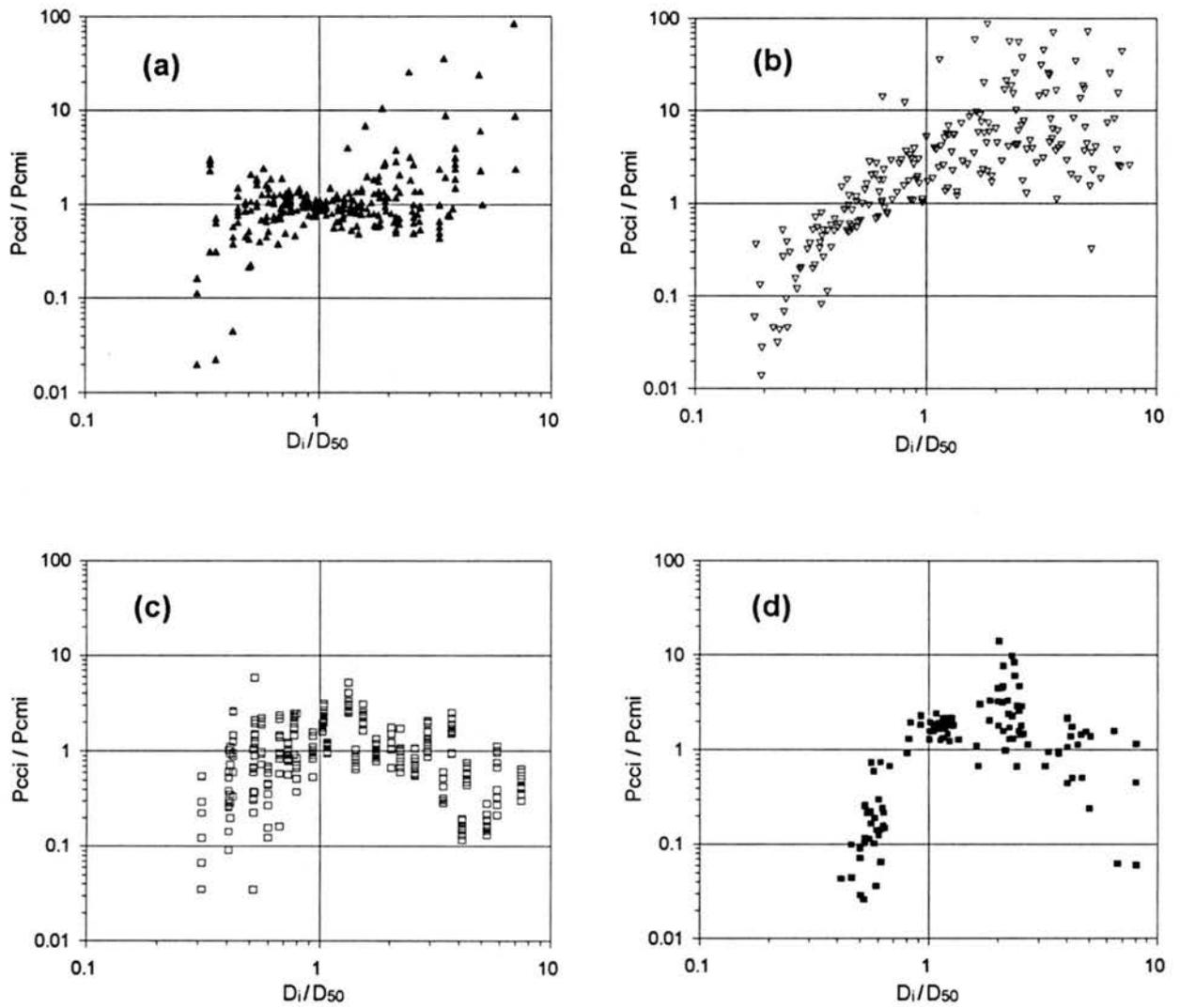


Fig. 6.32. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for the Einstein Equation (1950):  
 (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data;  
 (d) Niobrara River and Middle Loup River Data.

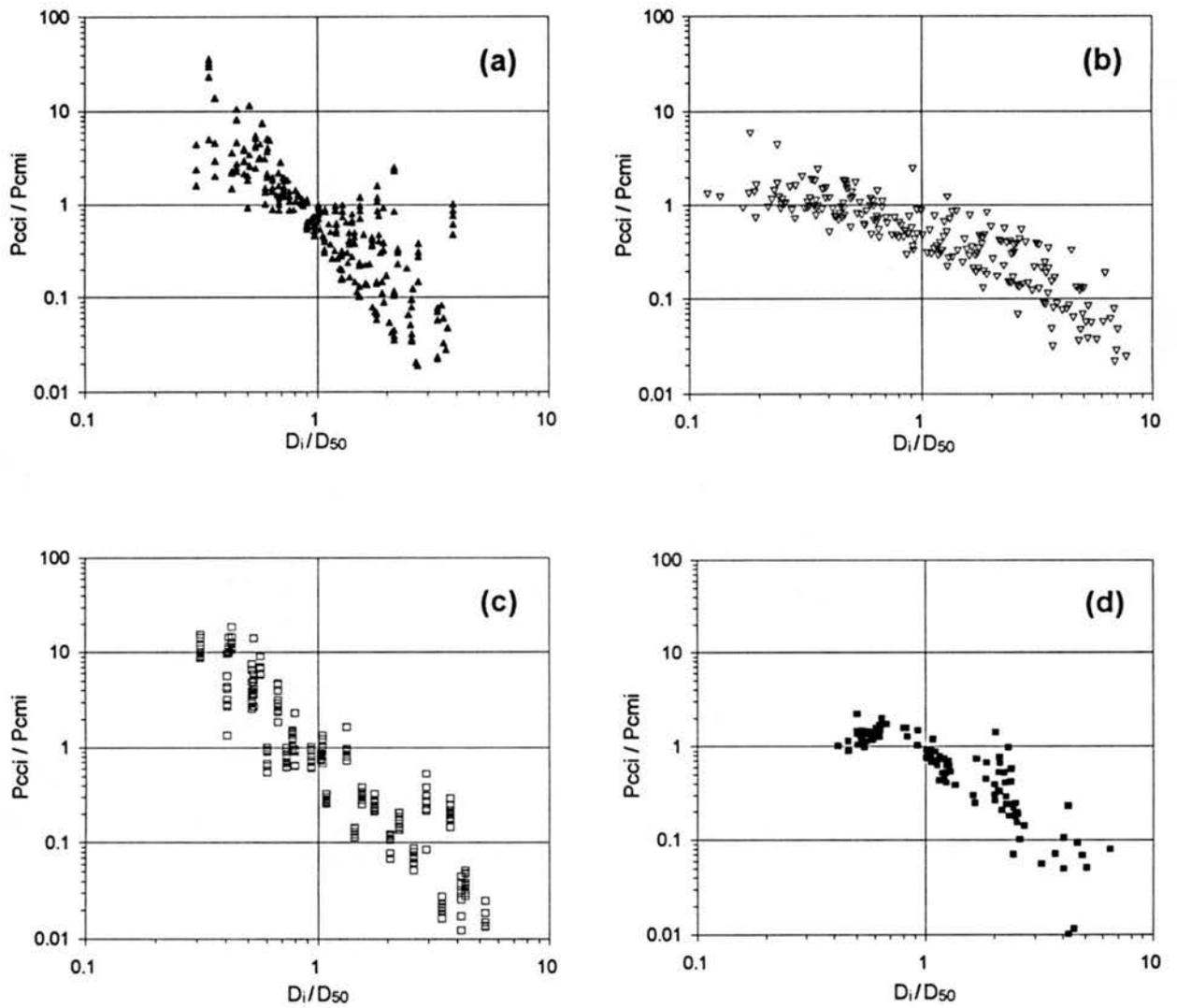


Fig. 6.33. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for the Laursen Equation (1958):  
 (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data;  
 (d) Niobrara River and Middle Loup River Data.

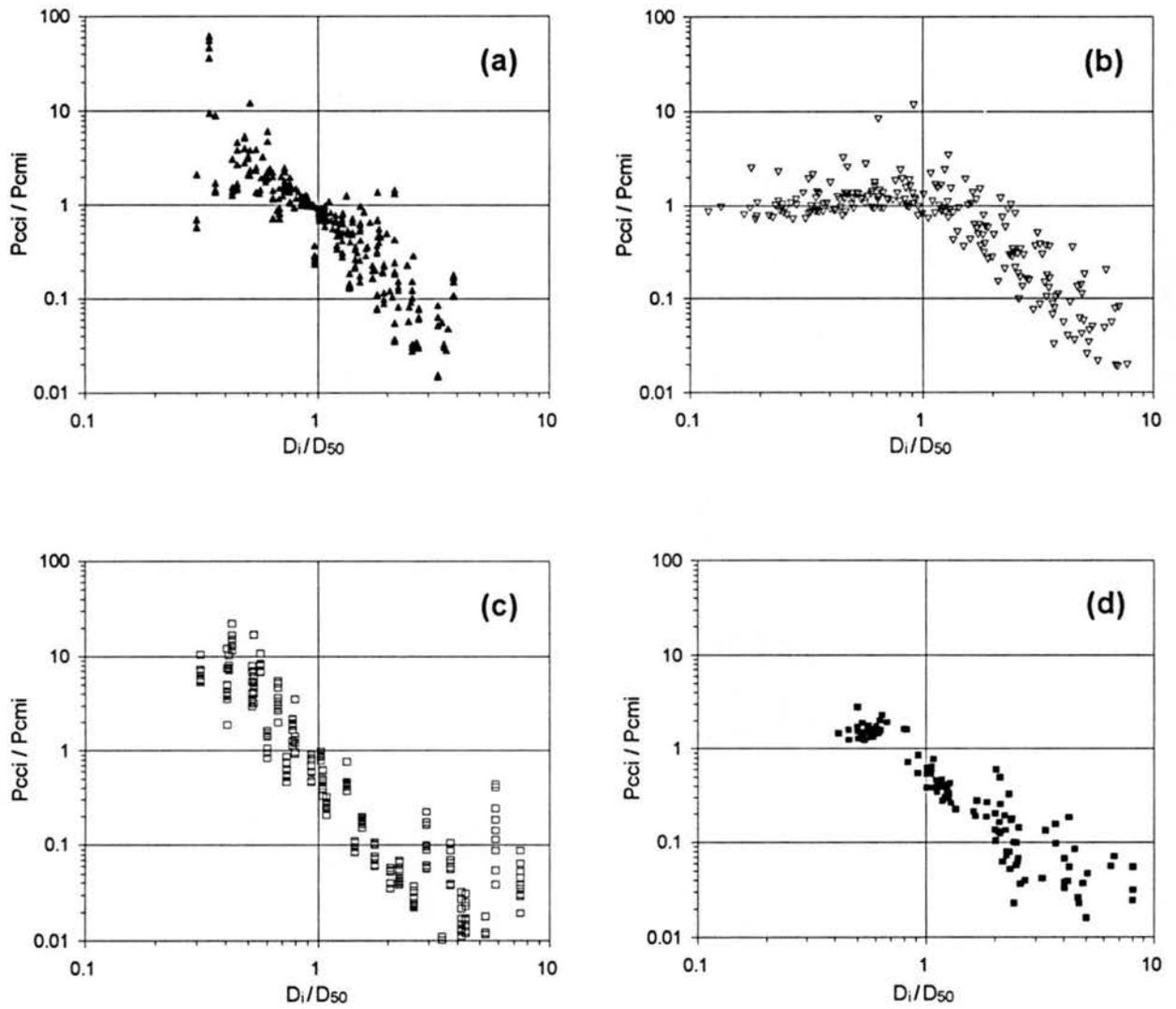


Fig. 6.34. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for the Toffaletti Equation (1968):  
 (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data;  
 (d) Niobrara River and Middle Loup River Data.

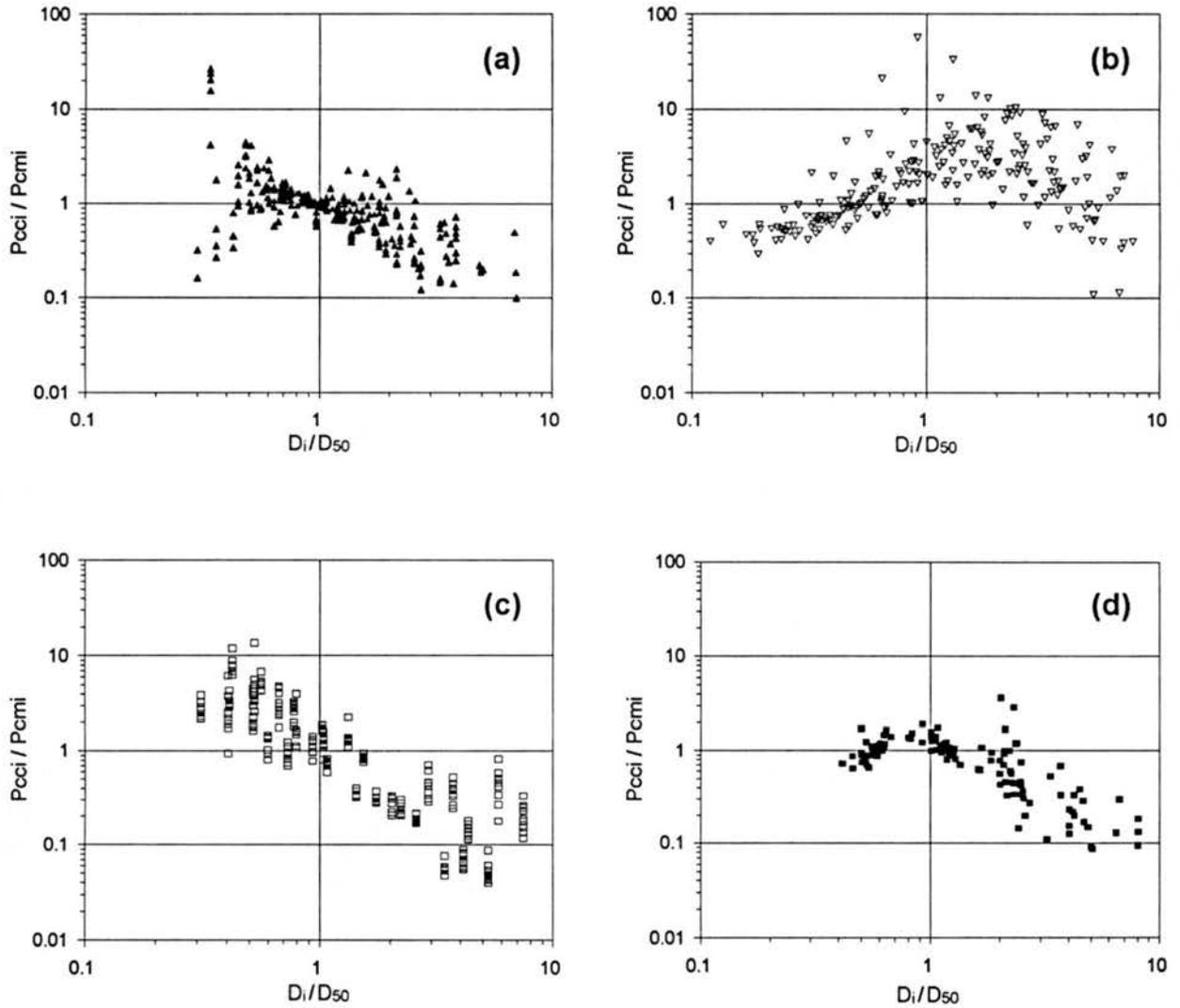


Fig. 6.35. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for the BMF Approach Using the Engelund and Hansen Transport Equation (1967): (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

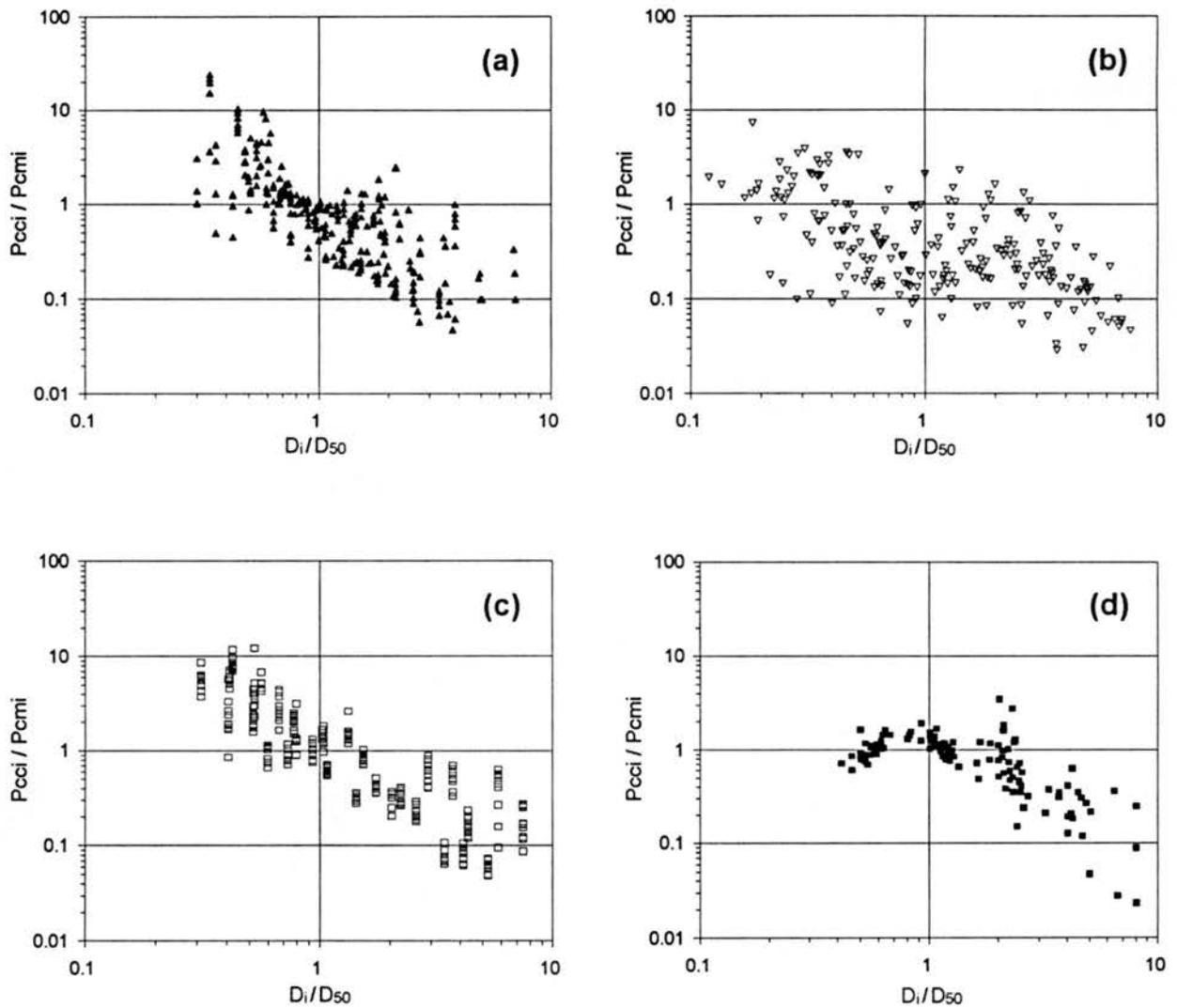


Fig. 6.36. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for the BMF Approach Using the Ackers and White Transport Equation (1973): (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

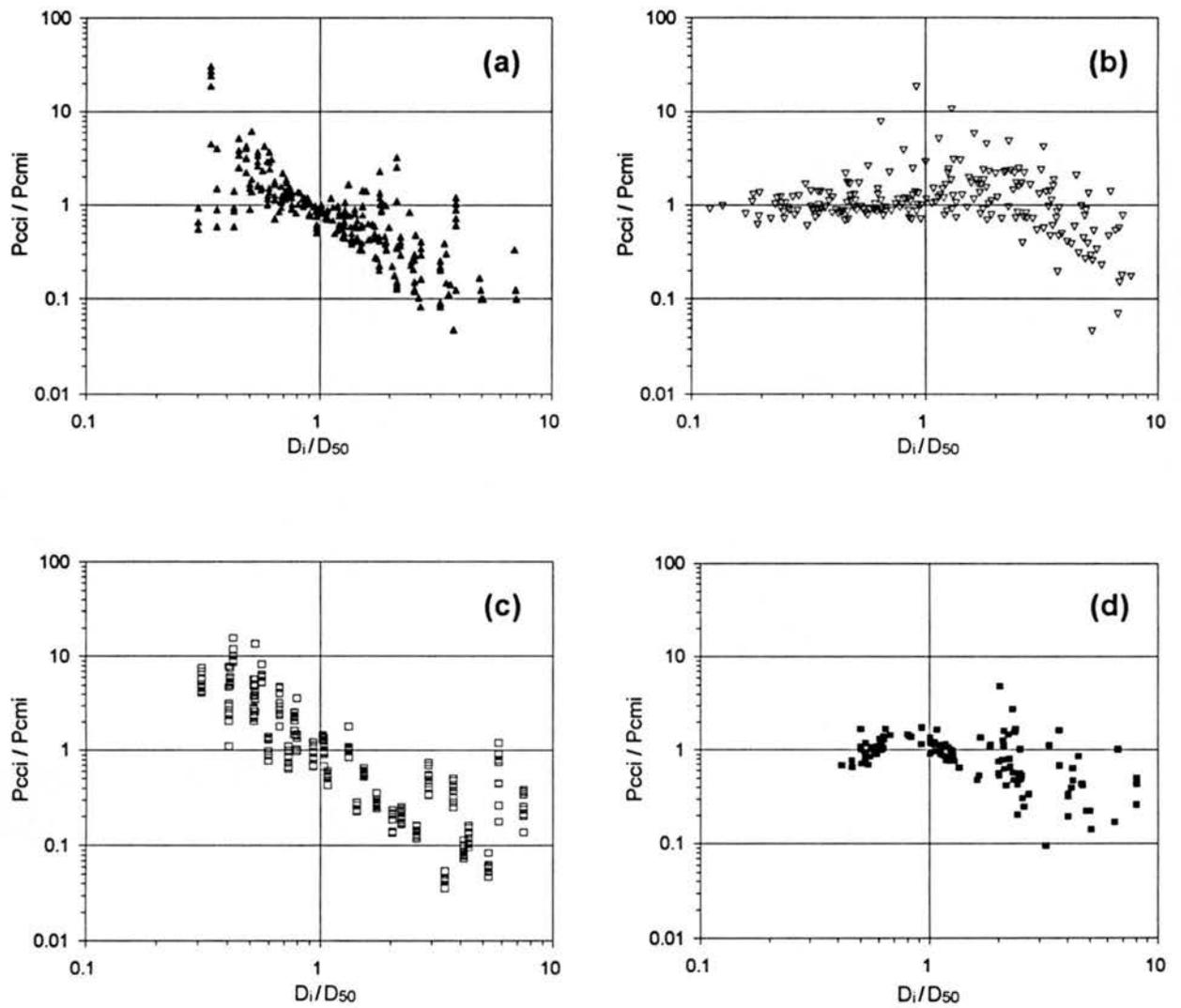


Fig. 6.37. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for the BMF Approach Using the Yang Transport Equation (1973): (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data

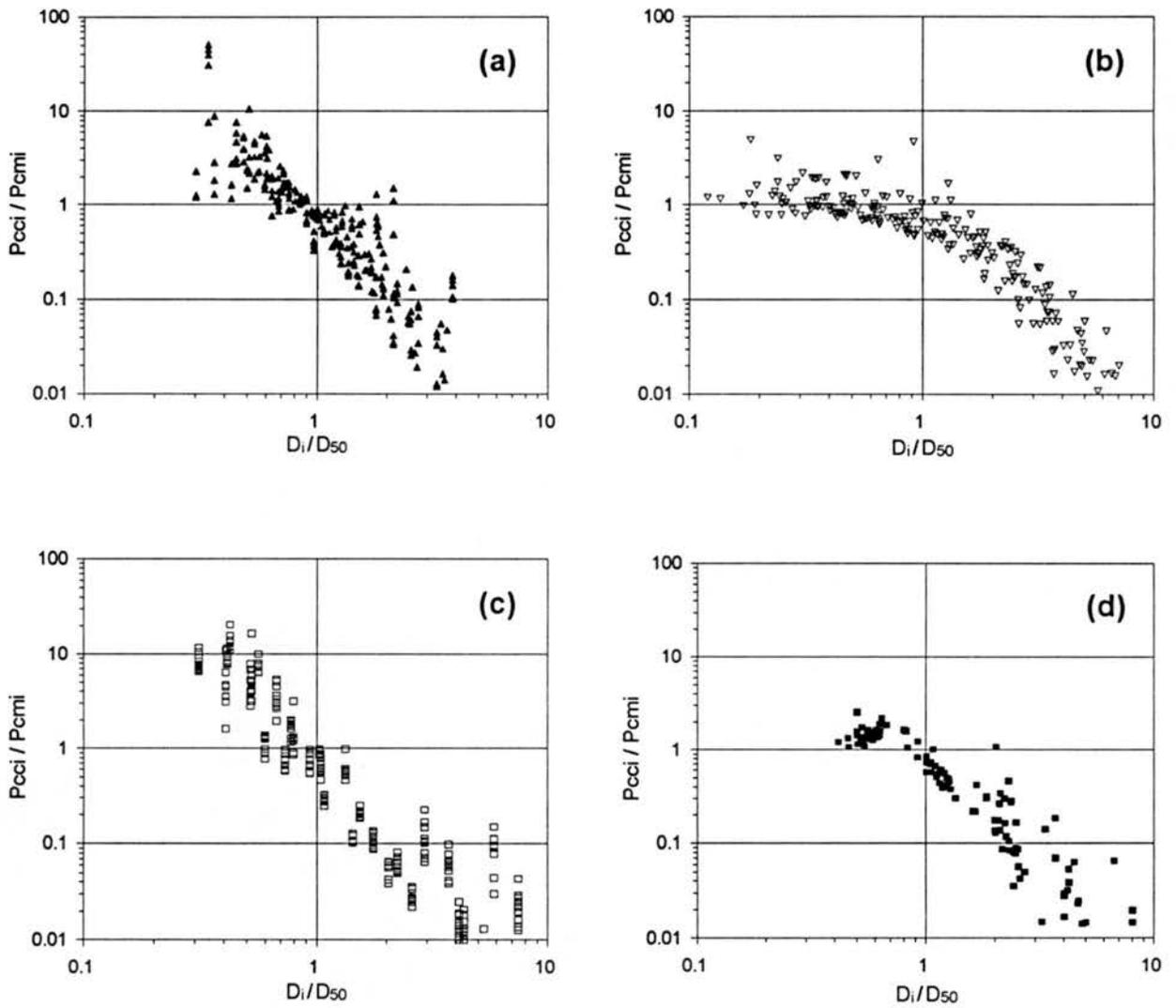


Fig. 6.38. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for Karim's Modified BMF Approach (1998): (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

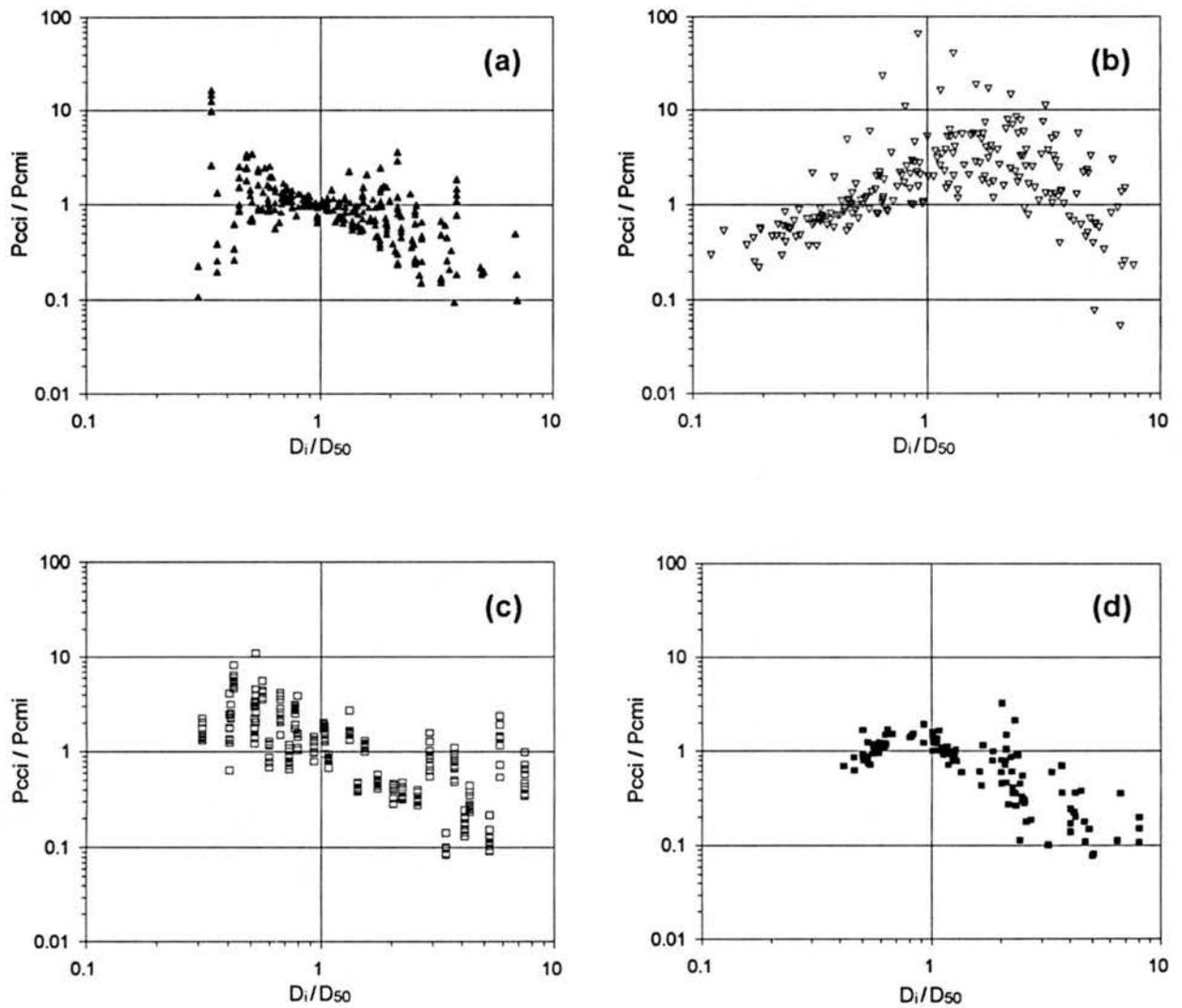


Fig. 6.39. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for the TCF Approach Using the Transport Capacity Distribution Function of Karim and Kennedy (1981): (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

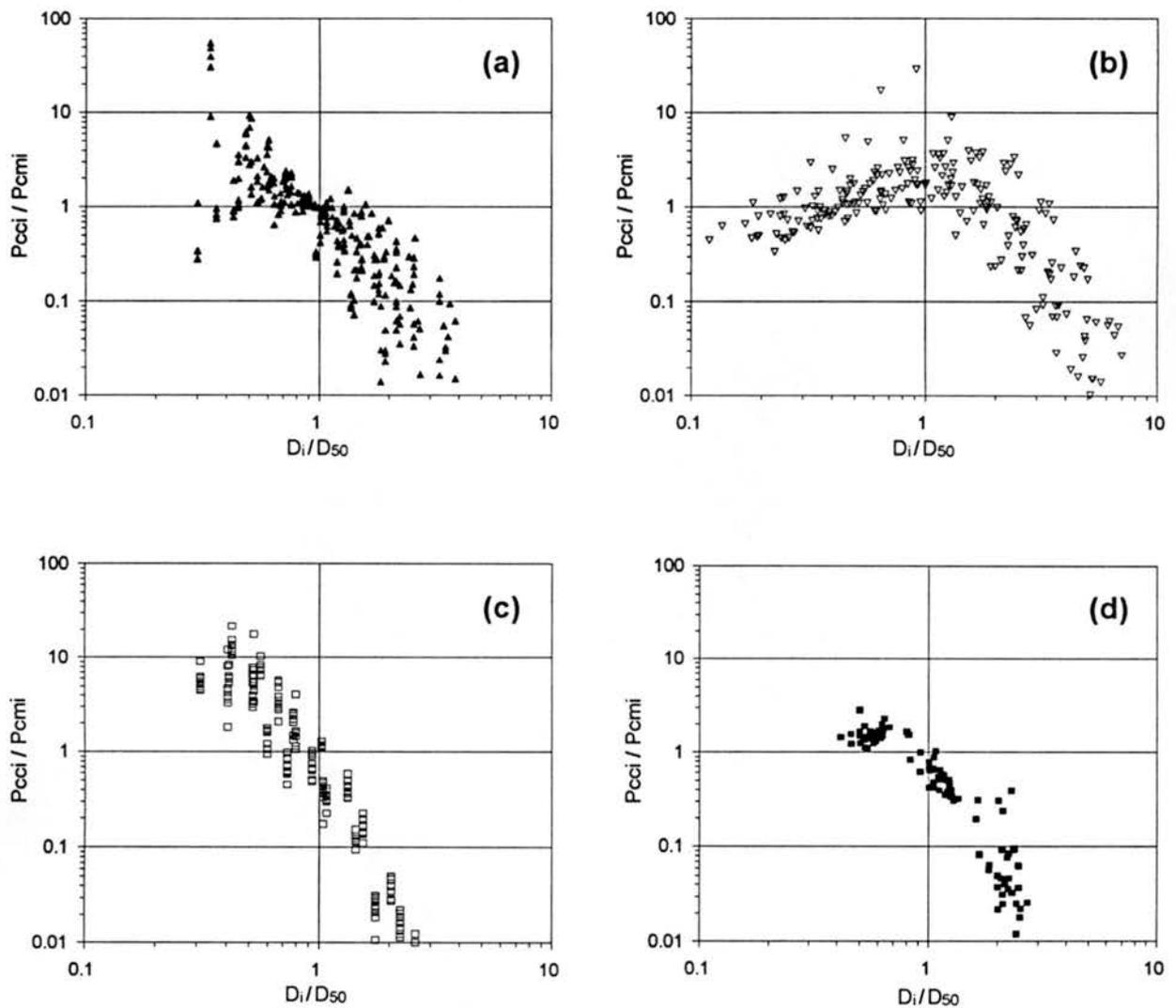


Fig. 6.40. Variation of  $P_{cci} / P_{cmi}$  versus  $D_i / D_{50}$  for the TCF Approach Using the Transport Capacity Distribution Function of Li (1988): (a) Einstein Data; (b) Einstein and Chien Data; (c) Samaga et al. Data; (d) Niobrara River and Middle Loup River Data.

## 6.4 VERIFICATION OF THE PROPOSED SEDIMENT TRANSPORT CAPACITY DISTRIBUTION FUNCTIONS

To verify the validity of sediment transport capacity distribution functions developed in Chapter 3, an independent data base with 48 sets of sediment transport data from flume and rivers are compiled. These data are given in Tables 6.3-6.5. The 20 sets of flume data given in Table 6.3 are derived from the laboratory experiments of White and Day (1982). It needs to be pointed out that the bed material size distribution information reported in White and Day's flume experiments are those of parent-bed material, which are generally different from the bed-surface materials. The 28 sets of river data given in Tables 6.4-6.5 include those measurements from the Rio Grande Conveyance Canal (Culbertson et al., 1972) and the Yellow River at Tuchengzi (Long and Liang, 1994). These field data contain complete flow and sediment information for each record, including the compositions for both bed material and sediment in transport.

Table 6.6 presents the statistical results for computed size fractions of bed-material load sediment in transport for the flume data of White and Day. The statistical results of mean normalized error, average geometric deviation, and discrepancy ratio indicate that the newly proposed transport distribution functions of Eq. (3.9) and (3.24) along with the function of Karim and Kennedy (1981) give the best prediction amongst all methods. It should be pointed out that the use of the size distribution from parent-bed material (instead of surface-bed material) may be favorable to the performance of Karim and Kennedy's function.

Table 6.7 gives the statistical results for computed size fractions of bed-material load sediment in transport for data of the Rio Grande Conveyance Canal and the Yellow River at

Tuchengzi. It can be seen that the mean normalized error was 51.0% and 78.7% for the newly proposed transport capacity distribution functions of Eqs. (3.9) and (3.24), respectively, while it was 65.7-183.6% for other methods. The average geometric deviation was 1.58 and 1.93 for Eqs. (3.9) and (3.24), respectively, while it was 2.75-39.6 for other methods. The percentage of data falling within the range of discrepancy ratios between 0.25 and 1.75 was 81.7% and 76.5% for Eq. (3.9) and (3.24), respectively, while it was 30.4-67.0% for other methods.

Table 6.3. Laboratory Data of White and Day (1982)

Data No. (1)	Run ID (2)	Flow Properties					Bed Material				Transported Sediment			
		Q (m <sup>3</sup> /s) (3)	W (m) (4)	d (m) (5)	S (m/m) (6)	T (°C) (7)	D <sub>50</sub> * (mm) (8)	D <sub>65</sub> * (mm) (9)	σ <sub>g</sub> * (10)	s <sub>g</sub> (11)	C <sub>T</sub> (kg/m <sup>3</sup> ) (12)	C <sub>T</sub> (PPM) (13)	D <sub>50t</sub> * (mm) (14)	σ <sub>gt</sub> * (15)
1	1	0.1990	2.46	0.1660	0.000680	12.0	2.065	3.470	3.887	2.65	0.0143	14.27	0.439	1.605
2	2	0.1990	2.46	0.1590	0.000770	12.0	2.065	3.470	3.887	2.65	0.0194	19.35	0.425	1.631
3	3	0.1990	2.46	0.1450	0.000790	12.0	2.065	3.470	3.887	2.65	0.0178	17.79	0.452	2.106
4	4	0.1970	2.46	0.1690	0.000660	12.0	2.065	3.470	3.887	2.65	0.0089	8.88	0.494	1.915
5	5	0.1960	2.46	0.1470	0.000890	12.0	2.065	3.470	3.887	2.65	0.0247	24.69	0.488	2.484
6	6	0.1970	2.46	0.1330	0.001090	12.0	2.065	3.470	3.887	2.65	0.0300	30.00	0.509	2.833
7	7	0.1970	2.46	0.1230	0.001700	12.0	2.065	3.470	3.887	2.65	0.0976	97.56	0.801	3.301
8	8	0.1970	2.46	0.1310	0.001530	12.0	2.065	3.470	3.887	2.65	0.0833	83.27	0.942	3.125
9	9	0.1960	2.46	0.1210	0.002490	12.0	2.065	3.470	3.887	2.65	0.1565	156.53	1.282	3.429
10	10	0.1980	2.46	0.1120	0.002890	12.0	2.065	3.470	3.887	2.65	0.4362	436.06	1.836	3.334
11	11	0.1930	2.46	0.1070	0.003660	12.0	2.065	3.470	3.887	2.65	0.8353	834.85	2.060	3.586
12	1	0.2030	2.46	0.1890	0.000445	12.0	1.745	2.273	2.983	2.65	0.0016	1.62	1.168	2.391
13	2	0.2100	2.46	0.1840	0.000446	12.0	1.745	2.273	2.983	2.65	0.0030	2.98	0.728	2.553
14	3	0.2020	2.46	0.1620	0.000722	12.5	1.745	2.273	2.983	2.65	0.0196	19.63	0.813	2.473
15	4	0.2020	2.46	0.1540	0.001616	12.0	1.745	2.273	2.983	2.65	0.0308	30.75	0.905	2.595
16	5	0.2000	2.46	0.1450	0.001769	12.0	1.745	2.273	2.983	2.65	0.0754	75.40	1.097	2.832
17	6	0.2020	2.46	0.1190	0.002262	12.0	1.745	2.273	2.983	2.65	0.7500	749.65	1.454	2.760
18	7	0.2040	2.46	0.1240	0.002188	12.0	1.745	2.273	2.983	2.65	0.6739	673.63	1.640	2.551
19	8	0.2000	2.46	0.1170	0.002614	13.0	1.745	2.273	2.983	2.65	0.8363	835.86	1.680	2.589
20	9	0.2010	2.46	0.1150	0.002987	12.0	1.745	2.273	2.983	2.65	1.0907	1089.95	1.691	2.550

Note: \* — Values computed from adjusted bed material size distribution (wash load materials excluded).

Table 6.3. Laboratory Data of White and Day (1982) (continued)

Data No. (1)	Run ID (2)	D <sub>c</sub> (mm) (16)	Size distribution of bed material*, finer than indicated diameters														
			Grp1 (%) (17)	Grp2 (%) (18)	Grp3 (%) (19)	Grp4 (%) (20)	Grp5 (%) (21)	Grp6 (%) (22)	Grp7 (%) (23)	Grp8 (%) (24)	Grp9 (%) (25)	Grp10 (%) (26)	Grp11 (%) (27)	Grp12 (%) (28)	Grp13 (%) (29)	Grp14 (%) (30)	Grp15 (%) (31)
			<b>0.18</b>	<b>0.25</b>	<b>0.355</b>	<b>0.5</b>	<b>0.71</b>	<b>1</b>	<b>1.4</b>	<b>1.7</b>	<b>2.36</b>	<b>3.35</b>	<b>4.76</b>	<b>6.35</b>	<b>7.85</b>	<b>9.52</b>	<b>15.6 mm</b>
1	1	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
2	2	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
3	3	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
4	4	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
5	5	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
6	6	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
7	7	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
8	8	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
9	9	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
10	10	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
11	11	0.25	1.3	5.46	16.46	27.89	34.89	39.66	44.64	47.74	54.7	64.8	74.68	85.43	92.3	95.65	98.28
12	1	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			
13	2	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			
14	3	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			
15	4	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			
16	5	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			
17	6	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			
18	7	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			
19	8	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			
20	9	0.25	2.13	6.83	18.6	27.9	33	39.14	46.11	51.01	67.94	81.41	96.41	97.88			

Note: \* - Parent-bed material.

Table 6.3. Laboratory Data of White and Day (1982) (continued)

Data No. (1)	Run ID (2)	Size distribution of transported sediment, finer than indicated diameters														
		Grp1 (%) (32)	Grp2 (%) (33)	Grp3 (%) (34)	Grp4 (%) (35)	Grp5 (%) (36)	Grp6 (%) (37)	Grp7 (%) (38)	Grp8 (%) (39)	Grp9 (%) (40)	Grp10 (%) (41)	Grp11 (%) (42)	Grp12 (%) (43)	Grp13 (%) (44)	Grp14 (%) (45)	Grp15 (%) (46)
		<b>0.18</b>	<b>0.25</b>	<b>0.355</b>	<b>0.5</b>	<b>0.71</b>	<b>1.0</b>	<b>1.4</b>	<b>1.7</b>	<b>2.36</b>	<b>3.35</b>	<b>4.76</b>	<b>6.35</b>	<b>7.85</b>	<b>9.52</b>	<b>15.6 mm</b>
1	1	0.67	6.28	32.72	65.84	82.56	90.81	94.60	96.08	97.90	99.13	99.92	100.00	100.00	100.00	100.00
2	2	1.00	8.58	37.73	69.09	83.21	89.91	93.25	94.58	96.57	98.26	99.73	100.00	100.00	100.00	100.00
3	3	1.33	9.97	37.33	62.37	75.12	82.01	86.21	88.26	92.42	96.90	100.01	100.01	100.01	100.01	100.01
4	4	0.61	5.33	27.19	53.67	70.53	81.71	88.66	91.65	95.80	98.43	100.10	100.10	100.10	100.10	100.10
5	5	0.99	8.34	32.83	56.02	68.33	76.03	81.29	83.94	89.06	94.96	100.45	100.45	100.45	100.45	100.45
6	6	0.93	8.11	32.06	53.50	64.57	71.53	76.51	79.06	84.25	91.29	96.63	99.36	99.91	100.03	100.03
7	7	1.02	7.28	25.53	41.65	51.21	58.49	64.35	67.53	73.96	83.36	92.27	97.98	99.83	100.18	100.24
8	8	0.72	5.55	20.12	35.72	46.05	54.06	60.73	64.49	72.75	83.45	93.09	97.99	99.53	99.75	99.78
9	9	0.65	4.83	17.48	31.16	40.21	47.59	54.21	58.00	66.53	76.64	86.79	94.97	98.78	99.82	100.12
10	10	0.47	3.76	13.66	23.96	31.37	38.08	45.17	49.60	59.41	72.11	84.80	94.34	98.70	99.79	100.03
11	11	0.56	3.90	14.32	24.81	31.06	37.06	43.09	46.96	55.50	67.22	79.90	91.28	97.69	99.59	100.01
12	1	1.01	4.13	13.57	25.76	35.57	46.53	58.55	66.29	84.20	95.06	99.09	100.00			
13	2	1.37	7.57	25.80	42.88	53.10	62.38	70.93	76.40	88.35	95.85	99.52	100.00			
14	3	0.60	4.07	18.44	35.22	47.85	58.78	67.81	73.53	87.54	97.17	100.27	100.27			
15	4	1.77	7.86	23.56	37.88	47.30	56.65	65.83	71.73	85.62	96.23	100.00	100.00			
16	5	1.77	7.71	23.07	35.85	43.81	51.61	60.05	65.27	79.42	91.37	99.64	100.17			
17	6	1.61	6.35	17.63	27.87	35.01	43.31	52.17	58.16	73.76	89.17	99.45	100.31			
18	7	0.70	3.28	10.83	19.73	27.18	36.22	46.09	52.89	70.21	87.89	99.09	99.97			
19	8	1.16	3.77	11.55	20.38	27.67	36.32	46.02	52.48	69.04	87.22	99.28	100.40			
20	9	0.50	2.43	9.51	18.40	25.90	34.83	44.70	51.40	68.27	86.69	98.99	100.00			

Table 6.4. Rio Grande Conveyance Channel Data of Culbertson, Scott, and Bennett (1972)

Data No. (1)	Survey Date (yymmdd) (2)	Flow Properties					Bed Material				Transported Sediment			
		Q (m <sup>3</sup> /s) (3)	W (m) (4)	d (m) (5)	S (m/m) (6)	T (°C) (7)	D <sub>50</sub> (mm) (8)	D <sub>65</sub> (mm) (9)	σ <sub>g</sub> (10)	s <sub>g</sub> (11)	C <sub>T</sub> (kg/m <sup>3</sup> ) (12)	C <sub>T</sub> (PPM) (13)	D <sub>50t</sub> (mm) (14)	σ <sub>gt</sub> (15)
1	650512	25.753	21.336	1.298	0.000650	15.0	0.270	-1	1.62	2.65	3.4200	3412.73	0.12	1.71
2	650512	25.753	22.860	1.374	0.000650	15.0	0.280	-1	1.75	2.65	3.4200	3412.73	0.12	1.71
3	650513	25.187	22.860	1.496	0.000650	15.0	0.210	-1	1.42	2.65	3.0200	3014.33	0.11	1.63
4	650513	25.187	20.117	1.247	0.000650	15.0	0.230	-1	1.42	2.65	3.0200	3014.33	0.11	1.63
5	650602	33.677	22.555	0.894	0.000730	17.0	0.200	-1	1.3	2.65	2.8100	2805.09	0.13	1.67
6	650603	36.507	27.432	0.887	0.000520	17.0	0.180	-1	1.34	2.65	2.9000	2894.77	0.11	1.46
7	651129	35.375	22.555	1.096	0.000660	4.0	0.180	-1	1.4	2.65	4.2200	4208.94	0.13	1.61
8	651130	35.375	22.555	1.108	0.000590	3.0	0.180	-1	1.42	2.65	4.5600	4547.09	0.16	1.63
9	660504	36.224	21.336	1.023	0.001110	18.0	0.210	-1	1.46	2.65	3.3500	3343.03	0.16	1.64

Table 6.4. Rio Grande Conveyance Channel Data of Culbertson, Scott, and Bennett (1972) (continued)

Data No. (1)	Survey Date (yymmdd) (2)	D <sub>c</sub> (mm) (16)	Size distribution of bed material, finer than indicated diameters						Size distribution of sediment load, finer than indicated diameters					
			Grp1 (%) (17)	Grp2 (%) (18)	Grp3 (%) (19)	Grp4 (%) (20)	Grp5 (%) (21)	Grp6 (%) (22)	Grp1 (%) (23)	Grp2 (%) (24)	Grp3 (%) (25)	Grp4 (%) (26)	Grp5 (%) (27)	Grp6 (%) (28)
			<b>0.062</b>	<b>0.125</b>	<b>0.25</b>	<b>0.5</b>	<b>1</b>	<b>2mm</b>	<b>0.062</b>	<b>0.125</b>	<b>0.25</b>	<b>0.5</b>	<b>1</b>	<b>2mm</b>
1	650512	0.062	0.0	4.0	43.0	87.0	98.0	100.0	70.0	86.0	97.0	100.0	100.0	100.0
2	650512	0.062	0.0	3.0	42.0	82.0	99.0	100.0	70.0	86.0	97.0	100.0	100.0	100.0
3	650513	0.062	0.0	10.0	71.0	98.0	100.0	100.0	70.0	87.0	97.0	100.0	100.0	100.0
4	650513	0.062	0.0	9.0	61.0	97.0	100.0	100.0	70.0	87.0	97.0	100.0	100.0	100.0
5	650602	0.062	0.0	4.0	75.0	98.0	100.0	100.0	52.0	75.0	94.0	99.0	100.0	100.0
6	650603	0.062	0.0	11.0	85.0	99.0	100.0	100.0	66.0	87.0	99.0	100.0	100.0	100.0
7	651129	0.062	0.0	12.0	82.0	99.0	100.0	100.0	41.0	68.0	93.0	100.0	100.0	100.0
8	651130	0.062	0.0	12.0	84.0	99.0	100.0	100.0	33.0	53.0	87.0	100.0	100.0	100.0
9	660504	0.062	1.0	8.0	66.0	98.0	100.0	100.0	26.0	48.0	86.0	99.0	100.0	100.0

Table 6.5. Yellow River Data at Tuchengzi of Long and Liang (1994)

Data No. (1)	Run ID (2)	Flow Properties					Bed Material				Transported Sediment			
		Q (m <sup>3</sup> /s) (3)	W (m) (4)	D (m) (5)	S (m/m) (6)	T (°C) (7)	D <sub>50</sub> <sup>*</sup> (mm) (8)	D <sub>65</sub> <sup>*</sup> (mm) (9)	σ <sub>g</sub> <sup>*</sup> (10)	S <sub>g</sub> (11)	C <sub>T</sub> (kg/m <sup>3</sup> ) (12)	C <sub>T</sub> (PPM) (13)	D <sub>50t</sub> <sup>*</sup> (mm) (14)	σ <sub>gt</sub> <sup>*</sup> (15)
1	570828	1173.33	572.67	1.447	0.0000940	28.4	0.056	0.071	1.812	2.65	14.94	14800.97	0.042	2.301
2	570918	1316.67	578.00	1.420	0.0000840	21.2	0.058	0.071	1.787	2.65	22.49	22183.81	0.050	2.073
3	570926	1273.33	539.67	1.437	0.0000900	18.6	0.064	0.078	1.709	2.65	26.57	26141.63	0.046	2.057
4	571008	1486.67	581.33	1.427	0.0000780	15.4	0.060	0.072	1.685	2.65	25.22	24831.72	0.037	2.121
5	571018	1233.33	566.00	1.417	0.0001000	15.2	0.065	0.077	1.620	2.65	23.26	22930.04	0.054	1.966
6	571027	848.00	607.53	1.443	0.0000920	14.1	0.070	0.083	1.707	2.65	20.40	20142.26	0.064	1.831
7	571104	1180.00	545.23	1.447	0.0001000	12.2	0.064	0.076	1.669	2.65	25.96	25545.09	0.058	1.811
8	580319	480.00	298.00	1.750	0.0000960	7.6	0.055	0.068	1.813	2.65	12.05	11956.98	0.052	1.949
9	580328	432.00	292.00	1.740	0.0000920	6.6	0.055	0.068	1.688	2.65	10.16	10099.79	0.062	1.842
10	580410	964.00	282.00	1.910	0.0000970	12.4	0.061	0.073	1.607	2.65	20.68	20417.87	0.054	1.784
11	580427	446.00	279.00	1.650	0.0000940	16.4	0.055	0.066	1.670	2.65	15.06	14921.68	0.065	1.673
12	580520	996.00	429.00	1.320	0.0001000	20.3	0.057	0.069	1.692	2.65	31.17	30574.14	0.062	1.816
13	580527	293.00	410.00	0.960	0.0001400	22.3	0.069	0.080	1.403	2.65	7.31	7274.65	0.066	1.599
14	580604	88.80	333.00	0.590	0.0001600	22.8	0.069	0.079	1.331	2.65	14.42	14290.44	0.075	1.496
15	580613	252.00	431.00	1.030	0.0001100	22.8	0.062	0.075	1.759	2.65	20.44	20182.67	0.070	1.375
16	580624	161.00	352.00	0.870	0.0001200	26.2	0.068	0.079	1.521	2.65	18.38	18174.34	0.074	1.682
17	580705	392.00	398.00	1.070	0.0000800	28.0	0.058	0.072	1.725	2.65	12.98	12877.14	0.066	1.584
18	580716	3880.00	794.00	1.740	0.0001000	26.9	0.064	0.074	1.595	2.65	101.61	95561.53	0.035	2.206
19	580814	3980.00	807.00	1.820	0.0001000	22.0	0.068	0.081	1.648	2.65	59.02	56926.68	0.038	2.179

Note: \* — Values computed from adjusted bed material size distribution (wash load materials excluded).

Table 6.5. Yellow River Data at Tuchengzi of Long and Liang (1994) (continued)

Data No. (1)	Run ID (2)	D <sub>c</sub> (mm) (16)	Size distribution of bed material, finer than indicated diameters								
			Grp1 (%) (17)	Grp2 (%) (18)	Grp3 (%) (19)	Grp4 (%) (20)	Grp5 (%) (21)	Grp6 (%) (22)	Grp7 (%) (23)	Grp8 (%) (24)	Grp9 (%) (25)
			<b>0.005</b>	<b>0.01</b>	<b>0.025</b>	<b>0.05</b>	<b>0.1</b>	<b>0.25</b>	<b>0.5</b>	<b>1</b>	<b>2mm</b>
1	570828	0.01	0.13	1.13	10.35	42.78	87.83	99.77	100.00	100.00	100.00
2	570918	0.01	0.00	1.07	10.58	39.63	89.57	99.87	100.00	100.00	100.00
3	570926	0.01	0.10	0.77	6.54	30.12	85.50	100.00	100.00	100.00	100.00
4	571008	0.01	0.13	0.60	6.59	35.47	92.80	99.87	100.00	100.00	100.00
5	571018	0.01	0.00	0.27	4.99	25.98	88.60	100.00	100.00	100.00	100.00
6	571027	0.01	0.00	0.17	4.03	21.57	81.23	97.03	100.00	100.00	100.00
7	571104	0.01	0.00	0.00	5.64	28.82	88.47	99.93	100.00	100.00	100.00
8	580319	0.01	0.00	2.80	14.50	45.50	91.80	99.90	100.00	100.00	100.00
9	580328	0.01	1.50	1.80	5.00	44.00	92.50	99.90	100.00	100.00	100.00
10	580410	0.01	0.00	0.00	1.00	32.00	93.00	99.90	100.00	100.00	100.00
11	580427	0.01	4.70	6.30	12.00	46.00	97.80	99.90	100.00	100.00	100.00
12	580520	0.01	0.00	0.00	7.50	38.50	96.00	99.90	100.00	100.00	100.00
13	580527	0.01	0.00	0.00	1.00	16.50	88.00	99.90	100.00	100.00	100.00
14	580604	0.01	0.00	0.00	0.00	11.00	93.50	99.90	100.00	100.00	100.00
15	580613	0.01	0.00	1.00	9.60	33.00	87.50	99.90	100.00	100.00	100.00
16	580624	0.01	0.00	0.00	2.00	21.00	87.50	99.90	100.00	100.00	100.00
17	580705	0.01	0.00	1.00	4.00	40.00	87.50	99.90	100.00	100.00	100.00
18	580716	0.01	0.00	1.20	5.50	28.00	93.50	99.90	100.00	100.00	100.00
19	580814	0.01	4.50	8.00	12.50	30.00	85.00	99.90	100.00	100.00	100.00

Table 6.5. Yellow River Data at Tuchengzi of Long and Liang (1994) (continued)

Data No. (1)	Run ID (2)	Size distribution of transported sediment, finer than indicated diameters								
		Grp1 (%) (26)	Grp2 (%) (27)	Grp3 (%) (28)	Grp4 (%) (29)	Grp5 (%) (30)	Grp6 (%) (31)	Grp7 (%) (32)	Grp8 (%) (33)	Grp9 (%) (34)
		<b>0.005</b>	<b>0.01</b>	<b>0.025</b>	<b>0.05</b>	<b>0.1</b>	<b>0.25</b>	<b>0.5</b>	<b>1</b>	<b>2mm</b>
1	570828	24.5	34.0	53.9	71.4	94.4	100.0	100.0	100.0	100.0
2	570918	20.9	28.5	42.7	64.1	93.0	100.0	100.0	100.0	100.0
3	570926	21.2	28.2	43.7	67.0	96.1	100.0	100.0	100.0	100.0
4	571008	17.0	25.0	46.4	74.8	97.2	100.0	100.0	100.0	100.0
5	571018	15.5	22.7	36.3	56.9	93.9	100.0	100.0	100.0	100.0
6	571027	11.4	16.5	25.8	42.1	86.9	100.0	100.0	100.0	100.0
7	571104	7.8	11.9	22.9	46.1	93.1	100.0	100.0	100.0	100.0
8	580319	5.8	11.3	27.9	53.2	97.6	99.9	100.0	100.0	100.0
9	580328	6.5	13.6	24.5	42.2	89.6	99.9	100.0	100.0	100.0
10	580410	4.9	8.3	19.8	49.6	95.6	99.9	100.0	100.0	100.0
11	580427	4.0	6.7	11.8	32.5	87.3	99.9	100.0	100.0	100.0
12	580520	14.2	19.7	28.1	46.9	88.9	99.9	100.0	100.0	100.0
13	580527	21.0	27.8	31.3	45.5	92.1	100.0	100.0	100.0	100.0
14	580604	1.5	2.0	2.4	9.9	80.3	99.8	100.0	100.0	100.0
15	580613	22.4	26.1	28.9	36.8	91.5	99.9	100.0	100.0	100.0
16	580624	24.6	27.4	29.5	39.3	82.0	99.9	100.0	100.0	100.0
17	580705	7.5	11.0	16.4	31.5	91.0	99.9	100.0	100.0	100.0
18	580716	22.7	33.2	55.6	78.2	97.0	99.9	100.0	100.0	100.0
19	580814	33.2	43.6	60.9	79.1	98.4	100.0	100.0	100.0	100.0

Table 6.6 Comparison between Computed and Measured Size Fractions of Sediment in Transport (*Bed-Material Load*) for the 20 Sets of Flume Data from White and Day (1982)

Fractional Bed-Material Load Computation Method (1)	Data in Range of Discrepancy Ratio, $R_i$ (%)				Mean Normalized Error, MNE (%) (6)	Average Geometric Deviation, AGD (7)	No. of Data Points (8)
	0.75-1.25 (2)	0.5-1.5 (3)	0.25-1.75 (4)	0.5-2.0 (5)			
<b>a) Direct computation by size fraction approach</b>							
Einstein's equation (1950)	4.7	8.0	14.6	12.7	187.3	4083.0	212
Laursen's equation (1958)	25.5	39.2	46.7	45.8	76.7	8.09	212
Toffaleti's equation (1968)							
<b>b) BMF approach using</b>							
Engelund and Hansen's equation (1967)	20.8	49.1	78.3	57.1	75.8	1.99	212
Ackers and White's equation (1973)	27.8	43.4	60.8	49.5	76.3	2.85	212
Yang's equation (1973)	30.2	56.6	64.2	62.7	60.1	4.03	212
Karim's modified BMF method (1998)	17.5	42.9	51.9	47.6	648.2	7.21	212
<b>c) TCF approach using</b>							
Function of Karim and Kennedy (1981)	50.5	78.3	82.1	83.0	162.4	1.62	212
Function of Li (1988)	2.0	5.2	9.9	7.1	130.1	117.1	212
Proposed function of Eq. (3.9)	34.0	64.2	78.8	73.6	66.4	1.89	212
Proposed function of Eq. (3.24)	32.1	63.7	77.8	69.8	86.1	1.92	212

Table 6.7. Comparison between Computed and Measured Size Fractions of Sediment in Transport (*Bed-Material Load*) for the 28 Sets of Data from Rio Grande Conveyance Canal and Yellow River at Tuchengzi.

Fractional Bed-Material Load Computation Method (1)	Data in Range of Discrepancy Ratio, $R_i$ (%)				Mean Normalized Error, MNE (%) (6)	Average Geometric Deviation, AGD (7)	No. of Data Points (8)
	0.75-1.25 (2)	0.5-1.5 (3)	0.25-1.75 (4)	0.5-2.0 (5)			
<b>a) Direct computation by size fraction approach</b>							
Einstein's equation (1950)	9.6	25.2	41.7	37.4	593.0	14.64	115
Laursen's equation (1958)	25.4	35.7	52.2	47.8	78.0	4.30	115
Toffaletti's equation (1968)	14.8	22.6	41.7	32.2	108.3	5.49	115
<b>b) BMF approach using</b>							
Engelund and Hansen's equation (1967)	22.6	45.2	67.0	54.8	66.7	2.28	115
Ackers and White's equation (1973)	9.6	19.1	35.7	27.0	179.3	8.39	115
Yang's equation (1973)	20.0	37.4	60.9	49.6	84.1	2.75	115
Karim's modified BMF method (1998)	17.4	27.0	42.6	34.8	115.7	6.62	115
<b>c) TCF approach using</b>							
Function of Karim and Kennedy (1981)	8.7	17.4	30.4	23.5	183.6	39.60	115
Function of Li (1988)	22.6	43.5	57.4	54.8	65.7	3.81	115
Proposed function of Eq. (3.9)	45.2	67.0	81.7	79.1	51.0	1.58	115
Proposed function of Eq. (3.24)	27.0	52.2	76.5	64.4	78.7	1.93	115

## CHAPTER 7

### SUMMARY AND CONCLUSIONS

#### 7.1 SUMMARY

In this dissertation, a new method for predicting fractional transport rates of bed-material load in sand-bed channels is presented. The proposed method is developed based on the concept of the transport capacity fraction (TCF) approach expressed by Eq. (2.8). The bed-material concentration for a given size fraction is obtained by weighting the bed-material concentration,  $C_b$ , with the newly developed transport capacity distribution function,  $P_{\omega}$ , from Eqs. (3.9) or (3.24). The first transport capacity distribution function given by Eq. (3.9) depends on relative fall velocity. The associated coefficients are given by Eqs. (3.12)-(3.14). The second transport capacity distribution function given by Eq. (3.24) depends on relative diameters. The associated coefficients are given by Eqs. (3.25)-(3.27). The procedure and a detailed example problem showing the use of the proposed method are provided.

For the computation of bed-material concentrations, the effect of size gradations on the transport of sediment mixtures is investigated in detail. First, Eq. (4.3) is proposed for predicting the median diameter,  $D_{50t}$ , of bed-material load. Then, the effect of size gradations on the transport of sediment mixtures is demonstrated by the use of Engelund and Hansen's transport function and Yang's unit stream power function. To compensate for the size gradation effect, the median diameter,  $D_{50t}$ , is proposed for use as the representative size for

bed-material load computations. For the existing bed-material load equations, an equivalent diameter,  $D_e$ , expressed by Eq. (4.6), is proposed. This equivalent diameter, which is related to  $D_{50}$ , is incorporated into the Engelund and Hansen, Ackers and White, and Yang formulas for the computation of bed-material concentrations.

A comprehensive comparison and evaluation of the proposed fractional load computation method with various existing fractional transport methods is conducted based on the available flume and field data. Statistical analysis and graphical comparisons are utilized to qualitatively and quantitatively demonstrate the performance and variations in different methods.

Sources of data and their flow and sediment properties used in the analysis are summarized as follows:

1) *Data for the development and verification of fractional bed-material load computation method*

For the development of new transport capacity distribution functions of Eqs. (3.9) and (3.24), a data base with 118 sets of flume and field data containing a total of 891 points is collected from different sources available in the literature. Each set of data contains a complete record for flow and sediment information, including the size distributions of bed material and transported sediments, pertaining to each measurement. This data base is limited to sand sizes with median diameter in the range of 0.10 to 0.90 mm, geometric standard deviation of bed material in the range of 1.30 to 3.0, flow discharge in the range of 0.0056 to 16.06 m<sup>3</sup>/s, flow velocity in the range of 0.49 to 1.41 m/s, flow depth in the range of 0.056 to 0.58 m, and slope in the range of 0.00093 to 0.013. A summary of these data is given in Table 3.1, and detailed information for each data set are provided in Tables 3.2-3.6.

In the verification of the proposed transport capacity distribution functions, an independent data base with 48 sets of flume and field data, which contain a total of 327 data points, was compiled. This independent data base covers flow and sediment conditions with median diameter of 0.055-2.06 mm, geometric standard deviation of 1.30-3.89, flow discharge of 0.19 - 3980.0 m<sup>3</sup>/s, flow velocity of 0.44 - 2.81 m/s, flow depth of 0.11-1.91 m, and slope of 0.000078-0.0037. Detailed data information are provided in Tables 6.3-6.5.

2) *Data for the analysis and verification of median diameters of sediment in transport and the effect of size gradations on the transport of sediment mixtures*

In the development of a prediction equation for median diameters of bed-material load and the analysis of the effect of size gradations on the transport of sediment mixtures, the 118 sets of data collected for the development of transport capacity distribution functions plus another 280 sets of flume data from CSU were used. For verifying the variation of median diameters of sediment in transport with size gradations, an independent database with 124 sets of flume and field data were compiled. These independent data cover flow and sediment conditions with median diameter of 0.055-2.10 mm, geometric standard deviation of 1.25-4.06, flow discharge of 0.0037-3980, flow velocity of 0.19-2.81, flow depth of 0.062-1.91, and slope of 0.000078-0.0039.

## 7.2 CONCLUSIONS

The following conclusions are drawn from this study:

1. Research on fractional sediment transport of nonuniform sediment mixtures can be classified into four categories: direct computation by size fraction approach; shear stress correction approach; the BMF approach; and the TCF approach. This

classification is very useful for the analysis and understanding of the problem. The TCF approach relates the fractional transport rate to the bed-material transport capacity and to the transport capacity distribution function. It computes sediment transport rate corresponding to each size group by weighting the bed-material concentration with the transport capacity distribution function. Through the use of the TCF formulations given by Eq. (2.8), discrepancies in computing bed-material load due to the distribution and number of class intervals can be avoided. Also due to the form of Eq. (2.8), errors made in fractional transport computations (e.g., extremely high transport rate for finest fractions) are limited.

2. The transport capacity distribution function,  $P_{ci}$ , in the TCF approach can be related to both hydraulic conditions and sediment properties. By introducing proper parameters in the determination of  $P_{ci}$ , the effects due to the presence of other size fractions in sediment mixtures on the transport of a given size fraction can be reflected. The literature review shows that the sheltering and exposure correction factor is mainly related to relative sediment sizes ( $D_i/D_{50}$ ,  $D_i/D_a$ ,  $D_i/D_A$ ,  $D_i/D_w$ ,  $D_i/D_n$ , etc.), size gradation ( $M$ ,  $\sigma_g$ ), and flow intensity ( $V/V_*$ ,  $d/D_{50}$ , etc.). This understanding is essential for incorporating the sheltering and exposure effects in the formulations of transport capacity distribution function,  $P_{ci}$ .
3. The proposed transport capacity distribution functions are derived from combinations of theoretical derivation and physical considerations. The first function of Eq. (3.9) depends on fall velocity, which is derived from the unit stream power theory and the concepts of the TCF approach and the bed material fraction (BMF) approach. The second function of Eq. (3.24) depends on relative diameter, which is derived from

Engelund and Hansen's transport relations and the concepts of the TCF approach and the BMF approach. The sheltering and exposure effects are considered in both functions by including a second term in their derivations.

4. Fractional bed-material concentration ( $C_b$ ) comparisons indicate that the proposed method gives the best predictions for the 118 sets of flume and field data (containing 891 points) used in the comparison. By using the Yang equation with  $D_e$  and the newly proposed transport capacity distribution functions of Eqs. (3.9) and (3.24), 78% and 77% of data can be accounted for within the range of discrepancy ratios between 0.25 and 1.75 (up from 36.0-68.5% for other methods), respectively. The discrepancy ratios for the proposed method are normally distributed and concentrated around the perfect agreement.
5. Transport capacity fraction ( $P_{ci}$ ) comparisons using the 118 sets of flume and field data indicate that the proposed method gives the best predictions for the data used in the comparison. 79% of data can be accounted for within the range of discrepancy ratios between 0.25 and 1.75 (compared to 46-70% for other methods) by the use of the proposed transport capacity distribution functions of Eqs. (3.9) and (3.24). A close agreement between computed and measured values is obtained for all ranges of  $P_{cni}$  values using both Eqs. (3.9) and (3.24). Overall, the Einstein method underpredicts the transport rate for finer sizes and overpredicts for the coarser size fractions, while the other methods overestimate the finer fractions and underestimate the coarser fractions. An independent test using another 48 sets of flume and field data (327 data points) shows that the proposed method ranks at the top among all the methods compared.

6. The size composition of sediment in transport is different from the size composition of bed-surface material. The median diameter of sediment in transport is generally finer than the median diameter of bed material, which results from selective transport of grains by flow. The median size of bed-material load,  $D_{50t}$ , can be predicted by Eq. (4.3), which is a function of not only median size of bed material, but also the size gradation of bed material and flow intensity. The relative median size of sediment in transport,  $D_{50t}/D_{50}$ , decreases as size gradation increases, and the relationship between them can be represented by Eq. (4.4).
7. Considering the physical processes governing the transport of sediment mixtures, the geometric standard deviation,  $\sigma_g$ , which represents the range of particle sizes present in the bed material, is found to be a significant additional parameter. For the same flow condition and the same  $D_{50}$ , the sediment size in transport and the transport rate of sediment mixtures are different for different sediment size gradations. For a given flow condition and median bed-material size, as the size gradation increases, the size of sediment in transport decreases, resulting in higher sediment transport rates.
8. The median diameter  $D_{50t}$  (equivalent diameter,  $D_e$ , for the existing bed-material load formulas) is a better indicator for nonuniform bed material. Using  $D_{50t}$  ( $D_e$ ) will produce a more accurate prediction of bed-material discharge for nonuniform sediment mixtures.
9. The effects of bed material size gradations on transport of sediment mixtures cannot be reflected appropriately by a single fixed size, such as  $D_{35}$  or  $D_{50}$ . Bed-material load formulas such as those developed by Engelund and Hansen, Ackers and White, and Yang are based on a single representative size of bed material and may generate

considerable scatter when applied to nonuniform sediment mixtures. By introducing  $D_e$ , which is related to  $D_{50t}$ , into bed-material load computations for the 118 data sets, the improvement in the range of discrepancy ratios between 0.25 and 1.75 was from 73% to 89% for the Engelund and Hansen formula, from 74% to 82% for the Ackers and White formula, and from 83% to 98% for the Yang formula. Independent verification using 54 sets of CSU flume data also shows significant improvement in bed-material load computations by the use of  $D_e$ .

### 7.3 RECOMMENDATIONS

Among the numerous research topics which are recommended for future studies, the author would like to concentrate on the following subjects:

1. Applicability of the transport capacity distribution functions proposed in the present study should be tested for a wider range of flow and sediment conditions, including highly graded bed material and large natural rivers.
2. The conceptual model and formulations proposed in this study should be extended to studies on the fractional transport of gravel bed material, where the sheltering and exposure effect are more pronounced. If necessary, the effective shear stress concept should be included.
3. Partial transport processes should be taken into account in the formulations of transport capacity distribution function in cases where the sediment particles are not fully mobilized.
4. The proposed method for fractional bed-material load computation should be incorporated into numerical models to simulate the change of bed composition and

hydraulic sorting for sand-bed materials.

5. Research for predicting bed-material transport rate for sand-bed materials by the use of predicted median diameters of sediment in transport as representative size should be conducted.

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