DEVELOPMENT OF A STREAM-AQUIFER MODEL SUITED FOR MANAGEMENT

by

H.J. Morel-Seytoux

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by

Dr. H. J. Morel-Seytoux
Department of Civil Engineering
Colorado State University

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COLORADO WATER RESOURCES RESEARCH INSTITUTE

Colorado State University
Fort Collins, Colorado

Norman A. Evans, Director

ABSTRACT

In most available computer models of stream-aquifer interaction, the river stages are treated as boundary conditions. However in reality for low flow conditions, the actual problem is the prediction of the river stages. In addition for management purposes it is essential to have *explicit* relations between the *decision* variables such as pumping rates or upstream flow releases and the resulting *states* of the system such as downstream river flows and aquifer drawdowns. By combining the *discrete kernel* approach for an isolated aquifer with the discrete kernel approach for an isolated river (treated as a cascade of Muskingum reaches) it was possible to derive the discrete kernels (*influence* coefficients) for all the states of interest as responses to all the relevant decision variables. The developed tool is well suited for management due to the explicit nature of the relations which makes it possible to formulate the management (optimization) problem as a Mathematical Programming problem for which efficient solution algorithms exist.

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RESEARCH OBJECTIVES

The first objective of this project was the development of a stream-aquifer model that portrays properly the *dynamics* of the interaction between a stream (or a canal) and the underlying water-table aquifer. The second objective was to design the model so that it can be used on a practical (i.e., inexpensive) basis for integrated management of a surface-groundwater system.

ACHIEVEMENTS OF CONTRACT

It is not desirable to repeat in this completion report all the results obtained over the past three years and the detailed procedures by which they were obtained. These results and procedures can (or will) be found in one dissertation (Illangasekare, 1978), one report (Illangaskare and Morel-Seytoux, 1978), one published paper (Illangasekare and Morel-Seytoux, 1978) and several papers in preparation.

Rather a brief review of the methods of attacks and a sample of results will be given. The emphasis is on the latest developments which have not been reported elsewhere and which hold promise for the future. Generally speaking the thrust of the research has been in the direction of development of new and imaginative methods that will reduce the cost of management studies of conjunctive use of surface and ground waters without significant reduction in accuracy. In this regard the project was fairly successful.

A. INTRODUCTION

In a system which includes *natural* components such as an aquifer and a stream in hydraulic connection with it and is subjected to man-made interferences such as stream diversions, pumping wells, field irrigation, etc., the future state of the system will depend on a number of *management* decisions. The state of the system can be described by water table drawdowns, aquifer return flows to the stream and river stage drawdowns. The evolution of these *states* depends upon the *decisions:* pumping rates from the wells, upstream inflows into the river, all of which can be controlled. The evolution of these states depends also on the initial conditions of the system such as initial water table elevations and initial river stages, which cannot be controlled.

The problem at hand is the development of a method of quantitative description of the states in terms of the decision and initial variables which is accurate, cost effective and explicit, so that the effect of various management strategies can be predicted properly, easily and inexpensively. The approach taken to develop these explicit relations was described in earlier studies (Morel-Seytoux et al, 1973, 1975; Morel-Seytoux, 1975, 1977; Morel-Seytoux and Daly, 1975; Rodriguez, 1976; Peters, 1977). It consists of describing separately the behaviour of the aquifer and of the stream (as if the other component did not exist) by a suitable linear model and then combining the two components in order to describe their actual dynamic interaction.

B. ISOLATED AQUIFER COMPONENT

It has been shown previously (e.g., Morel-Seytoux and Daly, 1975) that drawdown in an aquifer can be expressed in terms of the pumping rates by the relation:

$$s_w(n) = \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{wp}(n-\nu+1)Q_p(\nu) \tag{1}$$

where $s_w(n)$ is the drawdown at any point w at the end of the n^{th} time period, the $\delta_{wp}(\ )$ are the so-called *discrete pumping kernels* which are obtained from a finite-difference (Morel-Seytoux and Daly, 1975; Rodriguez, 1976) or finite element (Illangasekare and Morel-Seytoux, 1978; also Appendix 1 of this report) model, P is the total number of wells, and $Q_p(\nu)$ is the mean pumping rate from well p during the ν^{th} period (or pumped volume for the period).

If one does not treat the presence of the stream in an interactive way but as an imposed boundary condition, then in the presence of a stream Eq. (1) can be generalized to the form:

$$s_w(n) = \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{wp}(n-\nu+1)\ Q_p(\nu) + \sum_{r=1}^{R} \sum_{\nu=1}^{n} \delta_{wr}(n-\nu+1)\ Q_r(\nu) \tag{2}$$

where $Q_r(\nu)$ is the pumping rate from the r^{th} reach of the river and R is the number of reaches. The aquifer return flow is formally treated as a pumping rate from the river. However whereas pumping from a well can be effectively controlled the aquifer return flow cannot be because it depends itself upon the state of the aquifer. The solution provided by Eq. (2) is mathematically correct but only formal because $Q_r(\nu)$ though shown on the right-hand side is not a decision variable but is itself a state (dependent) variable.

Also Eqs. (1) and (2) are only valid for a system initially at rest. If initially the system is not at rest, even without pumping from wells or from the river the drawdowns will change in time, water moving from the high levels to the low ones. The *natural redistribution* of drawdowns naturally depends upon the initial conditions. A method of prediction was developed in earlier studies (Morel-Seytoux, 1975, 1977; Peters, 1977) consisting of recreating the initial conditions by artificially pumping from the system one period prior to the inital time. With this procedure the general behaviour of drawdowns is characterized by the expression:

$$s_w(n)-\bar{s} = \sum_{e=1}^{E} \sum_{\nu=1}^{n} \delta_{we}(n-\nu+1)Q_e(\nu) + \sum_{\pi=1}^{\Pi} \theta_{w\pi}(n) \ (s_\pi^\circ-\bar{s}) \tag{3}$$

where E is the total number of *excitation* points (wells and reaches), s_π° is the initial drawdown at point π, $\bar{s}$ is the initial average drawdown in the aquifer and the $\theta_{w\pi}(\ )$ are (*influence*) coefficients deduced from the $\delta_{we}(\ )$ coefficients by linear algebraic manipulations (Morel-Seytoux, 1975, 1977; Peters, 1977; Illangasekare, 1978).

C. ISOLATED RIVER COMPONENT

For modeling purposes the stream is divided into a set of R reaches (Fig. 1). The isolated stream is subjected to two types of excitations: (1) upstream inflows and (2) diversions. If S_r designates storage in reach r, R_r the inflow into reach r, R_{r+1} the outflow from the reach and W_r the net withdrawal (e.g., diversion-tributary inflow) from the reach, then the continuity equation can be written in the form:

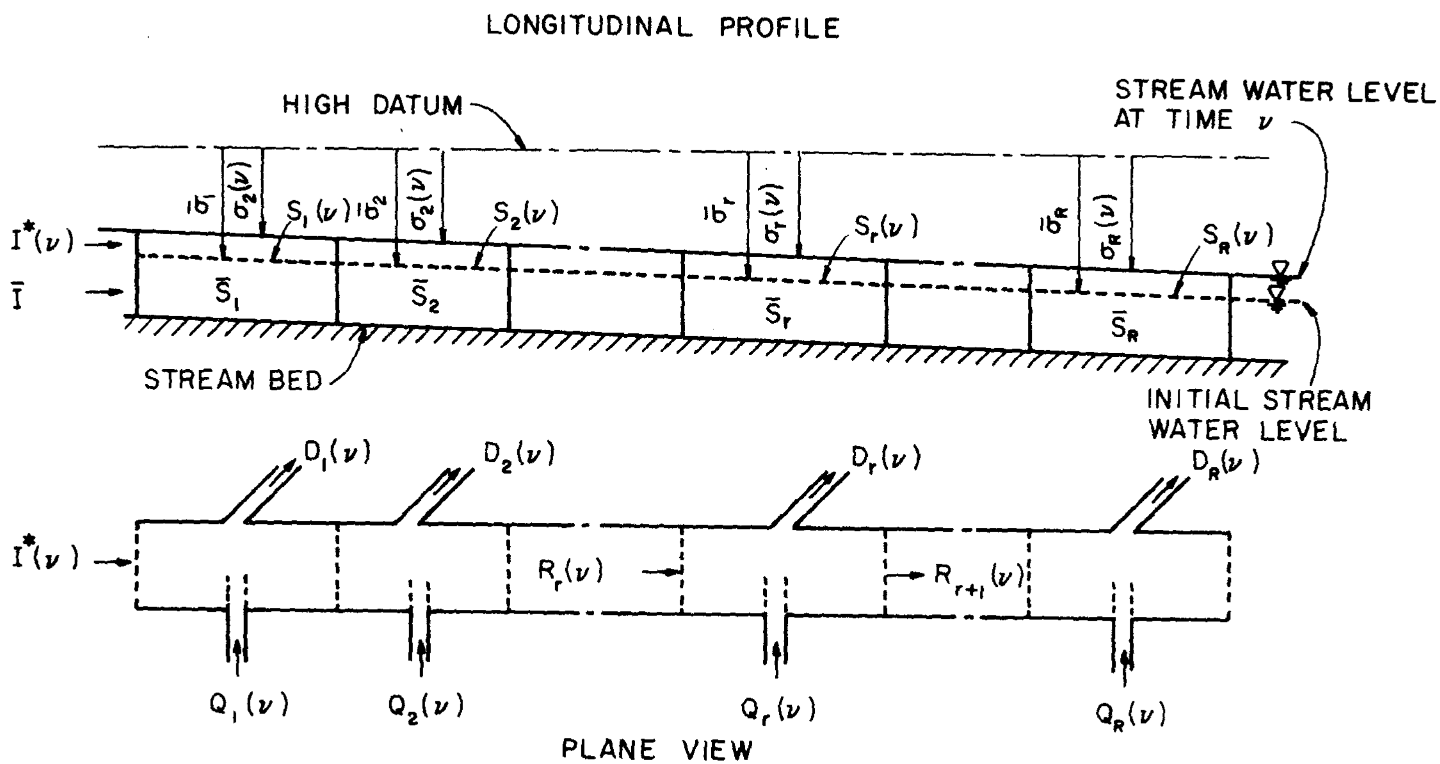

Fig. 1. Schematic stream routing model and definition of variables

$$\frac{dS_r}{dt} = R_r - W_r - R_{r+1} \tag{4}$$

The selected *pseudo*-momentum equation used to describe the stream evolution is the Muskingum equation:

$$S_r = K_r \left[\xi_r R_r + (1-\xi_r) R_{r+1}\right] \tag{5}$$

To maintain the convenience of dealing with linear systems it is assumed that for the expected range of fluctuations in river stage there exists a proportionality between storage and river stage, namely:

$$S_r = A_r(y_r - \bar{y}_r) \tag{6}$$

where S_r is *deviation* of storage in reach r from a *steady-state reference* level, A_r is the horizontal water surface area of the reach, y_r is river stage in the reach measured from river bed and $\bar{y}$ is the steady-state reference stage. Equivalently Eq. (6) can be written in terms of *river drawdown* rather than stage in the form

$$S_r = - A_r(\sigma_r - \bar{\sigma}_r) \tag{7}$$

where σ_r is river drawdown measured from a high datum and $\bar{\sigma}_r$ is the steady-state reference river drawdown.

It has been shown previously (Morel-Seytoux, 1975) using the system of Eqs. (4), (5) and (6) that the reach drawdown $\sigma_r(t)$ can be expressed as a function of the deviation of inflow from the steady-state reference level into the first reach denoted by $I^*(t)$ and stream withdrawals from the r^{th} reach and all the reaches upstream of it. The general solution is of the form:

$$\sigma_r(t) = \bar{\sigma}_r - \frac{1}{A_r} \sum_{\rho=1}^{r} \int_{o}^{t} \kappa_{r\rho}(t-\tau) \left\{- \frac{W_\rho(\tau)}{\eta_\rho} + b_\rho \; I^*(\tau)\right\} d\tau] \tag{8}$$

where b_ρ and η_ρ are functions of the Muskingum parameters K and ξ and $\kappa_{r\rho}(\ )$ is the routing kernel. The derivation of the expressions for the routing kernels has been given in earlier studies (Morel-Seytoux, 1975). In discrete form Eq. (8) becomes:

$$\sigma_r(n) = \bar{\sigma}_r - \frac{1}{A_r} \sum_{\rho=1}^{r} \sum_{\nu=1}^{n} \chi_{r\rho}(n-\nu+1) \left\{- \frac{W_\rho(\nu)}{\eta_\rho} + b_\rho \ I^*(\nu)\right\} \tag{9}$$

where $\sigma_r(n)$ is the river drawdown at the end of the n^{th} period, $W_\rho(\nu)$ is the volume of water withdrawn from the ρ^{th} sub-reach during the ν^{th} period and $I^*(\nu)$ is the deviation of upstream inflow from the steady state reference inflow during the ν^{th} period. The discrete routing kernel coefficients $\chi_{r\rho}(\nu)$ are defined as:

$$\chi_{r\rho}(\nu) = \int_0^1 \kappa_{r\rho}(\nu-\tau)d\tau \tag{10}$$

It has been shown (Morel-Seytoux, 1975) that the routing kernels are functions of the Muskingum parameters K and ξ of all the reaches. Hence, if the Muskingum parameters of the reaches are known the discrete routing kernels can be generated and saved. The coefficients in combination with the other fixed parameters of the routing model (A_r, η_r and b_r) can be used in Eq. (9) to derive the response $\sigma_r(n)$ due to upstream and withdrawal excitations $I(\nu)$ and $W(\nu)$.

In all the previous discussion it was tacitly assumed that there was no return flow or seepage taking place. However, Eq. (9) is still mathematically correct by interpreting the net withdrawal W_r as the difference between diversions D_r and return flows Q_r. Then Eq. (9) takes the more explicit form:

$$\sigma_r(n) = \bar{\sigma}_r - \frac{1}{A_r} \sum_{\rho=1}^{r} \sum_{\nu=1}^{n} \chi_{r\rho}(n-\nu+1) \left\{ \frac{Q_\rho(\nu) - D_\rho(\nu)}{\eta_\rho} + b_\rho I^*(\nu) \right\} \quad (10)$$

The upstream inflows (I^*) and the diversions D_ρ are controllable true decision variables whereas the aquifer return flows Q_ρ even though they appear on the right-hand side are not decision variables. Roughly speaking the system of Eqs. (2) and (10) provide two sets of equations for three sets of state variables, s_w, σ_r and Q_ρ. Another set of equations is needed to describe the stream-aquifer interaction.

Eq. (10) applies for a stream initially in a steady state at the reference level. If initially the system was not in steady state a *relaxation* term must be added on the right-hand side of Eq. (10). Let σ_r^o denote the initial river drawdowns. To represent the effect of these *initial condition* excitations one can assume that the system was in a steady state one period prior to initial time. One then chooses artificial diversions for the zeroth period in such a way that the prediction of drawdown by Eq. (9) with a summation index for time starting at the value 0 rather than 1 for $n=0$ be precisely the σ_r^o, namely:

$$\sigma_r^o = \bar{\sigma}_r + \frac{1}{A_r} \sum_{\rho=1}^{r} \chi_{r\rho}(1) \frac{D_\rho(o)}{\eta_\rho} \quad (11)$$

The system of Eqs. (11) provide R equations for the R unknowns $D_\rho(o)$ given the initial river drawdowns σ_r^o. Since the system of Eqs. (11) is linear the $D_\rho(o)$ can be expressed as linear combinations of the $(\sigma_r^o - \bar{\sigma}_r)$. Then substitution of these expressions for the $D_\rho(o)$ in Eq. (9) will yield the final expression for the river drawdown as:

$$\sigma_r(n) = \bar{\sigma}_r - \frac{1}{A_r} \sum_{\rho=1}^{r} \sum_{\nu=1}^{n} \chi_{r\rho}(n-\nu+1) \left\{ \frac{Q_\rho(\nu) - D_\rho(\nu)}{\eta_\rho} + b_\rho I^*(\nu) \right\}$$

$$- \frac{1}{A_r} \sum_{\rho=1}^{r} \sum_{i=1}^{\rho} \frac{\chi_{r\rho}(n+1)}{\eta_\rho} a^*_{\rho i} (\sigma^\circ_i - \bar{\sigma}_i) \qquad (12)$$

where the $a^*_{\rho i}$ are initial river drawdown coefficients which are functions of the routing discrete kernels and stream parameters.

The second term on the right-hand side represents the effect of diversions and aquifer return flows (*external* excitations) on the river drawdowns whereas the last term (the relaxation term) represents the effect of the initial stream conditions (*internal* excitations).

D. STREAM-AQUIFER INTERACTION

As discussed previously, Eqs. (2) and (10) apply respectively to the isolated aquifer and the isolated stream. In each case the aquifer return flow is treated as an imposed boundary condition. In reality it depends upon the state of the system. It has been shown that the aquifer return flow, Q_r, could be expressed in terms of reach drawdown and aquifer drawdown in the form:

$$Q_r = \Gamma_r (\sigma_r - s_r) \qquad (13)$$

where Γ_r is the reach transmissivity which can be expressed as a function of aquifer and river parameters as:

$$\Gamma = \frac{T}{e} L \left(\frac{\frac{W_p}{2} + e}{L + \frac{e}{2}} \right) \qquad (14)$$

where T is the aquifer conductivity, e the saturated thickness, L is the length of the reach, and W_p is the wetted perimeter. The validity of the derived Eq. (14) (Illangasekare, 1978) has been verified on a reach of the South Platte River (Morel-Seytoux, 1977; Peters, 1977). An alternate form of Eq. (13) is:

$$Q_r = \Gamma_r \left[\sigma_{br} + y_r - s_r\right] \tag{15}$$

where σ_{br} is the drawdown to river bed and y_r is river depth (or stage). With the system of Eqs. (13) the set of Eqs. (2) and (10) is now complete and it is possible to express the state variables of aquifer drawdown, river drawdown, reach storage, reach outflow and aquifer return flow in terms of the external excitations (pumping rates, diversions, upstream inflows) and of the internal excitations (initial aquifer and reach drawdowns). The coefficients in these linear relations are the *influence* coefficients. These relations can be written in the general format:

$$\begin{aligned} Y_{\cdot}(n) = {} & \sum_{\pi=1}^{\Pi} X^{(1)}_{\cdot\pi}(n)\, s^{\circ}_{\pi} + \sum_{\rho=1}^{R} X^{(2)}_{\cdot\rho}(n)\, \sigma^{\circ}_{\rho} \\ & + \sum_{\nu=1}^{n} X^{(3)}_{\cdot}(n-\nu+1)\, I(\nu) \\ & + \sum_{p=1}^{P} \sum_{\nu=1}^{n} X^{(4)}_{\cdot p}(n-\nu+1)\, Q_p(\nu) + \sum_{\rho=1}^{R} \sum_{\nu=1}^{n} X^{(5)}_{\cdot}(n-\nu+1) D_{\rho}(\nu) \\ & + X^{(6)}_{\cdot}(n)\, I^{\circ} + X^{(7)}_{\cdot}(n) \end{aligned} \tag{16}$$

where $Y_{\cdot}(n)$ is the state variable of interest and the $X^{(1)}(\ )$, $X^{(2)}(\ )$, $X^{(3)}(\ )$, $X^{(4)}(\ )$, $X^{(5)}(\ )$ and $X^{(6)}(\ )$ are the matrices or vectors of influence coefficients relating the state variable to the decision variables. The coefficient $X^{(7)}$ does not depend upon the excitations and may be called a constant in that sense though it varies with time. Table 1 provides the nomenclature used for all the particular

Table 1. Symbols Used for the Various Influence Coefficients

State Variable	Decision Variables						Constant Coefficients
	Initial Aquifer Drawdown s^{o}_{π}	Initial River Drawdown σ^{o}_{ρ}	Upstream Inflow $I(\nu)$	Pumping Rate $Q_p(\nu)$	Diversion $D_\rho(\nu)$	Initial Upstream Inflow I^{o}	
	Influence Coefficients						
$Y_{\cdot}(n)$	$X^{(1)}$	$X^{(2)}$	$X^{(3)}$	$X^{(4)}$	$X^{(5)}$	$X^{(6)}$	$X^{(7)}$
Reach return flow $Q_r(n)$	$\gamma_{r\pi}(\ )$	$\mu_{r\rho}(\ )$	$\psi_r(\ )$	$\varepsilon_{rp}(\ )$	$\zeta_{r\rho}(\ )$	$\Omega_r(\ )$	$\Lambda_t(\ )$
Reach drawdown $\sigma_r(n)$	$\bar{e}_{r\pi}(\ )$	$\bar{f}_{r\pi}(\ )$	$\bar{g}_r(\ )$	$\bar{d}_{rp}(\ )$	$\bar{h}_{r\rho}(\ )$	$\bar{c}_r(\ )$	$\bar{\ell}_r(\ )$
Reach storage $S_r(n)$	$e_{r\pi}(\ )$	$f_{r\rho}(\ )$	$g_r(\ )$	$d_{rp}(\ )$	$h_{r\rho}(\ )$	$c_r(\ )$	$\ell_r(\ )$
Reach flow $R_r(n)$	$\tilde{e}_{r\pi}(\ )$	$\tilde{f}_{r\rho}(\ )$	$\tilde{g}_r(\ )$	$\tilde{d}_{rp}(\ )$	$\tilde{h}_{r\rho}(\ )$	$\tilde{c}_r(\ )$	$\tilde{\ell}_r(\ )$
Aquifer drawdown $s_w(n)$	$\hat{e}_{w\pi}(\ )$	$\hat{f}_{w\rho}(\ )$	$\hat{g}_w(\ )$	$\hat{d}_{wp}(\ )$	$\hat{h}_{w\rho}(\ )$	$\hat{c}_w(\ )$	$\hat{\ell}_w(\ )$

coefficients. Programs have been written to generate systematically all the coefficients described in Table 1.

E. SAMPLE OF RESULTS

Figure 2 shows a small hypothetical stream-aquifer system. Figure 3(a) shows the calculated influence coefficients of aquifer return flows in reaches 1, 2 and 3 due to pumping at well p_1. Physically the coefficients $\varepsilon_{r_1p_1}$, $\varepsilon_{r_2p_1}$ and $\varepsilon_{r_3p_1}$ represent the actual return flows that would take place if well p_1 was pumped at a unit rate for one period (week) and then shut off permanently. According to Figure 3(a) for eight weeks the first reach loses water to the aquifer. This is understandable as the pumping from the nearby well lowers the aquifer level near the river. After a week the aquifer recovers because (1) the well is shut down and (2) it receives water from the stream. The first reach continues to lose water but less and less as time proceeds. In the case of the 3rd reach which is relatively far, the river gains water continuously. Because the first reach loses water, levels in reaches 2 and 3 drop. This drop in river stage is felt rapidly in reaches 2 and 3 because the *river drawdown wave* travels rapidly. On the other hand, the aquifer drawdown wave travels much less rapidly. Thus in reach 3 the water level in the river has dropped but the aquifer level has not, having not felt yet the impact of the pumping or not feeling it significantly being far from the pumping well 1. In the case of the second reach the situation is an intermediate one, the reach losing water at first, then gaining for about 2 weeks and then losing water again.

Figure 4 shows similar results for the case when well p_2 pumps for one week and is then shut down permanently. In this case the return flows are

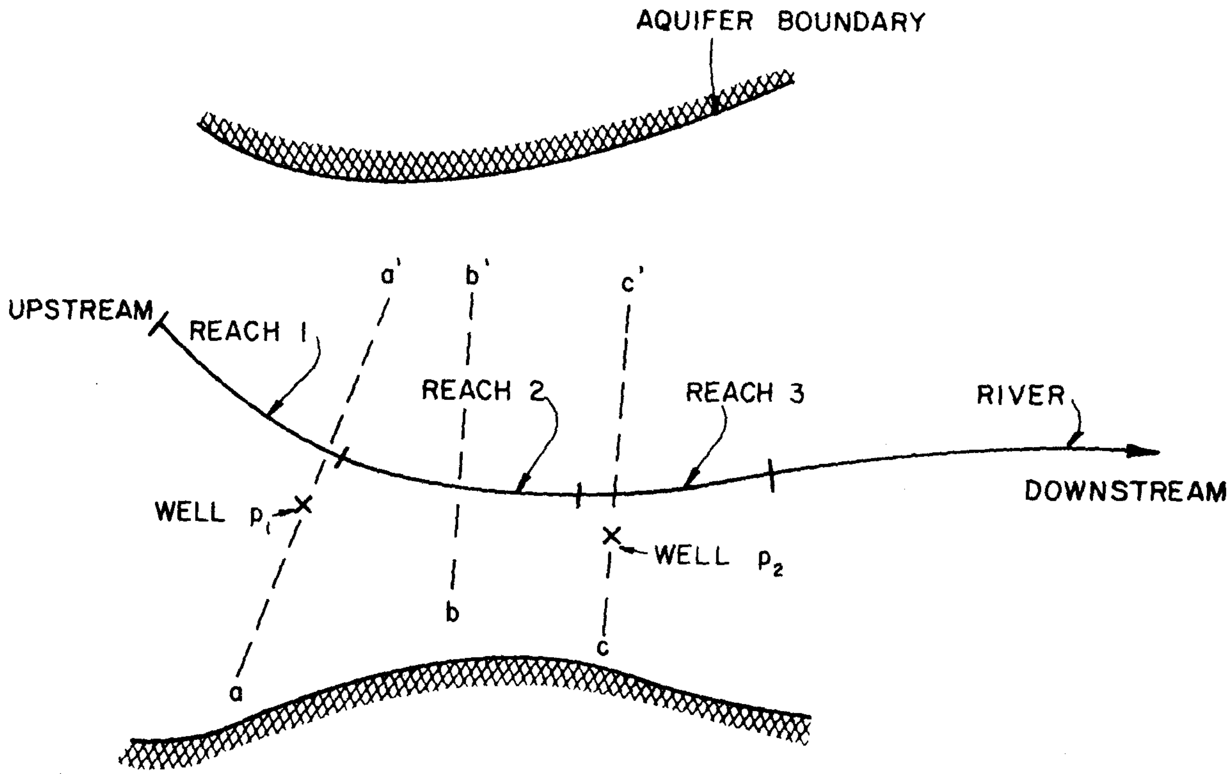

Fig. 2. Illustrative stream-aquifer system

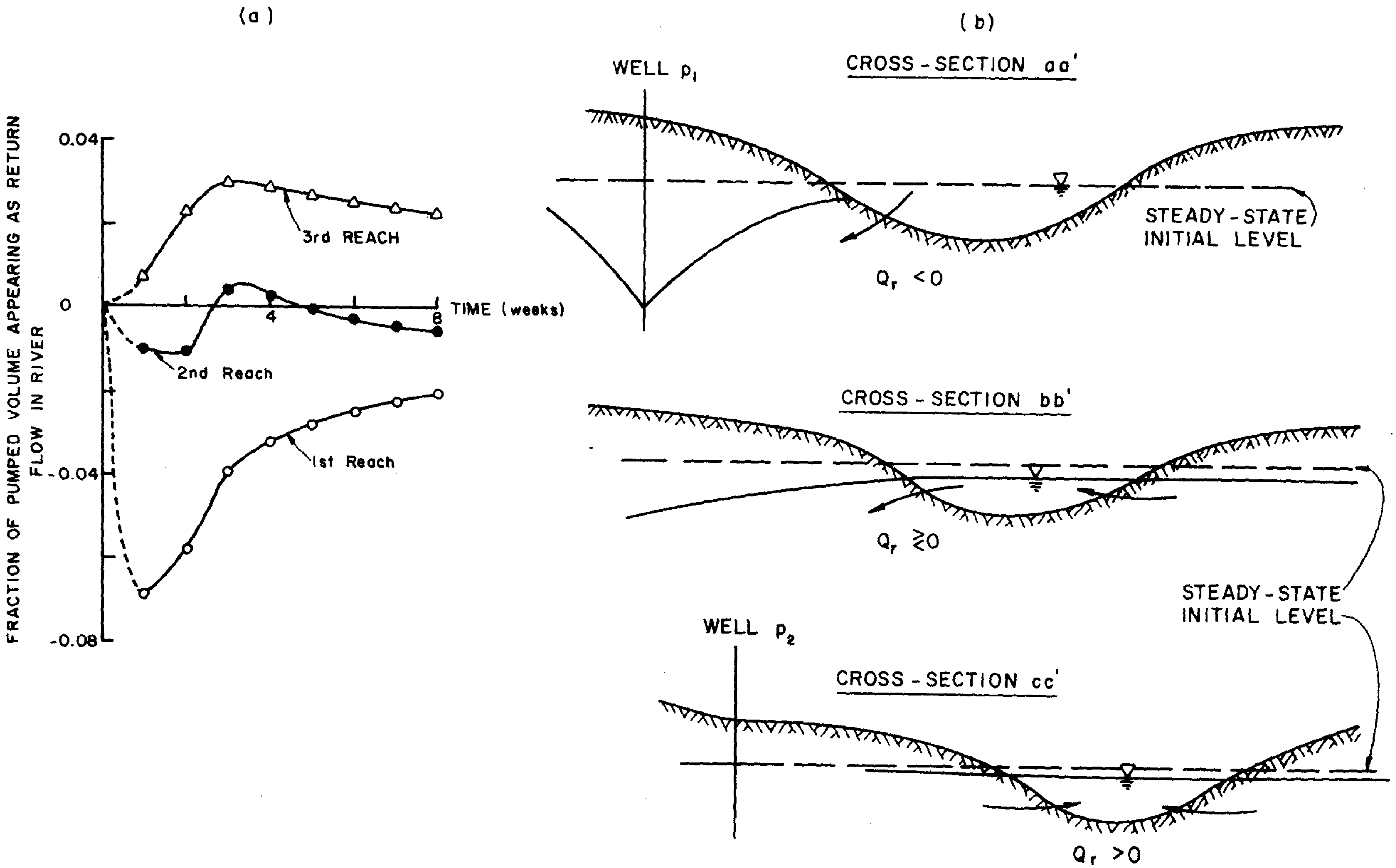

Fig. 3. Return flows in the reaches and profiles of water levels in river and in aquifer along several cross-sections, due to pumping at well p_1

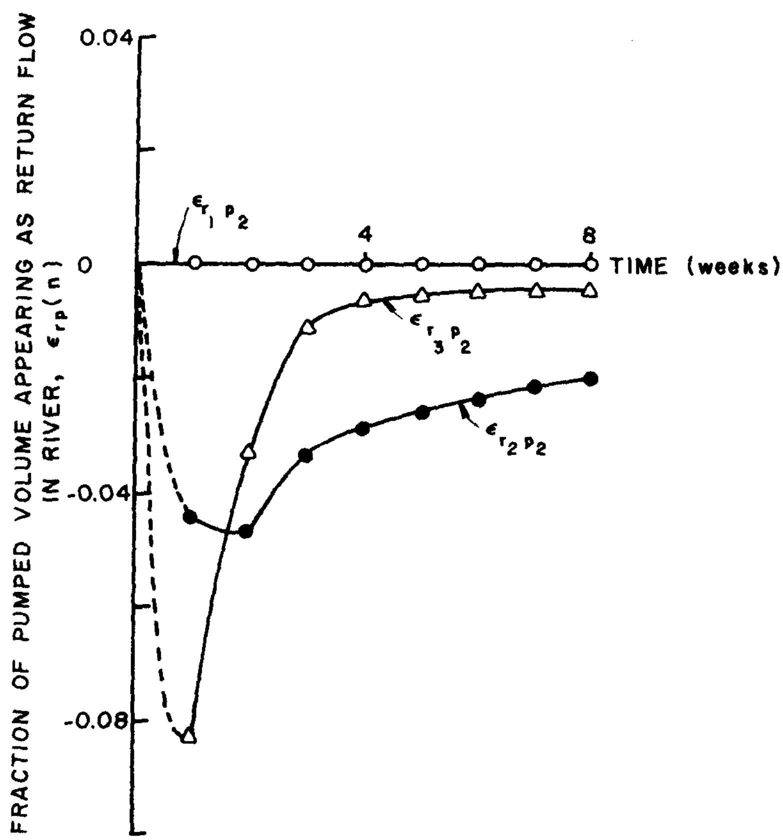

Fig. 4. Return flows in the reaches due to pumping at well p_2

all negative, all the reaches being upstream of well p_2. The reach nearest to the well experiences the greatest loss during the first week but later the upstream reach loses more water. This is due to the fact that during the later weeks the river stage in reach 3 drops due to losses from reach 2. That drop in river stage tends to compensate the aquifer drawdown near reach 3, whereas in reach 2 there is no drop in river level as there is no losses in reach 1, reach 1 being too far away from the well to feel any impact of significance for the first 8 weeks. Other curves of various influence coefficients can be found in a separate publication (Illangasekare, 1978).

F. CONCLUSIONS

New tools have been developed for the description of a stream-aquifer system. They appear to be well suited for studies of management and such studies are currently under way for the lower South Platte River in Colorado and for the lower Rio Grande River in the San Luis Valley in Colorado. These applications studies will be documented in subsequent reports.

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APPENDIX 1

A Finite Element "Discrete Kernel Generator" for Efficient Ground Water Management

To appear in Proceedings of the Second International Conference on Finite Elements in Water Resources, Imperial College, London, England, July 10-14, 1978.

A FINITE ELEMENT "DISCRETE KERNEL GENERATOR" FOR EFFICIENT GROUNDWATER MANAGEMENT

Tissa Illangasekare, H. J. Morel-Seytoux

Colorado State University, Fort Collins, Colorado 80523

INTRODUCTION

The constantly increasing demand for water in regions with already limited supply makes it imperative to manage the surface and groundwater supplies efficiently. In water scarce regions such as the Western United States, the water regulating agencies are facing the problem of regulating the water usage on a day-by-day basis during periods of high water demand. A typical case of a day-by-day controlled diversion of stream water to an irrigation district in the South Platte basin is shown on Figure 1. In the conventional approaches, for each set of decision variables such as aquifer pumping a simulation run has to be made and one must check whether the defined management objectives are met. Such an approach will be very inefficient in problems where large stream-aquifer systems are involved and a large number of decision variables has to be regulated on a day-by-day basis.

Most of the existing mathematical models of stream-aquifer systems are designed to predict the hydrologic behavior of the system in response to a particular set of numerical values of the excitations. An approach which makes use of the functional relation between the responses of the system to the excitation was presented by Morel-Seytoux (1973). The concept of the response function and the applicability of the approach to hydrologic modeling and simplified management problems were illustrated in a paper by Morel-Seytoux (1975). The use of these basic aquifer response functions in combination with a linearized stream routing model to predict the water table and river stage evolution was described briefly by Morel-Seytoux (1975).

Figure 2 schematically represents the operation of a management model using the influence coefficient approach. Figure 3 compares the conventional simulation approach and the suggested influence coefficient approach.

The models discussed by Morel-Seytoux (1975) and

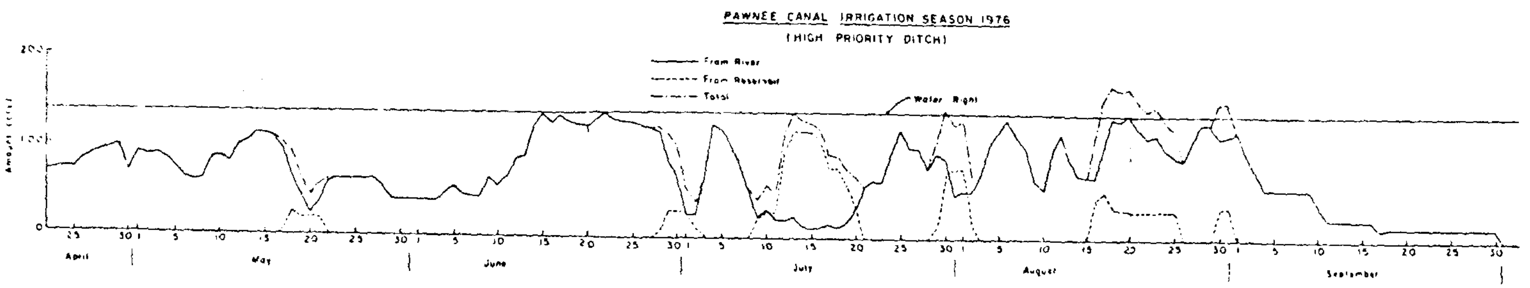

Figure 1 Daily diversion from stream, releases from reservoirs and total water delivery.

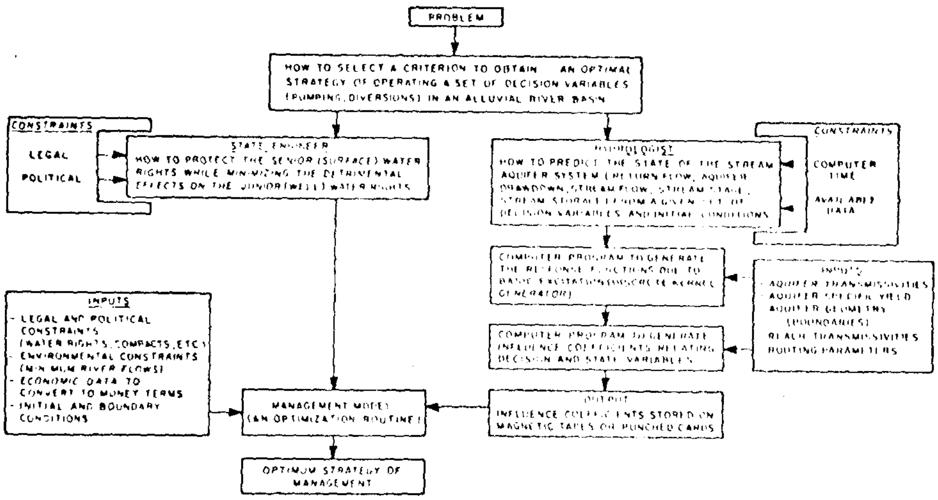

Figure 2 Schematic representation of the operation of management model using the influence coefficient approach.

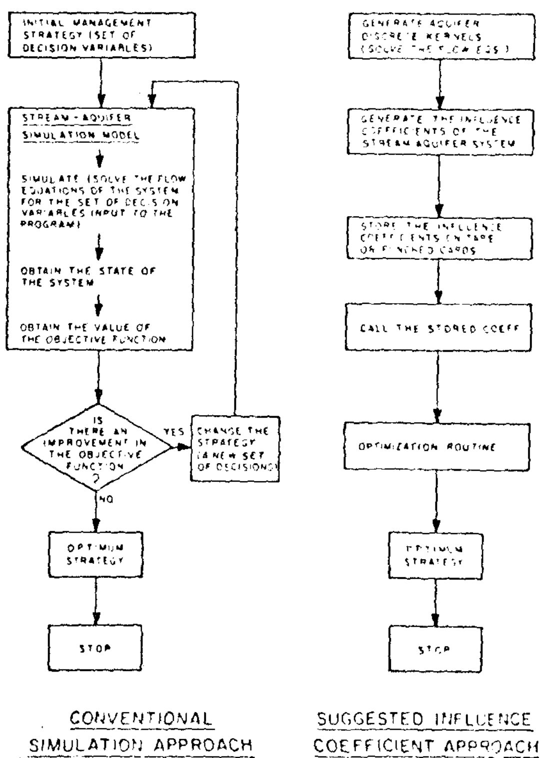

Figure 3 Comparison of conventional and new approach in deciding optimal management strategy.

Rodriguez-Amaya (1976) use two versions of the finite difference method to solve the basic saturated flow equation. A user oriented, (minimum input decisions taken by the user) storage efficient finite element model which generates the basic response functions without being limited by the size of the aquifer is presented in this paper.

DISCRETE KERNEL OF AQUIFER DRAWDOWN DUE TO PUMPING EXCITATION

The basic saturated flow equation is the Boussinesq equation:

$$\phi \frac{\partial s}{\partial t} - \frac{\partial}{\partial x} (T \frac{\partial s}{\partial x}) - \frac{\partial}{\partial y} (T \frac{\partial s}{\partial y}) = q_e \qquad (1)$$

where ϕ is the drainable (or effective) porosity, s is the drawdown measured positive downward from a (high) horizontal datum, t is time, x and y are the horizontal cartesian coordinates, T is the transmissivity, and q_e is the instantaneous pumping rate per unit area at excitation point e in the aquifer (chosen algebraically positive for a withdrawal excitation). It has been shown by Morel-Seytoux and Daly (1975) that the solution to Equation (1) can be expressed generally in the form:

$$s_{we}(n) = \sum_{\nu=1}^{n} \delta_{we}(n-\nu+1)Q_e(\nu) \qquad (2)$$

It is clear that once the discrete kernel coefficients $\delta_{we}(\nu)$, $\nu=1,2...n$ have been obtained, the drawdown response $s_{we}(n)$ can be obtained for any type of pumping schedule $Q_e(\nu)$, $\nu=1,2...n$ from Equation (2), whereas in the traditional simulation approach the total right-hand side of Equation (2) has to be computed for each given pumping schedule.

To generate the "discrete kernel" coefficients for an aquifer with given boundary conditions, Equation (1) has to be solved for a unit pulse excitation q_e on the right-hand side.

A USER ORIENTED COMPUTER-EFFICIENT MODEL

One of the shortcomings of existing numerical models of aquifer simulation is related to the decisions which has to be taken by the user with respect to the data inputs. The basic inputs needed for the numerical solution of Boussinesq's equation are the aquifer geometry, distribution of transmissivity, distribution of specific yield and the locations and values of net pumping excitations. In the "discrete kernel generator" the net pumping excitation is fixed as a unit pulse applied at a known node point. Thus all the inputs needed could be extracted from the basic data sources of maps defining aquifer boundaries, transmissivities and specific yields. A "user oriented" program is one such that the user does not have to make the decisions related to the type of mesh (or grid) system to be used in the numerical procedure, the spacing of nodes, the estimation of nodal transmissivity and specific yield values from data maps and time increment parameters, etc.

There are two aspects of efficiency which have to be considered in computer modeling. The efficiency with respect to the computer memory storage needed and the central processing time used. In solving problems associated with large stream-aquifer systems, the memory storage becomes a limiting factor. Even though the computing time which decides the computing cost becomes high for the generation of the discrete kernel coefficients, once they have been calculated and saved, simulation of aquifer behavior to any pumping excitation pattern can be obtained without ever making use any longer of the costly numerical model.

The "discrete kernel generator" developed in this study has the following features which makes it user oriented and storage efficient:

(1) The program uses the geometry of aquifer boundaries, contours of equal transmissivity and specific yields as inputs.

(2) The program generates a finite element mesh system to fit the given aquifer geometry.

(3) It defines a sub-mesh system to scan the total aquifer.

(4) The built-in time-parameters in the program guarantee numerical stability.

Moving sub-aquifer

The idea of the moving sub-aquifer is based on the fact that the aquifer drawdown (response) due to a pumping excitation at a node is significant only locally in the aquifer. Figure 4 shows the maximum value of the discrete kernels $(\delta_{we}(n))$ generated at different distances away from the excitation point e, for a homogeneous aquifer. For this case the response is significant only up to about the 8th node space. Hence for this particular case the width of the sub-aquifer within which the excitation is assumed to be felt is taken as 16 node spaces.

By assuming that the response is only significant locally in a region close to the excitation we are assuming that the initial zero gradient of the water table is not changed outside this region. Hence the boundary of this region which is also the boundary of the sub-aquifer (Figure 5) is assumed to have no flow boundary condition. The finite element equations are formulated for the sub-aquifer to solve Equation (1) for an excitation point on the excitation grid line EE (Figure 6). Once all the nodes on EE have been excited and the discrete kernels generated, the system moves by one grid space, making the next vertical grid line the excitation grid line. The sub-system scans the total aquifer by moving one grid space at

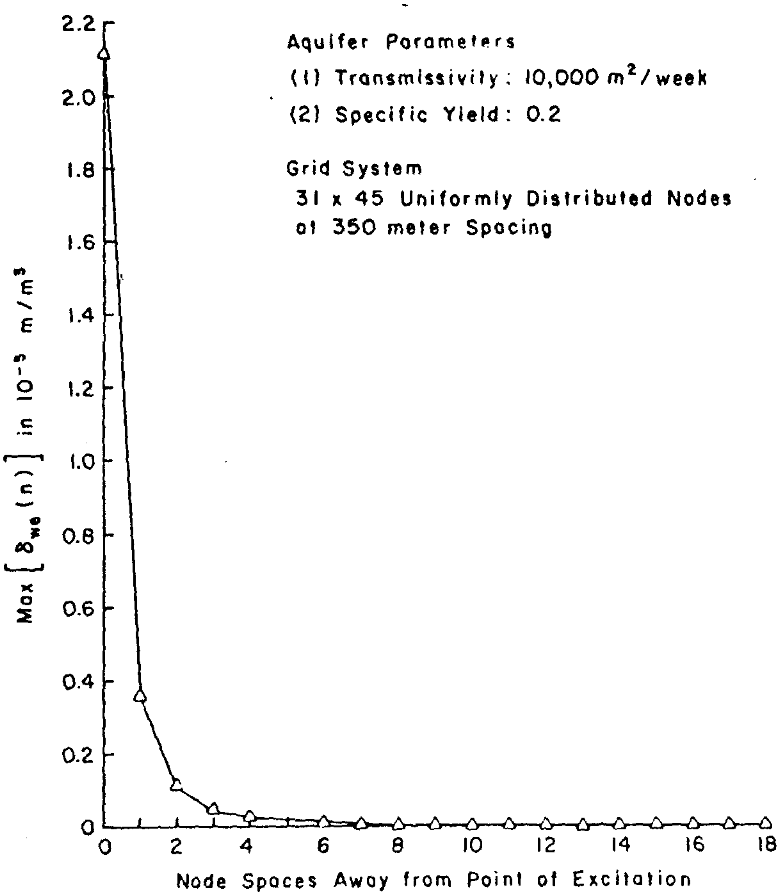

Figure 4 Maximum excitation response in a homogeneous aquifer.

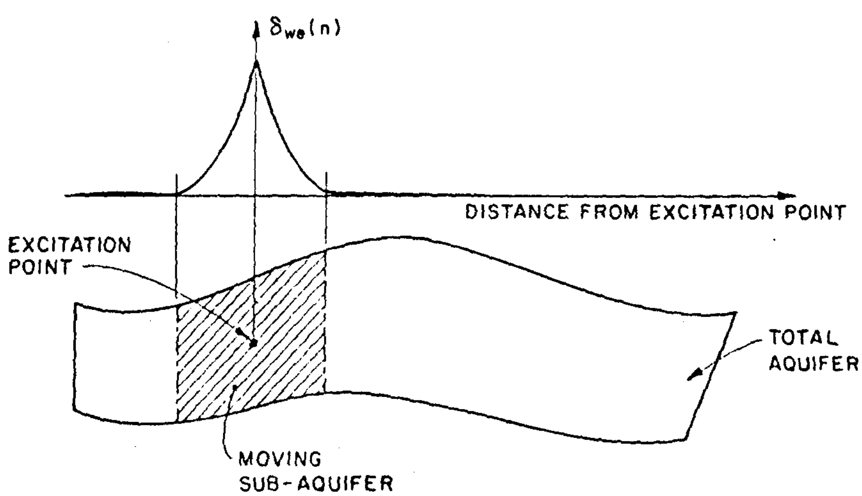

Figure 5 Moving sub-aquifer.

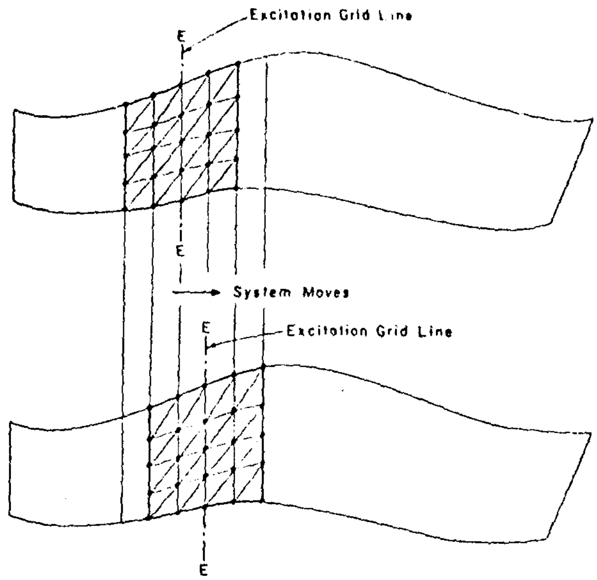

Figure 6 Moving sub-mesh.

a time till all the nodes have been excited.

FINITE ELEMENT FORMULATION

Using the Galerkin method on the operator defined by Equation (1) and following procedures described in various textbooks (e.g., Gray and Pinder (1974)) a matrix equation for the unknown drawdowns at the nodes is obtained. This equation is of the form:

$$[A]\ \{s\} = \{R\} \tag{3}$$

An element of the matrix [A] is given by,

$$A_{(i,k)(j,\ell)} = P_{ij}\ U_{k\ell} + Q_{ij}\ V_{k\ell} \tag{4}$$

where i,j are nodes defined on the x-y plane and k,ℓ are time nodes (Figure 7). The general expressions for the elements of matrices [P] and [Q] defined on the x-y plane and the elements of the matrices [U] and [V] defined on the time axis are given by:

$$P_{ij} = \sum_{m=1}^{K_i} \left\{ T_m \iint_{D^m_{xy}} \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) dxdy \right\} \tag{5}$$

$$Q_{ij} = \sum_{m=1}^{K_i} \left\{ \phi_m \iint_{D^m_{xy}} N_i\ N_j\ dxdy \right\} \tag{6}$$

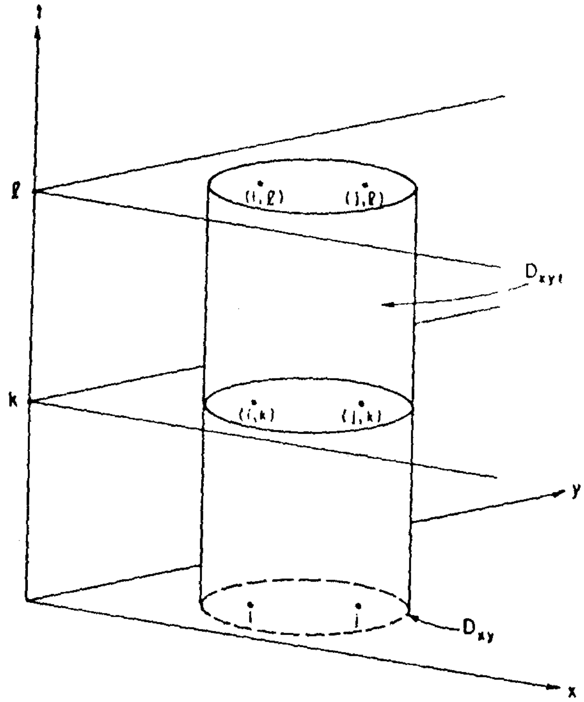

Figure 7 Definition of space and time nodes

$$U_{k\ell} = \int_{D_t} \gamma_k \, \gamma_\ell \, dt \tag{7}$$

$$V_{k\ell} = \int_{D_t} \gamma_k \frac{\partial \gamma_\ell}{\partial t} \, dt \tag{8}$$

where N_i and γ_k are the basis functions on space and time respectively, K_i is the number of elements sharing the space node i, T_m and ϕ_m are the constant transmissivity and specific yield in the m^{th} element, D^m_{xy} is the space domain of m^{th} element and D_t is the time domain.

An element of the right-hand side vector is given by,

$$R_{(i,\ell)} = \left\{ q_e \sum_{m=1}^{K_i} \iint_{D^m_{xy}} N_i \, dxdy \right\} \left\{ \int_{D_t} \gamma_\ell dt \right\} \tag{9}$$

The set of Equations (3) can be solved for $s(i,\ell)$, where (i,ℓ) is the node whose projection is the i^{th} node on the x-y plane and ℓ is the time level (Figure 7).

NUMERICAL SCHEME AND THE COMPUTER PROGRAM

The computer program developed for the generation of "discrete kernels" has three main components:

(1) a program which estimates the parameters and the size of the moving sub-mesh,
(2) a program which generates the mesh system and estimates the nodal values of transmissivity and specific yield from input data, and
(3) a program which, (a) computes and updates the "space" matrices [P] and [Q] as the system moves, (b) computes and updates the "time" matrices [U] and [V] as the time nodes move along the time axis, and (c) solves the system of Equations (3).

<u>Algebraic structure of the finite element equations</u>

The structure of the matrices defined by Equations (5) to (9) is determined by the type of elements and the basis functions used. In this study, simple triangular elements with linear basis functions $N_i(x,y)$ were used to generate the matrices [P] and [Q]. Along the time axis, two nodes per element with linear basis functions $\gamma_i(t)$ were used to generate the matrices [U] and [V].

In Equations (5), (6) and (9) the domains of integration D^m_{xy} become the triangular elements of the mesh system. The double integration was performed for each finite element to obtain the following expressions for an elemental setup shown in Figure 8.

$$[p] = \begin{bmatrix} \frac{-T_m(y_2-y_3)^2}{8A} & \frac{-T_m y_3(y_2-y_3)}{8A} & \frac{T_m y_2(y_2-y_3)}{8A} \\ \frac{-T_m y_3(y_2-y_3)}{8A} & \frac{-T_m(y_3^2 + \Delta L^2)}{8A} & \frac{T_m(y_2 y_3 + \Delta L^2)}{8A} \\ \frac{T_m y_2(y_2-y_3)}{8A} & \frac{T_m(y_2 y_3 + \Delta L^2)}{8A} & \frac{-T_m(y_2^2 + \Delta L^2)}{8A} \end{bmatrix} \qquad (10)$$

$$[q] = \begin{bmatrix} \phi_m \frac{A}{6} & \phi_m \frac{A}{12} & \phi_m \frac{A}{12} \\ \phi_m \frac{A}{12} & \phi_m \frac{A}{6} & \phi_m \frac{A}{12} \\ \phi_m \frac{A}{12} & \phi_m \frac{A}{12} & \phi_m \frac{A}{6} \end{bmatrix} \qquad (11)$$

$$\{r\} = \begin{bmatrix} \frac{A}{3} \\ \frac{A}{3} \\ \frac{A}{3} \end{bmatrix} \qquad (12)$$

where [p], [q] and {r} are the components of elements of [P], [Q] and {R} contributed by each finite element and A is the area of the triangle.

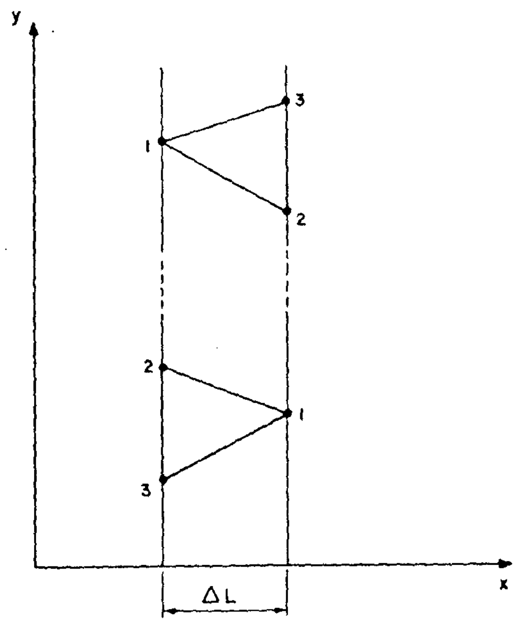

Figure 8 General element setup.

Using two nodes per element spaced at Δt, the following expressions were obtained for the "time" matrices.

$$[U] = \begin{bmatrix} \Delta t/3 & \Delta t/6 \\ \Delta t/6 & \Delta t/3 \end{bmatrix} \tag{13}$$

$$[V] = \begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix} \tag{14}$$

Parameters of the moving sub-system

At all stages of the computation the "space" matrices [P] and [Q] hold only the information of the sub-mesh system. Therefore the mesh system is generated only for the sub-aquifer. Two parameters define totally the configuration of the sub-aquifer mesh (Figure 9), namely the vertical grid line spacing (ΔL) which is kept constant all along the length of the aquifer and the number of vertical grid lines (NSUBV) in a sub-aquifer. The number of nodes (NW) on a vertical grid line is estimated using a user supplied average total aquifer width and the parameter ΔL. The NW value used will make the distance between nodes on a vertical, approximately equal to ΔL.

An empirical relationship which guarantees stability of solution was obtained using computer runs made for a homogeneous square aquifer whose nodes were spaced equally at a

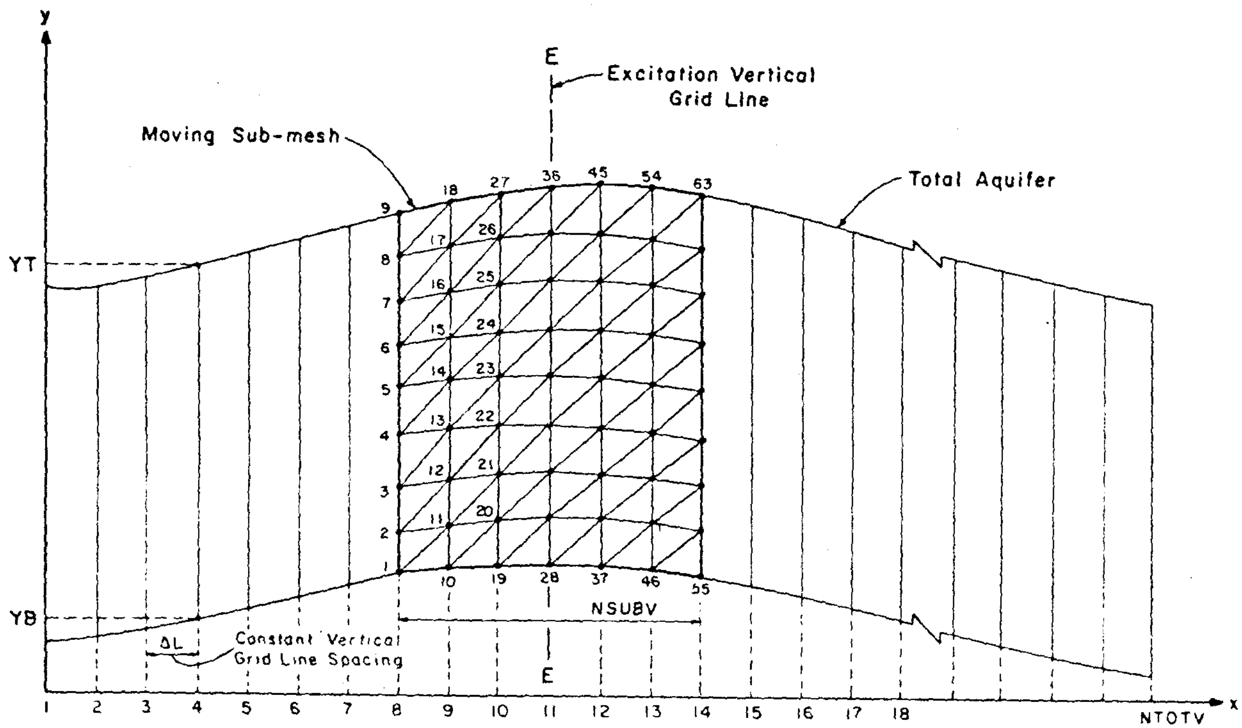

Figure 9 Finite element mesh system.

distance ΔL. The routine which solves the finite element equations was used for different combinations of T/ϕ, ΔL and time increments Δt and it was found that the following condition should be satisfied for stability of the numerical solution.

$$\frac{\phi}{T} \frac{(\Delta L)^2}{\Delta t} \leq 2.0 \tag{15}$$

Using the extreme case of $\Delta t = 0.5$ used during the first few periods,

$$\Delta L \leq \sqrt{\frac{T}{\phi}} \tag{16}$$

From user supplied values of average transmissivity and specific yield for the total aquifer, ΔL can be estimated from Equation (16). Using the same values of T and ϕ and a user supplied average aquifer width the analytical solution for a homogeneous aquifer is used to determine the number of grid spaces away from the excitation point at which the maximum value of the "discrete kernel" coefficients become insignificant. The above information is used to define the length of the sub-aquifer (NSUBV).

Finite element mesh generator

With a knowledge of the vertical grid line spacing ΔL the user can fit a vertical grid line system onto the total aquifer as shown in Figure 9. By superimposing the vertical grid line system onto the appropriate map the user prepares for each grid line vertical grid line information cards with the following information:

(a) the y-coordinates of the points at the intersection of the bottom and top boundaries of the aquifer. (YB and YT as shown on Figure 9),

(b) the contour values and the y-coordinates of the points of intersection of the contours of equal transmissivity,

(c) same as (b) for contours of equal specific yield.

Using the fixed number of nodes on a vertical (NW) the program distributes nodes at equal spaces on the vertical grid line between aquifer boundaries and the nodal values of transmissivity and specific yield are interpolated using the information given in (b) and (c). Each time the sub-aquifer moves a new vertical grid line information card is read and information on the leftmost grid line is dropped to minimize the computer storage requirements.

Finite element solution routine

Using the node numbering scheme shown on Figure 9, the symmetric "space" matrices [P] and [Q] becomes banded with a band width of NW + 2, where NW is the number of nodes on a vertical grid line. In addition to being banded the matrices are systematically sparse. That is, irrespective of the NW used, out of the NW+2 columns of the band all the columns have zero entries except columns 1, 2, NW+1 and NW+2 (Figure 10). This property makes it possible to save storage

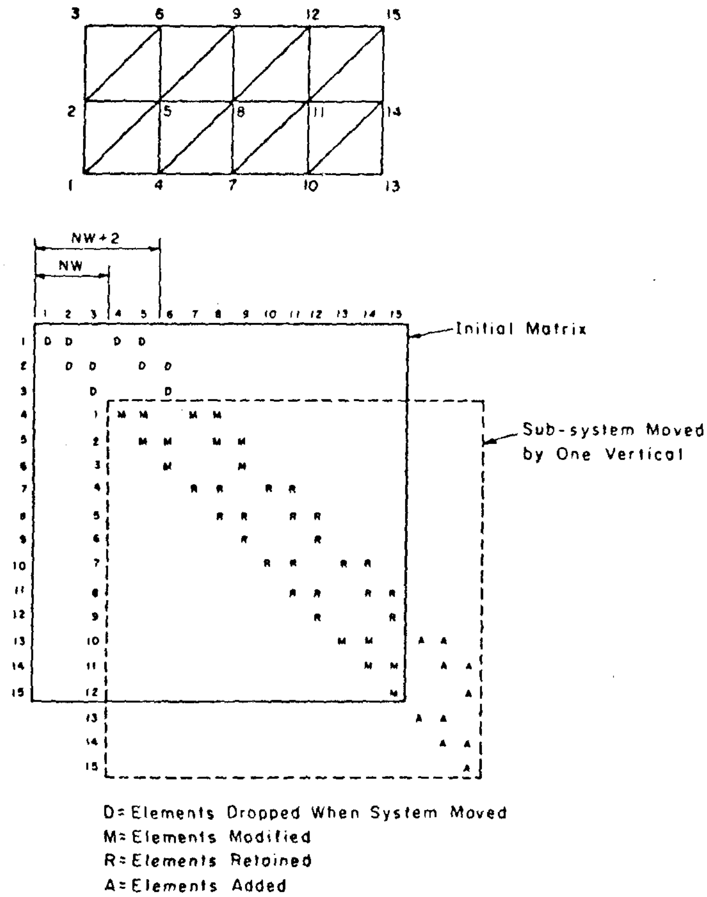

Figure 10 Structure of "space" matrices [P] and [Q]

as only these four columns have to be stored. In solving the system of equations given by Equation (3), the elements of matrix [A] are generated using Equation (4). Each time a node is excited the vector {R} is generated using Equation (9). The system of equations is solved using the Gauss-Seidel iterative scheme.

Once all the nodes on the excitation vertical grid line have been excited the sub-system is moved by one vertical. For the new sub-system the "space" matrices [P] and [Q] are modified by dropping the matrix elements corresponding to the set of nodes dropped and adding elements of the nodes of the added vertical. The elements to be dropped, modified, retained from the original matrix and added are shown in Figure 10 for an example problem.

The "discrete kernel" values generated are printed out (or stored on magnetic tape) at the end of each excitation.

RESULTS

The program developed was used to generate the discrete kernel coefficients for a homogeneous aquifer for which an analytical solution exists. A square aquifer of size 350 meters with no-flow boundary conditions and uniformly spaced nodes 350 meters apart was excited at the central node e (Figure 11). The analytical solutions and the program

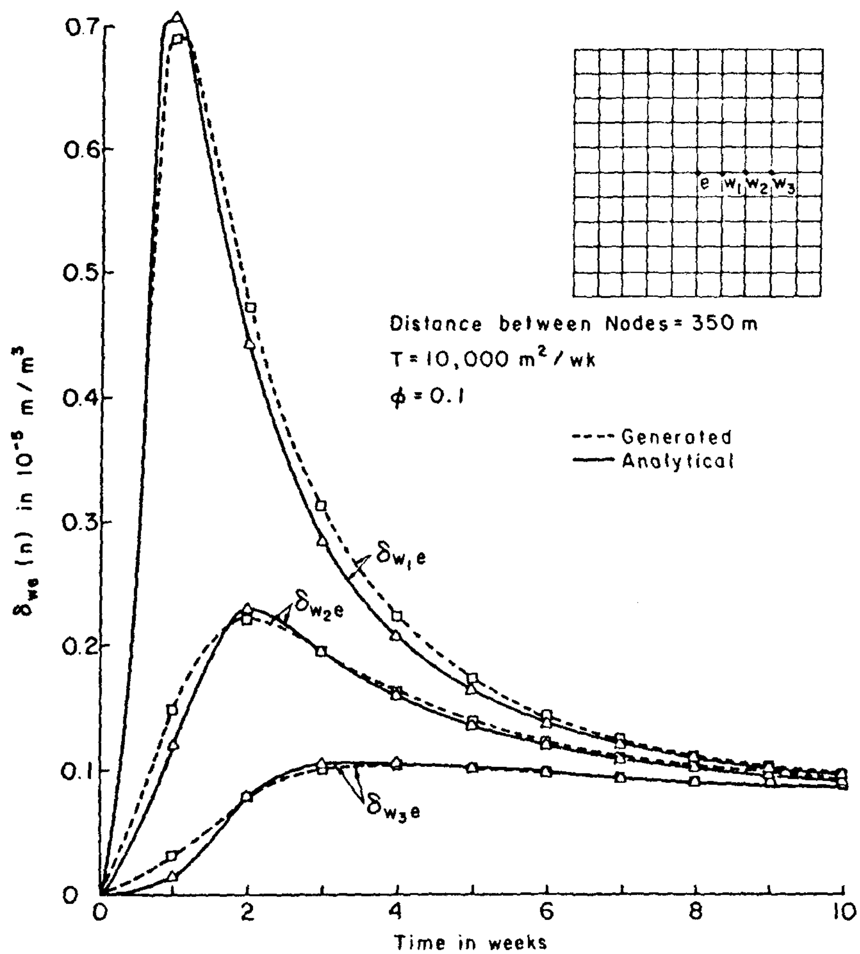

Figure 11 Comparison of program generated coefficients and analytical solutions - Case 1.

generated response functions $\delta_{we}(n)$ observed at the nodes w_1, w_2 and w_3 are compared on Figure 10. The comparison was done for a range of values of specific yield and the results are shown on Figures 12 and 13. The comparison shows that the analytical solutions and the program generated values are in very good agreement for a range of values of T/ϕ.

To demonstrate the steps involved in applying the model to real stream-aquifer systems, a sample reach was selected from the South Platte River (Figure 14).

Step 1: Using maps of aquifer geometry, transmissivity

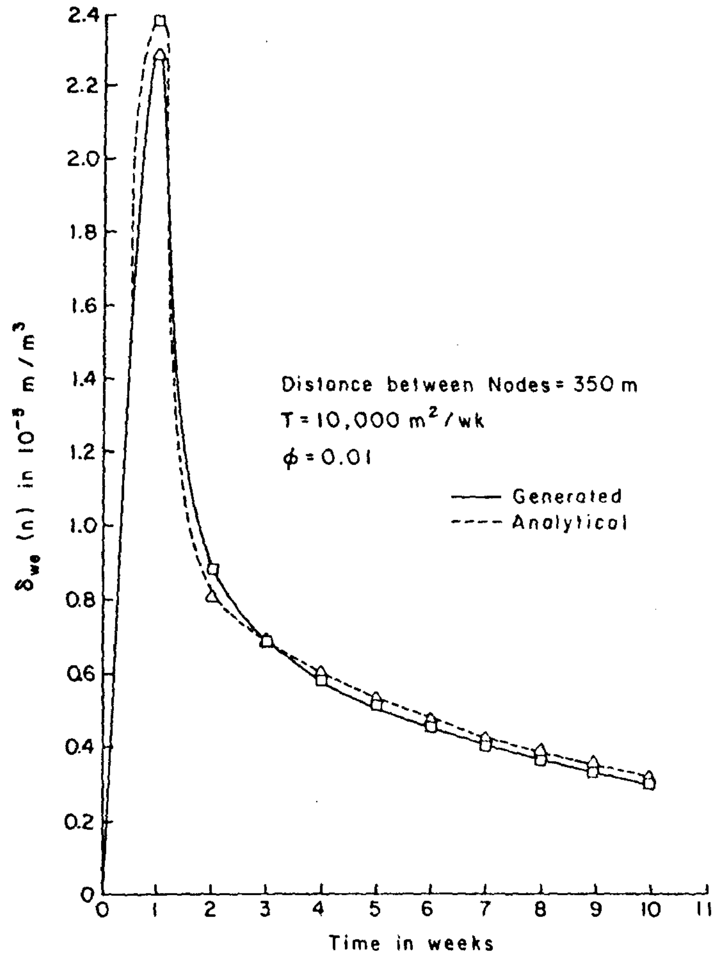

Figure 12 Comparison of program generated coefficients and analytical solutions - Case 2.

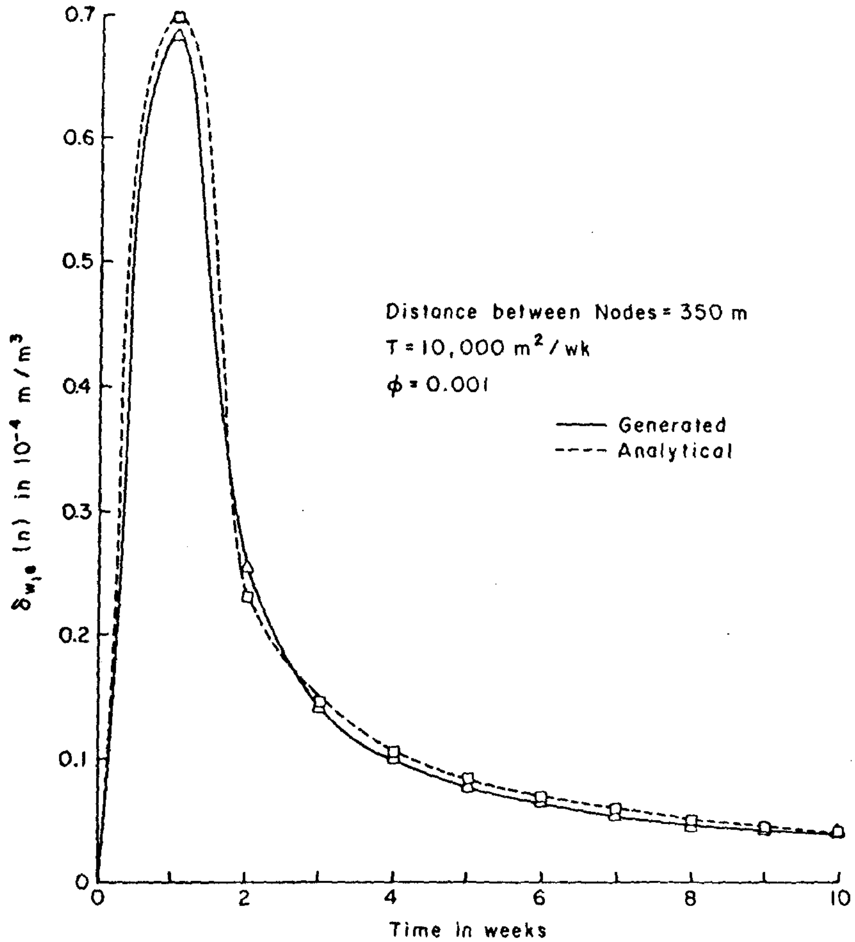

Figure 13 Comparison of program generated coefficients and analytical solutions - Case 3.

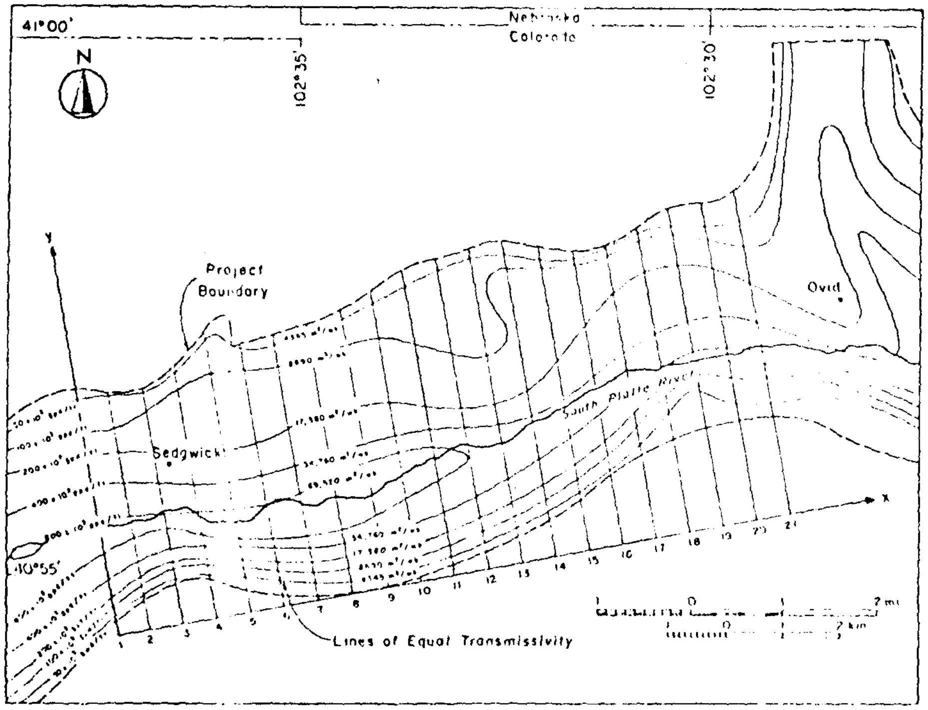

Figure 14 Sample reach from South Platte River.

and specific yield, estimate average values of aquifer width, transmissivity and specific yield.

average width = 2600 meters

T = 24,600 m^2/wk

ϕ = 0.2

From the relationship (16), using the average values of T and ϕ, the vertical grid line spacing ΔL is estimated.

ΔL = 350 meters

Step 2: Using the average values of aquifer width, T, ϕ and the estimated ΔL the program computes the moving-aquifer parameters.

length of the sub-system = 16 grid line spaces

number of nodes on a vertical = 9.

Step 3: The user defines the x,y axes (Figure 14) and fits a vertical grid line system spaced at 350 meters. The vertical grid line information cards are prepared for the 21 grid lines to be used as the inputs to the "discrete kernel generator" program.

CONCLUSION

A cost efficient method which has the potential to be used effectively in making day-by-day short term management decisions in stream-aquifer management problems was presented. The user oriented "kernel generator" program discussed simplifies the application of the model to real field problems. The agreement of the model generated values with analytical solutions shows that the model can be used to generate the "discrete kernels" to a reasonably good accuracy for different ranges of values of transmissivity and specific yield.

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