REQUIREMENTS FOR PRODUCTION OF A REPLICA SEA IN A MODEL BASIN

by

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ABSTRACT

An investigation is made of the boundary conditions to be imposed in a model basin in order to reproduce, to model scale, a previously observed prototype storm sea. If the wave motion is considered as a solution of Laplace's differential equation, subject to appropriate boundary conditions, it is concluded that it is only necessary to impose, around the whole perimeter of the model basin, wave heights representing, to model scale, those which were observed at corresponding locations in the prototype storm sea.

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Introduction

The following note concerns the possibility of generating, in a model basin, a sea which reproduces, to model scale, a storm sea observed on the open ocean. It is intended that the reproduction should be essentially true in time and space and should persist in the model tank for a sufficient time to permit significant model tests to be made. Such a model sea will be referred to hereafter as a "replica sea". It is possible to produce short crested seas in a model tank but if it is known that these are not identifiable with any actual sea then complications arise relative to the correlation of model test results and prototype behavior. A replica sea would not present these difficulties and if, in addition, it can be made to fill substantially the entire tank area it would provide a design tool of unusual value.

Prototype Data

It would be essential to have prototype data available if a replica sea is to be produced. These data should delineate the surface of a storm sea over a considerable area and for a sufficient period of time. The data should be taken in such a way that a closed boundary could be superimposed on the area mapped and referenced to points fixed with respect to the quiet water below the depths where wave disturbances are present.

Such data might be obtained from a timed sequence of simultaneous stereo-photos taken from two airplanes flying parallel courses upwind over a storm sea and provided with equipment for synchronizing camera shutters and for determining their distances apart as well as their distance above the sea. Reference markers should show on these photographs to indicate position with reference to the quiet water below the waves. The results of such observations could be shown on a series of maps of the area which would describe the sea surface at known instants by means of contours. The wind direction should also be shown on these maps.

Possible Method for Creating The Replica Sea

Suppose we have a model basin, which will be assumed to be rectangular in plan, in which the long dimension represents the downwind direction for the storm sea to be represented and that a model scale ratio has been chosen. It would then be possible to locate within the observed area a boundary representing, to prototype scale, the boundary of the model basin and to plot from the series of contour maps the variation of wave height with time for selected points on the boundary. Suppose now that the model tank is equipped with cam controlled wave generators around all four sides and that the above chosen points represent the centers of the wave generators. It will be assumed that there will be enough of these generators so that the length of each one will be short compared to the distances between crests of the sea to be reproduced. Each of the generators would be controlled by a mechanism[1] to cause it to maintain, at its location, the wave level corresponding to that of the prototype for the corresponding

[1] For one such mechanism see the "Brief Note on Mechanical Means of Generating a Confused Sea" by R. E. Glover Technical Note No. 3 Jan. 1957 for David Taylor Model Basin Under Contract Nonr 1610(02)

point. The wave height-time sequence to be maintained would be cut on the cam controlling each generator. There would be no two of these alike.

Type of Sea Generated

If the wave generators are put into operation as described, a disturbance will appear in the model basin. The question which it is important to answer is whether, after a sufficient time has elapsed for a wave to traverse the long dimension of the model tank, a replica sea will appear on the surface of the water in the tank.

Proposed Method of Investigation

In order to investigate this question it will be assumed that the disturbance which appears on the surface of the tank represents a solution of the appropriate hydrodynamical differential equation subject to the conditions imposed at the boundaries. If the wave generators impose, to scale, the conditions of the prototype over these boundaries and these are sufficient to render the solution unique then it may be expected that the replica sea will be realised in the model basin. There is an extensive literature on the mathematical aspects of this problem. This subject is known as Cauchy's problem, after one of the first mathematicians to investigate the combination of conditions required to obtain a unique solution of a differential equation.

In our case, if the water is considered to be both incompressible and the flow irrotational, the applicable differential equation is the Laplace equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \qquad (1)$$

Here x, y and z represent space coordinates and ϕ represents the velocity potential. In a physical sense this equation expresses the "condition of continuity". This is to say that for every element of volume dxdydz the amount of fluid which flows into it in any interval of time is just equal to the amount which flows out. This relationship must be satisfied whether the flow is steady or is changing with time. It is therefore always appropriate.

Boundary Conditions

The boundaries present in this case are represented by the bottom of the wave basin, the free surface at the air-water interface, and the walls of the tank. If the depth of water in the model basin is at least one half the distance between crests of the sea to be generated then the wave motion produced will be a true representation of deep water waves (3). In this case the distance between crests is to be measured in the down-wind direction. With this depth of water the appropriate boundary condition at the bottom of the tank will be met. At the surface of the water the requirement to be met is that the pressure should be atmospheric. It is obvious that we cannot impose any further conditions on this boundary, since any attempt to alter it will impose pressures which will violate the condition which we know must prevail there. This leaves the walls of the tank as the only boundary where control may be exercised. It is proposed to operate the wave generators at the walls to produce undulations which reproduce to model scale those found at the corresponding point in the storm sea. The question which now must be answered is; does this lead to a unique solution?

The Cauchy Problem

In the development of mathematics the question arose as to what requirements are necessary to render the solution of a differential equation unique. That is, what is necessary to be specified so that we get one, and only one, solution. The differential equation itself generally has many solutions. The additional conditions must therefore come from the initial and boundary conditions. If there are too few of these more than one solution can be obtained. If there are too many the problem is over determined and there are no solutions. Only with the proper number is a unique solution obtained.

Early investigators of these questions were Cauchy, Sophie, Kowalewsky and Darboux (1). The time was about 1840. Their work involved the restriction that the solution was to be analytic in form. They concluded that both a value of the dependent variable and a gradient had to be imposed at the boundary. In our case this would imply that we must specify both a wave height and a velocity normal to the wall of the tank, along the lines of wave generators, or some equivalent.

These questions have continued to interest mathematicians and an account of some of the developments since the time of these early investigators is contained in Hadamard's Lectures (1). Concerning the early results he has this to say:

> "The reasonings of Cauchy, Kowalewsky, and Darboux, the equivalent of which has been given above, are perfectly rigorous; nevertheless, their conclusion must not be considered as an entirely general one. The reason for this lies in the hypothesis, made above, that Cauchy's data, as well as the coefficients of the equations, are expressed by analytic functions; and the theorem is very often likely to be false when this hypothesis is not satisfied."

In a subsequent paragraph he continues:

> "If, in the first place, we take such a Cauchy Problem as was spoken of in paragraph 4 (Cauchy's problem with respect to $t = o$, for equations $(e_1)(e_2)$ and $(e_3)^1$), our above conclusions are valid, as we shall see as these lectures proceed, without the need of the hypothesis of analyticity.
>
> But the conclusions will be altogether different if, for instance, we deal with Laplace's classic equation of potentials
>
> $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$
>
> This will be immediately realised by comparison with another classic boundary problem; I mean Dirichet's problem. This consists as we know, in determining a solution of Laplace's equation within a given volume V, the value of u being given at every point of the boundary surface S of that volume. It is a known fact that this problem is correctly set: i.e. it has one (and only one) solution.
>
> This fact immediately appears as contradictory to Cauchy-Kowalewsky's theorem; for, if the numerical values of u at the points S (together with the partial differential equation) is by itself sufficient to determine the unknown function within V, we evidently have no right to impose upon u any additional condition and we cannot therefore, besides values of u, choose arbitarily those of $\frac{du}{dn}$.[2]
>
> Indeed there is, between these two sets of values, an infinity of relations which must be satisfied in order that a corresponding harmonic function should exist."

Since Laplace's equation applies to our case, it is inferred that when we have imposed the appropriate value of ϕ only around the walls of the tank, we have met the essential requirements. This consideration applies also to the boundary marked out on the prototype as well as at the wave basin walls.

[1] Differential equations for 1, 2, and 3 dimensional wave propagation in a compressible fluid medium.

[2] The derivative with respect to distance along the normal to the boundary.

Because of the boundary condition at the free surface, the wave heights and the potential ϕ are related. In each elementary component which contributes to the storm sea undulations, there is a relationship of the form. (Ref. 2, page 75)

$$\zeta = a \sin m(x - ct) \tag{2}$$

$$\phi = \frac{ga}{mc} \frac{\cosh m(z+h)}{\cosh mh} \cos m(x-ct) \tag{3}$$

Where: ζ represents the wave height and ϕ velocity potential

a represents a constant,

c the wave propagation velocity defined by,

$$c^2 = \frac{g\lambda}{2\pi} \tanh\left(\frac{2\pi h}{\lambda}\right) \tag{4}$$

g the acceleration of gravity,

h the depth of the water,

$m = \frac{2\pi}{\lambda}$, $\pi = 3.14159+$

λ the wave length

t time, and x, y, and z are space coordinates,

The x-y plane is the plane of the undisturbed water surface. The coordinate z is measured positive upward.

To establish ζ for this component it is necessary to fix a. When a is fixed so is ϕ. In view of the investigations described above, this should render the problem unique.

To propagate this wave, along, across the tank the wave generators would have to be set to maintain the appropriate values of ζ as a function of time along the walls. For a sum of such elements the wave generators would, in the same manner, be arranged to maintain the combined wave height at the walls. A replica sea would represent such a sum.

Since the differential equations designated by Hadamard as (e_1), (e_2), and (e_3),[3] differ from Laplace's equation by the inclusion of a term accounting for a compressibility it may be further inferred that the difference in the boundary requirements for a unique solution reflects the difference in the physical nature of the medium and that incompressibility restricts the freedoms which are present in a compressible medium.

[3] His equation (e_3) is of the form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{\omega^2}\frac{\partial^2 u}{\partial t^2} = 0.$$

Pertinent abstracts from J. Hadamard's "Lecons sur la Propagation Des Ondes." (8)

The following abstracts are of interest for our purposes. From his paragraph 23*

"We have, up to now, occupied ourselves with the problem in which the normal derivative was given on a limited surface. It should not be necessary to believe that this problem and that of Dirichlet, in which the values of the potential function itself are given on the entire surface, are the only ones which we will be able to solve. In the theory of heat one encounters an analogous problem in which enter the values of $\frac{dv}{dn} + hv$ (h being a negative number). But, even in hydrodynamics, one is in general led, not to the problem of Dirichlet or that which we have just treated, but a mixed problem in which the values of the harmonic function for which we are looking are given on a part of the surface and that of its normal derivative on the remaining part of the surface. As in the preceding case, this problem, if there is a solution, has only one."

From his paragraph 137*

Let us suppose now that the liquid has a free surface. In this case one no longer knows dp/dn.** But let us suppose that one is given at each point the value of p. We are thus led, this time, to the mixed problem presented in paragraphs 23 to 41. That is, p is

* Translation from the original French.

** The symbol p represents the pressure; the symbol n, distance along a normal. This is Hadamard's notation.

given on the free surface, dp/dn on the walls. It is then certain that the solution of the problem is unique . . . . "

These comments have two features of interest for our case because they show, first, that the case of the free surface has been given due consideration and, secondly, that as in the other cases, only one numerical datum is to be imposed on the boundaries. It is clear also that the conditions at a free surface are adequately accounted for if a pressure alone is prescribed there.

An Alternate Approach

What we need to prove is that if wave heights are specified, as a function of time, around the perimeter of a wave basin, the wave motion which appears in the basin will be the same as that within a similar perimeter marked out on an indefinitely extended water surface if the undulations which are imposed at the wave basin perimeter are the same as those which appear at corresponding points of the extended surface. For the sake of brevity we will refer to the extended surface as the prototype.

Suppose the wave basin perimeter is provided with a large number of wave generators, arranged with cam control, which will cause each of them to maintain a wave surface elevation at its location according to a prearranged program. It is further assumed that these prearranged levels will be maintained even though waves may arrive at its location from other points.

Consider a selected one of these generators and the corresponding point of the extended surface of the prototype. At the corresponding point we impose a disturbance in the form of a local displacement varying in a prescribed manner with time.

The wave motion produced by such a disturbance may be computed from the expressions

$$\phi = \zeta_0 \int_0^\infty \frac{g \sin \sqrt{\frac{g t^2 u}{a_1}}}{\sqrt{\frac{g u}{a_1}}}\, e^{\frac{uz}{a_1}}\, J_1(u)\, J_0\!\left(\frac{r}{a_1} u\right) du. \quad \ldots(1)$$

$$\zeta = \zeta_0 \int_0^\infty J_1(u)\, J_0\!\left(\frac{r}{a_1} u\right) \cos \sqrt{\frac{g t^2 u}{a_1}}\; du. \quad \ldots(2)$$

where:

a_1 represents a specified radius

g the acceleration of gravity

J_n a Bessel function of order n

r a radius

t time

u a variable of integration

z distance measured positive upward from the undisturbed surface

ζ a wave height

ζ_0 a specified initial elevation

ϕ the velocity potential

$e = 2.71828 +$ The base of the natural system of logarithms

The solution (1) represents the effect of releasing a right cylinder of water of radius a and height ζ_0 at the origin at time zero. The wave motion which ensues is given by (2) in terms of the wave height at the radius r at the time t.

This solution meets the boundary conditions

$$\zeta = \frac{1}{g}\frac{\partial \phi}{\partial t} \quad \text{at} \quad z = 0.$$

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial \phi}{\partial z} \quad \text{at} \quad z = 0. \tag{3}$$

$$\zeta = \zeta_0 \quad \text{for} \quad 0 < r < a_1 \quad \text{when} \quad t = 0.$$

$$\zeta = 0. \quad \text{for} \quad r > a_1 \quad \text{when} \quad t = 0.$$

The potential ϕ satisfies Laplace's equation:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0. \tag{4}$$

A plot of a wave profile computed from these relations is shown in figure 1. The volume of water $\pi a_1^2 \zeta_0$ can be considered as an elementary displacement produced by a wave generator. We may use these formulas to compute the wave motion at any point of the prototype water surface and, in particular, we may compute the wave motion at each of the points corresponding to the location of a wave generator on the perimeter of the wave basin. Suppose we do this and program each of the generators to maintain the computed levels at its location and at the same time we impose the arbitrary prescribed disturbance on the selected wave generator previously described. Under these conditions we may expect that the wave motion which appears in the wave basin will be exactly the same as that which appeared within the corresponding area of the prototype because the wave generators of the wave basin maintain the conditions which prevail at the corresponding points of the prototype and there is therefore nothing at the perimeter of the wave basin which departs from the conditions of the prototype.

If we select two generators and impose prescribed motions at the corresponding points of the prototype surface, we may compute the wave heights produced by each separately and superimpose them to find the disturbance produced when they act simultaneously. In this manner we may compute the wave heights at each of the generator locations as before and if we program each of them to maintain the computed wave heights at its location, we will again observe that the wave motion in the wave basin duplicates the prototype wave motion in the corresponding area. In this case, however, the motion imposed at the selected wave generators must be the sum of the prescribed wave motion and the undulations arriving at its location from the other selected generator. Then the wave heights at the boundary of the wave-basin again duplicate those at the corresponding prototype points and there is nowhere any departure from prototype conditions.

In this same manner we may impose on the prototype an arbitrary disturbance at each of the points corresponding to the position of a wave generator and compute the wave heights at any one generator as a sum of the wave heights produced by each separately. The computed wave heights will in each case include those produced by the arbitrary displacements imposed at its location. This is to say that the computed wave height at any location includes the effects of all of the generators. If we now compare the prototype and model basin wave motions, we find them again identical because the prototype conditions are maintained around the perimeter of the wave basin and there is nowhere any departure from prototype conditions.

In this case we have seen how a solution over an area may be constructed by imposing wave heights over the boundary of a closed perimeter and we note that the wave heights alone are sufficient for this purpose. Any attempt to add a flow across the boundary would have altered the boundary wave heights and, through them, the wave motion on the enclosed area. If the wave generators are set to reproduce the wave height at the boundaries of the wave basin which would be produced by a simple or complex wave train traveling over the prototype surface, we should expect to find a wave motion in the wave basin which would duplicate that of the original wave train of the prototype. This would be the case because the boundary wave heights alone are sufficient to determine the wave motion over the enclosed area.

For the sake of simplicity, these arguments have been presented as though the wave basin and the prototype had the same scale. If the appropriate similarity conditions are met, these arguments would apply where the model basin and prototype have different scales.

To complete this approach it may be worthwhile to consider whether the solution constructed in the manner described is unique. We may do this by supposing that there might be another solution. If this is possible it could be obtained by adding a solution of Laplace's equation to the one we now possess. The solution added would represent the difference between the solution obtained and the supposed alternate solution. The solution added must, of course, conform to the pressure conditions at the free surface. Because the proper boundary conditions were imposed in obtaining the first solution, it will not be permissible to add anything to them. This means that a condition of zero wave height would have to be imposed on the second solution around the walls of the model basin. This last restriction insures that the second solution must be zero everywhere because the boundary condition at the free surface and at the bottom of the tanks are inherently appropriate. If no wave motion can be generated around the walls of the tank, then no wave motion can be created anywhere. The second solution must therefore be nil. The solution originally obtained is unique because no alternate solution can be obtained which differs from it. It goes without saying, of course, that no pre-existing wave motion in the basin would be permissible. Any attempt to produce a specific type of wave motion in the presence of such interference would be futile.

Model Prototype Relationships

If the storm sea can be considered to be composed of elementary waves which are distributed in a random manner with respect to amplitude, length, phase and orientation, then each elementary deep water wave will be of the type:[2]

$$\zeta_p = a_p \sin \frac{2\pi}{\lambda_p}(x_p - c_p t_p). \qquad (5)'$$

$$\phi_p = \frac{a_p g \lambda_p}{2\pi c_p} e^{\frac{2\pi}{\lambda_p} z_p} \cos \frac{2\pi}{\lambda_p}(x_p - c_p t_p). \qquad (6)'$$

where the subscript p represents a prototype quantity. For model representation a scale ratio n, less than unity, is chosen so that the model wave amplitudes and wave lengths are n times those of the prototype. Then if the subscript m indicates a model quantity substitution of the relations:

$$n = \frac{\lambda_m}{\lambda_p} \qquad a_m = \frac{\lambda_m}{\lambda_p} a_p \qquad c_m = \sqrt{\frac{g\lambda_m}{2\pi}} \qquad c_p = \sqrt{\frac{g\lambda_p}{2\pi}}$$

$$t_m = \sqrt{\frac{\lambda_m}{\lambda_p}}\, t_p \qquad \zeta_m = \frac{\lambda_m}{\lambda_p} \zeta_p \qquad x_m = \frac{\lambda_m}{\lambda_p} x_p \qquad z_m = \frac{\lambda_m}{\lambda_p} z_p$$

$$\phi_m = \left(\frac{\lambda_m}{\lambda_p}\right)^{\frac{3}{2}} \phi_p . \qquad \frac{c_m t_m}{\lambda_m} = \frac{c_p t_p}{\lambda_p} \qquad (7)'$$

into the above equations will yield an identical set of equations with m subscripts. Note that these relationships apply to all of the components regardless of amplitude or wave length.

In both cases the Laplace differential equation applies. Then if we impose in the wave basin boundary conditions which correspond with those of the prototype and these boundary conditions render the solution unique, the wave motion in the wave basin must be a replica of the prototype sea.

Example

It is of interest to see how this would work out in a simple case. Suppose we wish to propagate a wave of the type of equation (2) down the length of the wave basin. This we know we can do because it has been done many times.

The horizontal velocity is:

$$u = \frac{\partial \phi}{\partial x} = \frac{g a_t}{c} \frac{\cosh m(z+h)}{\cosh mh} \sin m(x-ct). \tag{8}$$

If b represents the width of the wave basin the volume displacement D at any cross section is obtained by integration of the velocities in the manner:

$$D = b \int_{-h}^{0} u \, dz \tag{9}$$

If this integration is performed we obtain,

$$D = \frac{g a_t b}{mc} \frac{\sinh mh}{\cosh mh} \sin m(x-ct). \tag{10}$$

In this case we can generate the wave by generating the velocities u of Eq 8 or by creating the displacement D of Eq 10. These are equivalent because of the incompressibility factor. These are both equivalent to a generation of the wave by imposing a gradient $\frac{\partial \phi}{\partial x}$.

But this gradient, the potential ϕ and the wave height are connected through the factor a_t. They are not independent. We may not, therefore, impose both ϕ and $\frac{\partial \phi}{\partial n}$ at the boundary separately. Note that in this case a unique result is obtained by imposing a wave height alone at the boundary. This is in accord with the mathematical work which indicates that one boundary condition only can be imposed when Laplace's differential equation applies.

It is also of interest to consider the behavior of the wave generators in this case. The generators along the walls would not move because the water surface elevations called for by their cams would be satisfied. The generators at the downstream end of the tank would absorb the wave motion. This is because while the wave motion would correspond to the cam requirements at the time of arrival, the reflection of the wave at the end of the tank would tend to increase the water level above that called for. The wave generator would then act to prevent this. The type of wave generator described in reference (4) is assumed to be in operation here with the sleeve of Fig. 3, controlled by a float riding on the water on the tank side of the wave generator.

Conclusions

The evidence available leads to the conclusion that the fixing of wave heights completely around the boundary of an area will uniquely determine the wave motion over the enclosed area. This statement applies to model and prototype alike. If the wave motion around the walls of a model basin is made to be a replica of that observed at corresponding points of a storm sea the wave motion in the area within the wave basin boundaries also will reproduce the storm sea after a sufficient time has elapsed, after starting the wave generators, for a wave to travel the long dimension of the model basin. The replica sea so produced will represent the storm sea in both space and time if the time scale in the model is made appropriate for the scale factor chosen. The uniqueness considerations apply with equal force to the model and prototype conditions. If the proper similitude relationships are maintained then the mathematical studies would indicate that the replica sea must appear on the wave basin,

Comments

A person who saw the original storm sea and the replica sea produced in the way described might find the model sea somewhat unconvincing. This is because the driving effect of the wind would be absent. This factor produces many of the most impressive phenomena in an actual storm. The lack of the wind driving force would, in fact, produce an error in the replica sea of some magnitude. It can only be assumed that the storm sea observed had developed over a reach which was long compared to the prototype area equivalent to the model basin dimensions and that therefore the absence of a wind drive will not distort the waves seriously during the travel across the model basin. The wind drive could, of course, be arranged but it would involve expense and testing difficulties.

The accuracy of the replica sea could be checked by taking stereophotos at specific times and comparing them with the original stereos from which the storm sea data were derived.

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REFERENCES

1. Lectures on Cauchy's Problem in Linear Partial Differential Equations, by Jacques Hadamard, Dover.

2. Waves, by C. A. Coulson. Seventh Edition. Oliver and Boyd Lt'd.

3. Hydrodynamics, by Lamb Dover.

4. Brief note on Mechanical Means of Generating a Confused Sea, by R. E. Glover. Technical Report No. 3 prepared for David Taylor Model Basin under contract Nonr 1610(02). Jan. 1957.

5. Lecons sur la propagation des Ondes et les equations de l'Hydrodynamique by Jaques Hadamard, Librairie Scientifique A. Hermann; Paris 1903.

6. "Characteristics of Waves Generated by a Local Surface Disturbance" by J. E. Prins, University of California, Wave Research Laboratory, Series 99, Issue 1. August 19[illegible].

7. "Wave Machines" by Francis Biesel, Chapter 21 of "Ships and Waves" Published by Council on Wave Research and Society of Naval Architects and Marine Engineers, 1955.

8. "New Facilities of the Netherlands Ship Model Basin at Wageningen by Prof. Dr. W.P.A. Vanlammeren and Ir. G. Vossers, Publication No. 161[illegible] of the N.S.M.B., Reprinted from International Shipbuilding Progress Rotterdam. Vol. 4, No. 29, 1957.