

DISSERTATION

IMPULSIVE CONSUMERS AND OPTIMAL SOCIAL SECURITY

Submitted by

Thomas Scott Findley

Department of Economics

In partial fulfillment of the requirements

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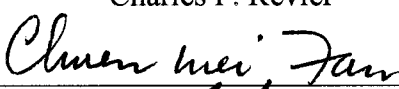
Committee on Graduate Work



Nancy A. Jarakoplos



Charles F. Revier



Chuen-mei Fan



W. Marshall Frasier



Advisor: Robert W. Kling



Department Head: Steven J. Shulman

ABSTRACT OF DISSERTATION

IMPULSIVE CONSUMERS AND OPTIMAL SOCIAL SECURITY

Unfunded social security programs are primarily justified on grounds that individuals have specific behavioral tendencies that lead to inadequate saving for retirement. To date, very little has been pursued in the way of theory to analyze this justification. I design a new model of consumer behavior that is consistent with many of the salient features of evidence on impulsive consumption behavior. In my model, “impulsive consumers” optimally formulate long-term plans, but often deviate from these consumption programs upon experiencing a psychological impulse to uncontrollably consume above and beyond. In order to examine how impulsive consumers fare in an unfunded social security program, I calibrate my model to match specific empirical features of aggregate life-cycle consumption. After calibrating the model, I employ dynamic welfare measures and find: i.) a significant welfare cost to consuming impulsively; ii.) a social security program (calibrated to the current U.S. program) does not generally improve the welfare of impulsive consumers; iii.) social security almost never improves the welfare of impulsive consumers under future demographics; and, iv.) the optimal social security tax rate is drastically smaller than the current U.S. rate.

Thomas Scott Findley
Department of Economics
Colorado State University
Fort Collins, CO 80523
Summer 2007

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Chapter One: Introduction

1.1 Motivation and Research Question

Upon signing the Social Security Act into law in 1935, President Franklin D. Roosevelt stated, “We have tried to frame a law which will give some measure of protection to the average citizen and to his family against the loss of a job and against poverty-ridden old age.”¹ This statement begs the question: Why might individuals be poverty-ridden during old age? Several non-exhaustive reasons could qualify, such as wage income uncertainty during the working years, disability and premature exit from the labor force, and outliving privately accumulated savings. Aside from these possibilities, George Akerlof (2002) stated in his 2001 acceptance lecture for receiving the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel, “It is common wisdom that people save too little. To compensate for this failure, most developed country governments heavily support the elderly during retirement.” Similarly, Kotlikoff et al (1982) assert

The essential premise underlying the Social Security system...is that left to their own devices, large numbers of people would fail to save adequately and find themselves destitute in their old age. While our Social Security and private pensions systems perform a number of functions in the area of income redistribution and insurance, their primary function is to force individuals to save for their old age. This, potentially prevents personal suffering and reduces the demand for general welfare.

So individuals are forced to save via a public pension program in order to prevent old-age destitution and the demand for state welfare on account of insufficient private saving.² But why might individuals fail to save adequately for retirement in the first place, and therefore need to be compelled? Kotlikoff et al (1982) cite myopia as one cause for inadequate private saving. In addition, Martin Feldstein (1985) states,

¹See *Social Security: A Primer* by the Congressional Budget Office (2001).

²Diamond (1977) states, “A forced savings program would be a way of making those with large lifetime incomes but small retirement incomes pay for their own welfare payments when old.”

“The principal rationale for such mandatory [social security] programs is that some individuals lack the foresight to save for their retirement years.” So a social security program is primarily justified on grounds that people do not privately save appropriate amounts because they are myopic and they lack the ability to foresee (or fully foresee) their consumption needs during retirement.

Several research questions logically follow in response to justifications that individuals are shortsighted and myopic. To what extent do individuals have to possess shortsightedness in order to benefit from a social security program? Furthermore, what is the optimal level of social security benefits provided by a pay-as-you-go system, and can the current size of the U.S. program be theoretically justified? Despite the fact that social security benefits represent the largest public program in most developed countries, and despite the fact that the principal justification for such programs relies on a foundation that individuals lack the foresight to adequately save for retirement, surprisingly little has been pursued in the way of theory that appropriately addresses these questions.

Among the few notable efforts, Feldstein (1985) designs a two-period overlapping generations model with shortsightedness, wherein the agent discounts the future and underestimates the level of social security benefits that he will receive during retirement. Feldstein’s principal finding is that “a social security pension may be inappropriate even if economic myopia is universal and very substantial”. Docquier (2002) examines the sensitivity of Feldstein’s conclusions by using a different social welfare function. In a working paper, Findley and Caliendo (2007) address how agents who have short (fixed) planning horizons fare in a pay-as-you-go social security program, and they generally find that social security is welfare-improving under this particular notion of shortsightedness within a partial-equilibrium framework.

Some parallel research questions also exist in regards to farsighted, non-myopic agents who optimally plan over lifetime horizons but who lack the self-control or com-

mitment to stick to such optimally-formulated plans. To what extent do farsighted individuals need to stray from optimal plans in order to justify an unfunded social security program? What is the optimal level of social security benefits, and can the size of the U.S. program be justified on behalf of such agents? İmrohorođlu et al (2003) address such welfare questions by engineering a 65-period overlapping generations model with agents who have hyperbolic discount functions (time-dependent and declining discount rates). Individuals with hyperbolic discount functions plan for the long term, but fail to actually implement and follow an optimally-formulated plan as time progresses. This is due to the agent's continual revision of formulated plans due to a time-dependent discount rate. The behavioral result is that agents consistently procrastinate saving to the later years of the work life. Such hyperbolic discounting behavior is supported by experimental evidence.³ İmrohorođlu et al (2003) conclude that social security is generally *not* welfare-improving, but they also provide extensive sensitivity analysis and discuss under what conditions social security can be welfare-improving within their setup.⁴

My dissertation also addresses the welfare implications of social security when agents are farsighted and plan optimally, but fail to stick to formulated plans. Yet this study is distinctly different in that agents don't always deviate from optimal plans, but can sometimes implement and actually follow through on plans. I introduce "impulsive consumers" in this dissertation, and I study how they fare in an unfunded social security program. An impulsive consumer is someone who sets optimal plans, sticks to these optimal plans for various periods of time, but is prone to impulsive spending fits. Such behavior is widely documented in the psychology, psychiatry, and consumer research literature, but has yet to be rigorously studied in economics.

³See Laibson (1997), Laibson et al (1998), and Angeletos et al (2001) for more on the hyperbolic discounting setup and motivating evidence.

⁴See Gul and Pesendorfer (2001, 2002a, and 2002b) for an axiomatic approach to temptation, self-control, and commitment devices. Gul and Pesendorfer (2002b) theoretically address the sensitivity of the results of İmrohorođlu et al (2003) from a time-consistent perspective.

1.2 The Objectives of the Study

The value of social security participation has already been addressed from the perspectives of underestimating benefits, hyperbolic discounting, and short planning horizons. Therefore, the purpose of my dissertation is to add to the literature by examining and assessing the normative implications of participation in a pay-as-you-go social security system when individuals are impulsive consumers. Several objectives are specifically pursued in studying this question.

The first objective is to construct and solve a model of the impulsive consumer. Such a model is a departure from the strict Neoclassical framework because impulsive consumers can undersave for retirement and because they display time-inconsistent behavior as they rationally revise previously-formulated consumption/saving plans after spells of impulsiveness. Although my model of impulsive consumers is a unique contribution to the consumption literature, it is conceptually similar to an early piece by Thaler and Shefrin (1981), who outline several features that a model of self-control (or lack of) would need to represent.⁵ They imagine that an individual is composed of two distinct “selves”.⁶ One self is a “planner”, and the other is a “doer”. Thaler and Shefrin (1981) indicate that the planner-self would be concerned with lifetime utility, while the doer-self would only be concerned with present circumstances. Moreover, the individual’s actual level of consumption and saving in each period of the life cycle would depend on which self wins this internal battle. Thaler and Shefrin (1981) stop short of actually providing a formal model that embodies these ideas. I devise a quantitative-theoretic framework that embeds the interplay between a long-term planner-self and a myopic doer-self. Specifically, my framework allows the agent to sometimes follow the life-cycle permanent-income consumption rule (long-term

⁵A model of impulsive consumption can be viewed as a particular case of self-control failure wherein the agent has difficulty refraining from spontaneous consumption desires.

⁶Thaler and Shefrin (1981) assert that models of two “selves” are distinctly different from models of the changing-tastes literature (such as hyperbolic discounting).

planner-self) and to sometimes impulsively consume an unplanned amount that is above and beyond the amount from optimally-formulated plans (doer-self).⁷ My model is general enough to allow for different frequencies and magnitudes of impulsiveness.

The second objective of this study is to calibrate the theoretical model for purposes of simulation and welfare analysis. Specific numerical values are selected for economic parameters based on well-known and widely-accepted empirical evidence in the macroeconomics literature. In addition, I am able to calibrate the behavioral impulsiveness parameters to specific simulation targets corresponding to the empirical data on aggregate life-cycle consumption. Alternatively stated, the model is calibrated to theoretically match the empirical data on aggregate life-cycle consumption with the consumption impulse parameters being the degrees of freedom in adjustment.

The last objective is to pursue the principal question motivating this study, namely whether impulsive consumers can benefit from participation in an unfunded social security program. In order to address this question, two states of the world are considered for an agent who impulsively consumes: A state of the world with participation in social security, and a state of the world without participation. “Critical values” of the consumption impulse parameters are determined, wherein a social planner (acting on the agent’s behalf from a lifetime utility perspective) is indifferent between these two states of the world. Dynamic welfare measures are used to quantify the lifetime utility differences across these states. Moreover, the optimal level of social security benefits (the optimal size of an unfunded program) is computed by maximizing the representative agent’s lifetime utility with respect to the social security tax rate. I study how wage heterogeneity affects the optimal tax rate. Sensitivity of these welfare results is also performed.

⁷The idea of impulsive consumption is related to Milton Friedman’s (1957) idea of *transitory consumption*. See Speight (1989) and Deaton (1992) for more on transitory consumption.

1.3 The Organization of the Study

I have organized the remainder of this study as follows. In Chapter Two, I review the relevant background literature on consumption, social security participation, aggregate consumption data, and bounded rationality. I introduce the representative-agent model of impulsive consumption in Chapter Three, and I derive solutions to the impulsive agent's intended and actual consumption programs. The theoretical framework represents an agent who can optimally plan and consume/save according to the life-cycle/permanent-income rule but who can also become impulsive in his consumption spending. In Chapter Four, I calibrate the model by finding values for the impulsive consumption parameters that generate simulated outcomes matching the empirical data on aggregate life-cycle consumption. I address the main research question of my dissertation in Chapter Five, namely whether impulsive consumption behavior (in isolation from other behavioral characteristics) can justify participation in a pay-as-you-go social security program from a lifetime welfare perspective. I outline and use dynamic welfare measures to meaningfully address this question. In addition, I explore the optimal size of a pay-as-you-go program for impulsive consumers. I conclude in Chapter Six by summarizing the primary findings of my dissertation, and I also critically assess the implications of my study with potential extensions for future work.

Chapter Two: Review of the Literature

2.1 Seminal Models of Consumption and Saving

2.1.1 Rule-of-Thumb Consumption and Saving

Keynes (1936/1964) outlined rule-of-thumb behavior that is able to explain some features of short-run aggregate consumption data fairly well.⁸ He hypothesized that agents consume a constant, fixed fraction of each additional dollar earned.⁹ This fixed fraction is known as the *Marginal Propensity to Consume*. In applying the rule-of-thumb idea to the lifetime of a representative agent, the work-life consumption program (for an agent who participates in a pay-as-you-go social security program) is denoted as

$$c(t) = (1 - s)(1 - \theta)w(t) \quad \forall t \in [0, T] \quad (1)$$

within a continuous-time setup.¹⁰ t represents time, wherein the agent enters the labor force at $t = 0$ and retires at $t = T$. Furthermore, $w(t)$ is the economy-wide wage earned at any instant, s is the constant (fixed) saving rate, and θ is the social security tax rate.¹¹ Wages grow at a constant rate δ such that $w(t) = w(0)e^{\delta t}$. It is evident from (1) that the *Marginal Propensity to Consume* is $\partial c(t)/\partial w(t) = (1 - s)(1 - \theta)$ for $t \in [0, T]$, meaning that a one unit increase in labor income generates an increase in period consumption equal to $(1 - s)(1 - \theta)$. A special case of rule-of-thumb behavior is that of “hand-to-mouth” consumers who don’t save any of their wage income (i.e., $s = 0$). For hand-to-mouth consumers the *Marginal Propensity to Consume* is equal to $1 - \theta$, meaning that period consumption is simply equal to disposable (after-tax)

⁸See Branson (1989), Dornbusch et al (1998), and Romer (2001).

⁹The constant fraction of each additional dollar saved is the agent’s time-independent saving rate.

¹⁰See Findley and Caliendo (Forthcoming) for a similar setup in a different application.

¹¹I abstract from all other forms of taxation (including capital taxation) throughout this dissertation. On a different note, (1) denotes that the rule-of-thumb agent saves from disposable (after-tax) wage income. Alternatively, (1) could be replaced with $c(t) = (1 - s - \theta)w(t)$ which would denote that the agent is following a rule-of-thumb applicable to pre-tax wage income.

wage income.

With the rule-of-thumb agent passing away at $t = \bar{T}$, consumption during the retirement phase of the life cycle is given as

$$c(t) = A + b(t) \quad \forall t \in [T, \bar{T}] \quad (2)$$

where A is a constant annuity paid out during retirement (from accumulated savings), and $b(t)$ is the social security benefit received at each instant. In a pay-as-you-go program, social security benefits are

$$b(t) = R\theta w(t) \quad \forall t \in [T, \bar{T}] \quad (3)$$

where R is the ratio of workers being taxed to retirees receiving benefits in the economy.¹² Consistent with this rule-of-thumb consumption/saving behavior, the agent's savings account, $k(t)$, earns interest at a constant rate, r , and evolves according to the following differential equations and boundary conditions

$$\frac{dk(t)}{dt} = rk(t) + s(1 - \theta)w(t) \quad \forall t \in [0, T] \quad (4)$$

$$\frac{dk(t)}{dt} = rk(t) - A \quad \forall t \in [T, \bar{T}] \quad (5)$$

$$k(0) = 0 \quad k(\bar{T}) = 0 \quad (6)$$

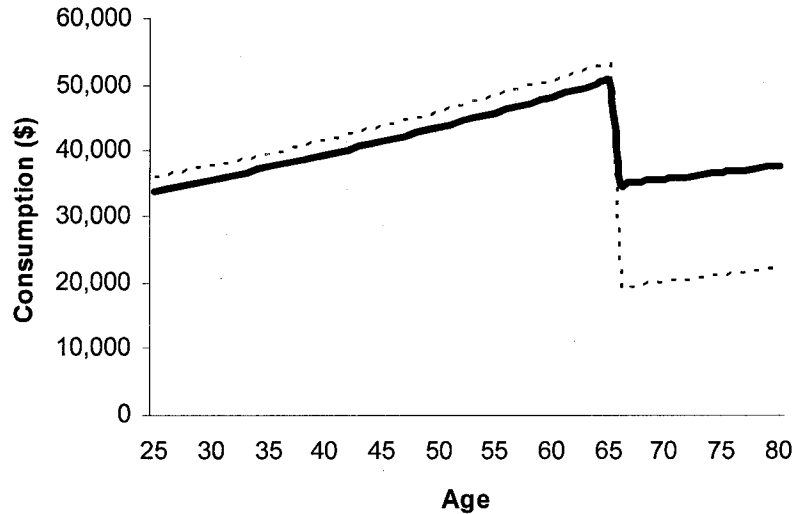
In order to pin down the rule-of-thumb agent's constant retirement annuity in (2), it can be shown from (4)–(6) that

¹²Equation (3) implies a balanced social security budget. The U.S. system has been collecting surpluses of tax revenues at least since 1983 (Smetters 2004). These surpluses have been lent to the U.S. Treasury at interest (the total sum of which has been labeled the “Social Security Trust Fund”). There is an ongoing debate about whether the Trust Fund can be considered a real asset reflecting increases to national saving. For more on this discussion see Elmendorf and Liebman (2000), Smetters (2003), Diamond and Orszag (2004), Smetters (2004), Diamond and Orszag (2005), and Diamond (2006). Equation (3), combined with (1), implies that the agent bears the full burden of the tax.

$$A = \frac{\int_0^T s(1 - \theta)w(t)e^{r(T-t)} dt}{\int_T^{\bar{T}} e^{r(T-t)} dt} \quad \forall t \in [T, \bar{T}] \quad (7)$$

The rule-of-thumb agent's consumption path over the life cycle is piece-wise continuous and exhibits a jump at retirement that depends on the amount of saving during the working years. The simulated life-cycle consumption paths of a rule-of-thumb agent who saves 5 percent of wage income (the solid line) and of a hand-to-mouth agent (the dashed line) are given in Figure 1 below.¹³ Both the rule-of-thumb agent and the hand-to-mouth agent participate in social security.

Figure 1. Rule-of-Thumb and Hand-to-Mouth Consumption



Note. The solid line denotes rule-of-thumb consumption at $s = 0.05$, and the dashed line is hand-to-mouth consumption (i.e., $s = 0$).

¹³The parameter values for these simulated consumption profiles are the relevant parameter values taken from the baseline calibration discussed in Chapter Four (with the addition of $\delta = 0.01$, and also $s = 0.05$ and $s = 0$ for the rule-of-thumb and hand-to-mouth agents respectively).

The hand-to-mouth agent enjoys a higher level of work life consumption compared to the agent who has a $MPC = 0.95$, yet during retirement the agent with $s = 0.05$ lives off of both private savings and social security benefits. In contrast, the hand-to-mouth agent subsists solely from social security benefits during retirement.¹⁴

2.1.2 Life-Cycle Consumption and the Permanent-Income Rule

The rule-of-thumb framework has not been able to account for several features of aggregate consumption data. In response, the *Life-Cycle Hypothesis* (Ando and Modigliani 1963) and the *Permanent-Income Hypothesis* (Friedman 1957, 1963) were separately offered as models of consumer behavior over the life cycle. The *Life-Cycle Hypothesis* essentially suggests that lifetime resources are taken into account when making consumption decisions, and the *Permanent-Income Hypothesis* suggests that consumption is primarily dependent on permanent (average) income over the planning horizon (and secondarily, on transitory deviations around permanent income).¹⁵ A life-cycle/permanent-income (LC/PI) agent optimally formulates a consumption program that equates the marginal utility of consumption across periods. Both of these hypotheses are represented below within a continuous-time optimal control setup.

INSTANTANEOUS UTILITY FUNCTION¹⁶ A formulation of the preference structure of an optimizing agent will first be examined as background. It is often convenient to work with an instantaneous utility function, $u[c(t)]$, that exhibits a constant elasticity of marginal utility of consumption.¹⁷ The constant elasticity of marginal utility of consumption, ϕ , can be expressed as

¹⁴Campbell and Mankiw (1989, 1990) econometrically estimate that roughly one-half of the U.S. population is comprised of hand-to-mouth consumers.

¹⁵See Branson (1989), Speight (1989), Deaton (1992), Dornbusch et al (1998), and Romer (2001) for more on this.

¹⁶This exposition follows Simon and Blume (1994).

¹⁷Following Romer (2001), De La Croix and Michel (2002), and Barro and Sala-i-Martin (2004), among others, this utility function will be of the CIES (constant intertemporal elasticity of substitution) variety. The usual name is the CRRA (constant relative risk aversion) utility function, but it will be referred to as CIES on account that there is no uncertainty in the models within this study.

$$\phi \equiv -\frac{u''(c)}{u'(c)}c = -\left(\frac{d[u'(c)]}{dc} \frac{c}{u'(c)}\right) \quad (8)$$

with the time arguments suppressed. It can be rewritten as

$$u''(c) = -\frac{\phi u'(c)}{c} \quad (9)$$

This second-order differential equation can be converted to a first-order differential equation by letting $v(c) \equiv u'(c)$ such that

$$\frac{d[v(c)]}{dc} = -\phi \frac{1}{c}v(c) \quad (10)$$

Algebraic separation of variables and integration of both sides of the equation gives

$$\int \frac{1}{v(c)}d[v(c)] = -\phi \int \frac{1}{c}dc \quad (11)$$

which is the same as

$$\ln[v(c)] = -\phi \ln[c] - \phi m_1 \quad (12)$$

where m_1 is a constant of integration. Exponentiating both sides of the equation yields

$$v(c) = m_2 c^{-\phi} \quad (13)$$

where $m_2 \equiv e^{-\phi m_1}$. Given $v(c) \equiv u'(c)$ from above

$$\frac{d[u(c)]}{dc} = m_2 c^{-\phi} \quad (14)$$

Separation of variables and integration of both sides of the equation gives

$$\int d[u(c)] = m_2 \int c^{-\phi}dc \quad (15)$$

The period utility function is therefore

$$u(c) = \begin{cases} m_2 \frac{c^{1-\phi}}{1-\phi} + m_3 & \text{for } \phi \neq 1 \\ m_2 \ln[c] + m_3 & \text{for } \phi = 1 \end{cases} \quad (16)$$

where m_3 is a constant of integration. With time arguments included, the utility function becomes

$$u[c(t)] = \begin{cases} \frac{c(t)^{1-\phi}}{1-\phi} & \text{for } \phi \neq 1 \\ \ln[c(t)] & \text{for } \phi = 1 \end{cases} \quad (17)$$

for the particular constants of integration, $m_2 = 1$ and $m_3 = 0$. This utility function possesses a constant elasticity of marginal utility of consumption since it is a particular solution to the second-order differential equation, (9), above.¹⁸ Because the agent's choices are invariant to affine transformations of the utility function, it is often convenient to succinctly represent (17) as

$$u[c(t)] = \frac{c(t)^{1-\phi} - 1}{1 - \phi} \quad (18)$$

wherein $u[c(t)] = \ln[c(t)]$ is just a special case of (18) as $\phi \rightarrow 1$ in the limit.¹⁹

A LC/PI OPTIMAL CONTROL PROBLEM A continuous-time version of the life-cycle/permanent-income framework (that is consistent with Browning and Crossley 2001, Bütler 2001, and Romer 2001, among others) is presented below using the variables introduced above. The agent enters the workforce at $t = 0$, retires at $t = T$, and dies at $t = \bar{T}$. The dates of retirement and death are exogenous. There is no uncertainty of any kind in the model. Wage income, $w(t)$, is earned for all $t \in [0, T]$, and grows at rate δ , again meaning that $w(t) = w(0)e^{\delta t}$. All wage income that is not consumed flows into the asset account, $k(t)$, which grows

¹⁸The constant value of ϕ denotes how willingly the agent intertemporally shifts consumption across different periods of the life cycle.

¹⁹This limiting result can be demonstrated using *L'Hôpital's Rule*.

at a rate of interest, r . The social security tax rate is θ . The agent receives social security benefits, $b(t)$, during the retirement phase of life, where $b(t) = R\theta w(t)$. The life-cycle/permanent-income agent has perfect foresight over the entire life-cycle, passes away with no assets, and constructs a consumption program that maximizes discounted life-cycle utility.

Combining the above information, the life-cycle/permanent-income optimal control problem is

$$\max \int_0^{\bar{T}} e^{-\rho t} \frac{c(t)^{1-\phi} - 1}{1-\phi} dt \quad (19)$$

where ρ is a constant rate of time preference. The life-cycle/permanent-income agent's asset account evolves according to the following equations of motion and boundary conditions.

$$\frac{dk(t)}{dt} = rk(t) + (1-\theta)w(t) - c(t) \quad \forall t \in [0, T] \quad (20)$$

$$\frac{dk(t)}{dt} = rk(t) + b(t) - c(t) \quad \forall t \in [T, \bar{T}] \quad (21)$$

$$k(0) = 0 \quad k(\bar{T}) = 0 \quad (22)$$

The control variable is consumption, and application of the *Maximum Principle* for fixed-endpoint two-stage optimal control problems will yield a set of first-order conditions.²⁰ Although the full set is not enumerated here for the sake of brevity, (23) will result from mathematical manipulation of the first-order conditions

$$c(t)^{-\phi} = j_1 e^{(\rho-r)t} \quad \forall t \in [0, \bar{T}] \quad (23)$$

where j_1 is a constant of integration. For the case where the rate of time preference is equal to the interest rate, $c(t)^{-\phi}$ is simply equal to j_1 meaning that the

²⁰Two-stage optimal control techniques are required due to a switch in the state equation at $t = T$. One-stage techniques could otherwise be used if all wage-income flows and social security benefits flows are compressed into an initial endowment.

agent's *marginal utility of consumption* is time-independent. This suggests that a life-cycle/permanent-income agent desires to smooth his marginal utility evenly across the life cycle.

How is marginal utility smoothing achieved? The first-order conditions can be solved to produce analytical solutions for the *optimal state path* and the *optimal control*. These solutions are respectively given as (24) for $t \in [0, T]$, (25) for $t \in [T, \bar{T}]$, and (26) for $t \in [0, \bar{T}]$ where the agent's growth rate of consumption is given as $g \equiv \frac{r-\rho}{\phi}$.

$$k^*(t) = \frac{w(0) [e^{rt} - e^{gt}] \{ (1 - \theta) [e^{(\delta-r)T} - 1] + R\theta [e^{(\delta-r)\bar{T}} - e^{(\delta-r)T}] \}}{(\delta - r) [e^{(g-r)\bar{T}} - 1]} + \frac{w(0) (1 - \theta) [e^{\delta t} - e^{rt}]}{(x - r)} \quad (24)$$

$$k^*(t) = \frac{w(0) [e^{(g-r)\bar{T}+rt} - e^{gt}] \{ (1 - \theta) [e^{(\delta-r)T} - 1] + R\theta [e^{(\delta-r)\bar{T}} - e^{(\delta-r)T}] \}}{(\delta - r) [e^{(g-r)\bar{T}} - 1]} + \frac{w(0)R\theta [e^{\delta t} - e^{(\delta-r)\bar{T}+rt}]}{(\delta - r)} \quad (25)$$

$$c^*(t) = \frac{w(0) \{ (1 - \theta) [e^{(\delta-r)T} - 1] + R\theta [e^{(\delta-r)\bar{T}} - e^{(\delta-r)T}] \} e^{r\bar{T}+gt} (g - r)}{(\delta - r) [e^{g\bar{T}} - e^{r\bar{T}}]} \quad (26)$$

It should be noted that (26) results in a time-independent (flat and smooth) consumption profile if $r = \rho$, meaning that $g = 0$.²¹ Therefore, maximal life-cycle utility and marginal utility smoothing are achieved by smoothing consumption when the interest rate and discount rate are equal.

THE ANNUITY VALUE OF LIFETIME WEALTH It is straightforward to show that a utility-maximizing agent's period consumption, as denoted by (26), is exactly equal to the annuity value of lifetime wealth (or permanent income) when the

²¹Consumption monotonically increases over the planning horizon if $r > \rho$ (i.e., the agent is relatively patient), and consumption monotonically decreases if $r < \rho$ (i.e., the agent is impatient).

interest rate equals the rate of time preference. To demonstrate this, $K(t)$ is defined as the agent's "wealth account" for $t \in [0, \bar{T}]$, and (27) is a differential equation that governs the evolution of this wealth account over the entire lifetime of the agent.

$$\frac{dK(t)}{dt} = rK(t) - A \quad \forall t \in [0, \bar{T}] \quad (27)$$

A is a constant annuity that exactly exhausts the agent's lifetime wealth at the date of death, meaning that $K(\bar{T}) = 0$. The initial condition is

$$K(0) = \int_0^T e^{-rt} w(t) dt - \int_0^T e^{-rt} \theta w(t) dt + \int_T^{\bar{T}} e^{-rt} b(t) dt \quad (28)$$

meaning that the agent is endowed up-front with the present value of his total lifetime wealth. The general solution to (27) is

$$K(t) = \left\{ j_2 - A \int^t e^{-r\tau} d\tau \right\} e^{rt} \quad \forall t \in [0, \bar{T}] \quad (29)$$

where j_2 is a constant of integration. Using the initial condition, the constant becomes

$$j_2 = K(0) + A \int_0^0 e^{-r\tau} d\tau \quad (30)$$

Employing the *Fundamental Theorem of Calculus*, the particular solution to (27) is

$$K(t) = \left\{ K(0) + A \int_t^0 e^{-r\tau} d\tau \right\} e^{rt} \quad \forall t \in [0, \bar{T}] \quad (31)$$

And using $K(\bar{T}) = 0$,

$$K(0) + A \int_{\bar{T}}^0 e^{-r\tau} d\tau = 0 \quad (32)$$

With some algebraic rearrangement,

$$A = \frac{K(0)}{\int_0^{\bar{T}} e^{-rt} dt} \quad (33)$$

where the denominator of (33) is the amount needed to sustain a one-dollar annuity over the entire lifetime of the agent. Therefore, (33) is the constant annuity value that (if consumed) exactly exhausts lifetime wealth at the date of death. Substituting in from (28) and integrating yields

$$A = \frac{rw(0) \{ (1 - \theta) [e^{(\delta-r)T} - 1] + R\theta [e^{(\delta-r)\bar{T}} - e^{(\delta-r)T}] \}}{(\delta - r) [1 - e^{-r\bar{T}}]} \quad (34)$$

(34) is exactly equal to (26) when $r = \rho$. Therefore, if the interest rate is equal to the constant discount rate, then the period consumption of the agent is simply equal to the annuity value of his lifetime wealth. This additionally suggests that the agent smooths out lifetime wealth evenly across the life cycle in order to achieve marginal utility smoothing and maximal lifetime utility.

PARTICIPATION IN PAY-AS-YOU-GO SOCIAL SECURITY In order to appropriately address how a life-cycle/permanent-income agent fares in a pay-as-you-go social security system, it is useful to calculate the internal rate of return (*IRR*) from participation even though social security taxes are not diverted into interest-earning accounts. By definition, the real (constant) internal rate of return, γ , solves the following equation

$$\int_0^T e^{\gamma(T-t)} \theta w(t) dt = \int_T^{\bar{T}} e^{\gamma(T-t)} b(t) dt \quad (35)$$

(35) calculates the longitudinal *IRR* for a representative agent in a life-cycle cohort.²² The left-hand side of (35) is the value of all social security taxes paid by the representative individual during the working period (valued at the date of retirement), and the right-hand side is the amount needed at retirement to sustain benefit withdrawals equal to $b(t)$ for $t \in [T, \bar{T}]$. With parameter values outlined in the baseline calibration from Chapter Four (and with the addition of $\delta = 0.01$), $\gamma = 0.017$ given 3.3 workers for every retiree receiving benefits (i.e., $R = 3.3$). In a future state of the world where the worker-to-retiree ratio is 2 (i.e., $R = 2$), the *IRR* drops slightly below zero to $\gamma = -0.007$.²³ Furthermore, the worker-to-retiree ratio would need to be around 5.5 in order for a pay-as-you-go program to equal the return from the market (i.e., if $R = 5.5$ then $r = \gamma = 0.035$). This longitudinal measure of the internal rate of return implicitly assumes that the economy is in a steady-state equilibrium with stable population growth.²⁴

The simulated consumption profiles for a life-cycle/permanent-income agent who participates in unfunded social security (the solid line) and for an otherwise identical agent who does not participate (the dashed line) are represented in Figure 2 below.²⁵

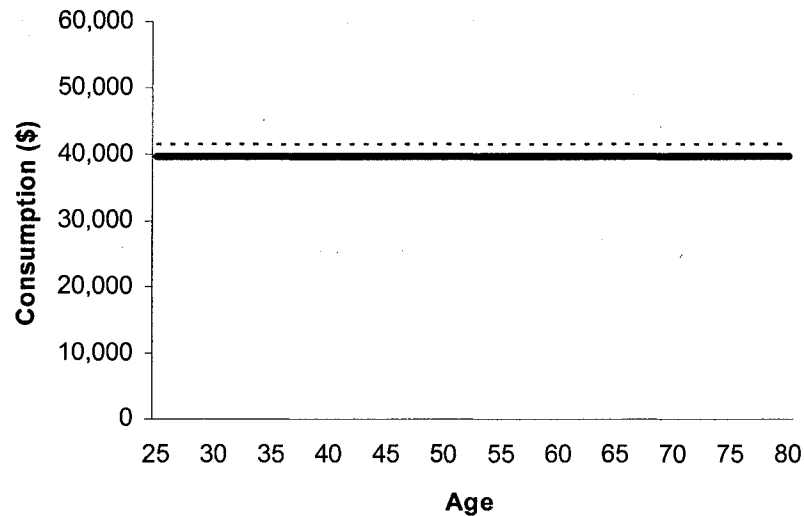
²²A longitudinal *IRR* is the rate of return that equates the flow of tax contributions from an individual to the flow of benefits payments that the individual receives. Alternatively stated, the longitudinal *IRR* is the rate that ensures that the net present value is equal to zero. A different measure is the cross-section *IRR* which is the rate of return that ensures that the net present value of an unfunded social security system is zero over a period of arbitrary length. See Settergren and Mikula (Forthcoming) for an expanded discussion of the cross-section *IRR*.

²³The drop in the ratio of workers to retirees from 3.3 to 2 reflects shifting U.S. demographics over the next several decades (Goss 2005/2006).

²⁴Samuelson (1958, 1975) shows that the *IRR* is a “biological interest rate” that equals the rate of population growth within a two-period overlapping generations framework. Feldstein (1985) also employs a two-period OLG model and shows that the *IRR* on unfunded social security equals the rate of real wage growth plus the rate of population growth plus the product of the two rates. Arthur and McNicoll (1978) design a continuous-time OLG model with a continuum of life-cycle cohorts, and they analytically demonstrate that life-cycle welfare can often decrease as the rate of population growth increases even in the face of intergenerational transfers (i.e., unfunded social security). (35) gives the longitudinal *IRR* for only a life-cycle cohort without any explicit assumption concerning population growth. However, $b(t)$ in (35) is a function of R , the worker-to-retiree ratio. The constant value of R implies that a time-independent rate of population growth exists to ensure that R remains constant across the life cycle. A change to a different constant value of R implies a uniquely different constant rate of population growth within a uniquely different steady state.

²⁵The parameter values for these simulated consumption profiles are also the relevant parameter values taken from the calibration that is presented in Chapter Four (with the addition of $\delta = 0.01$).

Figure 2. LC/PI Consumption with and without Social Security



Note. The solid line is life-cycle consumption with $\theta = 0.106$, and the dashed line is life-cycle consumption with $\theta = 0$.

The consumption profile of the agent who participates in social security (at today's demographics with $\gamma = 0.017$) is everywhere lower than the consumption profile of the identical agent who does not participate. If $\partial u / \partial c > 0$ (as is the case with the CIES utility function used above), then the agent who participates in social security has a strictly lower level of lifetime utility (compared to the agent who doesn't participate) because his consumption profile is everywhere lower over the life cycle. This results because the agent participates in a pay-as-you-go system that has an internal rate of return that is less than the interest rate, r . His lifetime wealth is smaller because $\gamma < r$, and therefore he smooths out this loss to lifetime wealth evenly across the life cycle.

Consequently, the question of how a perfect-foresight life-cycle/permanent-income agent fares in a pay-as-you-go program degenerates into a simple comparison of the internal rate of return from social security participation with the market rate of

return. If the internal rate of return is lower than the market interest rate (the case discussed above), then the life-cycle/permanent-income agent is strictly worse off from a lifetime utility perspective. If the *IRR* is greater than the market rate, then the agent is strictly better off (the consumption profile from this scenario would be everywhere higher compared to the dashed profile in Figure 2 above). Lastly, if the *IRR* is exactly equal to the market rate, then the agent is exactly indifferent between participating and not participating in a pay-as-you-go social security program (under this scenario the consumption profile would lie exactly on top of the dashed profile in Figure 2 above). When it comes to boundedly-rational impulsive consumers (who do not strictly behave according to the life-cycle/permanent-income rule over the entire life cycle), it is an incomplete exercise to simply compare the internal rate of return to the market rate of interest in order to determine lifetime welfare outcomes from social security participation. I revisit this topic in Chapter Five below.²⁶

²⁶I mention at this juncture some key details about my theoretical setup of unfunded social security. In the U.S., the actual amount of social security benefits for any particular individual depends on one's earnings history, age of retirement, and personal family demographics. More specifically, benefits are calculated from the *average indexed monthly earnings* (or AIME) which is an individual's average wages from qualified employment during the working years. Moreover, an individual's *primary insurance amount* (or PIA) is calculated from the AIME which is the basic social security benefit for a worker who retires with full eligibility (i.e., retires on or after age 67 if born in 1960 or later) and who does not have a dependent spouse or dependent children during the working years. The PIA is a specified fraction of the AIME which increases at a decreasing rate in the AIME (meaning that a worker with a lower AIME enjoys a relatively higher PIA than an otherwise identical worker with a higher AIME). An individual who retires early experiences a permanent cut in benefits whereas an otherwise identical individual who retires after the date of full eligibility receives a permanent increase in benefits. Lastly, family status also matters in calculating an individual's actual benefit, wherein a worker receives an additional fraction of the PIA if he has a dependent spouse or child. Throughout this dissertation, I abstract from retirement-date and household-size heterogeneity within my representative-agent framework. Otherwise, I would have one representative agent for each category of retirement-date and household-size possibilities within my model. Such heterogeneity would create accounting problems in my theoretical setup of an unfunded social security system (i.e., the social security budget within my framework would be extremely unmanageable to balance), and would likely render my analysis of pay-as-you-go social security intractable. I therefore analyze the welfare effects of unfunded social security for a life-cycle cohort with household-size and retirement-date homogeneity. Nevertheless, in Chapter Five I do entertain wage heterogeneity across a life-cycle cohort with a benefit rule that roughly proxies the redistributive characteristic of the actual PIA calculation. See Diamond and Orszag (2004) for more on the real-world details of social security benefit calculation in the U.S. system.

2.2 Aggregate Life-Cycle Consumption

2.2.1 Aggregate Work Life Consumption

Aggregate life-cycle consumption profiles in the U.S. (and other countries) are hump-shaped with a peak between 45–55 years of age (Thurow 1969; Browning and Crossley 2001; and Feigenbaum 2007). Furthermore, Feigenbaum (2007) reports that aggregate consumption profiles have a “peak size” between 1.1 to 1.5, meaning that peak aggregate consumption during the life cycle is 1.1 to 1.5 times that of initial consumption. These characteristics of aggregate consumption data are based on empirical work that uses synthetic-cohort techniques to econometrically construct consumption profiles that display the aforementioned features (Browning and Crossley 2001; Gourinchas and Parker 2002; Fernández-Villaverde and Krueger 2004; Bullard and Feigenbaum Forthcoming).²⁷

The outlined empirical findings (that aggregate consumption profiles are hump-shaped with the aforementioned ranges of location and peak size) have presented a prominent macroeconomic puzzle since the standard life-cycle/permanent-income framework predicts that aggregate consumption should be flat if the discount and interest rates are equal. Quoting Feigenbaum (2007), “The puzzle is not how the hump could possibly occur but rather which mechanism—or combination of mechanisms—can best account for the hump while also being consistent with other macroeconomic data.” A great deal of effort has been placed into solving this puzzle because macroeconomists care greatly about what is driving aggregate consumption which accounts for around 70 percent of Gross Domestic Product in the United States.

One possible explanation for the hump in aggregate consumption profiles is that current consumption is linked to labor earnings via a simple rule of thumb (Brown-

²⁷Currently there is insufficient panel data on consumption over the entire life cycle. Jianakoplos et al (1989), Deaton and Paxson (2000), among others, caution about using cross-sectional data to synthetically proxy life-cycle profiles that would ideally be generated from panel data sets. Nonetheless, the cited characteristics of aggregate life-cycle consumption data will be taken as given for the purposes of this study.

ing and Crossley 2001). A rule-of-thumb agent who consumes a constant fraction of current labor earnings will follow a life-cycle consumption program that is hump-shaped on account that labor-earnings profiles are hump-shaped.²⁸ Caliendo and Gahramanov (2007) theoretically study a mixed-economy of rule-of-thumb and life-cycle/permanent-income consumers, and they find that at least 30 percent of the population would need to consume according to a rule-of-thumb in order to generate an empirically-reasonable aggregate consumption profile. Given that Campbell and Mankiw (1989, 1990) empirically estimate that around one-half of the U.S. population lives hand-to-mouth (with the other one-half behaving according to the life-cycle/permanent-income rule), then aggregation of the consumption profiles of such rule-of-thumb agents (whose consumption profiles are essentially their wage-income profiles) would therefore produce an aggregate profile that is hump-shaped.

Several candidate solutions that augment the life-cycle/permanent-income framework have been offered to the consumption-hump puzzle. Adding liquidity constraints and precautionary saving against wage-income uncertainty has theoretically generated aggregate consumption profiles that match the empirical features (Nagatani 1972; Hubbard et al 1994; Carroll 1997; Gourinchas and Parker 2002). Accounting for differences in household size (Attanasio et al 1999), consumption-leisure tradeoffs (Heckman 1974; Bullard and Feigenbaum Forthcoming), mortality risk (Feigenbaum 2007), and securing loans with consumer durables (Fernández-Villaverde and Krueger 2001) have also produced empirically-reasonable hump-shaped profiles. Moreover, time-dependent discounting (Laibson 1997), short-term planning (Caliendo and Aadland 2007), and costs to implementing optimal saving plans (Caliendo and Huang 2007) have generated simulated profiles matching the aforementioned empirical data.

²⁸Competitively-determined real wages (per labor efficiency unit) grow exponentially at the rate of technology growth in a steady-state equilibrium, but age-productivity (labor efficiency unit) profiles are hump-shaped over the work life. Therefore, labor-earnings profiles are hump-shaped in growth-detrended data. See Browning and Crossley (2001), Gourinchas and Parker (2002), and Feigenbaum (2007) for more on this.

2.2.2 Aggregate Retirement Consumption

The standard life-cycle/permanent-income framework predicts that households will smooth consumption across the retirement threshold to the date of death. This is because period consumption is optimally set equal to the annuity value of lifetime wealth, and therefore, the exact timing of the receipt of income is irrelevant (assuming perfect capital markets). However, empirical work has documented that aggregate life-cycle consumption undergoes a discrete drop at the retirement threshold. Among several studies, Banks et al (1998) use synthetic-cohort methods and find a significant fall in consumption growth at retirement (reflecting a discrete drop in consumption) even after controlling for increased mortality risk, labor market status, work-related expenses, and household demographics. They suggest that this discrete fall in consumption reflects the arrival of unanticipated information that accumulated wealth is insufficient for consumption needs over the duration of retirement. After controlling for wealth heterogeneity, retirement timing, tastes for leisure, home production, work-related expenses, and bequest motives, Bernheim et al (2001) likewise find that consumption falls discretely at retirement with households experiencing a significantly larger drop in consumption if they arrive at retirement with lower levels of wealth. They conclude that the discrete drop reflects previous work life behavior that is outside the standard life-cycle/permanent-income framework.²⁹

As for the magnitude of the discrete fall in consumption at retirement, Ameriks et al (2007) report that households expect their consumption to fall discretely by 11 percent, yet consumption at retirement only falls by around 4–5 percent in their sample.³⁰ Hamermesh (1984) reports that consumption falls by approximately 9 percent

²⁹In addition, Hurst (2003) finds that if households experience a discrete drop in consumption at retirement, then such households were engaging in pre-retirement consumption behavior that is strictly inconsistent with the standard Neoclassical framework.

³⁰Around three-quarters of households hold advanced degrees in the Ameriks et al (2007) sample. They report that their results imply a 20 percent *expected* fall at retirement in samples that are more representative of the population. Therefore, it is likely that their results also imply that *actual* consumption falls by more than 5 percent at retirement in the population at large.

on average at retirement with over one-quarter of the sample reducing consumption by as much as 25 percent. Haider and Stephens (2007) document that consumption falls discretely by around 7–11 percent at retirement even after controlling for subjective retirement date expectations among many other things. Robb and Burbidge (1989) document that blue-collar workers experience a discrete drop of 15% at retirement as compared to no statistically-significant drop for white-collar workers. Hurd and Rohwedder (2005) report a 15–20 percent decline, which is consistent with the 10–20 percent estimated fall in Bernheim et al (2001). Banks et al (1998) estimate a discrete drop of 35 percent. Lastly, Mariger (1987) estimates that the drop is almost as high as 50 percent.³¹

Both Banks et al (1998) and Bernheim et al (2001) report that post-retirement consumption growth rates return to zero after the initial consumption correction at the retirement threshold. This implies that retirees smooth consumption over the remaining retirement years to the date of death. Browning and Crossley (2001) similarly state, “It appears that households at retirement recognize that they have made a mistake in consumption smoothing and that they need to scale back, but they do so fairly quickly and then smooth expenditures over the rest of life.” These empirical features will play a key role in the calibration of my theoretical model.

2.3 Boundedly-Rational Consumers

Models incorporating assumptions that agents are boundedly-rational have been proliferating in response to a large body of empirical/experimental work that has documented noticeable anomalies to standard Neoclassical economic theory. Thaler (2000) asserts that Neoclassical economic models consist of agents who are hyperrational in order to allow for analytical tractability, wherein “The IQ of Homo Economicus be-

³¹Robb and Burbidge (1989) illustrate a major methodological issue in Mariger’s functional specification of the *Euler Equation* that results in an overestimation of the magnitude of the discrete drop at retirement.

came bounded only by the IQ of the smartest economic theorist". There has been considerable theoretical and empirical effort that has borrowed findings from other fields (especially psychology) in order to provide alternative quantitative-theoretic explanations for behavioral anomalies. Thaler moreover asserts that bounded-rationality models will provide additional (more realistic) insight into economic behavior, especially as quantitative modeling techniques become increasingly sophisticated. Research within this framework has been dubbed "Behavioral Economics".³²

2.3.1 Saving and Consumption

Life-cycle consumption and saving behavior is no exception to behavioral anomalies, wherein it has been observed that agents often depart from consumption-smoothing behavior over the life cycle. In addition to the evidence discussed above, Thaler (1990), Carroll (1994), and Shea (1995) show that consumption is improperly sensitive to wealth and income. Choi et al (2006a) find that many future retirees would like to increase saving because they consider themselves to be inadequate savers. However, they also find that few possess the willpower to actually follow through and increase their saving. Poterba (1988) moreover finds that many need outside help in order to increase saving although certain people may already optimally save. This evidence is fundamentally inconsistent with the standard life-cycle/permanent-income framework. Akerlof (2002) states

For New Classical Economics, saving too little or too much, like involuntary unemployment, is an impossibility, a straightforward contradiction of the assumptions of the model. Since saving is the result of individual utility maximization, it must, absent externalities, be just right.

³²See Conlisk (1996) for a survey of bounded-rationality modeling. McCaffery and Slemrod (2006) recently called for the use and application of bounded-rationality models to resolve several puzzles and issues within the field of public finance. They labeled this sub-field "Behavioral Public Finance". The present study fits nicely into this new area of research by addressing the question of how impulsive consumers fare in an unfunded social security program.

The Save More TomorrowTM (or SMarT) plan of Thaler and Benartzi (2004) has provided remarkable success in providing a vehicle to help increase saving, especially among low-income households.³³ The SMarT plan simply has workers commit in the present to save a higher fraction of future wage increases. This is popular for several reasons. Workers gladly agree to procrastinate saving to the future (procrastination is a common symptom of present-biased preferences like hyperbolic discounting). Workers are not required to take a present reduction in their level of consumption which is appealing for reasons relating to loss aversion. Lastly, inertia causes savers to stick with the plan rather than expelling the effort to opt out. The SMarT plan has already made an impressive impact with potential enrollees.³⁴ Furthermore, the SMarT plan has been popular with the U.S. Congress, evidenced by recent legislation that encourages SMarT default rates in 401(k) saving plans (Moore 2006; U.S. Congress 2006).³⁵

Why is it that many consumers experience difficulty in following or sticking to the life-cycle/permanent-income program? Thaler (1994), Lusardi (1999), and Rabin (1999) assert that the permanent-income rule is an extremely difficult problem to solve/deduce, and furthermore, there aren't many opportunities for learning from one's mistakes when it comes to saving for retirement. Specific behavioral characteristics might also account for why consumption smoothing is often violated. Rule-of-thumb consumption and hyperbolic discounting behavior are possibilities. Another possibility is that of short planning horizons.³⁶ There has been considerable empirical evidence, such as the Health and Retirement Study and the Survey of Consumer

³³Save More Tomorrow is a registered trademark of Richard Thaler and Shlomo Benartzi.

³⁴Approximately 65 percent of workers in the Retirement Confidence Survey say they would like to participate in a SMarT plan (Helman et al 2006).

³⁵Another device that has helped increase saving is automatic enrollment (Madrian and Shea 2001; Choi et al 2006a; Choi et al 2006b). *The Pension Protection Act of 2006* also includes automatic enrollment default rules (U.S. Congress 2006).

³⁶Caliendo and Aadland (2007) offer a formal bounded-rationality model by incorporating a short planning horizon into the life-cycle framework, and they analyze aggregate life-cycle consumption implications under such behavior.

Finances, wherein individuals possess planning horizons that are much shorter than the lifetime planning horizons that are assumed in the standard framework. Carroll and Summers (1991) and Carroll (2001) report that Milton Friedman (1963) originally asserted and cited empirical evidence that the *MPC* out of transitory income is one-third (implying a short 3-year planning horizon). Furthermore, Lusardi (1999, 2003) finds a statistically significant link between planning horizon length and wealth accumulation. Munnell et al (2001/2002) also find that the length of the planning horizon is the single-most important factor determining 401(k) participation. In addition to rule-of-thumb behavior, time-dependent discounting, and short planning horizons, I propose the new possibility that consumers often have difficulty following (or sticking to) the life-cycle/permanent-income rule because they experience and succumb to consumption impulses. Before proceeding with theoretical analysis, I review a large literature and much evidence on impulsive consumer behavior from other academic fields.

2.3.2 Impulsive Consumers and Empirical Evidence

IMPULSIVE CONSUMPTION BEHAVIOR Impulsive consumption behavior is well documented in the psychology, psychiatry, and consumer research literature, but it has yet to be rigorously examined in the economics discipline. So what exactly is impulsive behavior with respect to consumption spending? Baumeister (2002) states,

Impulsive behavior is understood as behavior that is not regulated and that results from an unplanned, spontaneous impulse. In particular, impulsive purchasing involves getting a sudden urge to buy something, without advance intention or plan, and then acting on that impulse without carefully or thoroughly considering whether the purchase is consistent with one's long-range goals, ideals, resolves, and plans. Impulsive behavior is most interesting and relevant when it contradicts some of those long-term goals (such as saving money) because the person may regret having yielded to the impulse.

Moreover, Rook (1987) outlines several related sentiments that consumers often encounter upon experiencing a consumption impulse: Feeling an overwhelming force from a particular good or service; an intense feeling of immediacy; indifference toward any possible negative consequences of impulsively purchasing; feelings of euphoria; and, an inner tension between indulgence and self-control.

The idea of impulsive behavior dates back to Sigmund Freud (1911/1956) who describes impulses as being the products of a pleasure principle and a reality principle. The pleasure principle encourages immediate gratification but it is tempered by the reality principle (or rational deliberation).³⁷ These two forces counter each other, but at times the pleasure principle will beat out the reality principle, meaning that the individual will periodically, but temporarily, indulge impulses. In addition, an individual might lack sufficient physiological or psychological capability to consistently counter the pleasure principle with rational deliberation, and severe, chronic psychological problems can result in such acute cases (Kipnis 1971; Winshie 1977).

Wood (1998) delineates between two different types of consumption impulses that consumers face: *Compulsive impulses*, and *akratic impulses* with a strong and weak form. Consistent with the notion of one lacking sufficient capacity to regularly counter the pleasure principle, compulsive-consumption impulses tend to stem from an underlying physiological or psychological mechanism that generates unbearable urges to spend. Consumers experiencing compulsive impulses often feel as if they have been uncontrollably compelled by an outside force to purchase, or they feel as if they have an unendurable addiction to consume. In contrast, an akratic-consumption impulse is a “weakness of will” to abstain from unplanned purchases that are contrary to the consumer’s better judgement.³⁸ More specifically, an individual experiences a

³⁷Freud’s outline is conceptually similar to Thaler and Shefrin’s (1981) notion of a doer-self who is mitigated by a rational planner-self.

³⁸The idea of an akratic impulse being a “weakness of will” does not mean that the consumer is enslaved to the impulse without escape (as in the case of compulsive impulses). Succumbing to akratic impulses is similar to the idea of periodically, but temporarily, indulging impulses by giving in to the pleasure principle.

strong akratic impulse for consumption when he has the sudden, unplanned urge to buy something that is contemporaneously realized with regret to be contrary to his better judgement at the time of purchase. Alternatively, unplanned consumption is viewed as being the result of a *weak akratic impulse* when the unplanned purchase is only retrospectively judged with regret, such as an *ex post* realization that an item purchased on sale was not quite the bargain that it was made out to be at the time of the unplanned purchase. DeSarbo and Edwards (1996) reinforce the dichotomy between compulsive- and akratic-consumption impulses by asserting that compulsive impulses result from an “internal trigger” or psychological disorder from which impulsive spending becomes an escape, and that an akratic impulse is the result of an “external trigger” that stimulates the consumer to impulsively buy.

Impulsive consumers engage in time-inconsistent behavior. Hoch and Loewenstein (1991) offer a working definition of time inconsistency, wherein “A time-inconsistent choice is one that would not have been made if it had been contemplated from a removed, dispassionate perspective.” The notion of time-inconsistency or time-inconsistent preferences is closely related to the phenomenon of regret that impulsive consumers often experience. In highlighting that time-inconsistent behavior conflicts with Neoclassical economic theory, Bayley and Nancarrow (1998) state, “Economists argue that at the time of purchase the value of the impulse buy outweighs its perceived cost, but this switches post-purchase with consequent feelings of regret.” It therefore appears that if a consumer regrets a decision, then it is likely that the decision in question would not have been made as judged from a different frame of reference. In relating the two ideas of time-inconsistency and regret, Wood (1998) specifically states, “Consumer dissatisfaction and regret are the cognitive and experiential side of what economists behavioristically term time-inconsistent preferences.” A model of impulsive consumption behavior (that is consistent with the statements above) will need to incorporate intertemporal time inconsistency. An impulsive consumer could

sometimes follow an optimal program for an interval of time, and then impulsively consume an amount that is above the optimal program. The time-inconsistent behavior of such a consumer could be represented with a process of continual re-optimization (optimizing corrections) following periods of impulsiveness.

COMPULSIVE-IMPULSIVE CONSUMERS Underlying psychological problems can be manifest by compulsive-consumption impulses wherein individuals chronically and uncontrollably spend. O’Guinn and Faber (1989) state, “[Compulsions] are often excessive and ritualistic behaviors designed to alleviate tension, anxiety, or discomfort aroused by an obtrusive thought or obsession.” This suggests that consumption behavior is a vehicle through which compulsive impulses are repetitively alleviated. In fact, Black et al (1998) report that compulsive-buying behavior is officially classified as an Axis I Impulse Control Disorder Not Otherwise Specified in DSM-IV by the *American Psychiatric Association*. Furthermore, Christenson et al (1994) and Black et al (1998) find strong evidence indicating that compulsive-impulsive consumers are much more likely to have clinical lifetime psychiatric comorbidity with other Axis I mood disorders such as abnormally high levels of clinical depression, substance abuse, eating disorders, and clinical anxiety. In addition to Axis I disorders, Schlosser et al (1994) find that nearly 60 percent of subjects who experience compulsive-consumption impulses also meet the criteria for at least one Axis II personality disorder with obsessive-compulsive, avoidant, and borderline personality disorders being the most frequently associated types. This is supported by Frost et al (1998) who report a statistically-significant relationship between actual compulsive-impulsive consumption behavior and *Obsessive-Compulsive Disorder* or OCD. Their finding suggests that impulsive consumption is a compulsion (mechanism) through which psychological obsessions get alleviated.³⁹

³⁹Chamberlain and Sahakian (2007) assert that impulsive behavior is also linked to *Attention Deficit Hyperactivity Disorder* or *ADHD*, wherein they specifically state, “ADHD is regarded by

With respect to the genetic underpinnings of impulse control disorders (such as compulsive-impulsive consumption), Comings (1998) reports a significant genetic association between impulsive disorders and the dopamine D1 gene (or DRD1), dopamine D2 gene (or DRD2), dopamine D3 gene (or DRD3) and dopamine D4 gene (or DRD4). He concludes that this relationship indicates an inherited susceptibility to genetic abnormalities of the dopamine reward pathways, which abnormalities give rise to impulse control disorders.⁴⁰ This is supported by Chamberlain and Sahakian (2007) who report that neuroimaging techniques have isolated the *Right Pre-Frontal Cortex* area of the brain, specifically the *Right Inferior Frontal Gyrus*, as being the part of the brain that inhibits compulsive-impulsive behaviors. Individuals with damage to or lesions on the Right Inferior Frontal Gyrus of the Right Pre-Frontal Cortex of the brain display a diminished ability to control compulsive-impulsive behaviors compared with control groups. Furthermore, a negative relationship is also reported between the amount of damage to or lesions on the Right Inferior Frontal Gyrus and the ability to inhibit impulsive behavior.⁴¹

Estimates of the population-wide prevalence of compulsive-impulsive consumers range anywhere from 2-8 percent (Faber and O'Guinn 1992), to 5-8 percent (Black et al 1998), and to even as high as 16 percent of the adult U.S. population (Magee 1994). Koran et al (2006) use a large general population sample and find that 5.8 percent of the sample is clinically affected. Given such prevalence, can anything be done to help these compulsive-impulsive consumers who suffer from this clinical disorder? McElroy et al (1991), Black et al (1997), Bullock and Koran (2003), Aboujaoude et al (2003), Grant (2003), and Koran et al (2003) report clinical work that shows some promise in treating compulsive-impulsive consumers via professional psy-

many as an archetypal disorder of impulsivity...Into adulthood, ADHD is associated with impulsive phenomena..."

⁴⁰But on the other hand, Devor et al (1999) fail to find an underlying genetic association for compulsive spending impulses in their study.

⁴¹As an additional insight, transcranial magnetic stimulation of the Right Pre-Frontal Cortex temporarily impairs the inhibition of impulsive behavior in healthy subjects (Chambers et al 2006).

chiatric and pharmacological drug treatment.⁴² Black (2007) alternatively reports that the body of results to date have been mixed depending on the specific treatment regimen. This is supported by Chamberlain and Sahakian (2007) who report that serotonin-based medications have little effect on inhibiting impulsive behavior, but that certain psychostimulants have effectively inhibited impulsive behaviors by increasing noradrenaline and dopamine levels in the Pre-Frontal Cortex areas of the brain. In addition to psychotropic medication, cognitive-behavioral group therapy also appears helpful in treating compulsive-impulsive consumers (Black 2007).

AKRATIC-IMPULSIVE CONSUMERS It has been estimated that between 30 percent to 60 percent of all purchases are unplanned (Kollat and Willett 1967; Bellinger et al 1978; Cobb and Hoyer 1986; Han et al 1991; Wood 1998), wherein a sizable portion of these unplanned purchases possibly result from akratic-consumption impulses. Welles (1986) reports that most shoppers at least occasionally purchase on impulse, and Cobb and Hoyer (1986) report an upward trend in unplanned akratic-impulse purchases. Furthermore, 38 percent of adults in a national survey between 1975 and 1992 responded affirmatively to the question, "I am an impulse buyer" (Rook and Fisher 1995).⁴³ For expositional purposes, several anecdotal statements (as reported in Rook 1987) by consumers who experienced the akratic-consumption impulse are listed below.

- It's the feeling of "I want that, and by God I'm gonna get it!" (Female-48-describing a dress)
- For me it was a total mind filling experience. I could only think of one thing, and that was where I was going to put it when I got home. (male-68-describing a painting)
- The item you are sucked into stands out from the rest. As soon as you see it

⁴²Lorin Koran reports that one particular subject in the Koran et al (2003) study had purchased over 55 cameras, while a separate subject had bought over 2,000 wrenches (*Science Daily* 2003).

⁴³On account of the usual problems with self-reported data, it is very possible that the actual percentage of adults who are impulse buyers is actually higher.

you stop walking and stare at it for a few minutes, then it suddenly strikes your head and gives you goosebumps. (male-27)

- It felt like something that you had been looking for a long time had appeared before your eyes, and if you don't buy it now you won't have another chance. It is just the right place and time. (female-37-describing a pair of shoes)
- I was standing in the grocery store checkout line, and the candy bar was staring there at me. (male-26)
- The pants were shrieking "buy me", so I knew right then that I better walk away and get something else done. (female-35)
- I had gone on to a different department, but the sweater was following me. I felt like it was pulling me back to the men's department, where I finally bought it. (male-38)
- There is no stopping me. The urge just comes over me all at once and seems to take control. It is such an overwhelming feeling that I just have to go along with it. (male-52)
- The feeling I get when I suddenly have the urge to buy something is PANIC—rushing to get to a checkout stand before I change my mind. (female-30)
- Sometimes I get sick to my stomach. (male-29)
- It feels like a disease when you get it, because you can't stop or control it. (female-24)
- I feel like I'm doing something I'm not supposed to be doing, but am doing it anyway. What the heck! (male-54)
- To hell with everything else. I want it and I'm going to get it. (male-34)
- You know you shouldn't buy it, but it doesn't matter. (female-27)

Given the reported prevalence of akratic impulsiveness in consumption behavior, what are the demographic characteristics of such consumers? Income and wealth do not explain variation in this consumption behavior (O'Guinn and Faber 1989; Scherhorn et al 1990). Inexpensive goods/services are not the only goods impulsively purchased. Hausman (2000) observes that recent studies have reported that akratic-impulse purchases are frequently observed across a broad range of goods/services categories and across a wide variety of price ranges.⁴⁴ Hausman (2000) specifically

⁴⁴See Cobb and Hoyer (1986), Rook (1987), and Rook and Fisher (1995) for more on this.

states, "The pervasiveness of impulse buying, even for relatively expensive products, is counter-intuitive and has led to a few preliminary studies looking at impulse buying as an inherent individual trait, rather than a response to inexpensive product offerings." In reference to consumer debt, Livingstone and Lunt (1992) denote that people blame their debt on loss of personal self-control. They also state, "Those in debt...feel less in control of their finances, make more impulse purchases, and hence say they find it easier to get into debt." To no surprise, the frequency of shopping has been shown to increase impulsive buying (Herbig et al 1993). O'Guinn and Faber (1989), d'Astous (1990), Roberts (1998), and Wood (1998) find a negative relationship between age and akratic-impulsive consumption. But Wood (1998) specifically finds a non-monotonic relationship wherein akratic-impulsive consumption increases until around age 40 and then decreases thereafter. However, Scherhorn et al (1990) find no evidence of a relationship between age and impulsive consumption behavior.

What are the factors that contribute to an individual's propensity to succumb to an akratic impulse (and not just to compulsive-consumption impulses)? Some underlying psychological factors might contribute. Although impulse buying occurs under both negative and positive moods, Donovan et al (1994) and Beatty and Ferrell (1998) find that consumers' positive moods are more conducive to akratic-impulsive spending than negative moods. Separate from this, Tice et al (2001) find that an individual's psychological impulse controls break down during emotional distress, and that they indulge immediate impulses to make themselves feel better when they are emotionally upset. Baumeister (2002) extends this by asserting that individuals impulsively purchase when emotionally distressed. This idea is related to the findings of Mick and DeMoss (1990) wherein people often buy themselves gifts in order to improve their emotional outlook. This is also supported by Livingstone and Lunt (1992) who report that those in debt are more likely to be akratic-impulsive consumers and that they often express their social value by buying themselves presents. Hanley

and Wilhelm (1992) report that akratic-impulsive spenders are more likely to perceive money as possessing an intrinsic ability to enhance self-esteem; Dittmar et al (1995) assert that consumers engage in akratic-impulsive buying behavior in attempts to achieve an idealized self-image; and furthermore, Hirschmann (1992) finds individuals become impulsive consumers in order to fill an emotional void.

Rook and Fisher (1995) report that even highly impulsive consumers don't give in to every akratic-consumption impulse. So why do consumers sometimes give in to such impulses, and why do consumers sometimes abstain from these impulses? Rook and Fisher (1995) attribute this to normative influences and social norms that serve to moderate impulsive behavior. But these are not perfect filters for impulsive behavior, because individuals still engage in such behavior despite knowing that such behavior can be financially harmful. Rook and Fisher (1995) feel that some consumers get pleasure from the impulse itself, wherein they state, "Who has not heard someone characterize a shopping episode with the confession: 'I was bad today?' Sometimes consumers want to be bad."

The control of akratic-impulsive behavior is considered by some psychologists to be related to some type of cognitive resource stock. Baumeister et al (1998) and Muraven et al (1998) find that the exercise of self-control over akratic-impulsive behavior appears to deplete this resource stock such that the resource is diminished in its availability to provide assistance with successive tasks of exercising self-control. This is manifest in their empirical findings wherein the exercise of self-control is poorest among individuals who have already exercised a great deal of self-control.⁴⁵ They label this diminished capability for self-regulation as "ego depletion". Vohs et al (2007) moreover find that everyday decision-making also appears to cause ego depletion. Baumeister (2002) suggests that an individual's cognitive resource stock

⁴⁵Ozdenoren et al (2006) design a continuous-time optimal control model wherein the agent is endowed with a given resource stock of willpower to restrain excessive behavior. The law of motion denotes that the stock of willpower is depleted as self-control is exercised, and they analyze the implications of their model for explaining negative rates of time preference, among other things.

appears to be replenished on a daily basis during the normal sleep-cycle. Therefore, individuals making successive everyday decisions tend to experience ego depletion as the day wears on, and are therefore more susceptible to consumption impulses as the day advances. Baumeister (2002) specifically states, "Hardly anyone gets up and breaks a diet first thing in the morning...It is late at night that one yields to temptation. Similar patterns are found for addictive relapse, alcohol indulgence, impulsive crimes, regrettable sexual indiscretions, and the like." Such a process of ego depletion (and restoration) implies that individuals are more and more likely to become impulsive consumers as time advances through any given day and as they become more and more tired (such as at the end of a prolonged period of planned shopping).⁴⁶

The concept of ego depletion resulting from the exercise of self-control and everyday decision-making can be compounded by what Shiv and Fedorikhin (1999) observe. They find that excessive external stimuli can inhibit or constrain cognitive thought processes, and therefore, consumption choices will be based primarily on what they call affective (impulsive) reactions to goods/services. Indeed, they explicitly state,

The core finding related to the effects of constrained processing resources on the choice of affect-laden alternatives suggests that any factor that reduces the availability of processing resources in the shopping environment is likely to increase impulse buying by consumers. Marketers of affect-laden products could therefore benefit from actions designed to constrain processing resources such as having distracting music or displays in the shopping environment. Another tactic that derives from anecdotal evidence that we obtained from a local grocery-store manager is to reduce the checkout time so that consumers deliberate less about what is in their shopping carts and end up leaving the store with products they chose on impulse.

What are some of the primary consequences of being an akratic-impulsive consumer? It has been shown to cause financial hardship, product disappointment,

⁴⁶This might be a possible reason why stores target (or tempt) consumers with potential impulse purchases at the point-of-purchase.

psychological guilt, emotional trauma, and social disapproval (Rook 1987). With regards to mental consequences, akratic-impulsive consumers attempt to rationalize away feelings of guilt and regret in order to feel that an impulse purchase is appropriate. Nevertheless, guilt and regret often persist (Shiv and Fedorikhin 1999). DeSarbo and Edwards (1996) report on a severe psychological consequence of casually giving in to consumption impulses, wherein they assert that akratic-impulsive consumers can develop into compulsive-impulsive consumers given underlying predispositional factors. Akratic-impulsive consumers usually experience a gratifying sensation when making impulse purchases, but an anxiety overload can internally trigger an akratic-impulsive consumer to start purchasing compulsively as the primary method of coping with anxiety and maintaining euphoric emotions. DeSarbo and Edwards (1996) specifically state,

Many consumers have intermittent bouts of binge buying characterized by impulse buying, but they are not generally pathological in their spending. However, stressful life events or circumstances may cause such individuals to use shopping and spending as a release...as an escape from high levels of anxiety resulting from deep psychological problems. Hence, a behavior that originates as a response to specific circumstances, and is temporal in its manifestation, can eventually become addictive and predispositional in nature.

Institutional factors may be related to the prevalence of akratic-impulsive consumption. Rook (1987) and Kacen and Lee (2002) report that akratic consumption impulses are universal across cultures, but that local market structure and cultural norms affect (moderate) how consumers respond to specific consumption impulses. Rook and Fisher (1995) report that a culture of consumerism is breaking down and removing many of the normative influences and unwanted stigmas that deter impulsive consumption. The mechanisms through which this is accomplished is the increased prevalence of credit availability, 24-hour retailing, etc. McElroy et al (1994) similarly find a significant relationship between credit card usage and the degree of akratic-

impulsive consumption. Moreover, Roberts (1998) asserts that credit card usage might be the single most important factor that generates impulsive consumption behavior. Sellers decrease the perceived risk of impulse purchasing through convenient return policies, increased store hours, and credit availability (Rook and Fisher 1995). Furthermore, such a rise in consumer culture has served to reduce transaction costs which has resulted in more impulsive purchasing. Along these lines, Bayley and Nancarrow (1998) and Kacen and Lee (2002) assert that new avenues for purchasing (such as television shopping channels, telemarketing, and internet) are very conducive to and increasing the incidence of impulse buying. Indeed, Faber and O'Guinn (1989) find that the number of hours watching television increases impulsive consumption behavior. Moreover, Mogelonsky (1998) reports that stores consider the checkout lane as a "last chance" area to entice consumers to purchase small unplanned items such as candy, magazines, etc., because such impulse purchases account for approximately 1 percent of total annual store revenues. Around 46 percent of all shoppers make checkout-line impulse purchases at least once a week, but over two-thirds of these buyers think that these purchases are unnecessary. Lastly, based on psychological insights that individuals have difficult times making well-thought decisions in environments that contain massive external stimuli, Hausman (2000) states, "If retailers wish to promote impulse buying...retailers can make the environment more complex, further straining consumers' abilities to process information accurately. Such techniques as stocking more merchandise, creating stimulating atmospherics, and increasing information may be useful to stimulate impulse buying."

Chapter Three: Model of Impulsive Consumers

I now present the theoretical model of an impulsive consumer. I model an agent who intends to follow the life-cycle/permanent-income rule by setting period consumption equal to the annuity-value of lifetime wealth. But periodically, the agent impulsively consumes above his optimally-planned program.⁴⁷ A defining feature of the model is that the agent is free to re-optimize at every instant in time over the life cycle. Such time-inconsistent dynamic optimization is a response to his unplanned consumption behavior that lowers the value of his remaining lifetime wealth. In what follows, I outline starting assumptions, present the control problem, and solve for the impulsive agent's intended and actual consumption programs.

The representative agent begins working, retires, and dies at $t = 0$, $t = T$, and $t = \bar{T}$, respectively. The dates of retirement and death are exogenous. Labor is supplied inelastically, and wage income is earned at rate w for all $t \in [0, T]$. All wage income that is not consumed flows into the agent's saving account, $k(t)$, which grows at the rate of interest, r . The agent pays social security taxes at rate θ for all $t \in [0, T]$, and he receives benefits, $b = R\theta w$, for all $t \in [T, \bar{T}]$ from an unfunded social security program. Preferences are represented by the CIES utility function.

3.1 Consumption during the Work Life

3.1.1 Life-Cycle/Permanent-Income Behavior

At any particular instant $t_0 \in [0, T]$, the agent solves the following two-stage optimal control problem

$$\max_{t_0} \int_{t_0}^{\bar{T}} e^{-\rho(t-t_0)} \frac{c(t)^{1-\phi} - 1}{1-\phi} dt \quad (36)$$

⁴⁷The idea of impulsive consumption (interpreted as deviations from planned consumption) is closely related to Friedman's (1957) concept of *transitory consumption*. Modern interpretations of the *Permanent-Income Hypothesis* often omit this important feature of real-world behavior.

subject to the following laws of motion and boundary conditions

$$\frac{dk(t)}{dt} = rk(t) + (1 - \theta)w - c(t) \quad \forall t \in [t_0, T] \quad (37)$$

$$\frac{dk(t)}{dt} = rk(t) + b - c(t) \quad \forall t \in [T, \bar{T}] \quad (38)$$

$$k(t_0) \text{ given} \quad k(\bar{T}) = 0 \quad (39)$$

where ρ is the rate of time preference. Application of the *Maximum Principle* for two-stage optimal control problems with a fixed-endpoint constraint yields a set of necessary conditions.⁴⁸ With the state equations and boundary conditions given above, the optimal control must obey

$$e^{-\rho(t-t_0)}c(t)^{-\phi} = \lambda_1(t) \quad \forall t \in [t_0, T] \quad (40)$$

$$e^{-\rho(t-t_0)}c(t)^{-\phi} = \lambda_2(t) \quad \forall t \in [T, \bar{T}] \quad (41)$$

Furthermore, the costate variables obey

$$\frac{d\lambda_1(t)}{dt} = -r\lambda_1(t) \quad \forall t \in [t_0, T] \quad (42)$$

$$\frac{d\lambda_2(t)}{dt} = -r\lambda_2(t) \quad \forall t \in [T, \bar{T}] \quad (43)$$

$$\lambda_1(T) = \lambda_2(T) \quad (44)$$

where (44) is a switch-point (continuity) requirement for the costate variable.

The costate equations, (42) and (43), can be solved to obtain

$$\lambda_1(t) = a_1 e^{-rt} \quad \forall t \in [t_0, T] \quad (45)$$

$$\lambda_2(t) = a_2 e^{-rt} \quad \forall t \in [T, \bar{T}] \quad (46)$$

⁴⁸Two-stage optimal control techniques are required due to a switch in the laws of motion at $t = T$. For details about two-stage control techniques, see Kemp and Long (1977), Tomiyama (1985), Amit (1986), and Kamien and Schwartz (1991). My manipulations of the *two-stage maximum conditions* and *adjoint conditions* in (40)–(50) below parallel an outline in Caliendo and Aadland (2007).

where a_1 and a_2 are constants of integration. (45) and (46) can be evaluated at $t = T$.

$$\lambda_1(T) = a_1 e^{-rT} \quad (47)$$

$$\lambda_2(T) = a_2 e^{-rT} \quad (48)$$

Using (44), (47) and (48) can be set equal which results in $a_1 = a_2$. Therefore, the subscripts can be discarded and the costate variable can be concisely written as

$$\lambda(t) = a e^{-rt} \quad \forall t \in [t_0, \bar{T}] \quad (49)$$

Therefore, (40) and (41) can also be written as

$$e^{-\rho(t-t_0)} c(t)^{-\phi} = \lambda(t) \quad \forall t \in [t_0, \bar{T}] \quad (50)$$

Substituting (49) into (50) yields

$$e^{-\rho(t-t_0)} c(t)^{-\phi} = a e^{-rt} \quad \forall t \in [t_0, \bar{T}] \quad (51)$$

(51) can be solved for $c(t)$ which results in

$$c(t) = a^{-\frac{1}{\phi}} e^{gt + \frac{\rho}{\phi} t_0} \quad (52)$$

where $g \equiv \frac{r-\rho}{\phi}$ is the agent's planned growth rate of consumption.

The constant of integration, a , has to be determined in order to find a closed-form solution for $c(t)$. This will be achieved by first substituting (52) into (37).

$$\frac{dk(t)}{dt} = rk(t) + (1 - \theta)w - a^{-\frac{1}{\phi}} e^{gt + \frac{\rho}{\phi} t_0} \quad \forall t \in [t_0, T] \quad (53)$$

The general solution to (53) is

$$k(t) = \left\{ q_1 + \int^t \left[(1 - \theta) w - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \right\} e^{rt} \quad (54)$$

for $t \in [t_0, T]$ where q_1 is a constant and τ is a dummy variable of integration. Using the initial condition in (39),

$$q_1 = k(t_0) e^{-rt_0} - \int^{t_0} \left[(1 - \theta) w - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \quad (55)$$

Employing the *Fundamental Theorem of Calculus*, the particular solution is therefore

$$k(t) = \left\{ k(t_0) e^{-rt_0} + \int_{t_0}^t \left[(1 - \theta) w - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \right\} e^{rt} \quad (56)$$

(56) can be evaluated at $t = T$ which yields

$$k(T) = \left\{ k(t_0) e^{-rt_0} + \int_{t_0}^T \left[(1 - \theta) w - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \right\} e^{rT} \quad (57)$$

In addition, (52) has to be substituted into (38). This gives

$$\frac{dk(t)}{dt} = rk(t) + b - a^{-\frac{1}{\phi}} e^{gt + \frac{\rho}{\phi} t_0} \quad \forall t \in [T, \bar{T}] \quad (58)$$

with the general solution to this differential equation being

$$k(t) = \left\{ q_2 + \int^t \left[b - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \right\} e^{rt} \quad (59)$$

for $t \in [T, \bar{T}]$ where q_2 is a constant and τ is a dummy variable of integration. Using the endpoint-condition in (39),

$$q_2 = - \int^{\bar{T}} \left[b - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \quad (60)$$

Once again employing the *Fundamental Theorem of Calculus*, the particular solution to (58) is therefore

$$k(t) = e^{rt} \int^{\bar{T}} \left[b - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \quad (61)$$

for $t \in [T, \bar{T}]$. (61) can be evaluated at $t = T$ which yields

$$k(T) = e^{rT} \int^{\bar{T}} \left[b - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \quad (62)$$

In order to explicitly solve for a in closed-form, (57) can be set equal to (62). This is given below as the following

$$\begin{aligned} k(t_0)e^{r(T-t_0)} + e^{rT} \int_{t_0}^T \left[(1-\theta)w - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \\ = e^{rT} \int^{\bar{T}} \left[b - a^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \end{aligned} \quad (63)$$

With some algebra (63) can be rearranged as

$$\begin{aligned} k(t_0)e^{r(T-t_0)} + (1-\theta)we^{rT} \int_{t_0}^T e^{-r\tau} d\tau - a^{-\frac{1}{\phi}} e^{rT + \frac{\rho}{\phi} t_0} \int_{t_0}^T e^{(g-r)\tau} d\tau \\ = be^{rT} \int^{\bar{T}} e^{-r\tau} d\tau - a^{-\frac{1}{\phi}} e^{rT + \frac{\rho}{\phi} t_0} \int^{\bar{T}} e^{(g-r)\tau} d\tau \end{aligned} \quad (64)$$

Using some additional algebra and the *Fundamental Theorem of Calculus* produces

$$a^{-\frac{1}{\phi}} = \frac{k(t_0)e^{-rt_0} + (1-\theta)w \int_{t_0}^T e^{-rt} dt + b \int_T^{\bar{T}} e^{-rt} dt}{e^{\frac{\rho}{\phi}t_0} \int_{t_0}^{\bar{T}} e^{(g-r)t} dt} \quad (65)$$

Integrating (65) yields

$$a^{-\frac{1}{\phi}} = \frac{(g-r) \{k(t_0)e^{-rt_0} + (1-\theta)w [e^{-rt_0} - e^{-rT}] / r\}}{e^{\frac{\rho}{\phi}t_0} [e^{(g-r)\bar{T}} - e^{(g-r)t_0}]} + \frac{(g-r)b [e^{-rT} - e^{-r\bar{T}}] / r}{e^{\frac{\rho}{\phi}t_0} [e^{(g-r)\bar{T}} - e^{(g-r)t_0}]} \quad (66)$$

Substituting (66) back into (52) gives a closed-form solution for the *optimal control*.

$$c^*(t; t_0) = \frac{e^{gt}(g-r) \{k(t_0)e^{-rt_0} + \{(1-\theta)w [e^{-rt_0} - e^{-rT}] + b [e^{-rT} - e^{-r\bar{T}}]\} / r\}}{[e^{(g-r)\bar{T}} - e^{(g-r)t_0}]} \quad (67)$$

This is the optimal consumption program that the agent *intends* or *plans* to follow for all $t \in [t_0, \bar{T}]$, from the perspective of $t_0 \in [0, T]$ during the working period.

The agent's intended asset path for $t \in [t_0, T]$ can be derived from (56). This gives

$$k(t) = k(t_0)e^{r(t-t_0)} + (1-\theta)we^{rt} \int_{t_0}^t e^{-r\tau} d\tau - a^{-\frac{1}{\phi}}e^{rt+\frac{\rho}{\phi}t_0} \int_{t_0}^t e^{(g-r)\tau} d\tau \quad (68)$$

Integrating (68) yields

$$k(t) = k(t_0)e^{r(t-t_0)} + \frac{(1-\theta)we^{rt}}{r} [e^{-rt_0} - e^{-rt}] - \frac{e^{rt+\frac{\rho}{\phi}t_0}}{a^{\frac{1}{\phi}}(g-r)} [e^{(g-r)t} - e^{(g-r)t_0}] \quad (69)$$

Lastly, $k^*(t; t_0)$ for $t \in [t_0, T]$ is found by substituting (66) into (69). This produces

$$\begin{aligned}
k^*(t; t_0) &= k(t_0)e^{r(t-t_0)} + \frac{(1-\theta)w}{r} [e^{r(t-t_0)} - 1] \\
&\quad \frac{\{k(t_0)e^{-rt_0} + (1-\theta)w [e^{-rt_0} - e^{-rT}] / r\} [e^{gt} - e^{gt_0+r(t-t_0)}]}{[e^{(g-r)\bar{T}} - e^{(g-r)t_0}]} \\
&\quad \frac{\{b [e^{-rT} - e^{-r\bar{T}}] / r\} [e^{gt} - e^{gt_0+r(t-t_0)}]}{[e^{(g-r)\bar{T}} - e^{(g-r)t_0}]}
\end{aligned} \tag{70}$$

The intended asset path for $t \in [T, \bar{T}]$ can be derived from (61). This produces

$$k(t) = be^{rt} \int_{\bar{T}}^t e^{-r\tau} d\tau - a^{-\frac{1}{\phi}} e^{rt + \frac{\rho}{\phi} t_0} \int_{\bar{T}}^t e^{(g-r)\tau} d\tau \tag{71}$$

Integrating (71) yields

$$k(t) = \frac{be^{rt}}{r} [e^{-r\bar{T}} - e^{-rt}] - \frac{e^{rt + \frac{\rho}{\phi} t_0}}{a^{\frac{1}{\phi}} (g-r)} [e^{(g-r)t} - e^{(g-r)\bar{T}}] \tag{72}$$

And again, $k^*(t; t_0)$ for $t \in [T, \bar{T}]$ is found by substituting (66) into (72). This results in

$$\begin{aligned}
k^*(t; t_0) &= \frac{b}{r} [e^{r(t-\bar{T})} - 1] \\
&\quad \frac{\{k(t_0)e^{-rt_0} + (1-\theta)w [e^{-rt_0} - e^{-rT}] / r\} [e^{gt} - e^{g\bar{T}+r(t-\bar{T})}]}{[e^{(g-r)\bar{T}} - e^{(g-r)t_0}]} \\
&\quad \frac{\{b [e^{-rT} - e^{-r\bar{T}}] / r\} [e^{gt} - e^{g\bar{T}+r(t-\bar{T})}]}{[e^{(g-r)\bar{T}} - e^{(g-r)t_0}]}
\end{aligned} \tag{73}$$

3.1.2 The Consumption Impulse

$c^*(t; t_0)$, as given in (67), is the consumption program that the agent perceives to be optimal and intends to follow for all $t \in [t_0, \bar{T}]$ while standing from the perspective of any instant $t_0 \in [0, T]$. Under this optimally-formulated program, the agent intends

to consume the annuity value of his remaining lifetime income that is discounted back to his respective vantage point, $t_0 \in [0, T]$. Nevertheless, a defining feature of my setup is that at the same instant $t_0 \in [0, T]$, the agent might also experience a consumption impulse wherein he consumes an additional unplanned impulse amount, $I(t_0)$. Therefore, his actual consumption at each instant $t_0 \in [0, T]$ is

$$c_i(t_0) = c^*(t_0; t_0) + iI(t_0) \quad (74)$$

for $i \in \{0, 1\}$ where $i = 1$ denotes that the agent experiences a consumption impulse ($i = 0$ means that the agent is not impulsive), and where c_i denotes the actual consumption of the “impulsive consumer”.⁴⁹ The unplanned impulse amount is defined as

$$I(t_0) \equiv \psi \underbrace{[(1 - \theta)w - c^*(t_0; t_0)]}_{\equiv \Gamma} \quad (75)$$

where Γ is the agent’s transitory income, and ψ is the fraction of transitory income that is consumed when he experiences the consumption impulse.⁵⁰ Therefore, the agent’s actual consumption at any $t_0 \in [0, T]$ is the sum of his permanent income and some fraction of his transitory income at that instant.⁵¹

It should be mentioned again that t_0 was arbitrarily chosen to represent any point in time on $[0, T]$. This suggests that actual consumption can be found by replacing all t_0 in (74) with t .

$$c_i(t) = c^*(t; t) + iI(t) \quad (76)$$

⁴⁹The model is silent as to the specific source of the impulse, meaning that it could be a compulsive or an akratic impulse.

⁵⁰I define transitory income as the difference between current disposable labor income and permanent income. An alternative specification of transitory income could be the difference between all current income (including interest income from assets) and permanent income.

⁵¹Although the model is general enough to allow for alternative specifications of the impulse amount, the formulation in (75) provides a straightforward, intuitive specification wherein the agent can impulsively consume some fraction of the optimal saving flow.

for all $t \in [0, T]$ where $c^*(t; t)$ is just (67) with all t_0 replaced with t . (76) denotes that the actual consumption of an impulsive consumer is the sum of an optimally-planned amount and an unplanned impulse amount. (76) can be rewritten as

$$c_i(t) = \frac{e^{gt}(g-r) \{k(t)e^{-rt} + \{(1-\theta)w [e^{-rt} - e^{-rT}] + b [e^{-rT} - e^{-r\bar{T}]\} / r\}}{[e^{(g-r)\bar{T}} - e^{(g-r)t}]} + iI(t) \quad (77)$$

for all $t \in [0, T]$.

(77) is dependent on $k(t)$, and therefore, $k(t)$ has to be pinned down in closed-form in order to determine actual consumption. However, since $k(t)$ is not an integrable function (due to periodic impulsiveness), I find $k(t)$ by using a discretized differential-approximation method. With $c_i(t)$ given in (77), the law of motion for $t \in [0, T]$ is discretized

$$dk(t) = \{rk(t) + (1-\theta)w - c_i(t)\} dt \quad \forall t \in [0, T] \quad (78)$$

Using $k(0) = 0$ and with the approximation becoming more precise as $dt \rightarrow 0$,

$$k(t) = k(t-dt) + \{rk(t-dt) + (1-\theta)w - c_i(t-dt)\} dt \quad \forall t \in [0, T] \quad (79)$$

3.2 Consumption during Retirement

At any particular instant $t_0 \in [T, \bar{T}]$, the agent solves the following control problem

$$\max_{t_0} \int_{t_0}^{\bar{T}} e^{-\rho(t-t_0)} \frac{c(t)^{1-\phi} - 1}{1-\phi} dt \quad (80)$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + b - c(t) \quad \forall t \in [t_0, \bar{T}] \quad (81)$$

$$k(t_0) \text{ given} \quad k(\bar{T}) = 0 \quad (82)$$

Application of the *Maximum Principle* for finite-horizon control problems with a fixed-endpoint constraint yields a set of necessary conditions. The *maximum condition*, the *adjoint condition*, and the *law of motion* are given as (83), (84), and (81) respectively.

$$e^{-\rho(t-t_0)}c(t)^{-\phi} = \lambda(t) \quad \forall t \in [t_0, \bar{T}] \quad (83)$$

$$\frac{d\lambda(t)}{dt} = -r\lambda(t) \quad \forall t \in [t_0, \bar{T}] \quad (84)$$

The costate equation can be solved to obtain

$$\lambda(t) = ve^{-rt} \quad \forall t \in [t_0, \bar{T}] \quad (85)$$

where v is a constant of integration. Substituting (85) into (83) yields

$$e^{-\rho(t-t_0)}c(t)^{-\phi} = ve^{-rt} \quad \forall t \in [t_0, \bar{T}] \quad (86)$$

(86) can be solved for $c(t)$ which results in

$$c(t) = v^{-\frac{1}{\phi}}e^{gt + \frac{\rho}{\phi}t_0} \quad (87)$$

The constant of integration, v , has to be determined in order to derive a closed-form solution for $c(t)$. This will be achieved by first substituting (87) into (81).

$$\frac{dk(t)}{dt} = rk(t) + b - v^{-\frac{1}{\phi}}e^{gt + \frac{\rho}{\phi}t_0} \quad \forall t \in [t_0, \bar{T}] \quad (88)$$

The general solution to (88) is

$$k(t) = \left\{ q_4 + \int^t \left[b - v^{-\frac{1}{\phi}}e^{g\tau + \frac{\rho}{\phi}t_0} \right] e^{-r\tau} d\tau \right\} e^{rt} \quad (89)$$

for $t \in [t_0, \bar{T}]$ where q_4 is a constant and τ is a dummy variable of integration. Using the initial condition from (82),

$$q_4 = k(t_0)e^{-rt_0} - \int_{t_0}^{t_0} \left[b - v^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \quad (90)$$

Employing the *Fundamental Theorem of Calculus*, the particular solution is therefore

$$k(t) = \left\{ k(t_0)e^{-rt_0} + \int_{t_0}^t \left[b - v^{-\frac{1}{\phi}} e^{g\tau + \frac{\rho}{\phi} t_0} \right] e^{-r\tau} d\tau \right\} e^{rt} \quad (91)$$

Using the endpoint-condition from (82), (91) becomes

$$k(t_0)e^{-rt_0} + b \int_{t_0}^{\bar{T}} e^{-rt} dt - v^{-\frac{1}{\phi}} e^{\frac{\rho}{\phi} t_0} \int_{t_0}^{\bar{T}} e^{(g-r)t} dt = 0 \quad (92)$$

Rearranging (92) and integrating yields

$$v^{-\frac{1}{\phi}} = \frac{(g-r) \left\{ k(t_0)e^{-rt_0} + b \left[e^{-rt_0} - e^{-r\bar{T}} \right] / r \right\}}{e^{\frac{\rho}{\phi} t_0} \left[e^{(g-r)\bar{T}} - e^{(g-r)t_0} \right]} \quad (93)$$

Substituting (93) back into (87) generates a closed-form, analytical solution for the agent's planned consumption program for all $t \in [t_0, \bar{T}]$ from the perspective of any $t_0 \in [T, \bar{T}]$.

$$c^*(t; t_0) = \frac{e^{gt} (g-r) \left\{ k(t_0)e^{-rt_0} + b \left[e^{-rt_0} - e^{-r\bar{T}} \right] / r \right\}}{\left[e^{(g-r)\bar{T}} - e^{(g-r)t_0} \right]} \quad (94)$$

From the perspective of any $t_0 \in [T, \bar{T}]$, the intended state path for $t \in [t_0, \bar{T}]$ can be derived from (91).

$$k(t) = k(t_0)e^{r(t-t_0)} + be^{rt} \int_{t_0}^t e^{-r\tau} d\tau - v^{-\frac{1}{\phi}} e^{rt + \frac{\rho}{\phi} t_0} \int_{t_0}^t e^{(g-r)\tau} d\tau \quad (95)$$

Substituting (93) into (95) and integrating yields

$$k^*(t; t_0) = \frac{k(t_0)e^{r(t-t_0)} + b[e^{r(t-t_0)} - 1]/r}{\frac{\{k(t_0)e^{-rt_0} + b[e^{-rt_0} - e^{-r\bar{T}}]/r\} [e^{gt} - e^{gt_0+r(t-t_0)}]}{[e^{(g-r)\bar{T}} - e^{(g-r)t_0}]}} \quad (96)$$

for $t \in [t_0, \bar{T}]$

The model is general enough to allow for consumption impulses at any instant during retirement.⁵² But the objective of my dissertation is to examine the effects of impulsive consumption on wealth accumulation for retirement (and the welfare effects of social security participation given such wealth accumulation patterns), so I assume that the agent never faces a consumption impulse during the retirement years. Therefore, the agent simply follows the life-cycle/permanent-income rule by consuming the annuity-value of remaining lifetime wealth discounted back to $t = T$. This assumption is consistent with empirical evidence wherein consumers appear to smooth consumption during retirement after an initial downward correction at retirement (Banks et al 1998; Bernheim et al 2001; Browning and Crossley 2001). Given this assumption, the impulsive consumer's actual consumption during retirement is time-consistent for all $t \in [T, \bar{T}]$, and can be found by setting $t_0 = T$ in (94). This yields

$$c_i(t) = \frac{e^{gt} (g - r) \{k(T)e^{-rT} + b[e^{-rT} - e^{-r\bar{T}}]/r\}}{[e^{(g-r)\bar{T}} - e^{(g-r)T}]} \quad (97)$$

where the asset path is also found by setting $t_0 = T$ in (96). This gives

$$k(t) = \frac{k(T)e^{r(t-T)} + b[e^{r(t-T)} - 1]/r}{\frac{\{k(T)e^{-rT} + b[e^{-rT} - e^{-r\bar{T}}]/r\} [e^{gt} - e^{gT+r(t-T)}]}{[e^{(g-r)\bar{T}} - e^{(g-r)T}]}} \quad (98)$$

for $t \in [T, \bar{T}]$.

⁵²Such impulses would have to be defined with an alternative setup other than that of (75) since the agent does not earn any labor income during retirement by definition.

Chapter Four: Calibration of the Model

My baseline model of impulsive consumers has two key impulsiveness parameters that can be adjusted to analyze the welfare effects of unfunded social security: The frequency at which the agent experiences the consumption impulse (i.e., the frequency of $i = 1$), which I label $F \in [0, 1]$; and the fraction of transitory income that is impulsively consumed, ψ . In order to simulate the behavior of impulsive consumers, I must first select reasonable values for these impulsiveness parameters (and for the economic parameters in the model). The calibration of my model and these consumption-impulse parameters is the topic of this chapter in my dissertation.

I first select baseline values for the economic parameters of the model. These numerical values are based on widely-accepted empirical evidence in the macroeconomics literature. Secondly, I simulate multiple micro-level consumption profiles of impulsive consumers to generate an aggregate consumption profile. Thirdly, I adjust the consumption-impulse parameters in order to match specific simulation targets that correspond to empirical evidence on aggregate life-cycle consumption. Specifically, I locate the joint parameter space of F and ψ that generates a discrete drop in aggregate consumption at retirement that matches the drop in the data. Lastly, I examine the sensitivity of this consumption-impulse parameter space.

4.1 Baseline Parameter Values

Baseline values of the economic parameters are outlined in this section. These parameter values are summarized in Table 1 below. T is set to 40 and \bar{T} is set to 55 in order to simulate a consumer who starts work at age 25, retires at 65, and passes away at 80. Much of the recent macroeconomic literature on life-cycle consumption suggests a value of 3.5 percent for the real interest rate (Campbell 2001; Gourinchas and Parker 2002; Feigenbaum 2007; Bullard and Feigenbaum Forthcoming). Never-

theless, I intentionally select a lower baseline value of 2.5 percent for r in order to adjust for potential bias in my welfare analysis. The internal rate of return, γ , from an unfunded social security system is dependent on the rate of real wage growth, δ , among other things. For example, the *Implicit Function Theorem* can be applied to (35) to show that $\partial\gamma/\partial\delta = 1$, denoting the unitary co-movement of the *IRR* with the rate of real wage growth. The Bureau of Labor Statistics reports that real wages have grown at about 1 percent per year in the U.S. over the ten-year period ending in December 2004, but I have modeled an economy without growth in real wages. Therefore, I am effectively biasing the *IRR* from social security downward by 1 percentage-point. In order to adjust for this bias (i.e., correct for “stacking the cards against” social security), I use a baseline value of 2.5 percent (instead of 3.5 percent) for the real interest rate. Following convention, ρ is set equal to r .⁵³ The ratio of workers to retirees, R , has hovered around 3.3 in the U.S. economy since 1975 (Goss 2005/2006). In Chapter Five, I also consider an alternative steady state with $R = 2$ since the worker-to-retiree ratio is projected to fall to around 2 within the next few decades in the U.S. (Goss 2005/2006). The social security tax rate, θ , is set to 10.6 percent because the retirement part of the U.S. Social Security program (excluding disability components) is funded by payroll taxes equal to 10.6 percent of earnings. Employers and employees each legally pay 5.3 percent, but the agent pays the full tax in the model.⁵⁴ Attanasio (1999) reports that the body of microeconomic evidence on intertemporal consumption suggests that the curvature parameter, ϕ , is very close to 1. This means that the period utility function collapses to the natural logarithm as $\phi \rightarrow 1$. Lastly, the wage, w , is set to \$40,000 but my principal welfare results concerning social security participation are invariant to this particular simulation value.

⁵³This assumption also allows me to confine my analysis to empirically-reasonable consumption behavior, especially during the retirement phase.

⁵⁴This is a standard assumption (Kotlikoff 1979; Feldstein 1985; Jianakoplos and Bajtelsmit 2002).

Table 1. Baseline Parameter Values

<i>Parameter Name</i>	<i>Symbol</i>	<i>Value</i>
date of retirement	T	40
date of death	\bar{T}	55
real rate of return	r	0.025
rate of time preference	ρ	0.025
worker-to-retiree ratio	R	3.3
social security tax rate	θ	0.106
curvature parameter	ϕ	1
wage	w	\$40,000

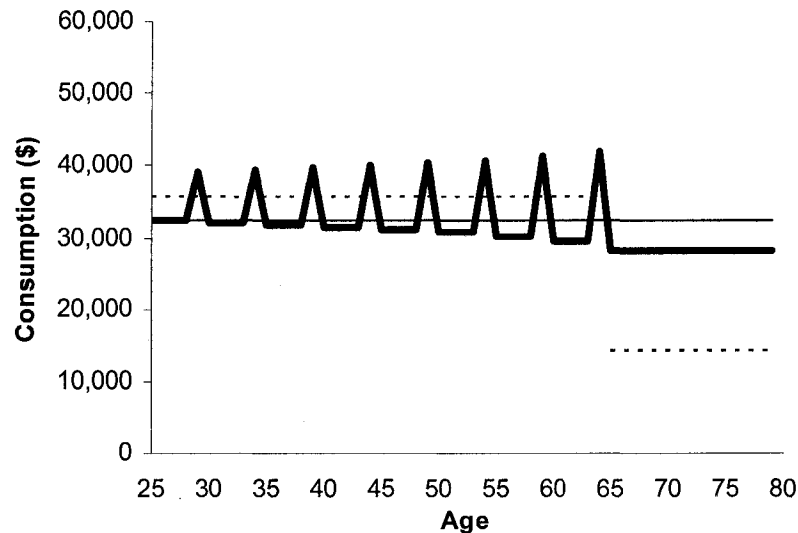
Throughout my calibration and welfare analysis, I simulate the consumption profiles of impulsive consumers at the time step-size of $dt = 0.1$, which roughly corresponds to monthly increments of time. I mention at this juncture a key assumption that I impose in order to drastically improve the tractability of the simulations of my model. Although it is possible that the exact timing of consumption impulses is irregular (and might even follow a random walk), I assume that the impulsive consumer experiences the consumption impulse in periodic intervals or sequences since otherwise there are a massive number of permutations of the timing sequences at which the individual might experience the consumption impulse over the working period.⁵⁵ For example, if the consumer is impulsive 10 percent of the time over his working period (i.e., $F = 0.1$), then he will experience the consumption impulse once out of every ten time-intervals. My welfare results in Chapter Five are very slightly, yet insignificantly, affected by whether the impulse is experienced at the first of ten time-intervals, the last of ten time-intervals, or anywhere in the middle. This holds true for $F \in [0.1, 1]$ at the step-size of $dt = 0.1$. The normative welfare calculations below are very sensitive to the frequency of impulsiveness, but not to the pattern of the sequence, so it is therefore reasonable to limit our attention to periodic impulse shocks rather than randomly-drawn shocks.

⁵⁵This assumption is plausible given that consumers often make impulsive purchases at the same time that regular, planned purchases are periodically made.

4.2 A Micro-Level Simulated Profile

Before proceeding with the calibration of the behavioral impulse parameters, I show in Figure 3 the consumption profile (the thick solid line) of an individual who impulsively consumes one-fifth of the time during the working period of the life cycle (i.e., $F = 0.2$). I also assume that $\psi = 2$, which means that the agent impulsively consumes about twice his optimal saving flow on top of his permanent income. The dashed lines are the real wage and social security benefits profiles. All other parameters are fixed at their baseline values, and this representation is at the $dt = 1$ time step-size for visual purposes only. The thin solid line is the consumption profile of a life-cycle/permanent-income consumer who never experiences the consumption impulse. I show this consumption profile at the same parameterization as a basis for visual comparison with impulsive consumption behavior.

Figure 3. An Impulsive Consumer



Note. The thick solid line corresponds to an impulsive consumer at $\psi = 2$; $F = 0.2$; $dt = 1$. The thin solid line is the profile of a LC/PI consumer. The dashed lines are the wage and benefits profiles.

I highlight several noticeable features in Figure 3. When the consumer experiences the consumption impulse, the spikes in his consumption exceed his disposable wage income because of the assumption $\psi = 2$. Moreover, the level or flat portions of the agent's consumption profile during the working period reflect consumption-smoothing behavior following re-optimization in response to impulsive consumption during the previous period. It is during such periods that the agent intends to follow the life-cycle/permanent-income rule over the remaining periods of his life cycle in order to maximize his utility. But such consumption-smoothing behavior abruptly ends when the agent again experiences the consumption impulse and consumes accordingly. It should be mentioned that each time the agent re-optimizes immediately after a period of impulsiveness (and returns to life-cycle/permanent-income smoothing behavior), the value of his remaining lifetime wealth is now lower as compared to the last time that he followed the life-cycle/permanent-income rule prior to becoming impulsive. Therefore, the agent must revise down his consumption program to smooth out the loss to wealth (in the form of forgone interest earnings) that was caused by the consumption impulse. This process is reflected by the observation in Figure 3 that any particular smoothed section of the consumption profile is consecutively lower than any previously smoothed portion of the agent's consumption profile.

4.3 Calibration of the Impulse Parameters

As depicted in Figure 3 above, I assumed for simple expositional purposes that the representative consumer experiences the consumption impulse 20 percent of the time over his working period. I also assumed that he consumes twice his optimal saving flow when he faces the consumption impulse. But are these assumptions reasonable? In addition, what is the parameter space for F and ψ that captures real-world impulsiveness? In this section, I take up the issue of how to appropriately simulate impulsive consumption behavior that is empirically plausible, meaning that I cali-

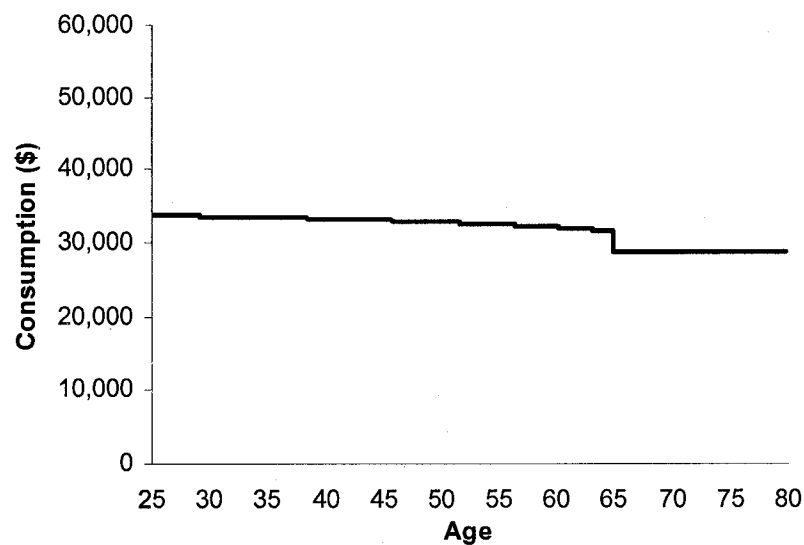
brate F and ψ to ensure that the model's output is consistent with key empirical features of real-world consumption data.

Several features of aggregate life-cycle consumption data have been thoroughly reported in the literature (see Chapter Two). I therefore aggregate the micro-level consumption profiles of representative impulsive consumers in order to get an aggregate-level consumption profile. In aggregating, I assume that all consumers share the same degree and frequency of impulsiveness, but the timing is random across consumers. For example, I aggregate across 5 types of representative agents (although there are Q consumers in the economy) who all share the same magnitude of impulsiveness, ψ , when $F = 0.2$. This suggests that 20 percent of the population of Q consumers is impulsive in any particular period.

I show in Figure 4 the aggregate life-cycle consumption profile for an economy populated with people who consume twice their optimal saving flow when periodically experiencing the consumption impulse about 20 percent of the time over their work years. This means that people spend an unplanned amount of \$1000 on average around twice a year. All other parameters are fixed at the baseline. I mention three prominent characteristics of this aggregate profile. First, the aggregate consumption profile is not hump-shaped, and in fact, the lack of a hump is robust across a wide range of economic and impulsiveness parameter values. It thus appears that impulsive consumption (in isolation) is not powerful enough to theoretically generate an aggregate life-cycle consumption profile that is hump-shaped and matches the empirical findings outlined in Chapter Two. Second, aggregate consumption falls discretely by 9 percent at the retirement threshold. This discrete drop results from my assumption that at least some fraction of the life-cycle population is impulsively consuming immediately prior to retirement. The magnitude of this discrete drop lies well within the range that is empirically reported in the literature. Third, the aggregate retirement profile is flat and smooth due to the modeling assumption that

individuals do not experience a consumption impulse during the retirement years.⁵⁶ Although it is possible that retirees are occasionally impulsive in reality, an aggregate consumption profile (with impulsive consumption during retirement) would monotonically decrease with the steepness of the profile being related to the degree and/or frequency of impulsiveness. Such an aggregate profile directly contradicts empirical evidence on retirement consumption smoothing (Banks et al 1998; Bernheim et al 2001; Browning and Crossley 2001), so I will avoid impulsiveness during retirement in order to keep the model's predictions consistent with the real world.⁵⁷

Figure 4. Aggregate Impulsive Consumption



Note. $F = 0.2; \psi = 2$. This roughly corresponds to agents impulsively consuming about \$1000 every six months. The magnitude of the discrete drop at retirement is 9 percent.

⁵⁶ Assuming that individuals never experience a consumption impulse during retirement means that they will simply consume the annuity value of their remaining lifetime wealth for all $t \in [T, \bar{T}]$. This is consistent with the conventional strategy of purchasing an annuity at retirement.

⁵⁷ The discrete drop in aggregate consumption at the retirement threshold could theoretically be removed by allowing for impulsive consumption behavior during retirement. Yet, the degree and frequency of impulsiveness would have to be so high such that the retirement consumption growth rate is very negative. Moreover, under a wide range of parameterizations aggregate consumption itself would become negative shortly thereafter (which is not empirically reasonable at all).

If I assume that impulsive consumption produced the discrete drop in aggregate consumption that is cited empirically, then I can isolate the range of numerical values for F and ψ that produce a simulated drop in aggregate consumption at retirement that matches empirical targets cited in the literature. I therefore calibrate my model of impulsive consumers by selecting only those values of F and ψ that produce a discrete drop in aggregate consumption at retirement between (and including) 4 percent to 35 percent.⁵⁸ Just for purposes of calibrating F and ψ to the empirical data on the discrete drop, I set $\theta = 0.09$ because this was the average tax rate over the timeframe of the cited data on the discrete drop.⁵⁹ I hold all other parameters fixed at the baseline, and I report the parameter space that is consistent with this empirical range in Table 2a.⁶⁰ The number reported in each cell is the simulated percentage drop in aggregate consumption at the retirement threshold. I also perform sensitivity analysis on this impulsiveness parameter space by adjusting the rate of return, r , by two percentage points in either direction away from its baseline value of 2.5 percent. This sensitivity is listed in Tables 2b and 2c. Lastly, I report in Table 3 the intersection and union of this parameter space, wherein \blacklozenge denotes the intersection of $F; \psi$ parameter space that generates a discrete drop in aggregate consumption that is within the target range for $r \in [0.005, 0.045]$. Likewise, \bullet denotes the union of this parameter space for $r \in [0.005, 0.045]$. Alternatively stated, \blacklozenge denotes the combinations of F and ψ that appear to robustly simulate impulsive consumption behavior, yet the combinations of F and ψ denoted by \bullet are also considered for sensitivity. Therefore, the range of $F; \psi$ parameter space denoted by \blacklozenge (and \bullet for sensitivity) is what I consider to be relevant in my assessment of unfunded social security in Chapter Five.

⁵⁸This appears to be the range of possibilities for the discrete drop in aggregate consumption as cited in the literature. See Chapter Two for an overview of this empirical characteristic of aggregate life-cycle consumption.

⁵⁹As outlined above, I use the baseline value $\theta = 0.106$ for welfare analysis in Chapter Five.

⁶⁰The worker-to-retiree ratio hovered around 3.3 (my baseline value) in the U.S. over the timeframe of this cited data on the discrete drop.

Table 2a. Drop (%) in Aggregate Consumption ($r = 0.025$)

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0	0	0	1	1	1	1	1	1	1
0.2	0	1	1	1	1	2	2	3	3	3
0.3	0	1	1	2	2	3	4	4	5	6
0.4	1	1	2	3	3	4	6	7	8	10
0.5	1	1	2	3	5	6	8	10	13	15
0.6	1	2	3	4	6	8	11	14	18	22
0.7	1	2	4	6	8	11	15	19	24	31
0.8	1	3	4	7	10	14	19	25	32	41
0.9	1	3	5	8	13	18	24	32	42	53
1.0	1	3	6	10	15	22	30	41	53	67
1.1	2	4	7	12	18	27	38	51	66	83
1.2	2	4	8	14	22	32	46	61	79	98
1.3	2	5	10	16	26	39	55	73	93	—
1.4	2	6	11	19	30	45	64	86	—	—
1.5	2	6	12	22	35	53	75	98	—	—
1.6	3	7	14	25	41	61	85	—	—	—
1.7	3	8	16	28	47	70	96	—	—	—
1.8	3	8	18	32	53	79	—	—	—	—
1.9	3	9	20	36	60	88	—	—	—	—
2.0	3	10	22	41	67	98	—	—	—	—
2.1	4	11	24	45	74	—	—	—	—	—
2.2	4	12	27	50	82	—	—	—	—	—
2.3	4	13	29	55	90	—	—	—	—	—
2.4	4	14	32	61	98	—	—	—	—	—
2.5	5	15	35	67	—	—	—	—	—	—
2.6	5	16	38	73	—	—	—	—	—	—
2.7	5	18	42	79	—	—	—	—	—	—
2.8	6	19	45	85	—	—	—	—	—	—
2.9	6	20	49	91	—	—	—	—	—	—
3.0	6	22	53	97	—	—	—	—	—	—
3.1	7	23	57	—	—	—	—	—	—	—
3.2	7	25	61	—	—	—	—	—	—	—
3.3	7	26	65	—	—	—	—	—	—	—
3.4	8	28	69	—	—	—	—	—	—	—
3.5	8	30	74	—	—	—	—	—	—	—
3.6	8	32	78	—	—	—	—	—	—	—
3.7	9	34	83	—	—	—	—	—	—	—
3.8	9	36	88	—	—	—	—	—	—	—

Note. $\theta = 0.09$ with all other parameters at the baseline. For the target range, ψ of 7.6 is the upperbound at F of 0.1.

Table 2b. Drop (%) in Aggregate Consumption ($r = 0.045$)

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0	0	0	0	0	0	1	1	1	1
0.2	0	0	0	1	1	1	1	2	2	2
0.3	0	0	1	1	1	2	2	3	3	4
0.4	0	1	1	2	2	3	4	5	6	7
0.5	0	1	1	2	3	4	5	7	9	11
0.6	0	1	2	3	4	6	8	10	13	17
0.7	1	1	2	4	5	8	11	15	19	25
0.8	1	2	3	5	7	10	15	20	27	36
0.9	1	2	3	6	9	13	19	27	38	50
1.0	1	2	4	7	11	17	25	36	50	67
1.1	1	2	5	9	14	22	33	47	65	87
1.2	1	3	6	10	17	27	41	60	83	—
1.3	1	3	7	12	21	34	52	75	—	—
1.4	1	4	8	14	25	41	63	91	—	—
1.5	1	4	9	17	30	50	77	—	—	—
1.6	2	5	10	20	36	60	91	—	—	—
1.7	2	5	12	23	43	71	—	—	—	—
1.8	2	6	13	27	50	82	—	—	—	—
1.9	2	6	15	31	58	94	—	—	—	—
2.0	2	7	17	36	67	—	—	—	—	—
2.1	2	8	19	41	76	—	—	—	—	—
2.2	2	8	22	46	87	—	—	—	—	—
2.3	3	9	24	53	96	—	—	—	—	—
2.4	3	10	27	59	—	—	—	—	—	—
2.5	3	11	30	66	—	—	—	—	—	—
2.6	3	12	33	74	—	—	—	—	—	—
2.7	3	13	37	82	—	—	—	—	—	—
2.8	4	14	41	90	—	—	—	—	—	—
2.9	4	16	45	98	—	—	—	—	—	—
3.0	4	17	49	—	—	—	—	—	—	—
3.1	4	18	54	—	—	—	—	—	—	—
3.2	5	20	59	—	—	—	—	—	—	—
3.3	5	21	64	—	—	—	—	—	—	—
3.4	5	23	69	—	—	—	—	—	—	—
3.5	5	25	75	—	—	—	—	—	—	—
3.6	6	27	81	—	—	—	—	—	—	—
3.7	6	29	87	—	—	—	—	—	—	—
3.8	6	31	93	—	—	—	—	—	—	—

Note. $\theta = 0.09$; $r = 0.045$ with all other parameters set to the baseline.

For the target range, ψ of 8.1 is the upperbound at F of 0.1.

Table 2c. Drop (%) in Aggregate Consumption ($r = 0.005$)

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0	0	1	1	1	1	2	2	2	2
0.2	0	1	1	2	2	3	3	4	5	5
0.3	1	1	2	3	4	5	6	7	8	9
0.4	1	2	3	4	5	7	8	10	12	14
0.5	1	2	4	5	7	9	12	14	17	20
0.6	1	3	5	7	9	12	15	19	23	27
0.7	2	3	6	8	12	15	19	24	30	36
0.8	2	4	7	10	14	19	24	31	38	45
0.9	2	5	8	12	17	23	30	38	46	56
1.0	2	5	9	14	20	27	36	45	56	67
1.1	3	6	11	16	24	32	42	54	66	79
1.2	3	7	12	19	27	38	49	62	77	91
1.3	3	8	14	21	31	43	57	72	87	—
1.4	3	8	15	24	36	49	65	81	97	—
1.5	4	9	17	27	40	56	73	91	—	—
1.6	4	10	19	30	45	63	81	—	—	—
1.7	4	11	21	34	50	69	89	—	—	—
1.8	5	12	23	37	56	77	97	—	—	—
1.9	5	13	25	41	61	84	—	—	—	—
2.0	5	14	27	45	67	91	—	—	—	—
2.1	6	15	30	49	73	97	—	—	—	—
2.2	6	16	32	53	79	—	—	—	—	—
2.3	6	17	35	58	85	—	—	—	—	—
2.4	7	19	37	62	90	—	—	—	—	—
2.5	7	20	40	67	96	—	—	—	—	—
2.6	7	21	43	72	—	—	—	—	—	—
2.7	8	23	46	76	—	—	—	—	—	—
2.8	8	24	49	81	—	—	—	—	—	—
2.9	9	25	52	86	—	—	—	—	—	—
3.0	9	27	55	90	—	—	—	—	—	—
3.1	10	29	59	95	—	—	—	—	—	—
3.2	10	30	63	99	—	—	—	—	—	—
3.3	10	32	66	—	—	—	—	—	—	—
3.4	11	34	69	—	—	—	—	—	—	—
3.5	11	35	72	—	—	—	—	—	—	—
3.6	12	37	76	—	—	—	—	—	—	—
3.7	12	39	80	—	—	—	—	—	—	—
3.8	13	41	83	—	—	—	—	—	—	—

Note. $\theta = 0.09$; $r = 0.005$ with all other parameters set to the baseline.

For the target range, ψ of 7.0 is the upperbound at F of 0.1.

Table 3. Relevant $F; \psi$ Parameter Space with Sensitivity

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	•	•	•
0.3	—	—	—	—	•	•	•	•	•	◆•
0.4	—	—	—	•	•	•	◆•	◆•	◆•	◆•
0.5	—	—	•	•	•	◆•	◆•	◆•	◆•	◆•
0.6	—	—	•	•	◆•	◆•	◆•	◆•	◆•	◆•
0.7	—	—	•	◆•	◆•	◆•	◆•	◆•	◆•	•
0.8	—	•	•	◆•	◆•	◆•	◆•	◆•	•	—
0.9	—	•	•	◆•	◆•	◆•	◆•	•	—	—
1.0	—	•	◆•	◆•	◆•	◆•	•	—	—	—
1.1	—	•	◆•	◆•	◆•	◆•	•	—	—	—
1.2	—	•	◆•	◆•	◆•	•	—	—	—	—
1.3	—	•	◆•	◆•	◆•	•	—	—	—	—
1.4	—	◆•	◆•	◆•	•	—	—	—	—	—
1.5	•	◆•	◆•	◆•	•	—	—	—	—	—
1.6	•	◆•	◆•	◆•	—	—	—	—	—	—
1.7	•	◆•	◆•	◆•	—	—	—	—	—	—
1.8	•	◆•	◆•	•	—	—	—	—	—	—
1.9	•	◆•	◆•	•	—	—	—	—	—	—
2.0	•	◆•	◆•	—	—	—	—	—	—	—
2.1	•	◆•	◆•	—	—	—	—	—	—	—
2.2	•	◆•	◆•	—	—	—	—	—	—	—
2.3	•	◆•	◆•	—	—	—	—	—	—	—
2.4	•	◆•	•	—	—	—	—	—	—	—
2.5	•	◆•	•	—	—	—	—	—	—	—
2.6	•	◆•	•	—	—	—	—	—	—	—
2.7	•	◆•	—	—	—	—	—	—	—	—
2.8	◆•	◆•	—	—	—	—	—	—	—	—
2.9	◆•	◆•	—	—	—	—	—	—	—	—
3.0	◆•	◆•	—	—	—	—	—	—	—	—
3.1	◆•	◆•	—	—	—	—	—	—	—	—
3.2	◆•	◆•	—	—	—	—	—	—	—	—
3.3	◆•	◆•	—	—	—	—	—	—	—	—
3.4	◆•	◆•	—	—	—	—	—	—	—	—
3.5	◆•	◆•	—	—	—	—	—	—	—	—
3.6	◆•	•	—	—	—	—	—	—	—	—
3.7	◆•	•	—	—	—	—	—	—	—	—
3.8	◆•	•	—	—	—	—	—	—	—	—

Note. ◆ denotes the \cap of $F; \psi$ space that produces a discrete drop within the target range for $r \in [0.005, 0.045]$. • denotes the \cup of this space. $\psi = 7.0$ is the upperbound of ◆ at $F = 0.1$.

Chapter Five: Welfare Analysis

5.1 The Welfare Cost of Being an Impulsive Consumer

At baseline parameter values, the present discounted value of lifetime wealth (from labor earnings and social security benefits) of both the life-cycle/permanent-income consumer and the impulsive consumer is approximately \$958,213. The LC/PI agent optimally expends the annuity value of his lifetime wealth (when the interest and discount rates are equal) in order to smooth marginal utility evenly across the planning horizon. Alternatively, the impulsive consumer intends to consume the annuity value of his lifetime wealth, but often gives in to unplanned urges to spend. Consequently, the impulsive consumer forgoes interest earnings on any impulse amount from thereon into the future. Therefore, he can never achieve the consumption program that the life-cycle/permanent-income consumer follows, even if he is impulsive only once!

So by how much is the impulsive consumer worse off in terms of lifetime welfare? Using a dynamic welfare metric, I examine the cost to agents who impulsively consume (as opposed to always being a life-cycle/permanent-income agent who never consumes impulsively) measured in terms of an annual (and lifetime) willingness to pay. To accomplish this, I augment the LC/PI setup outlined in Chapter Two for dynamic welfare analysis.⁶¹ With $c_p(t)$ denoting the consumption of a life-cycle/permanent-income agent, this augmented framework is given as

$$\max \int_0^{\bar{T}} e^{-\rho t} \frac{c_p(t)^{1-\phi} - 1}{1-\phi} dt \quad (99)$$

subject to

$$\frac{dK(t)}{dt} = rK(t) - c_p(t) \quad (100)$$

⁶¹I present this augmented LC/PI framework within a one-stage optimal control setup simply for convenience. It could also be presented within a two-stage setup.

$$K(0) = \int_0^T e^{-rt} (1 - \theta) w dt + \int_T^{\bar{T}} e^{-rt} b dt - \Omega \quad (101)$$

$$K(\bar{T}) = 0 \quad (102)$$

where

$$\Omega \equiv \pi \int_0^{\bar{T}} e^{-rt} dt \quad (103)$$

With $g \equiv \frac{r-\rho}{\phi}$, the solution to (99)–(102) is

$$c_p^*(t; \pi) = \frac{K(0)e^{r\bar{T}+gt} (g - r)}{eg\bar{T} - e^{r\bar{T}}} \quad (104)$$

As can be seen from (101) and (103), π is an annual amount that can be deducted from the life-cycle/permanent-income agent (while allowing him to re-optimize according to this lower level of income).

With $c_i(t)$ outlined in Chapter Three, I define (105) as the lifetime utility of an impulsive consumer from the perspective of a social planner, where η is the social discount rate that the social planner uses.⁶²

$$V_i \equiv \sum_{t=0}^{\bar{T}-\Delta t} \left\{ e^{-\eta t} \frac{c_i(t)^{1-\phi} - 1}{1 - \phi} \right\} \Delta t \quad (105)$$

Consistent with this specification of lifetime utility by the social planner, I use π to solve the following equation

$$\sum_{t=0}^{\bar{T}-\Delta t} \left\{ e^{-\eta t} \frac{c_p^*(t; \pi)^{1-\phi} - 1}{1 - \phi} \right\} \Delta t = \sum_{t=0}^{\bar{T}-\Delta t} \left\{ e^{-\eta t} \frac{c_i(t)^{1-\phi} - 1}{1 - \phi} \right\} \Delta t \quad (106)$$

⁶²I discretely approximate the lifetime utility of an impulsive consumer since the actual consumption program is not integrable. In addition, I use a baseline value of $\eta = 0.025$ throughout my welfare analysis suggesting that the social planner discounts at the private rate of time preference. Nevertheless, in several cases I analyze the sensitivity of my welfare results by setting $\eta = 0$ which is consistent with Feldstein (1985).

π now represents an annual amount of income that a social planner can deduct from the life-cycle/permanent-income consumer to force his lifetime utility to equal the lifetime utility of an otherwise identical impulsive consumer. π can therefore be interpreted as an annual willingness to pay to avoid consuming impulsively. π is labeled the “income equivalent”. Given (103), Ω is consequently the amount of lifetime wealth that must be taken from the life-cycle/permanent-income consumer in order to force his lifetime utility (after he has re-optimized according to the lower level of lifetime wealth) to be the same as that of an impulsive consumer. This “wealth equivalent”, Ω , can be interpreted as a lifetime willingness to pay to avoid being impulsive. I approximate the income equivalent with numerical techniques, wherein I specifically use a grid search to find the particular value of π (and the corresponding value of Ω) that equates the left-hand side of (106) with the right-hand side of (106).

In Table 4a, I report the income equivalent for the relevant values of F and ψ where all other economic parameters are held at the baseline (including $\eta = 0.025$). Table 4b gives the income equivalent corresponding to an alternative state of the world without participation in a pay-as-you-go social security program (i.e., $\theta = 0$). It is obvious to see from Table 4a and Table 4b that as the magnitude and/or the frequency of impulsiveness increases, the impulsive consumer becomes worse off from a perspective of annual income evaluation. So what exactly do these calculated values for the income equivalent suggest? For example, if the agent impulsively consumes at $F = 0.5$ and $\psi = 1.4$ with all other parameters held at baseline values, then $\pi = \$504$ as shown in Table 4a. This means that a life-cycle/permanent-income agent would be willing to annually pay \$504 in order to avoid succumbing to the consumption impulse every other month and consuming around 1.4 times his optimal saving flow (which roughly corresponds to impulsively spending an average of about \$1000 every two months during his working years). Alternatively, $\Omega = \$15,063$ wherein the life-

cycle/permanent-income agent would be willing to pay a lifetime amount of \$15,063 (which is 1.5 percent of his lifetime wealth) to avoid consuming impulsively per this specific example.

One interesting quantitative feature in both Table 4a and Table 4b is that the value of the income equivalent for this relevant behavioral parameter space is much higher as the frequency of impulsiveness decreases yet the magnitude of impulsiveness increases. For example, $\pi = \$1147$ (or $\Omega = \$34280$) corresponding to an individual who impulsively spends about \$3000 roughly once every 6 months ($F = 0.2$ and $\psi = 3.7$ in Table 4a). This impulsive consumer is much worse off compared to an otherwise identical agent who impulsively spends a little over \$500 every month for 6 consecutive months (i.e., $F = 1$ and $\psi = 0.7$ generating $\pi = \$284$ or $\Omega = \$8488$). Lifetime willingness to pay to avoid impulsiveness in the former case is 3.5 percent of lifetime wealth compared to only being 0.9 percent in the latter. It thus appears that being an impulsive consumer is more costly if one is infrequently impulsive, yet binges in relative terms. I believe that the intuition for this depends on the concavity of the period utility function. If the agent is impulsive more frequently, then he experiences a series of small gains in utility which collectively outweigh the utility gain from just one impulsive spending fit (holding the total dollar value of impulsive spending constant over a fixed period of time). Conversely, the frequently impulsive agent experiences a series of small utility losses (from a series of optimal downward revisions following impulsive spending episodes) which are collectively outweighed by the utility loss from just one optimal downward revision following a single impulsive spending episode over a fixed period of time (again holding the total dollar value of impulsive spending constant). Therefore, it appears that the curvature of the period utility function drives the outcome that it is less costly (in lifetime utility terms) to impulsively consume relatively smaller amounts more frequently as opposed to an identical total dollar amount less frequently.

In looking at economy-wide increases in the frequency of impulsiveness, F , or the magnitude of impulsiveness, ψ , the welfare cost of being impulsive increases at an increasing rate.⁶³ This convexity of the income equivalent is shown in Figure 5 and Figure 6 below, and might be of unique interest given the possibility of upward trends in economy-wide levels of impulsive consumption behavior.⁶⁴ For the particular frequency of impulsiveness, $F = 0.5$, both the solid and dashed lines in Figure 5 show that the income equivalent increases at an increasing rate with economy-wide increases in the magnitude of impulsiveness, ψ . The solid line corresponds to the income equivalent in a state of the world with participation in a pay-as-you-go social security program, and the dashed line corresponds to a state of the world without participation. For the specific impulse magnitude, $\psi = 0.8$, Figure 6 likewise depicts that the income equivalent increases at an increasing rate with economy-wide increases in the frequency of impulsiveness, F , where the solid and dashed lines again denote the income equivalent with social security participation and without. The convexity of the income equivalent in ψ or F is the result of concave utility, wherein high degrees of impulsiveness (either through a large F or ψ) generate periodic consumption booms that do not add much to lifetime utility in comparison to the subsequent loss in lifetime utility following a downward correction. Alternatively stated, the curvature of the period utility function dictates the curvature of the income equivalent across F or ψ . As the value of ϕ increases, the convexity of the income equivalent, π , (or the wealth equivalent, Ω) in either F or ψ increases. Likewise, as the value of ϕ decreases, the convexity of π (or Ω) in either F or ψ decreases. Indeed, the income equivalent is linear in either ψ or F as $\phi \rightarrow 0$.

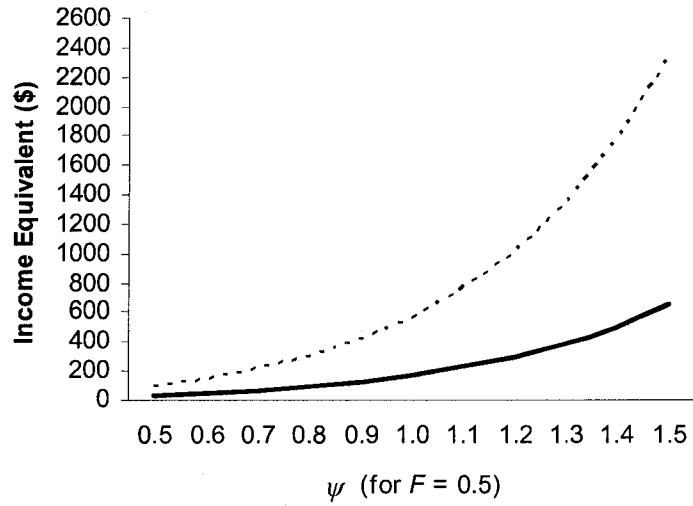
Lastly, I highlight the quantitative insight that the lifetime utility of impulsive consumers is considerably closer to the lifetime utility of otherwise identical life-

⁶³An alternative interpretation to economy-wide changes in either F or ψ is that of looking across otherwise identical economies that are heterogenous with respect to these parameters only.

⁶⁴See the motivating evidence on impulsive consumers in Chapter Two for more on this possibility.

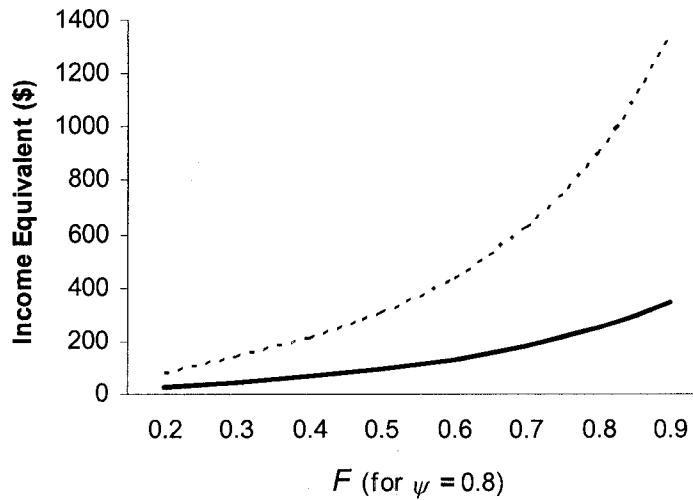
cycle/permanent-income consumers in a state of the world with participation in a pay-as-you-go social security program. This assertion is manifest by the value of the income equivalent across Table 4a and Table 4b for any same $F; \psi$ combination. This idea is also exhibited in both Figure 5 and Figure 6 for the represented $F; \psi$ combinations, wherein the dashed line representing the income equivalent (as a function of either F or ψ) in a state of the world without pay-as-you-go social security participation is strictly higher than the solid line representing the income equivalent in a state of the world with participation. Remember that the lifetime wealth of the life-cycle/permanent-income consumer is strictly lower from forced participation in social security that has an internal rate of return that is lower than the private interest rate (as is the case with the calibration in this study). He smooths out this loss to his lifetime wealth evenly across his planning horizon (which is the entire life cycle). Given that preferences are represented by the CIES utility function that exhibits $\partial u / \partial c > 0$, the life-cycle/permanent-income agent has a strictly lower level of lifetime utility since his consumption program is everywhere lower over the life cycle from social security participation. Therefore, the fact that the income equivalent (or the corresponding wealth equivalent) decreases for any particular $F; \psi$ combination in moving to a state of the world with an unfunded program denotes the quantitative result that even though impulsive consumers might be helped out in terms of an improvement to their own lifetime utility from social security participation, they are at least not hurt as much as otherwise identical life-cycle/permanent-income consumers get hurt by being forced to participate. So although life-cycle/permanent-income consumers are strictly worse off from participation (and the impulsive consumer might or might not be better off from a lifetime utility perspective), it appears that a social security program narrows the gap in well-being between the two types of consumers. Such an outcome could be loosely consistent with egalitarian ideals of equity if participation in social security does improve the lifetime utility of at least the impulsive consumer.

Figure 5. Income Equivalent across the Impulse Magnitude



Note. The solid line is with social security; the dotted line is without.

Figure 6. Income Equivalent across the Impulse Frequency



Note. The solid line is with social security; the dotted line is without.

5.2 Can Social Security Increase Retirement Consumption?

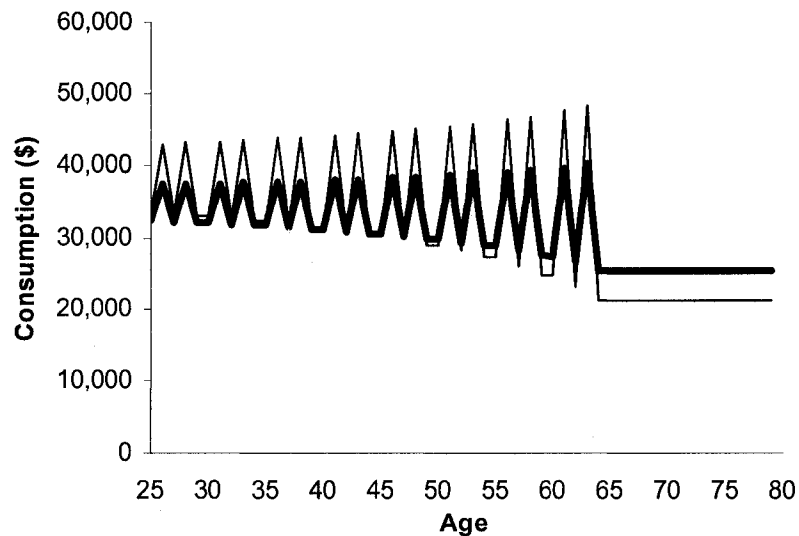
While the primary goal of a public pension system is to transfer an individual's resources from the working years to the retirement years, it is not clear whether social security really succeeds at this objective. Diamond (1977) concludes that social security does succeed at redistributing one's resources and mitigating retirement poverty. Kotlikoff et al (1982) also report evidence that social security is successful, wherein few individuals experience a drastic reduction in their standard of living that they otherwise would experience in the absence of social security. They specifically state,

To a large extent, Social Security contributed to the sound financial position of the aged population. If Social Security were removed, and not replaced by private accumulation, a large fraction of the aged population would face very sharp declines in living standards. Our econometric evidence indicates that persons receiving more social security benefits have relatively higher levels of sustainable consumption at retirement.

This evidence that social security successfully transfers one's resources to the retirement years directly contradicts the basic life-cycle/permanent-income framework if the *IRR* from social security is less than the private interest rate. Under this scenario, the LC/PI agent smooths out the loss to lifetime wealth over his lifetime horizon by an amount equal to the annuity value of the net present value of social security, wherein his consumption program is everywhere lower including during retirement (as depicted in Figure 2 above). However, my model is more consistent with the above empirical findings because social security can often succeed at transferring the agent's resources to retirement when consumers are impulsive and engage in time-inconsistent dynamic optimization. This is depicted in Figure 7 (at a step-size of $dt = 1$ for visual purposes). The thick line in Figure 7 is the consumption profile of an impulsive consumer who participates in social security calibrated to the current U.S. system, and the thin line represents an otherwise identical consumer in

a steady state without participation. For the corresponding step-size of $dt = 0.1$, the consumer who is participating in social security consumes an impulse amount of around \$1000 on average about once every three months (at $F = 0.4; \psi = 1.5$ in Figure 7), and the otherwise identical consumer who does not participate impulsively consumes an amount of about \$1800 on average about once every three months (at the same $F; \psi$ combination).

Figure 7. Life-Cycle Consumption and Social Security

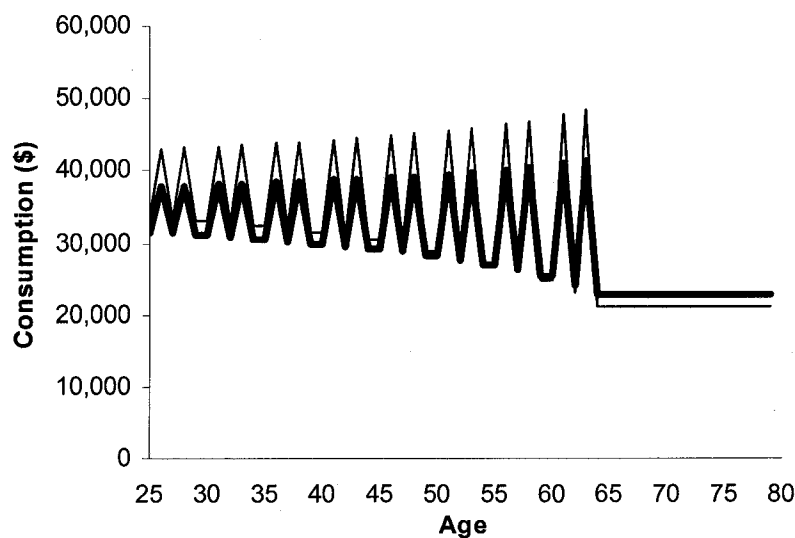


Note. $F = 0.4; \psi = 1.5; dt = 1$. The thick line is consumption at $\theta = 0.106$, and the thin line is life-cycle consumption at $\theta = 0$.

Figure 7 demonstrates that participation in a pay-as-you-go program (calibrated to the current U.S. system) forces the impulsive consumer to save for the retirement phase of his life cycle. This is manifest by the fact that his consumption profile during the work years is lower (especially when he experiences the consumption impulse) as compared to the state of the world without participation in social security. In the context of my theoretical model, a forced saving program successfully mitigates his consumption impulse amount, and therefore, the impulsive consumer's retirement

consumption program is everywhere higher as compared to the otherwise identical impulsive consumer who does not participate in social security. The percentage change in retirement consumption from participating in social security (calibrated to the current U.S. system) is reported in Table 5a for the relevant behavioral parameter space. For a broad range of impulsive behavior, social security can successfully transfer resources to the retirement years within my model. This is despite the fact that the system generates a below-market internal rate of return at the baseline calibration (i.e., $\gamma = 0.007$ for $R = 3.3$).⁶⁵ Under future population demographics of only two workers being taxed per each retiree receiving benefits (i.e., $R = 2$), the *IRR* is $\gamma = -0.01$. Yet, social security can still successfully transfer resources to retirement in my model for a decent range of relevant parameter space despite this negative *IRR*. Life-cycle consumption under this alternative steady state is depicted in Figure 8 below with the full percentage-change results listed in Table 5b.

Figure 8. Life-Cycle Consumption and Social Security ($R = 2$)



Note. $F = 0.4$; $\psi = 1.5$; $dt = 1$. The thick line is consumption at $\theta = 0.106$; $R = 2$, and the thin line is consumption at $\theta = 0$.

⁶⁵ $\gamma = 0.007$ results from my baseline calibration where $\delta = 0$ implicitly.

5.3 Is Social Security Utility-Improving?

The consumption profile of a LC/PI agent who participates in social security (at today's demographics with $\gamma = 0.007$) is everywhere lower as compared to the consumption profile of one who does not participate. This lower consumption program results since the agent participates in a system with an *IRR* that is less than the private interest rate (i.e., $\gamma = 0.007 < 0.025 = r$ at my baseline calibration). Because $\gamma < r$, the life-cycle/permanent-income agent's lifetime wealth is strictly smaller from participation, and therefore, he smooths out this loss to lifetime wealth evenly across the life cycle. Since $\partial u/\partial c > 0$ with the CIES utility function, the LC/PI agent who participates in social security has a strictly lower level of lifetime utility because his consumption profile is everywhere lower over the life cycle.

The question of whether unfunded social security improves lifetime utility is not quite straightforward for impulsive consumers who do not consistently consume the annuity value of remaining lifetime wealth. Since (105) denotes that $\partial u/\partial c > 0$, both Figure 7 and Figure 8 above (and Tables 5a and 5b below) illustrate the potential tradeoff from participation in social security. The consumption program of an impulsive consumer in social security is generally lower over the work years as compared to an otherwise identical impulsive consumer who does not participate. Therefore, there is a flow of utility losses. However, retirement consumption can be higher from participation in social security with a resulting flow of utility gains. Do the utility gains from social security participation where retirement consumption is higher offset the utility losses from forced saving? I now focus on answering this question.

5.3.1 Critical Impulse Magnitudes

If a consumer is never impulsive, then unfunded social security is never welfare-improving at a below-market *IRR*. But as individuals become more impulsive (either in F and/or ψ), then the prospect that social security can improve lifetime welfare

increases. What are the specific “critical” values of F or ψ at which social security becomes welfare-improving? For illustrative purposes, I define ψ_c as the particular value of the impulse magnitude (for a given frequency of consumption impulsiveness) such that a social planner is indifferent (on behalf of an impulsive consumer) between participation and no participation in pay-as-you-go social security calibrated to the U.S. program. Using (105) above, ψ_c (which I label the “critical impulse magnitude”) is the solution to

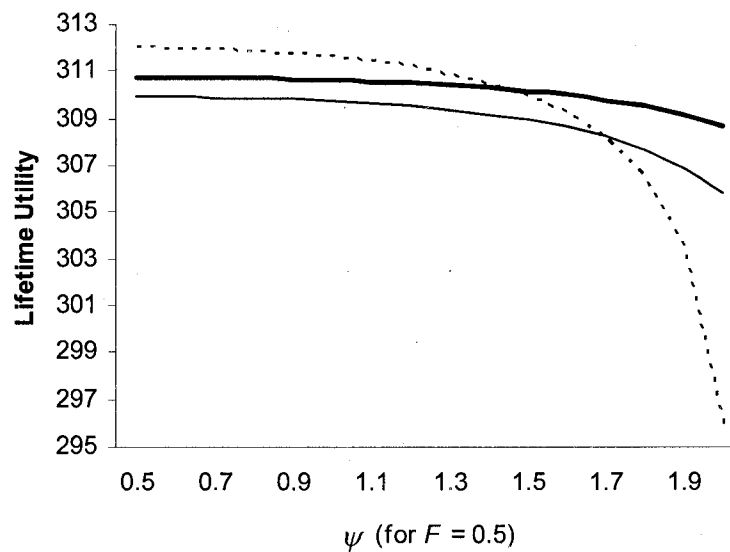
$$\sum_{t=0}^{\bar{T}-\Delta t} \left\{ \frac{e^{-\eta t} [c_i(t; \psi)|_{\theta=0.106}]^{1-\phi} - 1}{1-\phi} \right\} \Delta t = \sum_{t=0}^{\bar{T}-\Delta t} \left\{ \frac{e^{-\eta t} [c_i(t; \psi)|_{\theta=0}]^{1-\phi} - 1}{1-\phi} \right\} \Delta t \quad (107)$$

wherein lifetime welfare is equated by the social planner across the two states of the world regarding social security participation for otherwise identical impulsive consumers.

For expositional purposes only, I depict in Figure 9 below lifetime utility as a function of economy-wide changes in ψ (i.e., the fraction of optimal saving that is impulsively consumed) for an impulsive consumer across three different steady states of social security participation (where all other parameters are held at the baseline calibration). $F = 0.5$ across all of these steady states. The thick solid line is the lifetime utility of an impulsive consumer in a present state of the world with participation in social security at $R = 3.3$. The thin solid line is the lifetime utility of an otherwise identical agent in a future state of the world with participation in social security at $R = 2$. Lastly, the dashed line is the lifetime welfare of an impulsive consumer in a state of the world without participation in social security (i.e., $\theta = 0$). As can be seen in Figure 9, the thick solid line and the dashed line intersect at around an impulse magnitude of 1.5, meaning that the social planner acting on the agent’s behalf is indifferent between participation and no participation at that impulse magnitude. Therefore, $\psi_c|_{F=0.5} \cong 1.5$ under $R = 3.3$, and $\psi_c|_{F=0.5} \cong 1.7$ under $R = 2$.

Moreover, social security does not improve lifetime utility if the agent impulsively consumes at an impulse magnitude that is less than 1.5 at $R = 3.3$, yet social security is welfare-improving if he is impulsive at a magnitude greater than 1.5 at $R = 3.3$. In this example, it appears that this is a very high threshold of impulsiveness that the consumer has to experience in order to be able to benefit from social security, meaning that he has to spend 50 to 70 percent more than he planned to save every other paycheck in order to benefit. If he is even marginally less impulsive, then he would be better off without a social security program. Furthermore, $\psi_c|_{F=0.5} \cong 1.5$ under $R = 3.3$ borders on the fringe of impulsive behavior deemed relevant in Chapter Four. Indeed, $\psi_c|_{F=0.5} \cong 1.7$ under $R = 2$ falls outside the range of relevant behavior, so therefore, social security does not appear to be utility-improving meaning that participation in a pay-as-you-go social security program (calibrated to the U.S. system) strictly hurts an impulsive consumer in this specific example.

Figure 9. Lifetime Utility with and without Social Security



Note. $F = 0.5$ The thick solid line is V_i for $\theta = 0.106; R = 3.3$. The thin solid line is for $\theta = 0.106; R = 2$. The dashed line is V_i for $\theta = 0$.

5.3.2 Consumption-Equivalent Analysis

Of notable mention in Figure 9 is the difference in the curvature of lifetime utility as a function of economy-wide changes in ψ across the three steady states. Indeed, lifetime utility from the state of the world without participation in social security drops steeply for values of ψ that are greater than both $\psi_c|_{F=0.5,R=3.3} \cong 1.5$ and $\psi_c|_{F=0.5,R=2} \cong 1.7$. This suggests that the relative value from participating in social security can be drastically understated if individuals in the economy impulsively consume at magnitudes higher than any given ψ_c as compared to the loss to lifetime utility from impulsively consuming at magnitudes less than any particular ψ_c .⁶⁶ In order to meaningfully (and appropriately) quantify the difference in the curvature of these lifetime utility functions, I again employ a dynamic welfare measure labeled the “consumption equivalent”. This is an amount (measured in dollars) that is hypothetically subtracted by the social planner from the annual consumption of an impulsive consumer who participates in social security in order to force his lifetime welfare to equal that of an otherwise identical impulsive consumer who does not participate.

Formally, the consumption-equivalent is the specific value of χ that solves

$$\sum_{t=0}^{\bar{T}-\Delta t} \left\{ e^{-\eta t} \frac{[c_i(t)|_{\theta=0.106} - \chi]^{1-\phi} - 1}{1-\phi} \right\} \Delta t = \sum_{t=0}^{\bar{T}-\Delta t} \left\{ e^{-\eta t} \frac{[c_i(t)|_{\theta=0}]^{1-\phi} - 1}{1-\phi} \right\} \Delta t \quad (108)$$

As can be seen from (108), the value of χ can be interpreted as the benefit or cost from participation in an unfunded social security program measured in dollar terms of real annual consumption. Social security is utility-improving if $\chi > 0$, social security hurts lifetime welfare if $\chi < 0$ (i.e., the cost to the impulsive consumer of being forced to participate in social security), and it has no effect on welfare if $\chi = 0$. Of interest is quantifying the non-linearities in lifetime utility representatively shown in Figure 9 above. I employ numerical techniques (specifically a grid search) that

⁶⁶Although I cast this discussion in terms of ψ_c , the same concept applies for any particular F_c .

finds the particular value of the consumption equivalent, χ , that equates the left-hand side of (108) with the right-hand side of (108). I report the computed values of χ for the relevant $F; \psi$ combinations in Table 6a, Table 6b, Table 6c, and Table 6d. The values of χ in Table 6a correspond to the baseline calibration. Table 6b corresponds to a future steady state of $R = 2$. Table 6c corresponds to present demographics, but where the social planner does not give preferential treatment to any period of the impulsive consumer's life cycle over another (i.e., $\eta = 0$). Lastly, Table 6d corresponds to the steady state of $R = 2$ with $\eta = 0$.

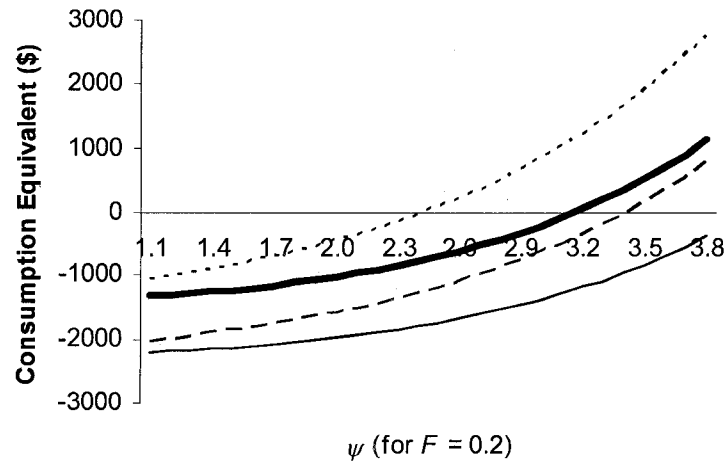
I want to point out in Tables 6a–6d the different possible critical impulse magnitudes, ψ_c , or alternatively, the different possible “critical impulse frequencies”, F_c . The critical values for ψ are denoted by where the consumption equivalent switches over from being negative to being positive in moving down any given column. Likewise, the critical values for F are denoted by where the value of χ switches over from being negative to being positive in moving across any given row. Again, it is at these values that the social planner (acting on the impulsive consumer's behalf) is indifferent about participation in unfunded social security.

For the relevant $F; \psi$ parameter space, it is apparent from Table 6a that social security (calibrated to the current U.S. system) is not generally welfare-improving at the baseline calibration of $\eta = 0.025 = \rho$. This is manifest by the predominantly negative sign for χ denoting the annual cost from social security participation measured in terms of real annual consumption. However, it appears that social security can improve an impulsive consumer's lifetime utility for a small subset of the relevant impulsiveness parameter space (where $\chi > 0$). Under future demographics when the ratio of workers to retirees is lower, it appears that social security will never improve the welfare of impulsive consumers given that the social planner discounts period utility at rate $\eta = 0.025$ (see Table 6b). If the social planner is patient and does not give preferential treatment to any period of the life cycle (i.e., $\eta = 0$) following

Feldstein (1985) among others, then social security can have a welfare-improving role for a much larger subset of relevant $F; \psi$ parameter space as denoted in Table 6c. Yet again, under the future steady state with $R = 2$ (i.e., after the so-called “social security crisis” has arrived), it appears that social security (calibrated to the U.S. system) cannot generally improve the lifetime welfare of impulsive consumers, even if the social planner is using a social discount rate of $\eta = 0$ (see Table 6d).

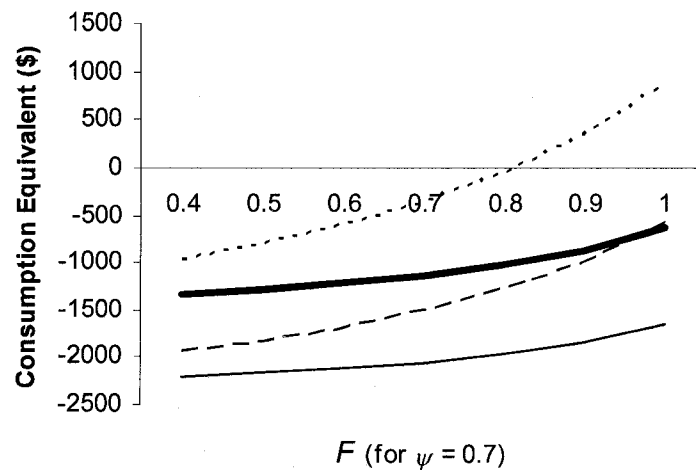
However, I want to highlight that economy-wide welfare gains from social security participation can exceed potential economy-wide welfare costs. This is manifest especially in Table 6c by moving across any given row or down any given column, wherein χ is increasing at an increasing rate for economy-wide increases in either F or ψ . For expositional purposes, I depict in Figure 10 the consumption equivalent as a function of economy-wide changes in ψ (for the case of $F = 0.2$ from Tables 6a–6d) across the two steady states of $R = 3.3$ and $R = 2$ when the social planner both discounts at $\eta = 0.025$ and $\eta = 0$. ψ_c is denoted in each respective case by where $\chi(\psi)$ intersects the horizontal axis. Of particular import is that mean-preserving economy-wide increases in the dispersion of ψ away from ψ_c generates larger welfare gains to an economy of impulsive consumers with $\psi > \psi_c$ as compared to the losses to lifetime utility experienced by an otherwise identical economy of impulsive consumers with $\psi < \psi_c$. For example, from Figure 10 it appears that for $F = 0.2$ and $\psi = 3.7$ (corresponding to an economy of agents who impulsively consume around \$3000 roughly once every six months) $\chi = \$2446$ at $R = 3.3; \eta = 0$. Alternatively, under the same scenario with an identical absolute-value decrease in the impulse magnitude away from ψ_c (i.e., $\psi = 1.2$ for $F = 0.2$ corresponding to an economy of individuals who impulsively consume around \$520 roughly once every six months), then $\chi = -\$1015$. Therefore, there is a net gain to lifetime welfare for economy-wide increases in ψ as compared to identical economy-wide decreases. In Figure 11, I depict the same exercise for economy-wide changes in F for $\psi = 0.7$.

Figure 10. Consumption Equivalent across ψ for $F = 0.2$



Note. $F = 0.2$ The thick solid line is for $R = 3.3$. The thin solid line is for $R = 2$. The dotted line is for $R = 3.3; \eta = 0$. The dashed line is for $R = 2; \eta = 0$.

Figure 11. Consumption Equivalent across F for $\psi = 0.7$



Note. $\psi = 0.7$ The thick solid line is for $R = 3.3$. The thin solid line is for $R = 2$. The dotted line is for $R = 3.3; \eta = 0$. The dashed line is for $R = 2; \eta = 0$.

5.4 Optimal Social Security

Given that participation in pay-as-you-go social security (calibrated to the current U.S. system with $\theta = 0.106$) is generally not welfare-improving under present demographics (and not at all under future demographics) for the relevant range of impulsive behavior, I now proceed to study the optimal size of an unfunded system for the same $F; \psi$ parameter space. I therefore define the optimal social security tax rate (from the perspective of a social planner) as the value of θ that maximizes the lifetime welfare of a representative impulsive consumer. This is formally given as

$$\theta^* = \arg \max_{\theta \in [0,1]} \left(\sum_{t=0}^{\bar{T}-\Delta t} \left\{ e^{-\eta t} \frac{c_i(t)^{1-\phi} - 1}{1-\phi} \right\} \Delta t \right) \quad (109)$$

θ^* can alternatively be interpreted as the optimal size of a pay-as-you-go program. I again use numerical techniques to find θ^* , wherein I employ a grid search that finds the particular value of θ that maximizes the representative impulsive consumer's (socially-discounted) lifetime utility. I report these results in Table 7a, Table 7b, Table 7c, and Table 7d for the range of relevant impulsive consumption behavior. Specifically, Table 7a denotes the values of θ^* under current demographics where the social planner discounts at the private discount rate. In Table 7b, I report the corresponding values of θ^* under $R = 2$ where again, $\eta = 0.025$. In Table 7c, I report θ^* for $R = 3.3; \eta = 0$, and in Table 7d I report the optimal tax rate for $R = 2; \eta = 0$.

I mention three noticeable features of importance from Tables 7a–7d. First, the optimal tax rate is generally zero but increases fairly quickly as individuals become more impulsive in their consumption behavior (with respect to either F or ψ). Second, the optimal size of a social security program is larger than the current U.S. program of $\theta = 0.106$ for only a small subset of the relevant $F; \psi$ parameter space under present demographics with a social planner who discounts at $\eta = 0.025$. However, θ^* is over twice as large as the current U.S. tax rate for a notable subset of

the relevant $F; \psi$ parameter space under $R = 3.3; \eta = 0$. Lastly, it appears that the optimal size of an unfunded social security program is to generally not have any program at all (and leave impulsive consumers to fend for themselves) under future demographics where $R = 2$ (see Tables 7b and 7d).

5.5 Optimal Social Security under Heterogenous Wages

In this section, I entertain the idea of optimal social security for an economy populated with impulsive consumers who are heterogeneous with respect to their wage. I therefore extend the baseline model by building in a wage distribution across a life-cycle cohort.⁶⁷ The economy has a constant population that is comprised of a continuum of identical life-cycle cohorts, wherein a cohort enters (is born) and exits the population (dies) at each instant. With “ n ” denoting agent heterogeneity with respect to wages across any particular cohort, agent n enters the workforce at $t = 0$, retires at $t = T$, and dies at time $t = \bar{T}$. The dates of retirement and death are exogenous and fixed. As such, there are T working cohorts, $(\bar{T} - T)$ retired cohorts, and \bar{T} total cohorts in the population at any instant. Agent n earns a constant wage, w^n , for all $t \in [0, T]$, and all wage income that is not consumed flows into the asset account, $k^n(t)$, which grows at rate r . The social security tax rate is θ , wherein social security benefits, b , are received during the retirement phase of the life cycle. Labor is supplied inelastically, and workers therefore bear the full burden of the social security tax. w^n is distributed uniformly across each respective life-cycle cohort with a mean of \bar{w} and support $[\bar{w} - \epsilon, \bar{w} + \epsilon]$. As such, $w^n \equiv \bar{w} - \epsilon + n$ for $n \in [0, 2\epsilon]$ with total wages across the cohort therefore given as $W \equiv 2\epsilon\bar{w}$. The social security budget balances at each instant in time such that $b = \frac{\theta WT}{\beta}$, where $\beta \equiv (\bar{T} - T)2\epsilon$ is the total number of retirees in the population. This benefit rule

⁶⁷My setup of wage heterogeneity for boundedly-rational, impulsive consumers follows Caliendo and Findley (2007) who model wage heterogeneity within the standard life-cycle/permanent-income framework when all consumers are fully rational.

has a redistributational component to it. Namely, all agents receive the same amount of social security benefits, but agents with wages higher than \bar{w} pay more in taxes, as opposed to agents on the lower end of the wage support who end up paying less in social security taxes. All cohorts are alike since ϵ and \bar{w} are autonomous.

Formally, the optimal control problem of agent n is

$$\max \int_{t_0}^{\bar{T}} e^{-\rho(t-t_0)} \frac{c^n(t)^{1-\phi} - 1}{1-\phi} dt \quad (110)$$

where the control variable is consumption, $c^n(t)$. Agent n is constrained by

$$\frac{dk^n(t)}{dt} = rk^n(t) + (1-\theta)w^n - c^n(t) \quad \forall t \in [t_0, T] \quad (111)$$

$$\frac{dk^n(t)}{dt} = rk^n(t) + b - c^n(t) \quad \forall t \in [T, \bar{T}] \quad (112)$$

$$k^n(t_0) \text{ given} \quad k^n(\bar{T}) = 0 \quad (113)$$

Application of the *Maximum Principle* for two-stage fixed-endpoint control problems yields the *optimal control* for agent n

$$c^{n*}(t; t_0) = \frac{e^{gt}(g-r)\{k^n(t_0)e^{-rt_0} + \{(1-\theta)w^n[e^{-rt_0} - e^{-rT}] + b[e^{-rT} - e^{-r\bar{T}]\}\}/r\}}{[e^{(g-r)\bar{T}} - e^{(g-r)t_0}]} \quad (114)$$

where $g \equiv \frac{(r-\rho)}{\phi}$ is the planned growth rate in consumption. This is the program that agent n intends to follow for $t \in [t_0, T]$ from the perspective of $t_0 \in [0, T]$.

Similar to the baseline model in Chapter Three, agent n periodically experiences the consumption impulse. His actual impulsive consumption program is therefore

$$c_i^n(t) = \frac{e^{gt}(g-r)\{k^n(t)e^{-rt} + \{(1-\theta)w^n[e^{-rt} - e^{-rT}] + b[e^{-rT} - e^{-r\bar{T}]\}\}/r\}}{[e^{(g-r)\bar{T}} - e^{(g-r)t}]} + iI^n(t) \quad (115)$$

for $t \in [0, T]$, where

$$I^n(t) \equiv \psi [(1 - \theta)w^n - c^{n*}(t; t)] \quad (116)$$

$$k^n(t) = k^n(t - dt) + \{rk^n(t - dt) + (1 - \theta)w^n - c_i^n(t - dt)\} dt \quad \forall t \in [0, T] \quad (117)$$

Consistent with the setup in Chapter Three, the actual retirement consumption program of impulsive consumer n is time-consistent and given as

$$c_i^n(t) = \frac{e^{gt}(g - r) \{k^n(T)e^{-rT} + b[e^{-rT} - e^{-r\bar{T}}]/r\}}{[e^{(g-r)\bar{T}} - e^{(g-r)T}]} \quad \forall t \in [T, \bar{T}] \quad (118)$$

with

$$k^n(t) = \frac{k^n(T)e^{r(t-T)} + b[e^{r(t-T)} - 1]/r}{\frac{\{k^n(T)e^{-rT} + b[e^{-rT} - e^{-r\bar{T}}]/r\} [e^{gt} - e^{gT+r(t-T)}]}{[e^{(g-r)\bar{T}} - e^{(g-r)T}]}} \quad (119)$$

for $t \in [T, \bar{T}]$.

Because all cohorts are identical, the social planner focuses on the welfare of a single cohort of impulsive consumers and optimally selects the social security tax rate such that

$$\theta^* = \arg \max_{\theta \in [0, 1]} \left(\sum_{n=0}^{2\epsilon - \Delta n} \{V_i^n\} \Delta n \right) \quad (120)$$

where

$$V_i^n \equiv \sum_{t=0}^{\bar{T} - \Delta t} \left\{ e^{-\eta t} \frac{c_i^n(t)^{1-\phi} - 1}{1 - \phi} \right\} \Delta t \quad (121)$$

Since the worker-to-retiree ratio is implicitly equal to $T/(\bar{T} - T)$ as denoted in the benefits formula above, I drop \bar{T} down from 55 (as in the baseline calibration) to 52 in order to produce a ratio that is equal to the current worker-to-retiree ratio of 3.3 in the U.S. I set \bar{w} to \$40,000 for consistency with my earlier results, and I hold all other variables fixed at their baseline values including $\eta = 0.025$. I use numerical methods to calculate θ^* . For the sake of comparison, I first consider the case of $\epsilon = 0.5$ which

roughly corresponds to my baseline setup of no wage heterogeneity. The results are given in Table 8a below. I also consider the case of $\epsilon = 20,000$ in Table 8b which means that the population is uniformly distributed from \$20,000 at the low wage end all the way up to \$60,000 at the high end. I am computationally unable to examine cases for $\epsilon > 20,000$ (with $\bar{w} = 40,000$ for comparison) on account that social welfare is undefined since low-end wage earners experience negative consumption for even low degrees of impulsive consumption behavior. Therefore, $\epsilon = 20,000$ can be considered a bound on my results.

For the relevant range of $F; \psi$ parameter space considered, an increase in the dispersion of wages away from an economy-wide wage of \$40,000 has the (expected) quantitative effect of increasing the optimal social security tax rate for this economy. This result is manifest by comparing the results from the same cells along the lower border of relevant $F; \psi$ ordered pairs in moving from Table 8a to Table 8b. For example, the optimal social security tax rate is only 3.3 percent for $F = 0.4$ and $\psi = 1.9$ with an economy-wide wage of \$40,000 (i.e., $\bar{w} = 40,000$ and $\epsilon = 0.5$). But with the introduction of a uniformly-distributed wage support of \$40,000 to an economy of impulsive consumers with a mean wage of \$40,000 (i.e., $\bar{w} = 40,000$ and $\epsilon = 20,000$), then the social security tax rate that maximizes social welfare across the economy increases by two percentage-points from 3.3 percent up to 5.3 percent. Indeed, it appears that the optimal tax rate generally increases by 2 to 3 percentage points with the introduction of this range of wage heterogeneity. What is surprising, however, is that all of these optimal tax rates are much lower than the current U.S. system of 10.6 percent (at least for the range of $F; \psi$ parameter space deemed relevant by the calibration in Chapter Four).

Table 4a. Income Equivalent (\$) with Social Security

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	12	13	13
0.3	—	—	—	—	13	16	18	20	23	26
0.4	—	—	—	17	21	25	30	36	43	51
0.5	—	—	19	25	32	40	50	62	76	93
0.6	—	—	25	35	46	61	78	101	129	164
0.7	—	—	34	48	66	90	120	160	213	284
0.8	—	28	44	65	93	130	180	249	347	—
0.9	—	35	57	85	127	184	265	382	—	—
1.0	—	42	72	112	171	256	385	—	—	—
1.1	—	51	90	145	227	354	554	—	—	—
1.2	—	62	110	184	298	483	—	—	—	—
1.3	—	73	135	232	389	657	—	—	—	—
1.4	—	86	163	289	504	—	—	—	—	—
1.5	43	101	196	358	649	—	—	—	—	—
1.6	47	117	234	441	—	—	—	—	—	—
1.7	54	135	277	541	—	—	—	—	—	—
1.8	60	155	327	660	—	—	—	—	—	—
1.9	67	176	384	802	—	—	—	—	—	—
2.0	74	200	449	—	—	—	—	—	—	—
2.1	82	227	522	—	—	—	—	—	—	—
2.2	91	256	606	—	—	—	—	—	—	—
2.3	99	287	701	—	—	—	—	—	—	—
2.4	108	322	809	—	—	—	—	—	—	—
2.5	118	359	932	—	—	—	—	—	—	—
2.6	129	400	1071	—	—	—	—	—	—	—
2.7	139	444	—	—	—	—	—	—	—	—
2.8	151	493	—	—	—	—	—	—	—	—
2.9	163	545	—	—	—	—	—	—	—	—
3.0	176	601	—	—	—	—	—	—	—	—
3.1	189	662	—	—	—	—	—	—	—	—
3.2	203	728	—	—	—	—	—	—	—	—
3.3	217	800	—	—	—	—	—	—	—	—
3.4	232	877	—	—	—	—	—	—	—	—
3.5	248	960	—	—	—	—	—	—	—	—
3.6	265	1050	—	—	—	—	—	—	—	—
3.7	282	1147	—	—	—	—	—	—	—	—
3.8	300	1252	—	—	—	—	—	—	—	—

Note. All parameters are set to baseline values. $\pi|_{F=0.1, \psi=7.0} = 1396$.

Table 4b. Income Equivalent (\$) without Social Security

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	29	32	36
0.3	—	—	—	—	36	44	51	60	70	81
0.4	—	—	—	48	61	76	93	113	137	165
0.5	—	—	53	73	96	124	159	201	253	318
0.6	—	—	74	106	145	194	257	339	446	590
0.7	—	—	101	149	211	293	404	556	771	1086
0.8	—	82	134	204	299	431	619	897	1329	—
0.9	—	102	174	274	414	621	937	1443	—	—
1.0	—	127	222	360	564	883	1411	—	—	—
1.1	—	154	279	466	759	1247	2133	—	—	—
1.2	—	186	345	596	1011	1755	—	—	—	—
1.3	—	222	423	754	1338	2482	—	—	—	—
1.4	—	262	513	948	1765	—	—	—	—	—
1.5	124	306	617	1183	2330	—	—	—	—	—
1.6	140	355	738	1470	—	—	—	—	—	—
1.7	158	410	877	1821	—	—	—	—	—	—
1.8	177	470	1036	2250	—	—	—	—	—	—
1.9	196	536	1219	2784	—	—	—	—	—	—
2.0	218	609	1429	—	—	—	—	—	—	—
2.1	240	688	1670	—	—	—	—	—	—	—
2.2	264	775	1947	—	—	—	—	—	—	—
2.3	289	870	2266	—	—	—	—	—	—	—
2.4	316	973	2635	—	—	—	—	—	—	—
2.5	344	1086	3063	—	—	—	—	—	—	—
2.6	373	1208	3561	—	—	—	—	—	—	—
2.7	404	1341	—	—	—	—	—	—	—	—
2.8	436	1486	—	—	—	—	—	—	—	—
2.9	470	1642	—	—	—	—	—	—	—	—
3.0	505	1813	—	—	—	—	—	—	—	—
3.1	542	1998	—	—	—	—	—	—	—	—
3.2	581	2199	—	—	—	—	—	—	—	—
3.3	621	2417	—	—	—	—	—	—	—	—
3.4	663	2655	—	—	—	—	—	—	—	—
3.5	707	2915	—	—	—	—	—	—	—	—
3.6	752	3196	—	—	—	—	—	—	—	—
3.7	799	3506	—	—	—	—	—	—	—	—
3.8	848	3845	—	—	—	—	—	—	—	—

Note. $\theta = 0$ with all other parameters at the baseline. $\pi|_{F=0.1, \psi=7.0} = 3782$.

Table 5a. Change (%) in Retirement Consumption

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	-1	-1	0
0.3	—	—	—	—	-2	-1	0	1	1	2
0.4	—	—	—	-1	0	1	2	3	5	6
0.5	—	—	-2	0	1	2	4	6	9	12
0.6	—	—	-1	1	2	5	7	11	15	21
0.7	—	—	0	2	4	7	11	17	25	36
0.8	—	-1	1	3	6	11	17	26	41	—
0.9	—	-1	1	5	9	15	25	40	—	—
1.0	—	0	2	6	12	21	36	—	—	—
1.1	—	0	3	8	16	29	55	—	—	—
1.2	—	1	5	11	21	40	—	—	—	—
1.3	—	1	6	13	27	58	—	—	—	—
1.4	—	2	7	17	36	—	—	—	—	—
1.5	-2	2	9	21	48	—	—	—	—	—
1.6	-1	3	11	26	—	—	—	—	—	—
1.7	-1	4	13	32	—	—	—	—	—	—
1.8	-1	5	15	40	—	—	—	—	—	—
1.9	-1	5	18	51	—	—	—	—	—	—
2.0	0	6	21	—	—	—	—	—	—	—
2.1	0	7	24	—	—	—	—	—	—	—
2.2	0	8	29	—	—	—	—	—	—	—
2.3	0	10	34	—	—	—	—	—	—	—
2.4	1	11	40	—	—	—	—	—	—	—
2.5	1	12	48	—	—	—	—	—	—	—
2.6	1	14	57	—	—	—	—	—	—	—
2.7	1	15	—	—	—	—	—	—	—	—
2.8	2	17	—	—	—	—	—	—	—	—
2.9	2	19	—	—	—	—	—	—	—	—
3.0	2	21	—	—	—	—	—	—	—	—
3.1	3	23	—	—	—	—	—	—	—	—
3.2	3	26	—	—	—	—	—	—	—	—
3.3	3	29	—	—	—	—	—	—	—	—
3.4	4	32	—	—	—	—	—	—	—	—
3.5	4	36	—	—	—	—	—	—	—	—
3.6	4	40	—	—	—	—	—	—	—	—
3.7	5	45	—	—	—	—	—	—	—	—
3.8	5	51	—	—	—	—	—	—	—	—

Note. All parameters are set to baseline values. Retirement consumption increases by 34 percent at $F = 0.1; \psi = 7.0$.

Table 5b. Change (%) in Retirement Consumption ($R = 2$)

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	-5	-5	-4
0.3	—	—	—	—	-5	-5	-4	-4	-3	-3
0.4	—	—	—	-5	-4	-4	-3	-2	-1	0
0.5	—	—	-5	-4	-4	-3	-2	0	1	3
0.6	—	—	-5	-4	-3	-1	0	2	5	9
0.7	—	—	-4	-3	-2	0	3	6	11	18
0.8	—	-5	-4	-2	0	2	6	12	20	—
0.9	—	-5	-3	-1	1	5	11	20	—	—
1.0	—	-4	-3	0	3	8	18	—	—	—
1.1	—	-4	-2	1	6	13	29	—	—	—
1.2	—	-4	-1	2	9	20	—	—	—	—
1.3	—	-3	-1	4	12	31	—	—	—	—
1.4	—	-3	0	6	18	—	—	—	—	—
1.5	-5	-3	1	8	25	—	—	—	—	—
1.6	-5	-2	2	11	—	—	—	—	—	—
1.7	-5	-2	4	15	—	—	—	—	—	—
1.8	-5	-1	5	20	—	—	—	—	—	—
1.9	-5	-1	7	27	—	—	—	—	—	—
2.0	-4	0	8	—	—	—	—	—	—	—
2.1	-4	0	11	—	—	—	—	—	—	—
2.2	-4	1	13	—	—	—	—	—	—	—
2.3	-4	2	16	—	—	—	—	—	—	—
2.4	-4	2	20	—	—	—	—	—	—	—
2.5	-4	3	25	—	—	—	—	—	—	—
2.6	-4	4	31	—	—	—	—	—	—	—
2.7	-3	5	—	—	—	—	—	—	—	—
2.8	-3	6	—	—	—	—	—	—	—	—
2.9	-3	7	—	—	—	—	—	—	—	—
3.0	-3	8	—	—	—	—	—	—	—	—
3.1	-3	10	—	—	—	—	—	—	—	—
3.2	-2	11	—	—	—	—	—	—	—	—
3.3	-2	13	—	—	—	—	—	—	—	—
3.4	-2	15	—	—	—	—	—	—	—	—
3.5	-2	18	—	—	—	—	—	—	—	—
3.6	-1	20	—	—	—	—	—	—	—	—
3.7	-1	23	—	—	—	—	—	—	—	—
3.8	-1	27	—	—	—	—	—	—	—	—

Note. $R = 2$ with all other parameters at the baseline. Retirement consumption increases by 16 percent at $F = 0.1; \psi = 7.0$.

Table 6a. Consumption Equivalent (\$) at Present Demographics

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	-1415	-1412	-1410
0.3	—	—	—	—	-1410	-1405	-1399	-1393	-1385	-1377
0.4	—	—	—	-1402	-1392	-1381	-1369	-1354	-1337	-1316
0.5	—	—	-1398	-1384	-1367	-1347	-1322	-1291	-1253	-1204
0.6	—	—	-1384	-1361	-1333	-1297	-1251	-1191	-1111	-1002
0.7	—	—	-1364	-1330	-1286	-1226	-1145	-1032	-868	-625
0.8	—	-1378	-1341	-1291	-1223	-1127	-988	-779	-444	—
0.9	—	-1365	-1313	-1243	-1141	-990	-754	-366	—	—
1.0	—	-1347	-1280	-1182	-1034	-799	-402	—	—	—
1.1	—	-1328	-1240	-1107	-895	-534	143	—	—	—
1.2	—	-1306	-1194	-1016	-714	-159	—	—	—	—
1.3	—	-1282	-1140	-904	-478	383	—	—	—	—
1.4	—	-1255	-1078	-768	-168	—	—	—	—	—
1.5	-1350	-1224	-1006	-602	243	—	—	—	—	—
1.6	-1339	-1191	-922	-399	—	—	—	—	—	—
1.7	-1327	-1154	-827	-151	—	—	—	—	—	—
1.8	-1314	-1113	-717	155	—	—	—	—	—	—
1.9	-1301	-1068	-592	535	—	—	—	—	—	—
2.0	-1288	-1019	-447	—	—	—	—	—	—	—
2.1	-1272	-965	-282	—	—	—	—	—	—	—
2.2	-1256	-907	-92	—	—	—	—	—	—	—
2.3	-1240	-844	126	—	—	—	—	—	—	—
2.4	-1222	-775	379	—	—	—	—	—	—	—
2.5	-1204	-700	672	—	—	—	—	—	—	—
2.6	-1184	-618	1016	—	—	—	—	—	—	—
2.7	-1164	-530	—	—	—	—	—	—	—	—
2.8	-1143	-434	—	—	—	—	—	—	—	—
2.9	-1121	-331	—	—	—	—	—	—	—	—
3.0	-1098	-218	—	—	—	—	—	—	—	—
3.1	-1074	-96	—	—	—	—	—	—	—	—
3.2	-1049	35	—	—	—	—	—	—	—	—
3.3	-1023	180	—	—	—	—	—	—	—	—
3.4	-996	337	—	—	—	—	—	—	—	—
3.5	-968	507	—	—	—	—	—	—	—	—
3.6	-939	693	—	—	—	—	—	—	—	—
3.7	-909	898	—	—	—	—	—	—	—	—
3.8	-878	1121	—	—	—	—	—	—	—	—

Note. All parameters are set to baseline values. $\chi|_{F=0.1, \psi=7.0} = 923$.

Table 6b. Consumption Equivalent (\$) at Future Demographics

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	-2268	-2266	-2264
0.3	—	—	—	—	-2264	-2260	-2256	-2251	-2246	-2239
0.4	—	—	—	-2258	-2251	-2242	-2233	-2222	-2208	-2192
0.5	—	—	-2255	-2245	-2232	-2216	-2197	-2174	-2144	-2106
0.6	—	—	-2244	-2227	-2206	-2178	-2143	-2096	-2033	-1948
0.7	—	—	-2230	-2205	-2170	-2124	-2061	-1973	-1844	-1650
0.8	—	-2241	-2212	-2174	-2122	-2048	-1940	-1775	-1510	—
0.9	—	-2230	-2190	-2137	-2059	-1942	-1758	-1452	—	—
1.0	—	-2217	-2166	-2091	-1977	-1795	-1484	—	—	—
1.1	—	-2203	-2136	-2035	-1871	-1590	-1056	—	—	—
1.2	—	-2186	-2101	-1965	-1732	-1299	—	—	—	—
1.3	—	-2168	-2061	-1880	-1551	-875	—	—	—	—
1.4	—	-2147	-2013	-1776	-1312	—	—	—	—	—
1.5	-2220	-2125	-1959	-1650	-994	—	—	—	—	—
1.6	-2211	-2099	-1896	-1495	—	—	—	—	—	—
1.7	-2203	-2072	-1824	-1306	—	—	—	—	—	—
1.8	-2193	-2041	-1741	-1072	—	—	—	—	—	—
1.9	-2183	-2008	-1647	-781	—	—	—	—	—	—
2.0	-2173	-1971	-1538	—	—	—	—	—	—	—
2.1	-2162	-1931	-1413	—	—	—	—	—	—	—
2.2	-2150	-1888	-1270	—	—	—	—	—	—	—
2.3	-2137	-1840	-1105	—	—	—	—	—	—	—
2.4	-2124	-1789	-914	—	—	—	—	—	—	—
2.5	-2111	-1733	-691	—	—	—	—	—	—	—
2.6	-2097	-1672	-431	—	—	—	—	—	—	—
2.7	-2082	-1607	—	—	—	—	—	—	—	—
2.8	-2066	-1535	—	—	—	—	—	—	—	—
2.9	-2050	-1458	—	—	—	—	—	—	—	—
3.0	-2033	-1375	—	—	—	—	—	—	—	—
3.1	-2015	-1284	—	—	—	—	—	—	—	—
3.2	-1997	-1185	—	—	—	—	—	—	—	—
3.3	-1977	-1078	—	—	—	—	—	—	—	—
3.4	-1957	-961	—	—	—	—	—	—	—	—
3.5	-1938	-835	—	—	—	—	—	—	—	—
3.6	-1916	-696	—	—	—	—	—	—	—	—
3.7	-1894	-543	—	—	—	—	—	—	—	—
3.8	-1871	-376	—	—	—	—	—	—	—	—

Note. $R = 2$ with all other parameters at the baseline. $\chi|_{F=0.1, \psi=7.0} = -556$.

Table 6c. Consumption Equivalent (\$) at Present Demographics ($\eta = 0$)

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	-1226	-1197	-1166
0.3	—	—	—	—	-1235	-1190	-1143	-1092	-1039	-980
0.4	—	—	—	-1215	-1151	-1082	-1008	-925	-834	-733
0.5	—	—	-1225	-1144	-1054	-953	-840	-711	-564	-392
0.6	—	—	-1171	-1063	-940	-797	-630	-431	-192	105
0.7	—	—	-1112	-971	-806	-606	-363	-55	341	874
0.8	—	-1195	-1046	-866	-647	-371	-14	467	1145	—
0.9	—	-1155	-973	-747	-458	-76	451	1221	—	—
1.0	—	-1112	-892	-609	-231	298	1087	—	—	—
1.1	—	-1065	-802	-450	43	780	1989	—	—	—
1.2	—	-1015	-703	-267	378	1414	—	—	—	—
1.3	—	-962	-592	-55	789	2296	—	—	—	—
1.4	—	-904	-469	193	1300	—	—	—	—	—
1.5	-1182	-842	-333	481	1946	—	—	—	—	—
1.6	-1159	-776	-181	819	—	—	—	—	—	—
1.7	-1135	-705	-12	1218	—	—	—	—	—	—
1.8	-1110	-629	176	1693	—	—	—	—	—	—
1.9	-1084	-548	387	2262	—	—	—	—	—	—
2.0	-1057	-461	622	—	—	—	—	—	—	—
2.1	-1030	-368	886	—	—	—	—	—	—	—
2.2	-1001	-268	1182	—	—	—	—	—	—	—
2.3	-970	-161	1516	—	—	—	—	—	—	—
2.4	-939	-47	1893	—	—	—	—	—	—	—
2.5	-907	75	2322	—	—	—	—	—	—	—
2.6	-874	205	2813	—	—	—	—	—	—	—
2.7	-839	345	—	—	—	—	—	—	—	—
2.8	-804	495	—	—	—	—	—	—	—	—
2.9	-767	655	—	—	—	—	—	—	—	—
3.0	-729	827	—	—	—	—	—	—	—	—
3.1	-690	1011	—	—	—	—	—	—	—	—
3.2	-649	1208	—	—	—	—	—	—	—	—
3.3	-608	1420	—	—	—	—	—	—	—	—
3.4	-565	1648	—	—	—	—	—	—	—	—
3.5	-520	1894	—	—	—	—	—	—	—	—
3.6	-475	2159	—	—	—	—	—	—	—	—
3.7	-428	2446	—	—	—	—	—	—	—	—
3.8	-379	2757	—	—	—	—	—	—	—	—

Note. $\eta = 0$ with all other parameters at the baseline. $\chi|_{F=0.1, \psi=7.0} = 2206$.

Table 6d. Consumption Equivalent (\$) at Future Demographics ($\eta = 0$)

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	-2136	-2116	-2094
0.3	—	—	—	—	-2142	-2110	-2077	-2041	-2003	-1962
0.4	—	—	—	-2127	-2083	-2033	-1980	-1921	-1856	-1783
0.5	—	—	-2135	-2077	-2013	-1941	-1860	-1766	-1659	-1534
0.6	—	—	-2096	-2019	-1931	-1828	-1707	-1562	-1386	-1167
0.7	—	—	-2054	-1953	-1834	-1689	-1511	-1285	-990	-590
0.8	—	-2113	-2006	-1877	-1718	-1517	-1254	-897	-386	—
0.9	—	-2084	-1953	-1790	-1580	-1300	-909	-331	—	—
1.0	—	-2053	-1895	-1690	-1414	-1023	-433	—	—	—
1.1	—	-2019	-1830	-1574	-1212	-665	246	—	—	—
1.2	—	-1983	-1758	-1440	-965	-191	—	—	—	—
1.3	—	-1945	-1677	-1284	-660	449	—	—	—	—
1.4	—	-1903	-1588	-1103	-280	—	—	—	—	—
1.5	-2103	-1848	-1488	-890	201	—	—	—	—	—
1.6	-2086	-1810	-1377	-641	—	—	—	—	—	—
1.7	-2069	-1759	-1254	-345	—	—	—	—	—	—
1.8	-2051	-1704	-1116	5	—	—	—	—	—	—
1.9	-2032	-1645	-962	427	—	—	—	—	—	—
2.0	-2013	-1581	-789	—	—	—	—	—	—	—
2.1	-1993	-1514	-595	—	—	—	—	—	—	—
2.2	-1972	-1441	-378	—	—	—	—	—	—	—
2.3	-1950	-1364	-133	—	—	—	—	—	—	—
2.4	-1928	-1281	144	—	—	—	—	—	—	—
2.5	-1905	-1192	460	—	—	—	—	—	—	—
2.6	-1881	-1097	821	—	—	—	—	—	—	—
2.7	-1856	-995	—	—	—	—	—	—	—	—
2.8	-1830	-887	—	—	—	—	—	—	—	—
2.9	-1804	-770	—	—	—	—	—	—	—	—
3.0	-1776	-645	—	—	—	—	—	—	—	—
3.1	-1748	-511	—	—	—	—	—	—	—	—
3.2	-1719	-368	—	—	—	—	—	—	—	—
3.3	-1689	-213	—	—	—	—	—	—	—	—
3.4	-1658	-47	—	—	—	—	—	—	—	—
3.5	-1626	132	—	—	—	—	—	—	—	—
3.6	-1593	325	—	—	—	—	—	—	—	—
3.7	-1559	534	—	—	—	—	—	—	—	—
3.8	-1524	761	—	—	—	—	—	—	—	—

Note. $R = 2; \eta = 0$ with all other parameters at the baseline. $\chi|_{F=0.1, \psi=7.0} = 334$.

Table 7a. Optimal Tax Rate (%) at Present Demographics

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	0.0	0.0	0.0
0.3	—	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0
0.4	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—
0.9	—	0.0	0.0	0.0	0.0	0.0	0.0	0.8	—	—
1.0	—	0.0	0.0	0.0	0.0	0.0	0.0	—	—	—
1.1	—	0.0	0.0	0.0	0.0	0.0	6.2	—	—	—
1.2	—	0.0	0.0	0.0	0.0	3.3	—	—	—	—
1.3	—	0.0	0.0	0.0	0.0	8.1	—	—	—	—
1.4	—	0.0	0.0	0.0	3.1	—	—	—	—	—
1.5	0.0	0.0	0.0	0.0	7.4	—	—	—	—	—
1.6	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.7	0.0	0.0	0.0	3.2	—	—	—	—	—	—
1.8	0.0	0.0	0.0	6.9	—	—	—	—	—	—
1.9	0.0	0.0	0.0	9.8	—	—	—	—	—	—
2.0	0.0	0.0	0.0	—	—	—	—	—	—	—
2.1	0.0	0.0	0.4	—	—	—	—	—	—	—
2.2	0.0	0.0	3.9	—	—	—	—	—	—	—
2.3	0.0	0.0	6.8	—	—	—	—	—	—	—
2.4	0.0	0.0	9.2	—	—	—	—	—	—	—
2.5	0.0	0.0	11.2	—	—	—	—	—	—	—
2.6	0.0	0.0	12.9	—	—	—	—	—	—	—
2.7	0.0	0.0	—	—	—	—	—	—	—	—
2.8	0.0	0.0	—	—	—	—	—	—	—	—
2.9	0.0	0.0	—	—	—	—	—	—	—	—
3.0	0.0	1.0	—	—	—	—	—	—	—	—
3.1	0.0	3.6	—	—	—	—	—	—	—	—
3.2	0.0	5.9	—	—	—	—	—	—	—	—
3.3	0.0	7.8	—	—	—	—	—	—	—	—
3.4	0.0	9.5	—	—	—	—	—	—	—	—
3.5	0.0	11.0	—	—	—	—	—	—	—	—
3.6	0.0	12.3	—	—	—	—	—	—	—	—
3.7	0.0	13.4	—	—	—	—	—	—	—	—
3.8	0.0	14.4	—	—	—	—	—	—	—	—

Note. All parameters are set to the baseline. $\theta^*|_{F=0.1, \psi=7.0} = 15.5$.

Table 7b. Optimal Tax Rate (%) at Future Demographics

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	0.0	0.0	0.0
0.3	—	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0
0.4	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—
0.9	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—	—
1.0	—	0.0	0.0	0.0	0.0	0.0	0.0	—	—	—
1.1	—	0.0	0.0	0.0	0.0	0.0	0.0	—	—	—
1.2	—	0.0	0.0	0.0	0.0	0.0	—	—	—	—
1.3	—	0.0	0.0	0.0	0.0	0.0	—	—	—	—
1.4	—	0.0	0.0	0.0	0.0	—	—	—	—	—
1.5	0.0	0.0	0.0	0.0	0.0	—	—	—	—	—
1.6	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.7	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.8	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.9	0.0	0.0	0.0	0.0	—	—	—	—	—	—
2.0	0.0	0.0	0.0	—	—	—	—	—	—	—
2.1	0.0	0.0	0.0	—	—	—	—	—	—	—
2.2	0.0	0.0	0.0	—	—	—	—	—	—	—
2.3	0.0	0.0	0.0	—	—	—	—	—	—	—
2.4	0.0	0.0	0.0	—	—	—	—	—	—	—
2.5	0.0	0.0	0.0	—	—	—	—	—	—	—
2.6	0.0	0.0	0.0	—	—	—	—	—	—	—
2.7	0.0	0.0	—	—	—	—	—	—	—	—
2.8	0.0	0.0	—	—	—	—	—	—	—	—
2.9	0.0	0.0	—	—	—	—	—	—	—	—
3.0	0.0	0.0	—	—	—	—	—	—	—	—
3.1	0.0	0.0	—	—	—	—	—	—	—	—
3.2	0.0	0.0	—	—	—	—	—	—	—	—
3.3	0.0	0.0	—	—	—	—	—	—	—	—
3.4	0.0	0.0	—	—	—	—	—	—	—	—
3.5	0.0	0.0	—	—	—	—	—	—	—	—
3.6	0.0	0.0	—	—	—	—	—	—	—	—
3.7	0.0	0.0	—	—	—	—	—	—	—	—
3.8	0.0	0.0	—	—	—	—	—	—	—	—

Note. $R = 2$ with all other parameters at the baseline.

$$\theta^*|_{F=0.1, \psi=7.0} = 0.0.$$

Table 7c. Optimal Tax Rate (%) at Present Demographics ($\eta = 0$)

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	0.0	0.0	0.0
0.3	—	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0
0.4	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.1	8.0
0.7	—	—	0.0	0.0	0.0	0.0	0.0	3.5	12.6	18.4
0.8	—	0.0	0.0	0.0	0.0	0.0	4.9	14.3	20.0	—
0.9	—	0.0	0.0	0.0	0.0	2.8	14.1	20.2	—	—
1.0	—	0.0	0.0	0.0	0.0	11.8	19.5	—	—	—
1.1	—	0.0	0.0	0.0	6.5	17.3	22.7	—	—	—
1.2	—	0.0	0.0	0.0	13.0	20.8	—	—	—	—
1.3	—	0.0	0.0	3.6	17.3	23.0	—	—	—	—
1.4	—	0.0	0.0	10.0	20.2	—	—	—	—	—
1.5	0.0	0.0	0.0	14.4	22.2	—	—	—	—	—
1.6	0.0	0.0	0.0	17.6	—	—	—	—	—	—
1.7	0.0	0.0	5.0	19.8	—	—	—	—	—	—
1.8	0.0	0.0	9.8	21.5	—	—	—	—	—	—
1.9	0.0	0.0	13.4	22.6	—	—	—	—	—	—
2.0	0.0	0.0	16.1	—	—	—	—	—	—	—
2.1	0.0	0.0	18.1	—	—	—	—	—	—	—
2.2	0.0	0.0	19.7	—	—	—	—	—	—	—
2.3	0.0	0.0	20.9	—	—	—	—	—	—	—
2.4	0.0	3.8	21.8	—	—	—	—	—	—	—
2.5	0.0	7.6	22.5	—	—	—	—	—	—	—
2.6	0.0	10.6	23.1	—	—	—	—	—	—	—
2.7	0.0	13.0	—	—	—	—	—	—	—	—
2.8	0.0	15.0	—	—	—	—	—	—	—	—
2.9	0.0	16.6	—	—	—	—	—	—	—	—
3.0	0.0	17.9	—	—	—	—	—	—	—	—
3.1	0.0	19.0	—	—	—	—	—	—	—	—
3.2	0.0	19.9	—	—	—	—	—	—	—	—
3.3	0.0	20.6	—	—	—	—	—	—	—	—
3.4	0.0	21.3	—	—	—	—	—	—	—	—
3.5	0.0	21.8	—	—	—	—	—	—	—	—
3.6	0.0	22.2	—	—	—	—	—	—	—	—
3.7	0.0	22.6	—	—	—	—	—	—	—	—
3.8	0.0	22.9	—	—	—	—	—	—	—	—

Note. $\eta = 0$ with all other parameters at the baseline.

$\theta^*|_{F=0.1, \psi=7.0}$ is undefined.

Table 7d. Optimal Tax Rate (%) at Future Demographics ($\eta = 0$)

ψ	F										
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
0.1	—	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	0.0	0.0	0.0	—
0.3	—	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	—	—
0.9	—	0.0	0.0	0.0	0.0	0.0	0.0	1.4	—	—	—
1.0	—	0.0	0.0	0.0	0.0	0.0	0.0	—	—	—	—
1.1	—	0.0	0.0	0.0	0.0	0.0	9.0	—	—	—	—
1.2	—	0.0	0.0	0.0	0.0	1.0	—	—	—	—	—
1.3	—	0.0	0.0	0.0	0.0	11.8	—	—	—	—	—
1.4	—	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.5	0.0	0.0	0.0	0.0	8.8	—	—	—	—	—	—
1.6	0.0	0.0	0.0	0.0	—	—	—	—	—	—	—
1.7	0.0	0.0	0.0	0.0	—	—	—	—	—	—	—
1.8	0.0	0.0	0.0	5.4	—	—	—	—	—	—	—
1.9	0.0	0.0	0.0	12.5	—	—	—	—	—	—	—
2.0	0.0	0.0	0.0	—	—	—	—	—	—	—	—
2.1	0.0	0.0	0.0	—	—	—	—	—	—	—	—
2.2	0.0	0.0	0.0	—	—	—	—	—	—	—	—
2.3	0.0	0.0	1.8	—	—	—	—	—	—	—	—
2.4	0.0	0.0	8.4	—	—	—	—	—	—	—	—
2.5	0.0	0.0	13.6	—	—	—	—	—	—	—	—
2.6	0.0	0.0	17.6	—	—	—	—	—	—	—	—
2.7	0.0	0.0	—	—	—	—	—	—	—	—	—
2.8	0.0	0.0	—	—	—	—	—	—	—	—	—
2.9	0.0	0.0	—	—	—	—	—	—	—	—	—
3.0	0.0	0.0	—	—	—	—	—	—	—	—	—
3.1	0.0	0.0	—	—	—	—	—	—	—	—	—
3.2	0.0	0.0	—	—	—	—	—	—	—	—	—
3.3	0.0	0.0	—	—	—	—	—	—	—	—	—
3.4	0.0	3.7	—	—	—	—	—	—	—	—	—
3.5	0.0	8.5	—	—	—	—	—	—	—	—	—
3.6	0.0	12.4	—	—	—	—	—	—	—	—	—
3.7	0.0	15.6	—	—	—	—	—	—	—	—	—
3.8	0.0	18.3	—	—	—	—	—	—	—	—	—

Note. $R = 2; \eta = 0$ with all other parameters at the baseline.

$\theta^*|_{F=0.1, \psi=7.0} = 14.8$.

Table 8a. Optimal Tax Rate (%) with no Wage Dispersion

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	0.0	0.0	0.0
0.3	—	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0
0.4	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—
0.9	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—	—
1.0	—	0.0	0.0	0.0	0.0	0.0	0.0	—	—	—
1.1	—	0.0	0.0	0.0	0.0	0.0	0.2	—	—	—
1.2	—	0.0	0.0	0.0	0.0	0.0	—	—	—	—
1.3	—	0.0	0.0	0.0	0.0	2.2	—	—	—	—
1.4	—	0.0	0.0	0.0	0.0	—	—	—	—	—
1.5	0.0	0.0	0.0	0.0	0.5	—	—	—	—	—
1.6	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.7	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.8	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.9	0.0	0.0	0.0	3.3	—	—	—	—	—	—
2.0	0.0	0.0	0.0	—	—	—	—	—	—	—
2.1	0.0	0.0	0.0	—	—	—	—	—	—	—
2.2	0.0	0.0	0.0	—	—	—	—	—	—	—
2.3	0.0	0.0	0.0	—	—	—	—	—	—	—
2.4	0.0	0.0	1.3	—	—	—	—	—	—	—
2.5	0.0	0.0	4.5	—	—	—	—	—	—	—
2.6	0.0	0.0	7.2	—	—	—	—	—	—	—
2.7	0.0	0.0	—	—	—	—	—	—	—	—
2.8	0.0	0.0	—	—	—	—	—	—	—	—
2.9	0.0	0.0	—	—	—	—	—	—	—	—
3.0	0.0	0.0	—	—	—	—	—	—	—	—
3.1	0.0	0.0	—	—	—	—	—	—	—	—
3.2	0.0	0.0	—	—	—	—	—	—	—	—
3.3	0.0	0.0	—	—	—	—	—	—	—	—
3.4	0.0	0.0	—	—	—	—	—	—	—	—
3.5	0.0	2.5	—	—	—	—	—	—	—	—
3.6	0.0	4.7	—	—	—	—	—	—	—	—
3.7	0.0	6.7	—	—	—	—	—	—	—	—
3.8	0.0	8.4	—	—	—	—	—	—	—	—

Note. $\epsilon = 0.5$; $\bar{T} = 52$ with all other parameters at the baseline.

Table 8b. Optimal Tax Rate (%) with Wage Dispersion

ψ	F									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	—	—	—	—	—	—	—	—	—	—
0.2	—	—	—	—	—	—	—	0.0	0.0	0.0
0.3	—	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0
0.4	—	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7	—	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—
0.9	—	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—	—
1.0	—	0.0	0.0	0.0	0.0	0.0	0.0	—	—	—
1.1	—	0.0	0.0	0.0	0.0	0.0	2.5	—	—	—
1.2	—	0.0	0.0	0.0	0.0	0.0	—	—	—	—
1.3	—	0.0	0.0	0.0	0.0	4.2	—	—	—	—
1.4	—	0.0	0.0	0.0	0.0	—	—	—	—	—
1.5	0.0	0.0	0.0	0.0	3.1	—	—	—	—	—
1.6	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.7	0.0	0.0	0.0	0.0	—	—	—	—	—	—
1.8	0.0	0.0	0.0	2.2	—	—	—	—	—	—
1.9	0.0	0.0	0.0	5.3	—	—	—	—	—	—
2.0	0.0	0.0	0.0	—	—	—	—	—	—	—
2.1	0.0	0.0	0.0	—	—	—	—	—	—	—
2.2	0.0	0.0	0.0	—	—	—	—	—	—	—
2.3	0.0	0.0	1.8	—	—	—	—	—	—	—
2.4	0.0	0.0	4.3	—	—	—	—	—	—	—
2.5	0.0	0.0	6.5	—	—	—	—	—	—	—
2.6	0.0	0.0	8.4	—	—	—	—	—	—	—
2.7	0.0	0.0	—	—	—	—	—	—	—	—
2.8	0.0	0.0	—	—	—	—	—	—	—	—
2.9	0.0	0.0	—	—	—	—	—	—	—	—
3.0	0.0	0.0	—	—	—	—	—	—	—	—
3.1	0.0	0.0	—	—	—	—	—	—	—	—
3.2	0.0	0.0	—	—	—	—	—	—	—	—
3.3	0.0	2.1	—	—	—	—	—	—	—	—
3.4	0.0	3.9	—	—	—	—	—	—	—	—
3.5	0.0	5.6	—	—	—	—	—	—	—	—
3.6	0.0	7.1	—	—	—	—	—	—	—	—
3.7	0.0	8.4	—	—	—	—	—	—	—	—
3.8	0.0	9.6	—	—	—	—	—	—	—	—

Note. $\epsilon = 20,000$; $\bar{T} = 52$ with all other parameters at the baseline.

Chapter Six: Concluding Remarks

Modern Neoclassical interpretations of the *Life-Cycle/Permanent-Income Hypothesis* suggest that individuals look forward across their lifetimes with perfect foresight and perfect information, and furthermore select consumption programs that maximize the present-discounted value of their respective lifetime utility. On the other hand, there is no shortage of literature and evidence that suggests that people have difficulty saving adequately (much less optimally) for retirement. Indeed, the existence of public pension programs, such as pay-as-you-go social security, is primarily justified on grounds that individuals have characteristic, behavioral tendencies that lead to inadequate saving for the retirement years of the life cycle. Despite this justification and despite the fact that public pension systems tend to be the largest government programs in most developed countries, very little has been pursued in the way of theory to examine this justification.

In the first study to theoretically analyze this broad justification for a social security program, Feldstein (1985) studies whether agents who underestimate their future social security benefits can have improved lifetime utility from unfunded social security participation. He addresses this question within the context of a two-period overlapping-generations model, and he concludes that agents do not generally benefit under this particular modeling definition of shortsightedness. Docquier (2002) revisits and studies the sensitivity of Feldstein's results by using a different social welfare function. İmrohoroğlu et al (2003) design a 65-period overlapping-generations model, and also address this question from the perspective of agents who have hyperbolic discount functions (time-inconsistent preferences) that lead to inadequate retirement saving. They also conclude that social security is generally not welfare-improving. Lastly, Findley and Caliendo (2007) examine the question of whether agents who have short, fixed planning horizons can benefit from an unfunded program, and they find that social security does improve welfare within a partial-equilibrium framework

when comparing their results to outside evidence. Other than these four studies, little else has been undertaken theoretically to address this justification.

In this dissertation, I also contribute to this research area by studying the question of whether impulsive consumption behavior (in isolation from other behavioral characteristics) can justify participation in a pay-as-you-go social security program. I accomplish this by designing a distinctly new model of consumer behavior that is consistent with many of the salient features of outside evidence on impulsive consumption, as documented in the psychology, psychiatry, and consumer research literature (but which has not yet been rigorously studied in the field of economics). In my model, “impulsive consumers” formulate plans for the long-term and attempt to set period consumption equal to the annuity value of remaining lifetime wealth. But they often deviate from these plans upon experiencing a psychological impulse to uncontrollably consume above and beyond their optimally-calculated consumption programs. Alternatively stated, an impulsive consumer is someone who sets optimal plans, sticks to these optimal plans for various periods of time, but is periodically prone to impulsive spending fits.

To examine impulsive consumers and optimal social security, I calibrate my model for simulation purposes. I first select numerical values for the economic parameters in my model based on well-documented macroeconomic evidence. I then calibrate the behavioral impulsiveness parameters such that simulated consumption outcomes are consistent with empirical evidence on aggregate life-cycle consumption. Specifically, I find the parameter space for the frequency and magnitude of impulsive consumption that generates a simulated discrete drop in aggregate life-cycle consumption that matches the magnitude of the drop observed in the data. After having fully calibrated my model, I employ dynamic welfare measures in order to examine the welfare effects of unfunded social security on impulsive consumers.

I now summarize my primary quantitative-theoretic findings. First, I find that

there is a significant welfare cost to being an impulsive consumer (as opposed to never succumbing to the consumption impulse). I use an “income equivalent” (willingness to pay measured in terms of annual income) to find that a life-cycle/permanent-income consumer would be willing to pay a significant fraction of his annual income (and corresponding lifetime wealth) in order to avoid being an impulsive consumer. I also find that participation in a pay-as-you-go social security program narrows the discrepancy in well-being between an impulsive consumer and a life-cycle/permanent-income consumer. This outcome can be generally consistent with egalitarian notions of fairness conditioned on the requirement that social security also succeeds at improving the welfare of at least the impulsive consumer. Second, I use a dynamic welfare measure labeled the “consumption equivalent” (willingness to pay measured in terms of real annual consumption), and I find that an unfunded social security program (calibrated to the U.S. program) does not generally improve the welfare of impulsive consumers for the types of impulsive consumer behavior deemed relevant by my calibration of the model. I also find that under a worker-to-retiree ratio of 2 in the economy (reflecting future population demographics in the U.S.), social security almost never improves the welfare of impulsive consumers even when the social planner does not give preferential treatment to any period of the impulsive consumer’s life cycle over another (i.e., the social discount rate is zero)! Lastly, I analyze the optimal tax rate of a pay-as-you-go social security program. For the most part, I find that the current size of the U.S. program is drastically larger than what would be warranted from the quantitative results of my model. This implication generally continues to hold even if the social planner does not discount, and even in the presence of wage heterogeneity (although the introduction of wage heterogeneity does appear to have the effect of increasing the optimal value for the tax rate as compared to a steady state with just an economy-wide wage).

I believe that it is prudent at this juncture that I address some specific features

of my theoretical setup that likely generate the aforementioned quantitative outcomes. Moreover, I also enumerate some possible extensions for future work that might alter my results. First, I believe that the major mechanism driving the fact that an unfunded social security program (calibrated to the U.S. program) is not welfare-improving within my model is that my impulsive consumer can engage in time-inconsistent dynamic optimization. This means that impulsive consumers can re-optimize immediately following a spell of impulsive consumption. As such, impulsive consumers can somewhat (but not fully) correct for their prior “bad” behavior. Therefore, it appears that my dynamically-inconsistent impulsive consumers do not generally benefit from unfunded social security (calibrated to the U.S. program) because they partially mitigate or partially correct their own impulsive behavior via re-optimization (recalculation of the life-cycle/permanent-income rule resulting in an upward revision in their optimal saving programs). It is highly possible that pay-as-you-go social security could improve the lifetime utility of impulsive consumers under an alternative setup wherein impulsive consumers do not re-optimize following binges of impulsive consumption.

Secondly, my impulsive agent consumes a constant, fixed fraction of his optimal saving flow upon experiencing (and succumbing to) the consumption impulse. Following periods of impulsive consumption, my agent re-optimizes and continually revises downward his consumption program meaning that he continually revises upward his optimal saving program. Therefore, the amount that my agent impulsively consumes grows over time because he consumes a constant fixed fraction of a growing optimal saving flow. This is a form of “anti-learning” behavior since my agent is impulsively consuming larger amounts as he ages. Therefore, this particular modeling specification behaviorally “stacks the cards” in favor of an unfunded social security program, yet I find (as outlined above) that my impulsive consumer does not generally benefit from such a program despite such anti-learning behavior. It is likely that the incor-

poration of learning to be less impulsive (with respect to the impulse amount) will erase any margin of possibility that impulsive consumption behavior (in isolation from other behavioral characteristics) can justify a pay-as-you-go social security program.

Thirdly, at any arbitrary instant during the working years, the agent impulsively consumes a fixed fraction of his optimal saving flow, which I have defined as the difference between current labor income and permanent income. The difference between current labor income and permanent income is one particular variation of what might be termed “transitory income”. Another modeling possibility is that the agent consumes a fixed fraction of the difference between his total current income (including both labor and interest income) and permanent income upon succumbing to the consumption impulse. This particular specification of transitory income would effectively increase the amount that the agent consumes upon experiencing the consumption impulse (compared to the current specification) when he has a positive asset account balance, yet this specification would also conversely mitigate the impulse amount (compared to the current specification) when the agent has a negative asset balance. It is *a priori* unclear how this alternative specification of transitory income might affect the principal welfare results in my dissertation.

Fourthly, the date of retirement is exogenous and fixed within my model of impulsive consumers. It is likely that augmenting my baseline model with a labor-leisure choice would affect the date at which my impulsive consumers would select to retire from working. It is possible (maybe even likely given certain assumptions as to how leisure enters the period utility function) that consumers would endogenously select to delay retirement as the degree of impulsive consumption behavior increases. A delay in retirement would decrease the prospect that social security is welfare-improving (all else equal) since individuals would be paying social security taxes for a longer period, while receiving benefits over a shorter period of time.

Fifthly, the quantitative-theoretic results of my dissertation tend to support the

theoretical conclusion of both Feldstein (1985) and İmrohoroğlu et al (2003) that unfunded social security is generally not welfare-improving despite a universal prevalence of “shortsightedness” throughout an economy. This highlights the apparent difficulty in finding whether a single behavioral mechanism (in isolation from other behavioral mechanisms) can theoretically justify a pay-as-you-go social security program (although Findley and Caliendo (2007) find that short planning horizons in isolation can justify social security within a partial-equilibrium framework). An interesting parallel research question is whether an unfunded social security program can improve the lifetime welfare of individuals who are plagued with several behavioral characteristics in tandem (such as consumption impulses coupled with hyperbolic discount functions, etc.). Furthermore, how might multiple behavioral characteristics interact with one another in affecting lifetime welfare?

Lastly, I have designed a model of impulsive consumers that appears to be consistent with the salient features of empirical evidence on impulsive consumption (as outlined in the psychology, psychiatry, and consumer research literature). I model an individual who attempts to optimally consume according to the life-cycle/permanent-income rule, yet who periodically succumbs to an *exogenous* consumption impulse. What if impulsive consumption behavior is an endogenous rational outcome from some larger optimization problem that has yet to be discovered? What are the macroeconomic consumption implications from such behavior, and is there a role for pay-as-you-go social security in an economy populated with such consumers? I leave this (along with the other aforementioned possibilities above) to future work.

References

- Aboujaoude, Elias, Nona Gamel, and Lorrin M. Koran. 2003. A 1-Year Naturalistic Follow-Up of Patients With Compulsive Shopping Disorder. *Journal of Clinical Psychiatry* 64(8):946-950.
- Akerlof, George A. 2002. Behavioral Macroeconomics and Macroeconomic Behavior. *The American Economic Review* 92(3): 411-433.
- Ameriks, John, Andrew Caplin, and John Leahy. 2007. Retirement Consumption: Insights from a Survey. *The Review of Economics and Statistics* 89(2): 265-274.
- Amit, Raphael. 1986. Petroleum Reservoir Exploitation: Switching from Primary to Secondary Recovery. *Operations Research* 34(4): 534-549.
- Ando, Albert, and Franco Modigliani. 1963. The 'Life Cycle' Hypothesis of Saving: Aggregate Implications and Tests. *The American Economic Review* 53(1): 55-84.
- Angeletos, George-Marios, David Laibson, Andrea Repetto, Jeremy Tobacman, and Stephen Weinberg. 2001. The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Evaluation. *The Journal of Economic Perspectives* 15(3): 47-68.
- Arthur, W. Brian, and Geoffrey McNicoll. 1978. Samuelson, Population and Intergenerational Transfers. *International Economic Review* 19(1): 241-246.
- Attanasio, Orazio. 1999. Consumption. In *Handbook of Macroeconomics* (Volume 1B), John B. Taylor and Michael Woodford, eds., 741-812. Amsterdam, The Netherlands: Elsevier Science B.V.
- Attanasio, Orazio P., James Banks, Costas Meghir, and Guglielmo Weber. 1999. Humps and Bumps in Lifetime Consumption. *Journal of Business & Economic Statistics* 17(1): 22-35.
- Banks, James, Richard Blundell, and Sarah Tanner. 1998. Is There a Retirement-Savings Puzzle? *The American Economic Review* 88(4): 769-788.
- Barro, Robert J., and Xavier Sala-i-Martin. 2004. *Economic Growth*. 2nd ed. Cambridge, MA: The MIT Press.
- Baumeister, Roy F. 2002. Yielding to Temptation: Self-Control Failure, Impulsive Purchasing, and Consumer Behavior. *The Journal of Consumer Research* 28(4): 670-676.
- Baumeister, Roy F., Ellen Bratslavsky, Mark Muraven, and Dianne M. Tice. 1998. Ego Depletion: Is the Active Self a Limited Resource? *Journal of Personality and Social Psychology* 74(5): 1252-1265.
- Bayley, Geoff, and Clive Nancarrow. 1998. Impulse Purchasing: A Qualitative Exploration of the Phenomenon. *Qualitative Market Research: An International Journal* 1(2): 99-114.

- Beatty, Sharon E., and M. Elizabeth Ferrell. 1998. Impulse Buying: Modeling its Precursors. *Journal of Retailing* 74(2): 169-191.
- Bellinger, Danny N., Dan H. Robertson, and Elizabeth C. Hirschman. 1978. Impulse Buying Varies by Product. *Journal of Advertising Research* 18(December): 15-18.
- Bernheim, B. Douglas, Jonathan Skinner, and Steven Weinberg. 2001. What Accounts for the Variation in Retirement Wealth among U.S. Households? *The American Economic Review* 91(4): 832-857.
- Black, Donald W. 2007. A Review of Compulsive Buying Disorder. *World Psychiatry* 6(1): 14-18.
- Black, Donald W., Patrick Monahan, and Janelle Gabel. 1997. Fluvoxamine in the Treatment of Compulsive Buying. *The Journal of Clinical Psychiatry* 58(4): 159-163.
- Black, Donald W., Susan Repertinger, Gary R. Gaffney, and Janelle Gabel. 1998. Family History and Psychiatric Comorbidity in Persons With Compulsive Buying: Preliminary Findings. *The American Journal of Psychiatry* 155(7): 960-963.
- Branson, William H. 1989. *Macroeconomic Theory and Policy*. 3rd ed. New York, NY: Harper & Row Publishers.
- Browning, Martin, and Thomas F. Crossley. 2001. The Life-cycle Model of Consumption and Saving. *Journal of Economic Perspectives* 15(3): 3-22.
- Bullard, James, and James Feigenbaum. Forthcoming. A Leisurely Reading of the Life-cycle Consumption Data. *Journal of Monetary Economics*
- Bullock, Kim, and Lorrin Koran. 2003. Psychopharmacology of Compulsive Buying. *Drugs of Today* 39(9): 695-700.
- Bütler, Monika. 2001. Neoclassical Life-cycle Consumption: A Textbook Example. *Economic Theory* 17(1): 209-221.
- Caliendo, Frank, and David Aadland. 2007. Short-Term Planning and the Life-Cycle Consumption Puzzle. *Journal of Economic Dynamics and Control* 31(4): 1392-1415.
- Caliendo, Frank, and T. Scott Findley. 2007. Optimal Social Security with Wage Heterogeneity when All Consumers are Rational. Working paper, Department of Economics, Colorado State University. Fort Collins, CO.
- Caliendo, Frank, and Emin Gahramanov. 2007. Revisiting the Rule-of-Thumb Explanation for the Consumption Hump. Working paper, Department of Economics, Colorado State University. Fort Collins, CO.
- Caliendo, Frank, and Kevin Huang. 2007. Costly Saving Implementation and Macroeconomic Consumption Puzzles. Working paper, Department of Economics, Colorado State University. Fort Collins, CO.

- Campbell, John Y. 2001. Forecasting U.S. Equity Returns in the 21st Century. In *Estimating the Real Rate of Return on Stocks Over the Long Term*, Social Security Advisory Board, 3-10. Washington, DC.
- Campbell, John Y., and N. Gregory Mankiw. 1989. Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence. In *NBER Macroeconomics Annual 1989*, O.J. Blanchard and S. Fischer, eds., 185-216. Cambridge, MA: The MIT Press.
- Campbell, John Y., and N. Gregory Mankiw. 1990. Permanent Income, Current Income, and Consumption. *Journal of Business & Economic Statistics* 8(3): 265-279.
- Carroll, Christopher D. 1994. How Does Future Income Affect Current Consumption? *The Quarterly Journal of Economics* 109(1): 111-147.
- Carroll, Christopher D. 1997. Buffer-Stock Saving and the Life-Cycle/Permanent Income Hypothesis. *The Quarterly Journal of Economics* 112(1): 1-55.
- Carroll, Christopher D. 2001. A Theory of the Consumption Function, with and without Liquidity Constraints. *Journal of Economic Perspectives* 15(3): 23-45.
- Carroll, Christopher D., and Lawrence H. Summers. 1991. Consumption Growth Parallels Income Growth: Some New Evidence. In *National Saving and Economic Performance*, B.D. Bernheim and J.B. Shoven, eds., 305-343. Chicago, IL: University of Chicago Press for NBER.
- Chamberlain, Samuel R., and Barbara J. Sahakian. 2007. The Neuropsychiatry of Impulsivity. *Current Opinion in Psychiatry* 20(3): 255-261.
- Chambers, Christopher D., Mark A. Bellgrove, Mark G. Stokes, Tracy R. Henderson, Hugh Garavan, Ian H. Robertson, Adam P. Morris, and Jason B. Mattingley. 2006. Executive "Brake Failure" following Deactivation of Human Frontal Lobe. *Journal of Cognitive Neuroscience* 18(3): 444-455.
- Choi, James J., David Laibson, Brigitte C. Madrian, and Andrew Metrick. 2006a. Saving for Retirement on the Path of Least Resistance. In *Behavioral Public Finance*, E.J. McCaffery and Joel Slemrod, eds., 304-352. New York, NY: Russell Sage Foundation Publications.
- Choi, James J., David Laibson, and Brigitte C. Madrian. 2006b. Reducing the Complexity Costs of 401(k) Participation Through Quick EnrollmentTM. *National Bureau of Economic Research Working Paper Series Working Paper No. 11979*.
- Christenson, Gary A., Ronald J. Faber, Martina de Zwaan, Nancy C. Raymond, Sheila M. Specker, Michael D. Ekern, Thomas B. Mackenzie, Ross D. Crosby, Scott J. Crow, Elke D. Eckert, Melissa P. Mussell, and James E. Mitchell. 1994. Compulsive Buying: Descriptive Characteristics and Psychiatric Comorbidity. *The Journal of Clinical Psychiatry* 55(1): 5-11.

- Cobb, Cathy J., and Wayne D. Hoyer. 1986. Planned versus impulse purchase behavior. *Journal of Retailing* 62(4): 384-409.
- Comings, David E. 1998. The molecular genetics of pathological gambling. *CNS Spectrums* 3(6): 20-37.
- Congressional Budget Office. 2001. *Social Security: A Primer*. Accessed online at www.cbo.gov
- Conlisk, John. 1996. Why Bounded Rationality? *Journal of Economic Literature* 34(2): 669-700.
- d'Astous, Alain. 1990. An Inquiry into the Compulsive Side of 'Normal' Consumers. *Journal of Consumer Policy* 13(1): 15-31.
- De La Croix, David, and Philippe Michel. 2002. *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*. New York, NY: Cambridge University Press.
- Deaton, Angus. 1992. *Understanding Consumption*. New York, NY: Oxford University Press.
- Deaton, Angus, and Christina Paxson. 2000. Growth and Saving among Individuals and Households. *The Review of Economics and Statistics* 82(2): 212-225.
- DeSarbo, Wayne S., and Elizabeth A. Edwards. 1996. Typologies of Compulsive Buying Behavior: A Constrained Clusterwise Regression Approach. *Journal of Consumer Psychology* 5(3): 231-262.
- Devor, Eric J., Harry J. Magee, Rebecca M. Dill-Devor, Janelle Gabel, and Donald W. Black. 1999. Serotonin Transporter Gene (5-HTT) Polymorphisms and Compulsive Buying. *American Journal of Medical Genetics (Neuropsychiatric Genetics)* 88(2): 123-125.
- Diamond, Peter. 1977. A Framework for Social Security Analysis. *Journal of Public Economics* 8(December): 275-298.
- Diamond, Peter. 2006. Social Security, the Government Budget, and National Savings. In *Samuelsonian Economics in the 21st Century*, M. Szenberg, ed., 54-65. New York, NY: Oxford University Press.
- Diamond, Peter, and Peter Orszag. 2004. *Saving Social Security: A Balanced Approach*. Washington D.C.: Brookings Institution Press.
- Diamond, Peter, and Peter Orszag. 2005. Saving Social Security. *Journal of Economic Perspectives* 19(2): 11-32.
- Dittmar, Helga, Jane Beattie, and Susanne Friese. 1995. Gender Identity and Material Symbols: Objects and Decision Considerations in Impulse Purchases. *Journal of Economic Psychology* 16(3): 491-511.

- Docquier, Frederic. 2002. On the Optimality of Public Pensions in an Economy with Life-Cyclers and Myopes. *Journal of Economic Behavior and Organization* 47(1): 121-140.
- Donovan, Robert J., John R. Rossiter, Gilian Marcoolyn, and Andrew Nesdale. 1994. Store Atmosphere and Purchasing Behavior. *Journal of Retailing* 70(3): 283-294.
- Dornbusch, Rudiger, Stanley Fischer, and Richard Startz. 1998. *Macroeconomics*. 7th ed. New York, NY: Irwin/McGraw-Hill.
- Elmendorf, Douglas, and Jeffrey Liebman. 2000. Social Security Reform and National Saving in an Era of Budget Surpluses. *Brookings Papers on Economic Activity* 2000(2): 1-71.
- Faber, Ronald J., and Thomas C. O'Guinn. 1992. A Clinical Screener for Compulsive Buying. *The Journal of Consumer Research* 19(3): 459-469.
- Feigenbaum, James. 2007. Can Mortality Risk Explain the Consumption Hump? Working paper, Department of Economics, University of Pittsburgh. Pittsburgh, PA.
- Feldstein, Martin. 1985. The Optimal Level of Social Security Benefits. *The Quarterly Journal of Economics* 100(2): 303-320.
- Fernández-Villaverde, Jesús, and Dirk Krueger. 2004. Consumption over the Life Cycle: Facts from Consumer Expenditure Survey Data. Working paper, Department of Economics, University of Pennsylvania. Philadelphia, PA.
- Findley, T. Scott, and Frank Caliendo. 2007. Short Horizons, Time Inconsistency, and Optimal Social Security. Working paper, Department of Economics, Colorado State University. Fort Collins, CO.
- Findley, T. Scott, and Frank Caliendo. Forthcoming. OutSMarTing the Social Security Crisis. *Public Finance Review*.
- Freud, Sigmund. 1911/1956. Formulations on the Two Principles of Mental Functioning. In *The Standard Edition of the Complete Psychological Works of Sigmund Freud* (Volume XII), J. Strachey and A. Freud, eds., 213-226. London, UK: Hogarth.
- Friedman, Milton. 1957. *A Theory of the Consumption Function*. Princeton, NJ: Princeton University Press.
- Friedman, Milton A. 1963. Windfalls, the 'Horizon,' and Related Concepts in the Permanent Income Hypothesis. In *Measurement in Economics*, Carl Christ et al, eds., 1-28. Stanford, CA: Stanford University Press.
- Frost, Randy O., Hyo-Jin Kim, Claire Morris, Cinnamon Bloss, Marta Murray-Close, and Gail Steketee. 1998. Hoarding, Compulsive Buying and Reasons for Saving. *Behaviour Research and Therapy* 36(7-8): 657-664.

- Goss, Stephen C. 2005/2006. The Financial Outlook for the Social Security Disability Insurance Program. *Social Security Bulletin* 66(3): 47-52.
- Gourinchas, Pierre-Olivier, and Jonathan A. Parker. 2002. Consumption Over the Life Cycle. *Econometrica* 70(1): 47-89.
- Grant, Jon E. 2003. Three Cases of Compulsive Buying Treated with Naltrexone. *International Journal of Psychiatry in Clinical Practice* 7(3): 223-225.
- Gul, Faruk, and Wolfgang Pesendorfer. 2001. Temptation and Self-Control. *Econometrica* 69(6): 1403-1436.
- Gul, Faruk, and Wolfgang Pesendorfer. 2002a. Self-Control and the Theory of Consumption. *Econometrica* 72(1): 119-158.
- Gul, Faruk, and Wolfgang Pesendorfer. 2002b. Self-Control, Revealed Preference and Consumption Choice. *Review of Economic Dynamics* 7(2): 243-264.
- Haider, Steven J., and Melvin Stephens, Jr. 2007. Is There a Retirement-Consumption Puzzle? Evidence Using Subjective Retirement Expectations. *The Review of Economics and Statistics* 89(2): 247-264.
- Hamermesh, Daniel S. 1984. Consumption During Retirement: The Missing Link in the Life Cycle. *The Review of Economics and Statistics* 66(1): 1-7.
- Han, Y.K., G.A. Morgan, A. Kotsiopoulos, and J. Kang-Park. 1991. Impulse buying Behavior of Apparel Purchers. *Clothing and Textile Research Journal* 9(3): 15-21.
- Hanley, Alice, and Mari S. Wilhelm. 1992. Compulsive Buying: An Exploration into Self-Esteem and Money Attitudes. *Journal of Economic Psychology* 13(1): 5-18.
- Hausman, Angela. 2000. A Multi-Method Investigation of Consumer Motivations in Impulse Buying Behavior. *Journal of Consumer Marketing* 17(5): 403-419.
- Heckman, James. 1974. Life Cycle Consumption and Labor Supply: An Explanation of the Relationship Between Income and Consumption Over the Life Cycle. *The American Economic Review* 64(1): 188-194.
- Helman, Ruth, Craig Copeland, and Jack VanDerhei. 2006. Will More of Us Be Working Forever? The 2006 Retirement Confidence Survey. *EBRI Issue Brief* 292(April): 1-28.
- Herbig, Paul, William Koehler, and Ken Day. 1993. Marketing to the Baby Bust Generation. *Journal of Consumer Marketing* 10(1): 4-9.
- Hirschman, Elizabeth C. 1992. The Consciousness of Addiction: Toward a General Theory of Compulsive Consumption. *Journal of Consumer Research* 19(2): 155-179.
- Hoch, Stephen J., and George F. Loewenstein. 1991. Time-Inconsistent Preferences and Consumer Self-Control. *The Journal of Consumer Research* 17(4): 492-507.

- Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes. 1994. The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving. *Carnegie-Rochester Conference Series on Public Policy* 40: 59-125.
- Hurd, Michael, and Susan Rohwedder. 2005. The Retirement-Consumption Puzzle: Anticipated and Actual Declines in Spending at Retirement. *RAND Labor and Population Working Paper Series*. Working paper WR-242.
- Hurst, Erik. 2003. Grasshoppers, Ants and Pre-Retirement Wealth: A Test of Permanent Income Consumers. *National Bureau of Economic Research Working Paper Series*, Working Paper No. 10098.
- İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas H. Joines. 2003. Time-Inconsistent Preferences and Social Security. *The Quarterly Journal of Economics* 118(2): 745-784.
- Jianakoplos, Nancy Ammon, and Vickie L. Bajtelsmit. 2002. Dual Private Pension Households and the Distribution of Wealth in the United States. *Journal of Pension Economics and Finance* 1(2): 131-155.
- Jianakoplos, Nancy A., Paul L. Menchik, and F. Owen Irvine. 1989. Using Panel Data to Assess the Bias in Cross-Sectional Inferences of Life-Cycle Changes in the Level and Composition of Household Wealth. In *Measurement of Savings, Investment and Wealth*, Robert E. Lipsey and Helen S. Tice, eds., 553-644. Chicago, IL: The University of Chicago Press.
- Kacen, Jacqueline J., and Julie Anne Lee. 2002. The Influence of Culture on Consumer Impulsive Buying Behavior. *Journal of Consumer Psychology* 12(2): 163-176.
- Kamien, Morton I., and Nancy L. Schwartz. 1991. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. 2nd ed. Amsterdam, The Netherlands: Elsevier North-Holland.
- Kemp, Murray C., and Ngo Van Long. 1977. On Optimal Control Problems with Integrands Discontinuous With Respect to Time. *Economic Record* 53: 405-420.
- Keynes, John Maynard. 1936/1964. *The General Theory of Employment, Interest, and Money*. New York, NY: Harcourt-Brace & Company.
- Kipnis, David. 1971. *Character Structure and Impulsiveness*. New York, NY: Academic Press.
- Kollat, David T., and Ronald P. Willett. 1967. Customer Impulse Purchasing Behavior. *Journal of Marketing Research* 4(1): 21-31.
- Koran, Lorrin M., Helen W. Chuong, Kim D. Bullock, and S. Christine Smith. 2003. Citalopram for Compulsive Shopping Disorder: An Open-Label Study Followed by Double-Blind Discontinuation. *The Journal of Clinical Psychiatry* 64(7): 793-798.

- Koran, Lorrin M., Ronald J. Faber, Elias Aboujaoude, Michael D. Large, and Richard T. Serpe. 2006. Estimated Prevalence of Compulsive Buying Behavior in the United States. *The American Journal of Psychiatry* 163(10): 1806-1812.
- Kotlikoff, Laurence J. 1979. Testing the Theory of Social Security and Life Cycle Accumulation. *The American Economic Review* 69(3): 396-410.
- Kotlikoff, Laurence J., Avia Spivak, and Lawrence H. Summers. 1982. The Adequacy of Savings. *The American Economic Review* 72(5): 1056-1069.
- Laibson, David. 1997. Golden Eggs and Hyperbolic Discounting. *The Quarterly Journal of Economics* 112(2): 443-477.
- Laibson, David I., Andrea Repetto, Jeremy Tobacman, Robert E. Hall, William G. Gale, and George A. Akerlof. 1998. Self-Control and Saving for Retirement. *Brookings Papers on Economic Activity* 1998(1): 91-196.
- Livingstone, Sonia M., and Peter K. Lunt. 1992. Predicting Personal Debt and Debt Repayment: Psychological, Social and Economic Determinants. *Journal of Economic Psychology* 13(1): 111-134.
- Lusardi, Annamaria. 1999. Information, Expectations, and Savings for Retirement. In *Behavioral Dimensions of Retirement Economics*, Henry Aaron, ed., 81-115. Washington, D.C.: Brookings Institution Press and Russell Sage Foundation.
- Lusardi, Annamaria. 2003. Planning and Saving for Retirement. Working paper, Department of Economics, Dartmouth College. Hanover, NH.
- Madrian, Brigitte C., and Dennis F. Shea. 2001. The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior. *The Quarterly Journal of Economics* 116(4): 1149-1187.
- Magee, Allison. 1994. Compulsive buying tendency as a predictor of attitudes and perceptions. *Advances in Consumer Research* 21(1): 590-594.
- Mariger, Randall P. 1987. A Life-Cycle Model with Liquidity Constraints: Theory and Empirical Results. *Econometrica* 55(3): 533-557.
- McCaffery, Edward J., and Joel Slemrod. 2006. Toward An Agenda for Behavioral Public Finance. In *Behavioral Public Finance*, E.J. McCaffery and Joel Slemrod, eds., 3-31. New York, NY: Russell Sage Foundation Publications.
- McElroy, S.L., A. Satlin, G.P. Harrison Jr., P.E. Keck Jr., and J.I. Hudson. 1991. Treatment of Compulsive Shopping with Antidepressants: A Report of Three Cases. *Annals of Clinical Psychiatry* 3: 199-204.
- McElroy, Susan L., Paul E. Kleck Jr., Harrison G. Pope Jr., Jacqueline M.R. Smith, and Stephen M. Strakowski. 1994. Compulsive Buying: A Report of 20 Cases. *Journal of Clinical Psychiatry* 55(6): 242-248.

- Mick, David G., and Michelle DeMoss. 1990. Self-Gifts: Phenomenological Insights from Four Contexts. *The Journal of Consumer Research* 17(3): 322-332.
- Mogelonsky, Marcia. 1998. Keep Candy in the Aisles. *American Demographics* 20: 32.
- Moore, Matt. 2006. Automatic 401(k)s: A Win for Workers. *National Center for Policy Analysis Brief Analysis* No. 567: 1-2.
- Munnell, Alicia H., Annika Sundén, and Catherine Taylor. 2001/2002. What Determines 401(k) Participation and Contributions? *Social Security Bulletin* 64(3): 64-75.
- Muraven, Mark, Dianne M. Tice, and Roy E. Baumeister. 1998. Self-Control as Limited Resource: Regulatory Depletion Patterns. *Journal of Personality and Social Psychology* 74(3): 774-789.
- Nagatani, Keizo. 1972. Life cycle Saving: Theory and Fact. *The American Economic Review* 62(3): 344-353.
- O'Guinn, Thomas C., and Ronald J. Faber. 1989. Compulsive Buying: A Phenomenological Exploration. *The Journal of Consumer Research* 16(2): 147-157.
- Ozdenoren, Emre, Stephen Salant, and Dan Silverman. 2006. Willpower and the Optimal Control of Visceral Urges. Working paper, Department of Economics, University of Michigan. Ann Arbor, MI.
- Poterba, James M. 1988. Are Consumers Forward Looking? Evidence from Fiscal Experiments. *The American Economic Review* 78(2): 413-418.
- Rabin, Matthew. 1999. Comment on: What, Me Worry? A Psychological Perspective on Economic Aspects of Retirement. In *Behavioral Dimensions of Retirement Economics*, Henry Aaron, ed., 247-251. Washington, D.C.: Brookings Institution Press and Russell Sage Foundation.
- Robb, A.L. and J.B. Burbidge. 1989. Consumption, Income, and Retirement. *The Canadian Journal of Economics* 22(3): 522-542.
- Roberts, James A. 1998. Compulsive Buying Among College Students: An Investigation of Its Antecedents, Consequences, and Implications for Public Policy. *Journal of Consumer Affairs* 32(2): 295-314.
- Romer, David. 2001. *Advanced Macroeconomics*. 2nd ed. Boston, MA: Irwin/McGraw-Hill.
- Rook, Dennis W. 1987. The Buying Impulse. *The Journal of Consumer Research* 14(2): 189-199.
- Rook, Dennis W., and Robert J. Fisher. 1995. Normative Influences on Impulsive Buying Behavior. *The Journal of Consumer Research* 22(3): 305-313.

- Samuelson, Paul A. 1958. An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *The Journal of Political Economy* 66(6): 467-482.
- Samuelson, Paul A. 1975. The Optimum Growth Rate for Population. *International Economic Review* 16(3): 531-538.
- Scherhorn, Gerhard, Lucia A. Reisch, and Gerhard Raab. 1990. Addictive Buying in West Germany: An Empirical Study. *Journal of Consumer Policy* 13(4): 355-387.
- Schlösser, Steven, Donald W. Black, Susan Repertinger and Daniel Freet. 1994. Compulsive buying: Demography, Phenomenology, and Comorbidity in 46 subjects. *General Hospital Psychiatry* 16(3): 205-212.
- Science Daily. 2003. *Antidepressant Helps Alleviate Compulsive Shopping Disorder, Stanford Researchers Find*. July 17, 2003. Accessed online at <http://www.sciencedaily.com/releases/2003/07/030717090529.htm>
- Settergren, Ole, and Boguslaw D. Mikula. Forthcoming. The Rate of Return of Pay-As-You-Go Pension Systems: A More Exact Consumption-Loan Model of Interest. In *Pension Reform through NDC's: Issues and Prospects for Non-Financial Defined Contribution Schemes*. R. Holzmann and E. Palmer, eds., World Bank.
- Shea, John. 1995. Union Contracts and the Life-Cycle/Permanent-Income Hypothesis. *The American Economic Review* 85(1): 186-200.
- Shiv, Baba, and Alexander Fedorikhin. 1999. Heart and Mind in Conflict: The Interplay of Affect and Cognition in Consumer Decision Making. *The Journal of Consumer Research* 26(3): 278-292.
- Simon, Carl P., and Lawrence Blume. 1994. *Mathematics for Economists*. New York, NY: W.W. Norton and Company.
- Smetters, Kent. 2003. Is the Social Security Trust Fund Worth Anything? *National Bureau of Economic Research Working Paper Series*. Working Paper No. 9845.
- Smetters, Kent. 2004. Is the Social Security Trust Fund a Store of Value? *The American Economic Review* 94(2): 176-181.
- Speight, Alan E. H. 1989. *Consumption, Rational Expectations and Liquidity*. New York, NY: St. Martin's Press.
- Thaler, Richard H. 1990. Saving, Fungibility, and Mental Accounts. *Journal of Economic Perspectives* 4(1): 193-205.
- Thaler, Richard H. 1994. Psychology and Savings Policies. *The American Economic Review* 84(2): 186-192.
- Thaler, Richard H. 2000. From Homo Economicus to Homo Sapiens. *The Journal of Economic Perspectives* 14(1): 133-141.

- Thaler, Richard H., and Shlomo Benartzi. 2004. Save More TomorrowTM: Using Behavioral Economics to Increase Employee Saving. *Journal of Political Economy* 112(1): 164-187.
- Thaler, Richard H., and H.M. Shefrin. 1981. An Economic Theory of Self-Control. *Journal of Political Economy* 89(2): 392-406.
- Thurow, Lester C. 1969. The Optimum Lifetime Distribution of Consumption Expenditures. *The American Economic Review* 59(3): 324-330.
- Tice, Dianne M., Ellen Bratslavsky, and Roy F. Baumeister. 2001. Emotional Distress Regulation Takes Precedence Over Impulse Control: If You Feel Bad, Do It! *Journal of Personality and Social Psychology* 80(1): 53-67.
- Tomiyama, Ken. 1985. Two-Stage Optimal Control Problems and Optimality Conditions. *Journal of Economic Dynamics and Control* 9(3): 317-337.
- U.S. Congress. 2006. Pension Protection Act of 2006. HR4. 109th Cong., 2nd sess. Washington, D.C.: GPO.
- Vohs, Kathleen D., Roy F. Baumeister, Jean M. Twenge, Brandon J. Schmeichel, and Dianne M. Tice. 2007. Decision Fatigue Exhausts Self-Regulatory Resources. Working paper, Department of Marketing, University of Minnesota. Minneapolis, MN
- Welles, G. 1986. We're in the Habit of Impulsive Buying. *USA Today*. May 21st, 1.
- Winshie, Howard. 1977. *The Impulsive Personality*. New York, NY: Plenum Press.
- Wood, Michael. 1998. Socio-Economic Status, Delay of Gratification, and Impulse Buying. *Journal of Economic Psychology* 19(3): 295-320.